Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/27-1.1.3.4-e-x-^m-a+b-x^n-^p-c+d-x^n-^q

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September 27, 2022 Compiled on September 27, 2022 at 11:07am

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [1081]. This is test number [27].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1081)	0.00 (0)
Mathematica	100.00 (1081)	0.00 (0)
Fricas	74.19 (802)	25.81 (279)
Maple	69.29 (749)	30.71 (332)
Giac	51.25 (554)	48.75 (527)
Mupad	49.12 (531)	50.88 (550)
Maxima	37.84 (409)	62.16 (672)
Sympy	36.54 (395)	63.46 (686)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

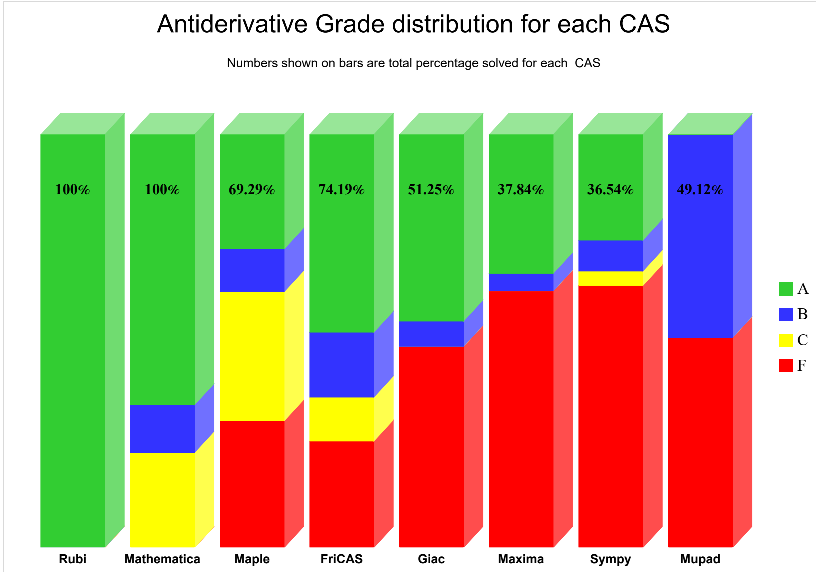
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

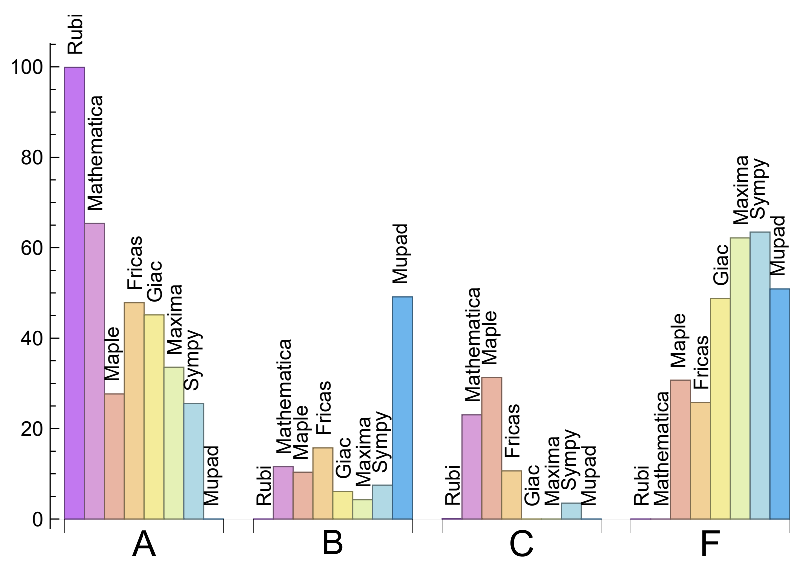
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.91	0.00	0.09	0.00
Mathematica	65.40	11.56	23.03	0.00
Fricas	47.83	15.73	10.64	25.81
Giac	45.14	6.11	0.00	48.75
Maxima	33.58	4.26	0.00	62.16
Maple	27.66	10.36	31.27	30.71
Sympy	25.53	7.49	3.52	63.46
Mupad	N/A	49.12	0.00	50.88

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	332	100.00 %	0.00 %	0.00 %
Fricas	279	36.92 %	60.93 %	2.15 %
Giac	527	93.55 %	1.14 %	5.31 %
Maxima	672	88.99 %	0.00 %	11.01 %
Sympy	686	80.17 %	16.18 %	3.64 %
Mupad	550	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

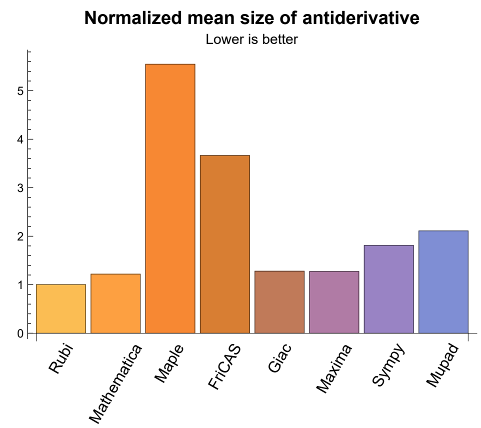
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

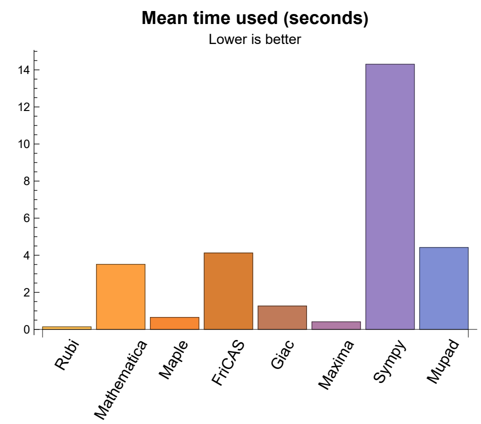
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.14	208.52	1.00	115.00	1.00
Mathematica	3.51	145.12	1.22	112.00	0.95
Maple	0.65	870.02	5.55	441.00	1.74
Maxima	0.41	123.39	1.27	97.00	1.02
Fricas	4.12	608.53	3.66	217.00	2.01
Sympy	14.30	197.09	1.81	107.00	1.03
Giac	1.26	151.15	1.28	112.50	1.05
Mupad	4.42	345.23	2.11	117.00	1.12

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
12
14
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{795, 796, 800, 818, 819, 821\}$

Mathematica $\{267, 268, 269, 279, 280, 322, 323, 324, 338, 339, 340, 341, 363, 365, 367, 373, 375, 377, 385, 387, 393, 395, 397, 421, 437, 438, 439, 440, 441, 455, 456, 457, 458, 459, 465, 467, 469, 475, 477, 479, 485, 487, 489, 495, 497, 499, 513, 515, 581, 582, 584, 585, 634, 636, 637, 672, 673, 674, 675, 676, 709, 710, 711, 712, 713, 743, 745, 746, 796, 798, 800, 815, 817, 818, 834, 835, 836, 837, 866, 883, 905, 926, 994, 1002, 1003, 1011\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

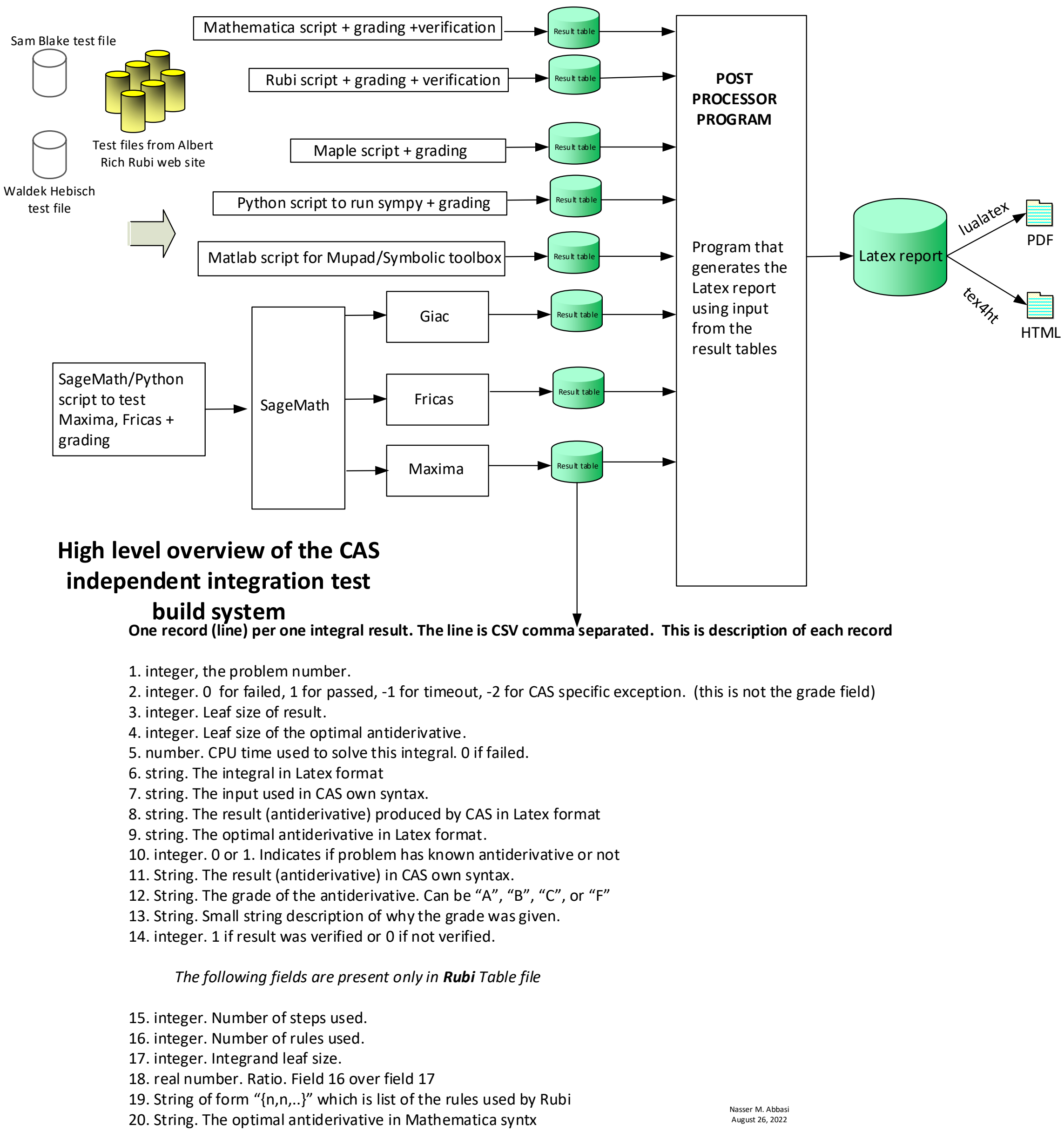
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55,
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B grade: { }

C grade: { 455 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 278, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 321, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 364, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 620, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 658, 659, 660, 661, 662, 663, 664, 677, 678, 679, 680, 681, 682, 683, 693, 696, 697, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 728, 729, 732, 733, 734, 735, 736, 737, 744, 747, 748, 749, 750, 751, 752, 753, 763, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 801, 802, 803, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 841, 842, 843, 844, 846, 847, 848, 849, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 903, 904, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1009, 1010, 1011, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081 }

B grade: { 30, 54, 267, 268, 269, 279, 280, 322, 323, 324, 338, 339, 340, 341, 363, 365, 366, 367, 373, 374, 375, 376, 377, 385, 386, 387, 393, 394, 395, 396, 397, 437, 438, 439, 440, 441, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 513, 514, 515, 672, 673, 674, 675, 676, 691, 692, 694, 695, 709, 710, 711, 712, 713, 727, 730, 731, 743, 745, 746, 761, 762, 764, 765, 766, 804, 845, 850, 866, 867, 868, 869, 880, 881, 882, 883, 884, 885, 886, 905, 906, 907, 924, 925, 926, 927, 928, 1004, 1005, 1006, 1007, 1008, 1012, 1013, 1014, 1015, 1016, 1017, 1051, 1052, 1053, 1065 }

C grade: { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 275, 276, 277, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 332, 333, 334, 335, 336, 337, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 449, 450, 451, 452, 453, 454, 455, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 581, 582, 584, 585, 600, 601, 602, 603, 604, 618, 619, 621, 622, 634, 635, 636, 637, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 684, 685, 686, 687, 688, 689, 690, 702, 703, 704, 705, 706, 707, 708, 721, 722, 723, 724, 725, 726, 738, 739, 740, 741, 742, 754, 755, 756, 757, 758, 759, 760, 795, 796, 797, 798, 799, 800, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 897, 898, 899, 900, 901, 902, 918, 919, 920, 921, 922, 923 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 557, 562, 565, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 929, 930, 931, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 945, 946, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 962, 963, 964, 965, 966, 967, 968, 969, 970, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 1002, 1003, 1004, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1029, 1036, 1039, 1040, 1041, 1042, 1043, 1044, 1046, 1047, 1048, 1049, 1050, 1056, 1057, 1058, 1059, 1078, 1079, 1080 }

B grade: { 27, 30, 54, 124, 125, 180, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 787, 788, 789, 790, 791, 792, 793, 794, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 932, 943, 944, 947, 948, 959, 960, 961, 971, 1005, 1014, 1023, 1045, 1051, 1052, 1053, 1054, 1060, 1061, 1062, 1063, 1065 }

C grade: { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 563, 564, 566, 605, 606, 607, 608, 609, 614, 615, 616, 617, 623, 624, 625, 626, 631, 632, 633, 634, 637, 638, 639, 640, 641, 642, 647, 648, 649, 650, 651, 795, 796, 797, 798, 799, 800, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 1064 }

F grade: { 127, 128, 129, 130, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 610, 611, 612, 613, 618, 619, 620, 621, 622, 627, 628, 629, 630, 635, 636, 643, 644, 645, 646, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 801, 802, 803, 804, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1024, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1055, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1081 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 228, 229, 230, 231, 244, 245, 246, 247, 248, 259, 260, 261, 270, 271, 272, 282, 283, 284, 285, 295, 296, 297, 298, 308, 309, 310, 311, 325, 326, 327, 328, 398, 399, 400, 401, 411, 412, 413, 414, 424, 425, 426, 427, 442, 443, 444, 445, 522, 525, 549, 554, 557, 559, 562, 565, 567, 568, 569, 570, 586, 587, 588, 589, 605, 606, 607, 608, 609, 623, 624, 625, 626, 638, 639, 640, 641, 642, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 964, 965, 966, 967, 968, 969, 970, 975, 976, 977, 978, 979, 980, 981, 982, 983, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1029, 1036, 1039, 1040, 1041, 1042, 1043, 1045, 1046, 1047, 1048, 1049, 1055, 1056, 1057, 1058, 1064, 1078, 1079, 1080, 1081 }

B grade: { 30, 54, 184, 217, 218, 232, 233, 516, 519, 527, 530, 533, 535, 538, 541, 543, 546, 551, 929, 930, 943, 944, 945, 959, 960, 961, 962, 963, 971, 972, 973, 974, 984, 985, 1014, 1044, 1050, 1051, 1052, 1053, 1054, 1059, 1060, 1061, 1062, 1063 }

C grade: { }

F grade: { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 262, 263, 264, 265, 266, 267, 268, 269, 273, 274, 275, 276, 277, 278, 279, 280, 281, 286, 287, 288, 289, 290, 291, 292, 293, 294, 299, 300, 301, 302, 303, 304, 305, 306, 307, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 402, 403, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 419, 420, 421, 422, 423, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 161, 163, 166, 169, 171, 174, 177, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 390, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 464, 470, 471, 472, 473, 474, 481, 483, 484, 506, 509, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 572, 573, 574, 575, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 605, 606, 607, 608, 609, 611, 612, 623, 624, 625, 626, 627, 628, 629, 630, 634, 637, 638, 639, 640, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 665, 667, 682, 683, 696, 697, 698, 699, 700, 701, 702, 703, 714, 715, 719, 720, 721, 722, 751, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 787, 788, 789, 790, 791, 792, 793, 794, 805, 806, 807, 808, 809, 810, 811, 814, 822, 824, 826, 827, 828, 829, 833, 853, 854, 855, 856, 857, 858, 859, 862, 871, 873, 874, 875, 879, 887, 888, 889, 890, 891, 892, 893, 896, 909, 911, 912, 913, 917, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1029, 1039, 1040, 1041, 1042, 1043, 1045, 1046, 1047, 1048, 1049, 1056, 1057, 1058, 1059, 1064, 1066, 1067, 1068, 1069, 1071, 1072, 1073, 1074, 1075, 1078, 1079, 1080, 1081 }

B grade: { 30, 54, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 124, 125, 126, 156, 157, 159, 160, 162, 164, 165, 167, 168, 170, 172, 173, 175, 176, 178, 267, 268, 269, 276, 278, 279, 280, 281, 317, 321, 322, 323, 324, 338, 339, 340, 341, 350, 351, 356, 357, 388, 389, 391, 392, 437, 438, 439, 440, 441, 456, 457, 458, 459, 463, 480, 482, 490, 491, 492, 493, 494, 507, 508, 571, 576, 577, 596, 614, 615, 616, 617, 620, 631, 632, 633, 641, 645, 646, 647, 648, 649, 650, 651, 666, 677, 678, 679, 680, 681, 684, 685, 686, 716, 717, 718, 732, 733, 734, 735, 736, 737, 738, 739, 747, 748, 749, 750, 752, 753, 754, 755, 779, 780, 781, 782, 783, 784, 785, 786, 812, 813, 823, 825, 830, 831, 832, 860, 861, 870, 872, 876, 877, 878, 894, 895, 908, 910, 914, 915, 916, 949, 975, 1014, 1036, 1044, 1050, 1051, 1052, 1053, 1054, 1055, 1060, 1061, 1062, 1063, 1065, 1070, 1076, 1077 }

C grade: { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 275, 277, 289, 290, 291, 292, 293, 294, 302, 303, 304, 306, 307, 315, 316, 318, 319, 320, 332, 333, 334, 335, 336, 337, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 449, 450, 451, 452, 453, 454, 455, 526, 550, 555, 556, 558, 560, 561, 563, 564, 566, 610, 613 }

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2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 114, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 199, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 271, 272, 273, 282, 283, 284, 285, 286, 295, 296, 297, 298, 299, 310, 311, 312, 327, 328, 329, 358, 359, 360, 361, 368, 369, 370, 371, 380, 381, 390, 391, 519, 522, 525, 543, 546, 549, 554, 557, 787, 789, 791, 807, 808, 855, 856, 889, 890, 930, 931, 932, 933, 934, 935, 936, 941, 942, 943, 947, 948, 949, 958, 962, 963, 964, 965, 966, 967, 969, 970, 971, 972, 973, 975, 976, 977, 978, 979, 982, 985, 1023, 1047, 1056, 1057, 1058, 1064 }

B grade: { 27, 30, 110, 113, 124, 125, 126, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 179, 180, 181, 184, 196, 197, 198, 201, 218, 232, 244, 245, 246, 248, 516, 530, 533, 538, 541, 562, 770, 929, 937, 938, 939, 940, 944, 945, 946, 950, 951, 952, 953, 954, 955, 956, 957, 959, 960, 961, 968, 974, 980, 981, 983, 984, 1024, 1029, 1039, 1040, 1041, 1045, 1046, 1051, 1052, 1053, 1059, 1060, 1061, 1062, 1078 }

C grade: { 127, 128, 500, 501, 502, 503, 504, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 553, 555, 556, 558, 841, 842, 843, 848, 849 }

F grade: { 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 107, 108, 109, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 129, 130, 163, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 263, 264, 265, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 281, 287, 288, 289, 290, 291, 292, 293, 294, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 362, 363, 364, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 379, 382, 383, 384, 385, 386, 387, 388, 389, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 527, 535, 551, 552, 559, 560, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 788, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 844, 845, 846, 847, 850, 851, 852, 853, 854, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1042, 1043, 1044, 1048, 1049, 1050, 1054, 1055, 1063, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1079, 1080, 1081 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 506, 507, 516, 519, 525, 543, 546, 549, 551, 554, 559, 562, 605, 606, 607, 608, 609, 610, 611, 623, 624, 625, 626, 627, 628, 638, 639, 640, 641, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 677, 678, 681, 682, 683, 696, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 732, 733, 734, 735, 736, 737, 747, 748, 749, 752, 753, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 793, 805, 806, 807, 808, 809, 812, 813, 822, 823, 824, 825, 826, 827, 853, 854, 855, 856, 857, 860, 870, 871, 872, 873, 874, 887, 888, 889, 890, 891, 894, 895, 908, 909, 910, 911, 912, 929, 930, 931, 937, 938, 939, 940, 942, 943, 944, 945, 946, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 968, 969, 971, 972, 973, 974, 975, 976, 980, 981, 982, 984, 985, 1002, 1003, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1051, 1056, 1057, 1058, 1060, 1061, 1062, 1064, 1078 }

B grade: { 27, 30, 54, 124, 125, 126, 508, 509, 527, 530, 535, 538, 679, 680, 697, 750, 751, 792, 794, 814, 828, 829, 830, 831, 832, 833, 859, 875, 877, 896, 913, 914, 915, 916, 917, 932, 933, 934, 935, 936, 941, 947, 948, 949, 950, 951, 952, 965, 966, 967, 970, 977, 978, 979, 983, 1004, 1005, 1007, 1008, 1014, 1052, 1053, 1055, 1079, 1080, 1081 }

C grade: { }

F grade: { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 517, 518, 520, 521, 522, 523, 524, 526, 528, 529, 531, 532, 533, 534, 536, 537, 539, 540, 541, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 557, 558, 560, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 738, 739, 740, 741, 742, 743, 744, 745, 746, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 788, 790, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 810, 811, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 858, 861, 862, 863, 864, 865, 866, 867, 868, 869, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 892, 893, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1006, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1054, 1059, 1063, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 276, 281, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 317, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 506, 507, 508, 509, 557, 562, 565, 567, 568, 569, 570, 571, 572, 573, 586, 587, 588, 589, 590, 591, 592, 605, 606, 607, 608, 609, 610, 611, 623, 624, 625, 626, 627, 628, 638, 639, 640, 641, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 677, 678, 679, 680, 681, 682, 683, 696, 697, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 732, 733, 734, 735, 736, 737, 747, 748, 749, 750, 751, 752, 753, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 793, 805, 806, 807, 808, 809, 822, 823, 824, 825, 826, 827, 853, 854, 855, 856, 857, 870, 871, 872, 873, 874, 887, 888, 889, 890, 891, 908, 909, 910, 911, 912, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 959, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 1002, 1003, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1014, 1015, 1016, 1017, 1023, 1024, 1029, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1078, 1079, 1080, 1081 }

C grade: { }

F grade: { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 267, 268, 269, 275, 277, 278, 279, 280, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 563, 564, 566, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 738, 739, 740, 741, 742, 743, 744, 745, 746, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 788, 790, 792, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 943, 944, 957, 958, 960, 961, 972, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1004, 1013, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.024	0.011	0.125	0.300	2.225	0.007	0.639	0.200

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.012	0.003	0.149	0.290	2.506	0.007	0.559	2.483

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.009	0.003	0.137	0.268	2.539	0.007	0.618	0.034

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	28	25	27	28	26
N.S.	1	1.00	1.00	0.97	0.97	0.86	0.93	0.97	0.90
time (sec)	N/A	0.015	0.007	0.074	0.290	2.261	0.098	0.560	0.035

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	27	29	26	29	28
N.S.	1	1.00	1.00	0.97	0.87	0.94	0.84	0.94	0.90
time (sec)	N/A	0.011	0.006	0.023	0.272	2.038	0.089	0.476	0.039

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	24	28	24	23	24
N.S.	1	1.00	1.00	0.86	0.86	1.00	0.86	0.82	0.86
time (sec)	N/A	0.012	0.005	0.020	0.309	2.429	0.083	0.590	2.337

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	28	30	26	40	25
N.S.	1	1.00	1.00	0.90	0.97	1.03	0.90	1.38	0.86
time (sec)	N/A	0.015	0.007	0.027	0.290	2.399	0.239	0.561	0.040

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	28	29	29	31	31	29
N.S.	1	1.00	1.03	0.90	0.94	0.94	1.00	1.00	0.94
time (sec)	N/A	0.011	0.006	0.023	0.313	2.205	0.311	0.500	0.033

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	25	27	29	29	29	28
N.S.	1	1.00	1.07	0.89	0.96	1.04	1.04	1.04	1.00
time (sec)	N/A	0.011	0.007	0.024	0.275	2.307	0.132	0.742	2.316

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	26	30	31	29	37	29
N.S.	1	1.00	1.07	0.90	1.03	1.07	1.00	1.28	1.00
time (sec)	N/A	0.015	0.010	0.023	0.270	2.252	0.276	0.595	0.052

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	51	51	54	53	51
N.S.	1	1.00	1.21	1.24	1.21	1.21	1.29	1.26	1.21
time (sec)	N/A	0.049	0.009	0.275	0.290	2.390	0.012	0.542	2.375

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	54	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.98	0.96	0.93
time (sec)	N/A	0.023	0.005	0.311	0.311	2.010	0.010	1.010	0.043

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	0.96
time (sec)	N/A	0.017	0.004	0.304	0.305	1.634	0.011	0.778	0.043

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	51	52	52	49	53	52	49
N.S.	1	1.00	1.11	1.13	1.13	1.07	1.15	1.13	1.07
time (sec)	N/A	0.023	0.009	0.250	0.275	2.231	0.050	0.612	0.038

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	53	51	53	49	52	50
N.S.	1	1.00	1.00	1.00	0.96	1.00	0.92	0.98	0.94
time (sec)	N/A	0.021	0.010	0.253	0.280	2.416	0.046	0.633	0.050

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	53	49	48	48
N.S.	1	1.00	1.00	0.98	0.96	1.06	0.98	0.96	0.96
time (sec)	N/A	0.018	0.009	0.244	0.293	1.965	0.048	0.748	0.046

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	49	52	54	51	69	49
N.S.	1	1.00	0.96	0.96	1.02	1.06	1.00	1.35	0.96
time (sec)	N/A	0.029	0.014	0.295	0.282	1.903	0.124	0.660	0.043

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	50	53	53	53	54	52
N.S.	1	1.00	0.96	0.94	1.00	1.00	1.00	1.02	0.98
time (sec)	N/A	0.019	0.009	0.254	0.283	1.873	0.147	0.632	0.049

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	46	51	53	53	51	50
N.S.	1	1.00	1.00	0.92	1.02	1.06	1.06	1.02	1.00
time (sec)	N/A	0.019	0.011	0.246	0.299	1.997	0.168	0.527	2.371

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	46	54	55	51	70	52
N.S.	1	1.00	1.00	0.90	1.06	1.08	1.00	1.37	1.02
time (sec)	N/A	0.025	0.012	0.253	0.288	1.973	0.402	0.538	2.359

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	48	54	53	58	56	53
N.S.	1	1.00	1.02	0.91	1.02	1.00	1.09	1.06	1.00
time (sec)	N/A	0.018	0.010	0.251	0.273	1.746	0.490	0.550	0.045

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	51	53	54	53	50
N.S.	1	1.00	1.00	0.90	1.02	1.06	1.08	1.06	1.00
time (sec)	N/A	0.018	0.013	0.257	0.287	3.084	0.546	0.518	2.342

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	119	136	125	107
N.S.	1	1.00	1.00	1.06	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.068	0.012	0.309	0.291	2.052	0.018	0.543	0.050

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	107	124	119	119	136	125	107
N.S.	1	1.00	1.13	1.31	1.25	1.25	1.43	1.32	1.13
time (sec)	N/A	0.174	0.014	0.299	0.296	2.460	0.020	1.165	2.338

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	119	134	125	107
N.S.	1	1.00	1.00	1.06	1.02	1.02	1.15	1.07	0.91
time (sec)	N/A	0.055	0.011	0.320	0.291	2.484	0.018	1.270	0.041

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	119	136	125	107
N.S.	1	1.00	1.00	1.06	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.049	0.010	0.311	0.299	2.708	0.017	0.867	0.041

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	107	124	119	119	138	125	107
N.S.	1	1.00	1.60	1.85	1.78	1.78	2.06	1.87	1.60
time (sec)	N/A	0.104	0.013	0.291	0.287	2.524	0.020	0.692	0.040

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	119	136	125	107
N.S.	1	1.00	1.00	1.06	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.047	0.009	0.302	0.295	2.868	0.018	1.357	0.041

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	119	133	124	107
N.S.	1	1.00	1.00	1.06	1.02	1.02	1.14	1.06	0.91
time (sec)	N/A	0.046	0.010	0.301	0.267	2.926	0.018	1.248	0.040

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	107	124	119	119	136	125	107
N.S.	1	1.00	2.55	2.95	2.83	2.83	3.24	2.98	2.55
time (sec)	N/A	0.049	0.012	0.299	0.289	3.498	0.021	1.051	0.041

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	119	134	124	106
N.S.	1	1.00	1.00	1.06	1.02	1.02	1.15	1.06	0.91
time (sec)	N/A	0.043	0.009	0.309	0.267	3.079	0.019	1.489	0.040

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	121	115	115	128	120	103
N.S.	1	1.00	1.00	1.11	1.06	1.06	1.17	1.10	0.94
time (sec)	N/A	0.039	0.008	0.319	0.279	2.289	0.018	1.570	0.041

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	113	124	120	117	134	124	105
N.S.	1	1.00	1.28	1.41	1.36	1.33	1.52	1.41	1.19
time (sec)	N/A	0.046	0.017	0.253	0.292	2.637	0.091	1.360	0.046

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	125	118	121	129	124	106
N.S.	1	1.00	1.00	1.12	1.05	1.08	1.15	1.11	0.95
time (sec)	N/A	0.043	0.017	0.270	0.287	2.188	0.093	1.214	0.042

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	120	116	121	128	119	104
N.S.	1	1.00	1.00	1.07	1.04	1.08	1.14	1.06	0.93
time (sec)	N/A	0.041	0.016	0.255	0.294	2.629	0.093	0.997	0.043

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	115	121	120	123	133	143	105
N.S.	1	1.00	1.02	1.07	1.06	1.09	1.18	1.27	0.93
time (sec)	N/A	0.081	0.023	0.255	0.290	2.879	0.483	0.838	2.348

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	115	123	121	121	133	127	109
N.S.	1	1.00	1.02	1.09	1.07	1.07	1.18	1.12	0.96
time (sec)	N/A	0.045	0.019	0.259	0.267	3.362	0.484	0.723	0.042

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	119	120	121	133	124	108
N.S.	1	1.00	1.00	1.05	1.06	1.07	1.18	1.10	0.96
time (sec)	N/A	0.042	0.019	0.260	0.271	3.429	0.400	0.705	0.043

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	106	117	122	123	131	148	113
N.S.	1	1.00	0.93	1.03	1.07	1.08	1.15	1.30	0.99
time (sec)	N/A	0.074	0.028	0.254	0.273	3.700	0.494	0.714	0.049

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	117	121	121	129	127	113
N.S.	1	1.00	1.00	1.06	1.10	1.10	1.17	1.15	1.03
time (sec)	N/A	0.045	0.020	0.256	0.301	2.788	0.596	0.941	2.341

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	114	120	121	133	124	111
N.S.	1	1.00	1.00	1.01	1.06	1.07	1.18	1.10	0.98
time (sec)	N/A	0.043	0.019	0.247	0.298	1.985	0.640	1.747	0.044

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	106	111	123	123	129	150	118
N.S.	1	1.00	0.93	0.97	1.08	1.08	1.13	1.32	1.04
time (sec)	N/A	0.072	0.030	0.265	0.272	2.016	1.495	1.555	0.050

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	111	122	121	131	127	118
N.S.	1	1.00	1.03	0.97	1.06	1.05	1.14	1.10	1.03
time (sec)	N/A	0.043	0.013	0.258	0.290	3.301	9.096	1.229	2.364

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	108	120	121	131	124	116
N.S.	1	1.00	1.00	0.99	1.10	1.11	1.20	1.14	1.06
time (sec)	N/A	0.044	0.021	0.255	0.269	2.039	30.089	0.751	0.067

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	118	106	123	123	129	149	122
N.S.	1	1.00	1.04	0.93	1.08	1.08	1.13	1.31	1.07
time (sec)	N/A	0.068	0.022	0.301	0.299	2.246	51.736	0.734	0.064

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	117	107	122	121	0	128	123
N.S.	1	1.00	1.02	0.93	1.06	1.05	0.00	1.11	1.07
time (sec)	N/A	0.044	0.019	0.270	0.268	2.015	0.000	0.576	2.375

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	102	119	121	0	123	120
N.S.	1	1.00	1.00	0.93	1.08	1.10	0.00	1.12	1.09
time (sec)	N/A	0.046	0.023	0.255	0.300	1.929	0.000	0.709	2.392

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	116	102	123	123	0	145	121
N.S.	1	1.00	1.03	0.90	1.09	1.09	0.00	1.28	1.07
time (sec)	N/A	0.060	0.032	0.266	0.266	1.695	0.000	1.411	0.079

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	104	122	121	0	128	121
N.S.	1	1.00	1.03	0.90	1.06	1.05	0.00	1.11	1.05
time (sec)	N/A	0.041	0.017	0.272	0.273	2.579	0.000	2.430	2.363

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	101	119	121	0	125	119
N.S.	1	1.00	1.00	0.92	1.08	1.10	0.00	1.14	1.08
time (sec)	N/A	0.052	0.024	0.250	0.285	2.351	0.000	1.296	0.077

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	121	102	123	123	0	136	121
N.S.	1	1.00	1.33	1.12	1.35	1.35	0.00	1.49	1.33
time (sec)	N/A	0.037	0.022	0.260	0.278	1.670	0.000	1.014	0.093

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	119	104	121	121	0	127	119
N.S.	1	1.00	1.05	0.92	1.07	1.07	0.00	1.12	1.05
time (sec)	N/A	0.042	0.018	0.260	0.276	2.088	0.000	0.906	2.369

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	121	121	0	127	121
N.S.	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.042	0.018	0.246	0.279	1.828	0.000	1.347	2.347

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	118	104	121	121	0	127	122
N.S.	1	1.00	2.46	2.17	2.52	2.52	0.00	2.65	2.54
time (sec)	N/A	0.024	0.017	0.268	0.309	2.252	0.000	2.614	2.370

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	121	121	0	127	121
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.043	0.024	0.256	0.294	1.455	0.000	2.064	0.064

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	171	151	182	167	114	217	164
N.S.	1	1.00	0.93	0.83	0.99	0.91	0.62	1.19	0.90
time (sec)	N/A	0.101	0.078	0.277	0.493	2.062	0.284	0.970	0.268

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	47	50	50	51	46	52	52
N.S.	1	1.00	0.87	0.93	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.040	0.013	0.333	0.281	1.337	0.219	1.273	0.079

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	154	131	157	162	114	207	144
N.S.	1	1.00	0.92	0.78	0.94	0.97	0.68	1.24	0.86
time (sec)	N/A	0.077	0.050	0.275	0.552	2.185	0.229	1.359	2.545

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	152	127	154	145	87	186	162
N.S.	1	1.00	0.94	0.78	0.95	0.90	0.54	1.15	1.00
time (sec)	N/A	0.075	0.049	0.291	0.498	2.104	0.255	0.814	2.610

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	32	31	30	27	32	31
N.S.	1	1.00	0.89	0.91	0.89	0.86	0.77	0.91	0.89
time (sec)	N/A	0.025	0.008	0.269	0.281	2.010	0.191	0.771	0.063

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	152	113	131	382	92	161	126
N.S.	1	1.00	1.01	0.75	0.87	2.55	0.61	1.07	0.84
time (sec)	N/A	0.062	0.028	0.272	0.507	1.584	0.197	1.308	2.566

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	129	110	128	369	71	133	123
N.S.	1	1.00	0.89	0.76	0.88	2.54	0.49	0.92	0.85
time (sec)	N/A	0.054	0.037	0.280	0.485	2.116	0.223	0.899	2.541

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	35	32	26	34	36
N.S.	1	1.00	1.00	0.97	1.03	0.94	0.76	1.00	1.06
time (sec)	N/A	0.024	0.008	0.274	0.308	1.887	0.545	1.199	0.105

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	134	114	140	372	90	155	126
N.S.	1	1.00	0.91	0.78	0.95	2.53	0.61	1.05	0.86
time (sec)	N/A	0.061	0.047	0.285	0.516	1.372	0.222	1.699	2.545

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	135	113	140	411	73	161	126
N.S.	1	1.00	0.91	0.76	0.94	2.76	0.49	1.08	0.85
time (sec)	N/A	0.062	0.058	0.295	0.531	2.124	0.258	1.353	0.244

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	46	48	47	41	69	46
N.S.	1	1.00	0.98	0.92	0.96	0.94	0.82	1.38	0.92
time (sec)	N/A	0.033	0.012	0.286	0.294	1.877	0.936	1.112	2.405

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	154	130	147	158	112	197	178
N.S.	1	1.00	0.93	0.79	0.89	0.96	0.68	1.19	1.08
time (sec)	N/A	0.073	0.067	0.290	0.524	1.929	0.694	0.861	2.591

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	154	130	148	176	99	176	145
N.S.	1	1.00	0.92	0.77	0.88	1.05	0.59	1.05	0.86
time (sec)	N/A	0.075	0.069	0.285	0.511	2.103	0.338	0.841	2.564

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	70	64	70	73	61	99	70
N.S.	1	1.00	1.01	0.93	1.01	1.06	0.88	1.43	1.01
time (sec)	N/A	0.042	0.016	0.283	0.297	1.794	0.622	0.989	0.127

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	173	150	178	180	139	216	161
N.S.	1	1.00	0.94	0.82	0.97	0.98	0.76	1.17	0.88
time (sec)	N/A	0.089	0.078	0.308	0.513	2.217	0.315	0.736	2.581

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	203	176	218	271	156	244	209
N.S.	1	1.00	0.87	0.76	0.94	1.16	0.67	1.05	0.90
time (sec)	N/A	0.096	0.081	0.282	0.488	2.033	0.559	0.804	2.620

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	76	82	121	82	106	86
N.S.	1	1.00	0.88	0.93	1.00	1.48	1.00	1.29	1.05
time (sec)	N/A	0.067	0.038	0.299	0.283	1.967	0.542	1.273	0.083

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	185	156	192	257	151	236	179
N.S.	1	1.00	0.86	0.73	0.89	1.20	0.70	1.10	0.83
time (sec)	N/A	0.092	0.075	0.289	0.498	1.826	0.595	1.706	0.268

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	181	151	187	240	126	211	193
N.S.	1	1.00	0.85	0.71	0.88	1.13	0.59	0.99	0.91
time (sec)	N/A	0.088	0.072	0.288	0.518	1.981	0.510	0.833	2.621

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	50	59	60	81	56	91	62
N.S.	1	1.00	0.83	0.98	1.00	1.35	0.93	1.52	1.03
time (sec)	N/A	0.041	0.021	0.276	0.300	1.719	0.464	0.770	0.081

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	165	138	162	578	126	189	158
N.S.	1	1.00	0.84	0.70	0.83	2.95	0.64	0.96	0.81
time (sec)	N/A	0.073	0.070	0.279	0.535	2.159	0.525	0.884	2.578

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	160	133	157	573	102	166	150
N.S.	1	1.00	0.84	0.70	0.83	3.02	0.54	0.87	0.79
time (sec)	N/A	0.075	0.069	0.277	0.505	1.742	1.142	0.725	2.577

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	38	40	44	36	65	37
N.S.	1	1.00	1.00	0.93	0.98	1.07	0.88	1.59	0.90
time (sec)	N/A	0.028	0.008	0.280	0.277	2.085	0.355	0.688	2.348

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	146	136	160	548	117	186	145
N.S.	1	1.00	0.85	0.80	0.94	3.20	0.68	1.09	0.85
time (sec)	N/A	0.057	0.055	0.272	0.518	2.042	0.363	0.615	0.250

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	134	158	537	97	160	143
N.S.	1	1.00	0.86	0.79	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.057	0.053	0.279	0.506	1.874	0.307	0.635	2.550

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	48	51	70	46	61	47
N.S.	1	1.00	0.90	0.94	1.00	1.37	0.90	1.20	0.92
time (sec)	N/A	0.032	0.018	0.283	0.304	1.934	0.268	0.747	0.141

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	196	164	139	166	570	122	180	156
N.S.	1	1.01	0.84	0.71	0.85	2.92	0.63	0.92	0.80
time (sec)	N/A	0.072	0.073	0.304	0.546	2.036	0.340	0.591	2.569

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	163	138	172	618	109	188	159
N.S.	1	1.00	0.83	0.70	0.88	3.15	0.56	0.96	0.81
time (sec)	N/A	0.072	0.075	0.290	0.527	3.284	0.374	0.802	2.565

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	76	76	118	70	80	78
N.S.	1	1.00	0.84	1.00	1.00	1.55	0.92	1.05	1.03
time (sec)	N/A	0.052	0.029	0.271	0.278	2.374	0.675	1.462	2.432

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	185	155	186	259	153	231	209
N.S.	1	1.00	0.86	0.72	0.87	1.20	0.71	1.07	0.97
time (sec)	N/A	0.082	0.083	0.292	0.497	2.394	0.403	2.056	2.615

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	183	154	186	277	138	206	176
N.S.	1	1.00	0.85	0.72	0.87	1.29	0.64	0.96	0.82
time (sec)	N/A	0.089	0.084	0.300	0.555	1.753	0.444	1.357	2.574

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	96	106	154	100	149	100
N.S.	1	1.00	0.88	0.99	1.09	1.59	1.03	1.54	1.03
time (sec)	N/A	0.072	0.057	0.288	0.274	2.043	0.776	1.568	0.144

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	94	102	115	179	112	131	117
N.S.	1	1.00	0.88	0.95	1.07	1.67	1.05	1.22	1.09
time (sec)	N/A	0.095	0.038	0.309	0.303	1.909	2.721	1.468	0.097

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	92	85	94	142	94	93	94
N.S.	1	1.00	1.05	0.97	1.07	1.61	1.07	1.06	1.07
time (sec)	N/A	0.103	0.024	0.299	0.273	2.178	1.403	1.539	2.402

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	61	72	89	70	61	70
N.S.	1	1.00	0.97	0.92	1.09	1.35	1.06	0.92	1.06
time (sec)	N/A	0.082	0.015	0.270	0.279	1.975	1.071	1.345	2.385

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	39	42	42	42	28	44
N.S.	1	1.00	0.94	1.22	1.31	1.31	1.31	0.88	1.38
time (sec)	N/A	0.027	0.009	0.268	0.272	1.669	0.440	1.217	2.332

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	59	63	77	119	75	74	71
N.S.	1	1.00	0.87	0.93	1.13	1.75	1.10	1.09	1.04
time (sec)	N/A	0.076	0.028	0.267	0.286	2.184	0.382	1.173	0.161

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	87	102	109	197	107	136	107
N.S.	1	1.00	0.86	1.01	1.08	1.95	1.06	1.35	1.06
time (sec)	N/A	0.079	0.035	0.292	0.276	1.952	0.806	1.471	2.458

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	108	123	136	229	133	131	130
N.S.	1	1.00	0.89	1.01	1.11	1.88	1.09	1.07	1.07
time (sec)	N/A	0.094	0.046	0.280	0.270	2.406	2.007	1.567	0.153

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	216	176	228	364	192	259	213
N.S.	1	1.00	0.88	0.72	0.93	1.48	0.78	1.05	0.87
time (sec)	N/A	0.106	0.097	0.285	0.491	1.855	10.258	1.110	2.582

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	210	171	223	347	163	234	227
N.S.	1	1.00	0.86	0.70	0.91	1.42	0.67	0.96	0.93
time (sec)	N/A	0.114	0.093	0.292	0.510	1.446	1.399	0.771	0.320

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	194	158	196	792	162	210	187
N.S.	1	1.00	0.87	0.71	0.88	3.57	0.73	0.95	0.84
time (sec)	N/A	0.094	0.093	0.287	0.510	2.329	8.393	0.946	2.558

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	188	153	191	789	141	187	183
N.S.	1	1.00	0.85	0.70	0.87	3.59	0.64	0.85	0.83
time (sec)	N/A	0.088	0.093	0.293	0.497	1.723	1.218	0.630	2.603

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	181	154	195	756	155	206	175
N.S.	1	1.00	0.90	0.77	0.97	3.76	0.77	1.02	0.87
time (sec)	N/A	0.074	0.099	0.280	0.507	1.495	6.225	0.700	0.267

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	178	152	193	743	136	187	173
N.S.	1	1.00	0.89	0.76	0.97	3.73	0.68	0.94	0.87
time (sec)	N/A	0.077	0.094	0.276	0.497	1.937	1.168	0.681	2.559

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	178	155	195	752	153	207	175
N.S.	1	1.00	0.89	0.77	0.97	3.74	0.76	1.03	0.87
time (sec)	N/A	0.078	0.074	0.302	0.490	1.651	1.082	0.703	0.268

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	175	153	192	743	133	180	173
N.S.	1	1.00	0.89	0.78	0.97	3.77	0.68	0.91	0.88
time (sec)	N/A	0.072	0.075	0.279	0.531	1.656	0.410	0.657	0.256

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	193	159	199	776	162	204	185
N.S.	1	1.00	0.85	0.70	0.88	3.42	0.71	0.90	0.81
time (sec)	N/A	0.086	0.096	0.300	0.488	1.702	0.480	0.659	2.598

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	189	158	201	812	143	209	188
N.S.	1	1.00	0.83	0.70	0.89	3.58	0.63	0.92	0.83
time (sec)	N/A	0.095	0.096	0.294	0.490	1.631	0.498	0.677	2.583

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	214	175	221	366	189	254	240
N.S.	1	1.00	0.87	0.71	0.90	1.49	0.77	1.03	0.98
time (sec)	N/A	0.109	0.104	0.291	0.548	1.895	0.534	0.617	2.637

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	210	174	221	384	173	229	207
N.S.	1	1.00	0.85	0.71	0.90	1.56	0.70	0.93	0.84
time (sec)	N/A	0.103	0.105	0.307	0.498	2.180	0.567	0.691	2.584

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	68	72	0	70	68
N.S.	1	1.00	0.94	0.93	0.97	1.03	0.00	1.00	0.97
time (sec)	N/A	0.051	0.021	0.342	0.272	1.927	0.000	0.734	2.844

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	242	228	324	273	0	311	1751
N.S.	1	1.00	0.80	0.76	1.08	0.91	0.00	1.03	5.82
time (sec)	N/A	0.214	0.084	0.398	0.508	2.013	0.000	0.657	11.360

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	238	225	349	228	0	308	873
N.S.	1	1.00	0.80	0.76	1.18	0.77	0.00	1.04	2.95
time (sec)	N/A	0.178	0.071	0.365	0.495	1.406	0.000	0.682	1.829

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	50	49	42	144	51	51
N.S.	1	1.00	0.81	0.94	0.92	0.79	2.72	0.96	0.96
time (sec)	N/A	0.035	0.013	0.342	0.281	2.374	5.877	0.628	0.313

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	207	289	244	0	286	1364
N.S.	1	1.00	0.78	0.72	1.00	0.85	0.00	0.99	4.74
time (sec)	N/A	0.103	0.053	0.358	0.490	1.875	0.000	0.715	9.046

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	207	317	199	342	278	1265
N.S.	1	1.00	0.78	0.72	1.10	0.69	1.19	0.97	4.39
time (sec)	N/A	0.103	0.051	0.358	0.517	2.233	138.078	0.608	8.117

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	31	42	41	31	138	51	602
N.S.	1	1.00	0.69	0.93	0.91	0.69	3.07	1.13	13.38
time (sec)	N/A	0.021	0.012	0.315	0.295	1.766	1.538	0.587	0.257

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	207	265	201	515	290	982
N.S.	1	1.00	0.78	0.72	0.92	0.70	1.79	1.01	3.41
time (sec)	N/A	0.098	0.058	0.347	0.483	1.448	73.690	0.608	5.415

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	207	293	254	0	278	1364
N.S.	1	1.00	0.78	0.72	1.02	0.88	0.00	0.97	4.74
time (sec)	N/A	0.102	0.059	0.294	0.494	2.060	0.000	0.701	9.007

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	59	61	54	0	71	58
N.S.	1	1.00	0.87	0.95	0.98	0.87	0.00	1.15	0.94
time (sec)	N/A	0.043	0.018	0.361	0.284	2.719	0.000	0.835	2.840

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	244	228	300	238	0	305	716
N.S.	1	1.00	0.82	0.76	1.00	0.80	0.00	1.02	2.39
time (sec)	N/A	0.198	0.082	0.370	0.501	2.100	0.000	1.206	3.854

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	259	228	328	301	0	309	1829
N.S.	1	1.00	0.86	0.76	1.09	1.00	0.00	1.03	6.08
time (sec)	N/A	0.177	0.093	0.374	0.488	3.831	0.000	1.013	11.834

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	83	87	99	0	111	87
N.S.	1	1.00	1.01	0.95	1.00	1.14	0.00	1.28	1.00
time (sec)	N/A	0.070	0.026	0.365	0.306	4.844	0.000	0.876	3.223

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	282	248	341	305	0	328	1734
N.S.	1	1.00	0.89	0.78	1.07	0.96	0.00	1.03	5.45
time (sec)	N/A	0.272	0.105	0.390	0.488	3.316	0.000	0.749	11.370

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	282	248	369	356	0	336	1860
N.S.	1	1.00	0.88	0.77	1.15	1.11	0.00	1.05	5.79
time (sec)	N/A	0.309	0.108	0.418	0.502	3.250	0.000	0.705	11.573

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	114	117	127	0	165	118
N.S.	1	1.00	1.00	0.96	0.98	1.07	0.00	1.39	0.99
time (sec)	N/A	0.089	0.034	0.385	0.269	8.964	0.000	0.925	3.212

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	304	279	376	332	0	377	1814
N.S.	1	1.00	0.86	0.79	1.07	0.94	0.00	1.07	5.15
time (sec)	N/A	0.339	0.122	0.405	0.507	2.190	0.000	1.252	11.909

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	137	1077	205	851	5418	1331	559
N.S.	1	1.00	0.93	7.28	1.39	5.75	36.61	8.99	3.78
time (sec)	N/A	0.071	0.456	0.274	0.336	1.663	1.815	2.637	3.213

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	261	91	215	1057	332	177
N.S.	1	1.00	0.93	3.68	1.28	3.03	14.89	4.68	2.49
time (sec)	N/A	0.027	0.076	0.292	0.277	2.890	0.570	1.328	2.718

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	53	53	92	410	143	95
N.S.	1	1.00	0.93	1.18	1.18	2.04	9.11	3.18	2.11
time (sec)	N/A	0.015	0.043	0.030	0.285	1.909	0.328	1.296	2.655

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	190	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	2.88	0.00	-0.02
time (sec)	N/A	0.023	0.097	0.056	0.000	0.000	8.353	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	1047	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	11.26	0.00	-0.01
time (sec)	N/A	0.029	0.167	0.055	0.000	0.000	142.313	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.220	0.038	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	86	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.089	0.101	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.011	0.019	0.110	0.293	2.973	1.107	0.974	0.053

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.011	0.018	0.113	0.271	1.848	0.743	0.620	2.560

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.012	0.015	0.110	0.292	2.610	0.477	0.610	0.044

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	30	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.77	1.18	0.74	0.79
time (sec)	N/A	0.011	0.017	0.106	0.301	2.181	1.220	0.579	0.041

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	28	27	29	44	29	31
N.S.	1	1.00	0.95	0.76	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.011	0.016	0.111	0.280	2.613	0.268	0.594	2.590

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	44	29	31
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.011	0.020	0.053	0.294	2.620	0.390	0.576	2.599

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	34	30	27	28	46	29	31
N.S.	1	1.00	0.87	0.77	0.69	0.72	1.18	0.74	0.79
time (sec)	N/A	0.012	0.021	0.061	0.280	1.860	0.447	0.540	0.040

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	42	29	30
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.14	0.78	0.81
time (sec)	N/A	0.011	0.021	0.062	0.272	2.206	0.558	0.529	0.042

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	51	56	80	53	51
N.S.	1	1.00	0.94	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.022	0.029	0.269	0.289	1.293	1.838	0.556	2.567

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.022	0.034	0.279	0.283	1.968	1.323	0.540	0.047

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.020	0.032	0.270	0.274	1.408	0.920	0.632	0.047

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	51	54	80	53	51
N.S.	1	1.00	0.94	0.83	0.81	0.86	1.27	0.84	0.81
time (sec)	N/A	0.020	0.032	0.263	0.270	1.472	1.770	0.573	0.049

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	52	51	53	78	53	51
N.S.	1	1.00	0.97	0.85	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.021	0.030	0.269	0.282	2.139	0.598	0.546	0.045

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	78	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.020	0.042	0.280	0.274	1.181	0.790	0.576	0.050

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	54	51	53	80	53	51
N.S.	1	1.00	0.94	0.86	0.81	0.84	1.27	0.84	0.81
time (sec)	N/A	0.020	0.034	0.269	0.277	2.138	0.875	0.569	0.048

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	76	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.25	0.87	0.84
time (sec)	N/A	0.019	0.038	0.268	0.277	1.817	1.054	0.540	0.048

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.029	0.041	0.270	0.275	1.321	2.923	0.546	2.519

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	91	76	73	78	114	77	69
N.S.	1	1.00	1.07	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.029	0.043	0.282	0.282	2.175	3.569	0.567	0.030

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.028	0.047	0.278	0.269	2.268	1.591	0.629	0.032

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	76	73	76	114	77	69
N.S.	1	1.00	0.94	0.89	0.86	0.89	1.34	0.91	0.81
time (sec)	N/A	0.029	0.040	0.265	0.271	1.685	2.493	0.600	0.033

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	80	76	73	75	112	77	69
N.S.	1	1.00	0.96	0.92	0.88	0.90	1.35	0.93	0.83
time (sec)	N/A	0.029	0.039	0.271	0.277	1.451	1.698	0.664	0.030

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	78	73	75	112	77	69
N.S.	1	1.00	1.00	0.94	0.88	0.90	1.35	0.93	0.83
time (sec)	N/A	0.028	0.053	0.270	0.275	1.534	2.036	0.625	0.033

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	78	73	75	112	77	69
N.S.	1	1.00	0.91	0.92	0.86	0.88	1.32	0.91	0.81
time (sec)	N/A	0.027	0.038	0.278	0.287	1.666	1.509	0.569	0.034

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	78	73	75	110	77	69
N.S.	1	1.00	0.94	0.94	0.88	0.90	1.33	0.93	0.83
time (sec)	N/A	0.030	0.050	0.272	0.275	1.535	2.115	0.601	0.032

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	67	58	58	143	428	64	111
N.S.	1	1.00	0.92	0.79	0.79	1.96	5.86	0.88	1.52
time (sec)	N/A	0.035	0.062	0.296	0.483	1.798	67.766	0.564	2.607

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	172	209	295	2433	605	289	1933
N.S.	1	1.00	0.60	0.73	1.02	8.45	2.10	1.00	6.71
time (sec)	N/A	0.371	0.183	0.375	0.505	2.004	30.202	0.589	2.889

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	153	191	212	3635	581	280	1640
N.S.	1	1.00	0.57	0.71	0.79	13.46	2.15	1.04	6.07
time (sec)	N/A	0.458	0.177	0.361	0.533	2.343	14.461	0.884	2.853

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	40	39	108	381	39	93
N.S.	1	1.00	1.00	0.75	0.74	2.04	7.19	0.74	1.75
time (sec)	N/A	0.024	0.046	0.292	0.478	1.513	5.114	0.580	2.600

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	152	191	278	2424	558	280	1915
N.S.	1	1.00	0.57	0.71	1.04	9.04	2.08	1.04	7.15
time (sec)	N/A	0.314	0.182	0.362	0.507	1.676	6.601	0.528	2.885

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	156	191	212	3663	561	280	1700
N.S.	1	1.00	0.58	0.71	0.79	13.67	2.09	1.04	6.34
time (sec)	N/A	0.392	0.205	0.359	0.500	2.554	9.239	0.607	2.856

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	40	39	120	371	39	102
N.S.	1	1.00	1.00	0.75	0.74	2.26	7.00	0.74	1.92
time (sec)	N/A	0.026	0.046	0.306	0.530	1.867	20.928	0.570	0.101

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	158	191	278	2424	586	280	2023
N.S.	1	1.00	0.59	0.71	1.03	8.98	2.17	1.04	7.49
time (sec)	N/A	0.322	0.173	0.346	0.495	1.975	53.258	0.610	2.913

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	77	65	68	222	0	68	116
N.S.	1	1.00	0.81	0.68	0.72	2.34	0.00	0.72	1.22
time (sec)	N/A	0.039	0.089	0.308	0.487	1.901	0.000	0.586	2.650

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	181	216	311	2566	1658	313	1884
N.S.	1	1.00	0.58	0.69	1.00	8.22	5.31	1.00	6.04
time (sec)	N/A	0.350	0.474	0.389	0.519	1.685	185.019	0.593	2.888

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	169	213	235	3787	1885	302	1578
N.S.	1	1.00	0.58	0.74	0.81	13.10	6.52	1.04	5.46
time (sec)	N/A	0.586	0.424	0.299	0.492	2.062	119.891	1.043	2.869

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	61	61	190	1042	63	115
N.S.	1	1.00	1.00	0.86	0.86	2.68	14.68	0.89	1.62
time (sec)	N/A	0.048	0.088	0.278	0.483	1.416	75.136	0.622	0.141

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	168	213	301	2555	1632	302	1922
N.S.	1	1.00	0.58	0.74	1.04	8.84	5.65	1.04	6.65
time (sec)	N/A	0.327	0.407	0.294	0.549	2.249	98.652	0.974	2.917

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	184	216	240	3798	2200	307	1757
N.S.	1	1.00	0.58	0.68	0.75	11.94	6.92	0.97	5.53
time (sec)	N/A	0.418	0.459	0.372	0.532	2.416	161.393	1.088	2.912

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	97	79	66	67	232	0	66	139
N.S.	1	1.01	0.82	0.69	0.70	2.42	0.00	0.69	1.45
time (sec)	N/A	0.040	0.091	0.317	0.497	1.292	0.000	0.958	0.152

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	188	217	312	2584	0	313	2080
N.S.	1	1.00	0.59	0.68	0.98	8.13	0.00	0.98	6.54
time (sec)	N/A	0.382	0.430	0.387	0.516	2.458	0.000	1.154	2.957

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	92	81	96	314	0	84	133
N.S.	1	1.00	0.88	0.78	0.92	3.02	0.00	0.81	1.28
time (sec)	N/A	0.042	0.141	0.291	0.508	2.023	0.000	0.971	2.761

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	192	234	341	2714	0	328	1944
N.S.	1	1.00	0.59	0.72	1.04	8.30	0.00	1.00	5.94
time (sec)	N/A	0.364	0.596	0.289	0.508	2.206	0.000	0.643	2.984

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	193	235	271	3951	0	328	1672
N.S.	1	1.00	0.59	0.72	0.83	12.08	0.00	1.00	5.11
time (sec)	N/A	0.408	0.584	0.293	0.489	2.597	0.000	1.265	2.893

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	92	82	96	313	0	84	136
N.S.	1	1.00	0.88	0.79	0.92	3.01	0.00	0.81	1.31
time (sec)	N/A	0.041	0.135	0.286	0.497	1.968	0.000	0.601	2.706

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	189	233	336	2674	0	322	1952
N.S.	1	1.00	0.59	0.73	1.05	8.33	0.00	1.00	6.08
time (sec)	N/A	0.343	0.565	0.287	0.494	1.988	0.000	0.618	2.950

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	208	236	273	3904	0	329	1786
N.S.	1	1.00	0.59	0.67	0.78	11.12	0.00	0.94	5.09
time (sec)	N/A	0.432	0.570	0.367	0.496	3.569	0.000	0.758	2.911

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	130	102	86	100	347	0	88	163
N.S.	1	1.01	0.79	0.67	0.78	2.69	0.00	0.68	1.26
time (sec)	N/A	0.049	0.137	0.316	0.638	1.968	0.000	0.698	2.728

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	209	237	346	2690	0	334	2500
N.S.	1	1.00	0.60	0.68	0.99	7.66	0.00	0.95	7.12
time (sec)	N/A	0.385	0.562	0.372	0.512	3.325	0.000	1.215	2.959

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	75	166	118	99	219	104	154
N.S.	1	1.00	0.73	1.61	1.15	0.96	2.13	1.01	1.50
time (sec)	N/A	0.057	0.052	0.303	0.292	2.974	0.366	1.529	2.717

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	126	84	75	168	73	114
N.S.	1	1.00	0.77	1.73	1.15	1.03	2.30	1.00	1.56
time (sec)	N/A	0.041	0.037	0.306	0.274	2.703	0.251	1.133	2.662

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	69	49	50	117	44	44
N.S.	1	1.00	0.74	1.50	1.07	1.09	2.54	0.96	0.96
time (sec)	N/A	0.028	0.024	0.306	0.380	2.569	0.141	1.114	2.598

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	61	50	67	125	76	61	80
N.S.	1	1.00	0.95	0.78	1.05	1.95	1.19	0.95	1.25
time (sec)	N/A	0.031	0.064	0.305	0.672	2.452	10.759	0.741	2.714

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	72	107	143	134	68	76
N.S.	1	1.00	0.77	0.86	1.27	1.70	1.60	0.81	0.90
time (sec)	N/A	0.045	0.101	0.347	0.500	2.973	18.668	0.649	2.933

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	78	96	158	172	160	120	93
N.S.	1	1.00	0.89	1.09	1.80	1.95	1.82	1.36	1.06
time (sec)	N/A	0.048	0.119	0.338	0.614	3.052	42.624	0.628	3.119

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	89	658	0	91	83	0	-1
N.S.	1	1.00	0.29	2.17	0.00	0.30	0.27	0.00	-0.00
time (sec)	N/A	0.102	3.865	0.316	0.000	0.636	1.248	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	75	618	0	67	82	0	-1
N.S.	1	1.00	0.28	2.31	0.00	0.25	0.31	0.00	-0.00
time (sec)	N/A	0.069	3.630	0.320	0.000	0.804	1.146	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	81	596	0	57	85	0	-1
N.S.	1	1.00	0.30	2.22	0.00	0.21	0.32	0.00	-0.00
time (sec)	N/A	0.070	3.956	0.329	0.000	0.461	1.343	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	80	616	0	64	94	0	-1
N.S.	1	1.00	0.29	2.26	0.00	0.24	0.35	0.00	-0.00
time (sec)	N/A	0.075	10.060	0.341	0.000	0.473	1.475	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	80	660	0	89	97	0	-1
N.S.	1	1.00	0.26	2.16	0.00	0.29	0.32	0.00	-0.00
time (sec)	N/A	0.101	10.058	0.344	0.000	0.321	1.667	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	91	966	0	102	83	0	-1
N.S.	1	1.00	0.16	1.66	0.00	0.18	0.14	0.00	-0.00
time (sec)	N/A	0.246	3.997	0.329	0.000	0.520	1.314	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	75	926	0	76	83	0	-1
N.S.	1	1.00	0.14	1.69	0.00	0.14	0.15	0.00	-0.00
time (sec)	N/A	0.177	3.656	0.316	0.000	0.540	1.204	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	81	902	0	64	85	0	-1
N.S.	1	1.00	0.15	1.66	0.00	0.12	0.16	0.00	-0.00
time (sec)	N/A	0.178	3.991	0.334	0.000	0.783	1.335	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	80	920	0	70	92	0	-1
N.S.	1	1.00	0.15	1.68	0.00	0.13	0.17	0.00	-0.00
time (sec)	N/A	0.174	7.623	0.331	0.000	0.550	1.405	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	80	964	0	97	97	0	-1
N.S.	1	1.00	0.14	1.66	0.00	0.17	0.17	0.00	-0.00
time (sec)	N/A	0.225	10.056	0.333	0.000	0.372	1.587	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	80	1006	0	123	97	0	-1
N.S.	1	1.00	0.13	1.64	0.00	0.20	0.16	0.00	-0.00
time (sec)	N/A	0.264	10.054	0.345	0.000	0.520	1.821	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	80	202	118	124	267	104	206
N.S.	1	1.00	0.78	1.96	1.15	1.20	2.59	1.01	2.00
time (sec)	N/A	0.054	0.063	0.332	0.313	2.295	0.601	0.563	2.655

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	162	84	99	216	73	211
N.S.	1	1.00	0.77	2.22	1.15	1.36	2.96	1.00	2.89
time (sec)	N/A	0.039	0.042	0.324	0.299	2.422	0.406	0.588	2.715

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	87	49	73	165	44	150
N.S.	1	1.00	0.74	1.89	1.07	1.59	3.59	0.96	3.26
time (sec)	N/A	0.026	0.026	0.328	0.295	1.453	0.265	0.630	3.346

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	85	66	80	172	82	80	131
N.S.	1	1.00	1.05	0.81	0.99	2.12	1.01	0.99	1.62
time (sec)	N/A	0.037	0.086	0.324	0.508	1.744	23.611	0.560	2.793

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	81	101	134	169	223	103	111
N.S.	1	1.00	0.74	0.92	1.22	1.54	2.03	0.94	1.01
time (sec)	N/A	0.054	0.102	0.368	0.513	2.117	19.175	0.841	3.379

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	107	171	191	243	131	110
N.S.	1	1.00	0.70	0.93	1.49	1.66	2.11	1.14	0.96
time (sec)	N/A	0.058	0.140	0.358	0.490	1.988	49.268	1.325	3.472

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	93	694	0	115	172	0	-1
N.S.	1	1.00	0.28	2.07	0.00	0.34	0.51	0.00	-0.00
time (sec)	N/A	0.122	5.515	0.328	0.000	0.586	2.203	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	77	654	0	91	170	0	-1
N.S.	1	1.00	0.26	2.19	0.00	0.30	0.57	0.00	-0.00
time (sec)	N/A	0.092	5.463	0.333	0.000	0.700	1.918	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	83	629	0	80	172	0	-1
N.S.	1	1.00	0.28	2.13	0.00	0.27	0.58	0.00	-0.00
time (sec)	N/A	0.089	5.594	0.338	0.000	0.412	2.306	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	82	626	0	67	184	0	-1
N.S.	1	1.00	0.28	2.11	0.00	0.23	0.62	0.00	-0.00
time (sec)	N/A	0.086	10.060	0.359	0.000	0.857	2.422	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	82	653	0	89	196	0	-1
N.S.	1	1.00	0.27	2.16	0.00	0.29	0.65	0.00	-0.00
time (sec)	N/A	0.094	10.049	0.337	0.000	0.605	2.772	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	96	1002	0	126	172	0	-1
N.S.	1	1.00	0.16	1.63	0.00	0.21	0.28	0.00	-0.00
time (sec)	N/A	0.266	5.811	0.332	0.000	0.419	2.292	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	78	962	0	100	172	0	-1
N.S.	1	1.00	0.13	1.66	0.00	0.17	0.30	0.00	-0.00
time (sec)	N/A	0.209	5.440	0.317	0.000	0.389	2.045	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	573	83	937	0	87	173	0	-1
N.S.	1	1.00	0.14	1.64	0.00	0.15	0.30	0.00	-0.00
time (sec)	N/A	0.215	5.614	0.335	0.000	0.403	2.343	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	85	932	0	77	182	0	-1
N.S.	1	1.00	0.15	1.61	0.00	0.13	0.31	0.00	-0.00
time (sec)	N/A	0.211	7.925	0.338	0.000	0.534	2.368	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	576	576	82	957	0	95	194	0	-1
N.S.	1	1.00	0.14	1.66	0.00	0.16	0.34	0.00	-0.00
time (sec)	N/A	0.216	10.048	0.322	0.000	0.477	2.651	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	82	1002	0	123	199	0	-1
N.S.	1	1.00	0.13	1.65	0.00	0.20	0.33	0.00	-0.00
time (sec)	N/A	0.258	10.056	0.341	0.000	0.521	3.062	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	80	132	118	76	175	101	104
N.S.	1	1.00	0.78	1.28	1.15	0.74	1.70	0.98	1.01
time (sec)	N/A	0.052	0.053	0.300	0.282	1.851	0.491	1.017	2.676

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	92	83	52	124	70	52
N.S.	1	1.00	0.77	1.26	1.14	0.71	1.70	0.96	0.71
time (sec)	N/A	0.037	0.037	0.302	0.275	2.073	0.369	1.400	2.649

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	52	48	29	75	38	29
N.S.	1	1.00	0.72	1.13	1.04	0.63	1.63	0.83	0.63
time (sec)	N/A	0.025	0.025	0.335	0.300	1.870	0.272	0.881	2.604

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	54	105	65	40	57
N.S.	1	1.00	1.00	0.77	1.12	2.19	1.35	0.83	1.19
time (sec)	N/A	0.023	0.041	0.321	0.565	1.898	4.686	1.137	2.715

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	62	109	126	80	62	67
N.S.	1	1.00	1.00	1.07	1.88	2.17	1.38	1.07	1.16
time (sec)	N/A	0.034	0.066	0.330	0.487	1.827	10.896	0.861	2.894

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	102	178	173	163	121	95
N.S.	1	1.00	0.87	1.13	1.98	1.92	1.81	1.34	1.06
time (sec)	N/A	0.053	0.114	0.352	0.492	1.617	25.943	1.053	2.995

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	89	624	0	67	80	0	-1
N.S.	1	1.00	0.33	2.31	0.00	0.25	0.30	0.00	-0.00
time (sec)	N/A	0.077	10.065	0.316	0.000	0.467	1.347	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	74	586	0	42	78	0	-1
N.S.	1	1.00	0.31	2.45	0.00	0.18	0.33	0.00	-0.00
time (sec)	N/A	0.044	10.025	0.322	0.000	0.285	0.989	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	78	587	0	50	82	0	-1
N.S.	1	1.00	0.32	2.42	0.00	0.21	0.34	0.00	-0.00
time (sec)	N/A	0.049	10.028	0.339	0.000	0.335	1.050	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	78	625	0	62	90	0	-1
N.S.	1	1.00	0.28	2.28	0.00	0.23	0.33	0.00	-0.00
time (sec)	N/A	0.073	10.027	0.342	0.000	0.486	1.243	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	91	932	0	78	80	0	-1
N.S.	1	1.00	0.17	1.70	0.00	0.14	0.15	0.00	-0.00
time (sec)	N/A	0.177	10.061	0.341	0.000	0.374	1.401	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	75	892	0	51	80	0	-1
N.S.	1	1.00	0.15	1.73	0.00	0.10	0.15	0.00	-0.00
time (sec)	N/A	0.152	10.039	0.336	0.000	0.313	1.270	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	77	891	0	54	82	0	-1
N.S.	1	1.00	0.15	1.75	0.00	0.11	0.16	0.00	-0.00
time (sec)	N/A	0.136	10.035	0.336	0.000	0.663	1.054	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	78	929	0	70	88	0	-1
N.S.	1	1.00	0.14	1.69	0.00	0.13	0.16	0.00	-0.00
time (sec)	N/A	0.172	10.034	0.350	0.000	0.611	1.171	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	78	970	0	97	94	0	-1
N.S.	1	1.00	0.13	1.67	0.00	0.17	0.16	0.00	-0.00
time (sec)	N/A	0.220	10.037	0.334	0.000	0.428	1.369	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	77	134	116	88	175	114	152
N.S.	1	1.00	0.75	1.30	1.13	0.85	1.70	1.11	1.48
time (sec)	N/A	0.053	0.054	0.348	0.277	1.907	0.601	0.778	2.771

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	94	81	63	124	77	60
N.S.	1	1.00	0.75	1.29	1.11	0.86	1.70	1.05	0.82
time (sec)	N/A	0.037	0.043	0.342	0.289	2.856	0.440	1.050	2.680

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	53	47	41	75	38	33
N.S.	1	1.00	0.72	1.15	1.02	0.89	1.63	0.83	0.72
time (sec)	N/A	0.025	0.029	0.338	0.282	2.831	0.328	1.284	2.615

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	57	70	170	56	53	65
N.S.	1	1.00	1.00	0.98	1.21	2.93	0.97	0.91	1.12
time (sec)	N/A	0.027	0.066	0.332	0.497	2.662	7.915	1.220	2.769

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	100	144	233	264	99	131
N.S.	1	1.00	0.90	1.16	1.67	2.71	3.07	1.15	1.52
time (sec)	N/A	0.046	0.128	0.357	0.496	1.911	25.847	1.179	2.929

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	100	141	215	289	192	137	167
N.S.	1	1.00	0.85	1.19	1.82	2.45	1.63	1.16	1.42
time (sec)	N/A	0.062	0.152	0.378	0.488	3.219	50.663	0.691	3.180

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	103	666	0	122	80	0	-1
N.S.	1	1.00	0.34	2.22	0.00	0.41	0.27	0.00	-0.00
time (sec)	N/A	0.098	10.065	0.349	0.000	0.507	11.861	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	78	627	0	95	80	0	-1
N.S.	1	1.00	0.29	2.33	0.00	0.35	0.30	0.00	-0.00
time (sec)	N/A	0.072	10.053	0.351	0.000	0.529	5.169	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	73	613	0	83	78	0	-1
N.S.	1	1.00	0.29	2.44	0.00	0.33	0.31	0.00	-0.00
time (sec)	N/A	0.047	10.028	0.342	0.000	0.466	3.160	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	86	631	0	102	82	0	-1
N.S.	1	1.00	0.32	2.32	0.00	0.38	0.30	0.00	-0.00
time (sec)	N/A	0.073	10.032	0.406	0.000	0.360	8.233	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	72	667	0	119	90	0	-1
N.S.	1	1.00	0.24	2.19	0.00	0.39	0.30	0.00	-0.00
time (sec)	N/A	0.098	10.036	0.438	0.000	0.468	19.453	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	79	937	0	104	80	0	-1
N.S.	1	1.00	0.14	1.71	0.00	0.19	0.15	0.00	-0.00
time (sec)	N/A	0.215	10.053	0.356	0.000	0.568	6.484	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	524	524	71	921	0	94	80	0	-1
N.S.	1	1.00	0.14	1.76	0.00	0.18	0.15	0.00	-0.00
time (sec)	N/A	0.141	10.039	0.333	0.000	0.800	3.191	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	72	939	0	106	82	0	-1
N.S.	1	1.00	0.13	1.71	0.00	0.19	0.15	0.00	-0.00
time (sec)	N/A	0.178	10.031	0.352	0.000	0.446	6.640	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	72	975	0	127	88	0	-1
N.S.	1	1.00	0.12	1.68	0.00	0.22	0.15	0.00	-0.00
time (sec)	N/A	0.216	10.032	0.359	0.000	0.519	14.679	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	611	72	1018	0	156	94	0	-1
N.S.	1	1.00	0.12	1.67	0.00	0.26	0.15	0.00	-0.00
time (sec)	N/A	0.257	10.027	0.347	0.000	0.483	30.162	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	73	150	116	98	338	104	145
N.S.	1	1.00	0.71	1.46	1.13	0.95	3.28	1.01	1.41
time (sec)	N/A	0.052	0.060	0.323	0.340	2.240	0.685	1.064	2.797

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	113	84	75	240	63	60
N.S.	1	1.00	0.77	1.55	1.15	1.03	3.29	0.86	0.82
time (sec)	N/A	0.039	0.044	0.327	0.283	1.832	0.519	1.382	2.761

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	64	49	52	144	32	33
N.S.	1	1.00	0.72	1.39	1.07	1.13	3.13	0.70	0.72
time (sec)	N/A	0.026	0.029	0.330	0.288	2.182	0.397	1.478	2.678

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	70	85	81	243	76	67	80
N.S.	1	1.00	0.91	1.10	1.05	3.16	0.99	0.87	1.04
time (sec)	N/A	0.035	0.091	0.342	0.502	3.181	12.192	0.987	2.782

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	99	157	170	351	1608	101	198
N.S.	1	1.00	0.88	1.39	1.50	3.11	14.23	0.89	1.75
time (sec)	N/A	0.060	0.121	0.362	0.495	2.914	87.162	1.464	2.971

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	108	683	0	153	80	0	-1
N.S.	1	1.00	0.36	2.28	0.00	0.51	0.27	0.00	-0.00
time (sec)	N/A	0.092	10.099	0.367	0.000	0.467	43.156	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	99	669	0	141	80	0	-1
N.S.	1	1.00	0.35	2.36	0.00	0.50	0.28	0.00	-0.00
time (sec)	N/A	0.074	10.080	0.309	0.000	0.514	29.408	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	103	674	0	145	78	0	-1
N.S.	1	1.00	0.36	2.38	0.00	0.51	0.28	0.00	-0.00
time (sec)	N/A	0.074	10.045	0.322	0.000	0.618	22.075	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	116	689	0	163	82	0	-1
N.S.	1	1.00	0.39	2.30	0.00	0.54	0.27	0.00	-0.00
time (sec)	N/A	0.097	10.055	0.359	0.000	0.415	55.628	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	83	722	0	178	90	0	-1
N.S.	1	1.00	0.25	2.16	0.00	0.53	0.27	0.00	-0.00
time (sec)	N/A	0.121	10.034	0.391	0.000	0.520	127.630	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	109	997	0	162	80	0	-1
N.S.	1	1.00	0.19	1.73	0.00	0.28	0.14	0.00	-0.00
time (sec)	N/A	0.227	10.095	0.381	0.000	0.585	53.888	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	559	559	92	981	0	154	80	0	-1
N.S.	1	1.00	0.16	1.75	0.00	0.28	0.14	0.00	-0.00
time (sec)	N/A	0.182	10.062	0.312	0.000	0.387	29.610	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	81	986	0	154	80	0	-1
N.S.	1	1.00	0.14	1.75	0.00	0.27	0.14	0.00	-0.00
time (sec)	N/A	0.180	10.058	0.319	0.000	0.577	20.512	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	86	1001	0	167	82	0	-1
N.S.	1	1.00	0.15	1.73	0.00	0.29	0.14	0.00	-0.00
time (sec)	N/A	0.211	10.031	0.351	0.000	0.521	41.398	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	83	1034	0	186	88	0	-1
N.S.	1	1.00	0.14	1.70	0.00	0.30	0.14	0.00	-0.00
time (sec)	N/A	0.251	10.039	0.370	0.000	0.527	97.768	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	503	69	156	85	82	109
N.S.	1	1.00	0.80	5.19	0.71	1.61	0.88	0.85	1.12
time (sec)	N/A	0.068	0.066	1.225	0.485	2.243	11.850	1.067	4.539

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	66	446	53	129	68	64	88
N.S.	1	1.00	0.87	5.87	0.70	1.70	0.89	0.84	1.16
time (sec)	N/A	0.042	0.049	0.404	0.518	2.406	6.451	1.517	4.278

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	425	42	110	51	44	71
N.S.	1	1.00	0.95	7.46	0.74	1.93	0.89	0.77	1.25
time (sec)	N/A	0.032	0.035	0.374	0.501	2.867	1.771	1.600	3.866

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	59	468	0	147	66	50	93
N.S.	1	1.00	0.91	7.20	0.00	2.26	1.02	0.77	1.43
time (sec)	N/A	0.040	0.039	0.520	0.000	2.397	3.004	0.707	4.658

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	511	0	194	0	72	113
N.S.	1	1.00	1.00	5.81	0.00	2.20	0.00	0.82	1.28
time (sec)	N/A	0.055	0.103	0.461	0.000	2.921	0.000	0.719	4.858

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	133	1309	0	3827	0	0	-1
N.S.	1	1.00	0.19	1.90	0.00	5.55	0.00	0.00	-0.00
time (sec)	N/A	0.325	4.101	0.413	0.000	19.975	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	63	848	0	3547	0	0	-1
N.S.	1	1.00	0.10	1.29	0.00	5.38	0.00	0.00	-0.00
time (sec)	N/A	0.122	6.231	0.336	0.000	12.057	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	697	697	136	1306	0	2319	0	0	-1
N.S.	1	1.00	0.20	1.87	0.00	3.33	0.00	0.00	-0.00
time (sec)	N/A	0.273	20.065	0.388	0.000	2.708	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	236	1003	0	3821	0	0	-1
N.S.	1	1.00	3.58	15.20	0.00	57.89	0.00	0.00	-0.02
time (sec)	N/A	0.036	3.876	0.400	0.000	18.905	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	165	696	0	2187	0	0	-1
N.S.	1	1.00	2.58	10.88	0.00	34.17	0.00	0.00	-0.02
time (sec)	N/A	0.019	8.832	0.354	0.000	4.253	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	244	1002	0	2372	0	0	-1
N.S.	1	1.00	3.70	15.18	0.00	35.94	0.00	0.00	-0.02
time (sec)	N/A	0.034	20.101	0.401	0.000	4.659	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	65	464	53	129	0	64	88
N.S.	1	1.00	0.83	5.95	0.68	1.65	0.00	0.82	1.13
time (sec)	N/A	0.052	0.053	0.386	0.499	2.812	0.000	1.116	5.383

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	56	425	43	112	65	49	71
N.S.	1	1.00	0.95	7.20	0.73	1.90	1.10	0.83	1.20
time (sec)	N/A	0.033	0.040	0.391	0.496	2.548	6.996	1.736	4.859

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	413	29	87	37	29	56
N.S.	1	1.00	1.00	10.32	0.72	2.18	0.92	0.72	1.40
time (sec)	N/A	0.024	0.026	0.338	0.495	2.601	5.182	1.684	5.210

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	59	433	0	148	63	53	94
N.S.	1	1.00	0.91	6.66	0.00	2.28	0.97	0.82	1.45
time (sec)	N/A	0.038	0.045	0.405	0.000	3.352	3.840	1.428	5.507

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	477	0	194	0	72	112
N.S.	1	1.00	1.00	5.42	0.00	2.20	0.00	0.82	1.27
time (sec)	N/A	0.054	0.086	0.446	0.000	3.689	0.000	1.265	5.719

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	667	667	67	848	0	3600	0	0	-1
N.S.	1	1.00	0.10	1.27	0.00	5.40	0.00	0.00	-0.00
time (sec)	N/A	0.160	10.027	0.365	0.000	15.486	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	67	416	0	2274	0	0	453
N.S.	1	1.00	0.33	2.02	0.00	11.04	0.00	0.00	2.20
time (sec)	N/A	0.024	10.029	0.332	0.000	4.026	0.000	0.000	25.804

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	697	697	136	874	0	2351	0	0	-1
N.S.	1	1.00	0.20	1.25	0.00	3.37	0.00	0.00	-0.00
time (sec)	N/A	0.282	20.062	0.420	0.000	3.161	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	2261	0	0	-1
N.S.	1	1.00	1.02	10.55	0.00	34.26	0.00	0.00	-0.02
time (sec)	N/A	0.037	10.026	0.357	0.000	3.799	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	165	416	0	2288	0	0	-1
N.S.	1	1.00	2.58	6.50	0.00	35.75	0.00	0.00	-0.02
time (sec)	N/A	0.022	10.037	0.342	0.000	3.764	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	243	722	0	2392	0	0	-1
N.S.	1	1.00	3.68	10.94	0.00	36.24	0.00	0.00	-0.02
time (sec)	N/A	0.039	20.125	0.412	0.000	6.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	1191	0	0	653
N.S.	1	1.00	0.22	1.29	0.00	9.38	0.00	0.00	5.14
time (sec)	N/A	0.015	10.029	11.081	0.000	3.144	0.000	0.000	3.419

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	82	582	96	169	99	100	118
N.S.	1	1.00	0.74	5.24	0.86	1.52	0.89	0.90	1.06
time (sec)	N/A	0.077	0.074	0.409	0.533	2.742	25.799	0.783	3.510

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	71	504	82	147	82	83	98
N.S.	1	1.00	0.79	5.60	0.91	1.63	0.91	0.92	1.09
time (sec)	N/A	0.081	0.060	0.399	0.480	2.353	12.258	0.859	3.404

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	59	446	66	121	65	65	78
N.S.	1	1.00	0.86	6.46	0.96	1.75	0.94	0.94	1.13
time (sec)	N/A	0.044	0.050	0.393	0.510	2.689	7.266	1.285	3.508

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	425	56	101	46	43	59
N.S.	1	1.00	0.94	8.50	1.12	2.02	0.92	0.86	1.18
time (sec)	N/A	0.030	0.032	0.359	0.567	2.622	2.059	1.546	3.496

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	468	0	138	60	48	125
N.S.	1	1.00	0.91	8.07	0.00	2.38	1.03	0.83	2.16
time (sec)	N/A	0.044	0.035	0.412	0.000	2.576	3.132	0.925	4.687

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	511	0	186	0	73	69
N.S.	1	1.00	1.00	6.31	0.00	2.30	0.00	0.90	0.85
time (sec)	N/A	0.051	0.077	0.419	0.000	2.800	0.000	1.063	3.748

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	95	574	0	188	0	100	83
N.S.	1	1.00	0.89	5.36	0.00	1.76	0.00	0.93	0.78
time (sec)	N/A	0.071	0.110	0.424	0.000	2.478	0.000	1.091	3.910

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	150	1788	0	3782	0	0	-1
N.S.	1	1.00	0.23	2.76	0.00	5.84	0.00	0.00	-0.00
time (sec)	N/A	0.679	4.204	0.404	0.000	34.860	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	130	1310	0	3756	0	0	-1
N.S.	1	1.00	0.21	2.10	0.00	6.02	0.00	0.00	-0.00
time (sec)	N/A	0.564	4.072	0.391	0.000	13.901	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	63	848	0	3481	0	0	-1
N.S.	1	1.00	0.10	1.41	0.00	5.79	0.00	0.00	-0.00
time (sec)	N/A	0.398	5.923	0.314	0.000	9.619	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	137	1306	0	2490	0	0	-1
N.S.	1	1.00	0.22	2.07	0.00	3.94	0.00	0.00	-0.00
time (sec)	N/A	0.530	20.072	0.394	0.000	2.617	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	654	654	153	1782	0	2612	0	0	-1
N.S.	1	1.00	0.23	2.72	0.00	3.99	0.00	0.00	-0.00
time (sec)	N/A	0.657	20.066	0.388	0.000	3.849	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	678	678	164	2280	0	2630	0	0	-1
N.S.	1	1.00	0.24	3.36	0.00	3.88	0.00	0.00	-0.00
time (sec)	N/A	1.071	10.064	0.411	0.000	5.247	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	93	634	110	191	131	117	135
N.S.	1	1.00	0.72	4.88	0.85	1.47	1.01	0.90	1.04
time (sec)	N/A	0.082	0.081	0.362	0.552	2.282	78.444	0.943	3.525

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	82	538	96	169	110	100	115
N.S.	1	1.00	0.75	4.94	0.88	1.55	1.01	0.92	1.06
time (sec)	N/A	0.075	0.076	0.380	0.525	2.246	48.891	1.740	3.495

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	71	462	82	147	90	83	95
N.S.	1	1.00	0.81	5.25	0.93	1.67	1.02	0.94	1.08
time (sec)	N/A	0.065	0.056	0.390	0.525	1.556	28.866	1.262	3.521

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	441	68	121	65	65	75
N.S.	1	1.00	0.88	6.58	1.01	1.81	0.97	0.97	1.12
time (sec)	N/A	0.061	0.053	0.339	0.496	1.514	12.277	0.963	3.449

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	500	0	152	73	61	89
N.S.	1	1.00	1.00	6.85	0.00	2.08	1.00	0.84	1.22
time (sec)	N/A	0.056	0.046	0.391	0.000	1.602	8.064	1.556	5.893

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	556	0	186	0	64	56
N.S.	1	1.00	1.00	7.13	0.00	2.38	0.00	0.82	0.72
time (sec)	N/A	0.062	0.072	0.415	0.000	1.720	0.000	0.827	3.524

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	95	617	0	218	0	101	87
N.S.	1	1.00	0.91	5.93	0.00	2.10	0.00	0.97	0.84
time (sec)	N/A	0.075	0.107	0.427	0.000	1.715	0.000	1.005	3.737

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	669	669	163	1840	0	3793	0	0	-1
N.S.	1	1.00	0.24	2.75	0.00	5.67	0.00	0.00	-0.00
time (sec)	N/A	0.764	5.834	0.370	0.000	51.206	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	645	150	1344	0	3774	0	0	-1
N.S.	1	1.00	0.23	2.08	0.00	5.85	0.00	0.00	-0.00
time (sec)	N/A	0.623	5.616	0.398	0.000	22.575	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	627	627	127	864	0	3708	0	0	-1
N.S.	1	1.00	0.20	1.38	0.00	5.91	0.00	0.00	-0.00
time (sec)	N/A	0.512	7.539	0.362	0.000	19.196	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	626	626	137	1339	0	0	0	0	-1
N.S.	1	1.00	0.22	2.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.514	10.057	0.415	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	651	651	154	1810	0	2585	0	0	-1
N.S.	1	1.00	0.24	2.78	0.00	3.97	0.00	0.00	-0.00
time (sec)	N/A	0.587	10.064	0.389	0.000	3.789	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	675	675	167	2306	0	2634	0	0	-1
N.S.	1	1.00	0.25	3.42	0.00	3.90	0.00	0.00	-0.00
time (sec)	N/A	0.659	10.069	0.428	0.000	4.910	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	69	528	82	146	0	82	98
N.S.	1	1.00	0.77	5.87	0.91	1.62	0.00	0.91	1.09
time (sec)	N/A	0.057	0.064	0.365	0.495	3.163	0.000	1.203	3.221

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	58	465	66	121	0	65	78
N.S.	1	1.00	0.82	6.55	0.93	1.70	0.00	0.92	1.10
time (sec)	N/A	0.048	0.051	0.389	0.506	2.656	0.000	1.879	3.393

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	425	56	103	61	48	60
N.S.	1	1.00	0.94	8.17	1.08	1.98	1.17	0.92	1.15
time (sec)	N/A	0.030	0.034	0.352	0.490	2.173	6.654	0.902	3.271

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	413	42	78	32	27	45
N.S.	1	1.00	1.00	12.52	1.27	2.36	0.97	0.82	1.36
time (sec)	N/A	0.023	0.024	0.304	0.510	2.951	3.925	0.797	3.233

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	433	0	139	58	54	47
N.S.	1	1.00	0.88	7.47	0.00	2.40	1.00	0.93	0.81
time (sec)	N/A	0.040	0.038	0.364	0.000	3.594	3.947	0.961	3.283

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	477	0	184	0	73	73
N.S.	1	1.00	1.00	5.89	0.00	2.27	0.00	0.90	0.90
time (sec)	N/A	0.054	0.067	0.411	0.000	3.156	0.000	0.751	3.417

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	95	540	0	217	0	101	94
N.S.	1	1.00	0.89	5.05	0.00	2.03	0.00	0.94	0.88
time (sec)	N/A	0.070	0.119	0.436	0.000	3.835	0.000	1.069	3.501

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	130	1308	0	3764	0	0	-1
N.S.	1	1.00	0.21	2.08	0.00	5.97	0.00	0.00	-0.00
time (sec)	N/A	0.509	10.065	0.344	0.000	28.186	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	67	848	0	3529	0	0	-1
N.S.	1	1.00	0.11	1.41	0.00	5.87	0.00	0.00	-0.00
time (sec)	N/A	0.421	10.044	0.357	0.000	14.669	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	67	416	0	2459	0	0	272
N.S.	1	1.00	0.48	2.95	0.00	17.44	0.00	0.00	1.93
time (sec)	N/A	0.284	10.032	0.297	0.000	6.487	0.000	0.000	40.219

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	137	874	0	2522	0	0	-1
N.S.	1	1.00	0.22	1.38	0.00	3.99	0.00	0.00	-0.00
time (sec)	N/A	0.491	20.068	0.377	0.000	4.145	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	654	654	152	1351	0	2614	0	0	-1
N.S.	1	1.00	0.23	2.07	0.00	4.00	0.00	0.00	-0.00
time (sec)	N/A	0.583	20.078	0.388	0.000	4.278	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	678	678	167	1849	0	2630	0	0	-1
N.S.	1	1.00	0.25	2.73	0.00	3.88	0.00	0.00	-0.00
time (sec)	N/A	0.658	10.079	0.389	0.000	8.583	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	2419	0	0	-1
N.S.	1	1.00	1.02	10.55	0.00	36.65	0.00	0.00	-0.02
time (sec)	N/A	0.038	10.029	0.323	0.000	4.750	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	166	416	0	2448	0	0	-1
N.S.	1	1.00	2.59	6.50	0.00	38.25	0.00	0.00	-0.02
time (sec)	N/A	0.020	10.105	0.302	0.000	6.356	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	242	722	0	2574	0	0	-1
N.S.	1	1.00	3.67	10.94	0.00	39.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	20.124	0.399	0.000	6.627	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	261	1047	0	2603	0	0	-1
N.S.	1	1.00	3.95	15.86	0.00	39.44	0.00	0.00	-0.02
time (sec)	N/A	0.038	20.137	0.362	0.000	9.511	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	70	560	82	189	0	82	95
N.S.	1	1.00	0.78	6.22	0.91	2.10	0.00	0.91	1.06
time (sec)	N/A	0.072	0.079	0.368	0.493	3.490	0.000	1.633	3.782

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	59	498	68	161	0	58	75
N.S.	1	1.00	0.83	7.01	0.96	2.27	0.00	0.82	1.06
time (sec)	N/A	0.050	0.067	0.417	0.491	2.476	0.000	0.748	3.712

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	456	56	149	58	47	60
N.S.	1	1.00	0.94	8.77	1.08	2.87	1.12	0.90	1.15
time (sec)	N/A	0.030	0.057	0.351	0.494	3.060	10.200	1.301	3.677

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	435	58	147	51	48	63
N.S.	1	1.00	0.95	7.91	1.05	2.67	0.93	0.87	1.15
time (sec)	N/A	0.032	0.056	0.306	0.495	2.729	8.372	1.498	3.630

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	69	485	0	213	78	68	68
N.S.	1	1.00	0.91	6.38	0.00	2.80	1.03	0.89	0.89
time (sec)	N/A	0.048	0.074	0.394	0.000	3.191	6.199	0.903	3.662

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	85	549	0	272	0	100	88
N.S.	1	1.00	0.85	5.49	0.00	2.72	0.00	1.00	0.88
time (sec)	N/A	0.065	0.135	0.436	0.000	2.453	0.000	0.788	3.801

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	100	636	0	303	0	118	112
N.S.	1	1.00	0.78	4.97	0.00	2.37	0.00	0.92	0.88
time (sec)	N/A	0.085	0.156	0.441	0.000	2.992	0.000	0.782	4.025

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	629	629	127	1810	0	3632	0	0	-1
N.S.	1	1.00	0.20	2.88	0.00	5.77	0.00	0.00	-0.00
time (sec)	N/A	0.505	6.859	0.329	0.000	26.536	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	635	635	126	1346	0	2638	0	0	-1
N.S.	1	1.00	0.20	2.12	0.00	4.15	0.00	0.00	-0.00
time (sec)	N/A	0.514	6.560	0.340	0.000	4.107	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	124	875	0	2638	0	0	-1
N.S.	1	1.00	0.20	1.38	0.00	4.17	0.00	0.00	-0.00
time (sec)	N/A	0.507	8.694	0.298	0.000	3.547	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	140	1361	0	2610	0	0	-1
N.S.	1	1.00	0.21	2.08	0.00	4.00	0.00	0.00	-0.00
time (sec)	N/A	0.596	20.085	0.404	0.000	3.114	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	675	675	153	1864	0	2708	0	0	-1
N.S.	1	1.00	0.23	2.76	0.00	4.01	0.00	0.00	-0.00
time (sec)	N/A	0.660	20.071	0.395	0.000	11.932	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	699	699	167	2389	0	2726	0	0	-1
N.S.	1	1.00	0.24	3.42	0.00	3.90	0.00	0.00	-0.00
time (sec)	N/A	0.748	10.075	0.397	0.000	16.063	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	233	1038	0	2628	0	0	-1
N.S.	1	1.00	3.53	15.73	0.00	39.82	0.00	0.00	-0.02
time (sec)	N/A	0.041	6.349	0.326	0.000	6.869	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	230	721	0	2572	0	0	-1
N.S.	1	1.00	3.59	11.27	0.00	40.19	0.00	0.00	-0.02
time (sec)	N/A	0.021	10.080	0.300	0.000	6.581	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	248	1053	0	2664	0	0	-1
N.S.	1	1.00	3.76	15.95	0.00	40.36	0.00	0.00	-0.02
time (sec)	N/A	0.040	20.122	0.396	0.000	11.034	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	261	1402	0	2697	0	0	-1
N.S.	1	1.00	3.95	21.24	0.00	40.86	0.00	0.00	-0.02
time (sec)	N/A	0.041	20.150	0.398	0.000	16.558	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	737	737	80	995	0	0	0	0	-1
N.S.	1	1.00	0.11	1.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	8.823	0.586	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	757	757	80	942	0	0	0	0	-1
N.S.	1	1.00	0.11	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.209	9.399	1.076	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	87	944	0	0	0	0	-1
N.S.	1	1.00	0.11	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	9.411	0.382	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	768	768	90	1001	0	0	0	0	-1
N.S.	1	1.00	0.12	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.196	9.343	0.362	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	738	738	80	995	0	0	0	0	-1
N.S.	1	1.00	0.11	1.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.178	8.783	0.598	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	758	758	80	942	0	0	0	0	-1
N.S.	1	1.00	0.11	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.176	9.305	0.538	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	89	944	0	0	0	0	-1
N.S.	1	1.00	0.11	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.190	9.413	0.370	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	768	768	89	1001	0	0	0	0	-1
N.S.	1	1.00	0.12	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	9.368	0.382	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	83	538	0	5012	0	0	-1
N.S.	1	1.00	0.26	1.69	0.00	15.76	0.00	0.00	-0.00
time (sec)	N/A	0.044	10.069	0.375	0.000	25.813	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	83	509	0	5036	0	0	-1
N.S.	1	1.00	0.26	1.57	0.00	15.54	0.00	0.00	-0.00
time (sec)	N/A	0.050	10.048	0.358	0.000	24.990	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	85	510	0	0	0	0	-1
N.S.	1	1.00	0.26	1.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.046	10.055	0.364	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	87	541	0	0	0	0	-1
N.S.	1	1.00	0.26	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.047	10.055	0.377	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	83	538	0	0	0	0	-1
N.S.	1	1.00	0.27	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.037	10.081	0.389	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	83	509	0	0	0	0	-1
N.S.	1	1.00	0.26	1.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.037	10.055	0.401	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	84	510	0	5060	0	0	-1
N.S.	1	1.00	0.26	1.59	0.00	15.81	0.00	0.00	-0.00
time (sec)	N/A	0.039	10.057	0.375	0.000	30.830	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	86	541	0	5060	0	0	-1
N.S.	1	1.00	0.27	1.68	0.00	15.71	0.00	0.00	-0.00
time (sec)	N/A	0.037	10.054	0.368	0.000	34.717	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	121	514	0	280	128	139	176
N.S.	1	1.00	0.97	4.11	0.00	2.24	1.02	1.11	1.41
time (sec)	N/A	0.094	0.223	0.378	0.000	3.230	12.256	0.519	6.171

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	458	0	195	95	96	136
N.S.	1	1.00	0.95	4.92	0.00	2.10	1.02	1.03	1.46
time (sec)	N/A	0.054	0.144	0.390	0.000	4.380	7.333	0.592	6.058

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	434	0	156	68	66	82
N.S.	1	1.00	1.00	6.20	0.00	2.23	0.97	0.94	1.17
time (sec)	N/A	0.041	0.071	0.342	0.000	3.553	2.630	0.590	6.156

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	476	0	383	85	79	114
N.S.	1	1.00	0.95	5.60	0.00	4.51	1.00	0.93	1.34
time (sec)	N/A	0.052	0.093	0.426	0.000	4.256	4.035	0.571	7.938

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	107	518	0	513	0	107	137
N.S.	1	1.00	0.93	4.50	0.00	4.46	0.00	0.93	1.19
time (sec)	N/A	0.084	0.282	0.414	0.000	3.336	0.000	0.617	5.130

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	241	1012	0	0	0	0	-1
N.S.	1	1.00	3.77	15.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	4.791	0.391	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	857	0	0	0	0	-1
N.S.	1	1.00	1.02	13.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	7.307	0.311	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	705	0	0	0	0	-1
N.S.	1	1.00	2.73	11.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	9.990	0.301	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	139	1314	0	0	0	0	-1
N.S.	1	1.00	2.24	21.19	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	10.076	0.411	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	335	1010	0	0	0	0	-1
N.S.	1	1.00	5.23	15.78	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	10.189	0.394	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	143	605	0	410	153	193	330
N.S.	1	1.00	0.93	3.93	0.00	2.66	0.99	1.25	2.14
time (sec)	N/A	0.110	0.337	0.355	0.000	2.283	53.165	1.437	6.122

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	111	531	0	297	116	151	215
N.S.	1	1.00	0.92	4.42	0.00	2.48	0.97	1.26	1.79
time (sec)	N/A	0.072	0.216	0.393	0.000	2.913	28.861	1.473	6.134

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	85	507	0	204	90	113	143
N.S.	1	1.00	0.89	5.28	0.00	2.12	0.94	1.18	1.49
time (sec)	N/A	0.056	0.181	0.333	0.000	3.033	14.368	1.332	5.908

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	106	565	0	486	102	112	155
N.S.	1	1.00	1.02	5.43	0.00	4.67	0.98	1.08	1.49
time (sec)	N/A	0.075	0.225	0.382	0.000	2.524	11.641	1.582	7.879

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	620	0	538	0	121	167
N.S.	1	1.00	0.93	5.34	0.00	4.64	0.00	1.04	1.44
time (sec)	N/A	0.098	0.254	0.441	0.000	2.881	0.000	1.096	9.517

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	280	1101	0	0	0	0	-1
N.S.	1	1.00	4.31	16.94	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	6.611	0.375	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	149	930	0	0	0	0	-1
N.S.	1	1.00	2.29	14.31	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	8.937	0.375	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	351	776	0	0	0	0	-1
N.S.	1	1.00	5.85	12.93	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	10.220	0.346	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	148	1404	0	0	0	0	-1
N.S.	1	1.00	2.35	22.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	10.091	0.405	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	343	1096	0	0	0	0	-1
N.S.	1	1.00	5.28	16.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	10.232	0.395	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	488	0	289	0	106	121
N.S.	1	1.00	0.88	4.69	0.00	2.78	0.00	1.02	1.16
time (sec)	N/A	0.079	0.204	0.347	0.000	3.486	0.000	1.084	5.426

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	75	448	0	205	0	64	86
N.S.	1	1.00	1.01	6.05	0.00	2.77	0.00	0.86	1.16
time (sec)	N/A	0.046	0.100	0.371	0.000	2.916	0.000	1.415	5.098

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	426	0	130	39	40	70
N.S.	1	1.00	1.00	8.35	0.00	2.55	0.76	0.78	1.37
time (sec)	N/A	0.034	0.042	0.318	0.000	3.466	5.231	1.645	5.888

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	453	0	431	70	71	114
N.S.	1	1.00	0.96	5.33	0.00	5.07	0.82	0.84	1.34
time (sec)	N/A	0.054	0.138	0.380	0.000	3.679	5.415	1.195	7.313

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	109	498	0	565	0	104	142
N.S.	1	1.00	0.93	4.26	0.00	4.83	0.00	0.89	1.21
time (sec)	N/A	0.084	0.269	0.411	0.000	4.078	0.000	0.937	8.424

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	719	0	0	0	0	-1
N.S.	1	1.00	1.02	11.23	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	10.036	0.342	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	429	0	0	0	0	-1
N.S.	1	1.00	1.02	6.70	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	10.027	0.309	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	429	0	0	0	0	-1
N.S.	1	1.00	2.73	7.27	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	10.041	0.310	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	890	0	0	0	0	-1
N.S.	1	1.00	2.27	14.35	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	10.077	0.397	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	339	738	0	0	0	0	-1
N.S.	1	1.00	5.30	11.53	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	10.180	0.394	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	111	527	0	440	0	103	115
N.S.	1	1.00	1.04	4.93	0.00	4.11	0.00	0.96	1.07
time (sec)	N/A	0.090	0.312	0.362	0.000	3.743	0.000	1.499	6.457

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	487	0	326	0	78	94
N.S.	1	1.00	0.98	5.94	0.00	3.98	0.00	0.95	1.15
time (sec)	N/A	0.050	0.171	0.363	0.000	2.572	0.000	1.783	5.990

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	463	0	236	66	73	89
N.S.	1	1.00	0.99	6.01	0.00	3.06	0.86	0.95	1.16
time (sec)	N/A	0.043	0.100	0.312	0.000	4.388	8.875	0.974	5.848

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	110	512	0	790	104	111	139
N.S.	1	1.00	0.96	4.49	0.00	6.93	0.91	0.97	1.22
time (sec)	N/A	0.080	0.394	0.395	0.000	4.848	7.307	1.045	8.444

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	142	575	0	1120	0	173	597
N.S.	1	1.00	0.90	3.64	0.00	7.09	0.00	1.09	3.78
time (sec)	N/A	0.156	0.432	0.447	0.000	3.884	0.000	1.346	10.472

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	231	1069	0	0	0	0	-1
N.S.	1	1.00	3.45	15.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	7.741	0.346	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	142	907	0	0	0	0	-1
N.S.	1	1.00	2.12	13.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	10.079	0.329	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	338	753	0	0	0	0	-1
N.S.	1	1.00	5.45	12.15	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	10.207	0.311	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	193	1392	0	0	0	0	-1
N.S.	1	1.00	2.97	21.42	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	10.133	0.425	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	425	1084	0	0	0	0	-1
N.S.	1	1.00	6.34	16.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	10.355	0.448	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	91	953	107	219	0	110	127
N.S.	1	1.00	0.78	8.15	0.91	1.87	0.00	0.94	1.09
time (sec)	N/A	0.063	0.163	0.417	0.484	2.106	0.000	1.373	4.087

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	79	893	91	191	0	93	107
N.S.	1	1.00	0.77	8.75	0.89	1.87	0.00	0.91	1.05
time (sec)	N/A	0.055	0.108	0.396	0.487	2.620	0.000	1.154	4.013

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	875	79	165	0	69	87
N.S.	1	1.00	0.84	10.67	0.96	2.01	0.00	0.84	1.06
time (sec)	N/A	0.043	0.087	0.397	0.495	3.132	0.000	0.671	3.985

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	61	440	66	149	0	53	72
N.S.	1	1.00	0.95	6.88	1.03	2.33	0.00	0.83	1.12
time (sec)	N/A	0.032	0.064	0.304	0.479	3.469	0.000	0.583	3.926

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	83	913	0	226	0	79	76
N.S.	1	1.00	0.94	10.38	0.00	2.57	0.00	0.90	0.86
time (sec)	N/A	0.050	0.093	0.375	0.000	4.212	0.000	0.519	3.981

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	97	958	0	278	0	113	117
N.S.	1	1.00	0.78	7.73	0.00	2.24	0.00	0.91	0.94
time (sec)	N/A	0.071	0.165	0.438	0.000	2.082	0.000	0.523	4.212

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	112	1021	0	310	0	105	154
N.S.	1	1.00	0.68	6.23	0.00	1.89	0.00	0.64	0.94
time (sec)	N/A	0.093	0.225	0.447	0.000	2.509	0.000	0.516	4.453

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	176	2199	0	3866	0	0	-1
N.S.	1	1.00	0.27	3.32	0.00	5.83	0.00	0.00	-0.00
time (sec)	N/A	0.810	5.367	0.412	0.000	34.314	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	641	641	167	1741	0	3633	0	0	-1
N.S.	1	1.00	0.26	2.72	0.00	5.67	0.00	0.00	-0.00
time (sec)	N/A	0.503	8.287	0.360	0.000	21.495	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	644	644	164	883	0	2613	0	0	-1
N.S.	1	1.00	0.25	1.37	0.00	4.06	0.00	0.00	-0.00
time (sec)	N/A	0.707	10.068	0.309	0.000	3.951	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	665	665	179	2194	0	2617	0	0	-1
N.S.	1	1.00	0.27	3.30	0.00	3.94	0.00	0.00	-0.00
time (sec)	N/A	0.567	20.085	0.410	0.000	5.271	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	199	2672	0	2721	0	0	-1
N.S.	1	1.00	0.29	3.89	0.00	3.96	0.00	0.00	-0.00
time (sec)	N/A	0.663	10.094	0.418	0.000	7.992	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	711	711	209	3170	0	2738	0	0	-1
N.S.	1	1.00	0.29	4.46	0.00	3.85	0.00	0.00	-0.00
time (sec)	N/A	0.735	10.117	0.428	0.000	9.367	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	103	999	119	239	0	127	147
N.S.	1	1.00	0.77	7.46	0.89	1.78	0.00	0.95	1.10
time (sec)	N/A	0.080	0.152	0.372	0.482	3.078	0.000	0.595	4.104

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	93	921	107	219	0	111	127
N.S.	1	1.00	0.78	7.74	0.90	1.84	0.00	0.93	1.07
time (sec)	N/A	0.066	0.132	0.410	0.489	2.389	0.000	0.606	4.052

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	81	903	93	192	0	93	107
N.S.	1	1.00	0.84	9.31	0.96	1.98	0.00	0.96	1.10
time (sec)	N/A	0.054	0.106	0.401	0.495	4.155	0.000	0.549	4.044

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	72	452	79	162	0	69	87
N.S.	1	1.00	0.94	5.87	1.03	2.10	0.00	0.90	1.13
time (sec)	N/A	0.043	0.085	0.356	0.489	3.948	0.000	0.629	3.987

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	957	0	220	0	70	101
N.S.	1	1.00	1.00	11.26	0.00	2.59	0.00	0.82	1.19
time (sec)	N/A	0.052	0.084	0.376	0.000	3.351	0.000	0.595	4.722

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	97	1015	0	280	0	114	110
N.S.	1	1.00	0.80	8.39	0.00	2.31	0.00	0.94	0.91
time (sec)	N/A	0.073	0.169	0.432	0.000	3.003	0.000	0.534	4.235

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	112	1076	0	310	0	129	151
N.S.	1	1.00	0.70	6.68	0.00	1.93	0.00	0.80	0.94
time (sec)	N/A	0.094	0.222	0.439	0.000	2.648	0.000	0.826	4.618

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	681	681	191	2224	0	3882	0	0	-1
N.S.	1	1.00	0.28	3.27	0.00	5.70	0.00	0.00	-0.00
time (sec)	N/A	0.643	6.847	0.394	0.000	58.724	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	657	657	176	1748	0	3858	0	0	-1
N.S.	1	1.00	0.27	2.66	0.00	5.87	0.00	0.00	-0.00
time (sec)	N/A	0.563	9.413	0.409	0.000	23.972	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	141	874	0	3583	0	0	-1
N.S.	1	1.00	0.22	1.37	0.00	5.62	0.00	0.00	-0.00
time (sec)	N/A	0.509	10.133	0.313	0.000	17.344	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	242	2218	0	68	0	0	-1
N.S.	1	1.00	0.46	4.25	0.00	0.13	0.00	0.00	-0.00
time (sec)	N/A	0.181	20.512	0.395	0.000	0.638	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	684	684	199	2691	0	2721	0	0	-1
N.S.	1	1.00	0.29	3.93	0.00	3.98	0.00	0.00	-0.00
time (sec)	N/A	0.660	10.101	0.427	0.000	5.893	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	708	708	212	3187	0	2738	0	0	-1
N.S.	1	1.00	0.30	4.50	0.00	3.87	0.00	0.00	-0.00
time (sec)	N/A	0.734	10.112	0.432	0.000	5.361	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	81	917	93	195	0	93	107
N.S.	1	1.00	0.85	9.65	0.98	2.05	0.00	0.98	1.13
time (sec)	N/A	0.055	0.106	0.371	0.481	3.207	0.000	1.349	4.061

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	70	875	79	167	0	69	87
N.S.	1	1.00	0.84	10.54	0.95	2.01	0.00	0.83	1.05
time (sec)	N/A	0.044	0.086	0.420	0.486	3.234	0.000	0.869	3.997

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	862	67	155	0	58	72
N.S.	1	1.00	0.98	13.47	1.05	2.42	0.00	0.91	1.12
time (sec)	N/A	0.035	0.073	0.376	0.494	3.155	0.000	1.464	4.009

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	443	72	153	0	59	75
N.S.	1	1.00	0.96	6.61	1.07	2.28	0.00	0.88	1.12
time (sec)	N/A	0.036	0.058	0.315	0.492	2.688	0.000	1.169	3.977

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	83	881	0	226	0	79	80
N.S.	1	1.00	0.94	10.01	0.00	2.57	0.00	0.90	0.91
time (sec)	N/A	0.055	0.091	0.413	0.000	1.821	0.000	1.014	4.008

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	97	927	0	280	0	114	117
N.S.	1	1.00	0.78	7.48	0.00	2.26	0.00	0.92	0.94
time (sec)	N/A	0.073	0.165	0.476	0.000	3.802	0.000	1.336	4.111

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	112	990	0	310	0	128	155
N.S.	1	1.00	0.68	6.04	0.00	1.89	0.00	0.78	0.95
time (sec)	N/A	0.097	0.240	0.507	0.000	3.973	0.000	1.704	4.368

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	641	641	167	1738	0	3641	0	0	-1
N.S.	1	1.00	0.26	2.71	0.00	5.68	0.00	0.00	-0.00
time (sec)	N/A	0.502	10.114	0.365	0.000	29.972	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	647	647	166	1305	0	2647	0	0	-1
N.S.	1	1.00	0.26	2.02	0.00	4.09	0.00	0.00	-0.00
time (sec)	N/A	0.519	10.077	0.390	0.000	3.503	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	644	644	164	883	0	2648	0	0	-1
N.S.	1	1.00	0.25	1.37	0.00	4.11	0.00	0.00	-0.00
time (sec)	N/A	0.505	10.067	0.346	0.000	3.101	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	665	665	180	1762	0	2618	0	0	-1
N.S.	1	1.00	0.27	2.65	0.00	3.94	0.00	0.00	-0.00
time (sec)	N/A	0.580	20.098	0.448	0.000	5.678	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	196	2241	0	2721	0	0	-1
N.S.	1	1.00	0.29	3.26	0.00	3.96	0.00	0.00	-0.00
time (sec)	N/A	0.668	10.107	0.436	0.000	12.467	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	711	711	212	2739	0	2738	0	0	-1
N.S.	1	1.00	0.30	3.85	0.00	3.85	0.00	0.00	-0.00
time (sec)	N/A	0.730	10.114	0.438	0.000	12.623	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	239	1432	0	2529	0	0	-1
N.S.	1	1.00	3.62	21.70	0.00	38.32	0.00	0.00	-0.02
time (sec)	N/A	0.039	10.206	0.373	0.000	6.861	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	237	1151	0	2637	0	0	-1
N.S.	1	1.00	3.59	17.44	0.00	39.95	0.00	0.00	-0.02
time (sec)	N/A	0.036	10.128	0.375	0.000	7.543	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	237	729	0	2582	0	0	-1
N.S.	1	1.00	3.70	11.39	0.00	40.34	0.00	0.00	-0.02
time (sec)	N/A	0.020	10.115	0.336	0.000	7.461	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	266	1456	0	2677	0	0	-1
N.S.	1	1.00	4.03	22.06	0.00	40.56	0.00	0.00	-0.02
time (sec)	N/A	0.036	10.122	0.448	0.000	11.241	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	279	1783	0	2709	0	0	-1
N.S.	1	1.00	4.23	27.02	0.00	41.05	0.00	0.00	-0.02
time (sec)	N/A	0.038	10.149	0.440	0.000	12.394	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	99	971	98	233	0	88	111
N.S.	1	1.00	1.04	10.22	1.03	2.45	0.00	0.93	1.17
time (sec)	N/A	0.054	0.095	0.421	0.491	2.385	0.000	0.992	4.383

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	927	81	223	0	67	94
N.S.	1	1.00	0.86	11.17	0.98	2.69	0.00	0.81	1.13
time (sec)	N/A	0.048	0.105	0.404	0.482	2.777	0.000	1.414	4.300

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	73	909	83	223	0	76	96
N.S.	1	1.00	0.86	10.69	0.98	2.62	0.00	0.89	1.13
time (sec)	N/A	0.047	0.093	0.383	0.495	3.146	0.000	1.325	4.264

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	73	464	85	219	0	72	97
N.S.	1	1.00	0.83	5.27	0.97	2.49	0.00	0.82	1.10
time (sec)	N/A	0.046	0.096	0.342	0.492	2.825	0.000	0.932	4.261

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	93	954	0	316	0	93	101
N.S.	1	1.00	0.88	9.00	0.00	2.98	0.00	0.88	0.95
time (sec)	N/A	0.069	0.140	0.456	0.000	2.655	0.000	0.944	4.332

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	109	1020	0	368	0	129	133
N.S.	1	1.00	0.76	7.13	0.00	2.57	0.00	0.90	0.93
time (sec)	N/A	0.087	0.228	0.500	0.000	3.334	0.000	1.318	4.562

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	123	1107	0	398	0	149	171
N.S.	1	1.00	0.66	5.98	0.00	2.15	0.00	0.81	0.92
time (sec)	N/A	0.113	0.282	0.537	0.000	2.674	0.000	1.835	4.763

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	668	168	2256	0	2746	0	0	-1
N.S.	1	1.00	0.25	3.38	0.00	4.11	0.00	0.00	-0.00
time (sec)	N/A	0.567	8.914	0.349	0.000	4.610	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	671	671	169	1789	0	2780	0	0	-1
N.S.	1	1.00	0.25	2.67	0.00	4.14	0.00	0.00	-0.00
time (sec)	N/A	0.590	10.074	0.385	0.000	3.765	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	665	665	167	904	0	2748	0	0	-1
N.S.	1	1.00	0.25	1.36	0.00	4.13	0.00	0.00	-0.00
time (sec)	N/A	0.568	10.085	0.346	0.000	6.335	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	686	686	180	2270	0	2717	0	0	-1
N.S.	1	1.00	0.26	3.31	0.00	3.96	0.00	0.00	-0.00
time (sec)	N/A	0.665	20.094	0.455	0.000	4.055	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	708	708	198	2775	0	2820	0	0	-1
N.S.	1	1.00	0.28	3.92	0.00	3.98	0.00	0.00	-0.00
time (sec)	N/A	0.728	10.112	0.467	0.000	7.968	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	732	732	210	3300	0	2837	0	0	-1
N.S.	1	1.00	0.29	4.51	0.00	3.88	0.00	0.00	-0.00
time (sec)	N/A	0.820	10.126	0.451	0.000	20.190	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	256	66	189	1792	0	89	0	0	-1
N.S.	1	0.26	0.74	7.00	0.00	0.35	0.00	0.00	-0.00
time (sec)	N/A	0.041	9.333	0.359	0.000	0.693	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	242	1479	0	2758	0	0	-1
N.S.	1	1.00	3.67	22.41	0.00	41.79	0.00	0.00	-0.02
time (sec)	N/A	0.041	10.193	0.369	0.000	6.460	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	253	748	0	2681	0	0	-1
N.S.	1	1.00	3.95	11.69	0.00	41.89	0.00	0.00	-0.02
time (sec)	N/A	0.023	10.171	0.320	0.000	6.988	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	259	1806	0	2771	0	0	-1
N.S.	1	1.00	3.92	27.36	0.00	41.98	0.00	0.00	-0.02
time (sec)	N/A	0.039	10.155	0.448	0.000	10.373	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	283	2157	0	2804	0	0	-1
N.S.	1	1.00	4.29	32.68	0.00	42.48	0.00	0.00	-0.02
time (sec)	N/A	0.040	10.142	0.461	0.000	17.166	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	126	917	0	469	0	136	202
N.S.	1	1.00	0.78	5.70	0.00	2.91	0.00	0.84	1.25
time (sec)	N/A	0.129	0.262	0.438	0.000	2.320	0.000	1.209	6.842

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	897	0	334	0	102	152
N.S.	1	1.00	0.72	6.60	0.00	2.46	0.00	0.75	1.12
time (sec)	N/A	0.075	0.193	0.461	0.000	2.175	0.000	1.991	6.087

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	453	0	255	0	79	125
N.S.	1	1.00	1.00	5.66	0.00	3.19	0.00	0.99	1.56
time (sec)	N/A	0.047	0.177	0.319	0.000	2.339	0.000	1.263	5.686

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	111	934	0	856	0	114	182
N.S.	1	1.00	0.92	7.72	0.00	7.07	0.00	0.94	1.50
time (sec)	N/A	0.081	0.460	0.428	0.000	1.936	0.000	1.322	8.281

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	132	978	0	870	0	183	438
N.S.	1	1.00	0.82	6.07	0.00	5.40	0.00	1.14	2.72
time (sec)	N/A	0.147	0.499	0.500	0.000	1.968	0.000	1.544	9.692

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	235	1468	0	0	0	0	-1
N.S.	1	1.00	3.67	22.94	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	10.153	0.348	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	153	908	0	0	0	0	-1
N.S.	1	1.00	2.39	14.19	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	10.076	0.379	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	232	753	0	0	0	0	-1
N.S.	1	1.00	3.93	12.76	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	10.149	0.339	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	172	2227	0	0	0	0	-1
N.S.	1	1.00	2.77	35.92	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	10.096	0.470	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	338	1768	0	0	0	0	-1
N.S.	1	1.00	5.28	27.62	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	10.194	0.464	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	162	1003	0	443	0	211	331
N.S.	1	1.00	0.86	5.31	0.00	2.34	0.00	1.12	1.75
time (sec)	N/A	0.164	0.341	0.407	0.000	2.218	0.000	1.772	7.752

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	125	983	0	314	0	173	229
N.S.	1	1.00	0.77	6.03	0.00	1.93	0.00	1.06	1.40
time (sec)	N/A	0.093	0.246	0.431	0.000	2.115	0.000	1.200	7.379

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	466	0	234	0	122	170
N.S.	1	1.00	1.00	4.96	0.00	2.49	0.00	1.30	1.81
time (sec)	N/A	0.058	0.199	0.383	0.000	1.591	0.000	1.214	7.354

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	122	1036	0	686	0	155	214
N.S.	1	1.00	0.93	7.91	0.00	5.24	0.00	1.18	1.63
time (sec)	N/A	0.100	0.280	0.458	0.000	2.082	0.000	1.748	9.143

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	1093	0	838	0	216	531
N.S.	1	1.00	0.91	6.43	0.00	4.93	0.00	1.27	3.12
time (sec)	N/A	0.174	0.567	0.465	0.000	2.309	0.000	1.740	10.816

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	338	1587	0	0	0	0	-1
N.S.	1	1.00	5.20	24.42	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	10.270	0.408	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	177	955	0	0	0	0	-1
N.S.	1	1.00	2.72	14.69	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	10.140	0.337	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	339	801	0	0	0	0	-1
N.S.	1	1.00	5.65	13.35	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	10.224	0.337	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	190	2364	0	0	0	0	-1
N.S.	1	1.00	3.02	37.52	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	10.152	0.436	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	370	1902	0	0	0	0	-1
N.S.	1	1.00	5.69	29.26	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	10.244	0.469	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	130	911	0	475	0	134	160
N.S.	1	1.00	1.06	7.41	0.00	3.86	0.00	1.09	1.30
time (sec)	N/A	0.099	0.365	0.383	0.000	2.308	0.000	0.817	7.290

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	100	892	0	348	0	116	111
N.S.	1	1.00	1.01	9.01	0.00	3.52	0.00	1.17	1.12
time (sec)	N/A	0.057	0.173	0.382	0.000	2.071	0.000	1.508	6.846

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	457	0	302	0	93	104
N.S.	1	1.00	0.99	5.25	0.00	3.47	0.00	1.07	1.20
time (sec)	N/A	0.051	0.155	0.333	0.000	1.896	0.000	1.290	6.373

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	124	915	0	862	0	139	162
N.S.	1	1.00	0.94	6.93	0.00	6.53	0.00	1.05	1.23
time (sec)	N/A	0.096	0.338	0.429	0.000	2.942	0.000	0.962	9.647

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	163	961	0	1236	0	257	355
N.S.	1	1.00	0.88	5.19	0.00	6.68	0.00	1.39	1.92
time (sec)	N/A	0.177	0.617	0.486	0.000	2.696	0.000	1.710	11.551

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	238	1207	0	0	0	0	-1
N.S.	1	1.00	3.72	18.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	10.137	0.356	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	172	923	0	0	0	0	-1
N.S.	1	1.00	2.69	14.42	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	10.106	0.327	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	392	769	0	0	0	0	-1
N.S.	1	1.00	6.64	13.03	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	10.167	0.337	0.000	0.000	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	226	1818	0	0	0	0	-1
N.S.	1	1.00	3.65	29.32	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	10.167	0.450	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	411	1512	0	0	0	0	-1
N.S.	1	1.00	6.42	23.62	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	10.360	0.433	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	149	133	978	0	746	0	195	367
N.S.	1	0.99	0.89	6.52	0.00	4.97	0.00	1.30	2.45
time (sec)	N/A	0.132	0.370	0.356	0.000	2.463	0.000	2.267	7.898

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	958	0	630	0	181	247
N.S.	1	1.00	0.82	7.15	0.00	4.70	0.00	1.35	1.84
time (sec)	N/A	0.078	0.282	0.375	0.000	2.921	0.000	1.181	7.704

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	101	485	0	450	0	153	199
N.S.	1	1.00	0.94	4.49	0.00	4.17	0.00	1.42	1.84
time (sec)	N/A	0.063	0.265	0.326	0.000	3.131	0.000	1.438	7.429

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	157	1002	0	1819	0	226	288
N.S.	1	1.00	0.91	5.83	0.00	10.58	0.00	1.31	1.67
time (sec)	N/A	0.165	0.763	0.447	0.000	3.701	0.000	1.278	12.182

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	223	1067	0	2384	0	367	2500
N.S.	1	1.00	0.93	4.43	0.00	9.89	0.00	1.52	10.37
time (sec)	N/A	0.515	0.962	0.601	0.000	4.001	0.000	1.381	19.626

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	381	1593	0	0	0	0	-1
N.S.	1	1.00	5.69	23.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	10.204	0.357	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	216	986	0	0	0	0	-1
N.S.	1	1.00	3.22	14.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	10.180	0.339	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	381	830	0	0	0	0	-1
N.S.	1	1.00	6.15	13.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	10.350	0.344	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	308	2383	0	0	0	0	-1
N.S.	1	1.00	4.74	36.66	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	10.270	0.485	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	515	1919	0	0	0	0	-1
N.S.	1	1.00	7.69	28.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	10.582	0.479	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	200	0	0	0	388	0	-1
N.S.	1	1.00	1.49	0.00	0.00	0.00	2.90	0.00	-0.01
time (sec)	N/A	0.060	0.814	0.038	0.000	0.000	26.098	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	149	0	0	0	252	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	1.91	0.00	-0.01
time (sec)	N/A	0.054	0.254	0.037	0.000	0.000	8.582	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	110	0	0	0	122	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.93	0.00	-0.01
time (sec)	N/A	0.053	0.146	0.039	0.000	0.000	2.625	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	110	0	0	0	119	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.91	0.00	-0.01
time (sec)	N/A	0.052	0.177	0.039	0.000	0.000	2.235	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	110	0	0	0	119	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.89	0.00	-0.01
time (sec)	N/A	0.055	0.309	0.041	0.000	0.000	26.136	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	110	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.714	0.039	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	256	0	104	283
N.S.	1	1.00	1.00	0.00	0.00	2.91	0.00	1.18	3.22
time (sec)	N/A	0.064	0.997	0.025	0.000	2.768	0.000	1.691	9.251

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	194	0	54	49
N.S.	1	1.00	1.00	0.00	0.00	4.04	0.00	1.12	1.02
time (sec)	N/A	0.041	0.633	0.085	0.000	3.450	0.000	1.205	5.043

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	204	0	89	136
N.S.	1	1.00	1.00	0.00	0.00	4.25	0.00	1.85	2.83
time (sec)	N/A	0.032	0.583	0.095	0.000	4.156	0.000	1.297	7.572

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	278	0	413	481
N.S.	1	1.00	1.00	0.00	0.00	3.05	0.00	4.54	5.29
time (sec)	N/A	0.051	1.004	0.032	0.000	5.589	0.000	1.740	10.775

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	2.296	0.081	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	1.892	0.079	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	1.819	0.080	0.000	0.000	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	170	0	0	0	0	0	-1
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	2.076	0.082	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	189	0	0	0	0	0	-1
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	2.341	0.027	0.000	0.000	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	365	0	0	0	0	0	-1
N.S.	1	1.00	4.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	2.488	0.028	0.000	0.000	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	122	7293	292	256	292	201	-1
N.S.	1	1.00	0.76	45.30	1.81	1.59	1.81	1.25	-0.01
time (sec)	N/A	0.080	0.267	1.614	0.495	2.277	82.272	1.081	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	112	4175	0	0	97	0	-1
N.S.	1	1.00	0.35	12.89	0.00	0.00	0.30	0.00	-0.00
time (sec)	N/A	0.229	10.097	0.431	0.000	0.000	26.981	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	94	5358	0	0	97	0	-1
N.S.	1	1.00	0.16	9.22	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.448	10.066	0.463	0.000	0.000	7.576	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	95	6858	207	206	201	105	-1
N.S.	1	1.00	0.79	56.68	1.71	1.70	1.66	0.87	-0.01
time (sec)	N/A	0.060	0.222	0.351	0.504	4.345	4.951	2.024	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	93	3721	0	0	97	0	-1
N.S.	1	1.00	0.33	13.01	0.00	0.00	0.34	0.00	-0.00
time (sec)	N/A	0.167	10.049	0.409	0.000	0.000	2.219	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	98	5736	0	0	100	0	-1
N.S.	1	1.00	0.17	9.89	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.433	10.038	0.416	0.000	0.000	2.397	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	81	6668	146	184	160	0	-1
N.S.	1	1.00	0.69	56.51	1.24	1.56	1.36	0.00	-0.01
time (sec)	N/A	0.060	0.220	0.480	0.497	1.394	5.203	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	97	3512	0	0	100	0	-1
N.S.	1	1.00	0.34	12.41	0.00	0.00	0.35	0.00	-0.00
time (sec)	N/A	0.168	10.045	0.464	0.000	0.000	15.703	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	564	81	5911	0	0	97	0	-1
N.S.	1	1.00	0.14	10.48	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.389	10.072	0.616	0.000	0.000	11.931	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	74	3759	81	180	131	109	-1
N.S.	1	1.00	0.94	47.58	1.03	2.28	1.66	1.38	-0.01
time (sec)	N/A	0.031	0.251	0.424	0.496	1.969	32.275	0.820	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	80	3690	0	76	97	0	-1
N.S.	1	1.00	0.30	13.72	0.00	0.28	0.36	0.00	-0.00
time (sec)	N/A	0.147	10.061	0.439	0.000	0.260	86.344	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	143	7705	369	310	0	303	-1
N.S.	1	1.00	0.71	38.33	1.84	1.54	0.00	1.51	-0.00
time (sec)	N/A	0.098	0.343	0.391	0.525	1.371	0.000	0.932	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	116	4619	0	0	199	0	-1
N.S.	1	1.00	0.32	12.69	0.00	0.00	0.55	0.00	-0.00
time (sec)	N/A	0.232	10.106	0.460	0.000	0.000	74.764	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	96	5790	0	0	199	0	-1
N.S.	1	1.00	0.15	9.32	0.00	0.00	0.32	0.00	-0.00
time (sec)	N/A	0.489	10.077	0.454	0.000	0.000	25.087	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	120	7290	290	258	335	272	-1
N.S.	1	1.00	0.75	45.28	1.80	1.60	2.08	1.69	-0.01
time (sec)	N/A	0.079	0.287	0.521	0.506	2.628	15.045	0.749	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	96	4173	0	0	199	0	-1
N.S.	1	1.00	0.30	12.88	0.00	0.00	0.61	0.00	-0.00
time (sec)	N/A	0.191	10.055	0.407	0.000	0.000	7.042	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	84	6142	0	0	202	0	-1
N.S.	1	1.00	0.14	10.00	0.00	0.00	0.33	0.00	-0.00
time (sec)	N/A	0.486	10.054	0.385	0.000	0.000	7.382	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	100	7108	223	232	289	0	-1
N.S.	1	1.00	0.66	46.76	1.47	1.53	1.90	0.00	-0.01
time (sec)	N/A	0.073	0.307	0.397	0.540	3.881	12.897	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	85	3966	0	0	202	0	-1
N.S.	1	1.00	0.27	12.63	0.00	0.00	0.64	0.00	-0.00
time (sec)	N/A	0.191	10.056	0.395	0.000	0.000	24.481	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	164	8117	443	358	0	437	-1
N.S.	1	1.00	0.68	33.68	1.84	1.49	0.00	1.81	-0.00
time (sec)	N/A	0.117	0.422	0.348	0.505	2.166	0.000	1.502	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	116	5063	0	0	308	0	-1
N.S.	1	1.00	0.29	12.53	0.00	0.00	0.76	0.00	-0.00
time (sec)	N/A	0.266	10.140	0.334	0.000	0.000	175.610	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	99	6202	0	0	308	0	-1
N.S.	1	1.00	0.15	9.38	0.00	0.00	0.47	0.00	-0.00
time (sec)	N/A	0.541	10.113	0.341	0.000	0.000	61.286	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	140	7702	364	308	413	383	-1
N.S.	1	1.00	0.70	38.32	1.81	1.53	2.05	1.91	-0.00
time (sec)	N/A	0.091	0.352	0.404	0.511	2.826	49.808	2.122	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	84	4617	0	0	308	0	-1
N.S.	1	1.00	0.23	12.68	0.00	0.00	0.85	0.00	-0.00
time (sec)	N/A	0.222	10.049	0.386	0.000	0.000	20.369	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	87	6530	0	0	311	0	-1
N.S.	1	1.00	0.13	10.05	0.00	0.00	0.48	0.00	-0.00
time (sec)	N/A	0.536	10.040	0.405	0.000	0.000	20.714	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	126	7544	306	286	403	0	-1
N.S.	1	1.00	0.67	40.13	1.63	1.52	2.14	0.00	-0.01
time (sec)	N/A	0.086	0.366	0.412	0.487	1.826	37.414	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	88	4422	0	0	311	0	-1
N.S.	1	1.00	0.25	12.56	0.00	0.00	0.88	0.00	-0.00
time (sec)	N/A	0.228	10.046	0.389	0.000	0.000	50.406	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	99	6861	214	212	194	90	-1
N.S.	1	1.00	0.82	56.70	1.77	1.75	1.60	0.74	-0.01
time (sec)	N/A	0.177	0.300	0.359	0.511	1.303	59.463	1.477	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	98	3723	0	0	94	0	-1
N.S.	1	1.00	0.34	13.02	0.00	0.00	0.33	0.00	-0.00
time (sec)	N/A	0.173	10.079	0.331	0.000	0.000	18.262	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	80	4914	0	0	94	0	-1
N.S.	1	1.00	0.15	9.05	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.381	10.069	0.333	0.000	0.000	5.537	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	6424	133	159	107	56	-1
N.S.	1	1.00	0.94	77.40	1.60	1.92	1.29	0.67	-0.01
time (sec)	N/A	0.044	0.231	0.386	0.492	1.192	3.146	0.799	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	80	3275	0	0	94	0	-1
N.S.	1	1.00	0.32	13.15	0.00	0.00	0.38	0.00	-0.00
time (sec)	N/A	0.141	10.046	0.374	0.000	0.000	1.668	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	83	5385	0	0	97	0	-1
N.S.	1	1.00	0.15	9.94	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.379	10.042	0.368	0.000	0.000	2.112	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	3397	68	160	60	76	-1
N.S.	1	1.00	0.87	45.29	0.91	2.13	0.80	1.01	-0.01
time (sec)	N/A	0.041	0.230	0.395	0.507	0.595	5.048	1.643	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	82	3303	0	53	97	0	-1
N.S.	1	1.00	0.33	13.43	0.00	0.22	0.39	0.00	-0.00
time (sec)	N/A	0.140	10.034	0.499	0.000	0.120	19.385	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	92	7016	175	264	0	85	-1
N.S.	1	1.00	0.77	58.47	1.46	2.20	0.00	0.71	-0.01
time (sec)	N/A	0.062	0.391	0.377	0.544	2.416	0.000	1.590	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	87	3760	0	0	0	0	-1
N.S.	1	1.00	0.30	13.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.170	10.090	0.352	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	77	5392	0	0	94	0	-1
N.S.	1	1.00	0.14	9.75	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.381	10.080	0.325	0.000	0.000	76.931	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	3654	87	217	95	59	-1
N.S.	1	1.00	0.94	42.99	1.02	2.55	1.12	0.69	-0.01
time (sec)	N/A	0.043	0.336	0.369	0.501	6.587	10.292	1.628	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	79	3565	0	88	94	0	-1
N.S.	1	1.00	0.31	13.82	0.00	0.34	0.36	0.00	-0.00
time (sec)	N/A	0.143	10.048	0.367	0.000	0.560	21.680	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	77	5563	0	100	97	0	-1
N.S.	1	1.00	0.13	9.51	0.00	0.17	0.17	0.00	-0.00
time (sec)	N/A	0.435	10.039	0.416	0.000	0.489	37.357	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	44	44	59	51	90	0	70
N.S.	1	1.00	0.66	0.66	0.88	0.76	1.34	0.00	1.04
time (sec)	N/A	0.019	0.259	0.342	0.280	8.143	76.340	0.000	4.747

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	95	3783	0	109	97	0	-1
N.S.	1	1.00	0.34	13.37	0.00	0.39	0.34	0.00	-0.00
time (sec)	N/A	0.167	10.035	0.394	0.000	0.873	152.285	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	99	7081	103	296	0	82	-1
N.S.	1	1.00	0.87	62.11	0.90	2.60	0.00	0.72	-0.01
time (sec)	N/A	0.055	0.504	0.325	0.497	3.858	0.000	1.480	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	108	7083	0	151	0	0	-1
N.S.	1	1.00	0.36	23.69	0.00	0.51	0.00	0.00	-0.00
time (sec)	N/A	0.172	10.146	0.339	0.000	0.521	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	86	10786	0	162	0	0	-1
N.S.	1	1.00	0.14	18.10	0.00	0.27	0.00	0.00	-0.00
time (sec)	N/A	0.429	10.076	0.337	0.000	0.717	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	44	39	54	59	168	51	73
N.S.	1	1.00	0.56	0.49	0.68	0.75	2.13	0.65	0.92
time (sec)	N/A	0.023	0.355	0.297	0.286	4.485	71.085	1.602	4.628

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	107	7077	0	145	0	0	-1
N.S.	1	1.00	0.36	23.83	0.00	0.49	0.00	0.00	-0.00
time (sec)	N/A	0.174	10.071	0.377	0.000	1.146	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	85	10961	0	156	0	0	-1
N.S.	1	1.00	0.14	17.57	0.00	0.25	0.00	0.00	-0.00
time (sec)	N/A	0.488	10.047	0.430	0.000	0.504	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	67	67	90	84	0	0	115
N.S.	1	1.00	0.64	0.64	0.87	0.81	0.00	0.00	1.11
time (sec)	N/A	0.031	0.355	0.348	0.286	2.794	0.000	0.000	4.804

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	121	7299	0	167	0	0	-1
N.S.	1	1.00	0.38	22.81	0.00	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.197	10.077	0.409	0.000	0.399	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	202	0	183	186	0	0	240
N.S.	1	1.00	0.92	0.00	0.83	0.85	0.00	0.00	1.09
time (sec)	N/A	0.174	0.231	0.011	0.510	3.159	0.000	0.000	4.809

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	210	0	155	174	0	0	219
N.S.	1	1.00	1.21	0.00	0.89	1.00	0.00	0.00	1.26
time (sec)	N/A	0.134	0.174	0.010	0.511	1.976	0.000	0.000	4.650

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	188	0	153	157	0	0	200
N.S.	1	1.00	1.09	0.00	0.89	0.91	0.00	0.00	1.16
time (sec)	N/A	0.107	0.153	0.009	0.524	4.498	0.000	0.000	4.661

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	169	0	139	144	0	0	194
N.S.	1	1.00	1.13	0.00	0.93	0.96	0.00	0.00	1.29
time (sec)	N/A	0.086	0.129	0.008	0.525	2.477	0.000	0.000	4.638

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	235	0	0	1046	0	0	345
N.S.	1	1.00	1.10	0.00	0.00	4.89	0.00	0.00	1.61
time (sec)	N/A	0.123	0.270	0.050	0.000	4.628	0.000	0.000	5.097

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	282	0	0	321	0	0	455
N.S.	1	1.00	1.05	0.00	0.00	1.20	0.00	0.00	1.70
time (sec)	N/A	0.179	0.438	0.010	0.000	2.441	0.000	0.000	5.345

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	316	0	0	345	0	0	490
N.S.	1	1.00	1.12	0.00	0.00	1.22	0.00	0.00	1.73
time (sec)	N/A	0.190	0.552	0.011	0.000	3.192	0.000	0.000	5.442

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	327	0	0	362	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	1.35	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.868	0.010	0.000	2.885	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	293	0	0	338	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	1.45	0.00	0.00	-0.00
time (sec)	N/A	0.118	0.598	0.008	0.000	3.137	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	265	0	0	313	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	1.56	0.00	0.00	-0.00
time (sec)	N/A	0.052	0.412	0.048	0.000	2.208	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	190	0	0	395	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	2.53	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.314	0.010	0.000	186.280	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	214	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.353	0.008	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	206	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.432	0.011	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	219	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	0.487	0.013	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	234	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.449	6.088	0.009	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	225	0	0	0	0	0	-1
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	5.541	0.005	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	431	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.158	2.550	0.043	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	231	0	0	0	0	0	-1
N.S.	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	20.187	0.008	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	243	0	0	0	0	0	-1
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.382	20.133	0.010	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	202	0	183	209	0	0	261
N.S.	1	1.00	0.91	0.00	0.82	0.94	0.00	0.00	1.17
time (sec)	N/A	0.152	0.276	0.010	0.535	3.598	0.000	0.000	4.846

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	210	0	155	197	0	0	206
N.S.	1	1.00	1.19	0.00	0.88	1.11	0.00	0.00	1.16
time (sec)	N/A	0.135	0.202	0.010	0.518	5.384	0.000	0.000	4.915

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	188	0	155	181	0	0	221
N.S.	1	1.00	1.07	0.00	0.89	1.03	0.00	0.00	1.26
time (sec)	N/A	0.109	0.181	0.008	0.503	2.660	0.000	0.000	4.839

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	170	0	140	167	0	0	186
N.S.	1	1.00	1.11	0.00	0.92	1.09	0.00	0.00	1.22
time (sec)	N/A	0.087	0.159	0.008	0.506	2.913	0.000	0.000	4.830

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	236	0	0	530	0	0	369
N.S.	1	1.00	1.10	0.00	0.00	2.48	0.00	0.00	1.72
time (sec)	N/A	0.125	0.272	0.046	0.000	2.543	0.000	0.000	5.855

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	282	0	0	612	0	0	490
N.S.	1	1.00	1.05	0.00	0.00	2.28	0.00	0.00	1.82
time (sec)	N/A	0.182	0.397	0.013	0.000	5.848	0.000	0.000	5.561

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	316	0	0	660	0	0	513
N.S.	1	1.00	1.11	0.00	0.00	2.32	0.00	0.00	1.81
time (sec)	N/A	0.193	0.522	0.011	0.000	3.732	0.000	0.000	5.455

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	325	0	0	701	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	2.66	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.808	0.012	0.000	6.432	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	291	0	0	653	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	2.85	0.00	0.00	-0.00
time (sec)	N/A	0.089	0.594	0.008	0.000	9.155	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	264	0	0	611	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	3.06	0.00	0.00	-0.00
time (sec)	N/A	0.045	0.422	0.043	0.000	6.656	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	195	0	0	434	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.324	0.010	0.000	82.536	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	216	0	0	0	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.357	0.010	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	206	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.432	0.012	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	219	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.486	0.012	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	147	0	0	0	0	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.362	7.180	0.009	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	127	0	0	0	0	0	-1
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	6.698	0.009	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	63	0	0	0	0	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.195	9.787	0.049	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	483	136	0	0	0	0	0	-1
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.286	20.073	0.008	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	148	0	0	0	0	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	20.071	0.010	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	145	509	119	118	0	134	133
N.S.	1	1.00	1.14	4.01	0.94	0.93	0.00	1.06	1.05
time (sec)	N/A	0.063	0.135	5.016	0.662	2.552	0.000	0.881	5.034

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	139	497	119	137	0	127	133
N.S.	1	1.00	1.09	3.88	0.93	1.07	0.00	0.99	1.04
time (sec)	N/A	0.063	0.118	5.068	0.589	2.378	0.000	0.682	4.730

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	127	493	97	106	0	98	111
N.S.	1	1.00	1.31	5.08	1.00	1.09	0.00	1.01	1.14
time (sec)	N/A	0.052	0.095	4.778	0.611	2.221	0.000	0.932	4.649

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	126	671	97	125	0	98	111
N.S.	1	1.00	1.29	6.85	0.99	1.28	0.00	1.00	1.13
time (sec)	N/A	0.043	0.090	4.824	0.648	3.039	0.000	0.962	4.685

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	104	760	86	90	0	87	100
N.S.	1	1.00	1.27	9.27	1.05	1.10	0.00	1.06	1.22
time (sec)	N/A	0.038	0.062	2.051	0.581	3.738	0.000	0.890	4.894

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	185	0	0	410	0	149	256
N.S.	1	1.00	1.35	0.00	0.00	2.99	0.00	1.09	1.87
time (sec)	N/A	0.061	0.173	0.050	0.000	8.055	0.000	2.016	4.798

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	202	0	0	187	0	163	382
N.S.	1	1.00	1.29	0.00	0.00	1.19	0.00	1.04	2.43
time (sec)	N/A	0.070	0.227	0.008	0.000	2.427	0.000	1.718	4.860

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	220	0	0	201	0	0	-1
N.S.	1	1.00	1.43	0.00	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.394	0.007	0.000	2.197	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	205	0	0	452	0	0	-1
N.S.	1	1.00	1.52	0.00	0.00	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.271	0.040	0.000	7.210	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	114	931	0	253	0	0	-1
N.S.	1	1.00	1.30	10.58	0.00	2.88	0.00	0.00	-0.01
time (sec)	N/A	0.008	0.192	1.601	0.000	7.578	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	141	931	0	307	0	0	-1
N.S.	1	1.00	1.34	8.87	0.00	2.92	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.219	11.166	0.000	5.916	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	142	955	0	283	0	0	-1
N.S.	1	1.00	1.15	7.70	0.00	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.245	11.068	0.000	8.035	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	154	645	0	320	0	0	-1
N.S.	1	1.00	1.09	4.57	0.00	2.27	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.255	11.656	0.000	7.846	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	40	0	0	0	0	0	-1
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	10.020	0.006	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	26	0	0	0	0	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.082	10.022	0.044	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	283	0	0	373	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	1.60	0.00	0.00	-0.00
time (sec)	N/A	0.073	0.750	0.043	0.000	7.976	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	67	0	0	0	0	0	-1
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.111	20.044	0.007	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	76	0	0	0	0	0	-1
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.161	20.043	0.009	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	137	758	119	142	0	127	135
N.S.	1	1.00	1.10	6.06	0.95	1.14	0.00	1.02	1.08
time (sec)	N/A	0.063	0.124	4.856	0.520	3.568	0.000	1.623	5.185

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	127	546	97	114	0	98	113
N.S.	1	1.00	1.30	5.57	0.99	1.16	0.00	1.00	1.15
time (sec)	N/A	0.052	0.103	5.651	0.535	3.143	0.000	1.068	4.976

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	126	708	97	130	0	98	113
N.S.	1	1.00	1.33	7.45	1.02	1.37	0.00	1.03	1.19
time (sec)	N/A	0.042	0.077	4.501	0.520	2.538	0.000	1.536	4.892

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	102	531	86	98	0	87	102
N.S.	1	1.00	1.23	6.40	1.04	1.18	0.00	1.05	1.23
time (sec)	N/A	0.038	0.072	2.435	0.536	2.505	0.000	1.295	5.052

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	185	0	0	182	0	149	344
N.S.	1	1.00	1.35	0.00	0.00	1.33	0.00	1.09	2.51
time (sec)	N/A	0.059	0.138	0.058	0.000	2.826	0.000	1.240	4.921

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	202	0	0	195	0	163	368
N.S.	1	1.00	1.28	0.00	0.00	1.23	0.00	1.03	2.33
time (sec)	N/A	0.070	0.193	0.010	0.000	2.991	0.000	0.896	5.073

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	222	0	0	232	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.408	0.008	0.000	3.284	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	206	0	0	197	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.275	0.048	0.000	2.703	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	114	954	0	283	0	0	-1
N.S.	1	1.00	1.30	10.84	0.00	3.22	0.00	0.00	-0.01
time (sec)	N/A	0.011	0.212	2.498	0.000	5.657	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	143	1162	0	272	0	0	-1
N.S.	1	1.00	1.39	11.28	0.00	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.285	14.109	0.000	8.339	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	141	1186	0	312	0	0	-1
N.S.	1	1.00	1.14	9.56	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.260	14.062	0.000	5.679	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	115	694	0	356	0	0	-1
N.S.	1	1.00	0.40	2.38	0.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.130	10.079	12.187	0.000	5.605	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	26	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.129	10.042	0.058	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	111	0	0	0	0	0	-1
N.S.	1	1.00	0.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.120	10.080	0.057	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	120	695	0	396	0	0	-1
N.S.	1	1.00	0.41	2.36	0.00	1.35	0.00	0.00	-0.00
time (sec)	N/A	0.118	20.089	35.431	0.000	5.797	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	142	510	128	140	0	136	148
N.S.	1	1.00	1.01	3.62	0.91	0.99	0.00	0.96	1.05
time (sec)	N/A	0.073	0.207	3.589	0.664	3.484	0.000	1.553	4.861

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	137	677	119	159	0	120	139
N.S.	1	1.00	1.05	5.21	0.92	1.22	0.00	0.92	1.07
time (sec)	N/A	0.071	0.169	6.070	0.617	4.261	0.000	1.674	5.150

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	132	672	108	130	0	109	128
N.S.	1	1.00	1.15	5.84	0.94	1.13	0.00	0.95	1.11
time (sec)	N/A	0.063	0.162	6.714	0.615	3.300	0.000	1.516	4.890

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	126	674	97	148	0	98	117
N.S.	1	1.00	1.26	6.74	0.97	1.48	0.00	0.98	1.17
time (sec)	N/A	0.047	0.146	3.771	0.713	2.670	0.000	1.726	4.846

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	127	674	97	125	0	98	117
N.S.	1	1.00	1.27	6.74	0.97	1.25	0.00	0.98	1.17
time (sec)	N/A	0.046	0.121	3.580	0.730	2.724	0.000	1.647	4.903

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	198	0	0	226	0	160	253
N.S.	1	1.00	1.29	0.00	0.00	1.47	0.00	1.04	1.64
time (sec)	N/A	0.072	0.261	0.008	0.000	3.304	0.000	1.786	5.399

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	209	0	0	238	0	181	399
N.S.	1	1.00	1.19	0.00	0.00	1.36	0.00	1.03	2.28
time (sec)	N/A	0.077	0.325	0.013	0.000	2.999	0.000	1.227	5.024

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	227	0	0	271	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.519	0.013	0.000	3.051	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	220	0	0	239	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.414	0.010	0.000	2.957	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	139	627	0	318	0	0	-1
N.S.	1	1.00	1.31	5.92	0.00	3.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.294	5.023	0.000	7.203	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	140	944	0	288	0	0	-1
N.S.	1	1.00	1.32	8.91	0.00	2.72	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.284	1.883	0.000	6.397	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	148	777	0	340	0	0	-1
N.S.	1	1.00	1.19	6.27	0.00	2.74	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.335	15.231	0.000	9.245	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	152	941	0	316	0	0	-1
N.S.	1	1.00	1.06	6.53	0.00	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.347	15.591	0.000	7.507	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	157	963	0	351	0	0	-1
N.S.	1	1.00	0.97	5.94	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.360	11.772	0.000	7.274	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	71	0	0	0	0	0	-1
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	10.057	0.011	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	66	0	0	0	0	0	-1
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.125	10.049	0.011	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	66	0	0	0	0	0	-1
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.122	10.055	0.010	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	45	0	0	0	0	0	-1
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.099	10.025	0.009	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	76	0	0	0	0	0	-1
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.140	20.057	0.012	0.000	0.000	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	79	0	0	0	0	0	-1
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	20.081	0.012	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	308	0	0	325	0	379	442
N.S.	1	1.00	1.17	0.00	0.00	1.23	0.00	1.44	1.67
time (sec)	N/A	0.407	0.571	0.013	0.000	3.451	0.000	0.924	4.984

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	265	0	0	282	0	320	336
N.S.	1	1.00	1.20	0.00	0.00	1.28	0.00	1.45	1.53
time (sec)	N/A	0.180	0.402	0.019	0.000	3.609	0.000	0.870	4.933

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	226	0	0	222	0	276	298
N.S.	1	1.00	1.22	0.00	0.00	1.19	0.00	1.48	1.60
time (sec)	N/A	0.133	0.242	0.009	0.000	2.707	0.000	0.849	4.617

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	205	0	0	206	0	223	249
N.S.	1	1.00	1.29	0.00	0.00	1.30	0.00	1.40	1.57
time (sec)	N/A	0.106	0.194	0.005	0.000	2.632	0.000	0.726	4.621

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	312	0	0	276	0	311	1607
N.S.	1	1.00	1.27	0.00	0.00	1.12	0.00	1.26	6.53
time (sec)	N/A	0.142	0.407	0.044	0.000	2.388	0.000	0.819	4.744

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	351	0	0	429	0	351	1917
N.S.	1	1.00	1.03	0.00	0.00	1.26	0.00	1.03	5.64
time (sec)	N/A	0.257	0.630	0.008	0.000	3.701	0.000	0.888	9.990

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	413	0	0	472	0	465	2767
N.S.	1	1.00	1.12	0.00	0.00	1.28	0.00	1.26	7.48
time (sec)	N/A	0.331	0.813	0.008	0.000	5.029	0.000	0.868	12.554

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	527	0	0	494	0	0	-1
N.S.	1	1.00	1.57	0.00	0.00	1.47	0.00	0.00	-0.00
time (sec)	N/A	0.257	4.107	0.009	0.000	4.978	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	467	0	0	452	0	0	-1
N.S.	1	1.00	1.69	0.00	0.00	1.64	0.00	0.00	-0.00
time (sec)	N/A	0.157	2.758	0.007	0.000	4.299	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	423	0	0	330	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	1.41	0.00	0.00	-0.00
time (sec)	N/A	0.056	2.289	0.042	0.000	2.945	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	309	0	0	0	0	0	-1
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	1.465	0.007	0.000	0.000	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	333	0	0	0	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.116	1.576	0.008	0.000	0.000	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	373	0	0	0	0	0	-1
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	1.868	0.008	0.000	0.000	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	419	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.291	2.078	0.010	0.000	0.000	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	281	0	0	0	0	0	-1
N.S.	1	1.00	4.39	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	6.434	0.007	0.000	0.000	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	240	0	0	0	0	0	-1
N.S.	1	1.00	3.75	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	5.535	0.007	0.000	0.000	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	-1
N.S.	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	10.120	0.002	0.000	0.000	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	327	0	0	0	0	0	-1
N.S.	1	1.00	5.11	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	10.181	0.007	0.000	0.000	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	289	0	0	0	0	0	-1
N.S.	1	1.00	4.52	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	10.245	0.008	0.000	0.000	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	310	0	0	455	0	409	490
N.S.	1	1.00	1.17	0.00	0.00	1.71	0.00	1.54	1.84
time (sec)	N/A	0.197	0.634	0.010	0.000	2.621	0.000	1.530	5.127

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	266	0	0	398	0	350	385
N.S.	1	1.00	1.19	0.00	0.00	1.78	0.00	1.57	1.73
time (sec)	N/A	0.174	0.411	0.009	0.000	3.327	0.000	1.147	5.105

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	228	0	0	353	0	306	302
N.S.	1	1.00	1.21	0.00	0.00	1.88	0.00	1.63	1.61
time (sec)	N/A	0.131	0.292	0.007	0.000	3.229	0.000	0.971	5.058

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	206	0	0	323	0	259	238
N.S.	1	1.00	1.27	0.00	0.00	1.99	0.00	1.60	1.47
time (sec)	N/A	0.107	0.180	0.005	0.000	3.817	0.000	1.049	5.049

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	310	0	0	425	0	341	1963
N.S.	1	1.00	1.27	0.00	0.00	1.73	0.00	1.39	8.01
time (sec)	N/A	0.149	0.412	0.053	0.000	3.258	0.000	1.872	4.770

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	352	0	0	1030	0	400	1908
N.S.	1	1.00	1.01	0.00	0.00	2.97	0.00	1.15	5.50
time (sec)	N/A	0.256	0.619	0.012	0.000	4.324	0.000	1.460	10.571

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	413	0	0	1151	0	493	2788
N.S.	1	1.00	1.12	0.00	0.00	3.11	0.00	1.33	7.54
time (sec)	N/A	0.339	0.835	0.011	0.000	6.156	0.000	1.157	15.187

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	527	0	0	1164	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	3.49	0.00	0.00	-0.00
time (sec)	N/A	0.254	4.222	0.009	0.000	6.835	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	466	0	0	1091	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	4.01	0.00	0.00	-0.00
time (sec)	N/A	0.120	2.898	0.008	0.000	3.274	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	423	0	0	469	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.048	2.496	0.002	0.000	5.435	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	309	0	0	0	0	0	-1
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	1.447	0.009	0.000	0.000	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	334	0	0	0	0	0	-1
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.110	1.610	0.011	0.000	0.000	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	374	0	0	0	0	0	-1
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	1.871	0.010	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	422	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	2.149	0.012	0.000	0.000	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0	-1
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	6.895	0.016	0.000	0.000	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	-1
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	6.645	0.007	0.000	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	9.624	0.044	0.000	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	138	0	0	0	0	0	-1
N.S.	1	1.00	2.23	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	10.064	0.006	0.000	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0	-1
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	10.115	0.008	0.000	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	308	0	0	369	0	394	477
N.S.	1	1.00	1.23	0.00	0.00	1.47	0.00	1.57	1.90
time (sec)	N/A	0.228	0.531	0.009	0.000	4.796	0.000	0.584	5.123

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	260	0	0	298	0	348	348
N.S.	1	1.00	1.23	0.00	0.00	1.41	0.00	1.65	1.65
time (sec)	N/A	0.165	0.334	0.005	0.000	3.959	0.000	0.559	5.057

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	221	0	0	246	0	297	304
N.S.	1	1.00	1.18	0.00	0.00	1.32	0.00	1.59	1.63
time (sec)	N/A	0.151	0.287	0.005	0.000	3.488	0.000	0.568	4.725

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	333	0	0	320	0	357	796
N.S.	1	1.00	1.28	0.00	0.00	1.23	0.00	1.37	3.05
time (sec)	N/A	0.204	0.589	0.052	0.000	3.814	0.000	0.794	6.080

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	353	0	0	383	0	394	2047
N.S.	1	1.00	0.88	0.00	0.00	0.96	0.00	0.99	5.13
time (sec)	N/A	0.330	0.768	0.010	0.000	3.969	0.000	0.770	10.881

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	413	0	0	503	0	481	2841
N.S.	1	1.00	0.94	0.00	0.00	1.14	0.00	1.09	6.46
time (sec)	N/A	0.424	0.959	0.008	0.000	8.396	0.000	0.735	13.254

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	526	0	0	550	0	0	-1
N.S.	1	1.00	1.57	0.00	0.00	1.65	0.00	0.00	-0.00
time (sec)	N/A	0.309	5.339	0.006	0.000	6.157	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	469	0	0	396	0	0	-1
N.S.	1	1.00	1.69	0.00	0.00	1.43	0.00	0.00	-0.00
time (sec)	N/A	0.159	3.779	0.004	0.000	3.850	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	457	0	0	0	0	0	-1
N.S.	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.137	2.993	0.007	0.000	0.000	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	328	0	0	0	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.120	1.693	0.008	0.000	0.000	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	369	0	0	0	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	1.866	0.008	0.000	0.000	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	419	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.318	2.076	0.010	0.000	0.000	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	478	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	2.641	0.011	0.000	0.000	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	343	0	0	0	0	0	-1
N.S.	1	1.00	5.28	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	8.245	0.007	0.000	0.000	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	280	0	0	0	0	0	-1
N.S.	1	1.00	4.31	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	7.701	0.007	0.000	0.000	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0	-1
N.S.	1	1.00	5.77	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	10.220	0.002	0.000	0.000	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	341	0	0	0	0	0	-1
N.S.	1	1.00	5.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	10.236	0.006	0.000	0.000	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	286	0	0	0	0	0	-1
N.S.	1	1.00	4.40	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	10.264	0.007	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	307	0	0	1004	0	454	438
N.S.	1	1.00	1.06	0.00	0.00	3.46	0.00	1.57	1.51
time (sec)	N/A	0.204	0.655	0.010	0.000	4.024	0.000	0.618	5.115

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	263	0	0	873	0	371	339
N.S.	1	1.00	1.08	0.00	0.00	3.58	0.00	1.52	1.39
time (sec)	N/A	0.174	0.418	0.009	0.000	3.343	0.000	0.724	5.090

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	231	0	0	768	0	313	267
N.S.	1	1.00	1.14	0.00	0.00	3.78	0.00	1.54	1.32
time (sec)	N/A	0.151	0.363	0.009	0.000	4.390	0.000	0.639	5.113

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	203	0	0	667	0	257	219
N.S.	1	1.00	1.21	0.00	0.00	3.97	0.00	1.53	1.30
time (sec)	N/A	0.109	0.160	0.005	0.000	5.105	0.000	0.576	5.097

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	162	0	0	592	0	226	208
N.S.	1	1.00	1.12	0.00	0.00	4.08	0.00	1.56	1.43
time (sec)	N/A	0.088	0.116	0.056	0.000	4.209	0.000	0.545	4.934

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	309	0	0	628	0	326	702
N.S.	1	1.00	1.27	0.00	0.00	2.57	0.00	1.34	2.88
time (sec)	N/A	0.145	0.422	0.062	0.000	5.517	0.000	1.292	6.436

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	353	0	0	837	0	383	1929
N.S.	1	1.00	1.19	0.00	0.00	2.83	0.00	1.29	6.52
time (sec)	N/A	0.219	0.823	0.009	0.000	3.230	0.000	1.697	11.355

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	466	0	0	826	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	3.03	0.00	0.00	-0.00
time (sec)	N/A	0.118	2.767	0.010	0.000	3.217	0.000	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	423	0	0	761	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	3.27	0.00	0.00	-0.00
time (sec)	N/A	0.053	1.883	0.059	0.000	3.801	0.000	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	255	0	0	0	0	0	-1
N.S.	1	1.00	1.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	1.073	0.003	0.000	0.000	0.000	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	314	0	0	0	0	0	-1
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	1.263	0.011	0.000	0.000	0.000	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	340	0	0	0	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.120	1.571	0.012	0.000	0.000	0.000	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	374	0	0	0	0	0	-1
N.S.	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.187	2.092	0.014	0.000	0.000	0.000	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	144	0	0	0	0	0	-1
N.S.	1	1.00	2.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	6.751	0.014	0.000	0.000	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	6.458	0.047	0.000	0.000	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	9.206	0.043	0.000	0.000	0.000	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	-1
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	10.072	0.007	0.000	0.000	0.000	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	183	0	0	0	0	0	-1
N.S.	1	1.00	2.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	10.123	0.008	0.000	0.000	0.000	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	264	0	0	1322	0	372	331
N.S.	1	1.00	1.10	0.00	0.00	5.49	0.00	1.54	1.37
time (sec)	N/A	0.209	0.484	0.010	0.000	3.629	0.000	0.640	5.012

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	231	0	0	1156	0	312	292
N.S.	1	1.00	1.15	0.00	0.00	5.75	0.00	1.55	1.45
time (sec)	N/A	0.164	0.369	0.009	0.000	2.910	0.000	0.602	4.688

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	202	0	0	1060	0	253	232
N.S.	1	1.00	1.22	0.00	0.00	6.42	0.00	1.53	1.41
time (sec)	N/A	0.115	0.168	0.006	0.000	3.009	0.000	0.564	4.684

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	162	0	0	927	0	221	213
N.S.	1	1.00	1.12	0.00	0.00	6.39	0.00	1.52	1.47
time (sec)	N/A	0.090	0.117	0.043	0.000	2.422	0.000	0.576	4.847

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	308	0	0	472	0	321	1413
N.S.	1	1.00	1.26	0.00	0.00	1.93	0.00	1.31	5.77
time (sec)	N/A	0.150	0.508	0.049	0.000	2.539	0.000	0.828	4.942

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	355	0	0	562	0	377	1959
N.S.	1	1.00	1.19	0.00	0.00	1.88	0.00	1.26	6.55
time (sec)	N/A	0.222	0.777	0.008	0.000	4.423	0.000	0.707	11.141

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	471	0	0	558	0	0	-1
N.S.	1	1.00	1.69	0.00	0.00	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.147	2.954	0.009	0.000	5.295	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	423	0	0	530	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	2.26	0.00	0.00	-0.00
time (sec)	N/A	0.061	2.066	0.046	0.000	2.635	0.000	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	255	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	1.144	0.040	0.000	0.000	0.000	0.000	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	308	0	0	0	0	0	-1
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	1.328	0.006	0.000	0.000	0.000	0.000	0.000

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	340	0	0	0	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.130	1.664	0.010	0.000	0.000	0.000	0.000	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	249	0	0	0	0	0	-1
N.S.	1	1.00	3.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	7.764	0.011	0.000	0.000	0.000	0.000	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	7.035	0.057	0.000	0.000	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	-1
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	10.040	0.003	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	338	0	0	0	0	0	-1
N.S.	1	1.00	5.28	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	10.174	0.010	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	349	0	0	1300	0	431	564
N.S.	1	1.00	1.01	0.00	0.00	3.75	0.00	1.24	1.63
time (sec)	N/A	0.503	0.952	0.013	0.000	3.012	0.000	0.772	5.189

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	298	0	0	1141	0	372	493
N.S.	1	1.00	1.18	0.00	0.00	4.51	0.00	1.47	1.95
time (sec)	N/A	0.208	0.668	0.011	0.000	2.186	0.000	1.405	5.160

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	254	0	0	1004	0	325	449
N.S.	1	1.00	1.25	0.00	0.00	4.95	0.00	1.60	2.21
time (sec)	N/A	0.161	0.533	0.012	0.000	1.774	0.000	1.029	4.986

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	227	0	0	872	0	301	412
N.S.	1	1.00	1.30	0.00	0.00	5.01	0.00	1.73	2.37
time (sec)	N/A	0.116	0.347	0.063	0.000	2.641	0.000	0.917	4.978

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	220	0	0	262	0	285	389
N.S.	1	1.00	1.32	0.00	0.00	1.57	0.00	1.71	2.33
time (sec)	N/A	0.112	0.259	0.061	0.000	1.787	0.000	0.693	4.923

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	350	0	0	975	0	389	2500
N.S.	1	1.00	1.29	0.00	0.00	3.60	0.00	1.44	9.23
time (sec)	N/A	0.218	0.978	0.062	0.000	2.490	0.000	0.794	5.342

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	392	0	0	1386	0	486	2500
N.S.	1	1.00	1.10	0.00	0.00	3.88	0.00	1.36	7.00
time (sec)	N/A	0.272	1.117	0.016	0.000	3.913	0.000	0.762	6.390

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	506	0	0	1329	0	0	-1
N.S.	1	1.00	1.57	0.00	0.00	4.13	0.00	0.00	-0.00
time (sec)	N/A	0.267	6.533	0.007	0.000	3.835	0.000	0.000	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	466	0	0	1127	0	0	-1
N.S.	1	1.00	1.79	0.00	0.00	4.33	0.00	0.00	-0.00
time (sec)	N/A	0.108	3.899	0.051	0.000	1.995	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	322	0	0	0	0	0	-1
N.S.	1	1.00	1.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	1.755	0.048	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	328	0	0	0	0	0	-1
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	1.994	0.002	0.000	0.000	0.000	0.000	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	358	0	0	0	0	0	-1
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.115	2.344	0.007	0.000	0.000	0.000	0.000	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	402	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.188	3.519	0.011	0.000	0.000	0.000	0.000	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	459	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	6.615	0.011	0.000	0.000	0.000	0.000	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	194	0	0	0	0	0	-1
N.S.	1	1.00	2.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	9.841	0.008	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	144	0	0	0	0	0	-1
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	9.614	0.051	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	129	0	0	0	0	0	-1
N.S.	1	1.00	1.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	9.290	0.048	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	141	0	0	0	0	0	-1
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	10.091	0.058	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	193	0	0	0	0	0	-1
N.S.	1	1.00	2.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	10.154	0.008	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	264	0	0	0	0	0	-1
N.S.	1	1.00	3.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	10.209	0.009	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	92	78	84	100	0	88	88
N.S.	1	1.00	1.02	0.87	0.93	1.11	0.00	0.98	0.98
time (sec)	N/A	0.066	0.036	0.376	0.277	10.163	0.000	0.824	5.880

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	68	72	0	70	68
N.S.	1	1.00	0.94	0.93	0.97	1.03	0.00	1.00	0.97
time (sec)	N/A	0.044	0.025	0.377	0.286	5.166	0.000	1.625	5.633

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	50	49	42	0	51	51
N.S.	1	1.00	0.81	0.94	0.92	0.79	0.00	0.96	0.96
time (sec)	N/A	0.034	0.017	0.378	0.285	3.822	0.000	1.521	5.096

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	31	42	41	31	138	51	1012
N.S.	1	1.00	0.69	0.93	0.91	0.69	3.07	1.13	22.49
time (sec)	N/A	0.021	0.015	3.593	0.300	2.421	0.979	1.239	4.987

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	59	61	54	0	73	58
N.S.	1	1.00	0.87	0.95	0.98	0.87	0.00	1.18	0.94
time (sec)	N/A	0.041	0.022	0.697	0.278	6.927	0.000	1.080	5.492

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	83	87	99	0	112	87
N.S.	1	1.00	1.01	0.95	1.00	1.14	0.00	1.29	1.00
time (sec)	N/A	0.066	0.030	0.844	0.352	20.180	0.000	1.264	6.214

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	104	98	100	576	0	112	532
N.S.	1	1.00	0.93	0.88	0.89	5.14	0.00	1.00	4.75
time (sec)	N/A	0.178	0.114	3.417	0.509	8.393	0.000	1.100	5.705

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	81	80	416	0	80	518
N.S.	1	1.00	0.89	0.88	0.87	4.52	0.00	0.87	5.63
time (sec)	N/A	0.078	0.096	2.659	0.503	4.471	0.000	1.239	5.724

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	60	59	325	0	59	379
N.S.	1	1.00	0.84	0.76	0.75	4.11	0.00	0.75	4.80
time (sec)	N/A	0.043	0.033	0.442	0.537	4.012	0.000	1.165	5.345

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	60	59	325	0	59	399
N.S.	1	1.00	0.84	0.76	0.75	4.11	0.00	0.75	5.05
time (sec)	N/A	0.033	0.035	0.438	0.549	4.277	0.000	1.215	5.295

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	169	81	80	432	0	80	354
N.S.	1	1.00	1.84	0.88	0.87	4.70	0.00	0.87	3.85
time (sec)	N/A	0.082	0.167	0.465	0.643	7.432	0.000	1.167	5.348

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	193	101	101	592	0	103	535
N.S.	1	1.00	1.72	0.90	0.90	5.29	0.00	0.92	4.78
time (sec)	N/A	0.145	0.150	0.447	0.523	9.637	0.000	1.415	5.530

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	377	234	375	1376	0	469	2500
N.S.	1	1.00	0.82	0.51	0.82	3.01	0.00	1.03	5.47
time (sec)	N/A	0.305	0.142	0.442	0.530	5.382	0.000	1.956	5.633

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	226	363	1374	0	453	2553
N.S.	1	1.00	0.76	0.50	0.81	3.06	0.00	1.01	5.69
time (sec)	N/A	0.195	0.073	0.421	0.512	3.515	0.000	1.380	5.725

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	218	361	1238	0	437	2500
N.S.	1	1.00	0.76	0.49	0.80	2.76	0.00	0.97	5.57
time (sec)	N/A	0.183	0.064	0.415	0.518	4.470	0.000	1.304	5.859

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	218	363	1274	0	477	2500
N.S.	1	1.00	0.76	0.49	0.81	2.84	0.00	1.06	5.57
time (sec)	N/A	0.187	0.083	0.427	0.509	3.846	0.000	1.482	5.497

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	226	365	1354	0	437	2500
N.S.	1	1.00	0.76	0.50	0.81	3.02	0.00	0.97	5.57
time (sec)	N/A	0.181	0.077	0.337	0.551	3.902	0.000	0.995	5.853

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	385	237	384	1407	0	488	2500
N.S.	1	1.00	0.84	0.52	0.83	3.06	0.00	1.06	5.43
time (sec)	N/A	0.316	0.141	0.437	0.510	4.115	0.000	1.160	6.084

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	406	241	390	1417	0	472	2500
N.S.	1	1.00	0.88	0.52	0.84	3.07	0.00	1.02	5.41
time (sec)	N/A	0.272	0.156	0.464	0.513	11.432	0.000	1.002	6.190

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	428	261	405	1468	0	483	2500
N.S.	1	1.00	0.89	0.54	0.85	3.06	0.00	1.01	5.22
time (sec)	N/A	0.408	0.172	0.453	0.499	17.300	0.000	1.429	6.012

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	1012	0	195	90	96	87
N.S.	1	1.00	0.95	10.88	0.00	2.10	0.97	1.03	0.94
time (sec)	N/A	0.061	0.130	0.415	0.000	3.006	7.096	1.254	4.688

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	138	1050	0	714	0	0	-1
N.S.	1	1.00	1.15	8.75	0.00	5.95	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.423	0.427	0.000	4.090	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	69	988	0	156	65	66	54
N.S.	1	1.00	0.99	14.11	0.00	2.23	0.93	0.94	0.77
time (sec)	N/A	0.043	0.084	0.391	0.000	2.298	3.559	1.077	4.660

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	106	1000	0	612	0	0	-1
N.S.	1	1.00	1.16	10.99	0.00	6.73	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.261	0.348	0.000	3.210	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	1039	0	383	82	79	199
N.S.	1	1.00	0.95	12.22	0.00	4.51	0.96	0.93	2.34
time (sec)	N/A	0.052	0.108	0.349	0.000	2.680	5.321	1.256	4.869

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	96	1070	0	281	0	121	-1
N.S.	1	1.00	1.26	14.08	0.00	3.70	0.00	1.59	-0.01
time (sec)	N/A	0.057	0.293	0.367	0.000	2.570	0.000	1.886	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	107	1108	0	513	0	107	269
N.S.	1	1.00	0.93	9.63	0.00	4.46	0.00	0.93	2.34
time (sec)	N/A	0.085	0.276	0.424	0.000	3.799	0.000	1.039	5.387

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	118	1093	0	329	0	225	-1
N.S.	1	1.00	1.07	9.94	0.00	2.99	0.00	2.05	-0.01
time (sec)	N/A	0.110	0.421	0.359	0.000	2.694	0.000	2.673	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	857	1067	141	421	0	0	0	0	-1
N.S.	1	1.25	0.16	0.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.325	10.103	0.405	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	700	904	241	368	0	0	0	0	-1
N.S.	1	1.29	0.34	0.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.917	10.316	0.394	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	786	1012	65	299	0	0	0	0	-1
N.S.	1	1.29	0.08	0.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.023	10.034	0.341	0.000	0.000	0.000	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	679	881	161	273	0	0	0	0	-1
N.S.	1	1.30	0.24	0.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.024	10.113	0.326	0.000	0.000	0.000	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	809	1031	138	421	0	0	0	0	-1
N.S.	1	1.27	0.17	0.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.149	10.067	0.398	0.000	0.000	0.000	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	703	893	333	370	0	0	0	0	-1
N.S.	1	1.27	0.47	0.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.906	10.191	0.396	0.000	0.000	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	10.054	0.071	0.000	0.000	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	10.033	0.064	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	10.050	0.079	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	143	0	0	0	0	0	-1
N.S.	1	1.00	2.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	10.088	0.015	0.000	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	369	0	289	0	106	102
N.S.	1	1.00	0.88	3.55	0.00	2.78	0.00	1.02	0.98
time (sec)	N/A	0.076	0.195	0.371	0.000	4.330	0.000	1.866	4.825

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	340	0	205	0	64	58
N.S.	1	1.00	0.99	4.59	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.047	0.100	0.365	0.000	2.645	0.000	1.867	4.729

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	316	0	130	37	40	40
N.S.	1	1.00	1.00	6.20	0.00	2.55	0.73	0.78	0.78
time (sec)	N/A	0.034	0.044	0.329	0.000	3.257	5.492	1.338	4.797

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	354	0	431	66	71	652
N.S.	1	1.00	0.94	4.16	0.00	5.07	0.78	0.84	7.67
time (sec)	N/A	0.055	0.124	0.339	0.000	2.801	7.367	1.451	5.028

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	109	407	0	565	0	104	396
N.S.	1	1.00	0.93	3.48	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.089	0.279	0.392	0.000	3.682	0.000	1.725	5.351

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	140	403	0	739	0	0	-1
N.S.	1	1.00	1.14	3.28	0.00	6.01	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.647	0.377	0.000	3.729	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	107	355	0	632	0	0	-1
N.S.	1	1.00	1.18	3.90	0.00	6.95	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.340	0.347	0.000	4.530	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	74	322	0	245	0	72	-1
N.S.	1	1.00	1.37	5.96	0.00	4.54	0.00	1.33	-0.02
time (sec)	N/A	0.030	0.210	0.329	0.000	4.057	0.000	1.758	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	100	349	0	332	0	116	-1
N.S.	1	1.00	1.25	4.36	0.00	4.15	0.00	1.45	-0.01
time (sec)	N/A	0.062	0.343	0.358	0.000	3.362	0.000	3.032	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	121	379	0	418	0	205	-1
N.S.	1	1.00	1.05	3.30	0.00	3.63	0.00	1.78	-0.01
time (sec)	N/A	0.122	0.641	0.370	0.000	3.214	0.000	3.860	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	872	872	249	363	0	0	0	0	-1
N.S.	1	1.00	0.29	0.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.804	10.229	0.355	0.000	0.000	0.000	0.000	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	638	837	65	265	0	0	0	0	-1
N.S.	1	1.31	0.10	0.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	10.058	0.351	0.000	0.000	0.000	0.000	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	742	161	191	0	0	0	0	-1
N.S.	1	1.16	0.25	0.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.498	10.040	0.339	0.000	0.000	0.000	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	677	864	337	288	0	0	0	0	-1
N.S.	1	1.28	0.50	0.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.784	10.211	0.385	0.000	0.000	0.000	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	804	982	65	292	0	0	0	0	-1
N.S.	1	1.22	0.08	0.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.746	10.045	0.330	0.000	0.000	0.000	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	656	756	65	191	0	0	0	0	-1
N.S.	1	1.15	0.10	0.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.543	10.026	0.321	0.000	0.000	0.000	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	833	1007	141	310	0	0	0	0	-1
N.S.	1	1.21	0.17	0.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.048	10.091	0.385	0.000	0.000	0.000	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	175	918	0	622	0	180	186
N.S.	1	1.00	1.00	5.25	0.00	3.55	0.00	1.03	1.06
time (sec)	N/A	0.125	0.443	0.394	0.000	3.169	0.000	2.223	5.191

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	130	887	0	475	0	134	144
N.S.	1	1.00	1.06	7.21	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.096	0.337	0.387	0.000	4.941	0.000	1.213	5.117

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	100	867	0	348	0	116	95
N.S.	1	1.00	1.01	8.76	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.106	0.185	0.346	0.000	3.747	0.000	1.265	4.932

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	541	0	302	0	93	84
N.S.	1	1.00	0.99	6.22	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.095	0.159	0.323	0.000	2.356	0.000	1.619	4.852

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	124	900	0	862	0	139	3017
N.S.	1	1.00	0.94	6.82	0.00	6.53	0.00	1.05	22.86
time (sec)	N/A	0.203	0.362	0.356	0.000	3.295	0.000	0.889	5.869

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	163	954	0	1236	0	257	2500
N.S.	1	1.00	0.88	5.16	0.00	6.68	0.00	1.39	13.51
time (sec)	N/A	0.173	0.651	0.429	0.000	4.157	0.000	1.087	7.047

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	188	1278	0	1386	0	337	-1
N.S.	1	1.00	0.98	6.69	0.00	7.26	0.00	1.76	-0.01
time (sec)	N/A	0.218	1.658	0.412	0.000	4.153	0.000	1.777	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	152	1228	0	1077	0	298	-1
N.S.	1	1.00	1.08	8.71	0.00	7.64	0.00	2.11	-0.01
time (sec)	N/A	0.116	1.297	0.344	0.000	4.296	0.000	1.266	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	112	1199	0	426	0	244	-1
N.S.	1	1.00	1.20	12.89	0.00	4.58	0.00	2.62	-0.01
time (sec)	N/A	0.064	0.763	0.352	0.000	3.857	0.000	3.071	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	124	867	0	467	0	237	-1
N.S.	1	1.00	1.19	8.34	0.00	4.49	0.00	2.28	-0.01
time (sec)	N/A	0.065	0.496	0.328	0.000	3.481	0.000	1.606	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	157	1221	0	612	0	418	-1
N.S.	1	1.00	1.05	8.19	0.00	4.11	0.00	2.81	-0.01
time (sec)	N/A	0.134	0.994	0.399	0.000	2.764	0.000	3.165	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	201	1252	0	760	0	395	-1
N.S.	1	1.00	0.97	6.02	0.00	3.65	0.00	1.90	-0.00
time (sec)	N/A	0.224	1.530	0.411	0.000	4.124	0.000	3.677	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	996	996	253	604	0	0	0	0	-1
N.S.	1	1.00	0.25	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.923	10.213	0.349	0.000	0.000	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	908	908	238	530	0	0	0	0	-1
N.S.	1	1.00	0.26	0.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.664	10.160	0.381	0.000	0.000	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	983	983	392	333	0	0	0	0	-1
N.S.	1	1.00	0.40	0.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.769	10.204	0.341	0.000	0.000	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1046	1046	408	626	0	0	0	0	-1
N.S.	1	1.00	0.39	0.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.387	10.374	0.451	0.000	0.000	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1146	1146	162	556	0	0	0	0	-1
N.S.	1	1.00	0.14	0.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.134	10.102	0.342	0.000	0.000	0.000	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1144	1144	172	359	0	0	0	0	-1
N.S.	1	1.00	0.15	0.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.058	10.134	0.346	0.000	0.000	0.000	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1225	1225	226	674	0	0	0	0	-1
N.S.	1	1.00	0.18	0.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.385	10.199	0.527	0.000	0.000	0.000	0.000	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	194	164	0	0	0	185	0	-1
N.S.	1	0.97	0.82	0.00	0.00	0.00	0.92	0.00	-0.00
time (sec)	N/A	0.161	7.749	0.062	0.000	0.000	7.597	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	115	110	0	0	0	119	0	-1
N.S.	1	0.93	0.89	0.00	0.00	0.00	0.97	0.00	-0.01
time (sec)	N/A	0.042	1.430	0.040	0.000	0.000	2.480	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	0	0	0	56	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	-0.01
time (sec)	N/A	0.013	0.539	0.038	0.000	0.000	0.503	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	125	0	0	0	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	2.878	0.075	0.000	0.000	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	179	0	0	0	0	0	-1
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	7.905	0.070	0.000	0.000	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	10.082	0.073	0.000	0.000	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	167	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	10.102	0.056	0.000	0.000	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	0	0	0	119	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.90	0.00	-0.01
time (sec)	N/A	0.046	3.511	0.040	0.000	0.000	22.145	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	0	0	0	56	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	-0.01
time (sec)	N/A	0.016	1.238	0.035	0.000	0.000	0.829	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	169	0	0	0	0	0	-1
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	10.105	0.064	0.000	0.000	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	10.076	0.071	0.000	0.000	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	10.077	0.067	0.000	0.000	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	0	0	288	0	106	103
N.S.	1	1.00	0.88	0.00	0.00	2.77	0.00	1.02	0.99
time (sec)	N/A	0.073	0.199	0.027	0.000	4.125	0.000	1.712	4.866

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	205	0	64	58
N.S.	1	1.00	0.99	0.00	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.044	0.102	0.015	0.000	5.026	0.000	1.420	4.745

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	130	37	40	40
N.S.	1	1.00	1.00	0.00	0.00	2.55	0.73	0.78	0.78
time (sec)	N/A	0.031	0.045	0.076	0.000	9.261	7.939	1.183	4.725

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	0	0	431	66	71	652
N.S.	1	1.00	0.94	0.00	0.00	5.07	0.78	0.84	7.67
time (sec)	N/A	0.049	0.127	0.086	0.000	5.164	9.445	1.341	5.191

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	109	0	0	565	0	104	396
N.S.	1	1.00	0.93	0.00	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.077	0.306	0.019	0.000	4.082	0.000	1.437	5.615

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	140	0	0	739	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	6.01	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.946	0.020	0.000	5.631	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	107	0	0	632	0	156	-1
N.S.	1	1.00	1.18	0.00	0.00	6.95	0.00	1.71	-0.01
time (sec)	N/A	0.056	0.461	0.082	0.000	2.397	0.000	2.236	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	74	0	0	245	0	72	-1
N.S.	1	1.00	1.37	0.00	0.00	4.54	0.00	1.33	-0.02
time (sec)	N/A	0.031	0.277	0.075	0.000	2.145	0.000	4.518	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	100	0	0	332	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	4.15	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.477	0.018	0.000	3.375	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	121	0	0	416	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	3.62	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.908	0.026	0.000	4.903	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	10.035	0.071	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	10.040	0.073	0.000	0.000	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	10.031	0.066	0.000	0.000	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	-1
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	10.145	0.071	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	-1
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	10.079	0.016	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	-1
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	10.080	0.018	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	-1
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	10.084	0.016	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	130	0	0	475	0	134	144
N.S.	1	1.00	1.06	0.00	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.094	0.386	0.020	0.000	5.347	0.000	1.811	5.093

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	100	0	0	348	0	116	95
N.S.	1	1.00	1.01	0.00	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.054	0.225	0.079	0.000	5.076	0.000	1.269	4.989

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	302	0	93	84
N.S.	1	1.00	0.99	0.00	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.046	0.165	0.069	0.000	5.693	0.000	1.740	4.929

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	124	0	0	862	0	139	3025
N.S.	1	1.00	0.94	0.00	0.00	6.53	0.00	1.05	22.92
time (sec)	N/A	0.090	0.374	0.086	0.000	4.671	0.000	1.445	6.226

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	163	0	0	1236	0	257	2500
N.S.	1	1.00	0.88	0.00	0.00	6.68	0.00	1.39	13.51
time (sec)	N/A	0.152	0.630	0.018	0.000	5.346	0.000	3.261	7.290

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	152	0	0	1077	0	343	-1
N.S.	1	1.00	1.08	0.00	0.00	7.64	0.00	2.43	-0.01
time (sec)	N/A	0.106	1.950	0.087	0.000	3.159	0.000	1.899	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	112	0	0	426	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	4.58	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.948	0.088	0.000	3.825	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	124	0	0	467	0	237	-1
N.S.	1	1.00	1.19	0.00	0.00	4.49	0.00	2.28	-0.01
time (sec)	N/A	0.064	0.611	0.075	0.000	3.899	0.000	0.887	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	157	0	0	612	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	4.11	0.00	0.00	-0.01
time (sec)	N/A	0.126	1.179	0.022	0.000	5.928	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	201	0	0	760	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	3.65	0.00	0.00	-0.00
time (sec)	N/A	0.205	2.008	0.025	0.000	7.553	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	169	0	0	0	0	0	-1
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	10.133	0.070	0.000	0.000	0.000	0.000	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	168	0	0	0	0	0	-1
N.S.	1	1.00	2.62	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.050	10.134	0.073	0.000	0.000	0.000	0.000	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	172	0	0	0	0	0	-1
N.S.	1	1.00	2.69	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	10.119	0.070	0.000	0.000	0.000	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	329	0	0	0	0	0	-1
N.S.	1	1.00	5.58	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	10.199	0.073	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0	-1
N.S.	1	1.00	3.65	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	10.220	0.021	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0	-1
N.S.	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	10.175	0.020	0.000	0.000	0.000	0.000	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	225	0	0	0	0	0	-1
N.S.	1	1.00	3.52	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	10.206	0.022	0.000	0.000	0.000	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	0	0	288	0	106	103
N.S.	1	1.00	0.88	0.00	0.00	2.77	0.00	1.02	0.99
time (sec)	N/A	0.068	0.187	0.027	0.000	3.413	0.000	2.757	4.678

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	205	0	64	58
N.S.	1	1.00	0.99	0.00	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.042	0.100	0.015	0.000	2.938	0.000	3.227	4.730

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	130	37	40	40
N.S.	1	1.00	1.00	0.00	0.00	2.55	0.73	0.78	0.78
time (sec)	N/A	0.031	0.044	0.071	0.000	3.241	18.502	2.699	4.586

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	0	0	431	66	71	652
N.S.	1	1.00	0.94	0.00	0.00	5.07	0.78	0.84	7.67
time (sec)	N/A	0.047	0.127	0.078	0.000	3.141	14.052	2.568	4.813

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	109	0	0	565	0	104	396
N.S.	1	1.00	0.93	0.00	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.074	0.289	0.017	0.000	2.931	0.000	1.818	5.507

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	140	0	0	739	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	6.01	0.00	0.00	-0.01
time (sec)	N/A	0.090	1.439	0.023	0.000	3.855	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	107	0	0	632	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	6.95	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.683	0.079	0.000	3.781	0.000	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	74	0	0	245	0	72	-1
N.S.	1	1.00	1.37	0.00	0.00	4.54	0.00	1.33	-0.02
time (sec)	N/A	0.032	0.350	0.070	0.000	2.754	0.000	1.802	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	100	0	0	332	0	116	-1
N.S.	1	1.00	1.25	0.00	0.00	4.15	0.00	1.45	-0.01
time (sec)	N/A	0.060	0.575	0.019	0.000	3.782	0.000	1.502	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	121	0	0	416	0	205	-1
N.S.	1	1.00	1.05	0.00	0.00	3.62	0.00	1.78	-0.01
time (sec)	N/A	0.102	1.495	0.024	0.000	3.012	0.000	4.424	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	851	851	65	0	0	0	0	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.794	10.074	0.083	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	754	754	65	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	10.041	0.076	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	878	878	141	0	0	0	0	0	-1
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.148	10.118	0.018	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1005	1005	65	0	0	0	0	0	-1
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.912	10.070	0.079	0.000	0.000	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	768	768	65	0	0	0	0	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.637	10.039	0.072	0.000	0.000	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1032	1032	141	0	0	0	0	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.165	10.104	0.014	0.000	0.000	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	10.055	0.070	0.000	0.000	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	10.045	0.073	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	-1
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	10.144	0.070	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	-1
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	10.113	0.016	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	-1
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	10.118	0.014	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	130	0	0	475	0	134	144
N.S.	1	1.00	1.06	0.00	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.094	0.381	0.020	0.000	2.141	0.000	0.833	5.021

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	100	0	0	348	0	116	95
N.S.	1	1.00	1.01	0.00	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.053	0.186	0.096	0.000	3.830	0.000	0.739	4.837

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	302	0	93	84
N.S.	1	1.00	0.99	0.00	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.046	0.164	0.084	0.000	2.191	0.000	1.187	4.797

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	124	0	0	862	0	139	3017
N.S.	1	1.00	0.94	0.00	0.00	6.53	0.00	1.05	22.86
time (sec)	N/A	0.088	0.377	0.100	0.000	2.938	0.000	1.588	5.820

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	163	0	0	1236	0	257	2500
N.S.	1	1.00	0.88	0.00	0.00	6.68	0.00	1.39	13.51
time (sec)	N/A	0.147	0.636	0.020	0.000	2.524	0.000	1.310	7.851

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	152	0	0	1077	0	298	-1
N.S.	1	1.00	1.08	0.00	0.00	7.64	0.00	2.11	-0.01
time (sec)	N/A	0.098	2.644	0.096	0.000	3.138	0.000	1.390	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	112	0	0	426	0	244	-1
N.S.	1	1.00	1.20	0.00	0.00	4.58	0.00	2.62	-0.01
time (sec)	N/A	0.054	1.272	0.085	0.000	3.265	0.000	3.296	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	124	0	0	467	0	237	-1
N.S.	1	1.00	1.19	0.00	0.00	4.49	0.00	2.28	-0.01
time (sec)	N/A	0.056	0.727	0.073	0.000	3.200	0.000	1.445	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	157	0	0	612	0	418	-1
N.S.	1	1.00	1.05	0.00	0.00	4.11	0.00	2.81	-0.01
time (sec)	N/A	0.117	1.429	0.022	0.000	3.007	0.000	2.623	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	201	0	0	760	0	395	-1
N.S.	1	1.00	0.97	0.00	0.00	3.65	0.00	1.90	-0.00
time (sec)	N/A	0.189	3.296	0.031	0.000	3.992	0.000	3.144	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	924	924	159	0	0	0	0	0	-1
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.898	10.122	0.082	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	999	999	169	0	0	0	0	0	-1
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.018	10.147	0.073	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1060	1060	225	0	0	0	0	0	-1
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.540	10.217	0.020	0.000	0.000	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1164	1164	159	0	0	0	0	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.372	10.123	0.085	0.000	0.000	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1162	1162	169	0	0	0	0	0	-1
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.345	10.142	0.078	0.000	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1243	1243	226	0	0	0	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.946	10.209	0.022	0.000	0.000	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0	-1
N.S.	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	10.143	0.075	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0	-1
N.S.	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	10.159	0.073	0.000	0.000	0.000	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	328	0	0	0	0	0	-1
N.S.	1	1.00	5.56	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	10.228	0.072	0.000	0.000	0.000	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0	-1
N.S.	1	1.00	3.65	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	10.216	0.020	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0	-1
N.S.	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	10.209	0.020	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	121	162	243	242	226	143	134
N.S.	1	1.00	0.98	1.32	1.98	1.97	1.84	1.16	1.09
time (sec)	N/A	0.059	0.138	0.065	0.779	2.837	69.776	2.006	5.713

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	97	122	159	191	144	105	93
N.S.	1	1.00	1.08	1.36	1.77	2.12	1.60	1.17	1.03
time (sec)	N/A	0.042	0.103	0.050	0.524	3.396	42.193	1.369	5.305

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	72	127	108	155	107	92	68
N.S.	1	1.00	0.86	1.51	1.29	1.85	1.27	1.10	0.81
time (sec)	N/A	0.039	0.125	0.057	0.557	2.300	54.640	1.788	5.104

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	82	110	67	166	75	163	57
N.S.	1	1.00	1.39	1.86	1.14	2.81	1.27	2.76	0.97
time (sec)	N/A	0.027	0.133	0.061	0.488	2.499	25.259	2.047	5.206

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	48	49	60	58	250	91
N.S.	1	1.00	1.02	1.04	1.07	1.30	1.26	5.43	1.98
time (sec)	N/A	0.025	0.092	0.057	0.394	2.691	4.814	2.349	4.817

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	69	70	84	85	78	310	126
N.S.	1	1.00	0.93	0.95	1.14	1.15	1.05	4.19	1.70
time (sec)	N/A	0.038	0.115	0.069	0.383	2.657	3.971	3.690	4.924

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	93	94	118	109	112	370	168
N.S.	1	1.00	0.89	0.90	1.13	1.05	1.08	3.56	1.62
time (sec)	N/A	0.053	0.141	0.086	0.366	2.473	5.703	4.381	5.224

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	113	118	152	133	146	430	210
N.S.	1	1.00	0.84	0.88	1.13	0.99	1.09	3.21	1.57
time (sec)	N/A	0.066	0.159	0.120	0.367	2.965	5.770	5.037	5.607

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	108	113	158	131	1386	175	117
N.S.	1	1.00	0.72	0.75	1.05	0.87	9.24	1.17	0.78
time (sec)	N/A	0.048	0.062	0.080	0.397	3.400	7.300	1.088	4.591

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	88	89	124	107	910	140	97
N.S.	1	1.00	0.75	0.76	1.06	0.91	7.78	1.20	0.83
time (sec)	N/A	0.039	0.052	0.060	0.365	3.838	4.382	1.453	4.518

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	64	65	90	82	422	105	77
N.S.	1	1.00	0.76	0.77	1.07	0.98	5.02	1.25	0.92
time (sec)	N/A	0.025	0.041	0.054	0.373	2.934	4.050	1.293	4.488

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	43	55	57	119	72	54
N.S.	1	1.00	0.79	0.81	1.04	1.08	2.25	1.36	1.02
time (sec)	N/A	0.016	0.026	0.049	0.465	2.077	3.797	1.639	4.442

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	68	83	75	156	107	116	80
N.S.	1	1.00	1.03	1.26	1.14	2.36	1.62	1.76	1.21
time (sec)	N/A	0.024	0.083	0.047	0.647	3.180	3.767	2.549	4.720

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	75	135	133	164	107	76	97
N.S.	1	1.00	0.88	1.59	1.56	1.93	1.26	0.89	1.14
time (sec)	N/A	0.030	0.129	0.066	0.539	3.045	5.714	1.494	5.108

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	175	193	194	144	130	-1
N.S.	1	1.00	1.00	1.92	2.12	2.13	1.58	1.43	-0.01
time (sec)	N/A	0.033	0.160	0.066	0.535	2.238	8.382	3.058	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	124	220	277	244	226	153	-1
N.S.	1	1.00	1.01	1.79	2.25	1.98	1.84	1.24	-0.01
time (sec)	N/A	0.053	0.187	0.082	0.512	2.452	19.954	1.837	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	123	162	240	243	253	144	130
N.S.	1	1.00	1.00	1.32	1.95	1.98	2.06	1.17	1.06
time (sec)	N/A	0.057	0.152	0.061	0.527	3.042	76.370	1.903	5.783

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	90	174	171	203	216	126	105
N.S.	1	1.00	0.78	1.51	1.49	1.77	1.88	1.10	0.91
time (sec)	N/A	0.054	0.164	0.067	0.543	2.581	70.566	1.559	5.703

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	216	134	195	187	225	95
N.S.	1	1.00	0.98	1.96	1.22	1.77	1.70	2.05	0.86
time (sec)	N/A	0.049	0.172	0.069	0.527	3.264	22.317	1.789	5.651

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	91	153	80	213	73	254	72
N.S.	1	1.00	1.20	2.01	1.05	2.80	0.96	3.34	0.95
time (sec)	N/A	0.034	0.210	0.078	0.494	3.935	23.388	4.860	5.831

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	48	49	84	138	370	122
N.S.	1	1.00	1.07	1.04	1.07	1.83	3.00	8.04	2.65
time (sec)	N/A	0.024	0.147	0.076	0.274	2.696	6.202	5.189	5.333

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	70	84	109	194	430	164
N.S.	1	1.00	0.96	0.95	1.14	1.47	2.62	5.81	2.22
time (sec)	N/A	0.036	0.181	0.092	0.290	3.124	7.226	6.389	5.759

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	94	94	118	134	262	490	206
N.S.	1	1.00	0.90	0.90	1.13	1.29	2.52	4.71	1.98
time (sec)	N/A	0.049	0.212	0.142	0.313	2.420	8.132	6.642	6.315

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	115	118	152	157	326	550	248
N.S.	1	1.00	0.86	0.88	1.13	1.17	2.43	4.10	1.85
time (sec)	N/A	0.060	0.258	0.257	0.291	2.788	8.986	9.164	6.812

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	110	115	158	155	3351	175	137
N.S.	1	1.00	0.73	0.77	1.05	1.03	22.34	1.17	0.91
time (sec)	N/A	0.049	0.069	0.085	0.309	2.667	5.944	1.112	4.662

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	89	91	124	132	2304	140	118
N.S.	1	1.00	0.76	0.78	1.06	1.13	19.69	1.20	1.01
time (sec)	N/A	0.037	0.058	0.079	0.283	1.723	4.655	1.458	4.569

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	66	67	90	106	1340	105	97
N.S.	1	1.00	0.79	0.80	1.07	1.26	15.95	1.25	1.15
time (sec)	N/A	0.026	0.046	0.067	0.286	2.645	3.576	0.850	4.551

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	45	55	80	498	72	77
N.S.	1	1.00	0.83	0.85	1.04	1.51	9.40	1.36	1.45
time (sec)	N/A	0.017	0.031	0.058	0.290	3.181	2.732	1.406	4.626

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	99	91	203	184	140	-1
N.S.	1	1.00	0.94	1.15	1.06	2.36	2.14	1.63	-0.01
time (sec)	N/A	0.034	0.118	0.049	0.509	3.325	2.497	2.030	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	105	170	163	190	202	115	-1
N.S.	1	1.00	0.87	1.40	1.35	1.57	1.67	0.95	-0.01
time (sec)	N/A	0.039	0.133	0.079	0.488	3.408	3.577	1.318	0.000

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	92	213	207	216	216	145	78
N.S.	1	1.00	0.82	1.90	1.85	1.93	1.93	1.29	0.70
time (sec)	N/A	0.040	0.190	0.080	0.505	4.015	5.924	2.284	5.858

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	117	259	275	246	253	173	-1
N.S.	1	1.00	0.95	2.11	2.24	2.00	2.06	1.41	-0.01
time (sec)	N/A	0.043	0.232	0.075	0.520	3.400	11.380	1.082	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	150	302	354	298	287	214	-1
N.S.	1	1.00	0.94	1.90	2.23	1.87	1.81	1.35	-0.01
time (sec)	N/A	0.058	0.234	0.101	0.518	3.042	33.353	1.296	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	97	129	178	192	150	111	99
N.S.	1	1.00	1.08	1.43	1.98	2.13	1.67	1.23	1.10
time (sec)	N/A	0.043	0.107	0.059	0.489	3.250	24.852	1.426	5.348

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	80	90	109	146	66	79	59
N.S.	1	1.00	1.36	1.53	1.85	2.47	1.12	1.34	1.00
time (sec)	N/A	0.027	0.061	0.059	0.508	3.738	29.769	0.860	5.077

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	76	69	54	130	63	66	35
N.S.	1	1.00	1.77	1.60	1.26	3.02	1.47	1.53	0.81
time (sec)	N/A	0.022	0.065	0.049	0.523	2.881	8.983	0.784	4.851

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	47	48	39	138	124	35
N.S.	1	1.00	0.91	1.09	1.12	0.91	3.21	2.88	0.81
time (sec)	N/A	0.023	0.074	0.050	0.275	3.250	3.070	0.831	4.565

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	70	83	62	204	180	58
N.S.	1	1.00	0.83	0.97	1.15	0.86	2.83	2.50	0.81
time (sec)	N/A	0.035	0.094	0.058	0.286	3.907	5.471	1.324	4.676

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	93	94	118	86	269	236	102
N.S.	1	1.00	0.92	0.93	1.17	0.85	2.66	2.34	1.01
time (sec)	N/A	0.047	0.115	0.062	0.288	3.010	7.798	2.076	4.716

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	56	67	85	59	338	99	53
N.S.	1	1.00	0.68	0.82	1.04	0.72	4.12	1.21	0.65
time (sec)	N/A	0.022	0.046	0.047	0.275	2.224	1.587	0.960	5.314

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	34	44	49	36	70	66	67
N.S.	1	1.00	0.67	0.86	0.96	0.71	1.37	1.29	1.31
time (sec)	N/A	0.013	0.029	0.044	0.276	3.526	1.248	1.017	4.917

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	71	73	58	131	39	80	65
N.S.	1	1.00	1.51	1.55	1.23	2.79	0.83	1.70	1.38
time (sec)	N/A	0.019	0.045	0.041	0.494	4.281	1.314	0.971	4.998

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	80	105	121	144	66	64	94
N.S.	1	1.00	1.31	1.72	1.98	2.36	1.08	1.05	1.54
time (sec)	N/A	0.023	0.078	0.054	0.495	3.261	2.252	0.920	5.529

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	101	146	200	201	150	125	-1
N.S.	1	1.00	1.09	1.57	2.15	2.16	1.61	1.34	-0.01
time (sec)	N/A	0.034	0.153	0.063	0.510	3.216	4.478	0.823	0.000

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	112	140	215	304	177	144	134
N.S.	1	1.00	0.95	1.19	1.82	2.58	1.50	1.22	1.14
time (sec)	N/A	0.057	0.158	0.088	0.497	4.577	38.192	0.935	6.339

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	88	114	144	249	264	105	90
N.S.	1	1.00	1.02	1.33	1.67	2.90	3.07	1.22	1.05
time (sec)	N/A	0.041	0.127	0.077	0.520	2.981	18.644	1.064	5.609

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	75	75	69	200	49	69	54
N.S.	1	1.00	1.44	1.44	1.33	3.85	0.94	1.33	1.04
time (sec)	N/A	0.027	0.082	0.050	0.522	3.502	7.568	1.483	5.058

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	46	46	46	68	66	46
N.S.	1	1.00	0.86	1.10	1.10	1.10	1.62	1.57	1.10
time (sec)	N/A	0.023	0.067	0.060	0.275	2.096	0.634	1.778	4.519

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	69	81	73	61	188	66
N.S.	1	1.00	0.88	1.01	1.19	1.07	0.90	2.76	0.97
time (sec)	N/A	0.036	0.099	0.071	0.285	4.171	4.426	2.082	4.642

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	81	94	116	98	90	303	91
N.S.	1	1.00	0.81	0.94	1.16	0.98	0.90	3.03	0.91
time (sec)	N/A	0.046	0.130	0.079	0.279	2.999	5.288	2.980	4.837

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	104	118	151	121	122	414	154
N.S.	1	1.00	0.83	0.94	1.20	0.96	0.97	3.29	1.22
time (sec)	N/A	0.060	0.155	0.090	0.279	3.067	6.445	4.846	4.920

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	80	91	128	95	561	146	79
N.S.	1	1.00	0.72	0.82	1.15	0.86	5.05	1.32	0.71
time (sec)	N/A	0.030	0.057	0.085	0.294	2.989	3.034	1.678	5.768

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	66	90	70	267	106	81
N.S.	1	1.00	0.72	0.84	1.14	0.89	3.38	1.34	1.03
time (sec)	N/A	0.020	0.044	0.072	0.293	4.055	2.947	2.266	5.195

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	33	43	53	47	65	62	38
N.S.	1	1.00	0.73	0.96	1.18	1.04	1.44	1.38	0.84
time (sec)	N/A	0.019	0.031	0.062	0.290	4.095	2.839	1.830	4.898

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	71	79	80	195	206	108	60
N.S.	1	1.00	1.20	1.34	1.36	3.31	3.49	1.83	1.02
time (sec)	N/A	0.021	0.066	0.051	0.495	3.483	4.408	1.729	5.106

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	91	131	162	248	262	107	-1
N.S.	1	1.00	0.99	1.42	1.76	2.70	2.85	1.16	-0.01
time (sec)	N/A	0.032	0.156	0.078	0.500	3.712	7.157	0.996	0.000

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	113	157	243	314	180	148	-1
N.S.	1	1.00	0.92	1.28	1.98	2.55	1.46	1.20	-0.01
time (sec)	N/A	0.045	0.167	0.089	0.504	2.976	12.493	0.753	0.000

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	101	115	0	0	0	0	0	-1
N.S.	1	0.96	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.122	0.043	0.000	0.000	0.000	0.000	0.000

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.101	0.014	0.000	0.000	0.000	0.000	0.000

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.092	0.013	0.000	0.000	0.000	0.000	0.000

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.094	0.016	0.000	0.000	0.000	0.000	0.000

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	100	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.092	0.015	0.000	0.000	0.000	0.000	0.000

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.089	0.004	0.000	0.000	0.000	0.000	0.000

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	95	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.089	0.017	0.000	0.000	0.000	0.000	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	106	0	0	0	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.106	0.016	0.000	0.000	0.000	0.000	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	110	0	0	0	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.063	0.017	0.000	0.000	0.000	0.000	0.000

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.154	0.014	0.000	0.000	0.000	0.000	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.115	0.021	0.000	0.000	0.000	0.000	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.111	0.023	0.000	0.000	0.000	0.000	0.000

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.107	0.020	0.000	0.000	0.000	0.000	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.113	0.023	0.000	0.000	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.116	0.020	0.000	0.000	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.129	0.020	0.000	0.000	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	99	75	57	62	0	162	831
N.S.	1	1.00	0.73	0.56	0.42	0.46	0.00	1.20	6.16
time (sec)	N/A	0.037	1.548	0.352	0.286	1.303	0.000	1.376	52.028

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	87	65	47	57	0	127	632
N.S.	1	1.00	0.84	0.62	0.45	0.55	0.00	1.22	6.08
time (sec)	N/A	0.028	1.145	0.334	0.295	1.274	0.000	2.843	31.390

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	404	52	37	52	0	92	-1
N.S.	1	1.00	5.53	0.71	0.51	0.71	0.00	1.26	-0.01
time (sec)	N/A	0.019	1.387	0.403	0.271	0.982	0.000	1.357	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	264	72	26	46	0	57	41
N.S.	1	1.00	7.14	1.95	0.70	1.24	0.00	1.54	1.11
time (sec)	N/A	0.012	1.135	0.332	0.275	2.629	0.000	2.736	5.069

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	184	47	27	55	0	0	129
N.S.	1	1.00	2.75	0.70	0.40	0.82	0.00	0.00	1.93
time (sec)	N/A	0.019	0.948	0.317	0.504	2.944	0.000	0.000	6.249

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	421	23	10	30	0	48	31
N.S.	1	1.00	13.58	0.74	0.32	0.97	0.00	1.55	1.00
time (sec)	N/A	0.006	1.768	0.352	0.501	1.971	0.000	2.053	5.257

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	693	28	21	37	0	90	43
N.S.	1	1.00	11.00	0.44	0.33	0.59	0.00	1.43	0.68
time (sec)	N/A	0.012	6.722	0.323	0.522	1.351	0.000	1.463	5.075

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	41	33	31	44	0	111	55
N.S.	1	1.00	0.44	0.35	0.33	0.47	0.00	1.18	0.59
time (sec)	N/A	0.019	10.040	0.330	0.541	2.147	0.000	2.252	5.038

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	46	38	41	49	0	132	67
N.S.	1	1.00	0.37	0.30	0.33	0.39	0.00	1.06	0.54
time (sec)	N/A	0.027	10.059	0.321	0.501	1.697	0.000	1.407	5.023

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	65	47	57	0	76	632
N.S.	1	1.00	0.85	0.62	0.45	0.55	0.00	0.73	6.08
time (sec)	N/A	0.029	1.174	0.328	0.292	1.474	0.000	2.049	27.092

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	406	55	37	52	0	59	429
N.S.	1	1.00	5.56	0.75	0.51	0.71	0.00	0.81	5.88
time (sec)	N/A	0.020	1.460	0.323	0.282	1.104	0.000	1.773	18.765

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	265	41	24	46	0	39	-1
N.S.	1	1.00	7.57	1.17	0.69	1.31	0.00	1.11	-0.03
time (sec)	N/A	0.013	1.156	0.414	0.294	0.595	0.000	1.910	0.000

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	38	40	16	27	0	20	6
N.S.	1	1.00	4.75	5.00	2.00	3.38	0.00	2.50	0.75
time (sec)	N/A	0.007	0.824	0.386	0.286	1.986	0.000	1.609	5.287

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	146	20	10	25	0	25	19
N.S.	1	1.00	5.03	0.69	0.34	0.86	0.00	0.86	0.66
time (sec)	N/A	0.006	0.877	0.333	0.527	2.099	0.000	2.737	5.561

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	407	25	21	34	0	48	33
N.S.	1	1.00	6.46	0.40	0.33	0.54	0.00	0.76	0.52
time (sec)	N/A	0.013	1.818	0.329	0.511	1.810	0.000	1.671	5.504

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	693	30	31	39	0	69	43
N.S.	1	1.00	7.37	0.32	0.33	0.41	0.00	0.73	0.46
time (sec)	N/A	0.020	6.933	0.399	0.537	2.465	0.000	1.058	5.658

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.054	0.133	0.000	0.000	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	72	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.038	0.148	0.000	0.000	0.000	0.000	0.000

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.036	0.132	0.000	0.000	0.000	0.000	0.000

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.048	0.134	0.000	0.000	0.000	0.000	0.000

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	78	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.040	0.148	0.000	0.000	0.000	0.000	0.000

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	36	15	11	10	16	11
N.S.	1	1.00	1.00	2.40	1.00	0.73	0.67	1.07	0.73
time (sec)	N/A	0.017	0.004	0.319	0.298	2.632	0.039	0.760	0.087

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	0	35	34	36	38	31
N.S.	1	1.00	0.82	0.00	1.59	1.55	1.64	1.73	1.41
time (sec)	N/A	0.018	0.156	0.093	0.347	1.692	2.257	0.941	4.942

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	88	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.093	0.114	0.000	0.000	0.000	0.000	0.000

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.077	0.125	0.000	0.000	0.000	0.000	0.000

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.070	0.123	0.000	0.000	0.000	0.000	0.000

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.061	0.002	0.000	0.000	0.000	0.000	0.000

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	64	69	58	332	0	162
N.S.	1	1.00	0.92	1.02	1.10	0.92	5.27	0.00	2.57
time (sec)	N/A	0.044	0.059	0.388	0.306	1.030	1.377	0.000	5.724

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	74	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.078	0.132	0.000	0.000	0.000	0.000	0.000

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	77	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.080	0.130	0.000	0.000	0.000	0.000	0.000

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	141	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.177	0.112	0.000	0.000	0.000	0.000	0.000

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	135	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.150	0.037	0.000	0.000	0.000	0.000	0.000

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	134	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.143	0.030	0.000	0.000	0.000	0.000	0.000

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	108	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.138	0.003	0.000	0.000	0.000	0.000	0.000

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	107	100	151	224	0	0	-1
N.S.	1	1.00	1.06	0.99	1.50	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.145	0.400	0.312	0.501	0.000	0.000	0.000

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	133	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.161	0.035	0.000	0.000	0.000	0.000	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	136	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.155	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	134	188	231	177	320	0	-1
N.S.	1	1.00	1.03	1.45	1.78	1.36	2.46	0.00	-0.01
time (sec)	N/A	0.093	0.126	0.383	0.307	3.410	70.042	0.000	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	118	150	108	202	0	-1
N.S.	1	1.00	0.97	1.31	1.67	1.20	2.24	0.00	-0.01
time (sec)	N/A	0.061	0.083	0.369	0.309	6.772	29.969	0.000	0.000

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	50	65	83	56	105	0	-1
N.S.	1	1.00	0.83	1.08	1.38	0.93	1.75	0.00	-0.02
time (sec)	N/A	0.035	0.050	0.359	0.298	5.146	12.466	0.000	0.000

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	44	59	60	45	0	0	-1
N.S.	1	1.00	0.81	1.09	1.11	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.050	0.399	0.321	2.754	0.000	0.000	0.000

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	107	121	120	0	0	-1
N.S.	1	1.00	1.00	1.43	1.61	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.087	0.398	0.307	3.526	0.000	0.000	0.000

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	100	157	243	267	0	0	-1
N.S.	1	1.00	0.95	1.50	2.31	2.54	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.123	0.408	0.314	2.232	0.000	0.000	0.000

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	185	342	286	230	401	0	-1
N.S.	1	1.00	1.17	2.16	1.81	1.46	2.54	0.00	-0.01
time (sec)	N/A	0.103	0.153	0.368	0.311	2.658	55.057	0.000	0.000

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	125	157	192	146	258	0	-1
N.S.	1	1.00	1.06	1.33	1.63	1.24	2.19	0.00	-0.01
time (sec)	N/A	0.074	0.109	0.368	0.327	2.607	27.035	0.000	0.000

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	76	91	112	82	139	0	-1
N.S.	1	1.00	0.88	1.06	1.30	0.95	1.62	0.00	-0.01
time (sec)	N/A	0.050	0.067	0.342	0.365	2.788	12.601	0.000	0.000

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	78	81	74	0	0	-1
N.S.	1	1.00	0.93	1.10	1.14	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.071	0.405	0.343	3.286	0.000	0.000	0.000

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	93	125	147	166	0	0	-1
N.S.	1	1.00	0.98	1.32	1.55	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.103	0.402	0.293	2.754	0.000	0.000	0.000

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	94	169	262	301	0	0	-1
N.S.	1	1.00	0.78	1.41	2.18	2.51	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.194	0.408	0.302	2.405	0.000	0.000	0.000

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	172	155	154	154	175	13	154
N.S.	1	1.00	12.29	11.07	11.00	11.00	12.50	0.93	11.00
time (sec)	N/A	0.001	0.004	0.284	0.331	1.396	0.031	0.754	0.156

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	156	156	182	156	156
N.S.	1	1.00	11.38	9.81	9.75	9.75	11.38	9.75	9.75
time (sec)	N/A	0.028	0.004	0.338	0.336	1.117	0.032	0.589	4.925

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	156	156	185	156	156
N.S.	1	1.00	11.62	9.81	9.75	9.75	11.56	9.75	9.75
time (sec)	N/A	0.025	0.004	0.332	0.307	0.940	0.032	0.604	4.772

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	189	0	0	229
N.S.	1	1.00	1.00	10.95	10.90	9.00	0.00	0.00	10.90
time (sec)	N/A	0.009	0.033	0.334	0.291	1.252	0.000	0.000	5.209

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	0	16	32	0	70	25
N.S.	1	1.00	1.00	0.00	1.23	2.46	0.00	5.38	1.92
time (sec)	N/A	0.009	0.143	0.085	0.415	1.384	0.000	0.569	4.810

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	9	9	9	9	10	8	11	8
N.S.	1	1.12	1.12	1.12	1.12	1.25	1.00	1.38	1.00
time (sec)	N/A	0.004	0.004	0.308	0.307	2.984	0.040	0.577	4.662

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	12	18	13
N.S.	1	1.00	1.00	0.93	1.13	0.87	0.80	1.20	0.87
time (sec)	N/A	0.010	0.005	0.447	0.305	2.257	0.069	0.557	0.060

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	12	15	13
N.S.	1	1.00	1.00	0.93	1.13	0.87	0.80	1.00	0.87
time (sec)	N/A	0.010	0.005	0.316	0.314	3.138	0.078	0.573	4.628

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	19	17	47	17	29	0	15
N.S.	1	1.00	1.27	1.13	3.13	1.13	1.93	0.00	1.00
time (sec)	N/A	0.011	0.025	0.330	0.427	2.570	0.277	0.000	4.723

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	177	81	81	87	13	12
N.S.	1	1.00	1.00	12.64	5.79	5.79	6.21	0.93	0.86
time (sec)	N/A	0.001	0.014	0.315	0.318	1.747	0.427	0.622	7.087

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.008	0.020	0.311	0.314	0.956	0.655	0.691	2.315

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.008	0.024	0.323	0.333	0.893	0.900	0.621	9.981

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	105	0	0	105
N.S.	1	1.00	1.00	9.67	29.14	5.00	0.00	0.00	5.00
time (sec)	N/A	0.009	0.070	0.377	0.377	1.221	0.000	0.000	4.987

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	85	53	46	76	56	37
N.S.	1	1.00	0.88	1.63	1.02	0.88	1.46	1.08	0.71
time (sec)	N/A	0.018	0.056	0.690	0.560	2.458	112.707	1.151	4.817

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	348	143	0	757	0	0	-1
N.S.	1	1.00	3.74	1.54	0.00	8.14	0.00	0.00	-0.01
time (sec)	N/A	0.060	1.544	0.078	0.000	1.320	0.000	0.000	0.000

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	223	0	0	607	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.156	1.459	0.120	0.000	1.460	0.000	0.000	0.000

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	178	0	0	469	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.635	0.111	0.000	0.974	0.000	0.000	0.000

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	141	0	0	359	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.379	0.118	0.000	1.246	0.000	0.000	0.000

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	123	0	0	281	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	3.16	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.233	0.115	0.000	1.035	0.000	0.000	0.000

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	105	0	0	408	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	4.48	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.335	0.111	0.000	1.088	0.000	0.000	0.000

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	57	0	0	135	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.645	0.112	0.000	0.883	0.000	0.000	0.000

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	274	0	0	771	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	2.15	0.00	0.00	-0.00
time (sec)	N/A	0.253	1.948	0.115	0.000	1.160	0.000	0.000	0.000

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	241	0	0	607	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	2.09	0.00	0.00	-0.00
time (sec)	N/A	0.200	0.757	0.139	0.000	3.022	0.000	0.000	0.000

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	191	0	0	471	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	2.13	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.422	0.126	0.000	5.298	0.000	0.000	0.000

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	157	0	0	361	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	2.41	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.303	0.131	0.000	7.323	0.000	0.000	0.000

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	185	0	0	540	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	4.06	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.503	0.113	0.000	2.979	0.000	0.000	0.000

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	136	0	0	769	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	5.23	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.816	0.112	0.000	4.435	0.000	0.000	0.000

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	29	25	46	35	22
N.S.	1	1.00	1.00	1.05	1.45	1.25	2.30	1.75	1.10
time (sec)	N/A	0.004	0.032	0.322	0.325	2.097	1.008	1.123	4.795

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	35	32	0	52	47
N.S.	1	1.00	0.96	0.96	1.30	1.19	0.00	1.93	1.74
time (sec)	N/A	0.006	0.065	0.340	0.350	1.655	0.000	0.683	4.882

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	35	32	0	56	47
N.S.	1	1.00	0.96	0.96	1.30	1.19	0.00	2.07	1.74
time (sec)	N/A	0.006	0.068	0.315	0.337	1.305	0.000	0.677	4.902

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	0	39	35	0	66	54
N.S.	1	1.00	0.96	0.00	1.44	1.30	0.00	2.44	2.00
time (sec)	N/A	0.008	0.108	0.078	0.360	2.864	0.000	0.652	4.863

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [345] had the largest ratio of [37]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	18	0.111
2	A	2	1	1.00	16	0.062
3	A	2	1	1.00	15	0.067
4	A	3	2	1.00	18	0.111
5	A	2	1	1.00	18	0.056
6	A	2	1	1.00	18	0.056
7	A	3	2	1.00	18	0.111
8	A	2	1	1.00	18	0.056
9	A	2	1	1.00	18	0.056
10	A	3	2	1.00	18	0.111
11	A	3	2	1.00	20	0.100
12	A	2	1	1.00	18	0.056
13	A	2	1	1.00	17	0.059
14	A	4	3	1.00	20	0.150
15	A	2	1	1.00	20	0.050
16	A	2	1	1.00	20	0.050
17	A	3	2	1.00	20	0.100
18	A	2	1	1.00	20	0.050
19	A	2	1	1.00	20	0.050
20	A	3	2	1.00	20	0.100
21	A	2	1	1.00	20	0.050
22	A	2	1	1.00	20	0.050
23	A	2	1	1.00	20	0.050
24	A	3	2	1.00	20	0.100
25	A	2	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	1	1.00	20	0.050
27	A	3	2	1.00	20	0.100
28	A	2	1	1.00	20	0.050
29	A	2	1	1.00	20	0.050
30	A	3	2	1.00	20	0.100
31	A	2	1	1.00	18	0.056
32	A	2	1	1.00	17	0.059
33	A	4	3	1.00	20	0.150
34	A	2	1	1.00	20	0.050
35	A	2	1	1.00	20	0.050
36	A	3	2	1.00	20	0.100
37	A	2	1	1.00	20	0.050
38	A	2	1	1.00	20	0.050
39	A	3	2	1.00	20	0.100
40	A	2	1	1.00	20	0.050
41	A	2	1	1.00	20	0.050
42	A	3	2	1.00	20	0.100
43	A	2	1	1.00	20	0.050
44	A	2	1	1.00	20	0.050
45	A	3	2	1.00	20	0.100
46	A	2	1	1.00	20	0.050
47	A	2	1	1.00	20	0.050
48	A	3	2	1.00	20	0.100
49	A	2	1	1.00	20	0.050
50	A	2	1	1.00	20	0.050
51	A	4	3	1.00	20	0.150
52	A	2	1	1.00	20	0.050
53	A	2	1	1.00	20	0.050
54	A	3	3	1.00	20	0.150
55	A	2	1	1.00	20	0.050
56	A	9	8	1.00	20	0.400
57	A	3	2	1.00	20	0.100
58	A	8	8	1.00	20	0.400
59	A	8	8	1.00	20	0.400
60	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	7	1.00	18	0.389
62	A	7	7	1.00	17	0.412
63	A	3	2	1.00	20	0.100
64	A	7	7	1.00	20	0.350
65	A	7	7	1.00	20	0.350
66	A	3	2	1.00	20	0.100
67	A	8	8	1.00	20	0.400
68	A	8	8	1.00	20	0.400
69	A	3	2	1.00	20	0.100
70	A	9	8	1.00	20	0.400
71	A	9	8	1.00	20	0.400
72	A	3	2	1.00	20	0.100
73	A	9	8	1.00	20	0.400
74	A	9	8	1.00	20	0.400
75	A	3	2	1.00	20	0.100
76	A	8	8	1.00	20	0.400
77	A	8	8	1.00	20	0.400
78	A	3	2	1.00	20	0.100
79	A	7	7	1.00	18	0.389
80	A	7	7	1.00	17	0.412
81	A	3	2	1.00	20	0.100
82	A	8	8	1.01	20	0.400
83	A	8	8	1.00	20	0.400
84	A	3	2	1.00	20	0.100
85	A	9	8	1.00	20	0.400
86	A	9	8	1.00	20	0.400
87	A	3	2	1.00	20	0.100
88	A	3	2	1.00	20	0.100
89	A	3	2	1.00	20	0.100
90	A	3	2	1.00	20	0.100
91	A	2	2	1.00	20	0.100
92	A	3	2	1.00	20	0.100
93	A	3	2	1.00	20	0.100
94	A	3	2	1.00	20	0.100
95	A	10	9	1.00	20	0.450

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	10	9	1.00	20	0.450
97	A	9	9	1.00	20	0.450
98	A	9	9	1.00	20	0.450
99	A	8	8	1.00	20	0.400
100	A	8	8	1.00	20	0.400
101	A	8	8	1.00	18	0.444
102	A	8	8	1.00	17	0.471
103	A	9	9	1.00	20	0.450
104	A	9	9	1.00	20	0.450
105	A	10	9	1.00	20	0.450
106	A	10	9	1.00	20	0.450
107	A	3	2	1.00	22	0.091
108	A	15	8	1.00	22	0.364
109	A	14	8	1.00	22	0.364
110	A	3	2	1.00	22	0.091
111	A	13	7	1.00	22	0.318
112	A	13	7	1.00	22	0.318
113	A	4	3	1.00	22	0.136
114	A	13	7	1.00	20	0.350
115	A	13	7	1.00	19	0.368
116	A	3	2	1.00	22	0.091
117	A	15	8	1.00	22	0.364
118	A	14	8	1.00	22	0.364
119	A	3	2	1.00	22	0.091
120	A	16	9	1.00	22	0.409
121	A	15	9	1.00	22	0.409
122	A	3	2	1.00	22	0.091
123	A	17	9	1.00	22	0.409
124	A	2	1	1.00	20	0.050
125	A	2	1	1.00	20	0.050
126	A	2	1	1.00	18	0.056
127	A	2	2	1.00	20	0.100
128	A	2	2	1.00	20	0.100
129	A	2	2	1.00	20	0.100
130	A	3	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	1	1.00	20	0.050
132	A	2	1	1.00	20	0.050
133	A	2	1	1.00	20	0.050
134	A	2	1	1.00	20	0.050
135	A	2	1	1.00	20	0.050
136	A	2	1	1.00	20	0.050
137	A	2	1	1.00	20	0.050
138	A	2	1	1.00	20	0.050
139	A	2	1	1.00	22	0.045
140	A	2	1	1.00	22	0.045
141	A	2	1	1.00	22	0.045
142	A	2	1	1.00	22	0.045
143	A	2	1	1.00	22	0.045
144	A	2	1	1.00	22	0.045
145	A	2	1	1.00	22	0.045
146	A	2	1	1.00	22	0.045
147	A	2	1	1.00	22	0.045
148	A	2	1	1.00	22	0.045
149	A	2	1	1.00	22	0.045
150	A	2	1	1.00	22	0.045
151	A	2	1	1.00	22	0.045
152	A	2	1	1.00	22	0.045
153	A	2	1	1.00	22	0.045
154	A	2	1	1.00	22	0.045
155	A	5	5	1.00	22	0.227
156	A	13	9	1.00	22	0.409
157	A	12	8	1.00	22	0.364
158	A	4	4	1.00	22	0.182
159	A	12	8	1.00	22	0.364
160	A	12	8	1.00	22	0.364
161	A	4	4	1.00	22	0.182
162	A	12	8	1.00	22	0.364
163	A	5	5	1.00	22	0.227
164	A	13	9	1.00	22	0.409
165	A	12	8	1.00	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	4	1.00	22	0.182
167	A	12	8	1.00	22	0.364
168	A	13	9	1.00	22	0.409
169	A	5	5	1.01	22	0.227
170	A	13	9	1.00	22	0.409
171	A	5	5	1.00	22	0.227
172	A	13	9	1.00	22	0.409
173	A	13	9	1.00	22	0.409
174	A	5	5	1.00	22	0.227
175	A	13	9	1.00	22	0.409
176	A	14	10	1.00	22	0.454
177	A	6	6	1.01	22	0.273
178	A	14	10	1.00	22	0.454
179	A	3	2	1.00	22	0.091
180	A	3	2	1.00	22	0.091
181	A	3	2	1.00	22	0.091
182	A	5	5	1.00	22	0.227
183	A	5	5	1.00	22	0.227
184	A	5	5	1.00	22	0.227
185	A	4	4	1.00	22	0.182
186	A	3	3	1.00	19	0.158
187	A	3	3	1.00	22	0.136
188	A	3	3	1.00	22	0.136
189	A	4	4	1.00	22	0.182
190	A	6	6	1.00	22	0.273
191	A	5	5	1.00	20	0.250
192	A	5	5	1.00	22	0.227
193	A	5	5	1.00	22	0.227
194	A	6	6	1.00	22	0.273
195	A	7	6	1.00	22	0.273
196	A	3	2	1.00	22	0.091
197	A	3	2	1.00	22	0.091
198	A	3	2	1.00	22	0.091
199	A	6	5	1.00	22	0.227
200	A	6	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	6	6	1.00	22	0.273
202	A	5	4	1.00	22	0.182
203	A	4	3	1.00	19	0.158
204	A	4	3	1.00	22	0.136
205	A	4	4	1.00	22	0.182
206	A	4	3	1.00	22	0.136
207	A	7	6	1.00	22	0.273
208	A	6	5	1.00	20	0.250
209	A	6	5	1.00	22	0.227
210	A	6	6	1.00	22	0.273
211	A	6	5	1.00	22	0.227
212	A	7	6	1.00	22	0.273
213	A	3	2	1.00	22	0.091
214	A	3	2	1.00	22	0.091
215	A	3	2	1.00	22	0.091
216	A	4	4	1.00	22	0.182
217	A	4	4	1.00	22	0.182
218	A	5	5	1.00	22	0.227
219	A	3	3	1.00	22	0.136
220	A	2	2	1.00	19	0.105
221	A	2	2	1.00	22	0.091
222	A	3	3	1.00	22	0.136
223	A	5	5	1.00	22	0.227
224	A	4	4	1.00	20	0.200
225	A	4	4	1.00	22	0.182
226	A	5	5	1.00	22	0.227
227	A	6	5	1.00	22	0.227
228	A	3	2	1.00	22	0.091
229	A	3	2	1.00	22	0.091
230	A	3	2	1.00	22	0.091
231	A	4	4	1.00	22	0.182
232	A	5	5	1.00	22	0.227
233	A	6	6	1.00	22	0.273
234	A	4	4	1.00	22	0.182
235	A	3	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	2	2	1.00	19	0.105
237	A	3	3	1.00	22	0.136
238	A	4	4	1.00	22	0.182
239	A	5	5	1.00	22	0.227
240	A	4	4	1.00	20	0.200
241	A	5	5	1.00	22	0.227
242	A	6	6	1.00	22	0.273
243	A	7	6	1.00	22	0.273
244	A	3	2	1.00	22	0.091
245	A	3	2	1.00	22	0.091
246	A	3	2	1.00	22	0.091
247	A	5	5	1.00	22	0.227
248	A	6	5	1.00	22	0.227
249	A	4	3	1.00	22	0.136
250	A	3	3	1.00	22	0.136
251	A	3	3	1.00	19	0.158
252	A	4	3	1.00	22	0.136
253	A	5	4	1.00	22	0.182
254	A	6	5	1.00	22	0.227
255	A	5	5	1.00	22	0.227
256	A	5	5	1.00	20	0.250
257	A	6	5	1.00	22	0.227
258	A	7	6	1.00	22	0.273
259	A	6	5	1.00	26	0.192
260	A	5	5	1.00	26	0.192
261	A	4	4	1.00	26	0.154
262	A	6	5	1.00	26	0.192
263	A	7	6	1.00	26	0.231
264	A	7	6	1.00	26	0.231
265	A	5	5	1.00	24	0.208
266	A	7	6	1.00	26	0.231
267	A	2	2	1.00	26	0.077
268	A	2	2	1.00	23	0.087
269	A	2	2	1.00	26	0.077
270	A	5	4	1.00	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	4	4	1.00	26	0.154
272	A	3	3	1.00	26	0.115
273	A	6	5	1.00	26	0.192
274	A	7	6	1.00	26	0.231
275	A	5	5	1.00	26	0.192
276	A	1	1	1.00	24	0.042
277	A	7	6	1.00	26	0.231
278	A	2	2	1.00	26	0.077
279	A	2	2	1.00	23	0.087
280	A	2	2	1.00	26	0.077
281	A	1	1	1.00	22	0.045
282	A	6	5	1.00	27	0.185
283	A	6	5	1.00	27	0.185
284	A	5	5	1.00	27	0.185
285	A	4	4	1.00	27	0.148
286	A	6	5	1.00	27	0.185
287	A	7	6	1.00	27	0.222
288	A	8	7	1.00	27	0.259
289	A	15	13	1.00	27	0.482
290	A	14	12	1.00	27	0.444
291	A	12	11	1.00	25	0.440
292	A	14	12	1.00	27	0.444
293	A	15	13	1.00	27	0.482
294	A	16	13	1.00	27	0.482
295	A	7	5	1.00	27	0.185
296	A	7	5	1.00	27	0.185
297	A	6	5	1.00	27	0.185
298	A	5	4	1.00	27	0.148
299	A	7	6	1.00	27	0.222
300	A	7	6	1.00	27	0.222
301	A	8	7	1.00	27	0.259
302	A	16	13	1.00	27	0.482
303	A	15	13	1.00	27	0.482
304	A	14	12	1.00	25	0.480
305	A	14	12	1.00	27	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	15	13	1.00	27	0.482
307	A	16	13	1.00	27	0.482
308	A	5	4	1.00	27	0.148
309	A	5	4	1.00	27	0.148
310	A	4	4	1.00	27	0.148
311	A	3	3	1.00	27	0.111
312	A	6	5	1.00	27	0.185
313	A	7	6	1.00	27	0.222
314	A	8	7	1.00	27	0.259
315	A	14	12	1.00	27	0.444
316	A	12	11	1.00	27	0.407
317	A	8	7	1.00	25	0.280
318	A	14	12	1.00	27	0.444
319	A	15	13	1.00	27	0.482
320	A	16	13	1.00	27	0.482
321	A	2	2	1.00	27	0.074
322	A	2	2	1.00	24	0.083
323	A	2	2	1.00	27	0.074
324	A	2	2	1.00	27	0.074
325	A	7	5	1.00	27	0.185
326	A	5	4	1.00	27	0.148
327	A	4	4	1.00	27	0.148
328	A	4	4	1.00	27	0.148
329	A	7	6	1.00	27	0.222
330	A	8	7	1.00	27	0.259
331	A	9	8	1.00	27	0.296
332	A	14	12	1.00	27	0.444
333	A	14	12	1.00	27	0.444
334	A	14	12	1.00	25	0.480
335	A	15	13	1.00	27	0.482
336	A	16	13	1.00	27	0.482
337	A	17	13	1.00	27	0.482
338	A	2	2	1.00	27	0.074
339	A	2	2	1.00	24	0.083
340	A	2	2	1.00	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	2	2	1.00	27	0.074
342	A	5	5	1.00	33	0.152
343	A	5	5	1.00	35	0.143
344	A	5	5	1.00	35	0.143
345	A	5	5	1.00	37	0.135
346	A	5	5	1.00	33	0.152
347	A	5	5	1.00	35	0.143
348	A	5	5	1.00	36	0.139
349	A	5	5	1.00	36	0.139
350	A	1	1	1.00	33	0.030
351	A	1	1	1.00	35	0.029
352	A	1	1	1.00	35	0.029
353	A	1	1	1.00	37	0.027
354	A	1	1	1.00	33	0.030
355	A	1	1	1.00	35	0.029
356	A	1	1	1.00	36	0.028
357	A	1	1	1.00	36	0.028
358	A	6	5	1.00	24	0.208
359	A	5	5	1.00	24	0.208
360	A	4	4	1.00	24	0.167
361	A	6	4	1.00	24	0.167
362	A	7	5	1.00	24	0.208
363	A	2	2	1.00	24	0.083
364	A	2	2	1.00	22	0.091
365	A	2	2	1.00	21	0.095
366	A	2	2	1.00	24	0.083
367	A	2	2	1.00	24	0.083
368	A	7	5	1.00	24	0.208
369	A	6	5	1.00	24	0.208
370	A	5	4	1.00	24	0.167
371	A	7	5	1.00	24	0.208
372	A	7	5	1.00	24	0.208
373	A	2	2	1.00	24	0.083
374	A	2	2	1.00	22	0.091
375	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	2	2	1.00	24	0.083
377	A	2	2	1.00	24	0.083
378	A	5	4	1.00	24	0.167
379	A	4	4	1.00	24	0.167
380	A	3	3	1.00	24	0.125
381	A	6	4	1.00	24	0.167
382	A	7	5	1.00	24	0.208
383	A	2	2	1.00	24	0.083
384	A	2	2	1.00	22	0.091
385	A	2	2	1.00	21	0.095
386	A	2	2	1.00	24	0.083
387	A	2	2	1.00	24	0.083
388	A	5	4	1.00	24	0.167
389	A	4	4	1.00	24	0.167
390	A	4	4	1.00	24	0.167
391	A	7	5	1.00	24	0.208
392	A	8	6	1.00	24	0.250
393	A	2	2	1.00	24	0.083
394	A	2	2	1.00	22	0.091
395	A	2	2	1.00	21	0.095
396	A	2	2	1.00	24	0.083
397	A	2	2	1.00	24	0.083
398	A	6	6	1.00	27	0.222
399	A	6	6	1.00	27	0.222
400	A	5	5	1.00	27	0.185
401	A	4	4	1.00	27	0.148
402	A	7	6	1.00	27	0.222
403	A	8	7	1.00	27	0.259
404	A	9	7	1.00	27	0.259
405	A	15	13	1.00	27	0.482
406	A	14	12	1.00	27	0.444
407	A	14	12	1.00	25	0.480
408	A	15	13	1.00	27	0.482
409	A	16	13	1.00	27	0.482
410	A	17	13	1.00	27	0.482

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	7	7	1.00	27	0.259
412	A	7	6	1.00	27	0.222
413	A	6	5	1.00	27	0.185
414	A	5	5	1.00	27	0.185
415	A	7	6	1.00	27	0.222
416	A	8	7	1.00	27	0.259
417	A	9	7	1.00	27	0.259
418	A	16	14	1.00	27	0.518
419	A	15	13	1.00	27	0.482
420	A	14	12	1.00	25	0.480
421	A	6	6	1.00	27	0.222
422	A	16	13	1.00	27	0.482
423	A	17	13	1.00	27	0.482
424	A	5	5	1.00	27	0.185
425	A	5	5	1.00	27	0.185
426	A	4	4	1.00	27	0.148
427	A	4	4	1.00	27	0.148
428	A	7	6	1.00	27	0.222
429	A	8	7	1.00	27	0.259
430	A	9	7	1.00	27	0.259
431	A	14	12	1.00	27	0.444
432	A	14	12	1.00	27	0.444
433	A	14	12	1.00	25	0.480
434	A	15	13	1.00	27	0.482
435	A	16	13	1.00	27	0.482
436	A	17	13	1.00	27	0.482
437	A	2	2	1.00	27	0.074
438	A	2	2	1.00	27	0.074
439	A	2	2	1.00	24	0.083
440	A	2	2	1.00	27	0.074
441	A	2	2	1.00	27	0.074
442	A	5	5	1.00	27	0.185
443	A	5	5	1.00	27	0.185
444	A	5	5	1.00	27	0.185
445	A	5	5	1.00	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	8	7	1.00	27	0.259
447	A	9	8	1.00	27	0.296
448	A	10	8	1.00	27	0.296
449	A	15	13	1.00	27	0.482
450	A	15	13	1.00	27	0.482
451	A	15	13	1.00	25	0.520
452	A	16	14	1.00	27	0.518
453	A	17	14	1.00	27	0.518
454	A	18	14	1.00	27	0.518
455	C	2	2	0.26	27	0.074
456	A	2	2	1.00	27	0.074
457	A	2	2	1.00	24	0.083
458	A	2	2	1.00	27	0.074
459	A	2	2	1.00	27	0.074
460	A	6	6	1.00	24	0.250
461	A	5	5	1.00	24	0.208
462	A	4	4	1.00	24	0.167
463	A	7	5	1.00	24	0.208
464	A	8	6	1.00	24	0.250
465	A	2	2	1.00	24	0.083
466	A	2	2	1.00	22	0.091
467	A	2	2	1.00	21	0.095
468	A	2	2	1.00	24	0.083
469	A	2	2	1.00	24	0.083
470	A	7	6	1.00	24	0.250
471	A	6	5	1.00	24	0.208
472	A	5	5	1.00	24	0.208
473	A	7	5	1.00	24	0.208
474	A	8	6	1.00	24	0.250
475	A	2	2	1.00	24	0.083
476	A	2	2	1.00	22	0.091
477	A	2	2	1.00	21	0.095
478	A	2	2	1.00	24	0.083
479	A	2	2	1.00	24	0.083
480	A	5	5	1.00	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	4	4	1.00	24	0.167
482	A	4	4	1.00	24	0.167
483	A	7	5	1.00	24	0.208
484	A	8	6	1.00	24	0.250
485	A	2	2	1.00	24	0.083
486	A	2	2	1.00	22	0.091
487	A	2	2	1.00	21	0.095
488	A	2	2	1.00	24	0.083
489	A	2	2	1.00	24	0.083
490	A	5	5	0.99	24	0.208
491	A	5	5	1.00	24	0.208
492	A	5	5	1.00	24	0.208
493	A	8	6	1.00	24	0.250
494	A	9	7	1.00	24	0.292
495	A	2	2	1.00	24	0.083
496	A	2	2	1.00	22	0.091
497	A	2	2	1.00	21	0.095
498	A	2	2	1.00	24	0.083
499	A	2	2	1.00	24	0.083
500	A	3	3	1.00	24	0.125
501	A	3	3	1.00	24	0.125
502	A	3	3	1.00	24	0.125
503	A	3	3	1.00	24	0.125
504	A	3	3	1.00	24	0.125
505	A	3	3	1.00	24	0.125
506	A	5	5	1.00	26	0.192
507	A	4	4	1.00	26	0.154
508	A	3	3	1.00	26	0.115
509	A	4	4	1.00	26	0.154
510	A	3	2	1.00	26	0.077
511	A	3	2	1.00	26	0.077
512	A	3	2	1.00	24	0.083
513	A	3	2	1.00	23	0.087
514	A	3	2	1.00	26	0.077
515	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	7	7	1.00	26	0.269
517	A	5	5	1.00	26	0.192
518	A	6	6	1.00	26	0.231
519	A	6	6	1.00	26	0.231
520	A	4	4	1.00	26	0.154
521	A	6	6	1.00	26	0.231
522	A	6	6	1.00	26	0.231
523	A	4	4	1.00	26	0.154
524	A	6	6	1.00	24	0.250
525	A	6	6	1.00	24	0.250
526	A	4	4	1.00	24	0.167
527	A	8	7	1.00	26	0.269
528	A	6	5	1.00	26	0.192
529	A	7	6	1.00	26	0.231
530	A	7	6	1.00	26	0.231
531	A	5	4	1.00	26	0.154
532	A	7	6	1.00	26	0.231
533	A	7	6	1.00	26	0.231
534	A	5	4	1.00	26	0.154
535	A	9	7	1.00	26	0.269
536	A	7	5	1.00	26	0.192
537	A	8	6	1.00	26	0.231
538	A	8	6	1.00	26	0.231
539	A	6	4	1.00	26	0.154
540	A	8	6	1.00	26	0.231
541	A	8	6	1.00	26	0.231
542	A	6	4	1.00	26	0.154
543	A	6	6	1.00	26	0.231
544	A	4	4	1.00	26	0.154
545	A	5	5	1.00	26	0.192
546	A	5	5	1.00	26	0.192
547	A	3	3	1.00	26	0.115
548	A	5	5	1.00	26	0.192
549	A	5	5	1.00	26	0.192
550	A	3	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	6	6	1.00	26	0.231
552	A	4	4	1.00	26	0.154
553	A	5	5	1.00	26	0.192
554	A	5	5	1.00	26	0.192
555	A	3	3	1.00	26	0.115
556	A	6	6	1.00	26	0.231
557	A	2	2	1.00	26	0.077
558	A	4	4	1.00	26	0.154
559	A	6	6	1.00	26	0.231
560	A	4	4	1.00	26	0.154
561	A	6	6	1.00	26	0.231
562	A	2	2	1.00	26	0.077
563	A	4	4	1.00	26	0.154
564	A	7	6	1.00	26	0.231
565	A	3	3	1.00	26	0.115
566	A	5	4	1.00	26	0.154
567	A	8	7	1.00	28	0.250
568	A	8	7	1.00	28	0.250
569	A	7	7	1.00	28	0.250
570	A	6	6	1.00	28	0.214
571	A	10	6	1.00	28	0.214
572	A	13	8	1.00	28	0.286
573	A	12	8	1.00	28	0.286
574	A	6	5	1.00	28	0.179
575	A	5	4	1.00	28	0.143
576	A	3	3	1.00	26	0.115
577	A	3	3	1.00	28	0.107
578	A	4	4	1.00	28	0.143
579	A	5	4	1.00	28	0.143
580	A	6	4	1.00	28	0.143
581	A	22	14	1.00	28	0.500
582	A	21	13	1.00	28	0.464
583	A	14	8	1.00	25	0.320
584	A	21	13	1.00	28	0.464
585	A	22	14	1.00	28	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
586	A	8	7	1.00	28	0.250
587	A	8	7	1.00	28	0.250
588	A	7	7	1.00	28	0.250
589	A	6	6	1.00	28	0.214
590	A	10	6	1.00	28	0.214
591	A	13	8	1.00	28	0.286
592	A	12	8	1.00	28	0.286
593	A	5	5	1.00	28	0.179
594	A	4	4	1.00	28	0.143
595	A	3	3	1.00	25	0.120
596	A	3	3	1.00	28	0.107
597	A	4	4	1.00	28	0.143
598	A	5	4	1.00	28	0.143
599	A	6	4	1.00	28	0.143
600	A	14	13	1.00	28	0.464
601	A	13	12	1.00	28	0.429
602	A	11	11	1.00	26	0.423
603	A	13	12	1.00	28	0.429
604	A	14	13	1.00	28	0.464
605	A	7	6	1.00	22	0.273
606	A	7	6	1.00	22	0.273
607	A	7	6	1.00	22	0.273
608	A	6	6	1.00	22	0.273
609	A	5	5	1.00	22	0.227
610	A	10	7	1.00	22	0.318
611	A	11	8	1.00	22	0.364
612	A	4	4	1.00	22	0.182
613	A	3	3	1.00	22	0.136
614	A	1	1	1.00	19	0.053
615	A	3	3	1.00	22	0.136
616	A	4	4	1.00	22	0.182
617	A	5	4	1.00	22	0.182
618	A	12	12	1.00	22	0.546
619	A	10	10	1.00	22	0.454
620	A	8	8	1.00	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	12	11	1.00	22	0.500
622	A	14	13	1.00	22	0.591
623	A	7	6	1.00	22	0.273
624	A	7	6	1.00	22	0.273
625	A	6	6	1.00	22	0.273
626	A	5	5	1.00	22	0.227
627	A	10	7	1.00	22	0.318
628	A	11	8	1.00	22	0.364
629	A	5	4	1.00	22	0.182
630	A	3	3	1.00	22	0.136
631	A	1	1	1.00	20	0.050
632	A	2	2	1.00	22	0.091
633	A	4	4	1.00	22	0.182
634	A	15	9	1.00	22	0.409
635	A	18	11	1.00	22	0.500
636	A	16	10	1.00	19	0.526
637	A	16	10	1.00	22	0.454
638	A	11	9	1.00	22	0.409
639	A	11	9	1.00	22	0.409
640	A	9	8	1.00	22	0.364
641	A	6	6	1.00	22	0.273
642	A	6	6	1.00	22	0.273
643	A	11	8	1.00	22	0.364
644	A	13	9	1.00	22	0.409
645	A	5	5	1.00	22	0.227
646	A	4	4	1.00	22	0.182
647	A	2	2	1.00	22	0.091
648	A	2	2	1.00	19	0.105
649	A	4	4	1.00	22	0.182
650	A	5	4	1.00	22	0.182
651	A	6	4	1.00	22	0.182
652	A	13	12	1.00	22	0.546
653	A	12	11	1.00	22	0.500
654	A	12	11	1.00	22	0.500
655	A	11	11	1.00	20	0.550

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	13	12	1.00	22	0.546
657	A	14	12	1.00	22	0.546
658	A	8	7	1.00	24	0.292
659	A	8	7	1.00	24	0.292
660	A	7	7	1.00	24	0.292
661	A	6	6	1.00	24	0.250
662	A	10	7	1.00	24	0.292
663	A	13	9	1.00	24	0.375
664	A	12	9	1.00	24	0.375
665	A	6	5	1.00	24	0.208
666	A	5	4	1.00	24	0.167
667	A	3	3	1.00	22	0.136
668	A	3	3	1.00	24	0.125
669	A	4	4	1.00	24	0.167
670	A	5	4	1.00	24	0.167
671	A	6	4	1.00	24	0.167
672	A	2	2	1.00	24	0.083
673	A	2	2	1.00	24	0.083
674	A	2	2	1.00	21	0.095
675	A	2	2	1.00	24	0.083
676	A	2	2	1.00	24	0.083
677	A	8	7	1.00	24	0.292
678	A	8	7	1.00	24	0.292
679	A	7	7	1.00	24	0.292
680	A	6	6	1.00	24	0.250
681	A	10	7	1.00	24	0.292
682	A	13	9	1.00	24	0.375
683	A	12	9	1.00	24	0.375
684	A	5	5	1.00	24	0.208
685	A	4	4	1.00	24	0.167
686	A	3	3	1.00	21	0.143
687	A	3	3	1.00	24	0.125
688	A	4	4	1.00	24	0.167
689	A	5	4	1.00	24	0.167
690	A	6	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	2	2	1.00	24	0.083
692	A	2	2	1.00	24	0.083
693	A	2	2	1.00	22	0.091
694	A	2	2	1.00	24	0.083
695	A	2	2	1.00	24	0.083
696	A	9	7	1.00	24	0.292
697	A	8	7	1.00	24	0.292
698	A	7	6	1.00	24	0.250
699	A	11	8	1.00	24	0.333
700	A	15	9	1.00	24	0.375
701	A	14	10	1.00	24	0.417
702	A	6	5	1.00	24	0.208
703	A	5	4	1.00	22	0.182
704	A	5	4	1.00	24	0.167
705	A	4	4	1.00	24	0.167
706	A	5	4	1.00	24	0.167
707	A	6	4	1.00	24	0.167
708	A	7	4	1.00	24	0.167
709	A	2	2	1.00	24	0.083
710	A	2	2	1.00	24	0.083
711	A	2	2	1.00	21	0.095
712	A	2	2	1.00	24	0.083
713	A	2	2	1.00	24	0.083
714	A	7	6	1.00	24	0.250
715	A	7	6	1.00	24	0.250
716	A	7	6	1.00	24	0.250
717	A	6	6	1.00	24	0.250
718	A	5	5	1.00	24	0.208
719	A	10	7	1.00	24	0.292
720	A	11	8	1.00	24	0.333
721	A	4	4	1.00	24	0.167
722	A	3	3	1.00	24	0.125
723	A	1	1	1.00	21	0.048
724	A	3	3	1.00	24	0.125
725	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	5	4	1.00	24	0.167
727	A	2	2	1.00	24	0.083
728	A	2	2	1.00	24	0.083
729	A	2	2	1.00	22	0.091
730	A	2	2	1.00	24	0.083
731	A	2	2	1.00	24	0.083
732	A	7	6	1.00	24	0.250
733	A	7	6	1.00	24	0.250
734	A	6	6	1.00	24	0.250
735	A	5	5	1.00	24	0.208
736	A	10	7	1.00	24	0.292
737	A	11	8	1.00	24	0.333
738	A	5	4	1.00	24	0.167
739	A	3	3	1.00	24	0.125
740	A	1	1	1.00	22	0.045
741	A	3	3	1.00	24	0.125
742	A	4	4	1.00	24	0.167
743	A	2	2	1.00	24	0.083
744	A	2	2	1.00	24	0.083
745	A	2	2	1.00	21	0.095
746	A	2	2	1.00	24	0.083
747	A	11	7	1.00	24	0.292
748	A	9	7	1.00	24	0.292
749	A	7	6	1.00	24	0.250
750	A	6	6	1.00	24	0.250
751	A	6	6	1.00	24	0.250
752	A	11	8	1.00	24	0.333
753	A	13	9	1.00	24	0.375
754	A	5	5	1.00	24	0.208
755	A	4	4	1.00	24	0.167
756	A	3	3	1.00	24	0.125
757	A	2	2	1.00	21	0.095
758	A	4	4	1.00	24	0.167
759	A	5	4	1.00	24	0.167
760	A	6	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	2	2	1.00	24	0.083
762	A	2	2	1.00	24	0.083
763	A	2	2	1.00	24	0.083
764	A	2	2	1.00	22	0.091
765	A	2	2	1.00	24	0.083
766	A	2	2	1.00	24	0.083
767	A	3	2	1.00	22	0.091
768	A	3	2	1.00	22	0.091
769	A	3	2	1.00	22	0.091
770	A	4	3	1.00	22	0.136
771	A	3	2	1.00	22	0.091
772	A	3	2	1.00	22	0.091
773	A	6	5	1.00	22	0.227
774	A	5	4	1.00	22	0.182
775	A	4	3	1.00	22	0.136
776	A	4	3	1.00	20	0.150
777	A	5	4	1.00	22	0.182
778	A	6	5	1.00	22	0.227
779	A	20	8	1.00	22	0.364
780	A	19	7	1.00	22	0.318
781	A	19	7	1.00	22	0.318
782	A	19	7	1.00	22	0.318
783	A	19	7	1.00	19	0.368
784	A	21	8	1.00	22	0.364
785	A	20	8	1.00	22	0.364
786	A	22	9	1.00	22	0.409
787	A	5	5	1.00	24	0.208
788	A	7	7	1.00	24	0.292
789	A	4	4	1.00	24	0.167
790	A	6	6	1.00	22	0.273
791	A	6	4	1.00	24	0.167
792	A	5	5	1.00	24	0.208
793	A	7	5	1.00	24	0.208
794	A	6	6	1.00	24	0.250
795	A	13	8	1.25	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	10	6	1.29	24	0.250
797	A	11	7	1.29	24	0.292
798	A	9	5	1.30	21	0.238
799	A	13	8	1.27	24	0.333
800	A	10	6	1.27	24	0.250
801	A	3	3	1.00	28	0.107
802	A	3	3	1.00	28	0.107
803	A	3	3	1.00	28	0.107
804	A	3	3	1.00	28	0.107
805	A	5	4	1.00	24	0.167
806	A	4	4	1.00	24	0.167
807	A	3	3	1.00	24	0.125
808	A	6	4	1.00	24	0.167
809	A	7	5	1.00	24	0.208
810	A	7	7	1.00	24	0.292
811	A	6	6	1.00	24	0.250
812	A	3	3	1.00	22	0.136
813	A	5	5	1.00	24	0.208
814	A	6	6	1.00	24	0.250
815	A	10	6	1.00	24	0.250
816	A	9	5	1.31	24	0.208
817	A	7	4	1.16	21	0.190
818	A	10	6	1.28	24	0.250
819	A	11	7	1.22	24	0.292
820	A	7	4	1.15	24	0.167
821	A	13	8	1.21	24	0.333
822	A	5	5	1.00	24	0.208
823	A	5	5	1.00	24	0.208
824	A	4	4	1.00	24	0.167
825	A	4	4	1.00	24	0.167
826	A	7	5	1.00	24	0.208
827	A	8	6	1.00	24	0.250
828	A	8	8	1.00	24	0.333
829	A	7	7	1.00	24	0.292
830	A	5	5	1.00	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
831	A	4	4	1.00	22	0.182
832	A	6	6	1.00	24	0.250
833	A	7	6	1.00	24	0.250
834	A	10	6	1.00	24	0.250
835	A	10	6	1.00	24	0.250
836	A	10	6	1.00	21	0.286
837	A	11	7	1.00	24	0.292
838	A	13	8	1.00	24	0.333
839	A	13	8	1.00	24	0.333
840	A	14	9	1.00	24	0.375
841	A	4	4	0.97	26	0.154
842	A	3	3	0.93	24	0.125
843	A	2	2	1.00	17	0.118
844	A	2	2	1.00	26	0.077
845	A	2	2	1.00	26	0.077
846	A	2	2	1.00	26	0.077
847	A	4	4	1.00	26	0.154
848	A	3	3	1.00	24	0.125
849	A	2	2	1.00	17	0.118
850	A	2	2	1.00	26	0.077
851	A	2	2	1.00	26	0.077
852	A	2	2	1.00	26	0.077
853	A	5	4	1.00	24	0.167
854	A	4	4	1.00	24	0.167
855	A	3	3	1.00	24	0.125
856	A	6	4	1.00	24	0.167
857	A	7	5	1.00	24	0.208
858	A	7	7	1.00	24	0.292
859	A	6	6	1.00	24	0.250
860	A	3	3	1.00	24	0.125
861	A	5	5	1.00	24	0.208
862	A	6	6	1.00	24	0.250
863	A	2	2	1.00	24	0.083
864	A	3	3	1.00	24	0.125
865	A	3	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	2	2	1.00	21	0.095
867	A	2	2	1.00	24	0.083
868	A	3	3	1.00	24	0.125
869	A	3	3	1.00	24	0.125
870	A	5	5	1.00	24	0.208
871	A	4	4	1.00	24	0.167
872	A	4	4	1.00	24	0.167
873	A	7	5	1.00	24	0.208
874	A	8	6	1.00	24	0.250
875	A	7	7	1.00	24	0.292
876	A	5	5	1.00	24	0.208
877	A	4	4	1.00	24	0.167
878	A	6	6	1.00	24	0.250
879	A	7	6	1.00	24	0.250
880	A	2	2	1.00	24	0.083
881	A	3	3	1.00	24	0.125
882	A	3	3	1.00	22	0.136
883	A	2	2	1.00	21	0.095
884	A	2	2	1.00	24	0.083
885	A	3	3	1.00	24	0.125
886	A	3	3	1.00	24	0.125
887	A	5	4	1.00	24	0.167
888	A	4	4	1.00	24	0.167
889	A	3	3	1.00	24	0.125
890	A	6	4	1.00	24	0.167
891	A	7	5	1.00	24	0.208
892	A	7	7	1.00	24	0.292
893	A	6	6	1.00	24	0.250
894	A	3	3	1.00	24	0.125
895	A	5	5	1.00	24	0.208
896	A	6	6	1.00	24	0.250
897	A	10	6	1.00	24	0.250
898	A	8	5	1.00	22	0.227
899	A	11	7	1.00	24	0.292
900	A	12	8	1.00	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
901	A	8	5	1.00	24	0.208
902	A	14	9	1.00	24	0.375
903	A	2	2	1.00	24	0.083
904	A	2	2	1.00	24	0.083
905	A	2	2	1.00	21	0.095
906	A	2	2	1.00	24	0.083
907	A	2	2	1.00	24	0.083
908	A	5	5	1.00	24	0.208
909	A	4	4	1.00	24	0.167
910	A	4	4	1.00	24	0.167
911	A	7	5	1.00	24	0.208
912	A	8	6	1.00	24	0.250
913	A	7	7	1.00	24	0.292
914	A	5	5	1.00	24	0.208
915	A	4	4	1.00	24	0.167
916	A	6	6	1.00	24	0.250
917	A	7	6	1.00	24	0.250
918	A	11	7	1.00	24	0.292
919	A	11	7	1.00	22	0.318
920	A	12	8	1.00	24	0.333
921	A	14	9	1.00	24	0.375
922	A	14	9	1.00	24	0.375
923	A	15	10	1.00	24	0.417
924	A	2	2	1.00	24	0.083
925	A	2	2	1.00	24	0.083
926	A	2	2	1.00	21	0.095
927	A	2	2	1.00	24	0.083
928	A	2	2	1.00	24	0.083
929	A	6	6	1.00	22	0.273
930	A	5	5	1.00	22	0.227
931	A	5	5	1.00	20	0.250
932	A	5	5	1.00	22	0.227
933	A	3	2	1.00	22	0.091
934	A	3	2	1.00	22	0.091
935	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	3	2	1.00	22	0.091
937	A	5	3	1.00	22	0.136
938	A	4	3	1.00	22	0.136
939	A	3	3	1.00	22	0.136
940	A	2	2	1.00	22	0.091
941	A	5	5	1.00	22	0.227
942	A	5	5	1.00	19	0.263
943	A	5	5	1.00	22	0.227
944	A	6	6	1.00	22	0.273
945	A	6	5	1.00	22	0.227
946	A	6	6	1.00	22	0.273
947	A	6	5	1.00	20	0.250
948	A	6	5	1.00	22	0.227
949	A	3	2	1.00	22	0.091
950	A	3	2	1.00	22	0.091
951	A	3	2	1.00	22	0.091
952	A	3	2	1.00	22	0.091
953	A	5	3	1.00	22	0.136
954	A	4	3	1.00	22	0.136
955	A	3	3	1.00	22	0.136
956	A	2	2	1.00	22	0.091
957	A	6	5	1.00	22	0.227
958	A	6	6	1.00	22	0.273
959	A	6	5	1.00	19	0.263
960	A	6	5	1.00	22	0.227
961	A	7	6	1.00	22	0.273
962	A	5	5	1.00	22	0.227
963	A	4	4	1.00	20	0.200
964	A	4	4	1.00	22	0.182
965	A	3	2	1.00	22	0.091
966	A	3	2	1.00	22	0.091
967	A	3	2	1.00	22	0.091
968	A	3	3	1.00	22	0.136
969	A	2	2	1.00	22	0.091
970	A	4	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
971	A	4	4	1.00	22	0.182
972	A	5	5	1.00	22	0.227
973	A	6	6	1.00	22	0.273
974	A	5	5	1.00	20	0.250
975	A	4	4	1.00	22	0.182
976	A	3	2	1.00	22	0.091
977	A	3	2	1.00	22	0.091
978	A	3	2	1.00	22	0.091
979	A	3	2	1.00	22	0.091
980	A	4	4	1.00	22	0.182
981	A	3	3	1.00	22	0.136
982	A	3	3	1.00	19	0.158
983	A	4	4	1.00	22	0.182
984	A	5	5	1.00	22	0.227
985	A	6	6	1.00	22	0.273
986	A	4	3	0.96	24	0.125
987	A	4	3	1.00	22	0.136
988	A	3	3	1.00	22	0.136
989	A	4	3	1.00	22	0.136
990	A	3	3	1.00	20	0.150
991	A	4	3	1.00	19	0.158
992	A	3	3	1.00	22	0.136
993	A	4	3	1.00	22	0.136
994	A	3	3	1.00	22	0.136
995	A	4	3	1.00	22	0.136
996	A	4	3	1.00	26	0.115
997	A	4	3	1.00	26	0.115
998	A	4	3	1.00	26	0.115
999	A	4	3	1.00	26	0.115
1000	A	4	3	1.00	26	0.115
1001	A	4	3	1.00	26	0.115
1002	A	6	4	1.00	28	0.143
1003	A	5	4	1.00	28	0.143
1004	A	4	4	1.00	28	0.143
1005	A	3	3	1.00	28	0.107

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1006	A	4	4	1.00	28	0.143
1007	A	1	1	1.00	28	0.036
1008	A	2	2	1.00	28	0.071
1009	A	3	2	1.00	28	0.071
1010	A	4	2	1.00	28	0.071
1011	A	5	3	1.00	28	0.107
1012	A	4	3	1.00	28	0.107
1013	A	3	3	1.00	28	0.107
1014	A	2	2	1.00	28	0.071
1015	A	1	1	1.00	28	0.036
1016	A	2	2	1.00	28	0.071
1017	A	3	2	1.00	28	0.071
1018	A	3	3	1.00	24	0.125
1019	A	3	3	1.00	22	0.136
1020	A	3	3	1.00	21	0.143
1021	A	4	4	1.00	24	0.167
1022	A	3	3	1.00	24	0.125
1023	A	3	2	1.00	18	0.111
1024	A	1	1	1.00	33	0.030
1025	A	3	2	1.00	24	0.083
1026	A	3	2	1.00	22	0.091
1027	A	3	2	1.00	20	0.100
1028	A	3	2	1.00	19	0.105
1029	A	3	2	1.00	22	0.091
1030	A	3	2	1.00	22	0.091
1031	A	3	2	1.00	22	0.091
1032	A	5	3	1.00	24	0.125
1033	A	5	3	1.00	22	0.136
1034	A	5	3	1.00	20	0.150
1035	A	4	3	1.00	19	0.158
1036	A	3	2	1.00	22	0.091
1037	A	5	3	1.00	22	0.136
1038	A	5	3	1.00	22	0.136
1039	A	3	2	1.00	26	0.077
1040	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1041	A	3	2	1.00	24	0.083
1042	A	3	2	1.00	26	0.077
1043	A	3	2	1.00	26	0.077
1044	A	3	2	1.00	26	0.077
1045	A	3	2	1.00	26	0.077
1046	A	3	2	1.00	26	0.077
1047	A	3	2	1.00	24	0.083
1048	A	3	2	1.00	26	0.077
1049	A	3	2	1.00	26	0.077
1050	A	3	2	1.00	26	0.077
1051	A	1	1	1.00	17	0.059
1052	A	2	2	1.00	21	0.095
1053	A	2	2	1.00	21	0.095
1054	A	2	2	1.00	25	0.080
1055	A	1	1	1.00	31	0.032
1056	A	2	1	1.12	17	0.059
1057	A	3	2	1.00	21	0.095
1058	A	3	2	1.00	21	0.095
1059	A	3	2	1.00	21	0.095
1060	A	1	1	1.00	17	0.059
1061	A	2	2	1.00	21	0.095
1062	A	2	2	1.00	21	0.095
1063	A	2	2	1.00	25	0.080
1064	A	5	5	1.00	22	0.227
1065	A	8	8	1.00	26	0.308
1066	A	8	6	1.00	30	0.200
1067	A	7	6	1.00	30	0.200
1068	A	6	6	1.00	30	0.200
1069	A	5	5	1.00	30	0.167
1070	A	5	5	1.00	30	0.167
1071	A	3	3	1.00	30	0.100
1072	A	9	7	1.00	30	0.233
1073	A	8	7	1.00	30	0.233
1074	A	7	7	1.00	30	0.233
1075	A	6	6	1.00	30	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1076	A	6	6	1.00	30	0.200
1077	A	6	6	1.00	30	0.200
1078	A	1	1	1.00	17	0.059
1079	A	1	1	1.00	27	0.037
1080	A	1	1	1.00	27	0.037
1081	A	1	1	1.00	27	0.037

Chapter 3

Listing of integrals

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3.43 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx$ . . . 414
3.44 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx$ . . . 417
3.45 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}}\,dx$ . . . 420
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3.49 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx$ . . . 433
3.50 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{18}}\,dx$ . . . 436
3.51 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx$ . . . 439
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3.56 $\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx$ . . . 456

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3.60 $\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx$ 474
3.61 $\int \frac{x(A+Bx^3)}{a+bx^3}\,dx$ 477
3.62 $\int \frac{A+Bx^3}{a+bx^3}\,dx$ 482
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3.64 $\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx$ 490
3.65 $\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx$ 495
3.66 $\int \frac{A+Bx^3}{x^4(a+bx^3)}\,dx$ 500
3.67 $\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$ 503
3.68 $\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx$ 508
3.69 $\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx$ 513
3.70 $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$ 516
3.71 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$ 521
3.72 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx$ 527
3.73 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$ 531
3.74 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$ 536
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3.76 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$ 545
3.77 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$ 551
3.78 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx$ 556
3.79 $\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$ 559
3.80 $\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx$ 564
3.81 $\int \frac{A+Bx^3}{x(a+bx^3)^2}\,dx$ 569
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3.87 $\int \frac{A+Bx^3}{x^7(a+bx^3)^2}\,dx$ 599
3.88 $\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx$ 603
3.89 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx$ 607

3.90 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx$ 611

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3.94 $\int \frac{A+Bx^3}{x^7(a+bx^3)^3}\,dx$ 626

3.95 $\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$ 630

3.96 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$ 636

3.97 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$ 642

3.98 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx$ 648

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3.102 $\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx$ 672

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3.104 $\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$ 684

3.105 $\int \frac{A+Bx^3}{x^5(a+bx^3)^3}\,dx$ 690

3.106 $\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$ 696

3.107 $\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx$ 702

3.108 $\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$ 705

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3.111 $\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$ 721

3.112 $\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$ 727

3.113 $\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx$ 733

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3.116 $\int \frac{1}{x(a+bx^3)(c+dx^3)}\,dx$ 749

3.117 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx$ 752

3.118 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$ 758

3.119 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx$ 764

3.120 $\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx$ 767

3.121 $\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx$ 774

3.122 $\int \frac{1}{x^7(a+bx^3)(c+dx^3)}\,dx$ 781

3.123 $\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx$ 784

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3.129 $\int \frac{x^m (A+Bx^3)}{(a+bx^3)^3}\, dx$ . . . 812
3.130 $\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\, dx$ . . . 815
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3.158 $\int \frac{\sqrt{x} (A+Bx^3)}{a+bx^3}\, dx$ . . . 911

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3.160 $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx$ . . . 923
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3.170 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx$ . . . 992
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3.179 $\int x^8\sqrt{a+bx^3}\,(A+Bx^3)\,dx$ . . . 1057
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3.184 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^7}\,dx$ . . . 1078
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3.186 $\int \sqrt{a+bx^3}\,(A+Bx^3)\,dx$ . . . 1088
3.187 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^3}\,dx$ . . . 1092
3.188 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^6}\,dx$ . . . 1097
3.189 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^9}\,dx$ . . . 1102

3.190 $\int x^4\sqrt{a+bx^3}\,(A+Bx^3)\,dx$ . . . 1107
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3.194 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^8}\,dx$ . . . 1131
3.195 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{11}}\,dx$ . . . 1137
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3.197 $\int x^5(a+bx^3)^{3/2}\,(A+Bx^3)\,dx$ . . . 1148
3.198 $\int x^2(a+bx^3)^{3/2}\,(A+Bx^3)\,dx$ . . . 1152
3.199 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx$ . . . 1156
3.200 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx$ . . . 1160
3.201 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx$ . . . 1165
3.202 $\int x^3(a+bx^3)^{3/2}\,(A+Bx^3)\,dx$ . . . 1170
3.203 $\int (a+bx^3)^{3/2}\,(A+Bx^3)\,dx$ . . . 1175
3.204 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx$ . . . 1179
3.205 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx$ . . . 1183
3.206 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx$ . . . 1188
3.207 $\int x^4(a+bx^3)^{3/2}\,(A+Bx^3)\,dx$ . . . 1192
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3.209 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx$ . . . 1205
3.210 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx$ . . . 1211
3.211 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx$ . . . 1217
3.212 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx$ . . . 1223
3.213 $\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$ . . . 1230
3.214 $\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$ . . . 1234
3.215 $\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$ . . . 1238
3.216 $\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx$ . . . 1241
3.217 $\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx$ . . . 1245
3.218 $\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx$ . . . 1249
3.219 $\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$ . . . 1254
3.220 $\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx$ . . . 1258

3.221 $\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx$ 1263

3.222 $\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx$ 1268

3.223 $\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$ 1273

3.224 $\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$ 1279

3.225 $\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$ 1285

3.226 $\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx$ 1291

3.227 $\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$ 1297

3.228 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$ 1303

3.229 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$ 1307

3.230 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$ 1311

3.231 $\int \frac{A+Bx^3}{x(a+bx^3)^{3/2}}\,dx$ 1314

3.232 $\int \frac{A+Bx^3}{x^4(a+bx^3)^{3/2}}\,dx$ 1318

3.233 $\int \frac{A+Bx^3}{x^7(a+bx^3)^{3/2}}\,dx$ 1323

3.234 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$ 1328

3.235 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$ 1334

3.236 $\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx$ 1338

3.237 $\int \frac{A+Bx^3}{x^3(a+bx^3)^{3/2}}\,dx$ 1343

3.238 $\int \frac{A+Bx^3}{x^6(a+bx^3)^{3/2}}\,dx$ 1348

3.239 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$ 1354

3.240 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$ 1360

3.241 $\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx$ 1366

3.242 $\int \frac{A+Bx^3}{x^5(a+bx^3)^{3/2}}\,dx$ 1372

3.243 $\int \frac{A+Bx^3}{x^8(a+bx^3)^{3/2}}\,dx$ 1378

3.244 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$ 1384

3.245 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$ 1388

3.246 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$ 1392

3.247 $\int \frac{A+Bx^3}{x(a+bx^3)^{5/2}}\,dx$ 1395

3.248 $\int \frac{A+Bx^3}{x^4(a+bx^3)^{5/2}}\,dx$ 1400

3.249 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$ 1406

3.250 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$ 1411

3.251 $\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx$ 1416

3.252 $\int \frac{A+Bx^3}{x^3(a+bx^3)^{5/2}}\,dx$ 1421

3.253 $\int \frac{A+Bx^3}{x^6(a+bx^3)^{5/2}}\,dx$ 1426

3.254 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$ 1432

3.255 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$ 1438

3.256 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$ 1444

3.257 $\int \frac{A+Bx^3}{x^2(a+bx^3)^{5/2}}\,dx$ 1450

3.258 $\int \frac{A+Bx^3}{x^5(a+bx^3)^{5/2}}\,dx$ 1456

3.259 $\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx$ 1462

3.260 $\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx$ 1466

3.261 $\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx$ 1470

3.262 $\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx$ 1474

3.263 $\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx$ 1479

3.264 $\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx$ 1485

3.265 $\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$ 1492

3.266 $\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx$ 1499

3.267 $\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$ 1507

3.268 $\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$ 1512

3.269 $\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx$ 1517

3.270 $\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$ 1522

3.271 $\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$ 1526

3.272 $\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$ 1530

3.273 $\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$ 1534

3.274 $\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$ 1539

3.275 $\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$ 1545

3.276 $\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$ 1552

3.277 $\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$ 1558

3.278 $\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$ 1565

3.279 $\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$ 1570

3.280 $\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$ 1576

3.281 $\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx$ 1581

3.282 $\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$ 1586

3.283 $\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx$ 1591

3.284 $\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx$ 1595

3.285 $\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx$ 1599

3.286 $\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)}\,dx$ 1603

3.287 $\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx$ 1608

3.288 $\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx$ 1614

3.289 $\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx$ 1621

3.290 $\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx$ 1631

3.291 $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$ 1640

3.292 $\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx$ 1648

3.293 $\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)}\,dx$ 1657

3.294 $\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx$ 1666

3.295 $\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$ 1676

3.296 $\int \frac{x^8\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$ 1681

3.297 $\int \frac{x^5\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$ 1686

3.298 $\int \frac{x^2\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$ 1690

3.299 $\int \frac{\left(c+dx^3\right)^{3/2}}{x(8c-dx^3)}\,dx$ 1694

3.300 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^4(8c-dx^3)}\,dx$ 1699

3.301 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^7(8c-dx^3)}\,dx$ 1705

3.302 $\int \frac{x^7\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$ 1712

3.303 $\int \frac{x^4\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$ 1722

3.304 $\int \frac{x\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$ 1731

3.305 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2(8c-dx^3)}\,dx$ 1740

3.306 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^5(8c-dx^3)}\,dx$ 1748

3.307 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^8(8c-dx^3)}\,dx$ 1757

3.308 $\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1767

3.309 $\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1771
3.310 $\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1775
3.311 $\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1779
3.312 $\int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1783
3.313 $\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1788
3.314 $\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1794
3.315 $\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1800
3.316 $\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1809
3.317 $\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1817
3.318 $\int \frac{1}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1824
3.319 $\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1833
3.320 $\int \frac{1}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1842
3.321 $\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1851
3.322 $\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1856
3.323 $\int \frac{1}{x^3(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1862
3.324 $\int \frac{1}{x^6(8c-dx^3)\sqrt{c+dx^3}}\,dx$ 1867
3.325 $\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1872
3.326 $\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1876
3.327 $\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1880
3.328 $\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1886
3.329 $\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1891
3.330 $\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1896
3.331 $\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1903
3.332 $\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1910
3.333 $\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1919
3.334 $\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1928
3.335 $\int \frac{1}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1937
3.336 $\int \frac{1}{x^5(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1946
3.337 $\int \frac{1}{x^8(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1955
3.338 $\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1965
3.339 $\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1971

3.340 $\int \frac{1}{x^3(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1976

3.341 $\int \frac{1}{x^6(8c-dx^3)(c+dx^3)^{3/2}}\,dx$ 1982

3.342 $\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$ 1988

3.343 $\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$ 1993

3.344 $\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$ 1998

3.345 $\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$ 2003

3.346 $\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$ 2009

3.347 $\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$ 2014

3.348 $\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$ 2019

3.349 $\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$ 2024

3.350 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$ 2029

3.351 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$ 2034

3.352 $\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$ 2040

3.353 $\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$ 2045

3.354 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$ 2049

3.355 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$ 2053

3.356 $\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$ 2058

3.357 $\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$ 2064

3.358 $\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$ 2069

3.359 $\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx$ 2074

3.360 $\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx$ 2079

3.361 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx$ 2083

3.362 $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx$ 2088

3.363 $\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$ . . . 2094

3.364 $\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx$ . . . 2098

3.365 $\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$ . . . 2102

3.366 $\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx$ . . . 2106

3.367 $\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx$ . . . 2110

3.368 $\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx$ . . . 2114

3.369 $\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx$ . . . 2119

3.370 $\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx$ . . . 2124

3.371 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx$ . . . 2128

3.372 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx$ . . . 2133

3.373 $\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx$ . . . 2139

3.374 $\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx$ . . . 2143

3.375 $\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx$ . . . 2147

3.376 $\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx$ . . . 2151

3.377 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx$ . . . 2155

3.378 $\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$ . . . 2159

3.379 $\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$ . . . 2163

3.380 $\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx$ . . . 2167

3.381 $\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx$ . . . 2172

3.382 $\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}}\,dx$ . . . 2177

3.383 $\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$ . . . 2182

3.384 $\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$ . . . 2186

3.385 $\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$ . . . 2190

3.386 $\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx$ . . . 2195

3.387 $\int \frac{1}{x^3(a+bx^3)\sqrt{c+dx^3}}\,dx$ . . . 2199

3.388 $\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$ . . . 2203

3.389 $\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$ . . . 2207

3.390 $\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$ . . . 2213

3.391 $\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx$ . . . 2218

3.392 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}}\,dx$ . . . 2224

3.393 $\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$ . . . 2232

3.394 $\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$ . . . 2237

3.395 $\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$ . . . 2241

3.396 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}}\,dx$ . . . 2245

3.397 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx$ . . . 2249

3.398 $\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$ . . . 2254

3.399 $\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$ . . . 2260

3.400 $\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$ . . . 2266

3.401 $\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$ . . . 2271

3.402 $\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2}\,dx$ . . . 2276

3.403 $\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)^2}\,dx$ . . . 2281

3.404 $\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)^2}\,dx$ . . . 2286

3.405 $\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$ . . . 2292

3.406 $\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$ . . . 2302

3.407 $\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$ . . . 2311

3.408 $\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)^2}\,dx$ . . . 2320

3.409 $\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)^2}\,dx$ . . . 2330

3.410 $\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)^2}\,dx$ . . . 2340

3.411 $\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$ . . . 2350

3.412 $\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$ . . . 2356

3.413 $\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$ . . . 2362

3.414 $\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$ . . . 2367

3.415 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx$ . . . 2372

3.416 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx$ . . . 2377

3.417 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx$ . . . 2383

3.418 $\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$ . . . 2389

3.419 $\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$ . . . 2399

3.420 $\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$ . . . 2409

3.421 $\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx$ 2418

3.422 $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx$ 2425

3.423 $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx$ 2435

3.424 $\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2445

3.425 $\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2450

3.426 $\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2455

3.427 $\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2460

3.428 $\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2465

3.429 $\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2470

3.430 $\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2476

3.431 $\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2482

3.432 $\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2491

3.433 $\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2500

3.434 $\int \frac{1}{x^2(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2509

3.435 $\int \frac{1}{x^5(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2518

3.436 $\int \frac{1}{x^8(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2528

3.437 $\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2538

3.438 $\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2544

3.439 $\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2550

3.440 $\int \frac{1}{x^3(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2555

3.441 $\int \frac{1}{x^6(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$ 2561

3.442 $\int \frac{x^{11}}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ 2567

3.443 $\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ 2572

3.444 $\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ 2577

3.445 $\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ 2582

3.446 $\int \frac{1}{x(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ 2587

3.447 $\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ 2593

3.448 $\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ 2599

3.449 $\int \frac{x^7}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ 2605

3.450 $\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ 2615

3.451 $\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ . . . 2625
3.452 $\int \frac{1}{x^2(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ . . . 2634
3.453 $\int \frac{1}{x^5(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ . . . 2644
3.454 $\int \frac{1}{x^8(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ . . . 2654
3.455 $\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ . . . 2664
3.456 $\int \frac{x^3}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ . . . 2669
3.457 $\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ . . . 2675
3.458 $\int \frac{1}{x^3(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ . . . 2680
3.459 $\int \frac{1}{x^6(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$ . . . 2686
3.460 $\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$ . . . 2693
3.461 $\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$ . . . 2699
3.462 $\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$ . . . 2704
3.463 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx$ . . . 2709
3.464 $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx$ . . . 2714
3.465 $\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$ . . . 2720
3.466 $\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$ . . . 2725
3.467 $\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$ . . . 2729
3.468 $\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)^2}\,dx$ . . . 2733
3.469 $\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)^2}\,dx$ . . . 2738
3.470 $\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$ . . . 2743
3.471 $\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$ . . . 2749
3.472 $\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$ . . . 2755
3.473 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx$ . . . 2760
3.474 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx$ . . . 2765
3.475 $\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$ . . . 2771
3.476 $\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$ . . . 2776
3.477 $\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$ . . . 2780
3.478 $\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx$ . . . 2784
3.479 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx$ . . . 2789

3.480 $\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$ 2794
3.481 $\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$ 2799
3.482 $\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$ 2804
3.483 $\int \frac{1}{x(a+bx^3)^2\sqrt{c+dx^3}}\,dx$ 2809
3.484 $\int \frac{1}{x^4(a+bx^3)^2\sqrt{c+dx^3}}\,dx$ 2814
3.485 $\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$ 2820
3.486 $\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$ 2825
3.487 $\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$ 2829
3.488 $\int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx$ 2833
3.489 $\int \frac{1}{x^3(a+bx^3)^2\sqrt{c+dx^3}}\,dx$ 2837
3.490 $\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$ 2842
3.491 $\int \frac{x^5}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$ 2848
3.492 $\int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$ 2853
3.493 $\int \frac{1}{x(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$ 2858
3.494 $\int \frac{1}{x^4(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$ 2864
3.495 $\int \frac{x^3}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$ 2872
3.496 $\int \frac{x}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$ 2877
3.497 $\int \frac{1}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$ 2881
3.498 $\int \frac{1}{x^2(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$ 2885
3.499 $\int \frac{1}{x^3(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$ 2890
3.500 $\int (ex)^m\,(a+bx^3)^{5/2}\,(A+Bx^3)\,dx$ 2895
3.501 $\int (ex)^m\,(a+bx^3)^{3/2}\,(A+Bx^3)\,dx$ 2899
3.502 $\int (ex)^m\sqrt{a+bx^3}\,(A+Bx^3)\,dx$ 2903
3.503 $\int \frac{(ex)^m\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$ 2907
3.504 $\int \frac{(ex)^m\,(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$ 2911
3.505 $\int \frac{(ex)^m\,(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$ 2915
3.506 $\int \frac{x^5}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$ 2918
3.507 $\int \frac{x^2}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$ 2922
3.508 $\int \frac{1}{x\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$ 2926
3.509 $\int \frac{1}{x^4\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$ 2930
3.510 $\int \frac{x^4}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$ 2934

3.511 $\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$ . . . 2937
3.512 $\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$ . . . 2940
3.513 $\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$ . . . 2943
3.514 $\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$ . . . 2946
3.515 $\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$ . . . 2949
3.516 $\int (ex)^{7/2}\sqrt{a+bx^3}\,(A+Bx^3)\,dx$ . . . 2952
3.517 $\int (ex)^{5/2}\sqrt{a+bx^3}\,(A+Bx^3)\,dx$ . . . 2957
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3.519 $\int \sqrt{ex}\,\sqrt{a+bx^3}\,(A+Bx^3)\,dx$ . . . 2968
3.520 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{\sqrt{ex}}\,dx$ . . . 2973
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3.522 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{(ex)^{5/2}}\,dx$ . . . 2984
3.523 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{(ex)^{7/2}}\,dx$ . . . 2989
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3.525 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{11/2}}\,dx$ . . . 3000
3.526 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{13/2}}\,dx$ . . . 3006
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3.569 $\int \frac{x^{5}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$ . . . 3240

3.570 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$ . . . 3245

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3.579 $\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)}\,dx$ . . . 3284

3.580 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx$ . . . 3288

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3.587 $\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$ . . . 3323

3.588 $\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$ . . . 3328

3.589 $\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$ . . . 3333

3.590 $\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx$ . . . 3337

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3.593 $\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$ . . . 3353

3.594 $\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$ . . . 3357

3.595 $\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$ . . . 3361

3.596 $\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx$ . . . 3365

3.597 $\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx$ . . . 3369

3.598 $\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx$ . . . 3373

3.599 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx$ 3377

3.600 $\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$ 3381

3.601 $\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$ 3386

3.602 $\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$ 3391

3.603 $\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx$ 3396

3.604 $\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx$ 3401

3.605 $\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3406

3.606 $\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3411

3.607 $\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3416

3.608 $\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3421

3.609 $\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3426

3.610 $\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3430

3.611 $\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3435

3.612 $\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3440

3.613 $\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3444

3.614 $\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3448

3.615 $\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3452

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3.619 $\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$ 3471

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3.623 $\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx$ 3489

3.624 $\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx$ 3494

3.625 $\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx$ 3499

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3.627 $\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx$ 3508

3.628 $\int \frac{1}{x^4(1-x^3)^{2/3}(1+x^3)}\,dx$ 3512

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3.633 $\int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)}\,dx$ . . . 3535
3.634 $\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx$ . . . 3540
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3.637 $\int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)}\,dx$ . . . 3554
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3.660 $\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$ . . . 3662

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3.713 $\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx$ . . . 3896
3.714 $\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ . . . 3899
3.715 $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ . . . 3905
3.716 $\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ . . . 3910
3.717 $\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ . . . 3915
3.718 $\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ . . . 3920

3.719 $\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3925
3.720 $\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3931
3.721 $\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3937
3.722 $\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3942
3.723 $\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3946
3.724 $\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3949
3.725 $\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3953
3.726 $\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3958
3.727 $\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3963
3.728 $\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3966
3.729 $\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3969
3.730 $\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3972
3.731 $\int \frac{1}{x^5\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$ 3975
3.732 $\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 3978
3.733 $\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 3984
3.734 $\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 3990
3.735 $\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 3995
3.736 $\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 4000
3.737 $\int \frac{1}{x^4(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 4006
3.738 $\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 4012
3.739 $\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 4017
3.740 $\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 4021
3.741 $\int \frac{1}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 4024
3.742 $\int \frac{1}{x^5(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 4028
3.743 $\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 4033
3.744 $\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 4036
3.745 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 4039
3.746 $\int \frac{1}{x^3(a+bx^3)^{2/3}(c+dx^3)}\,dx$ 4042
3.747 $\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ 4045
3.748 $\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ 4051
3.749 $\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ 4057

3.750 $\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4063
3.751 $\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4068
3.752 $\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4073
3.753 $\int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4080
3.754 $\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4088
3.755 $\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4093
3.756 $\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4098
3.757 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4102
3.758 $\int \frac{1}{x^3(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4106
3.759 $\int \frac{1}{x^6(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4110
3.760 $\int \frac{1}{x^9(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4114
3.761 $\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4118
3.762 $\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4121
3.763 $\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4124
3.764 $\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4127
3.765 $\int \frac{1}{x^2(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4130
3.766 $\int \frac{1}{x^5(a+bx^3)^{4/3}(c+dx^3)}\,dx$ . . . 4133
3.767 $\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx$ . . . 4136
3.768 $\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx$ . . . 4139
3.769 $\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx$ . . . 4142
3.770 $\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx$ . . . 4145
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3.774 $\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$ . . . 4160
3.775 $\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx$ . . . 4164
3.776 $\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx$ . . . 4168
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3.780 $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$ . . . 4189
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3.782 $\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$ . . . 4203
3.783 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$ . . . 4210
3.784 $\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx$ . . . 4218

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3.789 $\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx$ . . . 4253
3.790 $\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$ . . . 4257
3.791 $\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx$ . . . 4262
3.792 $\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)}\,dx$ . . . 4267
3.793 $\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx$ . . . 4272
3.794 $\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx$ . . . 4277
3.795 $\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$ . . . 4282
3.796 $\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$ . . . 4289
3.797 $\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$ . . . 4296
3.798 $\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$ . . . 4302
3.799 $\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx$ . . . 4307
3.800 $\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx$ . . . 4314
3.801 $\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx$ . . . 4320
3.802 $\int \frac{\sqrt{ex}\,\sqrt{c+dx^4}}{a+bx^4}\,dx$ . . . 4324
3.803 $\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx$ . . . 4328
3.804 $\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)}\,dx$ . . . 4332
3.805 $\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$ . . . 4336
3.806 $\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$ . . . 4341
3.807 $\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx$ . . . 4346
3.808 $\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx$ . . . 4350
3.809 $\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx$ . . . 4355
3.810 $\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$ . . . 4360
3.811 $\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$ . . . 4366
3.812 $\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx$ . . . 4371
3.813 $\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx$ . . . 4375

3.814 $\int \frac{1}{x^7(a+bx^4)\sqrt{c+dx^4}}\,dx$ 4380

3.815 $\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$ 4386

3.816 $\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$ 4393

3.817 $\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$ 4399

3.818 $\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx$ 4404

3.819 $\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$ 4410

3.820 $\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$ 4416

3.821 $\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx$ 4421

3.822 $\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4428

3.823 $\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4433

3.824 $\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4438

3.825 $\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4443

3.826 $\int \frac{1}{x(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4448

3.827 $\int \frac{1}{x^5(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4455

3.828 $\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4462

3.829 $\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4468

3.830 $\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4474

3.831 $\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4479

3.832 $\int \frac{1}{x^3(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4484

3.833 $\int \frac{1}{x^7(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4490

3.834 $\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4496

3.835 $\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4503

3.836 $\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4510

3.837 $\int \frac{1}{x^4(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4516

3.838 $\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4522

3.839 $\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4529

3.840 $\int \frac{1}{x^2(a+bx^4)^2\sqrt{c+dx^4}}\,dx$ 4536

3.841 $\int \frac{(ex)^m(a+bx^4)^2}{\sqrt{c+dx^4}}\,dx$ 4543

3.842 $\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}} \, dx$ 4547

3.843 $\int \frac{(ex)^m}{\sqrt{c+dx^4}} \, dx$ 4551

3.844 $\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}} \, dx$ 4554

3.845 $\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}} \, dx$ 4557

3.846 $\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}} \, dx$ 4560

3.847 $\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}} \, dx$ 4563

3.848 $\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}} \, dx$ 4567

3.849 $\int \frac{(ex)^m}{(c+dx^4)^{3/2}} \, dx$ 4571

3.850 $\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}} \, dx$ 4574

3.851 $\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}} \, dx$ 4577

3.852 $\int \frac{(ex)^m}{(a+bx^4)^3(c+dx^4)^{3/2}} \, dx$ 4580

3.853 $\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4583

3.854 $\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4587

3.855 $\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4591

3.856 $\int \frac{1}{x(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4595

3.857 $\int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4599

3.858 $\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4603

3.859 $\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4608

3.860 $\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4612

3.861 $\int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4616

3.862 $\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4620

3.863 $\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4625

3.864 $\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4628

3.865 $\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4631

3.866 $\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4634

3.867 $\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4637

3.868 $\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4640

3.869 $\int \frac{1}{x^5(a+bx^6)\sqrt{c+dx^6}} \, dx$ 4643

3.870 $\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}} \, dx$ 4646

3.871 $\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4650

3.872 $\int \frac{x^{5}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4654

3.873 $\int \frac{1}{x(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4658

3.874 $\int \frac{1}{x^7(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4664

3.875 $\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4671

3.876 $\int \frac{x^{8}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4676

3.877 $\int \frac{x^{2}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4680

3.878 $\int \frac{1}{x^4(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4684

3.879 $\int \frac{1}{x^{10}(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4689

3.880 $\int \frac{x^{4}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4694

3.881 $\int \frac{x^{3}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4697

3.882 $\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4700

3.883 $\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4703

3.884 $\int \frac{1}{x^2(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4706

3.885 $\int \frac{1}{x^3(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4709

3.886 $\int \frac{1}{x^5(a+bx^6)^2\sqrt{c+dx^6}}\,dx$ . . . 4712

3.887 $\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4715

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3.889 $\int \frac{x^{7}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4723

3.890 $\int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4727

3.891 $\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4731

3.892 $\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4735

3.893 $\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4740

3.894 $\int \frac{x^{3}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4744

3.895 $\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4748

3.896 $\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4752

3.897 $\int \frac{x^{9}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4757

3.898 $\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4762

3.899 $\int \frac{1}{x^7(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4767

3.900 $\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4773

3.901 $\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4779

3.902 $\int \frac{1}{x^3(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4784

3.903 $\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4790

3.904 $\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4793

3.905 $\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4796

3.906 $\int \frac{1}{x^2(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4799

3.907 $\int \frac{1}{x^4(a+bx^8)\sqrt{c+dx^8}}\,dx$ . . . 4802

3.908 $\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4805

3.909 $\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4809

3.910 $\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4813

3.911 $\int \frac{1}{x(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4817

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3.915 $\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4839

3.916 $\int \frac{1}{x^5(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4843

3.917 $\int \frac{1}{x^{13}(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4848

3.918 $\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4853

3.919 $\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4859

3.920 $\int \frac{1}{x^7(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4865

3.921 $\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4871

3.922 $\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4877

3.923 $\int \frac{1}{x^3(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4883

3.924 $\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4890

3.925 $\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4893

3.926 $\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4896

3.927 $\int \frac{1}{x^2(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4899

3.928 $\int \frac{1}{x^4(a+bx^8)^2\sqrt{c+dx^8}}\,dx$ . . . 4902

3.929 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^5\,dx$ 4905

3.930 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^3\,dx$ 4911

3.931 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x\,dx$ 4916

3.932 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx$ 4921

3.933 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx$ 4926

3.934 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$ 4930

3.935 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$ 4934

3.936 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$ 4938

3.937 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^{10}\,dx$ 4942

3.938 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^8\,dx$ 4947

3.939 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^6\,dx$ 4952

3.940 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^4\,dx$ 4956

3.941 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^2\,dx$ 4960

3.942 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx$ 4964

3.943 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$ 4969

3.944 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$ 4974

3.945 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^5\,dx$ 4980

3.946 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx$ 4985

3.947 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x\,dx$ 4991

3.948 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx$ 4996

3.949 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx$ 5001

3.950 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx$ 5005

3.951 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx$ 5009

3.952 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx$ 5013

3.953 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx$ 5017

3.954 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{10}\,dx$ 5023

3.955 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx$ 5028
3.956 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^6\,dx$ 5032
3.957 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx$ 5036
3.958 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx$ 5040
3.959 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx$ 5045
3.960 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$ 5050
3.961 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$ 5055
3.962 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$ 5061
3.963 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx$ 5067
3.964 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx$ 5072
3.965 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^3}\,dx$ 5076
3.966 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^5}\,dx$ 5080
3.967 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^7}\,dx$ 5084
3.968 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx$ 5088
3.969 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx$ 5092
3.970 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx$ 5096
3.971 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx$ 5100
3.972 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx$ 5105
3.973 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$ 5110
3.974 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$ 5116
3.975 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx$ 5122

3.976 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3} \, dx$ 5127
3.977 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5} \, dx$ 5131
3.978 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx$ 5135
3.979 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx$ 5139
3.980 $\int \frac{\left(a+\frac{b}{x^2}\right) x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}} \, dx$ 5143
3.981 $\int \frac{\left(a+\frac{b}{x^2}\right) x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}} \, dx$ 5148
3.982 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}} \, dx$ 5152
3.983 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} \, dx$ 5156
3.984 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4} \, dx$ 5161
3.985 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6} \, dx$ 5167
3.986 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx$ 5173
3.987 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 \, dx$ 5176
3.988 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \, dx$ 5179
3.989 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 \, dx$ 5182
3.990 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x \, dx$ 5185
3.991 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$ 5188
3.992 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x} \, dx$ 5191
3.993 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2} \, dx$ 5194
3.994 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} \, dx$ 5197
3.995 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx$ 5200
3.996 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2} \, dx$ 5203
3.997 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2} \, dx$ 5206
3.998 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex} \, dx$ 5209
3.999 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}} \, dx$ 5212
3.1000 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}} \, dx$ 5215
3.1001 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}} \, dx$ 5218
3.1002 $\int \sqrt{-1+\sqrt{x}} \sqrt{1+\sqrt{x}} \, x^{5/2} \, dx$ 5221

3.1003 $\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}\,dx$ 5226
3.1004 $\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}\,dx$ 5231
3.1005 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$ 5235
3.1006 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx$ 5239
3.1007 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx$ 5243
3.1008 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx$ 5246
3.1009 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx$ 5250
3.1010 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx$ 5254
3.1011 $\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$ 5258
3.1012 $\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$ 5262
3.1013 $\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$ 5266
3.1014 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx$ 5270
3.1015 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx$ 5273
3.1016 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx$ 5276
3.1017 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2}}\,dx$ 5280
3.1018 $\int x^2(-a+bx^n)^p\,(a+bx^n)^p\,dx$ 5284
3.1019 $\int x(-a+bx^n)^p\,(a+bx^n)^p\,dx$ 5287
3.1020 $\int (-a+bx^n)^p\,(a+bx^n)^p\,dx$ 5290
3.1021 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x}\,dx$ 5293
3.1022 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x^2}\,dx$ 5296
3.1023 $\int \frac{1+x^6}{x(1-x^6)}\,dx$ 5299
3.1024 $\int (ex)^m\,(a+bx^n)^p\,(a(1+m)+b(1+m+n+np)x^n)\,dx$ 5302
3.1025 $\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx$ 5305
3.1026 $\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx$ 5308
3.1027 $\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx$ 5311
3.1028 $\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$ 5314
3.1029 $\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx$ 5317
3.1030 $\int \frac{1}{x^2(a+bx^n)(c+dx^n)}\,dx$ 5321
3.1031 $\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx$ 5324
3.1032 $\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx$ 5327
3.1033 $\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx$ 5331
3.1034 $\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx$ 5335

3.1035 $\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$ 5339
3.1036 $\int \frac{1}{x(a+bx^n)^2(c+dx^n)}\,dx$ 5342
3.1037 $\int \frac{1}{x^2(a+bx^n)^2(c+dx^n)}\,dx$ 5345
3.1038 $\int \frac{1}{x^3(a+bx^n)^2(c+dx^n)}\,dx$ 5349
3.1039 $\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$ 5353
3.1040 $\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx$ 5357
3.1041 $\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx$ 5361
3.1042 $\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx$ 5365
3.1043 $\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx$ 5368
3.1044 $\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx$ 5371
3.1045 $\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$ 5375
3.1046 $\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx$ 5379
3.1047 $\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx$ 5383
3.1048 $\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx$ 5387
3.1049 $\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx$ 5390
3.1050 $\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx$ 5393
3.1051 $\int x^{13}(b+cx)^{13}(b+2cx)\,dx$ 5397
3.1052 $\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx$ 5400
3.1053 $\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx$ 5403
3.1054 $\int x^{-1+14n}\left(b+cx^n\right)^{13}\left(b+2cx^n\right)\,dx$ 5406
3.1055 $\int x^{-1+m}\left(a+bx^n\right)^{-1+p}\left(am+b(m+np)x^n\right)\,dx$ 5409
3.1056 $\int \frac{b+2cx}{x(b+cx)}\,dx$ 5412
3.1057 $\int \frac{b+2cx^2}{x(b+cx^2)}\,dx$ 5415
3.1058 $\int \frac{b+2cx^3}{x(b+cx^3)}\,dx$ 5418
3.1059 $\int \frac{b+2cx^n}{x(b+cx^n)}\,dx$ 5421
3.1060 $\int \frac{b+2cx}{x^8(b+cx)^8}\,dx$ 5424
3.1061 $\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8}\,dx$ 5427
3.1062 $\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8}\,dx$ 5430
3.1063 $\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx$ 5433
3.1064 $\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx$ 5437
3.1065 $\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx$ 5441
3.1066 $\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$ 5446
3.1067 $\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$ 5451

3.1068 $\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$ 5456
3.1069 $\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$ 5461
3.1070 $\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$ 5465
3.1071 $\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$ 5469
3.1072 $\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$ 5472
3.1073 $\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$ 5477
3.1074 $\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$ 5482
3.1075 $\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$ 5487
3.1076 $\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$ 5492
3.1077 $\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$ 5497
3.1078 $\int x^p(b+cx)^p(b+2cx)\,dx$ 5502
3.1079 $\int x^{-1+2(1+p)}(b+cx^2)^p\,(b+2cx^2)\,dx$ 5505
3.1080 $\int x^{-1+3(1+p)}(b+cx^3)^p\,(b+2cx^3)\,dx$ 5508
3.1081 $\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\,dx$ 5511

3.1 $\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{3}aAx^3+\frac{1}{6}(Ab+aB)x^6+\frac{1}{9}bBx^9$$

```
[Out] 1/3*a*A*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {455, 45}

$$\frac{1}{6}x^6(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{9}bBx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \frac{1}{3}\text{Subst}\left(\int (a+bx)(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(aA+(Ab+aB)x+bBx^2\right)\,dx,x,x^3\right)\\ &= \frac{1}{3}aAx^3+\frac{1}{6}(Ab+aB)x^6+\frac{1}{9}bBx^9\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{3}aAx^3 + \frac{1}{6}(Ab + aB)x^6 + \frac{1}{9}bBx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*(A + B*x^3),x]
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

Maple [A]
time = 0.12, size = 28, normalized size = 0.85

method	result	size
default	$\frac{aA\,x^3}{3} + \frac{(Ab+Ba)x^6}{6} + \frac{bB\,x^9}{9}$	28
norman	$\frac{bB\,x^9}{9} + \left(\frac{Ab}{6} + \frac{Ba}{6}\right)x^6 + \frac{aA\,x^3}{3}$	29
gosper	$\frac{1}{9}bB\,x^9 + \frac{1}{6}x^6Ab + \frac{1}{6}x^6Ba + \frac{1}{3}aA\,x^3$	30
risch	$\frac{1}{9}bB\,x^9 + \frac{1}{6}x^6Ab + \frac{1}{6}x^6Ba + \frac{1}{3}aA\,x^3$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 1/3*a*A*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

Maxima [A]
time = 0.30, size = 27, normalized size = 0.82

$$\frac{1}{9}\,Bbx^9 + \frac{1}{6}\,(Ba + Ab)x^6 + \frac{1}{3}\,Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3
```

Fricas [A]
time = 2.23, size = 27, normalized size = 0.82

$$\frac{1}{9}\,Bbx^9 + \frac{1}{6}\,(Ba + Ab)x^6 + \frac{1}{3}\,Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{Aax^3}{3} + \frac{Bbx^9}{9} + x^6\left(\frac{Ab}{6} + \frac{Ba}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)*(B*x**3+A),x)
[Out] A*a*x**3/3 + B*b*x**9/9 + x**6*(A*b/6 + B*a/6)
```

Giac [A]
time = 0.64, size = 29, normalized size = 0.88

$$\frac{1}{9}\,Bbx^9 + \frac{1}{6}\,Bax^6 + \frac{1}{6}\,Abx^6 + \frac{1}{3}\,Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
[Out] 1/9*B*b*x^9 + 1/6*B*a*x^6 + 1/6*A*b*x^6 + 1/3*A*a*x^3
```

Mupad [B]
time = 0.20, size = 28, normalized size = 0.85

$$\frac{B\,b\,x^9}{9} + \left(\frac{A\,b}{6} + \frac{B\,a}{6}\right)\,x^6 + \frac{A\,a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3),x)
[Out] x^6*((A*b)/6 + (B*a)/6) + (A*a*x^3)/3 + (B*b*x^9)/9
```

3.2 $\int x(a+bx^3)(A+Bx^3)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8$$

```
[Out] 1/2*a*A*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.062$, Rules used = $\{459\}$

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx+(Ab+aB)x^4+bBx^7\right)\,dx\\&=\frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 1.00

$$\frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

Maple [A]
time = 0.15, size = 28, normalized size = 0.85

method	result	size
default	$\frac{aA\,x^2}{2}+\frac{(Ab+Ba)x^5}{5}+\frac{bB\,x^8}{8}$	28
norman	$\frac{bB\,x^8}{8}+\left(\frac{Ab}{5}+\frac{Ba}{5}\right)x^5+\frac{aA\,x^2}{2}$	29
gosper	$\frac{1}{8}bB\,x^8+\frac{1}{5}x^5Ab+\frac{1}{5}aB\,x^5+\frac{1}{2}aA\,x^2$	30
risch	$\frac{1}{8}bB\,x^8+\frac{1}{5}x^5Ab+\frac{1}{5}aB\,x^5+\frac{1}{2}aA\,x^2$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 1/2*a*A*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.82

$$\frac{1}{8}\,Bbx^8+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2
```

Fricas [A]
time = 2.51, size = 27, normalized size = 0.82

$$\frac{1}{8}\,Bbx^8+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{Aax^2}{2}+\frac{Bbx^8}{8}+x^5\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] A*a*x**2/2 + B*b*x**8/8 + x**5*(A*b/5 + B*a/5)
```

Giac [**A**]
time = 0.56, size = 29, normalized size = 0.88

$$\frac{1}{8}Bbx^8 + \frac{1}{5}Bax^5 + \frac{1}{5}Abx^5 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/8*B*b*x^8 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/2*A*a*x^2
```

Mupad [**B**]
time = 2.48, size = 28, normalized size = 0.85

$$\frac{B\,b\,x^8}{8} + \left(\frac{A\,b}{5} + \frac{B\,a}{5}\right)\,x^5 + \frac{A\,a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] x^5*((A*b)/5 + (B*a)/5) + (A*a*x^2)/2 + (B*b*x^8)/8
```

3.3 $\int (a+bx^3)(A+Bx^3)\,dx$

Optimal. Leaf size=28

$$aAx+\frac{1}{4}(Ab+aB)x^4+\frac{1}{7}bBx^7$$

```
[Out] a*A*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {380}

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(A + B*x^3),x]
[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^3)(A+Bx^3)\,dx &= \int \left(aA+(Ab+aB)x^3+bBx^6\right)dx\\ &= aAx+\frac{1}{4}(Ab+aB)x^4+\frac{1}{7}bBx^7\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$aAx+\frac{1}{4}(Ab+aB)x^4+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(A + B*x^3),x]
[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

Maple [A]
time = 0.14, size = 25, normalized size = 0.89

method	result	size
default	$aAx+\frac{(Ab+Ba)x^4}{4}+\frac{bB\,x^7}{7}$	25
norman	$\frac{bB\,x^7}{7}+\left(\frac{Ab}{4}+\frac{Ba}{4}\right)x^4+aAx$	26
gosper	$\frac{1}{7}bB\,x^7+\frac{1}{4}x^4Ab+\frac{1}{4}Ba\,x^4+aAx$	27
risch	$\frac{1}{7}bB\,x^7+\frac{1}{4}x^4Ab+\frac{1}{4}Ba\,x^4+aAx$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] a*A*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.86

$$\frac{1}{7}Bbx^7+\frac{1}{4}(Ba+Ab)x^4+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x
```

Fricas [A]
time = 2.54, size = 24, normalized size = 0.86

$$\frac{1}{7}Bbx^7+\frac{1}{4}(Ba+Ab)x^4+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.93

$$Aax+\frac{Bbx^7}{7}+x^4\left(\frac{Ab}{4}+\frac{Ba}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A),x)
```

```
[Out] A*a*x + B*b*x**7/7 + x**4*(A*b/4 + B*a/4)
```

Giac [**A**]
time = 0.62, size = 26, normalized size = 0.93

$$\frac{1}{7}Bbx^7 + \frac{1}{4}Bax^4 + \frac{1}{4}Abx^4 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/7*B*b*x^7 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + A*a*x
```

Mupad [**B**]
time = 0.03, size = 25, normalized size = 0.89

$$\frac{B\,b\,x^7}{7} + \left(\frac{A\,b}{4} + \frac{B\,a}{4}\right)\,x^4 + A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3),x)
```

```
[Out] x^4*((A*b)/4 + (B*a)/4) + A*a*x + (B*b*x^7)/7
```

3.4 $\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx$

Optimal. Leaf size=29

$$\frac{1}{3}(Ab+aB)x^3+\frac{1}{6}bBx^6+aA\log(x)$$

```
[Out] 1/3*(A*b+B*a)*x^3+1/6*b*B*x^6+a*A*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {457, 77}

$$\frac{1}{3}x^3(aB+Ab)+aA\log(x)+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x,x]
```

```
[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(Ab+aB+\frac{aA}{x}+bBx\right)dx,x,x^3\right)\\ &= \frac{1}{3}(Ab+aB)x^3+\frac{1}{6}bBx^6+aA\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{3}(Ab + aB)x^3 + \frac{1}{6}bBx^6 + aA\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x,x]
```

```
[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

Maple [A]
time = 0.07, size = 28, normalized size = 0.97

method	result	size
norman	$\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^3+\frac{bB\,x^6}{6}+aA\ln\left(x\right)$	27
default	$\frac{bB\,x^6}{6}+\frac{Ab\,x^3}{3}+\frac{Ba\,x^3}{3}+aA\ln\left(x\right)$	28
risch	$\frac{bB\,x^6}{6}+\frac{Ab\,x^3}{3}+\frac{Ba\,x^3}{3}+\frac{b\,A^2}{6B}+\frac{Aa}{3}+\frac{B\,a^2}{6b}+aA\ln\left(x\right)$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b*B*x^6+1/3*A*b*x^3+1/3*B*a*x^3+a*A*ln(x)
```

Maxima [A]
time = 0.29, size = 28, normalized size = 0.97

$$\frac{1}{6}\,Bbx^6 + \frac{1}{3}\,(Ba + Ab)x^3 + \frac{1}{3}\,Aa\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + 1/3*A*a*log(x^3)
```

Fricas [A]
time = 2.26, size = 25, normalized size = 0.86

$$\frac{1}{6}\,Bbx^6 + \frac{1}{3}\,(Ba + Ab)x^3 + Aa\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] 1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + A*a*log(x)
```

<u>**Sympy**</u> [**A**]
time = 0.10, size = 27, normalized size = 0.93

$$Aa\log(x) + \frac{Bbx^6}{6} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x,x)
```

```
[Out] A*a*log(x) + B*b*x**6/6 + x**3*(A*b/3 + B*a/3)
```

<u>**Giac**</u> [**A**]
time = 0.56, size = 28, normalized size = 0.97

$$\frac{1}{6}Bbx^6 + \frac{1}{3}Bax^3 + \frac{1}{3}Abx^3 + Aa\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 1/6*B*b*x^6 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*log(abs(x))
```

<u>**Mupad**</u> [**B**]
time = 0.03, size = 26, normalized size = 0.90

$$x^3\left(\frac{A\,b}{3} + \frac{B\,a}{3}\right) + \frac{B\,b\,x^6}{6} + A\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x,x)
```

```
[Out] x^3*((A*b)/3 + (B*a)/3) + (B*b*x^6)/6 + A*a*log(x)
```

3.5 $\int \frac{(a+bx^3)(A+Bx^3)}{x^2} \, dx$

Optimal. Leaf size=31

$$-\frac{aA}{x}+\frac{1}{2}(Ab+aB)x^2+\frac{1}{5}bBx^5$$

```
[Out] -a*A/x+1/2*(A*b+B*a)*x^2+1/5*b*B*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{459\}$

$$\frac{1}{2}x^2(aB+Ab)-\frac{aA}{x}+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^2} \, dx &= \int \left(\frac{aA}{x^2}+(Ab+aB)x+bBx^4\right) dx\\ &= -\frac{aA}{x}+\frac{1}{2}(Ab+aB)x^2+\frac{1}{5}bBx^5\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$-\frac{aA}{x}+\frac{1}{2}(Ab+aB)x^2+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

Maple [A]
time = 0.02, size = 30, normalized size = 0.97

method	result	size
default	$\frac{bB\,x^{5}}{5}+\frac{Ab\,x^{2}}{2}+\frac{Ba\,x^{2}}{2}-\frac{aA}{x}$	30
norman	$\frac{\frac{bB\,x^{6}}{5}+\left(\frac{Ab}{2}+\frac{Ba}{2}\right)x^{3}-Aa}{x}$	30
risch	$\frac{bB\,x^{5}}{5}+\frac{Ab\,x^{2}}{2}+\frac{Ba\,x^{2}}{2}-\frac{aA}{x}$	30
gosper	$-\frac{-2bB\,x^{6}-5Ab\,x^{3}-5Ba\,x^{3}+10Aa}{10x}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/5*b*B*x^5+1/2*A*b*x^2+1/2*B*a*x^2-a*A/x
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.87

$$\frac{1}{5}\,Bbx^{5}+\frac{1}{2}\,(Ba+Ab)x^{2}-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="maxima")
[Out] 1/5*B*b*x^5 + 1/2*(B*a + A*b)*x^2 - A*a/x
```

Fricas [A]
time = 2.04, size = 29, normalized size = 0.94

$$\frac{2\,Bbx^{6}+5\,(Ba+Ab)x^{3}-10\,Aa}{10\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="fricas")
[Out] 1/10*(2*B*b*x^6 + 5*(B*a + A*b)*x^3 - 10*A*a)/x
```

Sympy [A]
time = 0.09, size = 26, normalized size = 0.84

$$-\frac{Aa}{x}+\frac{Bbx^{5}}{5}+x^{2}\left(\frac{Ab}{2}+\frac{Ba}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a/x + B*b*x**5/5 + x**2*(A*b/2 + B*a/2)
```

Giac [A]
time = 0.48, size = 29, normalized size = 0.94

$$\frac{1}{5}\,Bbx^5 + \frac{1}{2}\,Bax^2 + \frac{1}{2}\,Abx^2 - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*B*b*x^5 + 1/2*B*a*x^2 + 1/2*A*b*x^2 - A*a/x
```

Mupad [B]
time = 0.04, size = 28, normalized size = 0.90

$$x^2\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)-\frac{A\,a}{x}+\frac{B\,b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^2,x)
```

```
[Out] x^2*((A*b)/2 + (B*a)/2) - (A*a)/x + (B*b*x^5)/5
```

3.6 $\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx$

Optimal. Leaf size=28

$$-\frac{aA}{2x^2}+(Ab+aB)x+\frac{1}{4}bBx^4$$

```
[Out] -1/2*a*A/x^2+(A*b+B*a)*x+1/4*b*B*x^4
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$x(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a*A)/x^2 + (A*b + a*B)*x + (b*B*x^4)/4
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx &= \int \left(Ab\left(1+\frac{aB}{Ab}\right)+\frac{aA}{x^3}+bBx^3\right)dx\\ &= -\frac{aA}{2x^2}+(Ab+aB)x+\frac{1}{4}bBx^4\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.00

$$-\frac{aA}{2x^2}+(Ab+aB)x+\frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a*A)/x^2 + (A*b + a*B)*x + (b*B*x^4)/4
```

Maple [A]
time = 0.02, size = 24, normalized size = 0.86

method	result	size
default	$\frac{bB\,x^4}{4}+Abx+Bax-\frac{aA}{2x^2}$	24
risch	$\frac{bB\,x^4}{4}+Abx+Bax-\frac{aA}{2x^2}$	24
norman	$\frac{\frac{bB\,x^6}{4}+(Ab+Ba)x^3-\frac{Aa}{2}}{x^2}$	28
gosper	$-\frac{-bB\,x^6-4Ab\,x^3-4Ba\,x^3+2Aa}{4x^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*B*x^4+A*b*x+B*a*x-1/2*a*A/x^2
```

Maxima [A]
time = 0.31, size = 24, normalized size = 0.86

$$\frac{1}{4}\,Bbx^4+(Ba+Ab)x-\frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*b*x^4 + (B*a + A*b)*x - 1/2*A*a/x^2
```

Fricas [A]
time = 2.43, size = 28, normalized size = 1.00

$$\frac{Bbx^6+4\,(Ba+Ab)x^3-2\,Aa}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b*x^6 + 4*(B*a + A*b)*x^3 - 2*A*a)/x^2
```

Sympy [A]
time = 0.08, size = 24, normalized size = 0.86

$$-\frac{Aa}{2x^2}+\frac{Bbx^4}{4}+x(Ab+Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a/(2*x**2) + B*b*x**4/4 + x*(A*b + B*a)
```

Giac [A]
time = 0.59, size = 23, normalized size = 0.82

$$\frac{1}{4}Bbx^4 + Bax + Abx - \frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*B*b*x^4 + B*a*x + A*b*x - 1/2*A*a/x^2
```

Mupad [B]
time = 2.34, size = 24, normalized size = 0.86

$$x\left(A\,b + B\,a\right) - \frac{A\,a}{2\,x^2} + \frac{B\,b\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^3,x)
```

```
[Out] x*(A*b + B*a) - (A*a)/(2*x^2) + (B*b*x^4)/4
```

3.7 $\int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx$

Optimal. Leaf size=29

$$-\frac{aA}{3x^3}+\frac{1}{3}bBx^3+(Ab+aB)\log(x)$$

```
[Out] -1/3*a*A/x^3+1/3*b*B*x^3+(A*b+B*a)*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {457, 77}

$$\log(x)(aB+Ab)-\frac{aA}{3x^3}+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(bB+\frac{aA}{x^2}+\frac{Ab+aB}{x}\right)dx,x,x^3\right)\\ &= -\frac{aA}{3x^3}+\frac{1}{3}bBx^3+(Ab+aB)\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$-\frac{aA}{3x^3}+\frac{1}{3}bBx^3+(Ab+aB)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

Maple [A]
time = 0.03, size = 26, normalized size = 0.90

method	result	size
default	$-\frac{aA}{3x^3}+\frac{bB\,x^3}{3}+(Ab+Ba)\ln(x)$	26
risch	$-\frac{aA}{3x^3}+\frac{bB\,x^3}{3}+A\ln(x)\,b+B\ln(x)\,a$	26
norman	$\frac{-\frac{Aa}{3}+\frac{bB\,x^6}{3}}{x^3}+(Ab+Ba)\ln(x)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a*A/x^3+1/3*b*B*x^3+(A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.29, size = 28, normalized size = 0.97

$$\frac{1}{3}\,Bbx^3+\frac{1}{3}\,(Ba+Ab)\log\left(x^3\right)-\frac{Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*B*b*x^3 + 1/3*(B*a + A*b)*log(x^3) - 1/3*A*a/x^3
```

Fricas [A]
time = 2.40, size = 30, normalized size = 1.03

$$\frac{Bbx^6+3\,(Ba+Ab)x^3\log(x)-Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^6 + 3*(B*a + A*b)*x^3*log(x) - A*a)/x^3
```

Sympy [**A**]
time = 0.24, size = 26, normalized size = 0.90

$$-\frac{Aa}{3x^3} + \frac{Bbx^3}{3} + (Ab + Ba)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**4,x)
[Out] -A*a/(3*x**3) + B*b*x**3/3 + (A*b + B*a)*log(x)
```

Giac [**A**]
time = 0.56, size = 40, normalized size = 1.38

$$\frac{1}{3}Bbx^3 + (Ba + Ab)\log(|x|) - \frac{Bax^3 + Abx^3 + Aa}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="giac")
[Out] 1/3*B*b*x^3 + (B*a + A*b)*log(abs(x)) - 1/3*(B*a*x^3 + A*b*x^3 + A*a)/x^3
```

Mupad [**B**]
time = 0.04, size = 25, normalized size = 0.86

$$\ln(x)\,(A\,b + B\,a) - \frac{A\,a}{3\,x^3} + \frac{B\,b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^4,x)
[Out] log(x)*(A*b + B*a) - (A*a)/(3*x^3) + (B*b*x^3)/3
```

3.8 $\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx$

Optimal. Leaf size=31

$$-\frac{aA}{4x^4}-\frac{Ab+aB}{x}+\frac{1}{2}bBx^2$$

```
[Out] -1/4*a*A/x^4+(-A*b-B*a)/x+1/2*b*B*x^2
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$-\frac{aB+Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a*A)/x^4 - (A*b + a*B)/x + (b*B*x^2)/2
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx &= \int \left(\frac{aA}{x^5}+\frac{Ab+aB}{x^2}+bBx\right)dx\\ &= -\frac{aA}{4x^4}-\frac{Ab+aB}{x}+\frac{1}{2}bBx^2\end{aligned}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 1.03

$$-\frac{aA}{4x^4}+\frac{-Ab-aB}{x}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a*A)/x^4 + (-(A*b) - a*B)/x + (b*B*x^2)/2
```

Maple [A]
time = 0.02, size = 28, normalized size = 0.90

method	result	size
default	$\frac{bB\,x^2}{2}-\frac{aA}{4x^4}-\frac{Ab+Ba}{x}$	28
norman	$\frac{\frac{bB\,x^6}{2}+(-Ab-Ba)x^3-\frac{Aa}{4}}{x^4}$	30
gosper	$-\frac{-2bB\,x^6+4Ab\,x^3+4Ba\,x^3+Aa}{4x^4}$	31
risch	$\frac{bB\,x^2}{2}+\frac{(-Ab-Ba)x^3-\frac{Aa}{4}}{x^4}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
[Out] 1/2*b*B*x^2-1/4*a*A/x^4-(A*b+B*a)/x
```

Maxima [A]
time = 0.31, size = 29, normalized size = 0.94

$$\frac{1}{2}\,Bbx^2-\frac{4\,(Ba+Ab)x^3+Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="maxima")
[Out] 1/2*B*b*x^2 - 1/4*(4*(B*a + A*b)*x^3 + A*a)/x^4
```

Fricas [A]
time = 2.21, size = 29, normalized size = 0.94

$$\frac{2\,Bbx^6-4\,(Ba+Ab)x^3-Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="fricas")
[Out] 1/4*(2*B*b*x^6 - 4*(B*a + A*b)*x^3 - A*a)/x^4
```

Sympy [A]
time = 0.31, size = 31, normalized size = 1.00

$$\frac{Bbx^2}{2}+\frac{-Aa+x^3(-4Ab-4Ba)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**5,x)
```

```
[Out] B*b*x**2/2 + (-A*a + x**3*(-4*A*b - 4*B*a))/(4*x**4)
```

Giac [A]
time = 0.50, size = 31, normalized size = 1.00

$$\frac{1}{2}\,Bbx^2 - \frac{4\,Bax^3 + 4\,Abx^3 + Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*B*b*x^2 - 1/4*(4*B*a*x^3 + 4*A*b*x^3 + A*a)/x^4
```

Mupad [B]
time = 0.03, size = 29, normalized size = 0.94

$$\frac{B\,b\,x^2}{2} - \frac{(A\,b + B\,a)\,x^3 + \frac{A\,a}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^5,x)
```

```
[Out] (B*b*x^2)/2 - ((A*a)/4 + x^3*(A*b + B*a))/x^4
```

3.9 $\int \frac{(a+bx^3)(A+Bx^3)}{x^6} dx$

Optimal. Leaf size=28

$$-\frac{aA}{5x^5} - \frac{Ab+aB}{2x^2} + bBx$$

```
[Out] -1/5*a*A/x^5+1/2*(-A*b-B*a)/x^2+b*B*x
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {459}

$$-\frac{aB+Ab}{2x^2} - \frac{aA}{5x^5} + bBx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a*A)/x^5 - (A*b + a*B)/(2*x^2) + b*B*x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^6} dx &= \int \left(bB + \frac{aA}{x^6} + \frac{Ab+aB}{x^3}\right) dx \\ &= -\frac{aA}{5x^5} - \frac{Ab+aB}{2x^2} + bBx\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.07

$$-\frac{aA}{5x^5} + \frac{-Ab-aB}{2x^2} + bBx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

[Out] -1/5*(a*A)/x^5 + (-(A*b) - a*B)/(2*x^2) + b*B*x

Maple [A]
time = 0.02, size = 25, normalized size = 0.89

method	result	size
default	$bBx-\frac{aA}{5x^{5}}-\frac{Ab+Ba}{2x^{2}}$	25
risch	$bBx+\frac{\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^{3}-\frac{Aa}{5}}{x^{5}}$	28
norman	$\frac{bB\,x^{6}+\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^{3}-\frac{Aa}{5}}{x^{5}}$	29
gosper	$-\frac{-10bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+2Aa}{10x^{5}}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)

[Out] b*B*x-1/5*a*A/x^5-1/2*(A*b+B*a)/x^2

Maxima [A]
time = 0.28, size = 27, normalized size = 0.96

$$Bbx-\frac{5\,(Ba+Ab)x^{3}+2\,Aa}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="maxima")

[Out] B*b*x - 1/10*(5*(B*a + A*b)*x^3 + 2*A*a)/x^5

Fricas [A]
time = 2.31, size = 29, normalized size = 1.04

$$\frac{10\,Bbx^{6}-5\,(Ba+Ab)x^{3}-2\,Aa}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="fricas")

[Out] 1/10*(10*B*b*x^6 - 5*(B*a + A*b)*x^3 - 2*A*a)/x^5

Sympy [A]
time = 0.13, size = 29, normalized size = 1.04

$$Bbx+\frac{-2Aa+x^{3}(-5Ab-5Ba)}{10x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**6,x)
```

```
[Out] B*b*x + (-2*A*a + x**3*(-5*A*b - 5*B*a))/(10*x**5)
```

Giac [**A**]
time = 0.74, size = 29, normalized size = 1.04

$$Bbx - \frac{5\,Bax^3 + 5\,Abx^3 + 2\,Aa}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] B*b*x - 1/10*(5*B*a*x^3 + 5*A*b*x^3 + 2*A*a)/x^5
```

Mupad [**B**]
time = 2.32, size = 28, normalized size = 1.00

$$B\,b\,x - \frac{\left(\frac{A\,b}{2} + \frac{B\,a}{2}\right)\,x^3 + \frac{A\,a}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^6,x)
```

```
[Out] B*b*x - ((A*a)/5 + x^3*((A*b)/2 + (B*a)/2))/x^5
```

3.10 $\int \frac{(a+bx^3)(A+Bx^3)}{x^7} \, dx$

Optimal. Leaf size=29

$$-\frac{aA}{6x^6}-\frac{Ab+aB}{3x^3}+bB\log(x)$$

```
[Out] -1/6*a*A/x^6+1/3*(-A*b-B*a)/x^3+b*B*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {457, 77}

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{6x^6}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

```
[Out] -1/6*(a*A)/x^6 - (A*b + a*B)/(3*x^3) + b*B*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^7} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^3} \, dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{aA}{x^3}+\frac{Ab+aB}{x^2}+\frac{bB}{x}\right) dx, x, x^3\right)\\ &= -\frac{aA}{6x^6}-\frac{Ab+aB}{3x^3}+bB\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.07

$$-\frac{aA}{6x^6}+\frac{-Ab-aB}{3x^3}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^7,x]
[Out] -1/6*(a*A)/x^6 + (-(A*b) - a*B)/(3*x^3) + b*B*Log[x]
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.90

method	result	size
default	$-\frac{aA}{6x^6}-\frac{Ab+Ba}{3x^3}+bB\ln\left(x\right)$	26
norman	$\frac{\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^3-\frac{Aa}{6}}{x^6}+bB\ln\left(x\right)$	29
risch	$\frac{\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^3-\frac{Aa}{6}}{x^6}+bB\ln\left(x\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*a*A/x^6-1/3*(A*b+B*a)/x^3+b*B*ln(x)
```

Maxima [A]
time = 0.27, size = 30, normalized size = 1.03

$$\frac{1}{3}\,Bb\log\left(x^3\right)-\frac{2\,(Ba+Ab)x^3+Aa}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="maxima")
[Out] 1/3*B*b*log(x^3) - 1/6*(2*(B*a + A*b)*x^3 + A*a)/x^6
```

Fricas [A]
time = 2.25, size = 31, normalized size = 1.07

$$\frac{6\,Bbx^6\log\left(x\right)-2\,(Ba+Ab)x^3-Aa}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(6*B*b*x^6*log(x) - 2*(B*a + A*b)*x^3 - A*a)/x^6
```

Sympy [A]
time = 0.28, size = 29, normalized size = 1.00

$$Bb\log(x) + \frac{-Aa + x^3(-2Ab - 2Ba)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**7,x)
```

```
[Out] B*b*log(x) + (-A*a + x**3*(-2*A*b - 2*B*a))/(6*x**6)
```

Giac [A]
time = 0.59, size = 37, normalized size = 1.28

$$Bb\log(|x|) - \frac{3\,Bbx^6 + 2\,Bax^3 + 2\,Abx^3 + Aa}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] B*b*log(abs(x)) - 1/6*(3*B*b*x^6 + 2*B*a*x^3 + 2*A*b*x^3 + A*a)/x^6
```

Mupad [B]
time = 0.05, size = 29, normalized size = 1.00

$$B\,b\,\ln(x) - \frac{\left(\frac{A\,b}{3} + \frac{B\,a}{3}\right)\,x^3 + \frac{A\,a}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^7,x)
```

```
[Out] B*b*log(x) - ((A*a)/6 + x^3*((A*b)/3 + (B*a)/3))/x^6
```

3.11 $\int x^2(a+bx^3)^2\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=42

$$\frac{(Ab-aB)\left(a+bx^3\right)^3}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}$$

```
[Out] 1/9*(A*b-B*a)*(b*x^3+a)^3/b^2+1/12*B*(b*x^3+a)^4/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {455, 45}

$$\frac{\left(a+bx^3\right)^3(Ab-aB)}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^2*(A + B*x^3),x]
[Out] ((A*b - a*B)*(a + b*x^3)^3)/(9*b^2) + (B*(a + b*x^3)^4)/(12*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx &= \frac{1}{3}\text{Subst}\left(\int (a+bx)^2(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^2}{b}+\frac{B(a+bx)^3}{b}\right)\,dx,x,x^3\right)\\ &= \frac{(Ab-aB)\left(a+bx^3\right)^3}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.21

$$\frac{1}{36}x^3\left(12a^2A+6a(2Ab+aB)x^3+4b(Ab+2aB)x^6+3b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (x^3*(12*a^2*A + 6*a*(2*A*b + a*B)*x^3 + 4*b*(A*b + 2*a*B)*x^6 + 3*b^2*B*x^
9))/36
```

Maple [A]
time = 0.28, size = 52, normalized size = 1.24

method	result	size
default	$\frac{b^2Bx^{12}}{12}+\frac{(b^2A+2abB)x^9}{9}+\frac{(2abA+a^2B)x^6}{6}+\frac{a^2Ax^3}{3}$	52
norman	$\frac{b^2Bx^{12}}{12}+\left(\frac{1}{9}b^2A+\frac{2}{9}abB\right)x^9+\left(\frac{1}{3}abA+\frac{1}{6}a^2B\right)x^6+\frac{a^2Ax^3}{3}$	52
gosper	$\frac{1}{12}b^2Bx^{12}+\frac{1}{9}x^9b^2A+\frac{2}{9}x^9abB+\frac{1}{3}x^6abA+\frac{1}{6}x^6a^2B+\frac{1}{3}a^2Ax^3$	54
risch	$\frac{1}{12}b^2Bx^{12}+\frac{1}{9}x^9b^2A+\frac{2}{9}x^9abB+\frac{1}{3}x^6abA+\frac{1}{6}x^6a^2B+\frac{1}{3}a^2Ax^3$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^2*B*x^12+1/9*(A*b^2+2*B*a*b)*x^9+1/6*(2*A*a*b+B*a^2)*x^6+1/3*a^2*A*x
^3
```

Maxima [A]
time = 0.29, size = 51, normalized size = 1.21

$$\frac{1}{12}Bb^2x^{12}+\frac{1}{9}\left(2Bab+Ab^2\right)x^9+\frac{1}{6}\left(Ba^2+2Aab\right)x^6+\frac{1}{3}Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/12*B*b^2*x^12 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 + 1
/3*A*a^2*x^3
```

Fricas [A]
time = 2.39, size = 51, normalized size = 1.21

$$\frac{1}{12}Bb^2x^{12}+\frac{1}{9}\left(2Bab+Ab^2\right)x^9+\frac{1}{6}\left(Ba^2+2Aab\right)x^6+\frac{1}{3}Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/12*B*b^2*x^12 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 + 1
/3*A*a^2*x^3
```

Sympy [A]
time = 0.01, size = 54, normalized size = 1.29

$$\frac{Aa^2x^3}{3} + \frac{Bb^2x^{12}}{12} + x^9\left(\frac{Ab^2}{9} + \frac{2Bab}{9}\right) + x^6\left(\frac{Aab}{3} + \frac{Ba^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x**3/3 + B*b**2*x**12/12 + x**9*(A*b**2/9 + 2*B*a*b/9) + x**6*(A*a*b
/3 + B*a**2/6)
```

Giac [A]
time = 0.54, size = 53, normalized size = 1.26

$$\frac{1}{12}\,Bb^2x^{12} + \frac{2}{9}\,Babx^9 + \frac{1}{9}\,Ab^2x^9 + \frac{1}{6}\,Ba^2x^6 + \frac{1}{3}\,Aabx^6 + \frac{1}{3}\,Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/12*B*b^2*x^12 + 2/9*B*a*b*x^9 + 1/9*A*b^2*x^9 + 1/6*B*a^2*x^6 + 1/3*A*a*b
*x^6 + 1/3*A*a^2*x^3
```

Mupad [B]
time = 2.38, size = 51, normalized size = 1.21

$$x^6\left(\frac{B\,a^2}{6} + \frac{A\,b\,a}{3}\right) + x^9\left(\frac{A\,b^2}{9} + \frac{2\,B\,a\,b}{9}\right) + \frac{A\,a^2\,x^3}{3} + \frac{B\,b^2\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^6*((B*a^2)/6 + (A*a*b)/3) + x^9*((A*b^2)/9 + (2*B*a*b)/9) + (A*a^2*x^3)/3
 + (B*b^2*x^12)/12
```

3.12 $\int x(a+bx^3)^2\,(A+Bx^3)\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}a^2Ax^2+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{8}b(Ab+2aB)x^8+\frac{1}{11}b^2Bx^{11}$$

```
[Out] 1/2*a^2*A*x^2+1/5*a*(2*A*b+B*a)*x^5+1/8*b*(A*b+2*B*a)*x^8+1/11*b^2*B*x^11
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*
x^11)/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx &= \int \left(a^2Ax+a(2Ab+aB)x^4+b(Ab+2aB)x^7+b^2Bx^{10}\right)\,dx\\ &= \frac{1}{2}a^2Ax^2+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{8}b(Ab+2aB)x^8+\frac{1}{11}b^2Bx^{11}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 55, normalized size = 1.00

$$\frac{1}{2}a^2Ax^2+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{8}b(Ab+2aB)x^8+\frac{1}{11}b^2Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*
x^11)/11
```

Maple [A]
time = 0.31, size = 52, normalized size = 0.95

method	result	size
default	$\frac{b^{2} B \,x^{11}}{11}+\frac{\left(b^{2} A +2 a b B \right) x^{8}}{8}+\frac{\left(2 a b A +a^{2} B \right) x^{5}}{5}+\frac{a^{2} A \,x^{2}}{2}$	52
norman	$\frac{b^{2} B \,x^{11}}{11}+\left(\frac{1}{8} b^{2} A +\frac{1}{4} a b B \right) x^{8}+\left(\frac{2}{5} a b A +\frac{1}{5} a^{2} B \right) x^{5}+\frac{a^{2} A \,x^{2}}{2}$	52
gosper	$\frac{1}{11} b^{2} B \,x^{11}+\frac{1}{8} x^{8} b^{2} A +\frac{1}{4} x^{8} a b B +\frac{2}{5} x^{5} a b A +\frac{1}{5} a^{2} B \,x^{5}+\frac{1}{2} a^{2} A \,x^{2}$	54
risch	$\frac{1}{11} b^{2} B \,x^{11}+\frac{1}{8} x^{8} b^{2} A +\frac{1}{4} x^{8} a b B +\frac{2}{5} x^{5} a b A +\frac{1}{5} a^{2} B \,x^{5}+\frac{1}{2} a^{2} A \,x^{2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*B*x^11+1/8*(A*b^2+2*B*a*b)*x^8+1/5*(2*A*a*b+B*a^2)*x^5+1/2*a^2*A*x
^2
```

Maxima [A]
time = 0.31, size = 51, normalized size = 0.93

$$\frac{1}{11} \, B b^{2} x^{11} + \frac{1}{8} \, {\left(2 \, B a b + A b^{2}\right)} x^{8} + \frac{1}{5} \, {\left(B a^{2} + 2 \, A a b\right)} x^{5} + \frac{1}{2} \, A a^{2} x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 + 1
/2*A*a^2*x^2
```

Fricas [A]
time = 2.01, size = 51, normalized size = 0.93

$$\frac{1}{11} \, B b^{2} x^{11} + \frac{1}{8} \, {\left(2 \, B a b + A b^{2}\right)} x^{8} + \frac{1}{5} \, {\left(B a^{2} + 2 \, A a b\right)} x^{5} + \frac{1}{2} \, A a^{2} x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 + 1
/2*A*a^2*x^2
```

Sympy [A]
time = 0.01, size = 54, normalized size = 0.98

$$\frac{Aa^{2}x^{2}}{2} + \frac{Bb^{2}x^{11}}{11} + x^{8}\left(\frac{Ab^{2}}{8} + \frac{Bab}{4}\right) + x^{5}\cdot\left(\frac{2Aab}{5} + \frac{Ba^{2}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x**2/2 + B*b**2*x**11/11 + x**8*(A*b**2/8 + B*a*b/4) + x**5*(2*A*a*b
/5 + B*a**2/5)
```

Giac [A]
time = 1.01, size = 53, normalized size = 0.96

$$\frac{1}{11}Bb^2x^{11} + \frac{1}{4}Babx^8 + \frac{1}{8}Ab^2x^8 + \frac{1}{5}Ba^2x^5 + \frac{2}{5}Aabx^5 + \frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/11*B*b^2*x^11 + 1/4*B*a*b*x^8 + 1/8*A*b^2*x^8 + 1/5*B*a^2*x^5 + 2/5*A*a*b
*x^5 + 1/2*A*a^2*x^2
```

Mupad [B]
time = 0.04, size = 51, normalized size = 0.93

$$x^5\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^8\left(\frac{A\,b^2}{8}+\frac{B\,a\,b}{4}\right)+\frac{A\,a^2\,x^2}{2}+\frac{B\,b^2\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^8*((A*b^2)/8 + (B*a*b)/4) + (A*a^2*x^2)/2
 + (B*b^2*x^11)/11
```

3.13 $\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=50

$$a^2Ax+\frac{1}{4}a(2Ab+aB)x^4+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{10}b^2Bx^{10}$$

```
[Out] a^2*A*x+1/4*a*(2*A*b+B*a)*x^4+1/7*b*(A*b+2*B*a)*x^7+1/10*b^2*B*x^10
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$a^2Ax+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{10}b^2Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/
10
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx &= \int \left(a^2A+a(2Ab+aB)x^3+b(Ab+2aB)x^6+b^2Bx^9\right) \, dx\\ &= a^2Ax+\frac{1}{4}a(2Ab+aB)x^4+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{10}b^2Bx^{10}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 50, normalized size = 1.00

$$a^2Ax+\frac{1}{4}a(2Ab+aB)x^4+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{10}b^2Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/
10
```

Maple [A]
time = 0.30, size = 49, normalized size = 0.98

method	result	size
default	$\frac{b^2 B\,x^{10}}{10}+\frac{(b^2A+2abB)x^7}{7}+\frac{(2abA+a^2B)x^4}{4}+a^2Ax$	49
norman	$\frac{b^2 B\,x^{10}}{10}+\left(\frac{1}{7}b^2A+\frac{2}{7}abB\right)x^7+\left(\frac{1}{2}abA+\frac{1}{4}a^2B\right)x^4+a^2Ax$	49
gosper	$\frac{1}{10}b^2B\,x^{10}+\frac{1}{7}x^7b^2A+\frac{2}{7}x^7abB+\frac{1}{2}x^4abA+\frac{1}{4}a^2B\,x^4+a^2Ax$	51
risch	$\frac{1}{10}b^2B\,x^{10}+\frac{1}{7}x^7b^2A+\frac{2}{7}x^7abB+\frac{1}{2}x^4abA+\frac{1}{4}a^2B\,x^4+a^2Ax$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^2*B*x^10+1/7*(A*b^2+2*B*a*b)*x^7+1/4*(2*A*a*b+B*a^2)*x^4+a^2*A*x
```

Maxima [A]
time = 0.30, size = 48, normalized size = 0.96

$$\frac{1}{10}\,Bb^2x^{10}+\frac{1}{7}\,\left(2\,Bab+Ab^2\right)x^7+\frac{1}{4}\,\left(Ba^2+2\,Aab\right)x^4+Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 + A
*a^2*x
```

Fricas [A]
time = 1.63, size = 48, normalized size = 0.96

$$\frac{1}{10}\,Bb^2x^{10}+\frac{1}{7}\,\left(2\,Bab+Ab^2\right)x^7+\frac{1}{4}\,\left(Ba^2+2\,Aab\right)x^4+Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 + A
*a^2*x
```

Sympy [A]
time = 0.01, size = 51, normalized size = 1.02

$$Aa^2x+\frac{Bb^2x^{10}}{10}+x^7\left(\frac{Ab^2}{7}+\frac{2Bab}{7}\right)+x^4\left(\frac{Aab}{2}+\frac{Ba^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x + B*b**2*x**10/10 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**4*(A*a*b/2 +
B*a**2/4)
```

Giac [A]
time = 0.78, size = 50, normalized size = 1.00

$$\frac{1}{10}\,Bb^2x^{10}+\frac{2}{7}\,Babx^7+\frac{1}{7}\,Ab^2x^7+\frac{1}{4}\,Ba^2x^4+\frac{1}{2}\,Aabx^4+Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/10*B*b^2*x^10 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/4*B*a^2*x^4 + 1/2*A*a*b
*x^4 + A*a^2*x
```

Mupad [B]
time = 0.04, size = 48, normalized size = 0.96

$$x^4\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^7\left(\frac{A\,b^2}{7}+\frac{2\,B\,a\,b}{7}\right)+\frac{B\,b^2\,x^{10}}{10}+A\,a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^4*((B*a^2)/4 + (A*a*b)/2) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (B*b^2*x^10)/
10 + A*a^2*x
```

3.14 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x}\,dx$

Optimal. Leaf size=46

$$\frac{2}{3}aAbx^3+\frac{1}{6}Ab^2x^6+\frac{B(a+bx^3)^3}{9b}+a^2A\log(x)$$

```
[Out] 2/3*a*A*b*x^3+1/6*A*b^2*x^6+1/9*B*(b*x^3+a)^3/b+a^2*A*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {457, 81, 45}

$$a^2A\log(x)+\frac{2}{3}aAbx^3+\frac{B(a+bx^3)^3}{9b}+\frac{1}{6}Ab^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

```
[Out] (2*a*A*b*x^3)/3 + (A*b^2*x^6)/6 + (B*(a + b*x^3)^3)/(9*b) + a^2*A*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x} dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x} dx, x, x^3\right)$$
$$= \frac{B(a+bx^3)^3}{9b} + \frac{1}{3}A\text{Subst}\left(\int \frac{(a+bx)^2}{x} dx, x, x^3\right)$$
$$= \frac{B(a+bx^3)^3}{9b} + \frac{1}{3}A\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right) dx, x, x^3\right)$$
$$= \frac{2}{3}aAbx^3 + \frac{1}{6}Ab^2x^6 + \frac{B(a+bx^3)^3}{9b} + a^2A\log(x)$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.11

$$\frac{1}{3}a(2Ab+aB)x^3 + \frac{1}{6}b(Ab+2aB)x^6 + \frac{1}{9}b^2Bx^9 + a^2A\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

```
[Out] (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B*x^9)/9 + a^2*A*L
og[x]
```

Maple [A]
time = 0.25, size = 52, normalized size = 1.13

method	result	size
norman	$\left(\frac{1}{6}b^2A+\frac{1}{3}abB\right)x^6+\left(\frac{2}{3}abA+\frac{1}{3}a^2B\right)x^3+\frac{b^2Bx^9}{9}+a^2A\ln(x)$	50
default	$\frac{b^2Bx^9}{9}+\frac{Ab^2x^6}{6}+\frac{Babx^6}{3}+\frac{2aAbx^3}{3}+\frac{a^2Bx^3}{3}+a^2A\ln(x)$	52
risch	$\frac{b^2Bx^9}{9}+\frac{Ab^2x^6}{6}+\frac{Babx^6}{3}+\frac{2aAbx^3}{3}+\frac{a^2Bx^3}{3}+a^2A\ln(x)$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^2*B*x^9+1/6*A*b^2*x^6+1/3*B*a*b*x^6+2/3*a*A*b*x^3+1/3*a^2*B*x^3+a^2*A
*ln(x)
```

Maxima [A]
time = 0.27, size = 52, normalized size = 1.13

$$\frac{1}{9}Bb^2x^9 + \frac{1}{6}\left(2Bab+Ab^2\right)x^6 + \frac{1}{3}\left(Ba^2+2Aab\right)x^3 + \frac{1}{3}Aa^2\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + 1/3
*A*a^2*log(x^3)
```

Fricas [**A**]
time = 2.23, size = 49, normalized size = 1.07

$$\frac{1}{9}Bb^2x^9 + \frac{1}{6}\left(2Bab + Ab^2\right)x^6 + \frac{1}{3}\left(Ba^2 + 2Aab\right)x^3 + Aa^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] 1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + A*a
^2*log(x)
```

Sympy [**A**]
time = 0.05, size = 53, normalized size = 1.15

$$Aa^2\log\left(x\right) + \frac{Bb^2x^9}{9} + x^6\left(\frac{Ab^2}{6} + \frac{Bab}{3}\right) + x^3\cdot\left(\frac{2Aab}{3} + \frac{Ba^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*b**2*x**9/9 + x**6*(A*b**2/6 + B*a*b/3) + x**3*(2*A*a*b/3
 + B*a**2/3)
```

Giac [**A**]
time = 0.61, size = 52, normalized size = 1.13

$$\frac{1}{9}Bb^2x^9 + \frac{1}{3}Babx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{3}Ba^2x^3 + \frac{2}{3}Aabx^3 + Aa^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 1/9*B*b^2*x^9 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*B*a^2*x^3 + 2/3*A*a*b*x
^3 + A*a^2*log(abs(x))
```

Mupad [**B**]
time = 0.04, size = 49, normalized size = 1.07

$$x^3\left(\frac{B\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right) + x^6\left(\frac{A\,b^2}{6} + \frac{B\,a\,b}{3}\right) + \frac{B\,b^2\,x^9}{9} + A\,a^2\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x,x)
```

```
[Out] x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^6*((A*b^2)/6 + (B*a*b)/3) + (B*b^2*x^9)/9
 + A*a^2*log(x)
```

3.15 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2} dx$

Optimal. Leaf size=53

$$-\frac{a^2A}{x}+\frac{1}{2}a(2Ab+aB)x^2+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{8}b^2Bx^8$$

```
[Out] -a^2*A/x+1/2*a*(2*A*b+B*a)*x^2+1/5*b*(A*b+2*B*a)*x^5+1/8*b^2*B*x^8
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$-\frac{a^2A}{x}+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{8}b^2Bx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x
^8)/8
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2}\,dx &= \int \left(\frac{a^2A}{x^2}+a(2Ab+aB)x+b(Ab+2aB)x^4+b^2Bx^7\right)dx\\ &= -\frac{a^2A}{x}+\frac{1}{2}a(2Ab+aB)x^2+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{8}b^2Bx^8\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.00

$$-\frac{a^2A}{x}+\frac{1}{2}a(2Ab+aB)x^2+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{8}b^2Bx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x
^8)/8
```

Maple [A]
time = 0.25, size = 53, normalized size = 1.00

method	result	size
norman	$\frac{\frac{b^{2} B \,x^{9}}{8}+\left(\frac{1}{5} b^{2} A +\frac{2}{5} a b B \right) x^{6}+\left(a b A +\frac{1}{2} a^{2} B \right) x^{3}-a^{2} A}{x}$	52
default	$\frac{b^{2} B \,x^{8}}{8}+\frac{A \,b^{2} x^{5}}{5}+\frac{2 B a b \,x^{5}}{5}+a A b \,x^{2}+\frac{B \,a^{2} x^{2}}{2}-\frac{a^{2} A}{x}$	53
risch	$\frac{b^{2} B \,x^{8}}{8}+\frac{A \,b^{2} x^{5}}{5}+\frac{2 B a b \,x^{5}}{5}+a A b \,x^{2}+\frac{B \,a^{2} x^{2}}{2}-\frac{a^{2} A}{x}$	53
gosper	$-\frac{-5 b^{2} B \,x^{9}-8 A \,b^{2} x^{6}-16 B a b \,x^{6}-40 a A b \,x^{3}-20 a^{2} B \,x^{3}+40 a^{2} A}{40 x}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^2*B*x^8+1/5*A*b^2*x^5+2/5*B*a*b*x^5+a*A*b*x^2+1/2*B*a^2*x^2-a^2*A/x
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.96

$$\frac{1}{8} \, B b^{2} x^{8} + \frac{1}{5} \, {\left(2 \, B a b + A b^{2}\right)} x^{5} + \frac{1}{2} \, {\left(B a^{2} + 2 \, A a b\right)} x^{2} - \frac{A a^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^2*x^8 + 1/5*(2*B*a*b + A*b^2)*x^5 + 1/2*(B*a^2 + 2*A*a*b)*x^2 - A*a
^2/x
```

Fricas [A]
time = 2.42, size = 53, normalized size = 1.00

$$\frac{5 \, B b^{2} x^{9} + 8 \, {\left(2 \, B a b + A b^{2}\right)} x^{6} + 20 \, {\left(B a^{2} + 2 \, A a b\right)} x^{3} - 40 \, A a^{2}}{40 \, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*B*b^2*x^9 + 8*(2*B*a*b + A*b^2)*x^6 + 20*(B*a^2 + 2*A*a*b)*x^3 - 40
*A*a^2)/x
```

Sympy [A]
time = 0.05, size = 49, normalized size = 0.92

$$- \frac{A a^{2}}{x} + \frac{B b^{2} x^{8}}{8} + x^{5} \left(\frac{A b^{2}}{5} + \frac{2 B a b}{5}\right) + x^{2} \left(A a b + \frac{B a^{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*b**2*x**8/8 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**2*(A*a*b + B*a
**2/2)
```

Giac [A]
time = 0.63, size = 52, normalized size = 0.98

$$\frac{1}{8}Bb^2x^8+\frac{2}{5}Babx^5+\frac{1}{5}Ab^2x^5+\frac{1}{2}Ba^2x^2+Aabx^2-\frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*B*b^2*x^8 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/2*B*a^2*x^2 + A*a*b*x^2 -
 A*a^2/x
```

Mupad [B]
time = 0.05, size = 50, normalized size = 0.94

$$x^2\left(\frac{B\,a^2}{2}+A\,b\,a\right)+x^5\left(\frac{A\,b^2}{5}+\frac{2\,B\,a\,b}{5}\right)-\frac{A\,a^2}{x}+\frac{B\,b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^2,x)
```

```
[Out] x^2*((B*a^2)/2 + A*a*b) + x^5*((A*b^2)/5 + (2*B*a*b)/5) - (A*a^2)/x + (B*b^
2*x^8)/8
```

3.16 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3} dx$

Optimal. Leaf size=50

$$-\frac{a^2A}{2x^2}+a(2Ab+aB)x+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{7}b^2Bx^7$$

```
[Out] -1/2*a^2*A/x^2+a*(2*A*b+B*a)*x+1/4*b*(A*b+2*B*a)*x^4+1/7*b^2*B*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$-\frac{a^2A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a^2*A)/x^2 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7
)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx &= \int \left(a(2Ab+aB)+\frac{a^2A}{x^3}+b(Ab+2aB)x^3+b^2Bx^6\right)dx\\ &= -\frac{a^2A}{2x^2}+a(2Ab+aB)x+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{7}b^2Bx^7\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$-\frac{a^2A}{2x^2}+a(2Ab+aB)x+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a^2*A)/x^2 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7
)/7
```

Maple [A]
time = 0.24, size = 49, normalized size = 0.98

method	result	size
default	$\frac{b^2 B x^7}{7}+\frac{A b^2 x^4}{4}+\frac{B a b x^4}{2}+2abAx+a^2Bx-\frac{a^2 A}{2x^2}$	49
risch	$\frac{b^2 B x^7}{7}+\frac{A b^2 x^4}{4}+\frac{B a b x^4}{2}+2abAx+a^2Bx-\frac{a^2 A}{2x^2}$	49
norman	$\frac{\frac{b^2 B x^9}{7}+\left(\frac{1}{4}b^2A+\frac{1}{2}abB\right)x^6+\left(2abA+a^2B\right)x^3-\frac{a^2 A}{2}}{x^2}$	52
gosper	$-\frac{-4b^2 B x^9-7A b^2 x^6-14Bab x^6-56aAb x^3-28a^2 B x^3+14a^2 A}{28x^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^2*B*x^7+1/4*A*b^2*x^4+1/2*B*a*b*x^4+2*a*b*A*x+a^2*B*x-1/2*a^2*A/x^2
```

Maxima [A]
time = 0.29, size = 48, normalized size = 0.96

$$\frac{1}{7}\,Bb^2x^7+\frac{1}{4}\,\left(2\,Bab+Ab^2\right)x^4+\left(Ba^2+2\,Aab\right)x-\frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^2*x^7 + 1/4*(2*B*a*b + A*b^2)*x^4 + (B*a^2 + 2*A*a*b)*x - 1/2*A*a^2
/x^2
```

Fricas [A]
time = 1.97, size = 53, normalized size = 1.06

$$\frac{4\,Bb^2x^9+7\,(2\,Bab+Ab^2)x^6+28\,(Ba^2+2\,Aab)x^3-14\,Aa^2}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/28*(4*B*b^2*x^9 + 7*(2*B*a*b + A*b^2)*x^6 + 28*(B*a^2 + 2*A*a*b)*x^3 - 14
*A*a^2)/x^2
```

Sympy [A]
time = 0.05, size = 49, normalized size = 0.98

$$-\frac{Aa^2}{2x^2}+\frac{Bb^2x^7}{7}+x^4\left(\frac{Ab^2}{4}+\frac{Bab}{2}\right)+x\left(2Aab+Ba^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) + B*b**2*x**7/7 + x**4*(A*b**2/4 + B*a*b/2) + x*(2*A*a*b +
 B*a**2)
```

Giac [A]
time = 0.75, size = 48, normalized size = 0.96

$$\frac{1}{7}\,Bb^2x^7+\frac{1}{2}\,Babx^4+\frac{1}{4}\,Ab^2x^4+Ba^2x+2\,Aabx-\frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*B*b^2*x^7 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + B*a^2*x + 2*A*a*b*x - 1/2*A
*a^2/x^2
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.96

$$x^4\left(\frac{A\,b^2}{4}+\frac{B\,a\,b}{2}\right)+x\left(B\,a^2+2\,A\,b\,a\right)-\frac{A\,a^2}{2\,x^2}+\frac{B\,b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^3,x)
```

```
[Out] x^4*((A*b^2)/4 + (B*a*b)/2) + x*(B*a^2 + 2*A*a*b) - (A*a^2)/(2*x^2) + (B*b^
2*x^7)/7
```

3.17 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{3x^3}+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{6}b^2Bx^6+a(2Ab+aB)\log(x)$$

```
[Out] -1/3*a^2*A/x^3+1/3*b*(A*b+2*B*a)*x^3+1/6*b^2*B*x^6+a*(2*A*b+B*a)*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^2A}{3x^3}+\frac{1}{3}bx^3(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^6)/6 + a*(2*A*b + a*B
)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^2} dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(b(Ab+2aB) + \frac{a^2A}{x^2} + \frac{a(2Ab+aB)}{x} + b^2Bx\right) dx, x, x^3\right)$$
$$= -\frac{a^2A}{3x^3} + \frac{1}{3}b(Ab+2aB)x^3 + \frac{1}{6}b^2Bx^6 + a(2Ab+aB)\log(x)$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.96

$$\frac{1}{6}\left(-\frac{2a^2A}{x^3} + 2b(Ab+2aB)x^3 + b^2Bx^6 + 6a(2Ab+aB)\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
[Out] ((-2*a^2*A)/x^3 + 2*b*(A*b + 2*a*B)*x^3 + b^2*B*x^6 + 6*a*(2*A*b + a*B)*Log
[x])/6
```

Maple [A]
time = 0.30, size = 49, normalized size = 0.96

method	result	size
default	$\frac{b^2Bx^6}{6} + \frac{Ab^2x^3}{3} + \frac{2Babx^3}{3} - \frac{a^2A}{3x^3} + a(2Ab+Ba)\ln(x)$	49
norman	$\frac{\left(\frac{1}{3}b^2A+\frac{2}{3}abB\right)x^6 - \frac{a^2A}{3} + \frac{b^2Bx^9}{6}}{x^3} + (2abA+a^2B)\ln(x)$	52
risch	$\frac{b^2Bx^6}{6} + \frac{Ab^2x^3}{3} + \frac{2Babx^3}{3} + \frac{A^2b^2}{6B} + \frac{2abA}{3} + \frac{2a^2B}{3} - \frac{a^2A}{3x^3} + 2A\ln(x)ab + a^2B\ln(x)$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/6*b^2*B*x^6+1/3*A*b^2*x^3+2/3*B*a*b*x^3-1/3*a^2*A/x^3+a*(2*A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.28, size = 52, normalized size = 1.02

$$\frac{1}{6}Bb^2x^6 + \frac{1}{3}(2Bab+Ab^2)x^3 + \frac{1}{3}(Ba^2+2Aab)\log(x^3) - \frac{Aa^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^2*x^6 + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/3*(B*a^2 + 2*A*a*b)*log(x^3)
- 1/3*A*a^2/x^3
```

Fricas [A]
time = 1.90, size = 54, normalized size = 1.06

$$\frac{Bb^2x^9 + 2\,(2\,Bab + Ab^2)x^6 + 6\,(Ba^2 + 2\,Aab)x^3\log{(x)} - 2\,Aa^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^2*x^9 + 2*(2*B*a*b + A*b^2)*x^6 + 6*(B*a^2 + 2*A*a*b)*x^3*log(x) -
 2*A*a^2)/x^3
```

Sympy [A]
time = 0.12, size = 51, normalized size = 1.00

$$-\frac{Aa^2}{3x^3} + \frac{Bb^2x^6}{6} + a(2Ab + Ba)\log{(x)} + x^3\left(\frac{Ab^2}{3} + \frac{2Bab}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**4,x)
```

```
[Out] -A*a**2/(3*x**3) + B*b**2*x**6/6 + a*(2*A*b + B*a)*log(x) + x**3*(A*b**2/3
+ 2*B*a*b/3)
```

Giac [A]
time = 0.66, size = 69, normalized size = 1.35

$$\frac{1}{6}\,Bb^2x^6 + \frac{2}{3}\,Babx^3 + \frac{1}{3}\,Ab^2x^3 + \left(Ba^2 + 2\,Aab\right)\log{(|x|)} - \frac{Ba^2x^3 + 2\,Aabx^3 + Aa^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*B*b^2*x^6 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + (B*a^2 + 2*A*a*b)*log(abs(x
)) - 1/3*(B*a^2*x^3 + 2*A*a*b*x^3 + A*a^2)/x^3
```

Mupad [B]
time = 0.04, size = 49, normalized size = 0.96

$$x^3\,\left(\frac{A\,b^2}{3} + \frac{2\,B\,a\,b}{3}\right) + \ln{(x)}\,\left(B\,a^2 + 2\,A\,b\,a\right) - \frac{A\,a^2}{3\,x^3} + \frac{B\,b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^4,x)
```

```
[Out] x^3*((A*b^2)/3 + (2*B*a*b)/3) + log(x)*(B*a^2 + 2*A*a*b) - (A*a^2)/(3*x^3)
+ (B*b^2*x^6)/6
```

3.18 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{x}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{5}b^2Bx^5$$

```
[Out] -1/4*a^2*A/x^4-a*(2*A*b+B*a)/x+1/2*b*(A*b+2*B*a)*x^2+1/5*b^2*B*x^5
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^2A}{4x^4}+\frac{1}{2}bx^2(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{5}b^2Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a^2*A)/x^4 - (a*(2*A*b + a*B))/x + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x
^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx &= \int \left(\frac{a^2A}{x^5}+\frac{a(2Ab+aB)}{x^2}+b(Ab+2aB)x+b^2Bx^4\right)dx\\ &= -\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{x}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{5}b^2Bx^5\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 0.96

$$\frac{-5a^2A-20a(2Ab+aB)x^3+10b(Ab+2aB)x^6+4b^2Bx^9}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

```
[Out] (-5*a^2*A - 20*a*(2*A*b + a*B)*x^3 + 10*b*(A*b + 2*a*B)*x^6 + 4*b^2*B*x^9)/
(20*x^4)
```

Maple [A]
time = 0.25, size = 50, normalized size = 0.94

method	result	size
default	$\frac{b^2 B x^5}{5}+\frac{A b^2 x^2}{2}+B a b x^2-\frac{a^2 A}{4 x^4}-\frac{a(2 A b+B a)}{x}$	50
norman	$\frac{\frac{b^2 B x^9}{5}+\left(\frac{1}{2} b^2 A+a b B\right) x^6+\left(-2 a b A-a^2 B\right) x^3-\frac{a^2 A}{4}}{x^4}$	52
risch	$\frac{b^2 B x^5}{5}+\frac{A b^2 x^2}{2}+B a b x^2+\frac{\left(-2 a b A-a^2 B\right) x^3-\frac{a^2 A}{4}}{x^4}$	54
gosper	$-\frac{-4 b^2 B x^9-10 A b^2 x^6-20 B a b x^6+40 a A b x^3+20 a^2 B x^3+5 a^2 A}{20 x^4}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^2*B*x^5+1/2*A*b^2*x^2+B*a*b*x^2-1/4*a^2*A/x^4-a*(2*A*b+B*a)/x
```

Maxima [A]
time = 0.28, size = 53, normalized size = 1.00

$$\frac{1}{5} B b^2 x^5+\frac{1}{2}\left(2 B a b+A b^2\right) x^2-\frac{4\left(B a^2+2 A a b\right) x^3+A a^2}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^2*x^5 + 1/2*(2*B*a*b + A*b^2)*x^2 - 1/4*(4*(B*a^2 + 2*A*a*b)*x^3 +
A*a^2)/x^4
```

Fricas [A]
time = 1.87, size = 53, normalized size = 1.00

$$\frac{4 B b^2 x^9+10\left(2 B a b+A b^2\right) x^6-20\left(B a^2+2 A a b\right) x^3-5 A a^2}{20 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/20*(4*B*b^2*x^9 + 10*(2*B*a*b + A*b^2)*x^6 - 20*(B*a^2 + 2*A*a*b)*x^3 - 5
*A*a^2)/x^4
```

Sympy [A]
time = 0.15, size = 53, normalized size = 1.00

$$\frac{Bb^2x^5}{5} + x^2\left(\frac{Ab^2}{2} + Bab\right) + \frac{-Aa^2 + x^3(-8Aab - 4Ba^2)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**5,x)
```

```
[Out] B*b**2*x**5/5 + x**2*(A*b**2/2 + B*a*b) + (-A*a**2 + x**3*(-8*A*a*b - 4*B*a
**2))/(4*x**4)
```

Giac [A]
time = 0.63, size = 54, normalized size = 1.02

$$\frac{1}{5}\,Bb^2x^5 + Babx^2 + \frac{1}{2}\,Ab^2x^2 - \frac{4\,Ba^2x^3 + 8\,Aabx^3 + Aa^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/5*B*b^2*x^5 + B*a*b*x^2 + 1/2*A*b^2*x^2 - 1/4*(4*B*a^2*x^3 + 8*A*a*b*x^3
+ A*a^2)/x^4
```

Mupad [B]
time = 0.05, size = 52, normalized size = 0.98

$$x^2\left(\frac{A\,b^2}{2} + B\,a\,b\right) - \frac{x^3\,\left(B\,a^2 + 2\,A\,b\,a\right) + \frac{A\,a^2}{4}}{x^4} + \frac{B\,b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^5,x)
```

```
[Out] x^2*((A*b^2)/2 + B*a*b) - (x^3*(B*a^2 + 2*A*a*b) + (A*a^2)/4)/x^4 + (B*b^2*
x^5)/5
```

3.19 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6}\,dx$

Optimal. Leaf size=50

$$-\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{2x^2}+b(Ab+2aB)x+\frac{1}{4}b^2Bx^4$$

```
[Out] -1/5*a^2*A/x^5-1/2*a*(2*A*b+B*a)/x^2+b*(A*b+2*B*a)*x+1/4*b^2*B*x^4
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{2x^2}+bx(2aB+Ab)+\frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x
^4)/4
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6}\,dx &= \int \left(b(Ab+2aB)+\frac{a^2A}{x^6}+\frac{a(2Ab+aB)}{x^3}+b^2Bx^3\right)dx\\ &= -\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{2x^2}+b(Ab+2aB)x+\frac{1}{4}b^2Bx^4\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$-\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{2x^2}+b(Ab+2aB)x+\frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^6,x]

[Out] -1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x^4)/4

Maple [A]
time = 0.25, size = 46, normalized size = 0.92

method	result	size
default	$\frac{b^2 B x^4}{4}+b^2 A x+2 a b B x-\frac{a^2 A}{5 x^5}-\frac{a(2 A b+B a)}{2 x^2}$	46
risch	$\frac{b^2 B x^4}{4}+b^2 A x+2 a b B x+\frac{\left(-a b A-\frac{1}{2} a^2 B\right) x^3-\frac{a^2 A}{5}}{x^5}$	50
norman	$\frac{\frac{b^2 B x^9}{4}+\left(b^2 A+2 a b B\right) x^6+\left(-a b A-\frac{1}{2} a^2 B\right) x^3-\frac{a^2 A}{5}}{x^5}$	52
gosper	$-\frac{-5 b^2 B x^9-20 A b^2 x^6-40 B a b x^6+20 a A b x^3+10 a^2 B x^3+4 a^2 A}{20 x^5}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^2*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)

[Out] 1/4*b^2*B*x^4+b^2*A*x+2*a*b*B*x-1/5*a^2*A/x^5-1/2*a*(2*A*b+B*a)/x^2

Maxima [A]
time = 0.30, size = 51, normalized size = 1.02

$$\frac{1}{4} B b^2 x^4+\left(2 B a b+A b^2\right) x-\frac{5 \left(B a^2+2 A a b\right) x^3+2 A a^2}{10 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="maxima")

[Out] 1/4*B*b^2*x^4 + (2*B*a*b + A*b^2)*x - 1/10*(5*(B*a^2 + 2*A*a*b)*x^3 + 2*A*a^2)/x^5

Fricas [A]
time = 2.00, size = 53, normalized size = 1.06

$$\frac{5 B b^2 x^9+20 \left(2 B a b+A b^2\right) x^6-10 \left(B a^2+2 A a b\right) x^3-4 A a^2}{20 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="fricas")

[Out] 1/20*(5*B*b^2*x^9 + 20*(2*B*a*b + A*b^2)*x^6 - 10*(B*a^2 + 2*A*a*b)*x^3 - 4*A*a^2)/x^5

Sympy [A]
time = 0.17, size = 53, normalized size = 1.06

$$\frac{B b^2 x^4}{4}+x \left(A b^2+2 B a b\right)+\frac{-2 A a^2+x^3 \left(-10 A a b-5 B a^2\right)}{10 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**6,x)
```

```
[Out] B*b**2*x**4/4 + x*(A*b**2 + 2*B*a*b) + (-2*A*a**2 + x**3*(-10*A*a*b - 5*B*a
**2))/(10*x**5)
```

Giac [A]
time = 0.53, size = 51, normalized size = 1.02

$$\frac{1}{4}\,Bb^2x^4 + 2\,Babx + Ab^2x - \frac{5\,Ba^2x^3 + 10\,Aabx^3 + 2\,Aa^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*B*b^2*x^4 + 2*B*a*b*x + A*b^2*x - 1/10*(5*B*a^2*x^3 + 10*A*a*b*x^3 + 2*
A*a^2)/x^5
```

Mupad [B]
time = 2.37, size = 50, normalized size = 1.00

$$x\left(A\,b^2+2\,B\,a\,b\right)-\frac{x^3\,\left(\frac{B\,a^2}{2}+A\,b\,a\right)+\frac{A\,a^2}{5}}{x^5}+\frac{B\,b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^6,x)
```

```
[Out] x*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/2 + A*a*b) + (A*a^2)/5)/x^5 + (B*b^2*x^
4)/4
```

3.20 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7} dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{6x^6}-\frac{a(2Ab+aB)}{3x^3}+\frac{1}{3}b^2Bx^3+b(Ab+2aB)\log(x)$$

```
[Out] -1/6*a^2*A/x^6-1/3*a*(2*A*b+B*a)/x^3+1/3*b^2*B*x^3+b*(A*b+2*B*a)*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{3x^3}+b\log(x)(2aB+Ab)+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

```
[Out] -1/6*(a^2*A)/x^6 - (a*(2*A*b + a*B))/(3*x^3) + (b^2*B*x^3)/3 + b*(A*b + 2*a
*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(b^2B + \frac{a^2A}{x^3} + \frac{a(2Ab+aB)}{x^2} + \frac{b(Ab+2aB)}{x}\right)\,dx, x, x^3\right)$$
$$= -\frac{a^2A}{6x^6} - \frac{a(2Ab+aB)}{3x^3} + \frac{1}{3}b^2Bx^3 + b(Ab+2aB)\log(x)$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.00

$$\frac{1}{6}\left(-\frac{4aAb}{x^3} + 2b^2Bx^3 - \frac{a^2(A+2Bx^3)}{x^6} + 6b(Ab+2aB)\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
[Out] ((-4*a*A*b)/x^3 + 2*b^2*B*x^3 - (a^2*(A + 2*B*x^3))/x^6 + 6*b*(A*b + 2*a*B)
*Log[x])/6
```

Maple [A]
time = 0.25, size = 46, normalized size = 0.90

method	result	size
default	$-\frac{a^2A}{6x^6} - \frac{a(2Ab+Ba)}{3x^3} + \frac{b^2Bx^3}{3} + b(Ab+2Ba)\ln(x)$	46
norman	$\frac{(-\frac{2}{3}abA-\frac{1}{3}a^2B)x^3 - \frac{a^2A}{6} + \frac{b^2Bx^9}{3}}{x^6} + (b^2A+2abB)\ln(x)$	52
risch	$\frac{b^2Bx^3}{3} + \frac{(-\frac{2}{3}abA-\frac{1}{3}a^2B)x^3 - \frac{a^2A}{6}}{x^6} + A\ln(x)b^2 + 2B\ln(x)ab$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*a^2*A/x^6-1/3*a*(2*A*b+B*a)/x^3+1/3*b^2*B*x^3+b*(A*b+2*B*a)*ln(x)
```

Maxima [A]
time = 0.29, size = 54, normalized size = 1.06

$$\frac{1}{3}Bb^2x^3 + \frac{1}{3}\left(2Bab + Ab^2\right)\log\left(x^3\right) - \frac{2\left(Ba^2 + 2Aab\right)x^3 + Aa^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^2*x^3 + 1/3*(2*B*a*b + A*b^2)*log(x^3) - 1/6*(2*(B*a^2 + 2*A*a*b)*x
^3 + A*a^2)/x^6
```

Fricas [A]
time = 1.97, size = 55, normalized size = 1.08

$$\frac{2 Bb^2x^9 + 6 (2 Bab + Ab^2)x^6 \log (x) - 2 (Ba^2 + 2 Aab)x^3 - Aa^2}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*b^2*x^9 + 6*(2*B*a*b + A*b^2)*x^6*log(x) - 2*(B*a^2 + 2*A*a*b)*x^3
 - A*a^2)/x^6
```

Sympy [A]
time = 0.40, size = 51, normalized size = 1.00

$$\frac{Bb^2x^3}{3} + b(Ab + 2Ba) \log (x) + \frac{-Aa^2 + x^3(-4Aab - 2Ba^2)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**7,x)
```

```
[Out] B*b**2*x**3/3 + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x**3*(-4*A*a*b - 2*B*a*
*2))/(6*x**6)
```

Giac [A]
time = 0.54, size = 70, normalized size = 1.37

$$\frac{1}{3} Bb^2x^3 + \left(2 Bab + Ab^2\right) \log (|x|) - \frac{6 Babx^6 + 3 Ab^2x^6 + 2 Ba^2x^3 + 4 Aabx^3 + Aa^2}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*log(abs(x)) - 1/6*(6*B*a*b*x^6 + 3*A*b^2*
x^6 + 2*B*a^2*x^3 + 4*A*a*b*x^3 + A*a^2)/x^6
```

Mupad [B]
time = 2.36, size = 52, normalized size = 1.02

$$\ln (x)\, \left(A\, b^2 + 2\, B\, a\, b\right) - \frac{x^3\, \left(\frac{B\, a^2}{3} + \frac{2\, A\, b\, a}{3}\right) + \frac{A\, a^2}{6}}{x^6} + \frac{B\, b^2\, x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^7,x)
```

```
[Out] log(x)*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/6)/x^6
+ (B*b^2*x^3)/3
```

3.21 $\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8} dx$

Optimal. Leaf size=53

$$-\frac{a^2A}{7x^7}-\frac{a(2Ab+aB)}{4x^4}-\frac{b(Ab+2aB)}{x}+\frac{1}{2}b^2Bx^2$$

```
[Out] -1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{x}+\frac{1}{2}b^2Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

```
[Out] -1/7*(a^2*A)/x^7 - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/x + (b^2*B
*x^2)/2
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx &= \int \left(\frac{a^2A}{x^8}+\frac{a(2Ab+aB)}{x^5}+\frac{b(Ab+2aB)}{x^2}+b^2Bx\right)dx\\ &= -\frac{a^2A}{7x^7}-\frac{a(2Ab+aB)}{4x^4}-\frac{b(Ab+2aB)}{x}+\frac{1}{2}b^2Bx^2\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.02

$$-\frac{-14b^2x^6(-2A+Bx^3)+14abx^3(A+4Bx^3)+a^2(4A+7Bx^3)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

```
[Out] -1/28*(-14*b^2*x^6*(-2*A + B*x^3) + 14*a*b*x^3*(A + 4*B*x^3) + a^2*(4*A + 7
*B*x^3))/x^7
```

Maple [A]
time = 0.25, size = 48, normalized size = 0.91

method	result	size
default	$-\frac{a^{2} A}{7 x^{7}}-\frac{a(2 A b+B a)}{4 x^{4}}-\frac{b(A b+2 B a)}{x}+\frac{b^{2} B\, x^{2}}{2}$	48
norman	$\frac{\frac{b^{2} B\, x^{9}}{2}+\left(-b^{2} A-2 a b B\right) x^{6}+\left(-\frac{1}{2} a b A-\frac{1}{4} a^{2} B\right) x^{3}-\frac{a^{2} A}{7}}{x^{7}}$	53
risch	$\frac{b^{2} B\, x^{2}}{2}+\frac{\left(-b^{2} A-2 a b B\right) x^{6}+\left(-\frac{1}{2} a b A-\frac{1}{4} a^{2} B\right) x^{3}-\frac{a^{2} A}{7}}{x^{7}}$	54
gosper	$-\frac{-14 b^{2} B\, x^{9}+28 A\, b^{2} x^{6}+56 B a b\, x^{6}+14 a A b\, x^{3}+7 a^{2} B\, x^{3}+4 a^{2} A}{28 x^{7}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

Maxima [A]
time = 0.27, size = 54, normalized size = 1.02

$$\frac{1}{2}\, B b^{2} x^{2} - \frac{28\, {\left(2\, B a b + A b^{2}\right)} x^{6} + 7\, {\left(B a^{2} + 2\, A a b\right)} x^{3} + 4\, A a^{2}}{28\, x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*x^2 - 1/28*(28*(2*B*a*b + A*b^2)*x^6 + 7*(B*a^2 + 2*A*a*b)*x^3 +
4*A*a^2)/x^7
```

Fricas [A]
time = 1.75, size = 53, normalized size = 1.00

$$\frac{14\, B b^{2} x^{9} - 28\, {\left(2\, B a b + A b^{2}\right)} x^{6} - 7\, {\left(B a^{2} + 2\, A a b\right)} x^{3} - 4\, A a^{2}}{28\, x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(14*B*b^2*x^9 - 28*(2*B*a*b + A*b^2)*x^6 - 7*(B*a^2 + 2*A*a*b)*x^3 - 4
*A*a^2)/x^7
```

Sympy [A]
time = 0.49, size = 58, normalized size = 1.09

$$\frac{Bb^{2}x^{2}}{2} + \frac{-4Aa^{2} + x^{6}\left(-28Ab^{2} - 56Bab\right) + x^{3}\left(-14Aab - 7Ba^{2}\right)}{28x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**8,x)
```

```
[Out] B*b**2*x**2/2 + (-4*A*a**2 + x**6*(-28*A*b**2 - 56*B*a*b) + x**3*(-14*A*a*b
 - 7*B*a**2))/(28*x**7)
```

Giac [A]
time = 0.55, size = 56, normalized size = 1.06

$$\frac{1}{2}\,Bb^2x^2 - \frac{56\,Babx^6 + 28\,Ab^2x^6 + 7\,Ba^2x^3 + 14\,Aabx^3 + 4\,Aa^2}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*x^2 - 1/28*(56*B*a*b*x^6 + 28*A*b^2*x^6 + 7*B*a^2*x^3 + 14*A*a*b*
x^3 + 4*A*a^2)/x^7
```

Mupad [B]
time = 0.05, size = 53, normalized size = 1.00

$$\frac{B\,b^2\,x^2}{2} - \frac{x^3\,\left(\frac{B\,a^2}{4} + \frac{A\,b\,a}{2}\right) + x^6\,\left(A\,b^2 + 2\,B\,a\,b\right) + \frac{A\,a^2}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^8,x)
```

```
[Out] (B*b^2*x^2)/2 - (x^3*((B*a^2)/4 + (A*a*b)/2) + x^6*(A*b^2 + 2*B*a*b) + (A*a
^2)/7)/x^7
```

3.22 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx$

Optimal. Leaf size=50

$$-\frac{a^2A}{8x^8}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{2x^2}+b^2Bx$$

```
[Out] -1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$-\frac{a^2A}{8x^8}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{2x^2}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

```
[Out] -1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx &= \int \left(b^2B+\frac{a^2A}{x^9}+\frac{a(2Ab+aB)}{x^6}+\frac{b(Ab+2aB)}{x^3}\right)dx\\ &= -\frac{a^2A}{8x^8}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{2x^2}+b^2Bx\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$-\frac{a^2A}{8x^8}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{2x^2}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

```
[Out] -1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*x
```

Maple [A]
time = 0.26, size = 45, normalized size = 0.90

method	result	size
default	$-\frac{a^2 A}{8x^8}-\frac{a(2Ab+Ba)}{5x^5}-\frac{b(Ab+2Ba)}{2x^2}+b^2Bx$	45
risch	$b^2Bx+\frac{\left(-\frac{1}{2}b^2A-abB\right)x^6+\left(-\frac{2}{5}abA-\frac{1}{5}a^2B\right)x^3-\frac{a^2A}{8}}{x^8}$	51
norman	$\frac{b^2B\,x^9+\left(-\frac{1}{2}b^2A-abB\right)x^6+\left(-\frac{2}{5}abA-\frac{1}{5}a^2B\right)x^3-\frac{a^2A}{8}}{x^8}$	52
gosper	$-\frac{-40b^2B\,x^9+20A\,b^2x^6+40Bab\,x^6+16aAb\,x^3+8a^2B\,x^3+5a^2A}{40x^8}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

Maxima [A]
time = 0.29, size = 51, normalized size = 1.02

$$Bb^2x-\frac{20\,(2\,Bab+Ab^2)x^6+8\,(Ba^2+2\,Aab)x^3+5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] B*b^2*x - 1/40*(20*(2*B*a*b + A*b^2)*x^6 + 8*(B*a^2 + 2*A*a*b)*x^3 + 5*A*a^
2)/x^8
```

Fricas [A]
time = 3.08, size = 53, normalized size = 1.06

$$\frac{40\,Bb^2x^9-20\,(2\,Bab+Ab^2)x^6-8\,(Ba^2+2\,Aab)x^3-5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/40*(40*B*b^2*x^9 - 20*(2*B*a*b + A*b^2)*x^6 - 8*(B*a^2 + 2*A*a*b)*x^3 - 5
*A*a^2)/x^8
```

Sympy [A]
time = 0.55, size = 54, normalized size = 1.08

$$Bb^2x+\frac{-5Aa^2+x^6(-20Ab^2-40Bab)+x^3(-16Aab-8Ba^2)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**9,x)
```

```
[Out] B*b**2*x + (-5*A*a**2 + x**6*(-20*A*b**2 - 40*B*a*b) + x**3*(-16*A*a*b - 8*
B*a**2))/(40*x**8)
```

Giac [A]
time = 0.52, size = 53, normalized size = 1.06

$$Bb^2x - \frac{40\,Babx^6 + 20\,Ab^2x^6 + 8\,Ba^2x^3 + 16\,Aabx^3 + 5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] B*b^2*x - 1/40*(40*B*a*b*x^6 + 20*A*b^2*x^6 + 8*B*a^2*x^3 + 16*A*a*b*x^3 +
5*A*a^2)/x^8
```

Mupad [B]
time = 2.34, size = 50, normalized size = 1.00

$$B\,b^2\,x - \frac{x^3\,\left(\frac{B\,a^2}{5} + \frac{2\,A\,b\,a}{5}\right) + x^6\,\left(\frac{A\,b^2}{2} + B\,a\,b\right) + \frac{A\,a^2}{8}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^9,x)
```

```
[Out] B*b^2*x - (x^3*((B*a^2)/5 + (2*A*a*b)/5) + x^6*((A*b^2)/2 + B*a*b) + (A*a^2
)/8)/x^8
```

3.23 $\int x^9(a+bx^3)^5\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=117

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4(5Ab+aB)x^{13}+\frac{5}{16}a^3b(2Ab+aB)x^{16}+\frac{10}{19}a^2b^2(Ab+aB)x^{19}+\frac{5}{22}ab^3(Ab+2aB)x^{22}+\frac{1}{25}b^4(Ab+5$$

```
[Out] 1/10*a^5*A*x^10+1/13*a^4*(5*A*b+B*a)*x^13+5/16*a^3*b*(2*A*b+B*a)*x^16+10/19
*a^2*b^2*(A*b+B*a)*x^19+5/22*a*b^3*(A*b+2*B*a)*x^22+1/25*b^4*(A*b+5*B*a)*x^
25+1/28*b^5*B*x^28
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab)+\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{1}{25}b^4x^{25}(5aB+Ab)+\frac{5}{22}ab^3x^{22}(2aB+Ab)+\frac{1}{28}b^5Bx^{28}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16
)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 +
 (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \int\left(a^5Ax^9+a^4(5Ab+aB)x^{12}+5a^3b(2Ab+aB)x^{15}+10a^2b^2(Ab+aB)x^{18}\right.\\ &= \frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4(5Ab+aB)x^{13}+\frac{5}{16}a^3b(2Ab+aB)x^{16}+\frac{10}{19}a^2b^2(Ab+a\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4(5Ab+aB)x^{13}+\frac{5}{16}a^3b(2Ab+aB)x^{16}+\frac{10}{19}a^2b^2(Ab+aB)x^{19}+\frac{5}{22}ab^3(Ab+2aB)x^{22}+\frac{1}{25}b^4(Ab+5aB)x^{25}+\frac{1}{28}b^5Bx^{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16
)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 +
 (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

Maple [A]
time = 0.31, size = 124, normalized size = 1.06

method	result
norman	$\frac{a^5 A\,x^{10}}{10}+\left(\frac{5}{13}a^4bA+\frac{1}{13}a^5B\right)x^{13}+\left(\frac{5}{8}a^3b^2A+\frac{5}{16}a^4bB\right)x^{16}+\left(\frac{10}{19}a^2b^3A+\frac{10}{19}a^3b^2B\right)x^{19}+\left(\frac{5}{22}a\,b^4\right.$
default	$\frac{b^5B\,x^{28}}{28}+\frac{(b^5A+5a\,b^4B)x^{25}}{25}+\frac{(5a\,b^4A+10a^2b^3B)x^{22}}{22}+\frac{(10a^2b^3A+10a^3b^2B)x^{19}}{19}+\frac{(10a^3b^2A+5a^4bB)x^{16}}{16}+\frac{(5a^4bA+a^5}{13}$
gosper	$\frac{1}{10}a^5A\,x^{10}+\frac{5}{13}x^{13}a^4bA+\frac{1}{13}x^{13}a^5B+\frac{5}{8}x^{16}a^3b^2A+\frac{5}{16}x^{16}a^4bB+\frac{10}{19}x^{19}a^2b^3A+\frac{10}{19}x^{19}a^3b^2B+\frac{5}{22}$
risch	$\frac{1}{10}a^5A\,x^{10}+\frac{5}{13}x^{13}a^4bA+\frac{1}{13}x^{13}a^5B+\frac{5}{8}x^{16}a^3b^2A+\frac{5}{16}x^{16}a^4bB+\frac{10}{19}x^{19}a^2b^3A+\frac{10}{19}x^{19}a^3b^2B+\frac{5}{22}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*b^5*B*x^28+1/25*(A*b^5+5*B*a*b^4)*x^25+1/22*(5*A*a*b^4+10*B*a^2*b^3)*x
^22+1/19*(10*A*a^2*b^3+10*B*a^3*b^2)*x^19+1/16*(10*A*a^3*b^2+5*B*a^4*b)*x^1
6+1/13*(5*A*a^4*b+B*a^5)*x^13+1/10*a^5*A*x^10
```

Maxima [A]
time = 0.29, size = 119, normalized size = 1.02

$$\frac{1}{28}\,Bb^5x^{28}+\frac{1}{25}\,(5\,Bab^4+Ab^5)x^{25}+\frac{5}{22}\,(2\,Ba^2b^3+Aab^4)x^{22}+\frac{10}{19}\,(Ba^3b^2+Aa^2b^3)x^{19}+\frac{5}{16}\,(Ba^4b+2\,Aa^3b^2)x^{16}+\frac{1}{10}\,Aa^5x^{10}+\frac{1}{13}\,(Ba^5+5\,Aa^4b)x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/28*B*b^5*x^28 + 1/25*(5*B*a*b^4 + A*b^5)*x^25 + 5/22*(2*B*a^2*b^3 + A*a*b
^4)*x^22 + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^19 + 5/16*(B*a^4*b + 2*A*a^3*b^2
)*x^16 + 1/10*A*a^5*x^10 + 1/13*(B*a^5 + 5*A*a^4*b)*x^13
```

Fricas [A]
time = 2.05, size = 119, normalized size = 1.02

$$\frac{1}{28}\,Bb^5x^{28}+\frac{1}{25}\,(5\,Bab^4+Ab^5)x^{25}+\frac{5}{22}\,(2\,Ba^2b^3+Aab^4)x^{22}+\frac{10}{19}\,(Ba^3b^2+Aa^2b^3)x^{19}+\frac{5}{16}\,(Ba^4b+2\,Aa^3b^2)x^{16}+\frac{1}{10}\,Aa^5x^{10}+\frac{1}{13}\,(Ba^5+5\,Aa^4b)x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/28*B*b^5*x^28 + 1/25*(5*B*a*b^4 + A*b^5)*x^25 + 5/22*(2*B*a^2*b^3 + A*a*b
^4)*x^22 + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^19 + 5/16*(B*a^4*b + 2*A*a^3*b^2
)*x^16 + 1/10*A*a^5*x^10 + 1/13*(B*a^5 + 5*A*a^4*b)*x^13
```

Sympy [A]
time = 0.02, size = 136, normalized size = 1.16

$$\frac{Aa^5x^{10}}{10}+\frac{Bb^5x^{28}}{28}+x^{25}\left(\frac{Ab^5}{25}+\frac{Bab^4}{5}\right)+x^{22}\cdot\left(\frac{5Aab^4}{22}+\frac{5Ba^2b^3}{11}\right)+x^{19}\cdot\left(\frac{10Aa^2b^3}{19}+\frac{10Ba^3b^2}{19}\right)+x^{16}\cdot\left(\frac{5Aa^3b^2}{8}+\frac{5Ba^4b}{16}\right)+x^{13}\cdot\left(\frac{5Aa^4b}{13}+\frac{Ba^5}{13}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**10/10 + B*b**5*x**28/28 + x**25*(A*b**5/25 + B*a*b**4/5) + x**22*
(5*A*a*b**4/22 + 5*B*a**2*b**3/11) + x**19*(10*A*a**2*b**3/19 + 10*B*a**3*b
**2/19) + x**16*(5*A*a**3*b**2/8 + 5*B*a**4*b/16) + x**13*(5*A*a**4*b/13 +
B*a**5/13)
```

Giac [A]
time = 0.54, size = 125, normalized size = 1.07

$$\frac{1}{28}Bb^5x^{28}+\frac{1}{5}Bab^4x^{25}+\frac{1}{25}Ab^5x^{25}+\frac{5}{11}Ba^2b^3x^{22}+\frac{5}{22}Aab^4x^{22}+\frac{10}{19}Ba^3b^2x^{19}+\frac{10}{19}Aa^2b^3x^{19}+\frac{5}{16}Ba^4bx^{16}+\frac{5}{8}Aa^3b^2x^{16}+\frac{1}{13}Ba^5x^{13}+\frac{5}{13}Aa^4bx^{13}+\frac{1}{10}Aa^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/28*B*b^5*x^28 + 1/5*B*a*b^4*x^25 + 1/25*A*b^5*x^25 + 5/11*B*a^2*b^3*x^22
+ 5/22*A*a*b^4*x^22 + 10/19*B*a^3*b^2*x^19 + 10/19*A*a^2*b^3*x^19 + 5/16*B*
a^4*b*x^16 + 5/8*A*a^3*b^2*x^16 + 1/13*B*a^5*x^13 + 5/13*A*a^4*b*x^13 + 1/1
0*A*a^5*x^10
```

Mupad [B]
time = 0.05, size = 107, normalized size = 0.91

$$x^{13}\left(\frac{B\,a^5}{13}+\frac{5\,A\,b\,a^4}{13}\right)+x^{25}\left(\frac{A\,b^5}{25}+\frac{B\,a\,b^4}{5}\right)+\frac{A\,a^5\,x^{10}}{10}+\frac{B\,b^5\,x^{28}}{28}+\frac{10\,a^2\,b^2\,x^{19}\,\left(A\,b+B\,a\right)}{19}+\frac{5\,a^3\,b\,x^{16}\,\left(2\,A\,b+B\,a\right)}{16}+\frac{5\,a\,b^3\,x^{22}\,\left(A\,b+2\,B\,a\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^13*((B*a^5)/13 + (5*A*a^4*b)/13) + x^25*((A*b^5)/25 + (B*a*b^4)/5) + (A*a
^5*x^10)/10 + (B*b^5*x^28)/28 + (10*a^2*b^2*x^19*(A*b + B*a))/19 + (5*a^3*b
*x^16*(2*A*b + B*a))/16 + (5*a*b^3*x^22*(A*b + 2*B*a))/22
```

3.24 $\int x^8(a+bx^3)^5(A+Bx^3)\,dx$

Optimal. Leaf size=95

$$\frac{a^2(Ab-aB)(a+bx^3)^6}{18b^4}-\frac{a(2Ab-3aB)(a+bx^3)^7}{21b^4}+\frac{(Ab-3aB)(a+bx^3)^8}{24b^4}+\frac{B(a+bx^3)^9}{27b^4}$$

```
[Out] 1/18*a^2*(A*b-B*a)*(b*x^3+a)^6/b^4-1/21*a*(2*A*b-3*B*a)*(b*x^3+a)^7/b^4+1/2
4*(A*b-3*B*a)*(b*x^3+a)^8/b^4+1/27*B*(b*x^3+a)^9/b^4
```

Rubi [A]
time = 0.17, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$\frac{a^2(a+bx^3)^6(Ab-aB)}{18b^4}+\frac{(a+bx^3)^8(Ab-3aB)}{24b^4}-\frac{a(a+bx^3)^7(2Ab-3aB)}{21b^4}+\frac{B(a+bx^3)^9}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^3)^6)/(18*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^3)^7
)/(21*b^4) + ((A*b - 3*a*B)*(a + b*x^3)^8)/(24*b^4) + (B*(a + b*x^3)^9)/(27
*b^4)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^8 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \frac{1}{3}\text{Subst}\left(\int x^2(a+bx)^5(A+Bx)\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)(a+bx)^5}{b^3} + \frac{a(-2Ab+3aB)(a+bx)^6}{b^3} + \frac{(A\right.\right.$$
$$= \frac{a^2(Ab-aB)\left(a+bx^3\right)^6}{18b^4} - \frac{a(2Ab-3aB)\left(a+bx^3\right)^7}{21b^4} + \frac{(Ab-3aB)\left(a+bx\right.}{24b^4}$$

Mathematica [A]
time = 0.01, size = 107, normalized size = 1.13

$$\frac{x^9(168a^5A+126a^4(5Ab+aB)x^3+504a^3b(2Ab+aB)x^6+840a^2b^2(Ab+aB)x^9+360ab^3(Ab+2aB)x^{12}+63b^4(Ab+5aB)x^{15}+56b^5Bx^{18})}{1512}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^9*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^3 + 504*a^3*b*(2*A*b + a*B)*x^6 +
 840*a^2*b^2*(A*b + a*B)*x^9 + 360*a*b^3*(A*b + 2*a*B)*x^12 + 63*b^4*(A*b +
 5*a*B)*x^15 + 56*b^5*B*x^18))/1512
```

Maple [A]
time = 0.30, size = 124, normalized size = 1.31

method	result
norman	$\frac{a^5Ax^9}{9}+\left(\frac{5}{12}a^4bA+\frac{1}{12}a^5B\right)x^{12}+\left(\frac{2}{3}a^3b^2A+\frac{1}{3}a^4bB\right)x^{15}+\left(\frac{5}{9}a^2b^3A+\frac{5}{9}a^3b^2B\right)x^{18}+\left(\frac{5}{21}ab^4A+\right.$
default	$\frac{b^5Bx^{27}}{27}+\frac{(b^5A+5ab^4B)x^{24}}{24}+\frac{(5ab^4A+10a^2b^3B)x^{21}}{21}+\frac{(10a^2b^3A+10a^3b^2B)x^{18}}{18}+\frac{(10a^3b^2A+5a^4bB)x^{15}}{15}+\frac{(5a^4bA+a^5B}{12}$
gosper	$\frac{1}{9}a^5Ax^9+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+\frac{2}{3}x^{15}a^3b^2A+\frac{1}{3}x^{15}a^4bB+\frac{5}{9}x^{18}a^2b^3A+\frac{5}{9}x^{18}a^3b^2B+\frac{5}{21}x^{21}ab$
risch	$\frac{1}{9}a^5Ax^9+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+\frac{2}{3}x^{15}a^3b^2A+\frac{1}{3}x^{15}a^4bB+\frac{5}{9}x^{18}a^2b^3A+\frac{5}{9}x^{18}a^3b^2B+\frac{5}{21}x^{21}ab$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*b^5*B*x^27+1/24*(A*b^5+5*B*a*b^4)*x^24+1/21*(5*A*a*b^4+10*B*a^2*b^3)*x
^21+1/18*(10*A*a^2*b^3+10*B*a^3*b^2)*x^18+1/15*(10*A*a^3*b^2+5*B*a^4*b)*x^1
5+1/12*(5*A*a^4*b+B*a^5)*x^12+1/9*a^5*A*x^9
```

Maxima [A]
time = 0.30, size = 119, normalized size = 1.25

$$\frac{1}{27}Bb^5x^{27}+\frac{1}{24}(5Bab^4+Ab^5)x^{24}+\frac{5}{21}(2Ba^2b^3+Aab^4)x^{21}+\frac{5}{9}(Ba^3b^2+Aa^2b^3)x^{18}+\frac{1}{3}(Ba^4b+2Aa^3b^2)x^{15}+\frac{1}{9}Aa^5x^9+\frac{1}{12}(Ba^5+5Aa^4b)x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/27*B*b^5*x^27 + 1/24*(5*B*a*b^4 + A*b^5)*x^24 + 5/21*(2*B*a^2*b^3 + A*a*b
^4)*x^21 + 5/9*(B*a^3*b^2 + A*a^2*b^3)*x^18 + 1/3*(B*a^4*b + 2*A*a^3*b^2)*x
^15 + 1/9*A*a^5*x^9 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

Fricas [A]
time = 2.46, size = 119, normalized size = 1.25

$$\frac{1}{27}\,Bb^{5}x^{27}+\frac{1}{24}\,(5\,Bab^{4}+Ab^{5})x^{24}+\frac{5}{21}\,(2\,Ba^{2}b^{3}+Aab^{4})x^{21}+\frac{5}{9}\,(Ba^{3}b^{2}+Aa^{2}b^{3})x^{18}+\frac{1}{3}\,(Ba^{4}b+2\,Aa^{3}b^{2})x^{15}+\frac{1}{9}\,Aa^{5}x^{9}+\frac{1}{12}\,(Ba^{5}+5\,Aa^{4}b)x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/27*B*b^5*x^27 + 1/24*(5*B*a*b^4 + A*b^5)*x^24 + 5/21*(2*B*a^2*b^3 + A*a*b
^4)*x^21 + 5/9*(B*a^3*b^2 + A*a^2*b^3)*x^18 + 1/3*(B*a^4*b + 2*A*a^3*b^2)*x
^15 + 1/9*A*a^5*x^9 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

Sympy [A]
time = 0.02, size = 136, normalized size = 1.43

$$\frac{Aa^{5}x^{9}}{9}+\frac{Bb^{5}x^{27}}{27}+x^{24}\left(\frac{Ab^{5}}{24}+\frac{5Bab^{4}}{24}\right)+x^{21}\cdot\left(\frac{5Aab^{4}}{21}+\frac{10Ba^{2}b^{3}}{21}\right)+x^{18}\cdot\left(\frac{5Aa^{2}b^{3}}{9}+\frac{5Ba^{3}b^{2}}{9}\right)+x^{15}\cdot\left(\frac{2Aa^{3}b^{2}}{3}+\frac{Ba^{4}b}{3}\right)+x^{12}\cdot\left(\frac{5Aa^{4}b}{12}+\frac{Ba^{5}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**9/9 + B*b**5*x**27/27 + x**24*(A*b**5/24 + 5*B*a*b**4/24) + x**21
*(5*A*a*b**4/21 + 10*B*a**2*b**3/21) + x**18*(5*A*a**2*b**3/9 + 5*B*a**3*b*
*2/9) + x**15*(2*A*a**3*b**2/3 + B*a**4*b/3) + x**12*(5*A*a**4*b/12 + B*a**
5/12)
```

Giac [A]
time = 1.16, size = 125, normalized size = 1.32

$$\frac{1}{27}\,Bb^{5}x^{27}+\frac{5}{24}\,Bab^{4}x^{24}+\frac{1}{24}\,Ab^{5}x^{24}+\frac{10}{21}\,Ba^{2}b^{3}x^{21}+\frac{5}{21}\,Aab^{4}x^{21}+\frac{5}{9}\,Ba^{3}b^{2}x^{18}+\frac{5}{9}\,Aa^{2}b^{3}x^{18}+\frac{1}{3}\,Ba^{4}bx^{15}+\frac{2}{3}\,Aa^{3}b^{2}x^{15}+\frac{1}{12}\,Ba^{5}x^{12}+\frac{5}{12}\,Aa^{4}bx^{12}+\frac{1}{9}\,Aa^{5}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/27*B*b^5*x^27 + 5/24*B*a*b^4*x^24 + 1/24*A*b^5*x^24 + 10/21*B*a^2*b^3*x^2
1 + 5/21*A*a*b^4*x^21 + 5/9*B*a^3*b^2*x^18 + 5/9*A*a^2*b^3*x^18 + 1/3*B*a^4
*b*x^15 + 2/3*A*a^3*b^2*x^15 + 1/12*B*a^5*x^12 + 5/12*A*a^4*b*x^12 + 1/9*A*
a^5*x^9
```

Mupad [B]
time = 2.34, size = 107, normalized size = 1.13

$$x^{12}\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^{24}\left(\frac{A\,b^5}{24}+\frac{5\,B\,a\,b^4}{24}\right)+\frac{A\,a^5\,x^9}{9}+\frac{B\,b^5\,x^{27}}{27}+\frac{5\,a^2\,b^2\,x^{18}\,(A\,b+B\,a)}{9}+\frac{a^3\,b\,x^{15}\,(2\,A\,b+B\,a)}{3}+\frac{5\,a\,b^3\,x^{21}\,(A\,b+2\,B\,a)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^12*((B*a^5)/12 + (5*A*a^4*b)/12) + x^24*((A*b^5)/24 + (5*B*a*b^4)/24) + (
A*a^5*x^9)/9 + (B*b^5*x^27)/27 + (5*a^2*b^2*x^18*(A*b + B*a))/9 + (a^3*b*x^
15*(2*A*b + B*a))/3 + (5*a*b^3*x^21*(A*b + 2*B*a))/21
```

3.25 $\int x^7(a+bx^3)^5(A+Bx^3)\,dx$

Optimal. Leaf size=117

$$\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{10}{17}a^2b^2(Ab+aB)x^{17}+\frac{1}{4}ab^3(Ab+2aB)x^{20}+\frac{1}{23}b^4(Ab+5a$$

```
[Out] 1/8*a^5*A*x^8+1/11*a^4*(5*A*b+B*a)*x^11+5/14*a^3*b*(2*A*b+B*a)*x^14+10/17*a
^2*b^2*(A*b+B*a)*x^17+1/4*a*b^3*(A*b+2*B*a)*x^20+1/23*b^4*(A*b+5*B*a)*x^23+
1/26*b^5*B*x^26
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = $\{459\}$

$$\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+Ab)+\frac{1}{26}b^5Bx^{26}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/
14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4
*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \int \left(a^5Ax^7+a^4(5Ab+aB)x^{10}+5a^3b(2Ab+aB)x^{13}+10a^2b^2(Ab+aB)x\right.\\ &= \frac{1}{8}a^5Ax^8+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{10}{17}a^2b^2(Ab+aB\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{10}{17}a^2b^2(Ab+aB)x^{17}+\frac{1}{4}ab^3(Ab+2aB)x^{20}+\frac{1}{23}b^4(Ab+5aB)x^{23}+\frac{1}{26}b^5Bx^{26}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/
14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4
*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

Maple [A]
time = 0.32, size = 124, normalized size = 1.06

method	result
norman	$\frac{a^5 A x^8}{8} + \left(\frac{5}{11}a^4bA + \frac{1}{11}a^5B\right)x^{11} + \left(\frac{5}{7}a^3b^2A + \frac{5}{14}a^4bB\right)x^{14} + \left(\frac{10}{17}a^2b^3A + \frac{10}{17}a^3b^2B\right)x^{17} + \left(\frac{1}{4}a\,b^4A +\right.$
default	$\frac{b^5 B x^{26}}{26} + \frac{(b^5A+5a\,b^4B)x^{23}}{23} + \frac{(5a\,b^4A+10a^2b^3B)x^{20}}{20} + \frac{(10a^2b^3A+10a^3b^2B)x^{17}}{17} + \frac{(10a^3b^2A+5a^4bB)x^{14}}{14} + \frac{(5a^4bA+a^5B}{11}$
gosper	$\frac{1}{8}a^5A\,x^8 + \frac{5}{11}x^{11}a^4bA + \frac{1}{11}x^{11}a^5B + \frac{5}{7}x^{14}a^3b^2A + \frac{5}{14}x^{14}a^4bB + \frac{10}{17}x^{17}a^2b^3A + \frac{10}{17}x^{17}a^3b^2B + \frac{1}{4}x^{20}a$
risch	$\frac{1}{8}a^5A\,x^8 + \frac{5}{11}x^{11}a^4bA + \frac{1}{11}x^{11}a^5B + \frac{5}{7}x^{14}a^3b^2A + \frac{5}{14}x^{14}a^4bB + \frac{10}{17}x^{17}a^2b^3A + \frac{10}{17}x^{17}a^3b^2B + \frac{1}{4}x^{20}a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/26*b^5*B*x^26+1/23*(A*b^5+5*B*a*b^4)*x^23+1/20*(5*A*a*b^4+10*B*a^2*b^3)*x
^20+1/17*(10*A*a^2*b^3+10*B*a^3*b^2)*x^17+1/14*(10*A*a^3*b^2+5*B*a^4*b)*x^1
4+1/11*(5*A*a^4*b+B*a^5)*x^11+1/8*a^5*A*x^8
```

Maxima [A]
time = 0.29, size = 119, normalized size = 1.02

$$\frac{1}{26}Bb^5x^{26} + \frac{1}{23}\left(5Bab^4 + Ab^5\right)x^{23} + \frac{1}{4}\left(2Ba^2b^3 + Aab^4\right)x^{20} + \frac{10}{17}\left(Ba^3b^2 + Aa^2b^3\right)x^{17} + \frac{5}{14}\left(Ba^4b + 2Aa^3b^2\right)x^{14} + \frac{1}{8}Aa^5x^8 + \frac{1}{11}\left(Ba^5 + 5Aa^4b\right)x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/26*B*b^5*x^26 + 1/23*(5*B*a*b^4 + A*b^5)*x^23 + 1/4*(2*B*a^2*b^3 + A*a*b^
4)*x^20 + 10/17*(B*a^3*b^2 + A*a^2*b^3)*x^17 + 5/14*(B*a^4*b + 2*A*a^3*b^2)
*x^14 + 1/8*A*a^5*x^8 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

Fricas [A]
time = 2.48, size = 119, normalized size = 1.02

$$\frac{1}{26}Bb^5x^{26} + \frac{1}{23}\left(5Bab^4 + Ab^5\right)x^{23} + \frac{1}{4}\left(2Ba^2b^3 + Aab^4\right)x^{20} + \frac{10}{17}\left(Ba^3b^2 + Aa^2b^3\right)x^{17} + \frac{5}{14}\left(Ba^4b + 2Aa^3b^2\right)x^{14} + \frac{1}{8}Aa^5x^8 + \frac{1}{11}\left(Ba^5 + 5Aa^4b\right)x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/26*B*b^5*x^26 + 1/23*(5*B*a*b^4 + A*b^5)*x^23 + 1/4*(2*B*a^2*b^3 + A*a*b^
4)*x^20 + 10/17*(B*a^3*b^2 + A*a^2*b^3)*x^17 + 5/14*(B*a^4*b + 2*A*a^3*b^2)
*x^14 + 1/8*A*a^5*x^8 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

Sympy [A]
time = 0.02, size = 134, normalized size = 1.15

$$\frac{Aa^5x^8}{8}+\frac{Bb^5x^{26}}{26}+x^{23}\left(\frac{Ab^5}{23}+\frac{5Bab^4}{23}\right)+x^{20}\left(\frac{Aab^4}{4}+\frac{Ba^2b^3}{2}\right)+x^{17}\cdot\left(\frac{10Aa^2b^3}{17}+\frac{10Ba^3b^2}{17}\right)+x^{14}\cdot\left(\frac{5Aa^3b^2}{7}+\frac{5Ba^4b}{14}\right)+x^{11}\cdot\left(\frac{5Aa^4b}{11}+\frac{Ba^5}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**8/8 + B*b**5*x**26/26 + x**23*(A*b**5/23 + 5*B*a*b**4/23) + x**20
*(A*a*b**4/4 + B*a**2*b**3/2) + x**17*(10*A*a**2*b**3/17 + 10*B*a**3*b**2/1
7) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14) + x**11*(5*A*a**4*b/11 + B*a**
5/11)
```

Giac [A]
time = 1.27, size = 125, normalized size = 1.07

$$\frac{1}{26}Bb^5x^{26}+\frac{5}{23}Bab^4x^{23}+\frac{1}{23}Ab^5x^{23}+\frac{1}{2}Ba^2b^3x^{20}+\frac{1}{4}Aab^4x^{20}+\frac{10}{17}Ba^3b^2x^{17}+\frac{10}{17}Aa^2b^3x^{17}+\frac{5}{14}Ba^4bx^{14}+\frac{5}{7}Aa^3b^2x^{14}+\frac{1}{11}Ba^5x^{11}+\frac{5}{11}Aa^4bx^{11}+\frac{1}{8}Aa^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/26*B*b^5*x^26 + 5/23*B*a*b^4*x^23 + 1/23*A*b^5*x^23 + 1/2*B*a^2*b^3*x^20
+ 1/4*A*a*b^4*x^20 + 10/17*B*a^3*b^2*x^17 + 10/17*A*a^2*b^3*x^17 + 5/14*B*a
^4*b*x^14 + 5/7*A*a^3*b^2*x^14 + 1/11*B*a^5*x^11 + 5/11*A*a^4*b*x^11 + 1/8*
A*a^5*x^8
```

Mupad [B]
time = 0.04, size = 107, normalized size = 0.91

$$x^{11}\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^{23}\left(\frac{A\,b^5}{23}+\frac{5\,B\,a\,b^4}{23}\right)+\frac{A\,a^5\,x^8}{8}+\frac{B\,b^5\,x^{26}}{26}+\frac{10\,a^2\,b^2\,x^{17}\,(A\,b+B\,a)}{17}+\frac{5\,a^3\,b\,x^{14}\,(2\,A\,b+B\,a)}{14}+\frac{a\,b^3\,x^{20}\,(A\,b+2\,B\,a)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^11*((B*a^5)/11 + (5*A*a^4*b)/11) + x^23*((A*b^5)/23 + (5*B*a*b^4)/23) + (
A*a^5*x^8)/8 + (B*b^5*x^26)/26 + (10*a^2*b^2*x^17*(A*b + B*a))/17 + (5*a^3*
b*x^14*(2*A*b + B*a))/14 + (a*b^3*x^20*(A*b + 2*B*a))/4
```

3.26 $\int x^6(a+bx^3)^5 (A+Bx^3)\, dx$

Optimal. Leaf size=117

$$\frac{1}{7}a^5Ax^7+\frac{1}{10}a^4(5Ab+aB)x^{10}+\frac{5}{13}a^3b(2Ab+aB)x^{13}+\frac{5}{8}a^2b^2(Ab+aB)x^{16}+\frac{5}{19}ab^3(Ab+2aB)x^{19}+\frac{1}{22}b^4(Ab+5aB$$

```
[Out] 1/7*a^5*A*x^7+1/10*a^4*(5*A*b+B*a)*x^10+5/13*a^3*b*(2*A*b+B*a)*x^13+5/8*a^2
*b^2*(A*b+B*a)*x^16+5/19*a*b^3*(A*b+2*B*a)*x^19+1/22*b^4*(A*b+5*B*a)*x^22+1
/25*b^5*B*x^25
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$\frac{1}{7}a^5Ax^7+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{22}b^4x^{22}(5aB+Ab)+\frac{5}{19}ab^3x^{19}(2aB+Ab)+\frac{1}{25}b^5Bx^{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^
4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^6\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \int \left(a^5Ax^6+a^4(5Ab+aB)x^9+5a^3b(2Ab+aB)x^{12}+10a^2b^2(Ab+aB)x^{15}\right.\\ &= \frac{1}{7}a^5Ax^7+\frac{1}{10}a^4(5Ab+aB)x^{10}+\frac{5}{13}a^3b(2Ab+aB)x^{13}+\frac{5}{8}a^2b^2(Ab+aB)x\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{7}a^5Ax^7+\frac{1}{10}a^4(5Ab+aB)x^{10}+\frac{5}{13}a^3b(2Ab+aB)x^{13}+\frac{5}{8}a^2b^2(Ab+aB)x^{16}+\frac{5}{19}ab^3(Ab+2aB)x^{19}+\frac{1}{22}b^4(Ab+5aB)x^{22}+\frac{1}{25}b^5Bx^{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^
4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

Maple [A]
time = 0.31, size = 124, normalized size = 1.06

method	result
norman	$\frac{a^5 A x^7}{7} + \left(\frac{1}{2}a^4 bA + \frac{1}{10}a^5 B\right)x^{10} + \left(\frac{10}{13}a^3 b^2 A + \frac{5}{13}a^4 bB\right)x^{13} + \left(\frac{5}{8}a^2 b^3 A + \frac{5}{8}a^3 b^2 B\right)x^{16} + \left(\frac{5}{19}a\, b^4 A -\right.$
default	$\frac{b^5 B x^{25}}{25} + \frac{(b^5 A + 5a\, b^4 B)x^{22}}{22} + \frac{(5a\, b^4 A + 10a^2 b^3 B)x^{19}}{19} + \frac{(10a^2 b^3 A + 10a^3 b^2 B)x^{16}}{16} + \frac{(10a^3 b^2 A + 5a^4 bB)x^{13}}{13} + \frac{(5a^4 bA + a^5}{10}$
gosper	$\frac{1}{7}a^5 A\, x^7 + \frac{1}{2}x^{10}a^4 bA + \frac{1}{10}x^{10}a^5 B + \frac{10}{13}x^{13}a^3 b^2 A + \frac{5}{13}x^{13}a^4 bB + \frac{5}{8}x^{16}a^2 b^3 A + \frac{5}{8}x^{16}a^3 b^2 B + \frac{5}{19}x^{19}$
risch	$\frac{1}{7}a^5 A\, x^7 + \frac{1}{2}x^{10}a^4 bA + \frac{1}{10}x^{10}a^5 B + \frac{10}{13}x^{13}a^3 b^2 A + \frac{5}{13}x^{13}a^4 bB + \frac{5}{8}x^{16}a^2 b^3 A + \frac{5}{8}x^{16}a^3 b^2 B + \frac{5}{19}x^{19}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/25*b^5*B*x^25+1/22*(A*b^5+5*B*a*b^4)*x^22+1/19*(5*A*a*b^4+10*B*a^2*b^3)*x
^19+1/16*(10*A*a^2*b^3+10*B*a^3*b^2)*x^16+1/13*(10*A*a^3*b^2+5*B*a^4*b)*x^1
3+1/10*(5*A*a^4*b+B*a^5)*x^10+1/7*a^5*A*x^7
```

Maxima [A]
time = 0.30, size = 119, normalized size = 1.02

$$\frac{1}{25}Bb^5x^{25} + \frac{1}{22}\left(5\,Bab^4 + Ab^5\right)x^{22} + \frac{5}{19}\left(2\,Ba^2b^3 + Aab^4\right)x^{19} + \frac{5}{8}\left(Ba^3b^2 + Aa^2b^3\right)x^{16} + \frac{5}{13}\left(Ba^4b + 2\,Aa^3b^2\right)x^{13} + \frac{1}{7}Aa^5x^7 + \frac{1}{10}\left(Ba^5 + 5\,Aa^4b\right)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/25*B*b^5*x^25 + 1/22*(5*B*a*b^4 + A*b^5)*x^22 + 5/19*(2*B*a^2*b^3 + A*a*b
^4)*x^19 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 5/13*(B*a^4*b + 2*A*a^3*b^2)*
x^13 + 1/7*A*a^5*x^7 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

Fricas [A]
time = 2.71, size = 119, normalized size = 1.02

$$\frac{1}{25}Bb^5x^{25} + \frac{1}{22}\left(5\,Bab^4 + Ab^5\right)x^{22} + \frac{5}{19}\left(2\,Ba^2b^3 + Aab^4\right)x^{19} + \frac{5}{8}\left(Ba^3b^2 + Aa^2b^3\right)x^{16} + \frac{5}{13}\left(Ba^4b + 2\,Aa^3b^2\right)x^{13} + \frac{1}{7}Aa^5x^7 + \frac{1}{10}\left(Ba^5 + 5\,Aa^4b\right)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/25*B*b^5*x^25 + 1/22*(5*B*a*b^4 + A*b^5)*x^22 + 5/19*(2*B*a^2*b^3 + A*a*b
^4)*x^19 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 5/13*(B*a^4*b + 2*A*a^3*b^2)*
x^13 + 1/7*A*a^5*x^7 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

Sympy [A]
time = 0.02, size = 136, normalized size = 1.16

$$\frac{Aa^5x^7}{7} + \frac{Bb^5x^{25}}{25} + x^{22}\left(\frac{Ab^5}{22} + \frac{5Bab^4}{22}\right) + x^{19}\cdot\left(\frac{5Aab^4}{19} + \frac{10Ba^2b^3}{19}\right) + x^{16}\cdot\left(\frac{5Aa^2b^3}{8} + \frac{5Ba^3b^2}{8}\right) + x^{13}\cdot\left(\frac{10Aa^3b^2}{13} + \frac{5Ba^4b}{13}\right) + x^{10}\left(\frac{Aa^4b}{2} + \frac{Ba^5}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**7/7 + B*b**5*x**25/25 + x**22*(A*b**5/22 + 5*B*a*b**4/22) + x**19
*(5*A*a*b**4/19 + 10*B*a**2*b**3/19) + x**16*(5*A*a**2*b**3/8 + 5*B*a**3*b*
*2/8) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b/13) + x**10*(A*a**4*b/2 + B*a
**5/10)
```

Giac [A]
time = 0.87, size = 125, normalized size = 1.07

$$\frac{1}{25}Bb^5x^{25} + \frac{5}{22}Bab^4x^{22} + \frac{1}{22}Ab^5x^{22} + \frac{10}{19}Ba^2b^3x^{19} + \frac{5}{19}Aab^4x^{19} + \frac{5}{8}Ba^3b^2x^{16} + \frac{5}{8}Aa^2b^3x^{16} + \frac{5}{13}Ba^4bx^{13} + \frac{10}{13}Aa^3b^2x^{13} + \frac{1}{10}Ba^5x^{10} + \frac{1}{2}Aa^4bx^{10} + \frac{1}{7}Aa^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/25*B*b^5*x^25 + 5/22*B*a*b^4*x^22 + 1/22*A*b^5*x^22 + 10/19*B*a^2*b^3*x^1
9 + 5/19*A*a*b^4*x^19 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3*x^16 + 5/13*B*a^
4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/10*B*a^5*x^10 + 1/2*A*a^4*b*x^10 + 1/7*
A*a^5*x^7
```

Mupad [B]
time = 0.04, size = 107, normalized size = 0.91

$$x^{10}\left(\frac{B\,a^5}{10} + \frac{A\,b\,a^4}{2}\right) + x^{22}\left(\frac{A\,b^5}{22} + \frac{5\,B\,a\,b^4}{22}\right) + \frac{A\,a^5\,x^7}{7} + \frac{B\,b^5\,x^{25}}{25} + \frac{5\,a^2\,b^2\,x^{16}\,(A\,b + B\,a)}{8} + \frac{5\,a^3\,b\,x^{13}\,(2\,A\,b + B\,a)}{13} + \frac{5\,a\,b^3\,x^{19}\,(A\,b + 2\,B\,a)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^10*((B*a^5)/10 + (A*a^4*b)/2) + x^22*((A*b^5)/22 + (5*B*a*b^4)/22) + (A*a
^5*x^7)/7 + (B*b^5*x^25)/25 + (5*a^2*b^2*x^16*(A*b + B*a))/8 + (5*a^3*b*x^1
3*(2*A*b + B*a))/13 + (5*a*b^3*x^19*(A*b + 2*B*a))/19
```

3.27 $\int x^5(a+bx^3)^5(A+Bx^3)\,dx$

Optimal. Leaf size=67

$$-\frac{a(Ab-aB)(a+bx^3)^6}{18b^3}+\frac{(Ab-2aB)(a+bx^3)^7}{21b^3}+\frac{B(a+bx^3)^8}{24b^3}$$

```
[Out] -1/18*a*(A*b-B*a)*(b*x^3+a)^6/b^3+1/21*(A*b-2*B*a)*(b*x^3+a)^7/b^3+1/24*B*(
b*x^3+a)^8/b^3
```

Rubi [A]
time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {457, 77}

$$\frac{(a+bx^3)^7(Ab-2aB)}{21b^3}-\frac{a(a+bx^3)^6(Ab-aB)}{18b^3}+\frac{B(a+bx^3)^8}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] -1/18*(a*(A*b - a*B)*(a + b*x^3)^6)/b^3 + ((A*b - 2*a*B)*(a + b*x^3)^7)/(21
*b^3) + (B*(a + b*x^3)^8)/(24*b^3)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 (a+bx^3)^5 (A+Bx^3)\, dx = \frac{1}{3}\text{Subst}\left(\int x(a+bx)^5(A+Bx)\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^5}{b^2} + \frac{(Ab-2aB)(a+bx)^6}{b^2} + \frac{B(a+bx)}{b^2}\right.\right.$$
$$= -\frac{a(Ab-aB)(a+bx^3)^6}{18b^3} + \frac{(Ab-2aB)(a+bx^3)^7}{21b^3} + \frac{B(a+bx^3)^8}{24b^3}$$

Mathematica [A]
time = 0.01, size = 107, normalized size = 1.60

$$\frac{1}{504}x^6(84a^5A + 56a^4(5Ab+aB)x^3 + 210a^3b(2Ab+aB)x^6 + 336a^2b^2(Ab+aB)x^9 + 140ab^3(Ab+2aB)x^{12} + 24b^4(Ab+5aB)x^{15} + 21b^5Bx^{18})$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^6*(84*a^5*A + 56*a^4*(5*A*b + a*B)*x^3 + 210*a^3*b*(2*A*b + a*B)*x^6 + 3
36*a^2*b^2*(A*b + a*B)*x^9 + 140*a*b^3*(A*b + 2*a*B)*x^12 + 24*b^4*(A*b + 5
*a*B)*x^15 + 21*b^5*B*x^18))/504
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(61) = 122.
time = 0.29, size = 124, normalized size = 1.85

method	result
norman	$\frac{a^5Ax^6}{6} + \left(\frac{5}{9}a^4bA + \frac{1}{9}a^5B\right)x^9 + \left(\frac{5}{6}a^3b^2A + \frac{5}{12}a^4bB\right)x^{12} + \left(\frac{2}{3}a^2b^3A + \frac{2}{3}a^3b^2B\right)x^{15} + \left(\frac{5}{18}ab^4A + \frac{5}{9}a\right.$
default	$\frac{b^5Bx^{24}}{24} + \frac{(b^5A+5ab^4B)x^{21}}{21} + \frac{(5ab^4A+10a^2b^3B)x^{18}}{18} + \frac{(10a^2b^3A+10a^3b^2B)x^{15}}{15} + \frac{(10a^3b^2A+5a^4bB)x^{12}}{12} + \frac{(5a^4bA+a^5B}{9}$
gosper	$\frac{1}{6}a^5Ax^6 + \frac{5}{9}x^9a^4bA + \frac{1}{9}x^9a^5B + \frac{5}{6}x^{12}a^3b^2A + \frac{5}{12}x^{12}a^4bB + \frac{2}{3}x^{15}a^2b^3A + \frac{2}{3}x^{15}a^3b^2B + \frac{5}{18}x^{18}ab^4A$
risch	$\frac{1}{6}a^5Ax^6 + \frac{5}{9}x^9a^4bA + \frac{1}{9}x^9a^5B + \frac{5}{6}x^{12}a^3b^2A + \frac{5}{12}x^{12}a^4bB + \frac{2}{3}x^{15}a^2b^3A + \frac{2}{3}x^{15}a^3b^2B + \frac{5}{18}x^{18}ab^4A$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*b^5*B*x^24+1/21*(A*b^5+5*B*a*b^4)*x^21+1/18*(5*A*a*b^4+10*B*a^2*b^3)*x
^18+1/15*(10*A*a^2*b^3+10*B*a^3*b^2)*x^15+1/12*(10*A*a^3*b^2+5*B*a^4*b)*x^1
2+1/9*(5*A*a^4*b+B*a^5)*x^9+1/6*a^5*A*x^6
```

Maxima [A]
time = 0.29, size = 119, normalized size = 1.78

$$\frac{1}{24}Bb^5x^{24} + \frac{1}{21}(5Bab^4 + Ab^5)x^{21} + \frac{5}{18}(2Ba^2b^3 + Aab^4)x^{18} + \frac{2}{3}(Ba^3b^2 + Aa^2b^3)x^{15} + \frac{5}{12}(Ba^4b + 2Aa^3b^2)x^{12} + \frac{1}{6}Aa^5x^6 + \frac{1}{9}(Ba^5 + 5Aa^4b)x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/24*B*b^5*x^24 + 1/21*(5*B*a*b^4 + A*b^5)*x^21 + 5/18*(2*B*a^2*b^3 + A*a*b
^4)*x^18 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 5/12*(B*a^4*b + 2*A*a^3*b^2)*
x^12 + 1/6*A*a^5*x^6 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

Fricas [A]
time = 2.52, size = 119, normalized size = 1.78

$$\frac{1}{24}\,Bb^5x^{24}+\frac{1}{21}\,(5\,Bab^4+Ab^5)x^{21}+\frac{5}{18}\,(2\,Ba^2b^3+Aab^4)x^{18}+\frac{2}{3}\,(Ba^3b^2+Aa^2b^3)x^{15}+\frac{5}{12}\,(Ba^4b+2\,Aa^3b^2)x^{12}+\frac{1}{6}\,Aa^5x^6+\frac{1}{9}\,(Ba^5+5\,Aa^4b)x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/24*B*b^5*x^24 + 1/21*(5*B*a*b^4 + A*b^5)*x^21 + 5/18*(2*B*a^2*b^3 + A*a*b
^4)*x^18 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 5/12*(B*a^4*b + 2*A*a^3*b^2)*
x^12 + 1/6*A*a^5*x^6 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(60) = 120.
time = 0.02, size = 138, normalized size = 2.06

$$\frac{Aa^5x^6}{6}+\frac{Bb^5x^{24}}{24}+x^{21}\left(\frac{Ab^5}{21}+\frac{5Bab^4}{21}\right)+x^{18}\cdot\left(\frac{5Aab^4}{18}+\frac{5Ba^2b^3}{9}\right)+x^{15}\cdot\left(\frac{2Aa^2b^3}{3}+\frac{2Ba^3b^2}{3}\right)+x^{12}\cdot\left(\frac{5Aa^3b^2}{6}+\frac{5Ba^4b}{12}\right)+x^9\cdot\left(\frac{5Aa^4b}{9}+\frac{Ba^5}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**6/6 + B*b**5*x**24/24 + x**21*(A*b**5/21 + 5*B*a*b**4/21) + x**18
*(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*b**2
/3) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**9*(5*A*a**4*b/9 + B*a**5
/9)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(62) = 124.
time = 0.69, size = 125, normalized size = 1.87

$$\frac{1}{24}\,Bb^5x^{24}+\frac{5}{21}\,Bab^4x^{21}+\frac{1}{21}\,Ab^5x^{21}+\frac{5}{9}\,Ba^2b^3x^{18}+\frac{5}{18}\,Aab^4x^{18}+\frac{2}{3}\,Ba^3b^2x^{15}+\frac{2}{3}\,Aa^2b^3x^{15}+\frac{5}{12}\,Ba^4bx^{12}+\frac{5}{6}\,Aa^3b^2x^{12}+\frac{1}{9}\,Ba^5x^9+\frac{5}{9}\,Aa^4bx^9+\frac{1}{6}\,Aa^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/24*B*b^5*x^24 + 5/21*B*a*b^4*x^21 + 1/21*A*b^5*x^21 + 5/9*B*a^2*b^3*x^18
+ 5/18*A*a*b^4*x^18 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*b^3*x^15 + 5/12*B*a^4*
b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/9*B*a^5*x^9 + 5/9*A*a^4*b*x^9 + 1/6*A*a^5*x
^6
```

Mupad [B]
time = 0.04, size = 107, normalized size = 1.60

$$x^9\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)+x^{21}\left(\frac{A\,b^5}{21}+\frac{5\,B\,a\,b^4}{21}\right)+\frac{A\,a^5\,x^6}{6}+\frac{B\,b^5\,x^{24}}{24}+\frac{2\,a^2\,b^2\,x^{15}\,(A\,b+B\,a)}{3}+\frac{5\,a^3\,b\,x^{12}\,(2\,A\,b+B\,a)}{12}+\frac{5\,a\,b^3\,x^{18}\,(A\,b+2\,B\,a)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^9*((B*a^5)/9 + (5*A*a^4*b)/9) + x^21*((A*b^5)/21 + (5*B*a*b^4)/21) + (A*a
^5*x^6)/6 + (B*b^5*x^24)/24 + (2*a^2*b^2*x^15*(A*b + B*a))/3 + (5*a^3*b*x^1
2*(2*A*b + B*a))/12 + (5*a*b^3*x^18*(A*b + 2*B*a))/18
```

3.28 $\int x^4(a+bx^3)^5 (A+Bx^3)\, dx$

Optimal. Leaf size=117

$$\frac{1}{5}a^5Ax^5+\frac{1}{8}a^4(5Ab+aB)x^8+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{5}{7}a^2b^2(Ab+aB)x^{14}+\frac{5}{17}ab^3(Ab+2aB)x^{17}+\frac{1}{20}b^4(Ab+5aB)$$

```
[Out] 1/5*a^5*A*x^5+1/8*a^4*(5*A*b+B*a)*x^8+5/11*a^3*b*(2*A*b+B*a)*x^11+5/7*a^2*b
^2*(A*b+B*a)*x^14+5/17*a*b^3*(A*b+2*B*a)*x^17+1/20*b^4*(A*b+5*B*a)*x^20+1/2
3*b^5*B*x^23
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$\frac{1}{5}a^5Ax^5+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{23}b^5Bx^{23}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^4*
(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \int \left(a^5Ax^4+a^4(5Ab+aB)x^7+5a^3b(2Ab+aB)x^{10}+10a^2b^2(Ab+aB)x^1\right.\\ &= \frac{1}{5}a^5Ax^5+\frac{1}{8}a^4(5Ab+aB)x^8+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{5}{7}a^2b^2(Ab+aB)x\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{5}a^5Ax^5+\frac{1}{8}a^4(5Ab+aB)x^8+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{5}{7}a^2b^2(Ab+aB)x^{14}+\frac{5}{17}ab^3(Ab+2aB)x^{17}+\frac{1}{20}b^4(Ab+5aB)x^{20}+\frac{1}{23}b^5Bx^{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^4*
(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

Maple [A]
time = 0.30, size = 124, normalized size = 1.06

method	result
norman	$\frac{a^5 A x^5}{5} + \left(\frac{5}{8}a^4 bA + \frac{1}{8}a^5 B\right)x^8 + \left(\frac{10}{11}a^3b^2A + \frac{5}{11}a^4bB\right)x^{11} + \left(\frac{5}{7}a^2b^3A + \frac{5}{7}a^3b^2B\right)x^{14} + \left(\frac{5}{17}a\, b^4A + \frac{10}{17}\right.$
default	$\frac{b^5 B\, x^{23}}{23} + \frac{(b^5A+5a\, b^4B)x^{20}}{20} + \frac{(5a\, b^4A+10a^2b^3B)x^{17}}{17} + \frac{(10a^2b^3A+10a^3b^2B)x^{14}}{14} + \frac{(10a^3b^2A+5a^4bB)x^{11}}{11} + \frac{(5a^4bA+a^5B}{8}$
gosper	$\frac{1}{5}a^5A\, x^5 + \frac{5}{8}x^8a^4bA + \frac{1}{8}x^8a^5B + \frac{10}{11}x^{11}a^3b^2A + \frac{5}{11}x^{11}a^4bB + \frac{5}{7}x^{14}a^2b^3A + \frac{5}{7}x^{14}a^3b^2B + \frac{5}{17}x^{17}a\, b^4A$
risch	$\frac{1}{5}a^5A\, x^5 + \frac{5}{8}x^8a^4bA + \frac{1}{8}x^8a^5B + \frac{10}{11}x^{11}a^3b^2A + \frac{5}{11}x^{11}a^4bB + \frac{5}{7}x^{14}a^2b^3A + \frac{5}{7}x^{14}a^3b^2B + \frac{5}{17}x^{17}a\, b^4A$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/23*b^5*B*x^23+1/20*(A*b^5+5*B*a*b^4)*x^20+1/17*(5*A*a*b^4+10*B*a^2*b^3)*x
^17+1/14*(10*A*a^2*b^3+10*B*a^3*b^2)*x^14+1/11*(10*A*a^3*b^2+5*B*a^4*b)*x^1
1+1/8*(5*A*a^4*b+B*a^5)*x^8+1/5*a^5*A*x^5
```

Maxima [A]
time = 0.29, size = 119, normalized size = 1.02

$$\frac{1}{23}Bb^5x^{23} + \frac{1}{20}\left(5\,Bab^4 + Ab^5\right)x^{20} + \frac{5}{17}\left(2\,Ba^2b^3 + Aab^4\right)x^{17} + \frac{5}{7}\left(Ba^3b^2 + Aa^2b^3\right)x^{14} + \frac{5}{11}\left(Ba^4b + 2\,Aa^3b^2\right)x^{11} + \frac{1}{5}Aa^5x^5 + \frac{1}{8}\left(Ba^5 + 5\,Aa^4b\right)x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/23*B*b^5*x^23 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/17*(2*B*a^2*b^3 + A*a*b
^4)*x^17 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 5/11*(B*a^4*b + 2*A*a^3*b^2)*
x^11 + 1/5*A*a^5*x^5 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

Fricas [A]
time = 2.87, size = 119, normalized size = 1.02

$$\frac{1}{23}Bb^5x^{23} + \frac{1}{20}\left(5\,Bab^4 + Ab^5\right)x^{20} + \frac{5}{17}\left(2\,Ba^2b^3 + Aab^4\right)x^{17} + \frac{5}{7}\left(Ba^3b^2 + Aa^2b^3\right)x^{14} + \frac{5}{11}\left(Ba^4b + 2\,Aa^3b^2\right)x^{11} + \frac{1}{5}Aa^5x^5 + \frac{1}{8}\left(Ba^5 + 5\,Aa^4b\right)x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/23*B*b^5*x^23 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/17*(2*B*a^2*b^3 + A*a*b
^4)*x^17 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 5/11*(B*a^4*b + 2*A*a^3*b^2)*
x^11 + 1/5*A*a^5*x^5 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

Sympy [A]
time = 0.02, size = 136, normalized size = 1.16

$$\frac{Aa^5x^5}{5}+\frac{Bb^5x^{23}}{23}+x^{20}\left(\frac{Ab^5}{20}+\frac{Bab^4}{4}\right)+x^{17}\cdot\left(\frac{5Aab^4}{17}+\frac{10Ba^2b^3}{17}\right)+x^{14}\cdot\left(\frac{5Aa^2b^3}{7}+\frac{5Ba^3b^2}{7}\right)+x^{11}\cdot\left(\frac{10Aa^3b^2}{11}+\frac{5Ba^4b}{11}\right)+x^8\cdot\left(\frac{5Aa^4b}{8}+\frac{Ba^5}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**5/5 + B*b**5*x**23/23 + x**20*(A*b**5/20 + B*a*b**4/4) + x**17*(5
*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**14*(5*A*a**2*b**3/7 + 5*B*a**3*b**2/
7) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/11) + x**8*(5*A*a**4*b/8 + B*a**
5/8)
```

Giac [A]
time = 1.36, size = 125, normalized size = 1.07

$$\frac{1}{23}Bb^5x^{23}+\frac{1}{4}Bab^4x^{20}+\frac{1}{20}Ab^5x^{20}+\frac{10}{17}Ba^2b^3x^{17}+\frac{5}{17}Aab^4x^{17}+\frac{5}{7}Ba^3b^2x^{14}+\frac{5}{7}Aa^2b^3x^{14}+\frac{5}{11}Ba^4bx^{11}+\frac{10}{11}Aa^3b^2x^{11}+\frac{1}{8}Ba^5x^8+\frac{5}{8}Aa^4bx^8+\frac{1}{5}Aa^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/23*B*b^5*x^23 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 10/17*B*a^2*b^3*x^17
 + 5/17*A*a*b^4*x^17 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^3*x^14 + 5/11*B*a^4
*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/8*B*a^5*x^8 + 5/8*A*a^4*b*x^8 + 1/5*A*a^
5*x^5
```

Mupad [B]
time = 0.04, size = 107, normalized size = 0.91

$$x^8\left(\frac{B\,a^5}{8}+\frac{5\,A\,b\,a^4}{8}\right)+x^{20}\left(\frac{A\,b^5}{20}+\frac{B\,a\,b^4}{4}\right)+\frac{A\,a^5\,x^5}{5}+\frac{B\,b^5\,x^{23}}{23}+\frac{5\,a^2\,b^2\,x^{14}\,\left(A\,b+B\,a\right)}{7}+\frac{5\,a^3\,b\,x^{11}\,\left(2\,A\,b+B\,a\right)}{11}+\frac{5\,a\,b^3\,x^{17}\,\left(A\,b+2\,B\,a\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^8*((B*a^5)/8 + (5*A*a^4*b)/8) + x^20*((A*b^5)/20 + (B*a*b^4)/4) + (A*a^5*
x^5)/5 + (B*b^5*x^23)/23 + (5*a^2*b^2*x^14*(A*b + B*a))/7 + (5*a^3*b*x^11*(
2*A*b + B*a))/11 + (5*a*b^3*x^17*(A*b + 2*B*a))/17
```

3.29 $\int x^3(a+bx^3)^5\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=117

$$\frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{5}{16}ab^3(Ab+2aB)x^{16}+\frac{1}{19}b^4(Ab+5aB)x$$

```
[Out] 1/4*a^5*A*x^4+1/7*a^4*(5*A*b+B*a)*x^7+1/2*a^3*b*(2*A*b+B*a)*x^10+10/13*a^2*
b^2*(A*b+B*a)*x^13+5/16*a*b^3*(A*b+2*B*a)*x^16+1/19*b^4*(A*b+5*B*a)*x^19+1/
22*b^5*B*x^22
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\frac{1}{4}a^5Ax^4+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
(10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(
A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \int \left(a^5Ax^3+a^4(5Ab+aB)x^6+5a^3b(2Ab+aB)x^9+10a^2b^2(Ab+aB)x^{12}\right.\\ &= \frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{5}{16}ab^3(Ab+2aB)x^{16}+\frac{1}{19}b^4(Ab+5aB)x^{19}+\frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
(10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(
A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

Maple [A]
time = 0.30, size = 124, normalized size = 1.06

method	result
norman	$\frac{a^5 A x^4}{4} + \left(\frac{5}{7}a^4 bA + \frac{1}{7}a^5 B\right) x^7 + \left(a^3 b^2 A + \frac{1}{2}a^4 bB\right) x^{10} + \left(\frac{10}{13}a^2 b^3 A + \frac{10}{13}a^3 b^2 B\right) x^{13} + (\frac{5}{16}a\, b^4 A + \frac{5}{8}$
default	$\frac{b^5 B x^{22}}{22} + \frac{(b^5 A+5a\, b^4 B)x^{19}}{19} + \frac{(5a\, b^4 A+10a^2 b^3 B)x^{16}}{16} + \frac{(10a^2 b^3 A+10a^3 b^2 B)x^{13}}{13} + \frac{(10a^3 b^2 A+5a^4 bB)x^{10}}{10} + \frac{(5a^4 bA+a^5}{7}$
gosper	$\frac{1}{4}a^5 A\, x^4 + \frac{5}{7}x^7 a^4 bA + \frac{1}{7}x^7 a^5 B + x^{10} a^3 b^2 A + \frac{1}{2}x^{10} a^4 bB + \frac{10}{13}x^{13} a^2 b^3 A + \frac{10}{13}x^{13} a^3 b^2 B + \frac{5}{16}x^{16} a\, b^4 A$
risch	$\frac{1}{4}a^5 A\, x^4 + \frac{5}{7}x^7 a^4 bA + \frac{1}{7}x^7 a^5 B + x^{10} a^3 b^2 A + \frac{1}{2}x^{10} a^4 bB + \frac{10}{13}x^{13} a^2 b^3 A + \frac{10}{13}x^{13} a^3 b^2 B + \frac{5}{16}x^{16} a\, b^4 A$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/22*b^5*B*x^22+1/19*(A*b^5+5*B*a*b^4)*x^19+1/16*(5*A*a*b^4+10*B*a^2*b^3)*x
^16+1/13*(10*A*a^2*b^3+10*B*a^3*b^2)*x^13+1/10*(10*A*a^3*b^2+5*B*a^4*b)*x^1
0+1/7*(5*A*a^4*b+B*a^5)*x^7+1/4*a^5*A*x^4
```

Maxima [A]
time = 0.27, size = 119, normalized size = 1.02

$$\frac{1}{22}\,Bb^5x^{22} + \frac{1}{19}\,(5\,Bab^4 + Ab^5)x^{19} + \frac{5}{16}\,(2\,Ba^2b^3 + Aab^4)x^{16} + \frac{10}{13}\,(Ba^3b^2 + Aa^2b^3)x^{13} + \frac{1}{2}\,(Ba^4b + 2\,Aa^3b^2)x^{10} + \frac{1}{4}\,Aa^5x^4 + \frac{1}{7}\,(Ba^5 + 5\,Aa^4b)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/22*B*b^5*x^22 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/16*(2*B*a^2*b^3 + A*a*b
^4)*x^16 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/2*(B*a^4*b + 2*A*a^3*b^2)
*x^10 + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Fricas [A]
time = 2.93, size = 119, normalized size = 1.02

$$\frac{1}{22}\,Bb^5x^{22} + \frac{1}{19}\,(5\,Bab^4 + Ab^5)x^{19} + \frac{5}{16}\,(2\,Ba^2b^3 + Aab^4)x^{16} + \frac{10}{13}\,(Ba^3b^2 + Aa^2b^3)x^{13} + \frac{1}{2}\,(Ba^4b + 2\,Aa^3b^2)x^{10} + \frac{1}{4}\,Aa^5x^4 + \frac{1}{7}\,(Ba^5 + 5\,Aa^4b)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/22*B*b^5*x^22 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/16*(2*B*a^2*b^3 + A*a*b
^4)*x^16 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/2*(B*a^4*b + 2*A*a^3*b^2)
*x^10 + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Sympy [A]
time = 0.02, size = 133, normalized size = 1.14

$$\frac{Aa^5x^4}{4} + \frac{Bb^5x^{22}}{22} + x^{19}\left(\frac{Ab^5}{19} + \frac{5Bab^4}{19}\right) + x^{16}\cdot\left(\frac{5Aab^4}{16} + \frac{5Ba^2b^3}{8}\right) + x^{13}\cdot\left(\frac{10Aa^2b^3}{13} + \frac{10Ba^3b^2}{13}\right) + x^{10}\left(Aa^3b^2 + \frac{Ba^4b}{2}\right) + x^7\cdot\left(\frac{5Aa^4b}{7} + \frac{Ba^5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**4/4 + B*b**5*x**22/22 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**16
*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3*b
**2/13) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**7*(5*A*a**4*b/7 + B*a**5/7)
```

Giac [A]
time = 1.25, size = 124, normalized size = 1.06

$$\frac{1}{22}Bb^5x^{22} + \frac{5}{19}Bab^4x^{19} + \frac{1}{19}Ab^5x^{19} + \frac{5}{8}Ba^2b^3x^{16} + \frac{5}{16}Aab^4x^{16} + \frac{10}{13}Ba^3b^2x^{13} + \frac{10}{13}Aa^2b^3x^{13} + \frac{1}{2}Ba^4bx^{10} + Aa^3b^2x^{10} + \frac{1}{7}Ba^5x^7 + \frac{5}{7}Aa^4bx^7 + \frac{1}{4}Aa^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/22*B*b^5*x^22 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 5/8*B*a^2*b^3*x^16
+ 5/16*A*a*b^4*x^16 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2*b^3*x^13 + 1/2*B*a
^4*b*x^10 + A*a^3*b^2*x^10 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/4*A*a^5*x^
4
```

Mupad [B]
time = 0.04, size = 107, normalized size = 0.91

$$x^7\left(\frac{B\,a^5}{7} + \frac{5\,A\,b\,a^4}{7}\right) + x^{19}\left(\frac{A\,b^5}{19} + \frac{5\,B\,a\,b^4}{19}\right) + \frac{A\,a^5\,x^4}{4} + \frac{B\,b^5\,x^{22}}{22} + \frac{10\,a^2\,b^2\,x^{13}\,(A\,b + B\,a)}{13} + \frac{a^3\,b\,x^{10}\,(2\,A\,b + B\,a)}{2} + \frac{5\,a\,b^3\,x^{16}\,(A\,b + 2\,B\,a)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^7*((B*a^5)/7 + (5*A*a^4*b)/7) + x^19*((A*b^5)/19 + (5*B*a*b^4)/19) + (A*a
^5*x^4)/4 + (B*b^5*x^22)/22 + (10*a^2*b^2*x^13*(A*b + B*a))/13 + (a^3*b*x^1
0*(2*A*b + B*a))/2 + (5*a*b^3*x^16*(A*b + 2*B*a))/16
```

3.30 $\int x^2(a+bx^3)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=42

$$\frac{(Ab-aB)\left(a+bx^3\right)^6}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}$$

```
[Out] 1/18*(A*b-B*a)*(b*x^3+a)^6/b^2+1/21*B*(b*x^3+a)^7/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {455, 45}

$$\frac{\left(a+bx^3\right)^6(Ab-aB)}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^3)^6)/(18*b^2) + (B*(a + b*x^3)^7)/(21*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx&=\frac{1}{3}\text{Subst}\left(\int(a+bx)^5(A+Bx)\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^5}{b}+\frac{B(a+bx)^6}{b}\right)\,dx,x,x^3\right)\\&=\frac{(Ab-aB)\left(a+bx^3\right)^6}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(42) = 84.
time = 0.01, size = 107, normalized size = 2.55

$$\frac{1}{126}x^3(42a^5A + 21a^4(5Ab + aB)x^3 + 70a^3b(2Ab + aB)x^6 + 105a^2b^2(Ab + aB)x^9 + 42ab^3(Ab + 2aB)x^{12} + 7b^4(Ab + 5aB)x^{15} + 6b^5Bx^{18})$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^3*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^3 + 70*a^3*b*(2*A*b + a*B)*x^6 + 10
5*a^2*b^2*(A*b + a*B)*x^9 + 42*a*b^3*(A*b + 2*a*B)*x^12 + 7*b^4*(A*b + 5*a*
B)*x^15 + 6*b^5*B*x^18))/126
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(38) = 76.
time = 0.30, size = 124, normalized size = 2.95

method	result
norman	$\frac{a^5 A x^3}{3} + \left(\frac{5}{6}a^4bA + \frac{1}{6}a^5B\right)x^6 + \left(\frac{10}{9}a^3b^2A + \frac{5}{9}a^4bB\right)x^9 + \left(\frac{5}{6}a^2b^3A + \frac{5}{6}a^3b^2B\right)x^{12} + \left(\frac{1}{3}a\,b^4A + \frac{2}{3}a^2b\right.$
default	$\frac{b^5 B x^{21}}{21} + \frac{(b^5A+5a\,b^4B)x^{18}}{18} + \frac{(5a\,b^4A+10a^2b^3B)x^{15}}{15} + \frac{(10a^2b^3A+10a^3b^2B)x^{12}}{12} + \frac{(10a^3b^2A+5a^4bB)x^9}{9} + \frac{(5a^4bA+a^5B)}{6}$
gosper	$\frac{1}{3}a^5A\,x^3 + \frac{5}{6}x^6a^4bA + \frac{1}{6}x^6a^5B + \frac{10}{9}x^9a^3b^2A + \frac{5}{9}x^9a^4bB + \frac{5}{6}x^{12}a^2b^3A + \frac{5}{6}x^{12}a^3b^2B + \frac{1}{3}x^{15}a\,b^4A +$
risch	$\frac{1}{3}a^5A\,x^3 + \frac{5}{6}x^6a^4bA + \frac{1}{6}x^6a^5B + \frac{10}{9}x^9a^3b^2A + \frac{5}{9}x^9a^4bB + \frac{5}{6}x^{12}a^2b^3A + \frac{5}{6}x^{12}a^3b^2B + \frac{1}{3}x^{15}a\,b^4A +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*b^5*B*x^21+1/18*(A*b^5+5*B*a*b^4)*x^18+1/15*(5*A*a*b^4+10*B*a^2*b^3)*x
^15+1/12*(10*A*a^2*b^3+10*B*a^3*b^2)*x^12+1/9*(10*A*a^3*b^2+5*B*a^4*b)*x^9+
1/6*(5*A*a^4*b+B*a^5)*x^6+1/3*a^5*A*x^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(38) = 76.
time = 0.29, size = 119, normalized size = 2.83

$$\frac{1}{21}Bb^5x^{21} + \frac{1}{18}(5Bab^4 + Ab^5)x^{18} + \frac{1}{3}(2Ba^2b^3 + Aab^4)x^{15} + \frac{5}{6}(Ba^3b^2 + Aa^2b^3)x^{12} + \frac{5}{9}(Ba^4b + 2Aa^3b^2)x^9 + \frac{1}{3}Aa^5x^3 + \frac{1}{6}(Ba^5 + 5Aa^4b)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b^
4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*x^
9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(38) = 76$.

time = 3.50, size = 119, normalized size = 2.83

$$\frac{1}{21}Bb^5x^{21} + \frac{1}{18}(5Bab^4 + Ab^5)x^{18} + \frac{1}{3}(2Ba^2b^3 + Aab^4)x^{15} + \frac{5}{6}(Ba^3b^2 + Aa^2b^3)x^{12} + \frac{5}{9}(Ba^4b + 2Aa^3b^2)x^9 + \frac{1}{3}Aa^5x^3 + \frac{1}{6}(Ba^5 + 5Aa^4b)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b^
4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*x^
9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(36) = 72$.

time = 0.02, size = 136, normalized size = 3.24

$$\frac{Aa^5x^3}{3} + \frac{Bb^5x^{21}}{21} + x^{18}\left(\frac{Ab^5}{18} + \frac{5Bab^4}{18}\right) + x^{15}\left(\frac{Aab^4}{3} + \frac{2Ba^2b^3}{3}\right) + x^{12}\cdot\left(\frac{5Aa^2b^3}{6} + \frac{5Ba^3b^2}{6}\right) + x^9\cdot\left(\frac{10Aa^3b^2}{9} + \frac{5Ba^4b}{9}\right) + x^6\cdot\left(\frac{5Aa^4b}{6} + \frac{Ba^5}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**3/3 + B*b**5*x**21/21 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**15
*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b**2/6)
 + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**6*(5*A*a**4*b/6 + B*a**5/6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(38) = 76$.

time = 1.05, size = 125, normalized size = 2.98

$$\frac{1}{21}Bb^5x^{21} + \frac{5}{18}Bab^4x^{18} + \frac{1}{18}Ab^5x^{18} + \frac{2}{3}Ba^2b^3x^{15} + \frac{1}{3}Aab^4x^{15} + \frac{5}{6}Ba^3b^2x^{12} + \frac{5}{6}Aa^2b^3x^{12} + \frac{5}{9}Ba^4bx^9 + \frac{10}{9}Aa^3b^2x^9 + \frac{1}{6}Ba^5x^6 + \frac{5}{6}Aa^4bx^6 + \frac{1}{3}Aa^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/21*B*b^5*x^21 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 2/3*B*a^2*b^3*x^15
+ 1/3*A*a*b^4*x^15 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 5/9*B*a^4*b*
x^9 + 10/9*A*a^3*b^2*x^9 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/3*A*a^5*x^3
```

Mupad [B]

time = 0.04, size = 107, normalized size = 2.55

$$x^6\left(\frac{B\,a^5}{6} + \frac{5\,A\,b\,a^4}{6}\right) + x^{18}\left(\frac{A\,b^5}{18} + \frac{5\,B\,a\,b^4}{18}\right) + \frac{A\,a^5\,x^3}{3} + \frac{B\,b^5\,x^{21}}{21} + \frac{5\,a^2\,b^2\,x^{12}\,(A\,b + B\,a)}{6} + \frac{5\,a^3\,b\,x^9\,(2\,A\,b + B\,a)}{9} + \frac{a\,b^3\,x^{15}\,(A\,b + 2\,B\,a)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^6*((B*a^5)/6 + (5*A*a^4*b)/6) + x^18*((A*b^5)/18 + (5*B*a*b^4)/18) + (A*a
^5*x^3)/3 + (B*b^5*x^21)/21 + (5*a^2*b^2*x^12*(A*b + B*a))/6 + (5*a^3*b*x^9
*(2*A*b + B*a))/9 + (a*b^3*x^15*(A*b + 2*B*a))/3
```

3.31 $\int x(a+bx^3)^5 (A+Bx^3)\, dx$

Optimal. Leaf size=117

$$\frac{1}{2}a^5Ax^2+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{8}a^3b(2Ab+aB)x^8+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{14}ab^3(Ab+2aB)x^{14}+\frac{1}{17}b^4(Ab+5aB)x$$

```
[Out] 1/2*a^5*A*x^2+1/5*a^4*(5*A*b+B*a)*x^5+5/8*a^3*b*(2*A*b+B*a)*x^8+10/11*a^2*b
^2*(A*b+B*a)*x^11+5/14*a*b^3*(A*b+2*B*a)*x^14+1/17*b^4*(A*b+5*B*a)*x^17+1/2
0*b^5*B*x^20
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {459}

$$\frac{1}{2}a^5Ax^2+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)+\frac{1}{20}b^5Bx^{20}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*
(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \int \left(a^5Ax+a^4(5Ab+aB)x^4+5a^3b(2Ab+aB)x^7+10a^2b^2(Ab+aB)x^{10}+\right.\\ &= \frac{1}{2}a^5Ax^2+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{8}a^3b(2Ab+aB)x^8+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{2}a^5Ax^2+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{8}a^3b(2Ab+aB)x^8+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{14}ab^3(Ab+2aB)x^{14}+\frac{1}{17}b^4(Ab+5aB)x^{17}+\frac{1}{20}b^5Bx^{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*
(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

Maple [A]
time = 0.31, size = 124, normalized size = 1.06

method	result
norman	$\frac{a^5 A x^2}{2}+\left(a^4 b A+\frac{1}{5} a^5 B\right) x^5+\left(\frac{5}{4} a^3 b^2 A+\frac{5}{8} a^4 b B\right) x^8+\left(\frac{10}{11} a^2 b^3 A+\frac{10}{11} a^3 b^2 B\right) x^{11}+\left(\frac{5}{14} a\, b^4 A+\frac{5}{7} a\right.$
default	$\frac{b^5 B x^{20}}{20}+\frac{\left(b^5 A+5 a\, b^4 B\right) x^{17}}{17}+\frac{\left(5 a\, b^4 A+10 a^2 b^3 B\right) x^{14}}{14}+\frac{\left(10 a^2 b^3 A+10 a^3 b^2 B\right) x^{11}}{11}+\frac{\left(10 a^3 b^2 A+5 a^4 b B\right) x^8}{8}+\frac{\left(5 a^4 b A+a^5 B\right.}{5}$
gosper	$\frac{1}{2} a^5 A\, x^2+x^5 a^4 b A+\frac{1}{5} x^5 a^5 B+\frac{5}{4} x^8 a^3 b^2 A+\frac{5}{8} x^8 a^4 b B+\frac{10}{11} x^{11} a^2 b^3 A+\frac{10}{11} x^{11} a^3 b^2 B+\frac{5}{14} x^{14} a\, b^4 A$
risch	$\frac{1}{2} a^5 A\, x^2+x^5 a^4 b A+\frac{1}{5} x^5 a^5 B+\frac{5}{4} x^8 a^3 b^2 A+\frac{5}{8} x^8 a^4 b B+\frac{10}{11} x^{11} a^2 b^3 A+\frac{10}{11} x^{11} a^3 b^2 B+\frac{5}{14} x^{14} a\, b^4 A$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*b^5*B*x^20+1/17*(A*b^5+5*B*a*b^4)*x^17+1/14*(5*A*a*b^4+10*B*a^2*b^3)*x
^14+1/11*(10*A*a^2*b^3+10*B*a^3*b^2)*x^11+1/8*(10*A*a^3*b^2+5*B*a^4*b)*x^8+
1/5*(5*A*a^4*b+B*a^5)*x^5+1/2*a^5*A*x^2
```

Maxima [A]
time = 0.27, size = 119, normalized size = 1.02

$$\frac{1}{20} Bb^5x^{20}+\frac{1}{17}\left(5 Bab^4+Ab^5\right)x^{17}+\frac{5}{14}\left(2 Ba^2b^3+Aab^4\right)x^{14}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}+\frac{5}{8}\left(Ba^4b+2 Aa^3b^2\right)x^8+\frac{1}{2} Aa^5x^2+\frac{1}{5}\left(Ba^5+5 Aa^4b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/20*B*b^5*x^20 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 5/14*(2*B*a^2*b^3 + A*a*b
^4)*x^14 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 5/8*(B*a^4*b + 2*A*a^3*b^2)
*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Fricas [A]
time = 3.08, size = 119, normalized size = 1.02

$$\frac{1}{20} Bb^5x^{20}+\frac{1}{17}\left(5 Bab^4+Ab^5\right)x^{17}+\frac{5}{14}\left(2 Ba^2b^3+Aab^4\right)x^{14}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}+\frac{5}{8}\left(Ba^4b+2 Aa^3b^2\right)x^8+\frac{1}{2} Aa^5x^2+\frac{1}{5}\left(Ba^5+5 Aa^4b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/20*B*b^5*x^20 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 5/14*(2*B*a^2*b^3 + A*a*b
^4)*x^14 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 5/8*(B*a^4*b + 2*A*a^3*b^2)
*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Sympy [A]
time = 0.02, size = 134, normalized size = 1.15

$$\frac{Aa^5x^2}{2}+\frac{Bb^5x^{20}}{20}+x^{17}\left(\frac{Ab^5}{17}+\frac{5Bab^4}{17}\right)+x^{14}\cdot\left(\frac{5Aab^4}{14}+\frac{5Ba^2b^3}{7}\right)+x^{11}\cdot\left(\frac{10Aa^2b^3}{11}+\frac{10Ba^3b^2}{11}\right)+x^8\cdot\left(\frac{5Aa^3b^2}{4}+\frac{5Ba^4b}{8}\right)+x^5\left(Aa^4b+\frac{Ba^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**2/2 + B*b**5*x**20/20 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**14
*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3*b
**2/11) + x**8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**5*(A*a**4*b + B*a**5/5
)
```

Giac [A]
time = 1.49, size = 124, normalized size = 1.06

$$\frac{1}{20}Bb^5x^{20}+\frac{5}{17}Bab^4x^{17}+\frac{1}{17}Ab^5x^{17}+\frac{5}{7}Ba^2b^3x^{14}+\frac{5}{14}Aab^4x^{14}+\frac{10}{11}Ba^3b^2x^{11}+\frac{10}{11}Aa^2b^3x^{11}+\frac{5}{8}Ba^4bx^8+\frac{5}{4}Aa^3b^2x^8+\frac{1}{5}Ba^5x^5+Aa^4bx^5+\frac{1}{2}Aa^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/20*B*b^5*x^20 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 5/7*B*a^2*b^3*x^14
+ 5/14*A*a*b^4*x^14 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a^2*b^3*x^11 + 5/8*B*a
^4*b*x^8 + 5/4*A*a^3*b^2*x^8 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/2*A*a^5*x^2
```

Mupad [B]
time = 0.04, size = 106, normalized size = 0.91

$$x^5\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)+x^{17}\left(\frac{A\,b^5}{17}+\frac{5\,B\,a\,b^4}{17}\right)+\frac{A\,a^5\,x^2}{2}+\frac{B\,b^5\,x^{20}}{20}+\frac{10\,a^2\,b^2\,x^{11}\,(A\,b+B\,a)}{11}+\frac{5\,a^3\,b\,x^8\,(2\,A\,b+B\,a)}{8}+\frac{5\,a\,b^3\,x^{14}\,(A\,b+2\,B\,a)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^5*((B*a^5)/5 + A*a^4*b) + x^17*((A*b^5)/17 + (5*B*a*b^4)/17) + (A*a^5*x^2
)/2 + (B*b^5*x^20)/20 + (10*a^2*b^2*x^11*(A*b + B*a))/11 + (5*a^3*b*x^8*(2*
A*b + B*a))/8 + (5*a*b^3*x^14*(A*b + 2*B*a))/14
```

3.32 $\int (a+bx^3)^5 (A+Bx^3)\, dx$

Optimal. Leaf size=109

$$a^5Ax+\frac{1}{4}a^4(5Ab+aB)x^4+\frac{5}{7}a^3b(2Ab+aB)x^7+a^2b^2(Ab+aB)x^{10}+\frac{5}{13}ab^3(Ab+2aB)x^{13}+\frac{1}{16}b^4(Ab+5aB)x^{16}+$$

```
[Out] a^5*A*x+1/4*a^4*(5*A*b+B*a)*x^4+5/7*a^3*b*(2*A*b+B*a)*x^7+a^2*b^2*(A*b+B*a)
*x^10+5/13*a*b^3*(A*b+2*B*a)*x^13+1/16*b^4*(A*b+5*B*a)*x^16+1/19*b^5*B*x^19
```

Rubi [A]
time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$a^5Ax+\frac{1}{4}a^4x^4(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+a^2b^2x^{10}(aB+Ab)+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}b^5Bx^{19}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b
^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*
x^16)/16 + (b^5*B*x^19)/19
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (a+bx^3)^5 (A+Bx^3)\, dx = \int \left(a^5A+a^4(5Ab+aB)x^3+5a^3b(2Ab+aB)x^6+10a^2b^2(Ab+aB)x^9+5a\right.$$
$$= a^5Ax+\frac{1}{4}a^4(5Ab+aB)x^4+\frac{5}{7}a^3b(2Ab+aB)x^7+a^2b^2(Ab+aB)x^{10}+\frac{5}{13}a$$

Mathematica [A]
time = 0.01, size = 109, normalized size = 1.00

$$a^5Ax+\frac{1}{4}a^4(5Ab+aB)x^4+\frac{5}{7}a^3b(2Ab+aB)x^7+a^2b^2(Ab+aB)x^{10}+\frac{5}{13}ab^3(Ab+2aB)x^{13}+\frac{1}{16}b^4(Ab+5aB)x^{16}+\frac{1}{19}b^5Bx^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b
^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*
x^16)/16 + (b^5*B*x^19)/19
```

Maple [A]
time = 0.32, size = 121, normalized size = 1.11

method	result
norman	$a^5Ax + \left(\frac{5}{4}a^4bA + \frac{1}{4}a^5B\right)x^4 + \left(\frac{10}{7}a^3b^2A + \frac{5}{7}a^4bB\right)x^7 + \left(a^2b^3A + a^3b^2B\right)x^{10} + \left(\frac{5}{13}a\,b^4A + \frac{10}{13}a^2b^3\right.$
gosper	$a^5Ax + \frac{5}{4}x^4a^4bA + \frac{1}{4}x^4a^5B + \frac{10}{7}x^7a^3b^2A + \frac{5}{7}x^7a^4bB + A\,a^2b^3x^{10} + B\,a^3b^2x^{10} + \frac{5}{13}x^{13}a\,b^4A + \frac{10}{13}x$
default	$\frac{b^5Bx^{19}}{19} + \frac{(b^5A+5a\,b^4B)x^{16}}{16} + \frac{(5a\,b^4A+10a^2b^3B)x^{13}}{13} + \frac{(10a^2b^3A+10a^3b^2B)x^{10}}{10} + \frac{(10a^3b^2A+5a^4bB)x^7}{7} + \frac{(5a^4bA+a^5B)}{4}$
risch	$a^5Ax + \frac{5}{4}x^4a^4bA + \frac{1}{4}x^4a^5B + \frac{10}{7}x^7a^3b^2A + \frac{5}{7}x^7a^4bB + A\,a^2b^3x^{10} + B\,a^3b^2x^{10} + \frac{5}{13}x^{13}a\,b^4A + \frac{10}{13}x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/19*b^5*B*x^19+1/16*(A*b^5+5*B*a*b^4)*x^16+1/13*(5*A*a*b^4+10*B*a^2*b^3)*x
^13+1/10*(10*A*a^2*b^3+10*B*a^3*b^2)*x^10+1/7*(10*A*a^3*b^2+5*B*a^4*b)*x^7+
1/4*(5*A*a^4*b+B*a^5)*x^4+a^5*A*x
```

Maxima [A]
time = 0.28, size = 115, normalized size = 1.06

$$\frac{1}{19}Bb^5x^{19} + \frac{1}{16}\left(5\,Bab^4 + Ab^5\right)x^{16} + \frac{5}{13}\left(2\,Ba^2b^3 + Aab^4\right)x^{13} + \left(Ba^3b^2 + Aa^2b^3\right)x^{10} + \frac{5}{7}\left(Ba^4b + 2\,Aa^3b^2\right)x^7 + Aa^5x + \frac{1}{4}\left(Ba^5 + 5\,Aa^4b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/19*B*b^5*x^19 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/13*(2*B*a^2*b^3 + A*a*b
^4)*x^13 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7 +
 A*a^5*x + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Fricas [A]
time = 2.29, size = 115, normalized size = 1.06

$$\frac{1}{19}Bb^5x^{19} + \frac{1}{16}\left(5\,Bab^4 + Ab^5\right)x^{16} + \frac{5}{13}\left(2\,Ba^2b^3 + Aab^4\right)x^{13} + \left(Ba^3b^2 + Aa^2b^3\right)x^{10} + \frac{5}{7}\left(Ba^4b + 2\,Aa^3b^2\right)x^7 + Aa^5x + \frac{1}{4}\left(Ba^5 + 5\,Aa^4b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/19*B*b^5*x^19 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/13*(2*B*a^2*b^3 + A*a*b
^4)*x^13 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7 +
 A*a^5*x + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Sympy [A]
time = 0.02, size = 128, normalized size = 1.17

$$Aa^5x + \frac{Bb^5x^{19}}{19} + x^{16}\left(\frac{Ab^5}{16} + \frac{5Bab^4}{16}\right) + x^{13}\cdot\left(\frac{5Aab^4}{13} + \frac{10Ba^2b^3}{13}\right) + x^{10}\left(Aa^2b^3 + Ba^3b^2\right) + x^7\cdot\left(\frac{10Aa^3b^2}{7} + \frac{5Ba^4b}{7}\right) + x^4\cdot\left(\frac{5Aa^4b}{4} + \frac{Ba^5}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x + B*b**5*x**19/19 + x**16*(A*b**5/16 + 5*B*a*b**4/16) + x**13*(5*A
*a*b**4/13 + 10*B*a**2*b**3/13) + x**10*(A*a**2*b**3 + B*a**3*b**2) + x**7*
(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

Giac [A]
time = 1.57, size = 120, normalized size = 1.10

$$\frac{1}{19}Bb^5x^{19} + \frac{5}{16}Bab^4x^{16} + \frac{1}{16}Ab^5x^{16} + \frac{10}{13}Ba^2b^3x^{13} + \frac{5}{13}Aab^4x^{13} + Ba^3b^2x^{10} + Aa^2b^3x^{10} + \frac{5}{7}Ba^4bx^7 + \frac{10}{7}Aa^3b^2x^7 + \frac{1}{4}Ba^5x^4 + \frac{5}{4}Aa^4bx^4 + Aa^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/19*B*b^5*x^19 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 10/13*B*a^2*b^3*x^1
3 + 5/13*A*a*b^4*x^13 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 + 5/7*B*a^4*b*x^7 +
 10/7*A*a^3*b^2*x^7 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + A*a^5*x
```

Mupad [B]
time = 0.04, size = 103, normalized size = 0.94

$$x^4\left(\frac{B\,a^5}{4} + \frac{5\,A\,b\,a^4}{4}\right) + x^{16}\left(\frac{A\,b^5}{16} + \frac{5\,B\,a\,b^4}{16}\right) + \frac{B\,b^5\,x^{19}}{19} + A\,a^5\,x + a^2\,b^2\,x^{10}\,(A\,b + B\,a) + \frac{5\,a^3\,b\,x^7\,(2\,A\,b + B\,a)}{7} + \frac{5\,a\,b^3\,x^{13}\,(A\,b + 2\,B\,a)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^4*((B*a^5)/4 + (5*A*a^4*b)/4) + x^16*((A*b^5)/16 + (5*B*a*b^4)/16) + (B*b
^5*x^19)/19 + A*a^5*x + a^2*b^2*x^10*(A*b + B*a) + (5*a^3*b*x^7*(2*A*b + B*
a))/7 + (5*a*b^3*x^13*(A*b + 2*B*a))/13
```

3.33 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x} \, dx$

Optimal. Leaf size=88

$$\frac{5}{3}a^4Abx^3 + \frac{5}{3}a^3Ab^2x^6 + \frac{10}{9}a^2Ab^3x^9 + \frac{5}{12}aAb^4x^{12} + \frac{1}{15}Ab^5x^{15} + \frac{B(a+bx^3)^6}{18b} + a^5A\log(x)$$

```
[Out] 5/3*a^4*A*b*x^3+5/3*a^3*A*b^2*x^6+10/9*a^2*A*b^3*x^9+5/12*a*A*b^4*x^12+1/15
*A*b^5*x^15+1/18*B*(b*x^3+a)^6/b+a^5*A*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {457, 81, 45}

$$a^5A\log(x) + \frac{5}{3}a^4Abx^3 + \frac{5}{3}a^3Ab^2x^6 + \frac{10}{9}a^2Ab^3x^9 + \frac{5}{12}aAb^4x^{12} + \frac{B(a+bx^3)^6}{18b} + \frac{1}{15}Ab^5x^{15}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

```
[Out] (5*a^4*A*b*x^3)/3 + (5*a^3*A*b^2*x^6)/3 + (10*a^2*A*b^3*x^9)/9 + (5*a*A*b^4
*x^12)/12 + (A*b^5*x^15)/15 + (B*(a + b*x^3)^6)/(18*b) + a^5*A*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} \, dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x} \, dx, x, x^3\right)$$
$$= \frac{B(a+bx^3)^6}{18b} + \frac{1}{3}A\text{Subst}\left(\int \frac{(a+bx)^5}{x} \, dx, x, x^3\right)$$
$$= \frac{B(a+bx^3)^6}{18b} + \frac{1}{3}A\text{Subst}\left(\int \left(5a^4b + \frac{a^5}{x} + 10a^3b^2x + 10a^2b^3x^2 + 5ab^4x^3 + b\right.\right.$$
$$= \frac{5}{3}a^4Abx^3 + \frac{5}{3}a^3Ab^2x^6 + \frac{10}{9}a^2Ab^3x^9 + \frac{5}{12}aAb^4x^{12} + \frac{1}{15}Ab^5x^{15} + \frac{B(a+bx^3)}{18b}$$

Mathematica [A]
time = 0.02, size = 113, normalized size = 1.28

$$\frac{1}{3}a^4(5Ab+aB)x^3 + \frac{5}{6}a^3b(2Ab+aB)x^6 + \frac{10}{9}a^2b^2(Ab+aB)x^9 + \frac{5}{12}ab^3(Ab+2aB)x^{12} + \frac{1}{15}b^4(Ab+5aB)x^{15} + \frac{1}{18}b^5Bx^{18} + a^5A\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

```
[Out] (a^4*(5*A*b + a*B)*x^3)/3 + (5*a^3*b*(2*A*b + a*B)*x^6)/6 + (10*a^2*b^2*(A*
b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b + 5*a*B)*x^15
)/15 + (b^5*B*x^18)/18 + a^5*A*Log[x]
```

Maple [A]
time = 0.25, size = 124, normalized size = 1.41

method	result
norman	$\left(\frac{1}{15}b^5A+\frac{1}{3}a\,b^4B\right)x^{15}+\left(\frac{5}{12}a\,b^4A+\frac{5}{6}a^2b^3B\right)x^{12}+\left(\frac{10}{9}a^2b^3A+\frac{10}{9}a^3b^2B\right)x^9+\left(\frac{5}{3}a^3b^2A+\frac{5}{6}a^4bB\right.$
default	$\frac{b^5B\,x^{18}}{18}+\frac{A\,b^5x^{15}}{15}+\frac{Ba\,b^4x^{15}}{3}+\frac{5aA\,b^4x^{12}}{12}+\frac{5B\,a^2b^3x^{12}}{6}+\frac{10a^2A\,b^3x^9}{9}+\frac{10B\,a^3b^2x^9}{9}+\frac{5a^3A\,b^2x^6}{3}+\frac{5B\,a^4b\,x^6}{6}+$
risch	$\frac{b^5B\,x^{18}}{18}+\frac{A\,b^5x^{15}}{15}+\frac{Ba\,b^4x^{15}}{3}+\frac{5aA\,b^4x^{12}}{12}+\frac{5B\,a^2b^3x^{12}}{6}+\frac{10a^2A\,b^3x^9}{9}+\frac{10B\,a^3b^2x^9}{9}+\frac{5a^3A\,b^2x^6}{3}+\frac{5B\,a^4b\,x^6}{6}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*b^5*B*x^18+1/15*A*b^5*x^15+1/3*B*a*b^4*x^15+5/12*a*A*b^4*x^12+5/6*B*a^
2*b^3*x^12+10/9*a^2*A*b^3*x^9+10/9*B*a^3*b^2*x^9+5/3*a^3*A*b^2*x^6+5/6*B*a^
4*b*x^6+5/3*a^4*A*b*x^3+1/3*B*a^5*x^3+a^5*A*ln(x)
```

Maxima [A]
time = 0.29, size = 120, normalized size = 1.36

$$\frac{1}{18}Bb^5x^{18} + \frac{1}{15}(5Bab^4 + Ab^5)x^{15} + \frac{5}{12}(2Ba^2b^3 + Aab^4)x^{12} + \frac{10}{9}(Ba^3b^2 + Aa^2b^3)x^9 + \frac{5}{6}(Ba^4b + 2Aa^3b^2)x^6 + \frac{1}{3}Aa^5\log(x^3) + \frac{1}{3}(Ba^5 + 5Aa^4b)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 1/3*A*a^5*log(x^3) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Fricas [A]
time = 2.64, size = 117, normalized size = 1.33

$$\frac{1}{18}\,Bb^5x^{18}+\frac{1}{15}\,(5\,Bab^4+Ab^5)x^{15}+\frac{5}{12}\,(2\,Ba^2b^3+Aab^4)x^{12}+\frac{10}{9}\,(Ba^3b^2+Aa^2b^3)x^9+\frac{5}{6}\,(Ba^4b+2\,Aa^3b^2)x^6+Aa^5\log{(x)}+\frac{1}{3}\,(Ba^5+5\,Aa^4b)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] 1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + A*a^5*log(x) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Sympy [A]
time = 0.09, size = 134, normalized size = 1.52

$$Aa^5\log{(x)}+\frac{Bb^5x^{18}}{18}+x^{15}\left(\frac{Ab^5}{15}+\frac{Bab^4}{3}\right)+x^{12}\cdot\left(\frac{5Aab^4}{12}+\frac{5Ba^2b^3}{6}\right)+x^9\cdot\left(\frac{10Aa^2b^3}{9}+\frac{10Ba^3b^2}{9}\right)+x^6\cdot\left(\frac{5Aa^3b^2}{3}+\frac{5Ba^4b}{6}\right)+x^3\cdot\left(\frac{5Aa^4b}{3}+\frac{Ba^5}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x,x)
```

```
[Out] A*a**5*log(x) + B*b**5*x**18/18 + x**15*(A*b**5/15 + B*a*b**4/3) + x**12*(5
*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3*b**2/9
) + x**6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**3*(5*A*a**4*b/3 + B*a**5/3)
```

Giac [A]
time = 1.36, size = 124, normalized size = 1.41

$$\frac{1}{18}\,Bb^5x^{18}+\frac{1}{3}\,Bab^4x^{15}+\frac{1}{15}\,Ab^5x^{15}+\frac{5}{6}\,Ba^2b^3x^{12}+\frac{5}{12}\,Aab^4x^{12}+\frac{10}{9}\,Ba^3b^2x^9+\frac{10}{9}\,Aa^2b^3x^9+\frac{5}{6}\,Ba^4bx^6+\frac{5}{3}\,Aa^3b^2x^6+\frac{1}{3}\,Ba^5x^3+\frac{5}{3}\,Aa^4bx^3+Aa^5\log{(|x|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 1/18*B*b^5*x^18 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 5/6*B*a^2*b^3*x^12 +
 5/12*A*a*b^4*x^12 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/6*B*a^4*b*
x^6 + 5/3*A*a^3*b^2*x^6 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + A*a^5*log(abs(x
))
```

Mupad [B]
time = 0.05, size = 105, normalized size = 1.19

$$x^3\left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)+x^{15}\left(\frac{A\,b^5}{15}+\frac{B\,a\,b^4}{3}\right)+\frac{B\,b^5\,x^{18}}{18}+A\,a^5\,\ln{(x)}+\frac{10\,a^2\,b^2\,x^9\,(A\,b+B\,a)}{9}+\frac{5\,a^3\,b\,x^6\,(2\,A\,b+B\,a)}{6}+\frac{5\,a\,b^3\,x^{12}\,(A\,b+2\,B\,a)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x,x)
```

```
[Out] x^3*((B*a^5)/3 + (5*A*a^4*b)/3) + x^15*((A*b^5)/15 + (B*a*b^4)/3) + (B*b^5*
x^18)/18 + A*a^5*log(x) + (10*a^2*b^2*x^9*(A*b + B*a))/9 + (5*a^3*b*x^6*(2*
A*b + B*a))/6 + (5*a*b^3*x^12*(A*b + 2*B*a))/12
```

3.34 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx$

Optimal. Leaf size=112

$$-\frac{a^5A}{x}+\frac{1}{2}a^4(5Ab+aB)x^2+a^3b(2Ab+aB)x^5+\frac{5}{4}a^2b^2(Ab+aB)x^8+\frac{5}{11}ab^3(Ab+2aB)x^{11}+\frac{1}{14}b^4(Ab+5aB)x^{14}+\frac{1}{17}$$

```
[Out] -a^5*A/x+1/2*a^4*(5*A*b+B*a)*x^2+a^3*b*(2*A*b+B*a)*x^5+5/4*a^2*b^2*(A*b+B*a
)*x^8+5/11*a*b^3*(A*b+2*B*a)*x^11+1/14*b^4*(A*b+5*B*a)*x^14+1/17*b^5*B*x^17
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{x}+\frac{1}{2}a^4x^2(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{5}{4}a^2b^2x^8(aB+Ab)+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2
*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a
*B)*x^14)/14 + (b^5*B*x^17)/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx &= \int \left(\frac{a^5A}{x^2}+a^4(5Ab+aB)x+5a^3b(2Ab+aB)x^4+10a^2b^2(Ab+aB)x^7+5ab^3\right.\\ &= -\frac{a^5A}{x}+\frac{1}{2}a^4(5Ab+aB)x^2+a^3b(2Ab+aB)x^5+\frac{5}{4}a^2b^2(Ab+aB)x^8+\frac{5}{11}ab^3\end{aligned}$$

Mathematica [A]
time = 0.02, size = 112, normalized size = 1.00

$$-\frac{a^5A}{x}+\frac{1}{2}a^4(5Ab+aB)x^2+a^3b(2Ab+aB)x^5+\frac{5}{4}a^2b^2(Ab+aB)x^8+\frac{5}{11}ab^3(Ab+2aB)x^{11}+\frac{1}{14}b^4(Ab+5aB)x^{14}+\frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2
*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a
*B)*x^14)/14 + (b^5*B*x^17)/17
```

Maple [A]
time = 0.27, size = 125, normalized size = 1.12

method	result
norman	$\frac{-a^5A+\left(\frac{5}{2}a^4bA+\frac{1}{2}a^5B\right)x^3+\left(2a^3b^2A+a^4bB\right)x^6+\left(\frac{5}{4}a^2b^3A+\frac{5}{4}a^3b^2B\right)x^9+\left(\frac{5}{11}ab^4A+\frac{10}{11}a^2b^3B\right)x^{12}+\left(\frac{1}{14}b^5A+\frac{5}{14}ab^4B\right)x^{15}+\frac{b^5Bx^1}{17}}{x}$
default	$\frac{b^5Bx^{17}}{17}+\frac{Ab^5x^{14}}{14}+\frac{5Bab^4x^{14}}{14}+\frac{5Aab^4x^{11}}{11}+\frac{10Ba^2b^3x^{11}}{11}+\frac{5Aa^2b^3x^8}{4}+\frac{5Ba^3b^2x^8}{4}+2a^3Ab^2x^5+Ba^4bx^5$
risch	$\frac{b^5Bx^{17}}{17}+\frac{Ab^5x^{14}}{14}+\frac{5Bab^4x^{14}}{14}+\frac{5Aab^4x^{11}}{11}+\frac{10Ba^2b^3x^{11}}{11}+\frac{5Aa^2b^3x^8}{4}+\frac{5Ba^3b^2x^8}{4}+2a^3Ab^2x^5+Ba^4bx^5$
gosper	$-\frac{-308b^5Bx^{18}-374Ab^5x^{15}-1870Bab^4x^{15}-2380aAb^4x^{12}-4760Ba^2b^3x^{12}-6545a^2Ab^3x^9-6545Ba^3b^2x^9-10472a^3Ab^2x^6-5236}{5236x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*b^5*B*x^17+1/14*A*b^5*x^14+5/14*B*a*b^4*x^14+5/11*A*a*b^4*x^11+10/11*B
*a^2*b^3*x^11+5/4*A*a^2*b^3*x^8+5/4*B*a^3*b^2*x^8+2*a^3*A*b^2*x^5+B*a^4*b*x
^5+5/2*a^4*A*b*x^2+1/2*a^5*B*x^2-a^5*A/x
```

Maxima [A]
time = 0.29, size = 118, normalized size = 1.05

$$\frac{1}{17}Bb^5x^{17}+\frac{1}{14}\left(5Bab^4+Ab^5\right)x^{14}+\frac{5}{11}\left(2Ba^2b^3+Aab^4\right)x^{11}+\frac{5}{4}\left(Ba^3b^2+Aa^2b^3\right)x^8+\left(Ba^4b+2Aa^3b^2\right)x^5-\frac{Aa^5}{x}+\frac{1}{2}\left(Ba^5+5Aa^4b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/17*B*b^5*x^17 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/11*(2*B*a^2*b^3 + A*a*b
^4)*x^11 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + (B*a^4*b + 2*A*a^3*b^2)*x^5 -
A*a^5/x + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Fricas [A]
time = 2.19, size = 121, normalized size = 1.08

$$\frac{308\,Bb^5x^{18}+374\left(5\,Bab^4+Ab^5\right)x^{15}+2380\left(2\,Ba^2b^3+Aab^4\right)x^{12}+6545\left(Ba^3b^2+Aa^2b^3\right)x^9+5236\left(Ba^4b+2\,Aa^3b^2\right)x^6-5236\,Aa^5+2618\left(Ba^5+5\,Aa^4b\right)x^3}{5236\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5236*(308*B*b^5*x^18 + 374*(5*B*a*b^4 + A*b^5)*x^15 + 2380*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5236*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 5236*A*a^5 + 2618*(B*a^5 + 5*A*a^4*b)*x^3)/x
```

Sympy [A]
time = 0.09, size = 129, normalized size = 1.15

$$-\frac{Aa^5}{x} + \frac{Bb^5x^{17}}{17} + x^{14}\left(\frac{Ab^5}{14} + \frac{5Bab^4}{14}\right) + x^{11} \cdot \left(\frac{5Aab^4}{11} + \frac{10Ba^2b^3}{11}\right) + x^8 \cdot \left(\frac{5Aa^2b^3}{4} + \frac{5Ba^3b^2}{4}\right) + x^5 \cdot \left(2Aa^3b^2 + Ba^4b\right) + x^2 \cdot \left(\frac{5Aa^4b}{2} + \frac{Ba^5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a**5/x + B*b**5*x**17/17 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**11*(5*
A*a*b**4/11 + 10*B*a**2*b**3/11) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/4)
 + x**5*(2*A*a**3*b**2 + B*a**4*b) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

Giac [A]
time = 1.21, size = 124, normalized size = 1.11

$$\frac{1}{17}\,Bb^5x^{17} + \frac{5}{14}\,Bab^4x^{14} + \frac{1}{14}\,Ab^5x^{14} + \frac{10}{11}\,Ba^2b^3x^{11} + \frac{5}{11}\,Aab^4x^{11} + \frac{5}{4}\,Ba^3b^2x^8 + \frac{5}{4}\,Aa^2b^3x^8 + Ba^4bx^5 + 2\,Aa^3b^2x^5 + \frac{1}{2}\,Ba^5x^2 + \frac{5}{2}\,Aa^4bx^2 - \frac{Aa^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/17*B*b^5*x^17 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 10/11*B*a^2*b^3*x^1
1 + 5/11*A*a*b^4*x^11 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + B*a^4*b*x^5
 + 2*A*a^3*b^2*x^5 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 - A*a^5/x
```

Mupad [B]
time = 0.04, size = 106, normalized size = 0.95

$$x^2\left(\frac{B\,a^5}{2} + \frac{5\,A\,b\,a^4}{2}\right) + x^{14}\left(\frac{A\,b^5}{14} + \frac{5\,B\,a\,b^4}{14}\right) - \frac{A\,a^5}{x} + \frac{B\,b^5\,x^{17}}{17} + \frac{5\,a^2\,b^2\,x^8\,(A\,b + B\,a)}{4} + a^3\,b\,x^5\,(2\,A\,b + B\,a) + \frac{5\,a\,b^3\,x^{11}\,(A\,b + 2\,B\,a)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^2,x)
```

```
[Out] x^2*((B*a^5)/2 + (5*A*a^4*b)/2) + x^14*((A*b^5)/14 + (5*B*a*b^4)/14) - (A*a
^5)/x + (B*b^5*x^17)/17 + (5*a^2*b^2*x^8*(A*b + B*a))/4 + a^3*b*x^5*(2*A*b
+ B*a) + (5*a*b^3*x^11*(A*b + 2*B*a))/11
```

3.35 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} \, dx$

Optimal. Leaf size=112

$$-\frac{a^5 A}{2x^2}+a^4(5Ab+aB)x+\frac{5}{4}a^3b(2Ab+aB)x^4+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{1}{2}ab^3(Ab+2aB)x^{10}+\frac{1}{13}b^4(Ab+5aB)x^{13}+$$

```
[Out] -1/2*a^5*A/x^2+a^4*(5*A*b+B*a)*x+5/4*a^3*b*(2*A*b+B*a)*x^4+10/7*a^2*b^2*(A*
b+B*a)*x^7+1/2*a*b^3*(A*b+2*B*a)*x^10+1/13*b^4*(A*b+5*B*a)*x^13+1/16*b^5*B*
x^16
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5 A}{2x^2}+a^4x(aB+5Ab)+\frac{5}{4}a^3bx^4(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{1}{2}ab^3x^{10}(2aB+Ab)+\frac{1}{16}b^5Bx^{16}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a^5*A)/x^2 + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (1
0*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5
*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} \, dx = \int \left(a^4(5Ab+aB)+\frac{a^5A}{x^3}+5a^3b(2Ab+aB)x^3+10a^2b^2(Ab+aB)x^6+5ab\right.$$

$$= -\frac{a^5A}{2x^2}+a^4(5Ab+aB)x+\frac{5}{4}a^3b(2Ab+aB)x^4+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{1}{2}ab$$

Mathematica [A]
time = 0.02, size = 112, normalized size = 1.00

$$-\frac{a^5A}{2x^2}+a^4(5Ab+aB)x+\frac{5}{4}a^3b(2Ab+aB)x^4+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{1}{2}ab^3(Ab+2aB)x^{10}+\frac{1}{13}b^4(Ab+5aB)x^{13}+\frac{1}{16}b^5Bx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a^5*A)/x^2 + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (1
0*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5
*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

Maple [A]
time = 0.26, size = 120, normalized size = 1.07

method	result
default	$\frac{b^5 B x^{16}}{16}+\frac{A b^5 x^{13}}{13}+\frac{5Ba b^4 x^{13}}{13}+\frac{Aa b^4 x^{10}}{2}+B a^2 b^3 x^{10}+\frac{10a^2 A b^3 x^7}{7}+\frac{10B a^3 b^2 x^7}{7}+\frac{5a^3 A b^2 x^4}{2}+\frac{5B a^4 b x^4}{4}+5$
norman	$\frac{-\frac{a^5 A}{2}+(5a^4bA+a^5B)x^3+(\frac{5}{2}a^3b^2A+\frac{5}{4}a^4bB)x^6+(\frac{10}{7}a^2b^3A+\frac{10}{7}a^3b^2B)x^9+(\frac{1}{2}a b^4A+a^2b^3B)x^{12}+(\frac{1}{13}b^5A+\frac{5}{13}a b^4B)x^{15}+\frac{b^5 B x^{18}}{16}}{x^2}$
risch	$\frac{b^5 B x^{16}}{16}+\frac{A b^5 x^{13}}{13}+\frac{5Ba b^4 x^{13}}{13}+\frac{Aa b^4 x^{10}}{2}+B a^2 b^3 x^{10}+\frac{10a^2 A b^3 x^7}{7}+\frac{10B a^3 b^2 x^7}{7}+\frac{5a^3 A b^2 x^4}{2}+\frac{5B a^4 b x^4}{4}+5$
gosper	$-\frac{-91b^5 B x^{18}-112A b^5 x^{15}-560Ba b^4 x^{15}-728aA b^4 x^{12}-1456B a^2 b^3 x^{12}-2080a^2 A b^3 x^9-2080B a^3 b^2 x^9-3640a^3 A b^2 x^6-1820B a^4 b}{1456x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*b^5*B*x^16+1/13*A*b^5*x^13+5/13*B*a*b^4*x^13+1/2*A*a*b^4*x^10+B*a^2*b^
3*x^10+10/7*a^2*A*b^3*x^7+10/7*B*a^3*b^2*x^7+5/2*a^3*A*b^2*x^4+5/4*B*a^4*b*
x^4+5*a^4*b*A*x+a^5*B*x-1/2*a^5*A/x^2
```

Maxima [A]
time = 0.29, size = 116, normalized size = 1.04

$$\frac{1}{16}\,Bb^5x^{16}+\frac{1}{13}\,(5\,Bab^4+Ab^5)x^{13}+\frac{1}{2}\,(2\,Ba^2b^3+Aab^4)x^{10}+\frac{10}{7}\,(Ba^3b^2+Aa^2b^3)x^7+\frac{5}{4}\,(Ba^4b+2\,Aa^3b^2)x^4-\frac{Aa^5}{2\,x^2}+(Ba^5+5\,Aa^4b)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*B*b^5*x^16 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 1/2*(2*B*a^2*b^3 + A*a*b^
4)*x^10 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + 5/4*(B*a^4*b + 2*A*a^3*b^2)*x^
4 - 1/2*A*a^5/x^2 + (B*a^5 + 5*A*a^4*b)*x
```

Fricas [A]
time = 2.63, size = 121, normalized size = 1.08

$$\frac{91\,Bb^5x^{18}+112\,(5\,Bab^4+Ab^5)x^{15}+728\,(2\,Ba^2b^3+Aab^4)x^{12}+2080\,(Ba^3b^2+Aa^2b^3)x^9+1820\,(Ba^4b+2\,Aa^3b^2)x^6-728\,Aa^5+1456\,(Ba^5+5\,Aa^4b)x^3}{1456\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/1456*(91*B*b^5*x^18 + 112*(5*B*a*b^4 + A*b^5)*x^15 + 728*(2*B*a^2*b^3 + A
*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1820*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 728*A*a^5 + 1456*(B*a^5 + 5*A*a^4*b)*x^3)/x^2
```

Sympy [A]
time = 0.09, size = 128, normalized size = 1.14

$$-\frac{Aa^5}{2x^2}+\frac{Bb^5x^{16}}{16}+x^{13}\left(\frac{Ab^5}{13}+\frac{5Bab^4}{13}\right)+x^{10}\left(\frac{Aab^4}{2}+Ba^2b^3\right)+x^7\cdot\left(\frac{10Aa^2b^3}{7}+\frac{10Ba^3b^2}{7}\right)+x^4\cdot\left(\frac{5Aa^3b^2}{2}+\frac{5Ba^4b}{4}\right)+x\left(5Aa^4b+Ba^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a**5/(2*x**2) + B*b**5*x**16/16 + x**13*(A*b**5/13 + 5*B*a*b**4/13) + x*
*10*(A*a*b**4/2 + B*a**2*b**3) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/7)
 + x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x*(5*A*a**4*b + B*a**5)
```

Giac [A]
time = 1.00, size = 119, normalized size = 1.06

$$\frac{1}{16}Bb^5x^{16}+\frac{5}{13}Bab^4x^{13}+\frac{1}{13}Ab^5x^{13}+Ba^2b^3x^{10}+\frac{1}{2}Aab^4x^{10}+\frac{10}{7}Ba^3b^2x^7+\frac{10}{7}Aa^2b^3x^7+\frac{5}{4}Ba^4bx^4+\frac{5}{2}Aa^3b^2x^4+Ba^5x+5\,Aa^4bx-\frac{Aa^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*B*b^5*x^16 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + B*a^2*b^3*x^10 + 1/
2*A*a*b^4*x^10 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + 5/4*B*a^4*b*x^4
+ 5/2*A*a^3*b^2*x^4 + B*a^5*x + 5*A*a^4*b*x - 1/2*A*a^5/x^2
```

Mupad [B]
time = 0.04, size = 104, normalized size = 0.93

$$x\,\left(B\,a^5+5\,A\,b\,a^4\right)+x^{13}\,\left(\frac{A\,b^5}{13}+\frac{5\,B\,a\,b^4}{13}\right)-\frac{A\,a^5}{2\,x^2}+\frac{B\,b^5\,x^{16}}{16}+\frac{10\,a^2\,b^2\,x^7\,\left(A\,b+B\,a\right)}{7}+\frac{5\,a^3\,b\,x^4\,\left(2\,A\,b+B\,a\right)}{4}+\frac{a\,b^3\,x^{10}\,\left(A\,b+2\,B\,a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^3,x)
```

```
[Out] x*(B*a^5 + 5*A*a^4*b) + x^13*((A*b^5)/13 + (5*B*a*b^4)/13) - (A*a^5)/(2*x^2
) + (B*b^5*x^16)/16 + (10*a^2*b^2*x^7*(A*b + B*a))/7 + (5*a^3*b*x^4*(2*A*b
+ B*a))/4 + (a*b^3*x^10*(A*b + 2*B*a))/2
```

3.36 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^4}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{3x^3}+\frac{5}{3}a^3b(2Ab+aB)x^3+\frac{5}{3}a^2b^2(Ab+aB)x^6+\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{12}b^4(Ab+5aB)x^{12}+\frac{1}{15}b^5Bx^{15}+a^4(5Ab+a$$

```
[Out] -1/3*a^5*A/x^3+5/3*a^3*b*(2*A*b+B*a)*x^3+5/3*a^2*b^2*(A*b+B*a)*x^6+5/9*a*b^
3*(A*b+2*B*a)*x^9+1/12*b^4*(A*b+5*B*a)*x^12+1/15*b^5*B*x^15+a^4*(5*A*b+B*a)
*ln(x)
```

Rubi [A]
time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{3x^3}+a^4\log(x)(aB+5Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+\frac{5}{3}a^2b^2x^6(aB+Ab)+\frac{1}{12}b^4x^{12}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{15}b^5Bx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a^5*A)/x^3 + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x
^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*
B*x^15)/15 + a^4*(5*A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} \, dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^2} \, dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(5a^3b(2Ab+aB) + \frac{a^5A}{x^2} + \frac{a^4(5Ab+aB)}{x} + 10a^2b^2(Ab+aB)x\right.\right.$$
$$= -\frac{a^5A}{3x^3} + \frac{5}{3}a^3b(2Ab+aB)x^3 + \frac{5}{3}a^2b^2(Ab+aB)x^6 + \frac{5}{9}ab^3(Ab+2aB)x^9 + \frac{1}{12}$$

Mathematica [A]
time = 0.02, size = 115, normalized size = 1.02

$$-\frac{a^5A}{3x^3} + \frac{5}{3}a^3b(2Ab+aB)x^3 + \frac{5}{3}a^2b^2(Ab+aB)x^6 + \frac{5}{9}ab^3(Ab+2aB)x^9 + \frac{1}{12}b^4(Ab+5aB)x^{12} + \frac{1}{15}b^5Bx^{15} + \left(5a^4Ab+a^5B\right)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a^5*A)/x^3 + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x
^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*
B*x^15)/15 + (5*a^4*A*b + a^5*B)*Log[x]
```

Maple [A]
time = 0.26, size = 121, normalized size = 1.07

method	result
default	$\frac{b^5Bx^{15}}{15} + \frac{Ab^5x^{12}}{12} + \frac{5Bab^4x^{12}}{12} + \frac{5aAb^4x^9}{9} + \frac{10Ba^2b^3x^9}{9} + \frac{5a^2Ab^3x^6}{3} + \frac{5Ba^3b^2x^6}{3} + \frac{10Aa^3b^2x^3}{3} + \frac{5Ba^4bx^3}{3} -$
norman	$\frac{(\frac{1}{12}b^5A+\frac{5}{12}ab^4B)x^{15}+(\frac{5}{9}ab^4A+\frac{10}{9}a^2b^3B)x^{12}+(\frac{5}{3}a^2b^3A+\frac{5}{3}a^3b^2B)x^9+(\frac{10}{3}a^3b^2A+\frac{5}{3}a^4bB)x^6-\frac{a^5A}{3}+\frac{b^5Bx^{18}}{15}}{x^3} + (5a^4bA +$
risch	$\frac{b^5Bx^{15}}{15} + \frac{Ab^5x^{12}}{12} + \frac{5Bab^4x^{12}}{12} + \frac{5aAb^4x^9}{9} + \frac{10Ba^2b^3x^9}{9} + \frac{5a^2Ab^3x^6}{3} + \frac{5Ba^3b^2x^6}{3} + \frac{10Aa^3b^2x^3}{3} + \frac{5Ba^4bx^3}{3} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b^5*B*x^15+1/12*A*b^5*x^12+5/12*B*a*b^4*x^12+5/9*a*A*b^4*x^9+10/9*B*a^
2*b^3*x^9+5/3*a^2*A*b^3*x^6+5/3*B*a^3*b^2*x^6+10/3*A*a^3*b^2*x^3+5/3*B*a^4*
b*x^3-1/3*a^5*A/x^3+a^4*(5*A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.29, size = 120, normalized size = 1.06

$$\frac{1}{15}Bb^5x^{15} + \frac{1}{12}\left(5Bab^4 + Ab^5\right)x^{12} + \frac{5}{9}\left(2Ba^2b^3 + Aab^4\right)x^9 + \frac{5}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^6 + \frac{5}{3}\left(Ba^4b + 2Aa^3b^2\right)x^3 - \frac{Aa^5}{3x^3} + \frac{1}{3}\left(Ba^5 + 5Aa^4b\right)\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/15*B*b^5*x^15 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 5/9*(2*B*a^2*b^3 + A*a*b^
4)*x^9 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*x^3
- 1/3*A*a^5/x^3 + 1/3*(B*a^5 + 5*A*a^4*b)*log(x^3)
```

Fricas [A]
time = 2.88, size = 123, normalized size = 1.09

$$\frac{12\,Bb^5x^{18} + 15\,(5\,Bab^4 + Ab^5)x^{15} + 100\,(2\,Ba^2b^3 + Aab^4)x^{12} + 300\,(Ba^3b^2 + Aa^2b^3)x^9 + 300\,(Ba^4b + 2\,Aa^3b^2)x^6 - 60\,Aa^5 + 180\,(Ba^5 + 5\,Aa^4b)x^3\log(x)}{180\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/180*(12*B*b^5*x^18 + 15*(5*B*a*b^4 + A*b^5)*x^15 + 100*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 300*(B*a^4*b + 2*A*a^3*b^2)*
x^6 - 60*A*a^5 + 180*(B*a^5 + 5*A*a^4*b)*x^3*log(x))/x^3
```

Sympy [A]
time = 0.48, size = 133, normalized size = 1.18

$$-\frac{Aa^5}{3x^3} + \frac{Bb^5x^{15}}{15} + a^4\cdot(5Ab + Ba)\log(x) + x^{12}\left(\frac{Ab^5}{12} + \frac{5Bab^4}{12}\right) + x^9\cdot\left(\frac{5Aab^4}{9} + \frac{10Ba^2b^3}{9}\right) + x^6\cdot\left(\frac{5Aa^2b^3}{3} + \frac{5Ba^3b^2}{3}\right) + x^3\cdot\left(\frac{10Aa^3b^2}{3} + \frac{5Ba^4b}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**4,x)
```

```
[Out] -A*a**5/(3*x**3) + B*b**5*x**15/15 + a**4*(5*A*b + B*a)*log(x) + x**12*(A*b
**5/12 + 5*B*a*b**4/12) + x**9*(5*A*a*b**4/9 + 10*B*a**2*b**3/9) + x**6*(5*
A*a**2*b**3/3 + 5*B*a**3*b**2/3) + x**3*(10*A*a**3*b**2/3 + 5*B*a**4*b/3)
```

Giac [A]
time = 0.84, size = 143, normalized size = 1.27

$$\frac{1}{15}\,Bb^5x^{15} + \frac{5}{12}\,Bab^4x^{12} + \frac{1}{12}\,Ab^5x^{12} + \frac{10}{9}\,Ba^2b^3x^9 + \frac{5}{9}\,Aab^4x^9 + \frac{5}{3}\,Ba^3b^2x^6 + \frac{5}{3}\,Aa^2b^3x^6 + \frac{5}{3}\,Ba^4bx^3 + \frac{10}{3}\,Aa^3b^2x^3 + (Ba^5 + 5\,Aa^4b)\log(|x|) - \frac{Ba^5x^3 + 5\,Aa^4bx^3 + Aa^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/15*B*b^5*x^15 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + 10/9*B*a^2*b^3*x^9
+ 5/9*A*a*b^4*x^9 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + 5/3*B*a^4*b*x^3
 + 10/3*A*a^3*b^2*x^3 + (B*a^5 + 5*A*a^4*b)*log(abs(x)) - 1/3*(B*a^5*x^3 +
5*A*a^4*b*x^3 + A*a^5)/x^3
```

Mupad [B]
time = 2.35, size = 105, normalized size = 0.93

$$x^{12}\left(\frac{A\,b^5}{12} + \frac{5\,B\,a\,b^4}{12}\right) + \ln(x)\,\left(B\,a^5 + 5\,A\,b\,a^4\right) - \frac{A\,a^5}{3\,x^3} + \frac{B\,b^5\,x^{15}}{15} + \frac{5\,a^2\,b^2\,x^6\,(A\,b + B\,a)}{3} + \frac{5\,a^3\,b\,x^3\,(2\,A\,b + B\,a)}{3} + \frac{5\,a\,b^3\,x^9\,(A\,b + 2\,B\,a)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^4,x)
```

```
[Out] x^12*((A*b^5)/12 + (5*B*a*b^4)/12) + log(x)*(B*a^5 + 5*A*a^4*b) - (A*a^5)/(
3*x^3) + (B*b^5*x^15)/15 + (5*a^2*b^2*x^6*(A*b + B*a))/3 + (5*a^3*b*x^3*(2*
A*b + B*a))/3 + (5*a*b^3*x^9*(A*b + 2*B*a))/9
```

3.37 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{x}+\frac{5}{2}a^3b(2Ab+aB)x^2+2a^2b^2(Ab+aB)x^5+\frac{5}{8}ab^3(Ab+2aB)x^8+\frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{14}b^5$$

```
[Out] -1/4*a^5*A/x^4-a^4*(5*A*b+B*a)/x+5/2*a^3*b*(2*A*b+B*a)*x^2+2*a^2*b^2*(A*b+B
*a)*x^5+5/8*a*b^3*(A*b+2*B*a)*x^8+1/11*b^4*(A*b+5*B*a)*x^11+1/14*b^5*B*x^14
```

Rubi [A]
time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{x}+\frac{5}{2}a^3bx^2(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5Bx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a^5*A)/x^4 - (a^4*(5*A*b + a*B))/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a
*B)*x^11)/11 + (b^5*B*x^14)/14
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx &= \int \left(\frac{a^5A}{x^5}+\frac{a^4(5Ab+aB)}{x^2}+5a^3b(2Ab+aB)x+10a^2b^2(Ab+aB)x^4+5ab^3(\right.\\ &= -\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{x}+\frac{5}{2}a^3b(2Ab+aB)x^2+2a^2b^2(Ab+aB)x^5+\frac{5}{8}ab^3(Ab\end{aligned}$$

Mathematica [A]
time = 0.02, size = 115, normalized size = 1.02

$$-\frac{a^5A}{4x^4}+\frac{-5a^4Ab-a^5B}{x}+\frac{5}{2}a^3b(2Ab+aB)x^2+2a^2b^2(Ab+aB)x^5+\frac{5}{8}ab^3(Ab+2aB)x^8+\frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{14}b^5Bx^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a^5*A)/x^4 + (-5*a^4*A*b - a^5*B)/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*
a*B)*x^11)/11 + (b^5*B*x^14)/14
```

Maple [A]
time = 0.26, size = 123, normalized size = 1.09

method	result
norman	$\frac{-\frac{a^{5} A}{4}+\left(-5 a^{4} b A-a^{5} B\right) x^{3}+\left(5 a^{3} b^{2} A+\frac{5}{2} a^{4} b B\right) x^{6}+\left(2 a^{2} b^{3} A+2 a^{3} b^{2} B\right) x^{9}+\left(\frac{5}{8} a\, b^{4} A+\frac{5}{4} a^{2} b^{3} B\right) x^{12}+\left(\frac{1}{11} b^{5} A+\frac{5}{11} a\, b^{4} B\right) x^{15}+\frac{b^{5} B\, x^{18}}{14}}{x^{4}}$
default	$\frac{b^{5} B\, x^{14}}{14}+\frac{A\, b^{5} x^{11}}{11}+\frac{5 B a\, b^{4} x^{11}}{11}+\frac{5 a A\, b^{4} x^{8}}{8}+\frac{5 B\, a^{2} b^{3} x^{8}}{4}+2 A\, a^{2} b^{3} x^{5}+2 B\, a^{3} b^{2} x^{5}+5 A\, a^{3} b^{2} x^{2}+\frac{5 B\, a^{4} b\, x^{2}}{2}$
risch	$\frac{b^{5} B\, x^{14}}{14}+\frac{A\, b^{5} x^{11}}{11}+\frac{5 B a\, b^{4} x^{11}}{11}+\frac{5 a A\, b^{4} x^{8}}{8}+\frac{5 B\, a^{2} b^{3} x^{8}}{4}+2 A\, a^{2} b^{3} x^{5}+2 B\, a^{3} b^{2} x^{5}+5 A\, a^{3} b^{2} x^{2}+\frac{5 B\, a^{4} b\, x^{2}}{2}$
gosper	$-\frac{-44 b^{5} B\, x^{18}-56 A\, b^{5} x^{15}-280 B a\, b^{4} x^{15}-385 a A\, b^{4} x^{12}-770 B\, a^{2} b^{3} x^{12}-1232 a^{2} A\, b^{3} x^{9}-1232 B\, a^{3} b^{2} x^{9}-3080 a^{3} A\, b^{2} x^{6}-1540 B\, a^{4} b}{616 x^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*b^5*B*x^14+1/11*A*b^5*x^11+5/11*B*a*b^4*x^11+5/8*a*A*b^4*x^8+5/4*B*a^2
*b^3*x^8+2*A*a^2*b^3*x^5+2*B*a^3*b^2*x^5+5*A*a^3*b^2*x^2+5/2*B*a^4*b*x^2-1/
4*a^5*A/x^4-a^4*(5*A*b+B*a)/x
```

Maxima [A]
time = 0.27, size = 121, normalized size = 1.07

$$\frac{1}{14}\, Bb^{5}x^{14} + \frac{1}{11}\left(5\, Bab^{4} + Ab^{5}\right)x^{11} + \frac{5}{8}\left(2\, Ba^{2}b^{3} + Aab^{4}\right)x^{8} + 2\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{5} + \frac{5}{2}\left(Ba^{4}b + 2\, Aa^{3}b^{2}\right)x^{2} - \frac{Aa^{5} + 4\left(Ba^{5} + 5\, Aa^{4}b\right)x^{3}}{4\, x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/14*B*b^5*x^14 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/8*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*x^2 -
1/4*(A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

Fricas [A]
time = 3.36, size = 121, normalized size = 1.07

$$\frac{44\, Bb^{5}x^{18} + 56\left(5\, Bab^{4} + Ab^{5}\right)x^{15} + 385\left(2\, Ba^{2}b^{3} + Aab^{4}\right)x^{12} + 1232\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{9} + 1540\left(Ba^{4}b + 2\, Aa^{3}b^{2}\right)x^{6} - 154\, Aa^{5} - 616\left(Ba^{5} + 5\, Aa^{4}b\right)x^{3}}{616\, x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/616*(44*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 385*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1540*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 154*A*a^5 - 616*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

Sympy [A]
time = 0.48, size = 133, normalized size = 1.18

$$\frac{Bb^5x^{14}}{14}+x^{11}\left(\frac{Ab^5}{11}+\frac{5Bab^4}{11}\right)+x^8\cdot\left(\frac{5Aab^4}{8}+\frac{5Ba^2b^3}{4}\right)+x^5\cdot\left(2Aa^2b^3+2Ba^3b^2\right)+x^2\cdot\left(5Aa^3b^2+\frac{5Ba^4b}{2}\right)+\frac{-Aa^5+x^3(-20Aa^4b-4Ba^5)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**5,x)
```

```
[Out] B*b**5*x**14/14 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**8*(5*A*a*b**4/8 +
5*B*a**2*b**3/4) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**2*(5*A*a**3*b*
*2 + 5*B*a**4*b/2) + (-A*a**5 + x**3*(-20*A*a**4*b - 4*B*a**5))/(4*x**4)
```

Giac [A]
time = 0.72, size = 127, normalized size = 1.12

$$\frac{1}{14}\,Bb^5x^{14}+\frac{5}{11}\,Bab^4x^{11}+\frac{1}{11}\,Ab^5x^{11}+\frac{5}{4}\,Ba^2b^3x^8+\frac{5}{8}\,Aab^4x^8+2\,Ba^3b^2x^5+2\,Aa^2b^3x^5+\frac{5}{2}\,Ba^4bx^2+5\,Aa^3b^2x^2-\frac{4\,Ba^5x^3+20\,Aa^4bx^3+Aa^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/14*B*b^5*x^14 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 5/4*B*a^2*b^3*x^8 +
 5/8*A*a*b^4*x^8 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/2*B*a^4*b*x^2 + 5*
A*a^3*b^2*x^2 - 1/4*(4*B*a^5*x^3 + 20*A*a^4*b*x^3 + A*a^5)/x^4
```

Mupad [B]
time = 0.04, size = 109, normalized size = 0.96

$$x^{11}\left(\frac{A\,b^5}{11}+\frac{5\,B\,a\,b^4}{11}\right)-\frac{\frac{A\,a^5}{4}+x^3\left(B\,a^5+5\,A\,b\,a^4\right)}{x^4}+\frac{B\,b^5\,x^{14}}{14}+2\,a^2\,b^2\,x^5\left(A\,b+B\,a\right)+\frac{5\,a^3\,b\,x^2\left(2\,A\,b+B\,a\right)}{2}+\frac{5\,a\,b^3\,x^8\left(A\,b+2\,B\,a\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^5,x)
```

```
[Out] x^11*((A*b^5)/11 + (5*B*a*b^4)/11) - ((A*a^5)/4 + x^3*(B*a^5 + 5*A*a^4*b))/
x^4 + (B*b^5*x^14)/14 + 2*a^2*b^2*x^5*(A*b + B*a) + (5*a^3*b*x^2*(2*A*b + B
*a))/2 + (5*a*b^3*x^8*(A*b + 2*B*a))/8
```

3.38 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx$

Optimal. Leaf size=113

$$-\frac{a^5 A}{5x^5}-\frac{a^4(5Ab+aB)}{2x^2}+5a^3b(2Ab+aB)x+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{7}ab^3(Ab+2aB)x^7+\frac{1}{10}b^4(Ab+5aB)x^{10}+\frac{1}{13}b^5$$

```
[Out] -1/5*a^5*A/x^5-1/2*a^4*(5*A*b+B*a)/x^2+5*a^3*b*(2*A*b+B*a)*x+5/2*a^2*b^2*(A
*b+B*a)*x^4+5/7*a*b^3*(A*b+2*B*a)*x^7+1/10*b^4*(A*b+5*B*a)*x^10+1/13*b^5*B*
x^13
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5 A}{5x^5}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)+\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
(5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b +
 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx = \int \Bigg(5a^3b(2Ab+aB)+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^3}+10a^2b^2(Ab+aB)x^3+5ab^3(A$$

$$= -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{2x^2}+5a^3b(2Ab+aB)x+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{7}ab^3(A$$

Mathematica [A]
time = 0.02, size = 113, normalized size = 1.00

$$-\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{2x^2}+5a^3b(2Ab+aB)x+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{7}ab^3(Ab+2aB)x^7+\frac{1}{10}b^4(Ab+5aB)x^{10}+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
(5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b +
 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

Maple [A]
time = 0.26, size = 119, normalized size = 1.05

method	result
default	$\frac{b^5 B x^{13}}{13}+\frac{A b^5 x^{10}}{10}+\frac{B a b^4 x^{10}}{2}+\frac{5 A a b^4 x^7}{7}+\frac{10 B a^2 b^3 x^7}{7}+\frac{5 A a^2 b^3 x^4}{2}+\frac{5 B a^3 b^2 x^4}{2}+10 a^3 b^2 A x+5 a^4 b B x-\frac{a^5}{5x}$
norman	$\frac{-\frac{a^5 A}{5}+\left(-\frac{5}{2} a^4 b A-\frac{1}{2} a^5 B\right) x^3+\left(10 a^3 b^2 A+5 a^4 b B\right) x^6+\left(\frac{5}{2} a^2 b^3 A+\frac{5}{2} a^3 b^2 B\right) x^9+\left(\frac{5}{7} a b^4 A+\frac{10}{7} a^2 b^3 B\right) x^{12}+\left(\frac{1}{10} b^5 A+\frac{1}{2} a b^4 B\right) x^{15}+\frac{b^5 B x}{13}}{x^5}$
risch	$\frac{b^5 B x^{13}}{13}+\frac{A b^5 x^{10}}{10}+\frac{B a b^4 x^{10}}{2}+\frac{5 A a b^4 x^7}{7}+\frac{10 B a^2 b^3 x^7}{7}+\frac{5 A a^2 b^3 x^4}{2}+\frac{5 B a^3 b^2 x^4}{2}+10 a^3 b^2 A x+5 a^4 b B x+\frac{(-}{}$
gosper	$-\frac{-70 b^5 B x^{18}-91 A b^5 x^{15}-455 B a b^4 x^{15}-650 a A b^4 x^{12}-1300 B a^2 b^3 x^{12}-2275 a^2 A b^3 x^9-2275 B a^3 b^2 x^9-9100 a^3 A b^2 x^6-4550 B a^4 b x}{910 x^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^5*B*x^13+1/10*A*b^5*x^10+1/2*B*a*b^4*x^10+5/7*A*a*b^4*x^7+10/7*B*a^2
*b^3*x^7+5/2*A*a^2*b^3*x^4+5/2*B*a^3*b^2*x^4+10*a^3*b^2*A*x+5*a^4*b*B*x-1/5
*a^5*A/x^5-1/2*a^4*(5*A*b+B*a)/x^2
```

Maxima [A]
time = 0.27, size = 120, normalized size = 1.06

$$\frac{1}{13} B b^5 x^{13}+\frac{1}{10}\left(5 B a b^4+A b^5\right) x^{10}+\frac{5}{7}\left(2 B a^2 b^3+A a b^4\right) x^7+\frac{5}{2}\left(B a^3 b^2+A a^2 b^3\right) x^4+5\left(B a^4 b+2 A a^3 b^2\right) x-\frac{2 A a^5+5\left(B a^5+5 A a^4 b\right) x^3}{10 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/13*B*b^5*x^13 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/7*(2*B*a^2*b^3 + A*a*b^
4)*x^7 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 5*(B*a^4*b + 2*A*a^3*b^2)*x - 1/
10*(2*A*a^5 + 5*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

Fricas [A]
time = 3.43, size = 121, normalized size = 1.07

$$\frac{70 B b^5 x^{18}+91\left(5 B a b^4+A b^5\right) x^{15}+650\left(2 B a^2 b^3+A a b^4\right) x^{12}+2275\left(B a^3 b^2+A a^2 b^3\right) x^9+4550\left(B a^4 b+2 A a^3 b^2\right) x^6-182 A a^5-455\left(B a^5+5 A a^4 b\right) x^3}{910 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/910*(70*B*b^5*x^18 + 91*(5*B*a*b^4 + A*b^5)*x^15 + 650*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 4550*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 182*A*a^5 - 455*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

Sympy [A]
time = 0.40, size = 133, normalized size = 1.18

$$\frac{Bb^5x^{13}}{13} + x^{10}\left(\frac{Ab^5}{10} + \frac{Bab^4}{2}\right) + x^7\cdot\left(\frac{5Aab^4}{7} + \frac{10Ba^2b^3}{7}\right) + x^4\cdot\left(\frac{5Aa^2b^3}{2} + \frac{5Ba^3b^2}{2}\right) + x\left(10Aa^3b^2 + 5Ba^4b\right) + \frac{-2Aa^5 + x^3(-25Aa^4b - 5Ba^5)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**6,x)
```

```
[Out] B*b**5*x**13/13 + x**10*(A*b**5/10 + B*a*b**4/2) + x**7*(5*A*a*b**4/7 + 10*
B*a**2*b**3/7) + x**4*(5*A*a**2*b**3/2 + 5*B*a**3*b**2/2) + x*(10*A*a**3*b*
*2 + 5*B*a**4*b) + (-2*A*a**5 + x**3*(-25*A*a**4*b - 5*B*a**5))/(10*x**5)
```

Giac [A]
time = 0.70, size = 124, normalized size = 1.10

$$\frac{1}{13}Bb^5x^{13} + \frac{1}{2}Bab^4x^{10} + \frac{1}{10}Ab^5x^{10} + \frac{10}{7}Ba^2b^3x^7 + \frac{5}{7}Aab^4x^7 + \frac{5}{2}Ba^3b^2x^4 + \frac{5}{2}Aa^2b^3x^4 + 5Ba^4bx + 10Aa^3b^2x - \frac{5Ba^5x^3 + 25Aa^4bx^3 + 2Aa^5}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/13*B*b^5*x^13 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 10/7*B*a^2*b^3*x^7 +
 5/7*A*a*b^4*x^7 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5*B*a^4*b*x + 10
*A*a^3*b^2*x - 1/10*(5*B*a^5*x^3 + 25*A*a^4*b*x^3 + 2*A*a^5)/x^5
```

Mupad [B]
time = 0.04, size = 108, normalized size = 0.96

$$x^{10}\left(\frac{A\,b^5}{10} + \frac{B\,a\,b^4}{2}\right) - \frac{\frac{A\,a^5}{5} + x^3\left(\frac{B\,a^5}{2} + \frac{5\,A\,b\,a^4}{2}\right)}{x^5} + \frac{B\,b^5\,x^{13}}{13} + \frac{5\,a^2\,b^2\,x^4\,(A\,b + B\,a)}{2} + 5\,a^3\,b\,x\,(2\,A\,b + B\,a) + \frac{5\,a\,b^3\,x^7\,(A\,b + 2\,B\,a)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^6,x)
```

```
[Out] x^10*((A*b^5)/10 + (B*a*b^4)/2) - ((A*a^5)/5 + x^3*((B*a^5)/2 + (5*A*a^4*b)
/2))/x^5 + (B*b^5*x^13)/13 + (5*a^2*b^2*x^4*(A*b + B*a))/2 + 5*a^3*b*x*(2*A
*b + B*a) + (5*a*b^3*x^7*(A*b + 2*B*a))/7
```

3.39 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx$

Optimal. Leaf size=114

$$-\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{3x^3}+\frac{10}{3}a^2b^2(Ab+aB)x^3+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{9}b^4(Ab+5aB)x^9+\frac{1}{12}b^5Bx^{12}+5a^3b(2Ab+aB$$

```
[Out] -1/6*a^5*A/x^6-1/3*a^4*(5*A*b+B*a)/x^3+10/3*a^2*b^2*(A*b+B*a)*x^3+5/6*a*b^3
*(A*b+2*B*a)*x^6+1/9*b^4*(A*b+5*B*a)*x^9+1/12*b^5*B*x^12+5*a^3*b*(2*A*b+B*a
)*ln(x)
```

Rubi [A]

time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{6x^6}-\frac{a^4(aB+5Ab)}{3x^3}+5a^3b\log(x)(aB+2Ab)+\frac{10}{3}a^2b^2x^3(aB+Ab)+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{12}b^5Bx^{12}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

```
[Out] -1/6*(a^5*A)/x^6 - (a^4*(5*A*b + a*B))/(3*x^3) + (10*a^2*b^2*(A*b + a*B)*x^
3)/3 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^9)/9 + (b^5*B*x
^12)/12 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^3}+\frac{a^4(5Ab+aB)}{x^2}+\frac{5a^3b(2Ab+aB)}{x}\right.\right.\\ &= -\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{3x^3}+\frac{10}{3}a^2b^2(Ab+aB)x^3+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{9}b^4(\end{aligned}$$

Mathematica [A]
time = 0.03, size = 106, normalized size = 0.93

$$\frac{1}{36}\left(-\frac{6a^5A}{x^6}-\frac{12a^4(5Ab+aB)}{x^3}+120a^2b^2(Ab+aB)x^3+30ab^3(Ab+2aB)x^6+4b^4(Ab+5aB)x^9+3b^5Bx^{12}+180a^3b(2Ab+aB)\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

```
[Out] ((-6*a^5*A)/x^6 - (12*a^4*(5*A*b + a*B))/x^3 + 120*a^2*b^2*(A*b + a*B)*x^3
+ 30*a*b^3*(A*b + 2*a*B)*x^6 + 4*b^4*(A*b + 5*a*B)*x^9 + 3*b^5*B*x^12 + 180
*a^3*b*(2*A*b + a*B)*Log[x])/36
```

Maple [A]
time = 0.25, size = 117, normalized size = 1.03

method	result
default	$\frac{b^5Bx^{12}}{12}+\frac{Ab^5x^9}{9}+\frac{5Bab^4x^9}{9}+\frac{5Aab^4x^6}{6}+\frac{5Ba^2b^3x^6}{3}+\frac{10Aa^2b^3x^3}{3}+\frac{10Ba^3b^2x^3}{3}-\frac{a^5A}{6x^6}-\frac{a^4(5Ab+Ba)}{3x^3}+5a^3b$
norman	$\frac{(\frac{1}{9}b^5A+\frac{5}{9}ab^4B)x^{15}+(\frac{5}{6}ab^4A+\frac{5}{3}a^2b^3B)x^{12}+(\frac{10}{3}a^2b^3A+\frac{10}{3}a^3b^2B)x^9+(-\frac{5}{3}a^4bA-\frac{1}{3}a^5B)x^3-\frac{a^5A}{6}+\frac{b^5Bx^{18}}{12}}{x^6}+(10a^3b^2A+$
risch	$\frac{b^5Bx^{12}}{12}+\frac{Ab^5x^9}{9}+\frac{5Bab^4x^9}{9}+\frac{5Aab^4x^6}{6}+\frac{5Ba^2b^3x^6}{3}+\frac{10Aa^2b^3x^3}{3}+\frac{10Ba^3b^2x^3}{3}+\frac{(-\frac{5}{3}a^4bA-\frac{1}{3}a^5B)x^3-\frac{a^5A}{6}}{x^6}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^5*B*x^12+1/9*A*b^5*x^9+5/9*B*a*b^4*x^9+5/6*A*a*b^4*x^6+5/3*B*a^2*b^3
*x^6+10/3*A*a^2*b^3*x^3+10/3*B*a^3*b^2*x^3-1/6*a^5*A/x^6-1/3*a^4*(5*A*b+B*a
)/x^3+5*a^3*b*(2*A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.27, size = 122, normalized size = 1.07

$$\frac{1}{12}Bb^5x^{12}+\frac{1}{9}\left(5Bab^4+Ab^5\right)x^9+\frac{5}{6}\left(2Ba^2b^3+Aab^4\right)x^6+\frac{10}{3}\left(Ba^3b^2+Aa^2b^3\right)x^3+\frac{5}{3}\left(Ba^4b+2Aa^3b^2\right)\log\left(x^3\right)-\frac{Aa^5+2\left(Ba^5+5Aa^4b\right)x^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*B*b^5*x^12 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/6*(2*B*a^2*b^3 + A*a*b^4)
*x^6 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*log(x
^3) - 1/6*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

Fricas [A]
time = 3.70, size = 123, normalized size = 1.08

$$\frac{3\,Bb^{5}x^{18}+4\,(5\,Bab^{4}+Ab^{5})x^{15}+30\,(2\,Ba^{2}b^{3}+Aab^{4})x^{12}+120\,(Ba^{3}b^{2}+Aa^{2}b^{3})x^{9}+180\,(Ba^{4}b+2\,Aa^{3}b^{2})x^{6}\log\left(x\right)-6\,Aa^{5}-12\,(Ba^{5}+5\,Aa^{4}b)x^{3}}{36\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/36*(3*B*b^5*x^18 + 4*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^12 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 180*(B*a^4*b + 2*A*a^3*b^2)*x^6*
log(x) - 6*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

Sympy [A]
time = 0.49, size = 131, normalized size = 1.15

$$\frac{Bb^{5}x^{12}}{12}+5a^{3}b(2Ab+Ba)\log\left(x\right)+x^{9}\left(\frac{Ab^{5}}{9}+\frac{5Bab^{4}}{9}\right)+x^{6}\cdot\left(\frac{5Aab^{4}}{6}+\frac{5Ba^{2}b^{3}}{3}\right)+x^{3}\cdot\left(\frac{10Aa^{2}b^{3}}{3}+\frac{10Ba^{3}b^{2}}{3}\right)+\frac{-Aa^{5}+x^{3}(-10Aa^{4}b-2Ba^{5})}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**7,x)
```

```
[Out] B*b**5*x**12/12 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**9*(A*b**5/9 + 5*B*a*b*
*4/9) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b**3/3) + x**3*(10*A*a**2*b**3/3 + 10
*B*a**3*b**2/3) + (-A*a**5 + x**3*(-10*A*a**4*b - 2*B*a**5))/(6*x**6)
```

Giac [A]
time = 0.71, size = 148, normalized size = 1.30

$$\frac{1}{12}\,Bb^{5}x^{12}+\frac{5}{9}\,Bab^{4}x^{9}+\frac{1}{9}\,Ab^{5}x^{9}+\frac{5}{3}\,Ba^{2}b^{3}x^{6}+\frac{5}{6}\,Aab^{4}x^{6}+\frac{10}{3}\,Ba^{3}b^{2}x^{3}+\frac{10}{3}\,Aa^{2}b^{3}x^{3}+5\,(Ba^{4}b+2\,Aa^{3}b^{2})\log\left(|x|\right)-\frac{15\,Ba^{4}bx^{6}+30\,Aa^{3}b^{2}x^{6}+2\,Ba^{5}x^{3}+10\,Aa^{4}bx^{3}+Aa^{5}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*B*b^5*x^12 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 5/3*B*a^2*b^3*x^6 + 5/6
*A*a*b^4*x^6 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*(B*a^4*b + 2*A*a
^3*b^2)*log(abs(x)) - 1/6*(15*B*a^4*b*x^6 + 30*A*a^3*b^2*x^6 + 2*B*a^5*x^3
+ 10*A*a^4*b*x^3 + A*a^5)/x^6
```

Mupad [B]
time = 0.05, size = 113, normalized size = 0.99

$$\ln\left(x\right)\,\left(5\,B\,a^{4}\,b+10\,A\,a^{3}\,b^{2}\right)-\frac{\frac{A\,a^{5}}{6}+x^{3}\,\left(\frac{B\,a^{5}}{3}+\frac{5\,A\,b\,a^{4}}{3}\right)}{x^{6}}+x^{9}\,\left(\frac{A\,b^{5}}{9}+\frac{5\,B\,a\,b^{4}}{9}\right)+\frac{B\,b^{5}\,x^{12}}{12}+\frac{10\,a^{2}\,b^{2}\,x^{3}\,\left(A\,b+B\,a\right)}{3}+\frac{5\,a\,b^{3}\,x^{6}\,\left(A\,b+2\,B\,a\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^7,x)
```

```
[Out] log(x)*(10*A*a^3*b^2 + 5*B*a^4*b) - ((A*a^5)/6 + x^3*((B*a^5)/3 + (5*A*a^4*
b)/3))/x^6 + x^9*((A*b^5)/9 + (5*B*a*b^4)/9) + (B*b^5*x^12)/12 + (10*a^2*b^
2*x^3*(A*b + B*a))/3 + (5*a*b^3*x^6*(A*b + 2*B*a))/6
```

3.40 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx$

Optimal. Leaf size=110

$$-\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x}+5a^2b^2(Ab+aB)x^2+ab^3(Ab+2aB)x^5+\frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{11}b^5Bx^{11}$$

```
[Out] -1/7*a^5*A/x^7-1/4*a^4*(5*A*b+B*a)/x^4-5*a^3*b*(2*A*b+B*a)/x+5*a^2*b^2*(A*b
+B*a)*x^2+a*b^3*(A*b+2*B*a)*x^5+1/8*b^4*(A*b+5*B*a)*x^8+1/11*b^5*B*x^11
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$-\frac{a^5A}{7x^7}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{x}+5a^2b^2x^2(aB+Ab)+\frac{1}{8}b^4x^8(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

```
[Out] -1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
+ 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^8)/8 + (b^5*B*x^11)/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx &= \int \left(\frac{a^5A}{x^8}+\frac{a^4(5Ab+aB)}{x^5}+\frac{5a^3b(2Ab+aB)}{x^2}+10a^2b^2(Ab+aB)x+5ab^3(Ab\right.\\ &= -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x}+5a^2b^2(Ab+aB)x^2+ab^3(Ab+2\end{aligned}$$

Mathematica [A]
time = 0.02, size = 110, normalized size = 1.00

$$-\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x}+5a^2b^2(Ab+aB)x^2+ab^3(Ab+2aB)x^5+\frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

```
[Out] -1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
+ 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^8)/8 + (b^5*B*x^11)/11
```

Maple [A]
time = 0.26, size = 117, normalized size = 1.06

method	result
default	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^8}{8}+\frac{5 B a b^4 x^8}{8}+A a b^4 x^5+2 B a^2 b^3 x^5+5 A a^2 b^3 x^2+5 B a^3 b^2 x^2-\frac{a^4(5 A b+B a)}{4 x^4}-\frac{a^5 A}{7 x^7}-$
norman	$\frac{-\frac{a^5 A}{7}+(-\frac{5}{4} a^4 b A-\frac{1}{4} a^5 B) x^3+(-10 a^3 b^2 A-5 a^4 b B) x^6+(5 a^2 b^3 A+5 a^3 b^2 B) x^9+(a b^4 A+2 a^2 b^3 B) x^{12}+(\frac{1}{8} b^5 A+\frac{5}{8} a b^4 B) x^{15}+\frac{b^5 B x^{18}}{11}}{x^7}$
risch	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^8}{8}+\frac{5 B a b^4 x^8}{8}+A a b^4 x^5+2 B a^2 b^3 x^5+5 A a^2 b^3 x^2+5 B a^3 b^2 x^2+\frac{(-10 a^3 b^2 A-5 a^4 b B) x^6+(}{x}$
gosper	$-\frac{-56 b^5 B x^{18}-77 A b^5 x^{15}-385 B a b^4 x^{15}-616 a A b^4 x^{12}-1232 B a^2 b^3 x^{12}-3080 a^2 A b^3 x^9-3080 B a^3 b^2 x^9+6160 a^3 A b^2 x^6+3080 B a^4 b}{616 x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^5*B*x^11+1/8*A*b^5*x^8+5/8*B*a*b^4*x^8+A*a*b^4*x^5+2*B*a^2*b^3*x^5+5
*A*a^2*b^3*x^2+5*B*a^3*b^2*x^2-1/4*a^4*(5*A*b+B*a)/x^4-1/7*a^5*A/x^7-5*a^3*
b*(2*A*b+B*a)/x
```

Maxima [A]
time = 0.30, size = 121, normalized size = 1.10

$$\frac{1}{11} B b^5 x^{11}+\frac{1}{8}\left(5 B a b^4+A b^5\right) x^8+\left(2 B a^2 b^3+A a b^4\right) x^5+5\left(B a^3 b^2+A a^2 b^3\right) x^2-\frac{140\left(B a^4 b+2 A a^3 b^2\right) x^6+4 A a^5+7\left(B a^5+5 A a^4 b\right) x^3}{28 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/11*B*b^5*x^11 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + (2*B*a^2*b^3 + A*a*b^4)*x^5
 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 - 1/28*(140*(B*a^4*b + 2*A*a^3*b^2)*x^6 +
4*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

Fricas [A]
time = 2.79, size = 121, normalized size = 1.10

$$\frac{56 B b^5 x^{18}+77\left(5 B a b^4+A b^5\right) x^{15}+616\left(2 B a^2 b^3+A a b^4\right) x^{12}+3080\left(B a^3 b^2+A a^2 b^3\right) x^9-3080\left(B a^4 b+2 A a^3 b^2\right) x^6-88 A a^5-154\left(B a^5+5 A a^4 b\right) x^3}{616 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] 1/616*(56*B*b^5*x^18 + 77*(5*B*a*b^4 + A*b^5)*x^15 + 616*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 3080*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 88*A*a^5 - 154*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

Sympy [A]
time = 0.60, size = 129, normalized size = 1.17

$$\frac{Bb^5x^{11}}{11} + x^8\left(\frac{Ab^5}{8} + \frac{5Bab^4}{8}\right) + x^5\left(Aab^4 + 2Ba^2b^3\right) + x^2\cdot\left(5Aa^2b^3 + 5Ba^3b^2\right) + \frac{-4Aa^5 + x^6(-280Aa^3b^2 - 140Ba^4b) + x^3(-35Aa^4b - 7Ba^5)}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**8,x)
```

```
[Out] B*b**5*x**11/11 + x**8*(A*b**5/8 + 5*B*a*b**4/8) + x**5*(A*a*b**4 + 2*B*a**
2*b**3) + x**2*(5*A*a**2*b**3 + 5*B*a**3*b**2) + (-4*A*a**5 + x**6*(-280*A*
a**3*b**2 - 140*B*a**4*b) + x**3*(-35*A*a**4*b - 7*B*a**5))/(28*x**7)
```

Giac [A]
time = 0.94, size = 127, normalized size = 1.15

$$\frac{1}{11}\,Bb^5x^{11} + \frac{5}{8}\,Bab^4x^8 + \frac{1}{8}\,Ab^5x^8 + 2\,Ba^2b^3x^5 + Aab^4x^5 + 5\,Ba^3b^2x^2 + 5\,Aa^2b^3x^2 - \frac{140\,Ba^4bx^6 + 280\,Aa^3b^2x^6 + 7\,Ba^5x^3 + 35\,Aa^4bx^3 + 4\,Aa^5}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/11*B*b^5*x^11 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 2*B*a^2*b^3*x^5 + A*a*b
^4*x^5 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 - 1/28*(140*B*a^4*b*x^6 + 280*A*
a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 4*A*a^5)/x^7
```

Mupad [B]
time = 2.34, size = 113, normalized size = 1.03

$$x^8\left(\frac{A\,b^5}{8} + \frac{5\,B\,a\,b^4}{8}\right) - \frac{\frac{A\,a^5}{7} + x^6\,\left(5\,B\,a^4\,b + 10\,A\,a^3\,b^2\right) + x^3\,\left(\frac{B\,a^5}{4} + \frac{5\,A\,b\,a^4}{4}\right)}{x^7} + \frac{B\,b^5\,x^{11}}{11} + 5\,a^2\,b^2\,x^2\,\left(A\,b + B\,a\right) + a\,b^3\,x^5\,\left(A\,b + 2\,B\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^8,x)
```

```
[Out] x^8*((A*b^5)/8 + (5*B*a*b^4)/8) - ((A*a^5)/7 + x^6*(10*A*a^3*b^2 + 5*B*a^4*
b) + x^3*((B*a^5)/4 + (5*A*a^4*b)/4))/x^7 + (B*b^5*x^11)/11 + 5*a^2*b^2*x^2
*(A*b + B*a) + a*b^3*x^5*(A*b + 2*B*a)
```

3.41 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{2x^2}+10a^2b^2(Ab+aB)x+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{10}b^5B$$

```
[Out] -1/8*a^5*A/x^8-1/5*a^4*(5*A*b+B*a)/x^5-5/2*a^3*b*(2*A*b+B*a)/x^2+10*a^2*b^2
*(A*b+B*a)*x+5/4*a*b^3*(A*b+2*B*a)*x^4+1/7*b^4*(A*b+5*B*a)*x^7+1/10*b^5*B*x
^10
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{8x^8}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2x(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

```
[Out] -1/8*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*
b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^9}\,dx &= \int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^9}+\frac{a^4(5Ab+aB)}{x^6}+\frac{5a^3b(2Ab+aB)}{x^3}+5ab^3(Ab\right.\\ &= -\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{2x^2}+10a^2b^2(Ab+aB)x+\frac{5}{4}ab^3(Ab\end{aligned}$$

Mathematica [A]
time = 0.02, size = 113, normalized size = 1.00

$$-\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{2x^2}+10a^2b^2(Ab+aB)x+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

```
[Out] -1/8*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*
b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

Maple [A]
time = 0.25, size = 114, normalized size = 1.01

method	result
default	$\frac{b^5 B x^{10}}{10}+\frac{A b^5 x^7}{7}+\frac{5Ba b^4 x^7}{7}+\frac{5Aa b^4 x^4}{4}+\frac{5B a^2 b^3 x^4}{2}+10a^2b^3Ax+10a^3b^2Bx-\frac{a^4(5Ab+Ba)}{5x^5}-\frac{5a^3b(2Ab+Ba}{2x^2}$
norman	$\frac{-\frac{a^5 A}{8}+(-a^4bA-\frac{1}{5}a^5B)x^3+(-5a^3b^2A-\frac{5}{2}a^4bB)x^6+(10a^2b^3A+10a^3b^2B)x^9+(\frac{5}{4}a b^4A+\frac{5}{2}a^2b^3B)x^{12}+(\frac{1}{7}b^5A+\frac{5}{7}a b^4B)x^{15}+\frac{b^5 B x^{18}}{10}}{x^8}$
risch	$\frac{b^5 B x^{10}}{10}+\frac{A b^5 x^7}{7}+\frac{5Ba b^4 x^7}{7}+\frac{5Aa b^4 x^4}{4}+\frac{5B a^2 b^3 x^4}{2}+10a^2b^3Ax+10a^3b^2Bx+\frac{(-5a^3b^2A-\frac{5}{2}a^4bB)x^6+(-a^4bA}{x^8}$
gosper	$-\frac{-28b^5 B x^{18}-40A b^5 x^{15}-200Ba b^4 x^{15}-350aA b^4 x^{12}-700B a^2 b^3 x^{12}-2800a^2 A b^3 x^9-2800B a^3 b^2 x^9+1400a^3 A b^2 x^6+700B a^4 b x^6+}{280x^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^5*B*x^10+1/7*A*b^5*x^7+5/7*B*a*b^4*x^7+5/4*A*a*b^4*x^4+5/2*B*a^2*b^3
*x^4+10*a^2*b^3*A*x+10*a^3*b^2*B*x-1/5*a^4*(5*A*b+B*a)/x^5-5/2*a^3*b*(2*A*b
+B*a)/x^2-1/8*a^5*A/x^8
```

Maxima [A]
time = 0.30, size = 120, normalized size = 1.06

$$\frac{1}{10}\,Bb^5x^{10}+\frac{1}{7}\,(5\,Bab^4+Ab^5)x^7+\frac{5}{4}\,(2\,Ba^2b^3+Aab^4)x^4+10\,(Ba^3b^2+Aa^2b^3)x-\frac{100\,(Ba^4b+2\,Aa^3b^2)x^6+5\,Aa^5+8\,(Ba^5+5\,Aa^4b)x^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] 1/10*B*b^5*x^10 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + 5/4*(2*B*a^2*b^3 + A*a*b^4)
*x^4 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/40*(100*(B*a^4*b + 2*A*a^3*b^2)*x^6
 + 5*A*a^5 + 8*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

Fricas [A]
time = 1.98, size = 121, normalized size = 1.07

$$\frac{28\,Bb^5x^{18}+40\,(5\,Bab^4+Ab^5)x^{15}+350\,(2\,Ba^2b^3+Aab^4)x^{12}+2800\,(Ba^3b^2+Aa^2b^3)x^9-700\,(Ba^4b+2\,Aa^3b^2)x^6-35\,Aa^5-56\,(Ba^5+5\,Aa^4b)x^3}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/280*(28*B*b^5*x^18 + 40*(5*B*a*b^4 + A*b^5)*x^15 + 350*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 700*(B*a^4*b + 2*A*a^3*b^2)
*x^6 - 35*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

Sympy [A]
time = 0.64, size = 133, normalized size = 1.18

$$\frac{Bb^5x^{10}}{10} + x^7\left(\frac{Ab^5}{7} + \frac{5Bab^4}{7}\right) + x^4\cdot\left(\frac{5Aab^4}{4} + \frac{5Ba^2b^3}{2}\right) + x\left(10Aa^2b^3 + 10Ba^3b^2\right) + \frac{-5Aa^5 + x^6\left(-200Aa^3b^2 - 100Ba^4b\right) + x^3\left(-40Aa^4b - 8Ba^5\right)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**9,x)
```

```
[Out] B*b**5*x**10/10 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**4*(5*A*a*b**4/4 + 5*B
*a**2*b**3/2) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) + (-5*A*a**5 + x**6*(-2
00*A*a**3*b**2 - 100*B*a**4*b) + x**3*(-40*A*a**4*b - 8*B*a**5))/(40*x**8)
```

Giac [A]
time = 1.75, size = 124, normalized size = 1.10

$$\frac{1}{10}\,Bb^5x^{10} + \frac{5}{7}\,Bab^4x^7 + \frac{1}{7}\,Ab^5x^7 + \frac{5}{2}\,Ba^2b^3x^4 + \frac{5}{4}\,Aab^4x^4 + 10\,Ba^3b^2x + 10\,Aa^2b^3x - \frac{100\,Ba^4bx^6 + 200\,Aa^3b^2x^6 + 8\,Ba^5x^3 + 40\,Aa^4bx^3 + 5\,Aa^5}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] 1/10*B*b^5*x^10 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 5/2*B*a^2*b^3*x^4 + 5/4
*A*a*b^4*x^4 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/40*(100*B*a^4*b*x^6 + 20
0*A*a^3*b^2*x^6 + 8*B*a^5*x^3 + 40*A*a^4*b*x^3 + 5*A*a^5)/x^8
```

Mupad [B]
time = 0.04, size = 111, normalized size = 0.98

$$x^7\left(\frac{A\,b^5}{7} + \frac{5\,B\,a\,b^4}{7}\right) - \frac{\frac{A\,a^5}{8} + x^6\,\left(\frac{5\,B\,a^4\,b}{2} + 5\,A\,a^3\,b^2\right) + x^3\,\left(\frac{B\,a^5}{5} + A\,b\,a^4\right)}{x^8} + \frac{B\,b^5\,x^{10}}{10} + 10\,a^2\,b^2\,x\,\left(A\,b + B\,a\right) + \frac{5\,a\,b^3\,x^4\,\left(A\,b + 2\,B\,a\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^9,x)
```

```
[Out] x^7*((A*b^5)/7 + (5*B*a*b^4)/7) - ((A*a^5)/8 + x^6*(5*A*a^3*b^2 + (5*B*a^4*
b)/2) + x^3*((B*a^5)/5 + A*a^4*b))/x^8 + (B*b^5*x^10)/10 + 10*a^2*b^2*x*(A*
b + B*a) + (5*a*b^3*x^4*(A*b + 2*B*a))/4
```

3.42 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}} \, dx$

Optimal. Leaf size=114

$$-\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{3x^3}+\frac{5}{3}ab^3(Ab+2aB)x^3+\frac{1}{6}b^4(Ab+5aB)x^6+\frac{1}{9}b^5Bx^9+10a^2b^2(Ab+aB)\,\mathrm{lo}$$

```
[Out] -1/9*a^5*A/x^9-1/6*a^4*(5*A*b+B*a)/x^6-5/3*a^3*b*(2*A*b+B*a)/x^3+5/3*a*b^3*
(A*b+2*B*a)*x^3+1/6*b^4*(A*b+5*B*a)*x^6+1/9*b^5*B*x^9+10*a^2*b^2*(A*b+B*a)*
ln(x)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{9x^9}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{5a^3b(aB+2Ab)}{3x^3}+10a^2b^2\log(x)(aB+Ab)+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

```
[Out] -1/9*(a^5*A)/x^9 - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*
x^9)/9 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^4}+\frac{a^4(5Ab+aB)}{x^3}+\frac{5a^3b(2Ab+aB)}{x^2}+\right.\right.\\ &= -\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{3x^3}+\frac{5}{3}ab^3(Ab+2aB)x^3+\frac{1}{6}b^4(Ab\end{aligned}$$

Mathematica [A]
time = 0.03, size = 106, normalized size = 0.93

$$\frac{1}{18}\left(-\frac{2a^5A}{x^9}-\frac{3a^4(5Ab+aB)}{x^6}-\frac{30a^3b(2Ab+aB)}{x^3}+30ab^3(Ab+2aB)x^3+3b^4(Ab+5aB)x^6+2b^5Bx^9+180a^2b^2(Ab+aB)\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

```
[Out] ((-2*a^5*A)/x^9 - (3*a^4*(5*A*b + a*B))/x^6 - (30*a^3*b*(2*A*b + a*B))/x^3
+ 30*a*b^3*(A*b + 2*a*B)*x^3 + 3*b^4*(A*b + 5*a*B)*x^6 + 2*b^5*B*x^9 + 180*
a^2*b^2*(A*b + a*B)*Log[x])/18
```

Maple [A]
time = 0.26, size = 111, normalized size = 0.97

method	result
default	$\frac{b^5 B x^9}{9}+\frac{A b^5 x^6}{6}+\frac{5Ba b^4 x^6}{6}+\frac{5Aa b^4 x^3}{3}+\frac{10B a^2 b^3 x^3}{3}-\frac{a^5 A}{9x^9}-\frac{a^4(5Ab+Ba)}{6x^6}-\frac{5a^3b(2Ab+Ba)}{3x^3}+10a^2b^2(Ab+$
norman	$\frac{(\frac{1}{6}b^5A+\frac{5}{6}a b^4 B)x^{15}+(\frac{5}{3}a b^4 A+\frac{10}{3}a^2b^3B)x^{12}+(-\frac{10}{3}a^3b^2A-\frac{5}{3}a^4bB)x^6+(-\frac{5}{6}a^4bA-\frac{1}{6}a^5B)x^3-\frac{a^5A}{9}+\frac{b^5 B x^{18}}{9}}{x^9}+(10a^2b^3A+$
risch	$\frac{b^5 B x^9}{9}+\frac{A b^5 x^6}{6}+\frac{5Ba b^4 x^6}{6}+\frac{5Aa b^4 x^3}{3}+\frac{10B a^2 b^3 x^3}{3}+\frac{(-\frac{10}{3}a^3b^2A-\frac{5}{3}a^4bB)x^6+(-\frac{5}{6}a^4bA-\frac{1}{6}a^5B)x^3-\frac{a^5A}{9}}{x^9}+10A$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^5*B*x^9+1/6*A*b^5*x^6+5/6*B*a*b^4*x^6+5/3*A*a*b^4*x^3+10/3*B*a^2*b^3*
x^3-1/9*a^5*A/x^9-1/6*a^4*(5*A*b+B*a)/x^6-5/3*a^3*b*(2*A*b+B*a)/x^3+10*a^2*
b^2*(A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.27, size = 123, normalized size = 1.08

$$\frac{1}{9}Bb^5x^9+\frac{1}{6}\left(5Bab^4+Ab^5\right)x^6+\frac{5}{3}\left(2Ba^2b^3+Aab^4\right)x^3+\frac{10}{3}\left(Ba^3b^2+Aa^2b^3\right)\log\left(x^3\right)-\frac{30\left(Ba^4b+2Aa^3b^2\right)x^6+2Aa^5+3\left(Ba^5+5Aa^4b\right)x^3}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^5*x^9 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*x
^3 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*log(x^3) - 1/18*(30*(B*a^4*b + 2*A*a^3*b^
2)*x^6 + 2*A*a^5 + 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

Fricas [A]
time = 2.02, size = 123, normalized size = 1.08

$$\frac{2\,Bb^5x^{18}+3\,(5\,Bab^4+Ab^5)x^{15}+30\,(2\,Ba^2b^3+Aab^4)x^{12}+180\,(Ba^3b^2+Aa^2b^3)x^9\log{(x)}-30\,(Ba^4b+2\,Aa^3b^2)x^6-2\,Aa^5-3\,(Ba^5+5\,Aa^4b)x^3}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(2*B*b^5*x^18 + 3*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^12 + 180*(B*a^3*b^2 + A*a^2*b^3)*x^9*log(x) - 30*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 2*A*a^5 - 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

Sympy [A]
time = 1.50, size = 129, normalized size = 1.13

$$\frac{Bb^5x^9}{9}+10a^2b^2(Ab+Ba)\log{(x)}+x^6\left(\frac{Ab^5}{6}+\frac{5Bab^4}{6}\right)+x^3\cdot\left(\frac{5Aab^4}{3}+\frac{10Ba^2b^3}{3}\right)+\frac{-2Aa^5+x^6(-60Aa^3b^2-30Ba^4b)+x^3(-15Aa^4b-3Ba^5)}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**10,x)
```

```
[Out] B*b**5*x**9/9 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**6*(A*b**5/6 + 5*B*a*b*
*4/6) + x**3*(5*A*a*b**4/3 + 10*B*a**2*b**3/3) + (-2*A*a**5 + x**6*(-60*A*a
**3*b**2 - 30*B*a**4*b) + x**3*(-15*A*a**4*b - 3*B*a**5))/(18*x**9)
```

Giac [A]
time = 1.55, size = 150, normalized size = 1.32

$$\frac{1}{9}\,Bb^5x^9+\frac{5}{6}\,Bab^4x^6+\frac{1}{6}\,Ab^5x^6+\frac{10}{3}\,Ba^2b^3x^3+\frac{5}{3}\,Aab^4x^3+10\,(Ba^3b^2+Aa^2b^3)\log{(|x|)}-\frac{110\,Ba^3b^2x^9+110\,Aa^2b^3x^9+30\,Ba^4bx^6+60\,Aa^3b^2x^6+3\,Ba^5x^3+15\,Aa^4bx^3+2\,Aa^5}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="giac")
```

```
[Out] 1/9*B*b^5*x^9 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 10/3*B*a^2*b^3*x^3 + 5/3*
A*a*b^4*x^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*log(abs(x)) - 1/18*(110*B*a^3*b^2*
x^9 + 110*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 3*B*a^5*x^3 +
 15*A*a^4*b*x^3 + 2*A*a^5)/x^9
```

Mupad [B]
time = 0.05, size = 118, normalized size = 1.04

$$x^6\,\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)-\frac{\frac{A\,a^5}{9}+x^6\,\left(\frac{5\,B\,a^4\,b}{3}+\frac{10\,A\,a^3\,b^2}{3}\right)+x^3\,\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)}{x^9}+\ln\left(x\right)\,\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right)+\frac{B\,b^5\,x^9}{9}+\frac{5\,a\,b^3\,x^3\,\left(A\,b+2\,B\,a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^10,x)
```

```
[Out] x^6*((A*b^5)/6 + (5*B*a*b^4)/6) - ((A*a^5)/9 + x^6*((10*A*a^3*b^2)/3 + (5*B
*a^4*b)/3) + x^3*((B*a^5)/6 + (5*A*a^4*b)/6))/x^9 + log(x)*(10*A*a^2*b^3 +
10*B*a^3*b^2) + (B*b^5*x^9)/9 + (5*a*b^3*x^3*(A*b + 2*B*a))/3
```

3.43 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx$

Optimal. Leaf size=115

$$-\frac{a^5A}{10x^{10}}-\frac{a^4(5Ab+aB)}{7x^7}-\frac{5a^3b(2Ab+aB)}{4x^4}-\frac{10a^2b^2(Ab+aB)}{x}+\frac{5}{2}ab^3(Ab+2aB)x^2+\frac{1}{5}b^4(Ab+5aB)x^5+\frac{1}{8}b^5Bx$$

```
[Out] -1/10*a^5*A/x^10-1/7*a^4*(5*A*b+B*a)/x^7-5/4*a^3*b*(2*A*b+B*a)/x^4-10*a^2*b
^2*(A*b+B*a)/x+5/2*a*b^3*(A*b+2*B*a)*x^2+1/5*b^4*(A*b+5*B*a)*x^5+1/8*b^5*B*
x^8
```

Rubi [A]
time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{10x^{10}}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{10a^2b^2(aB+Ab)}{x}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

```
[Out] -1/10*(a^5*A)/x^10 - (a^4*(5*A*b + a*B))/(7*x^7) - (5*a^3*b*(2*A*b + a*B))/
(4*x^4) - (10*a^2*b^2*(A*b + a*B))/x + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4
*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^8)/8
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx = \int \left(\frac{a^5A}{x^{11}}+\frac{a^4(5Ab+aB)}{x^8}+\frac{5a^3b(2Ab+aB)}{x^5}+\frac{10a^2b^2(Ab+aB)}{x^2}+5ab^3(Ab\right.$$

$$= -\frac{a^5A}{10x^{10}}-\frac{a^4(5Ab+aB)}{7x^7}-\frac{5a^3b(2Ab+aB)}{4x^4}-\frac{10a^2b^2(Ab+aB)}{x}+\frac{5}{2}ab^3(Ab$$

Mathematica [A]
time = 0.01, size = 118, normalized size = 1.03

$$\frac{1400a^2b^3x^9(-2A+Bx^3)+140ab^4x^{12}(5A+2Bx^3)-700a^3b^2x^6(A+4Bx^3)+7b^5x^{15}(8A+5Bx^3)-50a^4bx^3(4A+7Bx^3)-4a^5(7A+10Bx^3)}{280x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

```
[Out] (1400*a^2*b^3*x^9*(-2*A + B*x^3) + 140*a*b^4*x^12*(5*A + 2*B*x^3) - 700*a^3
*b^2*x^6*(A + 4*B*x^3) + 7*b^5*x^15*(8*A + 5*B*x^3) - 50*a^4*b*x^3*(4*A + 7
*B*x^3) - 4*a^5*(7*A + 10*B*x^3))/(280*x^10)
```

Maple [A]
time = 0.26, size = 111, normalized size = 0.97

method	result
default	$\frac{b^5 B x^8}{8}+\frac{A b^5 x^5}{5}+B a b^4 x^5+\frac{5 A a b^4 x^2}{2}+5 B a^2 b^3 x^2-\frac{5 a^3 b (2 A b+B a)}{4 x^4}-\frac{a^4 (5 A b+B a)}{7 x^7}-\frac{a^5 A}{10 x^{10}}-\frac{10 a^2 b^2 (A b+B}{x}$
norman	$\frac{-\frac{a^5 A}{10}+\left(-\frac{5}{7} a^4 b A-\frac{1}{7} a^5 B\right) x^3+\left(-\frac{5}{2} a^3 b^2 A-\frac{5}{4} a^4 b B\right) x^6+\left(-10 a^2 b^3 A-10 a^3 b^2 B\right) x^9+\left(\frac{5}{2} a b^4 A+5 a^2 b^3 B\right) x^{12}+\left(\frac{1}{5} b^5 A+a b^4 B\right) x^{15}+ b^5 B}{x^{10}}$
risch	$\frac{b^5 B x^8}{8}+\frac{A b^5 x^5}{5}+B a b^4 x^5+\frac{5 A a b^4 x^2}{2}+5 B a^2 b^3 x^2+\frac{\left(-10 a^2 b^3 A-10 a^3 b^2 B\right) x^9+\left(-\frac{5}{2} a^3 b^2 A-\frac{5}{4} a^4 b B\right) x^6+\left(-\frac{5}{7} a^4 b A\right.}{x^{10}}$
gosper	$-\frac{-35 b^5 B x^{18}-56 A b^5 x^{15}-280 B a b^4 x^{15}-700 a A b^4 x^{12}-1400 B a^2 b^3 x^{12}+2800 a^2 A b^3 x^9+2800 B a^3 b^2 x^9+700 a^3 A b^2 x^6+350 B a^4 b x^6}{280 x^{10}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^5*B*x^8+1/5*A*b^5*x^5+B*a*b^4*x^5+5/2*A*a*b^4*x^2+5*B*a^2*b^3*x^2-5/4
*a^3*b*(2*A*b+B*a)/x^4-1/7*a^4*(5*A*b+B*a)/x^7-1/10*a^5*A/x^10-10*a^2*b^2*(
A*b+B*a)/x
```

Maxima [A]
time = 0.29, size = 122, normalized size = 1.06

$$\frac{1}{8} B b^5 x^8+\frac{1}{5} \left(5 B a b^4+A b^5\right) x^5+\frac{5}{2} \left(2 B a^2 b^3+A a b^4\right) x^2-\frac{1400 \left(B a^3 b^2+A a^2 b^3\right) x^9+175 \left(B a^4 b+2 A a^3 b^2\right) x^6+14 A a^5+20 \left(B a^5+5 A a^4 b\right) x^3}{140 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^5*x^8 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*x
^2 - 1/140*(1400*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 175*(B*a^4*b + 2*A*a^3*b^2)*
x^6 + 14*A*a^5 + 20*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

Fricas [A]
time = 3.30, size = 121, normalized size = 1.05

$$\frac{35 B b^5 x^{18}+56 \left(5 B a b^4+A b^5\right) x^{15}+700 \left(2 B a^2 b^3+A a b^4\right) x^{12}-2800 \left(B a^3 b^2+A a^2 b^3\right) x^9-350 \left(B a^4 b+2 A a^3 b^2\right) x^6-28 A a^5-40 \left(B a^5+5 A a^4 b\right) x^3}{280 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="fricas")
```

```
[Out] 1/280*(35*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 700*(2*B*a^2*b^3 + A*a
*b^4)*x^12 - 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 350*(B*a^4*b + 2*A*a^3*b^2)
*x^6 - 28*A*a^5 - 40*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

Sympy [A]
time = 9.10, size = 131, normalized size = 1.14

$$\frac{Bb^5x^8}{8}+x^5\left(\frac{Ab^5}{5}+Bab^4\right)+x^2\cdot\left(\frac{5Aab^4}{2}+5Ba^2b^3\right)+\frac{-14Aa^5+x^9(-1400Aa^2b^3-1400Ba^3b^2)+x^6(-350Aa^3b^2-175Ba^4b)+x^3(-100Aa^4b-20Ba^5)}{140x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**11,x)
```

```
[Out] B*b**5*x**8/8 + x**5*(A*b**5/5 + B*a*b**4) + x**2*(5*A*a*b**4/2 + 5*B*a**2*
b**3) + (-14*A*a**5 + x**9*(-1400*A*a**2*b**3 - 1400*B*a**3*b**2) + x**6*(-
350*A*a**3*b**2 - 175*B*a**4*b) + x**3*(-100*A*a**4*b - 20*B*a**5))/(140*x*
*10)
```

Giac [A]
time = 1.23, size = 127, normalized size = 1.10

$$\frac{1}{8}\,Bb^5x^8+Bab^4x^5+\frac{1}{5}\,Ab^5x^5+5\,Ba^2b^3x^2+\frac{5}{2}\,Aab^4x^2-\frac{1400\,Ba^3b^2x^9+1400\,Aa^2b^3x^9+175\,Ba^4bx^6+350\,Aa^3b^2x^6+20\,Ba^5x^3+100\,Aa^4bx^3+14\,Aa^5}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="giac")
```

```
[Out] 1/8*B*b^5*x^8 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 5*B*a^2*b^3*x^2 + 5/2*A*a*b^4
*x^2 - 1/140*(1400*B*a^3*b^2*x^9 + 1400*A*a^2*b^3*x^9 + 175*B*a^4*b*x^6 + 3
50*A*a^3*b^2*x^6 + 20*B*a^5*x^3 + 100*A*a^4*b*x^3 + 14*A*a^5)/x^10
```

Mupad [B]
time = 2.36, size = 118, normalized size = 1.03

$$x^5\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)-\frac{\frac{A\,a^5}{10}+x^6\left(\frac{5\,B\,a^4\,b}{4}+\frac{5\,A\,a^3\,b^2}{2}\right)+x^3\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^9\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right)}{x^{10}}+\frac{B\,b^5\,x^8}{8}+\frac{5\,a\,b^3\,x^2\,\left(A\,b+2\,B\,a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^11,x)
```

```
[Out] x^5*((A*b^5)/5 + B*a*b^4) - ((A*a^5)/10 + x^6*((5*A*a^3*b^2)/2 + (5*B*a^4*b
)/4) + x^3*((B*a^5)/7 + (5*A*a^4*b)/7) + x^9*(10*A*a^2*b^3 + 10*B*a^3*b^2))
/x^10 + (B*b^5*x^8)/8 + (5*a*b^3*x^2*(A*b + 2*B*a))/2
```

3.44 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx$

Optimal. Leaf size=109

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{8x^8}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{5a^2b^2(Ab+aB)}{x^2}+5ab^3(Ab+2aB)x+\frac{1}{4}b^4(Ab+5aB)x^4+\frac{1}{7}b^5Bx^7$$

```
[Out] -1/11*a^5*A/x^11-1/8*a^4*(5*A*b+B*a)/x^8-a^3*b*(2*A*b+B*a)/x^5-5*a^2*b^2*(A
*b+B*a)/x^2+5*a*b^3*(A*b+2*B*a)*x+1/4*b^4*(A*b+5*B*a)*x^4+1/7*b^5*B*x^7
```

Rubi [A]
time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{5a^2b^2(aB+Ab)}{x^2}+\frac{1}{4}b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

```
[Out] -1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^
5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^7)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx = \int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^{12}}+\frac{a^4(5Ab+aB)}{x^9}+\frac{5a^3b(2Ab+aB)}{x^6}+\frac{10a^2b^2(A$$
$$= -\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{8x^8}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{5a^2b^2(Ab+aB)}{x^2}+5ab^3(Ab+$$

Mathematica [A]
time = 0.02, size = 109, normalized size = 1.00

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{8x^8}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{5a^2b^2(Ab+aB)}{x^2}+5ab^3(Ab+2aB)x+\frac{1}{4}b^4(Ab+5aB)x^4+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

```
[Out] -1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^
5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^7)/7
```

Maple [A]
time = 0.26, size = 108, normalized size = 0.99

method	result
default	$\frac{b^5 B x^7}{7}+\frac{A b^5 x^4}{4}+\frac{5 B a b^4 x^4}{4}+5 a b^4 A x+10 a^2 b^3 B x-\frac{a^5 A}{11 x^{11}}-\frac{a^3 b (2 A b+B a)}{x^5}-\frac{5 a^2 b^2 (A b+B a)}{x^2}-\frac{a^4 (5 A b+B a)}{8 x^8}$
risch	$\frac{b^5 B x^7}{7}+\frac{A b^5 x^4}{4}+\frac{5 B a b^4 x^4}{4}+5 a b^4 A x+10 a^2 b^3 B x+\frac{(-5 a^2 b^3 A-5 a^3 b^2 B) x^9+(-2 a^3 b^2 A-a^4 b B) x^6+(-\frac{5}{8} a^4 b A-\frac{1}{8} a^5 \ldots}{x^{11}}$
norman	$\frac{-\frac{a^5 A}{11}+(-\frac{5}{8} a^4 b A-\frac{1}{8} a^5 B) x^3+(-2 a^3 b^2 A-a^4 b B) x^6+(-5 a^2 b^3 A-5 a^3 b^2 B) x^9+(5 a b^4 A+10 a^2 b^3 B) x^{12}+(\frac{1}{4} b^5 A+\frac{5}{4} a b^4 B) x^{15}+\frac{b^5 B x^{18}}{7} \ldots}{x^{11}}$
gosper	$-\frac{-88 b^5 B x^{18}-154 A b^5 x^{15}-770 B a b^4 x^{15}-3080 a A b^4 x^{12}-6160 B a^2 b^3 x^{12}+3080 a^2 A b^3 x^9+3080 B a^3 b^2 x^9+1232 a^3 A b^2 x^6+616 B a^4 b \ldots}{616 x^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^5*B*x^7+1/4*A*b^5*x^4+5/4*B*a*b^4*x^4+5*a*b^4*A*x+10*a^2*b^3*B*x-1/11
*a^5*A/x^11-a^3*b*(2*A*b+B*a)/x^5-5*a^2*b^2*(A*b+B*a)/x^2-1/8*a^4*(5*A*b+B*
a)/x^8
```

Maxima [A]
time = 0.27, size = 120, normalized size = 1.10

$$\frac{1}{7} B b^5 x^7+\frac{1}{4}\left(5 B a b^4+A b^5\right) x^4+5\left(2 B a^2 b^3+A a b^4\right) x-\frac{440\left(B a^3 b^2+A a^2 b^3\right) x^9+88\left(B a^4 b+2 A a^3 b^2\right) x^6+8 A a^5+11\left(B a^5+5 A a^4 b\right) x^3}{88 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^5*x^7 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5*(2*B*a^2*b^3 + A*a*b^4)*x -
 1/88*(440*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 88*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 8
*A*a^5 + 11*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

Fricas [A]
time = 2.04, size = 121, normalized size = 1.11

$$\frac{88 B b^5 x^{18}+154\left(5 B a b^4+A b^5\right) x^{15}+3080\left(2 B a^2 b^3+A a b^4\right) x^{12}-3080\left(B a^3 b^2+A a^2 b^3\right) x^9-616\left(B a^4 b+2 A a^3 b^2\right) x^6-56 A a^5-77\left(B a^5+5 A a^4 b\right) x^3}{616 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="fricas")
```

```
[Out] 1/616*(88*B*b^5*x^18 + 154*(5*B*a*b^4 + A*b^5)*x^15 + 3080*(2*B*a^2*b^3 + A
*a*b^4)*x^12 - 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 616*(B*a^4*b + 2*A*a^3*b^
2)*x^6 - 56*A*a^5 - 77*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

Sympy [A]
time = 30.09, size = 131, normalized size = 1.20

$$\frac{Bb^5x^7}{7} + x^4\left(\frac{Ab^5}{4} + \frac{5Bab^4}{4}\right) + x\left(5Aab^4 + 10Ba^2b^3\right) + \frac{-8Aa^5 + x^9\left(-440Aa^2b^3 - 440Ba^3b^2\right) + x^6\left(-176Aa^3b^2 - 88Ba^4b\right) + x^3\left(-55Aa^4b - 11Ba^5\right)}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**12,x)
```

```
[Out] B*b**5*x**7/7 + x**4*(A*b**5/4 + 5*B*a*b**4/4) + x*(5*A*a*b**4 + 10*B*a**2*
b**3) + (-8*A*a**5 + x**9*(-440*A*a**2*b**3 - 440*B*a**3*b**2) + x**6*(-176
*A*a**3*b**2 - 88*B*a**4*b) + x**3*(-55*A*a**4*b - 11*B*a**5))/(88*x**11)
```

Giac [A]
time = 0.75, size = 124, normalized size = 1.14

$$\frac{1}{7}\,Bb^5x^7 + \frac{5}{4}\,Bab^4x^4 + \frac{1}{4}\,Ab^5x^4 + 10\,Ba^2b^3x + 5\,Aab^4x - \frac{440\,Ba^3b^2x^9 + 440\,Aa^2b^3x^9 + 88\,Ba^4bx^6 + 176\,Aa^3b^2x^6 + 11\,Ba^5x^3 + 55\,Aa^4bx^3 + 8\,Aa^5}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="giac")
```

```
[Out] 1/7*B*b^5*x^7 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 10*B*a^2*b^3*x + 5*A*a*b^
4*x - 1/88*(440*B*a^3*b^2*x^9 + 440*A*a^2*b^3*x^9 + 88*B*a^4*b*x^6 + 176*A*
a^3*b^2*x^6 + 11*B*a^5*x^3 + 55*A*a^4*b*x^3 + 8*A*a^5)/x^11
```

Mupad [B]
time = 0.07, size = 116, normalized size = 1.06

$$x^4\left(\frac{A\,b^5}{4} + \frac{5\,B\,a\,b^4}{4}\right) - \frac{\frac{A\,a^5}{11} + x^6\left(B\,a^4\,b + 2\,A\,a^3\,b^2\right) + x^3\left(\frac{B\,a^5}{8} + \frac{5\,A\,b\,a^4}{8}\right) + x^9\left(5\,B\,a^3\,b^2 + 5\,A\,a^2\,b^3\right)}{x^{11}} + \frac{B\,b^5\,x^7}{7} + 5\,a\,b^3\,x\,\left(A\,b + 2\,B\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^12,x)
```

```
[Out] x^4*((A*b^5)/4 + (5*B*a*b^4)/4) - ((A*a^5)/11 + x^6*(2*A*a^3*b^2 + B*a^4*b)
 + x^3*((B*a^5)/8 + (5*A*a^4*b)/8) + x^9*(5*A*a^2*b^3 + 5*B*a^3*b^2))/x^11
+ (B*b^5*x^7)/7 + 5*a*b^3*x*(A*b + 2*B*a)
```

3.45 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}}\,dx$

Optimal. Leaf size=114

$$-\frac{a^5A}{12x^{12}}-\frac{a^4(5Ab+aB)}{9x^9}-\frac{5a^3b(2Ab+aB)}{6x^6}-\frac{10a^2b^2(Ab+aB)}{3x^3}+\frac{1}{3}b^4(Ab+5aB)x^3+\frac{1}{6}b^5Bx^6+5ab^3(Ab+2aB)\log(x)$$

```
[Out] -1/12*a^5*A/x^12-1/9*a^4*(5*A*b+B*a)/x^9-5/6*a^3*b*(2*A*b+B*a)/x^6-10/3*a^2
*b^2*(A*b+B*a)/x^3+1/3*b^4*(A*b+5*B*a)*x^3+1/6*b^5*B*x^6+5*a*b^3*(A*b+2*B*a
)*ln(x)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{12x^{12}}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{10a^2b^2(aB+Ab)}{3x^3}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{6}b^5Bx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

```
[Out] -1/12*(a^5*A)/x^12 - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))/
(6*x^6) - (10*a^2*b^2*(A*b + a*B))/(3*x^3) + (b^4*(A*b + 5*a*B)*x^3)/3 + (b
^5*B*x^6)/6 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(b^4(Ab+5aB) + \frac{a^5A}{x^5} + \frac{a^4(5Ab+aB)}{x^4} + \frac{5a^3b(2Ab+aB)}{x^3} + \frac{10\right.\right.$$
$$= -\frac{a^5A}{12x^{12}} - \frac{a^4(5Ab+aB)}{9x^9} - \frac{5a^3b(2Ab+aB)}{6x^6} - \frac{10a^2b^2(Ab+aB)}{3x^3} + \frac{1}{3}b^4(Ab$$

Mathematica [A]
time = 0.02, size = 118, normalized size = 1.04

$$-\frac{120a^2Ab^3x^9 - 60ab^4Bx^{15} - 6b^5x^{15}(2A+Bx^3) + 60a^3b^2x^6(A+2Bx^3) + 10a^4bx^3(2A+3Bx^3) + a^5(3A+4Bx^3) - 180ab^3(Ab+2aB)x^{12}\log(x)}{36x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

```
[Out] -1/36*(120*a^2*A*b^3*x^9 - 60*a*b^4*B*x^15 - 6*b^5*x^15*(2*A + B*x^3) + 60*
a^3*b^2*x^6*(A + 2*B*x^3) + 10*a^4*b*x^3*(2*A + 3*B*x^3) + a^5*(3*A + 4*B*x
^3) - 180*a*b^3*(A*b + 2*a*B)*x^12*Log[x])/x^12
```

Maple [A]
time = 0.30, size = 106, normalized size = 0.93

method	result
default	$\frac{b^5Bx^6}{6} + \frac{Ab^5x^3}{3} + \frac{5Bab^4x^3}{3} - \frac{a^4(5Ab+Ba)}{9x^9} - \frac{a^5A}{12x^{12}} - \frac{5a^3b(2Ab+Ba)}{6x^6} - \frac{10a^2b^2(Ab+Ba)}{3x^3} + 5ab^3(Ab+2Ba)\ln$
norman	$\frac{(\frac{1}{3}b^5A+\frac{5}{3}ab^4B)x^{15}+(-\frac{10}{3}a^2b^3A-\frac{10}{3}a^3b^2B)x^9+(-\frac{5}{3}a^3b^2A-\frac{5}{6}a^4bB)x^6+(-\frac{5}{9}a^4bA-\frac{1}{9}a^5B)x^3-\frac{a^5A}{12}+\frac{b^5Bx^{18}}{6}}{x^{12}} + (5ab^4A +$
risch	$\frac{b^5Bx^6}{6} + \frac{Ab^5x^3}{3} + \frac{5Bab^4x^3}{3} + \frac{b^5A^2}{6B} + \frac{5ab^4A}{3} + \frac{25a^2b^3B}{6} + \frac{(-\frac{10}{3}a^2b^3A-\frac{10}{3}a^3b^2B)x^9+(-\frac{5}{3}a^3b^2A-\frac{5}{6}a^4bB)x^6+(-\frac{5}{9}a^4}{x^{12}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^5*B*x^6+1/3*A*b^5*x^3+5/3*B*a*b^4*x^3-1/9*a^4*(5*A*b+B*a)/x^9-1/12*a^
5*A/x^12-5/6*a^3*b*(2*A*b+B*a)/x^6-10/3*a^2*b^2*(A*b+B*a)/x^3+5*a*b^3*(A*b+
2*B*a)*ln(x)
```

Maxima [A]
time = 0.30, size = 123, normalized size = 1.08

$$\frac{1}{6}Bb^5x^6 + \frac{1}{3}\left(5Bab^4 + Ab^5\right)x^3 + \frac{5}{3}\left(2Ba^2b^3 + Aab^4\right)\log\left(x^3\right) - \frac{120\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 30\left(Ba^4b + 2Aa^3b^2\right)x^6 + 3Aa^5 + 4\left(Ba^5 + 5Aa^4b\right)x^3}{36x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^5*x^6 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*l
og(x^3) - 1/36*(120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 30*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 3*A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

Fricas [A]
time = 2.25, size = 123, normalized size = 1.08

$$\frac{6\,Bb^5x^{18}+12\,(5\,Bab^4+Ab^5)x^{15}+180\,(2\,Ba^2b^3+Aab^4)x^{12}\log\left(x\right)-120\,(Ba^3b^2+Aa^2b^3)x^9-30\,(Ba^4b+2\,Aa^3b^2)x^6-3\,Aa^5-4\,(Ba^5+5\,Aa^4b)x^3}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="fricas")
```

```
[Out] 1/36*(6*B*b^5*x^18 + 12*(5*B*a*b^4 + A*b^5)*x^15 + 180*(2*B*a^2*b^3 + A*a*b
^4)*x^12*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 30*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 3*A*a^5 - 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

Sympy [A]
time = 51.74, size = 129, normalized size = 1.13

$$\frac{Bb^5x^6}{6}+5ab^3(Ab+2Ba)\log\left(x\right)+x^3\left(\frac{Ab^5}{3}+\frac{5Bab^4}{3}\right)+\frac{-3Aa^5+x^9(-120Aa^2b^3-120Ba^3b^2)+x^6(-60Aa^3b^2-30Ba^4b)+x^3(-20Aa^4b-4Ba^5)}{36x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**13,x)
```

```
[Out] B*b**5*x**6/6 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**3*(A*b**5/3 + 5*B*a*b**4
/3) + (-3*A*a**5 + x**9*(-120*A*a**2*b**3 - 120*B*a**3*b**2) + x**6*(-60*A*
a**3*b**2 - 30*B*a**4*b) + x**3*(-20*A*a**4*b - 4*B*a**5))/(36*x**12)
```

Giac [A]
time = 0.73, size = 149, normalized size = 1.31

$$\frac{1}{6}\,Bb^5x^6+\frac{5}{3}\,Bab^4x^3+\frac{1}{3}\,Ab^5x^3+5\,(2\,Ba^2b^3+Aab^4)\log\left(|x|\right)-\frac{250\,Ba^2b^3x^{12}+125\,Aab^4x^{12}+120\,Ba^3b^2x^9+120\,Aa^2b^3x^9+30\,Ba^4bx^6+60\,Aa^3b^2x^6+4\,Ba^5x^3+20\,Aa^4bx^3+3\,Aa^5}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="giac")
```

```
[Out] 1/6*B*b^5*x^6 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 5*(2*B*a^2*b^3 + A*a*b^4)
*log(abs(x)) - 1/36*(250*B*a^2*b^3*x^12 + 125*A*a*b^4*x^12 + 120*B*a^3*b^2*
x^9 + 120*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 4*B*a^5*x^3 +
 20*A*a^4*b*x^3 + 3*A*a^5)/x^12
```

Mupad [B]
time = 0.06, size = 122, normalized size = 1.07

$$\ln\left(x\right)\,\left(10\,B\,a^2\,b^3+5\,A\,a\,b^4\right)-\frac{\frac{A\,a^5}{12}+x^6\,\left(\frac{5\,B\,a^4\,b}{6}+\frac{5\,A\,a^3\,b^2}{3}\right)+x^3\,\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)+x^9\,\left(\frac{10\,B\,a^3\,b^2}{3}+\frac{10\,A\,a^2\,b^3}{3}\right)}{x^{12}}+x^3\,\left(\frac{A\,b^5}{3}+\frac{5\,B\,a\,b^4}{3}\right)+\frac{B\,b^5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^13,x)
```

```
[Out] log(x)*(10*B*a^2*b^3 + 5*A*a*b^4) - ((A*a^5)/12 + x^6*((5*A*a^3*b^2)/3 + (5
*B*a^4*b)/6) + x^3*((B*a^5)/9 + (5*A*a^4*b)/9) + x^9*((10*A*a^2*b^3)/3 + (1
0*B*a^3*b^2)/3))/x^12 + x^3*((A*b^5)/3 + (5*B*a*b^4)/3) + (B*b^5*x^6)/6
```

3.46 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx$

Optimal. Leaf size=115

$$-\frac{a^5A}{13x^{13}}-\frac{a^4(5Ab+aB)}{10x^{10}}-\frac{5a^3b(2Ab+aB)}{7x^7}-\frac{5a^2b^2(Ab+aB)}{2x^4}-\frac{5ab^3(Ab+2aB)}{x}+\frac{1}{2}b^4(Ab+5aB)x^2+\frac{1}{5}b^5Bx^5$$

```
[Out] -1/13*a^5*A/x^13-1/10*a^4*(5*A*b+B*a)/x^10-5/7*a^3*b*(2*A*b+B*a)/x^7-5/2*a^
2*b^2*(A*b+B*a)/x^4-5*a*b^3*(A*b+2*B*a)/x+1/2*b^4*(A*b+5*B*a)*x^2+1/5*b^5*B
*x^5
```

Rubi [A]
time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{13x^{13}}-\frac{a^4(aB+5Ab)}{10x^{10}}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{5a^2b^2(aB+Ab)}{2x^4}+\frac{1}{2}b^4x^2(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

```
[Out] -1/13*(a^5*A)/x^13 - (a^4*(5*A*b + a*B))/(10*x^10) - (5*a^3*b*(2*A*b + a*B)
)/(7*x^7) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/x + (
b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx = \int \left(\frac{a^5A}{x^{14}}+\frac{a^4(5Ab+aB)}{x^{11}}+\frac{5a^3b(2Ab+aB)}{x^8}+\frac{10a^2b^2(Ab+aB)}{x^5}+\frac{5ab^3(Ab}{x}\right.$$
$$= -\frac{a^5A}{13x^{13}}-\frac{a^4(5Ab+aB)}{10x^{10}}-\frac{5a^3b(2Ab+aB)}{7x^7}-\frac{5a^2b^2(Ab+aB)}{2x^4}-\frac{5ab^3(Ab+}{x}$$

Mathematica [A]
time = 0.02, size = 117, normalized size = 1.02

$$-\frac{-2275ab^4x^{12}(-2A+Bx^3)-91b^5x^{15}(5A+2Bx^3)+2275a^2b^3x^9(A+4Bx^3)+325a^3b^2x^6(4A+7Bx^3)+65a^4bx^3(7A+10Bx^3)+a^5(70A+91Bx^3)}{910x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

```
[Out] -1/910*(-2275*a*b^4*x^12*(-2*A + B*x^3) - 91*b^5*x^15*(5*A + 2*B*x^3) + 227
5*a^2*b^3*x^9*(A + 4*B*x^3) + 325*a^3*b^2*x^6*(4*A + 7*B*x^3) + 65*a^4*b*x^
3*(7*A + 10*B*x^3) + a^5*(70*A + 91*B*x^3))/x^13
```

Maple [A]
time = 0.27, size = 107, normalized size = 0.93

method	result
default	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^2}{2}+\frac{5 B a b^4 x^2}{2}-\frac{5 a^2 b^2 (A b+B a)}{2 x^4}-\frac{a^5 A}{13 x^{13}}-\frac{5 a^3 b (2 A b+B a)}{7 x^7}-\frac{a^4 (5 A b+B a)}{10 x^{10}}-\frac{5 a b^3 (A b+2 B a)}{x}$
norman	$\frac{-\frac{a^5 A}{13}+\left(-\frac{1}{2} a^4 b A-\frac{1}{10} a^5 B\right) x^3+\left(-\frac{10}{7} a^3 b^2 A-\frac{5}{7} a^4 b B\right) x^6+\left(-\frac{5}{2} a^2 b^3 A-\frac{5}{2} a^3 b^2 B\right) x^9+\left(-5 a b^4 A-10 a^2 b^3 B\right) x^{12}+\left(\frac{1}{2} b^5 A+\frac{5}{2} a b^4 B\right) x^{15}}{x^{13}}$
risch	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^2}{2}+\frac{5 B a b^4 x^2}{2}+\frac{\left(-5 a b^4 A-10 a^2 b^3 B\right) x^{12}+\left(-\frac{5}{2} a^2 b^3 A-\frac{5}{2} a^3 b^2 B\right) x^9+\left(-\frac{10}{7} a^3 b^2 A-\frac{5}{7} a^4 b B\right) x^6+\left(-\frac{1}{2} a^4 b A-\frac{1}{10} a\right.}{x^{13}}$
gosper	$-\frac{-182 b^5 B x^{18}-455 A b^5 x^{15}-2275 B a b^4 x^{15}+4550 a A b^4 x^{12}+9100 B a^2 b^3 x^{12}+2275 a^2 A b^3 x^9+2275 B a^3 b^2 x^9+1300 a^3 A b^2 x^6+650 B}{910 x^{13}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^5*B*x^5+1/2*A*b^5*x^2+5/2*B*a*b^4*x^2-5/2*a^2*b^2*(A*b+B*a)/x^4-1/13*
a^5*A/x^13-5/7*a^3*b*(2*A*b+B*a)/x^7-1/10*a^4*(5*A*b+B*a)/x^10-5*a*b^3*(A*b
+2*B*a)/x
```

Maxima [A]
time = 0.27, size = 122, normalized size = 1.06

$$\frac{1}{5} B b^5 x^5+\frac{1}{2} \left(5 B a b^4+A b^5\right) x^2-\frac{4550 \left(2 B a^2 b^3+A a b^4\right) x^{12}+2275 \left(B a^3 b^2+A a^2 b^3\right) x^9+650 \left(B a^4 b+2 A a^3 b^2\right) x^6+70 A a^5+91 \left(B a^5+5 A a^4 b\right) x^3}{910 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^5*x^5 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 - 1/910*(4550*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 650*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 70*A*a^5 + 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13
```

Fricas [A]
time = 2.02, size = 121, normalized size = 1.05

$$\frac{182 B b^5 x^{18}+455 \left(5 B a b^4+A b^5\right) x^{15}-4550 \left(2 B a^2 b^3+A a b^4\right) x^{12}-2275 \left(B a^3 b^2+A a^2 b^3\right) x^9-650 \left(B a^4 b+2 A a^3 b^2\right) x^6-70 A a^5-91 \left(B a^5+5 A a^4 b\right) x^3}{910 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="fricas")
```

```
[Out] 1/910*(182*B*b^5*x^18 + 455*(5*B*a*b^4 + A*b^5)*x^15 - 4550*(2*B*a^2*b^3 +
A*a*b^4)*x^12 - 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 650*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 70*A*a^5 - 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**14,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.58, size = 128, normalized size = 1.11

$$\frac{1}{5}Bb^5x^5 + \frac{5}{2}Bab^4x^2 + \frac{1}{2}Ab^5x^2 - \frac{9100\,Ba^2b^3x^{12} + 4550\,Aab^4x^{12} + 2275\,Ba^3b^2x^9 + 2275\,Aa^2b^3x^9 + 650\,Ba^4bx^6 + 1300\,Aa^3b^2x^6 + 91\,Ba^5x^3 + 455\,Aa^4bx^3 + 70\,Aa^5}{910\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="giac")
```

```
[Out] 1/5*B*b^5*x^5 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 - 1/910*(9100*B*a^2*b^3*x^1
2 + 4550*A*a*b^4*x^12 + 2275*B*a^3*b^2*x^9 + 2275*A*a^2*b^3*x^9 + 650*B*a^4
*b*x^6 + 1300*A*a^3*b^2*x^6 + 91*B*a^5*x^3 + 455*A*a^4*b*x^3 + 70*A*a^5)/x^
13
```

Mupad [**B**]
time = 2.37, size = 123, normalized size = 1.07

$$x^2\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right)-\frac{\frac{A\,a^5}{13}+x^{12}\,\left(10\,B\,a^2\,b^3+5\,A\,a\,b^4\right)+x^6\,\left(\frac{5\,B\,a^4\,b}{7}+\frac{10\,A\,a^3\,b^2}{7}\right)+x^3\,\left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+x^9\,\left(\frac{5\,B\,a^3\,b^2}{2}+\frac{5\,A\,a^2\,b^3}{2}\right)}{x^{13}}+\frac{B\,b^5\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^14,x)
```

```
[Out] x^2*((A*b^5)/2 + (5*B*a*b^4)/2) - ((A*a^5)/13 + x^12*(10*B*a^2*b^3 + 5*A*a*
b^4) + x^6*((10*A*a^3*b^2)/7 + (5*B*a^4*b)/7) + x^3*((B*a^5)/10 + (A*a^4*b)
/2) + x^9*((5*A*a^2*b^3)/2 + (5*B*a^3*b^2)/2))/x^13 + (B*b^5*x^5)/5
```

3.47 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}} dx$

Optimal. Leaf size=110

$$-\frac{a^5A}{14x^{14}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2aB)}{2x^2}+b^4(Ab+5aB)x+\frac{1}{4}b^5Bx^4$$

```
[Out] -1/14*a^5*A/x^14-1/11*a^4*(5*A*b+B*a)/x^11-5/8*a^3*b*(2*A*b+B*a)/x^8-2*a^2*
b^2*(A*b+B*a)/x^5-5/2*a*b^3*(A*b+2*B*a)/x^2+b^4*(A*b+5*B*a)*x+1/4*b^5*B*x^4
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{14x^{14}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{2a^2b^2(aB+Ab)}{x^5}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{2x^2}+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

```
[Out] -1/14*(a^5*A)/x^14 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) +
 b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx &= \int \left(b^4(Ab+5aB)+\frac{a^5A}{x^{15}}+\frac{a^4(5Ab+aB)}{x^{12}}+\frac{5a^3b(2Ab+aB)}{x^9}+\frac{10a^2b^2(Ab\right.\\ &= -\frac{a^5A}{14x^{14}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab}{2x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 110, normalized size = 1.00

$$-\frac{a^5A}{14x^{14}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2aB)}{2x^2}+b^4(Ab+5aB)x+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

```
[Out] -1/14*(a^5*A)/x^14 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) +
 b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

Maple [A]
time = 0.26, size = 102, normalized size = 0.93

method	result
default	$\frac{b^5 B x^4}{4}+b^5 A x+5 a b^4 B x-\frac{a^5 A}{14 x^{14}}-\frac{a^4(5 A b+B a)}{11 x^{11}}-\frac{2 a^2 b^2(A b+B a)}{x^5}-\frac{5 a b^3(A b+2 B a)}{2 x^2}-\frac{5 a^3 b(2 A b+B a)}{8 x^8}$
risch	$\frac{b^5 B x^4}{4}+b^5 A x+5 a b^4 B x+\frac{(-\frac{5}{2} a b^4 A-5 a^2 b^3 B) x^{12}+(-2 a^2 b^3 A-2 a^3 b^2 B) x^9+(-\frac{5}{4} a^3 b^2 A-\frac{5}{8} a^4 b B) x^6+(-\frac{5}{11} a^4 b A-\frac{1}{11} a^5 B}{x^{14}}$
norman	$\frac{-\frac{a^5 A}{14}+(-\frac{5}{11} a^4 b A-\frac{1}{11} a^5 B) x^3+(-\frac{5}{4} a^3 b^2 A-\frac{5}{8} a^4 b B) x^6+(-2 a^2 b^3 A-2 a^3 b^2 B) x^9+(-\frac{5}{2} a b^4 A-5 a^2 b^3 B) x^{12}+(b^5 A+5 a b^4 B) x^{15}+\frac{b^5 B}{4}}{x^{14}}$
gosper	$-\frac{-154 b^5 B x^{18}-616 A b^5 x^{15}-3080 B a b^4 x^{15}+1540 a A b^4 x^{12}+3080 B a^2 b^3 x^{12}+1232 a^2 A b^3 x^9+1232 B a^3 b^2 x^9+770 a^3 A b^2 x^6+385 B a^4}{616 x^{14}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^5*B*x^4+b^5*A*x+5*a*b^4*B*x-1/14*a^5*A/x^14-1/11*a^4*(5*A*b+B*a)/x^11
-2*a^2*b^2*(A*b+B*a)/x^5-5/2*a*b^3*(A*b+2*B*a)/x^2-5/8*a^3*b*(2*A*b+B*a)/x^
8
```

Maxima [A]
time = 0.30, size = 119, normalized size = 1.08

$$\frac{1}{4} B b^5 x^4+(5 B a b^4+A b^5) x-\frac{1540 (2 B a^2 b^3+A a b^4) x^{12}+1232 (B a^3 b^2+A a^2 b^3) x^9+385 (B a^4 b+2 A a^3 b^2) x^6+44 A a^5+56 (B a^5+5 A a^4 b) x^3}{616 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^5*x^4 + (5*B*a*b^4 + A*b^5)*x - 1/616*(1540*(2*B*a^2*b^3 + A*a*b^4)
*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 385*(B*a^4*b + 2*A*a^3*b^2)*x^6
+ 44*A*a^5 + 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

Fricas [A]
time = 1.93, size = 121, normalized size = 1.10

$$\frac{154 B b^5 x^{18}+616 (5 B a b^4+A b^5) x^{15}-1540 (2 B a^2 b^3+A a b^4) x^{12}-1232 (B a^3 b^2+A a^2 b^3) x^9-385 (B a^4 b+2 A a^3 b^2) x^6-44 A a^5-56 (B a^5+5 A a^4 b) x^3}{616 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="fricas")
```

```
[Out] 1/616*(154*B*b^5*x^18 + 616*(5*B*a*b^4 + A*b^5)*x^15 - 1540*(2*B*a^2*b^3 +
A*a*b^4)*x^12 - 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 385*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 44*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**15,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.71, size = 123, normalized size = 1.12

$$\frac{1}{4}Bb^5x^4 + 5Bab^4x + Ab^5x - \frac{3080Ba^2b^3x^{12} + 1540Aab^4x^{12} + 1232Ba^3b^2x^9 + 1232Aa^2b^3x^9 + 385Ba^4bx^6 + 770Aa^3b^2x^6 + 56Ba^5x^3 + 280Aa^4bx^3 + 44Aa^5}{616x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*x^4 + 5*B*a*b^4*x + A*b^5*x - 1/616*(3080*B*a^2*b^3*x^12 + 1540*A
*a*b^4*x^12 + 1232*B*a^3*b^2*x^9 + 1232*A*a^2*b^3*x^9 + 385*B*a^4*b*x^6 + 7
70*A*a^3*b^2*x^6 + 56*B*a^5*x^3 + 280*A*a^4*b*x^3 + 44*A*a^5)/x^14
```

Mupad [**B**]
time = 2.39, size = 120, normalized size = 1.09

$$x\left(Ab^5 + 5Bab^4\right) - \frac{\frac{Aa^5}{14} + x^{12}\left(5Ba^2b^3 + \frac{5Aab^4}{2}\right) + x^6\left(\frac{5Ba^4b}{8} + \frac{5Aa^3b^2}{4}\right) + x^3\left(\frac{Ba^5}{11} + \frac{5Aba^4}{11}\right) + x^9\left(2Ba^3b^2 + 2Aa^2b^3\right)}{x^{14}} + \frac{Bb^5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^15,x)
```

```
[Out] x*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/14 + x^12*(5*B*a^2*b^3 + (5*A*a*b^4)/2) +
x^6*((5*A*a^3*b^2)/4 + (5*B*a^4*b)/8) + x^3*((B*a^5)/11 + (5*A*a^4*b)/11) +
 x^9*(2*A*a^2*b^3 + 2*B*a^3*b^2))/x^14 + (B*b^5*x^4)/4
```

3.48 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{12x^{12}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{5a^2b^2(Ab+aB)}{3x^6}-\frac{5ab^3(Ab+2aB)}{3x^3}+\frac{1}{3}b^5Bx^3+b^4(Ab+5aB)\log(x)$$

```
[Out] -1/15*a^5*A/x^15-1/12*a^4*(5*A*b+B*a)/x^12-5/9*a^3*b*(2*A*b+B*a)/x^9-5/3*a^
2*b^2*(A*b+B*a)/x^6-5/3*a*b^3*(A*b+2*B*a)/x^3+1/3*b^5*B*x^3+b^4*(A*b+5*B*a)
*ln(x)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{15x^{15}}-\frac{a^4(aB+5Ab)}{12x^{12}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{5a^2b^2(aB+Ab)}{3x^6}+b^4\log(x)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{3x^3}+\frac{1}{3}b^5Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

```
[Out] -1/15*(a^5*A)/x^15 - (a^4*(5*A*b + a*B))/(12*x^12) - (5*a^3*b*(2*A*b + a*B)
)/(9*x^9) - (5*a^2*b^2*(A*b + a*B))/(3*x^6) - (5*a*b^3*(A*b + 2*a*B))/(3*x^
3) + (b^5*B*x^3)/3 + b^4*(A*b + 5*a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(b^5B+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^5}+\frac{5a^3b(2Ab+aB)}{x^4}+\frac{10a^2b^2(Ab\right.\right.\\ &= -\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{12x^{12}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{5a^2b^2(Ab+aB)}{3x^6}-\frac{5ab^3(Ab}{3x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 116, normalized size = 1.03

$$-\frac{300aAb^4x^{12}-60b^5Bx^{18}+300a^2b^3x^9(A+2Bx^3)+100a^3b^2x^6(2A+3Bx^3)+25a^4bx^3(3A+4Bx^3)+3a^5(4A+5Bx^3)}{180x^{15}}+b^4(Ab+5aB)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

```
[Out] -1/180*(300*a*A*b^4*x^12 - 60*b^5*B*x^18 + 300*a^2*b^3*x^9*(A + 2*B*x^3) +
100*a^3*b^2*x^6*(2*A + 3*B*x^3) + 25*a^4*b*x^3*(3*A + 4*B*x^3) + 3*a^5*(4*A
 + 5*B*x^3))/x^15 + b^4*(A*b + 5*a*B)*Log[x]
```

Maple [A]
time = 0.27, size = 102, normalized size = 0.90

method	result
default	$-\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+Ba)}{12x^{12}}-\frac{5a^3b(2Ab+Ba)}{9x^9}-\frac{5a^2b^2(Ab+Ba)}{3x^6}-\frac{5a\,b^3(Ab+2Ba)}{3x^3}+\frac{b^5B\,x^3}{3}+b^4(Ab+5Ba)\ln(x)$
norman	$\frac{(-\frac{5}{3}a\,b^4A-\frac{10}{3}a^2b^3B)x^{12}+(-\frac{5}{3}a^2b^3A-\frac{5}{3}a^3b^2B)x^9+(-\frac{10}{9}a^3b^2A-\frac{5}{9}a^4bB)x^6+(-\frac{5}{12}a^4bA-\frac{1}{12}a^5B)x^3-\frac{a^5A}{15}+\frac{b^5B\,x^{18}}{3}}{x^{15}}+(b^5A-$
risch	$\frac{b^5B\,x^3}{3}+\frac{-\frac{a^5A}{15}+(-\frac{5}{12}a^4bA-\frac{1}{12}a^5B)x^3+(-\frac{10}{9}a^3b^2A-\frac{5}{9}a^4bB)x^6+(-\frac{5}{3}a^2b^3A-\frac{5}{3}a^3b^2B)x^9+(-\frac{5}{3}a\,b^4A-\frac{10}{3}a^2b^3B)x^{12}}{x^{15}}+A\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*a^5*A/x^15-1/12*a^4*(5*A*b+B*a)/x^12-5/9*a^3*b*(2*A*b+B*a)/x^9-5/3*a^
2*b^2*(A*b+B*a)/x^6-5/3*a*b^3*(A*b+2*B*a)/x^3+1/3*b^5*B*x^3+b^4*(A*b+5*B*a)
*ln(x)
```

Maxima [A]
time = 0.27, size = 123, normalized size = 1.09

$$\frac{1}{3}\,Bb^5x^3+\frac{1}{3}\,(5\,Bab^4+Ab^5)\log\left(x^3\right)-\frac{300\,(2\,Ba^2b^3+Aab^4)x^{12}+300\,(Ba^3b^2+Aa^2b^3)x^9+100\,(Ba^4b+2\,Aa^3b^2)x^6+12\,Aa^5+15\,(Ba^5+5\,Aa^4b)x^3}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*x^3 + 1/3*(5*B*a*b^4 + A*b^5)*log(x^3) - 1/180*(300*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 100*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 12*A*a^5 + 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

Fricas [A]
time = 1.69, size = 123, normalized size = 1.09

$$\frac{60\,Bb^5x^{18}+180\,(5\,Bab^4+Ab^5)x^{15}\log(x)-300\,(2\,Ba^2b^3+Aab^4)x^{12}-300\,(Ba^3b^2+Aa^2b^3)x^9-100\,(Ba^4b+2\,Aa^3b^2)x^6-12\,Aa^5-15\,(Ba^5+5\,Aa^4b)x^3}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(60*B*b^5*x^18 + 180*(5*B*a*b^4 + A*b^5)*x^15*log(x) - 300*(2*B*a^2*b
^3 + A*a*b^4)*x^12 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 100*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 12*A*a^5 - 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**16,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.41, size = 145, normalized size = 1.28

$$\frac{1}{3}\,Bb^5x^3+(5\,Bab^4+Ab^5)\log(|x|)-\frac{685\,Bab^4x^{15}+137\,Ab^5x^{15}+600\,Ba^2b^3x^{12}+300\,Aab^4x^{12}+300\,Ba^3b^2x^9+300\,Aa^2b^3x^9+100\,Ba^4bx^6+200\,Aa^3b^2x^6+15\,Ba^5x^3+75\,Aa^4bx^3+12\,Aa^5}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="giac")
```

```
[Out] 1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*log(abs(x)) - 1/180*(685*B*a*b^4*x^15 +
 137*A*b^5*x^15 + 600*B*a^2*b^3*x^12 + 300*A*a*b^4*x^12 + 300*B*a^3*b^2*x^9
 + 300*A*a^2*b^3*x^9 + 100*B*a^4*b*x^6 + 200*A*a^3*b^2*x^6 + 15*B*a^5*x^3 +
 75*A*a^4*b*x^3 + 12*A*a^5)/x^15
```

Mupad [B]
time = 0.08, size = 121, normalized size = 1.07

$$\ln(x)\,\left(A\,b^5+5\,B\,a\,b^4\right)-\frac{\frac{A\,a^5}{15}+x^{12}\,\left(\frac{10\,B\,a^2\,b^3}{3}+\frac{5\,A\,a\,b^4}{3}\right)+x^6\,\left(\frac{5\,B\,a^4\,b}{9}+\frac{10\,A\,a^3\,b^2}{9}\right)+x^3\,\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^9\,\left(\frac{5\,B\,a^3\,b^2}{3}+\frac{5\,A\,a^2\,b^3}{3}\right)}{x^{15}}+\frac{B\,b^5\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^16,x)
```

```
[Out] log(x)*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/15 + x^12*((10*B*a^2*b^3)/3 + (5*A*a*
b^4)/3) + x^6*((10*A*a^3*b^2)/9 + (5*B*a^4*b)/9) + x^3*((B*a^5)/12 + (5*A*a
^4*b)/12) + x^9*((5*A*a^2*b^3)/3 + (5*B*a^3*b^2)/3))/x^15 + (B*b^5*x^3)/3
```

3.49 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx$

Optimal. Leaf size=115

$$-\frac{a^5A}{16x^{16}}-\frac{a^4(5Ab+aB)}{13x^{13}}-\frac{a^3b(2Ab+aB)}{2x^{10}}-\frac{10a^2b^2(Ab+aB)}{7x^7}-\frac{5ab^3(Ab+2aB)}{4x^4}-\frac{b^4(Ab+5aB)}{x}+\frac{1}{2}b^5Bx^2$$

```
[Out] -1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7*
a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5*B
*x^2
```

Rubi [A]
time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{16x^{16}}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{a^3b(aB+2Ab)}{2x^{10}}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{x}-\frac{5ab^3(2aB+Ab)}{4x^4}+\frac{1}{2}b^5Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

```
[Out] -1/16*(a^5*A)/x^16 - (a^4*(5*A*b + a*B))/(13*x^13) - (a^3*b*(2*A*b + a*B))/
(2*x^10) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (5*a*b^3*(A*b + 2*a*B))/(4*x^
4) - (b^4*(A*b + 5*a*B))/x + (b^5*B*x^2)/2
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = \int \left(\frac{a^5A}{x^{17}}+\frac{a^4(5Ab+aB)}{x^{14}}+\frac{5a^3b(2Ab+aB)}{x^{11}}+\frac{10a^2b^2(Ab+aB)}{x^8}+\frac{5ab^3(A$$
$$= -\frac{a^5A}{16x^{16}}-\frac{a^4(5Ab+aB)}{13x^{13}}-\frac{a^3b(2Ab+aB)}{2x^{10}}-\frac{10a^2b^2(Ab+aB)}{7x^7}-\frac{5ab^3(Ab}{4x}$$

Mathematica [A]
time = 0.02, size = 118, normalized size = 1.03

$$-\frac{-728b^5x^{15}(-2A+Bx^3)+1820ab^4x^{12}(A+4Bx^3)+520a^2b^3x^9(4A+7Bx^3)+208a^3b^2x^6(7A+10Bx^3)+56a^4bx^3(10A+13Bx^3)+7a^5(13A+16Bx^3)}{1456x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

```
[Out] -1/1456*(-728*b^5*x^15*(-2*A + B*x^3) + 1820*a*b^4*x^12*(A + 4*B*x^3) + 520
*a^2*b^3*x^9*(4*A + 7*B*x^3) + 208*a^3*b^2*x^6*(7*A + 10*B*x^3) + 56*a^4*b*
x^3*(10*A + 13*B*x^3) + 7*a^5*(13*A + 16*B*x^3))/x^16
```

Maple [A]
time = 0.27, size = 104, normalized size = 0.90

method	result
default	$-\frac{a^5 A}{16x^{16}}-\frac{a^4(5Ab+Ba)}{13x^{13}}-\frac{a^3b(2Ab+Ba)}{2x^{10}}-\frac{10a^2b^2(Ab+Ba)}{7x^7}-\frac{5a\,b^3(Ab+2Ba)}{4x^4}-\frac{b^4(Ab+5Ba)}{x}+\frac{b^5 B\,x^2}{2}$
norman	$\frac{-\frac{a^5 A}{16}+(-\frac{5}{13}a^4bA-\frac{1}{13}a^5B)x^3+(-a^3b^2A-\frac{1}{2}a^4bB)x^6+(-\frac{10}{7}a^2b^3A-\frac{10}{7}a^3b^2B)x^9+(-\frac{5}{4}a\,b^4A-\frac{5}{2}a^2b^3B)x^{12}+(-b^5A-5a\,b^4B)x^{15}+}{x^{16}}$
risch	$\frac{b^5 B\,x^2}{2}+\frac{-\frac{a^5 A}{16}+(-\frac{5}{13}a^4bA-\frac{1}{13}a^5B)x^3+(-a^3b^2A-\frac{1}{2}a^4bB)x^6+(-\frac{10}{7}a^2b^3A-\frac{10}{7}a^3b^2B)x^9+(-\frac{5}{4}a\,b^4A-\frac{5}{2}a^2b^3B)x^{12}+(-b^5A-5a}{x^{16}}$
gosper	$-\frac{-728b^5B\,x^{18}+1456A\,b^5x^{15}+7280Ba\,b^4x^{15}+1820aA\,b^4x^{12}+3640B\,a^2b^3x^{12}+2080a^2A\,b^3x^9+2080B\,a^3b^2x^9+1456a^3A\,b^2x^6+728B}{1456x^{16}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7*
a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5*B
*x^2
```

Maxima [A]
time = 0.27, size = 122, normalized size = 1.06

$$\frac{1}{2}\,Bb^5x^2-\frac{1456\,(5\,Bab^4+Ab^5)x^{15}+1820\,(2\,Ba^2b^3+Aab^4)x^{12}+2080\,(Ba^3b^2+Aa^2b^3)x^9+728\,(Ba^4b+2\,Aa^3b^2)x^6+91\,Aa^5+112\,(Ba^5+5\,Aa^4b)x^3}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*x^2 - 1/1456*(1456*(5*B*a*b^4 + A*b^5)*x^15 + 1820*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 728*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 91*A*a^5 + 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

Fricas [A]
time = 2.58, size = 121, normalized size = 1.05

$$\frac{728\,Bb^5x^{18}-1456\,(5\,Bab^4+Ab^5)x^{15}-1820\,(2\,Ba^2b^3+Aab^4)x^{12}-2080\,(Ba^3b^2+Aa^2b^3)x^9-728\,(Ba^4b+2\,Aa^3b^2)x^6-91\,Aa^5-112\,(Ba^5+5\,Aa^4b)x^3}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="fricas")
```

[Out] 1/1456*(728*B*b^5*x^18 - 1456*(5*B*a*b^4 + A*b^5)*x^15 - 1820*(2*B*a^2*b^3 + A*a*b^4)*x^12 - 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 728*(B*a^4*b + 2*A*a^3 *b^2)*x^6 - 91*A*a^5 - 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**17,x)

[Out] Timed out

Giac [**A**]
time = 2.43, size = 128, normalized size = 1.11

$$\frac{1}{2}Bb^5x^2 - \frac{7280\,Bab^4x^{15} + 1456\,Ab^5x^{15} + 3640\,Ba^2b^3x^{12} + 1820\,Aab^4x^{12} + 2080\,Ba^3b^2x^9 + 2080\,Aa^2b^3x^9 + 728\,Ba^4bx^6 + 1456\,Aa^3b^2x^6 + 112\,Ba^5x^3 + 560\,Aa^4bx^3 + 91\,Aa^5}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="giac")

[Out] 1/2*B*b^5*x^2 - 1/1456*(7280*B*a*b^4*x^15 + 1456*A*b^5*x^15 + 3640*B*a^2*b^3*x^12 + 1820*A*a*b^4*x^12 + 2080*B*a^3*b^2*x^9 + 2080*A*a^2*b^3*x^9 + 728*B*a^4*b*x^6 + 1456*A*a^3*b^2*x^6 + 112*B*a^5*x^3 + 560*A*a^4*b*x^3 + 91*A*a^5)/x^16

Mupad [**B**]
time = 2.36, size = 121, normalized size = 1.05

$$\frac{B\,b^5\,x^2}{2} - \frac{\frac{A\,a^5}{16} + x^6\left(\frac{B\,a^4\,b}{2} + A\,a^3\,b^2\right) + x^{12}\left(\frac{5\,B\,a^2\,b^3}{2} + \frac{5\,A\,a\,b^4}{4}\right) + x^3\left(\frac{B\,a^5}{13} + \frac{5\,A\,b\,a^4}{13}\right) + x^{15}\,(A\,b^5 + 5\,B\,a\,b^4) + x^9\left(\frac{10\,B\,a^3\,b^2}{7} + \frac{10\,A\,a^2\,b^3}{7}\right)}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^17,x)

[Out] (B*b^5*x^2)/2 - ((A*a^5)/16 + x^6*(A*a^3*b^2 + (B*a^4*b)/2) + x^12*((5*B*a^2*b^3)/2 + (5*A*a*b^4)/4) + x^3*((B*a^5)/13 + (5*A*a^4*b)/13) + x^15*(A*b^5 + 5*B*a*b^4) + x^9*((10*A*a^2*b^3)/7 + (10*B*a^3*b^2)/7))/x^16

3.50 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{18}}\,dx$

Optimal. Leaf size=110

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{14x^{14}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{5a^2b^2(Ab+aB)}{4x^8}-\frac{ab^3(Ab+2aB)}{x^5}-\frac{b^4(Ab+5aB)}{2x^2}+b^5Bx$$

```
[Out] -1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4*
a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B*x
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{b^4(5aB+Ab)}{2x^2}-\frac{ab^3(2aB+Ab)}{x^5}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

```
[Out] -1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 -
 (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{18}}\,dx &= \int \left(b^5B+\frac{a^5A}{x^{18}}+\frac{a^4(5Ab+aB)}{x^{15}}+\frac{5a^3b(2Ab+aB)}{x^{12}}+\frac{10a^2b^2(Ab+aB)}{x^9}+\frac{5a\ldots\right.\\ &= -\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{14x^{14}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{5a^2b^2(Ab+aB)}{4x^8}-\frac{ab^3(Ab+2\ldots}{x^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 110, normalized size = 1.00

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{14x^{14}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{5a^2b^2(Ab+aB)}{4x^8}-\frac{ab^3(Ab+2aB)}{x^5}-\frac{b^4(Ab+5aB)}{2x^2}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

```
[Out] -1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 -
 (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

Maple [A]
time = 0.25, size = 101, normalized size = 0.92

method	result
default	$-\frac{a^5 A}{17x^{17}}-\frac{a^4(5Ab+Ba)}{14x^{14}}-\frac{5a^3b(2Ab+Ba)}{11x^{11}}-\frac{5a^2b^2(Ab+Ba)}{4x^8}-\frac{a\,b^3(Ab+2Ba)}{x^5}-\frac{b^4(Ab+5Ba)}{2x^2}+b^5Bx$
risch	$b^5Bx+\frac{-\frac{a^5A}{17}+(-\frac{5}{14}a^4bA-\frac{1}{14}a^5B)x^3+(-\frac{10}{11}a^3b^2A-\frac{5}{11}a^4bB)x^6+(-\frac{5}{4}a^2b^3A-\frac{5}{4}a^3b^2B)x^9+(-a\,b^4A-2a^2b^3B)x^{12}+(-\frac{1}{2}b^5A-}{x^{17}}$
norman	$\frac{-\frac{a^5A}{17}+(-\frac{5}{14}a^4bA-\frac{1}{14}a^5B)x^3+(-\frac{10}{11}a^3b^2A-\frac{5}{11}a^4bB)x^6+(-\frac{5}{4}a^2b^3A-\frac{5}{4}a^3b^2B)x^9+(-a\,b^4A-2a^2b^3B)x^{12}+(-\frac{1}{2}b^5A-\frac{5}{2}a\,b^4B)x^1}{x^{17}}$
gosper	$-\frac{-5236b^5B\,x^{18}+2618A\,b^5x^{15}+13090Ba\,b^4x^{15}+5236aA\,b^4x^{12}+10472B\,a^2b^3x^{12}+6545a^2A\,b^3x^9+6545B\,a^3b^2x^9+4760a^3A\,b^2x^6+2}{5236x^{17}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^18,x,method=_RETURNVERBOSE)
```

```
[Out] -1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4*
a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B*x
```

Maxima [A]
time = 0.28, size = 119, normalized size = 1.08

$$Bb^5x-\frac{2618\,(5\,Bab^4+Ab^5)x^{15}+5236\,(2\,Ba^2b^3+Aab^4)x^{12}+6545\,(Ba^3b^2+Aa^2b^3)x^9+2380\,(Ba^4b+2\,Aa^3b^2)x^6+308\,Aa^5+374\,(Ba^5+5\,Aa^4b)x^3}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="maxima")
```

```
[Out] B*b^5*x - 1/5236*(2618*(5*B*a*b^4 + A*b^5)*x^15 + 5236*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 2380*(B*a^4*b + 2*A*a^3*b^2)*
x^6 + 308*A*a^5 + 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

Fricas [A]
time = 2.35, size = 121, normalized size = 1.10

$$\frac{5236\,Bb^5x^{18}-2618\,(5\,Bab^4+Ab^5)x^{15}-5236\,(2\,Ba^2b^3+Aab^4)x^{12}-6545\,(Ba^3b^2+Aa^2b^3)x^9-2380\,(Ba^4b+2\,Aa^3b^2)x^6-308\,Aa^5-374\,(Ba^5+5\,Aa^4b)x^3}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="fricas")
```

```
[Out] 1/5236*(5236*B*b^5*x^18 - 2618*(5*B*a*b^4 + A*b^5)*x^15 - 5236*(2*B*a^2*b^3
 + A*a*b^4)*x^12 - 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 2380*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 308*A*a^5 - 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**18,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.30, size = 125, normalized size = 1.14

$$Bb^5x - \frac{13090\,Bab^4x^{15} + 2618\,Ab^5x^{15} + 10472\,Ba^2b^3x^{12} + 5236\,Aab^4x^{12} + 6545\,Ba^3b^2x^9 + 6545\,Aa^2b^3x^9 + 2380\,Ba^4bx^6 + 4760\,Aa^3b^2x^6 + 374\,Ba^5x^3 + 1870\,Aa^4bx^3 + 308\,Aa^5}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="giac")
```

```
[Out] B*b^5*x - 1/5236*(13090*B*a*b^4*x^15 + 2618*A*b^5*x^15 + 10472*B*a^2*b^3*x^
12 + 5236*A*a*b^4*x^12 + 6545*B*a^3*b^2*x^9 + 6545*A*a^2*b^3*x^9 + 2380*B*a
^4*b*x^6 + 4760*A*a^3*b^2*x^6 + 374*B*a^5*x^3 + 1870*A*a^4*b*x^3 + 308*A*a^
5)/x^17
```

Mupad [**B**]
time = 0.08, size = 119, normalized size = 1.08

$$B\,b^5\,x - \frac{\frac{A\,a^5}{17} + x^{12}\,\left(2\,B\,a^2\,b^3 + A\,a\,b^4\right) + x^6\,\left(\frac{5\,B\,a^4\,b}{11} + \frac{10\,A\,a^3\,b^2}{11}\right) + x^3\,\left(\frac{B\,a^5}{14} + \frac{5\,A\,b\,a^4}{14}\right) + x^{15}\,\left(\frac{A\,b^5}{2} + \frac{5\,B\,a\,b^4}{2}\right) + x^9\,\left(\frac{5\,B\,a^3\,b^2}{4} + \frac{5\,A\,a^2\,b^3}{4}\right)}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^18,x)
```

```
[Out] B*b^5*x - ((A*a^5)/17 + x^12*(2*B*a^2*b^3 + A*a*b^4) + x^6*((10*A*a^3*b^2)/
11 + (5*B*a^4*b)/11) + x^3*((B*a^5)/14 + (5*A*a^4*b)/14) + x^15*((A*b^5)/2
+ (5*B*a*b^4)/2) + x^9*((5*A*a^2*b^3)/4 + (5*B*a^3*b^2)/4))/x^17
```

3.51 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}} \, dx$

Optimal. Leaf size=91

$$-\frac{a^5 B}{15 x^{15}}-\frac{5 a^4 b B}{12 x^{12}}-\frac{10 a^3 b^2 B}{9 x^9}-\frac{5 a^2 b^3 B}{3 x^6}-\frac{5 a b^4 B}{3 x^3}-\frac{A (a+b x^3)^6}{18 a x^{18}}+b^5 B \log (x)$$

```
[Out] -1/15*a^5*B/x^15-5/12*a^4*b*B/x^12-10/9*a^3*b^2*B/x^9-5/3*a^2*b^3*B/x^6-5/3
*a*b^4*B/x^3-1/18*A*(b*x^3+a)^6/a/x^18+b^5*B*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {457, 79, 45}

$$-\frac{a^5 B}{15 x^{15}}-\frac{5 a^4 b B}{12 x^{12}}-\frac{10 a^3 b^2 B}{9 x^9}-\frac{5 a^2 b^3 B}{3 x^6}-\frac{A (a+b x^3)^6}{18 a x^{18}}-\frac{5 a b^4 B}{3 x^3}+b^5 B \log (x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

```
[Out] -1/15*(a^5*B)/x^15 - (5*a^4*b*B)/(12*x^12) - (10*a^3*b^2*B)/(9*x^9) - (5*a^
2*b^3*B)/(3*x^6) - (5*a*b^4*B)/(3*x^3) - (A*(a + b*x^3)^6)/(18*a*x^18) + b^
5*B*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx,x,x^3\right)\\ &= -\frac{A(a+bx^3)^6}{18ax^{18}}+\frac{1}{3}B\text{Subst}\left(\int \frac{(a+bx)^5}{x^6}\,dx,x,x^3\right)\\ &= -\frac{A(a+bx^3)^6}{18ax^{18}}+\frac{1}{3}B\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)\right.\\ &= -\frac{a^5B}{15x^{15}}-\frac{5a^4bB}{12x^{12}}-\frac{10a^3b^2B}{9x^9}-\frac{5a^2b^3B}{3x^6}-\frac{5ab^4B}{3x^3}-\frac{A(a+bx^3)^6}{18ax^{18}}+b^5B\log(x\end{aligned}$$

Mathematica [A]
time = 0.02, size = 121, normalized size = 1.33

$$-\frac{60Ab^5x^{15}+150ab^4x^{12}(A+2Bx^3)+100a^2b^3x^9(2A+3Bx^3)+50a^3b^2x^6(3A+4Bx^3)+15a^4bx^3(4A+5Bx^3)+2a^5(5A+6Bx^3)-180b^5Bx^{18}\log(x)}{180x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

```
[Out] -1/180*(60*A*b^5*x^15 + 150*a*b^4*x^12*(A + 2*B*x^3) + 100*a^2*b^3*x^9*(2*A
 + 3*B*x^3) + 50*a^3*b^2*x^6*(3*A + 4*B*x^3) + 15*a^4*b*x^3*(4*A + 5*B*x^3)
 + 2*a^5*(5*A + 6*B*x^3) - 180*b^5*B*x^18*Log[x])/x^18
```

Maple [A]
time = 0.26, size = 102, normalized size = 1.12

method	result
default	$-\frac{a^4(5Ab+Ba)}{15x^{15}}-\frac{10a^2b^2(Ab+Ba)}{9x^9}-\frac{5a^3b(2Ab+Ba)}{12x^{12}}-\frac{5ab^3(Ab+2Ba)}{6x^6}-\frac{b^4(Ab+5Ba)}{3x^3}+b^5B\ln(x)-\frac{a^5A}{18x^{18}}$
norman	$\frac{(-\frac{1}{3}b^5A-\frac{5}{3}ab^4B)x^{15}+(-\frac{5}{6}ab^4A-\frac{5}{3}a^2b^3B)x^{12}+(-\frac{10}{9}a^2b^3A-\frac{10}{9}a^3b^2B)x^9+(-\frac{5}{6}a^3b^2A-\frac{5}{12}a^4bB)x^6+(-\frac{1}{3}a^4bA-\frac{1}{15}a^5B)x^3-\frac{a^5A}{18}}{x^{18}}$
risch	$\frac{(-\frac{1}{3}b^5A-\frac{5}{3}ab^4B)x^{15}+(-\frac{5}{6}ab^4A-\frac{5}{3}a^2b^3B)x^{12}+(-\frac{10}{9}a^2b^3A-\frac{10}{9}a^3b^2B)x^9+(-\frac{5}{6}a^3b^2A-\frac{5}{12}a^4bB)x^6+(-\frac{1}{3}a^4bA-\frac{1}{15}a^5B)x^3-\frac{a^5A}{18}}{x^{18}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^19,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*a^4*(5*A*b+B*a)/x^15-10/9*a^2*b^2*(A*b+B*a)/x^9-5/12*a^3*b*(2*A*b+B*a
)/x^12-5/6*a*b^3*(A*b+2*B*a)/x^6-1/3*b^4*(A*b+5*B*a)/x^3+b^5*B*ln(x)-1/18*a
^5*A/x^18
```

Maxima [A]
time = 0.28, size = 123, normalized size = 1.35

$$\frac{1}{3}\,Bb^{5}\log\left(x^{3}\right)-\frac{60\,(5\,Bab^{4}+Ab^{5})x^{15}+150\,(2\,Ba^{2}b^{3}+Aab^{4})x^{12}+200\,(Ba^{3}b^{2}+Aa^{2}b^{3})x^{9}+75\,(Ba^{4}b+2\,Aa^{3}b^{2})x^{6}+10\,Aa^{5}+12\,(Ba^{5}+5\,Aa^{4}b)x^{3}}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*log(x^3) - 1/180*(60*(5*B*a*b^4 + A*b^5)*x^15 + 150*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 75*(B*a^4*b + 2*A*a^3*b
^2)*x^6 + 10*A*a^5 + 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

Fricas [A]
time = 1.67, size = 123, normalized size = 1.35

$$\frac{180\,Bb^{5}x^{18}\log\left(x\right)-60\,(5\,Bab^{4}+Ab^{5})x^{15}-150\,(2\,Ba^{2}b^{3}+Aab^{4})x^{12}-200\,(Ba^{3}b^{2}+Aa^{2}b^{3})x^{9}-75\,(Ba^{4}b+2\,Aa^{3}b^{2})x^{6}-10\,Aa^{5}-12\,(Ba^{5}+5\,Aa^{4}b)x^{3}}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="fricas")
```

```
[Out] 1/180*(180*B*b^5*x^18*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^15 - 150*(2*B*a^2*b
^3 + A*a*b^4)*x^12 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 75*(B*a^4*b + 2*A*a^
3*b^2)*x^6 - 10*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**19,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.01, size = 136, normalized size = 1.49

$$Bb^{5}\log\left(|x|\right)-\frac{147\,Bb^{5}x^{18}+300\,Bab^{4}x^{15}+60\,Ab^{5}x^{15}+300\,Ba^{2}b^{3}x^{12}+150\,Aab^{4}x^{12}+200\,Ba^{3}b^{2}x^{9}+200\,Aa^{2}b^{3}x^{9}+75\,Ba^{4}bx^{6}+150\,Aa^{3}b^{2}x^{6}+12\,Ba^{5}x^{3}+60\,Aa^{4}bx^{3}+10\,Aa^{5}}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="giac")
```

```
[Out] B*b^5*log(abs(x)) - 1/180*(147*B*b^5*x^18 + 300*B*a*b^4*x^15 + 60*A*b^5*x^1
5 + 300*B*a^2*b^3*x^12 + 150*A*a*b^4*x^12 + 200*B*a^3*b^2*x^9 + 200*A*a^2*b
^3*x^9 + 75*B*a^4*b*x^6 + 150*A*a^3*b^2*x^6 + 12*B*a^5*x^3 + 60*A*a^4*b*x^3
 + 10*A*a^5)/x^18
```

Mupad [B]
time = 0.09, size = 121, normalized size = 1.33

$$B\,b^5\,\ln\left(x\right)-\frac{\frac{A\,a^5}{18}+x^{12}\,\left(\frac{5\,B\,a^2\,b^3}{3}+\frac{5\,A\,a\,b^4}{6}\right)+x^6\,\left(\frac{5\,B\,a^4\,b}{12}+\frac{5\,A\,a^3\,b^2}{6}\right)+x^3\,\left(\frac{B\,a^5}{15}+\frac{A\,b\,a^4}{3}\right)+x^{15}\,\left(\frac{A\,b^5}{3}+\frac{5\,B\,a\,b^4}{3}\right)+x^9\,\left(\frac{10\,B\,a^3\,b^2}{9}+\frac{10\,A\,a^2\,b^3}{9}\right)}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^19,x)
```

```
[Out] B*b^5*log(x) - ((A*a^5)/18 + x^12*((5*B*a^2*b^3)/3 + (5*A*a*b^4)/6) + x^6*(
(5*A*a^3*b^2)/6 + (5*B*a^4*b)/12) + x^3*((B*a^5)/15 + (A*a^4*b)/3) + x^15*(
(A*b^5)/3 + (5*B*a*b^4)/3) + x^9*((10*A*a^2*b^3)/9 + (10*B*a^3*b^2)/9))/x^1
8
```

3.52 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(5Ab+aB)}{16x^{16}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{a^2b^2(Ab+aB)}{x^{10}}-\frac{5ab^3(Ab+2aB)}{7x^7}-\frac{b^4(Ab+5aB)}{4x^4}-\frac{b^5B}{x}$$

```
[Out] -1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2*
b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*B/
x
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^2b^2(aB+Ab)}{x^{10}}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

```
[Out] -1/19*(a^5*A)/x^19 - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (a^2*b^2*(A*b + a*B))/x^10 - (5*a*b^3*(A*b + 2*a*B))/(7*x^7)
- (b^4*(A*b + 5*a*B))/(4*x^4) - (b^5*B)/x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}}\,dx &= \int \left(\frac{a^5A}{x^{20}}+\frac{a^4(5Ab+aB)}{x^{17}}+\frac{5a^3b(2Ab+aB)}{x^{14}}+\frac{10a^2b^2(Ab+aB)}{x^{11}}+\frac{5ab^3(A\\ &= -\frac{a^5A}{19x^{19}}-\frac{a^4(5Ab+aB)}{16x^{16}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{a^2b^2(Ab+aB)}{x^{10}}-\frac{5ab^3(Ab+}{7x^7}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 119, normalized size = 1.05

$$-\frac{6916b^5x^{15}(A+4Bx^3)+4940ab^4x^{12}(4A+7Bx^3)+3952a^2b^3x^9(7A+10Bx^3)+2128a^3b^2x^6(10A+13Bx^3)+665a^4bx^3(13A+16Bx^3)+91a^5(16A+19Bx^3)}{27664x^{19}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

```
[Out] -1/27664*(6916*b^5*x^15*(A + 4*B*x^3) + 4940*a*b^4*x^12*(4*A + 7*B*x^3) + 3
952*a^2*b^3*x^9*(7*A + 10*B*x^3) + 2128*a^3*b^2*x^6*(10*A + 13*B*x^3) + 665
*a^4*b*x^3*(13*A + 16*B*x^3) + 91*a^5*(16*A + 19*B*x^3))/x^19
```

Maple [A]
time = 0.26, size = 104, normalized size = 0.92

method	result
default	$-\frac{a^{5} A}{19 x^{19}}-\frac{a^{4}(5 A b+B a)}{16 x^{16}}-\frac{5 a^{3} b(2 A b+B a)}{13 x^{13}}-\frac{a^{2} b^{2}(A b+B a)}{x^{10}}-\frac{5 a\, b^{3}(A b+2 B a)}{7 x^{7}}-\frac{b^{4}(A b+5 B a)}{4 x^{4}}-\frac{b^{5} B}{x}$
norman	$\frac{-\frac{a^{5} A}{19}+\left(-\frac{5}{16} a^{4} b A-\frac{1}{16} a^{5} B\right) x^{3}+\left(-\frac{10}{13} a^{3} b^{2} A-\frac{5}{13} a^{4} b B\right) x^{6}+\left(-a^{2} b^{3} A-a^{3} b^{2} B\right) x^{9}+\left(-\frac{5}{7} a\, b^{4} A-\frac{10}{7} a^{2} b^{3} B\right) x^{12}+\left(-\frac{1}{4} b^{5} A-\frac{5}{4} a\, b^{4} B\right) x^{15}-}{x^{19}}$
risch	$\frac{-\frac{a^{5} A}{19}+\left(-\frac{5}{16} a^{4} b A-\frac{1}{16} a^{5} B\right) x^{3}+\left(-\frac{10}{13} a^{3} b^{2} A-\frac{5}{13} a^{4} b B\right) x^{6}+\left(-a^{2} b^{3} A-a^{3} b^{2} B\right) x^{9}+\left(-\frac{5}{7} a\, b^{4} A-\frac{10}{7} a^{2} b^{3} B\right) x^{12}+\left(-\frac{1}{4} b^{5} A-\frac{5}{4} a\, b^{4} B\right) x^{15}-}{x^{19}}$
gosper	$-\frac{27664 b^{5} B\, x^{18}+6916 A\, b^{5} x^{15}+34580 B a\, b^{4} x^{15}+19760 a A\, b^{4} x^{12}+39520 B\, a^{2} b^{3} x^{12}+27664 a^{2} A\, b^{3} x^{9}+27664 B\, a^{3} b^{2} x^{9}+21280 a^{3} A\, b^{2} x^{6}}{27664 x^{19}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^20,x,method=_RETURNVERBOSE)
```

```
[Out] -1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2*
b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*B/
x
```

Maxima [A]
time = 0.28, size = 121, normalized size = 1.07

$$-\frac{27664\, B b^{5} x^{18}+6916\,(5\, B a b^{4}+A b^{5}) x^{15}+19760\,(2\, B a^{2} b^{3}+A a b^{4}) x^{12}+27664\,(B a^{3} b^{2}+A a^{2} b^{3}) x^{9}+10640\,(B a^{4} b+2\, A a^{3} b^{2}) x^{6}+1456\, A a^{5}+1729\,(B a^{5}+5\, A a^{4} b) x^{3}}{27664\, x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="maxima")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^2
*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b +
 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

Fricas [A]
time = 2.09, size = 121, normalized size = 1.07

$$-\frac{27664\, B b^{5} x^{18}+6916\,(5\, B a b^{4}+A b^{5}) x^{15}+19760\,(2\, B a^{2} b^{3}+A a b^{4}) x^{12}+27664\,(B a^{3} b^{2}+A a^{2} b^{3}) x^{9}+10640\,(B a^{4} b+2\, A a^{3} b^{2}) x^{6}+1456\, A a^{5}+1729\,(B a^{5}+5\, A a^{4} b) x^{3}}{27664\, x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="fricas")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^2
*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b +
 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**20,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.91, size = 127, normalized size = 1.12

$$-\frac{27664\,Bb^5x^{18}+34580\,Bab^4x^{15}+6916\,Ab^5x^{15}+39520\,Ba^2b^3x^{12}+19760\,Aab^4x^{12}+27664\,Ba^3b^2x^9+27664\,Aa^2b^3x^9+10640\,Ba^4bx^6+21280\,Aa^3b^2x^6+1729\,Ba^5x^3+8645\,Aa^4bx^3+1456\,Aa^5}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="giac")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 34580*B*a*b^4*x^15 + 6916*A*b^5*x^15 + 39520*B
*a^2*b^3*x^12 + 19760*A*a*b^4*x^12 + 27664*B*a^3*b^2*x^9 + 27664*A*a^2*b^3*
x^9 + 10640*B*a^4*b*x^6 + 21280*A*a^3*b^2*x^6 + 1729*B*a^5*x^3 + 8645*A*a^4
*b*x^3 + 1456*A*a^5)/x^19
```

Mupad [**B**]
time = 2.37, size = 119, normalized size = 1.05

$$-\frac{\frac{A\,a^5}{19}+x^{12}\,\left(\frac{10\,B\,a^2\,b^3}{7}+\frac{5\,A\,a\,b^4}{7}\right)+x^6\,\left(\frac{5\,B\,a^4\,b}{13}+\frac{10\,A\,a^3\,b^2}{13}\right)+x^3\,\left(\frac{B\,a^5}{16}+\frac{5\,A\,b\,a^4}{16}\right)+x^{15}\,\left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right)+x^9\,\left(B\,a^3\,b^2+A\,a^2\,b^3\right)+B\,b^5\,x^{18}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^20,x)
```

```
[Out] -((A*a^5)/19 + x^12*((10*B*a^2*b^3)/7 + (5*A*a*b^4)/7) + x^6*((10*A*a^3*b^2
)/13 + (5*B*a^4*b)/13) + x^3*((B*a^5)/16 + (5*A*a^4*b)/16) + x^15*((A*b^5)/
4 + (5*B*a*b^4)/4) + x^9*(A*a^2*b^3 + B*a^3*b^2) + B*b^5*x^18)/x^19
```

3.53 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{20x^{20}}-\frac{a^4(5Ab+aB)}{17x^{17}}-\frac{5a^3b(2Ab+aB)}{14x^{14}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab+2aB)}{8x^8}-\frac{b^4(Ab+5aB)}{5x^5}-\frac{b^5B}{2x^2}$$

```
[Out] -1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^5-
1/2*b^5*B/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{20x^{20}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{8x^8}-\frac{b^5B}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

```
[Out] -1/20*(a^5*A)/x^20 - (a^4*(5*A*b + a*B))/(17*x^17) - (5*a^3*b*(2*A*b + a*B)
)/(14*x^14) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(8*x^8) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(2*x^2)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}}\,dx &= \int \left(\frac{a^5A}{x^{21}}+\frac{a^4(5Ab+aB)}{x^{18}}+\frac{5a^3b(2Ab+aB)}{x^{15}}+\frac{10a^2b^2(Ab+aB)}{x^{12}}+\frac{5ab^3(Ab\right.\\ &= -\frac{a^5A}{20x^{20}}-\frac{a^4(5Ab+aB)}{17x^{17}}-\frac{5a^3b(2Ab+aB)}{14x^{14}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab}{8x^8}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 121, normalized size = 1.03

$$-\frac{5236b^5x^{15}(2A+5Bx^3)+6545ab^4x^{12}(5A+8Bx^3)+5950a^2b^3x^9(8A+11Bx^3)+3400a^3b^2x^6(11A+14Bx^3)+1100a^4bx^3(14A+17Bx^3)+154a^5(17A+20Bx^3)}{52360x^{20}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

```
[Out] -1/52360*(5236*b^5*x^15*(2*A + 5*B*x^3) + 6545*a*b^4*x^12*(5*A + 8*B*x^3) +
 5950*a^2*b^3*x^9*(8*A + 11*B*x^3) + 3400*a^3*b^2*x^6*(11*A + 14*B*x^3) + 1
100*a^4*b*x^3*(14*A + 17*B*x^3) + 154*a^5*(17*A + 20*B*x^3))/x^20
```

Maple [A]
time = 0.25, size = 104, normalized size = 0.89

method	result
default	$-\frac{a^5 A}{20x^{20}}-\frac{a^4(5Ab+Ba)}{17x^{17}}-\frac{5a^3b(2Ab+Ba)}{14x^{14}}-\frac{10a^2b^2(Ab+Ba)}{11x^{11}}-\frac{5a\,b^3(Ab+2Ba)}{8x^8}-\frac{b^4(Ab+5Ba)}{5x^5}-\frac{b^5B}{2x^2}$
norman	$\frac{-\frac{a^5A}{20}+(-\frac{5}{17}a^4bA-\frac{1}{17}a^5B)x^3+(-\frac{5}{7}a^3b^2A-\frac{5}{14}a^4bB)x^6+(-\frac{10}{11}a^2b^3A-\frac{10}{11}a^3b^2B)x^9+(-\frac{5}{8}a\,b^4A-\frac{5}{4}a^2b^3B)x^{12}+(-\frac{1}{5}b^5A-a\,b^4B)x}{x^{20}}$
risch	$\frac{-\frac{a^5A}{20}+(-\frac{5}{17}a^4bA-\frac{1}{17}a^5B)x^3+(-\frac{5}{7}a^3b^2A-\frac{5}{14}a^4bB)x^6+(-\frac{10}{11}a^2b^3A-\frac{10}{11}a^3b^2B)x^9+(-\frac{5}{8}a\,b^4A-\frac{5}{4}a^2b^3B)x^{12}+(-\frac{1}{5}b^5A-a\,b^4B)x}{x^{20}}$
gosper	$-\frac{26180b^5Bx^{18}+10472Ab^5x^{15}+52360Ba\,b^4x^{15}+32725aA\,b^4x^{12}+65450B\,a^2b^3x^{12}+47600a^2A\,b^3x^9+47600B\,a^3b^2x^9+37400a^3A\,b^2}{52360x^{20}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^21,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^5-
1/2*b^5*B/x^2
```

Maxima [A]
time = 0.28, size = 121, normalized size = 1.03

$$-\frac{26180\,Bb^5x^{18}+10472\,(5\,Bab^4+Ab^5)x^{15}+32725\,(2\,Ba^2b^3+Aab^4)x^{12}+47600\,(Ba^3b^2+Aa^2b^3)x^9+18700\,(Ba^4b+2\,Aa^3b^2)x^6+2618\,Aa^5+3080\,(Ba^5+5\,Aa^4b)x^3}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="maxima")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

Fricas [A]
time = 1.83, size = 121, normalized size = 1.03

$$-\frac{26180\,Bb^5x^{18}+10472\,(5\,Bab^4+Ab^5)x^{15}+32725\,(2\,Ba^2b^3+Aab^4)x^{12}+47600\,(Ba^3b^2+Aa^2b^3)x^9+18700\,(Ba^4b+2\,Aa^3b^2)x^6+2618\,Aa^5+3080\,(Ba^5+5\,Aa^4b)x^3}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="fricas")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**21,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.35, size = 127, normalized size = 1.09

$$-\frac{26180\,Bb^5x^{18}+52360\,Bab^4x^{15}+10472\,Ab^5x^{15}+65450\,Ba^2b^3x^{12}+32725\,Aab^4x^{12}+47600\,Ba^3b^2x^9+47600\,Aa^2b^3x^9+18700\,Ba^4bx^6+37400\,Aa^3b^2x^6+3080\,Ba^5x^3+15400\,Aa^4bx^3+2618\,Aa^5}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="giac")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 52360*B*a*b^4*x^15 + 10472*A*b^5*x^15 + 65450*
B*a^2*b^3*x^12 + 32725*A*a*b^4*x^12 + 47600*B*a^3*b^2*x^9 + 47600*A*a^2*b^3
*x^9 + 18700*B*a^4*b*x^6 + 37400*A*a^3*b^2*x^6 + 3080*B*a^5*x^3 + 15400*A*a
^4*b*x^3 + 2618*A*a^5)/x^20
```

Mupad [**B**]
time = 2.35, size = 121, normalized size = 1.03

$$-\frac{\frac{A\,a^5}{20}+x^{12}\,\left(\frac{5\,B\,a^2\,b^3}{4}+\frac{5\,A\,a\,b^4}{8}\right)+x^6\,\left(\frac{5\,B\,a^4\,b}{14}+\frac{5\,A\,a^3\,b^2}{7}\right)+x^3\,\left(\frac{B\,a^5}{17}+\frac{5\,A\,b\,a^4}{17}\right)+x^{15}\,\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)+x^9\,\left(\frac{10\,B\,a^3\,b^2}{11}+\frac{10\,A\,a^2\,b^3}{11}\right)+\frac{B\,b^5\,x^{18}}{2}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^21,x)
```

```
[Out] -((A*a^5)/20 + x^12*((5*B*a^2*b^3)/4 + (5*A*a*b^4)/8) + x^6*((5*A*a^3*b^2)/
7 + (5*B*a^4*b)/14) + x^3*((B*a^5)/17 + (5*A*a^4*b)/17) + x^15*((A*b^5)/5 +
 B*a*b^4) + x^9*((10*A*a^2*b^3)/11 + (10*B*a^3*b^2)/11) + (B*b^5*x^18)/2)/x
^20
```

3.54 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx$

Optimal. Leaf size=48

$$-\frac{A(a+bx^3)^6}{21ax^{21}}+\frac{(Ab-7aB)\,(a+bx^3)^6}{126a^2x^{18}}$$

```
[Out] -1/21*A*(b*x^3+a)^6/a/x^21+1/126*(A*b-7*B*a)*(b*x^3+a)^6/a^2/x^18
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {457, 79, 37}

$$\frac{(a+bx^3)^6\,(Ab-7aB)}{126a^2x^{18}}-\frac{A(a+bx^3)^6}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

```
[Out] -1/21*(A*(a + b*x^3)^6)/(a*x^21) + ((A*b - 7*a*B)*(a + b*x^3)^6)/(126*a^2*x
^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{22}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx, x, x^3\right)\\
&= -\frac{A\left(a+bx^3\right)^6}{21ax^{21}} + \frac{(-Ab+7aB)\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^3\right)}{21a}\\
&= -\frac{A\left(a+bx^3\right)^6}{21ax^{21}} + \frac{(Ab-7aB)\left(a+bx^3\right)^6}{126a^2x^{18}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 118 vs. 2(48) = 96.
time = 0.02, size = 118, normalized size = 2.46

$$
-\frac{21b^5x^{15}(A+2Bx^3)+35ab^4x^{12}(2A+3Bx^3)+35a^2b^3x^9(3A+4Bx^3)+21a^3b^2x^6(4A+5Bx^3)+7a^4bx^3(5A+6Bx^3)+a^5(6A+7Bx^3)}{126x^{21}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

```
[Out] -1/126*(21*b^5*x^15*(A + 2*B*x^3) + 35*a*b^4*x^12*(2*A + 3*B*x^3) + 35*a^2*
b^3*x^9*(3*A + 4*B*x^3) + 21*a^3*b^2*x^6*(4*A + 5*B*x^3) + 7*a^4*b*x^3*(5*A
 + 6*B*x^3) + a^5*(6*A + 7*B*x^3))/x^21
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(44) = 88.
time = 0.27, size = 104, normalized size = 2.17

method	result
default	$-\frac{a^3b(2Ab+Ba)}{3x^{15}}-\frac{5ab^3(Ab+2Ba)}{9x^9}-\frac{5a^2b^2(Ab+Ba)}{6x^{12}}-\frac{b^4(Ab+5Ba)}{6x^6}-\frac{a^5A}{21x^{21}}-\frac{b^5B}{3x^3}-\frac{a^4(5Ab+Ba)}{18x^{18}}$
norman	$\frac{-\frac{a^5A}{21}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^3+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^6+(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B)x^9+(-\frac{5}{9}ab^4A-\frac{10}{9}a^2b^3B)x^{12}+(-\frac{1}{6}b^5A-\frac{5}{6}ab^4B)x^{15}}{x^{21}}$
risch	$\frac{-\frac{a^5A}{21}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^3+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^6+(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B)x^9+(-\frac{5}{9}ab^4A-\frac{10}{9}a^2b^3B)x^{12}+(-\frac{1}{6}b^5A-\frac{5}{6}ab^4B)x^{15}}{x^{21}}$
gosper	$-\frac{42b^5Bx^{18}+21Ab^5x^{15}+105Bab^4x^{15}+70aAb^4x^{12}+140Ba^2b^3x^{12}+105a^2Ab^3x^9+105Ba^3b^2x^9+84a^3Ab^2x^6+42Ba^4bx^6+35a^4Ab}{126x^{21}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^22,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^3*b*(2*A*b+B*a)/x^15-5/9*a*b^3*(A*b+2*B*a)/x^9-5/6*a^2*b^2*(A*b+B*a)
/x^12-1/6*b^4*(A*b+5*B*a)/x^6-1/21*a^5*A/x^21-1/3*b^5*B/x^3-1/18*a^4*(5*A*b
+B*a)/x^18
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(45) = 90$.
time = 0.31, size = 121, normalized size = 2.52

$$-\frac{42\,Bb^5x^{18}+21\,(5\,Bab^4+Ab^5)x^{15}+70\,(2\,Ba^2b^3+Aab^4)x^{12}+105\,(Ba^3b^2+Aa^2b^3)x^9+42\,(Ba^4b+2\,Aa^3b^2)x^6+6\,Aa^5+7\,(Ba^5+5\,Aa^4b)x^3}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="maxima")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(45) = 90$.
time = 2.25, size = 121, normalized size = 2.52

$$-\frac{42\,Bb^5x^{18}+21\,(5\,Bab^4+Ab^5)x^{15}+70\,(2\,Ba^2b^3+Aab^4)x^{12}+105\,(Ba^3b^2+Aa^2b^3)x^9+42\,(Ba^4b+2\,Aa^3b^2)x^6+6\,Aa^5+7\,(Ba^5+5\,Aa^4b)x^3}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**22,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(45) = 90$.
time = 2.61, size = 127, normalized size = 2.65

$$-\frac{42\,Bb^5x^{18}+105\,Bab^4x^{15}+21\,Ab^5x^{15}+140\,Ba^2b^3x^{12}+70\,Aab^4x^{12}+105\,Ba^3b^2x^9+105\,Aa^2b^3x^9+42\,Ba^4bx^6+84\,Aa^3b^2x^6+7\,Ba^5x^3+35\,Aa^4bx^3+6\,Aa^5}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="giac")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 105*B*a*b^4*x^15 + 21*A*b^5*x^15 + 140*B*a^2*b^3*x^
12 + 70*A*a*b^4*x^12 + 105*B*a^3*b^2*x^9 + 105*A*a^2*b^3*x^9 + 42*B*a^4*b*x
^6 + 84*A*a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 6*A*a^5)/x^21
```

Mupad [B]
time = 2.37, size = 122, normalized size = 2.54

$$-\frac{\frac{A\,a^5}{21}+x^6\,\left(\frac{B\,a^4\,b}{3}+\frac{2\,A\,a^3\,b^2}{3}\right)+x^{12}\,\left(\frac{10\,B\,a^2\,b^3}{9}+\frac{5\,A\,a\,b^4}{9}\right)+x^3\,\left(\frac{B\,a^5}{18}+\frac{5\,A\,b\,a^4}{18}\right)+x^{15}\,\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)+x^9\,\left(\frac{5\,B\,a^3\,b^2}{6}+\frac{5\,A\,a^2\,b^3}{6}\right)+\frac{B\,b^5\,x^{18}}{3}}{x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^22,x)
```

```
[Out] -((A*a^5)/21 + x^6*((2*A*a^3*b^2)/3 + (B*a^4*b)/3) + x^12*((10*B*a^2*b^3)/9
 + (5*A*a*b^4)/9) + x^3*((B*a^5)/18 + (5*A*a^4*b)/18) + x^15*((A*b^5)/6 + (
5*B*a*b^4)/6) + x^9*((5*A*a^2*b^3)/6 + (5*B*a^3*b^2)/6) + (B*b^5*x^18)/3)/x
^21
```

3.55 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}} dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{16x^{16}}-\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{ab^3(Ab+2aB)}{2x^{10}}-\frac{b^4(Ab+5aB)}{7x^7}-\frac{b^5B}{4x^4}$$

```
[Out] -1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/1
3*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x^7
-1/4*b^5*B/x^4
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^5B}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

```
[Out] -1/22*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2
*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}} dx = \int \left(\frac{a^5A}{x^{23}}+\frac{a^4(5Ab+aB)}{x^{20}}+\frac{5a^3b(2Ab+aB)}{x^{17}}+\frac{10a^2b^2(Ab+aB)}{x^{14}}+\frac{5ab^3(A\right.$$
$$= -\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{16x^{16}}-\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{ab^3(Ab}{2x}$$

Mathematica [A]
time = 0.02, size = 117, normalized size = 1.00

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{16x^{16}}-\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{ab^3(Ab+2aB)}{2x^{10}}-\frac{b^4(Ab+5aB)}{7x^7}-\frac{b^5B}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

```
[Out] -1/22*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2
*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

Maple [A]
time = 0.26, size = 104, normalized size = 0.89

method	result
default	$-\frac{a^5 A}{22x^{22}}-\frac{a^4(5Ab+Ba)}{19x^{19}}-\frac{5a^3b(2Ab+Ba)}{16x^{16}}-\frac{10a^2b^2(Ab+Ba)}{13x^{13}}-\frac{a\,b^3(Ab+2Ba)}{2x^{10}}-\frac{b^4(Ab+5Ba)}{7x^7}-\frac{b^5B}{4x^4}$
norman	$\frac{-\frac{a^5A}{22}+(-\frac{5}{19}a^4bA-\frac{1}{19}a^5B)x^3+(-\frac{5}{8}a^3b^2A-\frac{5}{16}a^4bB)x^6+(-\frac{10}{13}a^2b^3A-\frac{10}{13}a^3b^2B)x^9+(-\frac{1}{2}a\,b^4A-a^2b^3B)x^{12}+(-\frac{1}{7}b^5A-\frac{5}{7}a\,b^4B)x^{1}}{x^{22}}$
risch	$\frac{-\frac{a^5A}{22}+(-\frac{5}{19}a^4bA-\frac{1}{19}a^5B)x^3+(-\frac{5}{8}a^3b^2A-\frac{5}{16}a^4bB)x^6+(-\frac{10}{13}a^2b^3A-\frac{10}{13}a^3b^2B)x^9+(-\frac{1}{2}a\,b^4A-a^2b^3B)x^{12}+(-\frac{1}{7}b^5A-\frac{5}{7}a\,b^4B)x^{1}}{x^{22}}$
gosper	$-\frac{76076b^5B\,x^{18}+43472A\,b^5x^{15}+217360Ba\,b^4x^{15}+152152aA\,b^4x^{12}+304304B\,a^2b^3x^{12}+234080a^2A\,b^3x^9+234080B\,a^3b^2x^9+190190a}{304304x^{22}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^23,x,method=_RETURNVERBOSE)
```

```
[Out] -1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/1
3*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x^7
-1/4*b^5*B/x^4
```

Maxima [A]
time = 0.29, size = 121, normalized size = 1.03

$$-\frac{76076\,Bb^5x^{18}+43472\,(5\,Bab^4+Ab^5)x^{15}+152152\,(2\,Ba^2b^3+Aab^4)x^{12}+234080\,(Ba^3b^2+Aa^2b^3)x^9+95095\,(Ba^4b+2\,Aa^3b^2)x^6+13832\,Aa^5+16016\,(Ba^5+5\,Aa^4b)x^3}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="maxima")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B*
a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4
*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^22
```

Fricas [A]
time = 1.45, size = 121, normalized size = 1.03

$$-\frac{76076\,Bb^5x^{18}+43472\,(5\,Bab^4+Ab^5)x^{15}+152152\,(2\,Ba^2b^3+Aab^4)x^{12}+234080\,(Ba^3b^2+Aa^2b^3)x^9+95095\,(Ba^4b+2\,Aa^3b^2)x^6+13832\,Aa^5+16016\,(Ba^5+5\,Aa^4b)x^3}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="fricas")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B*
a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4
*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^22
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**23,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.06, size = 127, normalized size = 1.09

$$-\frac{76076\,Bb^5x^{18}+217360\,Bab^4x^{15}+43472\,Ab^5x^{15}+304304\,Ba^2b^3x^{12}+152152\,Aab^4x^{12}+234080\,Ba^3b^2x^9+234080\,Aa^2b^3x^9+95095\,Ba^4bx^6+190190\,Aa^3b^2x^6+16016\,Ba^5x^3+80080\,Aa^4bx^3+13832\,Aa^5}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="giac")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 217360*B*a*b^4*x^15 + 43472*A*b^5*x^15 + 3043
04*B*a^2*b^3*x^12 + 152152*A*a*b^4*x^12 + 234080*B*a^3*b^2*x^9 + 234080*A*a
^2*b^3*x^9 + 95095*B*a^4*b*x^6 + 190190*A*a^3*b^2*x^6 + 16016*B*a^5*x^3 + 8
0080*A*a^4*b*x^3 + 13832*A*a^5)/x^22
```

Mupad [**B**]
time = 0.06, size = 121, normalized size = 1.03

$$-\frac{\frac{A\,a^5}{22}+x^{12}\,\left(B\,a^2\,b^3+\frac{A\,a\,b^4}{2}\right)+x^6\,\left(\frac{5\,B\,a^4\,b}{16}+\frac{5\,A\,a^3\,b^2}{8}\right)+x^3\,\left(\frac{B\,a^5}{19}+\frac{5\,A\,b\,a^4}{19}\right)+x^{15}\,\left(\frac{A\,b^5}{7}+\frac{5\,B\,a\,b^4}{7}\right)+x^9\,\left(\frac{10\,B\,a^3\,b^2}{13}+\frac{10\,A\,a^2\,b^3}{13}\right)+\frac{B\,b^5\,x^{18}}{4}}{x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^23,x)
```

```
[Out] -((A*a^5)/22 + x^12*(B*a^2*b^3 + (A*a*b^4)/2) + x^6*((5*A*a^3*b^2)/8 + (5*B
*a^4*b)/16) + x^3*((B*a^5)/19 + (5*A*a^4*b)/19) + x^15*((A*b^5)/7 + (5*B*a*
b^4)/7) + x^9*((10*A*a^2*b^3)/13 + (10*B*a^3*b^2)/13) + (B*b^5*x^18)/4)/x^2
2
```

3.56 $\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=183

$$-\frac{a(Ab-aB)x}{b^3}+\frac{(Ab-aB)x^4}{4b^2}+\frac{Bx^7}{7b}-\frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{10/3}}+\frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3b^{10/3}}$$

```
[Out] -a*(A*b-B*a)*x/b^3+1/4*(A*b-B*a)*x^4/b^2+1/7*B*x^7/b+1/3*a^(4/3)*(A*b-B*a)*
ln(a^(1/3)+b^(1/3)*x)/b^(10/3)-1/6*a^(4/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/b^(10/3)-1/3*a^(4/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/b^(10/3)*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {470, 308, 206, 31, 648, 631, 210, 642}

$$-\frac{a^{4/3}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{10/3}}-\frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{10/3}}+\frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{10/3}}-\frac{ax(Ab-aB)}{b^3}+\frac{x^4(Ab-aB)}{4b^2}+\frac{Bx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^7)/(7*b) - (a^(
4/3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*b^(10/3)) + (a^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(10/3)) -
 (a^(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^
(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \frac{x^6}{a+bx^3}\,dx}{7b}$$

$$= \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2(a+bx^3)}\right)dx}{7b}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{(a^2(Ab-aB))\int \frac{1}{a+bx^3}\,dx}{b^3}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{\left(a^{4/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^3} + \frac{(a^{4/3}(A}{}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{10/3}} - \frac{(a^{4/3}(A}{}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{10/3}} - \frac{a^{4/3}(A}{}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} - \frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{10/3}} + \frac{a^{4/3}}{}$$

Mathematica [A]
time = 0.08, size = 171, normalized size = 0.93

$$\frac{84a\sqrt[3]{b}(-Ab+aB)x+21b^{4/3}(Ab-aB)x^4+12b^{7/3}Bx^7+28\sqrt{3}\,a^{4/3}(-Ab+aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-28a^{4/3}(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+14a^{4/3}(-Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (84*a*b^(1/3)*(-(A*b) + a*B)*x + 21*b^(4/3)*(A*b - a*B)*x^4 + 12*b^(7/3)*B*
x^7 + 28*Sqrt[3]*a^(4/3)*(-(A*b) + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]] - 28*a^(4/3)*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*
(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*b^(10/3)
)
```

Maple [A]
time = 0.28, size = 151, normalized size = 0.83

method	result

risch	$\frac{B\,x^7}{7b}+\frac{A\,x^4}{4b}-\frac{Ba\,x^4}{4b^2}-\frac{aAx}{b^2}+\frac{a^2Bx}{b^3}+\frac{a^2\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{3b^4}$
default	$-\frac{-\frac{1}{7}b^2B\,x^7-\frac{1}{4}A\,b^2x^4+\frac{1}{4}Bab\,x^4+abAx-a^2Bx}{b^3}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/7*b^2*B*x^7-1/4*A*b^2*x^4+1/4*B*a*b*x^4+a*b*A*x-a^2*B*x)+(1/3/b/
(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a
^2*(A*b-B*a)/b^3
```

Maxima [A]
time = 0.49, size = 182, normalized size = 0.99

$$\frac{4\,Bb^2x^7-7\,(Bab-Ab^2)x^4+28\,(Ba^2-Aab)x}{28\,b^3}-\frac{\sqrt{3}\,(Ba^3-Aa^2b)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba^3-Aa^2b)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba^3-Aa^2b)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*B*b^2*x^7 - 7*(B*a*b - A*b^2)*x^4 + 28*(B*a^2 - A*a*b)*x)/b^3 - 1/3
*sqrt(3)*(B*a^3 - A*a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(b^4*(a/b)^(2/3)) + 1/6*(B*a^3 - A*a^2*b)*log(x^2 - x*(a/b)^(1/3) + (a/
b)^(2/3))/(b^4*(a/b)^(2/3)) - 1/3*(B*a^3 - A*a^2*b)*log(x + (a/b)^(1/3))/(b
^4*(a/b)^(2/3))
```

Fricas [A]
time = 2.06, size = 167, normalized size = 0.91

$$\frac{12\,Bb^2x^7-21\,(Bab-Ab^2)x^4-28\,\sqrt{3}\,(Ba^2-Aab)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3a}\right)+14\,(Ba^2-Aab)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-28\,(Ba^2-Aab)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+84\,(Ba^2-Aab)x}{84\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/84*(12*B*b^2*x^7 - 21*(B*a*b - A*b^2)*x^4 - 28*sqrt(3)*(B*a^2 - A*a*b)*(a
/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(B*a^2
 - A*a*b)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(B*a^2 -
A*a*b)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(B*a^2 - A*a*b)*x)/b^3
```

Sympy [A]
time = 0.28, size = 114, normalized size = 0.62

$$\frac{Bx^7}{7b} + x^4\left(\frac{A}{4b} - \frac{Ba}{4b^2}\right) + x\left(-\frac{Aa}{b^2} + \frac{Ba^2}{b^3}\right) + \operatorname{RootSum}\left(27t^3b^{10} - A^3a^4b^3 + 3A^2Ba^5b^2 - 3AB^2a^6b + B^3a^7, \left(t \mapsto t\log\left(-\frac{3tb^3}{-Aab + Ba^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**7/(7*b) + x**4*(A/(4*b) - B*a/(4*b**2)) + x*(-A*a/b**2 + B*a**2/b**3)
+ RootSum(27*_t**3*b**10 - A**3*a**4*b**3 + 3*A**2*B*a**5*b**2 - 3*A*B**2*a
**6*b + B**3*a**7, Lambda(_t, _t*log(-3*_t*b**3/(-A*a*b + B*a**2) + x)))
```

Giac [A]
time = 0.97, size = 217, normalized size = 1.19

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2 - \left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^4} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2 - \left(-ab^2\right)^{\frac{1}{3}}Aab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4} + \frac{\left(Ba^3b^4 - Aa^2b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^7} + \frac{4Bb^6x^7 - 7Bab^5x^4 + 7Ab^6x^4 + 28Ba^2b^4x - 28Aab^5x}{28b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a^2 - (-a*b^2)^(1/3)*A*a*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*B*a^2 - (-a
*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/3*(B*a^
3*b^4 - A*a^2*b^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/28*(
4*B*b^6*x^7 - 7*B*a*b^5*x^4 + 7*A*b^6*x^4 + 28*B*a^2*b^4*x - 28*A*a*b^5*x)/
b^7
```

Mupad [B]
time = 0.27, size = 164, normalized size = 0.90

$$x^4\left(\frac{A}{4b} - \frac{Ba}{4b^2}\right) + \frac{Bx^7}{7b} + \frac{a^{4/3}\ln\left(b^{1/3}x + a^{1/3}\right)\left(Ab - Ba\right)}{3b^{10/3}} - \frac{ax\left(\frac{A}{b} - \frac{Ba}{b^2}\right)}{b} - \frac{a^{4/3}\ln\left(a^{1/3} - 2b^{1/3}x + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(Ab - Ba\right)}{3b^{10/3}} + \frac{a^{4/3}\ln\left(2b^{1/3}x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(Ab - Ba\right)}{3b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^4*(A/(4*b) - (B*a)/(4*b^2)) + (B*x^7)/(7*b) + (a^(4/3)*log(b^(1/3)*x + a^
(1/3))*(A*b - B*a))/(3*b^(10/3)) - (a*x*(A/b - (B*a)/b^2))/b - (a^(4/3)*log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b -
B*a))/(3*b^(10/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3
))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(10/3))
```

3.57 $\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=54

$$\frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^6}{6b}-\frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3}$$

```
[Out] 1/3*(A*b-B*a)*x^3/b^2+1/6*B*x^6/b-1/3*a*(A*b-B*a)*ln(b*x^3+a)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$-\frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3}+\frac{x^3(Ab-aB)}{3b^2}+\frac{Bx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x^3)/(3*b^2) + (B*x^6)/(6*b) - (a*(A*b - a*B)*Log[a + b*x^3])/
(3*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{a+bx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b^2}+\frac{Bx}{b}+\frac{a(-Ab+aB)}{b^2(a+bx)}\right)dx,x,x^3\right)\\
&= \frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^6}{6b}-\frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.87

$$
\frac{bx^3(2Ab-2aB+bBx^3)+2a(-Ab+aB)\log\left(a+bx^3\right)}{6b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(2*A*b - 2*a*B + b*B*x^3) + 2*a*(-(A*b) + a*B)*Log[a + b*x^3])/(6*b^
3)
```

Maple [A]
time = 0.33, size = 50, normalized size = 0.93

method	result	size
norman	$\frac{(Ab-Ba)x^3}{3b^2}+\frac{Bx^6}{6b}-\frac{a(Ab-Ba)\ln(bx^3+a)}{3b^3}$	49
default	$\frac{\frac{1}{2}bBx^6+Abx^3-Bax^3}{3b^2}-\frac{a(Ab-Ba)\ln(bx^3+a)}{3b^3}$	50
risch	$\frac{Bx^6}{6b}+\frac{Ax^3}{3b}-\frac{Bax^3}{3b^2}+\frac{A^2}{6bB}-\frac{Aa}{3b^2}+\frac{Ba^2}{6b^3}-\frac{a\ln(bx^3+a)A}{3b^2}+\frac{a^2\ln(bx^3+a)B}{3b^3}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b^2*(1/2*b*B*x^6+A*b*x^3-B*a*x^3)-1/3*a*(A*b-B*a)*ln(b*x^3+a)/b^3
```

Maple [A]
time = 0.28, size = 50, normalized size = 0.93

$$
\frac{Bbx^6-2\,(Ba-Ab)x^3}{6\,b^2}+\frac{(Ba^2-Aab)\log\left(bx^3+a\right)}{3\,b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(B*b*x^6 - 2*(B*a - A*b)*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(b*x^3 + a)/
b^3
```

Fricas [A]
time = 1.34, size = 51, normalized size = 0.94

$$\frac{Bb^2x^6 - 2\,(Bab - Ab^2)x^3 + 2\,(Ba^2 - Aab)\log\left(bx^3 + a\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^2*x^6 - 2*(B*a*b - A*b^2)*x^3 + 2*(B*a^2 - A*a*b)*log(b*x^3 + a))/
b^3
```

Sympy [A]
time = 0.22, size = 46, normalized size = 0.85

$$\frac{Bx^6}{6b} + \frac{a(-Ab + Ba)\log\left(a + bx^3\right)}{3b^3} + x^3\left(\frac{A}{3b} - \frac{Ba}{3b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**6/(6*b) + a*(-A*b + B*a)*log(a + b*x**3)/(3*b**3) + x**3*(A/(3*b) - B*
a/(3*b**2))
```

Giac [A]
time = 1.27, size = 52, normalized size = 0.96

$$\frac{Bbx^6 - 2\,Bax^3 + 2\,Abx^3}{6\,b^2} + \frac{(Ba^2 - Aab)\log\left(|bx^3 + a|\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(B*b*x^6 - 2*B*a*x^3 + 2*A*b*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(abs(b*x
^3 + a))/b^3
```

Mupad [B]
time = 0.08, size = 52, normalized size = 0.96

$$x^3\left(\frac{A}{3\,b} - \frac{B\,a}{3\,b^2}\right) + \frac{\ln\left(b\,x^3 + a\right)\,\left(B\,a^2 - A\,a\,b\right)}{3\,b^3} + \frac{B\,x^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^3*(A/(3*b) - (B*a)/(3*b^2)) + (log(a + b*x^3)*(B*a^2 - A*a*b))/(3*b^3) +
(B*x^6)/(6*b)
```

3.58 $\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=167

$$\frac{(Ab-aB)x^2}{2b^2}+\frac{Bx^5}{5b}+\frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}}+\frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{8/3}}-\frac{a^{2/3}(Ab-a}{}$$

```
[Out] 1/2*(A*b-B*a)*x^2/b^2+1/5*B*x^5/b+1/3*a^(2/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*
x)/b^(8/3)-1/6*a^(2/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
b^(8/3)+1/3*a^(2/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1
/2))/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {470, 327, 298, 31, 648, 631, 210, 642}

$$\frac{a^{2/3}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}}-\frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{8/3}}+\frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{8/3}}+\frac{x^2(Ab-aB)}{2b^2}+\frac{Bx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^5)/(5*b) + (a^(2/3)*(A*b - a*B)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (a^(2/3)*(A*b
- a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) - (a^(2/3)*(A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^5}{5b} - \frac{(-5Ab+5aB)\int \frac{x^4}{a+bx^3}\,dx}{5b}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} - \frac{(a(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{b^2}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{7/3}} - \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{\sqrt[3]{}}{a^{2/3}-\sqrt[3]{}}}{3b^{7/3}}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{8/3}} - \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{}}{a^{2/3}-}}{6b^{8/3}}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{8/3}} - \frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}\right.}{6b^{8/3}}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}\right.}{3b^{8/3}}$$

Mathematica [A]
time = 0.05, size = 154, normalized size = 0.92

$$\frac{15b^{2/3}(Ab-aB)x^2+6b^{5/3}Bx^5-10\sqrt{3}\,a^{2/3}(-Ab+aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-10a^{2/3}(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+5a^{2/3}(-Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{30b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (15*b^(2/3)*(A*b - a*B)*x^2 + 6*b^(5/3)*B*x^5 - 10*Sqrt[3]*a^(2/3)*(-(A*b)
+ a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-(A*b) + a
*B)*Log[a^(1/3) + b^(1/3)*x] + 5*a^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(30*b^(8/3))
```

Maple [A]
time = 0.28, size = 131, normalized size = 0.78

method	result	size
risch	$\frac{B\,x^5}{5b}+\frac{A\,x^2}{2b}-\frac{Ba\,x^2}{2b^2}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-Ab+Ba)\ln\left(x-\_R\right)}{\_R}\right)}{3b^3}$	65

default	$\frac{\frac{bB\,x^5}{5}+\frac{(Ab-Ba)x^2}{2}}{b^2}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a(Ab-Ba)}{b^2}$	131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/5*b*B*x^5+1/2*(A*b-B*a)*x^2)-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a*(A*b-B*a)/b^2
```

Maxima [A]
time = 0.55, size = 157, normalized size = 0.94

$$\frac{\sqrt{3}\,(Ba^2-Aab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\,Bbx^5-5\,(Ba-Ab)x^2}{10\,b^2}+\frac{(Ba^2-Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba^2-Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(b^3*(a/b)^(1/3)) + 1/10*(2*B*b*x^5 - 5*(B*a - A*b)*x^2)/b^2 + 1/6*(B*
a^2 - A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) - 1/3
*(B*a^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Fricas [A]
time = 2.18, size = 162, normalized size = 0.97

$$\frac{6\,Bbx^5-15\,(Ba-Ab)x^2+10\,\sqrt{3}\,(Ba-Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)+5\,(Ba-Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-10\,(Ba-Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{30\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*B*b*x^5 - 15*(B*a - A*b)*x^2 + 10*sqrt(3)*(B*a - A*b)*(a^2/b^2)^(1/
3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(B*a - A*b
)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 10
*(B*a - A*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b^2
```

Sympy [A]
time = 0.23, size = 114, normalized size = 0.68

$$\frac{Bx^5}{5b}+x^2\left(\frac{A}{2b}-\frac{Ba}{2b^2}\right)+\operatorname{RootSum}\left(27t^3b^8-A^3a^2b^3+3A^2Ba^3b^2-3AB^2a^4b+B^3a^5,\left(t\mapsto t\log\left(\frac{9t^2b^5}{A^2ab^2-2ABa^2b+B^2a^3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**5/(5*b) + x**2*(A/(2*b) - B*a/(2*b**2)) + RootSum(27*_t**3*b**8 - A**3
*a**2*b**3 + 3*A**2*B*a**3*b**2 - 3*A*B**2*a**4*b + B**3*a**5, Lambda(_t, _
t*log(9*_t**2*b**5/(A**2*a*b**2 - 2*A*B*a**2*b + B**2*a**3) + x)))
```

Giac [A]
time = 1.36, size = 207, normalized size = 1.24

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^4}+\frac{\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4}-\frac{\left(Ba^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Aab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^5}+\frac{2Bb^4x^5-5Bab^3x^2+5Ab^4x^2}{10b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/6*((-a*b^2)^(2/3)*B*a - (-a*b^2)^
(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2*b^3*(-
a/b)^(1/3) - A*a*b^4*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^5) + 1/10*(2*B*b^4*x^5 - 5*B*a*b^3*x^2 + 5*A*b^4*x^2)/b^5
```

Mupad [B]
time = 2.55, size = 144, normalized size = 0.86

$$x^2\left(\frac{A}{2b}-\frac{Ba}{2b^2}\right)+\frac{Bx^5}{5b}+\frac{a^{2/3}\ln\left(b^{1/3}x+a^{1/3}\right)(Ab-Ba)}{3b^{8/3}}+\frac{a^{2/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(Ab-Ba)}{3b^{8/3}}-\frac{a^{2/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(Ab-Ba)}{3b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^2*(A/(2*b) - (B*a)/(2*b^2)) + (B*x^5)/(5*b) + (a^(2/3)*log(b^(1/3)*x + a^
(1/3))*(A*b - B*a))/(3*b^(8/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/
3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(8/3)) - (a^(2/3)*
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b
 - B*a))/(3*b^(8/3))
```

3.59 $\int \frac{x^3(A+Bx^3)}{a+bx^3} dx$

Optimal. Leaf size=162

$$\frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}+\frac{\sqrt[3]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{\sqrt[3]{a}\,(Ab-}{}$$

```
[Out] (A*b-B*a)*x/b^2+1/4*B*x^4/b-1/3*a^(1/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(
7/3)+1/6*a^(1/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(7/3
)+1/3*a^(1/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b
^(7/3)*3^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {470, 327, 206, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{\sqrt[3]{a}\,(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{x(Ab-aB)}{b^2}+\frac{Bx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^4)/(4*b) + (a^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (a^(1/3)*(A*b - a*B)
*Log[a^(1/3) + b^(1/3)*x])/(3*b^(7/3)) + (a^(1/3)*(A*b - a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^4}{4b} - \frac{(-4Ab+4aB)\int \frac{x^3}{a+bx^3}\,dx}{4b}$$

$$= \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{(a(Ab-aB))\int \frac{1}{a+bx^3}\,dx}{b^2}$$

$$= \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{\left(\sqrt[3]{a}\,(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^2} - \frac{\left(\sqrt[3]{a}\,(Ab-aB)\right)\int \frac{2}{a^{2/3}-}}{3b^2}$$

$$= \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}} + \frac{\left(\sqrt[3]{a}\,(Ab-aB)\right)\int \frac{-}{a^{2/3}-}}{6b^{7/3}}$$

$$= \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}} + \frac{\sqrt[3]{a}\,(Ab-aB)\log\left(a^{2/3}\right.}{6b^{7}}$$

$$= \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} + \frac{\sqrt[3]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}} - \frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\right.}{3b^{7/3}}$$

Mathematica [A]
time = 0.05, size = 152, normalized size = 0.94

$$\frac{12\sqrt[3]{b}\,(Ab-aB)x+3b^{4/3}Bx^4-4\sqrt{3}\,\sqrt[3]{a}\,(-Ab+aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{a}\,(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt[3]{a}\,(-Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{12b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (12*b^(1/3)*(A*b - a*B)*x + 3*b^(4/3)*B*x^4 - 4*Sqrt[3]*a^(1/3)*(-(A*b) + a
*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-(A*b) + a*B)*
Log[a^(1/3) + b^(1/3)*x] - 2*a^(1/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(12*b^(7/3))
```

Maple [A]
time = 0.29, size = 127, normalized size = 0.78

method	result	size
risch	$\frac{B\,x^4}{4b}+\frac{Ax}{b}-\frac{Bax}{b^2}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-Ab+Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{3b^3}$	60

default	$\frac{\frac{1}{4}bB\,x^4+Abx-Bax}{b^2}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a(Ab-Ba)}{b^2}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/4*b*B*x^4+A*b*x-B*a*x)-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a*(A*b-B*a)/b^2
```

Maxima [A]
time = 0.50, size = 154, normalized size = 0.95

$$\frac{Bbx^4-4\,(Ba-Ab)x}{4\,b^2}+\frac{\sqrt{3}\,(Ba^2-Aab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba^2-Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba^2-Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b*x^4 - 4*(B*a - A*b)*x)/b^2 + 1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 1/6*(B*a^2 -
 A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(B*a
^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Fricas [A]
time = 2.10, size = 145, normalized size = 0.90

$$\frac{3\,Bbx^4-4\,\sqrt{3}\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+2\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)-4\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)-12\,(Ba-Ab)x}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*B*b*x^4 - 4*sqrt(3)*(B*a - A*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*
b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 2*(B*a - A*b)*(-a/b)^(1/3)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3)) - 4*(B*a - A*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/
3)) - 12*(B*a - A*b)*x)/b^2
```

Sympy [A]
time = 0.26, size = 87, normalized size = 0.54

$$\frac{Bx^4}{4b} + x\left(\frac{A}{b} - \frac{Ba}{b^2}\right) + \operatorname{RootSum}\left(27t^3b^7 + A^3ab^3 - 3A^2Ba^2b^2 + 3AB^2a^3b - B^3a^4, \left(t \mapsto t\log\left(\frac{3tb^2}{-Ab+Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**4/(4*b) + x*(A/b - B*a/b**2) + RootSum(27*_t**3*b**7 + A**3*a*b**3 - 3
*A**2*B*a**2*b**2 + 3*A*B**2*a**3*b - B**3*a**4, Lambda(_t, _t*log(3*_t*b**
2/(-A*b + B*a) + x)))
```

Giac [A]
time = 0.81, size = 186, normalized size = 1.15

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3} - \frac{\left(Ba^2b^2 - Aab^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^4} + \frac{Bb^3x^4 - 4Bab^2x + 4Ab^3x}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(
1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 - 1/3*(B*a^2*b^2 - A
*a*b^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 1/4*(B*b^3*x^4 -
4*B*a*b^2*x + 4*A*b^3*x)/b^4
```

Mupad [B]
time = 2.61, size = 162, normalized size = 1.00

$$x\left(\frac{A}{b} - \frac{Ba}{b^2}\right) + \frac{Bx^4}{4b} + \frac{(-a)^{1/3}\ln\left((-a)^{4/3} + ab^{1/3}x\right)(Ab - Ba)}{3b^{7/3}} - \frac{(-a)^{1/3}\ln\left(2ab^{1/3}x - (-a)^{4/3} - \sqrt{3}(-a)^{4/3}\,1\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(Ab - Ba)}{3b^{7/3}} + \frac{(-a)^{1/3}\ln\left(2ab^{1/3}x - (-a)^{4/3} + \sqrt{3}(-a)^{4/3}\,1\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(Ab - Ba)}{3b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x*(A/b - (B*a)/b^2) + (B*x^4)/(4*b) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3
)*x)*(A*b - B*a))/(3*b^(7/3)) - ((-a)^(1/3)*log(2*a*b^(1/3)*x - 3^(1/2)*(-a
)^(4/3)*1i - (-a)^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*b^(7/3)) +
((-a)^(1/3)*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/
2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(7/3))
```

3.60 $\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=35

$$\frac{Bx^3}{3b}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}$$

```
[Out] 1/3*B*x^3/b+1/3*(A*b-B*a)*ln(b*x^3+a)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}+\frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^3)/(3*b) + ((A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^3\right)}{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{a+bx}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{B}{b}+\frac{Ab-aB}{b(a+bx)}\right)\,dx,x,x^3\right)\\ &= \frac{Bx^3}{3b}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.89

$$\frac{bBx^3 + (Ab - aB)\log(a + bx^3)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3),x]
[Out] (b*B*x^3 + (A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

Maple [A]
time = 0.27, size = 32, normalized size = 0.91

method	result	size
default	$\frac{B\,x^{3}}{3b}+\frac{(Ab-Ba)\ln(b\,x^{3}+a)}{3b^{2}}$	32
norman	$\frac{B\,x^{3}}{3b}+\frac{(Ab-Ba)\ln(b\,x^{3}+a)}{3b^{2}}$	32
risch	$\frac{B\,x^{3}}{3b}+\frac{\ln(b\,x^{3}+a)A}{3b}-\frac{\ln(b\,x^{3}+a)Ba}{3b^{2}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*B*x^3/b+1/3*(A*b-B*a)*ln(b*x^3+a)/b^2
```

Maxima [A]
time = 0.28, size = 31, normalized size = 0.89

$$\frac{Bx^3}{3\,b} - \frac{(Ba - Ab)\log(bx^3 + a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*B*x^3/b - 1/3*(B*a - A*b)*log(b*x^3 + a)/b^2
```

Fricas [A]
time = 2.01, size = 30, normalized size = 0.86

$$\frac{Bbx^3 - (Ba - Ab)\log(bx^3 + a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
[Out] 1/3*(B*b*x^3 - (B*a - A*b)*log(b*x^3 + a))/b^2
```

Sympy [A]
time = 0.19, size = 27, normalized size = 0.77

$$\frac{Bx^3}{3b} - \frac{(-Ab + Ba)\log(a + bx^3)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a),x)
[Out] B*x**3/(3*b) - (-A*b + B*a)*log(a + b*x**3)/(3*b**2)
```

Giac [A]
time = 0.77, size = 32, normalized size = 0.91

$$\frac{Bx^3}{3\,b} - \frac{(Ba - Ab)\log(|bx^3 + a|)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
[Out] 1/3*B*x^3/b - 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/b^2
```

Mupad [B]
time = 0.06, size = 31, normalized size = 0.89

$$\frac{B\,x^3}{3\,b} + \frac{\ln(b\,x^3 + a)\,(A\,b - B\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3),x)
[Out] (B*x^3)/(3*b) + (log(a + b*x^3)*(A*b - B*a))/(3*b^2)
```

3.61 $\int \frac{x(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=150

$$\frac{Bx^2}{2b}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{5/3}}+\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+\right.}{6\sqrt[3]{a}\,b^{5/3}}$$

```
[Out] 1/2*B*x^2/b-1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(5/3)+1/6*(A*b-B*
a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(5/3)-1/3*(A*b-B*a)*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {470, 298, 31, 648, 631, 210, 642}

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{5/3}}-\frac{(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{5/3}}+\frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^2)/(2*b) - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*b^(5/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(6*a^(1/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2b} - \frac{(-2Ab+2aB)\int \frac{x}{a+bx^3}\,dx}{2b}$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,b^{4/3}} + \frac{(Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}\,b^{4/3}}$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-}{}$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{5/3}} +$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB}{}$$

Mathematica [A]
time = 0.03, size = 152, normalized size = 1.01

$$\frac{Bx^2}{2b} - \frac{(-Ab+aB)\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}} + \frac{(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{5/3}} - \frac{(-Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^2)/(2*b) - ((-(A*b) + a*B)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) + ((-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(1/3)*b^(5/3)) - ((-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(1/3)*b^(5/3))
```

Maple [A]
time = 0.27, size = 113, normalized size = 0.75

method	result	size
risch	$\frac{Bx^2}{2b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R}}{3b^2}$	45

default	$\frac{Bx^2}{2b}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(Ab-Ba)}{b}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/b+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1)))*(A*b-B*a)/b
```

<u>Maxima</u> [A]
time = 0.51, size = 131, normalized size = 0.87

$$\frac{Bx^2}{2\,b}-\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(b^
2*(a/b)^(1/3))
```

<u>Fricas</u> [A]
time = 1.58, size = 382, normalized size = 2.55

$$\left[\frac{3\,Bab^2x^2-3\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\,(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)-(-ab^2)^{\frac{2}{3}}(Ba-Ab)\log\left(b^2x^2+(-ab^2)^{\frac{1}{3}}bx+(-ab^2)^{\frac{2}{3}}\right)+2\,(-ab^2)^{\frac{2}{3}}(Ba-Ab)\log\left(bx-(-ab^2)^{\frac{1}{3}}\right)}{6\,ab^3},\ \frac{3\,Bab^2x^2-6\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right)-(-ab^2)^{\frac{2}{3}}(Ba-Ab)\log\left(b^2x^2+(-ab^2)^{\frac{1}{3}}bx+(-ab^2)^{\frac{2}{3}}\right)+2\,(-ab^2)^{\frac{2}{3}}(Ba-Ab)\log\left(bx-(-ab^2)^{\frac{1}{3}}\right)}{6\,ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*a*b^2*x^2 - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^
2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - (-a
*b^2)^(2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
```

```
+ 2*(-a*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3), 1/6*(3*B
*a*b^2*x^2 - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(-a*b^2)^(1/3)/a)*arctan
(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(
2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*(-a
*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3)]
```

Sympy [A]
time = 0.20, size = 92, normalized size = 0.61

$$\frac{Bx^2}{2b} + \operatorname{RootSum}\left(27t^3ab^5 + A^3b^3 - 3A^2Bab^2 + 3AB^2a^2b - B^3a^3, \left(t \mapsto t\log\left(\frac{9t^2ab^3}{A^2b^2 - 2ABab + B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**2/(2*b) + RootSum(27*_t**3*a*b**5 + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A*
B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(9*_t**2*a*b**3/(A**2*b**2 - 2*A*
B*a*b + B**2*a**2) + x)))
```

Giac [A]
time = 1.31, size = 161, normalized size = 1.07

$$\frac{Bx^2}{2\,b} - \frac{\sqrt{3}\,(Ba - Ab)\arctan\left(\frac{\sqrt{3}\,\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{1}{3}}\,b} + \frac{(Ba - Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{1}{3}}\,b} + \frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b) + 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-
a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
```

Mupad [B]
time = 2.57, size = 126, normalized size = 0.84

$$\frac{B\,x^2}{2\,b} - \frac{\ln\left(b^{1/3}\,x + a^{1/3}\right)\,\left(A\,b - B\,a\right)}{3\,a^{1/3}\,b^{5/3}} - \frac{\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b - B\,a\right)}{3\,a^{1/3}\,b^{5/3}} + \frac{\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b - B\,a\right)}{3\,a^{1/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] (B*x^2)/(2*b) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
- (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(
A*b - B*a))/(3*a^(1/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a
^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
```

3.62 $\int \frac{A+Bx^3}{a+bx^3} \, dx$

Optimal. Leaf size=145

$$\frac{Bx}{b}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b\right.}{6a^{2/3}b^{4/3}}$$

```
[Out] B*x/b+1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-1/6*(A*b-B*a)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/3*(A*b-B*a)*arctan
(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {396, 206, 31, 648, 631, 210, 642}

$$-\frac{(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3),x]
```

```
[Out] (B*x)/b - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
)*b^(4/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{Bx}{b} - \frac{(-Ab+aB)\int \frac{1}{a+bx^3}\,dx}{b}$$

$$= \frac{Bx}{b} + \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}b} + \frac{(Ab-aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b}$$

$$= \frac{Bx}{b} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}} + \frac{(Ab-aB}{}$$

$$= \frac{Bx}{b} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{(A}{}$$

$$= \frac{Bx}{b} - \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\log}{}$$

Mathematica [A]
time = 0.04, size = 129, normalized size = 0.89

$$\frac{6a^{2/3}\sqrt[3]{b}\,Bx - 2\sqrt{3}\,(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) - (Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3),x]
```

```
[Out] (6*a^(2/3)*b^(1/3)*B*x - 2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] + 2*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

Maple [A]
time = 0.28, size = 110, normalized size = 0.76

method	result	size
risch	$\frac{Bx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R^2}}{3b^2}$	42
default	$\frac{Bx}{b}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(Ab-Ba)}{b}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1)))*(A*b-B*a)/b
```

Maxima [A]
time = 0.48, size = 128, normalized size = 0.88

$$\frac{Bx}{b}-\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] B*x/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(b^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(b^2*(a/b
)^(2/3))
```

Fricas [A]
time = 2.12, size = 369, normalized size = 2.54

$$\left[\frac{6\,Ba^{2}bx - 3\,\sqrt{\frac{1}{3}}\,(Ba^{2}b - Aab^{2})\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^{3} - 3\,(a^{2}b)^{\frac{1}{3}}ax - a^{2} + 3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2} + (a^{2}b)^{\frac{2}{3}}x - (a^{2}b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right) + (a^{2}b)^{\frac{2}{3}}\,(Ba - Ab)\log\left(abx^{2} - (a^{2}b)^{\frac{2}{3}}x + (a^{2}b)^{\frac{1}{3}}a\right) - 2\,(a^{2}b)^{\frac{2}{3}}\,(Ba - Ab)\log\left(abx + (a^{2}b)^{\frac{2}{3}}\right)}{6\,a^{2}b^{2}}, \frac{6\,Ba^{2}bx - 6\,\sqrt{\frac{1}{3}}\,(Ba^{2}b - Aab^{2})\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\,\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^{2}b)^{\frac{2}{3}}x - (a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right) + (a^{2}b)^{\frac{2}{3}}\,(Ba - Ab)\log\left(abx^{2} - (a^{2}b)^{\frac{2}{3}}x + (a^{2}b)^{\frac{1}{3}}a\right) - 2\,(a^{2}b)^{\frac{2}{3}}\,(Ba - Ab)\log\left(abx + (a^{2}b)^{\frac{2}{3}}\right)}{6\,a^{2}b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*B*a^2*b*x - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(a^2*b)^(1/3)/b)*
log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + (a^2*b
)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(a
^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2), 1/6*(6*B*a^2
*b*x - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/
3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*
b)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(
a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2)]
```

Sympy [A]
time = 0.22, size = 71, normalized size = 0.49

$$\frac{Bx}{b} + \operatorname{RootSum}\left(27t^{3}a^{2}b^{4} - A^{3}b^{3} + 3A^{2}Bab^{2} - 3AB^{2}a^{2}b + B^{3}a^{3}, \left(t \mapsto t\log\left(-\frac{3tab}{-Ab+Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x/b + RootSum(27*_t**3*a**2*b**4 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B**2
*a**2*b + B**3*a**3, Lambda(_t, _t*log(-3*_t*a*b/(-A*b + B*a) + x)))
```

Giac [A]
time = 0.90, size = 133, normalized size = 0.92

$$\frac{\sqrt{3}\,(Ba - Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^{2}\right)^{\frac{2}{3}}} + \frac{(Ba - Ab)\log\left(x^{2} + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^{2}\right)^{\frac{2}{3}}} + \frac{Bx}{b} + \frac{(Ba - Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(-a*b^2)^(2/3) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/(-a*b^2)^(2/3) + B*x/b + 1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b)
```

Mupad [B]
time = 2.54, size = 123, normalized size = 0.85

$$\frac{B\,x}{b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3),x)
```

```
[Out] (B*x)/b + (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(2/3)*b^(4/3)) - (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b -
B*a))/(3*a^(2/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(2/3)*b^(4/3))
```

3.63 $\int \frac{A+Bx^3}{x(a+bx^3)}\,dx$

Optimal. Leaf size=34

$$\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^3\right)}{3ab}$$

```
[Out] A*ln(x)/a-1/3*(A*b-B*a)*ln(b*x^3+a)/a/b
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^3\right)}{3ab}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)),x]
```

```
[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^3])/(3*a*b)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x\left(a+bx^3\right)}\,dx&=\frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)}\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{ax}+\frac{-Ab+aB}{a(a+bx)}\right)\,dx,x,x^3\right)\\&=\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^3\right)}{3ab}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{A\log(x)}{a}+\frac{(-Ab+aB)\log\left(a+bx^3\right)}{3ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)),x]
[Out] (A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^3])/(3*a*b)
```

Maple [A]
time = 0.27, size = 33, normalized size = 0.97

method	result	size
default	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln(b\,x^3+a)}{3ab}$	33
norman	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln(b\,x^3+a)}{3ab}$	33
risch	$\frac{A\ln(x)}{a}-\frac{\ln(b\,x^3+a)A}{3a}+\frac{\ln(b\,x^3+a)B}{3b}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] A*ln(x)/a-1/3*(A*b-B*a)*ln(b*x^3+a)/a/b
```

Maxima [A]
time = 0.31, size = 35, normalized size = 1.03

$$\frac{A\log\left(x^3\right)}{3\,a}+\frac{(Ba-Ab)\log\left(bx^3+a\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*A*log(x^3)/a + 1/3*(B*a - A*b)*log(b*x^3 + a)/(a*b)
```

Fricas [A]
time = 1.89, size = 32, normalized size = 0.94

$$\frac{3\,Ab\log\left(x\right)+(Ba-Ab)\log\left(bx^3+a\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="fricas")
[Out] 1/3*(3*A*b*log(x) + (B*a - A*b)*log(b*x^3 + a))/(a*b)
```

Sympy [A]
time = 0.54, size = 26, normalized size = 0.76

$$\frac{A\log(x)}{a} + \frac{(-Ab + Ba)\log\left(\frac{a}{b} + x^3\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a),x)
```

```
[Out] A*log(x)/a + (-A*b + B*a)*log(a/b + x**3)/(3*a*b)
```

Giac [A]
time = 1.20, size = 34, normalized size = 1.00

$$\frac{A\log(|x|)}{a} + \frac{(Ba - Ab)\log(|bx^3 + a|)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] A*log(abs(x))/a + 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/(a*b)
```

Mupad [B]
time = 0.10, size = 36, normalized size = 1.06

$$\frac{B\,\ln\left(b\,x^3 + a\right)}{3\,b} - \frac{A\,\ln\left(b\,x^3 + a\right)}{3\,a} + \frac{A\,\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)),x)
```

```
[Out] (B*log(a + b*x^3))/(3*b) - (A*log(a + b*x^3))/(3*a) + (A*log(x))/a
```

3.64 $\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=147

$$-\frac{A}{ax}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}b^{2/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+\right.}{6a^{4/3}b^{2/3}}$$

```
[Out] -A/a/x+1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(2/3)-1/6*(A*b-B*a)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(2/3)+1/3*(A*b-B*a)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {464, 298, 31, 648, 631, 210, 642}

$$\frac{(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}b^{2/3}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(A/(a*x)) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(4/3)*b^(2/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(
4/3)*b^(2/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx = -\frac{A}{ax} - \frac{(Ab-aB)\int \frac{x}{a+bx^3}\,dx}{a}$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{4/3}\sqrt[3]{b}} - \frac{(Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{4/3}\sqrt[3]{b}}$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{4/3}b^{2/3}} - \frac{(Ab-}{}$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}} -$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)}{}$$

Mathematica [A]
time = 0.05, size = 134, normalized size = 0.91

$$\frac{-6\sqrt[3]{a}\,Ab^{2/3}+2\sqrt{3}\,(Ab-aB)x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(Ab-aB)x\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-(Ab-aB)x\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-6*a^(1/3)*A*b^(2/3) + 2*Sqrt[3]*(A*b - a*B)*x*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*(A*b - a*B)*x*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*x
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(2/3)*x)
```

Maple [A]
time = 0.28, size = 114, normalized size = 0.78

method	result

default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(Ab-Ba)}{a}-\frac{A}{ax}$
risch	$-\frac{A}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}b^{2}a^{4}-A^{3}b^{3}+3A^{2}Ba\,b^{2}-3A\,B^{2}a^{2}b+B^{3}a^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{4}b^{2}+3A^{3}b^{3}-9A^{2}Ba\,b^{2}+9A\,B^{2}a^{2}b-3B^{3}a^{3}\right.\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))*(A*b-B*a)/a-A/a/x
```

Maxima [A]
time = 0.52, size = 140, normalized size = 0.95

$$\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a*b*(a/b)^(1/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b*(a/b)^(1/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(1/3))
 - A/(a*x)
```

Fricas [A]
time = 1.37, size = 372, normalized size = 2.53

$$\left[-\frac{6\,Aab^{2}+3\sqrt{\frac{1}{3}}\,(Ba^{2}b-Aab^{2})x\sqrt{-\frac{(ab^{2})^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^{2}x^{3}-ab-3\sqrt{\frac{1}{3}}\left(abx+2\,(ab^{2})^{\frac{2}{3}}x^{2}-(ab^{2})^{\frac{1}{3}}a\right)\sqrt{-\frac{(ab^{2})^{\frac{1}{3}}}{a}}-3\,(ab^{2})^{\frac{2}{3}}x}{bx^{3}+a}\right)-(ab^{2})^{\frac{2}{3}}(Ba-Ab)x\log\left(b^{2}x^{2}-(ab^{2})^{\frac{1}{3}}bx+(ab^{2})^{\frac{2}{3}}\right)+2\,(ab^{2})^{\frac{2}{3}}(Ba-Ab)x\log\left(bx+(ab^{2})^{\frac{1}{3}}\right)}{6\,a^{2}b^{2}x},\ -\frac{6\,Aab^{2}+6\sqrt{\frac{1}{3}}\,(Ba^{2}b-Aab^{2})x\sqrt{\frac{(ab^{2})^{\frac{1}{3}}}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,bx-(ab^{2})^{\frac{1}{3}}\right)\sqrt{\frac{(ab^{2})^{\frac{1}{3}}}{a}}}{b}\right)-(ab^{2})^{\frac{2}{3}}(Ba-Ab)x\log\left(b^{2}x^{2}-(ab^{2})^{\frac{1}{3}}bx+(ab^{2})^{\frac{2}{3}}\right)+2\,(ab^{2})^{\frac{2}{3}}(Ba-Ab)x\log\left(bx+(ab^{2})^{\frac{1}{3}}\right)}{6\,a^{2}b^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*A*a*b^2 + 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt(-(a*b^2)^(1/3)/a)
*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^
(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - (a*b^2)
^(2/3)*(B*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(
a*b^2)^(2/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x), -1/6*(6*A
*a*b^2 + 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt((a*b^2)^(1/3)/a)*arctan(-sq
rt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - (a*b^2)^(2/3)*(B
*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(a*b^2)^(2
/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x)]
```

Sympy [A]
time = 0.22, size = 90, normalized size = 0.61

$$-\frac{A}{ax} + \operatorname{RootSum}\left(27t^3a^4b^2 - A^3b^3 + 3A^2Bab^2 - 3AB^2a^2b + B^3a^3, \left(t \mapsto t\log\left(\frac{9t^2a^3b}{A^2b^2 - 2ABab + B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a),x)
```

```
[Out] -A/(a*x) + RootSum(27*_t**3*a**4*b**2 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B
**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(9*_t**2*a**3*b/(A**2*b**2 - 2*A*B
*a*b + B**2*a**2) + x)))
```

Giac [A]
time = 1.70, size = 155, normalized size = 1.05

$$\frac{\sqrt{3}\,(Ba - Ab)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}a} - \frac{(Ba - Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}a} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^2} - \frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/((-a*b^2)^(1/3)*a) - 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(1/3)*a) - 1/3*(B*a*(-a/b)^(1/3) - A*b*(-a/b)^(1/3))*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - A/(a*x)
```

Mupad [B]
time = 2.54, size = 126, normalized size = 0.86

$$\frac{\ln\left(b^{1/3}\,x + a^{1/3}\right)\,\left(A\,b - B\,a\right)}{3\,a^{4/3}\,b^{2/3}} - \frac{A}{a\,x} + \frac{\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b - B\,a\right)}{3\,a^{4/3}\,b^{2/3}} - \frac{\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b - B\,a\right)}{3\,a^{4/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(4/3)*b^(2/3)) - A/(a*x) + (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b -
B*a))/(3*a^(4/3)*b^(2/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(4/3)*b^(2/3))
```

3.65 $\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=149

$$-\frac{A}{2ax^2}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{6a^{5/3}\sqrt[3]{b}}$$

```
[Out] -1/2*A/a/x^2-1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(1/3)+1/6*(A*b-B
*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(1/3)+1/3*(A*b-B*a)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {464, 206, 31, 648, 631, 210, 642}

$$\frac{(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*A/(a*x^2) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(Sqrt[3]*a^(5/3)*b^(1/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3
*a^(5/3)*b^(1/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(6*a^(5/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx = -\frac{A}{2ax^2} - \frac{(2Ab-2aB)\int \frac{1}{a+bx^3}\,dx}{2a}$$

$$= -\frac{A}{2ax^2} - \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{5/3}} - \frac{(Ab-aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{5/3}}$$

$$= -\frac{A}{2ax^2} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt[3]{b}} - \frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2a^{4/3}} + \frac{(A}{}$$

$$= -\frac{A}{2ax^2} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt[3]{b}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{A}{2ax^2} + \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt[3]{b}} + \frac{(Ab-}{}$$

Mathematica [A]
time = 0.06, size = 135, normalized size = 0.91

$$\frac{-\frac{3a^{2/3}A}{x^2} + \frac{2\sqrt{3}(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{2(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{6a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(2/3)*A)/x^2 + (2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3) + (2*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3
) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(
6*a^(5/3))
```

Maple [A]
time = 0.30, size = 113, normalized size = 0.76

method	result

default	$\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(-Ab+Ba)}{a}-\frac{A}{2a\,x^{2}}$
risch	$-\frac{A}{2a\,x^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}b\,a^{5}+A^{3}b^{3}-3A^{2}Ba\,b^{2}+3A\,B^{2}a^{2}b-B^{3}a^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{5}b-3A^{3}b^{3}+9A^{2}Ba\,b^{2}-9A\,B^{2}a^{2}b+3B^{3}a^{3}\right)\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1)))*(-A*b+B*a)/a-1/2*A/a/x^2
```

Maxima [A]
time = 0.53, size = 140, normalized size = 0.94

$$\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba-Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{A}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a*b*(a/b)^(2/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b*(a/b)^(2/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
 - 1/2*A/(a*x^2)
```

Fricas [A]
time = 2.12, size = 411, normalized size = 2.76

$$\left[-\frac{3\sqrt{\frac{1}{3}}\,(Ba^{2}b-Aab^{2})x^{2}\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}+3\,(-a^{2}b)^{\frac{1}{3}}ax-a^{2}-3\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(-a^{2}b)^{\frac{2}{3}}x+(-a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)+(-a^{2}b)^{\frac{2}{3}}(Ba-Ab)x^{2}\log\left(abx^{2}-(-a^{2}b)^{\frac{2}{3}}x-(-a^{2}b)^{\frac{1}{3}}a\right)-2\,(-a^{2}b)^{\frac{2}{3}}(Ba-Ab)x^{2}\log\left(abx+(-a^{2}b)^{\frac{2}{3}}\right)+3\,Aa^{2}b}{6\,a^{3}bx^{2}},\ \frac{6\sqrt{\frac{1}{3}}\,(Ba^{2}b-Aab^{2})x^{2}\sqrt{-\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(-a^{2}b)^{\frac{2}{3}}x+(-a^{2}b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-(-a^{2}b)^{\frac{2}{3}}(Ba-Ab)x^{2}\log\left(abx^{2}-(-a^{2}b)^{\frac{2}{3}}x-(-a^{2}b)^{\frac{1}{3}}a\right)+2\,(-a^{2}b)^{\frac{2}{3}}(Ba-Ab)x^{2}\log\left(abx+(-a^{2}b)^{\frac{2}{3}}\right)-3\,Aa^{2}b}{6\,a^{3}bx^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt((-a^2*b)^(1/3)/b)*log((2*a*
b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3
)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) + (-a^2*b)^(2/
3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*(
-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^(2/3)) + 3*A*a^2*b)/(a^3
*b*x^2), 1/6*(6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt(-(-a^2*b)^(1/3)/b)*a
rctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3
)/b)/a^2) - (-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x -
 (-a^2*b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^
(2/3)) - 3*A*a^2*b)/(a^3*b*x^2)]
```

Sympy [A]
time = 0.26, size = 73, normalized size = 0.49

$$-\frac{A}{2ax^2} + \operatorname{RootSum}\left(27t^3a^5b + A^3b^3 - 3A^2Bab^2 + 3AB^2a^2b - B^3a^3, \left(t \mapsto t\log\left(\frac{3ta^2}{-Ab+Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a),x)
```

```
[Out] -A/(2*a*x**2) + RootSum(27*_t**3*a**5*b + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A
*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(3*_t*a**2/(-A*b + B*a) + x)))
```

Giac [A]
time = 1.35, size = 161, normalized size = 1.08

$$-\frac{(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b} - \frac{A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*sqrt(3)*
((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))/(a^2*b) + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 1/2*A/(a*x^2)
```

Mupad [B]
time = 0.24, size = 126, normalized size = 0.85

$$-\frac{A}{2\,a\,x^2} - \frac{\ln\left(b^{1/3}\,x + a^{1/3}\right)\,\left(A\,b - B\,a\right)}{3\,a^{5/3}\,b^{1/3}} + \frac{\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b - B\,a\right)}{3\,a^{5/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b - B\,a\right)}{3\,a^{5/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b - B*a))/(3*a^(5/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a
^(5/3)*b^(1/3)) - A/(2*a*x^2) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(
1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(5/3)*b^(1/3))
```

3.66 $\int \frac{A+Bx^3}{x^4(a+bx^3)}\,dx$

Optimal. Leaf size=50

$$-\frac{A}{3ax^3}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log(a+bx^3)}{3a^2}$$

```
[Out] -1/3*A/a/x^3-(A*b-B*a)*ln(x)/a^2+1/3*(A*b-B*a)*ln(b*x^3+a)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\frac{(Ab-aB)\log(a+bx^3)}{3a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*A/(a*x^3) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])/(3
*a^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^4(a+bx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}-\frac{b(-Ab+aB)}{a^2(a+bx)}\right)dx,x,x^3\right)\\
&= -\frac{A}{3ax^3}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log(a+bx^3)}{3a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.98

$$
-\frac{A}{3ax^3}+\frac{(-Ab+aB)\log(x)}{a^2}+\frac{(Ab-aB)\log(a+bx^3)}{3a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*A/(a*x^3) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])
/(3*a^2)
```

Maple [A]
time = 0.29, size = 46, normalized size = 0.92

method	result	size
default	$\frac{(Ab-Ba)\ln(b\,x^3+a)}{3a^2}-\frac{A}{3a\,x^3}+\frac{(-Ab+Ba)\ln(x)}{a^2}$	46
norman	$-\frac{A}{3a\,x^3}-\frac{(Ab-Ba)\ln(x)}{a^2}+\frac{(Ab-Ba)\ln(b\,x^3+a)}{3a^2}$	47
risch	$-\frac{A}{3a\,x^3}-\frac{\ln(x)Ab}{a^2}+\frac{B\ln(x)}{a}+\frac{\ln(-b\,x^3-a)Ab}{3a^2}-\frac{\ln(-b\,x^3-a)B}{3a}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(A*b-B*a)*ln(b*x^3+a)/a^2-1/3*A/a/x^3+1/a^2*(-A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.29, size = 48, normalized size = 0.96

$$
-\frac{(Ba-Ab)\log(bx^3+a)}{3\,a^2}+\frac{(Ba-Ab)\log(x^3)}{3\,a^2}-\frac{A}{3\,ax^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*log(b*x^3 + a)/a^2 + 1/3*(B*a - A*b)*log(x^3)/a^2 - 1/3*A/
(a*x^3)
```

Fricas [A]
time = 1.88, size = 47, normalized size = 0.94

$$-\frac{(Ba-Ab)x^3\log\left(bx^3+a\right)-3\,(Ba-Ab)x^3\log\left(x\right)+Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*((B*a - A*b)*x^3*log(b*x^3 + a) - 3*(B*a - A*b)*x^3*log(x) + A*a)/(a^2
*x^3)
```

Sympy [A]
time = 0.94, size = 41, normalized size = 0.82

$$-\frac{A}{3ax^3}+\frac{(-Ab+Ba)\log\left(x\right)}{a^2}-\frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a),x)
```

```
[Out] -A/(3*a*x**3) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**3)/(3*
a**2)
```

Giac [A]
time = 1.11, size = 69, normalized size = 1.38

$$\frac{(Ba-Ab)\log\left(|x|\right)}{a^2}-\frac{(Bab-Ab^2)\log\left(|bx^3+a|\right)}{3\,a^2b}-\frac{Bax^3-Abx^3+Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] (B*a - A*b)*log(abs(x))/a^2 - 1/3*(B*a*b - A*b^2)*log(abs(b*x^3 + a))/(a^2*
b) - 1/3*(B*a*x^3 - A*b*x^3 + A*a)/(a^2*x^3)
```

Mupad [B]
time = 2.40, size = 46, normalized size = 0.92

$$\frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-B\,a\right)}{3\,a^2}-\frac{A}{3\,a\,x^3}-\frac{\ln\left(x\right)\,\left(A\,b-B\,a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)),x)
```

```
[Out] (log(a + b*x^3)*(A*b - B*a))/(3*a^2) - A/(3*a*x^3) - (log(x)*(A*b - B*a))/a
^2
```

3.67 $\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$

Optimal. Leaf size=165

$$-\frac{A}{4ax^4}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[3]{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}}+\frac{\sqrt[3]{b}\,(Ab-a\ldots}{}$$

```
[Out] -1/4*A/a/x^4+(A*b-B*a)/a^2/x-1/3*b^(1/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^
(7/3)+1/6*b^(1/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/
3)-1/3*b^(1/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(7/3)*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {464, 331, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt[3]{b}\,(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{7/3}}-\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}}+\frac{Ab-aB}{a^2x}-\frac{A}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

```
[Out] -1/4*A/(a*x^4) + (A*b - a*B)/(a^2*x) - (b^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) - (b^(1/3)*(A*b - a*B
)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)) + (b^(1/3)*(A*b - a*B)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx = -\frac{A}{4ax^4} - \frac{(4Ab-4aB)\int \frac{1}{x^2(a+bx^3)}\,dx}{4a}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} + \frac{(b(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{7/3}} + \frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{a}}}{3a^{7/3}}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}} + \frac{\left(\sqrt[3]{b}\,(Ab-aB)\right)\int \frac{-\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{}}}{6a^{7/3}}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}} + \frac{\sqrt[3]{b}\,(Ab-aB)\log\left(a^{2/3}-\right.}{6a^{7/3}}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}} - \frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}\right.}{3a^{7/3}}$$

Mathematica [A]
time = 0.07, size = 154, normalized size = 0.93

$$\frac{-\frac{3a^{4/3}A}{x^4}+\frac{12\sqrt[3]{a}\,(Ab-aB)}{x}-4\sqrt{3}\,\sqrt[3]{b}\,(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{b}\,(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2\sqrt[3]{b}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{12a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(4/3)*A)/x^4 + (12*a^(1/3)*(A*b - a*B))/x - 4*Sqrt[3]*b^(1/3)*(A*b -
 a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-(A*b) + a*B
)*Log[a^(1/3) + b^(1/3)*x] + 2*b^(1/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(12*a^(7/3))
```

Maple [A]
time = 0.29, size = 130, normalized size = 0.79

method	result

default	$\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b(Ab-Ba)}{a^2}-\frac{A}{4a\,x^4}-\frac{-Ab+Ba}{a^2x}$
risch	$\frac{\frac{(Ab-Ba)x^3}{a^2}-\frac{A}{4a}}{x^4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^7\_Z^3+A^3b^4-3A^2Ba\,b^3+3A\,B^2a^2b^2-B^3a^3b\right)}\_R\ln\left(\left(-4a^7\_R^3-3A^3b^4+9A^2Ba\,b^3-9A\,B^2a^2b^2\right.\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1)))*b*(A*b-B*a)/a^2-1/4*A/a/x^4-1/a^2*(-A*b+B*a)/x
```

Maxima [A]

time = 0.52, size = 147, normalized size = 0.89

$$-\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{4\,(Ba-Ab)x^3+Aa}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(1/3)
) - 1/4*(4*(B*a - A*b)*x^3 + A*a)/(a^2*x^4)
```

Fricas [A]

time = 1.93, size = 158, normalized size = 0.96

$$-\frac{4\sqrt{3}\,(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\,(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+4\,(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)+12\,(Ba-Ab)x^3+3\,Aa}{12\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*(B*a - A*b)*x^4*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(
1/3) + 1/3*sqrt(3)) - 2*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)
```

```
^(2/3) - a*(-b/a)^(1/3)) + 4*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x + a*(-b/a
)^(2/3)) + 12*(B*a - A*b)*x^3 + 3*A*a)/(a^2*x^4)
```

Sympy [A]
time = 0.69, size = 112, normalized size = 0.68

$$\mathrm{RootSum}\left(27t^3a^7 + A^3b^4 - 3A^2Bab^3 + 3AB^2a^2b^2 - B^3a^3b, \left(t \mapsto t\log\left(\frac{9t^2a^5}{A^2b^3 - 2ABab^2 + B^2a^2b} + x\right)\right)\right) + \frac{-Aa + x^3\cdot(4Ab - 4Ba)}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**7 + A**3*b**4 - 3*A**2*B*a*b**3 + 3*A*B**2*a**2*b**2 -
B**3*a**3*b, Lambda(_t, _t*log(9*_t**2*a**5/(A**2*b**3 - 2*A*B*a*b**2 + B**
2*a**2*b) + x))) + (-A*a + x**3*(4*A*b - 4*B*a))/(4*a**2*x**4)
```

Giac [A]
time = 0.86, size = 197, normalized size = 1.19

$$\frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3b} - \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3b} - \frac{4Bax^3 - 4Abx^3 + Aa}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/a^3 + 1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) - 1/6*((-a*b^2)^(
2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^
3*b) - 1/4*(4*B*a*x^3 - 4*A*b*x^3 + A*a)/(a^2*x^4)
```

Mupad [B]
time = 2.59, size = 178, normalized size = 1.08

$$\frac{(-b)^{1/3}\ln\left(a^{1/3}(-b)^{8/3} + b^3x\right)(Ab - Ba)}{3a^{7/3}} - \frac{\frac{A}{4a} - \frac{x^3(Ab - Ba)}{a^2}}{x^4} + \frac{(-b)^{1/3}\ln\left(a^{1/3}(-b)^{8/3} - 2b^3x + \sqrt{3}\,a^{1/3}(-b)^{8/3}\,1\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(Ab - Ba)}{3a^{7/3}} - \frac{(-b)^{1/3}\ln\left(2b^3x - a^{1/3}(-b)^{8/3} + \sqrt{3}\,a^{1/3}(-b)^{8/3}\,1\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(Ab - Ba)}{3a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)),x)
```

```
[Out] ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(A*b - B*a))/(3*a^(7/3)) - (A/(
4*a) - (x^3*(A*b - B*a))/a^2)/x^4 + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*
b^3*x + 3^(1/2)*a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/
(3*a^(7/3)) - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3
)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(7/3))
```

3.68 $\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx$

Optimal. Leaf size=168

$$-\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}-\frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}}-\frac{b^{2/3}(Ab-aB)}{}$$

```
[Out] -1/5*A/a/x^5+1/2*(A*b-B*a)/a^2/x^2+1/3*b^(2/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)
*x)/a^(8/3)-1/6*b^(2/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(8/3)-1/3*b^(2/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(
1/2))/a^(8/3)*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {464, 331, 206, 31, 648, 631, 210, 642}

$$-\frac{b^{2/3}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}-\frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{8/3}}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}}+\frac{Ab-aB}{2a^2x^2}-\frac{A}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

```
[Out] -1/5*A/(a*x^5) + (A*b - a*B)/(2*a^2*x^2) - (b^(2/3)*(A*b - a*B)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) + (b^(2/3)*(A*b -
 a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)) - (b^(2/3)*(A*b - a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx = -\frac{A}{5ax^5} - \frac{(5Ab-5aB)\int \frac{1}{x^3(a+bx^3)}\,dx}{5a}$$
$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{(b(Ab-aB))\int \frac{1}{a+bx^3}\,dx}{a^2}$$
$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{(b(Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{8/3}} + \frac{(b(Ab-aB))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\ldots}}{3a^{8/3}}$$
$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}} - \frac{(b^{2/3}(Ab-aB))\int \frac{-\sqrt[3]{a}\ldots}{a^{2/3}-\sqrt[3]{a}\ldots}}{6a^{8/3}}$$
$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}} - \frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\ldots\right.}{6a^{8/3}}$$
$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} - \frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\ldots\right.}{3a^{8/3}}$$

Mathematica [A]
time = 0.07, size = 154, normalized size = 0.92

$$\frac{-\frac{6a^{5/3}A}{x^5}+\frac{15a^{2/3}(Ab-aB)}{x^2}-10\sqrt{3}b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+5b^{2/3}(-Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{30a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

```
[Out] ((-6*a^(5/3)*A)/x^5 + (15*a^(2/3)*(A*b - a*B))/x^2 - 10*Sqrt[3]*b^(2/3)*(A*
b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(A*b - a*
B)*Log[a^(1/3) + b^(1/3)*x] + 5*b^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(8/3))
```

Maple [A]
time = 0.28, size = 130, normalized size = 0.77

method	result

default	$\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b(Ab-Ba)}{a^2}-\frac{A}{5a\,x^5}-\frac{-Ab+Ba}{2a^2x^2}$
risch	$\frac{\frac{(Ab-Ba)x^3}{2a^2}-\frac{A}{5a}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^8\_Z^3-A^3b^5+3A^2Ba\,b^4-3A\,B^2a^2b^3+B^3a^3b^2\right)}\_R\ln\left(\left(-4\_R^3a^8+3A^3b^5-9A^2Ba\,b^4+9A\,B^2a^2\right.\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1)))*b*(A*b-B*a)/a^2-1/5*A/a/x^5-1/2/a^2*(-A*b+B*a)/x^2
```

Maxima [A]
time = 0.51, size = 148, normalized size = 0.88

$$-\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,(Ba-Ab)x^3+2\,Aa}{10\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(2/3)
) - 1/10*(5*(B*a - A*b)*x^3 + 2*A*a)/(a^2*x^5)
```

Fricas [A]
time = 2.10, size = 176, normalized size = 1.05

$$-\frac{10\,\sqrt{3}\,(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-5\,(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+10\,(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+15\,(Ba-Ab)x^3+6\,Aa}{30\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x
*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*log(b^
2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*(B*a - A*b)*x^5*(
b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 15*(B*a - A*b)*x^3 + 6*A*a)/(
a^2*x^5)
```

Sympy [A]
time = 0.34, size = 99, normalized size = 0.59

$$\mathrm{RootSum}\left(27t^3a^8 - A^3b^5 + 3A^2Bab^4 - 3AB^2a^2b^3 + B^3a^3b^2,\ \left(t \mapsto t\log\left(-\frac{3ta^3}{-Ab^2 + Bab} + x\right)\right)\right) + \frac{-2Aa + x^3\cdot(5Ab - 5Ba)}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**8 - A**3*b**5 + 3*A**2*B*a*b**4 - 3*A*B**2*a**2*b**3 +
B**3*a**3*b**2, Lambda(_t, _t*log(-3*_t*a**3/(-A*b**2 + B*a*b) + x))) + (-2
*A*a + x**3*(5*A*b - 5*B*a))/(10*a**2*x**5)
```

Giac [A]
time = 0.84, size = 176, normalized size = 1.05

$$-\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}Ba - (-ab^2)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3} + \frac{(Bab - Ab^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3} - \frac{\left((-ab^2)^{\frac{1}{3}}Ba - (-ab^2)^{\frac{1}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3} - \frac{5Bax^3 - 5Abx^3 + 2Aa}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 + 1/3*(B*a*b - A*b^2)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/a^3 - 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^3 - 1/10*(5*B*a*x^3 - 5*A*b*x^
3 + 2*A*a)/(a^2*x^5)
```

Mupad [B]
time = 2.56, size = 145, normalized size = 0.86

$$\frac{b^{2/3}\ln\left(b^{1/3}x + a^{1/3}\right)(A\,b - B\,a)}{3\,a^{8/3}} - \frac{\frac{A}{5\,a} - \frac{x^3\,(A\,b - B\,a)}{2\,a^2}}{x^5} - \frac{b^{2/3}\ln\left(a^{1/3} - 2\,b^{1/3}x + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b - B\,a)}{3\,a^{8/3}} + \frac{b^{2/3}\ln\left(2\,b^{1/3}x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b - B\,a)}{3\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(8/3)) - (A/(5*a) - (x^
3*(A*b - B*a))/(2*a^2))/x^5 - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
 + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(8/3)) + (b^(2/3)*log(
3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B
*a))/(3*a^(8/3))
```

3.69 $\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx$

Optimal. Leaf size=69

$$-\frac{A}{6ax^6}+\frac{Ab-aB}{3a^2x^3}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log(a+bx^3)}{3a^3}$$

```
[Out] -1/6*A/a/x^6+1/3*(A*b-B*a)/a^2/x^3+b*(A*b-B*a)*ln(x)/a^3-1/3*b*(A*b-B*a)*ln
(b*x^3+a)/a^3
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$-\frac{b(Ab-aB)\log(a+bx^3)}{3a^3}+\frac{b\log(x)(Ab-aB)}{a^3}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

```
[Out] -1/6*A/(a*x^6) + (A*b - a*B)/(3*a^2*x^3) + (b*(A*b - a*B)*Log[x])/a^3 - (b*
(A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{ax^3}+\frac{-Ab+aB}{a^2x^2}-\frac{b(-Ab+aB)}{a^3x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)}\right)dx, x, x^3\right)$$
$$= -\frac{A}{6ax^6}+\frac{Ab-aB}{3a^2x^3}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log(a+bx^3)}{3a^3}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 1.01

$$\frac{-a(aA-2Abx^3+2aBx^3)+6b(Ab-aB)x^6\log(x)+2b(-Ab+aB)x^6\log(a+bx^3)}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

```
[Out] (-(a*(a*A - 2*A*b*x^3 + 2*a*B*x^3)) + 6*b*(A*b - a*B)*x^6*Log[x] + 2*b*(-(A
*b) + a*B)*x^6*Log[a + b*x^3])/(6*a^3*x^6)
```

Maple [A]
time = 0.28, size = 64, normalized size = 0.93

method	result	size
default	$-\frac{b(Ab-Ba)\ln(b\,x^3+a)}{3a^3}-\frac{A}{6a\,x^6}-\frac{-Ab+Ba}{3a^2x^3}+\frac{b(Ab-Ba)\ln(x)}{a^3}$	64
norman	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^3}{3a^2}}{x^6}+\frac{b(Ab-Ba)\ln(x)}{a^3}-\frac{b(Ab-Ba)\ln(b\,x^3+a)}{3a^3}$	66
risch	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^3}{3a^2}}{x^6}+\frac{b^2\ln(x)A}{a^3}-\frac{b\ln(x)B}{a^2}-\frac{b^2\ln(b\,x^3+a)A}{3a^3}+\frac{b\ln(b\,x^3+a)B}{3a^2}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b*(A*b-B*a)*ln(b*x^3+a)/a^3-1/6*A/a/x^6-1/3*(-A*b+B*a)/a^2/x^3+b*(A*b-
B*a)*ln(x)/a^3
```

Maxima [A]
time = 0.30, size = 70, normalized size = 1.01

$$\frac{(Bab-Ab^2)\log(bx^3+a)}{3\,a^3}-\frac{(Bab-Ab^2)\log(x^3)}{3\,a^3}-\frac{2\,(Ba-Ab)x^3+Aa}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(B*a*b - A*b^2)*log(b*x^3 + a)/a^3 - 1/3*(B*a*b - A*b^2)*log(x^3)/a^3 -
 1/6*(2*(B*a - A*b)*x^3 + A*a)/(a^2*x^6)
```

Fricas [A]

time = 1.79, size = 73, normalized size = 1.06

$$\frac{2\,(Bab-Ab^2)x^6\log{(bx^3+a)}-6\,(Bab-Ab^2)x^6\log{(x)}-2\,(Ba^2-Aab)x^3-Aa^2}{6\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(B*a*b - A*b^2)*x^6*log(b*x^3 + a) - 6*(B*a*b - A*b^2)*x^6*log(x) -
2*(B*a^2 - A*a*b)*x^3 - A*a^2)/(a^3*x^6)
```

Sympy [A]

time = 0.62, size = 61, normalized size = 0.88

$$\frac{-Aa+x^3\cdot(2Ab-2Ba)}{6a^2x^6}-\frac{b(-Ab+Ba)\log{(x)}}{a^3}+\frac{b(-Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a),x)
```

```
[Out] (-A*a + x**3*(2*A*b - 2*B*a))/(6*a**2*x**6) - b*(-A*b + B*a)*log(x)/a**3 +
b*(-A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

Giac [A]

time = 0.99, size = 99, normalized size = 1.43

$$-\frac{(Bab-Ab^2)\log{(|x|)}}{a^3}+\frac{(Bab^2-Ab^3)\log{(|bx^3+a|)}}{3\,a^3b}+\frac{3\,Babx^6-3\,Ab^2x^6-2\,Ba^2x^3+2\,Aabx^3-Aa^2}{6\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*log(abs(x))/a^3 + 1/3*(B*a*b^2 - A*b^3)*log(abs(b*x^3 + a)
)/(a^3*b) + 1/6*(3*B*a*b*x^6 - 3*A*b^2*x^6 - 2*B*a^2*x^3 + 2*A*a*b*x^3 - A*
a^2)/(a^3*x^6)
```

Mupad [B]

time = 0.13, size = 70, normalized size = 1.01

$$\frac{\ln{(x)}\,\left(A\,b^2-B\,a\,b\right)}{a^3}-\frac{\ln{(b\,x^3+a)}\,\left(A\,b^2-B\,a\,b\right)}{3\,a^3}-\frac{\frac{A}{6\,a}-\frac{x^3\,(A\,b-B\,a)}{3\,a^2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)),x)
```

```
[Out] (log(x)*(A*b^2 - B*a*b))/a^3 - (log(a + b*x^3)*(A*b^2 - B*a*b))/(3*a^3) - (
A/(6*a) - (x^3*(A*b - B*a))/(3*a^2))/x^6
```

3.70 $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$

Optimal. Leaf size=184

$$-\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{10/3}}-$$

```
[Out] -1/7*A/a/x^7+1/4*(A*b-B*a)/a^2/x^4-b*(A*b-B*a)/a^3/x+1/3*b^(4/3)*(A*b-B*a)*
ln(a^(1/3)+b^(1/3)*x)/a^(10/3)-1/6*b^(4/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(10/3)+1/3*b^(4/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {464, 331, 298, 31, 648, 631, 210, 642}

$$\frac{b^{4/3}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}}-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{10/3}}-\frac{b(Ab-aB)}{a^3x}+\frac{Ab-aB}{4a^2x^4}-\frac{A}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

```
[Out] -1/7*A/(a*x^7) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(a^3*x) + (b^(4/
3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(10/3)) + (b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)) - (
b^(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1
0/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx = -\frac{A}{7ax^7} - \frac{(7Ab-7aB)\int \frac{1}{x^5(a+bx^3)}\,dx}{7a}$$

$$= -\frac{A}{7ax^7} + \frac{Ab-aB}{4a^2x^4} + \frac{(b(Ab-aB))\int \frac{1}{x^2(a+bx^3)}\,dx}{a^2}$$

$$= -\frac{A}{7ax^7} + \frac{Ab-aB}{4a^2x^4} - \frac{b(Ab-aB)}{a^3x} - \frac{(b^2(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{A}{7ax^7} + \frac{Ab-aB}{4a^2x^4} - \frac{b(Ab-aB)}{a^3x} + \frac{\left(b^{5/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{10/3}} - \frac{\left(b^{5/3}(Ab-aB\right.}{}$$

$$= -\frac{A}{7ax^7} + \frac{Ab-aB}{4a^2x^4} - \frac{b(Ab-aB)}{a^3x} + \frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{10/3}} - \frac{\left(b^{4/3}(Ab-a\right.}{}$$

$$= -\frac{A}{7ax^7} + \frac{Ab-aB}{4a^2x^4} - \frac{b(Ab-aB)}{a^3x} + \frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{10/3}} - \frac{b^{4/3}(Ab-aB}{}$$

$$= -\frac{A}{7ax^7} + \frac{Ab-aB}{4a^2x^4} - \frac{b(Ab-aB)}{a^3x} + \frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}} + \frac{b^{4/3}(Ab-}{}$$

Mathematica [A]
time = 0.08, size = 173, normalized size = 0.94

$$\frac{-\frac{12a^{7/3}A}{x^7}+\frac{21a^{4/3}(Ab-aB)}{x^4}+\frac{84\sqrt[3]{a}\,b(-Ab+aB)}{x}+28\sqrt{3}\,b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+28b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+14b^{4/3}(-Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{84a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

```
[Out] ((-12*a^(7/3)*A)/x^7 + (21*a^(4/3)*(A*b - a*B))/x^4 + (84*a^(1/3)*b*(-(A*b)
 + a*B))/x + 28*Sqrt[3]*b^(4/3)*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 28*b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(4/3)
*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*a^(10/3
))
```

Maple [A]
time = 0.31, size = 150, normalized size = 0.82

method	result

default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{2}(Ab-Ba)}{a^{3}}-\frac{A}{7a\,x^{7}}-\frac{-Ab+Ba}{4a^{2}x^{4}}-\frac{b(Ab-}{a^{3}}$
risch	$\frac{-\frac{b(Ab-Ba)x^{6}}{a^{3}}+\frac{(Ab-Ba)x^{3}}{4a^{2}}-\frac{A}{7a}}{x^{7}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^{3}-A^{3}b^{7}+3A^{2}Ba\,b^{6}-3A\,B^{2}a^{2}b^{5}+B^{3}a^{3}b^{4}\right)}\_R\ln\left(\left(-4a^{10}\_R^{3}+3A^{3}b^{7}-9A\right.\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^8/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))*b^2*(A*b-B*a)/a^3-1/7*A/a/x^7-1/4*(-A*b+B*a)/a^2/x^4-b*(A*b-B*a)/a^
3/x
```

Maxima [A]

time = 0.51, size = 178, normalized size = 0.97

$$\frac{\sqrt{3}\,(Bab-Ab^{2})\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Bab-Ab^{2})\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Bab-Ab^{2})\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{28\,(Bab-Ab^{2})x^{6}-7\,(Ba^{2}-Aab)x^{3}-4\,Aa^{2}}{28\,a^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a*b - A*b^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(a^3*(a/b)^(1/3)) + 1/6*(B*a*b - A*b^2)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(1/3)) - 1/3*(B*a*b - A*b^2)*log(x + (a/b)^(1/3))/(a^3*
(a/b)^(1/3)) + 1/28*(28*(B*a*b - A*b^2)*x^6 - 7*(B*a^2 - A*a*b)*x^3 - 4*A*a
^2)/(a^3*x^7)
```

Fricas [A]

time = 2.22, size = 180, normalized size = 0.98

$$\frac{28\,\sqrt{3}\,(Bab-Ab^{2})x^{7}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+14\,(Bab-Ab^{2})x^{7}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-28\,(Bab-Ab^{2})x^{7}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+84\,(Bab-Ab^{2})x^{6}-21\,(Ba^{2}-Aab)x^{3}-12\,Aa^{2}}{84\,a^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/84*(28*sqrt(3)*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)
^(1/3) - 1/3*sqrt(3)) + 14*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x^2 - a*x*
(b/a)^(2/3) + a*(b/a)^(1/3)) - 28*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x +
 a*(b/a)^(2/3)) + 84*(B*a*b - A*b^2)*x^6 - 21*(B*a^2 - A*a*b)*x^3 - 12*A*a^
2)/(a^3*x^7)
```

Sympy [A]
time = 0.31, size = 139, normalized size = 0.76

$$\operatorname{RootSum}\left(27t^3a^{10} - A^3b^7 + 3A^2Bab^6 - 3AB^2a^2b^5 + B^3a^3b^4, \left(t \mapsto t\log\left(\frac{9t^2a^7}{A^2b^5 - 2ABab^4 + B^2a^2b^3} + x\right)\right)\right) + \frac{-4Aa^2 + x^6(-28Ab^2 + 28Bab) + x^3\cdot(7Aab - 7Ba^2)}{28a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**10 - A**3*b**7 + 3*A**2*B*a*b**6 - 3*A*B**2*a**2*b**5 +
 B**3*a**3*b**4, Lambda(_t, _t*log(9*_t**2*a**7/(A**2*b**5 - 2*A*B*a*b**4 +
 B**2*a**2*b**3) + x))) + (-4*A*a**2 + x**6*(-28*A*b**2 + 28*B*a*b) + x**3*
(7*A*a*b - 7*B*a**2))/(28*a**3*x**7)
```

Giac [A]
time = 0.74, size = 216, normalized size = 1.17

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^4} - \frac{\left(Bab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^4} + \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^4} + \frac{28\,Babx^6 - 28\,Ab^2x^6 - 7\,Ba^2x^3 + 7\,Aabx^3 - 4\,Aa^2}{28\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 - 1/3*(B*a*b^2*(-a/b)^(1/3) - A*b^3*(
-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 1/6*((-a*b^2)^(2
/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4
+ 1/28*(28*B*a*b*x^6 - 28*A*b^2*x^6 - 7*B*a^2*x^3 + 7*A*a*b*x^3 - 4*A*a^2)/
(a^3*x^7)
```

Mupad [B]
time = 2.58, size = 161, normalized size = 0.88

$$\frac{b^{4/3}\,\ln\left(b^{1/3}\,x + a^{1/3}\right)\,\left(A\,b - B\,a\right)}{3\,a^{10/3}} - \frac{\frac{A}{7\,a} - \frac{x^3\,\left(A\,b-B\,a\right)}{4\,a^2} + \frac{b\,x^6\,\left(A\,b-B\,a\right)}{a^3}}{x^7} + \frac{b^{4/3}\,\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b - B\,a\right)}{3\,a^{10/3}} - \frac{b^{4/3}\,\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b - B\,a\right)}{3\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^8*(a + b*x^3)),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(10/3)) - (A/(7*a) - (x
^3*(A*b - B*a))/(4*a^2) + (b*x^6*(A*b - B*a))/a^3)/x^7 + (b^(4/3)*log(3^(1/
2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/
(3*a^(10/3)) - (b^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3
^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(10/3))
```

3.71 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=233

$$-\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}+\frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)}-\frac{a^{4/3}(7Ab-10aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{3\sqrt{3}\,b^{13/3}}$$

```
[Out] -1/3*a*(7*A*b-10*B*a)*x/b^4+1/12*(7*A*b-10*B*a)*x^4/b^3-1/21*(7*A*b-10*B*a)
*x^7/a/b^2+1/3*(A*b-B*a)*x^10/a/b/(b*x^3+a)+1/9*a^(4/3)*(7*A*b-10*B*a)*ln(a
^(1/3)+b^(1/3)*x)/b^(13/3)-1/18*a^(4/3)*(7*A*b-10*B*a)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(13/3)-1/9*a^(4/3)*(7*A*b-10*B*a)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(13/3)*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {468, 308, 206, 31, 648, 631, 210, 642}

$$-\frac{a^{4/3}(7Ab-10aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{13/3}}-\frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{13/3}}+\frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{13/3}}-\frac{ax(7Ab-10aB)}{3b^4}+\frac{x^4(7Ab-10aB)}{12b^3}-\frac{x^7(7Ab-10aB)}{21ab^2}+\frac{x^{10}(Ab-aB)}{3ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(a*(7*A*b - 10*a*B)*x)/b^4 + ((7*A*b - 10*a*B)*x^4)/(12*b^3) - ((7*A*b
 - 10*a*B)*x^7)/(21*a*b^2) + ((A*b - a*B)*x^10)/(3*a*b*(a + b*x^3)) - (a^(4
/3)*(7*A*b - 10*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*
Sqrt[3]*b^(13/3)) + (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*
b^(13/3)) - (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(18*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)} + \frac{(-7Ab+10aB)\int \frac{x^9}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)} + \frac{(-7Ab+10aB)\int \left(\frac{a^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^6}{b}-\frac{a^3}{b^3(a+bx^3)}\right)dx}{3ab}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)} + \frac{(a^2(7\ldots}{}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)} + \frac{(a^{4/3}(\ldots}{}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)} + \frac{a^{4/3}(7\ldots}{}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)} + \frac{a^{4/3}(7\ldots}{}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)} - \frac{a^{4/3}(7\ldots}{}$$

Mathematica [A]
time = 0.08, size = 203, normalized size = 0.87

$$\frac{252a\sqrt[3]{b}\,(-2Ab+3aB)x+63b^{4/3}(Ab-2aB)x^4+36b^{7/3}Bx^7+\frac{84a^2\sqrt[3]{b}\,(-Ab+aB)x}{a+bx^3}+28\sqrt{3}\,a^{4/3}(-7Ab+10aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-28a^{4/3}(-7Ab+10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+14a^{4/3}(-7Ab+10aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{252b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (252*a*b^(1/3)*(-2*A*b + 3*a*B)*x + 63*b^(4/3)*(A*b - 2*a*B)*x^4 + 36*b^(7/
3)*B*x^7 + (84*a^2*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + 28*Sqrt[3]*a^(4/
3)*(-7*A*b + 10*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 28*a^(4/
3)*(-7*A*b + 10*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*(-7*A*b + 10*a*B
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(252*b^(13/3))
```

Maple [A]
time = 0.28, size = 176, normalized size = 0.76

method	result

risch	$\frac{B\,x^7}{7b^2}+\frac{A\,x^4}{4b^2}-\frac{Ba\,x^4}{2b^3}-\frac{2aAx}{b^3}+\frac{3a^2Bx}{b^4}+\frac{(-\frac{1}{3}A\,a^2b+\frac{1}{3}B\,a^3)x}{b^4(b\,x^3+a)}+\frac{a^2\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(7Ab-10Ba)\ln(x-\_R)}{\_R^2}\right)}{9b^5}$
default	$-\frac{-\frac{1}{7}b^2B\,x^7-\frac{1}{4}A\,b^2x^4+\frac{1}{2}Bab\,x^4+2abAx-3a^2Bx}{b^4}+\frac{a^2\left(\frac{(-\frac{Ab}{3}+\frac{Ba}{3})x}{b\,x^3+a}+\frac{(7Ab-10Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\ldots}{}\right)}{3}\right)}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/7*b^2*B*x^7-1/4*A*b^2*x^4+1/2*B*a*b*x^4+2*a*b*A*x-3*a^2*B*x)+a^2
/b^4*((-1/3*A*b+1/3*B*a)*x/(b*x^3+a)+1/3*(7*A*b-10*B*a)*(1/3/b/(a/b)^(2/3)*
ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]

time = 0.49, size = 218, normalized size = 0.94

$$\frac{(Ba^3-Aa^2b)x}{3\,(b^5x^3+ab^4)}+\frac{4\,Bb^2x^7-7\,(2\,Bab-Ab^2)x^4+28\,(3\,Ba^2-2\,Aab)x}{28\,b^4}-\frac{\sqrt{3}\,(10\,Ba^3-7\,Aa^2b)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(10\,Ba^3-7\,Aa^2b)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(10\,Ba^3-7\,Aa^2b)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a^3 - A*a^2*b)*x/(b^5*x^3 + a*b^4) + 1/28*(4*B*b^2*x^7 - 7*(2*B*a*b
- A*b^2)*x^4 + 28*(3*B*a^2 - 2*A*a*b)*x)/b^4 - 1/9*sqrt(3)*(10*B*a^3 - 7*A*
a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)
) + 1/18*(10*B*a^3 - 7*A*a^2*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5
*(a/b)^(2/3)) - 1/9*(10*B*a^3 - 7*A*a^2*b)*log(x + (a/b)^(1/3))/(b^5*(a/b)^
(2/3))
```

Fricas [A]
time = 2.03, size = 271, normalized size = 1.16

$$\frac{36\,Bb^3x^{10} - 9\,(10\,Bab^2 - 7\,Ab^3)x^7 + 63\,(10\,Ba^2b - 7\,Aab^2)x^4 - 28\,\sqrt{3}\,(10\,Ba^3 - 7\,Aa^2b + (10\,Ba^2b - 7\,Aab^2)x^3)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}\,a}{3\,a}\right) + 14\,(10\,Ba^3 - 7\,Aa^2b + (10\,Ba^2b - 7\,Aab^2)x^3)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 28\,(10\,Ba^3 - 7\,Aa^2b + (10\,Ba^2b - 7\,Aab^2)x^3)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 84\,(10\,Ba^3 - 7\,Aa^2b)x}{252\,(b^5x^3 + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(36*B*b^3*x^10 - 9*(10*B*a*b^2 - 7*A*b^3)*x^7 + 63*(10*B*a^2*b - 7*A*
a*b^2)*x^4 - 28*sqrt(3)*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^
3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(
10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^3)*(a/b)^(1/3)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A
*a*b^2)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(10*B*a^3 - 7*A*a^2*b)*x
)/(b^5*x^3 + a*b^4)
```

Sympy [A]
time = 0.56, size = 156, normalized size = 0.67

$$\frac{Bx^7}{7b^2} + x^4\left(\frac{A}{4b^2} - \frac{Ba}{2b^3}\right) + x\left(-\frac{2Aa}{b^3} + \frac{3Ba^2}{b^4}\right) + \frac{x(-Aa^2b + Ba^3)}{3ab^4 + 3b^5x^3} + \operatorname{RootSum}\left(729t^3b^{13} - 343A^3a^4b^3 + 1470A^2Ba^5b^2 - 2100AB^2a^6b + 1000B^3a^7, \left(t \mapsto t\log\left(-\frac{9tb^4}{-7Aab + 10Ba^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**7/(7*b**2) + x**4*(A/(4*b**2) - B*a/(2*b**3)) + x*(-2*A*a/b**3 + 3*B*a
**2/b**4) + x*(-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3) + RootSum(729*_
t**3*b**13 - 343*A**3*a**4*b**3 + 1470*A**2*B*a**5*b**2 - 2100*A*B**2*a**6*
b + 1000*B**3*a**7, Lambda(_t, _t*log(-9*_t*b**4/(-7*A*a*b + 10*B*a**2) + x
)))
```

Giac [A]
time = 0.80, size = 244, normalized size = 1.05

$$-\frac{\sqrt{3}\left(10\,(-ab^2)^{\frac{1}{3}}Ba^2 - 7\,(-ab^2)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5} + \frac{(10\,Ba^3 - 7\,Aa^2b)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4} - \frac{\left(10\,(-ab^2)^{\frac{1}{3}}Ba^2 - 7\,(-ab^2)^{\frac{1}{3}}Aab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5} + \frac{Ba^3x - Aa^2bx}{3\,(bx^3 + a)b^4} + \frac{4\,Bb^{12}x^7 - 14\,Bab^{11}x^4 + 7\,Ab^{12}x^4 + 84\,Ba^2b^{10}x - 56\,Aab^{11}x}{28\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(10*(-a*b^2)^(1/3)*B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 + 1/9*(10*B*a^3 - 7*A*a^2*b)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/18*(10*(-a*b^2)^(1/3)*
B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^
5 + 1/3*(B*a^3*x - A*a^2*b*x)/((b*x^3 + a)*b^4) + 1/28*(4*B*b^12*x^7 - 14*B
*a*b^11*x^4 + 7*A*b^12*x^4 + 84*B*a^2*b^10*x - 56*A*a*b^11*x)/b^14
```

Mupad [B]
time = 2.62, size = 209, normalized size = 0.90

$$x^4\left(\frac{A}{4\,b^2}-\frac{B\,a}{2\,b^3}\right)-x\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)+\frac{B\,x^7}{7\,b^2}+\frac{x\,\left(\frac{B\,a^3}{3}-\frac{A\,a^2\,b}{3}\right)}{b^5\,x^3+a\,b^4}+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(7\,A\,b-10\,B\,a\right)}{9\,b^{13/3}}-\frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,A\,b-10\,B\,a\right)}{9\,b^{13/3}}+\frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,A\,b-10\,B\,a\right)}{9\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^4*(A/(4*b^2) - (B*a)/(2*b^3)) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2
)/b^4) + (B*x^7)/(7*b^2) + (x*((B*a^3)/3 - (A*a^2*b)/3))/(a*b^4 + b^5*x^3)
+ (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(7*A*b - 10*B*a))/(9*b^(13/3)) - (a^(4/
3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
7*A*b - 10*B*a))/(9*b^(13/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)
*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 10*B*a))/(9*b^(13/3))
```

3.72 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=82

$$\frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^6}{6b^2}-\frac{a^2(Ab-aB)}{3b^4(a+bx^3)}-\frac{a(2Ab-3aB)\log(a+bx^3)}{3b^4}$$

```
[Out] 1/3*(A*b-2*B*a)*x^3/b^3+1/6*B*x^6/b^2-1/3*a^2*(A*b-B*a)/b^4/(b*x^3+a)-1/3*a
*(2*A*b-3*B*a)*ln(b*x^3+a)/b^4
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {457, 78}

$$-\frac{a^2(Ab-aB)}{3b^4(a+bx^3)}-\frac{a(2Ab-3aB)\log(a+bx^3)}{3b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^6)/(6*b^2) - (a^2*(A*b - a*B))/(3*b^4*(a
 + b*x^3)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-2aB}{b^3} + \frac{Bx}{b^2} - \frac{a^2(-Ab+aB)}{b^3(a+bx)^2} + \frac{a(-2Ab+3aB)}{b^3(a+bx)}\right) dx, x, x^3\right)$$
$$= \frac{(Ab-2aB)x^3}{3b^3} + \frac{Bx^6}{6b^2} - \frac{a^2(Ab-aB)}{3b^4\,(a+bx^3)} - \frac{a(2Ab-3aB)\log(a+bx^3)}{3b^4}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 0.88

$$\frac{2b(Ab-2aB)x^3 + b^2Bx^6 + \frac{2a^2(-Ab+aB)}{a+bx^3} + 2a(-2Ab+3aB)\log(a+bx^3)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (2*b*(A*b - 2*a*B)*x^3 + b^2*B*x^6 + (2*a^2*(-(A*b) + a*B))/(a + b*x^3) + 2
*a*(-2*A*b + 3*a*B)*Log[a + b*x^3])/(6*b^4)
```

Maple [A]
time = 0.30, size = 76, normalized size = 0.93

method	result
default	$\frac{(bB\,x^3+Ab-2Ba)^2}{6b^4B} - \frac{a\left(\frac{a(Ab-Ba)}{b(b\,x^3+a)} + \frac{(2Ab-3Ba)\ln(b\,x^3+a)}{b}\right)}{3b^3}$
norman	$\frac{-\frac{a(2abA-3a^2B)}{3b^4} + \frac{(2Ab-3Ba)x^6}{6b^2} + \frac{B\,x^9}{6b}}{b\,x^3+a} - \frac{a(2Ab-3Ba)\ln(b\,x^3+a)}{3b^4}$
risch	$\frac{B\,x^6}{6b^2} + \frac{A\,x^3}{3b^2} - \frac{2Ba\,x^3}{3b^3} + \frac{A^2}{6b^2B} - \frac{2Aa}{3b^3} + \frac{2B\,a^2}{3b^4} - \frac{a^2A}{3b^3(b\,x^3+a)} + \frac{a^3B}{3b^4(b\,x^3+a)} - \frac{2a\ln(b\,x^3+a)\,A}{3b^3} + \frac{a^2\ln(b\,x^3+a)\,B}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(B*b*x^3+A*b-2*B*a)^2/b^4/B-1/3*a/b^3*(a*(A*b-B*a)/b/(b*x^3+a)+(2*A*b-3
*B*a)/b*ln(b*x^3+a))
```

Maxima [A]
time = 0.28, size = 82, normalized size = 1.00

$$\frac{Ba^3 - Aa^2b}{3\,(b^5x^3 + ab^4)} + \frac{Bbx^6 - 2\,(2\,Ba - Ab)x^3}{6\,b^3} + \frac{(3\,Ba^2 - 2\,Aab)\log(bx^3 + a)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a^3 - A*a^2*b)/(b^5*x^3 + a*b^4) + 1/6*(B*b*x^6 - 2*(2*B*a - A*b)*x^
3)/b^3 + 1/3*(3*B*a^2 - 2*A*a*b)*log(b*x^3 + a)/b^4
```

Fricas [A]
time = 1.97, size = 121, normalized size = 1.48

$$\frac{Bb^3x^9 - (3\,Bab^2 - 2\,Ab^3)x^6 + 2\,Ba^3 - 2\,Aa^2b - 2\,(2\,Ba^2b - Aab^2)x^3 + 2\,(3\,Ba^3 - 2\,Aa^2b + (3\,Ba^2b - 2\,Aab^2)x^3)\log\left(bx^3 + a\right)}{6\,(b^5x^3 + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^3*x^9 - (3*B*a*b^2 - 2*A*b^3)*x^6 + 2*B*a^3 - 2*A*a^2*b - 2*(2*B*a
^2*b - A*a*b^2)*x^3 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x^3)
*log(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

Sympy [A]
time = 0.54, size = 82, normalized size = 1.00

$$\frac{Bx^6}{6b^2} + \frac{a(-2Ab + 3Ba)\log\left(a + bx^3\right)}{3b^4} + x^3\left(\frac{A}{3b^2} - \frac{2Ba}{3b^3}\right) + \frac{-Aa^2b + Ba^3}{3ab^4 + 3b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**6/(6*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**3)/(3*b**4) + x**3*(A/(3*
b**2) - 2*B*a/(3*b**3)) + (-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3)
```

Giac [A]
time = 1.27, size = 106, normalized size = 1.29

$$\frac{(3\,Ba^2 - 2\,Aab)\log\left(|bx^3 + a|\right)}{3\,b^4} + \frac{Bb^2x^6 - 4\,Babx^3 + 2\,Ab^2x^3}{6\,b^4} - \frac{3\,Ba^2bx^3 - 2\,Aab^2x^3 + 2\,Ba^3 - Aa^2b}{3\,(bx^3 + a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*B*a^2 - 2*A*a*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(B*b^2*x^6 - 4*B*a*b*
x^3 + 2*A*b^2*x^3)/b^4 - 1/3*(3*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 2*B*a^3 - A*a
^2*b)/((b*x^3 + a)*b^4)
```

Mupad [B]
time = 0.08, size = 86, normalized size = 1.05

$$x^3\left(\frac{A}{3\,b^2} - \frac{2\,B\,a}{3\,b^3}\right) + \frac{\ln\left(b\,x^3 + a\right)\,\left(3\,B\,a^2 - 2\,A\,a\,b\right)}{3\,b^4} + \frac{B\,x^6}{6\,b^2} + \frac{B\,a^3 - A\,a^2\,b}{3\,b\,\left(b^4\,x^3 + a\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) + (log(a + b*x^3)*(3*B*a^2 - 2*A*a*b))/(3
*b^4) + (B*x^6)/(6*b^2) + (B*a^3 - A*a^2*b)/(3*b*(a*b^3 + b^4*x^3))
```

3.73 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=215

$$\frac{(5Ab-8aB)x^2}{6b^3}-\frac{(5Ab-8aB)x^5}{15ab^2}+\frac{(Ab-aB)x^8}{3ab\,(a+bx^3)}+\frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{11/3}}+\frac{a^{2/3}(5Ab-8aB}{9}$$

```
[Out] 1/6*(5*A*b-8*B*a)*x^2/b^3-1/15*(5*A*b-8*B*a)*x^5/a/b^2+1/3*(A*b-B*a)*x^8/a/
b/(b*x^3+a)+1/9*a^(2/3)*(5*A*b-8*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(11/3)-1/18*a
^(2/3)*(5*A*b-8*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(11/3)+1/9
*a^(2/3)*(5*A*b-8*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^
(11/3)*3^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {468, 308, 298, 31, 648, 631, 210, 642}

$$\frac{a^{2/3}(5Ab-8aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{11/3}}-\frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{11/3}}+\frac{x^2(5Ab-8aB)}{6b^3}-\frac{x^5(5Ab-8aB)}{15ab^2}+\frac{x^8(Ab-aB)}{3ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((5*A*b - 8*a*B)*x^2)/(6*b^3) - ((5*A*b - 8*a*B)*x^5)/(15*a*b^2) + ((A*b -
a*B)*x^8)/(3*a*b*(a + b*x^3)) + (a^(2/3)*(5*A*b - 8*a*B)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(11/3)) + (a^(2/3)*(5*A*b - 8
*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(11/3)) - (a^(2/3)*(5*A*b - 8*a*B)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{(Ab-aB)x^8}{3ab\,(a+bx^3)} + \frac{(-5Ab+8aB)\int \frac{x^7}{a+bx^3}\,dx}{3ab} \\
&= \frac{(Ab-aB)x^8}{3ab\,(a+bx^3)} + \frac{(-5Ab+8aB)\int \left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2(a+bx^3)}\right)dx}{3ab} \\
&= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\,(a+bx^3)} - \frac{(a(5Ab-8aB))\int \frac{x}{a+bx^3}\,dx}{3b^3} \\
&= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\,(a+bx^3)} + \frac{\left(a^{2/3}(5Ab-8aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}}}{9b^{10/3}} \\
&= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\,(a+bx^3)} + \frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{\ }\right.}{9b^{11/3}} \\
&= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\,(a+bx^3)} + \frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{\ }\right.}{9b^{11/3}} \\
&= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\,(a+bx^3)} + \frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{3\sqrt{3}\,b^{11/3}}
\end{aligned}$$

Mathematica [**A**]
time = 0.07, size = 185, normalized size = 0.86

$$\frac{45b^{2/3}(Ab-2aB)x^2+18b^{5/3}Bx^5+\frac{30ab^{2/3}(Ab-aB)x^2}{a+bx^3}-10\sqrt{3}\,a^{2/3}(-5Ab+8aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-10a^{2/3}(-5Ab+8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+5a^{2/3}(-5Ab+8aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{90b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (45*b^(2/3)*(A*b - 2*a*B)*x^2 + 18*b^(5/3)*B*x^5 + (30*a*b^(2/3)*(A*b - a*B
)*x^2)/(a + b*x^3) - 10*Sqrt[3]*a^(2/3)*(-5*A*b + 8*a*B)*ArcTan[(1 - (2*b^(
1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(1/3) + b^(1/
3)*x] + 5*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(90*b^(11/3))
```

Maple [**A**]
time = 0.29, size = 156, normalized size = 0.73

method	result

risch	$\frac{Bx^5}{5b^2}+\frac{Ax^2}{2b^2}-\frac{Ba\,x^2}{b^3}+\frac{\left(\frac{1}{3}abA-\frac{1}{3}a^2B\right)x^2}{b^3(b\,x^3+a)}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-5Ab+8Ba)\ln\left(x-\_R\right)}{\_R}\right)}{9b^4}$
default	$\frac{\frac{bBx^5}{5}+\frac{(Ab-2Ba)x^2}{2}}{b^3}-\frac{a\left(\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x^2}{b\,x^3+a}+\left(\frac{5Ab}{3}-\frac{8Ba}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\cdots\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\cdots\right)\right)}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/5*b*B*x^5+1/2*(A*b-2*B*a)*x^2)-a/b^3*((-1/3*A*b+1/3*B*a)*x^2/(b*x^
3+a)+(5/3*A*b-8/3*B*a)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.50, size = 192, normalized size = 0.89

$$-\frac{(Ba^2-Aab)x^2}{3\,(b^4x^3+ab^3)}+\frac{\sqrt{3}\,(8\,Ba^2-5\,Aab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\,Bbx^5-5\,(2\,Ba-Ab)x^2}{10\,b^3}+\frac{(8\,Ba^2-5\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(8\,Ba^2-5\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a^2 - A*a*b)*x^2/(b^4*x^3 + a*b^3) + 1/9*sqrt(3)*(8*B*a^2 - 5*A*a*b
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) + 1
/10*(2*B*b*x^5 - 5*(2*B*a - A*b)*x^2)/b^3 + 1/18*(8*B*a^2 - 5*A*a*b)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) - 1/9*(8*B*a^2 - 5*A*a*b
)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

Fricas [A]
time = 1.83, size = 257, normalized size = 1.20

$$\frac{18\,Bb^2x^8-9\,(8\,Bab-5\,Ab^2)x^5-15\,(8\,Ba^2-5\,Aab)x^2+10\,\sqrt{3}\,((8\,Bab-5\,Ab^2)x^3+8\,Ba^2-5\,Aab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)+5\,((8\,Bab-5\,Ab^2)x^3+8\,Ba^2-5\,Aab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-10\,((8\,Bab-5\,Ab^2)x^3+8\,Ba^2-5\,Aab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{90\,(b^4x^3+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/90*(18*B*b^2*x^8 - 9*(8*B*a*b - 5*A*b^2)*x^5 - 15*(8*B*a^2 - 5*A*a*b)*x^2
 + 10*sqrt(3)*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)
*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*((8*B*a*b -
5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)
^(2/3) + a*(a^2/b^2)^(1/3)) - 10*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a
*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^4*x^3 + a*b^3)
```

Sympy [A]

time = 0.59, size = 151, normalized size = 0.70

$$\frac{Bx^5}{5b^2}+x^2\left(\frac{A}{2b^2}-\frac{Ba}{b^3}\right)+\frac{x^2(Aab-Ba^2)}{3ab^3+3b^4x^3}+\operatorname{RootSum}\left(729t^3b^{11}-125A^3a^2b^3+600A^2Ba^3b^2-960AB^2a^4b+512B^3a^5,\left(t\mapsto t\log\left(\frac{81t^2b^7}{25A^2ab^2-80ABa^2b+64B^2a^3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**5/(5*b**2) + x**2*(A/(2*b**2) - B*a/b**3) + x**2*(A*a*b - B*a**2)/(3*a
*b**3 + 3*b**4*x**3) + RootSum(729*_t**3*b**11 - 125*A**3*a**2*b**3 + 600*A
**2*B*a**3*b**2 - 960*A*B**2*a**4*b + 512*B**3*a**5, Lambda(_t, _t*log(81*_
t**2*b**7/(25*A**2*a*b**2 - 80*A*B*a**2*b + 64*B**2*a**3) + x)))
```

Giac [A]

time = 1.71, size = 236, normalized size = 1.10

$$-\frac{\left(8\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3}-\frac{\sqrt{3}\left(8\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5}-\frac{Ba^2x^2-Aabx^2}{3\,(bx^3+a)b^3}+\frac{\left(8\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5}+\frac{2\,Bb^8x^5-10\,Bab^7x^2+5\,Ab^8x^2}{10\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(8*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a*b^3) - 1/9*sqrt(3)*(8*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2
/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 1/3*(B
*a^2*x^2 - A*a*b*x^2)/((b*x^3 + a)*b^3) + 1/18*(8*(-a*b^2)^(2/3)*B*a - 5*(-
a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 + 1/10*(2*B*
b^8*x^5 - 10*B*a*b^7*x^2 + 5*A*b^8*x^2)/b^10
```

Mupad [B]

time = 0.27, size = 179, normalized size = 0.83

$$x^2\left(\frac{A}{2\,b^2}-\frac{B\,a}{b^3}\right)+\frac{B\,x^5}{5\,b^2}-\frac{x^2\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3}+\frac{a^{2/3}\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(5\,A\,b-8\,B\,a\right)}{9\,b^{11/3}}+\frac{a^{2/3}\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,A\,b-8\,B\,a\right)}{9\,b^{11/3}}-\frac{a^{2/3}\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,A\,b-8\,B\,a\right)}{9\,b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^2*(A/(2*b^2) - (B*a)/b^3) + (B*x^5)/(5*b^2) - (x^2*((B*a^2)/3 - (A*a*b)/3
))/(a*b^3 + b^4*x^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 8*B*a))/(
9*b^(11/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(5*A*b - 8*B*a))/(9*b^(11/3)) - (a^(2/3)*log(3^(1/2)*a^(
1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 8*B*a))/(9
*b^(11/3))
```

3.74 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=213

$$\frac{(4Ab-7aB)x}{3b^3}-\frac{(4Ab-7aB)x^4}{12ab^2}+\frac{(Ab-aB)x^7}{3ab\,(a+bx^3)}+\frac{\sqrt[3]{a}\,(4Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{10/3}}-\frac{\sqrt[3]{a}\,(4Ab-7aB)}{9b}$$

```
[Out] 1/3*(4*A*b-7*B*a)*x/b^3-1/12*(4*A*b-7*B*a)*x^4/a/b^2+1/3*(A*b-B*a)*x^7/a/b/
(b*x^3+a)-1/9*a^(1/3)*(4*A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(10/3)+1/18*a^(
1/3)*(4*A*b-7*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(10/3)+1/9*a
^(1/3)*(4*A*b-7*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(1
0/3)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {468, 308, 206, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{10/3}}+\frac{\sqrt[3]{a}\,(4Ab-7aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{10/3}}-\frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{10/3}}+\frac{x(4Ab-7aB)}{3b^3}-\frac{x^4(4Ab-7aB)}{12ab^2}+\frac{x^7(Ab-aB)}{3ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((4*A*b - 7*a*B)*x)/(3*b^3) - ((4*A*b - 7*a*B)*x^4)/(12*a*b^2) + ((A*b - a*
B)*x^7)/(3*a*b*(a + b*x^3)) + (a^(1/3)*(4*A*b - 7*a*B)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(10/3)) - (a^(1/3)*(4*A*b - 7*a
*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(10/3)) + (a^(1/3)*(4*A*b - 7*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^7}{3ab\,(a+bx^3)} + \frac{(-4Ab+7aB)\int \frac{x^6}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^7}{3ab\,(a+bx^3)} + \frac{(-4Ab+7aB)\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2(a+bx^3)}\right)dx}{3ab}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\,(a+bx^3)} - \frac{(a(4Ab-7aB))\int \frac{1}{a+bx^3}\,dx}{3b^3}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\,(a+bx^3)} - \frac{\left(\sqrt[3]{a}\,(4Ab-7aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}}{9b^3}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\,(a+bx^3)} - \frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{9b^{10/3}}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\,(a+bx^3)} - \frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{9b^{10/3}}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\,(a+bx^3)} + \frac{\sqrt[3]{a}\,(4Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\,\sqrt[3]{}}\right.}{3\sqrt{3}\,b^{10/3}}$$

Mathematica [A]
time = 0.07, size = 181, normalized size = 0.85

$$\frac{36\sqrt[3]{b}\,(Ab-2aB)x+9b^{4/3}Bx^4+\frac{12a\sqrt[3]{b}\,(Ab-aB)x}{a+bx^3}-4\sqrt{3}\,\sqrt[3]{a}\,(-4Ab+7aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{a}\,(-4Ab+7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt[3]{a}\,(-4Ab+7aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*(A*b - 2*a*B)*x + 9*b^(4/3)*B*x^4 + (12*a*b^(1/3)*(A*b - a*B)*x
)/(a + b*x^3) - 4*Sqrt[3]*a^(1/3)*(-4*A*b + 7*a*B)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(1/3) + b^(1/3)*x] -
 2*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(36*b^(10/3))
```

Maple [A]
time = 0.29, size = 151, normalized size = 0.71

method	result

risch	$\frac{B\,x^4}{4b^2}+\frac{Ax}{b^2}-\frac{2Bax}{b^3}+\frac{\left(\frac{1}{3}abA-\frac{1}{3}a^2B\right)x}{b^3(b\,x^3+a)}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-4Ab+7Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{9b^4}$
default	$\frac{\frac{1}{4}bB\,x^4+Abx-2Bax}{b^3}-\frac{a\left(\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b\,x^3+a}+\frac{(4Ab-7Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{3}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/4*b*B*x^4+A*b*x-2*B*a*x)-a/b^3*((-1/3*A*b+1/3*B*a)*x/(b*x^3+a)+1/3
*(4*A*b-7*B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.52, size = 187, normalized size = 0.88

$$-\frac{(Ba^2-Aab)x}{3\,(b^4x^3+ab^3)}+\frac{Bbx^4-4\,(2\,Ba-Ab)x}{4\,b^3}+\frac{\sqrt{3}\,(7\,Ba^2-4\,Aab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(7\,Ba^2-4\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(7\,Ba^2-4\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a^2 - A*a*b)*x/(b^4*x^3 + a*b^3) + 1/4*(B*b*x^4 - 4*(2*B*a - A*b)*x
)/b^3 + 1/9*sqrt(3)*(7*B*a^2 - 4*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/
3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) - 1/18*(7*B*a^2 - 4*A*a*b)*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/9*(7*B*a^2 - 4*A*a*b)*log(x
 + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Fricas [A]
time = 1.98, size = 240, normalized size = 1.13

$$\frac{9\,Bb^2x^7-9\,(7\,Bab-4\,Ab^2)x^4-4\,\sqrt{3}\,((7\,Bab-4\,Ab^2)x^3+7\,Ba^2-4\,Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+2\,((7\,Bab-4\,Ab^2)x^3+7\,Ba^2-4\,Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)-4\,((7\,Bab-4\,Ab^2)x^3+7\,Ba^2-4\,Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)-12\,(7\,Ba^2-4\,Aab)x}{36\,(b^4x^3+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(9*B*b^2*x^7 - 9*(7*B*a*b - 4*A*b^2)*x^4 - 4*sqrt(3)*((7*B*a*b - 4*A*b
^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^
(2/3) - sqrt(3)*a)/a) + 2*((7*B*a*b - 4*A*b^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a
/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 4*((7*B*a*b - 4*A*b^2)
*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 12*(7*B*a^2
- 4*A*a*b)*x)/(b^4*x^3 + a*b^3)
```

Sympy [A]
time = 0.51, size = 126, normalized size = 0.59

$$\frac{Bx^4}{4b^2}+x\left(\frac{A}{b^2}-\frac{2Ba}{b^3}\right)+\frac{x(Aab-Ba^2)}{3ab^3+3b^4x^3}+\operatorname{RootSum}\left(729t^3b^{10}+64A^3ab^3-336A^2Ba^2b^2+588AB^2a^3b-343B^3a^4,\left(t\mapsto t\log\left(\frac{9tb^3}{-4Ab+7Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**4/(4*b**2) + x*(A/b**2 - 2*B*a/b**3) + x*(A*a*b - B*a**2)/(3*a*b**3 +
3*b**4*x**3) + RootSum(729*_t**3*b**10 + 64*A**3*a*b**3 - 336*A**2*B*a**2*b
**2 + 588*A*B**2*a**3*b - 343*B**3*a**4, Lambda(_t, _t*log(9*_t*b**3/(-4*A*
b + 7*B*a) + x)))
```

Giac [A]
time = 0.83, size = 211, normalized size = 0.99

$$\frac{\sqrt{3}\left(7\left(-ab^2\right)^{\frac{1}{3}}Ba-4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4}-\frac{(7\,Ba^2-4\,Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3}+\frac{\left(7\left(-ab^2\right)^{\frac{1}{3}}Ba-4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4}-\frac{Ba^2x-Aabx}{3\,(bx^3+a)b^3}+\frac{Bb^6x^4-8\,Bab^5x+4\,Ab^6x}{4\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/9*(7*B*a^2 - 4*A*a*b)*(-a/b)^(
1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(7*(-a*b^2)^(1/3)*B*a - 4*(-
a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2
*x - A*a*b*x)/((b*x^3 + a)*b^3) + 1/4*(B*b^6*x^4 - 8*B*a*b^5*x + 4*A*b^6*x)
/b^8
```

Mupad [B]
time = 2.62, size = 193, normalized size = 0.91

$$x\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)-\frac{x\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3}+\frac{B\,x^4}{4\,b^2}+\frac{{(-a)}^{1/3}\,\ln\left({(-a)}^{4/3}+a\,b^{1/3}\,x\right)\,\left(4\,A\,b-7\,B\,a\right)}{9\,b^{10/3}}-\frac{{(-a)}^{1/3}\,\ln\left({(-a)}^{4/3}-2\,a\,b^{1/3}\,x+\sqrt{3}\,{(-a)}^{4/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,A\,b-7\,B\,a\right)}{9\,b^{10/3}}+\frac{{(-a)}^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-{(-a)}^{4/3}+\sqrt{3}\,{(-a)}^{4/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,A\,b-7\,B\,a\right)}{9\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x*(A/b^2 - (2*B*a)/b^3) - (x*((B*a^2)/3 - (A*a*b)/3))/(a*b^3 + b^4*x^3) + (
B*x^4)/(4*b^2) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(4*A*b - 7*B*a))
/(9*b^(10/3)) - ((-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a*b^
(1/3)*x)*((3^(1/2)*1i)/2 + 1/2)*(4*A*b - 7*B*a))/(9*b^(10/3)) + ((-a)^(1/3)
*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 -
1/2)*(4*A*b - 7*B*a))/(9*b^(10/3))
```

3.75 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal. Leaf size=60

$$\frac{Bx^3}{3b^2}+\frac{a(Ab-aB)}{3b^3\,(a+bx^3)}+\frac{(Ab-2aB)\log\left(a+bx^3\right)}{3b^3}$$

```
[Out] 1/3*B*x^3/b^2+1/3*a*(A*b-B*a)/b^3/(b*x^3+a)+1/3*(A*b-2*B*a)*ln(b*x^3+a)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{a(Ab-aB)}{3b^3\,(a+bx^3)}+\frac{(Ab-2aB)\log\left(a+bx^3\right)}{3b^3}+\frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (B*x^3)/(3*b^2) + (a*(A*b - a*B))/(3*b^3*(a + b*x^3)) + ((A*b - 2*a*B)*Log[
a + b*x^3])/(3*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{B}{b^2} + \frac{a(-Ab+aB)}{b^2(a+bx)^2} + \frac{Ab-2aB}{b^2(a+bx)}\right) dx, x, x^3\right)$$
$$= \frac{Bx^3}{3b^2} + \frac{a(Ab-aB)}{3b^3(a+bx^3)} + \frac{(Ab-2aB)\log(a+bx^3)}{3b^3}$$

Mathematica [**A**]
time = 0.02, size = 50, normalized size = 0.83

$$\frac{bBx^3 + \frac{a(Ab-aB)}{a+bx^3} + (Ab-2aB)\log(a+bx^3)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (b*B*x^3 + (a*(A*b - a*B))/(a + b*x^3) + (A*b - 2*a*B)*Log[a + b*x^3])/(3*b
^3)
```

Maple [**A**]
time = 0.28, size = 59, normalized size = 0.98

method	result	size
norman	$\frac{\frac{B x^6}{3b}+\frac{a(Ab-2Ba)}{3b^3}}{b x^3+a}+\frac{(Ab-2Ba)\ln\left(b x^3+a\right)}{3b^3}$	57
default	$\frac{B x^3}{3b^2}+\frac{\frac{a(Ab-Ba)}{b\left(b x^3+a\right)}+\frac{(Ab-2Ba)\ln\left(b x^3+a\right)}{b}}{3b^2}$	59
risch	$\frac{B x^3}{3b^2}+\frac{aA}{3b^2(b x^3+a)}-\frac{a^2 B}{3b^3(b x^3+a)}+\frac{\ln\left(b x^3+a\right) A}{3b^2}-\frac{2\ln\left(b x^3+a\right) Ba}{3b^3}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*B*x^3/b^2+1/3/b^2*(a*(A*b-B*a)/b/(b*x^3+a)+1/b*(A*b-2*B*a)*ln(b*x^3+a))
```

Maxima [**A**]
time = 0.30, size = 60, normalized size = 1.00

$$\frac{Bx^3}{3\,b^2} - \frac{Ba^2 - Aab}{3\,(b^4x^3 + ab^3)} - \frac{(2\,Ba - Ab)\log(bx^3 + a)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*B*x^3/b^2 - 1/3*(B*a^2 - A*a*b)/(b^4*x^3 + a*b^3) - 1/3*(2*B*a - A*b)*l
og(b*x^3 + a)/b^3
```

Fricas [A]
time = 1.72, size = 81, normalized size = 1.35

$$\frac{Bb^2x^6 + Babx^3 - Ba^2 + Aab - ((2\,Bab - Ab^2)x^3 + 2\,Ba^2 - Aab)\log\left(bx^3 + a\right)}{3\left(b^4x^3 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^6 + B*a*b*x^3 - B*a^2 + A*a*b - ((2*B*a*b - A*b^2)*x^3 + 2*B*a
^2 - A*a*b)*log(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A]
time = 0.46, size = 56, normalized size = 0.93

$$\frac{Bx^3}{3b^2} + \frac{Aab - Ba^2}{3ab^3 + 3b^4x^3} - \frac{(-Ab + 2Ba)\log\left(a + bx^3\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**3/(3*b**2) + (A*a*b - B*a**2)/(3*a*b**3 + 3*b**4*x**3) - (-A*b + 2*B*a
)*log(a + b*x**3)/(3*b**3)
```

Giac [A]
time = 0.77, size = 91, normalized size = 1.52

$$\frac{\frac{(bx^3+a)B}{b^2} + \frac{(2\,Ba-Ab)\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx^3+a} - \frac{Aab^2}{bx^3+a}}{b^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*((b*x^3 + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*ab
s(b)))/b^2 - (B*a^2*b/(b*x^3 + a) - A*a*b^2/(b*x^3 + a))/b^3)/b
```

Mupad [B]
time = 0.08, size = 62, normalized size = 1.03

$$\frac{B\,x^3}{3\,b^2} + \frac{\ln\left(b\,x^3 + a\right)\,\left(A\,b - 2\,B\,a\right)}{3\,b^3} - \frac{B\,a^2 - A\,a\,b}{3\,b\,\left(b^3\,x^3 + a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x^3)/(3*b^2) + (log(a + b*x^3)*(A*b - 2*B*a))/(3*b^3) - (B*a^2 - A*a*b)/
(3*b*(a*b^2 + b^3*x^3))
```

3.76 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=196

$$-\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\,(a+bx^3)}-\frac{(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{8/3}}+\cdots$$

```
[Out] -1/6*(2*A*b-5*B*a)*x^2/a/b^2+1/3*(A*b-B*a)*x^5/a/b/(b*x^3+a)-1/9*(2*A*b-5*B
*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(8/3)+1/18*(2*A*b-5*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(8/3)-1/9*(2*A*b-5*B*a)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {468, 327, 298, 31, 648, 631, 210, 642}

$$\frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18\sqrt[3]{a}\,b^{8/3}}-\frac{(2Ab-5aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{8/3}}-\frac{x^2(2Ab-5aB)}{6ab^2}+\frac{x^5(Ab-aB)}{3ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/6*((2*A*b - 5*a*B)*x^2)/(a*b^2) + ((A*b - a*B)*x^5)/(3*a*b*(a + b*x^3))
- ((2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sq
rt[3]*a^(1/3)*b^(8/3)) - ((2*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1
/3)*b^(8/3)) + ((2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(1/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^5}{3ab\,(a+bx^3)} + \frac{(-2Ab+5aB)\int \frac{x^4}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\,(a+bx^3)} + \frac{(2Ab-5aB)\int \frac{x}{a+bx^3}\,dx}{3b^2}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\,(a+bx^3)} - \frac{(2Ab-5aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9\sqrt[3]{a}\,b^{7/3}} + \frac{(2Ab-5aB)\int}{9}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\,(a+bx^3)} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{8/3}} + \frac{(2Ab-5aB}{}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\,(a+bx^3)} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{8/3}} + \frac{(2Ab-5aB}{}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\,(a+bx^3)} - \frac{(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}} - \frac{(2Ab-5a}{}$$

Mathematica [A]
time = 0.07, size = 165, normalized size = 0.84

$$\frac{9b^{2/3}Bx^2 - \frac{6b^{2/3}(Ab-aB)x^2}{a+bx^3} + \frac{2\sqrt{3}\,(-2Ab+5aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{2(-2Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}} + \frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{a}}}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (9*b^(2/3)*B*x^2 - (6*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3]*(-2
*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (2*(-2
*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + ((2*A*b - 5*a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(18*b^(8/3))
```

Maple [A]
time = 0.28, size = 138, normalized size = 0.70

method	result	size

risch	$\frac{B x^{2}}{2 b^{2}}+\frac{\left(-\frac{A b}{3}+\frac{B a}{3}\right) x^{2}}{b^{2}\left(b x^{3}+a\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)} \frac{(2 A b-5 B a) \ln \left(x-\_R\right)}{\_R}}{9 b^{3}}$	71
default	$\frac{B x^{2}}{2 b^{2}}+\frac{\frac{\left(-\frac{A b}{3}+\frac{B a}{3}\right) x^{2}}{b x^{3}+a}+\left(-\frac{5 B a}{3}+\frac{2 A b}{3}\right)\left(-\frac{\ln \left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\, \arctan \left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}$	138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/b^2+1/b^2*((-1/3*A*b+1/3*B*a)*x^2/(b*x^3+a)+(-5/3*B*a+2/3*A*b)*(-
1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))))
```

Maxima [A]

time = 0.53, size = 162, normalized size = 0.83

$$\frac{(Ba-Ab)x^{2}}{3\,(b^{3}x^{3}+ab^{2})}+\frac{Bx^{2}}{2\,b^{2}}-\frac{\sqrt{3}\,(5\,Ba-2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(5\,Ba-2\,Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(5\,Ba-2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x^2/(b^3*x^3 + a*b^2) + 1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a
- 2*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/
3)) - 1/18*(5*B*a - 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b
)^(1/3)) + 1/9*(5*B*a - 2*A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Fricas [A]

time = 2.16, size = 578, normalized size = 2.95

$$\left[\frac{9\,Bab^{3}x^{5}+3\,(5\,Ba^{2}b^{2}-2\,Aab^{3})x^{2}-3\sqrt{\frac{1}{3}}\,(5\,Ba^{3}b-2\,Aa^{2}b^{2}+(5\,Ba^{2}b^{2}-2\,Aab^{3})x^{3})\sqrt{\frac{(-ab^{2})^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^{2}x^{3}-ab+3\sqrt{\frac{1}{3}}\left(abx+2\,(-ab^{2})^{\frac{2}{3}}x^{2}+(-ab^{2})^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^{2})^{\frac{1}{3}}}{a}}-3\,(-ab^{2})^{\frac{2}{3}}x}{bx^{3}+a}\right)-((5\,Bab-2\,Ab^{2})x^{3}+5\,Ba^{2}-2\,Aab)(-ab^{2})^{\frac{2}{3}}\log\left(b^{2}x^{2}+(-ab^{2})^{\frac{1}{3}}bx+(-ab^{2})^{\frac{2}{3}}\right)+2\,((5\,Bab-2\,Ab^{2})x^{3}+5\,Ba^{2}-2\,Aab)(-ab^{2})^{\frac{2}{3}}\log\left(bx-(-ab^{2})^{\frac{1}{3}}\right)}{18\,(ab^{5}x^{3}+a^{2}b^{4})},\ \frac{9\,Bab^{3}x^{5}+3\,(5\,Ba^{2}b^{2}-2\,Aab^{3})x^{2}-6\sqrt{\frac{1}{3}}\,(5\,Ba^{3}b-2\,Aa^{2}b^{2}+(5\,Ba^{2}b^{2}-2\,Aab^{3})x^{3})\sqrt{-\frac{(-ab^{2})^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx+(-ab^{2})^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^{2})^{\frac{1}{3}}}{a}}}{b}\right)-((5\,Bab-2\,Ab^{2})x^{3}+5\,Ba^{2}-2\,Aab)(-ab^{2})^{\frac{2}{3}}\log\left(b^{2}x^{2}+(-ab^{2})^{\frac{1}{3}}bx+(-ab^{2})^{\frac{2}{3}}\right)+2\,((5\,Bab-2\,Ab^{2})x^{3}+5\,Ba^{2}-2\,Aab)(-ab^{2})^{\frac{2}{3}}\log\left(bx-(-ab^{2})^{\frac{1}{3}}\right)}{18\,(ab^{5}x^{3}+a^{2}b^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 3*sqrt(1/3)*(5*B*a
^3*b - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*
log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)
^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B
*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b
^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*
A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4), 1/1
8*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 6*sqrt(1/3)*(5*B*a^3*b
 - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arc
tan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - ((5*B*a
*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2
)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*
a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4)]
```

Sympy [A]
time = 0.52, size = 126, normalized size = 0.64

$$\frac{Bx^2}{2b^2} + \frac{x^2(-Ab+Ba)}{3ab^2+3b^3x^3} + \operatorname{RootSum}\left(729t^3ab^8 + 8A^3b^3 - 60A^2Bab^2 + 150AB^2a^2b - 125B^3a^3, \left(t \mapsto t\log\left(\frac{81t^2ab^5}{4A^2b^2 - 20ABab + 25B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**2/(2*b**2) + x**2*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*
_t**3*a*b**8 + 8*A**3*b**3 - 60*A**2*B*a*b**2 + 150*A*B**2*a**2*b - 125*B**
3*a**3, Lambda(_t, _t*log(81*_t**2*a*b**5/(4*A**2*b**2 - 20*A*B*a*b + 25*B*
*2*a**2) + x)))
```

Giac [A]
time = 0.88, size = 189, normalized size = 0.96

$$\frac{Bx^2}{2\,b^2} - \frac{\sqrt{3}\,(5\,Ba - 2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{1}{3}}\,b^2} + \frac{(5\,Ba - 2\,Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{1}{3}}\,b^2} + \frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2} + \frac{Bax^2 - Abx^2}{3\,(bx^3+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a - 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^2) + 1/18*(5*B*a - 2*A*b)*log(x^2
+ x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^2) + 1/9*(5*B*a*(-a/b)^(
1/3) - 2*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
+ 1/3*(B*a*x^2 - A*b*x^2)/((b*x^3 + a)*b^2)
```

Mupad [B]
time = 2.58, size = 158, normalized size = 0.81

$$\frac{B\,x^2}{2\,b^2} - \frac{x^2\,\left(\frac{A\,b}{3} - \frac{B\,a}{3}\right)}{b^3\,x^3 + a\,b^2} - \frac{\ln\left(b^{1/3}\,x + a^{1/3}\right)\,\left(2\,A\,b - 5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}} - \frac{\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b - 5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}} + \frac{\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b - 5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x^2)/(2*b^2) - (x^2*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (log(b^(1/3
)*x + a^(1/3))*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*
1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*A*b - 5*B*a))/(9*a^(1
/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 + 1/2)*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3))
```

3.77 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=190

$$-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\,(a+bx^3)}-\frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{7/3}}-\frac{(Ab-}{}$$

```
[Out] -1/3*(A*b-4*B*a)*x/a/b^2+1/3*(A*b-B*a)*x^4/a/b/(b*x^3+a)+1/9*(A*b-4*B*a)*ln
(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(7/3)-1/18*(A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(7/3)-1/9*(A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {468, 327, 206, 31, 648, 631, 210, 642}

$$-\frac{(Ab-4aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}-\frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{7/3}}-\frac{x(Ab-4aB)}{3ab^2}+\frac{x^4(Ab-aB)}{3ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*((A*b - 4*a*B)*x)/(a*b^2) + ((A*b - a*B)*x^4)/(3*a*b*(a + b*x^3)) - ((
A*b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*
a^(2/3)*b^(7/3)) + ((A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(7
/3)) - ((A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a
^(2/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^4}{3ab\,(a+bx^3)} + \frac{(-Ab+4aB)\int \frac{x^3}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\,(a+bx^3)} + \frac{(Ab-4aB)\int \frac{1}{a+bx^3}\,dx}{3b^2}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\,(a+bx^3)} + \frac{(Ab-4aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b^2} + \frac{(Ab-4aB)\int \frac{}{a^{2/3}-}}{9a^{2}}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\,(a+bx^3)} + \frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{7/3}} - \frac{(Ab-4aB)\int}{18}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\,(a+bx^3)} + \frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{7/3}} - \frac{(Ab-4aB)\log}{}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\,(a+bx^3)} - \frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}} + \frac{(Ab-4aB)\,\mathrm{l}}{9a}$$

Mathematica [A]
time = 0.07, size = 160, normalized size = 0.84

$$\frac{18\sqrt[3]{b}\,Bx - \frac{6\sqrt[3]{b}\,(Ab-aB)x}{a+bx^3} + \frac{2\sqrt{3}\,(-Ab+4aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}} + \frac{(-Ab+4aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}}}{18b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(1/3)*B*x - (6*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3) + (2*Sqrt[3]*(-(A*b
) + 4*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(A*b -
 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(A*b) + 4*a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(7/3))
```

Maple [A]
time = 0.28, size = 133, normalized size = 0.70

method	result	size
risch	$\frac{Bx}{b^2}+\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b^2(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-4Ba)\ln\left(x-\_R\right)}{\_R^2}}{9b^3}$	65

default	$\frac{Bx}{b^2}+\frac{\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b\,x^3+a}+\frac{(Ab-4Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}}{b^2}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b^2+1/b^2*((-1/3*A*b+1/3*B*a)*x/(b*x^3+a)+1/3*(A*b-4*B*a)*(1/3/b/(a/b)^
(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 157, normalized size = 0.83

$$\frac{(Ba-Ab)x}{3\left(b^3x^3+ab^2\right)}+\frac{Bx}{b^2}-\frac{\sqrt{3}\,(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(4\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(4\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x/(b^3*x^3 + a*b^2) + B*x/b^2 - 1/9*sqrt(3)*(4*B*a - A*b)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 1/18
*(4*B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 1
/9*(4*B*a - A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Fricas [A]
time = 1.74, size = 573, normalized size = 3.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*B*a^2*b^2*x^4 - 3*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2
 - A*a*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*
x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(
-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)
```

```
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a
*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
 + 6*(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3), 1/18*(18*B*a^2*b^2
*x^4 - 6*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2 - A*a*b^3)*x^3)*sq
rt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt((a^2*b)^(1/3)/b)/a^2) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2
*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a*b -
A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*
(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3)]
```

Sympy [A]
time = 1.14, size = 102, normalized size = 0.54

$$\frac{Bx}{b^2}+\frac{x(-Ab+Ba)}{3ab^2+3b^3x^3}+\operatorname{RootSum}\left(729t^3a^2b^7-A^3b^3+12A^2Bab^2-48AB^2a^2b+64B^3a^3,\left(t\mapsto t\log\left(-\frac{9tab^2}{-Ab+4Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x/b**2 + x*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*_t**3*a**2
*b**7 - A**3*b**3 + 12*A**2*B*a*b**2 - 48*A*B**2*a**2*b + 64*B**3*a**3, Lam
bda(_t, _t*log(-9*_t*a*b**2/(-A*b + 4*B*a) + x)))
```

Giac [A]
time = 0.72, size = 166, normalized size = 0.87

$$\frac{\sqrt{3}\,(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}\,b}+\frac{(4\,Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}\,b}+\frac{Bx}{b^2}+\frac{(4\,Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2}+\frac{Bax-Abx}{3\,(bx^3+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(2/3)*b) + 1/18*(4*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/((-a*b^2)^(2/3)*b) + B*x/b^2 + 1/9*(4*B*a - A*b)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*b^2)
```

Mupad [B]
time = 2.58, size = 150, normalized size = 0.79

$$\frac{B\,x}{b^2}-\frac{x\,\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x)/b^2 - (x*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (log(b^(1/3)*x + a^
(1/3))*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(
1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3))
 + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*
(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3))
```

3.78 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=41

$$\frac{-Ab+aB}{3b^2\left(a+bx^3\right)}+\frac{B\log\left(a+bx^3\right)}{3b^2}$$

```
[Out] 1/3*(-A*b+B*a)/b^2/(b*x^3+a)+1/3*B*ln(b*x^3+a)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\frac{B\log\left(a+bx^3\right)}{3b^2}-\frac{Ab-aB}{3b^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(A*b - a*B)/(b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^2}+\frac{B}{b(a+bx)}\right)\,dx,x,x^3\right)\\ &= -\frac{Ab-aB}{3b^2\left(a+bx^3\right)}+\frac{B\log\left(a+bx^3\right)}{3b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{-Ab+aB}{3b^2(a+bx^3)}+\frac{B\log(a+bx^3)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (-(A*b) + a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

Maple [A]
time = 0.28, size = 38, normalized size = 0.93

method	result	size
default	$-\frac{Ab-Ba}{3b^2(b\,x^3+a)}+\frac{B\ln(b\,x^3+a)}{3b^2}$	38
norman	$-\frac{Ab-Ba}{3b^2(b\,x^3+a)}+\frac{B\ln(b\,x^3+a)}{3b^2}$	38
risch	$-\frac{A}{3b(b\,x^3+a)}+\frac{Ba}{3b^2(b\,x^3+a)}+\frac{B\ln(b\,x^3+a)}{3b^2}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(A*b-B*a)/b^2/(b*x^3+a)+1/3*B*ln(b*x^3+a)/b^2
```

Maxima [A]
time = 0.28, size = 40, normalized size = 0.98

$$\frac{Ba-Ab}{3(b^3x^3+ab^2)}+\frac{B\log(bx^3+a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)/(b^3*x^3 + a*b^2) + 1/3*B*log(b*x^3 + a)/b^2
```

Fricas [A]
time = 2.09, size = 44, normalized size = 1.07

$$\frac{Ba-Ab+(Bbx^3+Ba)\log(bx^3+a)}{3(b^3x^3+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(B*a - A*b + (B*b*x^3 + B*a)*log(b*x^3 + a))/(b^3*x^3 + a*b^2)
```

Sympy [A]
time = 0.36, size = 36, normalized size = 0.88

$$\frac{B\log\left(a+bx^3\right)}{3b^2}+\frac{-Ab+Ba}{3ab^2+3b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*log(a + b*x**3)/(3*b**2) + (-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3)
```

Giac [A]
time = 0.69, size = 65, normalized size = 1.59

$$-\frac{B\left(\frac{\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b}-\frac{a}{(bx^3+a)b}\right)}{3\,b}-\frac{A}{3\,(bx^3+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*B*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/b
 - 1/3*A/((b*x^3 + a)*b)
```

Mupad [B]
time = 2.35, size = 37, normalized size = 0.90

$$\frac{B\,\ln\left(b\,x^3+a\right)}{3\,b^2}-\frac{A\,b-B\,a}{3\,b^2\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*log(a + b*x^3))/(3*b^2) - (A*b - B*a)/(3*b^2*(a + b*x^3))
```

3.79 $\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=171

$$\frac{(Ab-aB)x^2}{3ab\,(a+bx^3)}-\frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}-\frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}}+\frac{(Ab+2aB)\log\left(a^{2/3}-\right.}{18a^{4/3}}$$

[Out] 1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)-1/9*(A*b+2*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(5/3)+1/18*(A*b+2*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(5/3)-1/9*(A*b+2*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(5/3)*3^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {468, 298, 31, 648, 631, 210, 642}

$$-\frac{(2aB+Ab)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}}+\frac{x^2(Ab-aB)}{3ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x^3))/(a + b*x^3)^2,x]

[Out] ((A*b - a*B)*x^2)/(3*a*b*(a + b*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - ((A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(5/3)) + ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(5/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} + \frac{(Ab+2aB)\int \frac{x}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} - \frac{(Ab+2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{4/3}b^{4/3}} + \frac{(Ab+2aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{4/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}}{18a^{4/3}b^{5/3}}$$

$$= \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x\right.}{18a^{4/3}b^{5/3}}$$

$$= \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} - \frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}} - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}}$$

Mathematica [A]
time = 0.06, size = 146, normalized size = 0.85

$$\frac{-\frac{6\sqrt[3]{a}\,b^{2/3}(-Ab+aB)x^2}{a+bx^3} - 2\sqrt{3}\,(Ab+2aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + (Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(1/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(A*b + 2*a
*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(A*b + 2*a*B)*Log[a^(1/
3) + b^(1/3)*x] + (A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(4/3)*b^(5/3))
```

Maple [A]
time = 0.27, size = 136, normalized size = 0.80

method	result	size
risch	$\frac{(Ab-Ba)x^2}{3ab(b\,x^3+a)} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{(Ab+2Ba)\ln\left(x-\_R\right)}{\_R}}{9a\,b^2}$	67

default	$\frac{(Ab-Ba)x^2}{3ab(b\,x^3+a)}+\frac{(Ab+2Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab}$	136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)+1/3*(A*b+2*B*a)/a/b*(-1/3/b/(a/b)^(1/3)*ln(
x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1
/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
```

Maxima [A]
time = 0.52, size = 160, normalized size = 0.94

$$-\frac{(Ba-Ab)x^2}{3\,(ab^2x^3+a^2b)}+\frac{\sqrt{3}\,(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(2\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(2\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^2/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*B*a + A*b)*arctan
(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/18*(2
*B*a + A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/
9*(2*B*a + A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

Fricas [A]
time = 2.04, size = 548, normalized size = 3.20

$$\left[-\frac{6\,(Ba^2b^2-Aab^3)x^2-3\sqrt{\frac{1}{3}}\,(2\,Ba^3b+Aa^2b^2+(2\,Ba^2b^2+Aab^3)x^3)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3(-ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)-((2\,Bab+Ab^2)x^3+2\,Ba^2+Aab)(-ab^2)^{\frac{2}{3}}\log\left(b^2x^2+(-ab^2)^{\frac{1}{3}}bx+(-ab^2)^{\frac{2}{3}}\right)+2\,((2\,Bab+Ab^2)x^3+2\,Ba^2+Aab)(-ab^2)^{\frac{2}{3}}\log\left(bx-(-ab^2)^{\frac{1}{3}}\right)}{18\,(a^2b^4x^3+a^3b^3)},\ -\frac{6\,(Ba^2b^2-Aab^3)x^2-6\sqrt{\frac{1}{3}}\,(2\,Ba^3b+Aa^2b^2+(2\,Ba^2b^2+Aab^3)x^3)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right)-((2\,Bab+Ab^2)x^3+2\,Ba^2+Aab)(-ab^2)^{\frac{2}{3}}\log\left(b^2x^2+(-ab^2)^{\frac{1}{3}}bx+(-ab^2)^{\frac{2}{3}}\right)+2\,((2\,Bab+Ab^2)x^3+2\,Ba^2+Aab)(-ab^2)^{\frac{2}{3}}\log\left(bx-(-ab^2)^{\frac{1}{3}}\right)}{18\,(a^2b^4x^3+a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(6*(B*a^2*b^2 - A*a*b^3)*x^2 - 3*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 +
(2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b +
3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)
^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((2*B*a*b + A*b^2)*x^3 + 2*B
*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2
```

```
/3)) + 2*((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x -
 (-a*b^2)^(1/3)))/(a^2*b^4*x^3 + a^3*b^3), -1/18*(6*(B*a^2*b^2 - A*a*b^3)*x
^2 - 6*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 + (2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt
(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2
)^(1/3)/a)/b) - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((2*B*a*b + A*b^2)*x^3
 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x^3
+ a^3*b^3)]
```

Sympy [A]

time = 0.36, size = 117, normalized size = 0.68

$$\frac{x^2(Ab-Ba)}{3a^2b+3ab^2x^3}+\operatorname{RootSum}\left(729t^3a^4b^5+A^3b^3+6A^2Bab^2+12AB^2a^2b+8B^3a^3,\left(t\mapsto t\log\left(\frac{81t^2a^3b^3}{A^2b^2+4ABab+4B^2a^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] x**2*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**4*b**5 +
 A**3*b**3 + 6*A**2*B*a*b**2 + 12*A*B**2*a**2*b + 8*B**3*a**3, Lambda(_t, _
t*log(81*_t**2*a**3*b**3/(A**2*b**2 + 4*A*B*a*b + 4*B**2*a**2) + x)))
```

Giac [A]

time = 0.62, size = 186, normalized size = 1.09

$$\frac{\sqrt{3}\,(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{1}{3}}ab}-\frac{(2\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{1}{3}}ab}-\frac{\left(2\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b}-\frac{Bax^2-Abx^2}{3\,(bx^3+a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(1/3)*a*b) - 1/18*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b) - 1/9*(2*B*a*(-a/b)^(1/3) + A*b*(-a/b)^(1
/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x^2 - A*b*x
^2)/((b*x^3 + a)*a*b)
```

Mupad [B]

time = 0.25, size = 145, normalized size = 0.85

$$\frac{x^2\,(A\,b-B\,a)}{3\,a\,b\,\left(b\,x^3+a\right)}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a
^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log(b^
(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) + (x^2*(A*b - B*a))/(
3*a*b*(a + b*x^3))
```

3.80 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=169

$$\frac{(Ab-aB)x}{3ab\left(a+bx^3\right)}-\frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}}-\frac{(2Ab+aB)\log\left(a^{2/3}-\right.}{18a^{5/3}b}$$

```
[Out] 1/3*(A*b-B*a)*x/a/b/(b*x^3+a)+1/9*(2*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)
/b^(4/3)-1/18*(2*A*b+B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)
/b^(4/3)-1/9*(2*A*b+B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(5/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {393, 206, 31, 648, 631, 210, 642}

$$-\frac{(aB+2Ab)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}-\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x)/(3*a*b*(a + b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*A*b + a*B)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*A*b + a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x}{3ab\,(a+bx^3)} + \frac{(2Ab+aB)\int \frac{1}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x}{3ab\,(a+bx^3)} + \frac{(2Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{5/3}b} + \frac{(2Ab+aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{5/3}b}$$

$$= \frac{(Ab-aB)x}{3ab\,(a+bx^3)} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x}{3ab\,(a+bx^3)} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2}\right.}{18a^{5/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x}{3ab\,(a+bx^3)} - \frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}} - \frac{(}{}$$

Mathematica [A]
time = 0.05, size = 145, normalized size = 0.86

$$\frac{-\frac{6a^{2/3}\sqrt[3]{b}\,(-Ab+aB)x}{a+bx^3} - 2\sqrt{3}\,(2Ab+aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) - (2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*A*b + a*B)*Log[a^(1/3)
 + b^(1/3)*x] - (2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(4/3))
```

Maple [A]
time = 0.28, size = 134, normalized size = 0.79

method	result	size
risch	$\frac{(Ab-Ba)x}{3ab(b\,x^3+a)} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{(2Ab+Ba)\ln\left(x-\_R\right)}{\_R^2}}{9a\,b^2}$	65

default	$\frac{(Ab-Ba)x}{3ab(b\,x^{3}+a)}+\frac{(2Ab+Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3ab}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(A*b-B*a)*x/a/b/(b*x^3+a)+1/3*(2*A*b+B*a)/a/b*(1/3/b/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
```

Maxima [A]
time = 0.51, size = 158, normalized size = 0.93

$$-\frac{(Ba-Ab)x}{3\,(ab^{2}x^{3}+a^{2}b)}+\frac{\sqrt{3}\,(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba+2\,Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba+2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(B*a + 2*A*b)*arctan(1
/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(B*a
 + 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/9*
(B*a + 2*A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Fricas [A]
time = 1.87, size = 537, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\,(Ba^{3}b+2\,Aa^{2}b^{2}+(Ba^{2}b^{2}+2\,Aab^{3})x^{3})\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\,(a^{2}b)^{\frac{1}{3}}ax-a^{2}+3\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)-((Bab+2\,Ab^{2})x^{3}+Ba^{2}+2\,Aab)(a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(a^{2}b)^{\frac{2}{3}}x+(a^{2}b)^{\frac{1}{3}}a\right)+2\,((Bab+2\,Ab^{2})x^{3}+Ba^{2}+2\,Aab)(a^{2}b)^{\frac{2}{3}}\log\left(abx+(a^{2}b)^{\frac{2}{3}}\right)-6\,(Ba^{3}b-Aa^{2}b^{2})x}{18\,(a^{3}b^{3}x^{3}+a^{4}b^{2})},\ \frac{6\sqrt{\frac{1}{3}}\,(Ba^{3}b+2\,Aa^{2}b^{2}+(Ba^{2}b^{2}+2\,Aab^{3})x^{3})\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-((Bab+2\,Ab^{2})x^{3}+Ba^{2}+2\,Aab)(a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(a^{2}b)^{\frac{2}{3}}x+(a^{2}b)^{\frac{1}{3}}a\right)+2\,((Bab+2\,Ab^{2})x^{3}+Ba^{2}+2\,Aab)(a^{2}b)^{\frac{2}{3}}\log\left(abx+(a^{2}b)^{\frac{2}{3}}\right)-6\,(Ba^{3}b-Aa^{2}b^{2})x}{18\,(a^{3}b^{3}x^{3}+a^{4}b^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sq
rt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/
3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/
(b*x^3 + a)) - ((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B
```

```
*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a
^2*b^2)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^
2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b + 2
*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a^2*b^2)*x)/(a^3*b^3*x^3 +
a^4*b^2)]
```

Sympy [A]
time = 0.31, size = 97, normalized size = 0.57

$$\frac{x(Ab-Ba)}{3a^2b+3ab^2x^3}+\operatorname{RootSum}\left(729t^3a^5b^4-8A^3b^3-12A^2Bab^2-6AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{9ta^2b}{2Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] x*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - 8*
A**3*b**3 - 12*A**2*B*a*b**2 - 6*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*l
og(9*_t*a**2*b/(2*A*b + B*a) + x)))
```

Giac [A]
time = 0.64, size = 160, normalized size = 0.95

$$-\frac{\sqrt{3}\,(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a}-\frac{(Ba+2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a}-\frac{(Ba+2\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b}-\frac{Bax-Abx}{3\,(bx^3+a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*a) - 1/18*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(B*a + 2*A*b)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a*b)
```

Mupad [B]
time = 2.55, size = 143, normalized size = 0.85

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,A\,b+B\,a\right)}{9\,a^{5/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b+B\,a\right)}{9\,a^{5/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b+B\,a\right)}{9\,a^{5/3}\,b^{4/3}}+\frac{x\,\left(A\,b-B\,a\right)}{3\,a\,b\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^2,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a))/
(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) + (x*(A*b - B*a))/(3*
a*b*(a + b*x^3))
```

3.81 $\int \frac{A+Bx^3}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=51

$$\frac{Ab-aB}{3ab\left(a+bx^3\right)}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^3\right)}{3a^2}$$

[Out] 1/3*(A*b-B*a)/a/b/(b*x^3+a)+A*ln(x)/a^2-1/3*A*ln(b*x^3+a)/a^2

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$-\frac{A\log\left(a+bx^3\right)}{3a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x*(a + b*x^3)^2),x]

[Out] (A*b - a*B)/(3*a*b*(a + b*x^3)) + (A*Log[x])/a^2 - (A*Log[a + b*x^3])/(3*a^2)

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^2x}+\frac{-Ab+aB}{a(a+bx)^2}-\frac{Ab}{a^2(a+bx)}\right)\,dx,x,x^3\right)\\ &= \frac{Ab-aB}{3ab\left(a+bx^3\right)}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^3\right)}{3a^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.90

$$\frac{\frac{a(Ab-aB)}{b(a+bx^3)}+3A\log(x)-A\log\left(a+bx^3\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*(A*b - a*B))/(b*(a + b*x^3)) + 3*A*Log[x] - A*Log[a + b*x^3])/(3*a^2)
```

Maple [A]
time = 0.28, size = 48, normalized size = 0.94

method	result	size
default	$-\frac{-\frac{a(Ab-Ba)}{b\left(b\,x^{3}+a\right)}+A\ln\left(b\,x^{3}+a\right)}{3a^{2}}+\frac{A\ln(x)}{a^{2}}$	48
norman	$-\frac{(Ab-Ba)x^{3}}{3a^{2}(b\,x^{3}+a)}+\frac{A\ln(x)}{a^{2}}-\frac{A\ln\left(b\,x^{3}+a\right)}{3a^{2}}$	48
risch	$\frac{A}{3a(b\,x^{3}+a)}-\frac{B}{3b(b\,x^{3}+a)}+\frac{A\ln(x)}{a^{2}}-\frac{A\ln\left(b\,x^{3}+a\right)}{3a^{2}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2*(-a*(A*b-B*a)/b/(b*x^3+a)+A*ln(b*x^3+a))+A*ln(x)/a^2
```

Maxima [A]
time = 0.30, size = 51, normalized size = 1.00

$$-\frac{Ba-Ab}{3\left(ab^2x^3+a^2b\right)}-\frac{A\log\left(bx^3+a\right)}{3\,a^2}+\frac{A\log\left(x^3\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)/(a*b^2*x^3 + a^2*b) - 1/3*A*log(b*x^3 + a)/a^2 + 1/3*A*log
(x^3)/a^2
```

Fricas [A]
time = 1.93, size = 70, normalized size = 1.37

$$-\frac{Ba^2 - Aab + (Ab^2x^3 + Aab)\log(bx^3 + a) - 3(Ab^2x^3 + Aab)\log(x)}{3(a^2b^2x^3 + a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(B*a^2 - A*a*b + (A*b^2*x^3 + A*a*b)*log(b*x^3 + a) - 3*(A*b^2*x^3 + A
*a*b)*log(x))/(a^2*b^2*x^3 + a^3*b)
```

Sympy [A]
time = 0.27, size = 46, normalized size = 0.90

$$\frac{A\log(x)}{a^2} - \frac{A\log\left(\frac{a}{b} + x^3\right)}{3a^2} + \frac{Ab - Ba}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**2,x)
```

```
[Out] A*log(x)/a**2 - A*log(a/b + x**3)/(3*a**2) + (A*b - B*a)/(3*a**2*b + 3*a*b*
*2*x**3)
```

Giac [A]
time = 0.75, size = 61, normalized size = 1.20

$$-\frac{A\log(|bx^3 + a|)}{3a^2} + \frac{A\log(|x|)}{a^2} + \frac{Ab^2x^3 - Ba^2 + 2Aab}{3(bx^3 + a)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*A*log(abs(b*x^3 + a))/a^2 + A*log(abs(x))/a^2 + 1/3*(A*b^2*x^3 - B*a^2
 + 2*A*a*b)/((b*x^3 + a)*a^2*b)
```

Mupad [B]
time = 0.14, size = 47, normalized size = 0.92

$$\frac{A\ln(x)}{a^2} - \frac{A\ln(bx^3 + a)}{3a^2} + \frac{Ab - Ba}{3ab(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^2),x)
```

```
[Out] (A*log(x))/a^2 - (A*log(a + b*x^3))/(3*a^2) + (A*b - B*a)/(3*a*b*(a + b*x^3
))
```

3.82 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{-4Ab+aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}b^{2/3}}-\frac{(4Ab-a}{}$$

```
[Out] 1/3*(-4*A*b+B*a)/a^2/b/x+1/3*(A*b-B*a)/a/b/x/(b*x^3+a)+1/9*(4*A*b-B*a)*ln(a
^(1/3)+b^(1/3)*x)/a^(7/3)/b^(2/3)-1/18*(4*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(7/3)/b^(2/3)+1/9*(4*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 196, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {468, 331, 298, 31, 648, 631, 210, 642}

$$\frac{(4Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}-\frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}b^{2/3}}-\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -1/3*(4*A*b - a*B)/(a^2*b*x) + (A*b - a*B)/(3*a*b*x*(a + b*x^3)) + ((4*A*b
- a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3
)*b^(2/3)) + ((4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(2/3)) -
 ((4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)
*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx\left(a+bx^3\right)} + \frac{(4Ab-aB)\int \frac{1}{x^2(a+bx^3)}\,dx}{3ab}$$

$$= -\frac{4Ab-aB}{3a^2bx} + \frac{Ab-aB}{3abx\left(a+bx^3\right)} - \frac{(4Ab-aB)\int \frac{x}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{4Ab-aB}{3a^2bx} + \frac{Ab-aB}{3abx\left(a+bx^3\right)} + \frac{(4Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{7/3}\sqrt[3]{b}} - \cdots$$

$$= -\frac{4Ab-aB}{3a^2bx} + \frac{Ab-aB}{3abx\left(a+bx^3\right)} + \frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}b^{2/3}} - \cdots$$

$$= -\frac{4Ab-aB}{3a^2bx} + \frac{Ab-aB}{3abx\left(a+bx^3\right)} + \frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}b^{2/3}} - \cdots$$

$$= -\frac{4Ab-aB}{3a^2bx} + \frac{Ab-aB}{3abx\left(a+bx^3\right)} + \frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}} + \cdots$$

Mathematica [A]
time = 0.07, size = 164, normalized size = 0.84

$$\frac{-\frac{18\sqrt[3]{a}\,A}{x}+\frac{6\sqrt[3]{a}\,(-Ab+aB)x^2}{a+bx^3}+\frac{2\sqrt{3}\,(4Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{2(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{(-4Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}}{18a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(1/3)*A)/x + (6*a^(1/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3
]*(4*A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(
4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-4*A*b + a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^(7/3))
```

Maple [A]
time = 0.30, size = 139, normalized size = 0.71

method	result

default	$-\frac{\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x^2}{b\,x^3+a}+\left(\frac{4Ab}{3}-\frac{Ba}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2}-\frac{A}{a^2x}$
risch	$\frac{-\frac{(4Ab-Ba)x^3}{3a^2}-\frac{A}{a}}{x(b\,x^3+a)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^7b^2\_Z^3-64A^3b^3+48A^2Ba\,b^2-12A\,B^2a^2b+B^3a^3\right)}\_R\ln\left(\left(-4\_R^3a^7b^2+192A^3b^3-144A^2Ba\right.\right.\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((1/3*A*b-1/3*B*a)*x^2/(b*x^3+a)+(4/3*A*b-1/3*B*a)*(-1/3/b/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/
3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-A/a^2/x
```

Maxima [A]
time = 0.55, size = 166, normalized size = 0.85

$$\frac{(Ba-4\,Ab)x^3-3\,Aa}{3\,(a^2bx^4+a^3x)}+\frac{\sqrt{3}\,(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-4\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba-4\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 4*A*b)*x^3 - 3*A*a)/(a^2*b*x^4 + a^3*x) + 1/9*sqrt(3)*(B*a - 4*
A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)
) + 1/18*(B*a - 4*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^
(1/3)) - 1/9*(B*a - 4*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

Fricas [A]
time = 2.04, size = 570, normalized size = 2.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(18*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 3*sqrt(1/3)*((B*a^2*
b^2 - 4*A*a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*lo
```

```
g((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/
3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((B*a*b -
4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/
3)*b*x + (a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(
a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x), -1/18*(18
*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 6*sqrt(1/3)*((B*a^2*b^2 - 4*A*
a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(
1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - ((B*a*b - 4*A*b^2)*
x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x +
(a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2
/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x)]
```

Sympy [A]
time = 0.34, size = 122, normalized size = 0.63

$$\frac{-3Aa + x^3(-4Ab + Ba)}{3a^3x + 3a^2bx^4} + \operatorname{RootSum}\left(729t^3a^7b^2 - 64A^3b^3 + 48A^2Bab^2 - 12AB^2a^2b + B^3a^3, \left(t \mapsto t\log\left(\frac{81t^2a^5b}{16A^2b^2 - 8ABab + B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*A*a + x**3*(-4*A*b + B*a))/(3*a**3*x + 3*a**2*b*x**4) + RootSum(729*_t*
*3*a**7*b**2 - 64*A**3*b**3 + 48*A**2*B*a*b**2 - 12*A*B**2*a**2*b + B**3*a*
*3, Lambda(_t, _t*log(81*_t**2*a**5*b/(16*A**2*b**2 - 8*A*B*a*b + B**2*a**2
) + x)))
```

Giac [A]
time = 0.59, size = 180, normalized size = 0.92

$$\frac{\sqrt{3}\,(Ba - 4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{1}{3}}\,a^2} - \frac{(Ba - 4\,Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{1}{3}}\,a^2} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{Bax^3 - 4\,Abx^3 - 3\,Aa}{3\,(bx^4 + ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(1/3)*a^2) - 1/18*(B*a - 4*A*b)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2) - 1/9*(B*a*(-a/b)^(1/3) - 4*A*b*(-a/b)^(1
/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/3*(B*a*x^3 - 4*A*b*x^3
 - 3*A*a)/((b*x^4 + a*x)*a^2)
```

Mupad [B]
time = 2.57, size = 156, normalized size = 0.80

$$\frac{\ln\left(b^{1/3}\,x + a^{1/3}\right)\,\left(4\,A\,b - B\,a\right)}{9\,a^{7/3}\,b^{2/3}} - \frac{\frac{A}{a} + \frac{x^3\,(4\,A\,b - B\,a)}{3\,a^2}}{b\,x^4 + a\,x} + \frac{\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,A\,b - B\,a\right)}{9\,a^{7/3}\,b^{2/3}} - \frac{\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,A\,b - B\,a\right)}{9\,a^{7/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3)) - (A/a + (x^3*
(4*A*b - B*a))/(3*a^2))/(a*x + b*x^4) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3)) - (
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*A
*b - B*a))/(9*a^(7/3)*b^(2/3))
```

3.83 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=196

$$\frac{-5Ab+2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}+\frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}\sqrt[3]{b}}+\frac{(5A$$

```
[Out] 1/6*(-5*A*b+2*B*a)/a^2/b/x^2+1/3*(A*b-B*a)/a/b/x^2/(b*x^3+a)-1/9*(5*A*b-2*B
*a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(1/3)+1/18*(5*A*b-2*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(1/3)+1/9*(5*A*b-2*B*a)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {468, 331, 206, 31, 648, 631, 210, 642}

$$\frac{(5Ab-2aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}\sqrt[3]{b}}-\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/6*(5*A*b - 2*a*B)/(a^2*b*x^2) + (A*b - a*B)/(3*a*b*x^2*(a + b*x^3)) + ((
5*A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*a^(8/3)*b^(1/3)) - ((5*A*b - 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*
b^(1/3)) + ((5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(18*a^(8/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^2\left(a+bx^3\right)} + \frac{(5Ab-2aB)\int \frac{1}{x^3(a+bx^3)}\,dx}{3ab}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2} + \frac{Ab-aB}{3abx^2\left(a+bx^3\right)} - \frac{(5Ab-2aB)\int \frac{1}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2} + \frac{Ab-aB}{3abx^2\left(a+bx^3\right)} - \frac{(5Ab-2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{8/3}} - \frac{(5Ab-2aB)\int \frac{}{a^{2/3}}}{9a}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2} + \frac{Ab-aB}{3abx^2\left(a+bx^3\right)} - \frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}\sqrt[3]{b}} - \frac{(5Ab-2aB)\int}{}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2} + \frac{Ab-aB}{3abx^2\left(a+bx^3\right)} - \frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}\sqrt[3]{b}} + \frac{(5Ab-2aB)\,\mathrm{lo}}{}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2} + \frac{Ab-aB}{3abx^2\left(a+bx^3\right)} + \frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}} - \frac{(5Ab-2aB)}{9}$$

Mathematica [A]
time = 0.08, size = 163, normalized size = 0.83

$$\frac{-\frac{9a^{2/3}A}{x^2} + \frac{6a^{2/3}(-Ab+aB)x}{a+bx^3} + \frac{2\sqrt{3}\,(5Ab-2aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{2(-5Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} + \frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{18a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(2/3)*A)/x^2 + (6*a^(2/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + (2*Sqrt[3]
*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (2*
(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((5*A*b - 2*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(18*a^(8/3))
```

Maple [A]
time = 0.29, size = 138, normalized size = 0.70

method	result

default	$-\frac{\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x}{b\,x^{3}+a}+\frac{(5Ab-2Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}}{a^{2}}-\frac{A}{2a^{2}x^{2}}$
risch	$\frac{-\frac{(5Ab-2Ba)x^{3}}{6a^{2}}-\frac{A}{2a}}{x^{2}(b\,x^{3}+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{8}b\_Z^{3}+125A^{3}b^{3}-150A^{2}Ba\,b^{2}+60A\,B^{2}a^{2}b-8B^{3}a^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{8}b-375A^{3}b^{3}+450A^{2}B\right.\right.\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((1/3*A*b-1/3*B*a)*x/(b*x^3+a)+1/3*(5*A*b-2*B*a)*(1/3/b/(a/b)^(2/3)*
ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/2*A/a^2/x^
2
```

Maxima [A]
time = 0.53, size = 172, normalized size = 0.88

$$\frac{(2\,Ba-5\,Ab)x^{3}-3\,Aa}{6\,(a^{2}bx^{5}+a^{3}x^{2})}+\frac{\sqrt{3}\,(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(2\,Ba-5\,Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(2\,Ba-5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*((2*B*a - 5*A*b)*x^3 - 3*A*a)/(a^2*b*x^5 + a^3*x^2) + 1/9*sqrt(3)*(2*B*
a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)
^(2/3)) - 1/18*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*
b*(a/b)^(2/3)) + 1/9*(2*B*a - 5*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3
))
```

Fricas [A]
time = 3.28, size = 618, normalized size = 3.15

$$\left[-\frac{9\,Aa^{3}b-3\,(2\,Ba^{3}b-5\,Aa^{2}b^{2})x^{3}+3\,\sqrt{\frac{1}{3}}\,((2\,Ba^{2}b^{2}-5\,Aab^{3})x^{5}+(2\,Ba^{3}b-5\,Aa^{2}b^{2})x^{2})\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}+3\,(-a^{2}b)^{\frac{1}{3}}ax-a^{2}-3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(-a^{2}b)^{\frac{2}{3}}x+(-a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)+((2\,Bab-5\,Ab^{2})x^{5}+(2\,Ba^{2}-5\,Aab)x^{2})(-a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(-a^{2}b)^{\frac{2}{3}}x-(-a^{2}b)^{\frac{1}{3}}a\right)-2\,((2\,Bab-5\,Ab^{2})x^{5}+(2\,Ba^{2}-5\,Aab)x^{2})(-a^{2}b)^{\frac{2}{3}}\log\left(abx+(-a^{2}b)^{\frac{2}{3}}\right)}{18\,(a^{4}b^{2}x^{5}+a^{5}bx^{2})},\ -\frac{9\,Aa^{3}b-3\,(2\,Ba^{3}b-5\,Aa^{2}b^{2})x^{3}-6\,\sqrt{\frac{1}{3}}\,((2\,Ba^{2}b^{2}-5\,Aab^{3})x^{5}+(2\,Ba^{3}b-5\,Aa^{2}b^{2})x^{2})\sqrt{-\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(-a^{2}b)^{\frac{2}{3}}x+(-a^{2}b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right)+((2\,Bab-5\,Ab^{2})x^{5}+(2\,Ba^{2}-5\,Aab)x^{2})(-a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(-a^{2}b)^{\frac{2}{3}}x-(-a^{2}b)^{\frac{1}{3}}a\right)-2\,((2\,Bab-5\,Ab^{2})x^{5}+(2\,Ba^{2}-5\,Aab)x^{2})(-a^{2}b)^{\frac{2}{3}}\log\left(abx+(-a^{2}b)^{\frac{2}{3}}\right)}{18\,(a^{4}b^{2}x^{5}+a^{5}bx^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 + 3*sqrt(1/3)*((2*B*a^2
*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/
b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (
-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) +
((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*
x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (
2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^
2*x^5 + a^5*b*x^2), -1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 - 6*
sqrt(1/3)*((2*B*a^2*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*s
qrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3
)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5
*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)
*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*
log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^2*x^5 + a^5*b*x^2)]
```

Sympy [A]
time = 0.37, size = 109, normalized size = 0.56

$$\frac{-3Aa + x^3(-5Ab + 2Ba)}{6a^3x^2 + 6a^2bx^5} + \operatorname{RootSum}\left(729t^3a^8b + 125A^3b^3 - 150A^2Bab^2 + 60AB^2a^2b - 8B^3a^3, \left(t \mapsto t\log\left(\frac{9ta^3}{-5Ab + 2Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**2,x)
```

```
[Out] (-3*A*a + x**3*(-5*A*b + 2*B*a))/(6*a**3*x**2 + 6*a**2*b*x**5) + RootSum(72
9*_t**3*a**8*b + 125*A**3*b**3 - 150*A**2*B*a*b**2 + 60*A*B**2*a**2*b - 8*B
**3*a**3, Lambda(_t, _t*log(9*_t*a**3/(-5*A*b + 2*B*a) + x)))
```

Giac [A]
time = 0.80, size = 188, normalized size = 0.96

$$-\frac{(2\,Ba - 5\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}Ba - 5\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} + \frac{Bax - Abx}{3\,(bx^3 + a)a^2} + \frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}Ba - 5\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b} - \frac{A}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*B*a - 5*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/9*sqrt
(3)*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a^2
) + 1/18*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))/(a^3*b) - 1/2*A/(a^2*x^2)
```

Mupad [B]
time = 2.57, size = 159, normalized size = 0.81

$$-\frac{\frac{A}{2\,a} + \frac{x^3\,(5\,A\,b - 2\,B\,a)}{6\,a^2}}{b\,x^5 + a\,x^2} - \frac{\ln\left(b^{1/3}\,x + a^{1/3}\right)\,(5\,A\,b - 2\,B\,a)}{9\,a^{8/3}\,b^{1/3}} + \frac{\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b - 2\,B\,a)}{9\,a^{8/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b - 2\,B\,a)}{9\,a^{8/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*
A*b - 2*B*a))/(9*a^(8/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(5*A*b - 2*B*
a))/(9*a^(8/3)*b^(1/3)) - (A/(2*a) + (x^3*(5*A*b - 2*B*a))/(6*a^2))/(a*x^2
+ b*x^5) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2
 - 1/2)*(5*A*b - 2*B*a))/(9*a^(8/3)*b^(1/3))
```

3.84 $\int \frac{A+Bx^3}{x^4 (a+bx^3)^2} \, dx$

Optimal. Leaf size=76

$$-\frac{A}{3a^2x^3}-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}$$

```
[Out] -1/3*A/a^2/x^3+1/3*(-A*b+B*a)/a^2/(b*x^3+a)-(2*A*b-B*a)*ln(x)/a^3+1/3*(2*A*
b-B*a)*ln(b*x^3+a)/a^3
```

Rubi [A]

time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}-\frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/3*A/(a^2*x^3) - (A*b - a*B)/(3*a^2*(a + b*x^3)) - ((2*A*b - a*B)*Log[x])
/a^3 + ((2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^4(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^2x^2} + \frac{-2Ab+aB}{a^3x} - \frac{b(-Ab+aB)}{a^2(a+bx)^2} - \frac{b(-2Ab+aB)}{a^3(a+bx)}\right) dx, x, x^3\right)$$
$$= -\frac{A}{3a^2x^3} - \frac{Ab-aB}{3a^2(a+bx^3)} - \frac{(2Ab-aB)\log(x)}{a^3} + \frac{(2Ab-aB)\log(a+bx^3)}{3a^3}$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 0.84

$$\frac{-\frac{aA}{x^3} + \frac{a(-Ab+aB)}{a+bx^3} + 3(-2Ab+aB)\log(x) + (2Ab-aB)\log(a+bx^3)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((a*A)/x^3) + (a*(-(A*b) + a*B))/(a + b*x^3) + 3*(-2*A*b + a*B)*Log[x] +
(2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Maple [A]
time = 0.27, size = 76, normalized size = 1.00

method	result	size
default	$\frac{b\left(-\frac{a(Ab-Ba)}{b(bx^3+a)}+\frac{(2Ab-Ba)\ln(bx^3+a)}{b}\right)}{3a^3} - \frac{A}{3a^2x^3} + \frac{(-2Ab+Ba)\ln(x)}{a^3}$	76
norman	$\frac{-\frac{A}{3a}+\frac{b(2Ab-Ba)x^6}{3a^3}}{x^3(bx^3+a)} - \frac{(2Ab-Ba)\ln(x)}{a^3} + \frac{(2Ab-Ba)\ln(bx^3+a)}{3a^3}$	78
risch	$\frac{-\frac{(2Ab-Ba)x^3}{3a^2}-\frac{A}{3a}}{x^3(bx^3+a)} - \frac{2\ln(x)Ab}{a^3} + \frac{B\ln(x)}{a^2} + \frac{2\ln(-bx^3-a)Ab}{3a^3} - \frac{\ln(-bx^3-a)B}{3a^2}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^3*b*(-a*(A*b-B*a)/b/(b*x^3+a)+(2*A*b-B*a)/b*ln(b*x^3+a))-1/3*A/a^2/x^
3+(-2*A*b+B*a)/a^3*ln(x)
```

Maxima [A]
time = 0.28, size = 76, normalized size = 1.00

$$\frac{(Ba-2\,Ab)x^3 - Aa}{3\,(a^2bx^6 + a^3x^3)} - \frac{(Ba-2\,Ab)\log(bx^3+a)}{3\,a^3} + \frac{(Ba-2\,Ab)\log(x^3)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 2*A*b)*x^3 - A*a)/(a^2*b*x^6 + a^3*x^3) - 1/3*(B*a - 2*A*b)*log
(b*x^3 + a)/a^3 + 1/3*(B*a - 2*A*b)*log(x^3)/a^3
```

Fricas [A]
time = 2.37, size = 118, normalized size = 1.55

$$\frac{(Ba^2 - 2\,Aab)x^3 - Aa^2 - ((Bab - 2\,Ab^2)x^6 + (Ba^2 - 2\,Aab)x^3)\log\left(bx^3 + a\right) + 3\left((Bab - 2\,Ab^2)x^6 + (Ba^2 - 2\,Aab)x^3\right)\log\left(x\right)}{3\left(a^3bx^6 + a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((B*a^2 - 2*A*a*b)*x^3 - A*a^2 - ((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*
a*b)*x^3)*log(b*x^3 + a) + 3*((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*a*b)*x^3
)*log(x))/(a^3*b*x^6 + a^4*x^3)
```

Sympy [A]
time = 0.68, size = 70, normalized size = 0.92

$$\frac{-Aa + x^3(-2Ab + Ba)}{3a^3x^3 + 3a^2bx^6} + \frac{(-2Ab + Ba)\log\left(x\right)}{a^3} - \frac{(-2Ab + Ba)\log\left(\frac{a}{b} + x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**2,x)
```

```
[Out] (-A*a + x**3*(-2*A*b + B*a))/(3*a**3*x**3 + 3*a**2*b*x**6) + (-2*A*b + B*a)
*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

Giac [A]
time = 1.46, size = 80, normalized size = 1.05

$$\frac{(Ba - 2\,Ab)\log\left(|x|\right)}{a^3} + \frac{Bax^3 - 2\,Abx^3 - Aa}{3\,(bx^6 + ax^3)a^2} - \frac{(Bab - 2\,Ab^2)\log\left(|bx^3 + a|\right)}{3\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (B*a - 2*A*b)*log(abs(x))/a^3 + 1/3*(B*a*x^3 - 2*A*b*x^3 - A*a)/((b*x^6 + a
*x^3)*a^2) - 1/3*(B*a*b - 2*A*b^2)*log(abs(b*x^3 + a))/(a^3*b)
```

Mupad [B]
time = 2.43, size = 78, normalized size = 1.03

$$\frac{\ln\left(b\,x^3 + a\right)\,\left(2\,A\,b - B\,a\right)}{3\,a^3} - \frac{\frac{A}{3\,a} + \frac{x^3\,(2\,A\,b - B\,a)}{3\,a^2}}{b\,x^6 + a\,x^3} - \frac{\ln\left(x\right)\,\left(2\,A\,b - B\,a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (log(a + b*x^3)*(2*A*b - B*a))/(3*a^3) - (A/(3*a) + (x^3*(2*A*b - B*a))/(3*
a^2))/(a*x^3 + b*x^6) - (log(x)*(2*A*b - B*a))/a^3
```

3.85 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=215

$$\frac{-7Ab+4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{\sqrt[3]{b}\,(7Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{\sqrt[3]{b}\,(7Ab-4aB)\log}{9a^{10/3}}$$

[Out] 1/12*(-7*A*b+4*B*a)/a^2/b/x^4+1/3*(7*A*b-4*B*a)/a^3/x+1/3*(A*b-B*a)/a/b/x^4 /(b*x^3+a)-1/9*b^(1/3)*(7*A*b-4*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)+1/18*b^ (1/3)*(7*A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)-1/9* b^(1/3)*(7*A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)*3^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {468, 331, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt[3]{b}\,(7Ab-4aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}+\frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{10/3}}-\frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{10/3}}+\frac{7Ab-4aB}{3a^3x}-\frac{7Ab-4aB}{12a^2bx^4}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]

[Out] -1/12*(7*A*b - 4*a*B)/(a^2*b*x^4) + (7*A*b - 4*a*B)/(3*a^3*x) + (A*b - a*B) /(3*a*b*x^4*(a + b*x^3)) - (b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (b^(1/3)*(7*A*b - 4*a*B) *Log[a^(1/3) + b^(1/3)*x])/(9*a^(10/3)) + (b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2 /3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^4\left(a+bx^3\right)}+\frac{(7Ab-4aB)\int \frac{1}{x^5(a+bx^3)}\,dx}{3ab}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{(7Ab-4aB)\int \frac{1}{x^2(a+bx^3)}\,dx}{3a^2}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}+\frac{(b(7Ab-4aB))\int \frac{x}{a+bx^3}\,dx}{3a^3}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{\left(b^{2/3}(7Ab-4aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{10/3}} -$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{\sqrt[3]{b}(7Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}$$

Mathematica [A]
time = 0.08, size = 185, normalized size = 0.86

$$\frac{-\frac{9a^{4/3}A}{x^4}-\frac{36\sqrt[3]{a}(-2Ab+aB)}{x}-\frac{12\sqrt[3]{a}b(-Ab+aB)x^2}{a+bx^3}-4\sqrt{3}\sqrt[3]{b}(7Ab-4aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{b}(-7Ab+4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{36a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(4/3)*A)/x^4 - (36*a^(1/3)*(-2*A*b + a*B))/x - (12*a^(1/3)*b*(-(A*b)
 + a*B)*x^2)/(a + b*x^3) - 4*Sqrt[3]*b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-7*A*b + 4*a*B)*Log[a^(1/3) + b^
(1/3)*x] + 2*b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(36*a^(10/3))
```

Maple [A]
time = 0.29, size = 155, normalized size = 0.72

method	result

default	$\frac{b\left(\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x^2}{b\,x^3+a}+\left(\frac{7Ab}{3}-\frac{4Ba}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\right)}{a^3}-\frac{A}{4a^2x^4}-$
risch	$\frac{\frac{b(7Ab-4Ba)x^6}{3a^3}+\frac{(7Ab-4Ba)x^3}{4a^2}-\frac{A}{4a}}{x^4(b\,x^3+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^3+343A^3b^4-588A^2Ba\,b^3+336A\,B^2a^2b^2-64B^3a^3b\right)}\_R\ln\left(\left(-4a^{10}\_R^3-\right.\right.\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*b*((1/3*A*b-1/3*B*a)*x^2/(b*x^3+a)+(7/3*A*b-4/3*B*a)*(-1/3/b/(a/b)^(1
/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1
/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/4*A/a^
2/x^4-(-2*A*b+B*a)/a^3/x
```

Maxima [A]

time = 0.50, size = 186, normalized size = 0.87

$$-\frac{4\,(4\,Bab-7\,Ab^2)x^6+3\,(4\,Ba^2-7\,Aab)x^3+3\,Aa^2}{12\,(a^3bx^7+a^4x^4)}-\frac{\sqrt{3}\,(4\,Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(4\,Ba-7\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(4\,Ba-7\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(4*(4*B*a*b - 7*A*b^2)*x^6 + 3*(4*B*a^2 - 7*A*a*b)*x^3 + 3*A*a^2)/(a^
3*b*x^7 + a^4*x^4) - 1/9*sqrt(3)*(4*B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(1/3)) - 1/18*(4*B*a - 7*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1/3)) + 1/9*(4*B*a - 7*A*b)*log(
x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

Fricas [A]

time = 2.39, size = 259, normalized size = 1.20

$$-\frac{12\,(4\,Bab-7\,Ab^2)x^6+9\,(4\,Ba^2-7\,Aab)x^3+9\,Aa^2+4\,\sqrt{3}\,((4\,Bab-7\,Ab^2)x^7+(4\,Ba^2-7\,Aab)x^4)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-2\,((4\,Bab-7\,Ab^2)x^7+(4\,Ba^2-7\,Aab)x^4)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+4\,((4\,Bab-7\,Ab^2)x^7+(4\,Ba^2-7\,Aab)x^4)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{36\,(a^3bx^7+a^4x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*(4*B*a*b - 7*A*b^2)*x^6 + 9*(4*B*a^2 - 7*A*a*b)*x^3 + 9*A*a^2 + 4
*sqrt(3)*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*a
rctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 2*((4*B*a*b - 7*A*b^2)*x^
7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*
(-b/a)^(1/3)) + 4*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a
)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^3*b*x^7 + a^4*x^4)
```

Sympy [A]
time = 0.40, size = 153, normalized size = 0.71

$$\operatorname{RootSum}\left(729t^3a^{10} + 343A^3b^4 - 588A^2Bab^3 + 336AB^2a^2b^2 - 64B^3a^3b, \left(t \mapsto t\log\left(\frac{81t^2a^7}{49A^2b^3 - 56ABab^2 + 16B^2a^2b} + x\right)\right)\right) + \frac{-3Aa^2 + x^6\cdot(28Ab^2 - 16Bab) + x^3\cdot(21Aab - 12Ba^2)}{12a^4x^4 + 12a^3bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**10 + 343*A**3*b**4 - 588*A**2*B*a*b**3 + 336*A*B**2*a*
*2*b**2 - 64*B**3*a**3*b, Lambda(_t, _t*log(81*_t**2*a**7/(49*A**2*b**3 - 5
6*A*B*a*b**2 + 16*B**2*a**2*b) + x))) + (-3*A*a**2 + x**6*(28*A*b**2 - 16*B
*a*b) + x**3*(21*A*a*b - 12*B*a**2))/(12*a**4*x**4 + 12*a**3*b*x**7)
```

Giac [A]
time = 2.06, size = 231, normalized size = 1.07

$$\frac{\left(4\,Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 7\,Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4} + \frac{\sqrt{3}\left(4\left(-ab^2\right)^{\frac{2}{3}}Ba - 7\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b} - \frac{Babx^2 - Ab^2x^2}{3\,(bx^3+a)a^3} - \frac{\left(4\left(-ab^2\right)^{\frac{2}{3}}Ba - 7\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^4b} - \frac{4\,Bax^3 - 8\,Abx^3 + Aa}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(4*B*a*b*(-a/b)^(1/3) - 7*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/a^4 + 1/9*sqrt(3)*(4*(-a*b^2)^(2/3)*B*a - 7*(-a*b^2)^(2/3)*A
*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/3*(B*
a*b*x^2 - A*b^2*x^2)/((b*x^3 + a)*a^3) - 1/18*(4*(-a*b^2)^(2/3)*B*a - 7*(-a
*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/4*(4*
B*a*x^3 - 8*A*b*x^3 + A*a)/(a^3*x^4)
```

Mupad [B]
time = 2.62, size = 209, normalized size = 0.97

$$\frac{\frac{x^3\,(7\,A\,b-4\,B\,a)}{4\,a^2} - \frac{A}{4\,a} + \frac{b\,x^6\,(7\,A\,b-4\,B\,a)}{3\,a^3}}{b\,x^7 + a\,x^4} + \frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3} + b^3\,x\right)\,(7\,A\,b - 4\,B\,a)}{9\,a^{10/3}} + \frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3} - 2\,b^3\,x + \sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)(7\,A\,b - 4\,B\,a)}{9\,a^{10/3}} - \frac{(-b)^{1/3}\,\ln\left(2\,b^3\,x - a^{1/3}\,(-b)^{8/3} + \sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)(7\,A\,b - 4\,B\,a)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(7*A*b - 4*B*a))/(4*a^2) - A/(4*a) + (b*x^6*(7*A*b - 4*B*a))/(3*a^3))
/(a*x^4 + b*x^7) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(7*A*b - 4*B
*a))/(9*a^(10/3)) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/2)*
a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 4*B*a))/(9*a^(10/3))
 - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)^(8/3
)*1i)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - 4*B*a))/(9*a^(10/3))
```

3.86 $\int \frac{A+Bx^3}{x^6(a+bx^3)^2}\,dx$

Optimal. Leaf size=215

$$\frac{-8Ab+5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\,(a+bx^3)}-\frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}+\frac{b^{2/3}(8Ab-5aB)\log}{9a^{11/}}$$

```
[Out] 1/15*(-8*A*b+5*B*a)/a^2/b/x^5+1/6*(8*A*b-5*B*a)/a^3/x^2+1/3*(A*b-B*a)/a/b/x
^5/(b*x^3+a)+1/9*b^(2/3)*(8*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)-1/18*
b^(2/3)*(8*A*b-5*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)-1/
9*b^(2/3)*(8*A*b-5*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(11/3)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {468, 331, 206, 31, 648, 631, 210, 642}

$$-\frac{b^{2/3}(8Ab-5aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}-\frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{11/3}}+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{11/3}}+\frac{8Ab-5aB}{6a^3x^2}-\frac{8Ab-5aB}{15a^2bx^5}+\frac{Ab-aB}{3abx^5\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] -1/15*(8*A*b - 5*a*B)/(a^2*b*x^5) + (8*A*b - 5*a*B)/(6*a^3*x^2) + (A*b - a*
B)/(3*a*b*x^5*(a + b*x^3)) - (b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11/3)) + (b^(2/3)*(8*A*b - 5*a*
B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(11/3)) - (b^(2/3)*(8*A*b - 5*a*B)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} + \frac{(8Ab-5aB)\int \frac{1}{x^6(a+bx^3)}\,dx}{3ab}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} - \frac{(8Ab-5aB)\int \frac{1}{x^3(a+bx^3)}\,dx}{3a^2}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} + \frac{(b(8Ab-5aB))\int \frac{1}{a+bx^3}\,dx}{3a^3}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} + \frac{(b(8Ab-5aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{11/3}} +$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} + \frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{11/3}}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} + \frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{11/3}}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} - \frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}$$

Mathematica [A]
time = 0.08, size = 183, normalized size = 0.85

$$\frac{-\frac{18a^{5/3}A}{x^5}-\frac{45a^{2/3}(-2Ab+aB)}{x^2}-\frac{30a^{2/3}b(-Ab+aB)x}{a+bx^3}-10\sqrt{3}\,b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+5b^{2/3}(-8Ab+5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{90a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(5/3)*A)/x^5 - (45*a^(2/3)*(-2*A*b + a*B))/x^2 - (30*a^(2/3)*b*(-(A
*b) + a*B)*x)/(a + b*x^3) - 10*Sqrt[3]*b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(8*A*b - 5*a*B)*Log[a^(1/3) +
b^(1/3)*x] + 5*b^(2/3)*(-8*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(90*a^(11/3))
```

Maple [A]
time = 0.30, size = 154, normalized size = 0.72

method	result

default	$\frac{b\left(\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x}{b\,x^{3}+a}+\frac{(8Ab-5Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}\right)}{a^{3}}-\frac{A}{5a^{2}x^{5}}-\frac{-2Ab+B}{2a^{3}x^{2}}$
risch	$\frac{\frac{b(8Ab-5Ba)x^{6}}{6a^{3}}+\frac{(8Ab-5Ba)x^{3}}{10a^{2}}-\frac{A}{5a}}{x^{5}(b\,x^{3}+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^{3}-512A^{3}b^{5}+960A^{2}Ba\,b^{4}-600A\,B^{2}a^{2}b^{3}+125B^{3}a^{3}b^{2}\right)}\_R\ln\left(\left(-4\_R^{3}a^{11}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*b*((1/3*A*b-1/3*B*a)*x/(b*x^3+a)+1/3*(8*A*b-5*B*a)*(1/3/b/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/
b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))-1/5*A/a^2/x
^5-1/2*(-2*A*b+B*a)/a^3/x^2
```

Maxima [A]
time = 0.56, size = 186, normalized size = 0.87

$$-\frac{5\,(5\,Bab-8\,Ab^{2})x^{6}+3\,(5\,Ba^{2}-8\,Aab)x^{3}+6\,Aa^{2}}{30\,(a^{3}bx^{8}+a^{4}x^{5})}-\frac{\sqrt{3}\,(5\,Ba-8\,Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(5\,Ba-8\,Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(5\,Ba-8\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(5*(5*B*a*b - 8*A*b^2)*x^6 + 3*(5*B*a^2 - 8*A*a*b)*x^3 + 6*A*a^2)/(a^
3*b*x^8 + a^4*x^5) - 1/9*sqrt(3)*(5*B*a - 8*A*b)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(2/3)) + 1/18*(5*B*a - 8*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/3)) - 1/9*(5*B*a - 8*A*b)*log(
x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

Fricas [A]
time = 1.75, size = 277, normalized size = 1.29

$$-\frac{15\,(5\,Bab-8\,Ab^{2})x^{6}+9\,(5\,Ba^{2}-8\,Aab)x^{3}+18\,Aa^{2}+10\,\sqrt{3}\,((5\,Bab-8\,Ab^{2})x^{8}+(5\,Ba^{2}-8\,Aab)x^{5})\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-5\,((5\,Bab-8\,Ab^{2})x^{8}+(5\,Ba^{2}-8\,Aab)x^{5})\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}-abx\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)+10\,((5\,Bab-8\,Ab^{2})x^{8}+(5\,Ba^{2}-8\,Aab)x^{5})\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)}{90\,(a^{3}bx^{8}+a^{4}x^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/90*(15*(5*B*a*b - 8*A*b^2)*x^6 + 9*(5*B*a^2 - 8*A*a*b)*x^3 + 18*A*a^2 +
10*sqrt(3)*((5*B*a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1
/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*((5*B*a*b
 - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*
b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*((5*B*a*b - 8*A*b^2)*x^8 +
(5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)))/(a^3
*b*x^8 + a^4*x^5)
```

Sympy [A]
time = 0.44, size = 138, normalized size = 0.64

$$\operatorname{RootSum}\left(729t^{3}a^{11}-512A^{3}b^{5}+960A^{2}Bab^{4}-600AB^{2}a^{2}b^{3}+125B^{3}a^{3}b^{2},\left(t\mapsto t\log\left(-\frac{9ta^{4}}{-8Ab^{2}+5Bab}+x\right)\right)\right)+\frac{-6Aa^{2}+x^{6}\cdot(40Ab^{2}-25Bab)+x^{3}\cdot(24Aab-15Ba^{2})}{30a^{4}x^{5}+30a^{3}bx^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**11 - 512*A**3*b**5 + 960*A**2*B*a*b**4 - 600*A*B**2*a*
*2*b**3 + 125*B**3*a**3*b**2, Lambda(_t, _t*log(-9*_t*a**4/(-8*A*b**2 + 5*B
*a*b) + x))) + (-6*A*a**2 + x**6*(40*A*b**2 - 25*B*a*b) + x**3*(24*A*a*b -
15*B*a**2))/(30*a**4*x**5 + 30*a**3*b*x**8)
```

Giac [A]
time = 1.36, size = 206, normalized size = 0.96

$$-\frac{\sqrt{3}\left(5\left(-ab^{2}\right)^{\frac{1}{3}}Ba-8\left(-ab^{2}\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{4}}+\frac{(5\,Bab-8\,Ab^{2})\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{4}}-\frac{\left(5\left(-ab^{2}\right)^{\frac{1}{3}}Ba-8\left(-ab^{2}\right)^{\frac{1}{3}}Ab\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{4}}-\frac{Babx-Ab^{2}x}{3\,(bx^{3}+a)a^{3}}-\frac{5\,Bax^{3}-10\,Abx^{3}+2\,Aa}{10\,a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 + 1/9*(5*B*a*b - 8*A*b^2)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - 1/18*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b
^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4 - 1/3*(B*a*b*x
- A*b^2*x)/((b*x^3 + a)*a^3) - 1/10*(5*B*a*x^3 - 10*A*b*x^3 + 2*A*a)/(a^3*x
^5)
```

Mupad [B]

time = 2.57, size = 176, normalized size = 0.82

$$\frac{\frac{x^3\,(8\,A\,b-5\,B\,a)}{10\,a^2}-\frac{A}{5\,a}+\frac{b\,x^6\,(8\,A\,b-5\,B\,a)}{6\,a^3}}{b\,x^8+a\,x^5}+\frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(8\,A\,b-5\,B\,a)}{9\,a^{11/3}}-\frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(8\,A\,b-5\,B\,a)}{9\,a^{11/3}}+\frac{b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(8\,A\,b-5\,B\,a)}{9\,a^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(8*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (b*x^6*(8*A*b - 5*B*a))/(6*a^3)
)/(a*x^5 + b*x^8) + (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(8*A*b - 5*B*a))/(9*a
^(11/3)) - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/
2)*1i)/2 + 1/2)*(8*A*b - 5*B*a))/(9*a^(11/3)) + (b^(2/3)*log(3^(1/2)*a^(1/3
)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(8*A*b - 5*B*a))/(9*a^
(11/3))
```

3.87 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{A}{6a^2x^6}+\frac{2Ab-aB}{3a^3x^3}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}$$

```
[Out] -1/6*A/a^2/x^6+1/3*(2*A*b-B*a)/a^3/x^3+1/3*b*(A*b-B*a)/a^3/(b*x^3+a)+b*(3*A
*b-2*B*a)*ln(x)/a^4-1/3*b*(3*A*b-2*B*a)*ln(b*x^3+a)/a^4
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{2Ab-aB}{3a^3x^3}-\frac{A}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -1/6*A/(a^2*x^6) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(A*b - a*B))/(3*a^3*(a +
b*x^3)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^3
])/(3*a^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^2x^3}+\frac{-2Ab+aB}{a^3x^2}-\frac{b(-3Ab+2aB)}{a^4x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^2}+\frac{b^2(-3Ab+2}{a^4(a+bx}\right.\right.$$
$$= -\frac{A}{6a^2x^6}+\frac{2Ab-aB}{3a^3x^3}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a\right.}{3a^4}$$

Mathematica [A]
time = 0.06, size = 85, normalized size = 0.88

$$-\frac{\frac{a^2A}{x^6}+\frac{2a(-2Ab+aB)}{x^3}+\frac{2ab(-Ab+aB)}{a+bx^3}-6b(3Ab-2aB)\log(x)+2b(3Ab-2aB)\log\left(a+bx^3\right)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -1/6*((a^2*A)/x^6 + (2*a*(-2*A*b + a*B))/x^3 + (2*a*b*(-(A*b) + a*B))/(a +
b*x^3) - 6*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^3])/a
^4
```

Maple [A]
time = 0.29, size = 96, normalized size = 0.99

method	result	size
default	$-\frac{b^2\left(-\frac{a(Ab-Ba)}{b\left(b\,x^3+a\right)}+\frac{(3Ab-2Ba)\ln\left(b\,x^3+a\right)}{b}\right)}{3a^4}-\frac{A}{6a^2x^6}-\frac{-2Ab+Ba}{3a^3x^3}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}$	96
norman	$\frac{-\frac{A}{6a}+\frac{(3Ab-2Ba)x^3}{6a^2}-\frac{b\left(3b^2A-2abB\right)x^9}{3a^4}}{x^6\left(b\,x^3+a\right)}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}-\frac{b(3Ab-2Ba)\ln\left(b\,x^3+a\right)}{3a^4}$	99
risch	$\frac{\frac{b(3Ab-2Ba)x^6}{3a^3}+\frac{(3Ab-2Ba)x^3}{6a^2}-\frac{A}{6a}}{x^6\left(b\,x^3+a\right)}+\frac{3b^2\ln(x)A}{a^4}-\frac{2b\ln(x)B}{a^3}-\frac{b^2\ln\left(b\,x^3+a\right)A}{a^4}+\frac{2b\ln\left(b\,x^3+a\right)B}{3a^3}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^4*b^2*(-a*(A*b-B*a)/b/(b*x^3+a)+(3*A*b-2*B*a)/b*ln(b*x^3+a))-1/6*A/a
^2/x^6-1/3*(-2*A*b+B*a)/a^3/x^3+b*(3*A*b-2*B*a)*ln(x)/a^4
```

Maxima [A]
time = 0.27, size = 106, normalized size = 1.09

$$-\frac{2\left(2\,Bab-3\,Ab^2\right)x^6+\left(2\,Ba^2-3\,Aab\right)x^3+Aa^2}{6\left(a^3bx^9+a^4x^6\right)}+\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(bx^3+a\right)}{3\,a^4}-\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3 + A*a^2)/(a^3*b*x
^9 + a^4*x^6) + 1/3*(2*B*a*b - 3*A*b^2)*log(b*x^3 + a)/a^4 - 1/3*(2*B*a*b -
 3*A*b^2)*log(x^3)/a^4
```

Fricas [A]
time = 2.04, size = 154, normalized size = 1.59

$$-\frac{2\,(2\,Ba^2b-3\,Aab^2)x^6+Aa^3+(2\,Ba^3-3\,Aa^2b)x^3-2\,((2\,Bab^2-3\,Ab^3)x^9+(2\,Ba^2b-3\,Aab^2)x^6)\log\left(bx^3+a\right)+6\,((2\,Bab^2-3\,Ab^3)x^9+(2\,Ba^2b-3\,Aab^2)x^6)\log\left(x\right)}{6\,(a^4bx^9+a^5x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*B*a^2*b - 3*A*a*b^2)*x^6 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^3 - 2
*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(b*x^3 + a) +
 6*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(x))/(a^4*b
*x^9 + a^5*x^6)
```

Sympy [A]
time = 0.78, size = 100, normalized size = 1.03

$$\frac{-Aa^2+x^6\cdot(6Ab^2-4Bab)+x^3\cdot(3Aab-2Ba^2)}{6a^4x^6+6a^3bx^9}-\frac{b(-3Ab+2Ba)\log\left(x\right)}{a^4}+\frac{b(-3Ab+2Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**2,x)
```

```
[Out] (-A*a**2 + x**6*(6*A*b**2 - 4*B*a*b) + x**3*(3*A*a*b - 2*B*a**2))/(6*a**4*x
**6 + 6*a**3*b*x**9) - b*(-3*A*b + 2*B*a)*log(x)/a**4 + b*(-3*A*b + 2*B*a)*
log(a/b + x**3)/(3*a**4)
```

Giac [A]
time = 1.57, size = 149, normalized size = 1.54

$$-\frac{(2\,Bab-3\,Ab^2)\log\left(|x|\right)}{a^4}+\frac{(2\,Bab^2-3\,Ab^3)\log\left(|bx^3+a|\right)}{3\,a^4b}-\frac{2\,Bab^2x^3-3\,Ab^3x^3+3\,Ba^2b-4\,Aab^2}{3\,(bx^3+a)a^4}+\frac{6\,Babx^6-9\,Ab^2x^6-2\,Ba^2x^3+4\,Aabx^3-Aa^2}{6\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(2*B*a*b - 3*A*b^2)*log(abs(x))/a^4 + 1/3*(2*B*a*b^2 - 3*A*b^3)*log(abs(b*
x^3 + a))/(a^4*b) - 1/3*(2*B*a*b^2*x^3 - 3*A*b^3*x^3 + 3*B*a^2*b - 4*A*a*b^
2)/((b*x^3 + a)*a^4) + 1/6*(6*B*a*b*x^6 - 9*A*b^2*x^6 - 2*B*a^2*x^3 + 4*A*a
*b*x^3 - A*a^2)/(a^4*x^6)
```

Mupad [B]
time = 0.14, size = 100, normalized size = 1.03

$$\frac{\frac{x^3\,(3\,A\,b-2\,B\,a)}{6\,a^2}-\frac{A}{6\,a}+\frac{b\,x^6\,(3\,A\,b-2\,B\,a)}{3\,a^3}}{b\,x^9+a\,x^6}-\frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{3\,a^4}+\frac{\ln\left(x\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(3*A*b - 2*B*a))/(6*a^2) - A/(6*a) + (b*x^6*(3*A*b - 2*B*a))/(3*a^3))
/(a*x^6 + b*x^9) - (log(a + b*x^3)*(3*A*b^2 - 2*B*a*b))/(3*a^4) + (log(x)*(
3*A*b^2 - 2*B*a*b))/a^4
```

3.88 $\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=107

$$\frac{(Ab-3aB)x^3}{3b^4}+\frac{Bx^6}{6b^3}+\frac{a^3(Ab-aB)}{6b^5\left(a+bx^3\right)^2}-\frac{a^2(3Ab-4aB)}{3b^5\left(a+bx^3\right)}-\frac{a(Ab-2aB)\log\left(a+bx^3\right)}{b^5}$$

```
[Out] 1/3*(A*b-3*B*a)*x^3/b^4+1/6*B*x^6/b^3+1/6*a^3*(A*b-B*a)/b^5/(b*x^3+a)^2-1/3
*a^2*(3*A*b-4*B*a)/b^5/(b*x^3+a)-a*(A*b-2*B*a)*ln(b*x^3+a)/b^5
```

Rubi [A]
time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {457, 78}

$$\frac{a^3(Ab-aB)}{6b^5\left(a+bx^3\right)^2}-\frac{a^2(3Ab-4aB)}{3b^5\left(a+bx^3\right)}-\frac{a(Ab-2aB)\log\left(a+bx^3\right)}{b^5}+\frac{x^3(Ab-3aB)}{3b^4}+\frac{Bx^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^6)/(6*b^3) + (a^3*(A*b - a*B))/(6*b^5*(a
 + b*x^3)^2) - (a^2*(3*A*b - 4*a*B))/(3*b^5*(a + b*x^3)) - (a*(A*b - 2*a*B)
*Log[a + b*x^3])/b^5
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-3aB}{b^4} + \frac{Bx}{b^3} + \frac{a^3(-Ab+aB)}{b^4(a+bx)^3} - \frac{a^2(-3Ab+4aB)}{b^4(a+bx)^2} + \frac{3a(-Ab+2a}{b^4(a+bx)}\right.\right.$$

$$= \frac{(Ab-3aB)x^3}{3b^4} + \frac{Bx^6}{6b^3} + \frac{a^3(Ab-aB)}{6b^5(a+bx^3)^2} - \frac{a^2(3Ab-4aB)}{3b^5(a+bx^3)} - \frac{a(Ab-2aB)\log(a+bx}{b^5}$$

Mathematica [A]
time = 0.04, size = 94, normalized size = 0.88

$$\frac{2b(Ab-3aB)x^3 + b^2Bx^6 + \frac{a^3(Ab-aB)}{(a+bx^3)^2} + \frac{2a^2(-3Ab+4aB)}{a+bx^3} + 6a(-Ab+2aB)\log(a+bx^3)}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*b*(A*b - 3*a*B)*x^3 + b^2*B*x^6 + (a^3*(A*b - a*B))/(a + b*x^3)^2 + (2*a
^2*(-3*A*b + 4*a*B))/(a + b*x^3) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^3])/(6*
b^5)
```

Maple [A]
time = 0.31, size = 102, normalized size = 0.95

method	result
norman	$\frac{-\frac{a^2(3abA-6a^2B)}{2b^5}+\frac{Bx^{12}}{6b}+\frac{(Ab-2Ba)x^9}{3b^2}-\frac{2a(abA-2a^2B)x^3}{b^4}}{(bx^3+a)^2} - \frac{a(Ab-2Ba)\ln(bx^3+a)}{b^5}$
default	$\frac{(bBx^3+Ab-3Ba)^2}{6b^5B} - \frac{a\left(\frac{a(3Ab-4Ba)}{b(bx^3+a)} - \frac{a^2(Ab-Ba)}{2b(bx^3+a)^2} + \frac{(3Ab-6Ba)\ln(bx^3+a)}{b}\right)}{3b^4}$
risch	$\frac{Bx^6}{6b^3} + \frac{Ax^3}{3b^3} - \frac{Bax^3}{b^4} + \frac{A^2}{6b^3B} - \frac{Aa}{b^4} + \frac{3Ba^2}{2b^5} + \frac{(-Aa^2b+\frac{4}{3}Ba^3)x^3 - \frac{a^3(5Ab-7Ba)}{6b}}{b^4(bx^3+a)^2} - \frac{a\ln(bx^3+a)A}{b^4} + \frac{2a^2\ln(bx^3+a)B}{b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(B*b*x^3+A*b-3*B*a)^2/b^5/B-1/3*a/b^4*(a*(3*A*b-4*B*a)/b/(b*x^3+a)-1/2*
a^2*(A*b-B*a)/b/(b*x^3+a)^2+(3*A*b-6*B*a)/b*ln(b*x^3+a))
```

Maxima [A]
time = 0.30, size = 115, normalized size = 1.07

$$\frac{7\,Ba^4 - 5\,Aa^3b + 2\,(4\,Ba^3b - 3\,Aa^2b^2)x^3}{6\,(b^7x^6 + 2\,ab^6x^3 + a^2b^5)} + \frac{Bbx^6 - 2\,(3\,Ba - Ab)x^3}{6\,b^4} + \frac{(2\,Ba^2 - Aab)\log(bx^3 + a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^3)/(b^7*x^6 + 2*a*
b^6*x^3 + a^2*b^5) + 1/6*(B*b*x^6 - 2*(3*B*a - A*b)*x^3)/b^4 + (2*B*a^2 - A
*a*b)*log(b*x^3 + a)/b^5
```

Fricas [A]
time = 1.91, size = 179, normalized size = 1.67

$$\frac{Bb^4x^{12} - 2\,(2\,Bab^3 - Ab^4)x^9 - (11\,Ba^2b^2 - 4\,Aab^3)x^6 + 7\,Ba^4 - 5\,Aa^3b + 2\,(Ba^3b - 2\,Aa^2b^2)x^3 + 6\,((2\,Ba^2b^2 - Aab^3)x^6 + 2\,Ba^4 - Aa^3b + 2\,(2\,Ba^3b - Aa^2b^2)x^3)\log\left(bx^3 + a\right)}{6\,(b^7x^6 + 2\,ab^6x^3 + a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^4*x^12 - 2*(2*B*a*b^3 - A*b^4)*x^9 - (11*B*a^2*b^2 - 4*A*a*b^3)*x^
6 + 7*B*a^4 - 5*A*a^3*b + 2*(B*a^3*b - 2*A*a^2*b^2)*x^3 + 6*((2*B*a^2*b^2 -
 A*a*b^3)*x^6 + 2*B*a^4 - A*a^3*b + 2*(2*B*a^3*b - A*a^2*b^2)*x^3)*log(b*x^
3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

Sympy [A]
time = 2.72, size = 112, normalized size = 1.05

$$\frac{Bx^6}{6b^3} + \frac{a(-Ab + 2Ba)\log\left(a + bx^3\right)}{b^5} + x^3\left(\frac{A}{3b^3} - \frac{Ba}{b^4}\right) + \frac{-5Aa^3b + 7Ba^4 + x^3(-6Aa^2b^2 + 8Ba^3b)}{6a^2b^5 + 12ab^6x^3 + 6b^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**6/(6*b**3) + a*(-A*b + 2*B*a)*log(a + b*x**3)/b**5 + x**3*(A/(3*b**3)
- B*a/b**4) + (-5*A*a**3*b + 7*B*a**4 + x**3*(-6*A*a**2*b**2 + 8*B*a**3*b))
/(6*a**2*b**5 + 12*a*b**6*x**3 + 6*b**7*x**6)
```

Giac [A]
time = 1.47, size = 131, normalized size = 1.22

$$\frac{(2\,Ba^2 - Aab)\log\left(|bx^3 + a|\right)}{b^5} + \frac{Bb^3x^6 - 6\,Bab^2x^3 + 2\,Ab^3x^3}{6\,b^6} - \frac{18\,Ba^2b^2x^6 - 9\,Aab^3x^6 + 28\,Ba^3bx^3 - 12\,Aa^2b^2x^3 + 11\,Ba^4 - 4\,Aa^3b}{6\,(bx^3 + a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (2*B*a^2 - A*a*b)*log(abs(b*x^3 + a))/b^5 + 1/6*(B*b^3*x^6 - 6*B*a*b^2*x^3
+ 2*A*b^3*x^3)/b^6 - 1/6*(18*B*a^2*b^2*x^6 - 9*A*a*b^3*x^6 + 28*B*a^3*b*x^3
 - 12*A*a^2*b^2*x^3 + 11*B*a^4 - 4*A*a^3*b)/((b*x^3 + a)^2*b^5)
```

Mupad [B]
time = 0.10, size = 117, normalized size = 1.09

$$\frac{\frac{7\,B\,a^4-5\,A\,a^3\,b}{6\,b}+x^3\,\left(\frac{4\,B\,a^3}{3}-A\,a^2\,b\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x^3\,\left(\frac{A}{3\,b^3}-\frac{B\,a}{b^4}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(2\,B\,a^2-A\,a\,b\right)}{b^5}+\frac{B\,x^6}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((7*B*a^4 - 5*A*a^3*b)/(6*b) + x^3*((4*B*a^3)/3 - A*a^2*b))/(a^2*b^4 + b^6*
x^6 + 2*a*b^5*x^3) + x^3*(A/(3*b^3) - (B*a)/b^4) + (log(a + b*x^3)*(2*B*a^2
 - A*a*b))/b^5 + (B*x^6)/(6*b^3)
```

3.89 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal. Leaf size=88

$$\frac{Bx^3}{3b^3} - \frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2} + \frac{a(2Ab-3aB)}{3b^4(a+bx^3)} + \frac{(Ab-3aB)\log(a+bx^3)}{3b^4}$$

```
[Out] 1/3*B*x^3/b^3-1/6*a^2*(A*b-B*a)/b^4/(b*x^3+a)^2+1/3*a*(2*A*b-3*B*a)/b^4/(b*
x^3+a)+1/3*(A*b-3*B*a)*ln(b*x^3+a)/b^4
```

Rubi [A]
time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$-\frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2} + \frac{a(2Ab-3aB)}{3b^4(a+bx^3)} + \frac{(Ab-3aB)\log(a+bx^3)}{3b^4} + \frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (B*x^3)/(3*b^3) - (a^2*(A*b - a*B))/(6*b^4*(a + b*x^3)^2) + (a*(2*A*b - 3*a
*B))/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{B}{b^3} - \frac{a^2(-Ab+aB)}{b^3(a+bx)^3} + \frac{a(-2Ab+3aB)}{b^3(a+bx)^2} + \frac{Ab-3aB}{b^3(a+bx)}\right)dx, x, x^3\right)$$
$$= \frac{Bx^3}{3b^3} - \frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2} + \frac{a(2Ab-3aB)}{3b^4(a+bx^3)} + \frac{(Ab-3aB)\log(a+bx^3)}{3b^4}$$

Mathematica [A]
time = 0.02, size = 92, normalized size = 1.05

$$\frac{Bx^3}{3b^3} + \frac{-a^2Ab+a^3B}{6b^4(a+bx^3)^2} + \frac{2aAb-3a^2B}{3b^4(a+bx^3)} + \frac{(Ab-3aB)\log(a+bx^3)}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (B*x^3)/(3*b^3) + (-(a^2*A*b) + a^3*B)/(6*b^4*(a + b*x^3)^2) + (2*a*A*b - 3
*a^2*B)/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Maple [A]
time = 0.30, size = 85, normalized size = 0.97

method	result	size
norman	$\frac{\frac{a^2(Ab-3Ba)}{2b^4}+\frac{Bx^9}{3b}+\frac{2a(Ab-3Ba)x^3}{3b^3}}{(bx^3+a)^2}+\frac{(Ab-3Ba)\ln(bx^3+a)}{3b^4}$	76
default	$\frac{Bx^3}{3b^3}+\frac{\frac{a(2Ab-3Ba)}{b(bx^3+a)}-\frac{a^2(Ab-Ba)}{2b(bx^3+a)^2}+\frac{(Ab-3Ba)\ln(bx^3+a)}{b}}{3b^3}$	85
risch	$\frac{Bx^3}{3b^3}+\frac{(\frac{2}{3}abA-a^2B)x^3+\frac{(3Ab-5Ba)a^2}{6b}}{b^3(bx^3+a)^2}+\frac{\ln(bx^3+a)A}{3b^3}-\frac{\ln(bx^3+a)Ba}{b^4}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*B*x^3/b^3+1/3/b^3*(a*(2*A*b-3*B*a)/b/(b*x^3+a)-1/2*a^2*(A*b-B*a)/b/(b*x
^3+a)^2+1/b*(A*b-3*B*a)*ln(b*x^3+a))
```

Maxima [A]
time = 0.27, size = 94, normalized size = 1.07

$$\frac{Bx^3}{3\,b^3} - \frac{5\,Ba^3 - 3\,Aa^2b + 2\,(3\,Ba^2b - 2\,Aab^2)x^3}{6\,(b^6x^6 + 2\,ab^5x^3 + a^2b^4)} - \frac{(3\,Ba - Ab)\log(bx^3 + a)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*B*x^3/b^3 - 1/6*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^3)/(
b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) - 1/3*(3*B*a - A*b)*log(b*x^3 + a)/b^4
```

Fricas [A]
time = 2.18, size = 142, normalized size = 1.61

$$\frac{2\,Bb^3x^9+4\,Bab^2x^6-5\,Ba^3+3\,Aa^2b-4\,(Ba^2b-Aab^2)x^3-2\,((3\,Bab^2-Ab^3)x^6+3\,Ba^3-Aa^2b+2\,(3\,Ba^2b-Aab^2)x^3)\log\left(bx^3+a\right)}{6\,(b^6x^6+2\,ab^5x^3+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*b^3*x^9 + 4*B*a*b^2*x^6 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*b
^2)*x^3 - 2*((3*B*a*b^2 - A*b^3)*x^6 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b - A
*a*b^2)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A]
time = 1.40, size = 94, normalized size = 1.07

$$\frac{Bx^3}{3b^3}+\frac{3Aa^2b-5Ba^3+x^3\cdot(4Aab^2-6Ba^2b)}{6a^2b^4+12ab^5x^3+6b^6x^6}-\frac{(-Ab+3Ba)\log\left(a+bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**3/(3*b**3) + (3*A*a**2*b - 5*B*a**3 + x**3*(4*A*a*b**2 - 6*B*a**2*b))/
(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - (-A*b + 3*B*a)*log(a + b*x**
3)/(3*b**4)
```

Giac [A]
time = 1.54, size = 93, normalized size = 1.06

$$\frac{Bx^3}{3\,b^3}-\frac{(3\,Ba-Ab)\log\left(|bx^3+a|\right)}{3\,b^4}+\frac{9\,Bab^2x^6-3\,Ab^3x^6+12\,Ba^2bx^3-2\,Aab^2x^3+4\,Ba^3}{6\,(bx^3+a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*B*x^3/b^3 - 1/3*(3*B*a - A*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(9*B*a*b^2*
x^6 - 3*A*b^3*x^6 + 12*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 4*B*a^3)/((b*x^3 + a)^
2*b^4)
```

Mupad [B]
time = 2.40, size = 94, normalized size = 1.07

$$\frac{B\,x^3}{3\,b^3}-\frac{x^3\,\left(B\,a^2-\frac{2\,A\,a\,b}{3}\right)+\frac{5\,B\,a^3-3\,A\,a^2\,b}{6\,b}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-3\,B\,a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (B*x^3)/(3*b^3) - (x^3*(B*a^2 - (2*A*a*b)/3) + (5*B*a^3 - 3*A*a^2*b)/(6*b))
/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (log(a + b*x^3)*(A*b - 3*B*a))/(3*b^4)
```

3.90 $\int \frac{x^5 (A+Bx^3)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=66

$$\frac{a(Ab-aB)}{6b^3 (a+bx^3)^2}-\frac{Ab-2aB}{3b^3 (a+bx^3)}+\frac{B\log (a+bx^3)}{3b^3}$$

```
[Out] 1/6*a*(A*b-B*a)/b^3/(b*x^3+a)^2+1/3*(-A*b+2*B*a)/b^3/(b*x^3+a)+1/3*B*ln(b*x
^3+a)/b^3
```

Rubi [A]
time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$-\frac{Ab-2aB}{3b^3 (a+bx^3)}+\frac{a(Ab-aB)}{6b^3 (a+bx^3)^2}+\frac{B\log (a+bx^3)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (a*(A*b - a*B))/(6*b^3*(a + b*x^3)^2) - (A*b - 2*a*B)/(3*b^3*(a + b*x^3)) +
 (B*Log[a + b*x^3])/(3*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^3} + \frac{Ab-2aB}{b^2(a+bx)^2} + \frac{B}{b^2(a+bx)}\right)\,dx, x, x^3\right)$$
$$= \frac{a(Ab-aB)}{6b^3(a+bx^3)^2} - \frac{Ab-2aB}{3b^3(a+bx^3)} + \frac{B\log(a+bx^3)}{3b^3}$$

Mathematica [A]
time = 0.01, size = 64, normalized size = 0.97

$$\frac{3a^2B-2Ab^2x^3-ab(A-4Bx^3)+2B(a+bx^3)^2\log(a+bx^3)}{6b^3(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (3*a^2*B - 2*A*b^2*x^3 - a*b*(A - 4*B*x^3) + 2*B*(a + b*x^3)^2*Log[a + b*x^
3])/(6*b^3*(a + b*x^3)^2)
```

Maple [A]
time = 0.27, size = 61, normalized size = 0.92

method	result	size
norman	$\frac{-\frac{a(Ab-3Ba)}{6b^3}-\frac{(Ab-2Ba)x^3}{3b^2}}{(b\,x^3+a)^2}+\frac{B\ln(b\,x^3+a)}{3b^3}$	57
risch	$\frac{-\frac{a(Ab-3Ba)}{6b^3}-\frac{(Ab-2Ba)x^3}{3b^2}}{(b\,x^3+a)^2}+\frac{B\ln(b\,x^3+a)}{3b^3}$	57
default	$-\frac{Ab-2Ba}{3b^3(b\,x^3+a)}+\frac{a(Ab-Ba)}{6b^3(b\,x^3+a)^2}+\frac{B\ln(b\,x^3+a)}{3b^3}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(A*b-2*B*a)/b^3/(b*x^3+a)+1/6*a*(A*b-B*a)/b^3/(b*x^3+a)^2+1/3*B*ln(b*x
^3+a)/b^3
```

Maxima [A]
time = 0.28, size = 72, normalized size = 1.09

$$\frac{2\,(2\,Bab-Ab^2)x^3+3\,Ba^2-Aab}{6\,(b^5x^6+2\,ab^4x^3+a^2b^3)}+\frac{B\log(bx^3+a)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)/(b^5*x^6 + 2*a*b^4*x^3 + a^
2*b^3) + 1/3*B*log(b*x^3 + a)/b^3
```

Fricas [A]
time = 1.98, size = 89, normalized size = 1.35

$$\frac{2\,(2\,Bab - Ab^2)x^3 + 3\,Ba^2 - Aab + 2\,(Bb^2x^6 + 2\,Babx^3 + Ba^2)\log\left(bx^3 + a\right)}{6\,(b^5x^6 + 2\,ab^4x^3 + a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b + 2*(B*b^2*x^6 + 2*B*a*b*x^3
 + B*a^2)*log(b*x^3 + a))/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

Sympy [A]
time = 1.07, size = 70, normalized size = 1.06

$$\frac{B\log\left(a + bx^3\right)}{3b^3} + \frac{-Aab + 3Ba^2 + x^3(-2Ab^2 + 4Bab)}{6a^2b^3 + 12ab^4x^3 + 6b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*log(a + b*x**3)/(3*b**3) + (-A*a*b + 3*B*a**2 + x**3*(-2*A*b**2 + 4*B*a*b
))/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6)
```

Giac [A]
time = 1.34, size = 61, normalized size = 0.92

$$\frac{B\log\left(|bx^3 + a|\right)}{3\,b^3} + \frac{2\,(2\,Ba - Ab)x^3 + \frac{3\,Ba^2 - Aab}{b}}{6\,(bx^3 + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*B*log(abs(b*x^3 + a))/b^3 + 1/6*(2*(2*B*a - A*b)*x^3 + (3*B*a^2 - A*a*b
)/b)/((b*x^3 + a)^2*b^2)
```

Mupad [B]
time = 2.38, size = 70, normalized size = 1.06

$$\frac{\frac{3\,B\,a^2-A\,a\,b}{6\,b^3}-\frac{x^3\,(A\,b-2\,B\,a)}{3\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{B\,\ln\left(b\,x^3+a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((3*B*a^2 - A*a*b)/(6*b^3) - (x^3*(A*b - 2*B*a))/(3*b^2))/(a^2 + b^2*x^6 +
2*a*b*x^3) + (B*log(a + b*x^3))/(3*b^3)
```

3.91 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=32

$$-\frac{(A+Bx^3)^2}{6(Ab-aB)(a+bx^3)^2}$$

```
[Out] -1/6*(B*x^3+A)^2/(A*b-B*a)/(b*x^3+a)^2
```

Rubi [A]

time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {455, 37}

$$-\frac{(A+Bx^3)^2}{6(a+bx^3)^2(Ab-aB)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(A + B*x^3)^2/((A*b - a*B)*(a + b*x^3)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^3}\,dx, x, x^3\right)\\ &= -\frac{(A+Bx^3)^2}{6(Ab-aB)(a+bx^3)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.94

$$-\frac{Ab+B(a+2bx^3)}{6b^2(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(A*b + B*(a + 2*b*x^3))/(b^2*(a + b*x^3)^2)
```

Maple [A]
time = 0.27, size = 39, normalized size = 1.22

method	result	size
gosper	$-\frac{2bB\,x^3+Ab+Ba}{6b^2(b\,x^3+a)^2}$	29
norman	$\frac{-\frac{B\,x^3}{3b}-\frac{Ab+Ba}{6b^2}}{(b\,x^3+a)^2}$	33
risch	$\frac{-\frac{B\,x^3}{3b}-\frac{Ab+Ba}{6b^2}}{(b\,x^3+a)^2}$	33
default	$-\frac{B}{3b^2(b\,x^3+a)}-\frac{Ab-Ba}{6b^2(b\,x^3+a)^2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*B/b^2/(b*x^3+a)-1/6*(A*b-B*a)/b^2/(b*x^3+a)^2
```

Maxima [A]
time = 0.27, size = 42, normalized size = 1.31

$$-\frac{2\,Bbx^3+Ba+Ab}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Fricas [A]
time = 1.67, size = 42, normalized size = 1.31

$$-\frac{2\,Bbx^3+Ba+Ab}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Sympy [A]
time = 0.44, size = 42, normalized size = 1.31

$$\frac{-Ab - Ba - 2Bbx^3}{6a^2b^2 + 12ab^3x^3 + 6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (-A*b - B*a - 2*B*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

Giac [A]
time = 1.22, size = 28, normalized size = 0.88

$$-\frac{2\,Bbx^3 + Ba + Ab}{6\,(bx^3 + a)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/((b*x^3 + a)^2*b^2)
```

Mupad [B]
time = 2.33, size = 44, normalized size = 1.38

$$-\frac{\frac{A\,b+B\,a}{6\,b^2}+\frac{B\,x^3}{3\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] -((A*b + B*a)/(6*b^2) + (B*x^3)/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

3.92 $\int \frac{A+Bx^3}{x(a+bx^3)^3}\,dx$

Optimal. Leaf size=68

$$\frac{Ab-aB}{6ab\,(a+bx^3)^2}+\frac{A}{3a^2\,(a+bx^3)}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^3\right)}{3a^3}$$

```
[Out] 1/6*(A*b-B*a)/a/b/(b*x^3+a)^2+1/3*A/a^2/(b*x^3+a)+A*ln(x)/a^3-1/3*A*ln(b*x^
3+a)/a^3
```

Rubi [A]
time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {457, 78}

$$-\frac{A\log\left(a+bx^3\right)}{3a^3}+\frac{A\log(x)}{a^3}+\frac{A}{3a^2\,(a+bx^3)}+\frac{Ab-aB}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

```
[Out] (A*b - a*B)/(6*a*b*(a + b*x^3)^2) + A/(3*a^2*(a + b*x^3)) + (A*Log[x])/a^3
- (A*Log[a + b*x^3])/(3*a^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x} + \frac{-Ab+aB}{a(a+bx)^3} - \frac{Ab}{a^2(a+bx)^2} - \frac{Ab}{a^3(a+bx)}\right)dx, x, x^3\right)$$
$$= \frac{Ab-aB}{6ab\left(a+bx^3\right)^2} + \frac{A}{3a^2\left(a+bx^3\right)} + \frac{A\log(x)}{a^3} - \frac{A\log\left(a+bx^3\right)}{3a^3}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.87

$$\frac{\frac{a\left(3aAb-a^2B+2Ab^2x^3\right)}{b\left(a+bx^3\right)^2} + 6A\log(x) - 2A\log\left(a+bx^3\right)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((a*(3*a*A*b - a^2*B + 2*A*b^2*x^3))/(b*(a + b*x^3)^2) + 6*A*Log[x] - 2*A*L
og[a + b*x^3])/(6*a^3)
```

Maple [A]
time = 0.27, size = 63, normalized size = 0.93

method	result	size
risch	$\frac{\frac{Ab\,x^3}{3a^2}+\frac{3Ab-Ba}{6ab}}{\left(b\,x^3+a\right)^2}+\frac{A\ln(x)}{a^3}-\frac{A\ln\left(b\,x^3+a\right)}{3a^3}$	61
default	$-\frac{-\frac{Aa}{b\,x^3+a}-\frac{a^2(Ab-Ba)}{2b\left(b\,x^3+a\right)^2}+A\ln\left(b\,x^3+a\right)}{3a^3}+\frac{A\ln(x)}{a^3}$	63
norman	$\frac{-\frac{(2Ab-Ba)x^3}{3a^2}-\frac{b(3Ab-Ba)x^6}{6a^3}}{\left(b\,x^3+a\right)^2}+\frac{A\ln(x)}{a^3}-\frac{A\ln\left(b\,x^3+a\right)}{3a^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^3*(-A*a/(b*x^3+a)-1/2*a^2*(A*b-B*a)/b/(b*x^3+a)^2+A*ln(b*x^3+a))+A*l
n(x)/a^3
```

Maxima [A]
time = 0.29, size = 77, normalized size = 1.13

$$\frac{2\,Ab^2x^3 - Ba^2 + 3\,Aab}{6\left(a^2b^3x^6 + 2\,a^3b^2x^3 + a^4b\right)} - \frac{A\log\left(bx^3+a\right)}{3\,a^3} + \frac{A\log\left(x^3\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*A*b^2*x^3 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) -
 1/3*A*log(b*x^3 + a)/a^3 + 1/3*A*log(x^3)/a^3
```

Fricas [A]
time = 2.18, size = 119, normalized size = 1.75

$$\frac{2\,Aab^2x^3 - Ba^3 + 3\,Aa^2b - 2\,(Ab^3x^6 + 2\,Aab^2x^3 + Aa^2b)\log\left(bx^3 + a\right) + 6\,(Ab^3x^6 + 2\,Aab^2x^3 + Aa^2b)\log\left(x\right)}{6\,(a^3b^3x^6 + 2\,a^4b^2x^3 + a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*a*b^2*x^3 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a
^2*b)*log(b*x^3 + a) + 6*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*log(x))/(a^3
*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)
```

Sympy [A]
time = 0.38, size = 75, normalized size = 1.10

$$\frac{A\log\left(x\right)}{a^3} - \frac{A\log\left(\frac{a}{b} + x^3\right)}{3a^3} + \frac{3Aab + 2Ab^2x^3 - Ba^2}{6a^4b + 12a^3b^2x^3 + 6a^2b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**3,x)
```

```
[Out] A*log(x)/a**3 - A*log(a/b + x**3)/(3*a**3) + (3*A*a*b + 2*A*b**2*x**3 - B*a
**2)/(6*a**4*b + 12*a**3*b**2*x**3 + 6*a**2*b**3*x**6)
```

Giac [A]
time = 1.17, size = 74, normalized size = 1.09

$$-\frac{A\log\left(|bx^3 + a|\right)}{3\,a^3} + \frac{A\log\left(|x|\right)}{a^3} + \frac{3\,Ab^3x^6 + 8\,Aab^2x^3 - Ba^3 + 6\,Aa^2b}{6\,(bx^3 + a)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*A*log(abs(b*x^3 + a))/a^3 + A*log(abs(x))/a^3 + 1/6*(3*A*b^3*x^6 + 8*A
*a*b^2*x^3 - B*a^3 + 6*A*a^2*b)/((b*x^3 + a)^2*a^3*b)
```

Mupad [B]
time = 0.16, size = 71, normalized size = 1.04

$$\frac{\frac{3\,A\,b-B\,a}{6\,a\,b} + \frac{A\,b\,x^3}{3\,a^2}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6} - \frac{A\,\ln\left(b\,x^3 + a\right)}{3\,a^3} + \frac{A\,\ln\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^3),x)
```

```
[Out] ((3*A*b - B*a)/(6*a*b) + (A*b*x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) - (
A*log(a + b*x^3))/(3*a^3) + (A*log(x))/a^3
```

3.93 $\int \frac{A+Bx^3}{x^4(a+bx^3)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}$$

```
[Out] -1/3*A/a^3/x^3+1/6*(-A*b+B*a)/a^2/(b*x^3+a)^2+1/3*(-2*A*b+B*a)/a^3/(b*x^3+a
)-(3*A*b-B*a)*ln(x)/a^4+1/3*(3*A*b-B*a)*ln(b*x^3+a)/a^4
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/3*A/(a^3*x^3) - (A*b - a*B)/(6*a^2*(a + b*x^3)^2) - (2*A*b - a*B)/(3*a^3
*(a + b*x^3)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^3])
/(3*a^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x^2} + \frac{-3Ab+aB}{a^4x} - \frac{b(-Ab+aB)}{a^2(a+bx)^3} - \frac{b(-2Ab+aB)}{a^3(a+bx)^2} - \frac{b(-3Ab+aB)}{a^4(a+bx)}\right.\right.$$
$$= -\frac{A}{3a^3x^3} - \frac{Ab-aB}{6a^2\left(a+bx^3\right)^2} - \frac{2Ab-aB}{3a^3\left(a+bx^3\right)} - \frac{(3Ab-aB)\log(x)}{a^4} + \frac{(3Ab-aB)\log\left(a\right.}{3a^4}$$

Mathematica [A]
time = 0.03, size = 87, normalized size = 0.86

$$\frac{-\frac{2aA}{x^3} + \frac{a^2(-Ab+aB)}{(a+bx^3)^2} + \frac{2a(-2Ab+aB)}{a+bx^3} + 6(-3Ab+aB)\log(x) + 2(3Ab-aB)\log\left(a+bx^3\right)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a*A)/x^3 + (a^2*(-(A*b) + a*B))/(a + b*x^3)^2 + (2*a*(-2*A*b + a*B))/(
a + b*x^3) + 6*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^3])/(6*a
^4)
```

Maple [A]
time = 0.29, size = 102, normalized size = 1.01

method	result	size
norman	$\frac{-\frac{A}{3a}+\frac{2b(3Ab-Ba)x^6}{3a^3}+\frac{b^2(3Ab-Ba)x^9}{2a^4}}{x^3(b\,x^3+a)^2} - \frac{(3Ab-Ba)\ln(x)}{a^4} + \frac{(3Ab-Ba)\ln(b\,x^3+a)}{3a^4}$	98
default	$\frac{b\left(-\frac{a(2Ab-Ba)}{b\left(b\,x^3+a\right)}-\frac{a^2(Ab-Ba)}{2b\left(b\,x^3+a\right)^2}+\frac{(3Ab-Ba)\ln\left(b\,x^3+a\right)}{b}\right)}{3a^4} - \frac{A}{3a^3x^3} + \frac{(-3Ab+Ba)\ln(x)}{a^4}$	102
risch	$\frac{-\frac{b(3Ab-Ba)x^6}{3a^3}-\frac{(3Ab-Ba)x^3}{2a^2}-\frac{A}{3a}}{x^3(b\,x^3+a)^2} - \frac{3\ln(x)Ab}{a^4} + \frac{B\ln(x)}{a^3} + \frac{\ln(-b\,x^3-a)Ab}{a^4} - \frac{\ln(-b\,x^3-a)B}{3a^3}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^4*b*(-a*(2*A*b-B*a)/b/(b*x^3+a)-1/2*a^2*(A*b-B*a)/b/(b*x^3+a)^2+(3*A*
b-B*a)/b*ln(b*x^3+a))-1/3*A/a^3/x^3+(-3*A*b+B*a)/a^4*ln(x)
```

Maxima [A]
time = 0.28, size = 109, normalized size = 1.08

$$\frac{2\,(Bab-3\,Ab^2)x^6+3\,(Ba^2-3\,Aab)x^3-2\,Aa^2}{6\,(a^3b^2x^9+2\,a^4bx^6+a^5x^3)} - \frac{(Ba-3\,Ab)\log\left(bx^3+a\right)}{3\,a^4} + \frac{(Ba-3\,Ab)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(B*a*b - 3*A*b^2)*x^6 + 3*(B*a^2 - 3*A*a*b)*x^3 - 2*A*a^2)/(a^3*b^2*
x^9 + 2*a^4*b*x^6 + a^5*x^3) - 1/3*(B*a - 3*A*b)*log(b*x^3 + a)/a^4 + 1/3*(
B*a - 3*A*b)*log(x^3)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(89) = 178$.
time = 1.95, size = 197, normalized size = 1.95

$$\frac{2\left(Ba^2b-3\,Aab^2\right)x^6-2\,Aa^3+3\left(Ba^3-3\,Aa^2b\right)x^3-2\left(\left(Bab^2-3\,Ab^3\right)x^9+2\left(Ba^2b-3\,Aab^2\right)x^6+\left(Ba^3-3\,Aa^2b\right)x^3\right)\log\left(bx^3+a\right)+6\left(\left(Bab^2-3\,Ab^3\right)x^9+2\left(Ba^2b-3\,Aab^2\right)x^6+\left(Ba^3-3\,Aa^2b\right)x^3\right)\log\left(x\right)}{6\left(a^4b^2x^9+2\,a^5bx^6+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(B*a^2*b - 3*A*a*b^2)*x^6 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^3 - 2*
((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2)*x^6 + (B*a^3 - 3*A*a^2*b
)*x^3)*log(b*x^3 + a) + 6*((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2
)*x^6 + (B*a^3 - 3*A*a^2*b)*x^3)*log(x))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x
^3)
```

Sympy [A]
time = 0.81, size = 107, normalized size = 1.06

$$\frac{-2Aa^2+x^6(-6Ab^2+2Bab)+x^3(-9Aab+3Ba^2)}{6a^5x^3+12a^4bx^6+6a^3b^2x^9}+\frac{(-3Ab+Ba)\log{(x)}}{a^4}-\frac{(-3Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**3,x)
```

```
[Out] (-2*A*a**2 + x**6*(-6*A*b**2 + 2*B*a*b) + x**3*(-9*A*a*b + 3*B*a**2))/(6*a*
*5*x**3 + 12*a**4*b*x**6 + 6*a**3*b**2*x**9) + (-3*A*b + B*a)*log(x)/a**4 -
 (-3*A*b + B*a)*log(a/b + x**3)/(3*a**4)
```

Giac [A]
time = 1.47, size = 136, normalized size = 1.35

$$\frac{(Ba-3\,Ab)\log\left(|x|\right)}{a^4}-\frac{\left(Bab-3\,Ab^2\right)\log\left(|bx^3+a|\right)}{3\,a^4b}+\frac{3\,Bab^2x^6-9\,Ab^3x^6+8\,Ba^2bx^3-22\,Aab^2x^3+6\,Ba^3-14\,Aa^2b}{6\left(bx^3+a\right)^2a^4}-\frac{Bax^3-3\,Abx^3+Aa}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (B*a - 3*A*b)*log(abs(x))/a^4 - 1/3*(B*a*b - 3*A*b^2)*log(abs(b*x^3 + a))/(
a^4*b) + 1/6*(3*B*a*b^2*x^6 - 9*A*b^3*x^6 + 8*B*a^2*b*x^3 - 22*A*a*b^2*x^3
```

```
+ 6*B*a^3 - 14*A*a^2*b)/((b*x^3 + a)^2*a^4) - 1/3*(B*a*x^3 - 3*A*b*x^3 + A*
a)/(a^4*x^3)
```

Mupad [B]
time = 2.46, size = 107, normalized size = 1.06

$$\frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b-B\,a\right)}{3\,a^4}-\frac{\frac{A}{3\,a}+\frac{x^3\,\left(3\,A\,b-B\,a\right)}{2\,a^2}+\frac{b\,x^6\,\left(3\,A\,b-B\,a\right)}{3\,a^3}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9}-\frac{\ln\left(x\right)\,\left(3\,A\,b-B\,a\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] (log(a + b*x^3)*(3*A*b - B*a))/(3*a^4) - (A/(3*a) + (x^3*(3*A*b - B*a))/(2*
a^2) + (b*x^6*(3*A*b - B*a))/(3*a^3))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6) - (lo
g(x)*(3*A*b - B*a))/a^4
```

3.94 $\int \frac{A+Bx^3}{x^7(a+bx^3)^3}\,dx$

Optimal. Leaf size=122

$$-\frac{A}{6a^3x^6}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3(a+bx^3)^2}+\frac{b(3Ab-2aB)}{3a^4(a+bx^3)}+\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{b(2Ab-aB)\log(a+bx^3)}{a^5}$$

```
[Out] -1/6*A/a^3/x^6+1/3*(3*A*b-B*a)/a^4/x^3+1/6*b*(A*b-B*a)/a^3/(b*x^3+a)^2+1/3*
b*(3*A*b-2*B*a)/a^4/(b*x^3+a)+3*b*(2*A*b-B*a)*ln(x)/a^5-b*(2*A*b-B*a)*ln(b*
x^3+a)/a^5
```

Rubi [A]
time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$-\frac{b(2Ab-aB)\log(a+bx^3)}{a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}+\frac{b(3Ab-2aB)}{3a^4(a+bx^3)}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3(a+bx^3)^2}-\frac{A}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] -1/6*A/(a^3*x^6) + (3*A*b - a*B)/(3*a^4*x^3) + (b*(A*b - a*B))/(6*a^3*(a +
b*x^3)^2) + (b*(3*A*b - 2*a*B))/(3*a^4*(a + b*x^3)) + (3*b*(2*A*b - a*B)*Lo
g[x])/a^5 - (b*(2*A*b - a*B)*Log[a + b*x^3])/a^5
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x^3} + \frac{-3Ab+aB}{a^4x^2} - \frac{3b(-2Ab+aB)}{a^5x} + \frac{b^2(-Ab+aB)}{a^3(a+bx)^3} + \frac{b^2(-3Ab+}{a^4(a+b}\right.\right.$$
$$= -\frac{A}{6a^3x^6} + \frac{3Ab-aB}{3a^4x^3} + \frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2} + \frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)} + \frac{3b(2Ab-aB)\log(x)}{a^5} - \frac{b(}{}$$

Mathematica [A]
time = 0.05, size = 108, normalized size = 0.89

$$\frac{-\frac{a^2A}{x^6} - \frac{2a(-3Ab+aB)}{x^3} + \frac{a^2b(Ab-aB)}{(a+bx^3)^2} + \frac{2ab(3Ab-2aB)}{a+bx^3} + 18b(2Ab-aB)\log(x) + 6b(-2Ab+aB)\log\left(a+bx^3\right)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
[Out] (-((a^2*A)/x^6) - (2*a*(-3*A*b + a*B))/x^3 + (a^2*b*(A*b - a*B))/(a + b*x^3
)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^3) + 18*b*(2*A*b - a*B)*Log[x] + 6*b
*(-2*A*b + a*B)*Log[a + b*x^3])/(6*a^5)
```

Maple [A]
time = 0.28, size = 123, normalized size = 1.01

method	result	size
default	$-\frac{b^2\left(-\frac{a(3Ab-2Ba)}{b\left(b\,x^3+a\right)}-\frac{a^2(Ab-Ba)}{2b\left(b\,x^3+a\right)^2}+\frac{(6Ab-3Ba)\ln\left(b\,x^3+a\right)}{b}\right)}{3a^5}-\frac{A}{6a^3x^6}-\frac{-3Ab+Ba}{3a^4x^3}+\frac{3b(2Ab-Ba)\ln(x)}{a^5}$	123
norman	$\frac{-\frac{A}{6a}+\frac{(2Ab-Ba)x^3}{3a^2}-\frac{2b\left(2b^2A-abB\right)x^9}{a^4}-\frac{b^2\left(6b^2A-3abB\right)x^{12}}{2a^5}}{x^6\left(b\,x^3+a\right)^2}+\frac{3b(2Ab-Ba)\ln(x)}{a^5}-\frac{b(2Ab-Ba)\ln\left(b\,x^3+a\right)}{a^5}$	123
risch	$\frac{\frac{b^2(2Ab-Ba)x^9}{a^4}+\frac{3b(2Ab-Ba)x^6}{2a^3}+\frac{(2Ab-Ba)x^3}{3a^2}-\frac{A}{6a}}{x^6\left(b\,x^3+a\right)^2}+\frac{6b^2\ln(x)A}{a^5}-\frac{3b\ln(x)B}{a^4}-\frac{2b^2\ln\left(b\,x^3+a\right)A}{a^5}+\frac{b\ln\left(b\,x^3+a\right)B}{a^4}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/3/a^5*b^2*(-a*(3*A*b-2*B*a)/b/(b*x^3+a)-1/2*a^2*(A*b-B*a)/b/(b*x^3+a)^2+
(6*A*b-3*B*a)/b*ln(b*x^3+a))-1/6*A/a^3/x^6-1/3*(-3*A*b+B*a)/a^4/x^3+3*b*(2*
A*b-B*a)*ln(x)/a^5
```

Maxima [A]
time = 0.27, size = 136, normalized size = 1.11

$$-\frac{6\left(Bab^2-2\,Ab^3\right)x^9+9\left(Ba^2b-2\,Aab^2\right)x^6+Aa^3+2\left(Ba^3-2\,Aa^2b\right)x^3}{6\left(a^4b^2x^{12}+2\,a^5bx^9+a^6x^6\right)}+\frac{\left(Bab-2\,Ab^2\right)\log\left(bx^3+a\right)}{a^5}-\frac{\left(Bab-2\,Ab^2\right)\log\left(x^3\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(B*a*b^2 - 2*A*b^3)*x^9 + 9*(B*a^2*b - 2*A*a*b^2)*x^6 + A*a^3 + 2*(
B*a^3 - 2*A*a^2*b)*x^3)/(a^4*b^2*x^12 + 2*a^5*b*x^9 + a^6*x^6) + (B*a*b - 2
*A*b^2)*log(b*x^3 + a)/a^5 - (B*a*b - 2*A*b^2)*log(x^3)/a^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(110) = 220$.
time = 2.41, size = 229, normalized size = 1.88

$$-\frac{6\,(Ba^2b^2-2\,Aab^3)x^9+9\,(Ba^3b-2\,Aa^2b^2)x^6+Aa^4+2\,(Ba^4-2\,Aa^3b)x^3-6\,((Bab^3-2\,Ab^4)x^{12}+2\,(Ba^2b^2-2\,Aab^3)x^9+(Ba^3b-2\,Aa^2b^2)x^6)\log(bx^3+a)+18\,((Bab^3-2\,Ab^4)x^{12}+2\,(Ba^2b^2-2\,Aab^3)x^9+(Ba^3b-2\,Aa^2b^2)x^6)\log(x)}{6\,(a^5b^2x^{12}+2\,a^6bx^9+a^7x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^6 + A*a^4
 + 2*(B*a^4 - 2*A*a^3*b)*x^3 - 6*((B*a*b^3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 -
 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)*x^6)*log(b*x^3 + a) + 18*((B*a*b^
3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)
*x^6)*log(x))/(a^5*b^2*x^12 + 2*a^6*b*x^9 + a^7*x^6)
```

Sympy [A]
time = 2.01, size = 133, normalized size = 1.09

$$\frac{-Aa^3+x^9\cdot(12Ab^3-6Bab^2)+x^6\cdot(18Aab^2-9Ba^2b)+x^3\cdot(4Aa^2b-2Ba^3)}{6a^6x^6+12a^5bx^9+6a^4b^2x^{12}}-\frac{3b(-2Ab+Ba)\log(x)}{a^5}+\frac{b(-2Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**3,x)
```

```
[Out] (-A*a**3 + x**9*(12*A*b**3 - 6*B*a*b**2) + x**6*(18*A*a*b**2 - 9*B*a**2*b)
+ x**3*(4*A*a**2*b - 2*B*a**3))/(6*a**6*x**6 + 12*a**5*b*x**9 + 6*a**4*b**2
*x**12) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + b*(-2*A*b + B*a)*log(a/b + x**3)
/a**5
```

Giac [A]
time = 1.57, size = 131, normalized size = 1.07

$$-\frac{3\,(Bab-2\,Ab^2)\log(|x|)}{a^5}+\frac{(Bab^2-2\,Ab^3)\log(|bx^3+a|)}{a^5b}-\frac{6\,Bab^2x^9-12\,Ab^3x^9+9\,Ba^2bx^6-18\,Aab^2x^6+2\,Ba^3x^3-4\,Aa^2bx^3+Aa^3}{6\,(bx^6+ax^3)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -3*(B*a*b - 2*A*b^2)*log(abs(x))/a^5 + (B*a*b^2 - 2*A*b^3)*log(abs(b*x^3 +
a))/(a^5*b) - 1/6*(6*B*a*b^2*x^9 - 12*A*b^3*x^9 + 9*B*a^2*b*x^6 - 18*A*a*b^
2*x^6 + 2*B*a^3*x^3 - 4*A*a^2*b*x^3 + A*a^3)/((b*x^6 + a*x^3)^2*a^4)
```

Mupad [B]
time = 0.15, size = 130, normalized size = 1.07

$$\frac{\frac{x^3\,\left(2\,A\,b-B\,a\right)}{3\,a^2}-\frac{A}{6\,a}+\frac{b^2\,x^9\,\left(2\,A\,b-B\,a\right)}{a^4}+\frac{3\,b\,x^6\,\left(2\,A\,b-B\,a\right)}{2\,a^3}}{a^2\,x^6+2\,a\,b\,x^9+b^2\,x^{12}}-\frac{\ln\left(b\,x^3+a\right)\,\left(2\,A\,b^2-B\,a\,b\right)}{a^5}+\frac{\ln\left(x\right)\,\left(6\,A\,b^2-3\,B\,a\,b\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(2*A*b - B*a))/(3*a^2) - A/(6*a) + (b^2*x^9*(2*A*b - B*a))/a^4 + (3*b
*x^6*(2*A*b - B*a))/(2*a^3))/(a^2*x^6 + b^2*x^12 + 2*a*b*x^9) - (log(a + b*
x^3)*(2*A*b^2 - B*a*b))/a^5 + (log(x)*(6*A*b^2 - 3*B*a*b))/a^5
```

3.95 $\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=246

$$\frac{2(5Ab-11aB)x^2}{9b^4}-\frac{4(5Ab-11aB)x^5}{45ab^3}+\frac{(Ab-aB)x^{11}}{6ab\,(a+bx^3)^2}+\frac{(5Ab-11aB)x^8}{18ab^2\,(a+bx^3)}+\frac{4a^{2/3}(5Ab-11aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{9\sqrt{3}\,b^{14/3}}$$

```
[Out] 2/9*(5*A*b-11*B*a)*x^2/b^4-4/45*(5*A*b-11*B*a)*x^5/a/b^3+1/6*(A*b-B*a)*x^11
/a/b/(b*x^3+a)^2+1/18*(5*A*b-11*B*a)*x^8/a/b^2/(b*x^3+a)+4/27*a^(2/3)*(5*A*
b-11*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(14/3)-2/27*a^(2/3)*(5*A*b-11*B*a)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(14/3)+4/27*a^(2/3)*(5*A*b-11*B*a)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(14/3)*3^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {468, 294, 308, 298, 31, 648, 631, 210, 642}

$$\frac{4a^{2/3}(5Ab-11aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,b^{14/3}}-\frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27b^{14/3}}+\frac{2x^2(5Ab-11aB)}{9b^4}-\frac{4x^5(5Ab-11aB)}{45ab^3}+\frac{x^8(5Ab-11aB)}{18ab^2\,(a+bx^3)}+\frac{x^{11}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*(5*A*b - 11*a*B)*x^2)/(9*b^4) - (4*(5*A*b - 11*a*B)*x^5)/(45*a*b^3) + ((
A*b - a*B)*x^11)/(6*a*b*(a + b*x^3)^2) + ((5*A*b - 11*a*B)*x^8)/(18*a*b^2*(
a + b*x^3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(14/3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*Log[a^
(1/3) + b^(1/3)*x])/(27*b^(14/3)) - (2*a^(2/3)*(5*A*b - 11*a*B)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{(Ab-aB)x^{11}}{6ab\,(a+bx^3)^2}+\frac{(-5Ab+11aB)\int \frac{x^{10}}{(a+bx^3)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^{11}}{6ab\,(a+bx^3)^2}+\frac{(5Ab-11aB)x^8}{18ab^2\,(a+bx^3)}-\frac{(4(5Ab-11aB))\int \frac{x^7}{a+bx^3}\,dx}{9ab^2}\\
&= \frac{(Ab-aB)x^{11}}{6ab\,(a+bx^3)^2}+\frac{(5Ab-11aB)x^8}{18ab^2\,(a+bx^3)}-\frac{(4(5Ab-11aB))\int \left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2(a+bx^3)}\right)dx}{9ab^2}\\
&= \frac{2(5Ab-11aB)x^2}{9b^4}-\frac{4(5Ab-11aB)x^5}{45ab^3}+\frac{(Ab-aB)x^{11}}{6ab\,(a+bx^3)^2}+\frac{(5Ab-11aB)x^8}{18ab^2\,(a+bx^3)}-(4a(5\\
&= \frac{2(5Ab-11aB)x^2}{9b^4}-\frac{4(5Ab-11aB)x^5}{45ab^3}+\frac{(Ab-aB)x^{11}}{6ab\,(a+bx^3)^2}+\frac{(5Ab-11aB)x^8}{18ab^2\,(a+bx^3)}+(4a^{2/3}\\
&= \frac{2(5Ab-11aB)x^2}{9b^4}-\frac{4(5Ab-11aB)x^5}{45ab^3}+\frac{(Ab-aB)x^{11}}{6ab\,(a+bx^3)^2}+\frac{(5Ab-11aB)x^8}{18ab^2\,(a+bx^3)}+4a^{2/3}(\\
&= \frac{2(5Ab-11aB)x^2}{9b^4}-\frac{4(5Ab-11aB)x^5}{45ab^3}+\frac{(Ab-aB)x^{11}}{6ab\,(a+bx^3)^2}+\frac{(5Ab-11aB)x^8}{18ab^2\,(a+bx^3)}+4a^{2/3}(\\
&= \frac{2(5Ab-11aB)x^2}{9b^4}-\frac{4(5Ab-11aB)x^5}{45ab^3}+\frac{(Ab-aB)x^{11}}{6ab\,(a+bx^3)^2}+\frac{(5Ab-11aB)x^8}{18ab^2\,(a+bx^3)}+4a^{2/3}(
\end{aligned}
$$

Mathematica [**A**]
time = 0.10, size = 216, normalized size = 0.88

$$\frac{135b^{2/3}(Ab-3aB)x^2+54b^{5/3}Bx^5+\frac{45a^2b^{2/3}(-Ab+aB)x^2}{(a+bx^3)^2}+\frac{30ab^{2/3}(7Ab-10aB)x^2}{a+bx^3}-40\sqrt{3}\,a^{2/3}(-5Ab+11aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-40a^{2/3}(-5Ab+11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+20a^{2/3}(-5Ab+11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{270b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (135*b^(2/3)*(A*b - 3*a*B)*x^2 + 54*b^(5/3)*B*x^5 + (45*a^2*b^(2/3)*(-(A*b)
 + a*B)*x^2)/(a + b*x^3)^2 + (30*a*b^(2/3)*(7*A*b - 10*a*B)*x^2)/(a + b*x^3
) - 40*Sqrt[3]*a^(2/3)*(-5*A*b + 11*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]] - 40*a^(2/3)*(-5*A*b + 11*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*a^(2
/3)*(-5*A*b + 11*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(270*
b^(14/3))
```

Maple [A]
time = 0.28, size = 176, normalized size = 0.72

method	result
risch	$\frac{B\,x^{5}}{5 b^{3}}+\frac{A\,x^{2}}{2 b^{3}}-\frac{3 B a\,x^{2}}{2 b^{4}}+\frac{\left(\frac{7}{9} A a\,b^{2}-\frac{10}{9} B\,a^{2} b\right) x^{5}+\frac{a^{2} \left(11 A b -17 B a \right) x^{2}}{18}}{b^{4} \left(b\,x^{3}+a \right)^{2}}-\frac{4 a \left(\sum_{\_R =\mathrm{RootOf}\left(b\,\_Z^{3}+a \right)}\frac{\left(5 A b -11 B a \right) \ln \left(x -\_R \right)}{\_R}\right)}{27 b^{5}}$
default	$\frac{\frac{b B\,x^{5}}{5}+\frac{\left(A b -3 B a \right) x^{2}}{2}}{b^{4}}-\frac{a \left(\frac{\left(-\frac{7}{9} b^{2} A +\frac{10}{9} a b B \right) x^{5}-\frac{a \left(11 A b -17 B a \right) x^{2}}{18}}{\left(b\,x^{3}+a \right)^{2}}+\left(\frac{20 A b}{9}-\frac{44 B a}{9}\right) \left(-\frac{\ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\ldots \right)\right)}{b^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/5*b*B*x^5+1/2*(A*b-3*B*a)*x^2)-a/b^4*(((-7/9*b^2*A+10/9*a*b*B)*x^5
-1/18*a*(11*A*b-17*B*a)*x^2)/(b*x^3+a)^2+(20/9*A*b-44/9*B*a)*(-1/3/b/(a/b)^
(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.49, size = 228, normalized size = 0.93

$$-\frac{2 \,{\left(10 \, B a^{2} b - 7 \, A a b^{2}\right)} x^{5} + {\left(17 \, B a^{3} - 11 \, A a^{2} b\right)} x^{2}}{18 \,{\left(b^{6} x^{6} + 2 \, a b^{5} x^{3} + a^{2} b^{4}\right)}} + \frac{4 \, \sqrt{3} {\left(11 \, B a^{2} - 5 \, A a b\right)} \arctan\left(\frac{\sqrt{3} {\left(2 \, x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}}{3 \, \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 \, b^{5} \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2 \, B b x^{5} - 5 \,{\left(3 \, B a - A b\right)} x^{2}}{10 \, b^{4}} + \frac{2 \,{\left(11 \, B a^{2} - 5 \, A a b\right)} \log\left(x^{2} - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 \, b^{5} \left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4 \,{\left(11 \, B a^{2} - 5 \, A a b\right)} \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 \, b^{5} \left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(10*B*a^2*b - 7*A*a*b^2)*x^5 + (17*B*a^3 - 11*A*a^2*b)*x^2)/(b^6*x
^6 + 2*a*b^5*x^3 + a^2*b^4) + 4/27*sqrt(3)*(11*B*a^2 - 5*A*a*b)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/10*(2*B*b*x^
5 - 5*(3*B*a - A*b)*x^2)/b^4 + 2/27*(11*B*a^2 - 5*A*a*b)*log(x^2 - x*(a/b)^
(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(1/3)) - 4/27*(11*B*a^2 - 5*A*a*b)*log(x +
(a/b)^(1/3))/(b^5*(a/b)^(1/3))
```

Fricas [A]
time = 1.86, size = 364, normalized size = 1.48

$$\frac{54 \, B b^{3} x^{11} - 27 \,{\left(11 \, B a b^{2} - 5 \, A b^{3}\right)} x^{8} - 96 \,{\left(11 \, B a^{2} b - 5 \, A a b^{2}\right)} x^{5} - 60 \,{\left(11 \, B a^{3} - 5 \, A a^{2} b\right)} x^{2} + 40 \, \sqrt{3} {\left({\left(11 \, B a b^{2} - 5 \, A b^{3}\right)} x^{6} + 11 \, B a^{3} - 5 \, A a^{2} b + 2 \,{\left(11 \, B a^{2} b - 5 \, A a b^{2}\right)} x^{3}\right)} \left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}} \arctan\left(\frac{2 \, \sqrt{3} b x \left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}} - \sqrt{3} a}{3 \, a}\right) + 20 \,{\left({\left(11 \, B a b^{2} - 5 \, A b^{3}\right)} x^{6} + 11 \, B a^{3} - 5 \, A a^{2} b + 2 \,{\left(11 \, B a^{2} b - 5 \, A a b^{2}\right)} x^{3}\right)} \left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}} \log\left(a x^{2} - b x \left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}} + a \left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right) - 40 \,{\left({\left(11 \, B a b^{2} - 5 \, A b^{3}\right)} x^{6} + 11 \, B a^{3} - 5 \, A a^{2} b + 2 \,{\left(11 \, B a^{2} b - 5 \, A a b^{2}\right)} x^{3}\right)} \left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}} \log\left(a x + b \left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}\right)}{270 \,{\left(b^{6} x^{6} + 2 \, a b^{5} x^{3} + a^{2} b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/270*(54*B*b^3*x^11 - 27*(11*B*a*b^2 - 5*A*b^3)*x^8 - 96*(11*B*a^2*b - 5*A
*a*b^2)*x^5 - 60*(11*B*a^3 - 5*A*a^2*b)*x^2 + 40*sqrt(3)*((11*B*a*b^2 - 5*A
*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)*x^3)*(a^2/b^2
)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 20*((11
*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)
*x^3)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3))
- 40*((11*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5
*A*a*b^2)*x^3)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^6*x^6 + 2*a
*b^5*x^3 + a^2*b^4)
```

Sympy [A]
time = 10.26, size = 192, normalized size = 0.78

$$\frac{Bx^5}{5b^3} + x^2\left(\frac{A}{2b^3} - \frac{3Ba}{2b^4}\right) + \frac{x^5\cdot(14Aab^2 - 20Ba^2b) + x^2\cdot(11Aa^2b - 17Ba^3)}{18a^2b^4 + 36ab^5x^3 + 18b^6x^6} + \operatorname{RootSum}\left(19683t^3b^{14} - 8000A^3a^2b^3 + 52800A^2Ba^3b^2 - 116160AB^2a^4b + 85184B^3a^5, \left(t \mapsto t\log\left(\frac{729t^2b^9}{400A^2ab^2 - 1760ABa^2b + 1936B^2a^3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**5/(5*b**3) + x**2*(A/(2*b**3) - 3*B*a/(2*b**4)) + (x**5*(14*A*a*b**2 -
 20*B*a**2*b) + x**2*(11*A*a**2*b - 17*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x
**3 + 18*b**6*x**6) + RootSum(19683*_t**3*b**14 - 8000*A**3*a**2*b**3 + 528
00*A**2*B*a**3*b**2 - 116160*A*B**2*a**4*b + 85184*B**3*a**5, Lambda(_t, _t
*log(729*_t**2*b**9/(400*A**2*a*b**2 - 1760*A*B*a**2*b + 1936*B**2*a**3) +
x)))
```

Giac [A]
time = 1.11, size = 259, normalized size = 1.05

$$-\frac{4\left(11\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^4} - \frac{4\sqrt{3}\left(11\left(-ab^2\right)^{\frac{2}{3}}Ba - 5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^6} + \frac{2\left(11\left(-ab^2\right)^{\frac{2}{3}}Ba - 5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^6} - \frac{20\,Ba^2bx^5 - 14\,Aab^2x^5 + 17\,Ba^3x^2 - 11\,Aa^2bx^2}{18\left(bx^3 + a\right)^2b^4} + \frac{2\,Bb^{12}x^5 - 15\,Bab^{11}x^2 + 5\,Ab^{12}x^2}{10\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -4/27*(11*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a*b^4) - 4/27*sqrt(3)*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2
)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^6 + 2/
27*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/b^6 - 1/18*(20*B*a^2*b*x^5 - 14*A*a*b^2*x^5 + 17*B*a^3*x^2
- 11*A*a^2*b*x^2)/((b*x^3 + a)^2*b^4) + 1/10*(2*B*b^12*x^5 - 15*B*a*b^11*x^
2 + 5*A*b^12*x^2)/b^15
```

Mupad [B]
time = 2.58, size = 213, normalized size = 0.87

$$\frac{x^5\left(\frac{7\,A\,a\,b^2}{9} - \frac{10\,B\,a^2\,b}{9}\right) - x^2\left(\frac{17\,B\,a^3}{18} - \frac{11\,A\,a^2\,b}{18}\right)}{a^2\,b^4 + 2\,a\,b^5\,x^3 + b^6\,x^6} + x^2\left(\frac{A}{2\,b^3} - \frac{3\,B\,a}{2\,b^4}\right) + \frac{B\,x^5}{5\,b^3} + \frac{4\,a^{2/3}\,\ln\left(b^{1/3}\,x + a^{1/3}\right)\,\left(5\,A\,b - 11\,B\,a\right)}{27\,b^{14/3}} + \frac{4\,a^{2/3}\,\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,A\,b - 11\,B\,a\right)}{27\,b^{14/3}} - \frac{4\,a^{2/3}\,\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,A\,b - 11\,B\,a\right)}{27\,b^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^5*((7*A*a*b^2)/9 - (10*B*a^2*b)/9) - x^2*((17*B*a^3)/18 - (11*A*a^2*b)/1
8))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x^2*(A/(2*b^3) - (3*B*a)/(2*b^4)) +
 (B*x^5)/(5*b^3) + (4*a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 11*B*a))/(2
7*b^(14/3)) + (4*a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(5*A*b - 11*B*a))/(27*b^(14/3)) - (4*a^(2/3)*log(3^(1/
2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 11*B
*a))/(27*b^(14/3))
```

3.96 $\int \frac{x^9 (A+Bx^3)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=244

$$\frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab(a+bx^3)^2} + \frac{(2Ab-5aB)x^7}{9ab^2(a+bx^3)} + \frac{7\sqrt[3]{a}(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,b^{13/3}}$$

```
[Out] 7/9*(2*A*b-5*B*a)*x/b^4-7/36*(2*A*b-5*B*a)*x^4/a/b^3+1/6*(A*b-B*a)*x^10/a/b
/(b*x^3+a)^2+1/9*(2*A*b-5*B*a)*x^7/a/b^2/(b*x^3+a)-7/27*a^(1/3)*(2*A*b-5*B*
a)*ln(a^(1/3)+b^(1/3)*x)/b^(13/3)+7/54*a^(1/3)*(2*A*b-5*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(13/3)+7/27*a^(1/3)*(2*A*b-5*B*a)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(13/3)*3^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {468, 294, 308, 206, 31, 648, 631, 210, 642}

$$\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{13/3}} + \frac{7\sqrt[3]{a}(2Ab-5aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,b^{13/3}} - \frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{13/3}} + \frac{7x(2Ab-5aB)}{9b^4} - \frac{7x^4(2Ab-5aB)}{36ab^3} + \frac{x^7(2Ab-5aB)}{9ab^2(a+bx^3)} + \frac{x^{10}(Ab-aB)}{6ab(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (7*(2*A*b - 5*a*B)*x)/(9*b^4) - (7*(2*A*b - 5*a*B)*x^4)/(36*a*b^3) + ((A*b
- a*B)*x^10)/(6*a*b*(a + b*x^3)^2) + ((2*A*b - 5*a*B)*x^7)/(9*a*b^2*(a + b*
x^3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(9*Sqrt[3]*b^(13/3)) - (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(1/3) +
b^(1/3)*x])/(27*b^(13/3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{(Ab-aB)x^{10}}{6ab\,(a+bx^3)^2}+\frac{(-4Ab+10aB)\int \frac{x^9}{(a+bx^3)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^{10}}{6ab\,(a+bx^3)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\,(a+bx^3)}-\frac{(7(2Ab-5aB))\int \frac{x^6}{a+bx^3}\,dx}{9ab^2}\\
&= \frac{(Ab-aB)x^{10}}{6ab\,(a+bx^3)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\,(a+bx^3)}-\frac{(7(2Ab-5aB))\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2(a+bx^3)}\right)dx}{9ab^2}\\
&= \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\,(a+bx^3)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\,(a+bx^3)}-\frac{(7a(2Ab-\ldots}{}\\
&= \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\,(a+bx^3)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\,(a+bx^3)}-\frac{(7\sqrt[3]{a}\,(2Ab\ldots}{}\\
&= \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\,(a+bx^3)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\,(a+bx^3)}-\frac{7\sqrt[3]{a}\,(2Ab\ldots}{}\\
&= \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\,(a+bx^3)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\,(a+bx^3)}-\frac{7\sqrt[3]{a}\,(2Ab\ldots}{}\\
&= \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\,(a+bx^3)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\,(a+bx^3)}+\frac{7\sqrt[3]{a}\,(2Ab\ldots}{}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 210, normalized size = 0.86

$$\frac{108\sqrt[3]{b}\,(Ab-3aB)x+27b^{4/3}Bx^4+\frac{18a^2\sqrt[3]{b}\,(-Ab+aB)x}{(a+bx^3)^2}+\frac{6a\sqrt[3]{b}\,(13Ab-19aB)x}{a+bx^3}-28\sqrt{3}\,\sqrt[3]{a}\,(-2Ab+5aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+28\sqrt[3]{a}\,(-2Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-14\sqrt[3]{a}\,(-2Ab+5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{108b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (108*b^(1/3)*(A*b - 3*a*B)*x + 27*b^(4/3)*B*x^4 + (18*a^2*b^(1/3)*(-(A*b) +
 a*B)*x)/(a + b*x^3)^2 + (6*a*b^(1/3)*(13*A*b - 19*a*B)*x)/(a + b*x^3) - 28
*Sqrt[3]*a^(1/3)*(-2*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
```

```
]] + 28*a^(1/3)*(-2*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x] - 14*a^(1/3)*(-2*
A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*b^(13/3))
```

Maple [A]

time = 0.29, size = 171, normalized size = 0.70

method	result
risch	$\frac{B\,x^{4}}{4 b^{3}}+\frac{A x}{b^{3}}-\frac{3 B a x}{b^{4}}+\frac{\left(\frac{13}{18} A a\,b^{2}-\frac{19}{18} B\,a^{2} b\right) x^{4}+\frac{a^{2}(5 A b-8 B a) x}{9}}{b^{4}\left(b\,x^{3}+a\right)^{2}}-\frac{7 a \left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(2 A b-5 B a)\ln\left(x-\_R\right)}{\_R^{2}}\right)}{27 b^{5}}$
default	$\frac{\frac{1}{4} b B\,x^{4}+A b x-3 B a x}{b^{4}}-\frac{a\left(\frac{\left(-\frac{13}{18} b^{2} A+\frac{19}{18} a b B\right) x^{4}-\frac{a(5 A b-8 B a) x}{9}}{\left(b\,x^{3}+a\right)^{2}}+\frac{7(2 A b-5 B a)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\ \text{arcta}}{}\right.}{9}\right.}{b^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*b*B*x^4+A*b*x-3*B*a*x)-a/b^4*(((-13/18*b^2*A+19/18*a*b*B)*x^4-1/
9*a*(5*A*b-8*B*a)*x)/(b*x^3+a)^2+7/9*(2*A*b-5*B*a)*(1/3/b/(a/b)^(2/3)*ln(x+
(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]

time = 0.51, size = 223, normalized size = 0.91

$$-\frac{(19\,Ba^{2}b-13\,Aab^{2})x^{4}+2\,(8\,Ba^{3}-5\,Aa^{2}b)x}{18\,(b^{6}x^{6}+2\,ab^{5}x^{3}+a^{2}b^{4})}+\frac{Bbx^{4}-4\,(3\,Ba-Ab)x}{4\,b^{4}}+\frac{7\,\sqrt{3}\,(5\,Ba^{2}-2\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{7\,(5\,Ba^{2}-2\,Aab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{7\,(5\,Ba^{2}-2\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((19*B*a^2*b - 13*A*a*b^2)*x^4 + 2*(8*B*a^3 - 5*A*a^2*b)*x)/(b^6*x^6
+ 2*a*b^5*x^3 + a^2*b^4) + 1/4*(B*b*x^4 - 4*(3*B*a - A*b)*x)/b^4 + 7/27*sqr
t(3)*(5*B*a^2 - 2*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(b^5*(a/b)^(2/3)) - 7/54*(5*B*a^2 - 2*A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(b^5*(a/b)^(2/3)) + 7/27*(5*B*a^2 - 2*A*a*b)*log(x + (a/b)^(1/3)
)/(b^5*(a/b)^(2/3))
```

Fricas [A]
time = 1.45, size = 347, normalized size = 1.42

$$\frac{27\,Bb^3x^{10}-54\,(5\,Bab^2-2\,Ab^3)x^7-147\,(5\,Ba^2b-2\,Aab^2)x^4-28\,\sqrt{3}\,((5\,Bab^2-2\,Ab^3)x^6+5\,Ba^3-2\,Aa^2b+2\,(5\,Ba^2b-2\,Aab^2)x^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+14\,((5\,Bab^2-2\,Ab^3)x^6+5\,Ba^3-2\,Aa^2b+2\,(5\,Ba^2b-2\,Aab^2)x^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)-28\,((5\,Bab^2-2\,Ab^3)x^6+5\,Ba^3-2\,Aa^2b+2\,(5\,Ba^2b-2\,Aab^2)x^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)-84\,(5\,Ba^3-2\,Aa^2b)x}{108\,(b^6x^6+2\,ab^5x^3+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(27*B*b^3*x^10 - 54*(5*B*a*b^2 - 2*A*b^3)*x^7 - 147*(5*B*a^2*b - 2*A*
a*b^2)*x^4 - 28*sqrt(3)*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b +
2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b
)^(2/3) - sqrt(3)*a)/a) + 14*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2
*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3)) - 28*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b + 2*(5
*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 84*(5*B*a^3
 - 2*A*a^2*b)*x)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A]
time = 1.40, size = 163, normalized size = 0.67

$$\frac{Bx^4}{4b^3}+x\left(\frac{A}{b^3}-\frac{3Ba}{b^4}\right)+\frac{x^4\cdot(13Aab^2-19Ba^2b)+x(10Aa^2b-16Ba^3)}{18a^2b^4+36ab^5x^3+18b^6x^6}+\operatorname{RootSum}\left(19683t^3b^{13}+2744A^3ab^3-20580A^2Ba^2b^2+51450AB^2a^3b-42875B^3a^4,\left(t\mapsto t\log\left(\frac{27tb^4}{-14Ab+35Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**4/(4*b**3) + x*(A/b**3 - 3*B*a/b**4) + (x**4*(13*A*a*b**2 - 19*B*a**2*
b) + x*(10*A*a**2*b - 16*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x**3 + 18*b**6*
x**6) + RootSum(19683*_t**3*b**13 + 2744*A**3*a*b**3 - 20580*A**2*B*a**2*b*
*2 + 51450*A*B**2*a**3*b - 42875*B**3*a**4, Lambda(_t, _t*log(27*_t*b**4/(-
14*A*b + 35*B*a) + x)))
```

Giac [A]
time = 0.77, size = 234, normalized size = 0.96

$$\frac{7\,\sqrt{3}\left(5\,(-ab^2)^{\frac{1}{3}}\,Ba-2\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5}-\frac{7\,(5\,Ba^2-2\,Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^4}+\frac{7\left(5\,(-ab^2)^{\frac{1}{3}}\,Ba-2\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^5}-\frac{19\,Ba^2bx^4-13\,Aab^2x^4+16\,Ba^3x-10\,Aa^2bx}{18\,(bx^3+a)^2b^4}+\frac{Bb^9x^4-12\,Bab^8x+4\,Ab^9x}{4\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 7/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 2*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 7/27*(5*B*a^2 - 2*A*a*b)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 7/54*(5*(-a*b^2)^(1/3)*B*a - 2*
(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 - 1/18*(19
*B*a^2*b*x^4 - 13*A*a*b^2*x^4 + 16*B*a^3*x - 10*A*a^2*b*x)/((b*x^3 + a)^2*b
^4) + 1/4*(B*b^9*x^4 - 12*B*a*b^8*x + 4*A*b^9*x)/b^12
```

Mupad [B]
time = 0.32, size = 227, normalized size = 0.93

$$\frac{x^4\left(\frac{13\,A\,a\,b^2}{18}-\frac{19\,B\,a^2\,b}{18}\right)-x\left(\frac{8\,B\,a^3}{9}-\frac{5\,A\,a^2\,b}{9}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)+\frac{B\,x^4}{4\,b^3}+\frac{7\,{\left(-a\right)}^{1/3}\,\ln\left({\left(-a\right)}^{4/3}+a\,b^{1/3}\,x\right)\,\left(2\,A\,b-5\,B\,a\right)}{27\,b^{13/3}}-\frac{7\,{\left(-a\right)}^{1/3}\,\ln\left({\left(-a\right)}^{4/3}-2\,a\,b^{1/3}\,x+\sqrt{3}\,{\left(-a\right)}^{4/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b-5\,B\,a\right)}{27\,b^{13/3}}+\frac{7\,{\left(-a\right)}^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-{\left(-a\right)}^{4/3}+\sqrt{3}\,{\left(-a\right)}^{4/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b-5\,B\,a\right)}{27\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^4*((13*A*a*b^2)/18 - (19*B*a^2*b)/18) - x*((8*B*a^3)/9 - (5*A*a^2*b)/9))
/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x*(A/b^3 - (3*B*a)/b^4) + (B*x^4)/(4*b
^3) + (7*(-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(2*A*b - 5*B*a))/(27*b^(1
3/3)) - (7*(-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a*b^(1/3)*
x)*((3^(1/2)*1i)/2 + 1/2)*(2*A*b - 5*B*a))/(27*b^(13/3)) + (7*(-a)^(1/3)*lo
g(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2
)*(2*A*b - 5*B*a))/(27*b^(13/3))
```

3.97 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=222

$$-\frac{5(Ab-4aB)x^2}{18ab^3}+\frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2}+\frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)}-\frac{5(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,\sqrt[3]{a}\,b^{11/3}}-\frac{5(Ab-4aB)\log\left(\right.}{27\sqrt[3]{a}\,b^{1}}$$

```
[Out] -5/18*(A*b-4*B*a)*x^2/a/b^3+1/6*(A*b-B*a)*x^8/a/b/(b*x^3+a)^2+1/9*(A*b-4*B*
a)*x^5/a/b^2/(b*x^3+a)-5/27*(A*b-4*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(11
/3)+5/54*(A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(1
1/3)-5/27*(A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
1/3)/b^(11/3)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {468, 294, 327, 298, 31, 648, 631, 210, 642}

$$\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54\sqrt[3]{a}\,b^{11/3}}-\frac{5(Ab-4aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,\sqrt[3]{a}\,b^{11/3}}-\frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27\sqrt[3]{a}\,b^{11/3}}-\frac{5x^2(Ab-4aB)}{18ab^3}+\frac{x^5(Ab-4aB)}{9ab^2\,(a+bx^3)}+\frac{x^8(Ab-aB)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (-5*(A*b - 4*a*B)*x^2)/(18*a*b^3) + ((A*b - a*B)*x^8)/(6*a*b*(a + b*x^3)^2)
 + ((A*b - 4*a*B)*x^5)/(9*a*b^2*(a + b*x^3)) - (5*(A*b - 4*a*B)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(11/3)) - (5*(
A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*b^(11/3)) + (5*(A*b - 4*
a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2}+\frac{(-2Ab+8aB)\int \frac{x^7}{(a+bx^3)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2}+\frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)}-\frac{(5(Ab-4aB))\int \frac{x^4}{a+bx^3}\,dx}{9ab^2}\\
&= -\frac{5(Ab-4aB)x^2}{18ab^3}+\frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2}+\frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)}+\frac{(5(Ab-4aB))\int \frac{x}{a+bx^3}\,dx}{9b^3}\\
&= -\frac{5(Ab-4aB)x^2}{18ab^3}+\frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2}+\frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)}-\frac{(5(Ab-4aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27\sqrt[3]{a}\,b^{10/3}}\\
&= -\frac{5(Ab-4aB)x^2}{18ab^3}+\frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2}+\frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)}-\frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{27\sqrt[3]{a}\,b^{11/3}}\\
&= -\frac{5(Ab-4aB)x^2}{18ab^3}+\frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2}+\frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)}-\frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{27\sqrt[3]{a}\,b^{11/3}}\\
&= -\frac{5(Ab-4aB)x^2}{18ab^3}+\frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2}+\frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)}-\frac{5(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\,\sqrt[3]{a}}\right.}{9\sqrt{3}\,\sqrt[3]{a}\,b^{11/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 194, normalized size = 0.87

$$
\frac{27b^{2/3}Bx^2+\frac{9ab^{2/3}(Ab-aB)x^2}{(a+bx^3)^2}-\frac{6b^{2/3}(4Ab-7aB)x^2}{a+bx^3}+\frac{10\sqrt{3}\,(-Ab+4aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{10(-Ab+4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}}+\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{a}}}{54b^{11/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (27*b^(2/3)*B*x^2 + (9*a*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 - (6*b^(2/3
)*(4*A*b - 7*a*B)*x^2)/(a + b*x^3) + (10*Sqrt[3]*(-(A*b) + 4*a*B)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (10*(-(A*b) + 4*a*B)*Log[a^(1
/3) + b^(1/3)*x])/a^(1/3) + (5*(A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/a^(1/3))/(54*b^(11/3))
```

Maple [A]
time = 0.29, size = 158, normalized size = 0.71

method	result
risch	$\frac{B x^{2}}{2 b^{3}}+\frac{\left(-\frac{4}{9} b^{2} A+\frac{7}{9} a b B\right) x^{5}-\frac{a(5 A b-11 B a) x^{2}}{18}}{b^{3}\left(b x^{3}+a\right)^{2}}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)} \frac{(A b-4 B a) \ln \left(x-\_R\right)}{\_R}\right)}{27 b^{4}}$
default	$\frac{B x^{2}}{2 b^{3}}+\frac{\frac{\left(-\frac{4}{9} b^{2} A+\frac{7}{9} a b B\right) x^{5}-\frac{a(5 A b-11 B a) x^{2}}{18}}{\left(b x^{3}+a\right)^{2}}+\left(\frac{5 A b}{9}-\frac{20 B a}{9}\right)\left(-\frac{\ln \left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3} \arctan \left(\frac{\sqrt{3}\left(\ldots\right)}{3}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{1}{3}}} \ldots\right.}{b^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/b^3+1/b^3*(((-4/9*b^2*A+7/9*a*b*B)*x^5-1/18*a*(5*A*b-11*B*a)*x^2)
/(b*x^3+a)^2+(5/9*A*b-20/9*B*a)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b
/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.51, size = 196, normalized size = 0.88

$$\frac{2\,(7\,Bab-4\,Ab^2)x^5+(11\,Ba^2-5\,Aab)x^2}{18\,(b^5x^6+2\,ab^4x^3+a^2b^3)}+\frac{Bx^2}{2\,b^3}-\frac{5\,\sqrt{3}\,(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5\,(4\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,(4\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(7*B*a*b - 4*A*b^2)*x^5 + (11*B*a^2 - 5*A*a*b)*x^2)/(b^5*x^6 + 2*a*
b^4*x^3 + a^2*b^3) + 1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) - 5/54*(4*B*a -
A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) + 5/27*(4*B*a
 - A*b)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(184) = 368$.
time = 2.33, size = 792, normalized size = 3.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(27*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 -
A*a^2*b^3)*x^2 - 15*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*
a^3*b^2 + 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^
2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)
*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*((4*B*a*b^2
- A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2
/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*b^2 - A
*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)
*log(b*x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5), 1/54*(27
*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 - A*a^2*b^3
)*x^2 - 30*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*a^3*b^2 +
 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*
(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*((4*B*a*b^2 - A*b^3
)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log
(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*b^2 - A*b^3)*x
^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5)]
```

Sympy [A]
time = 8.39, size = 162, normalized size = 0.73

$$\frac{Bx^2}{2b^3} + \frac{x^5(-8Ab^2 + 14Bab) + x^2(-5Aab + 11Ba^2)}{18a^2b^3 + 36ab^4x^3 + 18b^5x^6} + \operatorname{RootSum}\left(19683t^3ab^{11} + 125A^3b^3 - 1500A^2Bab^2 + 6000AB^2a^2b - 8000B^3a^3, \left(t \mapsto t\log\left(\frac{729t^2ab^7}{25A^2b^2 - 200ABab + 400B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**2/(2*b**3) + (x**5*(-8*A*b**2 + 14*B*a*b) + x**2*(-5*A*a*b + 11*B*a**2
))/(18*a**2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a*b
**11 + 125*A**3*b**3 - 1500*A**2*B*a*b**2 + 6000*A*B**2*a**2*b - 8000*B**3*
a**3, Lambda(_t, _t*log(729*_t**2*a*b**7/(25*A**2*b**2 - 200*A*B*a*b + 400*
B**2*a**2) + x)))
```

Giac [A]
time = 0.95, size = 210, normalized size = 0.95

$$\frac{Bx^2}{2\,b^3} - \frac{5\,\sqrt{3}\,(4\,Ba - Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}\,b^3} + \frac{5\,(4\,Ba - Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}\,b^3} + \frac{5\left(4\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^3} + \frac{14\,Babx^5 - 8\,Ab^2x^5 + 11\,Ba^2x^2 - 5\,Aabx^2}{18\,\left(bx^3 + a\right)^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^3) + 5/54*(4*B*a - A*b)*log(x^2 + x
```

```
*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^3) + 5/27*(4*B*a*(-a/b)^(1/
3) - A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/
18*(14*B*a*b*x^5 - 8*A*b^2*x^5 + 11*B*a^2*x^2 - 5*A*a*b*x^2)/((b*x^3 + a)^2
*b^3)
```

Mupad [B]
time = 2.56, size = 187, normalized size = 0.84

$$\frac{x^2\left(\frac{11\,B\,a^2}{18}-\frac{5\,A\,a\,b}{18}\right)-x^5\left(\frac{4\,A\,b^2}{9}-\frac{7\,B\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{B\,x^2}{2\,b^3}-\frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}}-\frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}}+\frac{5\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^2*((11*B*a^2)/18 - (5*A*a*b)/18) - x^5*((4*A*b^2)/9 - (7*B*a*b)/9))/(a^2
*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (B*x^2)/(2*b^3) - (5*log(b^(1/3)*x + a^(1/3
))*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3)) - (5*log(3^(1/2)*a^(1/3)*1i - 2*b^(
1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3
)) + (5*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1
/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3))
```

3.98 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=220

$$-\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab\,(a+bx^3)^2}+\frac{(Ab-7aB)x^4}{18ab^2\,(a+bx^3)}-\frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{2/3}b^{10/3}}+\frac{2(Ab-7aB)\log\left(\cdot\right.}{27a^{2/3}b^{1\cdots}}$$

```
[Out] -2/9*(A*b-7*B*a)*x/a/b^3+1/6*(A*b-B*a)*x^7/a/b/(b*x^3+a)^2+1/18*(A*b-7*B*a)
*x^4/a/b^2/(b*x^3+a)+2/27*(A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10/3
)-1/27*(A*b-7*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(10/
3)-2/27*(A*b-7*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/
3)/b^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {468, 294, 327, 206, 31, 648, 631, 210, 642}

$$-\frac{2(Ab-7aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{2/3}b^{10/3}}-\frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{2/3}b^{10/3}}-\frac{2x(Ab-7aB)}{9ab^3}+\frac{x^4(Ab-7aB)}{18ab^2\,(a+bx^3)}+\frac{x^7(Ab-aB)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (-2*(A*b - 7*a*B)*x)/(9*a*b^3) + ((A*b - a*B)*x^7)/(6*a*b*(a + b*x^3)^2) +
((A*b - 7*a*B)*x^4)/(18*a*b^2*(a + b*x^3)) - (2*(A*b - 7*a*B)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(10/3)) + (2*(A*
b - 7*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(10/3)) - ((A*b - 7*a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(2/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^7}{6ab\,(a+bx^3)^2}+\frac{(-Ab+7aB)\int \frac{x^6}{(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^7}{6ab\,(a+bx^3)^2}+\frac{(Ab-7aB)x^4}{18ab^2\,(a+bx^3)}-\frac{(2(Ab-7aB))\int \frac{x^3}{a+bx^3}\,dx}{9ab^2}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab\,(a+bx^3)^2}+\frac{(Ab-7aB)x^4}{18ab^2\,(a+bx^3)}+\frac{(2(Ab-7aB))\int \frac{1}{a+bx^3}\,dx}{9b^3}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab\,(a+bx^3)^2}+\frac{(Ab-7aB)x^4}{18ab^2\,(a+bx^3)}+\frac{(2(Ab-7aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{2/3}b^3}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab\,(a+bx^3)^2}+\frac{(Ab-7aB)x^4}{18ab^2\,(a+bx^3)}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{27a^{2/3}b^{10/3}}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab\,(a+bx^3)^2}+\frac{(Ab-7aB)x^4}{18ab^2\,(a+bx^3)}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{27a^{2/3}b^{10/3}}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab\,(a+bx^3)^2}+\frac{(Ab-7aB)x^4}{18ab^2\,(a+bx^3)}-\frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\,\sqrt[3]{a}}\right.}{9\sqrt{3}\,a^{2/3}b^{10/3}}$$

Mathematica [A]
time = 0.09, size = 188, normalized size = 0.85

$$\frac{54\sqrt[3]{b}\,Bx+\frac{9a\sqrt[3]{b}\,(Ab-aB)x}{(a+bx^3)^2}-\frac{3\sqrt[3]{b}\,(7Ab-13aB)x}{a+bx^3}+\frac{4\sqrt{3}\,(-Ab+7aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{4(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}}+\frac{2(-Ab+7aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}}}{54b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(1/3)*B*x + (9*a*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 - (3*b^(1/3)*(7
*A*b - 13*a*B)*x)/(a + b*x^3) + (4*Sqrt[3]*(-(A*b) + 7*a*B)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*(A*b - 7*a*B)*Log[a^(1/3) + b^(1
/3)*x])/a^(2/3) + (2*(-(A*b) + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(2/3))/(54*b^(10/3))
```

Maple [A]
time = 0.29, size = 153, normalized size = 0.70

method	result
risch	$\frac{Bx}{b^3}+\frac{\left(-\frac{7}{18}b^2A+\frac{13}{18}abB\right)x^4-\frac{a(2Ab-5Ba)x}{9}}{b^3(b\,x^3+a)^2}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-7Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{27b^4}$
default	$\frac{Bx}{b^3}+\frac{\frac{\left(-\frac{7}{18}b^2A+\frac{13}{18}abB\right)x^4-\frac{a(2Ab-5Ba)x}{9}}{\left(b\,x^3+a\right)^2}+\frac{2(Ab-7Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b^3+1/b^3*(((-7/18*b^2*A+13/18*a*b*B)*x^4-1/9*a*(2*A*b-5*B*a)*x)/(b*x^3
+a)^2+2/9*(A*b-7*B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Maxima [A]
time = 0.50, size = 191, normalized size = 0.87

$$\frac{(13\,Bab-7\,Ab^2)x^4+2\,(5\,Ba^2-2\,Aab)x}{18\,(b^5x^6+2\,ab^4x^3+a^2b^3)}+\frac{Bx}{b^3}-\frac{2\,\sqrt{3}\,(7\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(7\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\,(7\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((13*B*a*b - 7*A*b^2)*x^4 + 2*(5*B*a^2 - 2*A*a*b)*x)/(b^5*x^6 + 2*a*b^
4*x^3 + a^2*b^3) + B*x/b^3 - 2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) + 1/27*(7*B*a - A*b)*log
(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 2/27*(7*B*a - A*b)*
log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(182) = 364.
time = 1.72, size = 789, normalized size = 3.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(54*B*a^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 6*sqrt(1/3)*((
7*B*a^2*b^3 - A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2
*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b
+ 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(7*
B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*B*
a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), 1/54*(54*B*a
^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 12*sqrt(1/3)*((7*B*a^2*b^3
- A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2*b^3)*x^3)*s
qrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*
b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*
x + (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(
7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*
B*a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4)]
```

Sympy [A]
time = 1.22, size = 141, normalized size = 0.64

$$\frac{Bx}{b^3} + \frac{x^4(-7Ab^2+13Bab)+x(-4Aab+10Ba^2)}{18a^2b^3+36ab^4x^3+18b^5x^6} + \operatorname{RootSum}\left(19683t^3a^2b^{10}-8A^3b^3+168A^2Bab^2-1176AB^2a^2b+2744B^3a^3, \left(t \mapsto t\log\left(-\frac{27tab^3}{-2Ab+14Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x/b**3 + (x**4*(-7*A*b**2 + 13*B*a*b) + x*(-4*A*a*b + 10*B*a**2))/(18*a**
2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a**2*b**10 -
8*A**3*b**3 + 168*A**2*B*a*b**2 - 1176*A*B**2*a**2*b + 2744*B**3*a**3, Lamb
da(_t, _t*log(-27*_t*a*b**3/(-2*A*b + 14*B*a) + x)))
```

Giac [A]
time = 0.63, size = 187, normalized size = 0.85

$$\frac{2\sqrt{3}\,(7\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}\,b^2} + \frac{(7\,Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}\,b^2} + \frac{Bx}{b^3} + \frac{2\,(7\,Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^3} + \frac{13\,Babx^4-7\,Ab^2x^4+10\,Ba^2x-4\,Aabx}{18\,(bx^3+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*b^2) + 1/27*(7*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) +
```

```
(-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + B*x/b^3 + 2/27*(7*B*a - A*b)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(13*B*a*b*x^4 - 7*A*b^2*x^4 +
 10*B*a^2*x - 4*A*a*b*x)/((b*x^3 + a)^2*b^3)
```

Mupad [B]
time = 2.60, size = 183, normalized size = 0.83

$$\frac{B\,x}{b^3}-\frac{x^4\,\left(\frac{7\,A\,b^2}{18}-\frac{13\,B\,a\,b}{18}\right)-x\,\left(\frac{5\,B\,a^2}{9}-\frac{2\,A\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-7\,B\,a\right)}{27\,a^{2/3}\,b^{10/3}}-\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-7\,B\,a\right)}{27\,a^{2/3}\,b^{10/3}}+\frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-7\,B\,a\right)}{27\,a^{2/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (B*x)/b^3 - (x^4*((7*A*b^2)/18 - (13*B*a*b)/18) - x*((5*B*a^2)/9 - (2*A*a*b
)/9))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (2*log(b^(1/3)*x + a^(1/3))*(A*b
- 7*B*a))/(27*a^(2/3)*b^(10/3)) - (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x +
 a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a))/(27*a^(2/3)*b^(10/3)) + (2*
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b
 - 7*B*a))/(27*a^(2/3)*b^(10/3))
```

3.99 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=201

$$\frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+5aB)x^2}{18ab^2\left(a+bx^3\right)}-\frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{8/3}}-\frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{8/3}}+\frac{(Ab+}{}$$

```
[Out] 1/6*(A*b-B*a)*x^5/a/b/(b*x^3+a)^2-1/18*(A*b+5*B*a)*x^2/a/b^2/(b*x^3+a)-1/27
*(A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(8/3)+1/54*(A*b+5*B*a)*ln(a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(8/3)-1/27*(A*b+5*B*a)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(8/3)*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {468, 294, 298, 31, 648, 631, 210, 642}

$$-\frac{(5aB+Ab)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{8/3}}+\frac{(5aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{8/3}}-\frac{x^2(5aB+Ab)}{18ab^2\left(a+bx^3\right)}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^5)/(6*a*b*(a + b*x^3)^2) - ((A*b + 5*a*B)*x^2)/(18*a*b^2*(a
+ b*x^3)) - ((A*b + 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(9*Sqrt[3]*a^(4/3)*b^(8/3)) - ((A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(
27*a^(4/3)*b^(8/3)) + ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(54*a^(4/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^5}{6ab\,(a+bx^3)^2} + \frac{(Ab+5aB)\int \frac{x^4}{(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^5}{6ab\,(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2\,(a+bx^3)} + \frac{(Ab+5aB)\int \frac{x}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^5}{6ab\,(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2\,(a+bx^3)} - \frac{(Ab+5aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{4/3}b^{7/3}} + \frac{(Ab+5aB)\int \frac{\quad}{a^{2/3}-\ldots}}{27a^{4/\ldots}}$$

$$= \frac{(Ab-aB)x^5}{6ab\,(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2\,(a+bx^3)} - \frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{8/3}} + \frac{(Ab+5aB)\int -\ldots}{54\ldots}$$

$$= \frac{(Ab-aB)x^5}{6ab\,(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2\,(a+bx^3)} - \frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{8/3}} + \frac{(Ab+5aB)\log\ldots}{\ldots}$$

$$= \frac{(Ab-aB)x^5}{6ab\,(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2\,(a+bx^3)} - \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{8/3}} - \frac{(Ab+5aB)\,\mathrm{l}\ldots}{27\ldots}$$

Mathematica [A]
time = 0.10, size = 181, normalized size = 0.90

$$\frac{-\frac{9b^{2/3}(Ab-aB)x^2}{(a+bx^3)^2}+\frac{6b^{2/3}(Ab-4aB)x^2}{a(a+bx^3)}-\frac{2\sqrt{3}\,(Ab+5aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-\frac{2(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{4/3}}+\frac{(Ab+5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{4/3}}}{54b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 + (6*b^(2/3)*(A*b - 4*a*B)*x^2)
/(a*(a + b*x^3)) - (2*Sqrt[3]*(A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(4/3) - (2*(A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3)
+ ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(
54*b^(8/3))
```

Maple [A]
time = 0.28, size = 154, normalized size = 0.77

method	result	size

risch	$\frac{\frac{(Ab-4Ba)x^5}{9ab}-\frac{(Ab+5Ba)x^2}{18b^2}}{(bx^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(Ab+5Ba)\ln(x-\_R)}{\_R}}{27b^3a}$	85
default	$\frac{\frac{(Ab-4Ba)x^5}{9ab}-\frac{(Ab+5Ba)x^2}{18b^2}}{(bx^3+a)^2}+\frac{(Ab+5Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^2a}$	154

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9*(A*b-4*B*a)/a/b*x^5-1/18*(A*b+5*B*a)/b^2*x^2)/(b*x^3+a)^2+1/9*(A*b+5*B
*a)/b^2/a*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1)))
```

Maxima [A]
time = 0.51, size = 195, normalized size = 0.97

$$-\frac{2\,(4\,Bab-Ab^2)x^5+(5\,Ba^2+Aab)x^2}{18\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)}+\frac{\sqrt{3}\,(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(5\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(5\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(4*B*a*b - A*b^2)*x^5 + (5*B*a^2 + A*a*b)*x^2)/(a*b^4*x^6 + 2*a^2*
b^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(1/3)) + 1/54*(5*B*a + A*b)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(1/3)) - 1/27*(5*B*a + A*b)*log(
x + (a/b)^(1/3))/(a*b^3*(a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(160) = 320.
time = 1.49, size = 756, normalized size = 3.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(6*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 3
*sqrt(1/3)*((5*B*a^2*b^3 + A*a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^
3*b^2 + A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqr
t(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^
3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a
*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 +
A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1
/3)))/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), -1/54*(6*(4*B*a^2*b^3 - A*a*
b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 6*sqrt(1/3)*((5*B*a^2*b^3 + A*
a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(
-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)
^(1/3)/a)/b) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b
+ A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^
(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*
a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^6 + 2*a^3*
b^5*x^3 + a^4*b^4)]
```

Sympy [A]
time = 6.23, size = 155, normalized size = 0.77

$$\frac{x^{5}\cdot\left(2Ab^{2}-8Bab\right)+x^{2}\left(-Aab-5Ba^{2}\right)}{18a^{3}b^{2}+36a^{2}b^{3}x^{3}+18ab^{4}x^{6}}+\operatorname{RootSum}\left(19683t^{3}a^{4}b^{8}+A^{3}b^{3}+15A^{2}Bab^{2}+75AB^{2}a^{2}b+125B^{3}a^{3},\left(t\mapsto t\log\left(\frac{729t^{2}a^{3}b^{5}}{A^{2}b^{2}+10ABab+25B^{2}a^{2}}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**5*(2*A*b**2 - 8*B*a*b) + x**2*(-A*a*b - 5*B*a**2))/(18*a**3*b**2 + 36*a
**2*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**4*b**8 + A**3*b**3
 + 15*A**2*B*a*b**2 + 75*A*B**2*a**2*b + 125*B**3*a**3, Lambda(_t, _t*log(7
29*_t**2*a**3*b**5/(A**2*b**2 + 10*A*B*a*b + 25*B**2*a**2) + x)))
```

Giac [A]
time = 0.70, size = 206, normalized size = 1.02

$$\frac{\sqrt{3}\left(5\,Ba+Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}}-\frac{\left(5\,Ba+Ab\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}}-\frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{2}b^{2}}-\frac{8\,Babx^{5}-2\,Ab^{2}x^{5}+5\,Ba^{2}x^{2}+Aabx^{2}}{18\left(bx^{3}+a\right)^{2}ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a*b^2) - 1/54*(5*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b^2) - 1/27*(5*B*a*(-a/b)^(1/3) + A*b*(-a
/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(8*B*a*
b*x^5 - 2*A*b^2*x^5 + 5*B*a^2*x^2 + A*a*b*x^2)/((b*x^3 + a)^2*a*b^2)
```

Mupad [B]
time = 0.27, size = 175, normalized size = 0.87

$$-\frac{\frac{x^2\,(A\,b+5\,B\,a)}{18\,b^2}-\frac{x^5\,(A\,b-4\,B\,a)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b+5\,B\,a)}{27\,a^{4/3}\,b^{8/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b+5\,B\,a)}{27\,a^{4/3}\,b^{8/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b+5\,B\,a)}{27\,a^{4/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b + 5*B*a))
/(27*a^(4/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - ((x^2*(A*b + 5*B
*a))/(18*b^2) - (x^5*(A*b - 4*B*a))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

3.100 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=199

$$\frac{(Ab-aB)x^4}{6ab\,(a+bx^3)^2}-\frac{(Ab+2aB)x}{9ab^2\,(a+bx^3)}-\frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{7/3}}+\frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{7/3}}-\frac{(Ab+$$

```
[Out] 1/6*(A*b-B*a)*x^4/a/b/(b*x^3+a)^2-1/9*(A*b+2*B*a)*x/a/b^2/(b*x^3+a)+1/27*(A
*b+2*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)-1/54*(A*b+2*B*a)*ln(a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)-1/27*(A*b+2*B*a)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {468, 294, 206, 31, 648, 631, 210, 642}

$$-\frac{(2aB+Ab)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{7/3}}-\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}}+\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{7/3}}-\frac{x(2aB+Ab)}{9ab^2\,(a+bx^3)}+\frac{x^4(Ab-aB)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^4)/(6*a*b*(a + b*x^3)^2) - ((A*b + 2*a*B)*x)/(9*a*b^2*(a + b
*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(9*Sqrt[3]*a^(5/3)*b^(7/3)) + ((A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*
a^(5/3)*b^(7/3)) - ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(5/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^4}{6ab\,(a+bx^3)^2} + \frac{(2Ab+4aB)\int \frac{x^3}{(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^4}{6ab\,(a+bx^3)^2} - \frac{(Ab+2aB)x}{9ab^2\,(a+bx^3)} + \frac{(Ab+2aB)\int \frac{1}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^4}{6ab\,(a+bx^3)^2} - \frac{(Ab+2aB)x}{9ab^2\,(a+bx^3)} + \frac{(Ab+2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{5/3}b^2} + \frac{(Ab+2aB)\int \frac{2\cdot}{a^{2/3}-\sqrt{}}}{27a^{5/}}$$

$$= \frac{(Ab-aB)x^4}{6ab\,(a+bx^3)^2} - \frac{(Ab+2aB)x}{9ab^2\,(a+bx^3)} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{7/3}} - \frac{(Ab+2aB)\int \frac{}{a^{2/}}}{54a}$$

$$= \frac{(Ab-aB)x^4}{6ab\,(a+bx^3)^2} - \frac{(Ab+2aB)x}{9ab^2\,(a+bx^3)} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{7/3}} - \frac{(Ab+2aB)\log(}{}$$

$$= \frac{(Ab-aB)x^4}{6ab\,(a+bx^3)^2} - \frac{(Ab+2aB)x}{9ab^2\,(a+bx^3)} - \frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{7/3}} + \frac{(Ab+2aB)\log}{27a^{}}$$

Mathematica [A]
time = 0.09, size = 178, normalized size = 0.89

$$\frac{-\frac{9\sqrt[3]{b}\,(Ab-aB)x}{(a+bx^3)^2} + \frac{3\sqrt[3]{b}\,(Ab-7aB)x}{a(a+bx^3)} - \frac{2\sqrt{3}\,(Ab+2aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}} - \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}}{54b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 + (3*b^(1/3)*(A*b - 7*a*B)*x)/(a*
(a + b*x^3)) - (2*Sqrt[3]*(A*b + 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/a^(5/3) + (2*(A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - ((
A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b
^(7/3))
```

Maple [A]
time = 0.28, size = 152, normalized size = 0.76

method	result	size

risch	$\frac{\frac{(Ab-7Ba)x^4}{18ab}-\frac{(Ab+2Ba)x}{9b^2}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab+2Ba)\ln\left(x-\_R\right)}{\_R^2}}{27b^3a}$	83
default	$\frac{\frac{(Ab-7Ba)x^4}{18ab}-\frac{(Ab+2Ba)x}{9b^2}}{(b\,x^3+a)^2}+\frac{(Ab+2Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9b^2a}$	152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/18*(A*b-7*B*a)/a/b*x^4-1/9*(A*b+2*B*a)/b^2*x)/(b*x^3+a)^2+1/9*(A*b+2*B*a
)/b^2/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1)))
```

Maxima [A]
time = 0.50, size = 193, normalized size = 0.97

$$-\frac{(7\,Bab-Ab^2)x^4+2\,(2\,Ba^2+Aab)x}{18\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)}+\frac{\sqrt{3}\,(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(2\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(2\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*B*a*b - A*b^2)*x^4 + 2*(2*B*a^2 + A*a*b)*x)/(a*b^4*x^6 + 2*a^2*b^
3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/54*(2*B*a + A*b)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/27*(2*B*a + A*b)*log(x
+ (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(158) = 316.
time = 1.94, size = 743, normalized size = 3.73

$$\left[-\frac{3\,(7\,Ba^3b^2-Aa^2b^3)x^4-3\sqrt{\frac{1}{3}}\,((2\,Ba^2b^3+Aab^4)x^6+2\,Ba^4b+Aa^3b^2+2\,(2\,Ba^3b^2+Aa^2b^3)x^3)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)+((2\,Bab^2+Ab^3)x^6+2\,Ba^3+Aa^2b+2\,(2\,Ba^2b+Aab^2)x^3)(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)-2\,((2\,Bab^2+Ab^3)x^6+2\,Ba^3+Aa^2b+2\,(2\,Ba^2b+Aab^2)x^3)(a^2b)^{\frac{2}{3}}\log\left(abx+(a^2b)^{\frac{2}{3}}\right)+6\,(2\,Ba^4b+Aa^3b^2)x}{54\,(a^3b^5x^6+2\,a^4b^4x^3+a^5b^3)},\ -\frac{3\,(7\,Ba^3b^2-Aa^2b^3)x^4-6\sqrt{\frac{1}{3}}\,((2\,Ba^2b^3+Aab^4)x^6+2\,Ba^4b+Aa^3b^2+2\,(2\,Ba^3b^2+Aa^2b^3)x^3)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)+((2\,Bab^2+Ab^3)x^6+2\,Ba^3+Aa^2b+2\,(2\,Ba^2b+Aab^2)x^3)(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)-2\,((2\,Bab^2+Ab^3)x^6+2\,Ba^3+Aa^2b+2\,(2\,Ba^2b+Aab^2)x^3)(a^2b)^{\frac{2}{3}}\log\left(abx+(a^2b)^{\frac{2}{3}}\right)+6\,(2\,Ba^4b+Aa^3b^2)x}{54\,(a^3b^5x^6+2\,a^4b^4x^3+a^5b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 3*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b
^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(-(a
^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*
a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3
 + a)) + ((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*
b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) -
2*((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^
3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/
(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), -1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)
*x^4 - 6*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2
*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(
a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b^2
+ A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^(2/
3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((2*B*a*b^2 + A*b^3
)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(
a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/(a^3*b^5*x^6 + 2*a^4*
b^4*x^3 + a^5*b^3)]
```

Sympy [A]
time = 1.17, size = 136, normalized size = 0.68

$$\frac{x^4(Ab^2-7Bab)+x(-2Aab-4Ba^2)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6}+\operatorname{RootSum}\left(19683t^3a^5b^7-A^3b^3-6A^2Bab^2-12AB^2a^2b-8B^3a^3,\left(t\mapsto t\log\left(\frac{27ta^2b^2}{Ab+2Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**4*(A*b**2 - 7*B*a*b) + x*(-2*A*a*b - 4*B*a**2))/(18*a**3*b**2 + 36*a**2
*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**5*b**7 - A**3*b**3 -
6*A**2*B*a*b**2 - 12*A*B**2*a**2*b - 8*B**3*a**3, Lambda(_t, _t*log(27*_t*a
**2*b**2/(A*b + 2*B*a) + x)))
```

Giac [A]
time = 0.68, size = 187, normalized size = 0.94

$$-\frac{\sqrt{3}\,(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,ab}-\frac{(2\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,ab}-\frac{(2\,Ba+Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^2}-\frac{7\,Babx^4-Ab^2x^4+4\,Ba^2x+2\,Aabx}{18\,(bx^3+a)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a*b) - 1/54*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) - 1/27*(2*B*a + A*b)*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(7*B*a*b*x^4 - A*b^2*x^4 + 4*B*a^2*x
 + 2*A*a*b*x)/((b*x^3 + a)^2*a*b^2)
```

Mupad [B]
time = 2.56, size = 173, normalized size = 0.87

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}}-\frac{\frac{x\,\left(A\,b+2\,B\,a\right)}{9\,b^2}-\frac{x^4\,\left(A\,b-7\,B\,a\right)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) - ((x*(A*b +
2*B*a))/(9*b^2) - (x^4*(A*b - 7*B*a))/(18*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3))
```

3.101 $\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=201

$$\frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2}+\frac{(2Ab+aB)x^2}{9a^2b\,(a+bx^3)}-\frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{5/3}}-\frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{5/3}}+\frac{(2Ab+\cdots}{}$$

```
[Out] 1/6*(A*b-B*a)*x^2/a/b/(b*x^3+a)^2+1/9*(2*A*b+B*a)*x^2/a^2/b/(b*x^3+a)-1/27*
(2*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(5/3)+1/54*(2*A*b+B*a)*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(5/3)-1/27*(2*A*b+B*a)*arctan(1
/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(5/3)*3^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {468, 296, 298, 31, 648, 631, 210, 642}

$$-\frac{(aB+2Ab)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{5/3}}+\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{5/3}}+\frac{x^2(aB+2Ab)}{9a^2b\,(a+bx^3)}+\frac{x^2(Ab-aB)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^2)/(6*a*b*(a + b*x^3)^2) + ((2*A*b + a*B)*x^2)/(9*a^2*b*(a +
 b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(9*Sqrt[3]*a^(7/3)*b^(5/3)) - ((2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(7/3)*b^(5/3)) + ((2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(54*a^(7/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2} + \frac{(4Ab+2aB)\int \frac{x}{(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b\,(a+bx^3)} + \frac{(2Ab+aB)\int \frac{x}{a+bx^3}\,dx}{9a^2b}$$

$$= \frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b\,(a+bx^3)} - \frac{(2Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{7/3}b^{4/3}} + \frac{(2Ab+aB)\int \frac{\sqrt[3]{\ldots}}{a^{2/3}-\sqrt[3]{\ldots}}}{27a^{7/3}b\ldots}$$

$$= \frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b\,(a+bx^3)} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\int \frac{-\ldots}{a^{2/3}\ldots}}{54a^{\ldots}}$$

$$= \frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b\,(a+bx^3)} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\log\left(a\ldots\right.}{5\ldots}$$

$$= \frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b\,(a+bx^3)} - \frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{5/3}} - \frac{(2Ab+aB)\log\ldots}{27a^{7\ldots}}$$

Mathematica [A]
time = 0.07, size = 178, normalized size = 0.89

$$\frac{-\frac{9a^{4/3}b^{2/3}(-Ab+aB)x^2}{(a+bx^3)^2} + \frac{6\sqrt[3]{a}\,b^{2/3}(2Ab+aB)x^2}{a+bx^3} - 2\sqrt{3}\,(2Ab+aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + (2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*a^(4/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1/3)*b^(2/3)
*(2*A*b + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B)*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] + (2*
A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(5
/3))
```

Maple [A]
time = 0.30, size = 155, normalized size = 0.77

method	result	size

risch	$\frac{\frac{(2Ab+Ba)x^5}{9a^2}+\frac{(7Ab-Ba)x^2}{18ab}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(2Ab+Ba)\ln\left(x-\_R\right)}{\_R}}{27a^2b^2}$	86
default	$\frac{\frac{(2Ab+Ba)x^5}{9a^2}+\frac{(7Ab-Ba)x^2}{18ab}}{(b\,x^3+a)^2}+\frac{(2Ab+Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b}$	155

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9*(2*A*b+B*a)/a^2*x^5+1/18*(7*A*b-B*a)/a/b*x^2)/(b*x^3+a)^2+1/9*(2*A*b+B
*a)/a^2/b*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1)))
```

Maxima [A]
time = 0.49, size = 195, normalized size = 0.97

$$\frac{2\,(Bab+2\,Ab^2)x^5-(Ba^2-7\,Aab)x^2}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}+\frac{\sqrt{3}\,(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba+2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba+2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(B*a*b + 2*A*b^2)*x^5 - (B*a^2 - 7*A*a*b)*x^2)/(a^2*b^3*x^6 + 2*a^3
*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/54*(B*a + 2*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(1/3)) - 1/27*(B*a + 2*A*b)*l
og(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(160) = 320$.
time = 1.65, size = 752, normalized size = 3.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(6*(B*a^2*b^3 + 2*A*a*b^4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 3*
sqrt(1/3)*((B*a^2*b^3 + 2*A*a*b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b
^2 + 2*A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)
/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 +
 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*
b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A
*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/
3)))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), 1/54*(6*(B*a^2*b^3 + 2*A*a*b^
4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 6*sqrt(1/3)*((B*a^2*b^3 + 2*A*a*
b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b^2 + 2*A*a^2*b^3)*x^3)*sqrt(-(
-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(
1/3)/a)/b) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*
A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2
/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*
b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^5*x^6 + 2*a^4*b^
4*x^3 + a^5*b^3)]
```

Sympy [**A**]
time = 1.08, size = 153, normalized size = 0.76

$$\frac{x^5\cdot(4Ab^2+2Bab)+x^2\cdot(7Aab-Ba^2)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}+\operatorname{RootSum}\left(19683t^3a^7b^5+8A^3b^3+12A^2Bab^2+6AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(\frac{729t^2a^5b^3}{4A^2b^2+4ABab+B^2a^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**5*(4*A*b**2 + 2*B*a*b) + x**2*(7*A*a*b - B*a**2))/(18*a**4*b + 36*a**3*
b**2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**7*b**5 + 8*A**3*b**
3 + 12*A**2*B*a*b**2 + 6*A*B**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(729*_
t**2*a**5*b**3/(4*A**2*b**2 + 4*A*B*a*b + B**2*a**2) + x)))
```

Giac [**A**]
time = 0.70, size = 207, normalized size = 1.03

$$\frac{\sqrt{3}\,(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,a^2b}-\frac{(Ba+2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}\,a^2b}-\frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b}+\frac{2\,Babx^5+4\,Ab^2x^5-Ba^2x^2+7\,Aabx^2}{18\,(bx^3+a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a^2*b) - 1/54*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b) - 1/27*(B*a*(-a/b)^(1/3) + 2*A*b*(-a
/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(2*B*a*b*
x^5 + 4*A*b^2*x^5 - B*a^2*x^2 + 7*A*a*b*x^2)/((b*x^3 + a)^2*a^2*b)
```

Mupad [B]
time = 0.27, size = 175, normalized size = 0.87

$$\frac{\frac{x^5\,(2\,A\,b+B\,a)}{9\,a^2}+\frac{x^2\,(7\,A\,b-B\,a)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^5*(2*A*b + B*a))/(9*a^2) + (x^2*(7*A*b - B*a))/(18*a*b))/(a^2 + b^2*x^6
 + 2*a*b*x^3) - (log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3
)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)
*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3))
```

3.102 $\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=197

$$\frac{(Ab-aB)x}{6ab\,(a+bx^3)^2}+\frac{(5Ab+aB)x}{18a^2b\,(a+bx^3)}-\frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{4/3}}+\frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{4/3}}-\frac{(5Ab$$

```
[Out] 1/6*(A*b-B*a)*x/a/b/(b*x^3+a)^2+1/18*(5*A*b+B*a)*x/a^2/b/(b*x^3+a)+1/27*(5*
A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(4/3)-1/54*(5*A*b+B*a)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(4/3)-1/27*(5*A*b+B*a)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(4/3)*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {393, 205, 206, 31, 648, 631, 210, 642}

$$-\frac{(aB+5Ab)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{4/3}}-\frac{(aB+5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}+\frac{(aB+5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{4/3}}+\frac{x(aB+5Ab)}{18a^2b\,(a+bx^3)}+\frac{x(Ab-aB)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x)/(6*a*b*(a + b*x^3)^2) + ((5*A*b + a*B)*x)/(18*a^2*b*(a + b*
x^3)) - ((5*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
9*Sqrt[3]*a^(8/3)*b^(4/3)) + ((5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a
^(8/3)*b^(4/3)) - ((5*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(54*a^(8/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x}{6ab\,(a+bx^3)^2} + \frac{(5Ab+aB)\int \frac{1}{(a+bx^3)^2}\,dx}{6ab}$$
$$= \frac{(Ab-aB)x}{6ab\,(a+bx^3)^2} + \frac{(5Ab+aB)x}{18a^2b\,(a+bx^3)} + \frac{(5Ab+aB)\int \frac{1}{a+bx^3}\,dx}{9a^2b}$$
$$= \frac{(Ab-aB)x}{6ab\,(a+bx^3)^2} + \frac{(5Ab+aB)x}{18a^2b\,(a+bx^3)} + \frac{(5Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{8/3}b} + \frac{(5Ab+aB)\int \frac{2\sqrt[}{}}{a^{2/3}-\sqrt[3]{}}}{27a^{8/3}}$$
$$= \frac{(Ab-aB)x}{6ab\,(a+bx^3)^2} + \frac{(5Ab+aB)x}{18a^2b\,(a+bx^3)} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{4/3}} - \frac{(5Ab+aB)\int \frac{-}{a^{2/3}-}}{54a^{8}}$$
$$= \frac{(Ab-aB)x}{6ab\,(a+bx^3)^2} + \frac{(5Ab+aB)x}{18a^2b\,(a+bx^3)} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}b^{4/3}} - \frac{(5Ab+aB)\log\left(a\right.}{5}$$
$$= \frac{(Ab-aB)x}{6ab\,(a+bx^3)^2} + \frac{(5Ab+aB)x}{18a^2b\,(a+bx^3)} - \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{4/3}} + \frac{(5Ab+aB)\log}{27a^{8/}}$$

Mathematica [A]

time = 0.08, size = 175, normalized size = 0.89

$$\frac{-\frac{9a^{5/3}\sqrt[3]{b}\,(-Ab+aB)x}{(a+bx^3)^2} + \frac{3a^{2/3}\sqrt[3]{b}\,(5Ab+aB)x}{a+bx^3} - 2\sqrt{3}\,(5Ab+aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) - (5Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^3,x]
```

```
[Out] ((-9*a^(5/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2/3)*b^(1/3)*(
5*A*b + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(5*A*b + a*B)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 2*(5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] - (5*A*b
+ a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(4/3))
```

Maple [A]

time = 0.28, size = 153, normalized size = 0.78

method	result	size
risch	$\frac{\frac{(5Ab+Ba)x^4}{18a^2}+\frac{(4Ab-Ba)x}{9ab}}{(b\,x^3+a)^2} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{(5Ab+Ba)\ln\left(x-\_R\right)}{\_R^2}}{27b^2a^2}$	84

default	$\frac{\frac{(5Ab+Ba)x^4}{18a^2}+\frac{(4Ab-Ba)x}{9ab}}{(b\,x^3+a)^2}+\frac{(5Ab+Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9b\,a^2}$	153

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] (1/18*(5*A*b+B*a)/a^2*x^4+1/9*(4*A*b-B*a)/a/b*x)/(b*x^3+a)^2+1/9/b/a^2*(5*A*b+B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))

Maxima [A]
time = 0.53, size = 192, normalized size = 0.97

$$\frac{(Bab+5\,Ab^2)x^4-2\,(Ba^2-4\,Aab)x}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}+\frac{\sqrt{3}\,(Ba+5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba+5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba+5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] 1/18*((B*a*b + 5*A*b^2)*x^4 - 2*(B*a^2 - 4*A*a*b)*x)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) - 1/54*(B*a + 5*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) + 1/27*(B*a + 5*A*b)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(156) = 312$.
time = 1.66, size = 743, normalized size = 3.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x^4 + 3*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt(-(a^

```
2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a
*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3
+ a)) - ((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b
^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2
*((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3
)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(
a^4*b^4*x^6 + 2*a^5*b^3*x^3 + a^6*b^2), 1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x
^4 + 6*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(
B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^
2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b^2 + 5*
A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)
*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b^2 + 5*A*b^3)*
x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*
b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(a^4*b^4*x^6 + 2*a^5*b^
3*x^3 + a^6*b^2)]
```

Sympy [A]
time = 0.41, size = 133, normalized size = 0.68

$$\frac{x^4\cdot(5Ab^2+Bab)+x(8Aab-2Ba^2)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}+\operatorname{RootSum}\left(19683t^3a^8b^4-125A^3b^3-75A^2Bab^2-15AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{27ta^3b}{5Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**4*(5*A*b**2 + B*a*b) + x*(8*A*a*b - 2*B*a**2))/(18*a**4*b + 36*a**3*b**
2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**8*b**4 - 125*A**3*b**3
 - 75*A**2*B*a*b**2 - 15*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(27*_t
*a**3*b/(5*A*b + B*a) + x)))
```

Giac [A]
time = 0.66, size = 180, normalized size = 0.91

$$-\frac{\sqrt{3}\,(Ba+5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}a^2}-\frac{(Ba+5\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{2}{3}}a^2}-\frac{(Ba+5\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b}+\frac{Babx^4+5\,Ab^2x^4-2\,Ba^2x+8\,Aabx}{18\,\left(bx^3+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(B*a + 5*A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(B*a + 5*A*b)*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(B*a*b*x^4 + 5*A*b^2*x^4 - 2*B*a^2*x +
 8*A*a*b*x)/((b*x^3 + a)^2*a^2*b)
```

Mupad [B]
time = 0.26, size = 173, normalized size = 0.88

$$\frac{\frac{x^4\,(5\,A\,b+B\,a)}{18\,a^2}+\frac{x\,(4\,A\,b-B\,a)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^3,x)
```

```
[Out] ((x^4*(5*A*b + B*a))/(18*a^2) + (x*(4*A*b - B*a))/(9*a*b))/(a^2 + b^2*x^6 +
 2*a*b*x^3) + (log(b^(1/3)*x + a^(1/3))*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3))
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(5*A*b + B*a))/(27*a^(8/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3))
```

3.103 $\int \frac{A+Bx^3}{x^2(a+bx^3)^3}\,dx$

Optimal. Leaf size=227

$$-\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\,(a+bx^3)^2}+\frac{7Ab-aB}{18a^2bx\,(a+bx^3)}+\frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\log\left(\right.}{27a^{10/3}}$$

```
[Out] -2/9*(7*A*b-B*a)/a^3/b/x+1/6*(A*b-B*a)/a/b/x/(b*x^3+a)^2+1/18*(7*A*b-B*a)/a
^2/b/x/(b*x^3+a)+2/27*(7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(2/3)-1/
27*(7*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(2/3)+2
/27*(7*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/
b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {468, 296, 331, 298, 31, 648, 631, 210, 642}

$$\frac{2(7Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{10/3}b^{2/3}}-\frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{10/3}b^{2/3}}-\frac{2(7Ab-aB)}{9a^3bx}+\frac{7Ab-aB}{18a^2bx\,(a+bx^3)}+\frac{Ab-aB}{6abx\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] (-2*(7*A*b - a*B))/(9*a^3*b*x) + (A*b - a*B)/(6*a*b*x*(a + b*x^3)^2) + (7*A
*b - a*B)/(18*a^2*b*x*(a + b*x^3)) + (2*(7*A*b - a*B)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(2/3)) + (2*(7*A*b - a*
B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(2/3)) - ((7*A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(10/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx &= \frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{(7Ab-aB)\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx}{6ab}\\
&= \frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}+\frac{(2(7Ab-aB))\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{9a^2b}\\
&= -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}-\frac{(2(7Ab-aB))\int \frac{x}{a+bx^3}\,dx}{9a^3}\\
&= -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}+\frac{(2(7Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,d}{27a^{10/3}\sqrt[3]{b}}\\
&= -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{27a^{10/3}b^{2/3}}\\
&= -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{27a^{10/3}b^{2/3}}\\
&= -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}+\frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\ldots}{\sqrt{3}\ldots}\right.}{9\sqrt{3}\,a^{10/3}b^{2/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 193, normalized size = 0.85

$$\frac{-\frac{54\sqrt[3]{a}\,A}{x}+\frac{9a^{4/3}(-Ab+aB)x^2}{\left(a+bx^3\right)^2}+\frac{6\sqrt[3]{a}\,(-5Ab+2aB)x^2}{a+bx^3}+\frac{4\sqrt{3}\,(7Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{4(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{2(-7Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}}{54a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(1/3)*A)/x + (9*a^(4/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1
/3)*(-5*A*b + 2*a*B)*x^2)/(a + b*x^3) + (4*Sqrt[3]*(7*A*b - a*B)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(7*A*b - a*B)*Log[a^(1/3) +
 b^(1/3)*x])/b^(2/3) + (2*(-7*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/b^(2/3))/(54*a^(10/3))
```

Maple [A]
time = 0.30, size = 159, normalized size = 0.70

method	result
default	$-\frac{\frac{\left(\frac{5}{9}b^2A-\frac{2}{9}abB\right)x^5+\frac{a(13Ab-7Ba)x^2}{18}}{\left(b\,x^3+a\right)^2}+\left(\frac{14Ab}{9}-\frac{2Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{a^3}$
risch	$\frac{-\frac{2b(7Ab-Ba)x^6}{9a^3}-\frac{7(7Ab-Ba)x^3}{18a^2}-\frac{A}{a}}{x(b\,x^3+a)^2}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}b^2\_Z^3-343A^3b^3+147A^2Ba\,b^2-21A\,B^2a^2b+B^3a^3\right)}\_R\ln\left(\left(-4\_R^3a^{10}b^2\right.\right.\right.}{27}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*(((5/9*b^2*A-2/9*a*b*B)*x^5+1/18*a*(13*A*b-7*B*a)*x^2)/(b*x^3+a)^2+(
14/9*A*b-2/9*B*a)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))))-A/a^3/x
```

Maxima [A]
time = 0.49, size = 199, normalized size = 0.88

$$\frac{4\,(Bab-7\,Ab^2)x^6+7\,(Ba^2-7\,Aab)x^3-18\,Aa^2}{18\,(a^3b^2x^7+2\,a^4bx^4+a^5x)}+\frac{2\,\sqrt{3}\,(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-7\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,(Ba-7\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*(B*a*b - 7*A*b^2)*x^6 + 7*(B*a^2 - 7*A*a*b)*x^3 - 18*A*a^2)/(a^3*b^
2*x^7 + 2*a^4*b*x^4 + a^5*x) + 2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(1/3)) + 1/27*(B*a - 7*A*b)
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(1/3)) - 2/27*(B*a - 7
*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(179) = 358.
time = 1.70, size = 776, normalized size = 3.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*
a^2*b^3)*x^3 - 6*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*
A*a^2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b
^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*
sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((B*a*b^2 - 7*
A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(
2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b
^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3
)*log(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x), 1/54
*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*a^2*b
^3)*x^3 - 12*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*A*a^
2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(
1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + 2*((B*a*b^2 - 7*A*b
^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3
)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b^3)
*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3)*l
og(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x)]
```

Sympy [A]
time = 0.48, size = 162, normalized size = 0.71

$$\frac{-18Aa^2 + x^6(-28Ab^2 + 4Bab) + x^3(-49Aab + 7Ba^2)}{18a^5x + 36a^4bx^4 + 18a^3b^2x^7} + \operatorname{RootSum}\left(19683t^3a^{10}b^2 - 2744A^3b^3 + 1176A^2Bab^2 - 168AB^2a^2b + 8B^3a^3, \left(t \mapsto t\log\left(\frac{729t^2a^7b}{196A^2b^2 - 56ABab + 4B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**3,x)
```

```
[Out] (-18*A*a**2 + x**6*(-28*A*b**2 + 4*B*a*b) + x**3*(-49*A*a*b + 7*B*a**2))/(1
8*a**5*x + 36*a**4*b*x**4 + 18*a**3*b**2*x**7) + RootSum(19683*_t**3*a**10*
b**2 - 2744*A**3*b**3 + 1176*A**2*B*a*b**2 - 168*A*B**2*a**2*b + 8*B**3*a**
3, Lambda(_t, _t*log(729*_t**2*a**7*b/(196*A**2*b**2 - 56*A*B*a*b + 4*B**2*
a**2) + x)))
```

Giac [A]
time = 0.66, size = 204, normalized size = 0.90

$$\frac{2\sqrt{3}\,(Ba - 7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,a^3} - \frac{(Ba - 7\,Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,a^3} - \frac{2\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 7\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4} - \frac{A}{a^3x} + \frac{4\,Babx^5 - 10\,Ab^2x^5 + 7\,Ba^2x^2 - 13\,Aabx^2}{18\,(bx^3 + a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a^3) - 1/27*(B*a - 7*A*b)*log(x^2 + x*(-a/b)^(1/3) +
```

```
(-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3) - 2/27*(B*a*(-a/b)^(1/3) - 7*A*b*(-a/b)^
(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - A/(a^3*x) + 1/18*(4*B*
a*b*x^5 - 10*A*b^2*x^5 + 7*B*a^2*x^2 - 13*A*a*b*x^2)/((b*x^3 + a)^2*a^3)
```

Mupad [B]
time = 2.60, size = 185, normalized size = 0.81

$$\frac{2\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}}-\frac{\frac{A}{a}+\frac{7\,x^3\,\left(7\,A\,b-B\,a\right)}{18\,a^2}+\frac{2\,b\,x^6\,\left(7\,A\,b-B\,a\right)}{9\,a^3}}{a^2\,x+2\,a\,b\,x^4+b^2\,x^7}+\frac{2\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}}-\frac{2\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] (2*log(b^(1/3)*x + a^(1/3))*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (A/a + (
7*x^3*(7*A*b - B*a))/(18*a^2) + (2*b*x^6*(7*A*b - B*a))/(9*a^3))/(a^2*x + b
^2*x^7 + 2*a*b*x^4) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (2*log(3^(1/2)*
a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*a))/(
27*a^(10/3)*b^(2/3))
```

3.104 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=227

$$-\frac{5(4Ab-aB)}{18a^3bx^2}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}+\frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)}+\frac{5(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{11/3}\sqrt[3]{b}}-\frac{5(4Ab-aB)\log}{27a^{11/3}}$$

```
[Out] -5/18*(4*A*b-B*a)/a^3/b/x^2+1/6*(A*b-B*a)/a/b/x^2/(b*x^3+a)^2+1/9*(4*A*b-B*
a)/a^2/b/x^2/(b*x^3+a)-5/27*(4*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(1
/3)+5/54*(4*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(
1/3)+5/27*(4*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
11/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {468, 296, 331, 206, 31, 648, 631, 210, 642}

$$\frac{5(4Ab-aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{11/3}\sqrt[3]{b}}+\frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}}-\frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}\sqrt[3]{b}}-\frac{5(4Ab-aB)}{18a^3bx^2}+\frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] (-5*(4*A*b - a*B))/(18*a^3*b*x^2) + (A*b - a*B)/(6*a*b*x^2*(a + b*x^3)^2) +
 (4*A*b - a*B)/(9*a^2*b*x^2*(a + b*x^3)) + (5*(4*A*b - a*B)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(1/3)) - (5*(4*A*
b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(1/3)) + (5*(4*A*b - a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{(8Ab-2aB)\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} + \frac{(5(4Ab-aB))\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{(5(4Ab-aB))\int \frac{1}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{(5(4Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}}{27a^{11/3}}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{27a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{27a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} + \frac{5(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2}{\sqrt{3}\,\sqrt[3]{}}\right.}{9\sqrt{3}\,a^{11/3}\sqrt[3]{b}}$$

Mathematica [A]
time = 0.10, size = 189, normalized size = 0.83

$$\frac{-\frac{27a^{2/3}A}{x^2} + \frac{9a^{5/3}(-Ab+aB)x}{\left(a+bx^3\right)^2} + \frac{3a^{2/3}(-11Ab+5aB)x}{a+bx^3} + \frac{10\sqrt{3}\,(4Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{10(-4Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} + \frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{54a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(2/3)*A)/x^2 + (9*a^(5/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2
/3)*(-11*A*b + 5*a*B)*x)/(a + b*x^3) + (10*Sqrt[3]*(4*A*b - a*B)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*(-4*A*b + a*B)*Log[a^(1/3)
 + b^(1/3)*x])/b^(1/3) + (5*(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/b^(1/3))/(54*a^(11/3))
```

Maple [A]
time = 0.29, size = 158, normalized size = 0.70

method	result
default	$-\frac{\frac{\left(\frac{11}{18}b^2A-\frac{5}{18}abB\right)x^4+\frac{a(7Ab-4Ba)x}{9}}{(b\,x^3+a)^2}+\frac{5(4Ab-Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9}}{a^3}-$
risch	$\frac{-\frac{5b(4Ab-Ba)x^6}{18a^3}-\frac{4(4Ab-Ba)x^3}{9a^2}-\frac{A}{2a}}{x^2(b\,x^3+a)^2}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}b\_Z^3+64A^3b^3-48A^2Ba\,b^2+12A\,B^2a^2b-B^3a^3\right)}\_R\ln\left(\left(-4\_R^3a^{11}b-19\right.\right.\right.}{27}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*(((11/18*b^2*A-5/18*a*b*B)*x^4+1/9*a*(7*A*b-4*B*a)*x)/(b*x^3+a)^2+5/
9*(4*A*b-B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))))-1/2*A/a^3/x^2
```

Maxima [A]
time = 0.49, size = 201, normalized size = 0.89

$$\frac{5\,(Bab-4\,Ab^2)x^6+8\,(Ba^2-4\,Aab)x^3-9\,Aa^2}{18\,(a^3b^2x^8+2\,a^4bx^5+a^5x^2)}+\frac{5\,\sqrt{3}\,(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,(Ba-4\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,(Ba-4\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(5*(B*a*b - 4*A*b^2)*x^6 + 8*(B*a^2 - 4*A*a*b)*x^3 - 9*A*a^2)/(a^3*b^2
*x^8 + 2*a^4*b*x^5 + a^5*x^2) + 5/27*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqrt(
3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) - 5/54*(B*a - 4*A*b
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) + 5/27*(B*a -
4*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(179) = 358.
time = 1.63, size = 812, normalized size = 3.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a^4*b - 4*A*a^
3*b^2)*x^3 - 15*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*a^3*b^2 - 4*A
*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*
a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((B*a*b^
2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(
-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*((B*a
*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2
)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^5*b^3*x^8 + 2*a^6*b^2*x^5
+ a^7*b*x^2), 1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a
^4*b - 4*A*a^3*b^2)*x^3 + 30*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*
a^3*b^2 - 4*A*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt(-(-a^2*b)^(1
/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2
*b)^(1/3)/b)/a^2) - 5*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^
5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x
- (-a^2*b)^(1/3)*a) + 10*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)
*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))
/(a^5*b^3*x^8 + 2*a^6*b^2*x^5 + a^7*b*x^2)]
```

Sympy [A]
time = 0.50, size = 143, normalized size = 0.63

$$\frac{-9Aa^2 + x^6(-20Ab^2 + 5Bab) + x^3(-32Aab + 8Ba^2)}{18a^5x^2 + 36a^4bx^5 + 18a^3b^2x^8} + \operatorname{RootSum}\left(19683t^3a^{11}b + 8000A^3b^3 - 6000A^2Bab^2 + 1500AB^2a^2b - 125B^3a^3, \left(t \mapsto t\log\left(\frac{27ta^4}{-20Ab + 5Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**3,x)
```

```
[Out] (-9*A*a**2 + x**6*(-20*A*b**2 + 5*B*a*b) + x**3*(-32*A*a*b + 8*B*a**2))/(18
*a**5*x**2 + 36*a**4*b*x**5 + 18*a**3*b**2*x**8) + RootSum(19683*_t**3*a**1
1*b + 8000*A**3*b**3 - 6000*A**2*B*a*b**2 + 1500*A*B**2*a**2*b - 125*B**3*a
**3, Lambda(_t, _t*log(27*_t*a**4/(-20*A*b + 5*B*a) + x)))
```

Giac [A]
time = 0.68, size = 209, normalized size = 0.92

$$-\frac{5\,(Ba - 4\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4} + \frac{5\,\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - 4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b} + \frac{5\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - 4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b} + \frac{5\,Babx^6 - 20\,Ab^2x^6 + 8\,Ba^2x^3 - 32\,Aabx^3 - 9\,Aa^2}{18\left(bx^4 + ax\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -5/27*(B*a - 4*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 5/27*sqrt
(3)*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (
```

```
-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) + 5/54*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^
(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) + 1/18*(5*B*a*b
*x^6 - 20*A*b^2*x^6 + 8*B*a^2*x^3 - 32*A*a*b*x^3 - 9*A*a^2)/((b*x^4 + a*x)^
2*a^3)
```

Mupad [B]
time = 2.58, size = 188, normalized size = 0.83

$$-\frac{\frac{A}{2\,a}+\frac{4\,x^3\,\left(4\,A\,b-B\,a\right)}{9\,a^2}+\frac{5\,b\,x^6\,\left(4\,A\,b-B\,a\right)}{18\,a^3}}{a^2\,x^2+2\,a\,b\,x^5+b^2\,x^8}-\frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(4\,A\,b-B\,a\right)}{27\,a^{11/3}\,b^{1/3}}+\frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{27\,a^{11/3}\,b^{1/3}}-\frac{5\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{27\,a^{11/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] (5*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
4*A*b - B*a))/(27*a^(11/3)*b^(1/3)) - (5*log(b^(1/3)*x + a^(1/3))*(4*A*b -
B*a))/(27*a^(11/3)*b^(1/3)) - (A/(2*a) + (4*x^3*(4*A*b - B*a))/(9*a^2) + (5
*b*x^6*(4*A*b - B*a))/(18*a^3))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5) - (5*log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*b - B
*a))/(27*a^(11/3)*b^(1/3))
```

3.105 $\int \frac{A+Bx^3}{x^5(a+bx^3)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{7(5Ab-2aB)}{36a^3bx^4}+\frac{7(5Ab-2aB)}{9a^4x}+\frac{Ab-aB}{6abx^4(a+bx^3)^2}+\frac{5Ab-2aB}{9a^2bx^4(a+bx^3)}-\frac{7\sqrt[3]{b}\,(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\,\sqrt[3]{a}}\right.}{9\sqrt{3}\,a^{13/3}}$$

```
[Out] -7/36*(5*A*b-2*B*a)/a^3/b/x^4+7/9*(5*A*b-2*B*a)/a^4/x+1/6*(A*b-B*a)/a/b/x^4
/(b*x^3+a)^2+1/9*(5*A*b-2*B*a)/a^2/b/x^4/(b*x^3+a)-7/27*b^(1/3)*(5*A*b-2*B*
a)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)+7/54*b^(1/3)*(5*A*b-2*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/3)-7/27*b^(1/3)*(5*A*b-2*B*a)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {468, 296, 331, 298, 31, 648, 631, 210, 642}

$$-\frac{7\sqrt[3]{b}\,(5Ab-2aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{13/3}}+\frac{7\sqrt[3]{b}\,(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{13/3}}-\frac{7\sqrt[3]{b}\,(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{13/3}}+\frac{7(5Ab-2aB)}{9a^4x}-\frac{7(5Ab-2aB)}{36a^3bx^4}+\frac{5Ab-2aB}{9a^2bx^4(a+bx^3)}+\frac{Ab-aB}{6abx^4(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] (-7*(5*A*b - 2*a*B))/(36*a^3*b*x^4) + (7*(5*A*b - 2*a*B))/(9*a^4*x) + (A*b
- a*B)/(6*a*b*x^4*(a + b*x^3)^2) + (5*A*b - 2*a*B)/(9*a^2*b*x^4*(a + b*x^3)
) - (7*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(13/3)) - (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(1/3) + b^(1
/3)*x])/(27*a^(13/3)) + (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{(10Ab-4aB)\int \frac{1}{x^5\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} + \frac{(7(5Ab-2aB))\int \frac{1}{x^5\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{(7(5Ab-2aB))\int \frac{1}{x^2\left(a+bx^3\right)}}{9a^3}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} + \frac{(7b(5Ab-}{}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{(7b^{2/3}(5A}{}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}\,(5Ab}{}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}\,(5Ab}{}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}\,(5Ab}{}$$

Mathematica [A]
time = 0.10, size = 214, normalized size = 0.87

$$\frac{-\frac{27a^{4/3}A}{x^4} - \frac{108\sqrt[3]{a}\,(-3Ab+aB)}{x} - \frac{18a^{4/3}b(-Ab+aB)x^2}{\left(a+bx^3\right)^2} - \frac{12\sqrt[3]{a}\,b(-8Ab+5aB)x^2}{a+bx^3} - 28\sqrt{3}\,\sqrt[3]{b}\,(5Ab-2aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 28\sqrt[3]{b}\,(-5Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + 14\sqrt[3]{b}\,(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{108a^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(4/3)*A)/x^4 - (108*a^(1/3)*(-3*A*b + a*B))/x - (18*a^(4/3)*b*(-(A*
b) + a*B)*x^2)/(a + b*x^3)^2 - (12*a^(1/3)*b*(-8*A*b + 5*a*B)*x^2)/(a + b*x
^3) - 28*Sqrt[3]*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
```

```
/Sqrt[3]] + 28*b^(1/3)*(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(1/
3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*a^(
13/3))
```

Maple [A]
time = 0.29, size = 175, normalized size = 0.71

method	result
default	$\frac{b\left(\frac{\left(\frac{8}{9}b^{2}A-\frac{5}{9}abB\right)x^{5}+\frac{a(19Ab-13Ba)x^{2}}{18}}{\left(b\,x^{3}+a\right)^{2}}+\left(\frac{35Ab}{9}-\frac{14Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\right)}{a^{4}}$
risch	$\frac{\frac{7b^{2}(5Ab-2Ba)x^{9}}{9a^{4}}+\frac{49b(5Ab-2Ba)x^{6}}{36a^{3}}+\frac{(5Ab-2Ba)x^{3}}{2a^{2}}-\frac{A}{4a}}{x^{4}\left(b\,x^{3}+a\right)^{2}}+\frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(a^{13}\_Z^{3}+125A^{3}b^{4}-150A^{2}Ba\,b^{3}+60A\,B^{2}a^{2}b^{2}-8B^{3}a^{3}b\right)}\_R\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*b*(((8/9*b^2*A-5/9*a*b*B)*x^5+1/18*a*(19*A*b-13*B*a)*x^2)/(b*x^3+a)^2
+(35/9*A*b-14/9*B*a)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))))-1/4*A/a^3/x^4-(-3*A*b+B*a)/a^4/x
```

Maxima [A]
time = 0.55, size = 221, normalized size = 0.90

$$-\frac{28\,(2\,Bab^{2}-5\,Ab^{3})x^{9}+49\,(2\,Ba^{2}b-5\,Aab^{2})x^{6}+9\,Aa^{3}+18\,(2\,Ba^{3}-5\,Aa^{2}b)x^{3}}{36\,(a^{4}b^{2}x^{10}+2\,a^{5}bx^{7}+a^{6}x^{4})}-\frac{7\,\sqrt{3}\,(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{4}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,(2\,Ba-5\,Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{4}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\,(2\,Ba-5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{4}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/36*(28*(2*B*a*b^2 - 5*A*b^3)*x^9 + 49*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 9*A*
a^3 + 18*(2*B*a^3 - 5*A*a^2*b)*x^3)/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x^4)
- 7/27*sqrt(3)*(2*B*a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(a^4*(a/b)^(1/3)) - 7/54*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(a^4*(a/b)^(1/3)) + 7/27*(2*B*a - 5*A*b)*log(x + (a/b)^(1/3))/
(a^4*(a/b)^(1/3))
```

Fricas [A]
time = 1.90, size = 366, normalized size = 1.49

$$-\frac{84\,(2\,Bab^2-5\,Ab^3)x^9+147\,(2\,Ba^2b-5\,Aab^2)x^6+27\,Aa^3+54\,(2\,Ba^3-5\,Aa^2b)x^3+28\,\sqrt{3}\,\left((2\,Bab^2-5\,Ab^3)x^{10}+2\,(2\,Ba^2b-5\,Aab^2)x^7+(2\,Ba^3-5\,Aa^2b)x^4\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-14\,\left((2\,Bab^2-5\,Ab^3)x^{10}+2\,(2\,Ba^2b-5\,Aab^2)x^7+(2\,Ba^3-5\,Aa^2b)x^4\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+28\,\left((2\,Bab^2-5\,Ab^3)x^{10}+2\,(2\,Ba^2b-5\,Aab^2)x^7+(2\,Ba^3-5\,Aa^2b)x^4\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{108\,(a^4b^2x^{10}+2\,a^5bx^7+a^6x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(84*(2*B*a*b^2 - 5*A*b^3)*x^9 + 147*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 27
*A*a^3 + 54*(2*B*a^3 - 5*A*a^2*b)*x^3 + 28*sqrt(3)*((2*B*a*b^2 - 5*A*b^3)*x
^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/a)^(1/
3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 14*((2*B*a*b^2 - 5*A*
b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/
a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 28*((2*B*a*b^2 -
5*A*b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*
(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x
^4)
```

Sympy [A]
time = 0.53, size = 189, normalized size = 0.77

$$\operatorname{RootSum}\left(19683t^3a^{13}+42875A^3b^4-51450A^2Bab^3+20580AB^2a^2b^2-2744B^3a^3b,\left(t\mapsto t\log\left(\frac{729t^2a^9}{1225A^2b^3-980ABab^2+196B^2a^2b}+x\right)\right)\right)+\frac{-9Aa^3+x^9\cdot(140Ab^3-56Bab^2)+x^6\cdot(245Aab^2-98Ba^2b)+x^3\cdot(90Aa^2b-36Ba^3)}{36a^6x^4+72a^5bx^7+36a^4b^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**13 + 42875*A**3*b**4 - 51450*A**2*B*a*b**3 + 20580*A
*B**2*a**2*b**2 - 2744*B**3*a**3*b, Lambda(_t, _t*log(729*_t**2*a**9/(1225*
A**2*b**3 - 980*A*B*a*b**2 + 196*B**2*a**2*b) + x))) + (-9*A*a**3 + x**9*(1
40*A*b**3 - 56*B*a*b**2) + x**6*(245*A*a*b**2 - 98*B*a**2*b) + x**3*(90*A*a
**2*b - 36*B*a**3))/(36*a**6*x**4 + 72*a**5*b*x**7 + 36*a**4*b**2*x**10)
```

Giac [A]
time = 0.62, size = 254, normalized size = 1.03

$$\frac{7\left(2\,Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5}+\frac{7\,\sqrt{3}\left(2\,(-ab^2)^{\frac{2}{3}}Ba-5\,(-ab^2)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b}-\frac{7\left(2\,(-ab^2)^{\frac{2}{3}}Ba-5\,(-ab^2)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b}-\frac{10\,Bab^2x^5-16\,Ab^3x^5+13\,Ba^2bx^2-19\,Aab^2x^2}{18\,(bx^3+a)^2a^4}-\frac{4\,Bax^3-12\,Abx^3+Aa}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 7/27*(2*B*a*b*(-a/b)^(1/3) - 5*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/a^5 + 7/27*sqrt(3)*(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)
*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 7/54*
(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a^5*b) - 1/18*(10*B*a*b^2*x^5 - 16*A*b^3*x^5 + 13*B*a^2*b*x^2
```

```
- 19*A*a*b^2*x^2)/((b*x^3 + a)^2*a^4) - 1/4*(4*B*a*x^3 - 12*A*b*x^3 + A*a)/
(a^4*x^4)
```

Mupad [B]
time = 2.64, size = 240, normalized size = 0.98

$$\frac{\frac{x^3\,\left(5\,A\,b-2\,B\,a\right)}{2\,a^2}-\frac{A}{4\,a}+\frac{7\,b^2\,x^9\,\left(5\,A\,b-2\,B\,a\right)}{9\,a^4}+\frac{49\,b\,x^6\,\left(5\,A\,b-2\,B\,a\right)}{36\,a^3}}{a^2\,x^4+2\,a\,b\,x^7+b^2\,x^{10}}+\frac{7\,{\left(-b\right)}^{1/3}\,\ln\left(a^{1/3}\,{\left(-b\right)}^{8/3}+b^3\,x\right)\,\left(5\,A\,b-2\,B\,a\right)}{27\,a^{13/3}}+\frac{7\,{\left(-b\right)}^{1/3}\,\ln\left(a^{1/3}\,{\left(-b\right)}^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\,{\left(-b\right)}^{8/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,A\,b-2\,B\,a\right)}{27\,a^{13/3}}-\frac{7\,{\left(-b\right)}^{1/3}\,\ln\left(2\,b^3\,x-a^{1/3}\,{\left(-b\right)}^{8/3}+\sqrt{3}\,a^{1/3}\,{\left(-b\right)}^{8/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,A\,b-2\,B\,a\right)}{27\,a^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(5*A*b - 2*B*a))/(2*a^2) - A/(4*a) + (7*b^2*x^9*(5*A*b - 2*B*a))/(9*a
^4) + (49*b*x^6*(5*A*b - 2*B*a))/(36*a^3))/(a^2*x^4 + b^2*x^10 + 2*a*b*x^7)
 + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(5*A*b - 2*B*a))/(27*a^(13
/3)) + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/2)*a^(1/3)*(-b
)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3)) - (7*(-b)
^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)^(8/3)*1i)*((
3^(1/2)*1i)/2 + 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3))
```

3.106 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{4(11Ab-5aB)}{45a^3bx^5}+\frac{2(11Ab-5aB)}{9a^4x^2}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}+\frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)}-\frac{4b^{2/3}(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{9\sqrt{3}\,a^{14/3}}$$

```
[Out] -4/45*(11*A*b-5*B*a)/a^3/b/x^5+2/9*(11*A*b-5*B*a)/a^4/x^2+1/6*(A*b-B*a)/a/b
/x^5/(b*x^3+a)^2+1/18*(11*A*b-5*B*a)/a^2/b/x^5/(b*x^3+a)+4/27*b^(2/3)*(11*A
*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)-2/27*b^(2/3)*(11*A*b-5*B*a)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)-4/27*b^(2/3)*(11*A*b-5*B*a)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {468, 296, 331, 206, 31, 648, 631, 210, 642}

$$-\frac{4b^{2/3}(11Ab-5aB)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{14/3}}-\frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{14/3}}+\frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{14/3}}+\frac{2(11Ab-5aB)}{9a^4x^2}-\frac{4(11Ab-5aB)}{45a^3bx^5}+\frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] (-4*(11*A*b - 5*a*B))/(45*a^3*b*x^5) + (2*(11*A*b - 5*a*B))/(9*a^4*x^2) + (
A*b - a*B)/(6*a*b*x^5*(a + b*x^3)^2) + (11*A*b - 5*a*B)/(18*a^2*b*x^5*(a +
b*x^3)) - (4*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3)) + (4*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3
) + b^(1/3)*x])/(27*a^(14/3)) - (2*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{(11Ab-5aB)\int \frac{1}{x^6\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4(11Ab-5aB))\int \frac{1}{x^6\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} - \frac{(4(11Ab-5aB))\int \frac{1}{x^3(a+\cdots}}{9a^3}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4b(11\cdots}{\cdots}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4b(11\cdots}{\cdots}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{4b^{2/3}(\cdots}{\cdots}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{4b^{2/3}(\cdots}{\cdots}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} - \frac{4b^{2/3}(\cdots}{\cdots}$$

Mathematica [A]
time = 0.11, size = 210, normalized size = 0.85

$$\frac{-\frac{54a^{5/3}A}{x^5} - \frac{135a^{2/3}(-3Ab+aB)}{x^2} - \frac{45a^{5/3}b(-Ab+aB)x}{\left(a+bx^3\right)^2} - \frac{15a^{2/3}b(-17Ab+11aB)x}{a+bx^3} - 40\sqrt{3}\,b^{2/3}(11Ab-5aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 40b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + 20b^{2/3}(-11Ab+5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{270a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(5/3)*A)/x^5 - (135*a^(2/3)*(-3*A*b + a*B))/x^2 - (45*a^(5/3)*b*(-(
A*b) + a*B)*x)/(a + b*x^3)^2 - (15*a^(2/3)*b*(-17*A*b + 11*a*B)*x)/(a + b*x
^3) - 40*Sqrt[3]*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
```

```
)/Sqrt[3]] + 40*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*b^(2
/3)*(-11*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(270*
a^(14/3))
```

Maple [A]

time = 0.31, size = 174, normalized size = 0.71

method	result
default	$\frac{b\left(\frac{\left(\frac{17}{18}b^2A-\frac{11}{18}abB\right)x^4+\frac{a(10Ab-7Ba)x}{9}}{\left(bx^3+a\right)^2}+\frac{4(11Ab-5Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9}\right)}{a^4}$
risch	$\frac{\frac{2b^2(11Ab-5Ba)x^9}{9a^4}+\frac{16b(11Ab-5Ba)x^6}{45a^3}+\frac{(11Ab-5Ba)x^3}{10a^2}-\frac{A}{5a}}{x^5\left(bx^3+a\right)^2}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(a^{14}\_Z^3-1331A^3b^5+1815A^2Bab^4-825AB^2a^2b^3+125B^3a\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*b*((((17/18*b^2*A-11/18*a*b*B)*x^4+1/9*a*(10*A*b-7*B*a)*x)/(b*x^3+a)^2
+4/9*(11*A*b-5*B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))))-1/5*A/a^3/x^5-1/2*(-3*A*b+B*a)/a^4/x^2
```

Maxima [A]

time = 0.50, size = 221, normalized size = 0.90

$$-\frac{20\,(5\,Bab^2-11\,Ab^3)x^9+32\,(5\,Ba^2b-11\,Aab^2)x^6+18\,Aa^3+9\,(5\,Ba^3-11\,Aa^2b)x^3}{90\,(a^4b^2x^{11}+2\,a^5bx^8+a^6x^5)}-\frac{4\,\sqrt{3}\,(5\,Ba-11\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,(5\,Ba-11\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{4\,(5\,Ba-11\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/90*(20*(5*B*a*b^2 - 11*A*b^3)*x^9 + 32*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 18
*A*a^3 + 9*(5*B*a^3 - 11*A*a^2*b)*x^3)/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^
5) - 4/27*sqrt(3)*(5*B*a - 11*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(
a/b)^(1/3))/(a^4*(a/b)^(2/3)) + 2/27*(5*B*a - 11*A*b)*log(x^2 - x*(a/b)^(1/
3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) - 4/27*(5*B*a - 11*A*b)*log(x + (a/b)^(
1/3))/(a^4*(a/b)^(2/3))
```

Fricas [A]
time = 2.18, size = 384, normalized size = 1.56

$$-\frac{60\,(5\,Bab^2-11\,Ab^3)x^9+96\,(5\,Ba^2b-11\,Aab^2)x^6+54\,Aa^3+27\,(5\,Ba^3-11\,Aa^2b)x^3+40\,\sqrt{3}\,((5\,Bab^2-11\,Ab^3)x^{11}+2\,(5\,Ba^2b-11\,Aab^2)x^8+(5\,Ba^3-11\,Aa^2b)x^5)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-20\,((5\,Bab^2-11\,Ab^3)x^{11}+2\,(5\,Ba^2b-11\,Aab^2)x^8+(5\,Ba^3-11\,Aa^2b)x^5)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+40\,((5\,Bab^2-11\,Ab^3)x^{11}+2\,(5\,Ba^2b-11\,Aab^2)x^8+(5\,Ba^3-11\,Aa^2b)x^5)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)}{270\,(a^4b^2x^{11}+2\,a^5bx^8+a^6x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/270*(60*(5*B*a*b^2 - 11*A*b^3)*x^9 + 96*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 5
4*A*a^3 + 27*(5*B*a^3 - 11*A*a^2*b)*x^3 + 40*sqrt(3)*((5*B*a*b^2 - 11*A*b^3
)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/
a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 20*(
(5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 1
1*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(
b^2/a^2)^(2/3)) + 40*((5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b
^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)))/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^5)
```

Sympy [A]
time = 0.57, size = 173, normalized size = 0.70

$$\operatorname{RootSum}\left(19683t^3a^{14}-85184A^3b^5+116160A^2Bab^4-52800AB^2a^2b^3+8000B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{27ta^5}{-44Ab^2+20Bab}+x\right)\right)\right)+\frac{-18Aa^3+x^9\cdot(220Ab^3-100Bab^2)+x^6\cdot(352Aab^2-160Ba^2b)+x^3\cdot(99Aa^2b-45Ba^3)}{90a^6x^5+180a^5bx^8+90a^4b^2x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**14 - 85184*A**3*b**5 + 116160*A**2*B*a*b**4 - 52800*
A*B**2*a**2*b**3 + 8000*B**3*a**3*b**2, Lambda(_t, _t*log(-27*_t*a**5/(-44*
A*b**2 + 20*B*a*b) + x))) + (-18*A*a**3 + x**9*(220*A*b**3 - 100*B*a*b**2)
+ x**6*(352*A*a*b**2 - 160*B*a**2*b) + x**3*(99*A*a**2*b - 45*B*a**3))/(90*
a**6*x**5 + 180*a**5*b*x**8 + 90*a**4*b**2*x**11)
```

Giac [A]
time = 0.69, size = 229, normalized size = 0.93

$$-\frac{4\,\sqrt{3}\left(5\,(-ab^2)^{\frac{1}{3}}\,Ba-11\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5}+\frac{4\,(5\,Bab-11\,Ab^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5}-\frac{2\left(5\,(-ab^2)^{\frac{1}{3}}\,Ba-11\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^5}-\frac{11\,Bab^2x^4-17\,Ab^3x^4+14\,Ba^2bx-20\,Aab^2x}{18\,(bx^3+a)^2a^4}-\frac{5\,Bax^3-15\,Abx^3+2\,Aa}{10\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -4/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 11*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^5 + 4/27*(5*B*a*b - 11*A*b^2)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 2/27*(5*(-a*b^2)^(1/3)*B*a - 11*
(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^5 - 1/18*(11
*B*a*b^2*x^4 - 17*A*b^3*x^4 + 14*B*a^2*b*x - 20*A*a*b^2*x)/((b*x^3 + a)^2*a
^4) - 1/10*(5*B*a*x^3 - 15*A*b*x^3 + 2*A*a)/(a^4*x^5)
```

Mupad [B]
time = 2.58, size = 207, normalized size = 0.84

$$\frac{\frac{x^3\,\left(11\,A\,b-5\,B\,a\right)}{10\,a^2}-\frac{A}{5\,a}+\frac{2\,b^2\,x^9\,\left(11\,A\,b-5\,B\,a\right)}{9\,a^4}+\frac{16\,b\,x^6\,\left(11\,A\,b-5\,B\,a\right)}{45\,a^3}}{a^2\,x^5+2\,a\,b\,x^8+b^2\,x^{11}}+\frac{4\,b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(11\,A\,b-5\,B\,a\right)}{27\,a^{14/3}}-\frac{4\,b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(11\,A\,b-5\,B\,a\right)}{27\,a^{14/3}}+\frac{4\,b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(11\,A\,b-5\,B\,a\right)}{27\,a^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(11*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (2*b^2*x^9*(11*A*b - 5*B*a))/(
9*a^4) + (16*b*x^6*(11*A*b - 5*B*a))/(45*a^3))/(a^2*x^5 + b^2*x^11 + 2*a*b*
x^8) + (4*b^(2/3)*log(b^(1/3)*x + a^(1/3))*(11*A*b - 5*B*a))/(27*a^(14/3))
- (4*b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/
2 + 1/2)*(11*A*b - 5*B*a))/(27*a^(14/3)) + (4*b^(2/3)*log(3^(1/2)*a^(1/3)*1
i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a))/(27*a^(
14/3))
```

3.107 $\int \frac{x^8}{(a+bx^3)(c+dx^3)} \, dx$

Optimal. Leaf size=70

$$\frac{x^3}{3bd} + \frac{a^2 \log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2 \log(c+dx^3)}{3d^2(bc-ad)}$$

```
[Out] 1/3*x^3/b/d+1/3*a^2*ln(b*x^3+a)/b^2/(-a*d+b*c)-1/3*c^2*ln(d*x^3+c)/d^2/(-a*
d+b*c)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 84}

$$\frac{a^2 \log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2 \log(c+dx^3)}{3d^2(bc-ad)} + \frac{x^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x^3/(3*b*d) + (a^2*Log[a + b*x^3])/(3*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^3
])/(3*d^2*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(a+bx^3)(c+dx^3)} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)} \, dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)}\right) dx, x, x^3\right) \\
&= \frac{x^3}{3bd} + \frac{a^2 \log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2 \log(c+dx^3)}{3d^2(bc-ad)}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.94

$$\frac{a^2d^2\log\left(a+bx^3\right)-b(d(-bc+ad)x^3+bc^2\log\left(c+dx^3\right))}{3b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^3] - b*(d*(-(b*c) + a*d)*x^3 + b*c^2*Log[c + d*x^3]))/
(3*b^2*d^2*(b*c - a*d))
```

Maple [A]
time = 0.34, size = 65, normalized size = 0.93

method	result	size
default	$\frac{x^3}{3bd}-\frac{a^2\ln(b\,x^3+a)}{3b^2(ad-bc)}+\frac{c^2\ln(d\,x^3+c)}{3d^2(ad-bc)}$	65
norman	$\frac{x^3}{3bd}-\frac{a^2\ln(b\,x^3+a)}{3b^2(ad-bc)}+\frac{c^2\ln(d\,x^3+c)}{3d^2(ad-bc)}$	65
risch	$\frac{x^3}{3bd}+\frac{c^2\ln(d\,x^3+c)}{3d^2(ad-bc)}-\frac{a^2\ln(-b\,x^3-a)}{3b^2(ad-bc)}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3/b/d-1/3*a^2/b^2/(a*d-b*c)*ln(b*x^3+a)+1/3*c^2/d^2/(a*d-b*c)*ln(d*x^
3+c)
```

Maxima [A]
time = 0.27, size = 68, normalized size = 0.97

$$\frac{a^2\log\left(bx^3+a\right)}{3\left(b^3c-ab^2d\right)}-\frac{c^2\log\left(dx^3+c\right)}{3\left(bcd^2-ad^3\right)}+\frac{x^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*log(b*x^3 + a)/(b^3*c - a*b^2*d) - 1/3*c^2*log(d*x^3 + c)/(b*c*d^2
- a*d^3) + 1/3*x^3/(b*d)
```

Fricas [A]
time = 1.93, size = 72, normalized size = 1.03

$$\frac{a^2d^2\log\left(bx^3+a\right)-b^2c^2\log\left(dx^3+c\right)+(b^2cd-abd^2)x^3}{3\left(b^3cd^2-ab^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*d^2*log(b*x^3 + a) - b^2*c^2*log(d*x^3 + c) + (b^2*c*d - a*b*d^2)*
x^3)/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.73, size = 70, normalized size = 1.00

$$\frac{a^2\log\left(|bx^3+a|\right)}{3\left(b^3c-ab^2d\right)}-\frac{c^2\log\left(|dx^3+c|\right)}{3\left(bcd^2-ad^3\right)}+\frac{x^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a^2*log(abs(b*x^3 + a))/(b^3*c - a*b^2*d) - 1/3*c^2*log(abs(d*x^3 + c))
/(b*c*d^2 - a*d^3) + 1/3*x^3/(b*d)
```

Mupad [B]
time = 2.84, size = 68, normalized size = 0.97

$$\frac{a^2\,\ln\left(b\,x^3+a\right)}{3\,b^3\,c-3\,a\,b^2\,d}+\frac{c^2\,\ln\left(d\,x^3+c\right)}{3\,a\,d^3-3\,b\,c\,d^2}+\frac{x^3}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (a^2*log(a + b*x^3))/(3*b^3*c - 3*a*b^2*d) + (c^2*log(c + d*x^3))/(3*a*d^3
- 3*b*c*d^2) + x^3/(3*b*d)
```

3.108 $\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=301

$$\frac{x^2}{2bd}-\frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}(bc-ad)}+\frac{c^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}(bc-ad)}+\frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right.}{3d^{5/3}(bc-ad)}$$

```
[Out] 1/2*x^2/b/d-1/3*a^(5/3)*ln(a^(1/3)+b^(1/3)*x)/b^(5/3)/(-a*d+b*c)+1/3*c^(5/3
)*ln(c^(1/3)+d^(1/3)*x)/d^(5/3)/(-a*d+b*c)+1/6*a^(5/3)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(5/3)/(-a*d+b*c)-1/6*c^(5/3)*ln(c^(2/3)-c^(1/3)*d^(
1/3)*x+d^(2/3)*x^2)/d^(5/3)/(-a*d+b*c)-1/3*a^(5/3)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)/(-a*d+b*c)*3^(1/2)+1/3*c^(5/3)*arctan(1/3
*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(5/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {490, 598, 298, 31, 648, 631, 210, 642}

$$-\frac{a^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}(bc-ad)}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}(bc-ad)}+\frac{c^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)}-\frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6d^{5/3}(bc-ad)}+\frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{5/3}(bc-ad)}+\frac{x^2}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x^2/(2*b*d) - (a^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^(5/3)*(b*c - a*d)) + (c^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*d^(5/3)*(b*c - a*d)) - (a^(5/3)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(5/3)*(b*c - a*d)) + (c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(
5/3)*(b*c - a*d)) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)*(b*c - a*d)) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2])/(6*d^(5/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \frac{x^2}{2bd} - \frac{\int \frac{x(2ac+2(bc+ad)x^3)}{(a+bx^3)(c+dx^3)}\,dx}{2bd}$$

$$= \frac{x^2}{2bd} - \frac{\int \left(\frac{2a^2dx}{(-bc+ad)(a+bx^3)} + \frac{2bc^2x}{(bc-ad)(c+dx^3)}\right)dx}{2bd}$$

$$= \frac{x^2}{2bd} + \frac{a^2\int \frac{x}{a+bx^3}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{x}{c+dx^3}\,dx}{d(bc-ad)}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{4/3}(bc-ad)} + \frac{a^{5/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3b^{4/3}(bc-ad)} + \frac{c^{5/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\ \cdots}}{3d^{4/3}(bc-a\cdots}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+\cdots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots}}{6b^{5/3}(bc-a\cdots}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\cdots\right.}{6b^{5/3}(bc\cdots}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}(bc-ad)} + \frac{c^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)} - \frac{a^{5/3}\log\left(\sqrt[3]{a}\cdots\right.}{3b^{5/3}(bc-\cdots}$$

Mathematica [A]
time = 0.08, size = 242, normalized size = 0.80

$$\frac{-\frac{3ax^2}{b}+\frac{3cx^2}{d}-\frac{2\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}}+\frac{2\sqrt{3}\,c^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{5/3}}-\frac{2a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{5/3}}+\frac{2c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{d^{5/3}}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{5/3}}-\frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{d^{5/3}}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((-3*a*x^2)/b + (3*c*x^2)/d - (2*Sqrt[3]*a^(5/3)*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]])/b^(5/3) + (2*Sqrt[3]*c^(5/3)*ArcTan[(1 - (2*d^(1/3)*x)/c
^(1/3))/Sqrt[3]])/d^(5/3) - (2*a^(5/3)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3) +
(2*c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/d^(5/3) + (a^(5/3)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/
3)*x + d^(2/3)*x^2])/d^(5/3))/(6*b*c - 6*a*d)
```

Maple [A]
time = 0.40, size = 228, normalized size = 0.76

method	result
default	$\frac{x^2}{2bd}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a^2}{b(ad-bc)}+\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\right.}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right.}{}$ …
risch	$\frac{x^2}{2bd}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(a^3b^2d^3-3a^2b^3c\,d^2+3a\,b^4c^2d-b^5c^3\right)\_Z^3-a^5d^3\right)}\_R\ln\left(\left(\left(\left(-a^5b^2c\,d^6+2a^4b^3c^2d^5-2a^3b^4c^3d^4+2a^2b^5c^4d^3-a\,b^6c^5d\right.\right.\right.\right.}{}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/b/d-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1)))*a^2/b/(a*d-b*c)+(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/
6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)
*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))*c^2/d/(a*d-b*c)
```

Maxima [A]
time = 0.51, size = 324, normalized size = 1.08

$$\frac{\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^3c-ab^2d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,c^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^2-ad^3\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{c^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{c^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}+\frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*c
 - a*b^2*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan(1/3*sqrt(3)*(2*x - (c/d)^
(1/3))/(c/d)^(1/3))/((b*c*d^2 - a*d^3)*(c/d)^(1/3)) + 1/6*a^2*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3)) - 1/6*c
^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3*(c/d
)^(1/3)) - 1/3*a^2*log(x + (a/b)^(1/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*(a/b)^
(1/3)) + 1/3*c^2*log(x + (c/d)^(1/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3*(c/d)^(1
/3)) + 1/2*x^2/(b*d)
```

Fricas [A]
time = 2.01, size = 273, normalized size = 0.91

$$\frac{2\,\sqrt{3}\,ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)-2\,\sqrt{3}\,bc\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,dx\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}\,c}{3\,c}\right)+ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+bc\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\log\left(cx^2-dx\left(-\frac{c^2}{d^2}\right)^{\frac{2}{3}}-c\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\right)-2\,ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)-2\,bc\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\log\left(cx+d\left(-\frac{c^2}{d^2}\right)^{\frac{2}{3}}\right)+3\,(bc-ad)x^2}{6\,(b^2cd-abd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*a*d*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3
) - sqrt(3)*a)/a) - 2*sqrt(3)*b*c*(-c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*
x*(-c^2/d^2)^(1/3) + sqrt(3)*c)/c) + a*d*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a
^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) + b*c*(-c^2/d^2)^(1/3)*log(c*x^2 - d*x*(
-c^2/d^2)^(2/3) - c*(-c^2/d^2)^(1/3)) - 2*a*d*(a^2/b^2)^(1/3)*log(a*x + b*(
a^2/b^2)^(2/3)) - 2*b*c*(-c^2/d^2)^(1/3)*log(c*x + d*(-c^2/d^2)^(2/3)) + 3*
(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.66, size = 311, normalized size = 1.03

$$-\frac{a^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(ab^2c-a^2bd)}+\frac{c^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2d-acd^2)}-\frac{(-ab^2)^{\frac{2}{3}}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^4c-\sqrt{3}\,ab^3d}+\frac{(-cd^2)^{\frac{2}{3}}\,c\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^3-\sqrt{3}\,ad^4}+\frac{(-ab^2)^{\frac{2}{3}}\,a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^4c-ab^3d)}-\frac{(-cd^2)^{\frac{2}{3}}\,c\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^3-ad^4)}+\frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3*
c^2*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) - (-a*b^2)^
(2/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^4*
c - sqrt(3)*a*b^3*d) + (-c*d^2)^(2/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-a*b^2)^(2/3)*a
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^4*c - a*b^3*d) - 1/6*(-c*d^2)^
(2/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^3 - a*d^4) + 1/2*x^
2/(b*d)
```

Mupad [**B**]
time = 11.36, size = 1751, normalized size = 5.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((((27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^3
*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))*(a^5/(b^5*(a*d
- b*c)^3))^(1/3))/3 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2
*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*d^2
+ b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^
2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3) + log(((((27*a^2*b*c^2*d*x*(a^2*d^2 +
b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-c^5/(d^
5*(a*d - b*c)^3))^(2/3))*(-c^5/(d^5*(a*d - b*c)^3))^(1/3))/3 - (9*(a*b^7*c^
8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(d^5*(a*d
- b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*
(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b*c*d^7))^(1
/3) - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(27*a^2*b*c^2*d*x*(a^2*d^
2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)
*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/4)*(a^5/(b^5*(a*d - b*c)^3)
)^(1/3))/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b
^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/36 + (a^4*c^4*x*(a^2*d^2 + b^2*c^
2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^2*b^6*c*
d^2 - 81*a*b^7*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(((3^(1/2)*1i - 1)^2
*(((3^(1/2)*1i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (
27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d
- b*c)^3))^(2/3))/4)*(a^5/(b^5*(a*d - b*c)^3))^(1/3))/6 - (9*(a*b^7*c^8 + a
^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)
^3))^(2/3))/36 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5
/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3)*(
3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(27*a^2*b*
c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1
)^2*(a*d + b*c)*(a*d - b*c)^4*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/4)*(-c^5/(d
^5*(a*d - b*c)^3))^(1/3))/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a
^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/36 + (a^4*c^4*x
*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d
^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b*c*d^7))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((
(3^(1/2)*1i - 1)^2*(((3^(1/2)*1i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)
*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)
^4*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/4)*(-c^5/(d^5*(a*d - b*c)^3))^(1/3))/6
 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(
-c^5/(d^5*(a*d - b*c)^3))^(2/3))/36 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c
*d))/(b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*
a^2*b*c*d^7))^(1/3)*(3^(1/2)*1i - 1))/2 + x^2/(2*b*d)
```

3.109 $\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=296

$$\frac{x}{bd}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}(bc-ad)}+\frac{c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}(bc-ad)}-\frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{4/3}(bc-ad)}$$

```
[Out] x/b/d+1/3*a^(4/3)*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)/(-a*d+b*c)-1/3*c^(4/3)*ln(c
^(1/3)+d^(1/3)*x)/d^(4/3)/(-a*d+b*c)-1/6*a^(4/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/b^(4/3)/(-a*d+b*c)+1/6*c^(4/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x
+d^(2/3)*x^2)/d^(4/3)/(-a*d+b*c)-1/3*a^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))/b^(4/3)/(-a*d+b*c)*3^(1/2)+1/3*c^(4/3)*arctan(1/3*(c^(1
/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(4/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {490, 536, 206, 31, 648, 631, 210, 642}

$$-\frac{a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}(bc-ad)}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}(bc-ad)}+\frac{c^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)}+\frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6d^{4/3}(bc-ad)}-\frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{4/3}(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x/(b*d) - (a^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*b^(4/3)*(b*c - a*d)) + (c^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]
*c^(1/3))])/(Sqrt[3]*d^(4/3)*(b*c - a*d)) + (a^(4/3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(4/3)*(b*c - a*d)) - (c^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(4/3)
*(b*c - a*d)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6
*b^(4/3)*(b*c - a*d)) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2])/(6*d^(4/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^3}{(a+bx^3)(c+dx^3)}\,dx}{bd}$$

$$= \frac{x}{bd} + \frac{a^2\int \frac{1}{a+bx^3}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{1}{c+dx^3}\,dx}{d(bc-ad)}$$

$$= \frac{x}{bd} + \frac{a^{4/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b(bc-ad)} + \frac{a^{4/3}\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3b(bc-ad)} - \frac{c^{4/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}}{3d(bc-ad)}$$

$$= \frac{x}{bd} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x-}}{6b^{4/3}(bc-ad}$$

$$= \frac{x}{bd} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\right.}{6b^{4/3}(bc-}$$

$$= \frac{x}{bd} - \frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}(bc-ad)} + \frac{c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\right.}{3b^{4/3}(bc-}$$

Mathematica [A]
time = 0.07, size = 238, normalized size = 0.80

$$\frac{-\frac{6ax}{b}+\frac{6cx}{d}-\frac{2\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}+\frac{2\sqrt{3}\,c^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{4/3}}+\frac{2a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{4/3}}-\frac{2c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{d^{4/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{4/3}}+\frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{d^{4/3}}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((-6*a*x)/b + (6*c*x)/d - (2*Sqrt[3]*a^(4/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(4/3) + (2*Sqrt[3]*c^(4/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/
3))/Sqrt[3]])/d^(4/3) + (2*a^(4/3)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) - (2*c
^(4/3)*Log[c^(1/3) + d^(1/3)*x])/d^(4/3) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2])/d^(4/3))/(6*b*c - 6*a*d)
```

Maple [A]
time = 0.36, size = 225, normalized size = 0.76

method	result

default	$\frac{x}{bd}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a^{2}}{b(ad-bc)}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right.}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}}{d(}$
risch	$\frac{x}{bd}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(d^{4}a^{3}-3a^{2}c\,d^{3}b+3a\,c^{2}d^{2}b^{2}-b^{3}c^{3}d\right)\_Z^{3}-b^{3}c^{4}\right)}\_R\ln\left(\left(-a^{5}bc\,d^{5}-a\,b^{5}c^{5}d\right)x+\left(-a^{5}b\,d^{6}+3d^{5}c\,b^{2}a^{4}-2d^{4}c^{2}b^{3}a^{3}-2d^{3}\right.\right.}{3bd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] x/b/d-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1)))/b*a^2/(a*d-b*c)+(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d
)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))/d*c^2/(a*d-b*c)
```

Maxima [A]

time = 0.49, size = 349, normalized size = 1.18

$$\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^{3}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^{2}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,c^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^{3}\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{a^{2}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{3}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{c^{2}\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{a^{2}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^{3}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{c^{2}\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*c
*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan(1
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d^2*(c/d)^(1/3) - a*d^3*(
c/d)^(1/3))*(c/d)^(1/3)) - 1/6*a^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) + 1/6*c^2*log(x^2 - x*(c/d)^(1/3)
+ (c/d)^(2/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + 1/3*a^2*log(x +
(a/b)^(1/3))/(b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) - 1/3*c^2*log(x + (c
/d)^(1/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + x/(b*d)
```

Fricas [A]

time = 1.41, size = 228, normalized size = 0.77

$$-\frac{2\,\sqrt{3}\,ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+2\,\sqrt{3}\,bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,dx\left(\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,c}{3\,c}\right)-ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)-bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)+2\,ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+2\,bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)-6\,(bc-ad)x}{6\,(b^{2}cd-abd^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a*d*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - s
qrt(3)*a)/a) + 2*sqrt(3)*b*c*(c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c/d)^(2
/3) - sqrt(3)*c)/c) - a*d*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3)) - b*c*(c/d)^(1/3)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)) + 2*a*d*(-a/b
)^(1/3)*log(x - (-a/b)^(1/3)) + 2*b*c*(c/d)^(1/3)*log(x + (c/d)^(1/3)) - 6*
(b*c - a*d)*x)/(b^2*c*d - a*b*d^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.68, size = 308, normalized size = 1.04

$$-\frac{a^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^{2}c-a^{2}bd\right)}+\frac{c^{2}\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^{2}d-acd^{2}\right)}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{3}c-\sqrt{3}\,ab^{2}d}-\frac{\left(-cd^{2}\right)^{\frac{1}{3}}c\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^{2}-\sqrt{3}\,ad^{3}}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}a\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{3}c-ab^{2}d\right)}-\frac{\left(-cd^{2}\right)^{\frac{1}{3}}c\log\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^{2}-ad^{3}\right)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3*
c^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (-a*b^2)^
(1/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*
c - sqrt(3)*a*b^2*d) - (-c*d^2)^(1/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) + 1/6*(-a*b^2)^(1/3)*a
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) - 1/6*(-c*d^2)^
(1/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3) + x/(b*d
)
```

Mupad [**B**]
time = 1.83, size = 873, normalized size = 2.95

$$\ln\left(a\,x+b^2\,c\,{\left(-\frac{a^4}{b^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}-a\,b\,d\,{\left(-\frac{a^4}{b^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}\right)\,{\left(\frac{a^4}{-27\,a^3\,b^4\,d^3+81\,a^2\,b^5\,c\,d^2-81\,a\,b^6\,c^2\,d+27\,b^7\,c^3}\right)}^{1/3}+\ln\left(c\,x+a\,d^2\,{\left(\frac{c^4}{d^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}-b\,c\,d\,{\left(\frac{c^4}{d^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}\right)\,{\left(\frac{c^4}{27\,a^3\,d^7-81\,a^2\,b\,c\,d^6+81\,a\,b^2\,c^2\,d^5-27\,b^3\,c^3\,d^4}\right)}^{1/3}+\frac{x}{b\,d}+\dots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(a*x + b^2*c*(-a^4/(b^4*(a*d - b*c)^3))^(1/3) - a*b*d*(-a^4/(b^4*(a*d -
b*c)^3))^(1/3))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a
*b^6*c^2*d))^(1/3) + log(c*x + a*d^2*(c^4/(d^4*(a*d - b*c)^3))^(1/3) - b*c*
d*(c^4/(d^4*(a*d - b*c)^3))^(1/3))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a
*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3) + x/(b*d) + (log((3*x*(a^2*b^4*c^6 +
a^6*c^2*d^4))/(b*d) - (3*a*c^2*(3^(1/2)*1i - 1)*(-a^4/(b^4*(a*d - b*c)^3))^
(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*d))*(a^4/(27*b^7*
c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a*b^6*c^2*d))^(1/3)*(3^(1/2)*1
i - 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) + (3*a*c^2*(3^(1/2
)*1i + 1)*(-a^4/(b^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d
 - a^4*b*c*d^4))/(2*d))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^
2 - 81*a*b^6*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((3*x*(a^2*b^4*c^6 + a
^6*c^2*d^4))/(b*d) + (3*a^2*c*(3^(1/2)*1i - 1)*(c^4/(d^4*(a*d - b*c)^3))^(1
/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*b))*(c^4/(27*a^3*d^
7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3)*(3^(1/2)*1i
- 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) - (3*a^2*c*(3^(1/2)*
1i + 1)*(c^4/(d^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d -
a^4*b*c*d^4))/(2*b))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 -
 81*a^2*b*c*d^6))^(1/3)*(3^(1/2)*1i + 1))/2
```

3.110 $\int \frac{x^5}{(a+bx^3)(c+dx^3)} dx$

Optimal. Leaf size=53

$$-\frac{a\log(a+bx^3)}{3b(bc-ad)}+\frac{c\log(c+dx^3)}{3d(bc-ad)}$$

```
[Out] -1/3*a*ln(b*x^3+a)/b/(-a*d+b*c)+1/3*c*ln(d*x^3+c)/d/(-a*d+b*c)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{c\log(c+dx^3)}{3d(bc-ad)}-\frac{a\log(a+bx^3)}{3b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/3*(a*Log[a + b*x^3])/(b*(b*c - a*d)) + (c*Log[c + d*x^3])/(3*d*(b*c - a*
d))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)} + \frac{c}{(bc-ad)(c+dx)}\right)dx, x, x^3\right)$$
$$= -\frac{a\log(a+bx^3)}{3b(bc-ad)} + \frac{c\log(c+dx^3)}{3d(bc-ad)}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.81

$$-\frac{ad\log(a+bx^3) - bc\log(c+dx^3)}{3b^2cd - 3abd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)),x]
[Out] -((a*d*Log[a + b*x^3] - b*c*Log[c + d*x^3])/(3*b^2*c*d - 3*a*b*d^2))
```

Maple [A]
time = 0.34, size = 50, normalized size = 0.94

method	result	size
default	$\frac{a\ln(b\,x^3+a)}{3(ad-bc)b} - \frac{c\ln(d\,x^3+c)}{3(ad-bc)d}$	50
norman	$\frac{a\ln(b\,x^3+a)}{3(ad-bc)b} - \frac{c\ln(d\,x^3+c)}{3(ad-bc)d}$	50
risch	$-\frac{c\ln(-d\,x^3-c)}{3d(ad-bc)} + \frac{a\ln(b\,x^3+a)}{3(ad-bc)b}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
[Out] 1/3*a/(a*d-b*c)/b*ln(b*x^3+a)-1/3*c/(a*d-b*c)/d*ln(d*x^3+c)
```

Maxima [A]
time = 0.28, size = 49, normalized size = 0.92

$$-\frac{a\log(bx^3+a)}{3(b^2c-abd)} + \frac{c\log(dx^3+c)}{3(bcd-ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*a*log(b*x^3 + a)/(b^2*c - a*b*d) + 1/3*c*log(d*x^3 + c)/(b*c*d - a*d^2
)
```

Fricas [A]
time = 2.37, size = 42, normalized size = 0.79

$$-\frac{ad\log\left(bx^3+a\right)-bc\log\left(dx^3+c\right)}{3\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/3*(a*d*log(b*x^3 + a) - b*c*log(d*x^3 + c))/(b^2*c*d - a*b*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(39) = 78$.
time = 5.88, size = 144, normalized size = 2.72

$$\frac{a\log\left(x^3+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{3b\left(ad-bc\right)}-\frac{c\log\left(x^3+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{3d\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] a*log(x**3 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2
/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(3*b*(a*d - b*c)) - c*log(x**3 + (-a**2*
c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)
))/(a*d + b*c))/(3*d*(a*d - b*c))
```

Giac [A]
time = 0.63, size = 51, normalized size = 0.96

$$-\frac{a\log\left(|bx^3+a|\right)}{3\left(b^2c-abd\right)}+\frac{c\log\left(|dx^3+c|\right)}{3\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a*log(abs(b*x^3 + a))/(b^2*c - a*b*d) + 1/3*c*log(abs(d*x^3 + c))/(b*c
*d - a*d^2)
```

Mupad [B]
time = 0.31, size = 51, normalized size = 0.96

$$-\frac{a\,\ln\left(b\,x^3+a\right)}{3\,b^2\,c-3\,a\,b\,d}-\frac{c\,\ln\left(d\,x^3+c\right)}{3\,a\,d^2-3\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] - (a*log(a + b*x^3))/(3*b^2*c - 3*a*b*d) - (c*log(c + d*x^3))/(3*a*d^2 - 3*
b*c*d)
```

3.111 $\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}(bc-ad)}-\frac{c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}(bc-ad)}-\frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{2/3}(bc-ad)}-\ldots$$

```
[Out] 1/3*a^(2/3)*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/(-a*d+b*c)-1/3*c^(2/3)*ln(c^(1/3)
+d^(1/3)*x)/d^(2/3)/(-a*d+b*c)-1/6*a^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/b^(2/3)/(-a*d+b*c)+1/6*c^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/d^(2/3)/(-a*d+b*c)+1/3*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/b^(2/3)/(-a*d+b*c)*3^(1/2)-1/3*c^(2/3)*arctan(1/3*(c^(1/3)-2*
d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(2/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {492, 298, 31, 648, 631, 210, 642}

$$\frac{a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}(bc-ad)}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}(bc-ad)}-\frac{c^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)}+\frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6d^{2/3}(bc-ad)}-\frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(2/3
)*(b*c - a*d)) - (c^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(2/3)*(b*c - a*d)) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
2/3)*(b*c - a*d)) - (c^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(2/3)*(b*c - a*
d)) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(
b*c - a*d)) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\int \frac{x}{a+bx^3}\,dx}{bc-ad} + \frac{c\int \frac{x}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{b}\,(bc-ad)} - \frac{a^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3\sqrt[3]{b}\,(bc-ad)} - \frac{c^{2/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3\sqrt[3]{d}\,(bc-ad)} +$$

$$= \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}}{6b^{2/3}(bc-ad)}$$

$$= \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x\right.}{6b^{2/3}(bc-ad)}$$

$$= \frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}(bc-ad)} - \frac{c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{3b^{2/3}(bc-ad)}$$

Mathematica [A]
time = 0.05, size = 224, normalized size = 0.78

$$\frac{\frac{2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} - \frac{2\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{2/3}} + \frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}} - \frac{2c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{d^{2/3}} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{d^{2/3}}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) -
(2*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(2/3) + (
2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (2*c^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/d^(2/3) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
b^(2/3) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(2/3))
/(6*b*c - 6*a*d)
```

Maple [A]
time = 0.36, size = 207, normalized size = 0.72

method	result

default	$\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{ad-bc}-\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{3}b^{2}d^{3}-3a^{2}b^{3}c\,d^{2}+3a\,b^{4}c^{2}d-b^{5}c^{3}\right)\_Z^{3}+a^{2}\right)}\_R\ln\left(\left(\left(-2a^{3}b^{2}c\,d^{4}+4a^{2}b^{3}c^{2}d^{3}-2a\,b^{4}c^{3}d^{2}\right)\_R^{3}-a^{2}cd-ab\,c^{2}\right)x+\left(-a\right.\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1)))*a/(a*d-b*c)-(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*
ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1
/2)*(2/(c/d)^(1/3)*x-1)))*c/(a*d-b*c)
```

Maxima [A]

time = 0.49, size = 289, normalized size = 1.00

$$-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^{2}c-abd\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd-ad^{2}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{a\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{c\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}+\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{c\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c
- a*b*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3)
)/(c/d)^(1/3))/((b*c*d - a*d^2)*(c/d)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) + 1/6*c*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3)) + 1/3*
a*log(x + (a/b)^(1/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) - 1/3*c*log(
x + (c/d)^(1/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3))
```

Fricas [A]

time = 1.88, size = 244, normalized size = 0.85

$$-\frac{2\sqrt{3}\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}+\sqrt{3}\,a}{3a}\right)-2\sqrt{3}\left(\frac{c^{2}}{d^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,dx\left(\frac{c^{2}}{d^{2}}\right)^{\frac{1}{3}}-\sqrt{3}\,c}{3c}\right)-\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax^{2}-bx\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}-a\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right)-\left(\frac{c^{2}}{d^{2}}\right)^{\frac{1}{3}}\log\left(cx^{2}-dx\left(\frac{c^{2}}{d^{2}}\right)^{\frac{2}{3}}+c\left(\frac{c^{2}}{d^{2}}\right)^{\frac{1}{3}}\right)+2\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax+b\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}\right)+2\left(\frac{c^{2}}{d^{2}}\right)^{\frac{1}{3}}\log\left(cx+d\left(\frac{c^{2}}{d^{2}}\right)^{\frac{2}{3}}\right)}{6\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3)
 + sqrt(3)*a)/a) - 2*sqrt(3)*(c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c^2
/d^2)^(1/3) - sqrt(3)*c)/c) - (-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(
2/3) - a*(-a^2/b^2)^(1/3)) - (c^2/d^2)^(1/3)*log(c*x^2 - d*x*(c^2/d^2)^(2/3
) + c*(c^2/d^2)^(1/3)) + 2*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3)) +
 2*(c^2/d^2)^(1/3)*log(c*x + d*(c^2/d^2)^(2/3)))/(b*c - a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.72, size = 286, normalized size = 0.99

$$\frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^{2}d\right)}-\frac{c\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^{2}-acd\right)}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{3}c-\sqrt{3}\,ab^{2}d}-\frac{\left(-cd^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^{2}-\sqrt{3}\,ad^{3}}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{3}c-ab^{2}d\right)}+\frac{\left(-cd^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^{2}-ad^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/d
)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(2/3)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*a*b
^2*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3)
)/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) + 1/6*(-c*d^2)^(2/3)*log(x^2 + x*(
-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3)
```

Mupad [**B**]
time = 9.05, size = 1364, normalized size = 4.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(a*x + b^3*c^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3) + a^2*b*d^2*(-a^2/(b^2*(
a*d - b*c)^3))^(2/3) - 2*a*b^2*c*d*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))*(a^2/(
```

```
27*b^5*c^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3) + l
og(c*x + a^2*d^3*(c^2/(d^2*(a*d - b*c)^3))^(2/3) + b^2*c^2*d*(c^2/(d^2*(a*d
 - b*c)^3))^(2/3) - 2*a*b*c*d^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3))*(c^2/(27*a
^3*d^5 - 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3) + (log(
((3^(1/2)*1i - 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(54
*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d
+ b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/4)*(-a^2/(b^2*(a*d -
 b*c)^3))^(1/3))/6 - 9*a^2*b^4*c^4*d^2 - 9*a^4*b^2*c^2*d^4 + 9*a*b^5*c^5*d
+ 9*a^5*b*c*d^5))/36 + a^2*b*c^2*d*x*(a*d + b*c))*(a^2/(27*b^5*c^3 - 27*a^3
*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)*(3^(1/2)*1i - 1))/2 -
(log(((3^(1/2)*1i + 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i + 1
)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^2*
(a*d + b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/4)*(-a^2/(b^2*(
a*d - b*c)^3))^(1/3))/6 + 9*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2*d^4 - 9*a*b^5*c
^5*d - 9*a^5*b*c*d^5))/36 - a^2*b*c^2*d*x*(a*d + b*c))*(a^2/(27*b^5*c^3 - 2
7*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)*(3^(1/2)*1i + 1))
/2 + (log(((3^(1/2)*1i - 1)^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i
 - 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1
)^2*(a*d + b*c)*(a*d - b*c)^4*(c^2/(d^2*(a*d - b*c)^3))^(2/3))/4)*(c^2/(d^2
*(a*d - b*c)^3))^(1/3))/6 - 9*a^2*b^4*c^4*d^2 - 9*a^4*b^2*c^2*d^4 + 9*a*b^5
*c^5*d + 9*a^5*b*c*d^5))/36 + a^2*b*c^2*d*x*(a*d + b*c))*(c^2/(27*a^3*d^5 -
 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3)*(3^(1/2)*1i - 1
))/2 - (log(((3^(1/2)*1i + 1)^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*
1i + 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i +
 1)^2*(a*d + b*c)*(a*d - b*c)^4*(c^2/(d^2*(a*d - b*c)^3))^(2/3))/4)*(c^2/(d
^2*(a*d - b*c)^3))^(1/3))/6 + 9*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2*d^4 - 9*a*b
^5*c^5*d - 9*a^5*b*c*d^5))/36 - a^2*b*c^2*d*x*(a*d + b*c))*(c^2/(27*a^3*d^5
 - 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3)*(3^(1/2)*1i +
 1))/2
```

3.112 $\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}+$$

```
[Out] -1/3*a^(1/3)*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/(-a*d+b*c)+1/3*c^(1/3)*ln(c^(1/3
)+d^(1/3)*x)/d^(1/3)/(-a*d+b*c)+1/6*a^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/b^(1/3)/(-a*d+b*c)-1/6*c^(1/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/d^(1/3)/(-a*d+b*c)+1/3*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/b^(1/3)/(-a*d+b*c)*3^(1/2)-1/3*c^(1/3)*arctan(1/3*(c^(1/3)-2
*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(1/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {492, 206, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)}-\frac{\sqrt[3]{c}\,\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)}-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3
)*(b*c - a*d)) - (c^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(1/3)*(b*c - a*d)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
1/3)*(b*c - a*d)) + (c^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(1/3)*(b*c - a*
d)) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(1/3)*(
b*c - a*d)) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(1/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\int \frac{1}{a+bx^3}\,dx}{bc-ad} + \frac{c\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= -\frac{\sqrt[3]{a}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3(bc-ad)} - \frac{\sqrt[3]{a}\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3(bc-ad)} + \frac{\sqrt[3]{c}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3(bc-ad)}$$

$$= -\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}\,(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3\sqrt[3]{d}\,(bc-ad)} - \frac{a^{2/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/}}}{2(bc-ad)}$$

$$= -\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}\,(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3\sqrt[3]{d}\,(bc-ad)} + \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}}{6\sqrt[3]{b}\,(bc-a}$$

$$= \frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}\,(bc-ad)} - \frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}\,(bc-ad)} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}}{3\sqrt[3]{b}\,(bc-ad)}$$

Mathematica [A]
time = 0.05, size = 224, normalized size = 0.78

$$\frac{\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - \frac{2\sqrt{3}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} + \frac{2\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt[3]{d}} + \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{\sqrt[3]{d}}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) -
(2*Sqrt[3]*c^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3) - (
2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (2*c^(1/3)*Log[c^(1/3) + d^(1
/3)*x])/d^(1/3) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
b^(1/3) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(1/3))
/(6*b*c - 6*a*d)
```

Maple [A]
time = 0.36, size = 207, normalized size = 0.72

method	result

default	$\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{ad-bc}-\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\cdots\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{3}b\,d^{3}-3c\,d^{2}a^{2}b^{2}+3c^{2}da\,b^{3}-b^{4}c^{3}\right)\_Z^{3}-a\right)}\_R\ln\left(\left(\left(a^{4}b\,d^{5}-4a^{3}b^{2}c\,d^{4}+6a^{2}b^{3}c^{2}d^{3}-4a\,b^{4}c^{3}d^{2}+b^{5}c^{4}d\right)\_R^{3}-a^{2}d^{2}-b^{2}\cdots\right.\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1)))*a/(a*d-b*c)-(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln
(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(c/d)^(1/3)*x-1)))*c/(a*d-b*c)
```

Maxima [A]
time = 0.52, size = 317, normalized size = 1.10

$$-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{a\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{c\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}-\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{c\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c*
(a/b)^(1/3) - a*b*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*sq
rt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1
/3))*(c/d)^(1/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c*(a/
b)^(2/3) - a*b*d*(a/b)^(2/3)) - 1/6*c*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)
)/(b*c*d*(c/d)^(2/3) - a*d^2*(c/d)^(2/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2
*c*(a/b)^(2/3) - a*b*d*(a/b)^(2/3)) + 1/3*c*log(x + (c/d)^(1/3))/(b*c*d*(c/
d)^(2/3) - a*d^2*(c/d)^(2/3))
```

Fricas [A]
time = 2.23, size = 199, normalized size = 0.69

$$-\frac{2\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3a}\right)+2\sqrt{3}\left(-\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,dx\left(-\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,c}{3c}\right)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)+2\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{6\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)
*a)/a) + 2*sqrt(3)*(-c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(-c/d)^(2/3) - sq
rt(3)*c)/c) - (a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - (-c/d)^(
1/3)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3)) + 2*(a/b)^(1/3)*log(x + (a/b)
^(1/3)) + 2*(-c/d)^(1/3)*log(x - (-c/d)^(1/3)))/(b*c - a*d)
```

Sympy [A]
time = 138.08, size = 342, normalized size = 1.19

$$\operatorname{RootSum}\left(t^{3}\cdot\left(27a^{3}d^{4}-81a^{2}bcd^{3}+81ab^{2}c^{2}d^{2}-27b^{3}c^{3}d\right)+c,\left(t\mapsto t\log\left(x+\frac{162t^{4}a^{4}bd^{5}-648t^{4}a^{3}b^{2}cd^{4}+972t^{4}a^{2}b^{3}c^{2}d^{3}-648t^{4}ab^{4}c^{3}d^{2}+162t^{4}b^{5}c^{4}d-3ta^{2}d^{2}+6tabcd-3tb^{2}c^{2}}{ad+bc}\right)\right)\right)+\operatorname{RootSum}\left(t^{3}\cdot\left(27a^{3}bd^{3}-81a^{2}b^{2}cd^{2}+81ab^{3}c^{2}d-27b^{4}c^{3}\right)-a,\left(t\mapsto t\log\left(x+\frac{162t^{4}a^{4}bd^{5}-648t^{4}a^{3}b^{2}cd^{4}+972t^{4}a^{2}b^{3}c^{2}d^{3}-648t^{4}ab^{4}c^{3}d^{2}+162t^{4}b^{5}c^{4}d-3ta^{2}d^{2}+6tabcd-3tb^{2}c^{2}}{ad+bc}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**3*d**4 - 81*a**2*b*c*d**3 + 81*a*b**2*c**2*d**2 - 27*b
**3*c**3*d) + c, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*d**5 - 648*_t**4*a
**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t**4*a*b**4*c**3*d**
2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d - 3*_t*b**2*c**2)
/(a*d + b*c)))) + RootSum(_t**3*(27*a**3*b*d**3 - 81*a**2*b**2*c*d**2 + 81*
a*b**3*c**2*d - 27*b**4*c**3) - a, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*
d**5 - 648*_t**4*a**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t*
*4*a*b**4*c**3*d**2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d
 - 3*_t*b**2*c**2)/(a*d + b*c))))
```

Giac [A]
time = 0.61, size = 278, normalized size = 0.97

$$\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^{2}d\right)}-\frac{c\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^{2}-acd\right)}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{2}c-\sqrt{3}\,abd}+\frac{\left(-cd^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd-\sqrt{3}\,ad^{2}}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{2}c-abd\right)}+\frac{\left(-cd^{2}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd-ad^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/d
)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(1/3)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*a*b
*d) + (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/
(sqrt(3)*b*c*d - sqrt(3)*a*d^2) - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/(b^2*c - a*b*d) + 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)^
(1/3) + (-c/d)^(2/3))/(b*c*d - a*d^2)
```

Mupad [B]
time = 8.12, size = 1265, normalized size = 4.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(x + a*d*(a/(b*(a*d - b*c)^3))^(1/3) - b*c*(a/(b*(a*d - b*c)^3))^(1/3))*
(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3)
+ log(x - a*d*(-c/(d*(a*d - b*c)^3))^(1/3) + b*c*(-c/(d*(a*d - b*c)^3))^(1/
3))*(-c/(27*a^3*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1
/3) + (log(((3^(1/2)*1i - 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)
^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d
+ b*c)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b*c)^3))^
(2/3))/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b
^3*c^2*d^4))/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i - 1)*(-a/
(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3))/2 -
 (log(((3^(1/2)*1i + 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(8
1*a*b^3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c
)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b*c)^3))^(2/3)
)/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^
2*d^4))/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i + 1)*(-a/(27*b
^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3))/2 + (log
(((3^(1/2)*1i - 1)*(-c/(d*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a*
b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a
*d - b*c)^4*(-c/(d*(a*d - b*c)^3))^(1/3))/2)*(-c/(d*(a*d - b*c)^3))^(2/3))/
36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*
d^4))/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i - 1)*(-c/(27*a^3
*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1/3))/2 - (log((
(3^(1/2)*1i + 1)*(-c/(d*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a*b^
3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-c/(d*(a*d - b*c)^3))^(1/3))/2)*(-c/(d*(a*d - b*c)^3))^(2/3))/36
 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*d^
4))/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i + 1)*(-c/(27*a^3*d
^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1/3))/2
```

3.113 $\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=45

$$\frac{\log(a+bx^3)}{3(bc-ad)}-\frac{\log(c+dx^3)}{3(bc-ad)}$$

```
[Out] 1/3*ln(b*x^3+a)/(-a*d+b*c)-1/3*ln(d*x^3+c)/(-a*d+b*c)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {455, 36, 31}

$$\frac{\log(a+bx^3)}{3(bc-ad)}-\frac{\log(c+dx^3)}{3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] Log[a + b*x^3]/(3*(b*c - a*d)) - Log[c + d*x^3]/(3*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)}\,dx, x, x^3\right)$$
$$= \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^3\right)}{3(bc-ad)} - \frac{d\text{Subst}\left(\int \frac{1}{c+dx}\,dx, x, x^3\right)}{3(bc-ad)}$$
$$= \frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.69

$$\frac{\log(a+bx^3)-\log(c+dx^3)}{3bc-3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (Log[a + b*x^3] - Log[c + d*x^3])/(3*b*c - 3*a*d)
```

Maple [A]
time = 0.32, size = 42, normalized size = 0.93

method	result	size
default	$-\frac{\ln(bx^3+a)}{3(ad-bc)}+\frac{\ln(dx^3+c)}{3ad-3bc}$	42
norman	$-\frac{\ln(bx^3+a)}{3(ad-bc)}+\frac{\ln(dx^3+c)}{3ad-3bc}$	42
risch	$\frac{\ln(dx^3+c)}{3ad-3bc}-\frac{\ln(-bx^3-a)}{3(ad-bc)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a*d-b*c)*ln(b*x^3+a)+1/3/(a*d-b*c)*ln(d*x^3+c)
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.91

$$\frac{\log(bx^3+a)}{3(bc-ad)} - \frac{\log(dx^3+c)}{3(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

[Out] 1/3*log(b*x^3 + a)/(b*c - a*d) - 1/3*log(d*x^3 + c)/(b*c - a*d)

Fricas [A]
time = 1.77, size = 31, normalized size = 0.69

$$\frac{\log(bx^3+a) - \log(dx^3+c)}{3(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(log(b*x^3 + a) - log(d*x^3 + c))/(b*c - a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(36) = 72$.
time = 1.54, size = 138, normalized size = 3.07

$$\frac{\log\left(x^3 + \frac{-\frac{a^2d^2}{ad-bc} + \frac{2abcd}{ad-bc} + ad - \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{3(ad-bc)} - \frac{\log\left(x^3 + \frac{\frac{a^2d^2}{ad-bc} - \frac{2abcd}{ad-bc} + ad + \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{3(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] log(x**3 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(3*(a*d - b*c)) - log(x**3 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(3*(a*d - b*c))
```

Giac [A]
time = 0.59, size = 51, normalized size = 1.13

$$\frac{b\log(|bx^3+a|)}{3(b^2c-abd)} - \frac{d\log(|dx^3+c|)}{3(bcd-ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b*log(abs(b*x^3 + a))/(b^2*c - a*b*d) - 1/3*d*log(abs(d*x^3 + c))/(b*c*
d - a*d^2)
```

Mupad [B]
time = 0.26, size = 602, normalized size = 13.38

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x^3\left(36cb^4d^3+36ab^3d^4\right)+\frac{x^3\left(54a^2b^3d^5+108ab^4cd^4+54b^5c^2d^3\right)+108ab^4c^2d^3+108a^2b^3cd^4}{3ad-3bc}+36ab^3cd^3}{3ad-3bc}+6b^3d^3x^3\right)1\mathrm{i}}{3ad-3bc}-\frac{\left(\frac{x^3\left(36cb^4d^3+36ab^3d^4\right)-\frac{x^3\left(54a^2b^3d^5+108ab^4cd^4+54b^5c^2d^3\right)+108ab^4c^2d^3+108a^2b^3cd^4}{3ad-3bc}+36ab^3cd^3}{3ad-3bc}-6b^3d^3x^3\right)1\mathrm{i}}{3ad-3bc}}{\frac{\frac{x^3\left(36cb^4d^3+36ab^3d^4\right)+\frac{x^3\left(54a^2b^3d^5+108ab^4cd^4+54b^5c^2d^3\right)+108ab^4c^2d^3+108a^2b^3cd^4}{3ad-3bc}+36ab^3cd^3}{3ad-3bc}+6b^3d^3x^3}{3ad-3bc}+\frac{\frac{x^3\left(36cb^4d^3+36ab^3d^4\right)-\frac{x^3\left(54a^2b^3d^5+108ab^4cd^4+54b^5c^2d^3\right)+108ab^4c^2d^3+108a^2b^3cd^4}{3ad-3bc}+36ab^3cd^3}{3ad-3bc}-6b^3d^3x^3}{3ad-3bc}}\right)2\mathrm{i}}{3ad-3bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] -(atan(((((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5 + 54*b^
5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a*
d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)*1i)/(3*a*d -
3*b*c) - (((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5 + 54*b
^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a
*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)*1i)/(3*a*d -
 3*b*c))/(((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5 + 54*b
^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a
*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)/(3*a*d - 3*b
*c) + ((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5 + 54*b^5*c
^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a*d -
 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)/(3*a*d - 3*b*c))
)*2i)/(3*a*d - 3*b*c)
```

3.114 $\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,(bc-ad)}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}\,(bc-ad)}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,(bc-ad)}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3\sqrt[3]{c}\,(bc-ad)}$$

```
[Out] -1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/(-a*d+b*c)+1/3*d^(1/3)*ln(c^(1/3
)+d^(1/3)*x)/c^(1/3)/(-a*d+b*c)+1/6*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/a^(1/3)/(-a*d+b*c)-1/6*d^(1/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/c^(1/3)/(-a*d+b*c)-1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(1/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(1/3)*arctan(1/3*(c^(1/3)-2
*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(1/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {493, 298, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,(bc-ad)}-\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,(bc-ad)}+\frac{\sqrt[3]{d}\,\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}\,(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6\sqrt[3]{c}\,(bc-ad)}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,(bc-ad)}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3\sqrt[3]{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1
/3)*(b*c - a*d))) + (d^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(1/3)*(b*c - a*d)) - (b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*(b*c - a*d)) + (d^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(1/3)*(b*c -
 a*d)) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3
)*(b*c - a*d)) - (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(1/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \frac{b\int \frac{x}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{x}{c+dx^3}\,dx}{bc-ad}$$

$$= -\frac{b^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,(bc-ad)} + \frac{b^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}\,(bc-ad)} + \frac{d^{2/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3\sqrt[3]{c}\,(bc-ad)}$$

$$= -\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,(bc-ad)} + \frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3\sqrt[3]{c}\,(bc-ad)} + \frac{\sqrt[3]{b}\,\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}}{6\sqrt[3]{a}\,(bc-ad)}$$

$$= -\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,(bc-ad)} + \frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3\sqrt[3]{c}\,(bc-ad)} + \frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\right.}{6\sqrt[3]{a}\,(bc-ad}$$

$$= -\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,(bc-ad)} + \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,\sqrt[3]{c}\,(bc-ad)} - \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3\sqrt[3]{a}\,(bc-ad)}$$

Mathematica [A]
time = 0.06, size = 224, normalized size = 0.78

$$\frac{\frac{2\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} - \frac{2\sqrt{3}\,\sqrt[3]{d}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{c}} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}} - \frac{2\sqrt[3]{d}\,\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt[3]{c}} - \frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{d}\,\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{\sqrt[3]{c}}}{-6bc+6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) -
(2*Sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(1/3) + (
2*b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) - (2*d^(1/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(1/3) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(1/3) + (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(1/3))
/(-6*b*c + 6*a*d)
```

Maple [A]
time = 0.35, size = 207, normalized size = 0.72

method	result

default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{ad-bc}+\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right.}{ad-}$
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{3}c\,d^{3}-3a^{2}b\,c^{2}d^{2}+3c^{3}da\,b^{2}-b^{3}c^{4}\right)\_Z^{3}+d\right)}\_R\ln\left(\left(\left(-a^{4}d^{4}+2a^{3}bc\,d^{3}-2a^{2}b^{2}c^{2}d^{2}+2a\,b^{3}c^{3}d-b^{4}c^{4}\right)\_R^{3}+bd\right)x+\left(-a^{6}\right.\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))*b/(a*d-b*c)+(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)
*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(
1/2)*(2/(c/d)^(1/3)*x-1)))*d/(a*d-b*c)
```

Maxima [A]

time = 0.48, size = 265, normalized size = 0.92

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc-ad\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc-ad\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c - a*d
)*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(
1/3))/((b*c - a*d)*(c/d)^(1/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(b*c*(a/b)^(1/3) - a*d*(a/b)^(1/3)) - 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)
^(2/3))/(b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3)) - 1/3*log(x + (a/b)^(1/3))/(b*c
*(a/b)^(1/3) - a*d*(a/b)^(1/3)) + 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(1/3)
 - a*d*(c/d)^(1/3))
```

Fricas [A]

time = 1.45, size = 201, normalized size = 0.70

$$\frac{2\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx^{2}-cx\left(-\frac{d}{c}\right)^{\frac{2}{3}}-c\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)-2\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)-2\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx+c\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3))
- 2*sqrt(3)*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/c)^(1/3) + 1/3*sqrt(3)) +
 (b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) + (-d/c)^(1/3)*lo
g(d*x^2 - c*x*(-d/c)^(2/3) - c*(-d/c)^(1/3)) - 2*(b/a)^(1/3)*log(b*x + a*(b
/a)^(2/3)) - 2*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)))/(b*c - a*d)
```

Sympy [A]
time = 73.69, size = 515, normalized size = 1.79

$$\operatorname{RootSum}\left(t^{3}\cdot\left(27a^{4}d^{3}-81a^{3}bcd^{2}+81a^{2}b^{2}c^{2}d-27ab^{3}c^{3}\right)-b,\left(t\mapsto t\log\left(x+\frac{243t^{5}a^{7}cd^{6}-1458t^{5}a^{6}bc^{2}d^{5}+3645t^{5}a^{5}b^{2}c^{3}d^{4}-4860t^{5}a^{4}b^{3}c^{4}d^{3}+3645t^{5}a^{3}b^{4}c^{5}d^{2}-1458t^{5}a^{2}b^{5}c^{6}d+243t^{5}ab^{6}c^{7}+9t^{2}a^{4}d^{4}-18t^{2}a^{3}bcd^{3}+18t^{2}a^{2}b^{2}c^{2}d^{2}-18t^{2}ab^{3}c^{3}d+9t^{2}b^{4}c^{4}}{abd^{2}+b^{2}cd}\right)\right)\right)+\operatorname{RootSum}\left(t^{3}\cdot\left(27a^{3}cd^{3}-81a^{2}bc^{2}d^{2}+81ab^{2}c^{3}d-27b^{3}c^{4}\right)+d,\left(t\mapsto t\log\left(x+\frac{243t^{5}a^{7}cd^{6}-1458t^{5}a^{6}bc^{2}d^{5}+3645t^{5}a^{5}b^{2}c^{3}d^{4}-4860t^{5}a^{4}b^{3}c^{4}d^{3}+3645t^{5}a^{3}b^{4}c^{5}d^{2}-1458t^{5}a^{2}b^{5}c^{6}d+243t^{5}ab^{6}c^{7}+9t^{2}a^{4}d^{4}-18t^{2}a^{3}bcd^{3}+18t^{2}a^{2}b^{2}c^{2}d^{2}-18t^{2}ab^{3}c^{3}d+9t^{2}b^{4}c^{4}}{abd^{2}+b^{2}cd}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**4*d**3 - 81*a**3*b*c*d**2 + 81*a**2*b**2*c**2*d - 27*a
*b**3*c**3) - b, Lambda(_t, _t*log(x + (243*_t**5*a**7*c*d**6 - 1458*_t**5*
a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860*_t**5*a**4*b**3*c*
*4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a**2*b**5*c**6*d + 24
3*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*b*c*d**3 + 18*_t**2
*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*b**4*c**4)/(a*b*d**
2 + b**2*c*d)))) + RootSum(_t**3*(27*a**3*c*d**3 - 81*a**2*b*c**2*d**2 + 81
*a*b**2*c**3*d - 27*b**3*c**4) + d, Lambda(_t, _t*log(x + (243*_t**5*a**7*c
*d**6 - 1458*_t**5*a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860
*_t**5*a**4*b**3*c**4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a*
*2*b**5*c**6*d + 243*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*
b*c*d**3 + 18*_t**2*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*
b**4*c**4)/(a*b*d**2 + b**2*c*d))))
```

Giac [A]
time = 0.61, size = 290, normalized size = 1.01

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^{2}d\right)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^{2}-acd\right)}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^{2}c-\sqrt{3}a^{2}bd}+\frac{\left(-cd^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^{2}d-\sqrt{3}acd^{2}}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ab^{2}c-a^{2}bd\right)}-\frac{\left(-cd^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^{2}d-acd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(2/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b^2*c - sqrt(3)*
a^2*b*d) + (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1
/3))/(sqrt(3)*b*c^2*d - sqrt(3)*a*c*d^2) + 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c - a^2*b*d) - 1/6*(-c*d^2)^(2/3)*log(x^
2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^2*d - a*c*d^2)
```

<u>Mupad</u> [B]
time = 5.42, size = 982, normalized size = 3.41

$$\ln\left(b\,x+a^3\,d^2\,{\left(\frac{b}{a\,{\left(a\,d-b\,c\right)}^3}\right)}^{2/3}+a\,b^2\,c^2\,{\left(\frac{b}{a\,{\left(a\,d-b\,c\right)}^3}\right)}^{2/3}-2\,a^2\,b\,c\,d\,{\left(\frac{b}{a\,{\left(a\,d-b\,c\right)}^3}\right)}^{2/3}\right)\,{\left(\frac{b}{27\,a^4\,d^3-81\,a^3\,b\,c\,d^2+81\,a^2\,b^2\,c^2\,d-27\,a\,b^3\,c^3}\right)}^{1/3}+\ln\left(d\,x+b^2\,c^3\,{\left(-\frac{d}{c\,{\left(a\,d-b\,c\right)}^3}\right)}^{2/3}+a^2\,c\,d^2\,{\left(-\frac{d}{c\,{\left(a\,d-b\,c\right)}^3}\right)}^{2/3}-2\,a\,b\,c^2\,d\,{\left(-\frac{d}{c\,{\left(a\,d-b\,c\right)}^3}\right)}^{2/3}\right)\,{\left(\frac{d}{-27\,a^3\,c\,d^3+81\,a^2\,b\,c^2\,d^2-81\,a\,b^2\,c^3\,d+27\,b^3\,c^4}\right)}^{1/3}+\frac{\ln\left(b^4\,d^4\,x-\frac{b\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^3\,\left(27\,b^3\,d^3\,x\,\left(a^2\,d^2+b^2\,c^2\right)\,{\left(a\,d-b\,c\right)}^2+\frac{27\,a\,b^3\,c\,d^3\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(a\,d+b\,c\right)\,{\left(a\,d-b\,c\right)}^4\,{\left(\frac{b}{a\,{\left(a\,d-b\,c\right)}^3}\right)}^{2/3}}{4}\right)}{216\,a\,{\left(a\,d-b\,c\right)}^3}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{b}{27\,a^4\,d^3-81\,a^3\,b\,c\,d^2+81\,a^2\,b^2\,c^2\,d-27\,a\,b^3\,c^3}\right)}^{1/3}}{2}-\frac{\ln\left(b^4\,d^4\,x+\frac{b\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^3\,\left(27\,b^3\,d^3\,x\,\left(a^2\,d^2+b^2\,c^2\right)\,{\left(a\,d-b\,c\right)}^2+\frac{27\,a\,b^3\,c\,d^3\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(a\,d+b\,c\right)\,{\left(a\,d-b\,c\right)}^4\,{\left(\frac{b}{a\,{\left(a\,d-b\,c\right)}^3}\right)}^{2/3}}{4}\right)}{216\,a\,{\left(a\,d-b\,c\right)}^3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{b}{27\,a^4\,d^3-81\,a^3\,b\,c\,d^2+81\,a^2\,b^2\,c^2\,d-27\,a\,b^3\,c^3}\right)}^{1/3}}{2}+\frac{\ln\left(b^4\,d^4\,x+\frac{d\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^3\,\left(27\,b^3\,d^3\,x\,\left(a^2\,d^2+b^2\,c^2\right)\,{\left(a\,d-b\,c\right)}^2+\frac{27\,a\,b^3\,c\,d^3\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(a\,d+b\,c\right)\,{\left(a\,d-b\,c\right)}^4\,{\left(-\frac{d}{c\,{\left(a\,d-b\,c\right)}^3}\right)}^{2/3}}{4}\right)}{216\,c\,{\left(a\,d-b\,c\right)}^3}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{d}{-27\,a^3\,c\,d^3+81\,a^2\,b\,c^2\,d^2-81\,a\,b^2\,c^3\,d+27\,b^3\,c^4}\right)}^{1/3}}{2}-\frac{\ln\left(b^4\,d^4\,x-\frac{d\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^3\,\left(27\,b^3\,d^3\,x\,\left(a^2\,d^2+b^2\,c^2\right)\,{\left(a\,d-b\,c\right)}^2+\frac{27\,a\,b^3\,c\,d^3\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(a\,d+b\,c\right)\,{\left(a\,d-b\,c\right)}^4\,{\left(-\frac{d}{c\,{\left(a\,d-b\,c\right)}^3}\right)}^{2/3}}{4}\right)}{216\,c\,{\left(a\,d-b\,c\right)}^3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{d}{-27\,a^3\,c\,d^3+81\,a^2\,b\,c^2\,d^2-81\,a\,b^2\,c^3\,d+27\,b^3\,c^4}\right)}^{1/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(b*x + a^3*d^2*(b/(a*(a*d - b*c)^3))^(2/3) + a*b^2*c^2*(b/(a*(a*d - b*c)
^3))^(2/3) - 2*a^2*b*c*d*(b/(a*(a*d - b*c)^3))^(2/3))*(b/(27*a^4*d^3 - 27*a
*b^3*c^3 + 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3) + log(d*x + b^2*c^3*(-
d/(c*(a*d - b*c)^3))^(2/3) + a^2*c*d^2*(-d/(c*(a*d - b*c)^3))^(2/3) - 2*a*b
*c^2*d*(-d/(c*(a*d - b*c)^3))^(2/3))*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 81*a^2
*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3) + (log(b^4*d^4*x - (b*(3^(1/2)*1i - 1)^
3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2
)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(2/3))/4))/(216
*a*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3 + 81*a^2*
b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 - (log(b^4*d^4*x + (b*(3^(1/2)*1i + 1
)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1
/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(2/3))/4))/(2
16*a*(a*d - b*c)^3))*(3^(1/2)*1i + 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3 + 81*a^
2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 + (log(b^4*d^4*x + (d*(3^(1/2)*1i -
 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^
(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c)^3))^(2/3))/4))
/(216*c*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 81
*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3))/2 - (log(b^4*d^4*x - (d*(3^(1/2)*1
i + 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*
(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c)^3))^(2/3))/
4))/(216*c*(a*d - b*c)^3))*(3^(1/2)*1i + 1)*(d/(27*b^3*c^4 - 27*a^3*c*d^3 +
 81*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3))/2
```

3.115 $\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=288

$$-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)}+\frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)}-$$

```
[Out] 1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(-a*d+b*c)-1/3*d^(2/3)*ln(c^(1/3)
+d^(1/3)*x)/c^(2/3)/(-a*d+b*c)-1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/a^(2/3)/(-a*d+b*c)+1/6*d^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/c^(2/3)/(-a*d+b*c)-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(2/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(c^(1/3)-2*
d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {400, 206, 31, 648, 631, 210, 642}

$$-\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)}+\frac{d^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \frac{b\int \frac{1}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}(bc-ad)} + \frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)} - \frac{d\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{2/3}(bc-ad)} - \frac{d\int \frac{}{c^{2/3}-}}{3}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}}{6a^{2/3}(bc-ad)}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x\right.}{6a^{2/3}(bc-ad)}$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)} + \frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3a^{2/3}(bc-ad)}$$

Mathematica [A]
time = 0.06, size = 224, normalized size = 0.78

$$\frac{\frac{2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{2\sqrt{3}\,d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{2/3}} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}} - \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{2/3}}}{-6bc+6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
(2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3))
/(-6*b*c + 6*a*d)
```

Maple [A]
time = 0.29, size = 207, normalized size = 0.72

method	result

default	$-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{ad-bc}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\cdots\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{5}d^{3}-3a^{4}bc\,d^{2}+3a^{3}b^{2}c^{2}d-b^{3}a^{2}c^{3}\right)\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(\left(-a^{5}d^{5}+3a^{4}bc\,d^{4}-2a^{3}b^{2}c^{2}d^{3}-2a^{2}b^{3}c^{3}d^{2}+3a\,b^{4}c^{4}d-b^{5}c^{5}\right)\_R^{3}\cdots\right.\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b/(a*d-b*c)+(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))*d/(a*d-b*c)

Maxima [A]

time = 0.49, size = 293, normalized size = 1.02

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b)^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)^(1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*d*(a/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a/b)^(2/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))

Fricas [A]

time = 2.06, size = 254, normalized size = 0.88

$$-\frac{2\sqrt{3}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3b}\right)+2\sqrt{3}\left(\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,cx\left(\frac{d^{2}}{c^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3d}\right)-\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}+abx\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)-\left(\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\log\left(d^{2}x^{2}-cdx\left(\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}+c^{2}\left(\frac{d^{2}}{c^{2}}\right)^{\frac{2}{3}}\right)+2\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)+2\left(\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\right)}{6\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3)
 - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2
/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^
2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2
/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a
^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.70, size = 278, normalized size = 0.97

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,abc-\sqrt{3}\,a^2d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^2-\sqrt{3}\,acd}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc-a^2d\right)}-\frac{\left(-cd^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2-acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)*a^
2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))
/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)
^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

Mupad [**B**]
time = 9.01, size = 1364, normalized size = 4.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a
*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d + b*
```

```
c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b^2/
(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3) +
log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a*
b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d + b*c)
*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-d^2/(27
*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3) + (lo
g(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/
2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(
1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)
*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*
a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81
*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x - ((3^(1/2)*1i
+ 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*b^3*d^3*x*(a
*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(
2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^
5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)
*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(d^2/(c^2*(a*d - b*c)^3)
)^(1/3)*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*
a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c
)^3))^(1/3))/2)*(d^2/(c^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6
*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*
b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(d^2/(c^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81
*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(1/3))/2)*(d^2/(c^2*(a*d -
 b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(
-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3)*(3^(1/2)*1i + 1))/2
```

3.116 $\int \frac{1}{x(a+bx^3)(c+dx^3)} \, dx$

Optimal. Leaf size=62

$$\frac{\log(x)}{ac} - \frac{b\log\left(a+bx^3\right)}{3a(bc-ad)} + \frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}$$

```
[Out] ln(x)/a/c-1/3*b*ln(b*x^3+a)/a/(-a*d+b*c)+1/3*d*ln(d*x^3+c)/c/(-a*d+b*c)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 84}

$$-\frac{b\log\left(a+bx^3\right)}{3a(bc-ad)} + \frac{d\log\left(c+dx^3\right)}{3c(bc-ad)} + \frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^3])/(3*a*(b*c - a*d)) + (d*Log[c + d*x^3])/(3
*c*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)} \, dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx} + \frac{b^2}{a(-bc+ad)(a+bx)} + \frac{d^2}{c(bc-ad)(c+dx)}\right) dx, x, x^3\right)\\
&= \frac{\log(x)}{ac} - \frac{b\log\left(a+bx^3\right)}{3a(bc-ad)} + \frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.87

$$\frac{3bc\log(x) - 3ad\log(x) - bc\log(a + bx^3) + ad\log(c + dx^3)}{3abc^2 - 3a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (3*b*c*Log[x] - 3*a*d*Log[x] - b*c*Log[a + b*x^3] + a*d*Log[c + d*x^3])/(3*
a*b*c^2 - 3*a^2*c*d)
```

Maple [A]
time = 0.36, size = 59, normalized size = 0.95

method	result	size
default	$\frac{b\ln(b\,x^3+a)}{3a(ad-bc)} - \frac{d\ln(d\,x^3+c)}{3c(ad-bc)} + \frac{\ln(x)}{ac}$	59
norman	$\frac{b\ln(b\,x^3+a)}{3a(ad-bc)} - \frac{d\ln(d\,x^3+c)}{3c(ad-bc)} + \frac{\ln(x)}{ac}$	59
risch	$\frac{\ln(x)}{ac} + \frac{b\ln(-b\,x^3-a)}{3a(ad-bc)} - \frac{d\ln(-d\,x^3-c)}{3c(ad-bc)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b/a/(a*d-b*c)*ln(b*x^3+a)-1/3*d/c/(a*d-b*c)*ln(d*x^3+c)+ln(x)/a/c
```

Maxima [A]
time = 0.28, size = 61, normalized size = 0.98

$$-\frac{b\log(bx^3 + a)}{3(abc - a^2d)} + \frac{d\log(dx^3 + c)}{3(bc^2 - acd)} + \frac{\log(x^3)}{3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*b*log(b*x^3 + a)/(a*b*c - a^2*d) + 1/3*d*log(d*x^3 + c)/(b*c^2 - a*c*d
) + 1/3*log(x^3)/(a*c)
```

Fricas [A]
time = 2.72, size = 54, normalized size = 0.87

$$-\frac{bc\log(bx^3 + a) - ad\log(dx^3 + c) - 3(bc - ad)\log(x)}{3(abc^2 - a^2cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/3*(b*c*log(b*x^3 + a) - a*d*log(d*x^3 + c) - 3*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.83, size = 71, normalized size = 1.15

$$-\frac{b^2 \log\left(|bx^3+a|\right)}{3\left(ab^2c-a^2bd\right)}+\frac{d^2 \log\left(|dx^3+c|\right)}{3\left(bc^2d-acd^2\right)}+\frac{\log\left(|x|\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^2*log(abs(b*x^3 + a))/(a*b^2*c - a^2*b*d) + 1/3*d^2*log(abs(d*x^3 +
c))/(b*c^2*d - a*c*d^2) + log(abs(x))/(a*c)
```

Mupad [**B**]
time = 2.84, size = 58, normalized size = 0.94

$$\frac{b\,\ln\left(b\,x^3+a\right)}{3\,a^2\,d-3\,a\,b\,c}+\frac{d\,\ln\left(d\,x^3+c\right)}{3\,b\,c^2-3\,a\,c\,d}+\frac{\ln\left(x\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (b*log(a + b*x^3))/(3*a^2*d - 3*a*b*c) + (d*log(c + d*x^3))/(3*b*c^2 - 3*a*
c*d) + log(x)/(a*c)
```

3.117 $\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=299

$$-\frac{1}{acx}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}(bc-ad)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{3c^{4/3}(bc-ad)}$$

```
[Out] -1/a/c/x+1/3*b^(4/3)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/(-a*d+b*c)-1/3*d^(4/3)*l
n(c^(1/3)+d^(1/3)*x)/c^(4/3)/(-a*d+b*c)-1/6*b^(4/3)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(4/3)/(-a*d+b*c)+1/6*d^(4/3)*ln(c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/c^(4/3)/(-a*d+b*c)+1/3*b^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(4/3)*arctan(1/3*(c
^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(4/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {491, 598, 298, 31, 648, 631, 210, 642}

$$\frac{b^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}(bc-ad)}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)}+\frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{4/3}(bc-ad)}-\frac{1}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -(1/(a*c*x)) + (b^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(4/3)*(b*c - a*d)) - (d^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sq
rt[3]*c^(1/3))])/(Sqrt[3]*c^(4/3)*(b*c - a*d)) + (b^(4/3)*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(4/3)*(b*c - a*d)) - (d^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^
(4/3)*(b*c - a*d)) - (b^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(6*a^(4/3)*(b*c - a*d)) + (d^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2])/(6*c^(4/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= -\frac{1}{acx}+\frac{\int \frac{x\left(-bc-ad-bdx^3\right)}{(a+bx^3)(c+dx^3)}\,dx}{ac} \\
&= -\frac{1}{acx}+\frac{\int \left(-\frac{b^2cx}{(bc-ad)(a+bx^3)}-\frac{ad^2x}{(-bc+ad)(c+dx^3)}\right)dx}{ac} \\
&= -\frac{1}{acx}-\frac{b^2\int \frac{x}{a+bx^3}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{x}{c+dx^3}\,dx}{c(bc-ad)} \\
&= -\frac{1}{acx}+\frac{b^{5/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{4/3}(bc-ad)}-\frac{b^{5/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{4/3}(bc-ad)}-\frac{d^{5/3}\int \frac{1}{\sqrt[3]{c}+\cdots}}{3c^{4/3}(bc-\cdots} \\
&= -\frac{1}{acx}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{4/3}(bc-ad)}-\frac{b^{4/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{\cdots}}{a^{2/3}-\sqrt[3]{a}\cdots}}{6a^{4/3}(bc\cdots} \\
&= -\frac{1}{acx}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{4/3}(bc-ad)}-\frac{b^{4/3}\log\left(a^{2/3}-\cdots\right.}{6a^{4/3}\cdots} \\
&= -\frac{1}{acx}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}(bc-ad)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{\cdots}\right.}{3a^{4/3}(b\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 244, normalized size = 0.82

$$
\frac{\frac{6b}{a}-\frac{6d}{c}-\frac{2\sqrt{3}\,b^{4/3}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}+\frac{2\sqrt{3}\,d^{4/3}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{4/3}}-\frac{2b^{4/3}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{4/3}}+\frac{2d^{4/3}x\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{4/3}}+\frac{b^{4/3}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{4/3}}-\frac{d^{4/3}x\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{4/3}}}{-6bcx+6adx}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/a - (6*d)/c - (2*Sqrt[3]*b^(4/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/a^(4/3) + (2*Sqrt[3]*d^(4/3)*x*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3
))/Sqrt[3]])/c^(4/3) - (2*b^(4/3)*x*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + (2*
d^(4/3)*x*Log[c^(1/3) + d^(1/3)*x])/c^(4/3) + (b^(4/3)*x*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) - (d^(4/3)*x*Log[c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2])/c^(4/3))/(-6*b*c*x + 6*a*d*x)
```

Maple [A]
time = 0.37, size = 228, normalized size = 0.76

method	result
default	$\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2}{a(ad-bc)}-\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right.}{c(ad}$
risch	$-\frac{1}{acx}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^3a^7-3c\,d^2a^6b+3c^2d\,a^5b^2-a^4b^3c^3\right)\_Z^3+b^4\right)}\_R\ln\left(\left(\left(-4a^{10}c^4d^6+22a^9b\,c^5d^5-52a^8b^2c^6d^4+68a^7b^3c^7d^3-\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1)))*b^2/a/(a*d-b*c)-(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1
/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*
3^(1/2)*(2/(c/d)^(1/3)*x-1)))*d^2/c/(a*d-b*c)-1/a/c/x
```

Maxima [A]

time = 0.50, size = 300, normalized size = 1.00

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(abc-a^2d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^2-acd\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{b\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{d\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}+\frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{d\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c
- a^2*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3)
)/(c/d)^(1/3))/((b*c^2 - a*c*d)*(c/d)^(1/3)) - 1/6*b*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) + 1/6*d*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) + 1/3*
b*log(x + (a/b)^(1/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) - 1/3*d*log(
x + (c/d)^(1/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) - 1/(a*c*x)
```

Fricas [A]

time = 2.10, size = 238, normalized size = 0.80

$$-\frac{2\sqrt{3}\,bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}\,adx\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)-adx\left(\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx^2-cx\left(\frac{d}{c}\right)^{\frac{2}{3}}+c\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)+2\,bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)+2\,adx\left(\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)+6\,bc-6\,ad}{6\left(abc^2-a^2cd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*b*c*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*
sqrt(3)) - 2*sqrt(3)*a*d*x*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(d/c)^(1/3) - 1
/3*sqrt(3)) - b*c*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1
/3)) - a*d*x*(d/c)^(1/3)*log(d*x^2 - c*x*(d/c)^(2/3) + c*(d/c)^(1/3)) + 2*b
*c*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) + 2*a*d*x*(d/c)^(1/3)*log(d*x +
 c*(d/c)^(2/3)) + 6*b*c - 6*a*d)/((a*b*c^2 - a^2*c*d)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.21, size = 305, normalized size = 1.02

$$\frac{b^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^2bc-a^3d\right)}-\frac{d^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2bc-\sqrt{3}\,a^3d}-\frac{\left(-cd^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^3-\sqrt{3}\,ac^2d}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc-a^3d\right)}+\frac{\left(-cd^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3-ac^2d\right)}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^2
*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) + (-a*b^2)^(2/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c - s
qrt(3)*a^3*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/
d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(2/3)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(2/3)*log(x
^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/(a*c*x)
```

Mupad [**B**]
time = 3.85, size = 716, normalized size = 2.39

$$\ln\left(b-a^2dx\left(-\frac{b^4}{a^4(ad-bc)^3}\right)^{1/3}+abcx\left(-\frac{b^4}{a^4(ad-bc)^3}\right)^{1/3}\right)\left(-\frac{b^4}{27a^7d^3-81a^6bcd^2+81a^5b^2c^2d-27a^4b^3c^3}\right)^{1/3}+\ln\left(d-bc^2x\left(\frac{d^4}{c^4(ad-bc)^3}\right)^{1/3}+acdx\left(\frac{d^4}{c^4(ad-bc)^3}\right)^{1/3}\right)\left(-\frac{d^4}{-27a^3c^4d^3+81a^2bc^5d^2-81ab^2c^6d+27b^3c^7}\right)^{1/3}-\frac{1}{acx}-\frac{\ln\left(b+2a^2dx\left(-\frac{b^4}{a^4(ad-bc)^3}\right)^{1/3}-2abcx\left(-\frac{b^4}{a^4(ad-bc)^3}\right)^{1/3}-\sqrt{3}\,b\,1{}\mathrm{i}\right)\left(-\frac{b^4}{27a^7d^3-81a^6bcd^2+81a^5b^2c^2d-27a^4b^3c^3}\right)^{1/3}\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+\frac{\ln\left(b+2a^2dx\left(-\frac{b^4}{a^4(ad-bc)^3}\right)^{1/3}-2abcx\left(-\frac{b^4}{a^4(ad-bc)^3}\right)^{1/3}+\sqrt{3}\,b\,1{}\mathrm{i}\right)\left(-\frac{b^4}{27a^7d^3-81a^6bcd^2+81a^5b^2c^2d-27a^4b^3c^3}\right)^{1/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-\frac{\ln\left(d+2bc^2x\left(\frac{d^4}{c^4(ad-bc)^3}\right)^{1/3}-2acdx\left(\frac{d^4}{c^4(ad-bc)^3}\right)^{1/3}-\sqrt{3}\,d\,1{}\mathrm{i}\right)\left(-\frac{d^4}{-27a^3c^4d^3+81a^2bc^5d^2-81ab^2c^6d+27b^3c^7}\right)^{1/3}\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+\frac{\ln\left(d+2bc^2x\left(\frac{d^4}{c^4(ad-bc)^3}\right)^{1/3}-2acdx\left(\frac{d^4}{c^4(ad-bc)^3}\right)^{1/3}+\sqrt{3}\,d\,1{}\mathrm{i}\right)\left(-\frac{d^4}{-27a^3c^4d^3+81a^2bc^5d^2-81ab^2c^6d+27b^3c^7}\right)^{1/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(b - a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) + a*b*c*x*(-b^4/(a^4*(a*d
- b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d - 8
```

```
1*a^6*b*c*d^2))^(1/3) + log(d - b*c^2*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3) + a
*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d^4/(27*b^3*c^7 - 27*a^3*c^4*d^3
+ 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3) - 1/(a*c*x) - (log(b - 3^(1/2)*
b*1i + 2*a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(a^4*(a
*d - b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d
- 81*a^6*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(b + 3^(1/2)*b*1i + 2*a^
2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(a^4*(a*d - b*c)^3
))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d - 81*a^6*b*
c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(d - 3^(1/2)*d*1i + 2*b*c^2*x*(d^4/
(c^4*(a*d - b*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d
^4/(27*b^3*c^7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3)
*(3^(1/2)*1i + 1))/2 + (log(d + 3^(1/2)*d*1i + 2*b*c^2*x*(d^4/(c^4*(a*d - b
*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d^4/(27*b^3*c^
7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3)*(3^(1/2)*1i
- 1))/2
```

3.118 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=301

$$-\frac{1}{2acx^2}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}(bc-ad)}-\frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}(bc-ad)}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{}\right.}{3c^{5/3}(bc-ad}$$

```
[Out] -1/2/a/c/x^2-1/3*b^(5/3)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/(-a*d+b*c)+1/3*d^(5/
3)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)+1/6*b^(5/3)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/(-a*d+b*c)-1/6*d^(5/3)*ln(c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/c^(5/3)/(-a*d+b*c)+1/3*b^(5/3)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(5/3)*arctan(1/
3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {491, 536, 206, 31, 648, 631, 210, 642}

$$\frac{b^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}(bc-ad)}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}(bc-ad)}-\frac{d^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{5/3}(bc-ad)}-\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{5/3}(bc-ad)}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{5/3}(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/2*1/(a*c*x^2) + (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(5/3)*(b*c - a*d)) - (d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)
/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(5/3)*(b*c - a*d)) - (b^(5/3)*Log[a^(1/3) +
 b^(1/3)*x])/(3*a^(5/3)*(b*c - a*d)) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(
3*c^(5/3)*(b*c - a*d)) + (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(6*a^(5/3)*(b*c - a*d)) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2])/(6*c^(5/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx &= -\frac{1}{2acx^2} + \frac{\int \frac{-2(bc+ad)-2bdx^3}{(a+bx^3)(c+dx^3)}\,dx}{2ac} \\
&= -\frac{1}{2acx^2} - \frac{b^2\int \frac{1}{a+bx^3}\,dx}{a(bc-ad)} + \frac{d^2\int \frac{1}{c+dx^3}\,dx}{c(bc-ad)} \\
&= -\frac{1}{2acx^2} - \frac{b^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{5/3}(bc-ad)} - \frac{b^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{5/3}(bc-ad)} + \frac{d^2\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,\ldots}}{3c^{5/3}(bc-a\ldots} \\
&= -\frac{1}{2acx^2} - \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}(bc-ad)} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{5/3}(bc-ad)} + \frac{b^{5/3}\int \frac{-\sqrt[3]{a}\ldots}{a^{2/3}-\sqrt[3]{a}\ldots}}{6a^{5/3}(\ldots} \\
&= -\frac{1}{2acx^2} - \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}(bc-ad)} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{5/3}(bc-ad)} + \frac{b^{5/3}\log\left(a^{2/3}\ldots\right.}{6a^{5\ldots}} \\
&= -\frac{1}{2acx^2} + \frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}(bc-ad)} - \frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{5/3}(bc-ad)} - \frac{b^{5/3}\log\left(\ldots\right.}{3a^{5/3}\ldots}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 259, normalized size = 0.86

$$
\frac{\frac{3b}{a}-\frac{3d}{c}-\frac{2\sqrt{3}\,b^{5/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{2\sqrt{3}\,d^{5/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}}+\frac{2b^{5/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}-\frac{2d^{5/3}x^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{5/3}}-\frac{b^{5/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{d^{5/3}x^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{5/3}}}{6(-bc+ad)x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((3*b)/a - (3*d)/c - (2*Sqrt[3]*b^(5/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(5/3) + (2*Sqrt[3]*d^(5/3)*x^2*ArcTan[(1 - (2*d^(1/3)*x)/c^
(1/3))/Sqrt[3]])/c^(5/3) + (2*b^(5/3)*x^2*Log[a^(1/3) + b^(1/3)*x])/a^(5/3)
 - (2*d^(5/3)*x^2*Log[c^(1/3) + d^(1/3)*x])/c^(5/3) - (b^(5/3)*x^2*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + (d^(5/3)*x^2*Log[c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(5/3))/(6*(-(b*c) + a*d)*x^2)
```

Maple [A]
time = 0.37, size = 228, normalized size = 0.76

method	result

default	$\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b^{2}}{a(ad-bc)}-\frac{1}{2c\,x^{2}a}-\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\right.}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right.}{}$
risch	$-\frac{1}{2c\,x^{2}a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(c^{5}d^{3}a^{3}-3a^{2}b\,c^{6}d^{2}+3a\,b^{2}c^{7}d-b^{3}c^{8}\right)\_Z^{3}+d^{5}\right)}\_R\ln\left(\left(\left(-4a^{11}c^{5}d^{6}+22a^{10}b\,c^{6}d^{5}-52a^{9}b^{2}c^{7}d^{4}+68a^{8}b^{3}c^{8}\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1)))/a*b^2/(a*d-b*c)-1/2/c/x^2/a-(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/
d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))/c*d^2/(a*d-b*c)
```

Maxima [A]
time = 0.49, size = 328, normalized size = 1.09

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{b\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{d\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}-acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}-\frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{d\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}-acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}-\frac{1}{2\,acx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c*
(a/b)^(1/3) - a^2*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sq
rt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1
/3))*(c/d)^(1/3)) + 1/6*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*c*(a/
b)^(2/3) - a^2*d*(a/b)^(2/3)) - 1/6*d*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)
)/(b*c^2*(c/d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/3*b*log(x + (a/b)^(1/3))/(a*b
*c*(a/b)^(2/3) - a^2*d*(a/b)^(2/3)) + 1/3*d*log(x + (c/d)^(1/3))/(b*c^2*(c/
d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/2/(a*c*x^2)
```

Fricas [A]
time = 3.83, size = 301, normalized size = 1.00

$$-\frac{2\sqrt{3}\,bcx^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3b}\right)+2\sqrt{3}\,adx^{2}\left(-\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,cx\left(-\frac{d^{2}}{c^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3d}\right)-bcx^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}-abx\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)-adx^{2}\left(-\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\log\left(d^{2}x^{2}+cdx\left(-\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}+c^{2}\left(-\frac{d^{2}}{c^{2}}\right)^{\frac{2}{3}}\right)+2\,bcx^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)+2\,adx^{2}\left(-\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\log\left(dx-c\left(-\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\right)+3\,bc-3\,ad}{6\left(abc^{2}-a^{2}cd\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*b*c*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)
^(2/3) - sqrt(3)*b)/b) + 2*sqrt(3)*a*d*x^2*(-d^2/c^2)^(1/3)*arctan(1/3*(2*s
qrt(3)*c*x*(-d^2/c^2)^(2/3) - sqrt(3)*d)/d) - b*c*x^2*(b^2/a^2)^(1/3)*log(b
^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) - a*d*x^2*(-d^2/c^2)^
(1/3)*log(d^2*x^2 + c*d*x*(-d^2/c^2)^(1/3) + c^2*(-d^2/c^2)^(2/3)) + 2*b*c*
x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 2*a*d*x^2*(-d^2/c^2)^(1/
3)*log(d*x - c*(-d^2/c^2)^(1/3)) + 3*b*c - 3*a*d)/((a*b*c^2 - a^2*c*d)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.01, size = 309, normalized size = 1.03

$$\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^2bc-a^3d\right)}-\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}-\frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2bc-\sqrt{3}\,a^3d}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^3-\sqrt{3}\,ac^2d}-\frac{\left(-ab^2\right)^{\frac{1}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc-a^3d\right)}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3-ac^2d\right)}-\frac{1}{2\,acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^2
*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) - (-a*b^2)^(1/3)
*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c -
 sqrt(3)*a^3*d) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/
(-c/d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(1/3)*b*log(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(1/3)
*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/2/(a*c*x^
2)
```

Mupad [**B**]
time = 11.83, size = 1829, normalized size = 6.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((b^5/(a^5*(a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(a
*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d - b
*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))*(b^5/(a^5*(a*d - b*c)^3))^(2/3))/9 + 9
*a^6*b^9*c^11*d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^4*c^
6*d^9))/3 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(b^5/(27*a^8*d^3 - 27*
a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2))^(1/3) + log(((-d^5/(c^5*(
a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^
5/(c^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2
+ b^2*c^2 + a*b*c*d))*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/9 + 9*a^6*b^9*c^11*
d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^4*c^6*d^9))/3 + 3*
a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8 - 27*a^3*c^5*d^3 +
81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3) + (log(((3^(1/2)*1i - 1)*(b^5/(a^
5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b
*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d) - (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i - 1
)*(a*d + b*c)*(a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3))/2)*(b^5/(a^5*(
a*d - b*c)^3))^(2/3))/36 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10
*b^5*c^7*d^8 - 9*a^11*b^4*c^6*d^9))/6 - 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*
c^2))*(b^5/(27*a^8*d^3 - 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2
))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)*(b^5/(a^5*(a*d - b*c)
^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^
2 + b^2*c^2 + a*b*c*d) + (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)
*(a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3))/2)*(b^5/(a^5*(a*d - b*c)^3)
)^(2/3))/36 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8
- 9*a^11*b^4*c^6*d^9))/6 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(b^5/(2
7*a^8*d^3 - 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2))^(1/3)*(3^(
1/2)*1i + 1))/2 + (log(((3^(1/2)*1i - 1)*(-d^5/(c^5*(a*d - b*c)^3))^(1/3)*(
((3^(1/2)*1i - 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2
+ a*b*c*d) - (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)
^4*(-d^5/(c^5*(a*d - b*c)^3))^(1/3))/2)*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/3
6 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8 - 9*a^11*b
^4*c^6*d^9))/6 - 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8
- 27*a^3*c^5*d^3 + 81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3)*(3^(1/2)*1i -
1))/2 - (log(((3^(1/2)*1i + 1)*(-d^5/(c^5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*
1i + 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d)
 + (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-d^5/(
c^5*(a*d - b*c)^3))^(1/3))/2)*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/36 - 9*a^6*
b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8 - 9*a^11*b^4*c^6*d^9
))/6 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8 - 27*a^3*c
^5*d^3 + 81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3)*(3^(1/2)*1i + 1))/2 - 1/
(2*a*c*x^2)
```

3.119 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=87

$$-\frac{1}{3acx^3}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}$$

```
[Out] -1/3/a/c/x^3-(a*d+b*c)*ln(x)/a^2/c^2+1/3*b^2*ln(b*x^3+a)/a^2/(-a*d+b*c)-1/3
*d^2*ln(d*x^3+c)/c^2/(-a*d+b*c)
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\frac{b^2\log\left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/3*1/(a*c*x^3) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^3])/(3
*a^2*(b*c - a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx}\right.\right.\\ &= -\frac{1}{3acx^3}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 1.01

$$-\frac{1}{3acx^3}+\frac{(-bc-ad)\log(x)}{a^2c^2}-\frac{b^2\log\left(a+bx^3\right)}{3a^2(-bc+ad)}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/3*1/(a*c*x^3) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^3])
/(3*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

Maple [A]
time = 0.36, size = 83, normalized size = 0.95

method	result	size
norman	$-\frac{1}{3ac\,x^3}-\frac{b^2\ln(b\,x^3+a)}{3a^2(ad-bc)}+\frac{d^2\ln(d\,x^3+c)}{3c^2(ad-bc)}-\frac{(ad+bc)\ln(x)}{a^2c^2}$	82
default	$-\frac{b^2\ln(b\,x^3+a)}{3a^2(ad-bc)}+\frac{d^2\ln(d\,x^3+c)}{3c^2(ad-bc)}-\frac{1}{3ac\,x^3}+\frac{(-ad-bc)\ln(x)}{a^2c^2}$	83
risch	$-\frac{1}{3ac\,x^3}-\frac{\ln(x)d}{a\,c^2}-\frac{\ln(x)b}{a^2c}+\frac{d^2\ln(d\,x^3+c)}{3c^2(ad-bc)}-\frac{b^2\ln(-b\,x^3-a)}{3(ad-bc)a^2}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^2/a^2/(a*d-b*c)*ln(b*x^3+a)+1/3*d^2/c^2/(a*d-b*c)*ln(d*x^3+c)-1/3/a/
c/x^3+1/a^2/c^2*(-a*d-b*c)*ln(x)
```

Maxima [A]
time = 0.31, size = 87, normalized size = 1.00

$$\frac{b^2\log\left(bx^3+a\right)}{3\left(a^2bc-a^3d\right)}-\frac{d^2\log\left(dx^3+c\right)}{3\left(bc^3-ac^2d\right)}-\frac{(bc+ad)\log\left(x^3\right)}{3\,a^2c^2}-\frac{1}{3\,acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*log(b*x^3 + a)/(a^2*b*c - a^3*d) - 1/3*d^2*log(d*x^3 + c)/(b*c^3 -
a*c^2*d) - 1/3*(b*c + a*d)*log(x^3)/(a^2*c^2) - 1/3/(a*c*x^3)
```

Fricas [A]
time = 4.84, size = 99, normalized size = 1.14

$$\frac{b^2c^2x^3\log\left(bx^3+a\right)-a^2d^2x^3\log\left(dx^3+c\right)-3\left(b^2c^2-a^2d^2\right)x^3\log\left(x\right)-abc^2+a^2cd}{3\left(a^2bc^3-a^3c^2d\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c^2*x^3*log(b*x^3 + a) - a^2*d^2*x^3*log(d*x^3 + c) - 3*(b^2*c^2 -
 a^2*d^2)*x^3*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.88, size = 111, normalized size = 1.28

$$\frac{b^3 \log \left(|bx^3+a|\right)}{3\left(a^2b^2c-a^3bd\right)}-\frac{d^3 \log \left(|dx^3+c|\right)}{3\left(bc^3d-ac^2d^2\right)}-\frac{(bc+ad)\log \left(|x|\right)}{a^2c^2}+\frac{bcx^3+adx^3-ac}{3\,a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^3*log(abs(b*x^3 + a))/(a^2*b^2*c - a^3*b*d) - 1/3*d^3*log(abs(d*x^3 +
 c))/(b*c^3*d - a*c^2*d^2) - (b*c + a*d)*log(abs(x))/(a^2*c^2) + 1/3*(b*c*x
^3 + a*d*x^3 - a*c)/(a^2*c^2*x^3)
```

Mupad [**B**]
time = 3.22, size = 87, normalized size = 1.00

$$-\frac{b^2\,\ln \left(b\,x^3+a\right)}{3\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{d^2\,\ln \left(d\,x^3+c\right)}{3\,\left(b\,c^3-a\,c^2\,d\right)}-\frac{1}{3\,a\,c\,x^3}-\frac{\ln \left(x\right)\,\left(a\,d+b\,c\right)}{a^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] - (b^2*log(a + b*x^3))/(3*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^3))/(3*(b*c
^3 - a*c^2*d)) - 1/(3*a*c*x^3) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

3.120 $\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=318

$$-\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}(bc-ad)}+\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}(bc-ad)}+\frac{d^{7/3}\log}{3c}$$

```
[Out] -1/4/a/c/x^4+(a*d+b*c)/a^2/c^2/x-1/3*b^(7/3)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/
(-a*d+b*c)+1/3*d^(7/3)*ln(c^(1/3)+d^(1/3)*x)/c^(7/3)/(-a*d+b*c)+1/6*b^(7/3)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/(-a*d+b*c)-1/6*d^(7/3)*l
n(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(7/3)/(-a*d+b*c)-1/3*b^(7/3)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/(-a*d+b*c)*3^(1/2)+1
/3*d^(7/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(7/3)/(-a*d+
b*c)*3^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {491, 597, 598, 298, 31, 648, 631, 210, 642}

$$-\frac{b^{7/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}(bc-ad)}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}(bc-ad)}+\frac{ad+bc}{a^2c^2x}+\frac{d^{7/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)}-\frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{7/3}(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/4*1/(a*c*x^4) + (b*c + a*d)/(a^2*c^2*x) - (b^(7/3)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*(b*c - a*d)) + (d^(7/3)*ArcT
an[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(7/3)*(b*c - a*d)
) - (b^(7/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)*(b*c - a*d)) + (d^(7/3)*L
og[c^(1/3) + d^(1/3)*x])/(3*c^(7/3)*(b*c - a*d)) + (b^(7/3)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*(b*c - a*d)) - (d^(7/3)*Log[c^(
2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(7/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= -\frac{1}{4acx^4}+\frac{\int \frac{-4(bc+ad)-4bdx^3}{x^2(a+bx^3)(c+dx^3)}\,dx}{4ac}\\
&= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{\int \frac{x\left(-4\left(b^2c^2+abcd+a^2d^2\right)-4bd(bc+ad)x^3\right)}{(a+bx^3)(c+dx^3)}\,dx}{4a^2c^2}\\
&= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{\int \left(-\frac{4b^3c^2x}{(bc-ad)(a+bx^3)}-\frac{4a^2d^3x}{(-bc+ad)(c+dx^3)}\right)dx}{4a^2c^2}\\
&= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}+\frac{b^3\int \frac{x}{a+bx^3}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{x}{c+dx^3}\,dx}{c^2(bc-ad)}\\
&= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{8/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{7/3}(bc-ad)}+\frac{b^{8/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{7/3}(bc-ad)}+\\
&= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{7/3}(bc-ad)}+\\
&= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{7/3}(bc-ad)}+\\
&= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}(bc-ad)}+\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 282, normalized size = 0.89

$$
\frac{\frac{3b}{a}-\frac{3d}{c}-\frac{12b^2x^3}{a^2}+\frac{12d^2x^3}{c^2}+\frac{4\sqrt{3}\,b^{7/3}x^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}}-\frac{4\sqrt{3}\,d^{7/3}x^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{7/3}}+\frac{4b^{7/3}x^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{7/3}}-\frac{4d^{7/3}x^4\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{7/3}}-\frac{2b^{7/3}x^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{7/3}}+\frac{2d^{7/3}x^4\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{7/3}}}{12(-bc+ad)x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((3*b)/a - (3*d)/c - (12*b^2*x^3)/a^2 + (12*d^2*x^3)/c^2 + (4*Sqrt[3]*b^(7/
3)*x^4*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (4*Sqrt[3]*d^
(7/3)*x^4*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(7/3) + (4*b^(7/3)
*x^4*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) - (4*d^(7/3)*x^4*Log[c^(1/3) + d^(1/
3)*x])/c^(7/3) - (2*b^(7/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/a^(7/3) + (2*d^(7/3)*x^4*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
])/c^(7/3))/(12*(-(b*c) + a*d)*x^4)
```

Maple [A]

time = 0.39, size = 248, normalized size = 0.78

method	result
default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{3}}{a^{2}(ad-bc)}+\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right.}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\cdots}{c^{2}(a\cdots}$
risch	$\frac{\frac{(ad+bc)x^{3}}{a^{2}c^{2}}-\frac{1}{4ac}}{x^{4}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(d^{3}a^{10}-3c\,d^{2}a^{9}b+3c^{2}d\,a^{8}b^{2}-a^{7}b^{3}c^{3}\right)\_Z^{3}-b^{7}\right)}\_R\ln\left(\left(\left(-4a^{13}c^{7}d^{6}+22a^{12}b\,c^{8}d^{5}-52a^{11}b^{2}c^{9}d^{4}+68\cdots\right.\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))*b^3/a^2/(a*d-b*c)+(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)
^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1
/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))*d^3/c^2/(a*d-b*c)-1/4/a/c/x^4-1/a^2/c^2*(-
a*d-b*c)/x
```

Maxima [A]

time = 0.49, size = 341, normalized size = 1.07

$$\frac{\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^{2}bc-a^{3}d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,d^{2}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^{3}-ac^{2}d\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{b^{2}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^{2}bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{d^{2}\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^{3}\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^{2}d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{b^{2}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^{2}bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{d^{2}\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^{3}\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^{2}d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}+\frac{4\,(bc+ad)x^{3}-ac}{4\,a^{2}c^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*b
*c - a^3*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan(1/3*sqrt(3)*(2*x - (c/d)^
```

```
(1/3))/(c/d)^(1/3))/((b*c^3 - a*c^2*d)*(c/d)^(1/3)) + 1/6*b^2*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3)) - 1/6*d
^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d*(c/d
)^(1/3)) - 1/3*b^2*log(x + (a/b)^(1/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*(a/b)^
(1/3)) + 1/3*d^2*log(x + (c/d)^(1/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d*(c/d)^(1
/3)) + 1/4*(4*(b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^4)
```

Fricas [**A**]
time = 3.32, size = 305, normalized size = 0.96

$$\frac{4\sqrt{3}\,b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-4\sqrt{3}\,a^2d^2x^4\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2\,b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)+2\,a^2d^2x^4\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx^2-cx\left(-\frac{d}{c}\right)^{\frac{2}{3}}-c\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)-4\,b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)-4\,a^2d^2x^4\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx+c\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)-3\,abc^2+3\,a^2cd+12\,(b^2c^2-a^2d^2)x^3}{12\,(a^2bc^3-a^3c^2d)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*b^2*c^2*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) -
1/3*sqrt(3)) - 4*sqrt(3)*a^2*d^2*x^4*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/
c)^(1/3) + 1/3*sqrt(3)) + 2*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(
2/3) + a*(b/a)^(1/3)) + 2*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x^2 - c*x*(-d/c)^(
2/3) - c*(-d/c)^(1/3)) - 4*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3))
 - 4*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)) - 3*a*b*c^2 + 3*a^2
*c*d + 12*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.75, size = 328, normalized size = 1.03

$$-\frac{b^3\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(a^3bc-a^4d)}+\frac{d^3\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)}-\frac{(-ab^2)^{\frac{2}{3}}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^3bc-\sqrt{3}\,a^4d}+\frac{(-cd^2)^{\frac{2}{3}}\,d\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^4-\sqrt{3}\,ac^3d}+\frac{(-ab^2)^{\frac{2}{3}}\,b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(a^3bc-a^4d)}-\frac{(-cd^2)^{\frac{2}{3}}\,d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^4-ac^3d)}+\frac{4\,bcx^3+4\,adx^3-ac}{4\,a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d^
3*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) - (-a*b^2)^(2/3
)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b*c
- sqrt(3)*a^4*d) + (-c*d^2)^(2/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))
/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(2/3)*b*log
```

```
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2)^(2/3
)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/4*(4*b*c
*x^3 + 4*a*d*x^3 - a*c)/(a^2*c^2*x^4)
```

Mupad [B]
time = 11.37, size = 1734, normalized size = 5.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b
^6*c^6)*(a*d - b*c)^2 + 27*a^19*b^3*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(b^7
/(a^7*(a*d - b*c)^3))^(2/3))*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13*b^
11*c^20*d^4 - 9*a^14*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13
*d^11))/9 + a^13*b^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 + 81*a^
8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3) + log(((-d^7/(c^7*(a*d - b*c)^3))^(2/3
)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + 27*a^19*b^3
*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^(2/3))*(-d^7
/(c^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13*b^11*c^20*d^4 - 9*a^14*b^10*c^19*d^
5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/9 + a^13*b^9*c^13*d^9*x)*
(d^7/(27*b^3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2*c^9*d))^(1
/3) - (1/(4*a*c) - (x^3*(a*d + b*c))/(a^2*c^2))/x^4 + (log(((3^(1/2)*1i - 1
)^2*(b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(27*a^14*b^3*c^14*d^
3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i -
 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b^7/(a^7*(a*d - b*c)^3))^(2/3))/4)*(b^7/(a
^7*(a*d - b*c)^3))^(1/3))/6 + 9*a^13*b^11*c^20*d^4 - 9*a^14*b^10*c^19*d^5 -
 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/36 + a^13*b^9*c^13*d^9*x)*(b
^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 + 81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3
)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(b^7/(a^7*(a*d - b*c)^3))^
(2/3)*(((3^(1/2)*1i + 1)*(27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d -
 b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^
4*(b^7/(a^7*(a*d - b*c)^3))^(2/3))/4)*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/6 -
9*a^13*b^11*c^20*d^4 + 9*a^14*b^10*c^19*d^5 + 9*a^19*b^5*c^14*d^10 - 9*a^20
*b^4*c^13*d^11))/36 - a^13*b^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c
^3 + 81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((
(3^(1/2)*1i - 1)^2*(-d^7/(c^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(27*
a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d
^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^
(2/3))/4)*(-d^7/(c^7*(a*d - b*c)^3))^(1/3))/6 + 9*a^13*b^11*c^20*d^4 - 9*a^
14*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/36 + a^13*
b^9*c^13*d^9*x)*(d^7/(27*b^3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*
a*b^2*c^9*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(-d^7/(c
^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i + 1)*(27*a^14*b^3*c^14*d^3*x*(a^6*d^
```

```
6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)^2*(a*d
+ b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^(2/3))/4)*(-d^7/(c^7*(a*d -
 b*c)^3))^(1/3))/6 - 9*a^13*b^11*c^20*d^4 + 9*a^14*b^10*c^19*d^5 + 9*a^19*b
^5*c^14*d^10 - 9*a^20*b^4*c^13*d^11))/36 - a^13*b^9*c^13*d^9*x)*(d^7/(27*b^
3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2*c^9*d))^(1/3)*(3^(1/2
)*1i + 1))/2
```

3.121 $\int \frac{1}{x^6(a+bx^3)(c+dx^3)} dx$

Optimal. Leaf size=321

$$-\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}-\frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)}+\frac{d^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{d^{8/3}\log}{3c^{8/}}$$

```
[Out] -1/5/a/c/x^5+1/2*(a*d+b*c)/a^2/c^2/x^2+1/3*b^(8/3)*ln(a^(1/3)+b^(1/3)*x)/a^
(8/3)/(-a*d+b*c)-1/3*d^(8/3)*ln(c^(1/3)+d^(1/3)*x)/c^(8/3)/(-a*d+b*c)-1/6*b
^(8/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/(-a*d+b*c)+1/6*d^(
8/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/(-a*d+b*c)-1/3*b^(8/
3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/(-a*d+b*c)*3^(
1/2)+1/3*d^(8/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(8/3)/
(-a*d+b*c)*3^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {491, 597, 536, 206, 31, 648, 631, 210, 642}

$$-\frac{b^{8/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)}-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{d^{8/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{8/3}(bc-ad)}+\frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{8/3}(bc-ad)}-\frac{1}{5acx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/5*1/(a*c*x^5) + (b*c + a*d)/(2*a^2*c^2*x^2) - (b^(8/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*(b*c - a*d)) + (d^(8/3)*
ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(8/3)*(b*c -
a*d)) + (b^(8/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)*(b*c - a*d)) - (d^(8/
3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(8/3)*(b*c - a*d)) - (b^(8/3)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*(b*c - a*d)) + (d^(8/3)*Log
[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(8/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= -\frac{1}{5acx^5}+\frac{\int \frac{-5(bc+ad)-5bdx^3}{x^3(a+bx^3)(c+dx^3)}\,dx}{5ac} \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}-\frac{\int \frac{-10\left(b^2c^2+abcd+a^2d^2\right)-10bd(bc+ad)x^3}{(a+bx^3)(c+dx^3)}\,dx}{10a^2c^2} \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\int \frac{1}{a+bx^3}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{1}{c+dx^3}\,dx}{c^2(bc-ad)} \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{8/3}(bc-ad)}+\frac{b^3\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{8/3}(bc-ad)}-\cdots \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{8/3}(bc-ad)}-\cdots \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{8/3}(bc-ad)}-\cdots \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}-\frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}(bc-ad)}+\frac{d^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{8/3}(bc-ad)}-\cdots
\end{aligned}
$$

Mathematica [**A**]
time = 0.11, size = 282, normalized size = 0.88

$$
\frac{\frac{6b}{a}-\frac{6d}{c}-\frac{15b^2x^3}{a^2}+\frac{15d^2x^3}{c^2}+\frac{10\sqrt{3}\,b^{8/3}x^5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}}-\frac{10\sqrt{3}\,d^{8/3}x^5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{8/3}}-\frac{10b^{8/3}x^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{8/3}}+\frac{10d^{8/3}x^5\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{8/3}}+\frac{5b^{8/3}x^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{8/3}}-\frac{5d^{8/3}x^5\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{8/3}}}{30(-bc+ad)x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/a - (6*d)/c - (15*b^2*x^3)/a^2 + (15*d^2*x^3)/c^2 + (10*Sqrt[3]*b^(8
/3)*x^5*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) - (10*Sqrt[3]*
```

```
d^(8/3)*x^5*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(8/3) - (10*b^(8
/3)*x^5*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) + (10*d^(8/3)*x^5*Log[c^(1/3) + d
^(1/3)*x])/c^(8/3) + (5*b^(8/3)*x^5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/a^(8/3) - (5*d^(8/3)*x^5*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/c^(8/3))/(30*(-(b*c) + a*d)*x^5)
```

Maple [A]
time = 0.42, size = 248, normalized size = 0.77

method	result
default	$-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b^{3}}{a^{2}(ad-bc)}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\cdots\right.}{c^{2}(ad\cdots}$
risch	$\frac{\frac{(ad+bc)x^{3}}{2a^{2}c^{2}}-\frac{1}{5ac}}{x^{5}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{3}a^{11}-3c\,d^{2}a^{10}b+3c^{2}d\,a^{9}b^{2}-a^{8}b^{3}c^{3}\right)\_Z^{3}+b^{8}\right)}\_R\ln\left(\left(\left(-4a^{14}c^{8}d^{6}+22a^{13}b\,c^{9}d^{5}-52a^{12}b^{2}c^{10}d^{4}\cdots\right.\right.\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1)))/a^2*b^3/(a*d-b*c)+(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(
2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(c/d)^(1/3)*x-1)))/c^2*d^3/(a*d-b*c)-1/5/a/c/x^5-1/2/a^2/c^2*(-
a*d-b*c)/x^2
```

Maxima [A]
time = 0.50, size = 369, normalized size = 1.15

$$\frac{\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^{2}bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,d^{2}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^{3}\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^{2}d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{b^{2}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^{2}bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{3}d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{d^{2}\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}-ac^{2}d\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{b^{2}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^{2}bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{3}d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{d^{2}\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}-ac^{2}d\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{5\,(bc+ad)x^{3}-2\,ac}{10\,a^{2}c^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*b
*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan(1
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^3*(c/d)^(1/3) - a*c^2*d*(
```

```
c/d)^(1/3))*(c/d)^(1/3)) - 1/6*b^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) + 1/6*d^2*log(x^2 - x*(c/d)^(1/3)
+ (c/d)^(2/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/3*b^2*log(x +
(a/b)^(1/3))/(a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) - 1/3*d^2*log(x + (c
/d)^(1/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/10*(5*(b*c + a*d)*
x^3 - 2*a*c)/(a^2*c^2*x^5)
```

<u>Fricas</u> [A]
time = 3.25, size = 356, normalized size = 1.11

$$-\frac{10\sqrt{3}\,b^2c^2x^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+10\sqrt{3}\,a^2d^2x^5\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3\,d}\right)-5\,b^2c^2x^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-5\,a^2d^2x^5\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2x^2-cdx\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}+c^2\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}\right)+10\,b^2c^2x^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+10\,a^2d^2x^5\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\right)+6\,abc^2-6\,a^2cd-15\,(b^2c^2-a^2d^2)x^3}{30\,(a^2bc^3-a^3c^2d)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-
b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 10*sqrt(3)*a^2*d^2*x^5*(d^2/c^2)^(1/3)*arc
tan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - 5*b^2*c^2*x^5*(-b^
2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) -
 5*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d
^2/c^2)^(2/3)) + 10*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/
3)) + 10*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) + 6*a*b*c
^2 - 6*a^2*c*d - 15*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^5)
```

<u>Sympy</u> [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A]
time = 0.70, size = 336, normalized size = 1.05

$$-\frac{b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(a^3bc-a^4d)}+\frac{d^3\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)}+\frac{(-ab^2)^{\frac{1}{3}}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^3bc-\sqrt{3}\,a^4d}-\frac{(-cd^2)^{\frac{1}{3}}\,d^2\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^4-\sqrt{3}\,ac^3d}+\frac{(-ab^2)^{\frac{1}{3}}\,b^2\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(a^3bc-a^4d)}-\frac{(-cd^2)^{\frac{1}{3}}\,d^2\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^4-ac^3d)}+\frac{5\,bcx^3+5\,adx^3-2\,ac}{10\,a^2c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d^
3*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) + (-a*b^2)^(1/3
)*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b*
c - sqrt(3)*a^4*d) - (-c*d^2)^(1/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
```

```
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(1/3)*b
^2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2
)^(1/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/
10*(5*b*c*x^3 + 5*a*d*x^3 - 2*a*c)/(a^2*c^2*x^5)
```

Mupad [B]
time = 11.57, size = 1860, normalized size = 5.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5 - 9*a^12*b^12*c
^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a^16*b^3*c^16*d^3
*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(-b^8/(a^8*(a*d - b*c)^3))^(1/3) + a^2*
d^2*x + b^2*c^2*x)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3)))/3 + 3*a^12*b^7*c^12*d
^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 27*a^8*b^3*c^3 + 81*a^9*b^2*
c^2*d - 81*a^10*b*c*d^2))^(1/3) - (1/(5*a*c) - (x^3*(a*d + b*c))/(2*a^2*c^2
))/x^5 + log(((d^8/(c^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5 - 9*a^1
2*b^12*c^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a^16*b^3*
c^16*d^3*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(d^8/(c^8*(a*d - b*c)^3))^(1/3)
 + a^2*d^2*x + b^2*c^2*x)*(d^8/(c^8*(a*d - b*c)^3))^(2/3)))/3 + 3*a^12*b^7*
c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^
2*b*c^9*d^2 - 81*a*b^2*c^10*d))^(1/3) + (log(((3^(1/2)*1i - 1)*(-b^8/(a^8*(
a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*
c)^4*(a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2) + (81*a^19*b^3*c^19*d^
3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^8/(a^8*(a*d - b*c)^3))^(1/
3))/2)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13
*b^11*c^19*d^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12))/6 + 3*a^12*b
^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 27*a^8*b^3*c^3 + 81
*a^9*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/
2)*1i + 1)*(-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^16*b
^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2)
 - (81*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^8/(
a^8*(a*d - b*c)^3))^(1/3))/2)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12
*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c
^12*d^12))/6 - 3*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^
3 - 27*a^8*b^3*c^3 + 81*a^9*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3)*(3^(1/2)*1i
 + 1))/2 + (log(((3^(1/2)*1i - 1)*(d^8/(c^8*(a*d - b*c)^3))^(1/3)*(((3^(1/2
)*1i - 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*d^3 + b^3*c^3 + a*b^
2*c^2*d + a^2*b*c*d^2) + (81*a^19*b^3*c^19*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)
*(a*d - b*c)^4*(d^8/(c^8*(a*d - b*c)^3))^(1/3))/2)*(d^8/(c^8*(a*d - b*c)^3)
)^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d^5 + 9*a^19*b^5*c^13
*d^11 - 9*a^20*b^4*c^12*d^12))/6 + 3*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4
```

```
))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^2*b*c^9*d^2 - 81*a*b^2*c^10*d
))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)*(d^8/(c^8*(a*d - b*c)
^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*
d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2) - (81*a^19*b^3*c^19*d^3*(3^(1/2)
*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(d^8/(c^8*(a*d - b*c)^3))^(1/3))/2)*(d^8
/(c^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d
^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12))/6 - 3*a^12*b^7*c^12*d^7*
x*(a^4*d^4 + b^4*c^4))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^2*b*c^9*d
^2 - 81*a*b^2*c^10*d))^(1/3)*(3^(1/2)*1i + 1))/2
```

3.122 $\int \frac{1}{x^7(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{6acx^6}+\frac{bc+ad}{3a^2c^2x^3}+\frac{(b^2c^2+abcd+a^2d^2)\log(x)}{a^3c^3}-\frac{b^3\log(a+bx^3)}{3a^3(bc-ad)}+\frac{d^3\log(c+dx^3)}{3c^3(bc-ad)}$$

```
[Out] -1/6/a/c/x^6+1/3*(a*d+b*c)/a^2/c^2/x^3+(a^2*d^2+a*b*c*d+b^2*c^2)*ln(x)/a^3/
c^3-1/3*b^3*ln(b*x^3+a)/a^3/(-a*d+b*c)+1/3*d^3*ln(d*x^3+c)/c^3/(-a*d+b*c)
```

Rubi [A]

time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 84}

$$-\frac{b^3\log(a+bx^3)}{3a^3(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{\log(x)(a^2d^2+abcd+b^2c^2)}{a^3c^3}+\frac{d^3\log(c+dx^3)}{3c^3(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/6*1/(a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*
d^2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^3])/(3*a^3*(b*c - a*d)) + (d^3*Lo
g[c + d*x^3])/(3*c^3*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7(a+bx^3)(c+dx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(a+bx)(c+dx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx^3}+\frac{-bc-ad}{a^2c^2x^2}+\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^4}{a^3(-bc+ad)(a+b\right.\right.\\ &= -\frac{1}{6acx^6}+\frac{bc+ad}{3a^2c^2x^3}+\frac{(b^2c^2+abcd+a^2d^2)\log(x)}{a^3c^3}-\frac{b^3\log(a+bx^3)}{3a^3(bc-ad)}+\frac{d^3\log}{3c^3(}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 119, normalized size = 1.00

$$-\frac{1}{6acx^6}+\frac{bc+ad}{3a^2c^2x^3}+\frac{(b^2c^2+abcd+a^2d^2)\log(x)}{a^3c^3}+\frac{b^3\log(a+bx^3)}{3a^3(-bc+ad)}+\frac{d^3\log(c+dx^3)}{3c^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/6*1/(a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*
d^2)*Log[x])/(a^3*c^3) + (b^3*Log[a + b*x^3])/(3*a^3*(-(b*c) + a*d)) + (d^3
*Log[c + d*x^3])/(3*c^3*(b*c - a*d))
```

Maple [A]
time = 0.38, size = 114, normalized size = 0.96

method	result	size
default	$\frac{b^3\ln(bx^3+a)}{3a^3(ad-bc)}-\frac{d^3\ln(dx^3+c)}{3c^3(ad-bc)}-\frac{1}{6acx^6}-\frac{-ad-bc}{3a^2c^2x^3}+\frac{(a^2d^2+abcd+b^2c^2)\ln(x)}{a^3c^3}$	114
norman	$\frac{-\frac{1}{6ac}+\frac{(ad+bc)x^3}{3a^2c^2}}{x^6}+\frac{(a^2d^2+abcd+b^2c^2)\ln(x)}{a^3c^3}+\frac{b^3\ln(bx^3+a)}{3a^3(ad-bc)}-\frac{d^3\ln(dx^3+c)}{3c^3(ad-bc)}$	114
risch	$\frac{-\frac{1}{6ac}+\frac{(ad+bc)x^3}{3a^2c^2}}{x^6}+\frac{\ln(x)d^2}{ac^3}+\frac{\ln(x)bd}{a^2c^2}+\frac{\ln(x)b^2}{a^3c}+\frac{b^3\ln(-bx^3-a)}{3(ad-bc)a^3}-\frac{d^3\ln(dx^3+c)}{3c^3(ad-bc)}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^3/a^3/(a*d-b*c)*ln(b*x^3+a)-1/3*d^3/c^3/(a*d-b*c)*ln(d*x^3+c)-1/6/a/c
/x^6-1/3*(-a*d-b*c)/a^2/c^2/x^3+(a^2*d^2+a*b*c*d+b^2*c^2)*ln(x)/a^3/c^3
```

Maxima [A]
time = 0.27, size = 117, normalized size = 0.98

$$-\frac{b^3\log(bx^3+a)}{3(a^3bc-a^4d)}+\frac{d^3\log(dx^3+c)}{3(bc^4-ac^3d)}+\frac{(b^2c^2+abcd+a^2d^2)\log(x^3)}{3a^3c^3}+\frac{2(bc+ad)x^3-ac}{6a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(b*x^3 + a)/(a^3*b*c - a^4*d) + 1/3*d^3*log(d*x^3 + c)/(b*c^4 -
 a*c^3*d) + 1/3*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^3)/(a^3*c^3) + 1/6*(2*(
b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^6)
```

Fricas [A]
time = 8.96, size = 127, normalized size = 1.07

$$-\frac{2b^3c^3x^6\log(bx^3+a)-2a^3d^3x^6\log(dx^3+c)-6(b^3c^3-a^3d^3)x^6\log(x)+a^2bc^3-a^3c^2d-2(ab^2c^3-a^3cd^2)x^3}{6(a^3bc^4-a^4c^3d)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3*x^6*log(b*x^3 + a) - 2*a^3*d^3*x^6*log(d*x^3 + c) - 6*(b^3*
c^3 - a^3*d^3)*x^6*log(x) + a^2*b*c^3 - a^3*c^2*d - 2*(a*b^2*c^3 - a^3*c*d^
2)*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.93, size = 165, normalized size = 1.39

$$-\frac{b^4\log\left(|bx^3+a|\right)}{3\left(a^3b^2c-a^4bd\right)}+\frac{d^4\log\left(|dx^3+c|\right)}{3\left(bc^4d-ac^3d^2\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(|x|\right)}{a^3c^3}-\frac{3\,b^2c^2x^6+3\,abcdx^6+3\,a^2d^2x^6-2\,abc^2x^3-2\,a^2cdx^3+a^2c^2}{6\,a^3c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^4*log(abs(b*x^3 + a))/(a^3*b^2*c - a^4*b*d) + 1/3*d^4*log(abs(d*x^3
+ c))/(b*c^4*d - a*c^3*d^2) + (b^2*c^2 + a*b*c*d + a^2*d^2)*log(abs(x))/(a^
3*c^3) - 1/6*(3*b^2*c^2*x^6 + 3*a*b*c*d*x^6 + 3*a^2*d^2*x^6 - 2*a*b*c^2*x^3
 - 2*a^2*c*d*x^3 + a^2*c^2)/(a^3*c^3*x^6)
```

Mupad [**B**]
time = 3.21, size = 118, normalized size = 0.99

$$\frac{b^3\,\ln\left(b\,x^3+a\right)}{3\,a^4\,d-3\,a^3\,b\,c}-\frac{\frac{1}{6\,a\,c}-\frac{x^3\,\left(a\,d+b\,c\right)}{3\,a^2\,c^2}}{x^6}+\frac{d^3\,\ln\left(d\,x^3+c\right)}{3\,b\,c^4-3\,a\,c^3\,d}+\frac{\ln\left(x\right)\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (b^3*log(a + b*x^3))/(3*a^4*d - 3*a^3*b*c) - (1/(6*a*c) - (x^3*(a*d + b*c))
/(3*a^2*c^2))/x^6 + (d^3*log(c + d*x^3))/(3*b*c^4 - 3*a*c^3*d) + (log(x)*(a
^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3)
```

3.123 $\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=352

$$-\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}(bc-ad)}-\frac{d^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{10/3}(bc-ad)}+\frac{b^{10/3}\log\left(\sqrt[3]{\cdots}\right.}{3a^{10/3}(\cdots}$$

```
[Out] -1/7/a/c/x^7+1/4*(a*d+b*c)/a^2/c^2/x^4+(-a^2*d^2-a*b*c*d-b^2*c^2)/a^3/c^3/x
+1/3*b^(10/3)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/(-a*d+b*c)-1/3*d^(10/3)*ln(c^(
1/3)+d^(1/3)*x)/c^(10/3)/(-a*d+b*c)-1/6*b^(10/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/a^(10/3)/(-a*d+b*c)+1/6*d^(10/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)
*x+d^(2/3)*x^2)/c^(10/3)/(-a*d+b*c)+1/3*b^(10/3)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(10/3)*arctan(1/3
*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(10/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {491, 597, 598, 298, 31, 648, 631, 210, 642}

$$\frac{b^{10/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}(bc-ad)}-\frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{10/3}(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}-\frac{d^{10/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{10/3}(bc-ad)}+\frac{d^{10/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{10/3}(bc-ad)}-\frac{d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{10/3}(bc-ad)}-\frac{1}{7acx^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/7*1/(a*c*x^7) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2*d
^2)/(a^3*c^3*x) + (b^(10/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(10/3)*(b*c - a*d)) - (d^(10/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*
x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(10/3)*(b*c - a*d)) + (b^(10/3)*Log[a^(1/
3) + b^(1/3)*x])/(3*a^(10/3)*(b*c - a*d)) - (d^(10/3)*Log[c^(1/3) + d^(1/3)
*x])/(3*c^(10/3)*(b*c - a*d)) - (b^(10/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(10/3)*(b*c - a*d)) + (d^(10/3)*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/(6*c^(10/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{7acx^7}+\frac{\int \frac{-7(bc+ad)-7bdx^3}{x^5(a+bx^3)(c+dx^3)}\,dx}{7ac}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{\int \frac{-28\left(b^2c^2+abcd+a^2d^2\right)-28bd(bc+ad)x^3}{x^2(a+bx^3)(c+dx^3)}\,dx}{28a^2c^2}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{\int \frac{x\left(-28(bc+ad)\left(b^2c^2+a^2d^2\right)-28bd\left(b^2c^2+a\right.\right.}{(a+bx^3)(c+dx^3)}}{28a^3c^3}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{\int \left(-\frac{28b^4c^3x}{(bc-ad)(a+bx^3)}-\frac{28a^3d^4x}{(-bc+ad)(c+dx^3}\right.}{28a^3c^3}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^4\int \frac{x}{a+bx^3}\,dx}{a^3(bc-ad)}+\frac{d^4\int \frac{x}{c+dx^3}\,dx}{c^3(bc-ad)}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{11/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{10/3}(bc-ad)}-\frac{b^{11/3}\int \frac{}{a^{2/}}}{3a}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{10/3}(bc-ad)}-\frac{d^{10/3}\,\mathrm{lo}}{3c}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{10/3}(bc-ad)}-\frac{d^{10/3}\,\mathrm{lo}}{3c}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}(bc-ad)}-\frac{d^{10/}}{}$$

Mathematica [A]
time = 0.12, size = 304, normalized size = 0.86

$$\frac{\frac{12b}{a}-\frac{12d}{c}-\frac{21b^2x^3}{a^2}+\frac{21d^2x^3}{c^2}+\frac{84b^3x^6}{a^3}-\frac{84d^3x^6}{c^3}-\frac{28\sqrt{3}\,b^{10/3}x^7\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{10/3}}+\frac{28\sqrt{3}\,d^{10/3}x^7\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{10/3}}-\frac{28b^{10/3}x^7\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{10/3}}+\frac{28d^{10/3}x^7\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{10/3}}+\frac{14b^{10/3}x^7\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{10/3}}-\frac{14d^{10/3}x^7\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{10/3}}}{84(-bc+ad)x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((12*b)/a - (12*d)/c - (21*b^2*x^3)/a^2 + (21*d^2*x^3)/c^2 + (84*b^3*x^6)/a
^3 - (84*d^3*x^6)/c^3 - (28*Sqrt[3]*b^(10/3)*x^7*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]])/a^(10/3) + (28*Sqrt[3]*d^(10/3)*x^7*ArcTan[(1 - (2*d^(1/
3)*x)/c^(1/3))/Sqrt[3]])/c^(10/3) - (28*b^(10/3)*x^7*Log[a^(1/3) + b^(1/3)*
x]])/a^(10/3) + (28*d^(10/3)*x^7*Log[c^(1/3) + d^(1/3)*x]])/c^(10/3) + (14*b^
(10/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/a^(10/3) - (14*d
^(10/3)*x^7*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]])/c^(10/3))/(84*(
-(b*c) + a*d)*x^7)
```

Maple [A]
time = 0.40, size = 279, normalized size = 0.79

method	result
default	$\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{4}}{a^{3}(ad-bc)}-\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\cdots\right.}{c^{3}(ad\cdots}$
risch	$\frac{-\frac{\left(a^{2}d^{2}+abcd+b^{2}c^{2}\right)x^{6}}{a^{3}c^{3}}+\frac{(ad+bc)x^{3}}{4a^{2}c^{2}}-\frac{1}{7ac}}{x^{7}}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{3}c^{10}a^{3}-3a^{2}b\,c^{11}d^{2}+3a\,b^{2}c^{12}d-b^{3}c^{13}\right)\_Z^{3}-d^{10}\right)}\_R\ln\left(\left(\left(-4a^{16}c^{10}d^{6}\cdots\right.\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1)))*b^4/a^3/(a*d-b*c)-(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^
(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/
3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))*d^4/c^3/(a*d-b*c)-1/7/a/c/x^7-1/4*(-a*d-b*c
)/a^2/c^2/x^4-(a^2*d^2+a*b*c*d+b^2*c^2)/a^3/c^3/x
```

Maxima [A]
time = 0.51, size = 376, normalized size = 1.07

$$-\frac{\sqrt{3}\,b^{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^{3}bc-a^{4}d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,d^{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^{4}-ac^{3}d\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{b^{3}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^{3}bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{4}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{d^{3}\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^{4}\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^{3}d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}+\frac{b^{3}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^{3}bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{4}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{d^{3}\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^{4}\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^{3}d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{28\left(b^{2}c^{2}+abcd+a^{2}d^{2}\right)x^{6}+4\,a^{2}c^{2}-7\left(abc^{2}+a^{2}cd\right)x^{3}}{28\,a^{3}c^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^3*
b*c - a^4*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d^3*arctan(1/3*sqrt(3)*(2*x - (c/d)
^(1/3))/(c/d)^(1/3))/((b*c^4 - a*c^3*d)*(c/d)^(1/3)) - 1/6*b^3*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d*(a/b)^(1/3)) + 1/6*
d^3*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^4*(c/d)^(1/3) - a*c^3*d*(c/
d)^(1/3)) + 1/3*b^3*log(x + (a/b)^(1/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d*(a/b)
^(1/3)) - 1/3*d^3*log(x + (c/d)^(1/3))/(b*c^4*(c/d)^(1/3) - a*c^3*d*(c/d)^(
1/3)) - 1/28*(28*(b^2*c^2 + a*b*c*d + a^2*d^2)*x^6 + 4*a^2*c^2 - 7*(a*b*c^2
 + a^2*c*d)*x^3)/(a^3*c^3*x^7)
```

Fricas [A]
time = 2.19, size = 332, normalized size = 0.94

$$-\frac{28\,\sqrt{3}\,b^{3}c^{3}x^{7}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-28\,\sqrt{3}\,a^{3}d^{3}x^{7}\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)-14\,b^{3}c^{3}x^{7}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)-14\,a^{3}d^{3}x^{7}\left(\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx^{2}-cx\left(\frac{d}{c}\right)^{\frac{2}{3}}+c\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)+28\,b^{3}c^{3}x^{7}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)+28\,a^{3}d^{3}x^{7}\left(\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)+84\,(b^{3}c^{3}-a^{3}d^{3})x^{6}+12\,a^{2}bc^{3}-12\,a^{3}c^{2}d-21\,(ab^{2}c^{3}-a^{3}cd^{2})x^{3}}{84\,(a^{3}bc^{4}-a^{4}c^{3}d)x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/84*(28*sqrt(3)*b^3*c^3*x^7*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3
) + 1/3*sqrt(3)) - 28*sqrt(3)*a^3*d^3*x^7*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x*
(d/c)^(1/3) - 1/3*sqrt(3)) - 14*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x^2 - a*x*(-
b/a)^(2/3) - a*(-b/a)^(1/3)) - 14*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x^2 - c*x*(
d/c)^(2/3) + c*(d/c)^(1/3)) + 28*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x + a*(-b/a
)^(2/3)) + 28*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x + c*(d/c)^(2/3)) + 84*(b^3*c^
3 - a^3*d^3)*x^6 + 12*a^2*b*c^3 - 12*a^3*c^2*d - 21*(a*b^2*c^3 - a^3*c*d^2)
*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^7)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.25, size = 377, normalized size = 1.07

$$\frac{b^{4}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(a^{4}bc-a^{5}d)}-\frac{d^{4}\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^{5}-ac^{4}d)}+\frac{(-ab^{2})^{\frac{2}{3}}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^{4}bc-\sqrt{3}\,a^{5}d}-\frac{(-cd^{2})^{\frac{2}{3}}\,d^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^{5}-\sqrt{3}\,ac^{4}d}-\frac{(-ab^{2})^{\frac{2}{3}}\,b^{2}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(a^{4}bc-a^{5}d)}+\frac{(-cd^{2})^{\frac{2}{3}}\,d^{2}\log\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^{5}-ac^{4}d)}-\frac{28\,b^{2}c^{2}x^{6}+28\,abcdx^{6}+28\,a^{2}d^{2}x^{6}-7\,abc^{2}x^{3}-7\,a^{2}cdx^{3}+4\,a^{2}c^{2}}{28\,a^{3}c^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^4*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b*c - a^5*d) - 1/3*d^4
*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^5 - a*c^4*d) + (-a*b^2)^(2/3)
```

```
*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^4*b*c
 - sqrt(3)*a^5*d) - (-c*d^2)^(2/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/
3))/(-c/d)^(1/3))/(sqrt(3)*b*c^5 - sqrt(3)*a*c^4*d) - 1/6*(-a*b^2)^(2/3)*b^
2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b*c - a^5*d) + 1/6*(-c*d^2)
^(2/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^5 - a*c^4*d) - 1/2
8*(28*b^2*c^2*x^6 + 28*a*b*c*d*x^6 + 28*a^2*d^2*x^6 - 7*a*b*c^2*x^3 - 7*a^2
*c*d*x^3 + 4*a^2*c^2)/(a^3*c^3*x^7)
```

Mupad [B]
time = 11.91, size = 1814, normalized size = 5.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8
+ b^8*c^8)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4*(
-b^10/(a^10*(a*d - b*c)^3))^(2/3))*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/3 -
9*a^19*b^14*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29
*b^4*c^19*d^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3
 - 27*a^10*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3) + log(((d^
10/(c^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8
)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^1
0*(a*d - b*c)^3))^(2/3))*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/3 - 9*a^19*b^14
*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d
^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c
^10*d^3 + 81*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3) - (1/(7*a*c) - (x^3*(
a*d + b*c))/(4*a^2*c^2) + (x^6*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3))/x^
7 - (log(((3^(1/2)*1i + 1)^2*(-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*
1i + 1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^2
8*b^3*c^28*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-b^10/(a^10*(a
*d - b*c)^3))^(2/3))/4)*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/6 + 9*a^19*b^14
*c^29*d^4 - 9*a^20*b^13*c^28*d^5 - 9*a^28*b^5*c^20*d^13 + 9*a^29*b^4*c^19*d
^14))/36 + a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3 - 27*a^10
*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2
+ (log(((3^(1/2)*1i - 1)^2*(-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i
 - 1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*
b^3*c^28*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-b^10/(a^10*(a*d
 - b*c)^3))^(2/3))/4)*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/6 - 9*a^19*b^14*c
^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d^1
4))/36 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3 - 27*a^10*b
^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 -
(log(((3^(1/2)*1i + 1)^2*(d^10/(c^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i +
1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*b^3
```

```
*c^28*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^10*(a*d - b
*c)^3))^(2/3))/4)*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/6 + 9*a^19*b^14*c^29*d
^4 - 9*a^20*b^13*c^28*d^5 - 9*a^28*b^5*c^20*d^13 + 9*a^29*b^4*c^19*d^14))/3
6 + a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c^10*d^
3 + 81*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(
((3^(1/2)*1i - 1)^2*(d^10/(c^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(2
7*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*b^3*c^28
*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^10*(a*d - b*c)^3
))^(2/3))/4)*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/6 - 9*a^19*b^14*c^29*d^4 +
9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d^14))/36 - a
^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c^10*d^3 + 8
1*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3)*(3^(1/2)*1i - 1))/2
```

3.124 $\int x^m(a+bx^3)^5\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=148

$$\frac{a^5Ax^{1+m}}{1+m}+\frac{a^4(5Ab+aB)x^{4+m}}{4+m}+\frac{5a^3b(2Ab+aB)x^{7+m}}{7+m}+\frac{10a^2b^2(Ab+aB)x^{10+m}}{10+m}+\frac{5ab^3(Ab+2aB)x^{13+m}}{13+m}+$$

```
[Out] a^5*A*x^(1+m)/(1+m)+a^4*(5*A*b+B*a)*x^(4+m)/(4+m)+5*a^3*b*(2*A*b+B*a)*x^(7+
m)/(7+m)+10*a^2*b^2*(A*b+B*a)*x^(10+m)/(10+m)+5*a*b^3*(A*b+2*B*a)*x^(13+m)/
(13+m)+b^4*(A*b+5*B*a)*x^(16+m)/(16+m)+b^5*B*x^(19+m)/(19+m)
```

Rubi [A]
time = 0.07, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {459}

$$\frac{a^5Ax^{m+1}}{m+1}+\frac{a^4x^{m+4}(aB+5Ab)}{m+4}+\frac{5a^3bx^{m+7}(aB+2Ab)}{m+7}+\frac{10a^2b^2x^{m+10}(aB+Ab)}{m+10}+\frac{b^4x^{m+16}(5aB+Ab)}{m+16}+\frac{5ab^3x^{m+13}(2aB+Ab)}{m+13}+\frac{b^5Bx^{m+19}}{m+19}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^(1 + m))/(1 + m) + (a^4*(5*A*b + a*B)*x^(4 + m))/(4 + m) + (5*a^3*
b*(2*A*b + a*B)*x^(7 + m))/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^(10 + m))/(1
0 + m) + (5*a*b^3*(A*b + 2*a*B)*x^(13 + m))/(13 + m) + (b^4*(A*b + 5*a*B)*x
^(16 + m))/(16 + m) + (b^5*B*x^(19 + m))/(19 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx&=\int\left(a^5Ax^m+a^4(5Ab+aB)x^{3+m}+5a^3b(2Ab+aB)x^{6+m}+10a^2b^2(Ab+\right.\\&=\frac{a^5Ax^{1+m}}{1+m}+\frac{a^4(5Ab+aB)x^{4+m}}{4+m}+\frac{5a^3b(2Ab+aB)x^{7+m}}{7+m}+\frac{10a^2b^2(Ab+}{10+}\end{aligned}$$

Mathematica [A]
time = 0.46, size = 137, normalized size = 0.93

$$x^{1+m}\left(\frac{a^5A}{1+m}+\frac{a^4(5Ab+aB)x^3}{4+m}+\frac{5a^3b(2Ab+aB)x^6}{7+m}+\frac{10a^2b^2(Ab+aB)x^9}{10+m}+\frac{5ab^3(Ab+2aB)x^{12}}{13+m}+\frac{b^4(Ab+5aB)x^{15}}{16+m}+\frac{b^5Bx^{18}}{19+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a^5*A)/(1 + m) + (a^4*(5*A*b + a*B)*x^3)/(4 + m) + (5*a^3*b*(2*
A*b + a*B)*x^6)/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^9)/(10 + m) + (5*a*b^3*
(A*b + 2*a*B)*x^12)/(13 + m) + (b^4*(A*b + 5*a*B)*x^15)/(16 + m) + (b^5*B*x
^18)/(19 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(148) = 296$.
time = 0.27, size = 1077, normalized size = 7.28 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] x*(B*b^5*m^6*x^18+51*B*b^5*m^5*x^18+1005*B*b^5*m^4*x^18+A*b^5*m^6*x^15+5*B*
a*b^4*m^6*x^15+9605*B*b^5*m^3*x^18+54*A*b^5*m^5*x^15+270*B*a*b^4*m^5*x^15+4
5474*B*b^5*m^2*x^18+1110*A*b^5*m^4*x^15+5550*B*a*b^4*m^4*x^15+95064*B*b^5*m
*x^18+5*A*a*b^4*m^6*x^12+10940*A*b^5*m^3*x^15+10*B*a^2*b^3*m^6*x^12+54700*B
*a*b^4*m^3*x^15+58240*B*b^5*x^18+285*A*a*b^4*m^5*x^12+52929*A*b^5*m^2*x^15+
570*B*a^2*b^3*m^5*x^12+264645*B*a*b^4*m^2*x^15+6165*A*a*b^4*m^4*x^12+112206
*A*b^5*m*x^15+12330*B*a^2*b^3*m^4*x^12+561030*B*a*b^4*m*x^15+10*A*a^2*b^3*m
^6*x^9+63355*A*a*b^4*m^3*x^12+69160*A*b^5*x^15+10*B*a^3*b^2*m^6*x^9+126710*
B*a^2*b^3*m^3*x^12+345800*B*a*b^4*x^15+600*A*a^2*b^3*m^5*x^9+316230*A*a*b^4
*m^2*x^12+600*B*a^3*b^2*m^5*x^9+632460*B*a^2*b^3*m^2*x^12+13740*A*a^2*b^3*m
^4*x^9+684360*A*a*b^4*m*x^12+13740*B*a^3*b^2*m^4*x^9+1368720*B*a^2*b^3*m*x^
12+10*A*a^3*b^2*m^6*x^6+149600*A*a^2*b^3*m^3*x^9+425600*A*a*b^4*x^12+5*B*a^
4*b*m^6*x^6+149600*B*a^3*b^2*m^3*x^9+851200*B*a^2*b^3*x^12+630*A*a^3*b^2*m^
5*x^6+783690*A*a^2*b^3*m^2*x^9+315*B*a^4*b*m^5*x^6+783690*B*a^3*b^2*m^2*x^9
+15330*A*a^3*b^2*m^4*x^6+1753800*A*a^2*b^3*m*x^9+7665*B*a^4*b*m^4*x^6+17538
00*B*a^3*b^2*m*x^9+5*A*a^4*b*m^6*x^3+179690*A*a^3*b^2*m^3*x^6+1106560*A*a^2
*b^3*x^9+B*a^5*m^6*x^3+89845*B*a^4*b*m^3*x^6+1106560*B*a^3*b^2*x^9+330*A*a^
4*b*m^5*x^3+1021860*A*a^3*b^2*m^2*x^6+66*B*a^5*m^5*x^3+510930*B*a^4*b*m^2*x
^6+8550*A*a^4*b*m^4*x^3+2437680*A*a^3*b^2*m*x^6+1710*B*a^5*m^4*x^3+1218840*
B*a^4*b*m*x^6+A*a^5*m^6+109300*A*a^4*b*m^3*x^3+1580800*A*a^3*b^2*x^6+21860*
B*a^5*m^3*x^3+790400*B*a^4*b*x^6+69*A*a^5*m^5+702645*A*a^4*b*m^2*x^3+140529
*B*a^5*m^2*x^3+1905*A*a^5*m^4+1984770*A*a^4*b*m*x^3+396954*B*a^5*m*x^3+2679
5*A*a^5*m^3+1383200*A*a^4*b*x^3+276640*B*a^5*x^3+201174*A*a^5*m^2+757896*A*
a^5*m+1106560*A*a^5)*x^m/(1+m)/(4+m)/(7+m)/(10+m)/(13+m)/(16+m)/(19+m)
```

Maxima [A]
time = 0.34, size = 205, normalized size = 1.39

$$\frac{Bb^5x^{m+19}}{m+19} + \frac{5\,Bab^4x^{m+16}}{m+16} + \frac{Ab^5x^{m+16}}{m+16} + \frac{10\,Ba^2b^3x^{m+13}}{m+13} + \frac{5\,Aab^4x^{m+13}}{m+13} + \frac{10\,Ba^3b^2x^{m+10}}{m+10} + \frac{10\,Aa^2b^3x^{m+10}}{m+10} + \frac{5\,Ba^4bx^{m+7}}{m+7} + \frac{10\,Aa^3b^2x^{m+7}}{m+7} + \frac{Ba^5x^{m+4}}{m+4} + \frac{5\,Aa^4bx^{m+4}}{m+4} + \frac{Aa^5x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] B*b^5*x^(m + 19)/(m + 19) + 5*B*a*b^4*x^(m + 16)/(m + 16) + A*b^5*x^(m + 16
)/(m + 16) + 10*B*a^2*b^3*x^(m + 13)/(m + 13) + 5*A*a*b^4*x^(m + 13)/(m + 1
3) + 10*B*a^3*b^2*x^(m + 10)/(m + 10) + 10*A*a^2*b^3*x^(m + 10)/(m + 10) +
5*B*a^4*b*x^(m + 7)/(m + 7) + 10*A*a^3*b^2*x^(m + 7)/(m + 7) + B*a^5*x^(m +
 4)/(m + 4) + 5*A*a^4*b*x^(m + 4)/(m + 4) + A*a^5*x^(m + 1)/(m + 1)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(148) = 296$.
time = 1.66, size = 851, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b^5*m^6 + 51*B*b^5*m^5 + 1005*B*b^5*m^4 + 9605*B*b^5*m^3 + 45474*B*b^5*
m^2 + 95064*B*b^5*m + 58240*B*b^5)*x^19 + ((5*B*a*b^4 + A*b^5)*m^6 + 345800
*B*a*b^4 + 69160*A*b^5 + 54*(5*B*a*b^4 + A*b^5)*m^5 + 1110*(5*B*a*b^4 + A*b
^5)*m^4 + 10940*(5*B*a*b^4 + A*b^5)*m^3 + 52929*(5*B*a*b^4 + A*b^5)*m^2 + 1
12206*(5*B*a*b^4 + A*b^5)*m)*x^16 + 5*((2*B*a^2*b^3 + A*a*b^4)*m^6 + 170240
*B*a^2*b^3 + 85120*A*a*b^4 + 57*(2*B*a^2*b^3 + A*a*b^4)*m^5 + 1233*(2*B*a^2
*b^3 + A*a*b^4)*m^4 + 12671*(2*B*a^2*b^3 + A*a*b^4)*m^3 + 63246*(2*B*a^2*b^
3 + A*a*b^4)*m^2 + 136872*(2*B*a^2*b^3 + A*a*b^4)*m)*x^13 + 10*((B*a^3*b^2
+ A*a^2*b^3)*m^6 + 110656*B*a^3*b^2 + 110656*A*a^2*b^3 + 60*(B*a^3*b^2 + A*
a^2*b^3)*m^5 + 1374*(B*a^3*b^2 + A*a^2*b^3)*m^4 + 14960*(B*a^3*b^2 + A*a^2*
b^3)*m^3 + 78369*(B*a^3*b^2 + A*a^2*b^3)*m^2 + 175380*(B*a^3*b^2 + A*a^2*b^
3)*m)*x^10 + 5*((B*a^4*b + 2*A*a^3*b^2)*m^6 + 158080*B*a^4*b + 316160*A*a^3
*b^2 + 63*(B*a^4*b + 2*A*a^3*b^2)*m^5 + 1533*(B*a^4*b + 2*A*a^3*b^2)*m^4 +
17969*(B*a^4*b + 2*A*a^3*b^2)*m^3 + 102186*(B*a^4*b + 2*A*a^3*b^2)*m^2 + 24
3768*(B*a^4*b + 2*A*a^3*b^2)*m)*x^7 + ((B*a^5 + 5*A*a^4*b)*m^6 + 276640*B*a
^5 + 1383200*A*a^4*b + 66*(B*a^5 + 5*A*a^4*b)*m^5 + 1710*(B*a^5 + 5*A*a^4*b
)*m^4 + 21860*(B*a^5 + 5*A*a^4*b)*m^3 + 140529*(B*a^5 + 5*A*a^4*b)*m^2 + 39
6954*(B*a^5 + 5*A*a^4*b)*m)*x^4 + (A*a^5*m^6 + 69*A*a^5*m^5 + 1905*A*a^5*m^
4 + 26795*A*a^5*m^3 + 201174*A*a^5*m^2 + 757896*A*a^5*m + 1106560*A*a^5)*x)
*x^m/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m^3 + 959070*m^2 + 18644
56*m + 1106560)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5418 vs. $2(138) = 276$.
time = 1.82, size = 5418, normalized size = 36.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**5*(B*x**3+A),x)

[Out] Piecewise((-A*a**5/(18*x**18) - A*a**4*b/(3*x**15) - 5*A*a**3*b**2/(6*x**12
) - 10*A*a**2*b**3/(9*x**9) - 5*A*a*b**4/(6*x**6) - A*b**5/(3*x**3) - B*a**
5/(15*x**15) - 5*B*a**4*b/(12*x**12) - 10*B*a**3*b**2/(9*x**9) - 5*B*a**2*b
**3/(3*x**6) - 5*B*a*b**4/(3*x**3) + B*b**5*log(x), Eq(m, -19)), (-A*a**5/(
15*x**15) - 5*A*a**4*b/(12*x**12) - 10*A*a**3*b**2/(9*x**9) - 5*A*a**2*b**3
/(3*x**6) - 5*A*a*b**4/(3*x**3) + A*b**5*log(x) - B*a**5/(12*x**12) - 5*B*a
**4*b/(9*x**9) - 5*B*a**3*b**2/(3*x**6) - 10*B*a**2*b**3/(3*x**3) + 5*B*a*b
**4*log(x) + B*b**5*x**3/3, Eq(m, -16)), (-A*a**5/(12*x**12) - 5*A*a**4*b/(
9*x**9) - 5*A*a**3*b**2/(3*x**6) - 10*A*a**2*b**3/(3*x**3) + 5*A*a*b**4*log
(x) + A*b**5*x**3/3 - B*a**5/(9*x**9) - 5*B*a**4*b/(6*x**6) - 10*B*a**3*b**
2/(3*x**3) + 10*B*a**2*b**3*log(x) + 5*B*a*b**4*x**3/3 + B*b**5*x**6/6, Eq(
m, -13)), (-A*a**5/(9*x**9) - 5*A*a**4*b/(6*x**6) - 10*A*a**3*b**2/(3*x**3)
 + 10*A*a**2*b**3*log(x) + 5*A*a*b**4*x**3/3 + A*b**5*x**6/6 - B*a**5/(6*x*
*6) - 5*B*a**4*b/(3*x**3) + 10*B*a**3*b**2*log(x) + 10*B*a**2*b**3*x**3/3 +
 5*B*a*b**4*x**6/6 + B*b**5*x**9/9, Eq(m, -10)), (-A*a**5/(6*x**6) - 5*A*a*
*4*b/(3*x**3) + 10*A*a**3*b**2*log(x) + 10*A*a**2*b**3*x**3/3 + 5*A*a*b**4*
x**6/6 + A*b**5*x**9/9 - B*a**5/(3*x**3) + 5*B*a**4*b*log(x) + 10*B*a**3*b*
*2*x**3/3 + 5*B*a**2*b**3*x**6/3 + 5*B*a*b**4*x**9/9 + B*b**5*x**12/12, Eq(
m, -7)), (-A*a**5/(3*x**3) + 5*A*a**4*b*log(x) + 10*A*a**3*b**2*x**3/3 + 5*
A*a**2*b**3*x**6/3 + 5*A*a*b**4*x**9/9 + A*b**5*x**12/12 + B*a**5*log(x) +
5*B*a**4*b*x**3/3 + 5*B*a**3*b**2*x**6/3 + 10*B*a**2*b**3*x**9/9 + 5*B*a*b*
*4*x**12/12 + B*b**5*x**15/15, Eq(m, -4)), (A*a**5*log(x) + 5*A*a**4*b*x**3
/3 + 5*A*a**3*b**2*x**6/3 + 10*A*a**2*b**3*x**9/9 + 5*A*a*b**4*x**12/12 + A
*b**5*x**15/15 + B*a**5*x**3/3 + 5*B*a**4*b*x**6/6 + 10*B*a**3*b**2*x**9/9
+ 5*B*a**2*b**3*x**12/6 + B*a*b**4*x**15/3 + B*b**5*x**18/18, Eq(m, -1)), (
A*a**5*m**6*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + 69*A*a**5*m**5*x*x**m/(m**7 + 70*m**6
 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560
) + 1905*A*a**5*m**4*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2279
69*m**3 + 959070*m**2 + 1864456*m + 1106560) + 26795*A*a**5*m**3*x*x**m/(m*
*7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456
*m + 1106560) + 201174*A*a**5*m**2*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 757896*A*a**5*
m*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 1106560*A*a**5*x*x**m/(m**7 + 70*m**6 + 1974*m*
*5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 5*A*a*
*4*b*m**6*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3
+ 959070*m**2 + 1864456*m + 1106560) + 330*A*a**4*b*m**5*x**4*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 8550*A*a**4*b*m**4*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700
*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 109300*A*a**4*b*
m**3*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959
070*m**2 + 1864456*m + 1106560) + 702645*A*a**4*b*m**2*x**4*x**m/(m**7 + 70
```

```
*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 11
06560) + 1984770*A*a**4*b*m*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m
**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1383200*A*a**4*b*x
**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 10*A*a**3*b**2*m**6*x**7*x**m/(m**7 + 70*m**6 +
 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560)
+ 630*A*a**3*b**2*m**5*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 +
 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 15330*A*a**3*b**2*m**4*
x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m
**2 + 1864456*m + 1106560) + 179690*A*a**3*b**2*m**3*x**7*x**m/(m**7 + 70*m
**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106
560) + 1021860*A*a**3*b**2*m**2*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 2437680*A*a**3
*b**2*m*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
959070*m**2 + 1864456*m + 1106560) + 1580800*A*a**3*b**2*x**7*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 10*A*a**2*b**3*m**6*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 600*A*a**2*b**
3*m**5*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
959070*m**2 + 1864456*m + 1106560) + 13740*A*a**2*b**3*m**4*x**10*x**m/(m**
7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*
m + 1106560) + 149600*A*a**2*b**3*m**3*x**10*x**m/(m**7 + 70*m**6 + 1974*m*
*5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 783690
*A*a**2*b**3*m**2*x**10*x**m/(m**7 + 70*m**6 + ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1331 vs. $2(148) = 296$.
time = 2.64, size = 1331, normalized size = 8.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b^5*m^6*x^19*x^m + 51*B*b^5*m^5*x^19*x^m + 1005*B*b^5*m^4*x^19*x^m + 5*B
*a*b^4*m^6*x^16*x^m + A*b^5*m^6*x^16*x^m + 9605*B*b^5*m^3*x^19*x^m + 270*B*
a*b^4*m^5*x^16*x^m + 54*A*b^5*m^5*x^16*x^m + 45474*B*b^5*m^2*x^19*x^m + 555
0*B*a*b^4*m^4*x^16*x^m + 1110*A*b^5*m^4*x^16*x^m + 95064*B*b^5*m*x^19*x^m +
 10*B*a^2*b^3*m^6*x^13*x^m + 5*A*a*b^4*m^6*x^13*x^m + 54700*B*a*b^4*m^3*x^1
6*x^m + 10940*A*b^5*m^3*x^16*x^m + 58240*B*b^5*x^19*x^m + 570*B*a^2*b^3*m^5
*x^13*x^m + 285*A*a*b^4*m^5*x^13*x^m + 264645*B*a*b^4*m^2*x^16*x^m + 52929*
A*b^5*m^2*x^16*x^m + 12330*B*a^2*b^3*m^4*x^13*x^m + 6165*A*a*b^4*m^4*x^13*x
^m + 561030*B*a*b^4*m*x^16*x^m + 112206*A*b^5*m*x^16*x^m + 10*B*a^3*b^2*m^6
*x^10*x^m + 10*A*a^2*b^3*m^6*x^10*x^m + 126710*B*a^2*b^3*m^3*x^13*x^m + 633
55*A*a*b^4*m^3*x^13*x^m + 345800*B*a*b^4*x^16*x^m + 69160*A*b^5*x^16*x^m +
```

```
600*B*a^3*b^2*m^5*x^10*x^m + 600*A*a^2*b^3*m^5*x^10*x^m + 632460*B*a^2*b^3*
m^2*x^13*x^m + 316230*A*a*b^4*m^2*x^13*x^m + 13740*B*a^3*b^2*m^4*x^10*x^m +
 13740*A*a^2*b^3*m^4*x^10*x^m + 1368720*B*a^2*b^3*m*x^13*x^m + 684360*A*a*b
^4*m*x^13*x^m + 5*B*a^4*b*m^6*x^7*x^m + 10*A*a^3*b^2*m^6*x^7*x^m + 149600*B
*a^3*b^2*m^3*x^10*x^m + 149600*A*a^2*b^3*m^3*x^10*x^m + 851200*B*a^2*b^3*x^
13*x^m + 425600*A*a*b^4*x^13*x^m + 315*B*a^4*b*m^5*x^7*x^m + 630*A*a^3*b^2*
m^5*x^7*x^m + 783690*B*a^3*b^2*m^2*x^10*x^m + 783690*A*a^2*b^3*m^2*x^10*x^m
 + 7665*B*a^4*b*m^4*x^7*x^m + 15330*A*a^3*b^2*m^4*x^7*x^m + 1753800*B*a^3*b
^2*m*x^10*x^m + 1753800*A*a^2*b^3*m*x^10*x^m + B*a^5*m^6*x^4*x^m + 5*A*a^4*
b*m^6*x^4*x^m + 89845*B*a^4*b*m^3*x^7*x^m + 179690*A*a^3*b^2*m^3*x^7*x^m +
1106560*B*a^3*b^2*x^10*x^m + 1106560*A*a^2*b^3*x^10*x^m + 66*B*a^5*m^5*x^4*
x^m + 330*A*a^4*b*m^5*x^4*x^m + 510930*B*a^4*b*m^2*x^7*x^m + 1021860*A*a^3*
b^2*m^2*x^7*x^m + 1710*B*a^5*m^4*x^4*x^m + 8550*A*a^4*b*m^4*x^4*x^m + 12188
40*B*a^4*b*m*x^7*x^m + 2437680*A*a^3*b^2*m*x^7*x^m + A*a^5*m^6*x*x^m + 2186
0*B*a^5*m^3*x^4*x^m + 109300*A*a^4*b*m^3*x^4*x^m + 790400*B*a^4*b*x^7*x^m +
 1580800*A*a^3*b^2*x^7*x^m + 69*A*a^5*m^5*x*x^m + 140529*B*a^5*m^2*x^4*x^m
+ 702645*A*a^4*b*m^2*x^4*x^m + 1905*A*a^5*m^4*x*x^m + 396954*B*a^5*m*x^4*x^
m + 1984770*A*a^4*b*m*x^4*x^m + 26795*A*a^5*m^3*x*x^m + 276640*B*a^5*x^4*x^
m + 1383200*A*a^4*b*x^4*x^m + 201174*A*a^5*m^2*x*x^m + 757896*A*a^5*m*x*x^m
 + 1106560*A*a^5*x*x^m)/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m^3 +
 959070*m^2 + 1864456*m + 1106560)
```

Mupad [B]
time = 3.21, size = 559, normalized size = 3.78

$$\frac{B\,b^5\,x^m\,x^{19}\,\left(m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{a^4\,x^m\,x^4\,\left(5\,A\,b+B\,a\right)\,\left(m^6+66\,m^5+1710\,m^4+21860\,m^3+140529\,m^2+396954\,m+276640\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{b^4\,x^m\,x^{16}\,\left(A\,b+5\,B\,a\right)\,\left(m^6+54\,m^5+1110\,m^4+10940\,m^3+52929\,m^2+112206\,m+69160\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{A\,a^5\,x\,x^m\,\left(m^6+69\,m^5+1905\,m^4+26795\,m^3+201174\,m^2+757896\,m+1106560\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{10\,a^2\,b^2\,x^m\,x^{10}\,\left(A\,b+B\,a\right)\,\left(m^6+60\,m^5+1374\,m^4+14960\,m^3+78369\,m^2+175380\,m+110656\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{5\,a\,b^3\,x^m\,x^{13}\,\left(A\,b+2\,B\,a\right)\,\left(m^6+57\,m^5+1233\,m^4+12671\,m^3+63246\,m^2+136872\,m+85120\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{5\,a^3\,b\,x^m\,x^7\,\left(2\,A\,b+B\,a\right)\,\left(m^6+63\,m^5+1533\,m^4+17969\,m^3+102186\,m^2+243768\,m+158080\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] (B*b^5*x^m*x^19*(95064*m + 45474*m^2 + 9605*m^3 + 1005*m^4 + 51*m^5 + m^6 +
 58240))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m
^6 + m^7 + 1106560) + (a^4*x^m*x^4*(5*A*b + B*a)*(396954*m + 140529*m^2 + 2
1860*m^3 + 1710*m^4 + 66*m^5 + m^6 + 276640))/(1864456*m + 959070*m^2 + 227
969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (b^4*x^m*x^16*(A
*b + 5*B*a)*(112206*m + 52929*m^2 + 10940*m^3 + 1110*m^4 + 54*m^5 + m^6 + 6
9160))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6
 + m^7 + 1106560) + (A*a^5*x*x^m*(757896*m + 201174*m^2 + 26795*m^3 + 1905*
m^4 + 69*m^5 + m^6 + 1106560))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700
*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (10*a^2*b^2*x^m*x^10*(A*b + B*a
)*(175380*m + 78369*m^2 + 14960*m^3 + 1374*m^4 + 60*m^5 + m^6 + 110656))/(1
864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 +
1106560) + (5*a*b^3*x^m*x^13*(A*b + 2*B*a)*(136872*m + 63246*m^2 + 12671*m^
3 + 1233*m^4 + 57*m^5 + m^6 + 85120))/(1864456*m + 959070*m^2 + 227969*m^3
+ 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (5*a^3*b*x^m*x^7*(2*A*b
```

```
+ B*a)*(243768*m + 102186*m^2 + 17969*m^3 + 1533*m^4 + 63*m^5 + m^6 + 15808
0))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 +
m^7 + 1106560)
```

3.125 $\int x^m (a+bx^3)^2 (A+Bx^3)\, dx$

Optimal. Leaf size=71

$$\frac{a^2 A x^{1+m}}{1+m} + \frac{a(2Ab+aB)x^{4+m}}{4+m} + \frac{b(Ab+2aB)x^{7+m}}{7+m} + \frac{b^2 B x^{10+m}}{10+m}$$

```
[Out] a^2*A*x^(1+m)/(1+m)+a*(2*A*b+B*a)*x^(4+m)/(4+m)+b*(A*b+2*B*a)*x^(7+m)/(7+m)
+b^2*B*x^(10+m)/(10+m)
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$\frac{a^2 A x^{m+1}}{m+1} + \frac{a x^{m+4}(aB+2Ab)}{m+4} + \frac{b x^{m+7}(2aB+Ab)}{m+7} + \frac{b^2 B x^{m+10}}{m+10}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(4 + m))/(4 + m) + (b*(A*b +
 2*a*B)*x^(7 + m))/(7 + m) + (b^2*B*x^(10 + m))/(10 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int x^m (a+bx^3)^2 (A+Bx^3)\, dx &= \int \left(a^2 A x^m + a(2Ab+aB)x^{3+m} + b(Ab+2aB)x^{6+m} + b^2 B x^{9+m}\right) dx \\ &= \frac{a^2 A x^{1+m}}{1+m} + \frac{a(2Ab+aB)x^{4+m}}{4+m} + \frac{b(Ab+2aB)x^{7+m}}{7+m} + \frac{b^2 B x^{10+m}}{10+m} \end{aligned}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.93

$$x^{1+m}\left(\frac{a^2 A}{1+m} + \frac{a(2Ab+aB)x^3}{4+m} + \frac{b(Ab+2aB)x^6}{7+m} + \frac{b^2 B x^9}{10+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^3)/(4 + m) + (b*(A*b + 2*a*
B)*x^6)/(7 + m) + (b^2*B*x^9)/(10 + m))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(71) = 142$.
time = 0.29, size = 261, normalized size = 3.68

method	result
risch	$\frac{x\left(B\,b^2m^3x^9+12B\,b^2m^2x^9+39m\,x^9b^2B+A\,b^2m^3x^6+2Bab\,m^3x^6+28b^2B\,x^9+15A\,b^2m^2x^6+30Bab\,m^2x^6+54A\,b^2x^6m+108Bab\,x^6m\right.}{}$
gosper	$\frac{x^{1+m}\left(B\,b^2m^3x^9+12B\,b^2m^2x^9+39m\,x^9b^2B+A\,b^2m^3x^6+2Bab\,m^3x^6+28b^2B\,x^9+15A\,b^2m^2x^6+30Bab\,m^2x^6+54A\,b^2x^6m+108Bab\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] x*(B*b^2*m^3*x^9+12*B*b^2*m^2*x^9+39*B*b^2*m*x^9+A*b^2*m^3*x^6+2*B*a*b*m^3*
x^6+28*B*b^2*x^9+15*A*b^2*m^2*x^6+30*B*a*b*m^2*x^6+54*A*b^2*m*x^6+108*B*a*b
*m*x^6+2*A*a*b*m^3*x^3+40*A*b^2*x^6+B*a^2*m^3*x^3+80*B*a*b*x^6+36*A*a*b*m^2
*x^3+18*B*a^2*m^2*x^3+174*A*a*b*m*x^3+87*B*a^2*m*x^3+A*a^2*m^3+140*A*a*b*x^
3+70*B*a^2*x^3+21*A*a^2*m^2+138*A*a^2*m+280*A*a^2)*x^m/(10+m)/(7+m)/(4+m)/(
1+m)
```

Maxima [**A**]
time = 0.28, size = 91, normalized size = 1.28

$$\frac{Bb^2x^{m+10}}{m+10}+\frac{2\,Babx^{m+7}}{m+7}+\frac{Ab^2x^{m+7}}{m+7}+\frac{Ba^2x^{m+4}}{m+4}+\frac{2\,Aabx^{m+4}}{m+4}+\frac{Aa^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] B*b^2*x^(m + 10)/(m + 10) + 2*B*a*b*x^(m + 7)/(m + 7) + A*b^2*x^(m + 7)/(m
+ 7) + B*a^2*x^(m + 4)/(m + 4) + 2*A*a*b*x^(m + 4)/(m + 4) + A*a^2*x^(m + 1
)/(m + 1)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(71) = 142$.
time = 2.89, size = 215, normalized size = 3.03

$$\frac{\left((Bb^2m^3+12\,Bb^2m^2+39\,Bb^2m+28\,Bb^2)x^{10}+((2\,Bab+Ab^2)m^3+80\,Bab+40\,Ab^2+15\,(2\,Bab+Ab^2)m^2+54\,(2\,Bab+Ab^2)m)x^7+((Ba^2+2\,Aab)m^3+70\,Ba^2+140\,Aab+18\,(Ba^2+2\,Aab)m^2+87\,(Ba^2+2\,Aab)m)x^4+(Aa^2m^3+21\,Aa^2m^2+138\,Aa^2m+280\,Aa^2)x\right)x^m}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")

[Out] ((B*b^2*m^3 + 12*B*b^2*m^2 + 39*B*b^2*m + 28*B*b^2)*x^10 + ((2*B*a*b + A*b^2)*m^3 + 80*B*a*b + 40*A*b^2 + 15*(2*B*a*b + A*b^2)*m^2 + 54*(2*B*a*b + A*b^2)*m)*x^7 + ((B*a^2 + 2*A*a*b)*m^3 + 70*B*a^2 + 140*A*a*b + 18*(B*a^2 + 2*A*a*b)*m^2 + 87*(B*a^2 + 2*A*a*b)*m)*x^4 + (A*a^2*m^3 + 21*A*a^2*m^2 + 138*A*a^2*m + 280*A*a^2)*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(63) = 126$.

time = 0.57, size = 1057, normalized size = 14.89

$$\begin{cases} -\frac{Aa^2}{9x^9} - \frac{Aab}{3x^6} - \frac{Ab^2}{3x^3} - \frac{Ba^2}{6x^6} - \frac{2Bab}{3x^3} + Bb^2\log(x) & \text{for}\: m = -10 \\ -\frac{Aa^2}{6x^6} - \frac{2Aab}{3x^3} + Ab^2\log(x) - \frac{Ba^2}{3x^3} + 2Bab\log(x) + \frac{Bb^2x^3}{3} & \text{for}\: m = -7 \\ -\frac{Aa^2}{3x^3} + 2Aab\log(x) + \frac{Ab^2x^3}{3} + Ba^2\log(x) + \frac{2Babx^3}{3} + \frac{Bb^2x^6}{6} & \text{for}\: m = -4 \\ Aa^2\log(x) + \frac{2Aabx^3}{3} + \frac{Ab^2x^6}{6} + \frac{Ba^2x^3}{3} + \frac{Babx^6}{3} + \frac{Bb^2x^9}{9} & \text{for}\: m = -1 \\ \frac{Aa^2m^3xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{21Aa^2m^2xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{138Aa^2mxx^m}{m^4+22m^3+159m^2+418m+280} + \frac{280Aa^2xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{2Aabm^3x^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{36Aabm^2x^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{174Aabmx^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{140Aabx^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{Ab^2m^3x^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{15Ab^2m^2x^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{54Ab^2mx^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{40Ab^2x^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{Ba^2m^3x^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{18Ba^2m^2x^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{87Ba^2mx^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{70Ba^2x^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{2Babm^3x^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{30Babm^2x^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{108Babmx^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{80Babx^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{Bb^2m^3x^{10}x^m}{m^4+22m^3+159m^2+418m+280} + \frac{12Bb^2m^2x^{10}x^m}{m^4+22m^3+159m^2+418m+280} + \frac{39Bb^2mx^{10}x^m}{m^4+22m^3+159m^2+418m+280} + \frac{28Bb^2x^{10}x^m}{m^4+22m^3+159m^2+418m+280} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*(b*x**3+a)**2*(B*x**3+A),x)

[Out] Piecewise((-A*a**2/(9*x**9) - A*a*b/(3*x**6) - A*b**2/(3*x**3) - B*a**2/(6*x**6) - 2*B*a*b/(3*x**3) + B*b**2*log(x), Eq(m, -10)), (-A*a**2/(6*x**6) - 2*A*a*b/(3*x**3) + A*b**2*log(x) - B*a**2/(3*x**3) + 2*B*a*b*log(x) + B*b**2*x**3/3, Eq(m, -7)), (-A*a**2/(3*x**3) + 2*A*a*b*log(x) + A*b**2*x**3/3 + B*a**2*log(x) + 2*B*a*b*x**3/3 + B*b**2*x**6/6, Eq(m, -4)), (A*a**2*log(x) + 2*A*a*b*x**3/3 + A*b**2*x**6/6 + B*a**2*x**3/3 + B*a*b*x**6/3 + B*b**2*x**9/9, Eq(m, -1)), (A*a**2*m**3*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 21*A*a**2*m**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 138*A*a**2*m*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 280*A*a**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 2*A*a*b*m**3*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 36*A*a*b*m**2*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 174*A*a*b*m*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 140*A*a*b*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + A*b**2*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 15*A*b**2*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 54*A*b**2*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 40*A*b**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*a**2*m**3*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 18*B*a**2*m**2*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 87*B*a**2*m*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 70*B*a**2*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 2*B*a*b*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 30*B*a*b*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 108*B*a*b*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 80*B*a*b*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*b**2*m**3*x**10*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 12*B*b**2*m**2*x**10*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 39*B*b**2*m*x**10*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 28*B*b**2*x**10*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(71) = 142.
time = 1.33, size = 332, normalized size = 4.68

$$\frac{Bb^2m^3x^{10}x^m + 12\,Bb^2m^2x^{10}x^m + 39\,Bb^2mx^{10}x^m + 2\,Babm^3x^7x^m + Ab^2m^3x^7x^m + 28\,Bb^2x^{10}x^m + 30\,Babm^2x^7x^m + 15\,Ab^2m^2x^7x^m + 108\,Babmx^7x^m + 54\,Ab^2mx^7x^m + Ba^2m^3x^4x^m + 2\,Aabm^3x^4x^m + 80\,Babx^7x^m + 40\,Ab^2x^7x^m + 18\,Ba^2m^2x^4x^m + 36\,Aabm^2x^4x^m + 87\,Ba^2mx^4x^m + 174\,Aabmx^4x^m + Aa^2m^3xx^m + 70\,Ba^2x^4x^m + 140\,Aabx^4x^m + 21\,Aa^2m^2xx^m + 138\,Aa^2mxx^m + 280\,Aa^2xx^m}{m^4 + 22\,m^3 + 159\,m^2 + 418\,m + 280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b^2*m^3*x^10*x^m + 12*B*b^2*m^2*x^10*x^m + 39*B*b^2*m*x^10*x^m + 2*B*a*b
*m^3*x^7*x^m + A*b^2*m^3*x^7*x^m + 28*B*b^2*x^10*x^m + 30*B*a*b*m^2*x^7*x^m
 + 15*A*b^2*m^2*x^7*x^m + 108*B*a*b*m*x^7*x^m + 54*A*b^2*m*x^7*x^m + B*a^2*
m^3*x^4*x^m + 2*A*a*b*m^3*x^4*x^m + 80*B*a*b*x^7*x^m + 40*A*b^2*x^7*x^m + 1
8*B*a^2*m^2*x^4*x^m + 36*A*a*b*m^2*x^4*x^m + 87*B*a^2*m*x^4*x^m + 174*A*a*b
*m*x^4*x^m + A*a^2*m^3*x*x^m + 70*B*a^2*x^4*x^m + 140*A*a*b*x^4*x^m + 21*A*
a^2*m^2*x*x^m + 138*A*a^2*m*x*x^m + 280*A*a^2*x*x^m)/(m^4 + 22*m^3 + 159*m^
2 + 418*m + 280)
```

Mupad [B]
time = 2.72, size = 177, normalized size = 2.49

$$x^m\left(\frac{B\,b^2\,x^{10}\,\left(m^3+12\,m^2+39\,m+28\right)}{m^4+22\,m^3+159\,m^2+418\,m+280}+\frac{A\,a^2\,x\,\left(m^3+21\,m^2+138\,m+280\right)}{m^4+22\,m^3+159\,m^2+418\,m+280}+\frac{a\,x^4\,\left(2\,A\,b+B\,a\right)\,\left(m^3+18\,m^2+87\,m+70\right)}{m^4+22\,m^3+159\,m^2+418\,m+280}+\frac{b\,x^7\,\left(A\,b+2\,B\,a\right)\,\left(m^3+15\,m^2+54\,m+40\right)}{m^4+22\,m^3+159\,m^2+418\,m+280}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^m*((B*b^2*x^10*(39*m + 12*m^2 + m^3 + 28))/(418*m + 159*m^2 + 22*m^3 + m^
4 + 280) + (A*a^2*x*(138*m + 21*m^2 + m^3 + 280))/(418*m + 159*m^2 + 22*m^3
 + m^4 + 280) + (a*x^4*(2*A*b + B*a)*(87*m + 18*m^2 + m^3 + 70))/(418*m + 1
59*m^2 + 22*m^3 + m^4 + 280) + (b*x^7*(A*b + 2*B*a)*(54*m + 15*m^2 + m^3 +
40))/(418*m + 159*m^2 + 22*m^3 + m^4 + 280))
```

3.126 $\int x^m(a+bx^3)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=45

$$\frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{4+m}}{4+m}+\frac{bBx^{7+m}}{7+m}$$

```
[Out] a*A*x^(1+m)/(1+m)+(A*b+B*a)*x^(4+m)/(4+m)+b*B*x^(7+m)/(7+m)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.056$, Rules used = {459}

$$\frac{x^{m+4}(aB+Ab)}{m+4}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(4 + m))/(4 + m) + (b*B*x^(7 + m))
/(7 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx^m+(Ab+aB)x^{3+m}+bBx^{6+m}\right)\,dx\\&=\frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{4+m}}{4+m}+\frac{bBx^{7+m}}{7+m}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 0.93

$$x^{1+m}\left(\frac{aA}{1+m}+\frac{(Ab+aB)x^3}{4+m}+\frac{bBx^6}{7+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a*A)/(1 + m) + ((A*b + a*B)*x^3)/(4 + m) + (b*B*x^6)/(7 + m))
```

Maple [A]
time = 0.03, size = 53, normalized size = 1.18

method	result	size
norman	$\frac{(Ab+Ba)x^4 e^{m\ln(x)}}{4+m}+\frac{Aax\,e^{m\ln(x)}}{1+m}+\frac{Bb\,x^7 e^{m\ln(x)}}{7+m}$	53
risch	$\frac{x\left(Bb\,m^2x^6+5Bbm\,x^6+4bB\,x^6+Ab\,m^2x^3+Ba\,m^2x^3+8Abm\,x^3+8Bam\,x^3+7Ab\,x^3+7Ba\,x^3+Aa\,m^2+11Aam+28Aa\right)x^m}{(7+m)(4+m)(1+m)}$	109
gosper	$\frac{x^{1+m}\left(Bb\,m^2x^6+5Bbm\,x^6+4bB\,x^6+Ab\,m^2x^3+Ba\,m^2x^3+8Abm\,x^3+8Bam\,x^3+7Ab\,x^3+7Ba\,x^3+Aa\,m^2+11Aam+28Aa\right)}{(7+m)(4+m)(1+m)}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] (A*b+B*a)/(4+m)*x^4*exp(m*ln(x))+A*a/(1+m)*x*exp(m*ln(x))+B*b/(7+m)*x^7*exp
(m*ln(x))
```

Maxima [A]
time = 0.28, size = 53, normalized size = 1.18

$$\frac{Bbx^{m+7}}{m+7}+\frac{Bax^{m+4}}{m+4}+\frac{Abx^{m+4}}{m+4}+\frac{Aax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] B*b*x^(m + 7)/(m + 7) + B*a*x^(m + 4)/(m + 4) + A*b*x^(m + 4)/(m + 4) + A*a
*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.
time = 1.91, size = 92, normalized size = 2.04

$$\frac{((Bbm^2+5\,Bbm+4\,Bb)x^7+((Ba+Ab)m^2+7\,Ba+7\,Ab+8\,(Ba+Ab)m)x^4+(Aam^2+11\,Aam+28\,Aa)x)x^m}{m^3+12\,m^2+39\,m+28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b*m^2 + 5*B*b*m + 4*B*b)*x^7 + ((B*a + A*b)*m^2 + 7*B*a + 7*A*b + 8*(B*
a + A*b)*m)*x^4 + (A*a*m^2 + 11*A*a*m + 28*A*a)*x)*x^m/(m^3 + 12*m^2 + 39*m
 + 28)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(37) = 74$.
time = 0.33, size = 410, normalized size = 9.11

$$\begin{cases} -\frac{Aa}{6x^6} - \frac{Ab}{3x^3} - \frac{Ba}{3x^3} + Bb\log(x) & \text{for } m = -7 \\ -\frac{Aa}{3x^3} + Ab\log(x) + Ba\log(x) + \frac{Bbx^3}{3} & \text{for } m = -4 \\ Aa\log(x) + \frac{Abx^3}{3} + \frac{Bax^3}{3} + \frac{Bbx^6}{6} & \text{for } m = -1 \\ \frac{Aam^2xx^m}{m^3+12m^2+39m+28} + \frac{11Aamxx^m}{m^3+12m^2+39m+28} + \frac{28Aaxx^m}{m^3+12m^2+39m+28} + \frac{Abm^2x^4x^m}{m^3+12m^2+39m+28} + \frac{8Abmx^4x^m}{m^3+12m^2+39m+28} + \frac{7Abx^4x^m}{m^3+12m^2+39m+28} + \frac{Bam^2x^4x^m}{m^3+12m^2+39m+28} + \frac{8Bamx^4x^m}{m^3+12m^2+39m+28} + \frac{7Bax^4x^m}{m^3+12m^2+39m+28} + \frac{Bbm^2x^7x^m}{m^3+12m^2+39m+28} + \frac{5Bbmx^7x^m}{m^3+12m^2+39m+28} + \frac{4Bbx^7x^m}{m^3+12m^2+39m+28} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a/(6*x**6) - A*b/(3*x**3) - B*a/(3*x**3) + B*b*log(x), Eq(m,
-7)), (-A*a/(3*x**3) + A*b*log(x) + B*a*log(x) + B*b*x**3/3, Eq(m, -4)), (A
*a*log(x) + A*b*x**3/3 + B*a*x**3/3 + B*b*x**6/6, Eq(m, -1)), (A*a*m**2*x*x
**m/(m**3 + 12*m**2 + 39*m + 28) + 11*A*a*m*x*x**m/(m**3 + 12*m**2 + 39*m +
 28) + 28*A*a*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + A*b*m**2*x**4*x**m/(m**
3 + 12*m**2 + 39*m + 28) + 8*A*b*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) +
 7*A*b*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*a*m**2*x**4*x**m/(m**3 +
12*m**2 + 39*m + 28) + 8*B*a*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 7*B
*a*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*b*m**2*x**7*x**m/(m**3 + 12*m
**2 + 39*m + 28) + 5*B*b*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*B*b*x
**7*x**m/(m**3 + 12*m**2 + 39*m + 28), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(45) = 90$.
time = 1.30, size = 143, normalized size = 3.18

$$\frac{Bbm^2x^7x^m + 5\,Bbmx^7x^m + 4\,Bbx^7x^m + Bam^2x^4x^m + Abm^2x^4x^m + 8\,Bamx^4x^m + 8\,Abmx^4x^m + 7\,Bax^4x^m + 7\,Abx^4x^m + Aam^2xx^m + 11\,Aamxx^m + 28\,Aaxx^m}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b*m^2*x^7*x^m + 5*B*b*m*x^7*x^m + 4*B*b*x^7*x^m + B*a*m^2*x^4*x^m + A*b*
m^2*x^4*x^m + 8*B*a*m*x^4*x^m + 8*A*b*m*x^4*x^m + 7*B*a*x^4*x^m + 7*A*b*x^4
*x^m + A*a*m^2*x*x^m + 11*A*a*m*x*x^m + 28*A*a*x*x^m)/(m^3 + 12*m^2 + 39*m
+ 28)
```

Mupad [**B**]
time = 2.65, size = 95, normalized size = 2.11

$$x^m\left(\frac{x^4\,\left(A\,b+B\,a\right)\,\left(m^2+8\,m+7\right)}{m^3+12\,m^2+39\,m+28}+\frac{B\,b\,x^7\,\left(m^2+5\,m+4\right)}{m^3+12\,m^2+39\,m+28}+\frac{A\,a\,x\,\left(m^2+11\,m+28\right)}{m^3+12\,m^2+39\,m+28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] x^m*((x^4*(A*b + B*a)*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (B*b*x^
7*(5*m + m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (A*a*x*(11*m + m^2 + 28))/(
39*m + 12*m^2 + m^3 + 28))
```

3.127 $\int \frac{x^m\left(A+Bx^3\right)}{a+bx^3}\,dx$

Optimal. Leaf size=66

$$\frac{Bx^{1+m}}{b(1+m)}+\frac{(Ab-aB)x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{ab(1+m)}$$

```
[Out] B*x^(1+m)/b/(1+m)+(A*b-B*a)*x^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b
*x^3/a)/a/b/(1+m)
```

Rubi [A]

time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {470, 371}

$$\frac{x^{m+1}(Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric2F1[1, (1
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(a*b*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^{1+m}}{b(1+m)} - \frac{(-Ab(1+m)+aB(1+m))\int \frac{x^m}{a+bx^3}\,dx}{b(1+m)}$$
$$= \frac{Bx^{1+m}}{b(1+m)} + \frac{(Ab-aB)x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{ab(1+m)}$$

Mathematica [A]
time = 0.10, size = 55, normalized size = 0.83

$$\frac{x^{1+m}\left(aB+(Ab-aB)\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)\right)}{ab(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3),x]
[Out] (x^(1 + m)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -(
(b*x^3)/a)]))/(a*b*(1 + m))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^m(B\,x^3+A)}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^3+A)/(b*x^3+a),x)
[Out] int(x^m*(B*x^3+A)/(b*x^3+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.35, size = 190, normalized size = 2.88

$$\frac{Amxx^{m}\Phi\left(\frac{bx^{3}e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{Axx^{m}\Phi\left(\frac{bx^{3}e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{Bmx^{4}x^{m}\Phi\left(\frac{bx^{3}e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}+\frac{4Bx^{4}x^{m}\Phi\left(\frac{bx^{3}e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] A*m*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3
)/(9*a*gamma(m/3 + 4/3)) + A*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m
/3 + 1/3)*gamma(m/3 + 1/3)/(9*a*gamma(m/3 + 4/3)) + B*m*x**4*x**m*lerchphi(
b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(9*a*gamma(m/3 + 7
/3)) + 4*B*x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma
(m/3 + 4/3)/(9*a*gamma(m/3 + 7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(B\,x^3+A\right)}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] int((x^m*(A + B*x^3))/(a + b*x^3), x)
```

3.128 $\int \frac{x^m \left(A+Bx^3\right)}{\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=93

$$\frac{(Ab-aB)x^{1+m}}{3ab\left(a+bx^3\right)}+\frac{(Ab(2-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{3a^2b(1+m)}$$

```
[Out] 1/3*(A*b-B*a)*x^(1+m)/a/b/(b*x^3+a)+1/3*(A*b*(-m+2)+a*B*(1+m))*x^(1+m)*hype
rgeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^2/b/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {468, 371}

$$\frac{x^{m+1}(aB(m+1)+Ab(2-m))\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{3a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(3*a*b*(a + b*x^3)) + ((A*b*(2 - m) + a*B*(1 + m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a^2*
b*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^{1+m}}{3ab\,(a+bx^3)} + \frac{(-Ab(-2+m)+aB(1+m))\int \frac{x^m}{a+bx^3}\,dx}{3ab}$$
$$= \frac{(Ab-aB)x^{1+m}}{3ab\,(a+bx^3)} + \frac{(Ab(2-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{3a^2b(1+m)}$$

Mathematica [A]
time = 0.17, size = 80, normalized size = 0.86

$$\frac{x^{1+m}\left(aB\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)+(Ab-aB)\,{}_2F_1\left(2,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)\right)}{a^2b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
(A*b - a*B)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a^2
*b*(1 + m))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^m(B\,x^3+A)}{(b\,x^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
[Out] int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*x^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 142.31, size = 1047, normalized size = 11.26

$$A\left(-\frac{am^2xx^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{amxx^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{3amxx^m\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{2axx^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{3axx^m\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}-\frac{bm^2x^4x^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{bmx^4x^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{2bx^4x^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}\right)+B\left(-\frac{am^2x^4x^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}-\frac{5amx^4x^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}+\frac{3amx^4x^m\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}-\frac{4ax^4x^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}+\frac{12ax^4x^m\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}-\frac{bm^2x^7x^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}-\frac{5bmx^7x^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}-\frac{4bx^7x^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] A*(-a*m**2*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/
3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + a*m
*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(
27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 3*a*m*x*x**m*
gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3
)) + 2*a*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3
+ 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 3*a*x
*x**m*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3
 + 4/3)) - b*m**2*x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3
)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4
/3)) + b*m*x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma
(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) +
2*b*x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 +
1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3))) + B*(-a*
m**2*x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 +
 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) - 5*a*m*
x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)
/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) + 3*a*m*x**4*
x**m*gamma(m/3 + 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3
+ 7/3)) - 4*a*x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*ga
mma(m/3 + 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3))
 + 12*a*x**4*x**m*gamma(m/3 + 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x*
*3*gamma(m/3 + 7/3)) - b*m**2*x**7*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a,
1, m/3 + 4/3)*gamma(m/3 + 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*g
amma(m/3 + 7/3)) - 5*b*m*x**7*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/
3 + 4/3)*gamma(m/3 + 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(
m/3 + 7/3)) - 4*b*x**7*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3
)*gamma(m/3 + 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7
/3)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,(B\,x^3+A)}{(b\,x^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] int((x^m*(A + B*x^3))/(a + b*x^3)^2, x)
```

3.129 $\int \frac{x^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=93

$$\frac{(Ab-aB)x^{1+m}}{6ab\left(a+bx^3\right)^2}+\frac{(Ab(5-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{6a^3b(1+m)}$$

```
[Out] 1/6*(A*b-B*a)*x^(1+m)/a/b/(b*x^3+a)^2+1/6*(A*b*(5-m)+a*B*(1+m))*x^(1+m)*hyp
ergeom([2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/b/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {468, 371}

$$\frac{x^{m+1}(aB(m+1)+Ab(5-m))\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{6a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(6*a*b*(a + b*x^3)^2) + ((A*b*(5 - m) + a*B*(1 + m)
)*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(6*a^
3*b*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^{1+m}}{6ab\,(a+bx^3)^2} + \frac{(-Ab(-5+m)+aB(1+m))\int \frac{x^m}{(a+bx^3)^2}\,dx}{6ab}$$
$$= \frac{(Ab-aB)x^{1+m}}{6ab\,(a+bx^3)^2} + \frac{(Ab(5-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{6a^3b(1+m)}$$

Mathematica [A]
time = 0.22, size = 80, normalized size = 0.86

$$\frac{x^{1+m}\left(aB\,{}_2F_1\left(2,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)+(Ab-aB)\,{}_2F_1\left(3,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)\right)}{a^3b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
(A*b - a*B)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a^3
*b*(1 + m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m(B\,x^3+A)}{(b\,x^3+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*x^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] int((x^m*(A + B*x^3))/(a + b*x^3)^3, x)
```

3.130 $\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=112

$$\frac{b(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a(bc-ad)e(1+m)}-\frac{d(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{dx^3}{c}\right)}{c(bc-ad)e(1+m)}$$

```
[Out] b*(e*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/(-a*d+b*c)/e
/(1+m)-d*(e*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-d*x^3/c)/c/(-a*d
+b*c)/e/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {493, 371}

$$\frac{b(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{dx^3}{c}\right)}{ce(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/
(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3
, (4 + m)/3, -((d*x^3)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \frac{b\int \frac{(ex)^m}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{(ex)^m}{c+dx^3}\,dx}{bc-ad}$$
$$= \frac{b(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a(bc-ad)e(1+m)} - \frac{d(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{dx^3}{c}\right)}{c(bc-ad)e(1+m)}$$

Mathematica [A]
time = 0.09, size = 86, normalized size = 0.77

$$\frac{x(ex)^m\left(-bc\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)+ad\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{dx^3}{c}\right)\right)}{ac(-bc+ad)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])
 + a*d*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((d*x^3)/c)]))/(a*c*(-(b
*c) + a*d)*(1 + m))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(b\,x^3+a)\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((x*e)^m/(b*d*x^6 + (b*c + a*d)*x^3 + a*c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((x*e)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{(b\,x^3+a)\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] int((e*x)^m/((a + b*x^3)*(c + d*x^3)), x)
```

3.131 $\int x^{7/2}(a+bx^3)(A+Bx^3)\,dx$

Optimal. Leaf size=39

$$\frac{2}{9}aAx^{9/2}+\frac{2}{15}(Ab+aB)x^{15/2}+\frac{2}{21}bBx^{21/2}$$

```
[Out] 2/9*a*A*x^(9/2)+2/15*(A*b+B*a)*x^(15/2)+2/21*b*B*x^(21/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {459}

$$\frac{2}{15}x^{15/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{21}bBx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(15/2))/15 + (2*b*B*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx^{7/2}+(Ab+aB)x^{13/2}+bBx^{19/2}\right)\,dx\\&=\frac{2}{9}aAx^{9/2}+\frac{2}{15}(Ab+aB)x^{15/2}+\frac{2}{21}bBx^{21/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.90

$$\frac{2}{315}x^{9/2}\left(35aA+21Abx^3+21aBx^3+15bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*x^(9/2)*(35*a*A + 21*A*b*x^3 + 21*a*B*x^3 + 15*b*B*x^6))/315
```

Maple [A]
time = 0.11, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{15}{2}}}{15}+\frac{2bB\,x^{\frac{21}{2}}}{21}$	28
default	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{15}{2}}}{15}+\frac{2bB\,x^{\frac{21}{2}}}{21}$	28
gosper	$\frac{2x^{\frac{9}{2}}\left(15bB\,x^{6}+21Ab\,x^{3}+21Ba\,x^{3}+35Aa\right)}{315}$	32
trager	$\frac{2x^{\frac{9}{2}}\left(15bB\,x^{6}+21Ab\,x^{3}+21Ba\,x^{3}+35Aa\right)}{315}$	32
risch	$\frac{2x^{\frac{9}{2}}\left(15bB\,x^{6}+21Ab\,x^{3}+21Ba\,x^{3}+35Aa\right)}{315}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 2/9*a*A*x^(9/2)+2/15*(A*b+B*a)*x^(15/2)+2/21*b*B*x^(21/2)
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.69

$$\frac{2}{21}\,Bbx^{\frac{21}{2}}+\frac{2}{15}\,(Ba+Ab)x^{\frac{15}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 2/21*B*b*x^(21/2) + 2/15*(B*a + A*b)*x^(15/2) + 2/9*A*a*x^(9/2)
```

Fricas [A]
time = 2.97, size = 32, normalized size = 0.82

$$\frac{2}{315}\left(15\,Bbx^{10}+21\,(Ba+Ab)x^{7}+35\,Aax^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 2/315*(15*B*b*x^10 + 21*(B*a + A*b)*x^7 + 35*A*a*x^4)*sqrt(x)
```

Sympy [A]
time = 1.11, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{9}{2}}}{9}+\frac{2Abx^{\frac{15}{2}}}{15}+\frac{2Bax^{\frac{15}{2}}}{15}+\frac{2Bbx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(9/2)/9 + 2*A*b*x**(15/2)/15 + 2*B*a*x**(15/2)/15 + 2*B*b*x**(21/2
)/21
```

Giac [A]
time = 0.97, size = 29, normalized size = 0.74

$$\frac{2}{21} Bbx^{\frac{21}{2}} + \frac{2}{15} Bax^{\frac{15}{2}} + \frac{2}{15} Abx^{\frac{15}{2}} + \frac{2}{9} Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/21*B*b*x^(21/2) + 2/15*B*a*x^(15/2) + 2/15*A*b*x^(15/2) + 2/9*A*a*x^(9/2)
```

Mupad [B]
time = 0.05, size = 31, normalized size = 0.79

$$\frac{2\,x^{9/2}\,\left(35\,A\,a+21\,A\,b\,x^3+21\,B\,a\,x^3+15\,B\,b\,x^6\right)}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] (2*x^(9/2)*(35*A*a + 21*A*b*x^3 + 21*B*a*x^3 + 15*B*b*x^6))/315
```

3.132 $\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{7}aAx^{7/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{19}bBx^{19/2}$$

```
[Out] 2/7*a*A*x^(7/2)+2/13*(A*b+B*a)*x^(13/2)+2/19*b*B*x^(19/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = $\{459\}$

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{19}bBx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx^{5/2}+(Ab+aB)x^{11/2}+bBx^{17/2}\right)\,dx\\&=\frac{2}{7}aAx^{7/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{19}bBx^{19/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.90

$$\frac{2x^{7/2}(247aA+133Abx^3+133aBx^3+91bBx^6)}{1729}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
```

[Out] (2*x^(7/2)*(247*a*A + 133*A*b*x^3 + 133*a*B*x^3 + 91*b*B*x^6))/1729

Maple [A]
time = 0.11, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{19}{2}}}{19}$	28
default	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{19}{2}}}{19}$	28
gosper	$\frac{2x^{\frac{7}{2}}\left(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa\right)}{1729}$	32
trager	$\frac{2x^{\frac{7}{2}}\left(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa\right)}{1729}$	32
risch	$\frac{2x^{\frac{7}{2}}\left(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa\right)}{1729}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)

[Out] 2/7*a*A*x^(7/2)+2/13*(A*b+B*a)*x^(13/2)+2/19*b*B*x^(19/2)

Maxima [A]
time = 0.27, size = 27, normalized size = 0.69

$$\frac{2}{19}\,Bbx^{\frac{19}{2}}+\frac{2}{13}\,(Ba+Ab)x^{\frac{13}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")

[Out] 2/19*B*b*x^(19/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/7*A*a*x^(7/2)

Fricas [A]
time = 1.85, size = 32, normalized size = 0.82

$$\frac{2}{1729}\left(91\,Bbx^{9}+133\,(Ba+Ab)x^{6}+247\,Aax^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")

[Out] 2/1729*(91*B*b*x^9 + 133*(B*a + A*b)*x^6 + 247*A*a*x^3)*sqrt(x)

Sympy [A]
time = 0.74, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{7}{2}}}{7}+\frac{2Abx^{\frac{13}{2}}}{13}+\frac{2Bax^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(7/2)/7 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*b*x**(19/2
)/19
```

Giac [A]
time = 0.62, size = 29, normalized size = 0.74

$$\frac{2}{19} Bbx^{\frac{19}{2}} + \frac{2}{13} Bax^{\frac{13}{2}} + \frac{2}{13} Abx^{\frac{13}{2}} + \frac{2}{7} Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/19*B*b*x^(19/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/7*A*a*x^(7/2)
```

Mupad [B]
time = 2.56, size = 31, normalized size = 0.79

$$\frac{2\,x^{7/2}\,\left(247\,A\,a+133\,A\,b\,x^3+133\,B\,a\,x^3+91\,B\,b\,x^6\right)}{1729}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] (2*x^(7/2)*(247*A*a + 133*A*b*x^3 + 133*B*a*x^3 + 91*B*b*x^6))/1729
```

3.133 $\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{5}aAx^{5/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{17}bBx^{17/2}$$

```
[Out] 2/5*a*A*x^(5/2)+2/11*(A*b+B*a)*x^(11/2)+2/17*b*B*x^(17/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {459}

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{17}bBx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx^{3/2}+(Ab+aB)x^{9/2}+bBx^{15/2}\right)\,dx\\&=\frac{2}{5}aAx^{5/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{17}bBx^{17/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.90

$$\frac{2}{935}x^{5/2}\left(187aA+85Abx^3+85aBx^3+55bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*x^(5/2)*(187*a*A + 85*A*b*x^3 + 85*a*B*x^3 + 55*b*B*x^6))/935
```

Maple [A]
time = 0.11, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
default	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
gosper	$\frac{2x^{\frac{5}{2}}\left(55bB\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa\right)}{935}$	32
trager	$\frac{2x^{\frac{5}{2}}\left(55bB\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa\right)}{935}$	32
risch	$\frac{2x^{\frac{5}{2}}\left(55bB\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa\right)}{935}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 2/5*a*A*x^(5/2)+2/11*(A*b+B*a)*x^(11/2)+2/17*b*B*x^(17/2)
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.69

$$\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{11}\,(Ba+Ab)x^{\frac{11}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 2/17*B*b*x^(17/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/5*A*a*x^(5/2)
```

Fricas [A]
time = 2.61, size = 32, normalized size = 0.82

$$\frac{2}{935}\left(55\,Bbx^{8}+85\,(Ba+Ab)x^{5}+187\,Aax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 2/935*(55*B*b*x^8 + 85*(B*a + A*b)*x^5 + 187*A*a*x^2)*sqrt(x)
```

Sympy [A]
time = 0.48, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{5}{2}}}{5}+\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Bax^{\frac{11}{2}}}{11}+\frac{2Bbx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(5/2)/5 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(17/2
)/17
```

Giac [A]
time = 0.61, size = 29, normalized size = 0.74

$$\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{11}\,Bax^{\frac{11}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/5*A*a*x^(5/2)
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{5/2}\,\left(187\,A\,a+85\,A\,b\,x^3+85\,B\,a\,x^3+55\,B\,b\,x^6\right)}{935}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] (2*x^(5/2)*(187*A*a + 85*A*b*x^3 + 85*B*a*x^3 + 55*B*b*x^6))/935
```

3.134 $\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{3}aAx^{3/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{15}bBx^{15/2}$$

```
[Out] 2/3*a*A*x^(3/2)+2/9*(A*b+B*a)*x^(9/2)+2/15*b*B*x^(15/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{15}bBx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int \left(aA\sqrt{x}+(Ab+aB)x^{7/2}+bBx^{13/2}\right)\,dx\\ &= \frac{2}{3}aAx^{3/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{15}bBx^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.90

$$\frac{2}{45}x^{3/2}\left(15aA+5Abx^3+5aBx^3+3bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*x^(3/2)*(15*a*A + 5*A*b*x^3 + 5*a*B*x^3 + 3*b*B*x^6))/45
```

Maple [A]
time = 0.11, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
default	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
gosper	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32
trager	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32
risch	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*a*A*x^(3/2)+2/9*(A*b+B*a)*x^(9/2)+2/15*b*B*x^(15/2)
```

Maxima [A]
time = 0.30, size = 27, normalized size = 0.69

$$\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{9}\,(Ba+Ab)x^{\frac{9}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
[Out] 2/15*B*b*x^(15/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/3*A*a*x^(3/2)
```

Fricas [A]
time = 2.18, size = 30, normalized size = 0.77

$$\frac{2}{45}\left(3\,Bbx^{7}+5\,(Ba+Ab)x^{4}+15\,Aax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*B*b*x^7 + 5*(B*a + A*b)*x^4 + 15*A*a*x)*sqrt(x)
```

Sympy [A]
time = 1.22, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{3}{2}}}{3}+\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Bax^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a*x**(3/2)/3 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(15/2)/15
```

Giac [A]
time = 0.58, size = 29, normalized size = 0.74

$$\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{9}\,Bax^{\frac{9}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/3*A*a*x^(3/2)
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{3/2}\,\left(15\,A\,a+5\,A\,b\,x^3+5\,B\,a\,x^3+3\,B\,b\,x^6\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] (2*x^(3/2)*(15*A*a + 5*A*b*x^3 + 5*B*a*x^3 + 3*B*b*x^6))/45
```

3.135 $\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx$

Optimal. Leaf size=37

$$2aA\sqrt{x}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{13}bBx^{13/2}$$

```
[Out] 2/7*(A*b+B*a)*x^(7/2)+2/13*b*B*x^(13/2)+2*a*A*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{459\}$

$$\frac{2}{7}x^{7/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{13}bBx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(13/2))/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx &= \int \left(\frac{aA}{\sqrt{x}}+(Ab+aB)x^{5/2}+bBx^{11/2}\right)dx\\ &= 2aA\sqrt{x}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{13}bBx^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.95

$$\frac{2}{91}\sqrt{x}\left(91aA+13Abx^3+13aBx^3+7bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
[Out] (2*Sqrt[x]*(91*a*A + 13*A*b*x^3 + 13*a*B*x^3 + 7*b*B*x^6))/91
```

Maple [A]
time = 0.11, size = 28, normalized size = 0.76

method	result	size
derivativedivides	$\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{13}{2}}}{13}+2aA\sqrt{x}$	28
default	$\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{13}{2}}}{13}+2aA\sqrt{x}$	28
trager	$\left(\frac{2}{13}bB\,x^{6}+\frac{2}{7}Ab\,x^{3}+\frac{2}{7}Ba\,x^{3}+2Aa\right)\sqrt{x}$	31
gosper	$\frac{2\sqrt{x}\,\left(7bB\,x^{6}+13Ab\,x^{3}+13Ba\,x^{3}+91Aa\right)}{91}$	32
risch	$\frac{2\sqrt{x}\,\left(7bB\,x^{6}+13Ab\,x^{3}+13Ba\,x^{3}+91Aa\right)}{91}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/7*(A*b+B*a)*x^(7/2)+2/13*b*B*x^(13/2)+2*a*A*x^(1/2)
```

Maxima [A]
time = 0.28, size = 27, normalized size = 0.73

$$\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{7}\,(Ba+Ab)x^{\frac{7}{2}}+2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
[Out] 2/13*B*b*x^(13/2) + 2/7*(B*a + A*b)*x^(7/2) + 2*A*a*sqrt(x)
```

Fricas [A]
time = 2.61, size = 29, normalized size = 0.78

$$\frac{2}{91}\left(7\,Bbx^{6}+13\,(Ba+Ab)x^{3}+91\,Aa\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
[Out] 2/91*(7*B*b*x^6 + 13*(B*a + A*b)*x^3 + 91*A*a)*sqrt(x)
```

Sympy [A]
time = 0.27, size = 44, normalized size = 1.19

$$2Aa\sqrt{x}+\frac{2Abx^{\frac{7}{2}}}{7}+\frac{2Bax^{\frac{7}{2}}}{7}+\frac{2Bbx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a*sqrt(x) + 2*A*b*x**(7/2)/7 + 2*B*a*x**(7/2)/7 + 2*B*b*x**(13/2)/13
```

Giac [A]
time = 0.59, size = 29, normalized size = 0.78

$$\frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{7}\,Bax^{\frac{7}{2}} + \frac{2}{7}\,Abx^{\frac{7}{2}} + 2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2*A*a*sqrt(x)
```

Mupad [B]
time = 2.59, size = 31, normalized size = 0.84

$$\frac{2\,\sqrt{x}\,\left(91\,A\,a+13\,A\,b\,x^3+13\,B\,a\,x^3+7\,B\,b\,x^6\right)}{91}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(91*A*a + 13*A*b*x^3 + 13*B*a*x^3 + 7*B*b*x^6))/91
```

3.136 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}} \, dx$

Optimal. Leaf size=37

$$-\frac{2aA}{\sqrt{x}}+\frac{2}{5}(Ab+aB)x^{5/2}+\frac{2}{11}bBx^{11/2}$$

```
[Out] 2/5*(A*b+B*a)*x^(5/2)+2/11*b*B*x^(11/2)-2*a*A/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$\frac{2}{5}x^{5/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{11}bBx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(11/2))/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx &= \int \left(\frac{aA}{x^{3/2}}+(Ab+aB)x^{3/2}+bBx^{9/2}\right)dx\\ &= -\frac{2aA}{\sqrt{x}}+\frac{2}{5}(Ab+aB)x^{5/2}+\frac{2}{11}bBx^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.95

$$-\frac{2(55aA-11Abx^3-11aBx^3-5bBx^6)}{55\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
[Out] (-2*(55*a*A - 11*A*b*x^3 - 11*a*B*x^3 - 5*b*B*x^6))/(55*Sqrt[x])
```

Maple [A]
time = 0.05, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{11}{2}}}{11}+\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2Ba\,x^{\frac{5}{2}}}{5}-\frac{2aA}{\sqrt{x}}$	30
default	$\frac{2bB\,x^{\frac{11}{2}}}{11}+\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2Ba\,x^{\frac{5}{2}}}{5}-\frac{2aA}{\sqrt{x}}$	30
gosper	$-\frac{2\left(-5bB\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa\right)}{55\sqrt{x}}$	32
trager	$-\frac{2\left(-5bB\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa\right)}{55\sqrt{x}}$	32
risch	$-\frac{2\left(-5bB\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa\right)}{55\sqrt{x}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/11*b*B*x^(11/2)+2/5*A*b*x^(5/2)+2/5*B*a*x^(5/2)-2*a*A/x^(1/2)
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.73

$$\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}}-\frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
[Out] 2/11*B*b*x^(11/2) + 2/5*(B*a + A*b)*x^(5/2) - 2*A*a/sqrt(x)
```

Fricas [A]
time = 2.62, size = 29, normalized size = 0.78

$$\frac{2\,(5\,Bbx^{6}+11\,(Ba+Ab)x^{3}-55\,Aa)}{55\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
[Out] 2/55*(5*B*b*x^6 + 11*(B*a + A*b)*x^3 - 55*A*a)/sqrt(x)
```

Sympy [A]
time = 0.39, size = 44, normalized size = 1.19

$$-\frac{2Aa}{\sqrt{x}} + \frac{2Abx^{\frac{5}{2}}}{5} + \frac{2Bax^{\frac{5}{2}}}{5} + \frac{2Bbx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(3/2),x)
[Out] -2*A*a/sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(11/2)/11
```

Giac [A]
time = 0.58, size = 29, normalized size = 0.78

$$\frac{2}{11} Bbx^{\frac{11}{2}} + \frac{2}{5} Bax^{\frac{5}{2}} + \frac{2}{5} Abx^{\frac{5}{2}} - \frac{2 Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="giac")
[Out] 2/11*B*b*x^(11/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) - 2*A*a/sqrt(x)
```

Mupad [B]
time = 2.60, size = 31, normalized size = 0.84

$$\frac{22\,A\,b\,x^3 - 110\,A\,a + 22\,B\,a\,x^3 + 10\,B\,b\,x^6}{55\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(3/2),x)
[Out] (22*A*b*x^3 - 110*A*a + 22*B*a*x^3 + 10*B*b*x^6)/(55*x^(1/2))
```

3.137 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}} dx$

Optimal. Leaf size=39

$$-\frac{2aA}{3x^{3/2}} + \frac{2}{3}(Ab + aB)x^{3/2} + \frac{2}{9}bBx^{9/2}$$

```
[Out] -2/3*a*A/x^(3/2)+2/3*(A*b+B*a)*x^(3/2)+2/9*b*B*x^(9/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$\frac{2}{3}x^{3/2}(aB + Ab) - \frac{2aA}{3x^{3/2}} + \frac{2}{9}bBx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a*A)/(3*x^(3/2)) + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(9/2))/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx &= \int \left(\frac{aA}{x^{5/2}} + (Ab+aB)\sqrt{x} + bBx^{7/2}\right) dx\\ &= -\frac{2aA}{3x^{3/2}} + \frac{2}{3}(Ab+aB)x^{3/2} + \frac{2}{9}bBx^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.87

$$\frac{2(-3aA + 3Abx^3 + 3aBx^3 + bBx^6)}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
[Out] (2*(-3*a*A + 3*A*b*x^3 + 3*a*B*x^3 + b*B*x^6))/(9*x^(3/2))
```

Maple [A]
time = 0.06, size = 30, normalized size = 0.77

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
default	$\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
gosper	$-\frac{2\left(-bB\,x^{6}-3Ab\,x^{3}-3Ba\,x^{3}+3Aa\right)}{9x^{\frac{3}{2}}}$	32
trager	$-\frac{2\left(-bB\,x^{6}-3Ab\,x^{3}-3Ba\,x^{3}+3Aa\right)}{9x^{\frac{3}{2}}}$	32
risch	$-\frac{2\left(-bB\,x^{6}-3Ab\,x^{3}-3Ba\,x^{3}+3Aa\right)}{9x^{\frac{3}{2}}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/9*b*B*x^(9/2)+2/3*A*b*x^(3/2)+2/3*B*a*x^(3/2)-2/3*a*A/x^(3/2)
```

Maxima [A]
time = 0.28, size = 27, normalized size = 0.69

$$\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{3}\,(Ba+Ab)x^{\frac{3}{2}}-\frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
[Out] 2/9*B*b*x^(9/2) + 2/3*(B*a + A*b)*x^(3/2) - 2/3*A*a/x^(3/2)
```

Fricas [A]
time = 1.86, size = 28, normalized size = 0.72

$$\frac{2\,(Bbx^6+3\,(Ba+Ab)x^3-3\,Aa)}{9\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
[Out] 2/9*(B*b*x^6 + 3*(B*a + A*b)*x^3 - 3*A*a)/x^(3/2)
```

Sympy [A]
time = 0.45, size = 46, normalized size = 1.18

$$-\frac{2Aa}{3x^{\frac{3}{2}}}+\frac{2Abx^{\frac{3}{2}}}{3}+\frac{2Bax^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a/(3*x**(3/2)) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(9/2)/
9
```

Giac [A]
time = 0.54, size = 29, normalized size = 0.74

$$\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{3}\,Bax^{\frac{3}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}-\frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2/3*A*a/x^(3/2)
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.79

$$\frac{6\,A\,b\,x^3-6\,A\,a+6\,B\,a\,x^3+2\,B\,b\,x^6}{9\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(5/2),x)
```

```
[Out] (6*A*b*x^3 - 6*A*a + 6*B*a*x^3 + 2*B*b*x^6)/(9*x^(3/2))
```

3.138 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{2aA}{5x^{5/2}}+2(Ab+aB)\sqrt{x}+\frac{2}{7}bBx^{7/2}$$

```
[Out] -2/5*a*A/x^(5/2)+2/7*b*B*x^(7/2)+2*(A*b+B*a)*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{5x^{5/2}}+\frac{2}{7}bBx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a*A)/(5*x^(5/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(7/2))/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx &= \int \left(\frac{aA}{x^{7/2}}+\frac{Ab+aB}{\sqrt{x}}+bBx^{5/2}\right)dx\\ &= -\frac{2aA}{5x^{5/2}}+2(Ab+aB)\sqrt{x}+\frac{2}{7}bBx^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.95

$$-\frac{2(7aA-35Abx^3-35aBx^3-5bBx^6)}{35x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
[Out] (-2*(7*a*A - 35*A*b*x^3 - 35*a*B*x^3 - 5*b*B*x^6))/(35*x^(5/2))
```

Maple [A]
time = 0.06, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{5x^{\frac{5}{2}}}$	30
default	$\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{5x^{\frac{5}{2}}}$	30
gosper	$-\frac{2(-5bB\,x^{6}-35Ab\,x^{3}-35Ba\,x^{3}+7Aa)}{35x^{\frac{5}{2}}}$	32
trager	$-\frac{2(-5bB\,x^{6}-35Ab\,x^{3}-35Ba\,x^{3}+7Aa)}{35x^{\frac{5}{2}}}$	32
risch	$-\frac{2(-5bB\,x^{6}-35Ab\,x^{3}-35Ba\,x^{3}+7Aa)}{35x^{\frac{5}{2}}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/7*b*B*x^(7/2)+2*A*b*x^(1/2)+2*B*a*x^(1/2)-2/5*a*A/x^(5/2)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.73

$$\frac{2}{7}\,Bbx^{\frac{7}{2}}+2\,(Ba+Ab)\sqrt{x}-\frac{2\,Aa}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
[Out] 2/7*B*b*x^(7/2) + 2*(B*a + A*b)*sqrt(x) - 2/5*A*a/x^(5/2)
```

Fricas [A]
time = 2.21, size = 29, normalized size = 0.78

$$\frac{2\,(5\,Bbx^{6}+35\,(Ba+Ab)x^{3}-7\,Aa)}{35\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
[Out] 2/35*(5*B*b*x^6 + 35*(B*a + A*b)*x^3 - 7*A*a)/x^(5/2)
```

Sympy [A]
time = 0.56, size = 42, normalized size = 1.14

$$-\frac{2Aa}{5x^{\frac{5}{2}}} + 2Ab\sqrt{x} + 2Ba\sqrt{x} + \frac{2Bbx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(7/2),x)
[Out] -2*A*a/(5*x**(5/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(7/2)/7
```

Giac [A]
time = 0.53, size = 29, normalized size = 0.78

$$\frac{2}{7}Bbx^{\frac{7}{2}} + 2Ba\sqrt{x} + 2Ab\sqrt{x} - \frac{2Aa}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="giac")
[Out] 2/7*B*b*x^(7/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/5*A*a/x^(5/2)
```

Mupad [B]
time = 0.04, size = 30, normalized size = 0.81

$$\frac{2Abx^3 - \frac{2Aa}{5} + 2Bax^3 + \frac{2Bbx^6}{7}}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(7/2),x)
[Out] (2*A*b*x^3 - (2*A*a)/5 + 2*B*a*x^3 + (2*B*b*x^6)/7)/x^(5/2)
```

3.139 $\int x^{7/2}(a+bx^3)^2 \left(A+Bx^3\right) dx$

Optimal. Leaf size=63

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{15}a(2Ab+aB)x^{15/2}+\frac{2}{21}b(Ab+2aB)x^{21/2}+\frac{2}{27}b^2Bx^{27/2}$$

```
[Out] 2/9*a^2*A*x^(9/2)+2/15*a*(2*A*b+B*a)*x^(15/2)+2/21*b*(A*b+2*B*a)*x^(21/2)+2
/27*b^2*B*x^(27/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{21}bx^{21/2}(2aB+Ab)+\frac{2}{15}ax^{15/2}(aB+2Ab)+\frac{2}{27}b^2Bx^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(15/2))/15 + (2*b*(A*b + 2*a*B)*
x^(21/2))/21 + (2*b^2*B*x^(27/2))/27
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)dx &= \int \left(a^2Ax^{7/2}+a(2Ab+aB)x^{13/2}+b(Ab+2aB)x^{19/2}+b^2Bx^{25/2}\right)dx\\ &= \frac{2}{9}a^2Ax^{9/2}+\frac{2}{15}a(2Ab+aB)x^{15/2}+\frac{2}{21}b(Ab+2aB)x^{21/2}+\frac{2}{27}b^2Bx^{27/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.94

$$\frac{2}{945}x^{9/2}\left(105a^2A+126aAbx^3+63a^2Bx^3+45Ab^2x^6+90abBx^6+35b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(9/2)*(105*a^2*A + 126*a*A*b*x^3 + 63*a^2*B*x^3 + 45*A*b^2*x^6 + 90*a*
b*B*x^6 + 35*b^2*B*x^9))/945
```

Maple [A]
time = 0.27, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{27}{2}}}{27}+\frac{2(b^2A+2abB)x^{\frac{21}{2}}}{21}+\frac{2(2abA+a^2B)x^{\frac{15}{2}}}{15}+\frac{2a^2Ax^{\frac{9}{2}}}{9}$	52
default	$\frac{2b^2Bx^{\frac{27}{2}}}{27}+\frac{2(b^2A+2abB)x^{\frac{21}{2}}}{21}+\frac{2(2abA+a^2B)x^{\frac{15}{2}}}{15}+\frac{2a^2Ax^{\frac{9}{2}}}{9}$	52
gosper	$\frac{2x^{\frac{9}{2}}(35b^2Bx^9+45Ab^2x^6+90Babx^6+126aAbx^3+63a^2Bx^3+105a^2A)}{945}$	56
trager	$\frac{2x^{\frac{9}{2}}(35b^2Bx^9+45Ab^2x^6+90Babx^6+126aAbx^3+63a^2Bx^3+105a^2A)}{945}$	56
risch	$\frac{2x^{\frac{9}{2}}(35b^2Bx^9+45Ab^2x^6+90Babx^6+126aAbx^3+63a^2Bx^3+105a^2A)}{945}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*b^2*B*x^(27/2)+2/21*(A*b^2+2*B*a*b)*x^(21/2)+2/15*(2*A*a*b+B*a^2)*x^(1
5/2)+2/9*a^2*A*x^(9/2)
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.81

$$\frac{2}{27}\,Bb^2x^{\frac{27}{2}}+\frac{2}{21}\,\left(2\,Bab+Ab^2\right)x^{\frac{21}{2}}+\frac{2}{15}\,\left(Ba^2+2\,Aab\right)x^{\frac{15}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/27*B*b^2*x^(27/2) + 2/21*(2*B*a*b + A*b^2)*x^(21/2) + 2/15*(B*a^2 + 2*A*a
*b)*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

Fricas [A]
time = 1.29, size = 56, normalized size = 0.89

$$\frac{2}{945}\,\left(35\,Bb^2x^{13}+45\,\left(2\,Bab+Ab^2\right)x^{10}+63\,\left(Ba^2+2\,Aab\right)x^7+105\,Aa^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^2*x^13 + 45*(2*B*a*b + A*b^2)*x^10 + 63*(B*a^2 + 2*A*a*b)*x^7
 + 105*A*a^2*x^4)*sqrt(x)
```

Sympy [A]
time = 1.84, size = 80, normalized size = 1.27

$$\frac{2Aa^{2}x^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{15}{2}}}{15}+\frac{2Ab^{2}x^{\frac{21}{2}}}{21}+\frac{2Ba^{2}x^{\frac{15}{2}}}{15}+\frac{4Babx^{\frac{21}{2}}}{21}+\frac{2Bb^{2}x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(15/2)/15 + 2*A*b**2*x**(21/2)/21 + 2*B*a*
*2*x**(15/2)/15 + 4*B*a*b*x**(21/2)/21 + 2*B*b**2*x**(27/2)/27
```

Giac [A]
time = 0.56, size = 53, normalized size = 0.84

$$\frac{2}{27}Bb^{2}x^{\frac{27}{2}}+\frac{4}{21}Babx^{\frac{21}{2}}+\frac{2}{21}Ab^{2}x^{\frac{21}{2}}+\frac{2}{15}Ba^{2}x^{\frac{15}{2}}+\frac{4}{15}Aabx^{\frac{15}{2}}+\frac{2}{9}Aa^{2}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/27*B*b^2*x^(27/2) + 4/21*B*a*b*x^(21/2) + 2/21*A*b^2*x^(21/2) + 2/15*B*a^
2*x^(15/2) + 4/15*A*a*b*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

Mupad [B]
time = 2.57, size = 51, normalized size = 0.81

$$x^{15/2}\left(\frac{2\,B\,a^2}{15}+\frac{4\,A\,b\,a}{15}\right)+x^{21/2}\left(\frac{2\,A\,b^2}{21}+\frac{4\,B\,a\,b}{21}\right)+\frac{2\,A\,a^2\,x^{9/2}}{9}+\frac{2\,B\,b^2\,x^{27/2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(15/2)*((2*B*a^2)/15 + (4*A*a*b)/15) + x^(21/2)*((2*A*b^2)/21 + (4*B*a*b)
/21) + (2*A*a^2*x^(9/2))/9 + (2*B*b^2*x^(27/2))/27
```

3.140 $\int x^{5/2}(a+bx^3)^2 (A+Bx^3)\, dx$

Optimal. Leaf size=63

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{13}a(2Ab+aB)x^{13/2}+\frac{2}{19}b(Ab+2aB)x^{19/2}+\frac{2}{25}b^2Bx^{25/2}$$

```
[Out] 2/7*a^2*A*x^(7/2)+2/13*a*(2*A*b+B*a)*x^(13/2)+2/19*b*(A*b+2*B*a)*x^(19/2)+2
/25*b^2*B*x^(25/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(19/2))/19 + (2*b^2*B*x^(25/2))/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx^3)^2 (A+Bx^3)\, dx &= \int \left(a^2Ax^{5/2}+a(2Ab+aB)x^{11/2}+b(Ab+2aB)x^{17/2}+b^2Bx^{23/2}\right) dx\\ &= \frac{2}{7}a^2Ax^{7/2}+\frac{2}{13}a(2Ab+aB)x^{13/2}+\frac{2}{19}b(Ab+2aB)x^{19/2}+\frac{2}{25}b^2Bx^{25/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.95

$$\frac{2x^{7/2}(475a^2(13A+7Bx^3)+350abx^3(19A+13Bx^3)+91b^2x^6(25A+19Bx^3))}{43225}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(7/2)*(475*a^2*(13*A + 7*B*x^3) + 350*a*b*x^3*(19*A + 13*B*x^3) + 91*b
^2*x^6*(25*A + 19*B*x^3)))/43225
```

Maple [A]
time = 0.28, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{25}{2}}}{25}+\frac{2(b^2A+2abB)x^{\frac{19}{2}}}{19}+\frac{2(2abA+a^2B)x^{\frac{13}{2}}}{13}+\frac{2a^2Ax^{\frac{7}{2}}}{7}$	52
default	$\frac{2b^2Bx^{\frac{25}{2}}}{25}+\frac{2(b^2A+2abB)x^{\frac{19}{2}}}{19}+\frac{2(2abA+a^2B)x^{\frac{13}{2}}}{13}+\frac{2a^2Ax^{\frac{7}{2}}}{7}$	52
gosper	$\frac{2x^{\frac{7}{2}}(1729b^2Bx^9+2275Ab^2x^6+4550Babx^6+6650aAbx^3+3325a^2Bx^3+6175a^2A)}{43225}$	56
trager	$\frac{2x^{\frac{7}{2}}(1729b^2Bx^9+2275Ab^2x^6+4550Babx^6+6650aAbx^3+3325a^2Bx^3+6175a^2A)}{43225}$	56
risch	$\frac{2x^{\frac{7}{2}}(1729b^2Bx^9+2275Ab^2x^6+4550Babx^6+6650aAbx^3+3325a^2Bx^3+6175a^2A)}{43225}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/25*b^2*B*x^(25/2)+2/19*(A*b^2+2*B*a*b)*x^(19/2)+2/13*(2*A*a*b+B*a^2)*x^(1
3/2)+2/7*a^2*A*x^(7/2)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.81

$$\frac{2}{25}\,Bb^2x^{\frac{25}{2}}+\frac{2}{19}\,(2\,Bab+Ab^2)x^{\frac{19}{2}}+\frac{2}{13}\,(Ba^2+2\,Aab)x^{\frac{13}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^2*x^(25/2) + 2/19*(2*B*a*b + A*b^2)*x^(19/2) + 2/13*(B*a^2 + 2*A*a
*b)*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

Fricas [A]
time = 1.97, size = 56, normalized size = 0.89

$$\frac{2}{43225}\,(1729\,Bb^2x^{12}+2275\,(2\,Bab+Ab^2)x^9+3325\,(Ba^2+2\,Aab)x^6+6175\,Aa^2x^3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/43225*(1729*B*b^2*x^12 + 2275*(2*B*a*b + A*b^2)*x^9 + 3325*(B*a^2 + 2*A*a
*b)*x^6 + 6175*A*a^2*x^3)*sqrt(x)
```

Sympy [A]
time = 1.32, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{4Aabx^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{19}{2}}}{19}+\frac{2Ba^2x^{\frac{13}{2}}}{13}+\frac{4Babx^{\frac{19}{2}}}{19}+\frac{2Bb^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(19/2)/19 + 2*B*a*
*2*x**(13/2)/13 + 4*B*a*b*x**(19/2)/19 + 2*B*b**2*x**(25/2)/25
```

Giac [A]
time = 0.54, size = 53, normalized size = 0.84

$$\frac{2}{25}Bb^2x^{\frac{25}{2}}+\frac{4}{19}Babx^{\frac{19}{2}}+\frac{2}{19}Ab^2x^{\frac{19}{2}}+\frac{2}{13}Ba^2x^{\frac{13}{2}}+\frac{4}{13}Aabx^{\frac{13}{2}}+\frac{2}{7}Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/25*B*b^2*x^(25/2) + 4/19*B*a*b*x^(19/2) + 2/19*A*b^2*x^(19/2) + 2/13*B*a^
2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,a^2}{13}+\frac{4\,A\,b\,a}{13}\right)+x^{19/2}\left(\frac{2\,A\,b^2}{19}+\frac{4\,B\,a\,b}{19}\right)+\frac{2\,A\,a^2\,x^{7/2}}{7}+\frac{2\,B\,b^2\,x^{25/2}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(13/2)*((2*B*a^2)/13 + (4*A*a*b)/13) + x^(19/2)*((2*A*b^2)/19 + (4*B*a*b)
/19) + (2*A*a^2*x^(7/2))/7 + (2*B*b^2*x^(25/2))/25
```

3.141 $\int x^{3/2}(a+bx^3)^2\left(A+Bx^3\right)dx$

Optimal. Leaf size=63

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{17}b(Ab+2aB)x^{17/2}+\frac{2}{23}b^2Bx^{23/2}$$

```
[Out] 2/5*a^2*A*x^(5/2)+2/11*a*(2*A*b+B*a)*x^(11/2)+2/17*b*(A*b+2*B*a)*x^(17/2)+2
/23*b^2*B*x^(23/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{23}b^2Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(17/2))/17 + (2*b^2*B*x^(23/2))/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)dx&=\int\left(a^2Ax^{3/2}+a(2Ab+aB)x^{9/2}+b(Ab+2aB)x^{15/2}+b^2Bx^{21/2}\right)dx\\&=\frac{2}{5}a^2Ax^{5/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{17}b(Ab+2aB)x^{17/2}+\frac{2}{23}b^2Bx^{23/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.95

$$\frac{2x^{5/2}(391a^2(11A+5Bx^3)+230abx^3(17A+11Bx^3)+55b^2x^6(23A+17Bx^3))}{21505}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(5/2)*(391*a^2*(11*A + 5*B*x^3) + 230*a*b*x^3*(17*A + 11*B*x^3) + 55*b
^2*x^6*(23*A + 17*B*x^3)))/21505
```

Maple [A]
time = 0.27, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{23}{2}}}{23}+\frac{2(b^2 A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2 B)x^{\frac{11}{2}}}{11}+\frac{2a^2 A x^{\frac{5}{2}}}{5}$	52
default	$\frac{2b^2 B x^{\frac{23}{2}}}{23}+\frac{2(b^2 A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2 B)x^{\frac{11}{2}}}{11}+\frac{2a^2 A x^{\frac{5}{2}}}{5}$	52
gosper	$\frac{2x^{\frac{5}{2}}(935b^2 B x^9+1265A b^2 x^6+2530Bab x^6+3910aAb x^3+1955a^2 B x^3+4301a^2 A)}{21505}$	56
trager	$\frac{2x^{\frac{5}{2}}(935b^2 B x^9+1265A b^2 x^6+2530Bab x^6+3910aAb x^3+1955a^2 B x^3+4301a^2 A)}{21505}$	56
risch	$\frac{2x^{\frac{5}{2}}(935b^2 B x^9+1265A b^2 x^6+2530Bab x^6+3910aAb x^3+1955a^2 B x^3+4301a^2 A)}{21505}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*b^2*B*x^(23/2)+2/17*(A*b^2+2*B*a*b)*x^(17/2)+2/11*(2*A*a*b+B*a^2)*x^(1
1/2)+2/5*a^2*A*x^(5/2)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.81

$$\frac{2}{23}\,Bb^2x^{\frac{23}{2}}+\frac{2}{17}\,\left(2\,Bab+Ab^2\right)x^{\frac{17}{2}}+\frac{2}{11}\,\left(Ba^2+2\,Aab\right)x^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^2*x^(23/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/11*(B*a^2 + 2*A*a
*b)*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

Fricas [A]
time = 1.41, size = 56, normalized size = 0.89

$$\frac{2}{21505}\,\left(935\,Bb^2x^{11}+1265\,\left(2\,Bab+Ab^2\right)x^8+1955\,\left(Ba^2+2\,Aab\right)x^5+4301\,Aa^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/21505*(935*B*b^2*x^11 + 1265*(2*B*a*b + A*b^2)*x^8 + 1955*(B*a^2 + 2*A*a*
b)*x^5 + 4301*A*a^2*x^2)*sqrt(x)
```

Sympy [A]
time = 0.92, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{5}{2}}}{5}+\frac{4Aabx^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{2Ba^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(17/2)/17 + 2*B*a*
*2*x**(11/2)/11 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(23/2)/23
```

Giac [A]
time = 0.63, size = 53, normalized size = 0.84

$$\frac{2}{23}Bb^2x^{\frac{23}{2}}+\frac{4}{17}Babx^{\frac{17}{2}}+\frac{2}{17}Ab^2x^{\frac{17}{2}}+\frac{2}{11}Ba^2x^{\frac{11}{2}}+\frac{4}{11}Aabx^{\frac{11}{2}}+\frac{2}{5}Aa^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/23*B*b^2*x^(23/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) + 2/11*B*a^
2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,B\,a^2}{11}+\frac{4\,A\,b\,a}{11}\right)+x^{17/2}\left(\frac{2\,A\,b^2}{17}+\frac{4\,B\,a\,b}{17}\right)+\frac{2\,A\,a^2\,x^{5/2}}{5}+\frac{2\,B\,b^2\,x^{23/2}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(17/2)*((2*A*b^2)/17 + (4*B*a*b)
/17) + (2*A*a^2*x^(5/2))/5 + (2*B*b^2*x^(23/2))/23
```

3.142 $\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{15}b(Ab+2aB)x^{15/2}+\frac{2}{21}b^2Bx^{21/2}$$

```
[Out] 2/3*a^2*A*x^(3/2)+2/9*a*(2*A*b+B*a)*x^(9/2)+2/15*b*(A*b+2*B*a)*x^(15/2)+2/2
1*b^2*B*x^(21/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x^
(15/2))/15 + (2*b^2*B*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx &= \int \left(a^2A\sqrt{x}+a(2Ab+aB)x^{7/2}+b(Ab+2aB)x^{13/2}+b^2Bx^{19/2}\right)\,dx\\ &= \frac{2}{3}a^2Ax^{3/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{15}b(Ab+2aB)x^{15/2}+\frac{2}{21}b^2Bx^{21/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.94

$$\frac{2}{315}x^{3/2}\left(35a^2\left(3A+Bx^3\right)+14abx^3\left(5A+3Bx^3\right)+3b^2x^6\left(7A+5Bx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(3/2)*(35*a^2*(3*A + B*x^3) + 14*a*b*x^3*(5*A + 3*B*x^3) + 3*b^2*x^6*(
7*A + 5*B*x^3)))/315
```

Maple [A]
time = 0.26, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^{2}B\,x^{\frac{21}{2}}}{21}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{15}{2}}}{15}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{9}{2}}}{9}+\frac{2a^{2}A\,x^{\frac{3}{2}}}{3}$	52
default	$\frac{2b^{2}B\,x^{\frac{21}{2}}}{21}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{15}{2}}}{15}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{9}{2}}}{9}+\frac{2a^{2}A\,x^{\frac{3}{2}}}{3}$	52
gosper	$\frac{2x^{\frac{3}{2}}\left(15b^{2}B\,x^{9}+21A\,b^{2}x^{6}+42Bab\,x^{6}+70aAb\,x^{3}+35a^{2}B\,x^{3}+105a^{2}A\right)}{315}$	56
trager	$\frac{2x^{\frac{3}{2}}\left(15b^{2}B\,x^{9}+21A\,b^{2}x^{6}+42Bab\,x^{6}+70aAb\,x^{3}+35a^{2}B\,x^{3}+105a^{2}A\right)}{315}$	56
risch	$\frac{2x^{\frac{3}{2}}\left(15b^{2}B\,x^{9}+21A\,b^{2}x^{6}+42Bab\,x^{6}+70aAb\,x^{3}+35a^{2}B\,x^{3}+105a^{2}A\right)}{315}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*b^2*B*x^(21/2)+2/15*(A*b^2+2*B*a*b)*x^(15/2)+2/9*(2*A*a*b+B*a^2)*x^(9/
2)+2/3*a^2*A*x^(3/2)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.81

$$\frac{2}{21}\,Bb^{2}x^{\frac{21}{2}}+\frac{2}{15}\,\left(2\,Bab+Ab^{2}\right)x^{\frac{15}{2}}+\frac{2}{9}\,\left(Ba^{2}+2\,Aab\right)x^{\frac{9}{2}}+\frac{2}{3}\,Aa^{2}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/9*(B*a^2 + 2*A*a*
b)*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

Fricas [A]
time = 1.47, size = 54, normalized size = 0.86

$$\frac{2}{315}\,\left(15\,Bb^{2}x^{10}+21\,\left(2\,Bab+Ab^{2}\right)x^{7}+35\,\left(Ba^{2}+2\,Aab\right)x^{4}+105\,Aa^{2}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^2*x^10 + 21*(2*B*a*b + A*b^2)*x^7 + 35*(B*a^2 + 2*A*a*b)*x^4
+ 105*A*a^2*x)*sqrt(x)
```

Sympy [A]
time = 1.77, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{3}{2}}}{3}+\frac{4Aabx^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{15}{2}}}{15}+\frac{2Ba^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{15}{2}}}{15}+\frac{2Bb^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a**2*x**(3/2)/3 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(15/2)/15 + 2*B*a**2
*x**(9/2)/9 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(21/2)/21
```

Giac [A]
time = 0.57, size = 53, normalized size = 0.84

$$\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{4}{15}\,Babx^{\frac{15}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}+\frac{2}{9}\,Ba^2x^{\frac{9}{2}}+\frac{4}{9}\,Aabx^{\frac{9}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) + 2/9*B*a^2
*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$x^{9/2}\left(\frac{2\,B\,a^2}{9}+\frac{4\,A\,b\,a}{9}\right)+x^{15/2}\left(\frac{2\,A\,b^2}{15}+\frac{4\,B\,a\,b}{15}\right)+\frac{2\,A\,a^2\,x^{3/2}}{3}+\frac{2\,B\,b^2\,x^{21/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(15/2)*((2*A*b^2)/15 + (4*B*a*b)/15
) + (2*A*a^2*x^(3/2))/3 + (2*B*b^2*x^(21/2))/21
```

3.143 $\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$2a^2A\sqrt{x}+\frac{2}{7}a(2Ab+aB)x^{7/2}+\frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{19}b^2Bx^{19/2}$$

```
[Out] 2/7*a*(2*A*b+B*a)*x^(7/2)+2/13*b*(A*b+2*B*a)*x^(13/2)+2/19*b^2*B*x^(19/2)+2
*a^2*A*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$2a^2A\sqrt{x}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^(13/
2))/13 + (2*b^2*B*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx &= \int \left(\frac{a^2A}{\sqrt{x}}+a(2Ab+aB)x^{5/2}+b(Ab+2aB)x^{11/2}+b^2Bx^{17/2}\right)dx\\ &= 2a^2A\sqrt{x}+\frac{2}{7}a(2Ab+aB)x^{7/2}+\frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{19}b^2Bx^{19/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.97

$$\frac{2\sqrt{x}\,(247a^2(7A+Bx^3)+38abx^3(13A+7Bx^3)+7b^2x^6(19A+13Bx^3))}{1729}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(247*a^2*(7*A + B*x^3) + 38*a*b*x^3*(13*A + 7*B*x^3) + 7*b^2*x^6
*(19*A + 13*B*x^3)))/1729
```

Maple [A]
time = 0.27, size = 52, normalized size = 0.85

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{19}{2}}}{19}+\frac{2(b^2A+2abB)x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2B)x^{\frac{7}{2}}}{7}+2a^2A\sqrt{x}$	52
default	$\frac{2b^2 B x^{\frac{19}{2}}}{19}+\frac{2(b^2A+2abB)x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2B)x^{\frac{7}{2}}}{7}+2a^2A\sqrt{x}$	52
trager	$\left(\frac{2}{19}b^2B\,x^9+\frac{2}{13}A\,b^2x^6+\frac{4}{13}Bab\,x^6+\frac{4}{7}aAb\,x^3+\frac{2}{7}a^2B\,x^3+2a^2A\right)\sqrt{x}$	55
gosper	$\frac{2\sqrt{x}\,(91b^2B\,x^9+133A\,b^2x^6+266Bab\,x^6+494aAb\,x^3+247a^2B\,x^3+1729a^2A)}{1729}$	56
risch	$\frac{2\sqrt{x}\,(91b^2B\,x^9+133A\,b^2x^6+266Bab\,x^6+494aAb\,x^3+247a^2B\,x^3+1729a^2A)}{1729}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*b^2*B*x^(19/2)+2/13*(A*b^2+2*B*a*b)*x^(13/2)+2/7*(2*A*a*b+B*a^2)*x^(7/
2)+2*a^2*A*x^(1/2)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.84

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{2}{13}\left(2\,Bab+Ab^2\right)x^{\frac{13}{2}}+\frac{2}{7}\left(Ba^2+2\,Aab\right)x^{\frac{7}{2}}+2\,Aa^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/7*(B*a^2 + 2*A*a*
b)*x^(7/2) + 2*A*a^2*sqrt(x)
```

Fricas [A]
time = 2.14, size = 53, normalized size = 0.87

$$\frac{2}{1729}\left(91\,Bb^2x^9+133\left(2\,Bab+Ab^2\right)x^6+247\left(Ba^2+2\,Aab\right)x^3+1729\,Aa^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1729*(91*B*b^2*x^9 + 133*(2*B*a*b + A*b^2)*x^6 + 247*(B*a^2 + 2*A*a*b)*x^
3 + 1729*A*a^2)*sqrt(x)
```

Sympy [A]
time = 0.60, size = 78, normalized size = 1.28

$$2Aa^2\sqrt{x} + \frac{4Aabx^{\frac{7}{2}}}{7} + \frac{2Ab^2x^{\frac{13}{2}}}{13} + \frac{2Ba^2x^{\frac{7}{2}}}{7} + \frac{4Babx^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a**2*sqrt(x) + 4*A*a*b*x**(7/2)/7 + 2*A*b**2*x**(13/2)/13 + 2*B*a**2*x*
*(7/2)/7 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(19/2)/19
```

Giac [A]
time = 0.55, size = 53, normalized size = 0.87

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}} + \frac{4}{13}\,Babx^{\frac{13}{2}} + \frac{2}{13}\,Ab^2x^{\frac{13}{2}} + \frac{2}{7}\,Ba^2x^{\frac{7}{2}} + \frac{4}{7}\,Aabx^{\frac{7}{2}} + 2\,Aa^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/7*B*a^2
*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2*A*a^2*sqrt(x)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.84

$$x^{7/2}\left(\frac{2\,B\,a^2}{7}+\frac{4\,A\,b\,a}{7}\right)+x^{13/2}\left(\frac{2\,A\,b^2}{13}+\frac{4\,B\,a\,b}{13}\right)+2\,A\,a^2\,\sqrt{x}+\frac{2\,B\,b^2\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(1/2),x)
```

```
[Out] x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(13/2)*((2*A*b^2)/13 + (4*B*a*b)/13
) + 2*A*a^2*x^(1/2) + (2*B*b^2*x^(19/2))/19
```

3.144 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{5}a(2Ab+aB)x^{5/2}+\frac{2}{11}b(Ab+2aB)x^{11/2}+\frac{2}{17}b^2Bx^{17/2}$$

```
[Out] 2/5*a*(2*A*b+B*a)*x^(5/2)+2/11*b*(A*b+2*B*a)*x^(11/2)+2/17*b^2*B*x^(17/2)-2
*a^2*A/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(
11/2))/11 + (2*b^2*B*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx &= \int \left(\frac{a^2A}{x^{3/2}}+a(2Ab+aB)x^{3/2}+b(Ab+2aB)x^{9/2}+b^2Bx^{15/2}\right)dx\\ &= -\frac{2a^2A}{\sqrt{x}}+\frac{2}{5}a(2Ab+aB)x^{5/2}+\frac{2}{11}b(Ab+2aB)x^{11/2}+\frac{2}{17}b^2Bx^{17/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.97

$$-\frac{2(935a^2A-374aAbx^3-187a^2Bx^3-85Ab^2x^6-170abBx^6-55b^2Bx^9)}{935\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*(935*a^2*A - 374*a*A*b*x^3 - 187*a^2*B*x^3 - 85*A*b^2*x^6 - 170*a*b*B*x
^6 - 55*b^2*B*x^9))/(935*Sqrt[x])
```

Maple [A]
time = 0.28, size = 54, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{17}{2}}}{17}+\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{11}{2}}}{11}+\frac{4Aabx^{\frac{5}{2}}}{5}+\frac{2Ba^2x^{\frac{5}{2}}}{5}-\frac{2a^2A}{\sqrt{x}}$	54
default	$\frac{2b^2Bx^{\frac{17}{2}}}{17}+\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{11}{2}}}{11}+\frac{4Aabx^{\frac{5}{2}}}{5}+\frac{2Ba^2x^{\frac{5}{2}}}{5}-\frac{2a^2A}{\sqrt{x}}$	54
gosper	$-\frac{2(-55b^2Bx^9-85Ab^2x^6-170Babx^6-374aAbx^3-187a^2Bx^3+935a^2A)}{935\sqrt{x}}$	56
trager	$-\frac{2(-55b^2Bx^9-85Ab^2x^6-170Babx^6-374aAbx^3-187a^2Bx^3+935a^2A)}{935\sqrt{x}}$	56
risch	$-\frac{2(-55b^2Bx^9-85Ab^2x^6-170Babx^6-374aAbx^3-187a^2Bx^3+935a^2A)}{935\sqrt{x}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*b^2*B*x^(17/2)+2/11*A*b^2*x^(11/2)+4/11*B*a*b*x^(11/2)+4/5*A*a*b*x^(5/
2)+2/5*B*a^2*x^(5/2)-2*a^2*A/x^(1/2)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.84

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{2}{11}\,\left(2\,Bab+Ab^2\right)x^{\frac{11}{2}}+\frac{2}{5}\,\left(Ba^2+2\,Aab\right)x^{\frac{5}{2}}-\frac{2\,Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/5*(B*a^2 + 2*A*a*
b)*x^(5/2) - 2*A*a^2/sqrt(x)
```

Fricas [A]
time = 1.18, size = 53, normalized size = 0.87

$$\frac{2\,\left(55\,Bb^2x^9+85\,(2\,Bab+Ab^2)x^6+187\,(Ba^2+2\,Aab)x^3-935\,Aa^2\right)}{935\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/935*(55*B*b^2*x^9 + 85*(2*B*a*b + A*b^2)*x^6 + 187*(B*a^2 + 2*A*a*b)*x^3
- 935*A*a^2)/sqrt(x)
```

Sympy [A]
time = 0.79, size = 78, normalized size = 1.28

$$-\frac{2Aa^2}{\sqrt{x}}+\frac{4Aabx^{\frac{5}{2}}}{5}+\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{2Ba^2x^{\frac{5}{2}}}{5}+\frac{4Babx^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(3/2),x)
```

```
[Out] -2*A*a**2/sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(11/2)/11 + 2*B*a**2*x
**(5/2)/5 + 4*B*a*b*x**(11/2)/11 + 2*B*b**2*x**(17/2)/17
```

Giac [A]
time = 0.58, size = 53, normalized size = 0.87

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{4}{11}\,Babx^{\frac{11}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{5}\,Ba^2x^{\frac{5}{2}}+\frac{4}{5}\,Aabx^{\frac{5}{2}}-\frac{2\,Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/5*B*a^2
*x^(5/2) + 4/5*A*a*b*x^(5/2) - 2*A*a^2/sqrt(x)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.84

$$x^{5/2}\,\left(\frac{2\,B\,a^2}{5}+\frac{4\,A\,b\,a}{5}\right)+x^{11/2}\,\left(\frac{2\,A\,b^2}{11}+\frac{4\,B\,a\,b}{11}\right)-\frac{2\,A\,a^2}{\sqrt{x}}+\frac{2\,B\,b^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(3/2),x)
```

```
[Out] x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(11/2)*((2*A*b^2)/11 + (4*B*a*b)/11
) - (2*A*a^2)/x^(1/2) + (2*B*b^2*x^(17/2))/17
```

3.145 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{15}b^2Bx^{15/2}$$

```
[Out] -2/3*a^2*A/x^(3/2)+2/3*a*(2*A*b+B*a)*x^(3/2)+2/9*b*(A*b+2*B*a)*x^(9/2)+2/15
*b^2*B*x^(15/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)
*x^(9/2))/9 + (2*b^2*B*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx &= \int \left(\frac{a^2A}{x^{5/2}}+a(2Ab+aB)\sqrt{x}+b(Ab+2aB)x^{7/2}+b^2Bx^{13/2}\right)dx\\ &= -\frac{2a^2A}{3x^{3/2}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{15}b^2Bx^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.94

$$\frac{2(-15a^2A+30aAbx^3+15a^2Bx^3+5Ab^2x^6+10abBx^6+3b^2Bx^9)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (2*(-15*a^2*A + 30*a*A*b*x^3 + 15*a^2*B*x^3 + 5*A*b^2*x^6 + 10*a*b*B*x^6 +
3*b^2*B*x^9))/(45*x^(3/2))
```

Maple [A]
time = 0.27, size = 54, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{15}{2}}}{15}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{2Ba^2x^{\frac{3}{2}}}{3}-\frac{2a^2A}{3x^{\frac{3}{2}}}$	54
default	$\frac{2b^2Bx^{\frac{15}{2}}}{15}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{2Ba^2x^{\frac{3}{2}}}{3}-\frac{2a^2A}{3x^{\frac{3}{2}}}$	54
gosper	$-\frac{2(-3b^2Bx^9-5Ab^2x^6-10Babx^6-30aAbx^3-15a^2Bx^3+15a^2A)}{45x^{\frac{3}{2}}}$	56
trager	$-\frac{2(-3b^2Bx^9-5Ab^2x^6-10Babx^6-30aAbx^3-15a^2Bx^3+15a^2A)}{45x^{\frac{3}{2}}}$	56
risch	$-\frac{2(-3b^2Bx^9-5Ab^2x^6-10Babx^6-30aAbx^3-15a^2Bx^3+15a^2A)}{45x^{\frac{3}{2}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*b^2*B*x^(15/2)+2/9*A*b^2*x^(9/2)+4/9*B*a*b*x^(9/2)+4/3*A*a*b*x^(3/2)+2
/3*B*a^2*x^(3/2)-2/3*a^2*A/x^(3/2)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.81

$$\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{2}{9}\left(2\,Bab+Ab^2\right)x^{\frac{9}{2}}+\frac{2}{3}\left(Ba^2+2\,Aab\right)x^{\frac{3}{2}}-\frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2/3*(B*a^2 + 2*A*a*b)
*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

Fricas [A]
time = 2.14, size = 53, normalized size = 0.84

$$\frac{2\,(3\,Bb^2x^9+5\,(2\,Bab+Ab^2)x^6+15\,(Ba^2+2\,Aab)x^3-15\,Aa^2)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^9 + 5*(2*B*a*b + A*b^2)*x^6 + 15*(B*a^2 + 2*A*a*b)*x^3 - 15
*A*a^2)/x^(3/2)
```

Sympy [A]
time = 0.87, size = 80, normalized size = 1.27

$$-\frac{2Aa^2}{3x^{\frac{3}{2}}} + \frac{4Aabx^{\frac{3}{2}}}{3} + \frac{2Ab^2x^{\frac{9}{2}}}{9} + \frac{2Ba^2x^{\frac{3}{2}}}{3} + \frac{4Babx^{\frac{9}{2}}}{9} + \frac{2Bb^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a**2/(3*x**(3/2)) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(9/2)/9 + 2*B*a**
2*x**(3/2)/3 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(15/2)/15
```

Giac [A]
time = 0.57, size = 53, normalized size = 0.84

$$\frac{2}{15}\,Bb^2x^{\frac{15}{2}} + \frac{4}{9}\,Babx^{\frac{9}{2}} + \frac{2}{9}\,Ab^2x^{\frac{9}{2}} + \frac{2}{3}\,Ba^2x^{\frac{3}{2}} + \frac{4}{3}\,Aabx^{\frac{3}{2}} - \frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/3*B*a^2*x^(
3/2) + 4/3*A*a*b*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$x^{3/2}\left(\frac{2\,B\,a^2}{3}+\frac{4\,A\,b\,a}{3}\right)+x^{9/2}\left(\frac{2\,A\,b^2}{9}+\frac{4\,B\,a\,b}{9}\right)-\frac{2\,A\,a^2}{3\,x^{3/2}}+\frac{2\,B\,b^2\,x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(5/2),x)
```

```
[Out] x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9) -
 (2*A*a^2)/(3*x^(3/2)) + (2*B*b^2*x^(15/2))/15
```

3.146 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{5x^{5/2}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{13}b^2Bx^{13/2}$$

```
[Out] -2/5*a^2*A/x^(5/2)+2/7*b*(A*b+2*B*a)*x^(7/2)+2/13*b^2*B*x^(13/2)+2*a*(2*A*b
+B*a)*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+2a\sqrt{x}\,(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a^2*A)/(5*x^(5/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(
7/2))/7 + (2*b^2*B*x^(13/2))/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx &= \int \left(\frac{a^2A}{x^{7/2}}+\frac{a(2Ab+aB)}{\sqrt{x}}+b(Ab+2aB)x^{5/2}+b^2Bx^{11/2}\right)dx\\ &= -\frac{2a^2A}{5x^{5/2}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{13}b^2Bx^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.97

$$-\frac{2(91a^2A-910aAbx^3-455a^2Bx^3-65Ab^2x^6-130abBx^6-35b^2Bx^9)}{455x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*(91*a^2*A - 910*a*A*b*x^3 - 455*a^2*B*x^3 - 65*A*b^2*x^6 - 130*a*b*B*x^
6 - 35*b^2*B*x^9))/(455*x^(5/2))
```

Maple [A]
time = 0.27, size = 54, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2 b^2 B\, x^{\frac{13}{2}}}{13}+\frac{2 A\, b^2 x^{\frac{7}{2}}}{7}+\frac{4 B a b\, x^{\frac{7}{2}}}{7}+4 a b A \sqrt{x}+2 a^2 B \sqrt{x}-\frac{2 a^2 A}{5 x^{\frac{5}{2}}}$	54
default	$\frac{2 b^2 B\, x^{\frac{13}{2}}}{13}+\frac{2 A\, b^2 x^{\frac{7}{2}}}{7}+\frac{4 B a b\, x^{\frac{7}{2}}}{7}+4 a b A \sqrt{x}+2 a^2 B \sqrt{x}-\frac{2 a^2 A}{5 x^{\frac{5}{2}}}$	54
gosper	$-\frac{2\left(-35 b^2 B\, x^9-65 A\, b^2 x^6-130 B a b\, x^6-910 a A b\, x^3-455 a^2 B\, x^3+91 a^2 A\right)}{455 x^{\frac{5}{2}}}$	56
trager	$-\frac{2\left(-35 b^2 B\, x^9-65 A\, b^2 x^6-130 B a b\, x^6-910 a A b\, x^3-455 a^2 B\, x^3+91 a^2 A\right)}{455 x^{\frac{5}{2}}}$	56
risch	$-\frac{2\left(-35 b^2 B\, x^9-65 A\, b^2 x^6-130 B a b\, x^6-910 a A b\, x^3-455 a^2 B\, x^3+91 a^2 A\right)}{455 x^{\frac{5}{2}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13*b^2*B*x^(13/2)+2/7*A*b^2*x^(7/2)+4/7*B*a*b*x^(7/2)+4*a*b*A*x^(1/2)+2*a
^2*B*x^(1/2)-2/5*a^2*A/x^(5/2)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.84

$$\frac{2}{13}\, B b^2 x^{\frac{13}{2}}+\frac{2}{7}\left(2\, B a b+A b^2\right) x^{\frac{7}{2}}+2\left(B a^2+2\, A a b\right)\sqrt{x}-\frac{2\, A a^2}{5\, x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) + 2*(B*a^2 + 2*A*a*b)*s
qrt(x) - 2/5*A*a^2/x^(5/2)
```

Fricas [A]
time = 1.82, size = 53, normalized size = 0.87

$$\frac{2\,(35\, B b^2 x^9+65\,(2\, B a b+A b^2) x^6+455\,(B a^2+2\, A a b) x^3-91\, A a^2)}{455\, x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/455*(35*B*b^2*x^9 + 65*(2*B*a*b + A*b^2)*x^6 + 455*(B*a^2 + 2*A*a*b)*x^3
- 91*A*a^2)/x^(5/2)
```

Sympy [A]
time = 1.05, size = 76, normalized size = 1.25

$$-\frac{2Aa^2}{5x^{\frac{5}{2}}}+4Aab\sqrt{x}+\frac{2Ab^2x^{\frac{7}{2}}}{7}+2Ba^2\sqrt{x}+\frac{4Babx^{\frac{7}{2}}}{7}+\frac{2Bb^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(7/2),x)
```

```
[Out] -2*A*a**2/(5*x**(5/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*s
qrt(x) + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(13/2)/13
```

Giac [A]
time = 0.54, size = 53, normalized size = 0.87

$$\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{4}{7}\,Babx^{\frac{7}{2}}+\frac{2}{7}\,Ab^2x^{\frac{7}{2}}+2\,Ba^2\sqrt{x}+4\,Aab\sqrt{x}-\frac{2\,Aa^2}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2*B*a^2*sqrt(
x) + 4*A*a*b*sqrt(x) - 2/5*A*a^2/x^(5/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.84

$$\sqrt{x}\,\left(2\,B\,a^2+4\,A\,b\,a\right)+x^{7/2}\,\left(\frac{2\,A\,b^2}{7}+\frac{4\,B\,a\,b}{7}\right)-\frac{2\,A\,a^2}{5\,x^{5/2}}+\frac{2\,B\,b^2\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(7/2),x)
```

```
[Out] x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(7/2)*((2*A*b^2)/7 + (4*B*a*b)/7) - (2*A*a^
2)/(5*x^(5/2)) + (2*B*b^2*x^(13/2))/13
```

3.147 $\int x^{7/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=85

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2(3Ab+aB)x^{15/2}+\frac{2}{7}ab(Ab+aB)x^{21/2}+\frac{2}{27}b^2(Ab+3aB)x^{27/2}+\frac{2}{33}b^3Bx^{33/2}$$

```
[Out] 2/9*a^3*A*x^(9/2)+2/15*a^2*(3*A*b+B*a)*x^(15/2)+2/7*a*b*(A*b+B*a)*x^(21/2)+
2/27*b^2*(A*b+3*B*a)*x^(27/2)+2/33*b^3*B*x^(33/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2x^{15/2}(aB+3Ab)+\frac{2}{27}b^2x^{27/2}(3aB+Ab)+\frac{2}{7}abx^{21/2}(aB+Ab)+\frac{2}{33}b^3Bx^{33/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(15/2))/15 + (2*a*b*(A*b + a*B
)*x^(21/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(27/2))/27 + (2*b^3*B*x^(33/2))/33
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx &= \int \left(a^3Ax^{7/2}+a^2(3Ab+aB)x^{13/2}+3ab(Ab+aB)x^{19/2}+b^2(Ab+3aB)x\right.\\ &= \frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2(3Ab+aB)x^{15/2}+\frac{2}{7}ab(Ab+aB)x^{21/2}+\frac{2}{27}b^2(Ab+3a\end{aligned}$$

Mathematica [A]
time = 0.04, size = 81, normalized size = 0.95

$$\frac{2x^{9/2}(231a^3(5A+3Bx^3)+297a^2bx^3(7A+5Bx^3)+165ab^2x^6(9A+7Bx^3)+35b^3x^9(11A+9Bx^3))}{10395}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]

[Out] (2*x^(9/2)*(231*a^3*(5*A + 3*B*x^3) + 297*a^2*b*x^3*(7*A + 5*B*x^3) + 165*a*b^2*x^6*(9*A + 7*B*x^3) + 35*b^3*x^9*(11*A + 9*B*x^3)))/10395

Maple [A]
time = 0.27, size = 76, normalized size = 0.89

method	result	siz
derivativedivides	$\frac{2b^3Bx^{\frac{33}{2}}}{33}+\frac{2(Ab^3+3Bab^2)x^{\frac{27}{2}}}{27}+\frac{2(3Aab^2+3Ba^2b)x^{\frac{21}{2}}}{21}+\frac{2(3Aa^2b+Ba^3)x^{\frac{15}{2}}}{15}+\frac{2a^3Ax^{\frac{9}{2}}}{9}$	76
default	$\frac{2b^3Bx^{\frac{33}{2}}}{33}+\frac{2(Ab^3+3Bab^2)x^{\frac{27}{2}}}{27}+\frac{2(3Aab^2+3Ba^2b)x^{\frac{21}{2}}}{21}+\frac{2(3Aa^2b+Ba^3)x^{\frac{15}{2}}}{15}+\frac{2a^3Ax^{\frac{9}{2}}}{9}$	76
gosper	$\frac{2x^{\frac{9}{2}}(315Bb^3x^{12}+385x^9Ab^3+1155x^9Bab^2+1485x^6Aab^2+1485x^6Ba^2b+2079x^3Aa^2b+693a^3Bx^3+1155Aa^3)}{10395}$	80
trager	$\frac{2x^{\frac{9}{2}}(315Bb^3x^{12}+385x^9Ab^3+1155x^9Bab^2+1485x^6Aab^2+1485x^6Ba^2b+2079x^3Aa^2b+693a^3Bx^3+1155Aa^3)}{10395}$	80
risch	$\frac{2x^{\frac{9}{2}}(315Bb^3x^{12}+385x^9Ab^3+1155x^9Bab^2+1485x^6Aab^2+1485x^6Ba^2b+2079x^3Aa^2b+693a^3Bx^3+1155Aa^3)}{10395}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)

[Out] 2/33*b^3*B*x^(33/2)+2/27*(A*b^3+3*B*a*b^2)*x^(27/2)+2/21*(3*A*a*b^2+3*B*a^2*b)*x^(21/2)+2/15*(3*A*a^2*b+B*a^3)*x^(15/2)+2/9*a^3*A*x^(9/2)

Maxima [A]
time = 0.27, size = 73, normalized size = 0.86

$$\frac{2}{33}Bb^3x^{\frac{33}{2}}+\frac{2}{27}\left(3Bab^2+Ab^3\right)x^{\frac{27}{2}}+\frac{2}{7}\left(Ba^2b+Aab^2\right)x^{\frac{21}{2}}+\frac{2}{9}Aa^3x^{\frac{9}{2}}+\frac{2}{15}\left(Ba^3+3Aa^2b\right)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")

[Out] 2/33*B*b^3*x^(33/2) + 2/27*(3*B*a*b^2 + A*b^3)*x^(27/2) + 2/7*(B*a^2*b + A*a*b^2)*x^(21/2) + 2/9*A*a^3*x^(9/2) + 2/15*(B*a^3 + 3*A*a^2*b)*x^(15/2)

Fricas [A]
time = 1.32, size = 78, normalized size = 0.92

$$\frac{2}{10395}\left(315Bb^3x^{16}+385\left(3Bab^2+Ab^3\right)x^{13}+1485\left(Ba^2b+Aab^2\right)x^{10}+1155Aa^3x^4+693\left(Ba^3+3Aa^2b\right)x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")

[Out] 2/10395*(315*B*b^3*x^16 + 385*(3*B*a*b^2 + A*b^3)*x^13 + 1485*(B*a^2*b + A*a*b^2)*x^10 + 1155*A*a^3*x^4 + 693*(B*a^3 + 3*A*a^2*b)*x^7)*sqrt(x)

Sympy [A]
time = 2.92, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{9}{2}}}{9}+\frac{2Aa^2bx^{\frac{15}{2}}}{5}+\frac{2Aab^2x^{\frac{21}{2}}}{7}+\frac{2Ab^3x^{\frac{27}{2}}}{27}+\frac{2Ba^3x^{\frac{15}{2}}}{15}+\frac{2Ba^2bx^{\frac{21}{2}}}{7}+\frac{2Bab^2x^{\frac{27}{2}}}{9}+\frac{2Bb^3x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(9/2)/9 + 2*A*a**2*b*x**(15/2)/5 + 2*A*a*b**2*x**(21/2)/7 + 2*A
*b**3*x**(27/2)/27 + 2*B*a**3*x**(15/2)/15 + 2*B*a**2*b*x**(21/2)/7 + 2*B*a
*b**2*x**(27/2)/9 + 2*B*b**3*x**(33/2)/33
```

Giac [A]
time = 0.55, size = 77, normalized size = 0.91

$$\frac{2}{33}Bb^3x^{\frac{33}{2}}+\frac{2}{9}Bab^2x^{\frac{27}{2}}+\frac{2}{27}Ab^3x^{\frac{27}{2}}+\frac{2}{7}Ba^2bx^{\frac{21}{2}}+\frac{2}{7}Aab^2x^{\frac{21}{2}}+\frac{2}{15}Ba^3x^{\frac{15}{2}}+\frac{2}{5}Aa^2bx^{\frac{15}{2}}+\frac{2}{9}Aa^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/33*B*b^3*x^(33/2) + 2/9*B*a*b^2*x^(27/2) + 2/27*A*b^3*x^(27/2) + 2/7*B*a^
2*b*x^(21/2) + 2/7*A*a*b^2*x^(21/2) + 2/15*B*a^3*x^(15/2) + 2/5*A*a^2*b*x^(
15/2) + 2/9*A*a^3*x^(9/2)
```

Mupad [B]
time = 2.52, size = 69, normalized size = 0.81

$$x^{15/2}\left(\frac{2\,B\,a^3}{15}+\frac{2\,A\,b\,a^2}{5}\right)+x^{27/2}\left(\frac{2\,A\,b^3}{27}+\frac{2\,B\,a\,b^2}{9}\right)+\frac{2\,A\,a^3\,x^{9/2}}{9}+\frac{2\,B\,b^3\,x^{33/2}}{33}+\frac{2\,a\,b\,x^{21/2}\,\left(A\,b+B\,a\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(15/2)*((2*B*a^3)/15 + (2*A*a^2*b)/5) + x^(27/2)*((2*A*b^3)/27 + (2*B*a*b
^2)/9) + (2*A*a^3*x^(9/2))/9 + (2*B*b^3*x^(33/2))/33 + (2*a*b*x^(21/2)*(A*b
 + B*a))/7
```

3.148 $\int x^{5/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=85

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2(3Ab+aB)x^{13/2}+\frac{6}{19}ab(Ab+aB)x^{19/2}+\frac{2}{25}b^2(Ab+3aB)x^{25/2}+\frac{2}{31}b^3Bx^{31/2}$$

```
[Out] 2/7*a^3*A*x^(7/2)+2/13*a^2*(3*A*b+B*a)*x^(13/2)+6/19*a*b*(A*b+B*a)*x^(19/2)
+2/25*b^2*(A*b+3*B*a)*x^(25/2)+2/31*b^3*B*x^(31/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{25}b^2x^{25/2}(3aB+Ab)+\frac{6}{19}abx^{19/2}(aB+Ab)+\frac{2}{31}b^3Bx^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/31
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx&=\int \left(a^3Ax^{5/2}+a^2(3Ab+aB)x^{11/2}+3ab(Ab+aB)x^{17/2}+b^2(Ab+3aB)\right.\\&=\frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2(3Ab+aB)x^{13/2}+\frac{6}{19}ab(Ab+aB)x^{19/2}+\frac{2}{25}b^2(Ab+\end{aligned}$$

Mathematica [A]
time = 0.04, size = 91, normalized size = 1.07

$$\frac{2}{91}a^3x^{7/2}\left(13A+7Bx^3\right)+\frac{6}{247}a^2bx^{13/2}\left(19A+13Bx^3\right)+\frac{6}{475}ab^2x^{19/2}\left(25A+19Bx^3\right)+\frac{2}{775}b^3x^{25/2}\left(31A+25Bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*x^(7/2)*(13*A + 7*B*x^3))/91 + (6*a^2*b*x^(13/2)*(19*A + 13*B*x^3))/
247 + (6*a*b^2*x^(19/2)*(25*A + 19*B*x^3))/475 + (2*b^3*x^(25/2)*(31*A + 25
*B*x^3))/775
```

Maple [A]
time = 0.28, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2b^3 B x^{\frac{31}{2}}}{31}+\frac{2(A b^3+3Ba b^2) x^{\frac{25}{2}}}{25}+\frac{2(3Aa b^2+3B a^2 b) x^{\frac{19}{2}}}{19}+\frac{2(3A a^2 b+B a^3) x^{\frac{13}{2}}}{13}+\frac{2a^3 A x^{\frac{7}{2}}}{7}$
default	$\frac{2b^3 B x^{\frac{31}{2}}}{31}+\frac{2(A b^3+3Ba b^2) x^{\frac{25}{2}}}{25}+\frac{2(3Aa b^2+3B a^2 b) x^{\frac{19}{2}}}{19}+\frac{2(3A a^2 b+B a^3) x^{\frac{13}{2}}}{13}+\frac{2a^3 A x^{\frac{7}{2}}}{7}$
gosper	$\frac{2x^{\frac{7}{2}} (43225B b^3 x^{12}+53599x^9 A b^3+160797x^9 Ba b^2+211575x^6 Aa b^2+211575x^6 B a^2 b+309225x^3 A a^2 b+103075a^3 B x^3+1}{1339975}$
trager	$\frac{2x^{\frac{7}{2}} (43225B b^3 x^{12}+53599x^9 A b^3+160797x^9 Ba b^2+211575x^6 Aa b^2+211575x^6 B a^2 b+309225x^3 A a^2 b+103075a^3 B x^3+1}{1339975}$
risch	$\frac{2x^{\frac{7}{2}} (43225B b^3 x^{12}+53599x^9 A b^3+160797x^9 Ba b^2+211575x^6 Aa b^2+211575x^6 B a^2 b+309225x^3 A a^2 b+103075a^3 B x^3+1}{1339975}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/31*b^3*B*x^(31/2)+2/25*(A*b^3+3*B*a*b^2)*x^(25/2)+2/19*(3*A*a*b^2+3*B*a^2
*b)*x^(19/2)+2/13*(3*A*a^2*b+B*a^3)*x^(13/2)+2/7*a^3*A*x^(7/2)
```

Maxima [A]
time = 0.28, size = 73, normalized size = 0.86

$$\frac{2}{31} Bb^3 x^{\frac{31}{2}} + \frac{2}{25} \left(3 Bab^2 + Ab^3\right) x^{\frac{25}{2}} + \frac{6}{19} \left(Ba^2 b + Aab^2\right) x^{\frac{19}{2}} + \frac{2}{7} Aa^3 x^{\frac{7}{2}} + \frac{2}{13} \left(Ba^3 + 3 Aa^2 b\right) x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/31*B*b^3*x^(31/2) + 2/25*(3*B*a*b^2 + A*b^3)*x^(25/2) + 6/19*(B*a^2*b + A
*a*b^2)*x^(19/2) + 2/7*A*a^3*x^(7/2) + 2/13*(B*a^3 + 3*A*a^2*b)*x^(13/2)
```

Fricas [A]
time = 2.17, size = 78, normalized size = 0.92

$$\frac{2}{1339975} \left(43225 Bb^3 x^{15} + 53599 \left(3 Bab^2 + Ab^3\right) x^{12} + 211575 \left(Ba^2 b + Aab^2\right) x^9 + 191425 Aa^3 x^3 + 103075 \left(Ba^3 + 3 Aa^2 b\right) x^6\right) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/1339975*(43225*B*b^3*x^15 + 53599*(3*B*a*b^2 + A*b^3)*x^12 + 211575*(B*a^
2*b + A*a*b^2)*x^9 + 191425*A*a^3*x^3 + 103075*(B*a^3 + 3*A*a^2*b)*x^6)*sqr
t(x)
```

Sympy [A]
time = 3.57, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{7}{2}}}{7}+\frac{6Aa^2bx^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{19}{2}}}{19}+\frac{2Ab^3x^{\frac{25}{2}}}{25}+\frac{2Ba^3x^{\frac{13}{2}}}{13}+\frac{6Ba^2bx^{\frac{19}{2}}}{19}+\frac{6Bab^2x^{\frac{25}{2}}}{25}+\frac{2Bb^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(19/2)/19 + 2
*A*b**3*x**(25/2)/25 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(19/2)/19 + 6*
B*a*b**2*x**(25/2)/25 + 2*B*b**3*x**(31/2)/31
```

Giac [A]
time = 0.57, size = 77, normalized size = 0.91

$$\frac{2}{31}Bb^3x^{\frac{31}{2}}+\frac{6}{25}Bab^2x^{\frac{25}{2}}+\frac{2}{25}Ab^3x^{\frac{25}{2}}+\frac{6}{19}Ba^2bx^{\frac{19}{2}}+\frac{6}{19}Aab^2x^{\frac{19}{2}}+\frac{2}{13}Ba^3x^{\frac{13}{2}}+\frac{6}{13}Aa^2bx^{\frac{13}{2}}+\frac{2}{7}Aa^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/31*B*b^3*x^(31/2) + 6/25*B*a*b^2*x^(25/2) + 2/25*A*b^3*x^(25/2) + 6/19*B*
a^2*b*x^(19/2) + 6/19*A*a*b^2*x^(19/2) + 2/13*B*a^3*x^(13/2) + 6/13*A*a^2*b
*x^(13/2) + 2/7*A*a^3*x^(7/2)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,a^3}{13}+\frac{6\,A\,b\,a^2}{13}\right)+x^{25/2}\left(\frac{2\,A\,b^3}{25}+\frac{6\,B\,a\,b^2}{25}\right)+\frac{2\,A\,a^3\,x^{7/2}}{7}+\frac{2\,B\,b^3\,x^{31/2}}{31}+\frac{6\,a\,b\,x^{19/2}\,(A\,b+B\,a)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(13/2)*((2*B*a^3)/13 + (6*A*a^2*b)/13) + x^(25/2)*((2*A*b^3)/25 + (6*B*a*
b^2)/25) + (2*A*a^3*x^(7/2))/7 + (2*B*b^3*x^(31/2))/31 + (6*a*b*x^(19/2)*(A
*b + B*a))/19
```

3.149 $\int x^{3/2}(a+bx^3)^3\,(A+Bx^3)\;dx$

Optimal. Leaf size=85

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}+\frac{6}{17}ab(Ab+aB)x^{17/2}+\frac{2}{23}b^2(Ab+3aB)x^{23/2}+\frac{2}{29}b^3Bx^{29/2}$$

```
[Out] 2/5*a^3*A*x^(5/2)+2/11*a^2*(3*A*b+B*a)*x^(11/2)+6/17*a*b*(A*b+B*a)*x^(17/2)
+2/23*b^2*(A*b+3*B*a)*x^(23/2)+2/29*b^3*B*x^(29/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{23}b^2x^{23/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{29}b^3Bx^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/29
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx^3)^3\,(A+Bx^3)\;dx&=\int \left(a^3Ax^{3/2}+a^2(3Ab+aB)x^{9/2}+3ab(Ab+aB)x^{15/2}+b^2(Ab+3aB)x^2\right.\\&=\frac{2}{5}a^3Ax^{5/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}+\frac{6}{17}ab(Ab+aB)x^{17/2}+\frac{2}{23}b^2(Ab+3\end{aligned}$$

Mathematica [A]
time = 0.05, size = 81, normalized size = 0.95

$$\frac{2x^{5/2}(11339a^3(11A+5Bx^3)+10005a^2bx^3(17A+11Bx^3)+4785ab^2x^6(23A+17Bx^3)+935b^3x^9(29A+23Bx^3))}{623645}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]

[Out] (2*x^(5/2)*(11339*a^3*(11*A + 5*B*x^3) + 10005*a^2*b*x^3*(17*A + 11*B*x^3) + 4785*a*b^2*x^6*(23*A + 17*B*x^3) + 935*b^3*x^9*(29*A + 23*B*x^3)))/623645

Maple [A]
time = 0.28, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2b^3 B x^{\frac{29}{2}}}{29}+\frac{2(A b^3+3B a b^2) x^{\frac{23}{2}}}{23}+\frac{2(3A a b^2+3B a^2 b) x^{\frac{17}{2}}}{17}+\frac{2(3A a^2 b+B a^3) x^{\frac{11}{2}}}{11}+\frac{2a^3 A x^{\frac{5}{2}}}{5}$
default	$\frac{2b^3 B x^{\frac{29}{2}}}{29}+\frac{2(A b^3+3B a b^2) x^{\frac{23}{2}}}{23}+\frac{2(3A a b^2+3B a^2 b) x^{\frac{17}{2}}}{17}+\frac{2(3A a^2 b+B a^3) x^{\frac{11}{2}}}{11}+\frac{2a^3 A x^{\frac{5}{2}}}{5}$
gosper	$\frac{2x^{\frac{5}{2}}(21505B b^3 x^{12}+27115x^9 A b^3+81345x^9 B a b^2+110055x^6 A a b^2+110055x^6 B a^2 b+170085x^3 A a^2 b+56695a^3 B x^3+12$ / 623645
trager	$\frac{2x^{\frac{5}{2}}(21505B b^3 x^{12}+27115x^9 A b^3+81345x^9 B a b^2+110055x^6 A a b^2+110055x^6 B a^2 b+170085x^3 A a^2 b+56695a^3 B x^3+12$ / 623645
risch	$\frac{2x^{\frac{5}{2}}(21505B b^3 x^{12}+27115x^9 A b^3+81345x^9 B a b^2+110055x^6 A a b^2+110055x^6 B a^2 b+170085x^3 A a^2 b+56695a^3 B x^3+12$ / 623645

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)

[Out] 2/29*b^3*B*x^(29/2)+2/23*(A*b^3+3*B*a*b^2)*x^(23/2)+2/17*(3*A*a*b^2+3*B*a^2*b)*x^(17/2)+2/11*(3*A*a^2*b+B*a^3)*x^(11/2)+2/5*a^3*A*x^(5/2)

Maxima [A]
time = 0.27, size = 73, normalized size = 0.86

$$\frac{2}{29}Bb^3x^{\frac{29}{2}}+\frac{2}{23}\left(3Bab^2+Ab^3\right)x^{\frac{23}{2}}+\frac{6}{17}\left(Ba^2b+Aab^2\right)x^{\frac{17}{2}}+\frac{2}{5}Aa^3x^{\frac{5}{2}}+\frac{2}{11}\left(Ba^3+3Aa^2b\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")

[Out] 2/29*B*b^3*x^(29/2) + 2/23*(3*B*a*b^2 + A*b^3)*x^(23/2) + 6/17*(B*a^2*b + A*a*b^2)*x^(17/2) + 2/5*A*a^3*x^(5/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)

Fricas [A]
time = 2.27, size = 78, normalized size = 0.92

$$\frac{2}{623645}\left(21505\,Bb^3x^{14}+27115\left(3\,Bab^2+Ab^3\right)x^{11}+110055\left(Ba^2b+Aab^2\right)x^8+124729\,Aa^3x^2+56695\left(Ba^3+3\,Aa^2b\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")

[Out] 2/623645*(21505*B*b^3*x^14 + 27115*(3*B*a*b^2 + A*b^3)*x^11 + 110055*(B*a^2*b + A*a*b^2)*x^8 + 124729*A*a^3*x^2 + 56695*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(x)

Sympy [A]
time = 1.59, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{6Aa^2bx^{\frac{11}{2}}}{11}+\frac{6Aab^2x^{\frac{17}{2}}}{17}+\frac{2Ab^3x^{\frac{23}{2}}}{23}+\frac{2Ba^3x^{\frac{11}{2}}}{11}+\frac{6Ba^2bx^{\frac{17}{2}}}{17}+\frac{6Bab^2x^{\frac{23}{2}}}{23}+\frac{2Bb^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(5/2)/5 + 6*A*a**2*b*x**(11/2)/11 + 6*A*a*b**2*x**(17/2)/17 + 2
*A*b**3*x**(23/2)/23 + 2*B*a**3*x**(11/2)/11 + 6*B*a**2*b*x**(17/2)/17 + 6*
B*a*b**2*x**(23/2)/23 + 2*B*b**3*x**(29/2)/29
```

Giac [A]
time = 0.63, size = 77, normalized size = 0.91

$$\frac{2}{29}Bb^3x^{\frac{29}{2}}+\frac{6}{23}Bab^2x^{\frac{23}{2}}+\frac{2}{23}Ab^3x^{\frac{23}{2}}+\frac{6}{17}Ba^2bx^{\frac{17}{2}}+\frac{6}{17}Aab^2x^{\frac{17}{2}}+\frac{2}{11}Ba^3x^{\frac{11}{2}}+\frac{6}{11}Aa^2bx^{\frac{11}{2}}+\frac{2}{5}Aa^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/29*B*b^3*x^(29/2) + 6/23*B*a*b^2*x^(23/2) + 2/23*A*b^3*x^(23/2) + 6/17*B*
a^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2*b
*x^(11/2) + 2/5*A*a^3*x^(5/2)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,B\,a^3}{11}+\frac{6\,A\,b\,a^2}{11}\right)+x^{23/2}\left(\frac{2\,A\,b^3}{23}+\frac{6\,B\,a\,b^2}{23}\right)+\frac{2\,A\,a^3\,x^{5/2}}{5}+\frac{2\,B\,b^3\,x^{29/2}}{29}+\frac{6\,a\,b\,x^{17/2}\,\left(A\,b+B\,a\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(11/2)*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^(23/2)*((2*A*b^3)/23 + (6*B*a*
b^2)/23) + (2*A*a^3*x^(5/2))/5 + (2*B*b^3*x^(29/2))/29 + (6*a*b*x^(17/2)*(A
*b + B*a))/17
```

3.150 $\int \sqrt{x} \left(a+bx^3\right)^3 \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=85

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2}+\frac{2}{5}ab(Ab+aB)x^{15/2}+\frac{2}{21}b^2(Ab+3aB)x^{21/2}+\frac{2}{27}b^3Bx^{27/2}$$

```
[Out] 2/3*a^3*A*x^(3/2)+2/9*a^2*(3*A*b+B*a)*x^(9/2)+2/5*a*b*(A*b+B*a)*x^(15/2)+2/
21*b^2*(A*b+3*B*a)*x^(21/2)+2/27*b^3*B*x^(27/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{27}b^3Bx^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (2*a*b*(A*b + a*B)*
x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(27/2))/27
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx &= \int \left(a^3A\sqrt{x}+a^2(3Ab+aB)x^{7/2}+3ab(Ab+aB)x^{13/2}+b^2(Ab+3aB)\right.\\ &= \frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2}+\frac{2}{5}ab(Ab+aB)x^{15/2}+\frac{2}{21}b^2(Ab+3a\end{aligned}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.94

$$\frac{2}{945}x^{3/2}\left(105a^3\left(3A+Bx^3\right)+63a^2bx^3\left(5A+3Bx^3\right)+27ab^2x^6\left(7A+5Bx^3\right)+5b^3x^9\left(9A+7Bx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*x^(3/2)*(105*a^3*(3*A + B*x^3) + 63*a^2*b*x^3*(5*A + 3*B*x^3) + 27*a*b^2
*x^6*(7*A + 5*B*x^3) + 5*b^3*x^9*(9*A + 7*B*x^3)))/945
```

Maple [A]
time = 0.26, size = 76, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2b^3 B x^{\frac{27}{2}}}{27}+\frac{2(A b^3+3Ba b^2)x^{\frac{21}{2}}}{21}+\frac{2(3Aa b^2+3B a^2 b)x^{\frac{15}{2}}}{15}+\frac{2(3A a^2 b+B a^3)x^{\frac{9}{2}}}{9}+\frac{2a^3 A x^{\frac{3}{2}}}{3}$	76
default	$\frac{2b^3 B x^{\frac{27}{2}}}{27}+\frac{2(A b^3+3Ba b^2)x^{\frac{21}{2}}}{21}+\frac{2(3Aa b^2+3B a^2 b)x^{\frac{15}{2}}}{15}+\frac{2(3A a^2 b+B a^3)x^{\frac{9}{2}}}{9}+\frac{2a^3 A x^{\frac{3}{2}}}{3}$	76
gosper	$\frac{2x^{\frac{3}{2}}(35B b^3 x^{12}+45x^9 A b^3+135x^9 Ba b^2+189x^6 Aa b^2+189x^6 B a^2 b+315x^3 A a^2 b+105a^3 B x^3+315A a^3)}{945}$	80
trager	$\frac{2x^{\frac{3}{2}}(35B b^3 x^{12}+45x^9 A b^3+135x^9 Ba b^2+189x^6 Aa b^2+189x^6 B a^2 b+315x^3 A a^2 b+105a^3 B x^3+315A a^3)}{945}$	80
risch	$\frac{2x^{\frac{3}{2}}(35B b^3 x^{12}+45x^9 A b^3+135x^9 Ba b^2+189x^6 Aa b^2+189x^6 B a^2 b+315x^3 A a^2 b+105a^3 B x^3+315A a^3)}{945}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*b^3*B*x^(27/2)+2/21*(A*b^3+3*B*a*b^2)*x^(21/2)+2/15*(3*A*a*b^2+3*B*a^2
*b)*x^(15/2)+2/9*(3*A*a^2*b+B*a^3)*x^(9/2)+2/3*a^3*A*x^(3/2)
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.86

$$\frac{2}{27}Bb^3x^{\frac{27}{2}}+\frac{2}{21}\left(3\,Bab^2+Ab^3\right)x^{\frac{21}{2}}+\frac{2}{5}\left(Ba^2b+Aab^2\right)x^{\frac{15}{2}}+\frac{2}{3}Aa^3x^{\frac{3}{2}}+\frac{2}{9}\left(Ba^3+3\,Aa^2b\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*B*b^3*x^(27/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 2/5*(B*a^2*b + A*
a*b^2)*x^(15/2) + 2/3*A*a^3*x^(3/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

Fricas [A]
time = 1.69, size = 76, normalized size = 0.89

$$\frac{2}{945}\left(35\,Bb^3x^{13}+45\left(3\,Bab^2+Ab^3\right)x^{10}+189\left(Ba^2b+Aab^2\right)x^7+315\,Aa^3x+105\left(Ba^3+3\,Aa^2b\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^3*x^13 + 45*(3*B*a*b^2 + A*b^3)*x^10 + 189*(B*a^2*b + A*a*b^2
)*x^7 + 315*A*a^3*x + 105*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

Sympy [A]
time = 2.49, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{3}{2}}}{3}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{2Aab^2x^{\frac{15}{2}}}{5}+\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{2Ba^2bx^{\frac{15}{2}}}{5}+\frac{2Bab^2x^{\frac{21}{2}}}{7}+\frac{2Bb^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a**3*x**(3/2)/3 + 2*A*a**2*b*x**(9/2)/3 + 2*A*a*b**2*x**(15/2)/5 + 2*A*
b**3*x**(21/2)/21 + 2*B*a**3*x**(9/2)/9 + 2*B*a**2*b*x**(15/2)/5 + 2*B*a*b*
*2*x**(21/2)/7 + 2*B*b**3*x**(27/2)/27
```

Giac [A]
time = 0.60, size = 77, normalized size = 0.91

$$\frac{2}{27}Bb^3x^{\frac{27}{2}}+\frac{2}{7}Bab^2x^{\frac{21}{2}}+\frac{2}{21}Ab^3x^{\frac{21}{2}}+\frac{2}{5}Ba^2bx^{\frac{15}{2}}+\frac{2}{5}Aab^2x^{\frac{15}{2}}+\frac{2}{9}Ba^3x^{\frac{9}{2}}+\frac{2}{3}Aa^2bx^{\frac{9}{2}}+\frac{2}{3}Aa^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*B*b^3*x^(27/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2) + 2/5*B*a^
2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*x^(9/
2) + 2/3*A*a^3*x^(3/2)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.81

$$x^{9/2}\left(\frac{2\,B\,a^3}{9}+\frac{2\,A\,b\,a^2}{3}\right)+x^{21/2}\left(\frac{2\,A\,b^3}{21}+\frac{2\,B\,a\,b^2}{7}\right)+\frac{2\,A\,a^3\,x^{3/2}}{3}+\frac{2\,B\,b^3\,x^{27/2}}{27}+\frac{2\,a\,b\,x^{15/2}\,\left(A\,b+B\,a\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(9/2)*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^(21/2)*((2*A*b^3)/21 + (2*B*a*b^2
)/7) + (2*A*a^3*x^(3/2))/3 + (2*B*b^3*x^(27/2))/27 + (2*a*b*x^(15/2)*(A*b +
 B*a))/5
```

3.151 $\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx$

Optimal. Leaf size=83

$$2a^3A\sqrt{x}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{19}b^2(Ab+3aB)x^{19/2}+\frac{2}{25}b^3Bx^{25/2}$$

```
[Out] 2/7*a^2*(3*A*b+B*a)*x^(7/2)+6/13*a*b*(A*b+B*a)*x^(13/2)+2/19*b^2*(A*b+3*B*a
)*x^(19/2)+2/25*b^3*B*x^(25/2)+2*a^3*A*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$2a^3A\sqrt{x}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(1
3/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx &= \int\left(\frac{a^3A}{\sqrt{x}}+a^2(3Ab+aB)x^{5/2}+3ab(Ab+aB)x^{11/2}+b^2(Ab+3aB)x^{17/2}+b^3\right.\\ &=2a^3A\sqrt{x}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{19}b^2(Ab+3aB)x^{19}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.96

$$\frac{2\sqrt{x}\,(6175a^3(7A+Bx^3)+1425a^2bx^3(13A+7Bx^3)+525ab^2x^6(19A+13Bx^3)+91b^3x^9(25A+19Bx^3))}{43225}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(6175*a^3*(7*A + B*x^3) + 1425*a^2*b*x^3*(13*A + 7*B*x^3) + 525*
a*b^2*x^6*(19*A + 13*B*x^3) + 91*b^3*x^9*(25*A + 19*B*x^3)))/43225
```

Maple [A]
time = 0.27, size = 76, normalized size = 0.92

method	result
derivativedivides	$\frac{2b^3 B x^{\frac{25}{2}}}{25}+\frac{2(A b^3+3Ba b^2)x^{\frac{19}{2}}}{19}+\frac{2(3Aa b^2+3B a^2 b)x^{\frac{13}{2}}}{13}+\frac{2(3A a^2 b+B a^3)x^{\frac{7}{2}}}{7}+2a^3 A\sqrt{x}$
default	$\frac{2b^3 B x^{\frac{25}{2}}}{25}+\frac{2(A b^3+3Ba b^2)x^{\frac{19}{2}}}{19}+\frac{2(3Aa b^2+3B a^2 b)x^{\frac{13}{2}}}{13}+\frac{2(3A a^2 b+B a^3)x^{\frac{7}{2}}}{7}+2a^3 A\sqrt{x}$
trager	$\left(\frac{2}{25}B b^3 x^{12}+\frac{2}{19}x^9 A b^3+\frac{6}{19}x^9 Ba b^2+\frac{6}{13}x^6 Aa b^2+\frac{6}{13}x^6 B a^2 b+\frac{6}{7}x^3 A a^2 b+\frac{2}{7}a^3 B x^3+\right.$
gosper	$\frac{2\sqrt{x}\,(1729B b^3 x^{12}+2275x^9 A b^3+6825x^9 Ba b^2+9975x^6 Aa b^2+9975x^6 B a^2 b+18525x^3 A a^2 b+6175a^3 B x^3+43225A a}{43225}$
risch	$\frac{2\sqrt{x}\,(1729B b^3 x^{12}+2275x^9 A b^3+6825x^9 Ba b^2+9975x^6 Aa b^2+9975x^6 B a^2 b+18525x^3 A a^2 b+6175a^3 B x^3+43225A a}{43225}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/25*b^3*B*x^(25/2)+2/19*(A*b^3+3*B*a*b^2)*x^(19/2)+2/13*(3*A*a*b^2+3*B*a^2
*b)*x^(13/2)+2/7*(3*A*a^2*b+B*a^3)*x^(7/2)+2*a^3*A*x^(1/2)
```

Maxima [A]
time = 0.28, size = 73, normalized size = 0.88

$$\frac{2}{25}\,Bb^3x^{\frac{25}{2}}+\frac{2}{19}\left(3\,Bab^2+Ab^3\right)x^{\frac{19}{2}}+\frac{6}{13}\left(Ba^2b+Aab^2\right)x^{\frac{13}{2}}+2\,Aa^3\sqrt{x}+\frac{2}{7}\left(Ba^3+3\,Aa^2b\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 6/13*(B*a^2*b + A
*a*b^2)*x^(13/2) + 2*A*a^3*sqrt(x) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

Fricas [A]
time = 1.45, size = 75, normalized size = 0.90

$$\frac{2}{43225}\left(1729\,Bb^3x^{12}+2275\left(3\,Bab^2+Ab^3\right)x^9+9975\left(Ba^2b+Aab^2\right)x^6+43225\,Aa^3+6175\left(Ba^3+3\,Aa^2b\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/43225*(1729*B*b^3*x^12 + 2275*(3*B*a*b^2 + A*b^3)*x^9 + 9975*(B*a^2*b + A
*a*b^2)*x^6 + 43225*A*a^3 + 6175*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

Sympy [A]
time = 1.70, size = 112, normalized size = 1.35

$$2Aa^3\sqrt{x} + \frac{6Aa^2bx^{\frac{7}{2}}}{7} + \frac{6Aab^2x^{\frac{13}{2}}}{13} + \frac{2Ab^3x^{\frac{19}{2}}}{19} + \frac{2Ba^3x^{\frac{7}{2}}}{7} + \frac{6Ba^2bx^{\frac{13}{2}}}{13} + \frac{6Bab^2x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(7/2)/7 + 6*A*a*b**2*x**(13/2)/13 + 2*A*b*
*3*x**(19/2)/19 + 2*B*a**3*x**(7/2)/7 + 6*B*a**2*b*x**(13/2)/13 + 6*B*a*b**
2*x**(19/2)/19 + 2*B*b**3*x**(25/2)/25
```

Giac [A]
time = 0.66, size = 77, normalized size = 0.93

$$\frac{2}{25}\,Bb^3x^{\frac{25}{2}} + \frac{6}{19}\,Bab^2x^{\frac{19}{2}} + \frac{2}{19}\,Ab^3x^{\frac{19}{2}} + \frac{6}{13}\,Ba^2bx^{\frac{13}{2}} + \frac{6}{13}\,Aab^2x^{\frac{13}{2}} + \frac{2}{7}\,Ba^3x^{\frac{7}{2}} + \frac{6}{7}\,Aa^2bx^{\frac{7}{2}} + 2\,Aa^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2) + 6/13*B*
a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*x^
(7/2) + 2*A*a^3*sqrt(x)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.83

$$x^{7/2}\left(\frac{2\,B\,a^3}{7}+\frac{6\,A\,b\,a^2}{7}\right)+x^{19/2}\left(\frac{2\,A\,b^3}{19}+\frac{6\,B\,a\,b^2}{19}\right)+2\,A\,a^3\,\sqrt{x}+\frac{2\,B\,b^3\,x^{25/2}}{25}+\frac{6\,a\,b\,x^{13/2}\,\left(A\,b+B\,a\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(1/2),x)
```

```
[Out] x^(7/2)*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^(19/2)*((2*A*b^3)/19 + (6*B*a*b^2
)/19) + 2*A*a^3*x^(1/2) + (2*B*b^3*x^(25/2))/25 + (6*a*b*x^(13/2)*(A*b + B*
a))/13
```

3.152 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2a^3A}{\sqrt{x}}+\frac{2}{5}a^2(3Ab+aB)x^{5/2}+\frac{6}{11}ab(Ab+aB)x^{11/2}+\frac{2}{17}b^2(Ab+3aB)x^{17/2}+\frac{2}{23}b^3Bx^{23/2}$$

```
[Out] 2/5*a^2*(3*A*b+B*a)*x^(5/2)+6/11*a*b*(A*b+B*a)*x^(11/2)+2/17*b^2*(A*b+3*B*a
)*x^(17/2)+2/23*b^3*B*x^(23/2)-2*a^3*A/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$-\frac{2a^3A}{\sqrt{x}}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (6*a*b*(A*b + a*B)*x
^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(23/2))/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx &= \int \left(\frac{a^3A}{x^{3/2}}+a^2(3Ab+aB)x^{3/2}+3ab(Ab+aB)x^{9/2}+b^2(Ab+3aB)x^{15/2}+b\right.\\ &= -\frac{2a^3A}{\sqrt{x}}+\frac{2}{5}a^2(3Ab+aB)x^{5/2}+\frac{6}{11}ab(Ab+aB)x^{11/2}+\frac{2}{17}b^2(Ab+3aB)x^{17/}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 1.00

$$-\frac{2(21505a^3A-12903a^2Abx^3-4301a^3Bx^3-5865aAb^2x^6-5865a^2bBx^6-1265Ab^3x^9-3795ab^2Bx^9-935b^3Bx^{12})}{21505\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]

[Out] (-2*(21505*a^3*A - 12903*a^2*A*b*x^3 - 4301*a^3*B*x^3 - 5865*a*A*b^2*x^6 - 5865*a^2*b*B*x^6 - 1265*A*b^3*x^9 - 3795*a*b^2*B*x^9 - 935*b^3*B*x^12))/(21505*Sqrt[x])

Maple [A]
time = 0.27, size = 78, normalized size = 0.94

method	result
derivativedivides	$\frac{2 b^{3} B \,x^{\frac{23}{2}}}{23}+\frac{2 A \,b^{3} x^{\frac{17}{2}}}{17}+\frac{6 B a \,b^{2} x^{\frac{17}{2}}}{17}+\frac{6 A a \,b^{2} x^{\frac{11}{2}}}{11}+\frac{6 B \,a^{2} b \,x^{\frac{11}{2}}}{11}+\frac{6 A \,a^{2} b \,x^{\frac{5}{2}}}{5}+\frac{2 B \,a^{3} x^{\frac{5}{2}}}{5}-\frac{2 a^{3} A}{\sqrt{x}}$
default	$\frac{2 b^{3} B \,x^{\frac{23}{2}}}{23}+\frac{2 A \,b^{3} x^{\frac{17}{2}}}{17}+\frac{6 B a \,b^{2} x^{\frac{17}{2}}}{17}+\frac{6 A a \,b^{2} x^{\frac{11}{2}}}{11}+\frac{6 B \,a^{2} b \,x^{\frac{11}{2}}}{11}+\frac{6 A \,a^{2} b \,x^{\frac{5}{2}}}{5}+\frac{2 B \,a^{3} x^{\frac{5}{2}}}{5}-\frac{2 a^{3} A}{\sqrt{x}}$
gosper	$-\frac{2\left(-935 B \,b^{3} x^{12}-1265 x^{9} A \,b^{3}-3795 x^{9} B a \,b^{2}-5865 x^{6} A a \,b^{2}-5865 x^{6} B \,a^{2} b-12903 x^{3} A \,a^{2} b-4301 a^{3} B \,x^{3}+21505 A \,a^{3}\right)}{21505 \sqrt{x}}$
trager	$-\frac{2\left(-935 B \,b^{3} x^{12}-1265 x^{9} A \,b^{3}-3795 x^{9} B a \,b^{2}-5865 x^{6} A a \,b^{2}-5865 x^{6} B \,a^{2} b-12903 x^{3} A \,a^{2} b-4301 a^{3} B \,x^{3}+21505 A \,a^{3}\right)}{21505 \sqrt{x}}$
risch	$-\frac{2\left(-935 B \,b^{3} x^{12}-1265 x^{9} A \,b^{3}-3795 x^{9} B a \,b^{2}-5865 x^{6} A a \,b^{2}-5865 x^{6} B \,a^{2} b-12903 x^{3} A \,a^{2} b-4301 a^{3} B \,x^{3}+21505 A \,a^{3}\right)}{21505 \sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/23*b^3*B*x^(23/2)+2/17*A*b^3*x^(17/2)+6/17*B*a*b^2*x^(17/2)+6/11*A*a*b^2*x^(11/2)+6/11*B*a^2*b*x^(11/2)+6/5*A*a^2*b*x^(5/2)+2/5*B*a^3*x^(5/2)-2*a^3*A/x^(1/2)

Maxima [A]
time = 0.28, size = 73, normalized size = 0.88

$$\frac{2}{23} Bb^{3}x^{\frac{23}{2}}+\frac{2}{17}\left(3 Bab^{2}+Ab^{3}\right)x^{\frac{17}{2}}+\frac{6}{11}\left(Ba^{2}b+Aab^{2}\right)x^{\frac{11}{2}}-\frac{2 Aa^{3}}{\sqrt{x}}+\frac{2}{5}\left(Ba^{3}+3 Aa^{2}b\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="maxima")

[Out] 2/23*B*b^3*x^(23/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/11*(B*a^2*b + A*a*b^2)*x^(11/2) - 2*A*a^3/sqrt(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)

Fricas [A]
time = 1.53, size = 75, normalized size = 0.90

$$\frac{2\left(935 Bb^{3}x^{12}+1265\left(3 Bab^{2}+Ab^{3}\right)x^{9}+5865\left(Ba^{2}b+Aab^{2}\right)x^{6}-21505 Aa^{3}+4301\left(Ba^{3}+3 Aa^{2}b\right)x^{3}\right)}{21505 \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21505*(935*B*b^3*x^12 + 1265*(3*B*a*b^2 + A*b^3)*x^9 + 5865*(B*a^2*b + A*
a*b^2)*x^6 - 21505*A*a^3 + 4301*(B*a^3 + 3*A*a^2*b)*x^3)/sqrt(x)
```

Sympy [A]
time = 2.04, size = 112, normalized size = 1.35

$$-\frac{2Aa^3}{\sqrt{x}}+\frac{6Aa^2bx^{\frac{5}{2}}}{5}+\frac{6Aab^2x^{\frac{11}{2}}}{11}+\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{2Ba^3x^{\frac{5}{2}}}{5}+\frac{6Ba^2bx^{\frac{11}{2}}}{11}+\frac{6Bab^2x^{\frac{17}{2}}}{17}+\frac{2Bb^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(3/2),x)
```

```
[Out] -2*A*a**3/sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 6*A*a*b**2*x**(11/2)/11 + 2*A*b
**3*x**(17/2)/17 + 2*B*a**3*x**(5/2)/5 + 6*B*a**2*b*x**(11/2)/11 + 6*B*a*b*
*2*x**(17/2)/17 + 2*B*b**3*x**(23/2)/23
```

Giac [A]
time = 0.62, size = 77, normalized size = 0.93

$$\frac{2}{23}Bb^3x^{\frac{23}{2}}+\frac{6}{17}Bab^2x^{\frac{17}{2}}+\frac{2}{17}Ab^3x^{\frac{17}{2}}+\frac{6}{11}Ba^2bx^{\frac{11}{2}}+\frac{6}{11}Aab^2x^{\frac{11}{2}}+\frac{2}{5}Ba^3x^{\frac{5}{2}}+\frac{6}{5}Aa^2bx^{\frac{5}{2}}-\frac{2\,Aa^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/11*B*
a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^
(5/2) - 2*A*a^3/sqrt(x)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.83

$$x^{5/2}\left(\frac{2\,B\,a^3}{5}+\frac{6\,A\,b\,a^2}{5}\right)+x^{17/2}\left(\frac{2\,A\,b^3}{17}+\frac{6\,B\,a\,b^2}{17}\right)-\frac{2\,A\,a^3}{\sqrt{x}}+\frac{2\,B\,b^3\,x^{23/2}}{23}+\frac{6\,a\,b\,x^{11/2}\,\left(A\,b+B\,a\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(3/2),x)
```

```
[Out] x^(5/2)*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^(17/2)*((2*A*b^3)/17 + (6*B*a*b^2
)/17) - (2*A*a^3)/x^(1/2) + (2*B*b^3*x^(23/2))/23 + (6*a*b*x^(11/2)*(A*b +
B*a))/11
```

3.153 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}} \, dx$

Optimal. Leaf size=85

$$-\frac{2a^3A}{3x^{3/2}}+\frac{2}{3}a^2(3Ab+aB)x^{3/2}+\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{15}b^2(Ab+3aB)x^{15/2}+\frac{2}{21}b^3Bx^{21/2}$$

```
[Out] -2/3*a^3*A/x^(3/2)+2/3*a^2*(3*A*b+B*a)*x^(3/2)+2/3*a*b*(A*b+B*a)*x^(9/2)+2/
15*b^2*(A*b+3*B*a)*x^(15/2)+2/21*b^3*B*x^(21/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$-\frac{2a^3A}{3x^{3/2}}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (2*a*b*(A*b + a*
B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx &= \int \left(\frac{a^3A}{x^{5/2}}+a^2(3Ab+aB)\sqrt{x}+3ab(Ab+aB)x^{7/2}+b^2(Ab+3aB)x^{13/2}+b^3\right.\\ &= -\frac{2a^3A}{3x^{3/2}}+\frac{2}{3}a^2(3Ab+aB)x^{3/2}+\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{15}b^2(Ab+3aB)x^{15/2}+\end{aligned}$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 0.91

$$\frac{2(-35a^3(A-Bx^3)+35a^2bx^3(3A+Bx^3)+7ab^2x^6(5A+3Bx^3)+b^3x^9(7A+5Bx^3))}{105x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (2*(-35*a^3*(A - B*x^3) + 35*a^2*b*x^3*(3*A + B*x^3) + 7*a*b^2*x^6*(5*A + 3
*B*x^3) + b^3*x^9*(7*A + 5*B*x^3)))/(105*x^(3/2))
```

Maple [A]
time = 0.28, size = 78, normalized size = 0.92

method	result	size
derivativedivides	$\frac{2b^3 B\,x^{\frac{21}{2}}}{21}+\frac{2A\,b^3 x^{\frac{15}{2}}}{15}+\frac{2Ba\,b^2 x^{\frac{15}{2}}}{5}+\frac{2Aa\,b^2 x^{\frac{9}{2}}}{3}+\frac{2B\,a^2 b\,x^{\frac{9}{2}}}{3}+2A\,a^2 b\,x^{\frac{3}{2}}+\frac{2B\,a^3 x^{\frac{3}{2}}}{3}-\frac{2a^3 A}{3x^{\frac{3}{2}}}$	78
default	$\frac{2b^3 B\,x^{\frac{21}{2}}}{21}+\frac{2A\,b^3 x^{\frac{15}{2}}}{15}+\frac{2Ba\,b^2 x^{\frac{15}{2}}}{5}+\frac{2Aa\,b^2 x^{\frac{9}{2}}}{3}+\frac{2B\,a^2 b\,x^{\frac{9}{2}}}{3}+2A\,a^2 b\,x^{\frac{3}{2}}+\frac{2B\,a^3 x^{\frac{3}{2}}}{3}-\frac{2a^3 A}{3x^{\frac{3}{2}}}$	78
gosper	$-\frac{2\left(-5B\,b^3 x^{12}-7x^9 A\,b^3-21x^9 Ba\,b^2-35x^6 Aa\,b^2-35x^6 B\,a^2 b-105x^3 A\,a^2 b-35a^3 B\,x^3+35A\,a^3\right)}{105x^{\frac{3}{2}}}$	80
trager	$-\frac{2\left(-5B\,b^3 x^{12}-7x^9 A\,b^3-21x^9 Ba\,b^2-35x^6 Aa\,b^2-35x^6 B\,a^2 b-105x^3 A\,a^2 b-35a^3 B\,x^3+35A\,a^3\right)}{105x^{\frac{3}{2}}}$	80
risch	$-\frac{2\left(-5B\,b^3 x^{12}-7x^9 A\,b^3-21x^9 Ba\,b^2-35x^6 Aa\,b^2-35x^6 B\,a^2 b-105x^3 A\,a^2 b-35a^3 B\,x^3+35A\,a^3\right)}{105x^{\frac{3}{2}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*b^3*B*x^(21/2)+2/15*A*b^3*x^(15/2)+2/5*B*a*b^2*x^(15/2)+2/3*A*a*b^2*x^
(9/2)+2/3*B*a^2*b*x^(9/2)+2*A*a^2*b*x^(3/2)+2/3*B*a^3*x^(3/2)-2/3*a^3*A/x^(
3/2)
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.86

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{2}{15}\left(3\,Bab^2+Ab^3\right)x^{\frac{15}{2}}+\frac{2}{3}\left(Ba^2b+Aab^2\right)x^{\frac{9}{2}}-\frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}+\frac{2}{3}\left(Ba^3+3\,Aa^2b\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 2/3*(B*a^2*b + A*
a*b^2)*x^(9/2) - 2/3*A*a^3/x^(3/2) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

Fricas [A]
time = 1.67, size = 75, normalized size = 0.88

$$\frac{2\left(5\,Bb^3x^{12}+7\left(3\,Bab^2+Ab^3\right)x^9+35\left(Ba^2b+Aab^2\right)x^6-35\,Aa^3+35\left(Ba^3+3\,Aa^2b\right)x^3\right)}{105\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(5*B*b^3*x^12 + 7*(3*B*a*b^2 + A*b^3)*x^9 + 35*(B*a^2*b + A*a*b^2)*x^
6 - 35*A*a^3 + 35*(B*a^3 + 3*A*a^2*b)*x^3)/x^(3/2)
```

Sympy [A]
time = 1.51, size = 112, normalized size = 1.32

$$-\frac{2Aa^3}{3x^{\frac{3}{2}}}+2Aa^2bx^{\frac{3}{2}}+\frac{2Aab^2x^{\frac{9}{2}}}{3}+\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Ba^3x^{\frac{3}{2}}}{3}+\frac{2Ba^2bx^{\frac{9}{2}}}{3}+\frac{2Bab^2x^{\frac{15}{2}}}{5}+\frac{2Bb^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a**3/(3*x**(3/2)) + 2*A*a**2*b*x**(3/2) + 2*A*a*b**2*x**(9/2)/3 + 2*A*
b**3*x**(15/2)/15 + 2*B*a**3*x**(3/2)/3 + 2*B*a**2*b*x**(9/2)/3 + 2*B*a*b**
2*x**(15/2)/5 + 2*B*b**3*x**(21/2)/21
```

Giac [A]
time = 0.57, size = 77, normalized size = 0.91

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{2}{5}\,Bab^2x^{\frac{15}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}+\frac{2}{3}\,Ba^2bx^{\frac{9}{2}}+\frac{2}{3}\,Aab^2x^{\frac{9}{2}}+\frac{2}{3}\,Ba^3x^{\frac{3}{2}}+2\,Aa^2bx^{\frac{3}{2}}-\frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 2/3*B*a^
2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2) -
 2/3*A*a^3/x^(3/2)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.81

$$x^{3/2}\left(\frac{2\,B\,a^3}{3}+2\,A\,b\,a^2\right)+x^{15/2}\left(\frac{2\,A\,b^3}{15}+\frac{2\,B\,a\,b^2}{5}\right)-\frac{2\,A\,a^3}{3\,x^{3/2}}+\frac{2\,B\,b^3\,x^{21/2}}{21}+\frac{2\,a\,b\,x^{9/2}\,\left(A\,b+B\,a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(5/2),x)
```

```
[Out] x^(3/2)*((2*B*a^3)/3 + 2*A*a^2*b) + x^(15/2)*((2*A*b^3)/15 + (2*B*a*b^2)/5)
 - (2*A*a^3)/(3*x^(3/2)) + (2*B*b^3*x^(21/2))/21 + (2*a*b*x^(9/2)*(A*b + B*
a))/3
```

3.154 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2a^3A}{5x^{5/2}}+2a^2(3Ab+aB)\sqrt{x}+\frac{6}{7}ab(Ab+aB)x^{7/2}+\frac{2}{13}b^2(Ab+3aB)x^{13/2}+\frac{2}{19}b^3Bx^{19/2}$$

```
[Out] -2/5*a^3*A/x^(5/2)+6/7*a*b*(A*b+B*a)*x^(7/2)+2/13*b^2*(A*b+3*B*a)*x^(13/2)+
2/19*b^3*B*x^(19/2)+2*a^2*(3*A*b+B*a)*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$-\frac{2a^3A}{5x^{5/2}}+2a^2\sqrt{x}\,(aB+3Ab)+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a^3*A)/(5*x^(5/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*x
^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3\,(A+Bx^3)}{x^{7/2}}\,dx &= \int \left(\frac{a^3A}{x^{7/2}}+\frac{a^2(3Ab+aB)}{\sqrt{x}}+3ab(Ab+aB)x^{5/2}+b^2(Ab+3aB)x^{11/2}+b^3Bx\right.\\ &= -\frac{2a^3A}{5x^{5/2}}+2a^2(3Ab+aB)\sqrt{x}+\frac{6}{7}ab(Ab+aB)x^{7/2}+\frac{2}{13}b^2(Ab+3aB)x^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 78, normalized size = 0.94

$$\frac{-3458a^3(A-5Bx^3)+7410a^2bx^3(7A+Bx^3)+570ab^2x^6(13A+7Bx^3)+70b^3x^9(19A+13Bx^3)}{8645x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-3458*a^3*(A - 5*B*x^3) + 7410*a^2*b*x^3*(7*A + B*x^3) + 570*a*b^2*x^6*(13
*A + 7*B*x^3) + 70*b^3*x^9*(19*A + 13*B*x^3))/(8645*x^(5/2))
```

Maple [A]
time = 0.27, size = 78, normalized size = 0.94

method	result	s
derivativedivides	$\frac{2b^3 B x^{\frac{19}{2}}}{19}+\frac{2A b^3 x^{\frac{13}{2}}}{13}+\frac{6Ba b^2 x^{\frac{13}{2}}}{13}+\frac{6Aa b^2 x^{\frac{7}{2}}}{7}+\frac{6B a^2 b x^{\frac{7}{2}}}{7}+6A a^2 b\sqrt{x}+2B a^3\sqrt{x}-\frac{2a^3 A}{5x^{\frac{5}{2}}}$	7
default	$\frac{2b^3 B x^{\frac{19}{2}}}{19}+\frac{2A b^3 x^{\frac{13}{2}}}{13}+\frac{6Ba b^2 x^{\frac{13}{2}}}{13}+\frac{6Aa b^2 x^{\frac{7}{2}}}{7}+\frac{6B a^2 b x^{\frac{7}{2}}}{7}+6A a^2 b\sqrt{x}+2B a^3\sqrt{x}-\frac{2a^3 A}{5x^{\frac{5}{2}}}$	7
gosper	$-\frac{2(-455B b^3 x^{12}-665x^9 A b^3-1995x^9 Ba b^2-3705x^6 Aa b^2-3705x^6 B a^2 b-25935x^3 A a^2 b-8645a^3 B x^3+1729A a^3)}{8645x^{\frac{5}{2}}}$	8
trager	$-\frac{2(-455B b^3 x^{12}-665x^9 A b^3-1995x^9 Ba b^2-3705x^6 Aa b^2-3705x^6 B a^2 b-25935x^3 A a^2 b-8645a^3 B x^3+1729A a^3)}{8645x^{\frac{5}{2}}}$	8
risch	$-\frac{2(-455B b^3 x^{12}-665x^9 A b^3-1995x^9 Ba b^2-3705x^6 Aa b^2-3705x^6 B a^2 b-25935x^3 A a^2 b-8645a^3 B x^3+1729A a^3)}{8645x^{\frac{5}{2}}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*b^3*B*x^(19/2)+2/13*A*b^3*x^(13/2)+6/13*B*a*b^2*x^(13/2)+6/7*A*a*b^2*x
^(7/2)+6/7*B*a^2*b*x^(7/2)+6*A*a^2*b*x^(1/2)+2*B*a^3*x^(1/2)-2/5*a^3*A/x^(5
/2)
```

Maxima [A]
time = 0.28, size = 73, normalized size = 0.88

$$\frac{2}{19}\,Bb^3x^{\frac{19}{2}}+\frac{2}{13}\left(3\,Bab^2+Ab^3\right)x^{\frac{13}{2}}+\frac{6}{7}\left(Ba^2b+Aab^2\right)x^{\frac{7}{2}}-\frac{2\,Aa^3}{5\,x^{\frac{5}{2}}}+2\left(Ba^3+3\,Aa^2b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 6/7*(B*a^2*b + A*
a*b^2)*x^(7/2) - 2/5*A*a^3/x^(5/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

Fricas [A]
time = 1.53, size = 75, normalized size = 0.90

$$\frac{2\,(455\,Bb^3x^{12}+665\,(3\,Bab^2+Ab^3)x^9+3705\,(Ba^2b+Aab^2)x^6-1729\,Aa^3+8645\,(Ba^3+3\,Aa^2b)x^3)}{8645\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/8645*(455*B*b^3*x^12 + 665*(3*B*a*b^2 + A*b^3)*x^9 + 3705*(B*a^2*b + A*a*
b^2)*x^6 - 1729*A*a^3 + 8645*(B*a^3 + 3*A*a^2*b)*x^3)/x^(5/2)
```

Sympy [A]
time = 2.11, size = 110, normalized size = 1.33

$$-\frac{2Aa^3}{5x^{\frac{5}{2}}}+6Aa^2b\sqrt{x}+\frac{6Aab^2x^{\frac{7}{2}}}{7}+\frac{2Ab^3x^{\frac{13}{2}}}{13}+2Ba^3\sqrt{x}+\frac{6Ba^2bx^{\frac{7}{2}}}{7}+\frac{6Bab^2x^{\frac{13}{2}}}{13}+\frac{2Bb^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(7/2),x)
```

```
[Out] -2*A*a**3/(5*x**(5/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(7/2)/7 + 2*A*b
**3*x**(13/2)/13 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*x*
*(13/2)/13 + 2*B*b**3*x**(19/2)/19
```

Giac [A]
time = 0.60, size = 77, normalized size = 0.93

$$\frac{2}{19}Bb^3x^{\frac{19}{2}}+\frac{6}{13}Bab^2x^{\frac{13}{2}}+\frac{2}{13}Ab^3x^{\frac{13}{2}}+\frac{6}{7}Ba^2bx^{\frac{7}{2}}+\frac{6}{7}Aab^2x^{\frac{7}{2}}+2\,Ba^3\sqrt{x}+6\,Aa^2b\sqrt{x}-\frac{2\,Aa^3}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 6/7*B*a
^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) -
2/5*A*a^3/x^(5/2)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.83

$$\sqrt{x}\,\left(2\,B\,a^3+6\,A\,b\,a^2\right)+x^{13/2}\,\left(\frac{2\,A\,b^3}{13}+\frac{6\,B\,a\,b^2}{13}\right)-\frac{2\,A\,a^3}{5\,x^{5/2}}+\frac{2\,B\,b^3\,x^{19/2}}{19}+\frac{6\,a\,b\,x^{7/2}\,\left(A\,b+B\,a\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(7/2),x)
```

```
[Out] x^(1/2)*(2*B*a^3 + 6*A*a^2*b) + x^(13/2)*((2*A*b^3)/13 + (6*B*a*b^2)/13) -
(2*A*a^3)/(5*x^(5/2)) + (2*B*b^3*x^(19/2))/19 + (6*a*b*x^(7/2)*(A*b + B*a))
/7
```

3.155 $\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=73

$$\frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{9/2}}{9b}-\frac{2\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}$$

```
[Out] 2/3*(A*b-B*a)*x^(3/2)/b^2+2/9*B*x^(9/2)/b-2/3*(A*b-B*a)*arctan(x^(3/2)*b^(1
/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A]

time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 327, 335, 281, 211}

$$-\frac{2\sqrt{a}\,(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}+\frac{2x^{3/2}(Ab-aB)}{3b^2}+\frac{2Bx^{9/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(9/2))/(9*b) - (2*Sqrt[a]*(A*b - a
*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx &= \frac{2Bx^{9/2}}{9b} - \frac{\left(2\left(-\frac{9Ab}{2}+\frac{9aB}{2}\right)\right)\int \frac{x^{7/2}}{a+bx^3}\,dx}{9b} \\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(a(Ab-aB))\int \frac{\sqrt{x}}{a+bx^3}\,dx}{b^2} \\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(2a(Ab-aB))\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(2a(Ab-aB))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{3b^2} \\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{2\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 67, normalized size = 0.92

$$\frac{2x^{3/2}(3Ab-3aB+bBx^3)}{9b^2} + \frac{2\sqrt{a}\,(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*x^(3/2)*(3*A*b - 3*a*B + b*B*x^3))/(9*b^2) + (2*Sqrt[a]*(-(A*b) + a*B)*A
rcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

Maple [A]
time = 0.30, size = 58, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}-\frac{2Ba\,x^{\frac{3}{2}}}{3}}{b^{2}}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b^{2}\sqrt{ab}}$	58
default	$\frac{\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}-\frac{2Ba\,x^{\frac{3}{2}}}{3}}{b^{2}}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b^{2}\sqrt{ab}}$	58
risch	$\frac{2x^{\frac{3}{2}}\left(bB\,x^{3}+3Ab-3Ba\right)}{9b^{2}}-\frac{2a\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)A}{3b\sqrt{ab}}+\frac{2a^{2}\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)B}{3b^{2}\sqrt{ab}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/b^2*(1/3*b*B*x^(9/2)+A*b*x^(3/2)-B*a*x^(3/2))-2/3*a*(A*b-B*a)/b^2/(a*b)
^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 58, normalized size = 0.79

$$\frac{2\left(Ba^{2}-Aab\right)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\,b^{2}}+\frac{2\left(Bbx^{\frac{9}{2}}-3\,(Ba-Ab)x^{\frac{3}{2}}\right)}{9\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b*
x^(9/2) - 3*(B*a - A*b)*x^(3/2))/b^2
```

Fricas [A]
time = 1.80, size = 143, normalized size = 1.96

$$\left[-\frac{3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^{3}-2\,bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}-a}{bx^{3}+a}\right)-2\,(Bbx^{4}-3\,(Ba-Ab)x)\sqrt{x}}{9\,b^{2}},\ \frac{2\left(3\,(Ba-Ab)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{a}\right)+(Bbx^{4}-3\,(Ba-Ab)x)\sqrt{x}\right)}{9\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*(B*a - A*b)*sqrt(-a/b)*log((b*x^3 - 2*b*x^(3/2)*sqrt(-a/b) - a)/(b
*x^3 + a)) - 2*(B*b*x^4 - 3*(B*a - A*b)*x)*sqrt(x))/b^2, 2/9*(3*(B*a - A*b)
```

```
*sqrt(a/b)*arctan(b*x^(3/2)*sqrt(a/b)/a) + (B*b*x^4 - 3*(B*a - A*b)*x)*sqrt
(x))/b^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(73) = 146$.
time = 67.77, size = 428, normalized size = 5.86

$$\begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{3}{2}}}{3} + \frac{2Bx^{\frac{9}{2}}}{9}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{\frac{2Ax^{\frac{9}{2}}}{9} + \frac{2Bx^{\frac{15}{2}}}{15}}{a} & \text{for } b = 0 \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3} + \frac{2Bx^{\frac{9}{2}}}{9}}{b} & \text{for } a = 0 \\ -\frac{Aa\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}} + \frac{Aa\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}} + \frac{Aa\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}} - \frac{Aa\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}} + \frac{2Ax^{\frac{3}{2}}}{3b} + \frac{Ba^2\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3b^3\sqrt{-\frac{a}{b}}} - \frac{Ba^2\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3b^3\sqrt{-\frac{a}{b}}} - \frac{Ba^2\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{3b^3\sqrt{-\frac{a}{b}}} + \frac{Ba^2\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{3b^3\sqrt{-\frac{a}{b}}} - \frac{2Bax^{\frac{3}{2}}}{3b^2} + \frac{2Bx^{\frac{9}{2}}}{9b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((2
*A*x**(9/2)/9 + 2*B*x**(15/2)/15)/a, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(
9/2)/9)/b, Eq(a, 0)), (-A*a*log(sqrt(x) - (-a/b)**(1/6))/(3*b**2*sqrt(-a/b)
) + A*a*log(sqrt(x) + (-a/b)**(1/6))/(3*b**2*sqrt(-a/b)) + A*a*log(-4*sqrt(
x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**2*sqrt(-a/b)) - A*a*log(4*s
qrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**2*sqrt(-a/b)) + 2*A*x**
(3/2)/(3*b) + B*a**2*log(sqrt(x) - (-a/b)**(1/6))/(3*b**3*sqrt(-a/b)) - B*a
**2*log(sqrt(x) + (-a/b)**(1/6))/(3*b**3*sqrt(-a/b)) - B*a**2*log(-4*sqrt(x
)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**3*sqrt(-a/b)) + B*a**2*log(4
*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**3*sqrt(-a/b)) - 2*B*a
*x**(3/2)/(3*b**2) + 2*B*x**(9/2)/(9*b), True))
```

Giac [A]
time = 0.56, size = 64, normalized size = 0.88

$$\frac{2\,(Ba^2 - Aab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b^2} + \frac{2\,\left(Bb^2x^{\frac{9}{2}} - 3\,Babx^{\frac{3}{2}} + 3\,Ab^2x^{\frac{3}{2}}\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b^
2*x^(9/2) - 3*B*a*b*x^(3/2) + 3*A*b^2*x^(3/2))/b^3
```

Mupad [B]
time = 2.61, size = 111, normalized size = 1.52

$$x^{3/2}\,\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)+\frac{2\,B\,x^{9/2}}{9\,b}-\frac{2\,\sqrt{a}\,\mathrm{atan}\left(\frac{72\,b^{3/2}\,x^{3/2}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{\sqrt{a}\,\left(72\,A\,a^2\,b^2-72\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)}\right)\,\left(A\,b-B\,a\right)}{3\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^(3/2)*((2*A)/(3*b) - (2*B*a)/(3*b^2)) + (2*B*x^(9/2))/(9*b) - (2*a^(1/2)*
atan((72*b^(3/2)*x^(3/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/(a^(1/2)*(7
2*A*a^2*b^2 - 72*B*a^3*b)*(A*b - B*a)))*(A*b - B*a))/(3*b^(5/2))
```

3.156 $\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=288

$$\frac{2(Ab-aB)\sqrt{x}}{b^2}+\frac{2Bx^{7/2}}{7b}+\frac{\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}}-\frac{\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}}{\sqrt[6]{a}}\right.}{3b^{13/6}}$$

```
[Out] 2/7*B*x^(7/2)/b-2/3*a^(1/6)*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/b^(13
/6)-1/3*a^(1/6)*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/b^(13/
6)-1/3*a^(1/6)*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/b^(13/6)
+1/6*a^(1/6)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2)
)/b^(13/6)*3^(1/2)-1/6*a^(1/6)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/
6)*3^(1/2)*x^(1/2))/b^(13/6)*3^(1/2)+2*(A*b-B*a)*x^(1/2)/b^2
```

Rubi [A]
time = 0.37, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {470, 327, 335, 215, 648, 632, 210, 642, 211}

$$\frac{\sqrt[6]{a}\,(Ab-aB)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}}-\frac{\sqrt[6]{a}\,(Ab-aB)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3b^{13/6}}-\frac{2\sqrt[6]{a}\,(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}}+\frac{\sqrt[6]{a}\,(Ab-aB)\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,b^{13/6}}-\frac{\sqrt[6]{a}\,(Ab-aB)\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,b^{13/6}}+\frac{2\sqrt{x}\,(Ab-aB)}{b^2}+\frac{2Bx^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(7/2))/(7*b) + (a^(1/6)*(A*b - a*B)*Ar
cTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (a^(1/6)*(A*b -
 a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (2*a^(1
/6)*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) + (a^(1/6)*
(A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*
Sqrt[3]*b^(13/6)) - (a^(1/6)*(A*b - a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*b^(13/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2Bx^{7/2}}{7b} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{x^{5/2}}{a+bx^3}\,dx}{7b}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(a(Ab-aB))\int \frac{1}{\sqrt{x}\,(a+bx^3)}\,dx}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(2a(Ab-aB))\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{\left(2\sqrt[6]{a}\,(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x \cdots}\right.}{3b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\left(\sqrt[6]{a}\,(Ab-aB\cdots\right.}{\cdots}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\sqrt[6]{a}\,(Ab-aB)\cdots}{\cdots}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} + \frac{\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{\sqrt[6]{a}\,(Ab\cdots}{\cdots}$$

Mathematica [A]
time = 0.18, size = 172, normalized size = 0.60

$$\frac{6\sqrt[6]{b}\,\sqrt{x}\,(7Ab-7aB+bBx^3)+14\sqrt[6]{a}\,(-Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)-7\sqrt[6]{a}\,(-Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}\right)+7\sqrt{3}\,\sqrt[6]{a}\,(-Ab+aB)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{21b^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (6*b^(1/6)*Sqrt[x]*(7*A*b - 7*a*B + b*B*x^3) + 14*a^(1/6)*(-(A*b) + a*B)*Ar
cTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - 7*a^(1/6)*(-(A*b) + a*B)*ArcTan[(a^(1/3)
- b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + 7*Sqrt[3]*a^(1/6)*(-(A*b) + a*B)*
ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(21*b^(13
/6))
```

Maple [A]
time = 0.38, size = 209, normalized size = 0.73

method	result
derivativedivides	$\frac{\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^{2}}-\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\text{arct}}{}\right.}{}$
default	$\frac{\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^{2}}-\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\text{arct}}{}\right.}{}$
risch	$\frac{2\left(bB\,x^{3}+7Ab-7Ba\right)\sqrt{x}}{7b^{2}}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)A}{6b}+\frac{a\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}\right.}{6b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/7*b*B*x^(7/2)+A*b*x^(1/2)-B*a*x^(1/2))-2*(1/3/a*(a/b)^(1/6)*arctan
(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(
1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6)
)+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+
1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))*a*(A*b-B*a)/b^2
```

Maxima [A]
time = 0.50, size = 295, normalized size = 1.02

$$\frac{\left(\frac{\sqrt{3}\,(Ba-Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(Ba-Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)a}{6\,b^{2}}+\frac{2\left(Bbx^{\frac{7}{2}}-7\,(Ba-Ab)\sqrt{x}\right)}{7\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/
6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A*b^
(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(
sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1
/3)*sqrt(a^(1/3)*b^(1/3))))*a/b^2 + 2/7*(B*b*x^(7/2) - 7*(B*a - A*b)*sqrt(x
))/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2433 vs. 2(204) = 408.
time = 2.00, size = 2433, normalized size = 8.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/42*(28*sqrt(3)*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*
A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(
1/6)*arctan(1/3*(2*sqrt(3)*sqrt(b^4*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4
*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*
a*b^6)/b^13)^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a*b^2 - A*b^3)*
sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^
3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6))*b^11*(-(
B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*
B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/6) + 2*sqrt(3)*(B*a*b^1
1 - A*b^12)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/
6) - sqrt(3)*(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4
*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6))/(B^6*a^7 - 6*A*B^
5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*
A^5*B*a^2*b^5 + A^6*a*b^6)) + 28*sqrt(3)*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 1
5*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b
^5 + A^6*a*b^6)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(b^4*(-(B^6*a^7 - 6*A
*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 -
 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)
*x - (B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*
b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6
)/b^13)^(1/6))*b^11*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/
6) + 2*sqrt(3)*(B*a*b^11 - A*b^12)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*
A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5
 + A^6*a*b^6)/b^13)^(5/6) + sqrt(3)*(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a
^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*
b^6))/(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 +
15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)) - 7*b^2*(-(B^6*a^7 - 6*A
*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 -
 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(4*b^4*(-(B^6*a^7 - 6*A*B^5*a^
6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*
B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x +
4*(B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2
 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b
^13)^(1/6)) + 7*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
```

```
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/
6)*log(4*b^4*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a
^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + 4*
(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7
- 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*
b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)) + 14*b^2*(-(B^6*a^7 - 6*A*B
^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6
*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b
+ 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^
2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*a - A*b)*sqrt(x)) - 14*b^2*(-(B^6*a^7 -
 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b
^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(-b^2*(-(B^6*a^7 - 6*A*B^5
*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A
^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*a - A*b)*sqrt(x)) - 12*(B*b*x^3
- 7*B*a + 7*A*b)*sqrt(x))/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(267) = 534$.
time = 30.20, size = 605, normalized size = 2.10

$$\begin{cases} \tilde{\infty}\left(2A\sqrt{x} + \frac{2Bx^{\frac{7}{2}}}{7}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{\frac{2Ax^{\frac{7}{2}}}{7} + \frac{2Bx^{\frac{13}{2}}}{13}}{a} & \text{for } b = 0 \\ \frac{2A\sqrt{x} + \frac{2Bx^{\frac{7}{2}}}{7}}{b} & \text{for } a = 0 \\ \frac{2A\sqrt{x}}{b} + \frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3b} - \frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3b} + \frac{A\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6b} - \frac{A\sqrt[6]{-\frac{a}{b}}\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6b} - \frac{\sqrt{3}A\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} - \frac{\sqrt{3}}{3}\right)}{3b} - \frac{\sqrt{3}A\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{3b} - \frac{2Ba\sqrt{x}}{b^2} - \frac{Ba\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3b^2} + \frac{Ba\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3b^2} - \frac{Ba\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6b^2} + \frac{Ba\sqrt[6]{-\frac{a}{b}}\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6b^2} + \frac{\sqrt{3}Ba\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} - \frac{\sqrt{3}}{3}\right)}{3b^2} + \frac{\sqrt{3}Ba\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{3b^2} + \frac{2Bx^{\frac{7}{2}}}{7b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((2*A*
x**(7/2)/7 + 2*B*x**(13/2)/13)/a, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2)/7
)/b, Eq(a, 0)), (2*A*sqrt(x)/b + A*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6
))/(3*b) - A*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*b) + A*(-a/b)**(
1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - A*(-a/b)
**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - sqrt(3
)*A*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*
b) - sqrt(3)*A*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqr
t(3)/3)/(3*b) - 2*B*a*sqrt(x)/b**2 - B*a*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)
**(1/6))/(3*b**2) + B*a*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*b**2)
 - B*a*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/
(6*b**2) + B*a*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**
(1/3))/(6*b**2) + sqrt(3)*B*a*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b
)**(1/6)) - sqrt(3)/3)/(3*b**2) + sqrt(3)*B*a*(-a/b)**(1/6)*atan(2*sqrt(3)*
sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b**2) + 2*B*x**(7/2)/(7*b), True)
)
```

Giac [A]
time = 0.59, size = 289, normalized size = 1.00

$$\frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b^3} - \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b^3} + \frac{\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} + 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b^3} + \frac{\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} - 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b^3} + \frac{2\left((ab^5)^{\frac{1}{6}}Ba - (ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b^3} + \frac{2\left(Bb^6x^{\frac{7}{2}} - 7Bab^5\sqrt{x} + 7Ab^6\sqrt{x}\right)}{7b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/
b)^(1/6) + x + (a/b)^(1/3))/b^3 - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^
(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/b^3 + 1/3*((
a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(
x))/(a/b)^(1/6))/b^3 + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(-
(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/b^3 + 2/3*((a*b^5)^(1/6)*B*a
 - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/b^3 + 2/7*(B*b^6*x^(7/2)
- 7*B*a*b^5*sqrt(x) + 7*A*b^6*sqrt(x))/b^7
```

Mupad [B]
time = 2.89, size = 1933, normalized size = 6.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^(1/2)*((2*A)/b - (2*B*a)/b^2) + (2*B*x^(7/2))/(7*b) + ((-a)^(1/6)*atan(((
(-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*
b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*(A*b - B*a)*(B
^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b
^(13/6)) + ((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*
A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*(A
*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/
6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*
a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(
-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*
b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a
^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^
3 + (96*(-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A
^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6))))*(A*b - B*a)*2i)/(3*b^(13/6)) + ((-
a)^(1/6)*atan((((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*
(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^
3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*
a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)) + ((
-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^
4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a
)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2
*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*((3^(1/
2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2
*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)
*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*
```

```
a^5*b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b
 - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a
^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b -
 B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))
/(3*b^(13/6))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*b^(13/6)) + ((-a)
^(1/6)*atan((((-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B
^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3)
)/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^
4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)) + ((-a
)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*
b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^
(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a
^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*((3^(1/2)
*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a
^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1
i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^
5*b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b -
 B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7
*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B
*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))/(
3*b^(13/6))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*b^(13/6))
```

3.157 $\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=270

$$\frac{2Bx^{5/2}}{5b}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}}+\frac{2(Ab-aB)\tan^{-1}}{3\sqrt[6]{a}\,b^{11}}$$

```
[Out] 2/5*B*x^(5/2)/b+2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(11
/6)+1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(11/
6)+1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(11/6)
+1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(1/6
)/b^(11/6)*3^(1/2)-1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/
2)*x^(1/2))/a^(1/6)/b^(11/6)*3^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {470, 335, 301, 648, 632, 210, 642, 211}

$$-\frac{(Ab-aB)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}}+\frac{(Ab-aB)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3\sqrt[6]{a}\,b^{11/6}}+\frac{2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}}+\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}\,b^{11/6}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}\,b^{11/6}}+\frac{2Bx^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*B*x^(5/2))/(5*b) - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt
[x])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1
/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(1/6)*b^(11/6)) - ((A*b - a*
B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a
^(1/6)*b^(11/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2Bx^{5/2}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{5b}$$

$$= \frac{2Bx^{5/2}}{5b} - \frac{\left(4\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{5b}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{3b^{5/3}} + \frac{(2(Ab-aB))\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\cdots}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\,x\cdots}\right.}{2\sqrt{3}\sqrt[6]{a}\,b^{11/6}}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}\cdots\right.}{2\sqrt{3}\sqrt[6]{a}\,b^{11/6}}$$

$$= \frac{2Bx^{5/2}}{5b} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\cdots}{\sqrt[6]{a}}\right.}{3\sqrt[6]{a}\,b^{11/6}}$$

Mathematica [A]
time = 0.18, size = 153, normalized size = 0.57

$$\frac{6\sqrt[6]{a}\,b^{5/6}Bx^{5/2}+10(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-5(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)-5\sqrt{3}(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{15\sqrt[6]{a}\,b^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (6*a^(1/6)*b^(5/6)*B*x^(5/2) + 10*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)] - 5*(A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])
] - 5*Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3
) + b^(1/3)*x)])/(15*a^(1/6)*b^(11/6))
```

Maple [A]
time = 0.36, size = 191, normalized size = 0.71

method	result

derivativedivides	$\frac{2Bx^{\frac{5}{2}}}{5b}+\frac{2\left(-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\,\mathrm{l}\ldots}{\ldots}\ldots}{b}$
default	$\frac{2Bx^{\frac{5}{2}}}{5b}+\frac{2\left(-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\,\mathrm{l}\ldots}{\ldots}\ldots}{b}$
risch	$\frac{2Bx^{\frac{5}{2}}}{5b}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)A}{6a}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)B}{6b}+\frac{\mathrm{arcta}\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*B*x^(5/2)/b+2*(-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(
1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+1
/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(5/6)*ln(
3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(-3^(1/2
)+2*x^(1/2)/(a/b)^(1/6)))*(A*b-B*a)/b
```

Maxima [A]
time = 0.53, size = 212, normalized size = 0.79

$$\frac{2\,Bx^{\frac{5}{2}}}{5\,b}+\frac{(Ba-Ab)\left(\frac{\sqrt{3}\,\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\,\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/5*B*x^(5/2)/b + 1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt
(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)
*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqr
t(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x
))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3
)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3635 vs. 2(188) = 376.
time = 2.34, size = 3635, normalized size = 13.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/30*(12*B*x^(5/2) - 20*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a
^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)
/(a*b^11))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^6*b^9 - 5*A*B^4*a^5*b^10
 + 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^1
4)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3
*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) + (B^1
0*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^
4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3
*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - (B^6*a^7*b^7 -
6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3
*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*
B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6
*b^6)/(a*b^11))^(2/3))*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
- 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^1
1))^(1/6) + 2*sqrt(3)*(B^5*a^5*b^2 - 5*A*B^4*a^4*b^3 + 10*A^2*B^3*a^3*b^4 -
 10*A^3*B^2*a^2*b^5 + 5*A^4*B*a*b^6 - A^5*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5
*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A
^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b +
15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^
5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^
3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - 20*sqrt(3)*b*(-(B^
6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^
2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*arctan(1/3*(sqrt(3)*sq
rt(-4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10 + 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^
3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^14)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b
+ 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*
b^5 + A^6*b^6)/(a*b^11))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8
*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5
+ 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B
*a*b^9 + A^10*b^10)*x - 4*(B^6*a^7*b^7 - 6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b
^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b
^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(2/3))*b^2*(-(B^6*
a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*
a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6) + 2*sqrt(3)*(B^5*a^5*b^2
 - 5*A*B^4*a^4*b^3 + 10*A^2*B^3*a^3*b^4 - 10*A^3*B^2*a^2*b^5 + 5*A^4*B*a*b^
6 - A^5*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1
/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^
3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a
^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5
```

*B*a*b^5 + A^6*b^6)) + 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log(a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log(-a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 5*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log(4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10 + 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^14)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^6*a^7*b^7 - 6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(2/3)) + 5*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log(-4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10 + 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5...

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(258) = 516$.
time = 14.46, size = 581, normalized size = 2.15

$$\begin{cases} \tilde{\infty} \left(- \frac{2A}{\sqrt{x}} + \frac{2Bx^{\frac{5}{2}}}{5}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5} + \frac{2Bx^{\frac{11}{2}}}{11}}{a} & \text{for } b = 0 \\ \frac{- \frac{2A}{\sqrt{x}} + \frac{2Bx^{\frac{5}{2}}}{5}}{b} & \text{for } a = 0 \\ \frac{A \log\left(\sqrt{x} - \sqrt[6]{- \frac{a}{b}}\right)}{3b \sqrt[6]{- \frac{a}{b}}} - \frac{A \log\left(\sqrt{x} + \sqrt[6]{- \frac{a}{b}}\right)}{3b \sqrt[6]{- \frac{a}{b}}} + \frac{A \log\left(-4\sqrt{x} \sqrt[6]{- \frac{a}{b}} + 4x + 4\sqrt[3]{- \frac{a}{b}}\right)}{6b \sqrt[6]{- \frac{a}{b}}} - \frac{A \log\left(4\sqrt{x} \sqrt[6]{- \frac{a}{b}} + 4x + 4\sqrt[3]{- \frac{a}{b}}\right)}{6b \sqrt[6]{- \frac{a}{b}}} + \frac{\sqrt{3} A \operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{- \frac{a}{b}}} - \frac{\sqrt{3}}{3}\right)}{3b \sqrt[6]{- \frac{a}{b}}} + \frac{\sqrt{3} A \operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{- \frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{3b \sqrt[6]{- \frac{a}{b}}} - \frac{Ba \log\left(\sqrt{x} - \sqrt[6]{- \frac{a}{b}}\right)}{3b^2 \sqrt[6]{- \frac{a}{b}}} + \frac{Ba \log\left(\sqrt{x} + \sqrt[6]{- \frac{a}{b}}\right)}{3b^2 \sqrt[6]{- \frac{a}{b}}} - \frac{Ba \log\left(-4\sqrt{x} \sqrt[6]{- \frac{a}{b}} + 4x + 4\sqrt[3]{- \frac{a}{b}}\right)}{6b^2 \sqrt[6]{- \frac{a}{b}}} + \frac{Ba \log\left(4\sqrt{x} \sqrt[6]{- \frac{a}{b}} + 4x + 4\sqrt[3]{- \frac{a}{b}}\right)}{6b^2 \sqrt[6]{- \frac{a}{b}}} - \frac{\sqrt{3} Ba \operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{- \frac{a}{b}}} - \frac{\sqrt{3}}{3}\right)}{3b^2 \sqrt[6]{- \frac{a}{b}}} - \frac{\sqrt{3} Ba \operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{- \frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{3b^2 \sqrt[6]{- \frac{a}{b}}} + \frac{2Bx^{\frac{5}{2}}}{5b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a),x)

[Out] Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*A*x**(5/2)/5 + 2*B*x**(11/2)/11)/a, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)/5)/b, Eq(a, 0)), (A*log(sqrt(x) - (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) - A*log(sqrt(x) + (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) + A*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) - A*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) + sqrt(3)*A*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b*(-a/b)**(1/6)) + sqrt(3)*A*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b*(-a/b)**(1/6))

```
- B*a*log(sqrt(x) - (-a/b)**(1/6))/(3*b**2*(-a/b)**(1/6)) + B*a*log(sqrt(x)
 + (-a/b)**(1/6))/(3*b**2*(-a/b)**(1/6)) - B*a*log(-4*sqrt(x)*(-a/b)**(1/6)
 + 4*x + 4*(-a/b)**(1/3))/(6*b**2*(-a/b)**(1/6)) + B*a*log(4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b**2*(-a/b)**(1/6)) - sqrt(3)*B*a*atan(
2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b**2*(-a/b)**(1/6)) - s
qrt(3)*B*a*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b**2*(-
a/b)**(1/6)) + 2*B*x**(5/2)/(5*b), True))
```

Giac [A]
time = 0.88, size = 280, normalized size = 1.04

$$\frac{2\,Bx^{\frac{5}{2}}}{5\,b}-\frac{2\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}-Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab}+\frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^{6}}-\frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^{6}}-\frac{\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^{6}}-\frac{\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/5*B*x^(5/2)/b - 2/3*(B*a*(a/b)^(5/6) - A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a
/b)^(1/6))/(a*b) + 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(
sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^6) - 1/6*sqrt(3)*((a*b^
5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a
/b)^(1/3))/(a*b^6) - 1/3*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan((sq
rt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^6) - 1/3*((a*b^5)^(5/6)*B*
a - (a*b^5)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6
))/(a*b^6)
```

Mupad [B]
time = 2.85, size = 1640, normalized size = 6.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] (2*B*x^(5/2))/(5*b) + (atan((((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 +
96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 +
 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(
1/3)*b^(11/3)) + ((A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5
*b + 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5
*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3
)))/(((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*
B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*
A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - ((A*b - B*
a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2 + (x^
(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(
27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3))))*(A*b - B*a)*2i)/(3*(-a)^(
1/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3
```

```
*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*
1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*
b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)) + (((3^(1/2)*1i)/
2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96
*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b
^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-
a)^(1/3)*b^(11/3)))/((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^
3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)
/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3
))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/
2)^2*(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B
*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 8
64*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*
b^(11/3))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(1/6)*b^(11/6))
+ (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a
^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A
*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^
(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(A*b
 - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2
+ (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^
5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/
3)))/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6
+ 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b
- B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/
6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a
)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2 + (x^(
1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2
- 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3))))*((3
^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(1/6)*b^(11/6))
```

3.158 $\int \frac{\sqrt{x}\,(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=53

$$\frac{2Bx^{3/2}}{3b}+\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{3/2}}$$

```
[Out] 2/3*B*x^(3/2)/b+2/3*(A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 335, 281, 211}

$$\frac{2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{3/2}}+\frac{2Bx^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) + (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*
Sqrt[a]*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}\,(A+Bx^3)}{a+bx^3}\,dx &= \frac{2Bx^{3/2}}{3b}-\frac{\left(2\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{3b}\\ &= \frac{2Bx^{3/2}}{3b}-\frac{\left(4\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{3b}\\ &= \frac{2Bx^{3/2}}{3b}+\frac{(2(Ab-aB))\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3b}\\ &= \frac{2Bx^{3/2}}{3b}+\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 53, normalized size = 1.00

$$\frac{2Bx^{3/2}}{3b}-\frac{2(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
[Out] (2*B*x^(3/2))/(3*b) - (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/
(3*Sqrt[a]*b^(3/2))
```

Maple [A]
time = 0.29, size = 40, normalized size = 0.75

method	result	size
derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3b}+\frac{2(Ab-Ba)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b\sqrt{ab}}$	40

default	$\frac{2B\,x^{\frac{3}{2}}}{3b}+\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b\sqrt{ab}}$	40
risch	$\frac{2B\,x^{\frac{3}{2}}}{3b}+\frac{2\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)A}{3\sqrt{ab}}-\frac{2\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)Ba}{3b\sqrt{ab}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*B*x^(3/2)/b+2/3*(A*b-B*a)/b/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 39, normalized size = 0.74

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b}-\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b)
```

Fricas [A]
time = 1.51, size = 108, normalized size = 2.04

$$\left[\frac{2\,Babx^{\frac{3}{2}}+(Ba-Ab)\sqrt{-ab}\,\log\left(\frac{bx^3-2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)}{3\,ab^2},\ \frac{2\left(Babx^{\frac{3}{2}}-(Ba-Ab)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)\right)}{3\,ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(2*B*a*b*x^(3/2) + (B*a - A*b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^
(3/2) - a)/(b*x^3 + a)))/(a*b^2), 2/3*(B*a*b*x^(3/2) - (B*a - A*b)*sqrt(a*b
)*arctan(sqrt(a*b)*x^(3/2)/a))/(a*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(51) = 102$.
time = 5.11, size = 381, normalized size = 7.19

$$\begin{cases}\tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}\right) & \text{for}\ a=0\wedge b=0\\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} & \text{for}\ a=0\\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{a} & \text{for}\ b=0\\ \frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}}-\frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}}-\frac{A\log\left(-4\sqrt{x}\,\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}}+\frac{A\log\left(4\sqrt{x}\,\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}}-\frac{Ba\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{Ba\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{Ba\log\left(-4\sqrt{x}\,\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}-\frac{Ba\log\left(4\sqrt{x}\,\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{2Bx^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)),
((-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3)/b, Eq(a, 0)), ((2*A*x**(3/2)/3 + 2*B*
x**(9/2)/9)/a, Eq(b, 0)), (A*log(sqrt(x) - (-a/b)**(1/6))/(3*b*sqrt(-a/b))
- A*log(sqrt(x) + (-a/b)**(1/6))/(3*b*sqrt(-a/b)) - A*log(-4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)) + A*log(4*sqrt(x)*(-a/b)*
*(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)) - B*a*log(sqrt(x) - (-a/b)
**(1/6))/(3*b**2*sqrt(-a/b)) + B*a*log(sqrt(x) + (-a/b)**(1/6))/(3*b**2*sqr
t(-a/b)) + B*a*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**
2*sqrt(-a/b)) - B*a*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3
*b**2*sqrt(-a/b)) + 2*B*x**(3/2)/(3*b), True))
```

Giac [A]
time = 0.58, size = 39, normalized size = 0.74

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b}-\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [B]
time = 2.60, size = 93, normalized size = 1.75

$$\frac{2\,B\,x^{3/2}}{3\,b}-\frac{2\,\mathrm{atan}\left(\frac{3\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,\left(24\,A^2\,b^3-48\,A\,B\,a\,b^2+24\,B^2\,a^2\,b\right)}{\left(72\,B\,a^2\,b^2-72\,A\,a\,b^3\right)\,\left(A\,b-B\,a\right)}\right)\,\left(A\,b-B\,a\right)}{3\,\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] (2*B*x^(3/2))/(3*b) - (2*atan((3*a^(1/2)*b^(3/2)*x^(3/2)*(24*A^2*b^3 + 24*B
^2*a^2*b - 48*A*B*a*b^2))/((72*B*a^2*b^2 - 72*A*a*b^3)*(A*b - B*a)))*(A*b -
 B*a))/(3*a^(1/2)*b^(3/2))
```

3.159 $\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx$

Optimal. Leaf size=268

$$\frac{2B\sqrt{x}}{b}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{2(Ab-aB)\tan^{-1}}{3a^{5/6}b^{7/}}$$

```
[Out] 2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)+1/3*(A*b-B*a)
*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)+1/3*(A*b-B*a)*a
rctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)-1/6*(A*b-B*a)*ln(a
^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(7/6)*3^(1/2)+1
/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/
b^(7/6)*3^(1/2)+2*B*x^(1/2)/b
```

Rubi [A]

time = 0.31, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {470, 335, 215, 648, 632, 210, 642, 211}

$$-\frac{(Ab-aB)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{(Ab-aB)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3a^{5/6}b^{7/6}}+\frac{2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}+\frac{2B\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)
])/(3*a^(5/6)*b^(7/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(5/6)*b^(7/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(5/6)*b^(7/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(
1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b^(7/6)) + ((A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b
^(7/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \frac{2B\sqrt{x}}{b} - \frac{\left(2\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\,(a+bx^3)}\,dx}{b}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{\left(4\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{3a^{5/6}b} + \frac{(2(Ab-$$

$$= \frac{2B\sqrt{x}}{b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x}\right.}{2\sqrt{3}\,a^{5/6}b^{7/6}}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}\right.}{2\sqrt{3}\,a^{5/6}b^{7/6}}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\right.}{3a^{5/6}b^{7/6}}$$

Mathematica [A]
time = 0.18, size = 152, normalized size = 0.57

$$\frac{6a^{5/6}\sqrt[6]{b}\,B\sqrt{x} + 2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right) - (Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}\right) + \sqrt{3}\,(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{3a^{5/6}b^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

```
[Out] (6*a^(5/6)*b^(1/6)*B*Sqrt[x] + 2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/
6)] - (A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] +
 Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b
^(1/3)*x)])/(3*a^(5/6)*b^(7/6))
```

Maple [A]
time = 0.36, size = 191, normalized size = 0.71

method	result
derivativedivides	$\frac{2B\sqrt{x}}{b} + \frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a} - \frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{6a}\right.}{b}$

default	$\frac{2B\sqrt{x}}{b}+\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a}\right.}{b}$
risch	$\frac{2B\sqrt{x}}{b}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)A}{6a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)B}{6b}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*B*x^(1/2)/b+2*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/
2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1
/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+
3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)
/(a/b)^(1/6)+3^(1/2)))*(A*b-B*a)/b
```

Maxima [A]
time = 0.51, size = 278, normalized size = 1.04

$$\frac{2B\sqrt{x}}{b}-\frac{\frac{\sqrt{3}\,(Ba-Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(Ba-Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*B*sqrt(x)/b - 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x
) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(
3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*
a*b^(1/3) - A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/
3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3
))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3
)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^
(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b
^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2424 vs. $2(188) = 376$.
time = 1.68, size = 2424, normalized size = 9.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B
^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)
*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4
*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^
6)/(a^5*b^7))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a^2*b - A*a*b^
2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3
*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6))*a^4*
b^6*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) + 2*sqrt(3)*
(B*a^5*b^6 - A*a^4*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(
a^5*b^7))^(5/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 2
0*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6
 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2
*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b +
 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^2*(-(B^6*a^
6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^
2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + (B^2*a^2 - 2*A*B*a*b +
A^2*b^2)*x - (B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^
2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A
^6*b^6)/(a^5*b^7))^(1/6))*a^4*b^6*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a
^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)
/(a^5*b^7))^(5/6) + 2*sqrt(3)*(B*a^5*b^6 - A*a^4*b^7)*sqrt(x)*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*
a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^
5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^
3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - b*(-(B^6*a
^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a
^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(4*a^2*b^2*(-(B^6*a^6
 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2
*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b +
 A^2*b^2)*x + 4*(B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15
*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
+ A^6*b^6)/(a^5*b^7))^(1/6)) + b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^
4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/
(a^5*b^7))^(1/6)*log(4*a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*
b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a
^5*b^7))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a^2*b - A*a*b^2
)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)) + 2*b
*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*
A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(a*b*(-(B^6*
a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*
```

a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a - A*b)*sqrt(x)) - 2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(-a*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^ 4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a - A*b)*sqr t(x)) + 12*B*sqrt(x))/b

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(257) = 514$.
time = 6.60, size = 558, normalized size = 2.08

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}}+2B\sqrt{x}\right) & \text{for } a=0 \wedge b=0 \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}+2B\sqrt{x}}{b} & \text{for } a=0 \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{7}{2}}}{7}}{a} & \text{for } b=0 \\ -\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3a}+\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3a}-\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a}+\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a}+\frac{\sqrt{3}A\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}}-\frac{\sqrt{3}}{3}\right)}{3a}+\frac{\sqrt{3}A\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{3a}+\frac{2B\sqrt{x}}{b}+\frac{B\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b}-\frac{B\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b}+\frac{B\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6b}-\frac{B\sqrt[6]{-\frac{a}{b}}\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6b}-\frac{\sqrt{3}B\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}}-\frac{\sqrt{3}}{3}\right)}{3b}-\frac{\sqrt{3}B\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/(b*x**3+a)/x**(1/2),x)

[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((- 2*A/(5*x**(5/2)) + 2*B*sqrt(x))/b, Eq(a, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2)/ 7)/a, Eq(b, 0)), (-A*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*a) + A*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*a) - A*(-a/b)**(1/6)*log(-4*sq rt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + A*(-a/b)**(1/6)*log(4* sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + sqrt(3)*A*(-a/b)**(1 /6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a) + sqrt(3)*A *(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a) + 2*B*sqrt(x)/b + B*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*b) - B*(- a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*b) + B*(-a/b)**(1/6)*log(-4*sqr t(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - B*(-a/b)**(1/6)*log(4*s qrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - sqrt(3)*B*(-a/b)**(1/ 6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b) - sqrt(3)*B* (-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b), True))

Giac [A]
time = 0.53, size = 280, normalized size = 1.04

$$\frac{2B\sqrt{x}}{b}-\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6ab^2}+\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6ab^2}-\frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3ab^2}-\frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3ab^2}-\frac{2\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="giac")

[Out] 2*B*sqrt(x)/b - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqr t(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^2) + 1/6*sqrt(3)*((a*b^5)^ (1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b) ^(1/3))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a -

```
(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/
(a*b^2) - 2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^
(1/6))/(a*b^2)
```

Mupad [B]
time = 2.88, size = 1915, normalized size = 7.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)),x)
```

```
[Out] (2*B*x^(1/2))/b + (atan((((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2
*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a*b
^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6
)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6)*b^(7/6)) + ((x^(1/2)*(96*A^4*b^5
+ 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2)
 + ((A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*
A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6)*b^(7/
6)))/(((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*
B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^
2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b -
B*a))/(3*(-a)^(5/6)*b^(7/6)) - ((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A
^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + ((A*b - B*a)*(288*A
^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a
)^(5/6)*b^(7/6)))*(A*b - B*a))/(3*(-a)^(5/6)*b^(7/6))))*(A*b - B*a)*2i)/(3*
(-a)^(5/6)*b^(7/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(
96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B
^3*a^3*b^2) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*
a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1
i)/(3*(-a)^(5/6)*b^(7/6)) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(9
6*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^
3*a^3*b^2) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a
^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i
)/(3*(-a)^(5/6)*b^(7/6)))/((((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(96
*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3
*a^3*b^2) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^
4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3
*(-a)^(5/6)*b^(7/6)) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4
*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3
*b^2) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^
2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a
)^(5/6)*b^(7/6))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(5/6)*b^(
7/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 9
6*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) -
```

```
(((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*
A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i)/(3*(-a)^(5/
6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96
*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + (
((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A
*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i)/(3*(-a)^(5/6
)*b^(7/6)))/((((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96*
B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((
(3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*
B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a)^(5/6)*b^
(7/6)) - (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96*B^4*
a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + (((3^(
1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*
a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a)^(5/6)*b^(7/6
))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(5/6)*b^(7/6))
```

3.160 $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx$

Optimal. Leaf size=268

$$-\frac{2A}{a\sqrt{x}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{2(Ab-aB)\tan^{-1}}{3a^{7/6}b^{5/}}$$

```
[Out] -2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/3*(A*b-B*a
)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/3*(A*b-B*a)*
arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/6*(A*b-B*a)*ln(
a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(5/6)*3^(1/2)+
1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)
/b^(5/6)*3^(1/2)-2*A/a/x^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {464, 335, 301, 648, 632, 210, 642, 211}

$$\frac{(Ab-aB)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{(Ab-aB)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3a^{7/6}b^{5/6}}-\frac{2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}-\frac{2A}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1
/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(7/6)*b^(5/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*
b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6)*b^(5/6)) + ((A*b - a*B)*Lo
g[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6
)*b^(5/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{a\sqrt{x}} - \frac{\left(2\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{a}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{\left(4\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{3a^{7/6}b^{2/3}} - \frac{(2(Ab-}{}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\,x}\right.}{2\sqrt{3}\,a^{7/6}b^{5/6}}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}\right.}{2\sqrt{3}\,a^{7/6}b^{5/6}}$$

$$= -\frac{2A}{a\sqrt{x}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{}}{\sqrt[6]{a}}\right.}{3a^{7/6}b^{5/6}}$$

Mathematica [A]
time = 0.20, size = 156, normalized size = 0.58

$$\frac{-\frac{6\sqrt[6]{a}\,A}{\sqrt{x}}+\frac{2(-Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{b^{5/6}}+\frac{\sqrt{3}(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{b^{5/6}}}{3a^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

```
[Out] ((-6*a^(1/6)*A)/Sqrt[x] + (2*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6
)])/b^(5/6) + ((A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sq
rt[x])])/b^(5/6) + (Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sq
rt[x])/(a^(1/3) + b^(1/3)*x)])/b^(5/6))/(3*a^(7/6))
```

Maple [A]
time = 0.36, size = 191, normalized size = 0.71

method	result

derivativedivides	$-\frac{2A}{a\sqrt{x}}-\frac{2\left(-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right.}{}\right.}{a}$
default	$-\frac{2A}{a\sqrt{x}}-\frac{2\left(-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right.}{}\right.}{a}$
risch	$-\frac{2A}{a\sqrt{x}}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)Ab}{6a^2}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)B}{6a}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*A/a/x^(1/2)-2*(-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1
/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+1/
3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(5/6)*ln(3
^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(-3^(1/2)
+2*x^(1/2)/(a/b)^(1/6)))*(A*b-B*a)/a
```

Maxima [A]
time = 0.50, size = 212, normalized size = 0.79

$$-\frac{(Ba-Ab)\left(\frac{\sqrt{3}\,\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\,\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{6a}-\frac{2A}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
 a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x)
+ b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6
) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3)
)) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b
^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^
(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2*A/(a*sqrt(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3663 vs. $2(188) = 376$.

time = 2.55, size = 3663, normalized size = 13.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3
*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/
6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*B^3
*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*sqrt(x)*(-(B
^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B
^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) + (B^10*a^10 - 10*A*
B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4
- 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*
B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - (B^6*a^11*b^3 - 6*A*B^5*a^10*
b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 - 6*A^5*
B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5
))^(2/3))*a*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*
a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6) +
2*sqrt(3)*(B^5*a^6*b - 5*A*B^4*a^5*b^2 + 10*A^2*B^3*a^4*b^3 - 10*A^3*B^2*a^
3*b^4 + 5*A^4*B*a^2*b^5 - A^5*a*b^6)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^
4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b
^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 1
5*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*a*x*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*arctan(1/3*(sqrt(3)*sqrt(-4*(
B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 +
 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2
*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^
6*b^6)/(a^7*b^5))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^
2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A
^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9
+ A^10*b^10)*x - 4*(B^6*a^11*b^3 - 6*A*B^5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 -
20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 - 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-
(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4
*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(2/3))*a*b*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^6*b - 5*A*
B^4*a^5*b^2 + 10*A^2*B^3*a^4*b^3 - 10*A^3*B^2*a^3*b^4 + 5*A^4*B*a^2*b^5 - A
^5*a*b^6)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*
B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6
```

```
) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5
*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B
*a*b^5 + A^6*b^6)) - 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
- 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b
^5))^(1/6)*log(a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20
*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))
^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3
 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^5))^(1/6)*log(-a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a^7*b^5))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 -
 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + a*x*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*log(4*(B^5*a^11*b^4 - 5*A*B^4
*a^10*b^5 + 10*A^2*B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5
*a^6*b^9)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*
B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6
) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^
3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A
^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^
6*a^11*b^3 - 6*A*B^5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 1
5*A^4*B^2*a^7*b^7 - 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5
*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B
*a*b^5 + A^6*b^6)/(a^7*b^5))^(2/3)) - a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a^7*b^5))^(1/6)*log(-4*(B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*
B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*...
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(257) = 514$.

time = 9.24, size = 561, normalized size = 2.09

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{\sqrt{x}}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{\sqrt{x}}}{b} & \text{for } a = 0 \\ \frac{-\frac{2A}{\sqrt{x}} + \frac{2Bx^{\frac{5}{2}}}{5}}{a} & \text{for } b = 0 \\ -\frac{A\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt[6]{-\frac{a}{b}}} + \frac{A\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt[6]{-\frac{a}{b}}} - \frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6a\sqrt[6]{-\frac{a}{b}}} + \frac{A\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6a\sqrt[6]{-\frac{a}{b}}} - \frac{\sqrt{3}A\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} - \frac{\sqrt{3}}{3}\right)}{3a\sqrt[6]{-\frac{a}{b}}} - \frac{\sqrt{3}A\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{3a\sqrt[6]{-\frac{a}{b}}} - \frac{2A}{a\sqrt{x}} + \frac{B\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt[6]{-\frac{a}{b}}} - \frac{B\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt[6]{-\frac{a}{b}}} + \frac{B\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6b\sqrt[6]{-\frac{a}{b}}} - \frac{B\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6b\sqrt[6]{-\frac{a}{b}}} + \frac{\sqrt{3}B\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} - \frac{\sqrt{3}}{3}\right)}{3b\sqrt[6]{-\frac{a}{b}}} + \frac{\sqrt{3}B\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{3b\sqrt[6]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(7*x**(7/2)) - 2*B/sqrt(x))/b, Eq(a, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)
/5)/a, Eq(b, 0)), (-A*log(sqrt(x) - (-a/b)**(1/6))/(3*a*(-a/b)**(1/6)) + A*
log(sqrt(x) + (-a/b)**(1/6))/(3*a*(-a/b)**(1/6)) - A*log(-4*sqrt(x)*(-a/b)*
*(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a*(-a/b)**(1/6)) + A*log(4*sqrt(x)*(-a/b
```

```
)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a*(-a/b)**(1/6)) - sqrt(3)*A*atan(2*sq
rt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a*(-a/b)**(1/6)) - sqrt(3)*
A*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a*(-a/b)**(1/6))
 - 2*A/(a*sqrt(x)) + B*log(sqrt(x) - (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) - B
*log(sqrt(x) + (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) + B*log(-4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) - B*log(4*sqrt(x)*(-a/
b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) + sqrt(3)*B*atan(2*s
qrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b*(-a/b)**(1/6)) + sqrt(3)
*B*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b*(-a/b)**(1/6)
), True))
```

Giac [A]
time = 0.61, size = 280, normalized size = 1.04

$$-\frac{2\,A}{a\sqrt{x}}-\frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^{2}b^{5}}+\frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^{2}b^{5}}+\frac{\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^{2}b^{5}}+\frac{\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^{2}b^{5}}+\frac{2\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^{2}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2*A/(a*sqrt(x)) - 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(
sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^5) + 1/6*sqrt(3)*((a*
b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x +
(a/b)^(1/3))/(a^2*b^5) + 1/3*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan
((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^5) + 1/3*((a*b^5)^(5
/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b
)^(1/6))/(a^2*b^5) + 2/3*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan(sqr
t(x)/(a/b)^(1/6))/(a^2*b^5)
```

Mupad [B]
time = 2.86, size = 1700, normalized size = 6.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)),x)
```

```
[Out] (atan((((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4
 + 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^1
2*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3
)) + ((A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 -
 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*
b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3))
)/(((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 9
6*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^
4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - ((
A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2
```

*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3))))*(A*b - B*a)*2i)/(3*(-a)^(7/6)*b^(5/6)) - (2*A)/(a*x^(1/2)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3)))/((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(7/6)*b^(5/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3)))/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(7/6)*b^(5/6))

3.161 $\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)}\,dx$

Optimal. Leaf size=53

$$-\frac{2A}{3ax^{3/2}}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}$$

```
[Out] -2/3*A/a/x^(3/2)-2/3*(A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(3/2)/b^(1
/2)
```

Rubi [A]

time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {464, 335, 281, 211}

$$-\frac{2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}-\frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) - (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3
*a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)}\,dx &= -\frac{2A}{3ax^{3/2}}-\frac{\left(2\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{3a}\\ &= -\frac{2A}{3ax^{3/2}}-\frac{\left(4\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{3a}\\ &= -\frac{2A}{3ax^{3/2}}-\frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3a}\\ &= -\frac{2A}{3ax^{3/2}}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 53, normalized size = 1.00

$$-\frac{2A}{3ax^{3/2}}+\frac{2(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
[Out] (-2*A)/(3*a*x^(3/2)) + (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])
/(3*a^(3/2)*Sqrt[b])
```

Maple [A]
time = 0.31, size = 40, normalized size = 0.75

method	result	size
derivativedivides	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a\sqrt{ab}}$	40

default	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a\sqrt{ab}}$	40
risch	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)Ab}{3a\sqrt{ab}}+\frac{2\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)B}{3\sqrt{ab}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] -2/3*A/a/x^(3/2)-2/3*(A*b-B*a)/a/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.53, size = 39, normalized size = 0.74

$$\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a}-\frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="maxima")
[Out] 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/2
))
```

Fricas [A]
time = 1.87, size = 120, normalized size = 2.26

$$\left[\frac{(Ba-Ab)\sqrt{-ab}\,x^2\log\left(\frac{bx^3+2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\,Aab\sqrt{x}}{3\,a^2bx^2},\ \frac{2\left((Ba-Ab)\sqrt{ab}\,x^2\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)-Aab\sqrt{x}\right)}{3\,a^2bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="fricas")
[Out] [1/3*((B*a - A*b)*sqrt(-a*b)*x^2*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*
x^3 + a)) - 2*A*a*b*sqrt(x))/(a^2*b*x^2), 2/3*((B*a - A*b)*sqrt(a*b)*x^2*ar
ctan(sqrt(a*b)*x^(3/2)/a) - A*a*b*sqrt(x))/(a^2*b*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(53)=106$.
time = 20.93, size = 371, normalized size = 7.00

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) & \text{for}\: a=0 \wedge b=0 \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} & \text{for}\: a=0 \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{a} & \text{for}\: b=0 \\ -\frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}+\frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}+\frac{A\log\left(-4\sqrt{x}\,\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}-\frac{A\log\left(4\sqrt{x}\,\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}-\frac{2A}{3ax^{\frac{3}{2}}}+\frac{B\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}}-\frac{B\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}}-\frac{B\log\left(-4\sqrt{x}\,\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}}+\frac{B\log\left(4\sqrt{x}\,\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)), ((-2*A/(3*x**(3/2))
 + 2*B*x**(3/2)/3)/a, Eq(b, 0)), (-A*log(sqrt(x) - (-a/b)**(1/6))/(3*a*sqrt
(-a/b)) + A*log(sqrt(x) + (-a/b)**(1/6))/(3*a*sqrt(-a/b)) + A*log(-4*sqrt(x
)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*a*sqrt(-a/b)) - A*log(4*sqrt(x)
*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*a*sqrt(-a/b)) - 2*A/(3*a*x**(3/2
)) + B*log(sqrt(x) - (-a/b)**(1/6))/(3*b*sqrt(-a/b)) - B*log(sqrt(x) + (-a/
b)**(1/6))/(3*b*sqrt(-a/b)) - B*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/
b)**(1/3))/(3*b*sqrt(-a/b)) + B*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b
)**(1/3))/(3*b*sqrt(-a/b)), True))
```

Giac [A]
time = 0.57, size = 39, normalized size = 0.74

$$\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a}-\frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/2
))
```

Mupad [B]
time = 0.10, size = 102, normalized size = 1.92

$$-\frac{2\,A}{3\,a\,x^{3/2}}-\frac{2\,\mathrm{atan}\left(\frac{3\,a^{3/2}\,\sqrt{b}\,x^{3/2}\,\left(24\,A^2\,a^3\,b^5-48\,A\,B\,a^4\,b^4+24\,B^2\,a^5\,b^3\right)}{\left(A\,b-B\,a\right)\,\left(72\,A\,a^5\,b^4-72\,B\,a^6\,b^3\right)}\right)\,\left(A\,b-B\,a\right)}{3\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)),x)
```

```
[Out] - (2*A)/(3*a*x^(3/2)) - (2*atan((3*a^(3/2)*b^(1/2)*x^(3/2)*(24*A^2*a^3*b^5
+ 24*B^2*a^5*b^3 - 48*A*B*a^4*b^4))/((A*b - B*a)*(72*A*a^5*b^4 - 72*B*a^6*b
^3)))*(A*b - B*a))/(3*a^(3/2)*b^(1/2))
```

3.162 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx$

Optimal. Leaf size=270

$$-\frac{2A}{5ax^{5/2}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}-\frac{2(Ab-aB)\tan^{-1}}{3a^{11/6}}$$

```
[Out] -2/5*A/a/x^(5/2)-2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(
1/6)-1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(1
/6)-1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(1/6
)+1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11
/6)/b^(1/6)*3^(1/2)-1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1
/2)*x^(1/2))/a^(11/6)/b^(1/6)*3^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {464, 335, 215, 648, 632, 210, 642, 211}

$$\frac{(Ab-aB)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3a^{11/6}\sqrt[6]{b}}-\frac{2(Ab-aB)\text{ArcTan}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}+\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}-\frac{2A}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(5*a*x^(5/2)) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^
(1/6)])/(3*a^(11/6)*b^(1/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqr
t[x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[
x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(11/6)*b^(1/6)) - ((A*b - a
*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*
a^(11/6)*b^(1/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx = -\frac{2A}{5ax^{5/2}} - \frac{\left(2\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\,(a+bx^3)}\,dx}{5a}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{\left(4\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{5a}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{3a^{11/6}} - \frac{(2(Ab-$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x}\right.}{6a^{5/3}}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}\right.}{2\sqrt{3}\,a^{11/6}\sqrt[6]{b}}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}}{\sqrt[6]{a}}\right.}{3a^{11/6}\sqrt[6]{b}}$$

Mathematica [A]
time = 0.17, size = 158, normalized size = 0.59

$$\frac{-\frac{6a^{5/6}A}{x^{5/2}}+\frac{10(-Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}}+\frac{5(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}\right)}{\sqrt[6]{b}}+\frac{5\sqrt{3}\,(-Ab+aB)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{\sqrt[6]{b}}}{15a^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

```
[Out] ((-6*a^(5/6)*A)/x^(5/2) + (10*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/
6)])/b^(1/6) + (5*(A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)
*Sqrt[x])])/b^(1/6) + (5*Sqrt[3]*(-(A*b) + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^
(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(1/6))/(15*a^(11/6))
```

Maple [A]
time = 0.35, size = 191, normalized size = 0.71

method	result

derivativedivides	$\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\cdots}{}\right.}{a}$
default	$\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\cdots}{}\right.}{a}$
risch	$-\frac{2A}{5a\,x^{\frac{5}{2}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)Ab}{6a^{2}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)B}{6a}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(1/6)
*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(-3^
(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^
(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3
^(1/2)))*(-A*b+B*a)/a-2/5*A/a/x^(5/2)
```

Maxima [A]
time = 0.49, size = 278, normalized size = 1.03

$$\frac{\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{6a}-\frac{2A}{5ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/
6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A*b^
(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(
sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1
/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2/5*A/(a*x^(5/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2424 vs. $2(188) = 376$.

time = 1.97, size = 2424, normalized size = 8.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/30*(20*sqrt(3)*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 2
0*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))
^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B
^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*
b^6)/(a^11*b))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a^3 - A*a^2*b
)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6))*a^9*b*
(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A
^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(5/6) + 2*sqrt(3)*(B*a^
10*b - A*a^9*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b
))^(5/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B
^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*
B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
6*A^5*B*a*b^5 + A^6*b^6)) + 20*sqrt(3)*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^11*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)
*x - (B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*
b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a
^11*b))^(1/6))*a^9*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(5
/6) + 2*sqrt(3)*(B*a^10*b - A*a^9*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b +
 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b
^5 + A^6*b^6)/(a^11*b))^(5/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B
^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*
b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - 5*a*x^3*(-(B^6*a^6 - 6*A*B
^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6
*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(4*a^4*(-(B^6*a^6 - 6*A*B^5*a^5*
b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*
a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + 4*
(B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b
))^(1/6)) + 5*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^
3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/
6)*log(4*a^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a
```

```
^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + 4*
(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6
- 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*
b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)) + 10*a*x^3*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(a^2*(-(B^6*a^6 - 6*A*B^5*a^5*
b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*
a*b^5 + A^6*b^6)/(a^11*b))^(1/6) - (B*a - A*b)*sqrt(x)) - 10*a*x^3*(-(B^6*a
^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a
^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(-a^2*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6) - (B*a - A*b)*sqrt(x)) + 12*A*sqr
t(x))/(a*x^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(258) = 516$.
time = 53.26, size = 586, normalized size = 2.17

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{11x^{\frac{11}{2}}} - \frac{2B}{5x^{\frac{5}{2}}}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{-\frac{2A}{11x^{\frac{11}{2}}} - \frac{2B}{5x^{\frac{5}{2}}}}{b} & \text{for}\: a = 0 \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}} + 2B\sqrt{x}}{a} & \text{for}\: b = 0 \\ -\frac{2A}{5ax^{\frac{5}{2}}} + \frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3a^2} - \frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3a^2} + \frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6a^2} - \frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6a^2} - \frac{\sqrt{3}Ab\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} - \frac{\sqrt{3}}{3}\right)}{3a^2} - \frac{\sqrt{3}Ab\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{3a^2} - \frac{B\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3a} + \frac{B\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3a} - \frac{B\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6a} + \frac{B\sqrt[6]{-\frac{a}{b}}\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6a} + \frac{\sqrt{3}B\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} - \frac{\sqrt{3}}{3}\right)}{3a} + \frac{\sqrt{3}B\sqrt[6]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3\sqrt[6]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b, 0
)), ((-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2)))/b, Eq(a, 0)), ((-2*A/(5*x**(5
/2)) + 2*B*sqrt(x))/a, Eq(b, 0)), (-2*A/(5*a*x**(5/2)) + A*b*(-a/b)**(1/6)*
log(sqrt(x) - (-a/b)**(1/6))/(3*a**2) - A*b*(-a/b)**(1/6)*log(sqrt(x) + (-a
/b)**(1/6))/(3*a**2) + A*b*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x
 + 4*(-a/b)**(1/3))/(6*a**2) - A*b*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6
) + 4*x + 4*(-a/b)**(1/3))/(6*a**2) - sqrt(3)*A*b*(-a/b)**(1/6)*atan(2*sqrt
(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a**2) - sqrt(3)*A*b*(-a/b)**(
1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a**2) - B*(-a
/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*a) + B*(-a/b)**(1/6)*log(sqrt(x)
 + (-a/b)**(1/6))/(3*a) - B*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*
x + 4*(-a/b)**(1/3))/(6*a) + B*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) +
4*x + 4*(-a/b)**(1/3))/(6*a) + sqrt(3)*B*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(
x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a) + sqrt(3)*B*(-a/b)**(1/6)*atan(2*sq
rt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a), True))
```

Giac [**A**]
time = 0.61, size = 280, normalized size = 1.04

$$\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^2b} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^2b} + \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} + 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a^2b} + \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} - 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a^2b} + \frac{2\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a^2b} - \frac{2A}{5ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/
b)^(1/6) + x + (a/b)^(1/3))/(a^2*b) - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b
^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b)
+ 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) +
 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A
*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 2/3*((
a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b) -
 2/5*A/(a*x^(5/2))
```

Mupad [B]
time = 2.91, size = 2023, normalized size = 7.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)),x)
```

```
[Out] - (2*A)/(5*a*x^(5/2)) - (atan((((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 +
 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*b - B*a
)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b
^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) + ((x
^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a
^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*
b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b
 - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/(((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^
9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*
b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*
B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6)) -
 ((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B
^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a
^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*
(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6))))*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^(1/
6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*
b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(
1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B
^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(
3*(-a)^(11/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 +
 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b
^8) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b
^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b
- B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A
^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384
*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 28
8*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(
```

```
1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6)) - (((3^(1/2)*1i)/2 - 1/2)*(x^(1
/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*
b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7
*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(
11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/
2)*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)
*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^
3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A
^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*
(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) + (((3^(1/2)*
1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^
7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(A*b -
 B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a
^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/
((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^
2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 +
 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 -
 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b
^(1/6)) - (((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5
 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2
)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*
a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)
^(11/6)*b^(1/6))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^
(1/6))
```

3.163 $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{5/2}}$$

```
[Out] -1/3*(A*b-3*B*a)*x^(3/2)/a/b^2+1/3*(A*b-B*a)*x^(9/2)/a/b/(b*x^3+a)+1/3*(A*b
-3*B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/b^(5/2)/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.227$,
Rules used = {468, 327, 335, 281, 211}

$$\frac{(Ab-3aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{5/2}}-\frac{x^{3/2}(Ab-3aB)}{3ab^2}+\frac{x^{9/2}(Ab-aB)}{3ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*((A*b - 3*a*B)*x^(3/2))/(a*b^2) + ((A*b - a*B)*x^(9/2))/(3*a*b*(a + b*
x^3)) + ((A*b - 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{\left(-\frac{3Ab}{2}+\frac{9aB}{2}\right)\int \frac{x^{7/2}}{a+bx^3}\,dx}{3ab}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{(Ab-3aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{(Ab-3aB)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{(Ab-3aB)\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 77, normalized size = 0.81

$$\frac{x^{3/2}(-Ab+3aB+2bBx^3)}{3b^2\,(a+bx^3)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (x^(3/2)*(-(A*b) + 3*a*B + 2*b*B*x^3))/(3*b^2*(a + b*x^3)) + ((A*b - 3*a*B)
*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2))
```

Maple [A]
time = 0.31, size = 65, normalized size = 0.68

method	result	size
derivativedivides	$\frac{2 B \,x^{\frac{3}{2}}}{3 b^{2}}+\frac{\frac{2 \left(-\frac{A b}{2}+\frac{B a}{2}\right) x^{\frac{3}{2}}}{3 \left(b \,x^{3}+a \right)}+\frac{\left(A b -3 B a \right) \arctan \left(\frac{b \,x^{\frac{3}{2}}}{\sqrt{a b}}\right)}{3 \sqrt{a b}}}{b^{2}}$	65
default	$\frac{2 B \,x^{\frac{3}{2}}}{3 b^{2}}+\frac{\frac{2 \left(-\frac{A b}{2}+\frac{B a}{2}\right) x^{\frac{3}{2}}}{3 \left(b \,x^{3}+a \right)}+\frac{\left(A b -3 B a \right) \arctan \left(\frac{b \,x^{\frac{3}{2}}}{\sqrt{a b}}\right)}{3 \sqrt{a b}}}{b^{2}}$	65
risch	$\frac{2 B \,x^{\frac{3}{2}}}{3 b^{2}}-\frac{x^{\frac{3}{2}} A}{3 b \left(b \,x^{3}+a \right)}+\frac{x^{\frac{3}{2}} B a}{3 b^{2} \left(b \,x^{3}+a \right)}+\frac{\arctan \left(\frac{b \,x^{\frac{3}{2}}}{\sqrt{a b}}\right) A}{3 b \sqrt{a b}}-\frac{\arctan \left(\frac{b \,x^{\frac{3}{2}}}{\sqrt{a b}}\right) B a}{b^{2} \sqrt{a b}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*B*x^(3/2)/b^2+2/3/b^2*((-1/2*A*b+1/2*B*a)*x^(3/2)/(b*x^3+a)+1/2*(A*b-3*
B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 68, normalized size = 0.72

$$\frac{(Ba-Ab)x^{\frac{3}{2}}}{3\,(b^3x^3+ab^2)}+\frac{2\,Bx^{\frac{3}{2}}}{3\,b^2}-\frac{(3\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x^(3/2)/(b^3*x^3 + a*b^2) + 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a
- A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 1.90, size = 222, normalized size = 2.34

$$\left[\frac{((3\,Bab-Ab^2)x^3+3\,Ba^2-Aab)\sqrt{-ab}\,\log\left(\frac{bx^3-2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\,(2\,Bab^2x^4+(3\,Ba^2b-Aab^2)x)\sqrt{x}}{6\,(ab^4x^3+a^2b^3)},-\frac{((3\,Bab-Ab^2)x^3+3\,Ba^2-Aab)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)-(2\,Bab^2x^4+(3\,Ba^2b-Aab^2)x)\sqrt{x}}{3\,(ab^4x^3+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((3*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)*sqrt(-a*b)*log((b*x^3 - 2*s
qrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2*(2*B*a*b^2*x^4 + (3*B*a^2*b - A*a*b
^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3), -1/3*(((3*B*a*b - A*b^2)*x^3 + 3*B*a
^2 - A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*B*a*b^2*x^4 + (3*B*a
^2*b - A*a*b^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 68, normalized size = 0.72

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b^2}-\frac{(3\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b^2}+\frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{3\,(bx^3+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b
)*b^2) + 1/3*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^3 + a)*b^2)
```

Mupad [B]
time = 2.65, size = 116, normalized size = 1.22

$$\frac{2\,B\,x^{3/2}}{3\,b^2}-\frac{x^{3/2}\,\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2}+\frac{\mathrm{atan}\left(\frac{36\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,\left(A^2\,b^2-6\,A\,B\,a\,b+9\,B^2\,a^2\right)}{\left(A\,b-3\,B\,a\right)\,\left(36\,A\,a\,b^2-108\,B\,a^2\,b\right)}\right)\,\left(A\,b-3\,B\,a\right)}{3\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (2*B*x^(3/2))/(3*b^2) - (x^(3/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (
atan((36*a^(1/2)*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 - 6*A*B*a*b))/((A*b -
 3*B*a)*(36*A*a*b^2 - 108*B*a^2*b)))*(A*b - 3*B*a))/(3*a^(1/2)*b^(5/2))
```

3.164 $\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=312

$$-\frac{(Ab-7aB)\sqrt{x}}{3ab^2}+\frac{(Ab-aB)x^{7/2}}{3ab(a+bx^3)}-\frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}}$$

```
[Out] 1/3*(A*b-B*a)*x^(7/2)/a/b/(b*x^3+a)+1/9*(A*b-7*B*a)*arctan(b^(1/6)*x^(1/2)/
a^(1/6))/a^(5/6)/b^(13/6)+1/18*(A*b-7*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2
)/a^(1/6))/a^(5/6)/b^(13/6)+1/18*(A*b-7*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/
2)/a^(1/6))/a^(5/6)/b^(13/6)-1/36*(A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*
b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(13/6)*3^(1/2)+1/36*(A*b-7*B*a)*ln(a^(1/
3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(13/6)*3^(1/2)-1/3*
(A*b-7*B*a)*x^(1/2)/a/b^2
```

Rubi [A]
time = 0.35, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 327, 335, 215, 648, 632, 210, 642, 211}

$$-\frac{(Ab-7aB)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\text{ArcTan}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}}-\frac{(Ab-7aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}-\frac{\sqrt{x}(Ab-7aB)}{3ab^2}+\frac{x^{7/2}(Ab-aB)}{3ab(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*((A*b - 7*a*B)*Sqrt[x])/(a*b^2) + ((A*b - a*B)*x^(7/2))/(3*a*b*(a + b*
x^3)) - ((A*b - 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a
^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1
/6)])/(18*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/
6)])/(9*a^(5/6)*b^(13/6)) - ((A*b - 7*a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^
(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*
Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(
5/6)*b^(13/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{\left(-\frac{Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{(Ab-7aB)\int \frac{1}{\sqrt{x}\,(a+bx^3)}\,dx}{6b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{(Ab-7aB)\mathrm{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{(Ab-7aB)\mathrm{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{\cdots}}\right.}{9a^{5/6}b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab-7a\cdots}{\cdots}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab-7a\cdots}{\cdots}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} - \frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}} + \frac{(\cdots}{\cdots}$$

Mathematica [A]
time = 0.47, size = 181, normalized size = 0.58

$$\frac{\frac{6\sqrt[6]{b}\,\sqrt{x}\,(-Ab+7aB+6bBx^3)}{a+bx^3}+\frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{a^{5/6}}+\frac{(-Ab+7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}\right)}{a^{5/6}}+\frac{\sqrt{3}\,(Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{a^{5/6}}}{18b^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((6*b^(1/6)*Sqrt[x]*(-(A*b) + 7*a*B + 6*b*B*x^3))/(a + b*x^3) + (2*(A*b - 7
*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/a^(5/6) + ((-(A*b) + 7*a*B)*ArcTan
```

```
[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/a^(5/6) + (Sqrt[3]*(A*b
- 7*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/
a^(5/6))/(18*b^(13/6))
```

Maple [A]
time = 0.39, size = 216, normalized size = 0.69

method	result
derivativedivides	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{6}+\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(Ab-7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\cdots\right)}{\cdots}}{\cdots}$
default	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{6}+\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(Ab-7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\cdots\right)}{\cdots}}{\cdots}$
risch	$\frac{2B\sqrt{x}}{b^2}-\frac{\sqrt{x}\,A}{3b(b\,x^3+a)}+\frac{\sqrt{x}\,Ba}{3b^2(b\,x^3+a)}+\frac{A\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36ba}+\frac{A\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\cdots\right)}{18ba}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/b^2*x^(1/2)+2/b^2*((-1/6*A*b+1/6*B*a)*x^(1/2)/(b*x^3+a)+1/6*(A*b-7*B*a)
*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(1/6)*
ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(-3^(
1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(
1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^
(1/2))))
```

Maxima [A]
time = 0.52, size = 311, normalized size = 1.00

$$\frac{(Ba-Ab)\sqrt{x}}{3\,(b^3x^3+ab^2)}+\frac{2\,B\sqrt{x}}{b^2}-\frac{\frac{\sqrt{3}\,(7\,Ba-Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(7\,Ba-Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(7\,Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(7\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(7\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{36\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*sqrt(x)/(b^3*x^3 + a*b^2) + 2*B*sqrt(x)/b^2 - 1/36*(sqrt(3)
*(7*B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(
a^(5/6)*b^(1/6)) - sqrt(3)*(7*B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(
x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) - A*b^(4/3))
```

```
*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3
)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a
^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(sq
rt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3
)*sqrt(a^(1/3)*b^(1/3))))/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2566 vs. $2(232) = 464$.
time = 1.69, size = 2566, normalized size = 8.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

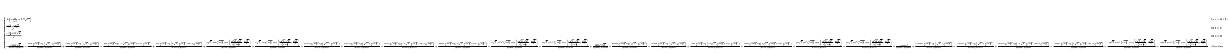
```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b +
36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5
*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^4*(-
(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3
*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3
) + (49*B^2*a^2 - 14*A*B*a*b + A^2*b^2)*x + (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)
*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*
B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(
1/6))*a^4*b^11*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b
^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)
/(a^5*b^13))^(5/6) + 2*sqrt(3)*(7*B*a^5*b^11 - A*a^4*b^12)*sqrt(x)*(-(11764
9*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b
^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) - sq
rt(3)*(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A
^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6))/(117649*B
^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3
 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*(b^3*x^3 + a
*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860
*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^1
3))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^4*(-(117649*B^6*a^6 - 100842*A*B
^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b
^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^2 - 14*A*B*a*b
 + A^2*b^2)*x - (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6*a^6 - 100842*
A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^
2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6))*a^4*b^11*(-(117649*B^6
*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) + 2*sqrt(
3)*(7*B*a^5*b^11 - A*a^4*b^12)*sqrt(x)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5
*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 4
```

```
2*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) + sqrt(3)*(117649*B^6*a^6 - 1008
42*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2
*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6))/(117649*B^6*a^6 - 100842*A*B^5*a^5*b
+ 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A
^5*B*a*b^5 + A^6*b^6)) - (b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5
*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4
 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(a^2*b^4*(-(117649*B^6*a^
6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735
*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^
2 - 14*A*B*a*b + A^2*b^2)*x + (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6
*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) + (b^3*x
^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2
- 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a
^5*b^13))^(1/6)*log(a^2*b^4*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*
A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^2 - 14*A*B*a*b + A^2*b^2)*x - (
7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 360
15*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*
a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) + 2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6
 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*
A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(a*b^2*(-(
117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*
a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)
 - (7*B*a - A*b)*sqrt(x)) - 2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*
A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^
2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(-a*b^2*(-(117649*B^
6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6) - (7*B*a
 - A*b)*sqrt(x)) + 12*(6*B*b*x^3 + 7*B*a - A*b)*sqrt(x))/(b^3*x^3 + a*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. $2(299) = 598$.

time = 185.02, size = 1658, normalized size = 5.31



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2
*A*x**(7/2)/7 + 2*B*x**(13/2)/13)/a**2, Eq(b, 0)), ((-2*A/(5*x**(5/2)) + 2*
B*sqrt(x))/b**2, Eq(a, 0)), (-12*A*a*b*sqrt(x)/(36*a**2*b**2 + 36*a*b**3*x*
*3) - 2*A*a*b*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2 + 36
*a*b**3*x**3) + 2*A*a*b*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**2
```

```
*b**2 + 36*a*b**3*x**3) - A*a*b*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6)
+ 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3) + A*a*b*(-a/b)**(1
/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36
*a*b**3*x**3) + 2*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a
/b)**(1/6)) - sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*sqrt(3)*A*a*b*
(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**
2*b**2 + 36*a*b**3*x**3) - 2*A*b**2*x**3*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)
**(1/6))/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*A*b**2*x**3*(-a/b)**(1/6)*log(
sqrt(x) + (-a/b)**(1/6))/(36*a**2*b**2 + 36*a*b**3*x**3) - A*b**2*x**3*(-a/
b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b*
*2 + 36*a*b**3*x**3) + A*b**2*x**3*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6
) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*sqrt(3)*A*b*
*2*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)
/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*sqrt(3)*A*b**2*x**3*(-a/b)**(1/6)*atan
(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3
*x**3) + 84*B*a**2*sqrt(x)/(36*a**2*b**2 + 36*a*b**3*x**3) + 14*B*a**2*(-a/
b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2 + 36*a*b**3*x**3) - 14
*B*a**2*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**2*b**2 + 36*a*b**
3*x**3) + 7*B*a**2*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a
/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3) - 7*B*a**2*(-a/b)**(1/6)*log(4*
sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x*
*3) - 14*sqrt(3)*B*a**2*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/
6)) - sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*x**3) - 14*sqrt(3)*B*a**2*(-a/b)
**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**2*b**2
 + 36*a*b**3*x**3) + 72*B*a*b*x**(7/2)/(36*a**2*b**2 + 36*a*b**3*x**3) + 14
*B*a*b*x**3*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2 + 36*a
*b**3*x**3) - 14*B*a*b*x**3*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*
a**2*b**2 + 36*a*b**3*x**3) + 7*B*a*b*x**3*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a
/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3) - 7*B*a
*b*x**3*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/
(36*a**2*b**2 + 36*a*b**3*x**3) - 14*sqrt(3)*B*a*b*x**3*(-a/b)**(1/6)*atan(
2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*
x**3) - 14*sqrt(3)*B*a*b*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b
)**(1/6)) + sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*x**3), True))
```

Giac [A]
time = 0.59, size = 313, normalized size = 1.00

$$\frac{2\,B\sqrt{x}}{b^2}-\frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3}+\frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3}+\frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\,(bx^3+a)b^2}-\frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,ab^3}-\frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,ab^3}-\frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^2 - 1/36*sqrt(3)*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*lo
g(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^3) + 1/36*sqrt(3)*(7*
```

```
(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x
 + (a/b)^(1/3))/(a*b^3) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*b^2)
 - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/
6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^
(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) -
 1/9*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/
(a*b^3)
```

Mupad [B]
time = 2.89, size = 1884, normalized size = 6.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (2*B*x^(1/2))/b^2 - (x^(1/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (atan
(((((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a
^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^
3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7
*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 +
 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*(A
*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))
/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((
2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b
- 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 1
47*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)
)/(18*(-a)^(5/6)*b^(13/6)) - (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2
*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*(A*b - 7*B
*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a
)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*(A*b - 7*B*a)*
1i)/(9*(-a)^(5/6)*b^(13/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*((2*x^(1/2)*(A
^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a
*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*B^3*a^4 - A^
3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A
*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((3^(1/2)*1i)/2 - 1/2)*((2*x^(1
/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A
^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*B^3*a^
4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6
)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((3^(1/2)*1i)/2 - 1/2)*((
2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b
- 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*
B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b
^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6)) - (((3^(1/2)*1i)/2 - 1/2)
*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3
```

```
*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(3
43*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6
)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*((3^(1/2)*1i)/2 - 1/
2)*(A*b - 7*B*a)*1i)/(9*(-a)^(5/6)*b^(13/6)) - (atan(((((3^(1/2)*1i)/2 + 1/
2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a
^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a)*
(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5
/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((3^(1/2)*1i)/
2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A
*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7
*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(
-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((3^(1/2
)*1i)/2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 -
1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 + 1/2)*(A
*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))
/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6)) - (((3^(
1/2)*1i)/2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2
 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 + 1/2)
*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^
2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*((3
^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a)*1i)/(9*(-a)^(5/6)*b^(13/6))
```

3.165 $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=289

$$\frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)}-\frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}}+\frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}}+\frac{(Ab+5aB}{}$$

```
[Out] 1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/9*(A*b+5*B*a)*arctan(b^(1/6)*x^(1/2)/
a^(1/6))/a^(7/6)/b^(11/6)+1/18*(A*b+5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2
)/a^(1/6))/a^(7/6)/b^(11/6)+1/18*(A*b+5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/
2)/a^(1/6))/a^(7/6)/b^(11/6)+1/36*(A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*
b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(11/6)*3^(1/2)-1/36*(A*b+5*B*a)*ln(a^(1/
3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(11/6)*3^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {468, 335, 301, 648, 632, 210, 642, 211}

$$-\frac{(5aB+Ab)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}}+\frac{(5aB+Ab)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{7/6}b^{11/6}}+\frac{(5aB+Ab)\text{ArcTan}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}}+\frac{(5aB+Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}+\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(3*a*b*(a + b*x^3)) - ((A*b + 5*a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5*a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5
*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(7/6)*b^(11/6)) + ((A*b + 5*a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(7/6)*b^(11/6)) - ((A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(7/6)*b^(11/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^{5/2}}{3ab\,(a+bx^3)} + \frac{\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\,(a+bx^3)} + \frac{\left(2\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\,(a+bx^3)} + \frac{(Ab+5aB)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{9a^{7/6}b^{5/3}} + \text{—}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\,(a+bx^3)} + \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{a}-\sqrt{3}}\right.}{12\sqrt{3}\,a^{7/6}}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\,(a+bx^3)} + \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\right.}{12\sqrt{3}\,a^{7/6}b^{11/6}}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\,(a+bx^3)} - \frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}\right.}{18a^{7/6}b^{11/6}}$$

Mathematica [A]
time = 0.42, size = 169, normalized size = 0.58

$$\frac{-\frac{6\sqrt[6]{a}\,b^{5/6}(-Ab+aB)x^{5/2}}{a+bx^3}+2(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)-\sqrt{3}\,(Ab+5aB)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{18a^{7/6}b^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(1/6)*b^(5/6)*(-(A*b) + a*B)*x^(5/2))/(a + b*x^3) + 2*(A*b + 5*a*B)*
ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (A*b + 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)
*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] - Sqrt[3]*(A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(18*a^(7/6)*b^(11/6))
```

Maple [A]
time = 0.30, size = 213, normalized size = 0.74

method	result

derivativedivides	$\frac{(Ab-Ba)x^{\frac{5}{2}}}{3ab(bx^3+a)}+\frac{(Ab+5Ba)\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{3ab}$
default	$\frac{(Ab-Ba)x^{\frac{5}{2}}}{3ab(bx^3+a)}+\frac{(Ab+5Ba)\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{3ab}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/3*(A*b+5*B*a)/a/b*(-1/12/a*3^(1/2)*(a
/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*a
rctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)
^(1/6))+1/12/a*3^(1/2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(
1/3))+1/6/b/(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6)))
```

Maxima [A]
time = 0.49, size = 235, normalized size = 0.81

$$-\frac{(Ba-Ab)x^{\frac{5}{2}}}{3\,(ab^2x^3+a^2b)}-\frac{(5\,Ba+Ab)\left(\frac{\sqrt{3}\,\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\,\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{36\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^(5/2)/(a*b^2*x^3 + a^2*b) - 1/36*(5*B*a + A*b)*(sqrt(3)*
log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)
) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^
(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqr
t(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^
(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(
1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3))))/(b^(2/3)*s
qrt(a^(1/3)*b^(1/3))))/(a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3787 vs. $2(207) = 414$.
time = 2.06, size = 3787, normalized size = 13.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*(B*a - A*b)*x^(5/2) + 4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*
a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375
*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*arctan(1/3*(
2*sqrt(3)*sqrt((3125*B^5*a^11*b^9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9
*b^11 + 250*A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-
(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^
3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) +
 (9765625*B^10*a^10 + 19531250*A*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 937
5000*A^3*B^7*a^7*b^3 + 3281250*A^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 1
31250*A^6*B^4*a^4*b^6 + 15000*A^7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A
^9*B*a*b^9 + A^10*b^10)*x - (15625*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 93
75*A^2*B^4*a^9*b^9 + 2500*A^3*B^3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*
B*a^6*b^12 + A^6*a^5*b^13)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*
B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 +
 A^6*b^6)/(a^7*b^11))^(2/3))*a*b^2*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9
375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B
*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5*a^6*b^2 + 3125*A*
B^4*a^5*b^3 + 1250*A^2*B^3*a^4*b^4 + 250*A^3*B^2*a^3*b^5 + 25*A^4*B*a^2*b^6
 + A^5*a*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a
^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*
b^6)/(a^7*b^11))^(1/6) + sqrt(3)*(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*
A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b
^5 + A^6*b^6))/(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 +
2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)) + 4
*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^
2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^11))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(3125*B^5*a^11*b^
9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250*A^3*B^2*a^8*b^12 + 2
5*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5
*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30
*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) + (9765625*B^10*a^10 + 19531250*A
*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7*a^7*b^3 + 3281250*A
^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^4*a^4*b^6 + 15000*A^
7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 + A^10*b^10)*x - (156
25*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a^9*b^9 + 2500*A^3*B^
3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 + A^6*a^5*b^13)*(-(15
625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b
^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(2/3))*a*b
^2*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B
^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1
/6) - 2*sqrt(3)*(3125*B^5*a^6*b^2 + 3125*A*B^4*a^5*b^3 + 1250*A^2*B^3*a^4*b
```

```
^4 + 250*A^3*B^2*a^3*b^5 + 25*A^4*B*a^2*b^6 + A^5*a*b^7)*sqrt(x)*(-(15625*B
^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 +
375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6) - sqrt(3)
*(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a
^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6))/(15625*B^6*a^6 +
18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B
^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)) - 2*(a*b^2*x^3 + a^2*b)*(-(15625*B^
6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 3
75*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(a^6*b^
9*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^
3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/
6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*
a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) + 2*(a*b^2*x^3 + a^2*b)*(-(156
25*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^
3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(-
a^6*b^9*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*
A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11
))^(5/6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^
3*B^2*a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) - (a*b^2*x^3 + a^2*b)*(-
(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^
3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*l
og((3125*B^5*a^11*b^9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250*
A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-(15625*B^6*a
^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*
A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) + (9765625*B^
10*a^10 + 19531250*A*B^9*a^9*b + 17578125*A^2*B...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1885 vs. $2(277) = 554$.
time = 119.89, size = 1885, normalized size = 6.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2
*A*x**(5/2)/5 + 2*B*x**(11/2)/11)/a**2, Eq(b, 0)), ((-2*A/(7*x**(7/2)) - 2*
B/sqrt(x))/b**2, Eq(a, 0)), (2*A*a*b*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*
b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - 2*A*a*b*log(sqrt(x) +
(-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6))
+ A*a*b*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2
*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - A*a*b*log(4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x*
*3*(-a/b)**(1/6)) + 2*sqrt(3)*A*a*b*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)
```

```
) - sqrt(3)/3)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6))
+ 2*sqrt(3)*A*a*b*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36
*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 12*A*b**2*x**(5/
2)*(-a/b)**(1/6)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)
) + 2*A*b**2*x**3*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6)
+ 36*a*b**3*x**3*(-a/b)**(1/6)) - 2*A*b**2*x**3*log(sqrt(x) + (-a/b)**(1/6)
)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + A*b**2*x**3
*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)
**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - A*b**2*x**3*log(4*sqrt(x)*(-a/b)*
*(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**
3*(-a/b)**(1/6)) + 2*sqrt(3)*A*b**2*x**3*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**
(1/6)) - sqrt(3)/3)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1
/6)) + 2*sqrt(3)*A*b**2*x**3*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqr
t(3)/3)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 10*B*
a**2*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x
**3*(-a/b)**(1/6)) - 10*B*a**2*log(sqrt(x) + (-a/b)**(1/6))/(36*a**2*b**2*(
-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 5*B*a**2*log(-4*sqrt(x)*(-a/
b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*
x**3*(-a/b)**(1/6)) - 5*B*a**2*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)
**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 10*s
qrt(3)*B*a**2*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**
2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 10*sqrt(3)*B*a**2*at
an(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**2*b**2*(-a/b)**(
1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - 12*B*a*b*x**(5/2)*(-a/b)**(1/6)/(36*
a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 10*B*a*b*x**3*log
(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/
b)**(1/6)) - 10*B*a*b*x**3*log(sqrt(x) + (-a/b)**(1/6))/(36*a**2*b**2*(-a/b
)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 5*B*a*b*x**3*log(-4*sqrt(x)*(-a/
b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*
x**3*(-a/b)**(1/6)) - 5*B*a*b*x**3*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-
a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) +
10*sqrt(3)*B*a*b*x**3*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)
/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 10*sqrt(3)*B
*a*b*x**3*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**2*b*
*2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)), True))
```

Giac [A]
time = 1.04, size = 302, normalized size = 1.04

$$\frac{\left(5\,Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}+Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^{2}b}-\frac{Bax^{\frac{5}{2}}-Abx^{\frac{5}{2}}}{3\,(bx^{3}+a)ab}-\frac{\sqrt{3}\left(5\,(ab^{5})^{\frac{5}{6}}\,Ba+(ab^{5})^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^{2}b^{6}}+\frac{\sqrt{3}\left(5\,(ab^{5})^{\frac{5}{6}}\,Ba+(ab^{5})^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^{2}b^{6}}+\frac{\left(5\,(ab^{5})^{\frac{5}{6}}\,Ba+(ab^{5})^{\frac{5}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^{2}b^{6}}+\frac{\left(5\,(ab^{5})^{\frac{5}{6}}\,Ba+(ab^{5})^{\frac{5}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^{2}b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(5*B*a*(a/b)^(5/6) + A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*
b) - 1/3*(B*a*x^(5/2) - A*b*x^(5/2))/((b*x^3 + a)*a*b) - 1/36*sqrt(3)*(5*(a
```

```
*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x +
(a/b)^(1/3))/(a^2*b^6) + 1/36*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*
A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^6) + 1/18*(
5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sq
rt(x))/(a/b)^(1/6))/(a^2*b^6) + 1/18*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A
*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^6)
```

Mupad [B]
time = 2.87, size = 1578, normalized size = 5.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (x^(5/2)*(A*b - B*a))/(3*a*b*(a + b*x^3)) - (atan(((((3^(1/2)*1i)/2 - 1/2)^
2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A
^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 +
600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)
^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3
 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((3^(1/2)*
1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^
3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))/((((3^(1/2)*1i
)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a
^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A
^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(
324*(-a)^(7/3)*b^(11/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^
3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((
3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*
B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3))))*((3^(1/2
)*1i)/2 - 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^(7/6)*b^(11/6)) - (atan(((((3^(1/2
)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B
^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(
24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))
)*1i)/(324*(-a)^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2
*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(
1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 +
 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))
/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/
3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b
 + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6
)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(A*b +
5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^
2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a
^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11
```

```
/3))))*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^(7/6)*b^(11/6)) - (
atan((((A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b +
 20*A^2*B*a*b^2 - (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 +
240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)) -
 ((A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A
^2*B*a*b^2 + (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A
*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))/(((A*
b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*
a*b^2 - (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^
2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3)) + ((A*b + 5*B*
a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 +
(x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/
(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3))))*(A*b + 5*B*a)*1i)/(9
*(-a)^(7/6)*b^(11/6))
```

3.166 $\int \frac{\sqrt{x}\,(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=71

$$\frac{(Ab-aB)x^{3/2}}{3ab\,(a+bx^3)}+\frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}$$

```
[Out] 1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3*(A*b+B*a)*arctan(x^(3/2)*b^(1/2)/a^
(1/2))/a^(3/2)/b^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {468, 335, 281, 211}

$$\frac{(aB+Ab)\text{ArcTan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}+\frac{x^{3/2}(Ab-aB)}{3ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^
(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}\,(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{(Ab-aB)x^{3/2}}{3ab\,(a+bx^3)}+\frac{(Ab+aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2ab}\\
&= \frac{(Ab-aB)x^{3/2}}{3ab\,(a+bx^3)}+\frac{(Ab+aB)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{ab}\\
&= \frac{(Ab-aB)x^{3/2}}{3ab\,(a+bx^3)}+\frac{(Ab+aB)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3ab}\\
&= \frac{(Ab-aB)x^{3/2}}{3ab\,(a+bx^3)}+\frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 71, normalized size = 1.00

$$
-\frac{(-Ab+aB)x^{3/2}}{3ab\,(a+bx^3)}+\frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*((-(A*b) + a*B)*x^(3/2))/(a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt
[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

Maple [A]
time = 0.28, size = 61, normalized size = 0.86

method	result	size
derivativedivides	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{3ab(b\,x^3+a)}+\frac{(Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3ab\sqrt{ab}}$	61

default	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{3ab(b\,x^3+a)}+\frac{(Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3ab\sqrt{ab}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3*(A*b+B*a)/a/b/(a*b)^(1/2)*arctan(b*
x^(3/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 61, normalized size = 0.86

$$-\frac{(Ba-Ab)x^{\frac{3}{2}}}{3\left(ab^2x^3+a^2b\right)}+\frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^(3/2)/(a*b^2*x^3 + a^2*b) + 1/3*(B*a + A*b)*arctan(b*x^(
3/2)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 1.42, size = 190, normalized size = 2.68

$$\left[-\frac{2\left(Ba^2b-Aab^2\right)x^{\frac{3}{2}}+\left(\left(Bab+Ab^2\right)x^3+Ba^2+Aab\right)\sqrt{-ab}\,\log\left(\frac{bx^3-2\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)}{6\left(a^2b^3x^3+a^3b^2\right)},-\frac{\left(Ba^2b-Aab^2\right)x^{\frac{3}{2}}-\left(\left(Bab+Ab^2\right)x^3+Ba^2+Aab\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)}{3\left(a^2b^3x^3+a^3b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(B*a^2*b - A*a*b^2)*x^(3/2) + ((B*a*b + A*b^2)*x^3 + B*a^2 + A*a*b
)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)))/(a^2*b^3*
x^3 + a^3*b^2), -1/3*((B*a^2*b - A*a*b^2)*x^(3/2) - ((B*a*b + A*b^2)*x^3 +
B*a^2 + A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a))/(a^2*b^3*x^3 + a^3*b^
2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. 2(61) = 122.
time = 75.14, size = 1042, normalized size = 14.68

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) & \text{for}\: a=0 \wedge b=0 \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{a^2} & \text{for}\: b=0 \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b^2} & \text{for}\: a=0 \\ \frac{2Aabx^{\frac{3}{2}}}{6a^3b+6a^2b^2x^3} - \frac{Aab\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} + \frac{Aab\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} + \frac{Aab\sqrt{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} - \frac{Aab\sqrt{-\frac{a}{b}}\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} - \frac{Ab^2x^3\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} + \frac{Ab^2x^3\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} + \frac{Ab^2x^3\sqrt{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} - \frac{Ab^2x^3\sqrt{-\frac{a}{b}}\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} - \frac{2Ba^2x^{\frac{3}{2}}}{6a^3b+6a^2b^2x^3} - \frac{Ba^2\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} + \frac{Ba^2\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} + \frac{Ba^2\sqrt{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} - \frac{Ba^2\sqrt{-\frac{a}{b}}\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} - \frac{Babx^3\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} + \frac{Babx^3\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} + \frac{Babx^3\sqrt{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} - \frac{Babx^3\sqrt{-\frac{a}{b}}\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((2*A*x**(3/2)/3 + 2*B*x**(9/2)/9)/a**2, Eq(b, 0)), ((-2*A/(9*x**(9/2)) -
 2*B/(3*x**(3/2)))/b**2, Eq(a, 0)), (2*A*a*b*x**(3/2)/(6*a**3*b + 6*a**2*b*
*2*x**3) - A*a*b*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2
*b**2*x**3) + A*a*b*sqrt(-a/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b + 6*a
**2*b**2*x**3) + A*a*b*sqrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-
a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - A*a*b*sqrt(-a/b)*log(4*sqrt(x)
*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - A*b
**2*x**3*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x*
*3) + A*b**2*x**3*sqrt(-a/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b + 6*a**
2*b**2*x**3) + A*b**2*x**3*sqrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x +
4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - A*b**2*x**3*sqrt(-a/b)*log
(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x
**3) - 2*B*a**2*x**(3/2)/(6*a**3*b + 6*a**2*b**2*x**3) - B*a**2*sqrt(-a/b)*
log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3) + B*a**2*sqrt(-a
/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3) + B*a**2*sqr
t(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6
*a**2*b**2*x**3) - B*a**2*sqrt(-a/b)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*
(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - B*a*b*x**3*sqrt(-a/b)*log(sq
rt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3) + B*a*b*x**3*sqrt(-a/b
)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3) + B*a*b*x**3*s
qrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b +
 6*a**2*b**2*x**3) - B*a*b*x**3*sqrt(-a/b)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*
x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3), True))
```

Giac [A]
time = 0.62, size = 63, normalized size = 0.89

$$\frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\,ab}-\frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{3\,(bx^3+a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/3*(B*a*x^(3
/2) - A*b*x^(3/2))/((b*x^3 + a)*a*b)
```

Mupad [B]
time = 0.14, size = 115, normalized size = 1.62

$$\frac{B\,a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,b^2\,x^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,a\,b\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,\sqrt{a}\,b^{3/2}\,x^{3/2}-B\,a^{3/2}\,\sqrt{b}\,x^{3/2}+B\,a\,b\,x^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\,a^{5/2}\,b^{3/2}+3\,a^{3/2}\,b^{5/2}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*a^2*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*b^2*x^3*atan((b^(1/2)*x^(3/2))/a
^(1/2)) + A*a*b*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*a^(1/2)*b^(3/2)*x^(3/2)
 - B*a^(3/2)*b^(1/2)*x^(3/2) + B*a*b*x^3*atan((b^(1/2)*x^(3/2))/a^(1/2)))/(
3*a^(5/2)*b^(3/2) + 3*a^(3/2)*b^(5/2)*x^3)
```

3.167 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=289

$$\frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)}-\frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}}+\frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}}+\frac{(5Ab+aB}{}$$

```
[Out] 1/9*(5*A*b+B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)+1/18*(5*A*
b+B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)+1/18*(5*
A*b+B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)-1/36*(5
*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11/6)/b^
(7/6)*3^(1/2)+1/36*(5*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)
*x^(1/2))/a^(11/6)/b^(7/6)*3^(1/2)+1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)
```

Rubi [A]
time = 0.33, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {468, 335, 215, 648, 632, 210, 642, 211}

$$-\frac{(aB+5Ab)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\text{ArcTan}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}}-\frac{(aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}+\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(3*a*b*(a + b*x^3)) - ((5*A*b + a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b +
 a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(11/6)*b^(7/6)) - ((5*A*b + a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(11/6)*b^(7/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \frac{(Ab-aB)\sqrt{x}}{3ab\,(a+bx^3)} + \frac{\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\int \frac{1}{\sqrt{x}\,(a+bx^3)}\,dx}{3ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\,(a+bx^3)} + \frac{\left(2\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\,(a+bx^3)} + \frac{(5Ab+aB)\mathrm{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{9a^{11/6}b} + \ldots$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\,(a+bx^3)} + \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} - \frac{(5Ab+aB)\mathrm{Subst}\left(\int \frac{-\sqrt{3}\,\ldots}{\sqrt[3]{a}-\sqrt{3}\,\ldots}\ldots\right)}{12\sqrt{3}\,a^{11\ldots}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\,(a+bx^3)} + \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} - \frac{(5Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\ldots\right)}{12\sqrt{3}\,a^{11/6}b^{7/\ldots}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\,(a+bx^3)} - \frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}\ldots\right.}{18a^{11/6}b^{7/\ldots}}$$

Mathematica [A]
time = 0.41, size = 168, normalized size = 0.58

$$\frac{-\frac{6a^{5/6}\sqrt[6]{b}\,(-Ab+aB)\sqrt{x}}{a+bx^3}+2(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)-(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}\right)+\sqrt{3}\,(5Ab+aB)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{18a^{11/6}b^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a^(5/6)*b^(1/6)*(-(A*b) + a*B)*Sqrt[x])/(a + b*x^3) + 2*(5*A*b + a*B)*
ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (5*A*b + a*B)*ArcTan[(a^(1/3) - b^(1/3)
*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + Sqrt[3]*(5*A*b + a*B)*ArcTanh[(Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(18*a^(11/6)*b^(7/6))
```

Maple [A]
time = 0.29, size = 213, normalized size = 0.74

method	result

derivativedivides	$\frac{(Ab-Ba)\sqrt{x}}{3ab(b\,x^3+a)}+\frac{(5Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\cdots\right.}{\cdots}\right.}{3\cdots}$
default	$\frac{(Ab-Ba)\sqrt{x}}{3ab(b\,x^3+a)}+\frac{(5Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\cdots\right.}{\cdots}\right.}{3\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)+1/3*(5*A*b+B*a)/a/b*(1/3/a*(a/b)^(1/6)*
arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/
6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)
^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(
1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

Maxima [A]
time = 0.55, size = 301, normalized size = 1.04

$$-\frac{(Ba-Ab)\sqrt{x}}{3\,(ab^2x^3+a^2b)}+\frac{\frac{\sqrt{3}\,(Ba+5\,Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(Ba+5\,Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}+5\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}+5\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}+5\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{36\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*sqrt(x)/(a*b^2*x^3 + a^2*b) + 1/36*(sqrt(3)*(B*a + 5*A*b)*
log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)
) - sqrt(3)*(B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x
+ a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(1/3)
*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2
*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6)
+ 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3)
)) + 2*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b
^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*
b^(1/3))))/(a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2555 vs. 2(207) = 414.
time = 2.25, size = 2555, normalized size = 8.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="fricas")

[Out] 1/36*(4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x + (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6))*a^9*b^6*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(5/6) - 2*sqrt(3)*(B*a^10*b^6 + 5*A*a^9*b^7)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(5/6) + sqrt(3)*(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6))/(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)) + 4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x - (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6))*a^9*b^6*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(5/6) - 2*sqrt(3)*(B*a^10*b^6 + 5*A*a^9*b^7)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(5/6) - sqrt(3)*(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6))/(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)) + (a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x + (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)) - (a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A

^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*
a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*
a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 +
 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a
*b + 25*A^2*b^2)*x - (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*
a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 +
 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)) + 2*(a*b^2*x^3 + a^2
b)(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^
3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(
1/6)*log(a^2*b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3
*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a
^11*b^7))^(1/6) + (B*a + 5*A*b)*sqrt(x)) - 2*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6
 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B
^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log(-a^2*
b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3
+ 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/
6) + (B*a + 5*A*b)*sqrt(x)) - 12*(B*a - A*b)*sqrt(x))/(a*b^2*x^3 + a^2*b)

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1632 vs. $2(277) = 554$.
time = 98.65, size = 1632, normalized size = 5.65

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)/(b*x**3+a)**2/x**(1/2),x)

[Out] Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b, 0
)), ((-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2)))/b**2, Eq(a, 0)), ((2*A*sqrt(x
) + 2*B*x**(7/2)/7)/a**2, Eq(b, 0)), (12*A*a*b*sqrt(x)/(36*a**3*b + 36*a**2
*b**2*x**3) - 10*A*a*b*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**3*
b + 36*a**2*b**2*x**3) + 10*A*a*b*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6)
)/(36*a**3*b + 36*a**2*b**2*x**3) - 5*A*a*b*(-a/b)**(1/6)*log(-4*sqrt(x)*(-
a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*x**3) + 5*A*
a*b*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*
a**3*b + 36*a**2*b**2*x**3) + 10*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)
sqrt(x)/(3(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) + 1
0*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sq
rt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) - 10*A*b**2*x**3*(-a/b)**(1/6)*log
(sqrt(x) - (-a/b)**(1/6))/(36*a**3*b + 36*a**2*b**2*x**3) + 10*A*b**2*x**3*
(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b + 36*a**2*b**2*x**3)
- 5*A*b**2*x**3*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)
(1/3))/(36*a3*b + 36*a**2*b**2*x**3) + 5*A*b**2*x**3*(-a/b)**(1/6)*log(
4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*
x**3) + 10*sqrt(3)*A*b**2*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/

```
b)**(1/6)) - sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) + 10*sqrt(3)*A*b**2
*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(
36*a**3*b + 36*a**2*b**2*x**3) - 12*B*a**2*sqrt(x)/(36*a**3*b + 36*a**2*b**
2*x**3) - 2*B*a**2*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**3*b +
36*a**2*b**2*x**3) + 2*B*a**2*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3
6*a**3*b + 36*a**2*b**2*x**3) - B*a**2*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)*
*(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*x**3) + B*a**2*(-
a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b
 + 36*a**2*b**2*x**3) + 2*sqrt(3)*B*a**2*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(
x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) + 2*sqrt(
3)*B*a**2*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/
3)/(36*a**3*b + 36*a**2*b**2*x**3) - 2*B*a*b*x**3*(-a/b)**(1/6)*log(sqrt(x)
 - (-a/b)**(1/6))/(36*a**3*b + 36*a**2*b**2*x**3) + 2*B*a*b*x**3*(-a/b)**(1
/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b + 36*a**2*b**2*x**3) - B*a*b*x*
*3*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*
a**3*b + 36*a**2*b**2*x**3) + B*a*b*x**3*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*x**3) + 2*sqrt(3
)*B*a*b*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(
3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) + 2*sqrt(3)*B*a*b*x**3*(-a/b)**(1/6)*
atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**3*b + 36*a**2*
b**2*x**3), True))
```

Giac [A]
time = 0.97, size = 302, normalized size = 1.04

$$\frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{1}{6}}Ba+5\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^{2}b^{2}}-\frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{1}{6}}Ba+5\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^{2}b^{2}}-\frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\left(bx^{3}+a\right)ab}+\frac{\left(\left(ab^{5}\right)^{\frac{1}{6}}Ba+5\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^{2}b^{2}}+\frac{\left(\left(ab^{5}\right)^{\frac{1}{6}}Ba+5\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^{2}b^{2}}+\frac{\left(\left(ab^{5}\right)^{\frac{1}{6}}Ba+5\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*
(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^2) - 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a
+ 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/
(a^2*b^2) - 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a*b) + 1/18*((a*b^
5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x)
)/(a/b)^(1/6))/(a^2*b^2) + 1/18*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*a
rctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^2) + 1/9*((a*b
^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^2)
```

Mupad [B]
time = 2.92, size = 1922, normalized size = 6.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^2),x)
```

```
[Out] (atan(((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^
3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A^3*b^5 + B
^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*
(5*A*b + B*a)*1i)/(18*(-a)^(11/6)*b^(7/6)) + (((2*x^(1/2)*(625*A^4*b^5 + B^
4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^
4) + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^
2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a)*1i)/(18*(-a)^(11/6)*b^(
7/6)))/((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A
^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A^3*b^5 +
B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))
*(5*A*b + B*a))/(18*(-a)^(11/6)*b^(7/6)) - (((2*x^(1/2)*(625*A^4*b^5 + B^4*
a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4)
 + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*
a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a))/(18*(-a)^(11/6)*b^(7/6))
))*(5*A*b + B*a)*1i)/(9*(-a)^(11/6)*b^(7/6)) + (atan(((((3^(1/2)*1i)/2 - 1/
2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3
 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*((3^(1/2)*1i)/2 - 1/2
)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*
b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(11/6)*b^(7/6)) + (((3^(1/2)*1
i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^
2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) + (2*((3^(1/2)*1i
)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A
*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(11/6)*b^(7/6)))/((((
3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 1
50*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*((3
^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b
^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6))))/(18*(-a)^(11/6)*b^(7/6))
 - (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4
*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) +
(2*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2
*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6))))/(18*(-a)^(11/6)*b^
(7/6))))*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*1i)/(9*(-a)^(11/6)*b^(7/6)) +
 (atan(((((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^
4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^
4) - (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 7
5*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(
11/6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4
*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2
))/(27*a^4) + (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^
3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(
18*(-a)^(11/6)*b^(7/6)))/((((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)
*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^
3*a^3*b^2))/(27*a^4) - (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5
 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6
))))/(18*(-a)^(11/6)*b^(7/6)) - (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x
```

```
^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 2
0*A*B^3*a^3*b^2))/(27*a^4) + (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A
^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*
b^(7/6))))/(18*(-a)^(11/6)*b^(7/6))))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*
1i)/(9*(-a)^(11/6)*b^(7/6)) + (x^(1/2)*(A*b - B*a))/(3*a*b*(a + b*x^3))
```

3.168 $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^2}\,dx$

Optimal. Leaf size=318

$$-\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)}+\frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}}{\sqrt[6]{a}}\right.}{18a^{13/6}b^{5/6}}$$

```
[Out] -1/9*(7*A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/18*(7*A
*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/18*(7
*A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/36*(
7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(13/6)/b
^(5/6)*3^(1/2)+1/36*(7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2
)*x^(1/2))/a^(13/6)/b^(5/6)*3^(1/2)+1/3*(-7*A*b+B*a)/a^2/b/x^(1/2)+1/3*(A*b
-B*a)/a/b/(b*x^3+a)/x^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 331, 335, 301, 648, 632, 210, 642, 211}

$$\frac{(7Ab-aB)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\text{ArcTan}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}-\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

```
[Out] -1/3*(7*A*b - a*B)/(a^2*b*Sqrt[x]) + (A*b - a*B)/(3*a*b*Sqrt[x]*(a + b*x^3)
) + ((7*A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(13
/6)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)]
)/(18*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])
/(9*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6
)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6)*b^(5/6)) + ((7*A*b - a*B)*Log[
a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6
)*b^(5/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^2}\,dx = \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} + \frac{\left(\frac{7Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}(a+bx^3)}\,dx}{3ab}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} - \frac{(7Ab-aB)\int \frac{x^{3/2}}{a+bx^3}\,dx}{6a^2}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} - \frac{(7Ab-aB)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} - \frac{(7Ab-aB)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\right.}{9a^{13/6}b^{2/3}}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} - \frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\text{S}}{}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} - \frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\,\text{l}}{}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} + \frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}} - \frac{(7Ab}{}$$

Mathematica [A]
time = 0.46, size = 184, normalized size = 0.58

$$\frac{\frac{6\sqrt[6]{a}\,(-6aA-7Abx^3+aBx^3)}{\sqrt{x}\,(a+bx^3)} + \frac{2(-7Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}} + \frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}\right)}{b^{5/6}} + \frac{\sqrt{3}\,(7Ab-aB)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{b^{5/6}}}{18a^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

```
[Out] ((6*a^(1/6)*(-6*a*A - 7*A*b*x^3 + a*B*x^3))/(Sqrt[x]*(a + b*x^3)) + (2*(-7*
A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/b^(5/6) + ((7*A*b - a*B)*ArcT
an[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(5/6) + (Sqrt[3]*(7*
A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)]
)/b^(5/6))/(18*a^(13/6))
```

Maple [A]
time = 0.37, size = 216, normalized size = 0.68

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)x^{\frac{5}{2}}}{b\,x^{3}+a}+\left(\frac{7Ab}{6}-\frac{Ba}{6}\right)\left(-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\arctan\left(\cdots\right)}{3b\left(\cdots\right)}\cdots\right.\right.}{a^{2}}$
default	$-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)x^{\frac{5}{2}}}{b\,x^{3}+a}+\left(\frac{7Ab}{6}-\frac{Ba}{6}\right)\left(-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\arctan\left(\cdots\right)}{3b\left(\cdots\right)}\cdots\right.\right.}{a^{2}}$
risch	$-\frac{2A}{a^{2}\sqrt{x}}-\frac{x^{\frac{5}{2}}Ab}{3a^{2}(b\,x^{3}+a)}+\frac{x^{\frac{5}{2}}B}{3a(b\,x^{3}+a)}+\frac{7Ab\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^{3}}-\frac{7A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\cdots\right)}{18a^{2}\left(\frac{a}{b}\right)\cdots}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^2*((1/6*A*b-1/6*B*a)*x^(5/2)/(b*x^3+a)+(7/6*A*b-1/6*B*a)*(-1/12/a*3^(1
/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(
1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)
/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(
a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))))-2*A/
a^2/x^(1/2)
```

Maxima [A]
time = 0.53, size = 240, normalized size = 0.75

$$\frac{(Ba-7\,Ab)x^{3}-6\,Aa}{3\left(a^{2}bx^{\frac{7}{2}}+a^{3}\sqrt{x}\right)}-\frac{(Ba-7\,Ab)\left(\frac{\sqrt{3}\,\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\,\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{36\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 7*A*b)*x^3 - 6*A*a)/(a^2*b*x^(7/2) + a^3*sqrt(x)) - 1/36*(B*a -
 7*A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)
*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(
1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*ar
ctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/
(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(
1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3798 vs. $2(226) = 452$.

time = 2.42, size = 3798, normalized size = 11.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B
^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*
b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^1
6*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 +
12005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^
5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 -
100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*A*
B^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a
^6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*
B^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249
*A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5 -
6860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 + 11
7649*A^6*a^9*b^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*
A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b
^6)/(a^13*b^5))^(2/3))*a^2*b*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*
b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 1
17649*A^6*b^6)/(a^13*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^7*b - 35*A*B^4*a^6*b^2
+ 490*A^2*B^3*a^5*b^3 - 3430*A^3*B^2*a^4*b^4 + 12005*A^4*B*a^3*b^5 - 16807*
A^5*a^2*b^6)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 68
60*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^
6*b^6)/(a^13*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*
a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5
 + 117649*A^6*b^6))/(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*
A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b
^6)) + 4*sqrt(3)*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*
B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a
*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(B^5*a
```

```
^16*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7
+ 12005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*
a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4
- 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*
A*B^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6
*a^6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^
7*B^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 2824752
49*A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5
- 6860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 +
117649*A^6*a^9*b^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 686
0*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6
*b^6)/(a^13*b^5))^(2/3))*a^2*b*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^
4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 +
 117649*A^6*b^6)/(a^13*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^7*b - 35*A*B^4*a^6*b^
2 + 490*A^2*B^3*a^5*b^3 - 3430*A^3*B^2*a^4*b^4 + 12005*A^4*B*a^3*b^5 - 1680
7*A^5*a^2*b^6)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 -
6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*
A^6*b^6)/(a^13*b^5))^(1/6) + sqrt(3)*(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^
4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b
^5 + 117649*A^6*b^6))/(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 686
0*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6
*b^6)) - 2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^
4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 +
 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(a^11*b^4*(-(B^6*a^6 - 42*A*B^5*a^5*b
 + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100
842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5 - 35*A*B^4*a
^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*B*a*b^4 - 168
07*A^5*b^5)*sqrt(x)) + 2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b +
735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(-a^11*b^4*(-(B^6*a^6 -
 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^
2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a
^5 - 35*A*B^4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^
4*B*a*b^4 - 16807*A^5*b^5)*sqrt(x)) + (a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A
*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2
*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log((B^5*a^16
*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 + 1
2005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5
*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 1
00842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2200 vs. $2(299) = 598$.

time = 161.39, size = 2200, normalized size = 6.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*A/(13*x**(13/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b, 0
)), ((-2*A/(13*x**(13/2)) - 2*B/(7*x**(7/2)))/b**2, Eq(a, 0)), ((-2*A/sqrt(
x) + 2*B*x**(5/2)/5)/a**2, Eq(b, 0)), (-14*A*a*b*sqrt(x)*log(sqrt(x) - (-a/
b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)*
*(1/6)) + 14*A*a*b*sqrt(x)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b*sqrt(x)*
(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 7*A*a*b*sqrt(x)*log(
-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)
**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 7*A*a*b*sqrt(x)*log(4*sqrt
(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6)
 + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 14*sqrt(3)*A*a*b*sqrt(x)*atan(2*s
qrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)**(1
/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 14*sqrt(3)*A*a*b*sqrt(x)*atan(
2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)*
*(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 72*A*a*b*(-a/b)**(1/6)/(36*
a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 14*A*
b**2*x**(7/2)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6)
 + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 14*A*b**2*x**(7/2)*log(sqrt(x) +
(-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a
/b)**(1/6)) - 7*A*b**2*x**(7/2)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/
b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)*
*(1/6)) + 7*A*b**2*x**(7/2)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(
1/3))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6
)) - 14*sqrt(3)*A*b**2*x**(7/2)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) -
sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)*
*(1/6)) - 14*sqrt(3)*A*b**2*x**(7/2)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6
)) + sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-
a/b)**(1/6)) - 84*A*b**2*x**3*(-a/b)**(1/6)/(36*a**3*b*sqrt(x)*(-a/b)**(1/6
) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*B*a**2*sqrt(x)*log(sqrt(x) - (
-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/
b)**(1/6)) - 2*B*a**2*sqrt(x)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b*sqrt(
x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + B*a**2*sqrt(x)*lo
g(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/
b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - B*a**2*sqrt(x)*log(4*sqr
t(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6
) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*sqrt(3)*B*a**2*sqrt(x)*atan(2*
sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)**(
1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*sqrt(3)*B*a**2*sqrt(x)*atan
(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)
**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*B*a*b*x**(7/2)*log(sqrt(
x) - (-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2
)*(-a/b)**(1/6)) - 2*B*a*b*x**(7/2)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b
```

```
*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + B*a*b*x**(7
/2)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x
)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - B*a*b*x**(7/2)*log
(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)
**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*sqrt(3)*B*a*b*x**(7/2)*a
tan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a
/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*sqrt(3)*B*a*b*x**(7/2
)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**3*b*sqrt(x)*
(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 12*B*a*b*x**3*(-a/b)
**(1/6)/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1
/6)), True))
```

Giac [A]
time = 1.09, size = 307, normalized size = 0.97

$$\frac{Bax^3 - 7\,Abx^3 - 6\,Aa}{3\left(bx^{\frac{7}{2}} + a\sqrt{x}\right)a^2} - \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba - 7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b^5} + \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba - 7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b^5} + \frac{\left((ab^5)^{\frac{5}{6}}\,Ba - 7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}} + 2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^3b^5} + \frac{\left((ab^5)^{\frac{5}{6}}\,Ba - 7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}} - 2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^3b^5} + \frac{\left((ab^5)^{\frac{5}{6}}\,Ba - 7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a*x^3 - 7*A*b*x^3 - 6*A*a)/((b*x^(7/2) + a*sqrt(x))*a^2) - 1/36*sqrt
(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/
6) + x + (a/b)^(1/3))/(a^3*b^5) + 1/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^
5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^5)
 + 1/18*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/
6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^5) + 1/18*((a*b^5)^(5/6)*B*a - 7*(a*b^5
)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^
5) + 1/9*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/
6))/(a^3*b^5)
```

Mupad [B]
time = 2.91, size = 1757, normalized size = 5.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x)
```

```
[Out] (atan((((7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*
a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17
*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/
6)))*1i)/((-a)^(13/3)*b^(5/3)) + ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*
B^3*a^18*b^3 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b
 - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5
))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/(((7*A*b - B*a)^2
*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*
```

a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*
b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5
/3)) - ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*
a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17
*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/
6))))/((-a)^(13/3)*b^(5/3))))*(7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6)) - (
(2*A)/a + (x^3*(7*A*b - B*a))/(3*a^2))/(a*x^(1/2) + b*x^(7/2)) + (atan(((((
3^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^
6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 -
 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*
A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)) + (((3
^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3
 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 -
1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A
*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/((((3^
(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6
- 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1
/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*
B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3)) - (((3^(1/2
)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 17
01*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*
(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^
18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3))))*((3^(1/2)*1i)
/2 - 1/2)*(7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6)) + (atan(((((3^(1/2)*1i)
/2 + 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*
B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*
b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^
5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)) + (((3^(1/2)*1i)/
2 + 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B
^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b
 - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5
))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/((((3^(1/2)*1i)/2
 + 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^
2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b
- B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5)
)/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3)) - (((3^(1/2)*1i)/2 + 1
/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*a^
17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*
a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5
832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(
7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6))

3.169 $\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^2}\,dx$

Optimal. Leaf size=96

$$\frac{-3Ab+aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}$$

```
[Out] 1/3*(-3*A*b+B*a)/a^2/b/x^(3/2)+1/3*(A*b-B*a)/a/b/x^(3/2)/(b*x^3+a)-1/3*(3*A
*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.227$, Rules used = {468, 331, 335, 281, 211}

$$-\frac{(3Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}-\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

```
[Out] -1/3*(3*A*b - a*B)/(a^2*b*x^(3/2)) + (A*b - a*B)/(3*a*b*x^(3/2)*(a + b*x^3)
) - ((3*A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^2}\,dx &= \frac{Ab-aB}{3abx^{3/2}(a+bx^3)}+\frac{\left(\frac{9Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}(a+bx^3)}\,dx}{3ab}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}(a+bx^3)}-\frac{(3Ab-aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2a^2}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}(a+bx^3)}-\frac{(3Ab-aB)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}(a+bx^3)}-\frac{(3Ab-aB)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3a^2}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}(a+bx^3)}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 79, normalized size = 0.82

$$
\frac{-2aA-3Abx^3+aBx^3}{3a^2x^{3/2}(a+bx^3)}+\frac{(-3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

```
[Out] (-2*a*A - 3*A*b*x^3 + a*B*x^3)/(3*a^2*x^(3/2)*(a + b*x^3)) + ((-3*A*b + a*B
)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])
```

Maple [A]
time = 0.32, size = 66, normalized size = 0.69

method	result	size
derivativedivides	$-\frac{2A}{3a^{2}x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x^{\frac{3}{2}}}{b\,x^{3}+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{3a^{2}}$	66
default	$-\frac{2A}{3a^{2}x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x^{\frac{3}{2}}}{b\,x^{3}+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{3a^{2}}$	66
risch	$-\frac{2A}{3a^{2}x^{\frac{3}{2}}}-\frac{x^{\frac{3}{2}}Ab}{3a^{2}(b\,x^{3}+a)}+\frac{x^{\frac{3}{2}}B}{3a(b\,x^{3}+a)}-\frac{\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)Ab}{a^{2}\sqrt{ab}}+\frac{\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)B}{3a\sqrt{ab}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*A/a^2/x^(3/2)-2/3/a^2*((1/2*A*b-1/2*B*a)*x^(3/2)/(b*x^3+a)+1/2*(3*A*b-
B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 67, normalized size = 0.70

$$\frac{(Ba-3\,Ab)x^{3}-2\,Aa}{3\left(a^{2}bx^{\frac{9}{2}}+a^{3}x^{\frac{3}{2}}\right)}+\frac{(Ba-3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 3*A*b)*x^3 - 2*A*a)/(a^2*b*x^(9/2) + a^3*x^(3/2)) + 1/3*(B*a -
3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 1.29, size = 232, normalized size = 2.42

$$\left[\frac{((Bab-3\,Ab^{2})x^{5}+(Ba^{2}-3\,Aab)x^{2})\sqrt{-ab}\,\log\left(\frac{bx^{3}+2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^{3}+a}\right)-2\,(2\,Aa^{2}b-(Ba^{2}b-3\,Aab^{2})x^{3})\sqrt{x}}{6\,(a^{3}b^{2}x^{5}+a^{4}bx^{2})},\frac{((Bab-3\,Ab^{2})x^{5}+(Ba^{2}-3\,Aab)x^{2})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)-(2\,Aa^{2}b-(Ba^{2}b-3\,Aab^{2})x^{3})\sqrt{x}}{3\,(a^{3}b^{2}x^{5}+a^{4}bx^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((B*a*b - 3*A*b^2)*x^5 + (B*a^2 - 3*A*a*b)*x^2)*sqrt(-a*b)*log((b*x^3
 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2*(2*A*a^2*b - (B*a^2*b - 3*A*a
*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2), 1/3*(((B*a*b - 3*A*b^2)*x^5
+ (B*a^2 - 3*A*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*A*a^2*b
 - (B*a^2*b - 3*A*a*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.96, size = 66, normalized size = 0.69

$$\frac{(Ba - 3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a^2} + \frac{Bax^3 - 3\,Abx^3 - 2\,Aa}{3\left(bx^{\frac{9}{2}} + ax^{\frac{3}{2}}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a - 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(B*a*x^
3 - 3*A*b*x^3 - 2*A*a)/((b*x^(9/2) + a*x^(3/2))*a^2)
```

Mupad [**B**]
time = 0.15, size = 139, normalized size = 1.45

$$-\frac{2\,A\,a^{3/2}\,\sqrt{b} - B\,a^2\,x^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right) + 3\,A\,b^2\,x^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right) + 3\,A\,\sqrt{a}\,b^{3/2}\,x^3 - B\,a^{3/2}\,\sqrt{b}\,x^3 + 3\,A\,a\,b\,x^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right) - B\,a\,b\,x^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\,a^{7/2}\,\sqrt{b}\,x^{3/2} + 3\,a^{5/2}\,b^{3/2}\,x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x)
```

```
[Out] -(2*A*a^(3/2)*b^(1/2) - B*a^2*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*A
*b^2*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*A*a^(1/2)*b^(3/2)*x^3 - B*
a^(3/2)*b^(1/2)*x^3 + 3*A*a*b*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) - B*a
*b*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)))/(3*a^(7/2)*b^(1/2)*x^(3/2) + 3*
a^(5/2)*b^(3/2)*x^(9/2))
```

3.170 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx$

Optimal. Leaf size=318

$$-\frac{11Ab-5aB}{15a^2bx^{5/2}}+\frac{Ab-aB}{3abx^{5/2}(a+bx^3)}+\frac{(11Ab-5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\tan^{-1}\left(\sqrt{3}+\right.}{18a^{17/6}\sqrt[6]{b}}$$

```
[Out] 1/15*(-11*A*b+5*B*a)/a^2/b/x^(5/2)+1/3*(A*b-B*a)/a/b/x^(5/2)/(b*x^3+a)-1/9*
(11*A*b-5*B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)-1/18*(11*A*
b-5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)-1/18*(
11*A*b-5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)+1/
36*(11*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(
17/6)/b^(1/6)*3^(1/2)-1/36*(11*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1
/6)*3^(1/2)*x^(1/2))/a^(17/6)/b^(1/6)*3^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 331, 335, 215, 648, 632, 210, 642, 211}

$$\frac{(11Ab-5aB)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\text{ArcTan}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}}+\frac{(11Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}-\frac{11Ab-5aB}{15a^2bx^{5/2}}+\frac{Ab-aB}{3abx^{5/2}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

```
[Out] -1/15*(11*A*b - 5*a*B)/(a^2*b*x^(5/2)) + (A*b - a*B)/(3*a*b*x^(5/2)*(a + b*
x^3)) + ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(1
8*a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x]
)/a^(1/6)])/(18*a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[
x])/a^(1/6)])/(9*a^(17/6)*b^(1/6)) + ((11*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3
]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(17/6)*b^(1/6)) - ((1
1*A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/
(12*Sqrt[3]*a^(17/6)*b^(1/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} + \frac{\left(\frac{11Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}(a+bx^3)}\,dx}{3ab}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\int \frac{1}{\sqrt{x}\,(a+bx^3)}\,dx}{6a^2}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}\ldots}\right.}{9a^{17/6}}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5\ldots}{}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} + \frac{(11Ab-5\ldots}{}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} + \frac{(11Ab-5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(1\ldots}{}$$

Mathematica [A]
time = 0.43, size = 188, normalized size = 0.59

$$\frac{\frac{6a^{5/6}\left(-6aA-11Abx^3+5aBx^3\right)}{x^{5/2}(a+bx^3)} + \frac{10(-11Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}} + \frac{5(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{\sqrt[6]{b}} + \frac{5\sqrt{3}(-11Ab+5aB)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{\sqrt[6]{b}}}{90a^{17/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

```
[Out] ((6*a^(5/6)*(-6*a*A - 11*A*b*x^3 + 5*a*B*x^3))/(x^(5/2)*(a + b*x^3)) + (10*
(-11*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/b^(1/6) + (5*(11*A*b -
```

```
5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(1/6) +
(5*Sqrt[3]*(-11*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(
1/3) + b^(1/3)*x)])/b^(1/6))/(90*a^(17/6))
```

Maple [A]

time = 0.39, size = 217, normalized size = 0.68

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)\sqrt{x}}{b\,x^{3}+a}+\frac{(11Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\mathrm{arct}}{}\right.}{}\right.}{a^{2}}$
default	$-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)\sqrt{x}}{b\,x^{3}+a}+\frac{(11Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\mathrm{arct}}{}\right.}{}\right.}{a^{2}}$
risch	$-\frac{2A}{5a^{2}x^{\frac{5}{2}}}-\frac{\sqrt{x}\,Ab}{3a^{2}(b\,x^{3}+a)}+\frac{\sqrt{x}\,B}{3a(b\,x^{3}+a)}-\frac{11Ab\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^{3}}-\frac{11Ab\left(\frac{a}{b}\right)^{\frac{1}{6}}\mathrm{arcta}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^2*((1/6*A*b-1/6*B*a)*x^(1/2)/(b*x^3+a)+1/6*(11*A*b-5*B*a)*(1/3/a*(a/b)
^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a
/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2
)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+
(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))))-2/5*
A/a^2/x^(5/2)
```

Maxima [A]

time = 0.52, size = 312, normalized size = 0.98

$$\frac{(5\,Ba-11\,Ab)x^{3}-6\,Aa}{15\left(a^{2}bx^{\frac{11}{2}}+a^{3}x^{\frac{5}{2}}\right)}+\frac{\frac{\sqrt{3}\,(5\,Ba-11\,Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(5\,Ba-11\,Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(5\,Bab^{\frac{1}{3}}-11\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(5\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}-11\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(5\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}-11\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{36\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*((5*B*a - 11*A*b)*x^3 - 6*A*a)/(a^2*b*x^(11/2) + a^3*x^(5/2)) + 1/36*(
sqrt(3)*(5*B*a - 11*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 11*A*b)*log(-sqrt(3)*a^(1/6)*
b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(1/3)
 - 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(
1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*A*a^(1/3)*b^(4/3)
)*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)
))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*A*a^(1/3
)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/
3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. $2(232) = 464$.
time = 2.46, size = 2584, normalized size = 8.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/180*(20*sqrt(3)*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^
5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a
^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*arctan(1/3*
(2*sqrt(3)*sqrt(a^6*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4
*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*
B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 12
1*A^2*b^2)*x + (5*B*a^4 - 11*A*a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B
^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*
B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6))*a^14*
b*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500
*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*
A^6*b^6)/(a^17*b))^(5/6) + 2*sqrt(3)*(5*B*a^15*b - 11*A*a^14*b^2)*sqrt(x)*(
-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^
3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6
*b^6)/(a^17*b))^(5/6) - sqrt(3)*(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 11343
75*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 48
31530*A^5*B*a*b^5 + 1771561*A^6*b^6))/(15625*B^6*a^6 - 206250*A*B^5*a^5*b +
 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^
4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)) + 20*sqrt(3)*(a^2*b*x^6 + a^3*x
^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 33275
00*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 177156
1*A^6*b^6)/(a^17*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^6*(-(15625*B^6*a^6
```

- 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 121*A^2*b^2)*x - (5*B*a^4 - 11*A*a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6))*a^14*b*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(5/6) + 2*sqrt(3)*(5*B*a^15*b - 11*A*a^14*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(5/6) + sqrt(3)*(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6))/(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)) - 5*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^6*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 121*A^2*b^2)*x + (5*B*a^4 - 11*A*a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)) + 5*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^6*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 121*A^2*b^2)*x - (5*B*a^4 - 11*A*a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)) + 10*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^3*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6) - (5*B*a - 11*A*b)*sqrt(x)) - 10*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(-a^3*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6) - (5*B*a - 11*A*b)*sqrt(x)) - 12*((5*B*a - 11*A*b)*x^3 - 6*A*a)*sqrt(x))/(a^2*b*x^6 + a^3*x^3)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.15, size = 313, normalized size = 0.98

$$\frac{\sqrt{3}\left(5\left(ab^{5}\right)^{\frac{1}{6}}Ba-11\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^{3}b}-\frac{\sqrt{3}\left(5\left(ab^{5}\right)^{\frac{1}{6}}Ba-11\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^{3}b}+\frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\left(bx^{3}+a\right)a^{2}}+\frac{\left(5\left(ab^{5}\right)^{\frac{1}{6}}Ba-11\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^{3}b}+\frac{\left(5\left(ab^{5}\right)^{\frac{1}{6}}Ba-11\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^{3}b}+\frac{\left(5\left(ab^{5}\right)^{\frac{1}{6}}Ba-11\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^{3}b}-\frac{2\,A}{5\,a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b) - 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B
*a - 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/
3))/(a^3*b) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a^2) + 1/18*(5*(
a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sq
rt(x))/(a/b)^(1/6))/(a^3*b) + 1/18*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*
A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b) + 1/9*(
5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^
3*b) - 2/5*A/(a^2*x^(5/2))
```

Mupad [B]
time = 2.96, size = 2080, normalized size = 6.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x)
```

```
[Out] - ((2*A)/(5*a) + (x^3*(11*A*b - 5*B*a))/(15*a^2))/(a*x^(5/2) + b*x^(11/2))
- (atan((((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*
A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - ((11
*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^
2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b -
5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)) + ((x^(1/2)*(21346578*A^4*a^10*b^9 + 91
1250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38
811960*A^3*B*a^11*b^8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500
*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a
```

)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)))/(((x^(1/2
)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7
- 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - ((11*A*b - 5*B*a)*(34
930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 4763
2860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^
(17/6)*b^(1/6)) - ((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 +
26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^
8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 2165
1300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(
11*A*b - 5*B*a))/(18*(-a)^(17/6)*b^(1/6))))*(11*A*b - 5*B*a)*1i)/(9*(-a)^(1
7/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*
b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*
b^6 - 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(
34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47
632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*
(-a)^(17/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*
b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*
b^6 - 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(
34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47
632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*
(-a)^(17/6)*b^(1/6)))/((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*
b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*
b^6 - 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(
34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47
632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a
)^(17/6)*b^(1/6)) - (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
 - 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(349
30764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632
860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(
17/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*1i)/(9*(-a)^(17/6)
*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
+ 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
- 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493
0764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 476328
60*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)
^(17/6)*b^(1/6)) + (((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
+ 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
- 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493
0764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 476328
60*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)
^(17/6)*b^(1/6)))/((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
+ 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
- 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493
0764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 476328

```
60*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(1
7/6)*b^(1/6)) - ((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9 + 9
11250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 3
8811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493076
4*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*
A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(17/6
)*b^(1/6))))*((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*1i)/(9*(-a)^(17/6)*b^(
1/6))
```

3.171 $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=104

$$\frac{(Ab-aB)x^{9/2}}{6ab\,(a+bx^3)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\,(a+bx^3)}+\frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

```
[Out] 1/6*(A*b-B*a)*x^(9/2)/a/b/(b*x^3+a)^2-1/12*(A*b+3*B*a)*x^(3/2)/a/b^2/(b*x^3
+a)+1/12*(A*b+3*B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {468, 294, 335, 281, 211}

$$\frac{(3aB+Ab)\text{ArcTan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}-\frac{x^{3/2}(3aB+Ab)}{12ab^2\,(a+bx^3)}+\frac{x^{9/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(9/2))/(6*a*b*(a + b*x^3)^2) - ((A*b + 3*a*B)*x^(3/2))/(12*a
*b^2*(a + b*x^3)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(3/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{(Ab-aB)x^{9/2}}{6ab\,(a+bx^3)^2} + \frac{\left(\frac{3Ab}{2}+\frac{9aB}{2}\right)\int \frac{x^{7/2}}{(a+bx^3)^2}\,dx}{6ab} \\
&= \frac{(Ab-aB)x^{9/2}}{6ab\,(a+bx^3)^2} - \frac{(Ab+3aB)x^{3/2}}{12ab^2\,(a+bx^3)} + \frac{(Ab+3aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8ab^2} \\
&= \frac{(Ab-aB)x^{9/2}}{6ab\,(a+bx^3)^2} - \frac{(Ab+3aB)x^{3/2}}{12ab^2\,(a+bx^3)} + \frac{(Ab+3aB)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{4ab^2} \\
&= \frac{(Ab-aB)x^{9/2}}{6ab\,(a+bx^3)^2} - \frac{(Ab+3aB)x^{3/2}}{12ab^2\,(a+bx^3)} + \frac{(Ab+3aB)\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{12ab^2} \\
&= \frac{(Ab-aB)x^{9/2}}{6ab\,(a+bx^3)^2} - \frac{(Ab+3aB)x^{3/2}}{12ab^2\,(a+bx^3)} + \frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 92, normalized size = 0.88

$$-\frac{x^{3/2}(aAb+3a^2B-Ab^2x^3+5abBx^3)}{12ab^2\,(a+bx^3)^2}+\frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/12*(x^(3/2)*(a*A*b + 3*a^2*B - A*b^2*x^3 + 5*a*b*B*x^3))/(a*b^2*(a + b*x
^3)^2) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(3/2)*b^(5
/2))
```

Maple [A]
time = 0.29, size = 81, normalized size = 0.78

method	result	size
derivativedivides	$\frac{\frac{(Ab-5Ba)x^{\frac{9}{2}}}{12ab}-\frac{(Ab+3Ba)x^{\frac{3}{2}}}{12b^2}}{(b\,x^3+a)^2}+\frac{(Ab+3Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12b^2a\sqrt{ab}}$	81
default	$\frac{\frac{(Ab-5Ba)x^{\frac{9}{2}}}{12ab}-\frac{(Ab+3Ba)x^{\frac{3}{2}}}{12b^2}}{(b\,x^3+a)^2}+\frac{(Ab+3Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12b^2a\sqrt{ab}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1/8*(A*b-5*B*a)/a/b*x^(9/2)-1/8*(A*b+3*B*a)/b^2*x^(3/2))/(b*x^3+a)^2+1
/12*(A*b+3*B*a)/b^2/a/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 96, normalized size = 0.92

$$-\frac{(5\,Bab-Ab^2)x^{\frac{9}{2}}+(3\,Ba^2+Aab)x^{\frac{3}{2}}}{12\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)}+\frac{(3\,Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*((5*B*a*b - A*b^2)*x^(9/2) + (3*B*a^2 + A*a*b)*x^(3/2))/(a*b^4*x^6 +
2*a^2*b^3*x^3 + a^3*b^2) + 1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(
sqrt(a*b)*a*b^2)
```

Fricas [A]
time = 2.02, size = 314, normalized size = 3.02

$$\left[-\frac{((3\,Bab^2+Ab^3)x^6+3\,Ba^3+Aa^2b+2\,(3\,Ba^2b+Aab^2)x^3)\sqrt{-ab}\,\log\left(\frac{bx^3-2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\,((5\,Ba^2b^2-Aab^3)x^4+(3\,Ba^3b+Aa^2b^2)x)\sqrt{x}}{24\,(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3)},\ \frac{((3\,Bab^2+Ab^3)x^6+3\,Ba^3+Aa^2b+2\,(3\,Ba^2b+Aab^2)x^3)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)-((5\,Ba^2b^2-Aab^3)x^4+(3\,Ba^3b+Aa^2b^2)x)\sqrt{x}}{12\,(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b
^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2
```

```
*((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a^2*b^
5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), 1/12*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3
+ A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)
/a) - ((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a
^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.97, size = 84, normalized size = 0.81

$$\frac{(3\,Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,ab^2}-\frac{5\,Babx^{\frac{9}{2}}-Ab^2x^{\frac{9}{2}}+3\,Ba^2x^{\frac{3}{2}}+Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/12*(5*
B*a*b*x^(9/2) - A*b^2*x^(9/2) + 3*B*a^2*x^(3/2) + A*a*b*x^(3/2))/((b*x^3 +
a)^2*a*b^2)
```

Mupad [**B**]
time = 2.76, size = 133, normalized size = 1.28

$$\frac{\mathrm{atan}\left(\frac{9\,b^{3/2}\,x^{3/2}\,\left(A^2\,b^2+6\,A\,B\,a\,b+9\,B^2\,a^2\right)}{\sqrt{a}\,\left(9\,A\,b^2+27\,B\,a\,b\right)\,\left(A\,b+3\,B\,a\right)}\right)\,\left(A\,b+3\,B\,a\right)}{12\,a^{3/2}\,b^{5/2}}-\frac{\frac{x^{3/2}\,\left(A\,b+3\,B\,a\right)}{12\,b^2}-\frac{x^{9/2}\,\left(A\,b-5\,B\,a\right)}{12\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (atan((9*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 + 6*A*B*a*b))/(a^(1/2)*(9*A*b
^2 + 27*B*a*b)*(A*b + 3*B*a)))*(A*b + 3*B*a))/(12*a^(3/2)*b^(5/2)) - ((x^(3
/2)*(A*b + 3*B*a))/(12*b^2) - (x^(9/2)*(A*b - 5*B*a))/(12*a*b))/(a^2 + b^2*
x^6 + 2*a*b*x^3)
```

3.172 $\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=327

$$\frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2}-\frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)}-\frac{(5Ab+7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}}+\frac{(5Ab+7aB)\tan^{-1}\left(\sqrt{3}+\right.}{216a^{11/6}b^{13/6}}$$

```
[Out] 1/6*(A*b-B*a)*x^(7/2)/a/b/(b*x^3+a)^2+1/108*(5*A*b+7*B*a)*arctan(b^(1/6)*x^
(1/2)/a^(1/6))/a^(11/6)/b^(13/6)+1/216*(5*A*b+7*B*a)*arctan(-3^(1/2)+2*b^(1
/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(13/6)+1/216*(5*A*b+7*B*a)*arctan(3^(1/2)+2
*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(13/6)-1/432*(5*A*b+7*B*a)*ln(a^(1/3)+
b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11/6)/b^(13/6)*3^(1/2)+1/432*
(5*A*b+7*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11/6
)/b^(13/6)*3^(1/2)-1/36*(5*A*b+7*B*a)*x^(1/2)/a/b^2/(b*x^3+a)
```

Rubi [A]

time = 0.36, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 294, 335, 215, 648, 632, 210, 642, 211}

$$-\frac{(7aB+5Ab)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\text{ArcTan}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}}-\frac{(7aB+5Ab)\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{144\sqrt{3}\,a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{144\sqrt{3}\,a^{11/6}b^{13/6}}-\frac{\sqrt{x}\,(7aB+5Ab)}{36ab^2\,(a+bx^3)}+\frac{x^{7/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(7/2))/(6*a*b*(a + b*x^3)^2) - ((5*A*b + 7*a*B)*Sqrt[x])/(36
*a*b^2*(a + b*x^3)) - ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(11/6)*b^(13/6)) - ((5*A*b + 7*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11
/6)*b^(13/6)) + ((5*A*b + 7*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11/6)*b^(13/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} + \frac{\left(\frac{5Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} + \frac{(5Ab+7aB)\int \frac{1}{\sqrt{x}\,(a+bx^3)}\,dx}{72ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} + \frac{(5Ab+7aB)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{36ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} + \frac{(5Ab+7aB)\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{\ldots}}\right.}{108a^{11/6}b^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} + \frac{(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7\ldots}{\ldots}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} + \frac{(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7\ldots}{\ldots}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} - \frac{(5Ab+7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}} + \ldots$$

Mathematica [A]
time = 0.60, size = 192, normalized size = 0.59

$$\frac{-\frac{6a^{5/6}\sqrt[6]{b}\,\sqrt{x}\,(7a^2B-Ab^2x^3+ab(5A+13Bx^3))}{(a+bx^3)^2}+2(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)-(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}\right)+\sqrt{3}\,(5Ab+7aB)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{216a^{11/6}b^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-6*a^(5/6)*b^(1/6)*Sqrt[x]*(7*a^2*B - A*b^2*x^3 + a*b*(5*A + 13*B*x^3)))/
(a + b*x^3)^2 + 2*(5*A*b + 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (5*A*
```

```
b + 7*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + Sqrt[3
]*(5*A*b + 7*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1
/3)*x)])/(216*a^(11/6)*b^(13/6))
```

Maple [A]
time = 0.29, size = 234, normalized size = 0.72

method	result
derivativedivides	$\frac{\frac{(Ab-13Ba)x^{\frac{7}{2}}}{36ab}-\frac{(5Ab+7Ba)\sqrt{x}}{36b^2}}{(bx^3+a)^2}+\frac{(5Ab+7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}\cdots\right.}{\cdots}$
default	$\frac{\frac{(Ab-13Ba)x^{\frac{7}{2}}}{36ab}-\frac{(5Ab+7Ba)\sqrt{x}}{36b^2}}{(bx^3+a)^2}+\frac{(5Ab+7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/72*(A*b-13*B*a)/a/b*x^(7/2)-1/72*(5*A*b+7*B*a)/b^2*x^(1/2))/(b*x^3+a)^
2+1/36*(5*A*b+7*B*a)/b^2/a*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1
/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6
/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^
(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arcta
n(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

Maxima [A]
time = 0.51, size = 341, normalized size = 1.04

$$-\frac{(13\,Bab-Ab^2)x^{\frac{7}{2}}+(7\,Ba^2+5\,Aab)\sqrt{x}}{36\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)}+\frac{\frac{\sqrt{3}\,(7\,Ba+5\,Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(7\,Ba+5\,Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(7\,Bab^{\frac{1}{3}}+5\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(7\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}+5\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(7\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}+5\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{432\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/36*((13*B*a*b - A*b^2)*x^(7/2) + (7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b^4*x^6
 + 2*a^2*b^3*x^3 + a^3*b^2) + 1/432*(sqrt(3)*(7*B*a + 5*A*b)*log(sqrt(3)*a^
(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(7
*B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(
a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/s
qrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(7*B*a^(4
/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1
```

```
/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(
7*B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6)
 - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3
))))/(a*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2714 vs. $2(241) = 482$.
time = 2.21, size = 2714, normalized size = 8.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432*(4*sqrt(3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 +
504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 4593
75*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)
*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b
+ 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4
+ 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70
*A*B*a*b + 25*A^2*b^2)*x + (7*B*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(1/6))*a^9*b^11*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*
a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*
b^5 + 15625*A^6*b^6)/(a^11*b^13))^(5/6) - 2*sqrt(3)*(7*B*a^10*b^11 + 5*A*a^
9*b^12)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
 + 15625*A^6*b^6)/(a^11*b^13))^(5/6) + sqrt(3)*(117649*B^6*a^6 + 504210*A*B
^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2
*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6))/(117649*B^6*a^6 + 504210*A*
B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^
2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)) + 4*sqrt(3)*(a*b^4*x^6 + 2
*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2
*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5
*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4
*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 8575
00*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^
6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^2)*x - (7*B
*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 90
0375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 13
1250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6))*a^9*b^11*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(5/6) - 2*sqrt(3)*(7*B*a^10*b^11 + 5*A*a^9*b^12)*sqrt(x)*(-(117649*B^6*a
```

```
^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 +
 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^
(5/6) - sqrt(3)*(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b
^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 +
 15625*A^6*b^6))/(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*
b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
+ 15625*A^6*b^6)) + (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6
 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 4
59375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1
/6)*log(a^4*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
 + 15625*A^6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^
2)*x + (7*B*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5
*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a
^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)) - (a*b^4*x
^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 9003
75*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 1312
50*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log(a^4*b^4*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^2)*x - (7*B*a^3*b^2 + 5*A*a^
2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*
b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
+ 15625*A^6*b^6)/(a^11*b^13))^(1/6)) + 2*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b
^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 85750
0*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6
*b^6)/(a^11*b^13))^(1/6)*log(a^2*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b
 + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4
 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)
*sqrt(x)) - 2*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504
210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*
A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*lo
g(-a^2*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2
+ 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15
625*A^6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)*sqrt(x)) - 12*((13*B*a*b
- A*b^2)*x^3 + 7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**3,x)
[Out] Timed out
```

Giac [A]
time = 0.64, size = 328, normalized size = 1.00

$$\frac{\sqrt{3}\left(7\left(ab^{5}\right)^{\frac{1}{6}}Ba+5\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^{2}b^{3}}-\frac{\sqrt{3}\left(7\left(ab^{5}\right)^{\frac{1}{6}}Ba+5\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^{2}b^{3}}+\frac{\left(7\left(ab^{5}\right)^{\frac{1}{6}}Ba+5\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^{2}b^{3}}+\frac{\left(7\left(ab^{5}\right)^{\frac{1}{6}}Ba+5\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^{2}b^{3}}+\frac{\left(7\left(ab^{5}\right)^{\frac{1}{6}}Ba+5\left(ab^{5}\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^{2}b^{3}}-\frac{13\,Babx^{\frac{7}{2}}-Ab^{2}x^{\frac{7}{2}}+7\,Ba^{2}\sqrt{x}+5\,Aab\sqrt{x}}{36\left(bx^{3}+a\right)^{2}ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/432*sqrt(3)*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^3) - 1/432*sqrt(3)*(7*(a*b^5)^(1/6
)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(
1/3))/(a^2*b^3) + 1/216*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(
(sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^3) + 1/216*(7*(a*b^5)
^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))
/(a/b)^(1/6))/(a^2*b^3) + 1/108*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)
*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^3) - 1/36*(13*B*a*b*x^(7/2) - A*b^2*x^(
7/2) + 7*B*a^2*sqrt(x) + 5*A*a*b*sqrt(x))/((b*x^3 + a)^2*a*b^2)
```

Mupad [B]
time = 2.98, size = 1944, normalized size = 5.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (atan((((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 52
5*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 240
1*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(2
79936*a^4*b^3))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)) - ((((5*A*b
+ 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(
279936*(-a)^(23/6)*b^(19/6)) + (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*
A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(
5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/(((((5*A*b + 7*B*a)*(125*A^3
*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6
)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 +
 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b + 7*B*a))/(
216*(-a)^(11/6)*b^(13/6)) + ((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 +
 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)) + (x^(1/
2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b +
3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(1
3/6))))*(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)*b^(13/6)) - ((x^(1/2)*(5*A*b +
 7*B*a))/(36*b^2) - (x^(7/2)*(A*b - 13*B*a))/(36*a*b))/(a^2 + b^2*x^6 + 2*a
*b*x^3) + (atan(((((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*
a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*
```

```
a^4*b^3) - (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a
^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*
A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)) + (((3^(1/2)*1i)/2 - 1/2)*((x^(
1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b
+ 3500*A^3*B*a*b^3))/(279936*a^4*b^3) + (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*
B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(2799
36*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/(
(((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^
2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(
1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^
2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(21
6*(-a)^(11/6)*b^(13/6)) - (((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 +
2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))
/(279936*a^4*b^3) + (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 +
343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19
/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(13/6))))*((3^(1/2)*1i)/2 - 1/2)*
(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)*b^(13/6)) + (atan(((((3^(1/2)*1i)/2 +
1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*
B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(1/2)*1i)/2 + 1/2)*(
5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b
^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^
(13/6)) + (((3^(1/2)*1i)/2 + 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7
350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3
) + (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 73
5*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7
*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/((((3^(1/2)*1i)/2 + 1/2)*((x^(1/2)*(6
25*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*
A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*(1
25*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)
^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(13/6)) - (((3^(1/2)
*1i)/2 + 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2
+ 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) + (((3^(1/2)*1i)/2
 + 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*
A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11
/6)*b^(13/6))))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)
*b^(13/6))
```

3.173 $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=327

$$\frac{(Ab-aB)x^{5/2}}{6ab\,(a+bx^3)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\,(a+bx^3)}-\frac{(7Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}}+\frac{(7Ab+5aB)\tan^{-1}\left(\sqrt{3}+\right.}{216a^{13/6}b^{11/6}}$$

```
[Out] 1/6*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)^2+1/36*(7*A*b+5*B*a)*x^(5/2)/a^2/b/(b*x
^3+a)+1/108*(7*A*b+5*B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(11/6)
+1/216*(7*A*b+5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^
(11/6)+1/216*(7*A*b+5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/
6)/b^(11/6)+1/432*(7*A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2
)*x^(1/2))/a^(13/6)/b^(11/6)*3^(1/2)-1/432*(7*A*b+5*B*a)*ln(a^(1/3)+b^(1/3)
*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(13/6)/b^(11/6)*3^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 296, 335, 301, 648, 632, 210, 642, 211}

$$-\frac{(5aB+7Ab)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}}+\frac{(5aB+7Ab)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{13/6}b^{11/6}}+\frac{(5aB+7Ab)\text{ArcTan}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}}+\frac{(5aB+7Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}+\frac{x^{5/2}(5aB+7Ab)}{36a^2b\,(a+bx^3)}+\frac{x^{5/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(6*a*b*(a + b*x^3)^2) + ((7*A*b + 5*a*B)*x^(5/2))/(36
*a^2*b*(a + b*x^3)) - ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13
/6)*b^(11/6)) - ((7*A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13/6)*b^(11/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{(Ab-aB)x^{5/2}}{6ab\,(a+bx^3)^2} + \frac{\left(\frac{7Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{(a+bx^3)^2}\,dx}{6ab} \\
&= \frac{(Ab-aB)x^{5/2}}{6ab\,(a+bx^3)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\,(a+bx^3)} + \frac{(7Ab+5aB)\int \frac{x^{3/2}}{a+bx^3}\,dx}{72a^2b} \\
&= \frac{(Ab-aB)x^{5/2}}{6ab\,(a+bx^3)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\,(a+bx^3)} + \frac{(7Ab+5aB)\mathrm{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{36a^2b} \\
&= \frac{(Ab-aB)x^{5/2}}{6ab\,(a+bx^3)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\,(a+bx^3)} + \frac{(7Ab+5aB)\mathrm{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{\cdots}}\cdots\right.}{108a^{13/6}b^{5/3}} \\
&= \frac{(Ab-aB)x^{5/2}}{6ab\,(a+bx^3)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\,(a+bx^3)} + \frac{(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(7Ab+5a\cdots}{\cdots} \\
&= \frac{(Ab-aB)x^{5/2}}{6ab\,(a+bx^3)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\,(a+bx^3)} + \frac{(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(7Ab+5a\cdots}{\cdots} \\
&= \frac{(Ab-aB)x^{5/2}}{6ab\,(a+bx^3)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\,(a+bx^3)} - \frac{(7Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}} + \frac{(7\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.58, size = 193, normalized size = 0.59

$$\frac{\frac{6\sqrt[6]{a}\,b^{5/6}x^{5/2}\left(-a^2B+7Ab^2x^3+ab(13A+5Bx^3)\right)}{(a+bx^3)^2}+2(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)-(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}\right)-\sqrt{3}\,(7Ab+5aB)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{216a^{13/6}b^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((6*a^(1/6)*b^(5/6)*x^(5/2)*(-(a^2*B) + 7*A*b^2*x^3 + a*b*(13*A + 5*B*x^3))
)/(a + b*x^3)^2 + 2*(7*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (7*
A*b + 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] - Sqrt
[3]*(7*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^
(1/3)*x)])/(216*a^(13/6)*b^(11/6))
```

Maple [A]
time = 0.29, size = 235, normalized size = 0.72

method	result
derivativedivides	$\frac{\frac{(7Ab+5Ba)x^{\frac{11}{2}}}{36a^2}+\frac{(13Ab-Ba)x^{\frac{5}{2}}}{36ab}}{(bx^3+a)^2}+\frac{(7Ab+5Ba)\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{}$
default	$\frac{\frac{(7Ab+5Ba)x^{\frac{11}{2}}}{36a^2}+\frac{(13Ab-Ba)x^{\frac{5}{2}}}{36ab}}{(bx^3+a)^2}+\frac{(7Ab+5Ba)\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/72*(7*A*b+5*B*a)/a^2*x^(11/2)+1/72*(13*A*b-B*a)/a/b*x^(5/2))/(b*x^3+a)
^2+1/36*(7*A*b+5*B*a)/a^2/b*(-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)
^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+
3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)
^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arct
an(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6)))
```

Maxima [A]
time = 0.49, size = 271, normalized size = 0.83

$$\frac{(5\,Bab+7\,Ab^2)x^{\frac{11}{2}}-(Ba^2-13\,Aab)x^{\frac{5}{2}}}{36\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}-\frac{(5\,Ba+7\,Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{432\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*((5*B*a*b + 7*A*b^2)*x^(11/2) - (B*a^2 - 13*A*a*b)*x^(5/2))/(a^2*b^3*x
^6 + 2*a^3*b^2*x^3 + a^4*b) - 1/432*(5*B*a + 7*A*b)*(sqrt(3)*log(sqrt(3)*a^
(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*lo
g(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6))
 - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1
```

```
/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6)
- 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3)))
 - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^
(1/3))))/(a^2*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3951 vs. $2(241) = 482$.
time = 2.60, size = 3951, normalized size = 12.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/432*(4*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 +
131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 9003
75*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6
)*arctan(1/3*(2*sqrt(3)*sqrt((3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10 + 6
1250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^13 +
16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375
*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10 + 13
6718750*A*B^9*a^9*b + 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^7*b^
3 + 7878281250*A^4*B^6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431250*
A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*b^8
+ 2017680350*A^9*B*a*b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7 + 1
31250*A*B^5*a^14*b^8 + 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^10 +
 900375*A^4*B^2*a^11*b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^13)*(
-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*
B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)
/(a^13*b^11))^(2/3))*a^2*b^2*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375
*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5*a^7
*b^2 + 21875*A*B^4*a^6*b^3 + 61250*A^2*B^3*a^5*b^4 + 85750*A^3*B^2*a^4*b^5
+ 60025*A^4*B*a^3*b^6 + 16807*A^5*a^2*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 13125
0*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^
4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) + s
qrt(3)*(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 85750
0*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^
6*b^6))/(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8575
00*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A
^6*b^6)) + 4*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6
 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 9
00375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(
1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10
```

```
 + 61250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^1
3 + 16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 45
9375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 50
4210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10
+ 136718750*A*B^9*a^9*b + 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^
7*b^3 + 7878281250*A^4*B^6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431
250*A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*
b^8 + 2017680350*A^9*B*a*b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7
 + 131250*A*B^5*a^14*b^8 + 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^
10 + 900375*A^4*B^2*a^11*b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^1
3)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*
A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*
b^6)/(a^13*b^11))^(2/3))*a^2*b^2*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 45
9375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 50
4210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5
*a^7*b^2 + 21875*A*B^4*a^6*b^3 + 61250*A^2*B^3*a^5*b^4 + 85750*A^3*B^2*a^4*
b^5 + 60025*A^4*B*a^3*b^6 + 16807*A^5*a^2*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 1
31250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90037
5*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)
 - sqrt(3)*(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8
57500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 11764
9*A^6*b^6))/(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 +
857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 1176
49*A^6*b^6)) - 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 1
31250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90037
5*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)
*log(a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^
2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 +
117649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 61
250*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5
*b^5)*sqrt(x)) + 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 +
 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900
375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/
6)*log(-a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5
 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B...
```

Sympy **[F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]

time = 1.27, size = 328, normalized size = 1.00

$$\frac{\left(5\,Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}+7\,Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^3b}+\frac{5\,Babx^{\frac{11}{2}}+7\,Ab^2x^{\frac{11}{2}}-Ba^2x^{\frac{5}{2}}+13\,Aabx^{\frac{5}{2}}}{36\,(bx^3+a)^2a^2b}-\frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^6}+\frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^6}+\frac{\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^6}+\frac{\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/108*(5*B*a*(a/b)^(5/6) + 7*A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(
a^3*b) + 1/36*(5*B*a*b*x^(11/2) + 7*A*b^2*x^(11/2) - B*a^2*x^(5/2) + 13*A*a
*b*x^(5/2))/((b*x^3 + a)^2*a^2*b) - 1/432*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + 7*
(a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*
b^6) + 1/432*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*log(-sqrt(
3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^6) + 1/216*(5*(a*b^5)^(5/6
)*B*a + 7*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)
^(1/6))/(a^3*b^6) + 1/216*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*arcta
n(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^6)
```

Mupad [B]

time = 2.89, size = 1672, normalized size = 5.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^(11/2)*(7*A*b + 5*B*a))/(36*a^2) + (x^(5/2)*(13*A*b - B*a))/(36*a*b))/(
a^2 + b^2*x^6 + 2*a*b*x^3) + (atan(((((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^
2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*A*b + 5*B*a)*(49*A^2*b^
4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*(7*A*b + 5
*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)) - (((343*A^3*b^3 + 125*B^3*a^3 + 5
25*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*(7*A*b + 5*B*a)*(49
*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*(7*
A*b + 5*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((((343*A^3*b^3 + 125*B^3*
a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*A*b + 5*B
*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6
)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3)) + (((343*A^3*b^3 + 125*
B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*(7*A*b +
 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(
11/6)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3))))*(7*A*b + 5*B*a)*1
i)/(108*(-a)^(13/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5
*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1
296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25
*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^
(13/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^
```

```
3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*
((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*
B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((
((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 52
5*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 - 1/
2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)
^(19/6)*b^(11/6))))/(46656*(-a)^(13/3)*b^(11/3)) + (((3^(1/2)*1i)/2 - 1/2)^
2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2
*B*a*b^2)/(1296*a^3) + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*
A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6))))/(46
656*(-a)^(13/3)*b^(11/3))))*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*1i)/(108
*(-a)^(13/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2
*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3
) - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^
2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*
b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125
*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*((3^(1/
2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3
))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((((3^(1/
2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2
*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A
*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)
*b^(11/6))))/(46656*(-a)^(13/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(7*A*
b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^
2)/(1296*a^3) + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4
 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6))))/(46656*(-a
)^(13/3)*b^(11/3))))*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*1i)/(108*(-a)^(
13/6)*b^(11/6))
```

3.174 $\int \frac{\sqrt{x}\,(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=104

$$\frac{(Ab-aB)x^{3/2}}{6ab\,(a+bx^3)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\,(a+bx^3)}+\frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

```
[Out] 1/6*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)^2+1/12*(3*A*b+B*a)*x^(3/2)/a^2/b/(b*x^3
+a)+1/12*(3*A*b+B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {468, 296, 335, 281, 211}

$$\frac{(aB+3Ab)\text{ArcTan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}+\frac{x^{3/2}(aB+3Ab)}{12a^2b\,(a+bx^3)}+\frac{x^{3/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(6*a*b*(a + b*x^3)^2) + ((3*A*b + a*B)*x^(3/2))/(12*a
^2*b*(a + b*x^3)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(5/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}\,(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{(Ab-aB)x^{3/2}}{6ab\,(a+bx^3)^2}+\frac{\left(\frac{9Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{x}}{(a+bx^3)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\,(a+bx^3)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\,(a+bx^3)}+\frac{(3Ab+aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8a^2b}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\,(a+bx^3)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\,(a+bx^3)}+\frac{(3Ab+aB)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{4a^2b}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\,(a+bx^3)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\,(a+bx^3)}+\frac{(3Ab+aB)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{12a^2b}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\,(a+bx^3)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\,(a+bx^3)}+\frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 92, normalized size = 0.88

$$-\frac{x^{3/2}(-5aAb+a^2B-3Ab^2x^3-abBx^3)}{12a^2b\,(a+bx^3)^2}+\frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/12*(x^(3/2)*(-5*a*A*b + a^2*B - 3*A*b^2*x^3 - a*b*B*x^3))/(a^2*b*(a + b*
x^3)^2) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(5/2)*b^(
3/2))
```

Maple [A]
time = 0.29, size = 82, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\frac{(3Ab+Ba)x^{\frac{9}{2}}}{12a^{2}}+\frac{(5Ab-Ba)x^{\frac{3}{2}}}{12ab}}{(b\,x^{3}+a)^{2}}+\frac{(3Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12a^{2}b\sqrt{ab}}$	82
default	$\frac{\frac{(3Ab+Ba)x^{\frac{9}{2}}}{12a^{2}}+\frac{(5Ab-Ba)x^{\frac{3}{2}}}{12ab}}{(b\,x^{3}+a)^{2}}+\frac{(3Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12a^{2}b\sqrt{ab}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1/8*(3*A*b+B*a)/a^2*x^(9/2)+1/8*(5*A*b-B*a)/a/b*x^(3/2))/(b*x^3+a)^2+1
/12*(3*A*b+B*a)/a^2/b/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 96, normalized size = 0.92

$$\frac{(Bab+3\,Ab^{2})x^{\frac{9}{2}}-(Ba^{2}-5\,Aab)x^{\frac{3}{2}}}{12\,(a^{2}b^{3}x^{6}+2\,a^{3}b^{2}x^{3}+a^{4}b)}+\frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*((B*a*b + 3*A*b^2)*x^(9/2) - (B*a^2 - 5*A*a*b)*x^(3/2))/(a^2*b^3*x^6 +
 2*a^3*b^2*x^3 + a^4*b) + 1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(s
qrt(a*b)*a^2*b)
```

Fricas [A]
time = 1.97, size = 313, normalized size = 3.01

$$\left[-\frac{((Bab^{2}+3\,Ab^{3})x^{6}+Ba^{3}+3\,Aa^{2}b+2\,(Ba^{2}b+3\,Aab^{2})x^{3})\sqrt{-ab}\,\log\left(\frac{bx^{3}-2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^{3}+a}\right)-2\,((Ba^{2}b^{2}+3\,Aab^{3})x^{4}-(Ba^{3}b-5\,Aa^{2}b^{2})x)\sqrt{x}}{24\,(a^{3}b^{4}x^{6}+2\,a^{4}b^{3}x^{3}+a^{5}b^{2})},\ \frac{((Bab^{2}+3\,Ab^{3})x^{6}+Ba^{3}+3\,Aa^{2}b+2\,(Ba^{2}b+3\,Aab^{2})x^{3})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)+((Ba^{2}b^{2}+3\,Aab^{3})x^{4}-(Ba^{3}b-5\,Aa^{2}b^{2})x)\sqrt{x}}{12\,(a^{3}b^{4}x^{6}+2\,a^{4}b^{3}x^{3}+a^{5}b^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b
^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2
*((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a^3*b^
4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2), 1/12*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 +
3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)
/a) + ((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a
^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 84, normalized size = 0.81

$$\frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,a^2b}+\frac{Babx^{\frac{9}{2}}+3\,Ab^2x^{\frac{9}{2}}-Ba^2x^{\frac{3}{2}}+5\,Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/12*(B*
a*b*x^(9/2) + 3*A*b^2*x^(9/2) - B*a^2*x^(3/2) + 5*A*a*b*x^(3/2))/((b*x^3 +
a)^2*a^2*b)
```

Mupad [B]
time = 2.71, size = 136, normalized size = 1.31

$$\frac{\frac{x^{9/2}\,(3\,A\,b+B\,a)}{12\,a^2}+\frac{x^{3/2}\,(5\,A\,b-B\,a)}{12\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{\mathrm{atan}\left(\frac{b^{3/2}\,x^{3/2}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{\sqrt{a}\,\left(3\,A\,b+B\,a\right)\,\left(3\,A\,b^3+B\,a\,b^2\right)}\right)\,\left(3\,A\,b+B\,a\right)}{12\,a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^(9/2)*(3*A*b + B*a))/(12*a^2) + (x^(3/2)*(5*A*b - B*a))/(12*a*b))/(a^2
+ b^2*x^6 + 2*a*b*x^3) + (atan((b^(3/2)*x^(3/2)*(9*A^2*b^3 + B^2*a^2*b + 6*
A*B*a*b^2))/(a^(1/2)*(3*A*b + B*a)*(3*A*b^3 + B*a*b^2)))*(3*A*b + B*a))/(12
*a^(5/2)*b^(3/2))
```

3.175 $\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx$

Optimal. Leaf size=321

$$\frac{(Ab-aB)\sqrt{x}}{6ab\,(a+bx^3)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\,(a+bx^3)}-\frac{5(11Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}}+\frac{5(11Ab+aB)\tan^{-1}\left(\sqrt{3}\right.}{216a^{17/6}b^{7/6}}$$

```
[Out] 5/108*(11*A*b+B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/6)+5/216*(
11*A*b+B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/6)+5/2
16*(11*A*b+B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/6)-
5/432*(11*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^
(17/6)/b^(7/6)*3^(1/2)+5/432*(11*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1
/6)*3^(1/2)*x^(1/2))/a^(17/6)/b^(7/6)*3^(1/2)+1/6*(A*b-B*a)*x^(1/2)/a/b/(b*
x^3+a)^2+1/36*(11*A*b+B*a)*x^(1/2)/a^2/b/(b*x^3+a)
```

Rubi [A]
time = 0.34, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 296, 335, 215, 648, 632, 210, 642, 211}

$$-\frac{5(aB+11Ab)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\text{ArcTan}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}}-\frac{5(aB+11Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{144\sqrt{3}\,a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{144\sqrt{3}\,a^{17/6}b^{7/6}}+\frac{\sqrt{x}\,(aB+11Ab)}{36a^2b\,(a+bx^3)}+\frac{\sqrt{x}\,(Ab-aB)}{6ab\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(6*a*b*(a + b*x^3)^2) + ((11*A*b + a*B)*Sqrt[x])/(36*
a^2*b*(a + b*x^3)) - (5*(11*A*b + a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(17/6)*b^(7/6)) - (5*(11*A*b + a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17
/6)*b^(7/6)) + (5*(11*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17/6)*b^(7/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx &= \frac{(Ab-aB)\sqrt{x}}{6ab\,(a+bx^3)^2} + \frac{(11Ab+aB)\int \frac{1}{\sqrt{x}\,(a+bx^3)^2}\,dx}{12ab} \\
&= \frac{(Ab-aB)\sqrt{x}}{6ab\,(a+bx^3)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\,(a+bx^3)} + \frac{(5(11Ab+aB))\int \frac{1}{\sqrt{x}\,(a+bx^3)}\,dx}{72a^2b} \\
&= \frac{(Ab-aB)\sqrt{x}}{6ab\,(a+bx^3)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\,(a+bx^3)} + \frac{(5(11Ab+aB))\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{36a^2b} \\
&= \frac{(Ab-aB)\sqrt{x}}{6ab\,(a+bx^3)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\,(a+bx^3)} + \frac{(5(11Ab+aB))\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{\cdots}}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\cdots}\cdots\right.}{108a^{17/6}b} \\
&= \frac{(Ab-aB)\sqrt{x}}{6ab\,(a+bx^3)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\,(a+bx^3)} + \frac{5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{(5(11A\cdots}{\cdots} \\
&= \frac{(Ab-aB)\sqrt{x}}{6ab\,(a+bx^3)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\,(a+bx^3)} + \frac{5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{5(11Ab\cdots}{\cdots} \\
&= \frac{(Ab-aB)\sqrt{x}}{6ab\,(a+bx^3)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\,(a+bx^3)} - \frac{5(11Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} + \cdots
\end{aligned}$$

Mathematica [A]
time = 0.56, size = 189, normalized size = 0.59

$$\frac{\frac{6a^{5/6}\sqrt[6]{b}\,\sqrt{x}\,(-5a^2B+11Ab^2x^3+ab(17A+Bx^3))}{(a+bx^3)^2} + 10(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right) - 5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}\right) + 5\sqrt{3}\,(11Ab+aB)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{216a^{17/6}b^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

```
[Out] ((6*a^(5/6)*b^(1/6)*Sqrt[x]*(-5*a^2*B + 11*A*b^2*x^3 + a*b*(17*A + B*x^3)))
/(a + b*x^3)^2 + 10*(11*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - 5*(1
```

```
1*A*b + a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + 5*Sq
rt[3]*(11*A*b + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b
^(1/3)*x)])/(216*a^(17/6)*b^(7/6))
```

Maple [A]
time = 0.29, size = 233, normalized size = 0.73

method	result
derivativedivides	$\frac{\frac{(11Ab+Ba)x^{\frac{7}{2}}}{36a^2}+\frac{(17Ab-5Ba)\sqrt{x}}{36ab}}{(bx^3+a)^2}+\frac{5(11Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{12a}\right.}{}$
default	$\frac{\frac{(11Ab+Ba)x^{\frac{7}{2}}}{36a^2}+\frac{(17Ab-5Ba)\sqrt{x}}{36ab}}{(bx^3+a)^2}+\frac{5(11Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{12a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/72*(11*A*b+B*a)/a^2*x^(7/2)+1/72*(17*A*b-5*B*a)/a/b*x^(1/2))/(b*x^3+a)
^2+5/36*(11*A*b+B*a)/a^2/b*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1
/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6
/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^
(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arcta
n(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

Maxima [A]
time = 0.49, size = 336, normalized size = 1.05

$$\frac{(Bab+11\,Ab^2)x^{\frac{7}{2}}-(5\,Ba^2-17\,Aab)\sqrt{x}}{36\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}+\frac{5\left(\frac{\sqrt{3}\,(Ba+11\,Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(Ba+11\,Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}+11\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}+11\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}+11\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{432\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/36*((B*a*b + 11*A*b^2)*x^(7/2) - (5*B*a^2 - 17*A*a*b)*sqrt(x))/(a^2*b^3*x
^6 + 2*a^3*b^2*x^3 + a^4*b) + 5/432*(sqrt(3)*(B*a + 11*A*b)*log(sqrt(3)*a^(
1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*
a + 11*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^
(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt
(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b
^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*
```

```
sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^
(4/3)*b^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*
b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/
(a^2*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2674 vs. $2(235) = 470$.

time = 1.99, size = 2674, normalized size = 8.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*(20*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B
^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^
2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*arctan(1/3*
(2*sqrt(3)*sqrt(a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2
+ 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 177
1561*A^6*b^6)/(a^17*b^7))^(1/3) + (B^2*a^2 + 22*A*B*a*b + 121*A^2*b^2)*x +
(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4
*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*
b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6))*a^14*b^6*(-(B^6*a^6 + 66*A*B^5*a^
5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4
 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) - 2*sqrt(3)*(B*a
^15*b^6 + 11*A*a^14*b^7)*sqrt(x)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4
*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*
b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) + sqrt(3)*(B^6*a^6 + 66*A*B^5*a^5*
b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
 966306*A^5*B*a*b^5 + 1771561*A^6*b^6))/(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^
2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5
*B*a*b^5 + 1771561*A^6*b^6)) + 20*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^
4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3
*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17
*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b
 + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + (B^2*a^2 + 22*A*B
*a*b + 121*A^2*b^2)*x - (B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6))*a^14*b^6*
(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3
+ 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7)
)^(5/6) - 2*sqrt(3)*(B*a^15*b^6 + 11*A*a^14*b^7)*sqrt(x)*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) - sqrt(3)*
```

```
(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 +
219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6))/(B^6*a^6 +
66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*
B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)) + 5*(a^2*b^3*x^6 + 2*a
^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26
620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561
*A^6*b^6)/(a^17*b^7))^(1/6)*log(25*a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b + 18
15*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 96630
6*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + 25*(B^2*a^2 + 22*A*B*a
*b + 121*A^2*b^2)*x + 25*(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A
*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*
a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)) - 5*(a^2
*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^
4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a
*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(25*a^6*b^2*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + 25*(B^2*
a^2 + 22*A*B*a*b + 121*A^2*b^2)*x - 25*(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(
B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2
19615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(
1/6)) + 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*
b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(5*a^3*b*(-(B^6
*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2196
15*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6
) + 5*(B*a + 11*A*b)*sqrt(x)) - 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-
(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 +
219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^
(1/6)*log(-5*a^3*b*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 266
20*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*
A^6*b^6)/(a^17*b^7))^(1/6) + 5*(B*a + 11*A*b)*sqrt(x)) + 12*((B*a*b + 11*A*
b^2)*x^3 - 5*B*a^2 + 17*A*a*b)*sqrt(x))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*
b)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.62, size = 322, normalized size = 1.00

$$\frac{5\sqrt{3}\left((ab^5)^{\frac{1}{6}}Ba+11\,(ab^5)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^2}-\frac{5\sqrt{3}\left((ab^5)^{\frac{1}{6}}Ba+11\,(ab^5)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^2}+\frac{5\left((ab^5)^{\frac{1}{6}}Ba+11\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^2}+\frac{5\left((ab^5)^{\frac{1}{6}}Ba+11\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^2}+\frac{5\left((ab^5)^{\frac{1}{6}}Ba+11\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^3b^2}+\frac{Babx^{\frac{7}{2}}+11\,Ab^2x^{\frac{7}{2}}-5\,Ba^2\sqrt{x}+17\,Aab\sqrt{x}}{36\,(bx^3+a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 5/432*sqrt(3)*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x
)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^2) - 5/432*sqrt(3)*((a*b^5)^(1/6)*B
*a + 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/
3))/(a^3*b^2) + 5/216*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan((sq
rt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^2) + 5/216*((a*b^5)^(1/6
)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/
b)^(1/6))/(a^3*b^2) + 5/108*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arct
an(sqrt(x)/(a/b)^(1/6))/(a^3*b^2) + 1/36*(B*a*b*x^(7/2) + 11*A*b^2*x^(7/2)
- 5*B*a^2*sqrt(x) + 17*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^2*b)
```

Mupad [B]
time = 2.95, size = 1952, normalized size = 6.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^3),x)
```

```
[Out] ((x^(7/2)*(11*A*b + B*a))/(36*a^2) + (x^(1/2)*(17*A*b - 5*B*a))/(36*a*b))/(
a^2 + b^2*x^6 + 2*a*b*x^3) - (atan(((((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4
*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^
8) - (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33
*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)
^(17/6)*b^(7/6)) + (((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*
a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b
 + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/
(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)^(17/6)*b^(7/6)))
/((5*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*
A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*(11*A*b + B*a)*(1331*A
^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(4
7/6)*b^(7/6)))*(11*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6)) - (5*((625*x^(1/2)
*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A
*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b
^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11
*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6))))*(11*A*b + B*a)*5i)/(108*(-a)^(17/6
)*b^(7/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(1
4641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^
3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*(1331
```

*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6)) + (((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6)))/((5*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))))/(216*(-a)^(17/6)*b^(7/6)) - (5*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))))/(216*(-a)^(17/6)*b^(7/6))))*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*5i)/(108*(-a)^(17/6)*b^(7/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6)))/((5*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))))/(216*(-a)^(17/6)*b^(7/6)) - (5*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))))/(216*(-a)^(17/6)*b^(7/6))))*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*5i)/(108*(-a)^(17/6)*b^(7/6))

3.176 $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3}\,dx$

Optimal. Leaf size=351

$$-\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\,(a+bx^3)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\,(a+bx^3)}+\frac{7(13Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}}-\frac{7(1$$

```
[Out] -7/108*(13*A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5/6)-7/216*
(13*A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5/6)-7/
216*(13*A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5/6)
-7/432*(13*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a
^(19/6)/b^(5/6)*3^(1/2)+7/432*(13*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(
1/6)*3^(1/2)*x^(1/2))/a^(19/6)/b^(5/6)*3^(1/2)-7/36*(13*A*b-B*a)/a^3/b/x^(1
/2)+1/6*(A*b-B*a)/a/b/(b*x^3+a)^2/x^(1/2)+1/36*(13*A*b-B*a)/a^2/b/(b*x^3+a)
/x^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {468, 296, 331, 335, 301, 648, 632, 210, 642, 211}

$$\frac{7(13Ab-aB)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)\text{ArcTan}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}+\frac{7(13Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{13Ab-aB}{36a^2b\sqrt{x}\,(a+bx^3)}+\frac{Ab-aB}{6ab\sqrt{x}\,(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

```
[Out] (-7*(13*A*b - a*B))/(36*a^3*b*Sqrt[x]) + (A*b - a*B)/(6*a*b*Sqrt[x]*(a + b*
x^3)^2) + (13*A*b - a*B)/(36*a^2*b*Sqrt[x]*(a + b*x^3)) + (7*(13*A*b - a*B)
*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6)*b^(5/6)) - (7
*(13*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6
)*b^(5/6)) - (7*(13*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(1
9/6)*b^(5/6)) - (7*(13*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqr
t[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6)*b^(5/6)) + (7*(13*A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6
)*b^(5/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
```

```
m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{\left(\frac{13Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}+\frac{(7(13Ab-aB))\int \frac{1}{x^{3/2}(a+bx^3)}\,dx}{72a^2b}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}-\frac{(7(13Ab-aB))\int \frac{x}{a-}}{72a^3}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}-\frac{(7(13Ab-aB))\text{Subs}}{3}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}-\frac{(7(13Ab-aB))\text{Subs}}{}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}-\frac{7(13Ab-aB)\tan^{-1}}{108a^{19/6}b^{5}}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}-\frac{7(13Ab-aB)\tan^{-1}}{108a^{19/6}b^{5}}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}+\frac{7(13Ab-aB)\tan^{-1}}{216a^{19}}$$

Mathematica [A]
time = 0.57, size = 208, normalized size = 0.59

$$\frac{-\frac{6\sqrt[6]{a}\left(91Ab^2x^6+a^2(72A-13Bx^3)+abx^3(169A-7Bx^3)\right)}{\sqrt{x}\,(a+bx^3)^2}+\frac{14(-13Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}}+\frac{7(13Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{b^{5/6}}+\frac{7\sqrt{3}(13Ab-aB)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{b^{5/6}}}{216a^{19/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

```
[Out] ((-6*a^(1/6)*(91*A*b^2*x^6 + a^2*(72*A - 13*B*x^3) + a*b*x^3*(169*A - 7*B*x
^3)))/(Sqrt[x]*(a + b*x^3)^2) + (14*(-13*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x]
)/a^(1/6)])/b^(5/6) + (7*(13*A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/
6)*b^(1/6)*Sqrt[x])])/b^(5/6) + (7*Sqrt[3]*(13*A*b - a*B)*ArcTanh[(Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(5/6))/(216*a^(19/6))
```

Maple [A]
time = 0.37, size = 236, normalized size = 0.67

method	result
derivativedivides	$-\frac{2A}{a^3\sqrt{x}}-\frac{2\left(\frac{\left(\frac{19}{72}b^2A-\frac{7}{72}abB\right)x^{\frac{11}{2}}+\frac{a(25Ab-13Ba)x^{\frac{5}{2}}}{72}}{\left(bx^3+a\right)^2}+\left(\frac{91Ab}{72}-\frac{7Ba}{72}\right)\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right.\cdots\right.}{12a}\cdots\right.\right.}{\cdots}$
default	$-\frac{2A}{a^3\sqrt{x}}-\frac{2\left(\frac{\left(\frac{19}{72}b^2A-\frac{7}{72}abB\right)x^{\frac{11}{2}}+\frac{a(25Ab-13Ba)x^{\frac{5}{2}}}{72}}{\left(bx^3+a\right)^2}+\left(\frac{91Ab}{72}-\frac{7Ba}{72}\right)\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right.\cdots\right.}{12a}\cdots\right.\right.}{\cdots}$
risch	$-\frac{2A}{a^3\sqrt{x}}-\frac{19x^{\frac{11}{2}}b^2A}{36a^3(bx^3+a)^2}+\frac{7x^{\frac{11}{2}}bB}{36a^2(bx^3+a)^2}-\frac{25Ax^{\frac{5}{2}}b}{36a^2(bx^3+a)^2}+\frac{13Bx^{\frac{5}{2}}}{36a(bx^3+a)^2}+\frac{91Ab\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{\cdots}\right.}{432a\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*A/a^3/x^(1/2)-2/a^3*(((19/72*b^2*A-7/72*a*b*B)*x^(11/2)+1/72*a*(25*A*b-1
3*B*a)*x^(5/2))/(b*x^3+a)^2+(91/72*A*b-7/72*B*a)*(-1/12/a*3^(1/2)*(a/b)^(5/
6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2
*x^(1/2)/(a/b)^(1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))
+1/12/a*3^(1/2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1
/6/b/(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))))
```

Maxima [A]

time = 0.50, size = 273, normalized size = 0.78

$$\frac{7\,(Bab-13\,Ab^2)x^6+13\,(Ba^2-13\,Aab)x^3-72\,Aa^2}{36\left(a^3b^2x^{\frac{13}{2}}+2\,a^4bx^{\frac{7}{2}}+a^5\sqrt{x}\right)}-\frac{7\,(Ba-13\,Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{432\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(7*(B*a*b - 13*A*b^2)*x^6 + 13*(B*a^2 - 13*A*a*b)*x^3 - 72*A*a^2)/(a^3
*b^2*x^(13/2) + 2*a^4*b*x^(7/2) + a^5*sqrt(x)) - 7/432*(B*a - 13*A*b)*(sqrt
(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(
5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))
/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3
)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt
(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/
3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3904 vs. $2(254) = 508$.
time = 3.57, size = 3904, normalized size = 11.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432*(28*sqrt(3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5
*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*
b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*arctan(1/3*(
2*sqrt(3)*sqrt((B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1690*A^2*B^3*a^19*b^6 -
21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371293*A^5*a^16*b^9)*sqrt(
x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b
^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*
b^5))^(5/6) + (B^10*a^10 - 130*A*B^9*a^9*b + 7605*A^2*B^8*a^8*b^2 - 263640*
A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4 - 93565836*A^5*B^5*a^5*b^5 + 1013
629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^3*a^3*b^7 + 36707882445*A^8*B^2*a
^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858491849*A^10*b^10)*x - (B^6*a^19*b
^3 - 78*A*B^5*a^18*b^4 + 2535*A^2*B^4*a^17*b^5 - 43940*A^3*B^3*a^16*b^6 + 4
28415*A^4*B^2*a^15*b^7 - 2227758*A^5*B*a^14*b^8 + 4826809*A^6*a^13*b^9)*(-(
B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 4
28415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^
(2/3))*a^3*b*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3
*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b
^6)/(a^19*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^8*b - 65*A*B^4*a^7*b^2 + 1690*A^2*
B^3*a^6*b^3 - 21970*A^3*B^2*a^5*b^4 + 142805*A^4*B*a^4*b^5 - 371293*A^5*a^3
*b^6)*sqrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^
3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*
b^6)/(a^19*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a
^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b
^5 + 4826809*A^6*b^6))/(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 4
3940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 48268
09*A^6*b^6)) + 28*sqrt(3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 -
78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*
B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*arct
an(1/50421*(2*sqrt(3)*sqrt(-282475249*(B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1
690*A^2*B^3*a^19*b^6 - 21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371
293*A^5*a^16*b^9)*sqrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^
2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 +
4826809*A^6*b^6)/(a^19*b^5))^(5/6) + 282475249*(B^10*a^10 - 130*A*B^9*a^9*b
 + 7605*A^2*B^8*a^8*b^2 - 263640*A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4
- 93565836*A^5*B^5*a^5*b^5 + 1013629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^
3*a^3*b^7 + 36707882445*A^8*B^2*a^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858
```

```
491849*A^10*b^10)*x - 282475249*(B^6*a^19*b^3 - 78*A*B^5*a^18*b^4 + 2535*A^
2*B^4*a^17*b^5 - 43940*A^3*B^3*a^16*b^6 + 428415*A^4*B^2*a^15*b^7 - 2227758
*A^5*B*a^14*b^8 + 4826809*A^6*a^13*b^9)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*
A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*
A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(2/3))*a^3*b*(-(B^6*a^6 - 78*A*B
^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^
2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6) + 33614*sq
rt(3)*(B^5*a^8*b - 65*A*B^4*a^7*b^2 + 1690*A^2*B^3*a^6*b^3 - 21970*A^3*B^2*
a^5*b^4 + 142805*A^4*B*a^4*b^5 - 371293*A^5*a^3*b^6)*sqrt(x)*(-(B^6*a^6 - 7
8*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B
^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6) + 168
07*sqrt(3)*(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3
*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6))
/(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 +
 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)) - 14*(a^3
*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*
a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*
b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(16807*a^16*b^4*(-(B^6*a^6 - 78
*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^
2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 1680
7*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3
+ 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) + 14*(a^3*b^2*x^7 + 2*a^4*b
*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^
3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*
b^6)/(a^19*b^5))^(1/6)*log(-16807*a^16*b^4*(-(B^6*a^6 - 78*A*B^5*a^5*b + 25
35*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 22277
58*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 16807*(B^5*a^5 - 65*A
*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*
b^4 - 371293*A^5*b^5)*sqrt(x)) + 7*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B
^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 42
8415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 48...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.76, size = 329, normalized size = 0.94

$$-\frac{2\,A}{a^3\sqrt{x}}+\frac{7\,Babx^{\frac{11}{2}}-19\,Ab^2x^{\frac{11}{2}}+13\,Ba^2x^{\frac{5}{2}}-25\,Aabx^{\frac{5}{2}}}{36\,(bx^3+a)^2a^3}-\frac{7\,\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba-13\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b^5}+\frac{7\,\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba-13\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b^5}+\frac{7\left((ab^5)^{\frac{5}{6}}\,Ba-13\,(ab^5)^{\frac{5}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^4b^5}+\frac{7\left((ab^5)^{\frac{5}{6}}\,Ba-13\,(ab^5)^{\frac{5}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^4b^5}+\frac{7\left((ab^5)^{\frac{5}{6}}\,Ba-13\,(ab^5)^{\frac{5}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="giac")

[Out] -2*A/(a^3*sqrt(x)) + 1/36*(7*B*a*b*x^(11/2) - 19*A*b^2*x^(11/2) + 13*B*a^2*x^(5/2) - 25*A*a*b*x^(5/2))/((b*x^3 + a)^2*a^3) - 7/432*sqrt(3)*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b^5) + 7/432*sqrt(3)*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b^5) + 7/216*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^4*b^5) + 7/216*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^4*b^5) + 7/108*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^4*b^5)

Mupad [**B**]
time = 2.91, size = 1786, normalized size = 5.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x)

[Out] (atan((((13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)) + ((13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/(((13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3)) - ((13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*(13*A*b - B*a)*7i)/(108*(-a)^(19/6)*b^(5/6)) - ((2*A)/a + (13*x^3*(13*A*b - B*a))/(36*a^2) + (7*b*x^6*(13*A*b - B*a))/(36*a^3))/(a^2*x^(1/2) + b^2*x^(13/2) + 2*a*b*x^(7/2)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2

*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))
/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)) + (((3^(1/2)*1i)
/2 - 1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3
*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 +
(343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*
a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/
(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/((((3^(1/2)*1i)/
2 - 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*
a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (
343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a
^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(
10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3)) - (((3^(1/2)*1i)/2 -
1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24
*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*
x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*
b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(1007
7696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*((3^(1/2)*1i)/2 - 1/2)*
(13*A*b - B*a)*7i)/(108*(-a)^(19/6)*b^(5/6)) + (atan(((((3^(1/2)*1i)/2 + 1/
2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b
^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^
(1/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^
6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(100776
96*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)) + (((3^(1/2)*1i)/2 + 1/2
)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^
3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(
1/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6
 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(1007769
6*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/((((3^(1/2)*1i)/2 + 1/2)
^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6
 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1
/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6
+ 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696
*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(
13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1
100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*
((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 82
9406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a
)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(13*A*b
- B*a)*7i)/(108*(-a)^(19/6)*b^(5/6))

3.177 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=129

$$\frac{-5Ab+aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

```
[Out] 1/4*(-5*A*b+B*a)/a^3/b/x^(3/2)+1/6*(A*b-B*a)/a/b/x^(3/2)/(b*x^3+a)^2+1/12*(
5*A*b-B*a)/a^2/b/x^(3/2)/(b*x^3+a)-1/4*(5*A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a
^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {468, 296, 331, 335, 281, 211}

$$-\frac{(5Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}-\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

```
[Out] -1/4*(5*A*b - a*B)/(a^3*b*x^(3/2)) + (A*b - a*B)/(6*a*b*x^(3/2)*(a + b*x^3)
^2) + (5*A*b - a*B)/(12*a^2*b*x^(3/2)*(a + b*x^3)) - ((5*A*b - a*B)*ArcTan[
(Sqrt[b]*x^(3/2))/Sqrt[a]])/(4*a^(7/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx=\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{\left(\frac{15Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$=\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}+\frac{(3(5Ab-aB))\int \frac{1}{x^{5/2}\left(a+bx^3\right)}\,dx}{8a^2b}$$

$$=-\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(3(5Ab-aB))\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8a^3}$$

$$=-\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(3(5Ab-aB))\text{Subst}\left(\int \frac{}{a}\right.}{4a^3}$$

$$=-\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(5Ab-aB)\text{Subst}\left(\int \frac{1}{a+bx}\right.}{4a^3}$$

$$=-\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right.}{4a^{7/2}\sqrt{b}}$$

Mathematica [A]
time = 0.14, size = 102, normalized size = 0.79

$$\frac{-8a^2A-25aAbx^3+5a^2Bx^3-15Ab^2x^6+3abBx^6}{12a^3x^{3/2}\left(a+bx^3\right)^2}+\frac{(-5Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
[Out] (-8*a^2*A - 25*a*A*b*x^3 + 5*a^2*B*x^3 - 15*A*b^2*x^6 + 3*a*b*B*x^6)/(12*a^
3*x^(3/2)*(a + b*x^3)^2) + ((-5*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]
])/(4*a^(7/2)*Sqrt[b])
```

Maple [A]
time = 0.32, size = 86, normalized size = 0.67

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{7}{8}b^2A-\frac{3}{8}abB\right)x^{\frac{9}{2}}+\frac{a(9Ab-5Ba)x^{\frac{3}{2}}}{8}}{\left(b\,x^3+a\right)^2}+\frac{3(5Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{3a^3}-\frac{2A}{3a^3x^{\frac{3}{2}}}$

default	$-\frac{2\left(\frac{\left(\frac{7}{8}b^{2}A-\frac{3}{8}abB\right)x^{\frac{9}{2}}+\frac{a(9Ab-5Ba)x^{\frac{3}{2}}}{8}}{\left(b\,x^{3}+a\right)^{2}}+\frac{3(5Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{3a^{3}}-\frac{2A}{3a^{3}x^{\frac{3}{2}}}$
risch	$-\frac{2A}{3a^{3}x^{\frac{3}{2}}}-\frac{7x^{\frac{9}{2}}b^{2}A}{12a^{3}(b\,x^{3}+a)^{2}}+\frac{x^{\frac{9}{2}}bB}{4a^{2}(b\,x^{3}+a)^{2}}-\frac{3A\,x^{\frac{3}{2}}b}{4a^{2}(b\,x^{3}+a)^{2}}+\frac{5B\,x^{\frac{3}{2}}}{12a(b\,x^{3}+a)^{2}}-\frac{5\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)Ab}{4a^{3}\sqrt{ab}}+$ arct…

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a^3*(((7/8*b^2*A-3/8*a*b*B)*x^(9/2)+1/8*a*(9*A*b-5*B*a)*x^(3/2))/(b*x^
3+a)^2+3/8*(5*A*b-B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))-2/3*A/a^3
/x^(3/2)
```

Maxima [A]
time = 0.64, size = 100, normalized size = 0.78

$$\frac{3\,(Bab-5\,Ab^{2})x^{6}+5\,(Ba^{2}-5\,Aab)x^{3}-8\,Aa^{2}}{12\left(a^{3}b^{2}x^{\frac{15}{2}}+2\,a^{4}bx^{\frac{9}{2}}+a^{5}x^{\frac{3}{2}}\right)}+\frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(B*a*b - 5*A*b^2)*x^6 + 5*(B*a^2 - 5*A*a*b)*x^3 - 8*A*a^2)/(a^3*b^2
*x^(15/2) + 2*a^4*b*x^(9/2) + a^5*x^(3/2)) + 1/4*(B*a - 5*A*b)*arctan(b*x^(
3/2)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 1.97, size = 347, normalized size = 2.69

$$\left[\frac{3\,((Bab^{2}-5\,Ab^{3})x^{8}+2\,(Ba^{2}b-5\,Aab^{2})x^{5}+(Ba^{3}-5\,Aa^{2}b)x^{2})\sqrt{-ab}\,\log\left(\frac{bx^{3}+2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^{3}+a}\right)+2\,(3\,(Ba^{2}b^{2}-5\,Aab^{3})x^{6}-8\,Aa^{3}b+5\,(Ba^{3}b-5\,Aa^{2}b^{2})x^{3})\sqrt{x}}{24\,(a^{4}b^{3}x^{8}+2\,a^{5}b^{2}x^{5}+a^{6}bx^{2})},\ \frac{3\,((Bab^{2}-5\,Ab^{3})x^{8}+2\,(Ba^{2}b-5\,Aab^{2})x^{5}+(Ba^{3}-5\,Aa^{2}b)x^{2})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)+(3\,(Ba^{2}b^{2}-5\,Aab^{3})x^{6}-8\,Aa^{3}b+5\,(Ba^{3}b-5\,Aa^{2}b^{2})x^{3})\sqrt{x}}{12\,(a^{4}b^{3}x^{8}+2\,a^{5}b^{2}x^{5}+a^{6}bx^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(3*((B*a*b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 -
5*A*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 +
a)) + 2*(3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b
^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2*x^5 + a^6*b*x^2), 1/12*(3*((B*a*
b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 - 5*A*a^2*b)*x^2)
*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) + (3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8
```

```
*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2
*x^5 + a^6*b*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.70, size = 88, normalized size = 0.68

$$\frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^3}-\frac{2\,A}{3\,a^3x^{\frac{3}{2}}}+\frac{3\,Babx^{\frac{9}{2}}-7\,Ab^2x^{\frac{9}{2}}+5\,Ba^2x^{\frac{3}{2}}-9\,Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(B*a - 5*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/3*A/(a^3*
x^(3/2)) + 1/12*(3*B*a*b*x^(9/2) - 7*A*b^2*x^(9/2) + 5*B*a^2*x^(3/2) - 9*A*
a*b*x^(3/2))/((b*x^3 + a)^2*a^3)
```

Mupad [B]
time = 2.73, size = 163, normalized size = 1.26

$$-\frac{\frac{2\,A}{3\,a}+\frac{5\,x^3\,(5\,A\,b-B\,a)}{12\,a^2}+\frac{b\,x^6\,(5\,A\,b-B\,a)}{4\,a^3}}{a^2\,x^{3/2}+b^2\,x^{15/2}+2\,a\,b\,x^{9/2}}-\frac{\mathrm{atan}\left(\frac{8\,a^{7/2}\,\sqrt{b}\,x^{3/2}\,(86400\,A^2\,a^9\,b^5-34560\,A\,B\,a^{10}\,b^4+3456\,B^2\,a^{11}\,b^3)}{(5\,A\,b-B\,a)\,(138240\,A\,a^{13}\,b^4-27648\,B\,a^{14}\,b^3)}\right)\,(5\,A\,b-B\,a)}{4\,a^{7/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x)
```

```
[Out] - ((2*A)/(3*a) + (5*x^3*(5*A*b - B*a))/(12*a^2) + (b*x^6*(5*A*b - B*a))/(4*
a^3))/(a^2*x^(3/2) + b^2*x^(15/2) + 2*a*b*x^(9/2)) - (atan((8*a^(7/2)*b^(1/
2)*x^(3/2)*(86400*A^2*a^9*b^5 + 3456*B^2*a^11*b^3 - 34560*A*B*a^10*b^4))/((
5*A*b - B*a)*(138240*A*a^13*b^4 - 27648*B*a^14*b^3)))*(5*A*b - B*a))/(4*a^(
7/2)*b^(1/2))
```

3.178 $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=351

$$-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}+\frac{11(17Ab-5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}}-$$

```
[Out] -11/180*(17*A*b-5*B*a)/a^3/b/x^(5/2)+1/6*(A*b-B*a)/a/b/x^(5/2)/(b*x^3+a)^2+
1/36*(17*A*b-5*B*a)/a^2/b/x^(5/2)/(b*x^3+a)-11/108*(17*A*b-5*B*a)*arctan(b^
(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)-11/216*(17*A*b-5*B*a)*arctan(-3^(1/
2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)-11/216*(17*A*b-5*B*a)*arctan
(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)+11/432*(17*A*b-5*B*a)*
ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(23/6)/b^(1/6)*3^(1
/2)-11/432*(17*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1
/2))/a^(23/6)/b^(1/6)*3^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {468, 296, 331, 335, 215, 648, 632, 210, 642, 211}

$$\frac{11(17Ab-5aB)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\text{ArcTan}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\text{ArcTan}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}}+\frac{11(17Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

```
[Out] (-11*(17*A*b - 5*a*B))/(180*a^3*b*x^(5/2)) + (A*b - a*B)/(6*a*b*x^(5/2)*(a
+ b*x^3)^2) + (17*A*b - 5*a*B)/(36*a^2*b*x^(5/2)*(a + b*x^3)) + (11*(17*A*b
 - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(23/6)*b^(1
/6)) - (11*(17*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/
(216*a^(23/6)*b^(1/6)) - (11*(17*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)])/(108*a^(23/6)*b^(1/6)) + (11*(17*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(23/6)*b^(1/6)) - (11*(
17*A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])
/(144*Sqrt[3]*a^(23/6)*b^(1/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{\left(\frac{17Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}+\frac{(11(17Ab-5aB))\int \frac{1}{x^{7/2}\left(a+bx^3\right)}\,dx}{72a^2b}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{(11(17Ab-5aB))}{72a}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{(11(17Ab-5aB))}{}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{(11(17Ab-5aB))}{}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)\,\mathrm{ta}}{108a^{23/}}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)\,\mathrm{ta}}{108a^{23/}}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}+\frac{11(17Ab-5aB)\,\mathrm{ta}}{216}$$

Mathematica [A]
time = 0.56, size = 209, normalized size = 0.60

$$\frac{-\frac{6a^{5/6}\left(187Ab^2x^6+a^2\left(72A-85Bx^3\right)+abx^3\left(289A-55Bx^3\right)\right)}{x^{5/2}\left(a+bx^3\right)^2}+\frac{110(-17Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}}+\frac{55(17Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{\sqrt[6]{b}}+\frac{55\sqrt{3}(-17Ab+5aB)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{\sqrt[6]{b}}}{1080a^{23/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

```
[Out] ((-6*a^(5/6)*(187*A*b^2*x^6 + a^2*(72*A - 85*B*x^3) + a*b*x^3*(289*A - 55*B
*x^3)))/(x^(5/2)*(a + b*x^3)^2) + (110*(-17*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sq
rt[x])/a^(1/6)])/b^(1/6) + (55*(17*A*b - 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x
)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(1/6) + (55*Sqrt[3]*(-17*A*b + 5*a*B)*ArcTa
nh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(1/6))/(1080
*a^(23/6))
```

Maple [A]
time = 0.37, size = 237, normalized size = 0.68

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{23}{72}b^2A-\frac{11}{72}abB\right)x^{\frac{7}{2}}+\frac{a(29Ab-17Ba)\sqrt{x}}{72}}{\left(b\,x^3+a\right)^2}+\frac{11(17Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{\right.}}{12a}}{}\right.}{}$
default	$-\frac{2\left(\frac{\left(\frac{23}{72}b^2A-\frac{11}{72}abB\right)x^{\frac{7}{2}}+\frac{a(29Ab-17Ba)\sqrt{x}}{72}}{\left(b\,x^3+a\right)^2}+\frac{11(17Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{\right.}}{12a}}{}\right.}{}$
risch	$-\frac{2A}{5a^3x^{\frac{5}{2}}}-\frac{23x^{\frac{7}{2}}b^2A}{36a^3(b\,x^3+a)^2}+\frac{11x^{\frac{7}{2}}bB}{36a^2(b\,x^3+a)^2}-\frac{29A\sqrt{x}\,b}{36a^2(b\,x^3+a)^2}+\frac{17B\sqrt{x}}{36a(b\,x^3+a)^2}-\frac{187Ab\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{\right.}}{432a^{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^3*(((23/72*b^2*A-11/72*a*b*B)*x^(7/2)+1/72*a*(29*A*b-17*B*a)*x^(1/2))/
(b*x^3+a)^2+11/72*(17*A*b-5*B*a)*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1
/6))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3
))+1/6/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*
(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)
*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))))-2/5*A/a^3/x^(5/2)
```

Maxima [A]

time = 0.51, size = 346, normalized size = 0.99

$$\frac{11\,(5\,Bab-17\,Ab^2)x^6+17\,(5\,Ba^2-17\,Aab)x^3-72\,Aa^2}{180\left(a^3b^2x^{\frac{17}{2}}+2\,a^4bx^{\frac{11}{2}}+a^5x^{\frac{5}{2}}\right)}+\frac{11\left(\frac{\sqrt{3}\,(5\,Ba-17\,Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(5\,Ba-17\,Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(5\,Bab^{\frac{1}{3}}-17\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(5\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}-17\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(5\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}-17\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{432\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/180*(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^2
)/(a^3*b^2*x^(17/2) + 2*a^4*b*x^(11/2) + a^5*x^(5/2)) + 11/432*(sqrt(3)*(5*
```

```
B*a - 17*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a
^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 17*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqr
t(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(1/3) - 17*A*b^(
4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a
^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*arctan((s
qrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/
3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*
arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3))
)/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2690 vs. $2(261) = 522$.
time = 3.32, size = 2690, normalized size = 7.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2160*(220*sqrt(3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6
 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3
+ 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23
*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a
^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^
2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + (25*
B^2*a^2 - 170*A*B*a*b + 289*A^2*b^2)*x + (5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(
15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3
*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A
^6*b^6)/(a^23*b))^(1/6))*a^19*b*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 270
9375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4
- 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + 2*sqrt(3)*(5*B
*a^20*b - 17*A*a^19*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 27
09375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4
 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) - sqrt(3)*(1562
5*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3
*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b
^6))/(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 122825
00*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 2413
7569*A^6*b^6)) + 220*sqrt(3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625
*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*
a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^
6)/(a^23*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^8*(-(15625*B^6*a^6 - 318750
*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 3132037
5*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3
) + (25*B^2*a^2 - 170*A*B*a*b + 289*A^2*b^2)*x - (5*B*a^5 - 17*A*a^4*b)*sqr
```

```
t(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 1228
2500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24
137569*A^6*b^6)/(a^23*b))^(1/6))*a^19*b*(-(15625*B^6*a^6 - 318750*A*B^5*a^5
*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*
a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + 2*sqrt
(3)*(5*B*a^20*b - 17*A*a^19*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^
5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2
*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + sqrt(
3)*(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500
*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 241375
69*A^6*b^6))/(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2
- 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^
5 + 24137569*A^6*b^6)) - 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*
B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a
^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6
)/(a^23*b))^(1/6)*log(121*a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 27093
75*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 -
42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + 121*(25*B^2*a^2
- 170*A*B*a*b + 289*A^2*b^2)*x + 121*(5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(1562
5*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3
*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b
^6)/(a^23*b))^(1/6)) + 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^
6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3
*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/
(a^23*b))^(1/6)*log(121*a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375
*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42
595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + 121*(25*B^2*a^2 -
170*A*B*a*b + 289*A^2*b^2)*x - 121*(5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(15625*
B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a
^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6
)/(a^23*b))^(1/6)) + 110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6
*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*
b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(
a^23*b))^(1/6)*log(11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A
^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 4259
5710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*
sqrt(x)) - 110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6 - 318
750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 3132
0375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(
1/6)*log(-11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^
4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.21, size = 334, normalized size = 0.95

$$\frac{11\sqrt{3}\left(5\,(ab^5)^{\frac{1}{6}}Ba-17\,(ab^5)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b}-\frac{11\sqrt{3}\left(5\,(ab^5)^{\frac{1}{6}}Ba-17\,(ab^5)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b}+\frac{11\left(5\,(ab^5)^{\frac{1}{6}}Ba-17\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^4b}+\frac{11\left(5\,(ab^5)^{\frac{1}{6}}Ba-17\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^4b}+\frac{11\left(5\,(ab^5)^{\frac{1}{6}}Ba-17\,(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^4b}+\frac{11\,Babx^{\frac{7}{2}}-23\,Ab^2x^{\frac{7}{2}}+17\,Ba^2\sqrt{x}-29\,Aab\sqrt{x}}{36\,(bx^3+a)^2a^3}-\frac{2\,A}{5\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 11/432*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqr
t(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b) - 11/432*sqrt(3)*(5*(a*b^5)^(1/
6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a^4*b) + 11/216*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arcta
n((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^4*b) + 11/216*(5*(a*b^5
)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x
))/(a/b)^(1/6))/(a^4*b) + 11/108*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*
b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^4*b) + 1/36*(11*B*a*b*x^(7/2) - 23*A*b^2*
x^(7/2) + 17*B*a^2*sqrt(x) - 29*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^3) - 2/5*A/
(a^3*x^(5/2))
```

Mupad [B]
time = 2.96, size = 2500, normalized size = 7.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x)
```

```
[Out] - ((2*A)/(5*a) + (17*x^3*(17*A*b - 5*B*a))/(180*a^2) + (11*b*x^6*(17*A*b -
5*B*a))/(180*a^3))/(a^2*x^(5/2) + b^2*x^(17/2) + 2*a*b*x^(11/2)) - (atan(((
(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 +
 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5
21928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b - 5*B*a)*(51243917694905548
8*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*
a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(
17*A*b - 5*B*a)*11i)/(216*(-a)^(23/6)*b^(1/6)) + ((x^(1/2)*(443639472636450
816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B
^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a
^16*b^8) + (11*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837
801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 45215221495504
8960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a)*11i)/(216
```

```
*(-a)^(23/6)*b^(1/6)))/((11*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331
9819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549
423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b
 - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5
 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/
(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a))/(216*(-a)^(23/6)*b^(1/6)) - (1
1*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5
 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 -
 521928791337000960*A^3*B*a^16*b^8) + (11*(17*A*b - 5*B*a)*(512439176949055
488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^
2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))
*(17*A*b - 5*B*a))/(216*(-a)^(23/6)*b^(1/6))))*(17*A*b - 5*B*a)*11i)/(108*(
-a)^(23/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1
/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 2302
62702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928
791337000960*A^3*B*a^16*b^8) - (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*
(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 1329859
45575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^
(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(
17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331981981056000
0*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*
B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) + (11*((3^(1/2)*1i)/2 - 1
/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B
^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*
a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)))/((11*
((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^
15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^
7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) -
 (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b
^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 4
52152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6))))/(216*(-a)^(2
3/6)*b^(1/6)) - (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(44363
9472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 23026270206044
1600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5219287913370009
60*A^3*B*a^16*b^8) + (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(512439176
949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 13298594557501440
0*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(
1/6))))/(216*(-a)^(23/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)
*11i)/(108*(-a)^(23/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)*(17*A*b -
5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^1
9*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*
b^6 - 521928791337000960*A^3*B*a^16*b^8) - (11*((3^(1/2)*1i)/2 + 1/2)*(17*A
*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b
^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7)
)/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)) + (((3^(1/2)*1i
```

```
)/2 + 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331
9819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549
423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) + (11*((3^(1/
2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 1303783
7801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 4521522149550
48960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(
1/6)))/((11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(4436394726364
50816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2
*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^...
```

3.179 $\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=103

$$\frac{2a^2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

```
[Out] 2/9*a^2*(A*b-B*a)*(b*x^3+a)^(3/2)/b^4-2/15*a*(2*A*b-3*B*a)*(b*x^3+a)^(5/2)/
b^4+2/21*(A*b-3*B*a)*(b*x^3+a)^(7/2)/b^4+2/27*B*(b*x^3+a)^(9/2)/b^4
```

Rubi [A]
time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{2a^2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^4}+\frac{2\left(a+bx^3\right)^{7/2}(Ab-3aB)}{21b^4}-\frac{2a\left(a+bx^3\right)^{5/2}(2Ab-3aB)}{15b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b
*x^3)^(5/2))/(15*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(7/2))/(21*b^4) + (2*B
*(a + b*x^3)^(9/2))/(27*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{1}{3}\text{Subst}\left(\int x^2\sqrt{a+bx}\,(A+Bx)\,dx,x,x^3\right)$$
$$=\frac{1}{3}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)\sqrt{a+bx}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{3/2}}{b^3}+\frac{(\right.\right.$$
$$=\frac{2a^2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2(Ab-3aB)\left(\right.}{21b^4}$$

Mathematica [A]
time = 0.05, size = 75, normalized size = 0.73

$$\frac{2\left(a+bx^3\right)^{3/2}\left(-16a^3B+24a^2b\left(A+Bx^3\right)-6ab^2x^3\left(6A+5Bx^3\right)+5b^3x^6\left(9A+7Bx^3\right)\right)}{945b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^3) - 6*a*b^2*x^3*(6*A +
 5*B*x^3) + 5*b^3*x^6*(9*A + 7*B*x^3)))/(945*b^4)
```

Maple [A]
time = 0.30, size = 166, normalized size = 1.61

method	result
gosper	$\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(35B\,x^{9}b^{3}+45A\,b^{3}x^{6}-30Ba\,b^{2}x^{6}-36Aa\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+24A\,a^{2}b-16B\,a^{3}\right)}{945b^{4}}$
trager	$\frac{2\left(35B\,b^{4}x^{12}+45A\,b^{4}x^{9}+5Ba\,b^{3}x^{9}+9a\,b^{3}A\,x^{6}-6B\,a^{2}b^{2}x^{6}-12A\,a^{2}b^{2}x^{3}+8B\,a^{3}b\,x^{3}+24A\,a^{3}b-16B\,a^{4}\right)\sqrt{b\,x^{3}+a}}{945b^{4}}$
risch	$\frac{2\left(35B\,b^{4}x^{12}+45A\,b^{4}x^{9}+5Ba\,b^{3}x^{9}+9a\,b^{3}A\,x^{6}-6B\,a^{2}b^{2}x^{6}-12A\,a^{2}b^{2}x^{3}+8B\,a^{3}b\,x^{3}+24A\,a^{3}b-16B\,a^{4}\right)\sqrt{b\,x^{3}+a}}{945b^{4}}$
elliptic	$\frac{2B\,x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{2\left(Ab+\frac{Ba}{9}\right)x^{9}\sqrt{b\,x^{3}+a}}{21b}+\frac{2\left(Aa-\frac{6a\left(Ab+\frac{Ba}{9}\right)}{7b}\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\left(Aa-\frac{6a\left(Ab+\frac{Ba}{9}\right)}{7b}\right)x^{3}\sqrt{}}{45b^{2}}$
default	$B\left(\frac{2x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{2a\,x^{9}\sqrt{b\,x^{3}+a}}{189b}-\frac{4a^{2}x^{6}\sqrt{b\,x^{3}+a}}{315b^{2}}+\frac{16a^{3}x^{3}\sqrt{b\,x^{3}+a}}{945b^{3}}-\frac{32a^{4}\sqrt{b\,x^{3}+a}}{945b^{4}}\right)+A$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/27*x^12*(b*x^3+a)^(1/2)+2/189*a/b*x^9*(b*x^3+a)^(1/2)-4/315*a^2/b^2*x^
6*(b*x^3+a)^(1/2)+16/945*a^3/b^3*x^3*(b*x^3+a)^(1/2)-32/945*a^4/b^4*(b*x^3+
a)^(1/2))+A*(2/21*x^9*(b*x^3+a)^(1/2)+2/105*a/b*x^6*(b*x^3+a)^(1/2)-8/315*a
^2/b^2*x^3*(b*x^3+a)^(1/2)+16/315*a^3/b^3*(b*x^3+a)^(1/2))
```

Maxima [A]
time = 0.29, size = 118, normalized size = 1.15

$$\frac{2}{945}\,B\left(\frac{35\,(bx^3+a)^{\frac{9}{2}}}{b^4}-\frac{135\,(bx^3+a)^{\frac{7}{2}}a}{b^4}+\frac{189\,(bx^3+a)^{\frac{5}{2}}a^2}{b^4}-\frac{105\,(bx^3+a)^{\frac{3}{2}}a^3}{b^4}\right)+\frac{2}{315}\,A\left(\frac{15\,(bx^3+a)^{\frac{7}{2}}}{b^3}-\frac{42\,(bx^3+a)^{\frac{5}{2}}a}{b^3}+\frac{35\,(bx^3+a)^{\frac{3}{2}}a^2}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/945*B*(35*(b*x^3 + a)^(9/2)/b^4 - 135*(b*x^3 + a)^(7/2)*a/b^4 + 189*(b*x^
3 + a)^(5/2)*a^2/b^4 - 105*(b*x^3 + a)^(3/2)*a^3/b^4) + 2/315*A*(15*(b*x^3
+ a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3 + a)^(3/2)*a^2/b^3)
```

Fricas [A]
time = 2.97, size = 99, normalized size = 0.96

$$\frac{2\,(35\,Bb^4x^{12}+5\,(Bab^3+9\,Ab^4)x^9-3\,(2\,Ba^2b^2-3\,Aab^3)x^6-16\,Ba^4+24\,Aa^3b+4\,(2\,Ba^3b-3\,Aa^2b^2)x^3)\sqrt{bx^3+a}}{945\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^4*x^12 + 5*(B*a*b^3 + 9*A*b^4)*x^9 - 3*(2*B*a^2*b^2 - 3*A*a*b
^3)*x^6 - 16*B*a^4 + 24*A*a^3*b + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^3)*sqrt(b*x
^3 + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(100) = 200$.
time = 0.37, size = 219, normalized size = 2.13

$$\begin{cases}\frac{16Aa^3\sqrt{a+bx^3}}{315b^3}-\frac{8Aa^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Aax^6\sqrt{a+bx^3}}{105b}+\frac{2Ax^9\sqrt{a+bx^3}}{21}-\frac{32Ba^4\sqrt{a+bx^3}}{945b^4}+\frac{16Ba^3x^3\sqrt{a+bx^3}}{945b^3}-\frac{4Ba^2x^6\sqrt{a+bx^3}}{315b^2}+\frac{2Bax^9\sqrt{a+bx^3}}{189b}+\frac{2Bx^{12}\sqrt{a+bx^3}}{27} & \text{for } b\neq 0\\ \sqrt{a}\left(\frac{Ax^9}{9}+\frac{Bx^{12}}{12}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*A*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*A*a**2*x**3*sqrt(a + b
*x**3)/(315*b**2) + 2*A*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*A*x**9*sqrt(a +
 b*x**3)/21 - 32*B*a**4*sqrt(a + b*x**3)/(945*b**4) + 16*B*a**3*x**3*sqrt(a
 + b*x**3)/(945*b**3) - 4*B*a**2*x**6*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x
**9*sqrt(a + b*x**3)/(189*b) + 2*B*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (s
qrt(a)*(A*x**9/9 + B*x**12/12), True))
```

Giac [A]
time = 1.53, size = 104, normalized size = 1.01

$$\frac{2\left(35\,(bx^3+a)^{\frac{9}{2}}B-135\,(bx^3+a)^{\frac{7}{2}}Ba+189\,(bx^3+a)^{\frac{5}{2}}Ba^2-105\,(bx^3+a)^{\frac{3}{2}}Ba^3+45\,(bx^3+a)^{\frac{7}{2}}Ab-126\,(bx^3+a)^{\frac{5}{2}}Aab+105\,(bx^3+a)^{\frac{3}{2}}Aa^2b\right)}{945\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2)*B - 135*(b*x^3 + a)^(7/2)*B*a + 189*(b*x^3 + a)
^(5/2)*B*a^2 - 105*(b*x^3 + a)^(3/2)*B*a^3 + 45*(b*x^3 + a)^(7/2)*A*b - 126
*(b*x^3 + a)^(5/2)*A*a*b + 105*(b*x^3 + a)^(3/2)*A*a^2*b)/b^4
```

Mupad [B]
time = 2.72, size = 154, normalized size = 1.50

$$\frac{2\,B\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{x^9\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{21\,b}+\frac{8\,a^2\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{45\,b^3}+\frac{x^6\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{15\,b}-\frac{4\,a\,x^3\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (2*B*x^12*(a + b*x^3)^(1/2))/27 + (x^9*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/9
))/(21*b) + (8*a^2*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^3)^(1
/2))/(45*b^3) + (x^6*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^3)^
(1/2))/(15*b) - (4*a*x^3*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x
^3)^(1/2))/(45*b^2)
```

3.180 $\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=73

$$-\frac{2a(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

```
[Out] -2/9*a*(A*b-B*a)*(b*x^3+a)^(3/2)/b^3+2/15*(A*b-2*B*a)*(b*x^3+a)^(5/2)/b^3+2
/21*B*(b*x^3+a)^(7/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-2aB)}{15b^3}-\frac{2a\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)
^(5/2))/(15*b^3) + (2*B*(a + b*x^3)^(7/2))/(21*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\sqrt{a+bx^3}\,(A+Bx^3)\,dx = \frac{1}{3}\text{Subst}\left(\int x\sqrt{a+bx}\,(A+Bx)\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)\sqrt{a+bx}}{b^2} + \frac{(Ab-2aB)(a+bx)^{3/2}}{b^2} + \frac{B(a+bx}{b^2}\right.\right.$$
$$= -\frac{2a(Ab-aB)\,(a+bx^3)^{3/2}}{9b^3} + \frac{2(Ab-2aB)\,(a+bx^3)^{5/2}}{15b^3} + \frac{2B(a+bx^3)^{7/2}}{21b^3}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.77

$$\frac{2(a+bx^3)^{3/2}\,(-14aAb+8a^2B+21Ab^2x^3-12abBx^3+15b^2Bx^6)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(-14*a*A*b + 8*a^2*B + 21*A*b^2*x^3 - 12*a*b*B*x^3 + 1
5*b^2*B*x^6))/(315*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(61) = 122.
time = 0.31, size = 126, normalized size = 1.73

method	result
gosper	$-\frac{2(b\,x^3+a)^{\frac{3}{2}}(-15b^2B\,x^6-21A\,b^2x^3+12Bab\,x^3+14abA-8a^2B)}{315b^3}$
trager	$-\frac{2(-15B\,x^9b^3-21A\,b^3x^6-3Ba\,b^2x^6-7Aa\,b^2x^3+4B\,a^2b\,x^3+14A\,a^2b-8B\,a^3)\sqrt{b\,x^3+a}}{315b^3}$
risch	$-\frac{2(-15B\,x^9b^3-21A\,b^3x^6-3Ba\,b^2x^6-7Aa\,b^2x^3+4B\,a^2b\,x^3+14A\,a^2b-8B\,a^3)\sqrt{b\,x^3+a}}{315b^3}$
elliptic	$\frac{2B\,x^9\sqrt{b\,x^3+a}}{21}+\frac{2\left(Ab+\frac{Ba}{7}\right)x^6\sqrt{b\,x^3+a}}{15b}+\frac{2\left(Aa-\frac{4a\left(Ab+\frac{Ba}{7}\right)}{5b}\right)x^3\sqrt{b\,x^3+a}}{9b}-\frac{4a\left(Aa-\frac{4a\left(Ab+\frac{Ba}{7}\right)}{5b}\right)\sqrt{b\,x}}{9b^2}$
default	$B\left(\frac{2x^9\sqrt{b\,x^3+a}}{21}+\frac{2a\,x^6\sqrt{b\,x^3+a}}{105b}-\frac{8a^2x^3\sqrt{b\,x^3+a}}{315b^2}+\frac{16a^3\sqrt{b\,x^3+a}}{315b^3}\right)+A\left(\frac{2x^6\sqrt{b\,x^3+a}}{15}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/21*x^9*(b*x^3+a)^(1/2)+2/105*a/b*x^6*(b*x^3+a)^(1/2)-8/315*a^2/b^2*x^3
*(b*x^3+a)^(1/2)+16/315*a^3/b^3*(b*x^3+a)^(1/2))+A*(2/15*x^6*(b*x^3+a)^(1/2
)+2/45*a/b*x^3*(b*x^3+a)^(1/2)-4/45*a^2/b^2*(b*x^3+a)^(1/2))
```

Maxima [A]
time = 0.27, size = 84, normalized size = 1.15

$$\frac{2}{315}B\left(\frac{15(bx^3+a)^{\frac{7}{2}}}{b^3}-\frac{42(bx^3+a)^{\frac{5}{2}}a}{b^3}+\frac{35(bx^3+a)^{\frac{3}{2}}a^2}{b^3}\right)+\frac{2}{45}A\left(\frac{3(bx^3+a)^{\frac{5}{2}}}{b^2}-\frac{5(bx^3+a)^{\frac{3}{2}}a}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*B*(15*(b*x^3 + a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3
+ a)^(3/2)*a^2/b^3) + 2/45*A*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)
*a/b^2)
```

Fricas [A]
time = 2.70, size = 75, normalized size = 1.03

$$\frac{2\left(15\,Bb^3x^9+3\,(Bab^2+7\,Ab^3)x^6+8\,Ba^3-14\,Aa^2b-(4\,Ba^2b-7\,Aab^2)x^3\right)\sqrt{bx^3+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^3*x^9 + 3*(B*a*b^2 + 7*A*b^3)*x^6 + 8*B*a^3 - 14*A*a^2*b - (4
*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(70) = 140.
time = 0.25, size = 168, normalized size = 2.30

$$\begin{cases} -\frac{4Aa^2\sqrt{a+bx^3}}{45b^2}+\frac{2Aax^3\sqrt{a+bx^3}}{45b}+\frac{2Ax^6\sqrt{a+bx^3}}{15}+\frac{16Ba^3\sqrt{a+bx^3}}{315b^3}-\frac{8Ba^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Bax^6\sqrt{a+bx^3}}{105b}+\frac{2Bx^9\sqrt{a+bx^3}}{21} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-4*A*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*A*a*x**3*sqrt(a + b*x**
3)/(45*b) + 2*A*x**6*sqrt(a + b*x**3)/15 + 16*B*a**3*sqrt(a + b*x**3)/(315*
b**3) - 8*B*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x**6*sqrt(a + b*x
**3)/(105*b) + 2*B*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*(A*x**6/6
+ B*x**9/9), True))
```

Giac [A]
time = 1.13, size = 73, normalized size = 1.00

$$\frac{2\left(15\,(bx^3+a)^{\frac{7}{2}}B-42\,(bx^3+a)^{\frac{5}{2}}Ba+35\,(bx^3+a)^{\frac{3}{2}}Ba^2+21\,(bx^3+a)^{\frac{5}{2}}Ab-35\,(bx^3+a)^{\frac{3}{2}}Aab\right)}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(15*(b*x^3 + a)^(7/2)*B - 42*(b*x^3 + a)^(5/2)*B*a + 35*(b*x^3 + a)^(
3/2)*B*a^2 + 21*(b*x^3 + a)^(5/2)*A*b - 35*(b*x^3 + a)^(3/2)*A*a*b)/b^3
```

Mupad [B]
time = 2.66, size = 114, normalized size = 1.56

$$\frac{2\,B\,x^9\,\sqrt{b\,x^3+a}}{21}+\frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{15\,b}-\frac{2\,a\,\left(2\,A\,a-\frac{4\,a\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b^2}+\frac{x^3\,\left(2\,A\,a-\frac{4\,a\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (2*B*x^9*(a + b*x^3)^(1/2))/21 + (x^6*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/7)
)/(15*b) - (2*a*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(1/2)
)/(9*b^2) + (x^3*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(1/2
))/(9*b)
```

3.181 $\int x^2\sqrt{a+bx^3}\,(A+Bx^3)\,dx$

Optimal. Leaf size=46

$$\frac{2(Ab-aB)\,(a+bx^3)^{3/2}}{9b^2}+\frac{2B(a+bx^3)^{5/2}}{15b^2}$$

[Out] 2/9*(A*b-B*a)*(b*x^3+a)^(3/2)/b^2+2/15*B*(b*x^3+a)^(5/2)/b^2

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {455, 45}

$$\frac{2(a+bx^3)^{3/2}\,(Ab-aB)}{9b^2}+\frac{2B(a+bx^3)^{5/2}}{15b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]

[Out] (2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^2) + (2*B*(a + b*x^3)^(5/2))/(15*b^2
)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx^3}\,(A+Bx^3)\,dx&=\frac{1}{3}\text{Subst}\left(\int\sqrt{a+bx}\,(A+Bx)\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int\left(\frac{(Ab-aB)\sqrt{a+bx}}{b}+\frac{B(a+bx)^{3/2}}{b}\right)dx,x,x^3\right)\\&=\frac{2(Ab-aB)\,(a+bx^3)^{3/2}}{9b^2}+\frac{2B(a+bx^3)^{5/2}}{15b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.74

$$\frac{2(a+bx^3)^{3/2}(5Ab-2aB+3bBx^3)}{45b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^3))/(45*b^2)
```

Maple [A]
time = 0.31, size = 69, normalized size = 1.50

method	result	size
gosper	$\frac{2(b\,x^3+a)^{\frac{3}{2}}(3bB\,x^3+5Ab-2Ba)}{45b^2}$	31
trager	$\frac{2(3b^2B\,x^6+5A\,b^2x^3+Bab\,x^3+5abA-2a^2B)\sqrt{b\,x^3+a}}{45b^2}$	52
risch	$\frac{2(3b^2B\,x^6+5A\,b^2x^3+Bab\,x^3+5abA-2a^2B)\sqrt{b\,x^3+a}}{45b^2}$	52
default	$B\left(\frac{2x^6\sqrt{b\,x^3+a}}{15}+\frac{2a\,x^3\sqrt{b\,x^3+a}}{45b}-\frac{4a^2\sqrt{b\,x^3+a}}{45b^2}\right)+\frac{2A(b\,x^3+a)^{\frac{3}{2}}}{9b}$	69
elliptic	$\frac{2B\,x^6\sqrt{b\,x^3+a}}{15}+\frac{2\left(Ab+\frac{Ba}{5}\right)x^3\sqrt{b\,x^3+a}}{9b}+\frac{2\left(Aa-\frac{2a\left(Ab+\frac{Ba}{5}\right)}{3b}\right)\sqrt{b\,x^3+a}}{3b}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/15*x^6*(b*x^3+a)^(1/2)+2/45*a/b*x^3*(b*x^3+a)^(1/2)-4/45*a^2/b^2*(b*x^
3+a)^(1/2))+2/9*A*(b*x^3+a)^(3/2)/b
```

Maxima [A]
time = 0.38, size = 49, normalized size = 1.07

$$\frac{2}{45}\,B\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^2}-\frac{5\,(bx^3+a)^{\frac{3}{2}}a}{b^2}\right)+\frac{2\,(bx^3+a)^{\frac{3}{2}}A}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)*a/b^2) + 2/9*(b*x^3 +
 a)^(3/2)*A/b
```

Fricas [A]
time = 2.57, size = 50, normalized size = 1.09

$$\frac{2\,(3\,Bb^2x^6 + (Bab + 5\,Ab^2)x^3 - 2\,Ba^2 + 5\,Aab)\sqrt{bx^3 + a}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^6 + (B*a*b + 5*A*b^2)*x^3 - 2*B*a^2 + 5*A*a*b)*sqrt(b*x^3 +
 a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(44) = 88$.
time = 0.14, size = 117, normalized size = 2.54

$$\begin{cases} \frac{2Aa\sqrt{a+bx^3}}{9b} + \frac{2Ax^3\sqrt{a+bx^3}}{9} - \frac{4Ba^2\sqrt{a+bx^3}}{45b^2} + \frac{2Bax^3\sqrt{a+bx^3}}{45b} + \frac{2Bx^6\sqrt{a+bx^3}}{15} & \text{for}\ b \neq 0 \\ \sqrt{a}\left(\frac{Ax^3}{3} + \frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((2*A*a*sqrt(a + b*x**3)/(9*b) + 2*A*x**3*sqrt(a + b*x**3)/9 - 4*B
*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*B*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*B
*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**6/6), True)
)
```

Giac [A]
time = 1.11, size = 44, normalized size = 0.96

$$\frac{2\left(3\,(bx^3 + a)^{\frac{5}{2}}B - 5\,(bx^3 + a)^{\frac{3}{2}}Ba + 5\,(bx^3 + a)^{\frac{3}{2}}Ab\right)}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(3*(b*x^3 + a)^(5/2)*B - 5*(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)
*A*b)/b^2
```

Mupad [B]
time = 2.60, size = 44, normalized size = 0.96

$$\frac{6\,B\,(b\,x^3 + a)^{5/2} + 10\,A\,b\,(b\,x^3 + a)^{3/2} - 10\,B\,a\,(b\,x^3 + a)^{3/2}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (6*B*(a + b*x^3)^(5/2) + 10*A*b*(a + b*x^3)^(3/2) - 10*B*a*(a + b*x^3)^(3/2
))/(45*b^2)
```

3.182 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x}\,dx$

Optimal. Leaf size=64

$$\frac{2}{3}A\sqrt{a+bx^3}+\frac{2B(a+bx^3)^{3/2}}{9b}-\frac{2}{3}\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/3*
A*(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 81, 52, 65, 214}

$$\frac{2}{3}A\sqrt{a+bx^3}-\frac{2}{3}\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2B(a+bx^3)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

```
[Out] (2*A*Sqrt[a + b*x^3])/3 + (2*B*(a + b*x^3)^(3/2))/(9*b) - (2*Sqrt[a]*A*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x}\,dx, x, x^3\right)\\ &= \frac{2B(a+bx^3)^{3/2}}{9b}+\frac{1}{3}A\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)\\ &= \frac{2}{3}A\sqrt{a+bx^3}+\frac{2B(a+bx^3)^{3/2}}{9b}+\frac{1}{3}(aA)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{2}{3}A\sqrt{a+bx^3}+\frac{2B(a+bx^3)^{3/2}}{9b}+\frac{(2aA)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\ &= \frac{2}{3}A\sqrt{a+bx^3}+\frac{2B(a+bx^3)^{3/2}}{9b}-\frac{2}{3}\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 0.95

$$\frac{2\sqrt{a+bx^3}\,(3Ab+aB+bBx^3)}{9b}-\frac{2}{3}\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(3*A*b + a*B + b*B*x^3))/(9*b) - (2*Sqrt[a]*A*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/3
```

Maple [A]
time = 0.30, size = 50, normalized size = 0.78

method	result	size
default	$\frac{2B\left(b\,x^{3}+a\right)^{\frac{3}{2}}}{9b}+A\left(-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\sqrt{a}}{3}+\frac{2\sqrt{b\,x^{3}+a}}{3}\right)$	50
elliptic	$\frac{2B\,x^{3}\sqrt{b\,x^{3}+a}}{9}+\frac{2\left(Ab+\frac{Ba}{3}\right)\sqrt{b\,x^{3}+a}}{3b}-\frac{2A\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\sqrt{a}}{3}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b+A*(-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/
3*(b*x^3+a)^(1/2))
```

Maxima [A]
time = 0.67, size = 67, normalized size = 1.05

$$\frac{1}{3}\left(\sqrt{a}\,\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)+2\,\sqrt{bx^{3}+a}\right)A+\frac{2\,(bx^{3}+a)^{\frac{3}{2}}B}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) +
 2*sqrt(b*x^3 + a))*A + 2/9*(b*x^3 + a)^(3/2)*B/b
```

Fricas [A]
time = 2.45, size = 125, normalized size = 1.95

$$\left[\frac{3\,A\sqrt{a}\,b\log\left(\frac{bx^{3}-2\,\sqrt{bx^{3}+a}\,\sqrt{a}+2\,a}{x^{3}}\right)+2\,(Bbx^{3}+Ba+3\,Ab)\sqrt{bx^{3}+a}}{9\,b},\frac{2\left(3\,A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{bx^{3}+a}\,\sqrt{-a}}{a}\right)+(Bbx^{3}+Ba+3\,Ab)\sqrt{bx^{3}+a}\right)}{9\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/9*(3*A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*
(B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b, 2/9*(3*A*sqrt(-a)*b*arctan(sqrt
(b*x^3 + a)*sqrt(-a)/a) + (B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b]
```

Sympy [A]
time = 10.76, size = 76, normalized size = 1.19

$$-\frac{A\left(-\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2\sqrt{a+bx^3}\right)}{3}-\frac{B\left(\begin{cases}-\sqrt{a}\,x^3 & \text{for } b=0\\ -\frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x,x)
```

```
[Out] -A*(-2*a*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a) - 2*sqrt(a + b*x**3))/3 -
 B*Piecewise((-sqrt(a)*x**3, Eq(b, 0)), (-2*(a + b*x**3)**(3/2)/(3*b), True
))/3
```

Giac [A]
time = 0.74, size = 61, normalized size = 0.95

$$\frac{2\,Aa\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}+\frac{2\,\left(\left(bx^3+a\right)^{\frac{3}{2}}Bb^2+3\,\sqrt{bx^3+a}\,Ab^3\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*A*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/9*((b*x^3 + a)^(3/2)*
B*b^2 + 3*sqrt(b*x^3 + a)*A*b^3)/b^3
```

Mupad [B]
time = 2.71, size = 80, normalized size = 1.25

$$\frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9}+\frac{\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{3}\right)}{3\,b}+\frac{A\,\sqrt{a}\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x,x)
```

```
[Out] (2*B*x^3*(a + b*x^3)^(1/2))/9 + ((a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/3))/(3*
b) + (A*a^(1/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a
^(1/2)))/x^6))/3
```

3.183 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^4}\,dx$

Optimal. Leaf size=84

$$\frac{(Ab+2aB)\sqrt{a+bx^3}}{3a}-\frac{A(a+bx^3)^{3/2}}{3ax^3}-\frac{(Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -1/3*A*(b*x^3+a)^(3/2)/a/x^3-1/3*(A*b+2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2
))/a^(1/2)+1/3*(A*b+2*B*a)*(b*x^3+a)^(1/2)/a
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 52, 65, 214}

$$\frac{\sqrt{a+bx^3}\,(2aB+Ab)}{3a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{A(a+bx^3)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(3*a) - (A*(a + b*x^3)^(3/2))/(3*a*x^3) - (
(A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^4}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{A(a+bx^3)^{3/2}}{3ax^3} + \frac{(Ab+2aB)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)}{6a}$$

$$= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a} - \frac{A(a+bx^3)^{3/2}}{3ax^3} + \frac{1}{6}(Ab+2aB)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.$$

$$= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a} - \frac{A(a+bx^3)^{3/2}}{3ax^3} + \frac{(Ab+2aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \right.}{3b}$$

$$= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a} - \frac{A(a+bx^3)^{3/2}}{3ax^3} - \frac{(Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Mathematica [A]
time = 0.10, size = 65, normalized size = 0.77

$$\frac{\sqrt{a+bx^3}\,(-A+2Bx^3)}{3x^3} + \frac{(-Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-A + 2*B*x^3))/(3*x^3) + ((-(A*b) - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Maple [A]
time = 0.35, size = 72, normalized size = 0.86

method	result
risch	$-\frac{A\sqrt{b\,x^{3}+a}}{3x^{3}}+\frac{2B\sqrt{b\,x^{3}+a}}{3}-\frac{(Ab+2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{3x^{3}}+\frac{2B\sqrt{b\,x^{3}+a}}{3}-\frac{2\left(\frac{Ab}{2}+Ba\right)\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$
default	$A\left(-\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt{b\,x^{3}+a}}{3x^{3}}\right)+B\left(-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\sqrt{a}}{3}+\frac{2\sqrt{b\,x^{3}+a}}{3}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)
+B*(-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/3*(b*x^3+a)^(1/2))
```

Maxima [A]
time = 0.50, size = 107, normalized size = 1.27

$$\frac{1}{6}\left(\frac{b\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{bx^{3}+a}}{x^{3}}\right)A+\frac{1}{3}\left(\sqrt{a}\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)+2\sqrt{bx^{3}+a}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
 - 2*sqrt(b*x^3 + a)/x^3)*A + 1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/
(sqrt(b*x^3 + a) + sqrt(a))) + 2*sqrt(b*x^3 + a))*B
```

Fricas [A]
time = 2.97, size = 143, normalized size = 1.70

$$\left[\frac{(2\,Ba+Ab)\sqrt{a}\,x^{3}\log\left(\frac{bx^{3}-2\sqrt{bx^{3}+a}\sqrt{a}+2\,a}{x^{3}}\right)+2\,(2\,Bax^{3}-Aa)\sqrt{bx^{3}+a}}{6\,ax^{3}},\frac{(2\,Ba+Ab)\sqrt{-a}\,x^{3}\arctan\left(\frac{\sqrt{bx^{3}+a}\sqrt{-a}}{a}\right)+(2\,Bax^{3}-Aa)\sqrt{bx^{3}+a}}{3\,ax^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*((2*B*a + A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*
a)/x^3) + 2*(2*B*a*x^3 - A*a)*sqrt(b*x^3 + a))/(a*x^3), 1/3*((2*B*a + A*b)*
sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (2*B*a*x^3 - A*a)*sqrt(b*
x^3 + a))/(a*x^3)]
```

Sympy [A]
time = 18.67, size = 134, normalized size = 1.60

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}-\frac{2B\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3}+\frac{2Ba}{3\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2B\sqrt{b}\,x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*b*asinh(sqrt(a)/(sqrt(b)*x
**(3/2)))/(3*sqrt(a)) - 2*B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2
*B*a/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*B*sqrt(b)*x**(3/2)/(3*sq
rt(a/(b*x**3) + 1))
```

Giac [A]
time = 0.65, size = 68, normalized size = 0.81

$$\frac{2\sqrt{bx^3+a}\,Bb+\frac{(2\,Bab+Ab^2)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx^3+a}\,Ab}{x^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(b*x^3 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(
-a))/sqrt(-a) - sqrt(b*x^3 + a)*A*b/x^3)/b
```

Mupad [B]
time = 2.93, size = 76, normalized size = 0.90

$$\frac{2\,B\,\sqrt{b\,x^3+a}}{3}-\frac{A\,\sqrt{b\,x^3+a}}{3\,x^3}+\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(\frac{A\,b}{2}+B\,a\right)}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^4,x)
```

```
[Out] (2*B*(a + b*x^3)^(1/2))/3 - (A*(a + b*x^3)^(1/2))/(3*x^3) + (log((((a + b*x
^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*((A*b)/2 + B*a))
/(3*a^(1/2))
```

3.184 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^7}\,dx$

Optimal. Leaf size=88

$$\frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3}-\frac{A(a+bx^3)^{3/2}}{6ax^6}+\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

```
[Out] -1/6*A*(b*x^3+a)^(3/2)/a/x^6+1/12*b*(A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(
1/2))/a^(3/2)+1/12*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 43, 65, 214}

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{\sqrt{a+bx^3}\,(Ab-4aB)}{12ax^3}-\frac{A(a+bx^3)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a*x^3) - (A*(a + b*x^3)^(3/2))/(6*a*x^6
) + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^7}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^3}\,dx, x, x^3\right)$$

$$= -\frac{A(a+bx^3)^{3/2}}{6ax^6} + \frac{\left(-\frac{Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^3\right)}{6a}$$

$$= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6} - \frac{(b(Ab-4aB))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} \ldots\right.}{24a}$$

$$= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6} - \frac{(Ab-4aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x \ldots\right.}{12a}$$

$$= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6} + \frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

Mathematica [A]
time = 0.12, size = 78, normalized size = 0.89

$$\frac{\sqrt{a+bx^3}\,(-2aA-Abx^3-4aBx^3)}{12ax^6} - \frac{b(-Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*a*A - A*b*x^3 - 4*a*B*x^3))/(12*a*x^6) - (b*(-(A*b) +
4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

Maple [A]
time = 0.34, size = 96, normalized size = 1.09

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(A b\,x^{3}+4 B a\,x^{3}+2 A a\right)}{12 x^{6} a}+\frac{b \left(A b-4 B a\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{3}{2}}}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{6 x^{6}}-\frac{\left(A b+4 B a\right)\sqrt{b\,x^{3}+a}}{12 a\,x^{3}}+\frac{b \left(A b-4 B a\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{3}{2}}}$
default	$A\left(\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{3}+a}}{6 x^{6}}-\frac{b\sqrt{b\,x^{3}+a}}{12 a\,x^{3}}\right)+B\left(-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt{b\,x^{3}+}}{3x^{3}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] A*(1/12*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/6*(b*x^3+a)^(1/2)/x^
6-1/12*b*(b*x^3+a)^(1/2)/a/x^3)+B*(-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/
a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(74) = 148$.
time = 0.61, size = 158, normalized size = 1.80

$$-\frac{1}{24}\left(\frac{b^{2}\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left((bx^{3}+a)^{\frac{3}{2}}b^{2}+\sqrt{bx^{3}+a}\,ab^{2}\right)}{(bx^{3}+a)^{2}a-2\,(bx^{3}+a)a^{2}+a^{3}}\right)A+\frac{1}{6}\left(\frac{b\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{bx^{3}+a}}{x^{3}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/24*(b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(
3/2) + 2*((b*x^3 + a)^(3/2)*b^2 + sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a -
 2*(b*x^3 + a)*a^2 + a^3))*A + 1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt
(b*x^3 + a) + sqrt(a)))/sqrt(a) - 2*sqrt(b*x^3 + a)/x^3)*B
```

Fricas [A]
time = 3.05, size = 172, normalized size = 1.95

$$\left[-\frac{(4\,Bab-Ab^{2})\sqrt{a}\,x^{6}\log\left(\frac{bx^{3}+2\,\sqrt{bx^{3}+a}\,\sqrt{a}+2\,a}{x^{3}}\right)+2\,((4\,Ba^{2}+Aab)x^{3}+2\,Aa^{2})\sqrt{bx^{3}+a}}{24\,a^{2}x^{6}},\frac{(4\,Bab-Ab^{2})\sqrt{-a}\,x^{6}\arctan\left(\frac{\sqrt{bx^{3}+a}\,\sqrt{-a}}{a}\right)-((4\,Ba^{2}+Aab)x^{3}+2\,Aa^{2})\sqrt{bx^{3}+a}}{12\,a^{2}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/24*((4*B*a*b - A*b^2)*sqrt(a)*x^6*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a
) + 2*a)/x^3) + 2*((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x
^6), 1/12*((4*B*a*b - A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a
) - ((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(76) = 152$.
time = 42.62, size = 160, normalized size = 1.82

$$-\frac{Aa}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{A\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^{\frac{3}{2}}}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] -A*a/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*sqrt(b)/(4*x**(9/2)*sqr
t(a/(b*x**3) + 1)) - A*b**(3/2)/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + A*b*
*2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x*
*3) + 1)/(3*x**(3/2)) - B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

Giac [A]
time = 0.63, size = 120, normalized size = 1.36

$$\frac{\frac{(4\,Bab^2-Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{4\,(bx^3+a)^{\frac{3}{2}}Bab^2-4\sqrt{bx^3+a}\,Ba^2b^2+(bx^3+a)^{\frac{3}{2}}Ab^3+\sqrt{bx^3+a}\,Aab^3}{ab^2x^6}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - (
4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 + (b*x^3 + a)^(3/
2)*A*b^3 + sqrt(b*x^3 + a)*A*a*b^3)/(a*b^2*x^6))/b
```

Mupad [B]
time = 3.12, size = 93, normalized size = 1.06

$$\frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)\,(A\,b-4\,B\,a)}{24\,a^{3/2}}-\frac{(4\,B\,a^2+A\,b\,a)\,\sqrt{b\,x^3+a}}{12\,a^2\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^7,x)
```

```
[Out] (b*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)
*(A*b - 4*B*a))/(24*a^(3/2)) - ((4*B*a^2 + A*a*b)*(a + b*x^3)^(1/2))/(12*a^
2*x^3) - (A*(a + b*x^3)^(1/2))/(6*x^6)
```

3.185 $\int x^3\sqrt{a+bx^3}\,(A+Bx^3)\,dx$

Optimal. Leaf size=303

$$\frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2}+\frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b}+\frac{2Bx^4(a+bx^3)^{3/2}}{17b}-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2(17Ab}{}$$

```
[Out] 2/17*B*x^4*(b*x^3+a)^(3/2)/b+6/935*a*(17*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+2
/187*(17*A*b-8*B*a)*x^4*(b*x^3+a)^(1/2)/b-4/935*3^(3/4)*a^2*(17*A*b-8*B*a)*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {470, 285, 327, 224}

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6ax\sqrt{a+bx^3}(17Ab-8aB)}{935b^2}+\frac{2x^4\sqrt{a+bx^3}(17Ab-8aB)}{187b}+\frac{2Bx^4(a+bx^3)^{3/2}}{17b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (6*a*(17*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(935*b^2) + (2*(17*A*b - 8*a*B)*x^
4*Sqrt[a + b*x^3])/(187*b) + (2*B*x^4*(a + b*x^3)^(3/2))/(17*b) - (4*3^(3/4
)*Sqrt[2 + Sqrt[3]]*a^2*(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^3\sqrt{a+bx^3}\,(A+Bx^3)\;dx = \frac{2Bx^4(a+bx^3)^{3/2}}{17b} - \frac{\left(2\left(-\frac{17Ab}{2}+4aB\right)\right)\int x^3\sqrt{a+bx^3}\;dx}{17b}$$

$$= \frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b} + \frac{2Bx^4(a+bx^3)^{3/2}}{17b} + \frac{(3a(17Ab-8aB))\int}{187b}$$

$$= \frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2} + \frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b} + \frac{2Bx^4(a+}{17b}$$

$$= \frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2} + \frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b} + \frac{2Bx^4(a+}{17b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.86, size = 89, normalized size = 0.29

$$\frac{2x\sqrt{a+bx^3}\left(-\left((a+bx^3)\left(-17Ab+8aB-11bBx^3\right)\right)+\frac{a(-17Ab+8aB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{187b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-17*A*b + 8*a*B - 11*b*B*x^3)) + (a*(-
17*A*b + 8*a*B)*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (
b*x^3)/a]))/(187*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 657 vs. 2(236) = 472.
time = 0.32, size = 658, normalized size = 2.17 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/17*x^7*(b*x^3+a)^(1/2)+6/187*a*x^4*(b*x^3+a)^(1/2)/b-48/935*a^2*x*(b*x
^3+a)^(1/2)/b^2-32/935*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
```

```
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)))+A*(2/11*x^4*(b*x^3+a)^(1/2)+6/55*a*x*(b*x^3+a)^(1/2)/b
+4/55*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.64, size = 91, normalized size = 0.30

$$\frac{2\left(6\left(8Ba^3-17Aa^2b\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\left(55Bb^3x^7+5\left(3Bab^2+17Ab^3\right)x^4-3\left(8Ba^2b-17Aab^2\right)x\right)\sqrt{bx^3+a}\right)}{935b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/935*(6*(8*B*a^3 - 17*A*a^2*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) +
 (55*B*b^3*x^7 + 5*(3*B*a*b^2 + 17*A*b^3)*x^4 - 3*(8*B*a^2*b - 17*A*a*b^2)*
x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A]
time = 1.25, size = 83, normalized size = 0.27

$$\frac{A\sqrt{a}\,x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{B\sqrt{a}\,x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3)) + B*sqrt(a)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^3*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

3.186 $\int \sqrt{a+bx^3}\,(A+Bx^3)\,dx$

Optimal. Leaf size=268

$$\frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b}+\frac{2Bx(a+bx^3)^{3/2}}{11b}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\cdots}{\left(\left(1+\cdots\right.\right.\cdots}}\cdots}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\cdots\right.}{\left(\left(1+\sqrt{3}\cdots\right.\right.}}\cdots}$$

```
[Out] 2/11*B*x*(b*x^3+a)^(3/2)/b+2/55*(11*A*b-2*B*a)*x*(b*x^3+a)^(1/2)/b+2/55*3^(
3/4)*a*(11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {396, 201, 224}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(11Ab-2aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\sqrt{a+bx^3}\,(11Ab-2aB)}{55b}+\frac{2Bx(a+bx^3)^{3/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(11*A*b - 2*a*B)*x*Sqrt[a + b*x^3])/(55*b) + (2*B*x*(a + b*x^3)^(3/2))/(
11*b) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*b^(4/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2Bx\left(a+bx^3\right)^{3/2}}{11b} - \frac{\left(2\left(-\frac{11Ab}{2}+aB\right)\right)\int \sqrt{a+bx^3}\,dx}{11b}$$

$$= \frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b} + \frac{2Bx\left(a+bx^3\right)^{3/2}}{11b} + \frac{(3a(11Ab-2aB))\int \frac{\ }{\sqrt{a-}}}{55b}$$

$$= \frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b} + \frac{2Bx\left(a+bx^3\right)^{3/2}}{11b} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a(11Ab\ }{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.63, size = 75, normalized size = 0.28

$$\frac{2x\sqrt{a+bx^3}\left(B\left(a+bx^3\right)+\frac{(11Ab-2aB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3) + ((11*A*b - 2*a*B)*Hypergeometric2F1[-
1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(11*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(205) = 410$.
time = 0.32, size = 618, normalized size = 2.31 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/11*x^4*(b*x^3+a)^(1/2)+6/55*a*x*(b*x^3+a)^(1/2)/b+4/55*I/b^2*a^2*3^(1/
2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/5*x*(b*x^3+a
)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 67, normalized size = 0.25

$$-\frac{2\left(3\left(2\,Ba^2-11\,Aab\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(5\,Bb^2x^4+\left(3\,Bab+11\,Ab^2\right)x\right)\sqrt{bx^3+a}\right)}{55\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/55*(3*(2*B*a^2 - 11*A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - (
5*B*b^2*x^4 + (3*B*a*b + 11*A*b^2)*x)*sqrt(b*x^3 + a))/b^2
```

Sympy [A]
time = 1.15, size = 82, normalized size = 0.31

$$\frac{A\sqrt{a}\,x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{B\sqrt{a}\,x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) + B*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(a + b*x^3)^(1/2), x)
```

3.187 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^3}\,dx$

Optimal. Leaf size=269

$$\frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a}-\frac{A(a+bx^3)^{3/2}}{2ax^2}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x \cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} \cdots\right.}}}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} \cdots\right.}}}$$

```
[Out] -1/2*A*(b*x^3+a)^(3/2)/a/x^2+1/10*(5*A*b+4*B*a)*x*(b*x^3+a)^(1/2)/a+1/10*3^
(3/4)*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1
/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 201, 224}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(4aB+5Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{x\sqrt{a+bx^3}\,(4aB+5Ab)}{10a}-\frac{A(a+bx^3)^{3/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
```

```
[Out] ((5*A*b + 4*a*B)*x*Sqrt[a + b*x^3])/(10*a) - (A*(a + b*x^3)^(3/2))/(2*a*x^2
) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(5*A*b + 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^3}\,dx = -\frac{A(a+bx^3)^{3/2}}{2ax^2} - \frac{\left(-\frac{5Ab}{2}-2aB\right)\int \sqrt{a+bx^3}\,dx}{2a}$$

$$= \frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a} - \frac{A(a+bx^3)^{3/2}}{2ax^2} + \frac{1}{20}(3(5Ab+4aB))\int \frac{1}{\sqrt{a+bx^3}}$$

$$= \frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a} - \frac{A(a+bx^3)^{3/2}}{2ax^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(5Ab+4aB)\Big(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 3.96, size = 81, normalized size = 0.30

$$\frac{\sqrt{a+bx^3}\left(-A(a+bx^3)+\frac{(5Ab+4aB)x^3\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) + ((5*A*b + 4*a*B)*x^3*Hypergeometric2F1
[-1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(206) = 412.
time = 0.33, size = 596, normalized size = 2.22

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-4B\,x^3+5A)}{10x^2}-\frac{2i\left(\frac{3Ab}{4}+\frac{3Ba}{5}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\cdots}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{2x^2}+\frac{2Bx\sqrt{b\,x^3+a}}{5}-\frac{2i\left(\frac{3Ab}{4}+\frac{3Ba}{5}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-\cdots}}\cdots}{\cdots}$

default	$B\left(\frac{2x\sqrt{b\,x^{3}+a}}{5}-\frac{2ia\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)))+A*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-a*b^2)^(
1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 57, normalized size = 0.21

$$\frac{3\,(4\,Ba+5\,Ab)\sqrt{b}\,x^2\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(4\,Bbx^3-5\,Ab)\sqrt{bx^3+a}}{10\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/10*(3*(4*B*a + 5*A*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) + (4*
B*b*x^3 - 5*A*b)*sqrt(b*x^3 + a))/(b*x^2)
```

Sympy [A]
time = 1.34, size = 85, normalized size = 0.32

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{B\sqrt{a}\,x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] A*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3)) + B*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^3,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^3, x)
```

3.188 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^6}\,dx$

Optimal. Leaf size=272

$$\frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2}-\frac{A(a+bx^3)^{3/2}}{5ax^5}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}\cdots}{20a\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}\cdots}$$

```
[Out] -1/5*A*(b*x^3+a)^(3/2)/a/x^5+1/20*(A*b-10*B*a)*(b*x^3+a)^(1/2)/a/x^2-1/20*3
^(3/4)*b^(2/3)*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 283, 224}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(Ab-10aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20a\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{\sqrt{a+bx^3}\,(Ab-10aB)}{20ax^2}-\frac{A(a+bx^3)^{3/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

```
[Out] ((A*b - 10*a*B)*Sqrt[a + b*x^3])/(20*a*x^2) - (A*(a + b*x^3)^(3/2))/(5*a*x^
5) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(20*a*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^6}\,dx = -\frac{A(a+bx^3)^{3/2}}{5ax^5} - \frac{\left(\frac{Ab}{2}-5aB\right)\int \frac{\sqrt{a+bx^3}}{x^3}\,dx}{5a}$$

$$= \frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2} - \frac{A(a+bx^3)^{3/2}}{5ax^5} - \frac{(3b(Ab-10aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{40a}$$

$$= \frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2} - \frac{A(a+bx^3)^{3/2}}{5ax^5} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}(Ab-10aB)\Bigg(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 80, normalized size = 0.29

$$\frac{\sqrt{a+bx^3}\left(-2A(a+bx^3)+\frac{\left(\frac{Ab}{2}-5aB\right)x^3\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{10ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + (((A*b)/2 - 5*a*B)*x^3*Hypergeometric2
F1[-2/3, -1/2, 1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(10*a*x^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(209) = 418.
time = 0.34, size = 616, normalized size = 2.26

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(3Ab\,x^3+10Ba\,x^3+4Aa)}{20x^5a}+\frac{i(Ab-10Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\ \sqrt{\cdots}\cdots}{\cdots}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{5x^5}-\frac{(3Ab+10Ba)\sqrt{b\,x^3+a}}{20a\,x^2}-\frac{2i\left(Bb-\frac{b(3Ab+10Ba)}{40a}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\cdots\right)\cdots}{(-a\,b^2)^{\frac{1}{3}}}}\cdots}{\cdots}$

default	$A\left(-\frac{\sqrt{b\,x^3+a}}{5x^5}-\frac{3b\sqrt{b\,x^3+a}}{20a\,x^2}+\frac{i b\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(\cdots}{}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\cdots}}}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b*(b*x^3+a)^(1/2)/a/x^2+1/20*I*b/a*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+B*(-1/2*(b*x^3+a)^(
1/2)/x^2-1/2*I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 64, normalized size = 0.24

$$\frac{3\,(10\,Ba - Ab)\sqrt{b}\,x^5\text{weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right) - \left((10\,Ba + 3\,Ab)x^3 + 4\,Aa\right)\sqrt{bx^3 + a}}{20\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(3*(10*B*a - A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) - ((10
*B*a + 3*A*b)*x^3 + 4*A*a)*sqrt(b*x^3 + a))/(a*x^5)
```

Sympy [A]
time = 1.48, size = 94, normalized size = 0.35

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\sqrt{a}\,\Gamma\left(-\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**6,x)
```

```
[Out] A*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3)) + B*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\sqrt{b\,x^3 + a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^6,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^6, x)
```

3.189 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^9}\,dx$

Optimal. Leaf size=305

$$\frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5}+\frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2}-\frac{A(a+bx^3)^{3/2}}{8ax^8}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}(7Ab-16aB)\,\Big(\sqrt[3]{}\cdots}{\cdots}$$

```
[Out] -1/8*A*(b*x^3+a)^(3/2)/a/x^8+1/80*(7*A*b-16*B*a)*(b*x^3+a)^(1/2)/a/x^5+3/32
0*b*(7*A*b-16*B*a)*(b*x^3+a)^(1/2)/a^2/x^2+1/320*3^(3/4)*b^(5/3)*(7*A*b-16*
B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^2
/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 283, 331, 224}

$$\frac{3b\sqrt{a+bx^3}\,(7Ab-16aB)}{320a^2x^2}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-16aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{a+bx^3}\,(7Ab-16aB)}{80ax^5}-\frac{A(a+bx^3)^{3/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
```

```
[Out] ((7*A*b - 16*a*B)*Sqrt[a + b*x^3])/(80*a*x^5) + (3*b*(7*A*b - 16*a*B)*Sqrt[
a + b*x^3])/(320*a^2*x^2) - (A*(a + b*x^3)^(3/2))/(8*a*x^8) + (3^(3/4)*Sqrt
[2 + Sqrt[3]]*b^(5/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(320*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^9}\,dx = -\frac{A(a+bx^3)^{3/2}}{8ax^8} - \frac{\left(\frac{7Ab}{2}-8aB\right)\int \frac{\sqrt{a+bx^3}}{x^6}\,dx}{8a}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} - \frac{A(a+bx^3)^{3/2}}{8ax^8} - \frac{(3b(7Ab-16aB))\int \frac{1}{x^3\sqrt{a+bx^3}}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} + \frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2} - \frac{A(a+bx^3)^{3/2}}{8ax^8} + \frac{(3b}{}$$

$$3^{3/}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} + \frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2} - \frac{A(a+bx^3)^{3/2}}{8ax^8} + \text{—}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 80, normalized size = 0.26

$$\frac{\sqrt{a+bx^3}\left(-5A(a+bx^3)+\frac{\left(\frac{7Ab}{2}-8aB\right)x^3\,{}_2F_1\left(-\frac{5}{3},-\frac{1}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{40ax^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + (((7*A*b)/2 - 8*a*B)*x^3*Hypergeometri
c2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*a*x^8)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(238) = 476.
time = 0.34, size = 660, normalized size = 2.16 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b*(b*x^3+a)^(1/2)/a/x^2+1/20*I*b/a*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
```

```
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-1/8*(b*x^3+a)^(
1/2)/x^8-3/80*b*(b*x^3+a)^(1/2)/a/x^5+21/320*b^2*(b*x^3+a)^(1/2)/a^2/x^2-7/
320*I*b^2/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 89, normalized size = 0.29

$$-\frac{3\,(16\,Bab-7\,Ab^2)\sqrt{b}\,x^8\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(3\,(16\,Bab-7\,Ab^2)x^6+4\,(16\,Ba^2+3\,Aab)x^3+40\,Aa^2\right)\sqrt{bx^3+a}}{320\,a^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="fricas")
[Out] -1/320*(3*(16*B*a*b - 7*A*b^2)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x
) + (3*(16*B*a*b - 7*A*b^2)*x^6 + 4*(16*B*a^2 + 3*A*a*b)*x^3 + 40*A*a^2)*sq
rt(b*x^3 + a))/(a^2*x^8)
```

Sympy [A]
time = 1.67, size = 97, normalized size = 0.32

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**9,x)
```

```
[Out] A*sqrt(a)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**8*gamma(-5/3)) + B*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^9,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^9, x)
```

3.190 $\int x^4\sqrt{a+bx^3}\,(A+Bx^3)\,dx$

Optimal. Leaf size=581

$$\frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2}+\frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b}-\frac{24a^2(19Ab-10aB)\sqrt{a+bx^3}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2Bx}{}$$

```
[Out] 2/19*B*x^5*(b*x^3+a)^(3/2)/b+6/1729*a*(19*A*b-10*B*a)*x^2*(b*x^3+a)^(1/2)/b
^2+2/247*(19*A*b-10*B*a)*x^5*(b*x^3+a)^(1/2)/b-24/1729*a^2*(19*A*b-10*B*a)*
(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-8/1729*3^(3/4)*a^(7
/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)+12/1729*3^(1/4)*a^(7/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*El
lipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {470, 285, 327, 309, 224, 1891}

$$-\frac{8\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(19Ab-10aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(19Ab-10aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{24a^2\sqrt{a+bx^3}\,(19Ab-10aB)}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{6ax^2\sqrt{a+bx^3}\,(19Ab-10aB)}{1729b^2}+\frac{2x^5\sqrt{a+bx^3}\,(19Ab-10aB)}{247b}+\frac{2Bx^5(a+bx^3)^{3/2}}{19b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (6*a*(19*A*b - 10*a*B)*x^2*Sqrt[a + b*x^3])/(1729*b^2) + (2*(19*A*b - 10*a*
B)*x^5*Sqrt[a + b*x^3])/(247*b) - (24*a^2*(19*A*b - 10*a*B)*Sqrt[a + b*x^3]
)/(1729*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*B*x^5*(a + b*x^3)
^(3/2))/(19*b) + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*A*b - 10*a*B)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(
8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
```

```
x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*3^(3/4)*a^(7/3)*(19*A*b - 10*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(8/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}-\frac{\left(2\left(-\frac{19Ab}{2}+5aB\right)\right)\int x^4\sqrt{a+bx^3}\,dx}{19b}$$

$$=\frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}+\frac{(3a(19Ab-10aB))}{247b}$$

$$=\frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2}+\frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b}+\frac{2Bx^5(a}{}$$

$$=\frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2}+\frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b}+\frac{2Bx^5(a}{}$$

$$=\frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2}+\frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b}-\frac{24a^2}{1729b^{8/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 4.00, size = 91, normalized size = 0.16

$$\frac{2x^2\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\left(-19Ab+10aB-13bBx^3\right)\right)+\frac{a(-19Ab+10aB)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{247b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-19*A*b + 10*a*B - 13*b*B*x^3)) + (a
*(-19*A*b + 10*a*B)*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1
 + (b*x^3)/a]))/(247*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 965 vs. $2(439) = 878$.

time = 0.33, size = 966, normalized size = 1.66

method	result
risch	$\frac{2x^2(91b^2Bx^6+133Ab^2x^3+21Babx^3+57abA-30a^2B)\sqrt{bx^3+a}}{1729b^2}+\frac{8ia^2(19Ab-10Ba)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{\cdots}}{\cdots}\right)}{(-a\cdots}}\cdots}{\cdots}$
elliptic	$\frac{2Bx^8\sqrt{bx^3+a}}{19}+\frac{2\left(Ab+\frac{3Ba}{19}\right)x^5\sqrt{bx^3+a}}{13b}+\frac{2\left(Aa-\frac{10a\left(Ab+\frac{3Ba}{19}\right)}{13b}\right)x^2\sqrt{bx^3+a}}{7b}+\frac{8ia\left(Aa-\frac{10a\left(Ab+\frac{3Ba}{19}\right)}{13b}\right)\cdots}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/19*x^8*(b*x^3+a)^(1/2)+6/247*a*x^5*(b*x^3+a)^(1/2)/b-60/1729*a^2*x^2*(
b*x^3+a)^(1/2)/b^2-80/1729*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))))+A*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a*x^2*(b*x^3+a)^(1/2)/b+8/91*I/b^2*a^
2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 102, normalized size = 0.18

$$-\frac{2\left(12\,(10\,Ba^3-19\,Aa^2b)\sqrt{b}\,\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(91\,Bb^3x^8+7\,(3\,Bab^2+19\,Ab^3)x^5-3\,(10\,Ba^2b-19\,Aab^2)x^2)\sqrt{bx^3+a}\right)}{1729\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/1729*(12*(10*B*a^3 - 19*A*a^2*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)) - (91*B*b^3*x^8 + 7*(3*B*a*b^2 + 19*A*b^3)*x
^5 - 3*(10*B*a^2*b - 19*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/b^3
```

Sympy [A]
time = 1.31, size = 83, normalized size = 0.14

$$\frac{A\sqrt{a}\,x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{B\sqrt{a}\,x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(8/3)) + B*sqrt(a)*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^4*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

3.191 $\int x\sqrt{a+bx^3}\,(A+Bx^3)\,dx$

Optimal. Leaf size=548

$$\frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b}+\frac{6a(13Ab-4aB)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2Bx^2(a+bx^3)^{3/2}}{13b}-\frac{3\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,a^{4/3}(\cdots}{\cdots}$$

```
[Out] 2/13*B*x^2*(b*x^3+a)^(3/2)/b+2/91*(13*A*b-4*B*a)*x^2*(b*x^3+a)^(1/2)/b+6/91
*a*(13*A*b-4*B*a)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+2
/91*3^(3/4)*a^(4/3)*(13*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)-3/91*3^(1/4)*a^(4/3)*(13*A*b-4*B*a)*(a^(1/3)+b^
(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {470, 285, 309, 224, 1891}

$$\frac{2\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(13Ab-4aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(13Ab-4aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{6a\sqrt{a+bx^3}\,(13Ab-4aB)}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x^2\sqrt{a+bx^3}\,(13Ab-4aB)}{91b}+\frac{2Bx^2(a+bx^3)^{3/2}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(13*A*b - 4*a*B)*x^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*A*b - 4*a*B)*Sqr
t[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*B*x^2*(
a + b*x^3)^(3/2))/(13*b) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b - 4
*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*3^(3/4)*a^(4/3)*(13*A*b - 4*a*B)*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
```

```
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(
5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2Bx^2(a+bx^3)^{3/2}}{13b}-\frac{\left(2\left(-\frac{13Ab}{2}+2aB\right)\right)\int x\sqrt{a+bx^3}\,dx}{13b}$$

$$=\frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b}+\frac{2Bx^2(a+bx^3)^{3/2}}{13b}+\frac{(3a(13Ab-4aB))\int -}{91b}$$

$$=\frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b}+\frac{2Bx^2(a+bx^3)^{3/2}}{13b}+\frac{(3a(13Ab-4aB))\int}{91b^4}$$

$$=\frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b}+\frac{6a(13Ab-4aB)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2Bx^2(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.66, size = 75, normalized size = 0.14

$$\frac{x^2\sqrt{a+bx^3}\left(4B(a+bx^3)+\frac{(13Ab-4aB)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{26b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3) + ((13*A*b - 4*a*B)*Hypergeometric2F1
[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(26*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(410) = 820.
time = 0.32, size = 926, normalized size = 1.69

method	result

risch	$\frac{2x^2(7bB\,x^3+13Ab+3Ba)\sqrt{b\,x^3+a}}{91b} - \frac{2ia(13Ab-4Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}$
elliptic	$\frac{2B\,x^5\sqrt{b\,x^3+a}}{13}+\frac{2\left(Ab+\frac{3Ba}{13}\right)x^2\sqrt{b\,x^3+a}}{7b} - \frac{2i\left(Aa-\frac{4a\left(Ab+\frac{3Ba}{13}\right)}{7b}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\cdots}{2b}\right)\cdots}{(-a\,b^2)^{\frac{1}{3}}}}\cdots}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a*x^2*(b*x^3+a)^(1/2)/b+8/91*I/b^2*a^2*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
```

```
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*
3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 76, normalized size = 0.14

$$\frac{2\left(3\,(4\,Ba^2-13\,Aab)\sqrt{b}\,\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(7\,Bb^2x^5+(3\,Bab+13\,Ab^2)x^2\right)\sqrt{bx^3+a}\right)}{91\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/91*(3*(4*B*a^2 - 13*A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrass
PInverse(0, -4*a/b, x)) + (7*B*b^2*x^5 + (3*B*a*b + 13*A*b^2)*x^2)*sqrt(b*x
^3 + a))/b^2
```

Sympy [A]
time = 1.20, size = 83, normalized size = 0.15

$$\frac{A\sqrt{a}\,x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{B\sqrt{a}\,x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3)) + B*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

3.192 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^2}\,dx$

Optimal. Leaf size=545

$$\frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a}+\frac{3(7Ab+2aB)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{A(a+bx^3)^{3/2}}{ax}-\frac{3\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,(7Ab+$$

```
[Out] -A*(b*x^3+a)^(3/2)/a/x+1/7*(7*A*b+2*B*a)*x^2*(b*x^3+a)^(1/2)/a+3/7*(7*A*b+2
*B*a)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/7*3^(3/4)*a
^(1/3)*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)-3/14*3^(1/4)*a^(1/3)*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*Ellipt
icE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/
2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 285, 309, 224, 1891}

$$\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2aB+7Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2aB+7Ab)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{3\sqrt{a+bx^3}\,(2aB+7Ab)}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{x^2\sqrt{a+bx^3}\,(2aB+7Ab)}{7a}-\frac{A(a+bx^3)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

```
[Out] ((7*A*b + 2*a*B)*x^2*Sqrt[a + b*x^3])/(7*a) + (3*(7*A*b + 2*a*B)*Sqrt[a + b
*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/
2))/(a*x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b + 2*a*B)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (Sqrt[2]*3^(3/4)*a^(1/3)*(7*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*
```

```
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
```

```
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^2}\,dx = -\frac{A(a+bx^3)^{3/2}}{ax} - \frac{\left(-\frac{7Ab}{2}-aB\right)\int x\sqrt{a+bx^3}\,dx}{a}$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} - \frac{A(a+bx^3)^{3/2}}{ax} + \frac{1}{14}(3(7Ab+2aB))\int \frac{x}{\sqrt{a+bx}}$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} - \frac{A(a+bx^3)^{3/2}}{ax} + \frac{(3(7Ab+2aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{a+b}}}{14\sqrt[3]{b}}$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} + \frac{3(7Ab+2aB)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{A(a+bx^3)^{3}}{ax}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.99, size = 81, normalized size = 0.15

$$\frac{\sqrt{a+bx^3}\left(-2A(a+bx^3)+\frac{(7Ab+2aB)x^3\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{2ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + ((7*A*b + 2*a*B)*x^3*Hypergeometric2F1
[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 901 vs. 2(409) = 818.
time = 0.33, size = 902, normalized size = 1.66

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-2B\,x^3+7A)}{7x}-\frac{2i\left(\frac{3Ab}{2}+\frac{3Ba}{7}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{x}+\frac{2B\,x^2\sqrt{b\,x^3+a}}{7}-\frac{2i\left(\frac{3Ab}{2}+\frac{3Ba}{7}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
```

```
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))))+A*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b
*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 64, normalized size = 0.12

$$-\frac{3\,(2\,Ba+7\,Ab)\sqrt{b}\,x\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-(2\,Bbx^3-7\,Ab)\sqrt{bx^3+a}}{7\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/7*(3*(2*B*a + 7*A*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPIn
verse(0, -4*a/b, x)) - (2*B*b*x^3 - 7*A*b)*sqrt(b*x^3 + a))/(b*x)
```

Sympy [A]
time = 1.34, size = 85, normalized size = 0.16

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}+\frac{B\sqrt{a}\,x^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] A*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x*gamma(2/3)) + B*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^2,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^2, x)
```

3.193 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^5}\,dx$

Optimal. Leaf size=546

$$-\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax}+\frac{3\sqrt[3]{b}\,(Ab+8aB)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{A(a+bx^3)^{3/2}}{4ax^4}-\frac{3\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,(Ab+8aB)}{}$$

```
[Out] -1/4*A*(b*x^3+a)^(3/2)/a/x^4-1/8*(A*b+8*B*a)*(b*x^3+a)^(1/2)/a/x+3/8*b^(1/3
)*(A*b+8*B*a)*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/8*3^(3/4)
*b^(1/3)*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)*2^(1/
2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)-3/16*3^(1/4)*b^(1/3)*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*Elliptic
E((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 283, 309, 224, 1891}

$$\frac{3^{3/4}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(8aB+Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(8aB+Ab)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}\,(8aB+Ab)}{8ax}+\frac{3\sqrt[3]{b}\sqrt{a+bx^3}\,(8aB+Ab)}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{A(a+bx^3)^{3/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

```
[Out] -1/8*((A*b + 8*a*B)*Sqrt[a + b*x^3])/(a*x) + (3*b^(1/3)*(A*b + 8*a*B)*Sqrt[
a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2
))/(4*a*x^4) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(A*b + 8*a*B)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) + (3^(3/4)*b^(1/3)*(A*b + 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
```

```
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
```

```
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^5}\,dx = -\frac{A(a+bx^3)^{3/2}}{4ax^4} - \frac{\left(-\frac{Ab}{2}-4aB\right)\int \frac{\sqrt{a+bx^3}}{x^2}\,dx}{4a}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} - \frac{A(a+bx^3)^{3/2}}{4ax^4} + \frac{(3b(Ab+8aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{16a}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} - \frac{A(a+bx^3)^{3/2}}{4ax^4} + \frac{\left(3b^{2/3}(Ab+8aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt{a+}}}{16a}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}\,(Ab+8aB)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{A(a+bx^3)^{3/2}}{4ax^4} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.62, size = 80, normalized size = 0.15

$$\frac{\sqrt{a+bx^3}\left(-A(a+bx^3) - \frac{(Ab+8aB)x^3\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((A*b + 8*a*B)*x^3*Hypergeometric2F1[-
1/2, -1/3, 2/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(4*a*x^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(408) = 816.
time = 0.33, size = 920, normalized size = 1.68

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(3Ab\,x^3+8Ba\,x^3+2Aa)}{8x^4a} - \frac{i(Ab+8Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-\frac{3(-a\cdots}{2\cdots}}}\cdots}{\cdots}$ $2i\left(Bb+\frac{b(3Ab+8Ba)}{16a}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\cdots}{(-a\,b^2)^{\frac{1}{3}}}}\cdots$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{4x^4} - \frac{(3Ab+8Ba)\sqrt{b\,x^3+a}}{8ax} - \cdots$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4*(b*x^3+a)^(1/2)/x^4-3/8*b*(b*x^3+a)^(1/2)/a/x-1/8*I*b/a*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
```

```
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))))+B*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 70, normalized size = 0.13

$$-\frac{3\,(8\,Ba+Ab)\sqrt{b}\,x^4\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left((8\,Ba+3\,Ab)x^3+2\,Aa\right)\sqrt{bx^3+a}}{8\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="fricas")
[Out] -1/8*(3*(8*B*a + A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPIn
verse(0, -4*a/b, x)) + ((8*B*a + 3*A*b)*x^3 + 2*A*a)*sqrt(b*x^3 + a))/(a*x^
4)
```

Sympy [A]
time = 1.41, size = 92, normalized size = 0.17

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**5,x)
```

```
[Out] A*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3)) + B*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^5,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^5, x)
```

3.194 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^8}\,dx$

Optimal. Leaf size=581

$$\frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4}+\frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x}-\frac{3b^{4/3}(5Ab-14aB)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{A(a+bx^3)^{3/2}}{7ax^7}+$$

```
[Out] -1/7*A*(b*x^3+a)^(3/2)/a/x^7+1/56*(5*A*b-14*B*a)*(b*x^3+a)^(1/2)/a/x^4+3/11
2*b*(5*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^2/x-3/112*b^(4/3)*(5*A*b-14*B*a)*(b*x^
3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-1/112*3^(3/4)*b^(4/3)*(5*A*b
-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)*2^(1/2)/(b*x^3+a)
^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)+3/224*3^(1/4)*b^(4/3)*(5*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {464, 283, 331, 309, 224, 1891}

$$-\frac{3^{3/4}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(5Ab-14aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(5Ab-14aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3b^{4/3}\sqrt{a+bx^3}(5Ab-14aB)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{3b\sqrt{a+bx^3}(5Ab-14aB)}{112a^2x}+\frac{\sqrt{a+bx^3}(5Ab-14aB)}{56ax^4}-\frac{A(a+bx^3)^{3/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

```
[Out] ((5*A*b - 14*a*B)*Sqrt[a + b*x^3])/(56*a*x^4) + (3*b*(5*A*b - 14*a*B)*Sqrt[
a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))/(7*a*x^7)
 + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3
```

```
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3]) - (3^(3/4)*b^(4/3)*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^8}\,dx = -\frac{A(a+bx^3)^{3/2}}{7ax^7} - \frac{\left(\frac{5Ab}{2}-7aB\right)\int \frac{\sqrt{a+bx^3}}{x^5}\,dx}{7a}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} - \frac{A(a+bx^3)^{3/2}}{7ax^7} - \frac{(3b(5Ab-14aB))\int \frac{1}{x^2\sqrt{a+bx}}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{A(a+bx^3)^{3/2}}{7ax^7} - $$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{A(a+bx^3)^{3/2}}{7ax^7} - $$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{3b^{4/3}(5Ab-14a}{112a^2\left(\left(1+\sqrt{3}\right)\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 80, normalized size = 0.14

$$\frac{\sqrt{a+bx^3}\left(-4A(a+bx^3)+\frac{\left(\frac{5Ab}{2}-7aB\right)x^3\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{28ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-4*A*(a + b*x^3) + (((5*A*b)/2 - 7*a*B)*x^3*Hypergeometri
c2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(28*a*x^7)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(439) = 878$.
time = 0.33, size = 964, normalized size = 1.66

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-15A\,b^{2}x^{6}+42Bab\,x^{6}+6aAb\,x^{3}+28a^{2}B\,x^{3}+16a^{2}A\right)}{112x^{7}a^{2}}+\frac{ib(5Ab-14Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\cdots}{\cdots}\right)}{\left(-a\,b^{2}\right)\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{7x^{7}}-\frac{(3Ab+14Ba)\sqrt{b\,x^{3}+a}}{56a\,x^{4}}+\frac{3b(5Ab-14Ba)\sqrt{b\,x^{3}+a}}{112a^{2}x}+\frac{ib(5Ab-14Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\cdots\right)}{\cdots}}\cdots}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/4*(b*x^3+a)^(1/2)/x^4-3/8*b*(b*x^3+a)^(1/2)/a/x-1/8*I*b/a*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-1/7*(b*x^3+a)^(1/2)/x^7-3/56*b/a*(b*x^3
+a)^(1/2)/x^4+15/112*b^2/a^2*(b*x^3+a)^(1/2)/x+5/112*I*b^2/a^2*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 97, normalized size = 0.17

$$-\frac{3\,(14\,Bab-5\,Ab^2)\sqrt{b}\,x^7\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(3\,(14\,Bab-5\,Ab^2)x^6+2\,(14\,Ba^2+3\,Aab)x^3+16\,Aa^2\right)\sqrt{bx^3+a}}{112\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/112*(3*(14*B*a*b - 5*A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)) + (3*(14*B*a*b - 5*A*b^2)*x^6 + 2*(14*B*a^2 +
 3*A*a*b)*x^3 + 16*A*a^2)*sqrt(b*x^3 + a))/(a^2*x^7)
```

Sympy [A]
time = 1.59, size = 97, normalized size = 0.17

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**8,x)
```

```
[Out] A*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**7*gamma(-4/3)) + B*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^8,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^8, x)
```

3.195 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{11}}\,dx$

Optimal. Leaf size=614

$$\frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7}+\frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4}-\frac{3b^2(11Ab-20aB)\sqrt{a+bx^3}}{448a^3x}+\frac{3b^{7/3}(11Ab-20}{448a^3\left(\left(1+\sqrt{3}\right.\right.}$$

```
[Out] -1/10*A*(b*x^3+a)^(3/2)/a/x^10+1/140*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a/x^7+
3/1120*b*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a^2/x^4-3/448*b^2*(11*A*b-20*B*a)*
(b*x^3+a)^(1/2)/a^3/x+3/448*b^(7/3)*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a^3/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/448*3^(3/4)*b^(7/3)*(11*A*b-20*B*a)*(a^(1/3)
+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(8/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/896*3^(1/4)*b
^(7/3)*(11*A*b-20*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {464, 283, 331, 309, 224, 1891}

$$\frac{3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-20aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224\sqrt{2}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-20aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{896a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3b^{7/3}\sqrt{a+bx^3}(11Ab-20aB)}{448a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{3b^2\sqrt{a+bx^3}(11Ab-20aB)}{448a^3x}+\frac{3b\sqrt{a+bx^3}(11Ab-20aB)}{1120a^2x^4}+\frac{\sqrt{a+bx^3}(11Ab-20aB)}{140ax^7}-\frac{A(a+bx^3)^{3/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
```

```
[Out] ((11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(140*a*x^7) + (3*b*(11*A*b - 20*a*B)*Sq
rt[a + b*x^3])/(1120*a^2*x^4) - (3*b^2*(11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(
448*a^3*x) + (3*b^(7/3)*(11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(448*a^3*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))/(10*a*x^10) - (3*3^(1
/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(11*A*b - 20*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
```

```
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
3^(3/4)*b^(7/3)*(11*A*b - 20*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(224*Sqrt[2]*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{11}}\,dx &= -\frac{A(a+bx^3)^{3/2}}{10ax^{10}} - \frac{\left(\frac{11Ab}{2}-10aB\right)\int \frac{\sqrt{a+bx^3}}{x^8}\,dx}{10a} \\
&= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} - \frac{A(a+bx^3)^{3/2}}{10ax^{10}} - \frac{(3b(11Ab-20aB))\int \frac{1}{x^5\sqrt{a+}}}{280a} \\
&= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{A(a+bx^3)^{3/2}}{10ax^{10}} + \\
&= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{3b^2(11Ab-20a}{448a} \\
&= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{3b^2(11Ab-20a}{448a} \\
&= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{3b^2(11Ab-20a}{448a}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 80, normalized size = 0.13

$$\frac{\sqrt{a+bx^3}\left(-7A(a+bx^3)+\frac{\left(\frac{11Ab}{2}-10aB\right)x^3\,{}_2F_1\left(-\frac{7}{3},-\frac{1}{2};-\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{70ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-7*A*(a + b*x^3) + (((11*A*b)/2 - 10*a*B)*x^3*Hypergeomet
ric2F1[-7/3, -1/2, -4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(70*a*x^10)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1005 vs. 2(468) = 936.
time = 0.34, size = 1006, normalized size = 1.64

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(165x^9Ab^3-300x^9Bab^2-66x^6Aab^2+120x^6Ba^2b+48x^3Aa^2b+320a^3Bx^3+224Aa^3)}{2240x^{10}a^3}-\frac{ib^2(11Ab-20Ba)\sqrt{3}\,(\ldots}{}$

	ib^{2}
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{10x^{10}}-\frac{(3Ab+20Ba)\sqrt{b\,x^{3}+a}}{140a\,x^{7}}+\frac{3b(11Ab-20Ba)\sqrt{b\,x^{3}+a}}{1120a^{2}x^{4}}-\frac{3b^{2}(11Ab-20Ba)\sqrt{b\,x^{3}+a}}{448a^{3}x}-$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x,method=_RETURNVERBOSE)

[Out] B*(-1/7*(b*x^3+a)^(1/2)/x^7-3/56*b/a*(b*x^3+a)^(1/2)/x^4+15/112*b^2/a^2*(b*x^3+a)^(1/2)/x+5/112*I*b^2/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-1/10*(b*x^3+a)^(1/2)/x^10-3/140*b/a*(b*x^3+a)^(1/2)/x^7+33/1120*b^2/a^2*(b*x^3+a)^(1/2)/x^4-33/448*b^3/a^3*(b*x^3+a)^(1/2)/x-11/448*I*b^3/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*

```
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 123, normalized size = 0.20

$$\frac{15\,(20\,Bab^2-11\,Ab^3)\sqrt{b}\,x^{10}\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(15\,(20\,Bab^2-11\,Ab^3)x^9-6\,(20\,Ba^2b-11\,Aab^2)x^6-224\,Aa^3-16\,(20\,Ba^3+3\,Aa^2b)x^3\right)\sqrt{bx^3+a}}{2240\,a^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/2240*(15*(20*B*a*b^2 - 11*A*b^3)*sqrt(b)*x^10*weierstrassZeta(0, -4*a/b,
weierstrassPInverse(0, -4*a/b, x)) + (15*(20*B*a*b^2 - 11*A*b^3)*x^9 - 6*(2
0*B*a^2*b - 11*A*a*b^2)*x^6 - 224*A*a^3 - 16*(20*B*a^3 + 3*A*a^2*b)*x^3)*sq
rt(b*x^3 + a))/(a^3*x^10)
```

Sympy [A]
time = 1.82, size = 97, normalized size = 0.16

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**11,x)
```

```
[Out] A*sqrt(a)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**10*gamma(-7/3)) + B*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^11,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^11, x)
```

3.196 $\int x^8(a+bx^3)^{3/2}\left(A+Bx^3\right)dx$

Optimal. Leaf size=103

$$\frac{2a^2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{9/2}}{27b^4}+\frac{2B\left(a+bx^3\right)^{11/2}}{33b^4}$$

```
[Out] 2/15*a^2*(A*b-B*a)*(b*x^3+a)^(5/2)/b^4-2/21*a*(2*A*b-3*B*a)*(b*x^3+a)^(7/2)
/b^4+2/27*(A*b-3*B*a)*(b*x^3+a)^(9/2)/b^4+2/33*B*(b*x^3+a)^(11/2)/b^4
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\frac{2a^2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^4}+\frac{2\left(a+bx^3\right)^{9/2}(Ab-3aB)}{27b^4}-\frac{2a\left(a+bx^3\right)^{7/2}(2Ab-3aB)}{21b^4}+\frac{2B\left(a+bx^3\right)^{11/2}}{33b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
b*x^3)^(7/2))/(21*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(9/2))/(27*b^4) + (2*
B*(a + b*x^3)^(11/2))/(33*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^8 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{1}{3}\text{Subst}\left(\int x^2(a+bx)^{3/2}(A+Bx) \, dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)(a+bx)^{3/2}}{b^3} + \frac{a(-2Ab+3aB)(a+bx)^{5/2}}{b^3}\right.\right.$$
$$= \frac{2a^2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^4} - \frac{2a(2Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4} + \frac{2(Ab-3a}{}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 0.78

$$\frac{2(a+bx^3)^{5/2}\left(88a^2Ab-48a^3B-220aAb^2x^3+120a^2bBx^3+385Ab^3x^6-210ab^2Bx^6+315b^3Bx^9\right)}{10395b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
[Out] (2*(a + b*x^3)^(5/2)*(88*a^2*A*b - 48*a^3*B - 220*a*A*b^2*x^3 + 120*a^2*b*B
*x^3 + 385*A*b^3*x^6 - 210*a*b^2*B*x^6 + 315*b^3*B*x^9))/(10395*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(87) = 174.
time = 0.33, size = 202, normalized size = 1.96

method	result
gosper	$\frac{2(bx^3+a)^{\frac{5}{2}}(315Bx^9b^3+385Ab^3x^6-210Bab^2x^6-220Aab^2x^3+120Ba^2bx^3+88Aa^2b-48Ba^3)}{10395b^4}$
trager	$\frac{2(315b^5Bx^{15}+385b^5Ax^{12}+420ab^4Bx^{12}+550ab^4Ax^9+15a^2b^3Bx^9+33a^2Ab^3x^6-18a^3b^2Bx^6-44Aa^3b^2x^3+24Ba^4bx^3+88a^4bA}{10395b^4}$
risch	$\frac{2(315b^5Bx^{15}+385b^5Ax^{12}+420ab^4Bx^{12}+550ab^4Ax^9+15a^2b^3Bx^9+33a^2Ab^3x^6-18a^3b^2Bx^6-44Aa^3b^2x^3+24Ba^4bx^3+88a^4bA}{10395b^4}$
default	$B\left(\frac{2bx^{15}\sqrt{bx^3+a}}{33}+\frac{8ax^{12}\sqrt{bx^3+a}}{99}+\frac{2a^2x^9\sqrt{bx^3+a}}{693b}-\frac{4a^3x^6\sqrt{bx^3+a}}{1155b^2}+\frac{16a^4x^3\sqrt{bx^3+a}}{3465b^3}\right.$
elliptic	$\frac{2Bbx^{15}\sqrt{bx^3+a}}{33}+\frac{2(b^2A+\frac{12}{11}abB)x^{12}\sqrt{bx^3+a}}{27b}+\frac{2\left(2abA+a^2B-\frac{8a(b^2A+\frac{12}{11}abB)}{9b}\right)x^9\sqrt{bx^3+a}}{21b}+\frac{2\left(a^2A-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] B*(2/33*b*x^15*(b*x^3+a)^(1/2)+8/99*a*x^12*(b*x^3+a)^(1/2)+2/693/b*a^2*x^9*
(b*x^3+a)^(1/2)-4/1155*a^3/b^2*x^6*(b*x^3+a)^(1/2)+16/3465*a^4/b^3*x^3*(b*x
```

```
^3+a)^(1/2)-32/3465*a^5/b^4*(b*x^3+a)^(1/2))+A*(2/27*b*x^12*(b*x^3+a)^(1/2)
+20/189*a*x^9*(b*x^3+a)^(1/2)+2/315/b*a^2*x^6*(b*x^3+a)^(1/2)-8/945*a^3/b^2
*x^3*(b*x^3+a)^(1/2)+16/945*a^4/b^3*(b*x^3+a)^(1/2))
```

Maxima [A]
time = 0.31, size = 118, normalized size = 1.15

$$\frac{2}{945}\left(\frac{35\,(bx^3+a)^{\frac{9}{2}}}{b^3}-\frac{90\,(bx^3+a)^{\frac{7}{2}}a}{b^3}+\frac{63\,(bx^3+a)^{\frac{5}{2}}a^2}{b^3}\right)A+\frac{2}{3465}\left(\frac{105\,(bx^3+a)^{\frac{11}{2}}}{b^4}-\frac{385\,(bx^3+a)^{\frac{9}{2}}a}{b^4}+\frac{495\,(bx^3+a)^{\frac{7}{2}}a^2}{b^4}-\frac{231\,(bx^3+a)^{\frac{5}{2}}a^3}{b^4}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 +
a)^(5/2)*a^2/b^3)*A + 2/3465*(105*(b*x^3 + a)^(11/2)/b^4 - 385*(b*x^3 + a)^
(9/2)*a/b^4 + 495*(b*x^3 + a)^(7/2)*a^2/b^4 - 231*(b*x^3 + a)^(5/2)*a^3/b^4
)*B
```

Fricas [A]
time = 2.29, size = 124, normalized size = 1.20

$$\frac{2\,(315\,Bb^5x^{15}+35\,(12\,Bab^4+11\,Ab^5)x^{12}+5\,(3\,Ba^2b^3+110\,Aab^4)x^9-3\,(6\,Ba^3b^2-11\,Aa^2b^3)x^6-48\,Ba^5+88\,Aa^4b+4\,(6\,Ba^4b-11\,Aa^3b^2)x^3)\sqrt{bx^3+a}}{10395\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/10395*(315*B*b^5*x^15 + 35*(12*B*a*b^4 + 11*A*b^5)*x^12 + 5*(3*B*a^2*b^3
+ 110*A*a*b^4)*x^9 - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*x^6 - 48*B*a^5 + 88*A*a
^4*b + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(100) = 200$.
time = 0.60, size = 267, normalized size = 2.59

$$\begin{cases} \frac{16Aa^4\sqrt{a+bx^3}}{945b^3}-\frac{8Aa^3x^3\sqrt{a+bx^3}}{945b^2}+\frac{2Aa^2x^6\sqrt{a+bx^3}}{315b}+\frac{20Aax^9\sqrt{a+bx^3}}{189}+\frac{2Abx^{12}\sqrt{a+bx^3}}{27}-\frac{32Ba^5\sqrt{a+bx^3}}{3465b^4}+\frac{16Ba^4x^3\sqrt{a+bx^3}}{3465b^3}-\frac{4Ba^3x^6\sqrt{a+bx^3}}{1155b^2}+\frac{2Ba^2x^9\sqrt{a+bx^3}}{693b}+\frac{8Bax^{12}\sqrt{a+bx^3}}{99}+\frac{2Bbx^{15}\sqrt{a+bx^3}}{33} & \text{for}\ b\neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^9}{9}+\frac{Bx^{12}}{12}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((16*A*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*A*a**3*x**3*sqrt(a + b
*x**3)/(945*b**2) + 2*A*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*A*a*x**9*sq
rt(a + b*x**3)/189 + 2*A*b*x**12*sqrt(a + b*x**3)/27 - 32*B*a**5*sqrt(a + b
*x**3)/(3465*b**4) + 16*B*a**4*x**3*sqrt(a + b*x**3)/(3465*b**3) - 4*B*a**3
*x**6*sqrt(a + b*x**3)/(1155*b**2) + 2*B*a**2*x**9*sqrt(a + b*x**3)/(693*b)
 + 8*B*a*x**12*sqrt(a + b*x**3)/99 + 2*B*b*x**15*sqrt(a + b*x**3)/33, Ne(b,
 0)), (a**(3/2)*(A*x**9/9 + B*x**12/12), True))
```

Giac [A]
time = 0.56, size = 104, normalized size = 1.01

$$\frac{2\left(315\,(bx^3+a)^{\frac{11}{2}}B-1155\,(bx^3+a)^{\frac{9}{2}}Ba+1485\,(bx^3+a)^{\frac{7}{2}}Ba^2-693\,(bx^3+a)^{\frac{5}{2}}Ba^3+385\,(bx^3+a)^{\frac{9}{2}}Ab-990\,(bx^3+a)^{\frac{7}{2}}Aab+693\,(bx^3+a)^{\frac{5}{2}}Aa^2b\right)}{10395\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/10395*(315*(b*x^3 + a)^(11/2)*B - 1155*(b*x^3 + a)^(9/2)*B*a + 1485*(b*x^
3 + a)^(7/2)*B*a^2 - 693*(b*x^3 + a)^(5/2)*B*a^3 + 385*(b*x^3 + a)^(9/2)*A*
b - 990*(b*x^3 + a)^(7/2)*A*a*b + 693*(b*x^3 + a)^(5/2)*A*a^2*b)/b^4
```

Mupad [B]
time = 2.65, size = 206, normalized size = 2.00

$$\frac{20\,A\,a\,x^9\,\sqrt{b\,x^3+a}}{189}+\frac{2\,A\,b\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{8\,B\,a\,x^{12}\,\sqrt{b\,x^3+a}}{99}+\frac{2\,B\,b\,x^{15}\,\sqrt{b\,x^3+a}}{33}+\frac{16\,A\,a^4\,\sqrt{b\,x^3+a}}{945\,b^3}-\frac{32\,B\,a^5\,\sqrt{b\,x^3+a}}{3465\,b^4}-\frac{8\,A\,a^3\,x^3\,\sqrt{b\,x^3+a}}{945\,b^2}+\frac{2\,A\,a^2\,x^6\,\sqrt{b\,x^3+a}}{315\,b}+\frac{16\,B\,a^4\,x^3\,\sqrt{b\,x^3+a}}{3465\,b^3}-\frac{4\,B\,a^3\,x^6\,\sqrt{b\,x^3+a}}{1155\,b^2}+\frac{2\,B\,a^2\,x^9\,\sqrt{b\,x^3+a}}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] (20*A*a*x^9*(a + b*x^3)^(1/2))/189 + (2*A*b*x^12*(a + b*x^3)^(1/2))/27 + (8
*B*a*x^12*(a + b*x^3)^(1/2))/99 + (2*B*b*x^15*(a + b*x^3)^(1/2))/33 + (16*A
*a^4*(a + b*x^3)^(1/2))/(945*b^3) - (32*B*a^5*(a + b*x^3)^(1/2))/(3465*b^4)
 - (8*A*a^3*x^3*(a + b*x^3)^(1/2))/(945*b^2) + (2*A*a^2*x^6*(a + b*x^3)^(1/
2))/(315*b) + (16*B*a^4*x^3*(a + b*x^3)^(1/2))/(3465*b^3) - (4*B*a^3*x^6*(a
 + b*x^3)^(1/2))/(1155*b^2) + (2*B*a^2*x^9*(a + b*x^3)^(1/2))/(693*b)
```

3.197 $\int x^5(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal. Leaf size=73

$$-\frac{2a(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{7/2}}{21b^3}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^3}$$

```
[Out] -2/15*a*(A*b-B*a)*(b*x^3+a)^(5/2)/b^3+2/21*(A*b-2*B*a)*(b*x^3+a)^(7/2)/b^3+
2/27*B*(b*x^3+a)^(9/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{2\left(a+bx^3\right)^{7/2}(Ab-2aB)}{21b^3}-\frac{2a\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^3}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3
)^(7/2))/(21*b^3) + (2*B*(a + b*x^3)^(9/2))/(27*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{1}{3}\text{Subst}\left(\int x(a+bx)^{3/2}(A+Bx) \, dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^{3/2}}{b^2} + \frac{(Ab-2aB)(a+bx)^{5/2}}{b^2} + \frac{B(a$$
$$= -\frac{2a(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^3} + \frac{2(Ab-2aB)\left(a+bx^3\right)^{7/2}}{21b^3} + \frac{2B(a+bx^3)}{27b^3}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.77

$$\frac{2(a+bx^3)^{5/2}\left(-18aAb+8a^2B+45Ab^2x^3-20abBx^3+35b^2Bx^6\right)}{945b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
[Out] (2*(a + b*x^3)^(5/2)*(-18*a*A*b + 8*a^2*B + 45*A*b^2*x^3 - 20*a*b*B*x^3 + 3
5*b^2*B*x^6))/(945*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(61) = 122.
time = 0.32, size = 162, normalized size = 2.22

method	result
gosper	$-\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}\left(-35b^2B\,x^6-45A\,b^2x^3+20Bab\,x^3+18abA-8a^2B\right)}{945b^3}$
trager	$-\frac{2\left(-35B\,b^4x^{12}-45A\,b^4x^9-50Ba\,b^3x^9-72a\,b^3A\,x^6-3B\,a^2b^2x^6-9A\,a^2b^2x^3+4B\,a^3b\,x^3+18A\,a^3b-8B\,a^4\right)\sqrt{b\,x^3+a}}{945b^3}$
risch	$-\frac{2\left(-35B\,b^4x^{12}-45A\,b^4x^9-50Ba\,b^3x^9-72a\,b^3A\,x^6-3B\,a^2b^2x^6-9A\,a^2b^2x^3+4B\,a^3b\,x^3+18A\,a^3b-8B\,a^4\right)\sqrt{b\,x^3+a}}{945b^3}$
default	$B\left(\frac{2b\,x^{12}\sqrt{b\,x^3+a}}{27}+\frac{20a\,x^9\sqrt{b\,x^3+a}}{189}+\frac{2a^2x^6\sqrt{b\,x^3+a}}{315b}-\frac{8a^3x^3\sqrt{b\,x^3+a}}{945b^2}+\frac{16a^4\sqrt{b\,x^3+a}}{945b^3}\right)$
elliptic	$\frac{2Bb\,x^{12}\sqrt{b\,x^3+a}}{27}+\frac{2\left(b^2A+\frac{10}{9}abB\right)x^9\sqrt{b\,x^3+a}}{21b}+\frac{2\left(2abA+a^2B-\frac{6a\left(b^2A+\frac{10}{9}abB\right)}{7b}\right)x^6\sqrt{b\,x^3+a}}{15b}+\frac{2\left(a^2A-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] B*(2/27*b*x^12*(b*x^3+a)^(1/2)+20/189*a*x^9*(b*x^3+a)^(1/2)+2/315/b*a^2*x^6
*(b*x^3+a)^(1/2)-8/945*a^3/b^2*x^3*(b*x^3+a)^(1/2)+16/945*a^4/b^3*(b*x^3+a)
```

```
^(1/2))+A*(2/21*b*x^9*(b*x^3+a)^(1/2)+16/105*a*x^6*(b*x^3+a)^(1/2)+2/105/b*
a^2*x^3*(b*x^3+a)^(1/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2))
```

Maxima [A]
time = 0.30, size = 84, normalized size = 1.15

$$\frac{2}{105}\left(\frac{5\,(bx^3+a)^{\frac{7}{2}}}{b^2}-\frac{7\,(bx^3+a)^{\frac{5}{2}}a}{b^2}\right)A+\frac{2}{945}\left(\frac{35\,(bx^3+a)^{\frac{9}{2}}}{b^3}-\frac{90\,(bx^3+a)^{\frac{7}{2}}a}{b^3}+\frac{63\,(bx^3+a)^{\frac{5}{2}}a^2}{b^3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)^(5/2)*a/b^2)*A + 2/945*(35*(
b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 + a)^(5/2)*a^
2/b^3)*B
```

Fricas [A]
time = 2.42, size = 99, normalized size = 1.36

$$\frac{2\,(35\,Bb^4x^{12}+5\,(10\,Bab^3+9\,Ab^4)x^9+3\,(Ba^2b^2+24\,Aab^3)x^6+8\,Ba^4-18\,Aa^3b-(4\,Ba^3b-9\,Aa^2b^2)x^3)\sqrt{bx^3+a}}{945\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^4*x^12 + 5*(10*B*a*b^3 + 9*A*b^4)*x^9 + 3*(B*a^2*b^2 + 24*A*a
*b^3)*x^6 + 8*B*a^4 - 18*A*a^3*b - (4*B*a^3*b - 9*A*a^2*b^2)*x^3)*sqrt(b*x^
3 + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(70) = 140$.
time = 0.41, size = 216, normalized size = 2.96

$$\begin{cases} -\frac{4Aa^3\sqrt{a+bx^3}}{105b^2}+\frac{2Aa^2x^3\sqrt{a+bx^3}}{105b}+\frac{16Aax^6\sqrt{a+bx^3}}{105}+\frac{2Abx^9\sqrt{a+bx^3}}{21}+\frac{16Ba^4\sqrt{a+bx^3}}{945b^3}-\frac{8Ba^3x^3\sqrt{a+bx^3}}{945b^2}+\frac{2Ba^2x^6\sqrt{a+bx^3}}{315b}+\frac{20Bax^9\sqrt{a+bx^3}}{189}+\frac{2Bbx^{12}\sqrt{a+bx^3}}{27} & \text{for } b\neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((-4*A*a**3*sqrt(a + b*x**3)/(105*b**2) + 2*A*a**2*x**3*sqrt(a + b
*x**3)/(105*b) + 16*A*a*x**6*sqrt(a + b*x**3)/105 + 2*A*b*x**9*sqrt(a + b*x
**3)/21 + 16*B*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*B*a**3*x**3*sqrt(a + b*
x**3)/(945*b**2) + 2*B*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*B*a*x**9*sqr
t(a + b*x**3)/189 + 2*B*b*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (a**(3/2)*(
A*x**6/6 + B*x**9/9), True))
```

Giac [A]
time = 0.59, size = 73, normalized size = 1.00

$$\frac{2\left(35\left(bx^3+a\right)^{\frac{9}{2}}B-90\left(bx^3+a\right)^{\frac{7}{2}}Ba+63\left(bx^3+a\right)^{\frac{5}{2}}Ba^2+45\left(bx^3+a\right)^{\frac{7}{2}}Ab-63\left(bx^3+a\right)^{\frac{5}{2}}Aab\right)}{945\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2)*B - 90*(b*x^3 + a)^(7/2)*B*a + 63*(b*x^3 + a)^(
5/2)*B*a^2 + 45*(b*x^3 + a)^(7/2)*A*b - 63*(b*x^3 + a)^(5/2)*A*a*b)/b^3
```

Mupad [B]
time = 2.72, size = 211, normalized size = 2.89

$$\frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\,\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{15\,b}-\frac{2\,a\,\left(2\,A\,a^2-\frac{4\,a\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\,\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b^2}+\frac{2\,B\,b\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{x^3\,\left(2\,A\,a^2-\frac{4\,a\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\,\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b}+\frac{x^9\,\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)\,\sqrt{b\,x^3+a}}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] (x^6*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 + (20*B*a*b)/9))/
(7*b)))/(15*b) - (2*a*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 +
(20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b^2) + (2*B*b*x^12*(a +
 b*x^3)^(1/2))/27 + (x^3*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2
 + (20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b) + (x^9*(2*A*b^2 +
 (20*B*a*b)/9)*(a + b*x^3)^(1/2))/(21*b)
```

3.198 $\int x^2(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal. Leaf size=46

$$\frac{2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}$$

```
[Out] 2/15*(A*b-B*a)*(b*x^3+a)^(5/2)/b^2+2/21*B*(b*x^3+a)^(7/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {455, 45}

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^2) + (2*B*(a + b*x^3)^(7/2))/(21*b^
2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx&=\frac{1}{3}\text{Subst}\left(\int(a+bx)^{3/2}(A+Bx)\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^{3/2}}{b}+\frac{B(a+bx)^{5/2}}{b}\right)\,dx,x,x^3\right)\\&=\frac{2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.74

$$\frac{2(a+bx^3)^{5/2}\left(7Ab-2aB+5bBx^3\right)}{105b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^3))/(105*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(38) = 76.
time = 0.33, size = 87, normalized size = 1.89

method	result
gosper	$\frac{2\left(b\,x^{3}+a\right)^{\frac{5}{2}}\left(5bB\,x^{3}+7Ab-2Ba\right)}{105b^{2}}$
trager	$\frac{2\left(5B\,x^{9}b^{3}+7A\,b^{3}x^{6}+8Ba\,b^{2}x^{6}+14Aa\,b^{2}x^{3}+B\,a^{2}b\,x^{3}+7A\,a^{2}b-2B\,a^{3}\right)\sqrt{b\,x^{3}+a}}{105b^{2}}$
risch	$\frac{2\left(5B\,x^{9}b^{3}+7A\,b^{3}x^{6}+8Ba\,b^{2}x^{6}+14Aa\,b^{2}x^{3}+B\,a^{2}b\,x^{3}+7A\,a^{2}b-2B\,a^{3}\right)\sqrt{b\,x^{3}+a}}{105b^{2}}$
default	$B\left(\frac{2b\,x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{16a\,x^{6}\sqrt{b\,x^{3}+a}}{105}+\frac{2a^{2}x^{3}\sqrt{b\,x^{3}+a}}{105b}-\frac{4a^{3}\sqrt{b\,x^{3}+a}}{105b^{2}}\right)+\frac{2A\left(b\,x^{3}+a\right)^{\frac{5}{2}}}{15b}$
elliptic	$\frac{2Bb\,x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2\left(b^{2}A+\frac{8}{7}abB\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}+\frac{2\left(2abA+a^{2}B-\frac{4a\left(b^{2}A+\frac{8}{7}abB\right)}{5b}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(a^{2}A-\frac{2a\left(\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/21*b*x^9*(b*x^3+a)^(1/2)+16/105*a*x^6*(b*x^3+a)^(1/2)+2/105/b*a^2*x^3*
(b*x^3+a)^(1/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2))+2/15*A*(b*x^3+a)^(5/2)/b
```

Maxima [A]
time = 0.30, size = 49, normalized size = 1.07

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}A}{15\,b}+\frac{2}{105}\left(\frac{5\left(bx^3+a\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(bx^3+a\right)^{\frac{5}{2}}a}{b^2}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*A/b + 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)
^(5/2)*a/b^2)*B
```

Fricas [A]
time = 1.45, size = 73, normalized size = 1.59

$$\frac{2\,(5\,Bb^3x^9+(8\,Bab^2+7\,Ab^3)x^6-2\,Ba^3+7\,Aa^2b+(Ba^2b+14\,Aab^2)x^3)\sqrt{bx^3+a}}{105\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/105*(5*B*b^3*x^9 + (8*B*a*b^2 + 7*A*b^3)*x^6 - 2*B*a^3 + 7*A*a^2*b + (B*a
^2*b + 14*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(44) = 88$.
time = 0.27, size = 165, normalized size = 3.59

$$\begin{cases} \frac{2Aa^2\sqrt{a+bx^3}}{15b}+\frac{4Aax^3\sqrt{a+bx^3}}{15}+\frac{2Abx^6\sqrt{a+bx^3}}{15}-\frac{4Ba^3\sqrt{a+bx^3}}{105b^2}+\frac{2Ba^2x^3\sqrt{a+bx^3}}{105b}+\frac{16Bax^6\sqrt{a+bx^3}}{105}+\frac{2Bbx^9\sqrt{a+bx^3}}{21} & \text{for } b\neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((2*A*a**2*sqrt(a + b*x**3)/(15*b) + 4*A*a*x**3*sqrt(a + b*x**3)/1
5 + 2*A*b*x**6*sqrt(a + b*x**3)/15 - 4*B*a**3*sqrt(a + b*x**3)/(105*b**2) +
 2*B*a**2*x**3*sqrt(a + b*x**3)/(105*b) + 16*B*a*x**6*sqrt(a + b*x**3)/105
+ 2*B*b*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B*x**6/6
), True))
```

Giac [A]
time = 0.63, size = 44, normalized size = 0.96

$$\frac{2\left(5\,(bx^3+a)^{\frac{7}{2}}B-7\,(bx^3+a)^{\frac{5}{2}}Ba+7\,(bx^3+a)^{\frac{5}{2}}Ab\right)}{105\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/105*(5*(b*x^3 + a)^(7/2)*B - 7*(b*x^3 + a)^(5/2)*B*a + 7*(b*x^3 + a)^(5/2
)*A*b)/b^2
```

Mupad [B]
time = 3.35, size = 150, normalized size = 3.26

$$\frac{\left(2\,A\,a^2-\frac{2\,a\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{4\,a\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)}{5\,b}\right)}{3\,b}\right)\,\sqrt{b\,x^3+a}}{3\,b}+\frac{x^3\,\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{4\,a\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)}{5\,b}\right)}{9\,b}+\frac{2\,B\,b\,x^9\,\sqrt{b\,x^3+a}}{21}+\frac{x^6\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)\,\sqrt{b\,x^3+a}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] ((2*A*a^2 - (2*a*(2*B*a^2 + 4*A*a*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b))
)/(3*b))*(a + b*x^3)^(1/2))/(3*b) + (x^3*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a
*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b)))/(9*b) + (2*B*b*x^9*(a + b*x^3)^
(1/2))/21 + (x^6*(2*A*b^2 + (16*B*a*b)/7)*(a + b*x^3)^(1/2))/(15*b)
```

3.199 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x} \, dx$

Optimal. Leaf size=81

$$\frac{2}{3}aA\sqrt{a+bx^3}+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}-\frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

```
[Out] 2/9*A*(b*x^3+a)^(3/2)+2/15*B*(b*x^3+a)^(5/2)/b-2/3*a^(3/2)*A*arctanh((b*x^3
+a)^(1/2)/a^(1/2))+2/3*a*A*(b*x^3+a)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 81, 52, 65, 214}

$$-\frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2}{3}aA\sqrt{a+bx^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

```
[Out] (2*a*A*Sqrt[a + b*x^3])/3 + (2*A*(a + b*x^3)^(3/2))/9 + (2*B*(a + b*x^3)^(5
/2))/(15*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx, x, x^3\right)\\
&= \frac{2B(a+bx^3)^{5/2}}{15b} + \frac{1}{3}A\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^3\right)\\
&= \frac{2}{9}A(a+bx^3)^{3/2} + \frac{2B(a+bx^3)^{5/2}}{15b} + \frac{1}{3}(aA)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)\\
&= \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2}{9}A(a+bx^3)^{3/2} + \frac{2B(a+bx^3)^{5/2}}{15b} + \frac{1}{3}(a^2A)\,\text{Subst}\left(\int \frac{}{x}\right.\\
&= \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2}{9}A(a+bx^3)^{3/2} + \frac{2B(a+bx^3)^{5/2}}{15b} + \frac{(2a^2A)\,\text{Subst}\left(\int \frac{}{-\frac{a}{b}}\right.}{}\\
&= \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2}{9}A(a+bx^3)^{3/2} + \frac{2B(a+bx^3)^{5/2}}{15b} - \frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{}}{}\right.
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 85, normalized size = 1.05

$$\frac{2\sqrt{a+bx^3}\,(20aAb+3a^2B+5Ab^2x^3+6abBx^3+3b^2Bx^6)}{45b} - \frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(20*a*A*b + 3*a^2*B + 5*A*b^2*x^3 + 6*a*b*B*x^3 + 3*b^2*
B*x^6))/(45*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Maple [A]
time = 0.32, size = 66, normalized size = 0.81

method	result
default	$\frac{2B\left(b\,x^{3}+a\right)^{\frac{5}{2}}}{15b}+A\left(\frac{2b\,x^{3}\sqrt{b\,x^{3}+a}}{9}+\frac{8a\sqrt{b\,x^{3}+a}}{9}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}\right)$
elliptic	$\frac{2Bb\,x^{6}\sqrt{b\,x^{3}+a}}{15}+\frac{2\left(b^{2}A+\frac{6}{5}abB\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(2abA+a^{2}B-\frac{2\left(b^{2}A+\frac{6}{5}abB\right)a}{3b}\right)\sqrt{b\,x^{3}+a}}{3b}-\frac{2a^{\frac{3}{2}}A\operatorname{arctanh}\left(\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*B*(b*x^3+a)^(5/2)/b+A*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)
-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.51, size = 80, normalized size = 0.99

$$\frac{2\left(bx^{3}+a\right)^{\frac{5}{2}}B}{15\,b}+\frac{1}{9}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)+2\left(bx^{3}+a\right)^{\frac{3}{2}}+6\sqrt{bx^{3}+a}\,a\right)A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*B/b + 1/9*(3*a^(3/2)*log((sqrt(b*x^3 + a) - sqrt(a))
/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)^(3/2) + 6*sqrt(b*x^3 + a)*a)*
A
```

Fricas [A]
time = 1.74, size = 172, normalized size = 2.12

$$\left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(\frac{bx^{3}-2\sqrt{bx^{3}+a}\sqrt{a}+2\,a}{x^{3}}\right)+2\left(3\,Bb^{2}x^{6}+\left(6\,Bab+5\,Ab^{2}\right)x^{3}+3\,Ba^{2}+20\,Aab\right)\sqrt{bx^{3}+a}}{45\,b},\frac{2\left(15\,A\sqrt{-a}\,ab\arctan\left(\frac{\sqrt{bx^{3}+a}\sqrt{-a}}{a}\right)+\left(3\,Bb^{2}x^{6}+\left(6\,Bab+5\,Ab^{2}\right)x^{3}+3\,Ba^{2}+20\,Aab\right)\sqrt{bx^{3}+a}\right)}{45\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] [1/45*(15*A*a^(3/2)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) +
2*(3*B*b^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 +
```

```
a))/b, 2/45*(15*A*sqrt(-a)*a*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*B*b
^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 + a))/b]
```

Sympy [A]
time = 23.61, size = 82, normalized size = 1.01

$$\frac{2Aa^2\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3\sqrt{-a}}+\frac{2Aa\sqrt{a+bx^3}}{3}+\frac{2A(a+bx^3)^{\frac{3}{2}}}{9}+\frac{2B(a+bx^3)^{\frac{5}{2}}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x,x)
```

```
[Out] 2*A*a**2*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*sqrt(-a)) + 2*A*a*sqrt(a + b*x*
*3)/3 + 2*A*(a + b*x**3)**(3/2)/9 + 2*B*(a + b*x**3)**(5/2)/(15*b)
```

Giac [A]
time = 0.56, size = 80, normalized size = 0.99

$$\frac{2\,Aa^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}+\frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}Bb^4+5\,(bx^3+a)^{\frac{3}{2}}Ab^5+15\,\sqrt{bx^3+a}\,Aab^5\right)}{45\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 2/3*A*a^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/45*(3*(b*x^3 + a)^(
5/2)*B*b^4 + 5*(b*x^3 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^3 + a)*A*a*b^5)/b^5
```

Mupad [B]
time = 2.79, size = 131, normalized size = 1.62

$$\frac{A\,a^{3/2}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}+\frac{\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{2\,a\,\left(2\,A\,b^2+\frac{12\,B\,a\,b}{5}\right)}{3\,b}\right)}{3\,b}+\frac{2\,B\,b\,x^6\,\sqrt{b\,x^3+a}}{15}+\frac{x^3\,\left(2\,A\,b^2+\frac{12\,B\,a\,b}{5}\right)\,\sqrt{b\,x^3+a}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x,x)
```

```
[Out] (A*a^(3/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2
)))/x^6))/3 + ((a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (2*a*(2*A*b^2 + (12*B
*a*b)/5))/(3*b)))/(3*b) + (2*B*b*x^6*(a + b*x^3)^(1/2))/15 + (x^3*(2*A*b^2
+ (12*B*a*b)/5)*(a + b*x^3)^(1/2))/(9*b)
```

3.200 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4} dx$

Optimal. Leaf size=110

$$\frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3}+\frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a}-\frac{A\left(a+bx^3\right)^{5/2}}{3ax^3}-\frac{1}{3}\sqrt{a}\,(3Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

```
[Out] 1/9*(3*A*b+2*B*a)*(b*x^3+a)^(3/2)/a-1/3*A*(b*x^3+a)^(5/2)/a/x^3-1/3*(3*A*b+
2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+1/3*(3*A*b+2*B*a)*(b*x^3+a)
^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 52, 65, 214}

$$\frac{\left(a+bx^3\right)^{3/2}(2aB+3Ab)}{9a}+\frac{1}{3}\sqrt{a+bx^3}\,(2aB+3Ab)-\frac{1}{3}\sqrt{a}\,(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{A\left(a+bx^3\right)^{5/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^3])/3 + ((3*A*b + 2*a*B)*(a + b*x^3)^(3/2))/(
9*a) - (A*(a + b*x^3)^(5/2))/(3*a*x^3) - (Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{A(a+bx^3)^{5/2}}{3ax^3} + \frac{\left(\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^3\right)}{3a}$$

$$= \frac{(3Ab+2aB)(a+bx^3)^{3/2}}{9a} - \frac{A(a+bx^3)^{5/2}}{3ax^3} + \frac{1}{6}(3Ab+2aB)\text{Subst}\left(\int \frac{\sqrt{a}\cdots}{\cdots}\right.$$

$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)(a+bx^3)^{3/2}}{9a} - \frac{A(a+bx^3)^{5/2}}{3ax^3} + \frac{1}{6}\cdots$$

$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)(a+bx^3)^{3/2}}{9a} - \frac{A(a+bx^3)^{5/2}}{3ax^3} + \cdots$$

$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)(a+bx^3)^{3/2}}{9a} - \frac{A(a+bx^3)^{5/2}}{3ax^3} - \frac{1}{3}\cdots$$

Mathematica [A]
time = 0.10, size = 81, normalized size = 0.74

$$\frac{\sqrt{a+bx^3}\,(-3aA+6Abx^3+8aBx^3+2bBx^6)}{9x^3} - \frac{1}{3}\sqrt{a}\,(3Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-3*a*A + 6*A*b*x^3 + 8*a*B*x^3 + 2*b*B*x^6))/(9*x^3) - (S
qrt[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Maple [A]
time = 0.37, size = 101, normalized size = 0.92

method	result
elliptic	$-\frac{aA\sqrt{b\,x^3+a}}{3x^3}+\frac{2Bb\,x^3\sqrt{b\,x^3+a}}{9}+\frac{2\left(b^2A+\frac{4}{3}abB\right)\sqrt{b\,x^3+a}}{3b}-\frac{2\left(\frac{3}{2}abA+a^2B\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$
default	$A\left(-\frac{a\sqrt{b\,x^3+a}}{3x^3}+\frac{2b\sqrt{b\,x^3+a}}{3}-b\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)\sqrt{a}\right)+B\left(\frac{2b\,x^3\sqrt{b\,x^3+a}}{9}+\frac{8a\sqrt{b}\right.$
risch	$-\frac{aA\sqrt{b\,x^3+a}}{3x^3}+b^2B\left(\frac{2x^3\sqrt{b\,x^3+a}}{9b}-\frac{4a\sqrt{b\,x^3+a}}{9b^2}\right)+\frac{2bA\sqrt{b\,x^3+a}}{3}+\frac{4aB\sqrt{b\,x^3+a}}{3}-\frac{(3Ab+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-b*arctanh((b*x^3+a)^(1/
2)/a^(1/2))*a^(1/2))+B*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3
*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.51, size = 134, normalized size = 1.22

$$\frac{1}{6}\left(3\sqrt{a}\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+4\sqrt{bx^3+a}\,b-\frac{2\sqrt{bx^3+a}\,a}{x^3}\right)A+\frac{1}{9}\left(3a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2\left(bx^3+a\right)^{\frac{3}{2}}+6\sqrt{bx^3+a}\,a\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)
)) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3 + a)*a/x^3)*A + 1/9*(3*a^(3/2)*log(
(sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)^(3
/2) + 6*sqrt(b*x^3 + a)*a)*B
```

Fricas [A]
time = 2.12, size = 169, normalized size = 1.54

$$\left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}\,x^3\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(2\,Bbx^6+2\,(4\,Ba+3\,Ab)x^3-3\,Aa)\sqrt{bx^3+a}}{18\,x^3},\ \frac{3\,(2\,Ba+3\,Ab)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+(2\,Bbx^6+2\,(4\,Ba+3\,Ab)x^3-3\,Aa)\sqrt{bx^3+a}}{9\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] [1/18*(3*(2*B*a + 3*A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a)
 + 2*a)/x^3) + 2*(2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a
))/x^3, 1/9*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + (2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a))/x^3]
```

Sympy [A]
time = 19.17, size = 223, normalized size = 2.03

$$-A\sqrt{a}\,b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)-\frac{Aa\sqrt{b}\,\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}+\frac{2Aa\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Ab^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}-\frac{2Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3}+\frac{2Ba^2}{3\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Ba\sqrt{b}\,x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}+Bb\left(\begin{cases}\frac{\sqrt{a}\,x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**4,x)
```

```
[Out] -A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - A*a*sqrt(b)*sqrt(a/(b*x**3
) + 1)/(3*x**(3/2)) + 2*A*a*sqrt(b)/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*A
*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - 2*B*a**(3/2)*asinh(sqrt(a)/(s
qrt(b)*x**(3/2)))/3 + 2*B*a**2/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) +
2*B*a*sqrt(b)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + B*b*Piecewise((sqrt(a)*x*
*3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Giac [A]
time = 0.84, size = 103, normalized size = 0.94

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}Bb+6\sqrt{bx^3+a}\,Bab+6\sqrt{bx^3+a}\,Ab^2+\frac{3\left(2\,Ba^2b+3\,Aab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\sqrt{bx^3+a}\,Aab}{x^3}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(2*(b*x^3 + a)^(3/2)*B*b + 6*sqrt(b*x^3 + a)*B*a*b + 6*sqrt(b*x^3 + a)*
A*b^2 + 3*(2*B*a^2*b + 3*A*a*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a)
 - 3*sqrt(b*x^3 + a)*A*a*b/x^3)/b
```

Mupad [B]
time = 3.38, size = 111, normalized size = 1.01

$$\frac{\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(3\,A\,b+2\,B\,a\right)\,\sqrt{\frac{a}{4}}}{3}+\frac{\left(2\,A\,b^2+\frac{8\,B\,a\,b}{3}\right)\,\sqrt{b\,x^3+a}}{3\,b}-\frac{A\,a\,\sqrt{b\,x^3+a}}{3\,x^3}+\frac{2\,B\,b\,x^3\,\sqrt{b\,x^3+a}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^4,x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(
3*A*b + 2*B*a)*(a/4)^(1/2))/3 + ((2*A*b^2 + (8*B*a*b)/3)*(a + b*x^3)^(1/2))
/(3*b) - (A*a*(a + b*x^3)^(1/2))/(3*x^3) + (2*B*b*x^3*(a + b*x^3)^(1/2))/9
```

3.201 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx$

Optimal. Leaf size=115

$$\frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a}-\frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3}-\frac{A(a+bx^3)^{5/2}}{6ax^6}-\frac{b(Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

```
[Out] -1/12*(A*b+4*B*a)*(b*x^3+a)^(3/2)/a/x^3-1/6*A*(b*x^3+a)^(5/2)/a/x^6-1/4*b*(
A*b+4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+1/4*b*(A*b+4*B*a)*(b*x^
3+a)^(1/2)/a
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 79, 43, 52, 65, 214}

$$-\frac{(a+bx^3)^{3/2}(4aB+Ab)}{12ax^3}+\frac{b\sqrt{a+bx^3}(4aB+Ab)}{4a}-\frac{b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{A(a+bx^3)^{5/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

```
[Out] (b*(A*b + 4*a*B)*Sqrt[a + b*x^3])/(4*a) - ((A*b + 4*a*B)*(a + b*x^3)^(3/2))
/(12*a*x^3) - (A*(a + b*x^3)^(5/2))/(6*a*x^6) - (b*(A*b + 4*a*B)*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx, x, x^3\right)\\
&= -\frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{(Ab+4aB)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^3\right)}{12a}\\
&= -\frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{(b(Ab+4aB))\text{Subst}\left(\int \frac{\sqrt{a}}{}\right.}{8a}\\
&= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{1}{8}(b\\
&= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{1}{4}(A\\
&= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} - \frac{b(A}{}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 81, normalized size = 0.70

$$
\frac{\sqrt{a+bx^3}\left(-2aA-5Abx^3-4aBx^3+8bBx^6\right)}{12x^6} - \frac{b(Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*a*A - 5*A*b*x^3 - 4*a*B*x^3 + 8*b*B*x^6))/(12*x^6) - (
b*(A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

Maple [A]
time = 0.36, size = 107, normalized size = 0.93

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(5Ab\,x^3+4Ba\,x^3+2Aa\right)}{12x^6}+\frac{b\left(\frac{16B\sqrt{b\,x^3+a}}{3}-\frac{2(3Ab+12Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}\right)}{8}$
elliptic	$-\frac{Aa\sqrt{b\,x^3+a}}{6x^6}-\frac{\left(\frac{5Ab}{4}+Ba\right)\sqrt{b\,x^3+a}}{3x^3}+\frac{2Bb\sqrt{b\,x^3+a}}{3}-\frac{2\left(\frac{3}{8}b^2A+\frac{3}{2}abB\right)\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$

default	$A\left(-\frac{a\sqrt{b\,x^3+a}}{6x^6}-\frac{5b\sqrt{b\,x^3+a}}{12x^3}-\frac{b^2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{4\sqrt{a}}\right)+B\left(-\frac{a\sqrt{b\,x^3+a}}{3x^3}+\frac{2b\sqrt{b\,x^3+a}}{3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2*arctanh((b
*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+B*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+
a)^(1/2)-b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2))
```

Maxima [A]
time = 0.49, size = 171, normalized size = 1.49

$$\frac{1}{24}\left(\frac{3\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\left(5\,(bx^3+a)^{\frac{3}{2}}b^2-3\,\sqrt{bx^3+a}\,ab^2\right)}{(bx^3+a)^2-2\,(bx^3+a)a+a^2}\right)A+\frac{1}{6}\left(3\,\sqrt{a}\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+4\,\sqrt{bx^3+a}\,b-\frac{2\,\sqrt{bx^3+a}\,a}{x^3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sq
rt(a) - 2*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^
2 - 2*(b*x^3 + a)*a + a^2))*A + 1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqr
t(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3 + a
)*a/x^3)*B
```

Fricas [A]
time = 1.99, size = 191, normalized size = 1.66

$$\left[\frac{3\,(4\,Bab+Ab^2)\sqrt{a}\,x^6\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\,(8\,Babx^6-(4\,Ba^2+5\,Aab)x^3-2\,Aa^2)\sqrt{bx^3+a}}{24\,ax^6},\ \frac{3\,(4\,Bab+Ab^2)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+(8\,Babx^6-(4\,Ba^2+5\,Aab)x^3-2\,Aa^2)\sqrt{bx^3+a}}{12\,ax^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(
a) + 2*a)/x^3) + 2*(8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt(b
*x^3 + a))/(a*x^6), 1/12*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^
3 + a)*sqrt(-a)/a) + (8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt
(b*x^3 + a))/(a*x^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(102) = 204$.
time = 49.27, size = 243, normalized size = 2.11

$$-\frac{Aa^2}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Aa\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Ab^{\frac{3}{2}}}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{4\sqrt{a}}-B\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)-\frac{Ba\sqrt{b}\,\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}+\frac{2Ba\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Bb^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**7,x)
```

```
[Out] -A*a**2/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*a*sqrt(b)/(4*x**(9/2
)*sqrt(a/(b*x**3) + 1)) - A*b**(3/2)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*
b**(3/2)/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A*b**2*asinh(sqrt(a)/(sqrt(b)
*x**(3/2)))/(4*sqrt(a)) - B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - B
*a*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*B*a*sqrt(b)/(3*x**(3/2)*sq
rt(a/(b*x**3) + 1)) + 2*B*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))
```

Giac [A]
time = 1.33, size = 131, normalized size = 1.14

$$\frac{8\sqrt{bx^3+a}\,Bb^2 + \frac{3\,(4\,Bab^2+Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4\,(bx^3+a)^{\frac{3}{2}}Bab^2-4\sqrt{bx^3+a}\,Ba^2b^2+5\,(bx^3+a)^{\frac{3}{2}}Ab^3-3\sqrt{bx^3+a}\,Aab^3}{b^2x^6}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(8*sqrt(b*x^3 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x^3 + a
)/sqrt(-a))/sqrt(-a) - (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a
^2*b^2 + 5*(b*x^3 + a)^(3/2)*A*b^3 - 3*sqrt(b*x^3 + a)*A*a*b^3)/(b^2*x^6))/
b
```

Mupad [B]
time = 3.47, size = 110, normalized size = 0.96

$$\frac{2\,B\,b\,\sqrt{b\,x^3+a}}{3}-\frac{\sqrt{b\,x^3+a}\,\left(4\,B\,a^3+5\,A\,b\,a^2\right)}{12\,a^2\,x^3}-\frac{A\,a\,\sqrt{b\,x^3+a}}{6\,x^6}+\frac{b\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(A\,b+4\,B\,a\right)}{8\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^7,x)
```

```
[Out] (2*B*b*(a + b*x^3)^(1/2))/3 - ((a + b*x^3)^(1/2)*(4*B*a^3 + 5*A*a^2*b))/(12
*a^2*x^3) - (A*a*(a + b*x^3)^(1/2))/(6*x^6) + (b*log((((a + b*x^3)^(1/2) -
a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(A*b + 4*B*a))/(8*a^(1/2))
```

3.202 $\int x^3(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=336

$$\frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2}+\frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b}+\frac{2(23Ab-8aB)x^4(a+bx^3)^{3/2}}{391b}+\frac{2Bx^4(a+b$$

```
[Out] 2/391*(23*A*b-8*B*a)*x^4*(b*x^3+a)^(3/2)/b+2/23*B*x^4*(b*x^3+a)^(5/2)/b+54/
21505*a^2*(23*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+18/4301*a*(23*A*b-8*B*a)*x^4
*(b*x^3+a)^(1/2)/b-36/21505*3^(3/4)*a^3*(23*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*
EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 285, 327, 224}

$$\frac{54a^2x\sqrt{a+bx^3}(23Ab-8aB)}{21505b^2}-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(23Ab-8aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2x^4(a+bx^3)^{3/2}(23Ab-8aB)}{391b}+\frac{18ax^4\sqrt{a+bx^3}(23Ab-8aB)}{4301b}+\frac{2Bx^4(a+bx^3)^{5/2}}{23b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (54*a^2*(23*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(21505*b^2) + (18*a*(23*A*b - 8
*a*B)*x^4*Sqrt[a + b*x^3])/(4301*b) + (2*(23*A*b - 8*a*B)*x^4*(a + b*x^3)^(
3/2))/(391*b) + (2*B*x^4*(a + b*x^3)^(5/2))/(23*b) - (36*3^(3/4)*Sqrt[2 + S
qrt[3]]*a^3*(23*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(21505*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2Bx^4(a+bx^3)^{5/2}}{23b} - \frac{\left(2\left(-\frac{23Ab}{2}+4aB\right)\right)\int x^3(a+bx^3)^{3/2}\,dx}{23b}$$

$$= \frac{2(23Ab-8aB)x^4(a+bx^3)^{3/2}}{391b} + \frac{2Bx^4(a+bx^3)^{5/2}}{23b} + \frac{(9a(23Ab-8aB))}{391}$$

$$= \frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b} + \frac{2(23Ab-8aB)x^4(a+bx^3)^{3/2}}{391b} + \frac{2Bx^4(}{}$$

$$= \frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2} + \frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b} + \frac{2(23}{}$$

$$= \frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2} + \frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b} + \frac{2(23}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.51, size = 93, normalized size = 0.28

$$\frac{2x\sqrt{a+bx^3}\left(-(a+bx^3)^2(-23Ab+8aB-17bBx^3)+\frac{a^2(-23Ab+8aB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{391b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-23*A*b + 8*a*B - 17*b*B*x^3)) + (a^
2*(-23*A*b + 8*a*B)*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1
 + (b*x^3)/a]))/(391*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(265) = 530.
time = 0.33, size = 694, normalized size = 2.07 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/23*b*x^10*(b*x^3+a)^(1/2)+52/391*a*x^7*(b*x^3+a)^(1/2)+54/4301*a^2*x^4
*(b*x^3+a)^(1/2)/b-432/21505*a^3*x*(b*x^3+a)^(1/2)/b^2-288/21505*I*a^4/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/17*b*x^
7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935*a^2*x*(b*x^3+a)^(1/2)
/b+36/935*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 115, normalized size = 0.34

$$\frac{2\left(54\,(8\,Ba^4 - 23\,Aa^3b)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right) + (935\,Bb^4x^{10} + 55\,(26\,Bab^3 + 23\,Ab^4)x^7 + 5\,(27\,Ba^2b^2 + 460\,Aab^3)x^4 - 27\,(8\,Ba^3b - 23\,Aa^2b^2)x)\sqrt{bx^3 + a}\right)}{21505\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/21505*(54*(8*B*a^4 - 23*A*a^3*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x
) + (935*B*b^4*x^10 + 55*(26*B*a*b^3 + 23*A*b^4)*x^7 + 5*(27*B*a^2*b^2 + 46
0*A*a*b^3)*x^4 - 27*(8*B*a^3*b - 23*A*a^2*b^2)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A]
time = 2.20, size = 172, normalized size = 0.51

$$\frac{Aa^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{A\sqrt{a}\,bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{B\sqrt{a}\,bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{10}{3} \\ \frac{13}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + A*sqrt(a)*b*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + B*a**(3/2)*x**7*gamma(7/3)*h
yper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + B*sq
rt(a)*b*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(13/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^3*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.203 $\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=299

$$\frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b}+\frac{2(17Ab-2aB)x\left(a+bx^3\right)^{3/2}}{187b}+\frac{2Bx\left(a+bx^3\right)^{5/2}}{17b}+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\ a^2(17A}{}$$

```
[Out] 2/187*(17*A*b-2*B*a)*x*(b*x^3+a)^(3/2)/b+2/17*B*x*(b*x^3+a)^(5/2)/b+18/935*
a*(17*A*b-2*B*a)*x*(b*x^3+a)^(1/2)/b+18/935*3^(3/4)*a^2*(17*A*b-2*B*a)*(a^(
1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {396, 201, 224}

$$\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\ a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(17Ab-2aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\left(a+bx^3\right)^{3/2}(17Ab-2aB)}{187b}+\frac{18ax\sqrt{a+bx^3}\,(17Ab-2aB)}{935b}+\frac{2Bx\left(a+bx^3\right)^{5/2}}{17b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (18*a*(17*A*b - 2*a*B)*x*Sqrt[a + b*x^3])/(935*b) + (2*(17*A*b - 2*a*B)*x*(
a + b*x^3)^(3/2))/(187*b) + (2*B*x*(a + b*x^3)^(5/2))/(17*b) + (18*3^(3/4)*
Sqrt[2 + Sqrt[3]]*a^2*(17*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{2Bx\left(a+bx^3\right)^{5/2}}{17b} - \frac{\left(2\left(-\frac{17Ab}{2}+aB\right)\right)\int \left(a+bx^3\right)^{3/2}\,dx}{17b}$$

$$= \frac{2(17Ab-2aB)x\left(a+bx^3\right)^{3/2}}{187b} + \frac{2Bx\left(a+bx^3\right)^{5/2}}{17b} + \frac{(9a(17Ab-2aB))\int \sqrt{a}}{187b}$$

$$= \frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b} + \frac{2(17Ab-2aB)x\left(a+bx^3\right)^{3/2}}{187b} + \frac{2Bx(a+b}{17b}$$

$$= \frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b} + \frac{2(17Ab-2aB)x\left(a+bx^3\right)^{3/2}}{187b} + \frac{2Bx(a+b}{17b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 5.46, size = 77, normalized size = 0.26

$$\frac{2x\sqrt{a+bx^3}\left(B(a+bx^3)^2-\frac{a\left(-\frac{17Ab}{2}+aB\right){}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{17b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2 - (a*((-17*A*b)/2 + a*B)*Hypergeometr
ic2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(17*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(232) = 464.
time = 0.33, size = 654, normalized size = 2.19 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935*a^2*x*(b*
x^3+a)^(1/2)/b+36/935*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-
1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)))+A*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)
-18/55*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 91, normalized size = 0.30

$$-\frac{2\left(27\left(2\,Ba^{3}-17\,Aa^{2}b\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(55\,Bb^{3}x^{7}+5\left(20\,Bab^{2}+17\,Ab^{3}\right)x^{4}+\left(27\,Ba^{2}b+238\,Aab^{2}\right)x\right)\sqrt{bx^{3}+a}\right)}{935\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
[Out] -2/935*(27*(2*B*a^3 - 17*A*a^2*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x)
 - (55*B*b^3*x^7 + 5*(20*B*a*b^2 + 17*A*b^3)*x^4 + (27*B*a^2*b + 238*A*a*b^
2)*x)*sqrt(b*x^3 + a))/b^2
```

Sympy [A]
time = 1.92, size = 170, normalized size = 0.57

$$\frac{Aa^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{A\sqrt{a}\,bx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{B\sqrt{a}\,bx^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A),x)
[Out] A*a**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + A*sqrt(a)*b*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*a**(3/2)*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*sqrt(a)*b
*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3)^(3/2),x)
[Out] int((A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.204 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx$

Optimal. Leaf size=295

$$\frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3}+\frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a}-\frac{A(a+bx^3)^{5/2}}{2ax^2}+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,a(11Ab+4a}{}$$

```
[Out] 1/22*(11*A*b+4*B*a)*x*(b*x^3+a)^(3/2)/a-1/2*A*(b*x^3+a)^(5/2)/a/x^2+9/110*(
11*A*b+4*B*a)*x*(b*x^3+a)^(1/2)+9/110*3^(3/4)*a*(11*A*b+4*B*a)*(a^(1/3)+b^(
1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1
/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 201, 224}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+11Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{110\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{x(a+bx^3)^{3/2}(4aB+11Ab)}{22a}+\frac{9}{110}x\sqrt{a+bx^3}(4aB+11Ab)-\frac{A(a+bx^3)^{5/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
```

```
[Out] (9*(11*A*b + 4*a*B)*x*Sqrt[a + b*x^3])/110 + ((11*A*b + 4*a*B)*x*(a + b*x^3
)^(3/2))/(22*a) - (A*(a + b*x^3)^(5/2))/(2*a*x^2) + (9*3^(3/4)*Sqrt[2 + Sqr
t[3]]*a*(11*A*b + 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(110*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = -\frac{A(a+bx^3)^{5/2}}{2ax^2} - \frac{\left(-\frac{11Ab}{2}-2aB\right)\int (a+bx^3)^{3/2}\,dx}{2a}$$

$$= \frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a} - \frac{A(a+bx^3)^{5/2}}{2ax^2} + \frac{1}{44}(9(11Ab+4aB))\int \sqrt{a+}$$

$$= \frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3} + \frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a} - \frac{A(a+bx^3)^{5/}}{2ax^2}$$

$$= \frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3} + \frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a} - \frac{A(a+bx^3)^{5/}}{2ax^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 5.59, size = 83, normalized size = 0.28

$$-\frac{A(a+bx^3)^{5/2}}{2ax^2}-\frac{\left(-\frac{11Ab}{2}-2aB\right)x\sqrt{a+bx^3}\ {}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(A*(a + b*x^3)^(5/2))/(a*x^2) - (((-11*A*b)/2 - 2*a*B)*x*Sqrt[a + b*x^
3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 628 vs. 2(228) = 456.
time = 0.34, size = 629, normalized size = 2.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-1/2*a*(b*x^3+
a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 80, normalized size = 0.27

$$\frac{27\,(4\,Ba^2+11\,Aab)\sqrt{b}\,x^2\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(20\,Bb^2x^6+4\,(14\,Bab+11\,Ab^2)x^3-55\,Aab\right)\sqrt{bx^3+a}}{110\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/110*(27*(4*B*a^2 + 11*A*a*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x
) + (20*B*b^2*x^6 + 4*(14*B*a*b + 11*A*b^2)*x^3 - 55*A*a*b)*sqrt(b*x^3 + a)
)/(b*x^2)
```

Sympy [A]
time = 2.31, size = 172, normalized size = 0.58

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{A\sqrt{a}\,bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{Ba^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{B\sqrt{a}\,bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**3,x)
```

```
[Out] A*a**(3/2)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + A*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*a**(3/2)*x*gamma(1/3)*hyper((
-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*sqrt(a)*b*
x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3+A)\,{(b\,x^3+a)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^3,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^3, x)
```

3.205 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx$

Optimal. Leaf size=297

$$\frac{9b(Ab+2aB)x\sqrt{a+bx^3}}{20a}-\frac{(Ab+2aB)\left(a+bx^3\right)^{3/2}}{4ax^2}-\frac{A(a+bx^3)^{5/2}}{5ax^5}+\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}\ b^{2/3}(Ab+2aB)\left(\sqrt[3]{\right.}}{}$$

```
[Out] -1/4*(A*b+2*B*a)*(b*x^3+a)^(3/2)/a/x^2-1/5*A*(b*x^3+a)^(5/2)/a/x^5+9/20*b*(
A*b+2*B*a)*x*(b*x^3+a)^(1/2)/a+9/20*3^(3/4)*b^(2/3)*(A*b+2*B*a)*(a^(1/3)+b^
(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 283, 201, 224}

$$\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}\ b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2aB+Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{9bx\sqrt{a+bx^3}\,(2aB+Ab)}{20a}-\frac{(a+bx^3)^{3/2}(2aB+Ab)}{4ax^2}-\frac{A(a+bx^3)^{5/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```

```
[Out] (9*b*(A*b + 2*a*B)*x*Sqrt[a + b*x^3])/(20*a) - ((A*b + 2*a*B)*(a + b*x^3)^(
3/2))/(4*a*x^2) - (A*(a + b*x^3)^(5/2))/(5*a*x^5) + (9*3^(3/4)*Sqrt[2 + Sqr
t[3]]*b^(2/3)*(A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(20*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = -\frac{A(a+bx^3)^{5/2}}{5ax^5} + -\frac{\left(-\frac{5Ab}{2}-5aB\right)\int \frac{(a+bx^3)^{3/2}}{x^3}\,dx}{5a}$$

$$= -\frac{(Ab+2aB)(a+bx^3)^{3/2}}{4ax^2} - \frac{A(a+bx^3)^{5/2}}{5ax^5} + \frac{(9b(Ab+2aB))\int \sqrt{a+bx^3}$$

$$= \frac{9b(Ab+2aB)x\sqrt{a+bx^3}}{20a} - \frac{(Ab+2aB)(a+bx^3)^{3/2}}{4ax^2} - \frac{A(a+bx^3)^{5/2}}{5ax^5} + \frac{}{4}$$

9

$$= \frac{9b(Ab+2aB)x\sqrt{a+bx^3}}{20a} - \frac{(Ab+2aB)(a+bx^3)^{3/2}}{4ax^2} - \frac{A(a+bx^3)^{5/2}}{5ax^5} + -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 82, normalized size = 0.28

$$\frac{\sqrt{a+bx^3}\left(-\frac{2A(a+bx^3)^2}{a}-\frac{5(Ab+2aB)x^3\,{}_2F_1\left(-\frac{3}{2},-\frac{2}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-2*A*(a + b*x^3)^2)/a - (5*(A*b + 2*a*B)*x^3*Hypergeomet
ric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(10*x^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(230) = 460$.
time = 0.36, size = 626, normalized size = 2.11 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
```

```
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+B*(-1/2*a*(b*x^3+a)^
(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.86, size = 67, normalized size = 0.23

$$\frac{27\,(2\,Ba+Ab)\sqrt{b}\,x^5\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(8\,Bbx^6-(10\,Ba+13\,Ab)x^3-4\,Aa\right)\sqrt{bx^3+a}}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(27*(2*B*a + A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) + (8*B
*b*x^6 - (10*B*a + 13*A*b)*x^3 - 4*A*a)*sqrt(b*x^3 + a))/x^5
```

Sympy [A]
time = 2.42, size = 184, normalized size = 0.62

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{A\sqrt{a}\,b\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{B\sqrt{a}\,bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**6,x)
```

```
[Out] A*a**(3/2)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + A*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,
```

```
), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + B*a**(3/2)*gamma(-2/3)*h
yper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) +
B*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3+A)\,\left(b\,x^3+a\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^6,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^6, x)
```

3.206 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx$

Optimal. Leaf size=302

$$\frac{9b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2}+\frac{(Ab-16aB)\left(a+bx^3\right)^{3/2}}{80ax^5}-\frac{A(a+bx^3)^{5/2}}{8ax^8}-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}(Ab-16aB)\left(\cdots\right.}{\cdots}$$

```
[Out] 1/80*(A*b-16*B*a)*(b*x^3+a)^(3/2)/a/x^5-1/8*A*(b*x^3+a)^(5/2)/a/x^8+9/320*b
*(A*b-16*B*a)*(b*x^3+a)^(1/2)/a/x^2-9/320*3^(3/4)*b^(5/3)*(A*b-16*B*a)*(a^(
1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^
(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 283, 224}

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(Ab-16aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{320a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{(a+bx^3)^{3/2}(Ab-16aB)}{80ax^5}+\frac{9b\sqrt{a+bx^3}\,(Ab-16aB)}{320ax^2}-\frac{A(a+bx^3)^{5/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
```

```
[Out] (9*b*(A*b - 16*a*B)*Sqrt[a + b*x^3])/(320*a*x^2) + ((A*b - 16*a*B)*(a + b*x
^3)^(3/2))/(80*a*x^5) - (A*(a + b*x^3)^(5/2))/(8*a*x^8) - (9*3^(3/4)*Sqrt[2
 + Sqrt[3]]*b^(5/3)*(A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(320*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx &= -\frac{A(a+bx^3)^{5/2}}{8ax^8} - \frac{\left(\frac{Ab}{2}-8aB\right)\int \frac{(a+bx^3)^{3/2}}{x^6}\,dx}{8a} \\
&= \frac{(Ab-16aB)(a+bx^3)^{3/2}}{80ax^5} - \frac{A(a+bx^3)^{5/2}}{8ax^8} - \frac{(9b(Ab-16aB))\int \frac{\sqrt{a+bx^3}}{x^3}
\end{aligned}$$

$$= \frac{9b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2} + \frac{(Ab-16aB)(a+bx^3)^{3/2}}{80ax^5} - \frac{A(a+bx^3)^{5/2}}{8ax^8} -$$

$$= \frac{9b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2} + \frac{(Ab-16aB)(a+bx^3)^{3/2}}{80ax^5} - \frac{A(a+bx^3)^{5/2}}{8ax^8} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 82, normalized size = 0.27

$$\frac{\sqrt{a+bx^3}\left(-\frac{5A(a+bx^3)^2}{a}+\frac{\left(\frac{Ab}{2}-8aB\right)x^3\,{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + (((A*b)/2 - 8*a*B)*x^3*Hypergeom
etric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*x^8)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 652 vs. 2(235) = 470.
time = 0.34, size = 653, normalized size = 2.16 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-1/8*a*(b*x^3+a)^
(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320*b^2/a*(b*x^3+a)^(1/2)/x^2+9/32
0*I*b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="maxima")
```

[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 89, normalized size = 0.29

$$\frac{27\,(16\,Bab-Ab^2)\sqrt{b}\,x^8\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left((208\,Bab+27\,Ab^2)x^6+4\,(16\,Ba^2+19\,Aab)x^3+40\,Aa^2\right)\sqrt{bx^3+a}}{320\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="fricas")

[Out] 1/320*(27*(16*B*a*b - A*b^2)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x) - ((208*B*a*b + 27*A*b^2)*x^6 + 4*(16*B*a^2 + 19*A*a*b)*x^3 + 40*A*a^2)*sqrt(b*x^3 + a))/(a*x^8)

Sympy [A]
time = 2.77, size = 196, normalized size = 0.65

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)}+\frac{A\sqrt{a}\,b\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\sqrt{a}\,b\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**9,x)

[Out] A*a**(3/2)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + A*sqrt(a)*b*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + B*a**(3/2)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + B*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="giac")

[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^9,x)

[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^9, x)

3.207 $\int x^4(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=614

$$\frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2}+\frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b}-\frac{216a^3(5Ab-2aB)\sqrt{a+bx^3}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2(5Ab$$

[Out] 2/95*(5*A*b-2*B*a)*x^5*(b*x^3+a)^(3/2)/b+2/25*B*x^5*(b*x^3+a)^(5/2)/b+54/86
45*a^2*(5*A*b-2*B*a)*x^2*(b*x^3+a)^(1/2)/b^2+18/1235*a*(5*A*b-2*B*a)*x^5*(b
*x^3+a)^(1/2)/b-216/8645*a^3*(5*A*b-2*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)
x+a^(1/3)(1+3^(1/2)))-72/8645*3^(3/4)*a^(10/3)*(5*A*b-2*B*a)*(a^(1/3)+b^(
1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1
/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+108/8645*3^(1/4)*a^
(10/3)*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(
1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.27, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {470, 285, 327, 309, 224, 1891}

$$-\frac{72\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(5Ab-2aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(5Ab-2aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{216a^3\sqrt{a+bx^3}(5Ab-2aB)}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{54a^2x^2\sqrt{a+bx^3}(5Ab-2aB)}{8645b^2}+\frac{2x^5\left(a+bx^3\right)^{3/2}(5Ab-2aB)}{95b}+\frac{18ax^5\sqrt{a+bx^3}(5Ab-2aB)}{1235b}+\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

Antiderivative was successfully verified.

[In] Int[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]

[Out] (54*a^2*(5*A*b - 2*a*B)*x^2*Sqrt[a + b*x^3])/(8645*b^2) + (18*a*(5*A*b - 2*
a*B)*x^5*Sqrt[a + b*x^3])/(1235*b) - (216*a^3*(5*A*b - 2*a*B)*Sqrt[a + b*x^
3])/(8645*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(5*A*b - 2*a*B)
x^5(a + b*x^3)^(3/2))/(95*b) + (2*B*x^5*(a + b*x^3)^(5/2))/(25*b) + (108*
3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[

```
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
 - (72*Sqrt[2]*3^(3/4)*a^(10/3)*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b} - \frac{\left(2\left(-\frac{25Ab}{2}+5aB\right)\right)\int x^4\left(a+bx^3\right)^{3/2}\,dx}{25b}$$

$$= \frac{2(5Ab-2aB)x^5\left(a+bx^3\right)^{3/2}}{95b} + \frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b} + \frac{(9a(5Ab-2aB))\int x}{95b}$$

$$= \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} + \frac{2(5Ab-2aB)x^5\left(a+bx^3\right)^{3/2}}{95b} + \frac{2Bx^5(a}{25}$$

$$= \frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} + \frac{2(5Ab}{}$$

$$= \frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} + \frac{2(5Ab}{}$$

$$= \frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} - \frac{216}{8645b^8}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 5.81, size = 96, normalized size = 0.16

$$\frac{2x^2\sqrt{a+bx^3}\left(-(a+bx^3)^2(-25Ab+10aB-19bBx^3)+\frac{5a^2(-5Ab+2aB)\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{475b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-25*A*b + 10*a*B - 19*b*B*x^3)) +
(5*a^2*(-5*A*b + 2*a*B)*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sq
rt[1 + (b*x^3)/a]))/(475*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(468) = 936.
time = 0.33, size = 1002, normalized size = 1.63

method	result
risch	$\frac{2x^2(1729B\,x^9b^3+2275A\,b^3x^6+2548Ba\,b^2x^6+3850Aa\,b^2x^3+189B\,a^2b\,x^3+675A\,a^2b-270B\,a^3)\sqrt{b\,x^3+a}}{43225b^2}+\frac{72ia^3(5Ab-2Ba)}{}$

elliptic	$\frac{2Bb\,x^{11}\sqrt{b\,x^3+a}}{25}+\frac{2(b^2A+\frac{28}{25}abB)x^8\sqrt{b\,x^3+a}}{19b}+\frac{2\left(2abA+a^2B-\frac{16a\left(b^2A+\frac{28}{25}abB\right)}{19b}\right)x^5\sqrt{b\,x^3+a}}{13b}+\cdots$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/25*b*x^11*(b*x^3+a)^(1/2)+56/475*a*x^8*(b*x^3+a)^(1/2)+54/6175*a^2*x^5
*(b*x^3+a)^(1/2)/b-108/8645*a^3*x^2*(b*x^3+a)^(1/2)/b^2-144/8645*I*a^4/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(2/19*b*x^8*(b*x^3+a)^(1/2)+44
/247*a*x^5*(b*x^3+a)^(1/2)+54/1729*a^2*x^2*(b*x^3+a)^(1/2)/b+72/1729*I/b^2*
a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
```

```
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 126, normalized size = 0.21

$$-\frac{2\left(540\left(2\,Ba^{4}-5\,Aa^{3}b\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(1729\,Bb^{4}x^{11}+91\left(28\,Bab^{3}+25\,Ab^{4}\right)x^{8}+7\left(27\,Ba^{2}b^{2}+550\,Aab^{3}\right)x^{5}-135\left(2\,Ba^{3}b-5\,Aa^{2}b^{2}\right)x^{2}\right)\sqrt{bx^{3}+a}\right)}{43225\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
[Out] -2/43225*(540*(2*B*a^4 - 5*A*a^3*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)) - (1729*B*b^4*x^11 + 91*(28*B*a*b^3 + 25*A*b
^4)*x^8 + 7*(27*B*a^2*b^2 + 550*A*a*b^3)*x^5 - 135*(2*B*a^3*b - 5*A*a^2*b^2
)*x^2)*sqrt(b*x^3 + a))/b^3
```

Sympy [A]
time = 2.29, size = 172, normalized size = 0.28

$$\frac{Aa^{\frac{3}{2}}x^{5}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{A\sqrt{a}\,bx^{8}\Gamma\left(\frac{8}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^{8}\Gamma\left(\frac{8}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}+\frac{B\sqrt{a}\,bx^{11}\Gamma\left(\frac{11}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{3}\\\frac{14}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{14}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(3/2)*(B*x**3+A),x)
[Out] A*a**(3/2)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(8/3)) + A*sqrt(a)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + B*a**(3/2)*x**8*gamma(8/3)*h
yper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + B*sq
rt(a)*b*x**11*gamma(11/3)*hyper((-1/2, 11/3), (14/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(14/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^4*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.208 $\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal. Leaf size=581

$$\frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b}+\frac{54a^2(19Ab-4aB)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2(19Ab-4aB)x^2(a+bx^3)^{3/2}}{247b}+\frac{2Bx}{}$$

```
[Out] 2/247*(19*A*b-4*B*a)*x^2*(b*x^3+a)^(3/2)/b+2/19*B*x^2*(b*x^3+a)^(5/2)/b+18/
1729*a*(19*A*b-4*B*a)*x^2*(b*x^3+a)^(1/2)/b+54/1729*a^2*(19*A*b-4*B*a)*(b*x
^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+18/1729*3^(3/4)*a^(7/3)
*(19*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b
*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)-27/1729*3^(1/4)*a^(7/3)*(19*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*Ellipti
cE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {470, 285, 309, 224, 1891}

$$\frac{18\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(19Ab-4aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(19Ab-4aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{54a^2\sqrt{a+bx^3}(19Ab-4aB)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x^2(a+bx^3)^{3/2}(19Ab-4aB)}{247b}+\frac{18ax^2\sqrt{a+bx^3}(19Ab-4aB)}{1729b}+\frac{2Bx^2(a+bx^3)^{5/2}}{19b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (18*a*(19*A*b - 4*a*B)*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*A*b - 4*
a*B)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) +
(2*(19*A*b - 4*a*B)*x^2*(a + b*x^3)^(3/2))/(247*b) + (2*B*x^2*(a + b*x^3)^(
5/2))/(19*b) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*A*b - 4*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
```

```
2]*Sqrt[a + b*x^3]) + (18*Sqrt[2]*3^(3/4)*a^(7/3)*(19*A*b - 4*a*B)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
```

```
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{2Bx^2(a+bx^3)^{5/2}}{19b} - \frac{\left(2\left(-\frac{19Ab}{2}+2aB\right)\right)\int x\left(a+bx^3\right)^{3/2}\,dx}{19b}$$

$$= \frac{2(19Ab-4aB)x^2(a+bx^3)^{3/2}}{247b} + \frac{2Bx^2(a+bx^3)^{5/2}}{19b} + \frac{(9a(19Ab-4aB))}{247}$$

$$= \frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b} + \frac{2(19Ab-4aB)x^2(a+bx^3)^{3/2}}{247b} + \frac{2Bx^2}{}$$

$$= \frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b} + \frac{2(19Ab-4aB)x^2(a+bx^3)^{3/2}}{247b} + \frac{2Bx^2}{}$$

$$= \frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b} + \frac{54a^2(19Ab-4aB)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.44, size = 78, normalized size = 0.13

$$\frac{x^2\sqrt{a+bx^3}\left(4B\left(a+bx^3\right)^2+\frac{a(19Ab-4aB)\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{38b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
[Out] (x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3)^2 + (a*(19*A*b - 4*a*B)*Hypergeometri
c2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(38*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(439) = 878.
time = 0.32, size = 962, normalized size = 1.66

method	result
risch	$\frac{2x^2(91b^2Bx^6+133Ab^2x^3+154Babx^3+304abA+27a^2B)\sqrt{bx^3+a}}{1729b} - \frac{18ia^2(19Ab-4Ba)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\cdots\right.}{(\cdots}}}{\cdots}$
elliptic	$\frac{2Bbx^8\sqrt{bx^3+a}}{19}+\frac{2(b^2A+\frac{22}{19}abB)x^5\sqrt{bx^3+a}}{13b}+\frac{2\left(2abA+a^2B-\frac{10a\left(b^2A+\frac{22}{19}abB\right)}{13b}\right)x^2\sqrt{bx^3+a}}{7b}-\frac{2i\left(a^2A-\cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729*a^2*x^2*
(b*x^3+a)^(1/2)/b+72/1729*I/b^2*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
```

```
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)))+A*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3
^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 100, normalized size = 0.17

$$\frac{2\left(27\,(4\,Ba^3-19\,Aa^2b)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(91\,Bb^3x^8+7\,(22\,Bab^2+19\,Ab^3)x^5+(27\,Ba^2b+304\,Aab^2)x^2\right)\sqrt{bx^3+a}\right)}{1729\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/1729*(27*(4*B*a^3 - 19*A*a^2*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weiers
trassPInverse(0, -4*a/b, x)) + (91*B*b^3*x^8 + 7*(22*B*a*b^2 + 19*A*b^3)*x^
5 + (27*B*a^2*b + 304*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/b^2
```

Sympy [A]
time = 2.04, size = 172, normalized size = 0.30

$$\frac{Aa^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{A\sqrt{a}\,bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{B\sqrt{a}\,bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3)) + A*sqrt(a)*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*a**(3/2)*x**5*gamma(5/3)*hyp
er((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*sqrt(a
)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(11/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.209 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx$

Optimal. Leaf size=573

$$\frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3}+\frac{27a(13Ab+2aB)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{(13Ab+2aB)x^2(a+bx^3)^{3/2}}{13a}-\frac{A(a+bx^3}{ax}$$

```
[Out] 1/13*(13*A*b+2*B*a)*x^2*(b*x^3+a)^(3/2)/a-A*(b*x^3+a)^(5/2)/a/x+9/91*(13*A*
b+2*B*a)*x^2*(b*x^3+a)^(1/2)+27/91*a*(13*A*b+2*B*a)*(b*x^3+a)^(1/2)/b^(2/3)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/91*3^(3/4)*a^(4/3)*(13*A*b+2*B*a)*(a^(1/
3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/182*3^(1/4
)*a^(4/3)*(13*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 285, 309, 224, 1891}

$$\frac{9\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(2aB+13Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(2aB+13Ab)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{27a\sqrt{a+bx^3}(2aB+13Ab)}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{x^2(a+bx^3)^{3/2}(2aB+13Ab)}{13a}+\frac{9}{91}x^2\sqrt{a+bx^3}(2aB+13Ab)-\frac{A(a+bx^3)^{5/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

```
[Out] (9*(13*A*b + 2*a*B)*x^2*Sqrt[a + b*x^3])/91 + (27*a*(13*A*b + 2*a*B)*Sqrt[a
 + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + ((13*A*b + 2*
a*B)*x^2*(a + b*x^3)^(3/2))/(13*a) - (A*(a + b*x^3)^(5/2))/(a*x) - (27*3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9
```

```
*Sqrt[2]*3^(3/4)*a^(4/3)*(13*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
```

```
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = -\frac{A(a+bx^3)^{5/2}}{ax} - \frac{\left(-\frac{13Ab}{2}-aB\right)\int x(a+bx^3)^{3/2}\,dx}{a}$$

$$= \frac{(13Ab+2aB)x^2(a+bx^3)^{3/2}}{13a} - \frac{A(a+bx^3)^{5/2}}{ax} + \frac{1}{26}(9(13Ab+2aB))\int x\sqrt{}$$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{(13Ab+2aB)x^2(a+bx^3)^{3/2}}{13a} - \frac{A(a+bx^3}{ax}$$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{(13Ab+2aB)x^2(a+bx^3)^{3/2}}{13a} - \frac{A(a+bx^3}{ax}$$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{27a(13Ab+2aB)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{(13A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.61, size = 83, normalized size = 0.14

$$-\frac{A(a+bx^3)^{5/2}}{ax} - \frac{\left(-\frac{13Ab}{2}-aB\right)x^2\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

```
[Out] -((A*(a + b*x^3)^(5/2))/(a*x)) - (((-13*A*b)/2 - a*B)*x^2*Sqrt[a + b*x^3]*H
ypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 936 vs. 2(433) = 866.
time = 0.34, size = 937, normalized size = 1.64

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-14bB\,x^{6}-26Ab\,x^{3}-32Ba\,x^{3}+91Aa\right)}{91x}-\frac{9ia(13Ab+2Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{x}+\frac{2Bb\,x^{5}\sqrt{b\,x^{3}+a}}{13}+\frac{2\left(b^{2}A+\frac{16}{13}abB\right)x^{2}\sqrt{b\,x^{3}+a}}{7b}-\frac{2i\left(\frac{5abA}{2}+a^{2}B-\frac{4a\left(b^{2}A+\frac{16}{13}abB\right)}{7b}\right)\sqrt{3}\,(-}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
```

```
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(
b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 87, normalized size = 0.15

$$-\frac{27\,(2\,Ba^2+13\,Aab)\sqrt{b}\;x\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-(14\,Bb^2x^6+2\,(16\,Bab+13\,Ab^2)x^3-91\,Aab)\sqrt{bx^3+a}}{91\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] -1/91*(27*(2*B*a^2 + 13*A*a*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierst
rassPInverse(0, -4*a/b, x)) - (14*B*b^2*x^6 + 2*(16*B*a*b + 13*A*b^2)*x^3 -
 91*A*a*b)*sqrt(b*x^3 + a))/(b*x)
```

Sympy [A]
time = 2.34, size = 173, normalized size = 0.30

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}+\frac{A\sqrt{a}\,bx^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{B\sqrt{a}\,bx^5\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**2,x)
```

```
[Out] A*a**(3/2)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x*gamma(2/3)) + A*sqrt(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*a**(3/2)*x**2*gamma(2/3)*hype
r((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*sqrt(a)
*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^2,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^2, x)
```

3.210 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx$

Optimal. Leaf size=578

$$\frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a}+\frac{27\sqrt[3]{b}\,(7Ab+8aB)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{(7Ab+8aB)\left(a+bx^3\right)^{3/2}}{8ax}-\frac{A(a+bx^3)^{5/2}}{4ax^4}-\cdots$$

2

```
[Out] -1/8*(7*A*b+8*B*a)*(b*x^3+a)^(3/2)/a/x-1/4*A*(b*x^3+a)^(5/2)/a/x^4+9/56*b*(
7*A*b+8*B*a)*x^2*(b*x^3+a)^(1/2)/a+27/56*b^(1/3)*(7*A*b+8*B*a)*(b*x^3+a)^(1
/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/56*3^(3/4)*a^(1/3)*b^(1/3)*(7*A*b+8*B
*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/112*3^(1
/4)*a^(1/3)*b^(1/3)*(7*A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {464, 283, 285, 309, 224, 1891}

$$\frac{9\,3^{3/4}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(8aB+7Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(8aB+7Ab)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{(a+bx^3)^{3/2}(8aB+7Ab)}{8ax}+\frac{27\sqrt[3]{b}\sqrt{a+bx^3}\,(8aB+7Ab)}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{9bx^2\sqrt{a+bx^3}\,(8aB+7Ab)}{56a}-\frac{A(a+bx^3)^{5/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

```
[Out] (9*b*(7*A*b + 8*a*B)*x^2*Sqrt[a + b*x^3])/(56*a) + (27*b^(1/3)*(7*A*b + 8*a
*B)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - ((7*A*b + 8
*a*B)*(a + b*x^3)^(3/2))/(8*a*x) - (A*(a + b*x^3)^(5/2))/(4*a*x^4) - (27*3^
(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*A*b + 8*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
```

```
9*3^(3/4)*a^(1/3)*b^(1/3)*(7*A*b + 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = -\frac{A(a+bx^3)^{5/2}}{4ax^4} - \frac{\left(-\frac{7Ab}{2}-4aB\right)\int \frac{(a+bx^3)^{3/2}}{x^2}\,dx}{4a}$$

$$= -\frac{(7Ab+8aB)(a+bx^3)^{3/2}}{8ax} - \frac{A(a+bx^3)^{5/2}}{4ax^4} + \frac{(9b(7Ab+8aB))\int x\sqrt{a+}}{16a}$$

$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} - \frac{(7Ab+8aB)(a+bx^3)^{3/2}}{8ax} - \frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} - \frac{(7Ab+8aB)(a+bx^3)^{3/2}}{8ax} - \frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} + \frac{27\sqrt[3]{b}(7Ab+8aB)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(7Ab+8a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.93, size = 85, normalized size = 0.15

$$-\frac{A(a+bx^3)^{5/2}}{4ax^4} + \frac{\left(-\frac{7Ab}{2}-4aB\right)\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{4x\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

[Out] -1/4*(A*(a + b*x^3)^(5/2))/(a*x^4) + (((-7*A*b)/2 - 4*a*B)*Sqrt[a + b*x^3]*
Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(4*x*Sqrt[1 + (b*x^3)/a])

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(436) = 872$.
time = 0.34, size = 932, normalized size = 1.61

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-16 b B\,x^{6}+77 A b\,x^{3}+56 B a\,x^{3}+14 A a\right)}{56 x^{4}}-\frac{9 i (7 A b+8 B a)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{\ }}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$ $2 i\left(\frac{27}{16} b^{2} A+\frac{27}{14} a b B\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\right.}{2}\right.}{}}$
elliptic	$-\frac{A a\sqrt{b\,x^{3}+a}}{4 x^{4}}-\frac{\left(\frac{11 A b}{8}+B a\right)\sqrt{b\,x^{3}+a}}{x}+\frac{2 B b\,x^{2}\sqrt{b\,x^{3}+a}}{7}-$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)

[Out] A*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/

```
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))))+B*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^
(1/2)-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 77, normalized size = 0.13

$$-\frac{27\,(8\,Ba+7\,Ab)\sqrt{b}\,x^4\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(16\,Bbx^6-7\,(8\,Ba+11\,Ab)x^3-14\,Aa\right)\sqrt{bx^3+a}}{56\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] -1/56*(27*(8*B*a + 7*A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstras
sPInverse(0, -4*a/b, x)) - (16*B*b*x^6 - 7*(8*B*a + 11*A*b)*x^3 - 14*A*a)*s
qrt(b*x^3 + a))/x^4
```

Sympy [A]
time = 2.37, size = 182, normalized size = 0.31

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{A\sqrt{a}\,b\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}+\frac{B\sqrt{a}\,bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**5,x)
```

```
[Out] A*a**(3/2)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**4*gamma(-1/3)) + A*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*a**(3/2)*gamma(-1/3)*hype
r((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*sqrt
(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^5,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^5, x)
```

3.211 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx$

Optimal. Leaf size=576

$$-\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax}+\frac{27b^{4/3}(Ab+14aB)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{(Ab+14aB)\left(a+bx^3\right)^{3/2}}{56ax^4}-\frac{A(a+bx^3)^{5/2}}{7ax^7}-$$

```
[Out] -1/56*(A*b+14*B*a)*(b*x^3+a)^(3/2)/a/x^4-1/7*A*(b*x^3+a)^(5/2)/a/x^7-9/112*
b*(A*b+14*B*a)*(b*x^3+a)^(1/2)/a/x+27/112*b^(4/3)*(A*b+14*B*a)*(b*x^3+a)^(1
/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/112*3^(3/4)*b^(4/3)*(A*b+14*B*a)*(a
^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/224*3^
(1/4)*b^(4/3)*(A*b+14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 576, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 283, 309, 224, 1891}

$$\frac{9\,3^{3/4}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(14aB+Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(14aB+Ab)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{27b^{4/3}\sqrt{a+bx^3}(14aB+Ab)}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{9b\sqrt{a+bx^3}(14aB+Ab)}{112ax}-\frac{(a+bx^3)^{3/2}(14aB+Ab)}{56ax^4}-\frac{A(a+bx^3)^{5/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

```
[Out] (-9*b*(A*b + 14*a*B)*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(A*b + 14*a*B
)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - ((A*b + 14
*a*B)*(a + b*x^3)^(3/2))/(56*a*x^4) - (A*(a + b*x^3)^(5/2))/(7*a*x^7) - (27
*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(A*b + 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) +
```

```
(9*3^(3/4)*b^(4/3)*(A*b + 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
```

```
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = -\frac{A(a+bx^3)^{5/2}}{7ax^7} - \frac{\left(-\frac{Ab}{2}-7aB\right)\int \frac{(a+bx^3)^{3/2}}{x^5}\,dx}{7a}$$

$$= -\frac{(Ab+14aB)(a+bx^3)^{3/2}}{56ax^4} - \frac{A(a+bx^3)^{5/2}}{7ax^7} + \frac{(9b(Ab+14aB))\int \frac{\sqrt{a+b}}{x^2}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax} - \frac{(Ab+14aB)(a+bx^3)^{3/2}}{56ax^4} - \frac{A(a+bx^3)^{5/2}}{7ax^7}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax} - \frac{(Ab+14aB)(a+bx^3)^{3/2}}{56ax^4} - \frac{A(a+bx^3)^{5/2}}{7ax^7}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax} + \frac{27b^{4/3}(Ab+14aB)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{(Ab+14}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 82, normalized size = 0.14

$$\frac{\sqrt{a+bx^3}\left(-\frac{4A(a+bx^3)^2}{a} - \frac{(Ab+14aB)x^3\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-4*A*(a + b*x^3)^2)/a - ((A*b + 14*a*B)*x^3*Hypergeometr
ic2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(28*x^7)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 956 vs. 2(434) = 868.
time = 0.32, size = 957, normalized size = 1.66

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(27 A\,b^{2} x^{6}+154 B a b\,x^{6}+34 a A b\,x^{3}+28 a^{2} B\,x^{3}+16 a^{2} A\right)}{112 x^{7} a}-\frac{9 i b\left(A b+14 B a\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}}{\cdots}\right)}{\left(-a\,b\cdots\right.}}\cdots - 2 i\left(b^{2} B+\frac{b^{2}\left(27 A b+154 B a\right)}{224 a}\right)\sqrt{3}\,\left(-a\,b^{2}\cdots\right.}{\cdots}$
elliptic	$-\frac{A a\sqrt{b\,x^{3}+a}}{7 x^{7}}-\frac{\left(\frac{17 A b}{14}+B a\right)\sqrt{b\,x^{3}+a}}{4 x^{4}}-\frac{b\left(27 A b+154 B a\right)\sqrt{b\,x^{3}+a}}{112 a x}-\cdots$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
```

```
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3
+a)^(1/2)/x^4-27/112*b^2/a*(b*x^3+a)^(1/2)/x-9/112*I*b^2/a*3^(1/2)*(-a*b^2)
^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 95, normalized size = 0.16

$$-\frac{27\,(14\,Bab+Ab^2)\sqrt{b}\,x^7\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left((154\,Bab+27\,Ab^2)x^6+2\,(14\,Ba^2+17\,Aab)x^3+16\,Aa^2\right)\sqrt{bx^3+a}}{112\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] -1/112*(27*(14*B*a*b + A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weiers
trassPInverse(0, -4*a/b, x)) + ((154*B*a*b + 27*A*b^2)*x^6 + 2*(14*B*a^2 +
17*A*a*b)*x^3 + 16*A*a^2)*sqrt(b*x^3 + a))/(a*x^7)
```

Sympy [A]
time = 2.65, size = 194, normalized size = 0.34

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{A\sqrt{a}\,b\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\sqrt{a}\,b\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**8,x)
```

```
[Out] A*a**(3/2)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**7*gamma(-4/3)) + A*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + B*a**(3/2)*gamma(-4/3)
*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)
) + B*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x*gamma(2/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^8,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^8, x)
```

3.212 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx$

Optimal. Leaf size=608

$$\frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4}+\frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x}-\frac{27b^{7/3}(Ab-4aB)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{(Ab-4aB)\,(a+$$

```
[Out] 1/28*(A*b-4*B*a)*(b*x^3+a)^(3/2)/a/x^7-1/10*A*(b*x^3+a)^(5/2)/a/x^10+9/224*
b*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/x^4+27/448*b^2*(A*b-4*B*a)*(b*x^3+a)^(1/2)/
a^2/x-27/448*b^(7/3)*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))-9/448*3^(3/4)*b^(7/3)*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((
b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)/a^(5/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+27/896*3^(1/4)*b^(7/3)*(A*b-4*B*a)*(a^
(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {464, 283, 331, 309, 224, 1891}

$$-\frac{9\,3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(Ab-4aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224\sqrt{2}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(Ab-4aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27b^{7/3}\sqrt{a+bx^3}\,(Ab-4aB)}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{27b^2\sqrt{a+bx^3}\,(Ab-4aB)}{448a^2x}+\frac{(a+bx^3)^{3/2}(Ab-4aB)}{28ax^7}+\frac{9b\sqrt{a+bx^3}\,(Ab-4aB)}{224ax^4}-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
```

```
[Out] (9*b*(A*b - 4*a*B)*Sqrt[a + b*x^3])/(224*a*x^4) + (27*b^2*(A*b - 4*a*B)*Sqr
t[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(A*b - 4*a*B)*Sqrt[a + b*x^3])/(448
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + ((A*b - 4*a*B)*(a + b*x^3)^(3/2
))/(28*a*x^7) - (A*(a + b*x^3)^(5/2))/(10*a*x^10) + (27*3^(1/4)*Sqrt[2 - Sq
rt[3]]*b^(7/3)*(A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
```

```
x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*b^(7/3)*(
A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(224*Sqrt[2]*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = -\frac{A(a+bx^3)^{5/2}}{10ax^{10}} - \frac{\left(\frac{5Ab}{2}-10aB\right)\int \frac{(a+bx^3)^{3/2}}{x^8}\,dx}{10a}$$

$$= \frac{(Ab-4aB)(a+bx^3)^{3/2}}{28ax^7} - \frac{A(a+bx^3)^{5/2}}{10ax^{10}} - \frac{(9b(Ab-4aB))\int \frac{\sqrt{a+bx^3}}{x^5}\,d}{56a}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{(Ab-4aB)(a+bx^3)^{3/2}}{28ax^7} - \frac{A(a+bx^3)^{5/2}}{10ax^{10}} - \frac{(2}{}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x} + \frac{(Ab-4aB)(a+}{28ax^7}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x} + \frac{(Ab-4aB)(a+}{28ax^7}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x} - \frac{27b^{7/3}(Ab-4a}{448a^2\left(\left(1+\sqrt{3}\right.\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 82, normalized size = 0.13

$$\frac{\sqrt{a+bx^3}\left(-\frac{7A(a+bx^3)^2}{a}+\frac{5(Ab-4aB)x^3\,{}_2F_1\left(-\frac{7}{3},-\frac{3}{2};-\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{70x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-7*A*(a + b*x^3)^2)/a + (5*(A*b - 4*a*B)*x^3*Hypergeomet
ric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(70*x^10)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(462) = 924.
time = 0.34, size = 1002, normalized size = 1.65

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-135x^9 A\,b^3+540x^9 Ba\,b^2+54x^6 Aa\,b^2+680x^6 B\,a^2 b+368x^3 A\,a^2 b+320a^3 B\,x^3+224A\,a^3\right)}{2240x^{10}a^2}+\frac{9ib^2(Ab-4Ba)\sqrt{3}\,(}{}$

	$9ib^2$
elliptic	$-\frac{Aa\sqrt{b\,x^3+a}}{10x^{10}}-\frac{\left(\frac{23Ab}{20}+Ba\right)\sqrt{b\,x^3+a}}{7x^7}-\frac{b(27Ab+340Ba)\sqrt{b\,x^3+a}}{1120a\,x^4}+\frac{27b^2(Ab-4Ba)\sqrt{b\,x^3+a}}{448a^2x}+$ —
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3+a)^(1/2)/x^4-27/112*b^2/a*(b*x
^3+a)^(1/2)/x-9/112*I*b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*
(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*
(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-
1/10*a*(b*x^3+a)^(1/2)/x^10-23/140*b*(b*x^3+a)^(1/2)/x^7-27/1120*b^2/a*(b*x
^3+a)^(1/2)/x^4+27/448*b^3/a^2*(b*x^3+a)^(1/2)/x+9/448*I*b^3/a^2*3^(1/2)*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
```

```
1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 123, normalized size = 0.20

$$-\frac{135\,(4\,Bab^2-Ab^3)\sqrt{b}\,x^{10}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(135\,(4\,Bab^2-Ab^3)x^9+2\,(340\,Ba^2b+27\,Aab^2)x^6+224\,Aa^3+16\,(20\,Ba^3+23\,Aa^2b)x^3\right)\sqrt{bx^3+a}}{2240\,a^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="fricas")
```

```
[Out] -1/2240*(135*(4*B*a*b^2 - A*b^3)*sqrt(b)*x^10*weierstrassZeta(0, -4*a/b, we
ierstrassPInverse(0, -4*a/b, x)) + (135*(4*B*a*b^2 - A*b^3)*x^9 + 2*(340*B*
a^2*b + 27*A*a*b^2)*x^6 + 224*A*a^3 + 16*(20*B*a^3 + 23*A*a^2*b)*x^3)*sqrt(
b*x^3 + a))/(a^2*x^10)
```

Sympy [A]
time = 3.06, size = 199, normalized size = 0.33

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)}+\frac{A\sqrt{a}\,b\Gamma\left(-\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)}+\frac{B\sqrt{a}\,b\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**11,x)
```

```
[Out] A*a**(3/2)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**10*gamma(-7/3)) + A*sqrt(a)*b*gamma(-7/3)*hyper((-7/3, -1/2), (-
4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + B*a**(3/2)*gamma(-7
/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4
/3)) + B*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*x**4*gamma(-1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3+A)\,\left(b\,x^3+a\right)^{3/2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^11,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^11, x)
```

3.213 $\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=103

$$\frac{2a^2(Ab-aB)\sqrt{a+bx^3}}{3b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

```
[Out] -2/9*a*(2*A*b-3*B*a)*(b*x^3+a)^(3/2)/b^4+2/15*(A*b-3*B*a)*(b*x^3+a)^(5/2)/b
^4+2/21*B*(b*x^3+a)^(7/2)/b^4+2/3*a^2*(A*b-B*a)*(b*x^3+a)^(1/2)/b^4
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{2a^2\sqrt{a+bx^3}\,(Ab-aB)}{3b^4}+\frac{2\left(a+bx^3\right)^{5/2}(Ab-3aB)}{15b^4}-\frac{2a\left(a+bx^3\right)^{3/2}(2Ab-3aB)}{9b^4}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*a^2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*x
^3)^(3/2))/(9*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(5/2))/(15*b^4) + (2*B*(a
 + b*x^3)^(7/2))/(21*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3\sqrt{a+bx}} + \frac{a(-2Ab+3aB)\sqrt{a+bx}}{b^3} + \frac{(Ab-3aB)(a+bx)^{3/}}{b^3}\right.\right.$$

$$= \frac{2a^2(Ab-aB)\sqrt{a+bx^3}}{3b^4} - \frac{2a(2Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4} + \frac{2(Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}$$

Mathematica [A]
time = 0.05, size = 80, normalized size = 0.78

$$\frac{2\sqrt{a+bx^3}\,(56a^2Ab-48a^3B-28aAb^2x^3+24a^2bBx^3+21Ab^3x^6-18ab^2Bx^6+15b^3Bx^9)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
[Out] (2*Sqrt[a + b*x^3]*(56*a^2*A*b - 48*a^3*B - 28*a*A*b^2*x^3 + 24*a^2*b*B*x^3
 + 21*A*b^3*x^6 - 18*a*b^2*B*x^6 + 15*b^3*B*x^9))/(315*b^4)
```

Maple [A]
time = 0.30, size = 132, normalized size = 1.28

method	result
gosper	$\frac{2\sqrt{b\,x^3+a}\,\left(15B\,x^9b^3+21A\,b^3x^6-18Ba\,b^2x^6-28Aa\,b^2x^3+24B\,a^2b\,x^3+56A\,a^2b-48B\,a^3\right)}{315b^4}$
trager	$\frac{2\sqrt{b\,x^3+a}\,\left(15B\,x^9b^3+21A\,b^3x^6-18Ba\,b^2x^6-28Aa\,b^2x^3+24B\,a^2b\,x^3+56A\,a^2b-48B\,a^3\right)}{315b^4}$
risch	$\frac{2\sqrt{b\,x^3+a}\,\left(15B\,x^9b^3+21A\,b^3x^6-18Ba\,b^2x^6-28Aa\,b^2x^3+24B\,a^2b\,x^3+56A\,a^2b-48B\,a^3\right)}{315b^4}$
elliptic	$\frac{2B\,x^9\sqrt{b\,x^3+a}}{21b}+\frac{2\left(A-\frac{6aB}{7b}\right)x^6\sqrt{b\,x^3+a}}{15b}-\frac{8a\left(A-\frac{6aB}{7b}\right)x^3\sqrt{b\,x^3+a}}{45b^2}+\frac{16a^2\left(A-\frac{6aB}{7b}\right)\sqrt{b\,x^3+a}}{45b^3}$
default	$B\left(\frac{2x^9\sqrt{b\,x^3+a}}{21b}-\frac{4a\,x^6\sqrt{b\,x^3+a}}{35b^2}+\frac{16a^2x^3\sqrt{b\,x^3+a}}{105b^3}-\frac{32a^3\sqrt{b\,x^3+a}}{105b^4}\right)+A\left(\frac{2x^6\sqrt{b\,x^3+a}}{15b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] B*(2/21*x^9*(b*x^3+a)^(1/2)/b-4/35*a/b^2*x^6*(b*x^3+a)^(1/2)+16/105*a^2/b^3
*x^3*(b*x^3+a)^(1/2)-32/105*a^3*(b*x^3+a)^(1/2)/b^4)+A*(2/15*x^6*(b*x^3+a)^
(1/2)/b-8/45*a/b^2*x^3*(b*x^3+a)^(1/2)+16/45*a^2*(b*x^3+a)^(1/2)/b^3)
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.15

$$\frac{2}{105}\,B\left(\frac{5\,(bx^3+a)^{\frac{7}{2}}}{b^4}-\frac{21\,(bx^3+a)^{\frac{5}{2}}a}{b^4}+\frac{35\,(bx^3+a)^{\frac{3}{2}}a^2}{b^4}-\frac{35\,\sqrt{bx^3+a}\,a^3}{b^4}\right)+\frac{2}{45}\,A\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^3}-\frac{10\,(bx^3+a)^{\frac{3}{2}}a}{b^3}+\frac{15\,\sqrt{bx^3+a}\,a^2}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*B*(5*(b*x^3 + a)^(7/2)/b^4 - 21*(b*x^3 + a)^(5/2)*a/b^4 + 35*(b*x^3 +
 a)^(3/2)*a^2/b^4 - 35*sqrt(b*x^3 + a)*a^3/b^4) + 2/45*A*(3*(b*x^3 + a)^(5/
2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^3 + a)*a^2/b^3)
```

Fricas [A]
time = 1.85, size = 76, normalized size = 0.74

$$\frac{2\,(15\,Bb^3x^9-3\,(6\,Bab^2-7\,Ab^3)x^6-48\,Ba^3+56\,Aa^2b+4\,(6\,Ba^2b-7\,Aab^2)x^3)\sqrt{bx^3+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^3*x^9 - 3*(6*B*a*b^2 - 7*A*b^3)*x^6 - 48*B*a^3 + 56*A*a^2*b +
 4*(6*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

Sympy [A]
time = 0.49, size = 175, normalized size = 1.70

$$\begin{cases}\frac{16Aa^2\sqrt{a+bx^3}}{45b^3}-\frac{8Aax^3\sqrt{a+bx^3}}{45b^2}+\frac{2Ax^6\sqrt{a+bx^3}}{15b}-\frac{32Ba^3\sqrt{a+bx^3}}{105b^4}+\frac{16Ba^2x^3\sqrt{a+bx^3}}{105b^3}-\frac{4Bax^6\sqrt{a+bx^3}}{35b^2}+\frac{2Bx^9\sqrt{a+bx^3}}{21b} & \text{for } b\neq 0\\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{\sqrt{a}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*A*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*A*a*x**3*sqrt(a + b*x**
3)/(45*b**2) + 2*A*x**6*sqrt(a + b*x**3)/(15*b) - 32*B*a**3*sqrt(a + b*x**3
)/(105*b**4) + 16*B*a**2*x**3*sqrt(a + b*x**3)/(105*b**3) - 4*B*a*x**6*sqrt
(a + b*x**3)/(35*b**2) + 2*B*x**9*sqrt(a + b*x**3)/(21*b), Ne(b, 0)), ((A*x
**9/9 + B*x**12/12)/sqrt(a), True))
```

Giac [A]
time = 1.02, size = 101, normalized size = 0.98

$$-\frac{2\,(Ba^3-Aa^2b)\sqrt{bx^3+a}}{3\,b^4}+\frac{2\left(15\,(bx^3+a)^{\frac{7}{2}}B-63\,(bx^3+a)^{\frac{5}{2}}Ba+105\,(bx^3+a)^{\frac{3}{2}}Ba^2+21\,(bx^3+a)^{\frac{5}{2}}Ab-70\,(bx^3+a)^{\frac{3}{2}}Aab\right)}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(B*a^3 - A*a^2*b)*sqrt(b*x^3 + a)/b^4 + 2/315*(15*(b*x^3 + a)^(7/2)*B
- 63*(b*x^3 + a)^(5/2)*B*a + 105*(b*x^3 + a)^(3/2)*B*a^2 + 21*(b*x^3 + a)^(
5/2)*A*b - 70*(b*x^3 + a)^(3/2)*A*a*b)/b^4
```

Mupad [B]
time = 2.68, size = 104, normalized size = 1.01

$$\frac{8\,a^2\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^3}+\frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{15\,b}+\frac{2\,B\,x^9\,\sqrt{b\,x^3+a}}{21\,b}-\frac{4\,a\,x^3\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] (8*a^2*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^3) + (x^6*(a + b*x^3
)^(1/2)*(2*A - (12*B*a)/(7*b)))/(15*b) + (2*B*x^9*(a + b*x^3)^(1/2))/(21*b)
 - (4*a*x^3*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^2)
```

3.214 $\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$-\frac{2a(Ab-aB)\sqrt{a+bx^3}}{3b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2B(a+bx^3)^{5/2}}{15b^3}$$

```
[Out] 2/9*(A*b-2*B*a)*(b*x^3+a)^(3/2)/b^3+2/15*B*(b*x^3+a)^(5/2)/b^3-2/3*a*(A*b-B
*a)*(b*x^3+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{2(a+bx^3)^{3/2}\left(Ab-2aB\right)}{9b^3}-\frac{2a\sqrt{a+bx^3}\left(Ab-aB\right)}{3b^3}+\frac{2B(a+bx^3)^{5/2}}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*a*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)^(
3/2))/(9*b^3) + (2*B*(a + b*x^3)^(5/2))/(15*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2\sqrt{a+bx}} + \frac{(Ab-2aB)\sqrt{a+bx}}{b^2} + \frac{B(a+bx)^{3/2}}{b^2}\right) dx, x, x^3\right)$$
$$= -\frac{2a(Ab-aB)\sqrt{a+bx^3}}{3b^3} + \frac{2(Ab-2aB)\left(a+bx^3\right)^{3/2}}{9b^3} + \frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.77

$$\frac{2\sqrt{a+bx^3}\,(-10aAb+8a^2B+5Ab^2x^3-4abBx^3+3b^2Bx^6)}{45b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-10*a*A*b + 8*a^2*B + 5*A*b^2*x^3 - 4*a*b*B*x^3 + 3*b^2
*B*x^6))/(45*b^3)
```

Maple [A]
time = 0.30, size = 92, normalized size = 1.26

method	result	s
gosper	$-\frac{2\sqrt{b\,x^3+a}\,\left(-3b^2B\,x^6-5A\,b^2x^3+4Bab\,x^3+10abA-8a^2B\right)}{45b^3}$	5
trager	$-\frac{2\sqrt{b\,x^3+a}\,\left(-3b^2B\,x^6-5A\,b^2x^3+4Bab\,x^3+10abA-8a^2B\right)}{45b^3}$	5
risch	$-\frac{2\sqrt{b\,x^3+a}\,\left(-3b^2B\,x^6-5A\,b^2x^3+4Bab\,x^3+10abA-8a^2B\right)}{45b^3}$	5
elliptic	$\frac{2B\,x^6\sqrt{b\,x^3+a}}{15b}+\frac{2\left(A-\frac{4aB}{5b}\right)x^3\sqrt{b\,x^3+a}}{9b}-\frac{4a\left(A-\frac{4aB}{5b}\right)\sqrt{b\,x^3+a}}{9b^2}$	7
default	$B\left(\frac{2x^6\sqrt{b\,x^3+a}}{15b}-\frac{8a\,x^3\sqrt{b\,x^3+a}}{45b^2}+\frac{16a^2\sqrt{b\,x^3+a}}{45b^3}\right)+A\left(\frac{2x^3\sqrt{b\,x^3+a}}{9b}-\frac{4a\sqrt{b\,x^3+a}}{9b^2}\right)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/15*x^6*(b*x^3+a)^(1/2)/b-8/45*a/b^2*x^3*(b*x^3+a)^(1/2)+16/45*a^2*(b*x
^3+a)^(1/2)/b^3)+A*(2/9*x^3*(b*x^3+a)^(1/2)/b-4/9*a*(b*x^3+a)^(1/2)/b^2)
```

Maxima [A]
time = 0.27, size = 83, normalized size = 1.14

$$\frac{2}{45}\,B\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^3}-\frac{10\,(bx^3+a)^{\frac{3}{2}}a}{b^3}+\frac{15\,\sqrt{bx^3+a}\,a^2}{b^3}\right)+\frac{2}{9}\,A\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^2}-\frac{3\,\sqrt{bx^3+a}\,a}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^
3 + a)*a^2/b^3) + 2/9*A*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2)
```

Fricas [A]
time = 2.07, size = 52, normalized size = 0.71

$$\frac{2\,(3\,Bb^2x^6-(4\,Bab-5\,Ab^2)x^3+8\,Ba^2-10\,Aab)\sqrt{bx^3+a}}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^6 - (4*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 10*A*a*b)*sqrt(b*x^
3 + a)/b^3
```

Sympy [A]
time = 0.37, size = 124, normalized size = 1.70

$$\begin{cases} -\frac{4Aa\sqrt{a+bx^3}}{9b^2}+\frac{2Ax^3\sqrt{a+bx^3}}{9b}+\frac{16Ba^2\sqrt{a+bx^3}}{45b^3}-\frac{8Bax^3\sqrt{a+bx^3}}{45b^2}+\frac{2Bx^6\sqrt{a+bx^3}}{15b} & \text{for } b\neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-4*A*a*sqrt(a + b*x**3)/(9*b**2) + 2*A*x**3*sqrt(a + b*x**3)/(9*
b) + 16*B*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*B*a*x**3*sqrt(a + b*x**3)/(45
*b**2) + 2*B*x**6*sqrt(a + b*x**3)/(15*b), Ne(b, 0)), ((A*x**6/6 + B*x**9/9
)/sqrt(a), True))
```

Giac [A]
time = 1.40, size = 70, normalized size = 0.96

$$\frac{2\,\sqrt{bx^3+a}\,(Ba^2-Aab)}{3\,b^3}+\frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}B-10\,(bx^3+a)^{\frac{3}{2}}Ba+5\,(bx^3+a)^{\frac{3}{2}}Ab\right)}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*(B*a^2 - A*a*b)/b^3 + 2/45*(3*(b*x^3 + a)^(5/2)*B - 10*
(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)*A*b)/b^3
```

Mupad [B]
time = 2.65, size = 52, normalized size = 0.71

$$\frac{2\,\sqrt{b\,x^3+a}\,\left(8\,B\,a^2-4\,B\,a\,b\,x^3-10\,A\,a\,b+3\,B\,b^2\,x^6+5\,A\,b^2\,x^3\right)}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] (2*(a + b*x^3)^(1/2)*(8*B*a^2 + 5*A*b^2*x^3 + 3*B*b^2*x^6 - 10*A*a*b - 4*B*
a*b*x^3))/(45*b^3)
```

3.215 $\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=46

$$\frac{2(Ab-aB)\sqrt{a+bx^3}}{3b^2}+\frac{2B(a+bx^3)^{3/2}}{9b^2}$$

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b^2+2/3*(A*b-B*a)*(b*x^3+a)^(1/2)/b^2
```

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {455, 45}

$$\frac{2\sqrt{a+bx^3}\,(Ab-aB)}{3b^2}+\frac{2B(a+bx^3)^{3/2}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^2) + (2*B*(a + b*x^3)^(3/2))/(9*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b}\right)dx,x,x^3\right)\\ &= \frac{2(Ab-aB)\sqrt{a+bx^3}}{3b^2}+\frac{2B(a+bx^3)^{3/2}}{9b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.72

$$\frac{2\sqrt{a+bx^3}\,(3Ab-2aB+bBx^3)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(3*A*b - 2*a*B + b*B*x^3))/(9*b^2)
```

Maple [A]
time = 0.34, size = 52, normalized size = 1.13

method	result	size
gosper	$\frac{2\sqrt{b\,x^{3}+a}\,\left(bB\,x^{3}+3Ab-2Ba\right)}{9b^{2}}$	30
trager	$\frac{2\sqrt{b\,x^{3}+a}\,\left(bB\,x^{3}+3Ab-2Ba\right)}{9b^{2}}$	30
risch	$\frac{2\sqrt{b\,x^{3}+a}\,\left(bB\,x^{3}+3Ab-2Ba\right)}{9b^{2}}$	30
elliptic	$\frac{2B\,x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(A-\frac{2aB}{3b}\right)\sqrt{b\,x^{3}+a}}{3b}$	43
default	$B\left(\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\sqrt{b\,x^{3}+a}}{9b^{2}}\right)+\frac{2A\sqrt{b\,x^{3}+a}}{3b}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/9*x^3*(b*x^3+a)^(1/2)/b-4/9*a*(b*x^3+a)^(1/2)/b^2)+2/3*A*(b*x^3+a)^(1/
2)/b
```

Maxima [A]
time = 0.30, size = 48, normalized size = 1.04

$$\frac{2}{9}\,B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^2}-\frac{3\,\sqrt{bx^3+a}\,a}{b^2}\right)+\frac{2\,\sqrt{bx^3+a}\,A}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2) + 2/3*sqrt(b*x^3 +
a)*A/b
```

Fricas [A]
time = 1.87, size = 29, normalized size = 0.63

$$\frac{2\,(Bbx^3 - 2\,Ba + 3\,Ab)\sqrt{bx^3 + a}}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 2/9*(B*b*x^3 - 2*B*a + 3*A*b)*sqrt(b*x^3 + a)/b^2
```

Sympy [A]
time = 0.27, size = 75, normalized size = 1.63

$$\begin{cases} \frac{2A\sqrt{a+bx^3}}{3b} - \frac{4Ba\sqrt{a+bx^3}}{9b^2} + \frac{2Bx^3\sqrt{a+bx^3}}{9b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3} + \frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(1/2),x)
[Out] Piecewise((2*A*sqrt(a + b*x**3)/(3*b) - 4*B*a*sqrt(a + b*x**3)/(9*b**2) + 2
*B*x**3*sqrt(a + b*x**3)/(9*b), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/sqrt(a),
True))
```

Giac [A]
time = 0.88, size = 38, normalized size = 0.83

$$\frac{2\,(bx^3 + a)^{\frac{3}{2}}B}{9\,b^2} - \frac{2\,\sqrt{bx^3 + a}\,(Ba - Ab)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] 2/9*(b*x^3 + a)^(3/2)*B/b^2 - 2/3*sqrt(b*x^3 + a)*(B*a - A*b)/b^2
```

Mupad [B]
time = 2.60, size = 29, normalized size = 0.63

$$\frac{2\,\sqrt{b\,x^3 + a}\,\left(B\,b\,x^3 + 3\,A\,b - 2\,B\,a\right)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(1/2),x)
[Out] (2*(a + b*x^3)^(1/2)*(3*A*b - 2*B*a + B*b*x^3))/(9*b^2)
```

3.216 $\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=48

$$\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {457, 81, 65, 214}

$$\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx,x,x^3\right)\\ &= \frac{2B\sqrt{a+bx^3}}{3b}+\frac{1}{3}A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)\\ &= \frac{2B\sqrt{a+bx^3}}{3b}+\frac{(2A)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b}\\ &= \frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 1.00

$$\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

Maple [A]
time = 0.32, size = 37, normalized size = 0.77

method	result	size
default	$-\frac{2A\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2B\sqrt{b\,x^3+a}}{3b}$	37

elliptic	$-\frac{2A\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2B\sqrt{b\,x^3+a}}{3b}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b
```

Maxima [A]
time = 0.57, size = 54, normalized size = 1.12

$$\frac{A\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2\sqrt{bx^3+a}\,B}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] 1/3*A*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
+ 2/3*sqrt(b*x^3 + a)*B/b
```

Fricas [A]
time = 1.90, size = 105, normalized size = 2.19

$$\left[\frac{A\sqrt{a}\,b\log\left(\frac{bx^3-2\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\sqrt{bx^3+a}\,Ba}{3\,ab},\ \frac{2\left(A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx^3+a}\,Ba\right)}{3\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] [1/3*(A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*sq
rt(b*x^3 + a)*B*a)/(a*b), 2/3*(A*sqrt(-a)*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + sqrt(b*x^3 + a)*B*a)/(a*b)]
```

Sympy [A]
time = 4.69, size = 65, normalized size = 1.35

$$\frac{2A\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+bx^3}}\right)}{3a\sqrt{-\frac{1}{a}}}-\frac{B\left(\begin{cases}-\frac{x^3}{\sqrt{a}} & \text{for } b=0\\ -\frac{2\sqrt{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(1/2),x)
```

```
[Out] 2*A*atan(1/(sqrt(-1/a)*sqrt(a + b*x**3)))/(3*a*sqrt(-1/a)) - B*Piecewise((-
x**3/sqrt(a), Eq(b, 0)), (-2*sqrt(a + b*x**3)/b, True))/3
```

Giac [A]
time = 1.14, size = 40, normalized size = 0.83

$$\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}+\frac{2\,\sqrt{bx^3+a}\,B}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*x^3 + a)*B/b
```

Mupad [B]
time = 2.72, size = 57, normalized size = 1.19

$$\frac{2\,B\,\sqrt{b\,x^3+a}}{3\,b}+\frac{A\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(1/2)),x)
```

```
[Out] (2*B*(a + b*x^3)^(1/2))/(3*b) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a
 + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(1/2))
```

3.217 $\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=58

$$-\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

```
[Out] 1/3*(A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*A*(b*x^3+a)^(1
/2)/a/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {457, 79, 65, 214}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{A\sqrt{a+bx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/3*(A*Sqrt[a + b*x^3])/(a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/(3*a^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx, x, x^3\right) \\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3} + \frac{\left(-\frac{Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{3a} \\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3} + \frac{\left(2\left(-\frac{Ab}{2}+aB\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3ab} \\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3} + \frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 58, normalized size = 1.00

$$-\frac{A\sqrt{a+bx^3}}{3ax^3} + \frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/3*(A*Sqrt[a + b*x^3])/(a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/(3*a^(3/2))
```

Maple [A]
time = 0.33, size = 62, normalized size = 1.07

method	result	size

risch	$\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{A\sqrt{b\,x^3+a}}{3a\,x^3}$	47
elliptic	$\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{A\sqrt{b\,x^3+a}}{3a\,x^3}$	47
default	$A\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^3+a}}{3a\,x^3}\right)-\frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*(b*x^3+a)^(1/2)/a/x^3
)-2/3*B*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(47) = 94$.
time = 0.49, size = 109, normalized size = 1.88

$$-\frac{1}{6}\,A\left(\frac{2\,\sqrt{bx^3+a}\,b}{(bx^3+a)a-a^2}+\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)+\frac{B\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*A*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
- sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/3*B*log((sqrt(b*x^3 +
a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
```

Fricas [A]
time = 1.83, size = 126, normalized size = 2.17

$$\left[-\frac{(2\,Ba-Ab)\sqrt{a}\,x^3\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\,\sqrt{bx^3+a}\,Aa}{6\,a^2x^3},\ \frac{(2\,Ba-Ab)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)-\sqrt{bx^3+a}\,Aa}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*B*a - A*b)*sqrt(a)*x^3*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2
*a)/x^3) + 2*sqrt(b*x^3 + a)*A*a)/(a^2*x^3), 1/3*((2*B*a - A*b)*sqrt(-a)*x^
3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^3 + a)*A*a)/(a^2*x^3)]
```

Sympy [A]
time = 10.90, size = 80, normalized size = 1.38

$$-\frac{A\sqrt{b}\,\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}}+\frac{Ab\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}-\frac{2B\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*x**(3/2)) + A*b*asinh(sqrt(a)/(sqrt(b)
*x**(3/2)))/(3*a**(3/2)) - 2*B*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)
)
```

Giac [A]
time = 0.86, size = 62, normalized size = 1.07

$$\frac{\frac{(2\,Bab-Ab^2)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{\sqrt{bx^3+a}\,Ab}{ax^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*B*a*b - A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - sqrt
(b*x^3 + a)*A*b/(a*x^3))/b
```

Mupad [B]
time = 2.89, size = 67, normalized size = 1.16

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(A\,b-2\,B\,a)}{6\,a^{3/2}}-\frac{A\,\sqrt{b\,x^3+a}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(1/2)),x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
A*b - 2*B*a))/(6*a^(3/2)) - (A*(a + b*x^3)^(1/2))/(3*a*x^3)
```

3.218 $\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=90

$$-\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}$$

```
[Out] -1/12*b*(3*A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-1/6*A*(b*x^3
+a)^(1/2)/a/x^6+1/12*(3*A*b-4*B*a)*(b*x^3+a)^(1/2)/a^2/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 44, 65, 214}

$$-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}+\frac{\sqrt{a+bx^3}\,(3Ab-4aB)}{12a^2x^3}-\frac{A\sqrt{a+bx^3}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/6*(A*Sqrt[a + b*x^3])/(a*x^6) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a^
2*x^3) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx,x,x^3\right)$$

$$= -\frac{A\sqrt{a+bx^3}}{6ax^6} + \frac{\left(-\frac{3Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^3\right)}{6a}$$

$$= -\frac{A\sqrt{a+bx^3}}{6ax^6} + \frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3} + \frac{(b(3Ab-4aB))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\right.}{24a^2}$$

$$= -\frac{A\sqrt{a+bx^3}}{6ax^6} + \frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3} + \frac{(3Ab-4aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\right.}{12a^2}$$

$$= -\frac{A\sqrt{a+bx^3}}{6ax^6} + \frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3} - \frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}$$

Mathematica [A]
time = 0.11, size = 78, normalized size = 0.87

$$\frac{\sqrt{a+bx^3}\,(-2aA+3Abx^3-4aBx^3)}{12a^2x^6} + \frac{b(-3Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*a*A + 3*A*b*x^3 - 4*a*B*x^3))/(12*a^2*x^6) + (b*(-3*A*
b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))
```

Maple [A]
time = 0.35, size = 102, normalized size = 1.13

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-3 A b\,x^{3}+4 B a\,x^{3}+2 A a \right)}{12 a^{2} x^{6}}-\frac{b \left(3 A b -4 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{5}{2}}}$
elliptic	$-\frac{b \left(3 A b -4 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{5}{2}}}-\frac{A \sqrt{b\,x^{3}+a}}{6 x^{6} a}+\frac{\left(3 A b -4 B a \right) \sqrt{b\,x^{3}+a}}{12 a^{2} x^{3}}$
default	$A \left(-\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4 a^{\frac{5}{2}}}-\frac{\sqrt{b\,x^{3}+a}}{6 x^{6} a}+\frac{b \sqrt{b\,x^{3}+a}}{4 a^{2} x^{3}}\right)+B \left(\frac{b \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3 a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{3}}}{3 a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-1/6*(b*x^3+a)^(1/2)/x^
6/a+1/4*b*(b*x^3+a)^(1/2)/a^2/x^3)+B*(1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)
)/a^(3/2)-1/3*(b*x^3+a)^(1/2)/a/x^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(74) = 148$.
time = 0.49, size = 178, normalized size = 1.98

$$-\frac{1}{6}\,B\left(\frac{2\,\sqrt{bx^3+a}\,b}{(bx^3+a)a-a^2}+\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)+\frac{1}{24}\,A\left(\frac{3\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\,(bx^3+a)^{\frac{3}{2}}b^2-5\,\sqrt{bx^3+a}\,ab^2\right)}{(bx^3+a)^2a^2-2\,(bx^3+a)a^3+a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*B*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
- sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/24*A*(3*b^2*log((sqrt(
b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*(b*x^3 +
a)^(3/2)*b^2 - 5*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a^2 - 2*(b*x^3 + a)*
a^3 + a^4))
```

Fricas [A]
time = 1.62, size = 173, normalized size = 1.92

$$\left[-\frac{(4\,Bab-3\,Ab^2)\sqrt{a}\,x^6\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\,((4\,Ba^2-3\,Aab)x^3+2\,Aa^2)\sqrt{bx^3+a}}{24\,a^3x^6},\ -\frac{(4\,Bab-3\,Ab^2)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+((4\,Ba^2-3\,Aab)x^3+2\,Aa^2)\sqrt{bx^3+a}}{12\,a^3x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt
(a) + 2*a)/x^3) + 2*((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a
^3*x^6), -1/12*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqr
t(-a)/a) + ((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(80) = 160$.
time = 25.94, size = 163, normalized size = 1.81

$$-\frac{A}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{A\sqrt{b}}{12ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{Ab^{\frac{3}{2}}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{4a^{\frac{5}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}}+\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**(1/2),x)
```

```
[Out] -A/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + A*sqrt(b)/(12*a*x**(9/2)*sq
rt(a/(b*x**3) + 1)) + A*b**(3/2)/(4*a**2*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A
*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*
x**3) + 1)/(3*a*x**(3/2)) + B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*a**(3/
2))
```

Giac [A]
time = 1.05, size = 121, normalized size = 1.34

$$-\frac{\frac{(4\,Bab^2-3\,Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{4\,(bx^3+a)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^3+a}\,Ba^2b^2-3\,(bx^3+a)^{\frac{3}{2}}Ab^3+5\,\sqrt{bx^3+a}\,Aab^3}{a^2b^2x^6}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2
) + (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 - 3*(b*x^3 +
 a)^(3/2)*A*b^3 + 5*sqrt(b*x^3 + a)*A*a*b^3)/(a^2*b^2*x^6))/b
```

Mupad [B]
time = 2.99, size = 95, normalized size = 1.06

$$\frac{\sqrt{b\,x^3+a}\,\left(3\,A\,b-4\,B\,a\right)}{12\,a^2\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,a\,x^6}+\frac{b\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(3\,A\,b-4\,B\,a\right)}{24\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^(1/2)),x)
```

```
[Out] ((a + b*x^3)^(1/2)*(3*A*b - 4*B*a))/(12*a^2*x^3) - (A*(a + b*x^3)^(1/2))/(6
*a*x^6) + (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1
/2)))/x^6)*(3*A*b - 4*B*a))/(24*a^(5/2))
```

3.219 $\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=270

$$\frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2}+\frac{2Bx^4\sqrt{a+bx^3}}{11b}-\frac{4\sqrt{2+\sqrt{3}}\,a(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{\ldots}}{\left(\left(1+\sqrt{3}\right)\ldots\right.}}}{55\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{\ldots}\right.}{\left(\left(1+\sqrt{3}\right)\ldots\right.}}}$$

```
[Out] 2/55*(11*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+2/11*B*x^4*(b*x^3+a)^(1/2)/b-4/16
5*a*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/
2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)*3^(3/4)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {470, 327, 224}

$$-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(11Ab-8aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2x\sqrt{a+bx^3}\,(11Ab-8aB)}{55b^2}+\frac{2Bx^4\sqrt{a+bx^3}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(11*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(55*b^2) + (2*B*x^4*Sqrt[a + b*x^3])
/(11*b) - (4*Sqrt[2 + Sqrt[3]]*a*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{\left(2\left(-\frac{11Ab}{2}+4aB\right)\right)\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{11b}$$

$$= \frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2} + \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{(2a(11Ab-8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{55b^2}$$

$$= \frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2} + \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{4\sqrt{2+\sqrt{3}}\,a(11Ab-8aB)\left(\sqrt[3]{a}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 89, normalized size = 0.33

$$\frac{2x\left(-\left(\left(a+bx^3\right)\left(-11Ab+8aB-5bBx^3\right)\right)+a(-11Ab+8aB)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{55b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x*(-((a + b*x^3)*(-11*A*b + 8*a*B - 5*b*B*x^3)) + a*(-11*A*b + 8*a*B)*Sq
rt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(55*b^2*
Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(207) = 414$.
time = 0.32, size = 624, normalized size = 2.31 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/11*x^4*(b*x^3+a)^(1/2)/b-16/55*a*x*(b*x^3+a)^(1/2)/b^2-32/165*I*a^2/b^
3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/5*x*(
b*x^3+a)^(1/2)/b+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 67, normalized size = 0.25

$$\frac{2\left(2\left(8\,Ba^2-11\,Aab\right)\sqrt{b}\,\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\left(5\,Bb^2x^4-\left(8\,Bab-11\,Ab^2\right)x\right)\sqrt{bx^3+a}\right)}{55\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/55*(2*(8*B*a^2 - 11*A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (5
*B*b^2*x^4 - (8*B*a*b - 11*A*b^2)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A]
time = 1.35, size = 80, normalized size = 0.30

$$\frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(B\,x^3+A\right)}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] int((x^3*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

3.220 $\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=239

$$\frac{2Bx\sqrt{a+bx^3}}{5b}+\frac{2\sqrt{2+\sqrt{3}}\,(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{\cdots}\right.}{\left(1+\sqrt{\cdots}\right.}\right.\right.}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] 2/5*B*x*(b*x^3+a)^(1/2)/b+2/15*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {396, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(5Ab-2aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2Bx\sqrt{a+bx^3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*x*Sqrt[a + b*x^3])/(5*b) + (2*Sqrt[2 + Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^
(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{2Bx\sqrt{a+bx^3}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+aB\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{5b}$$

$$= \frac{2Bx\sqrt{a+bx^3}}{5b} + \frac{2\sqrt{2+\sqrt{3}}\,(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 74, normalized size = 0.31

$$\frac{2Bx(a+bx^3)+(5Ab-2aB)x\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*x*(a + b*x^3) + (5*A*b - 2*a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F
1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(5*b*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(180) = 360.
time = 0.32, size = 586, normalized size = 2.45

method	result
risch	$\frac{2Bx\sqrt{b\,x^{3}+a}}{5b}-\frac{2i(5Ab-2Ba)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2}}{2b}}}}{}$
elliptic	$\frac{2Bx\sqrt{b\,x^{3}+a}}{5b}-\frac{2i\left(A-\frac{2aB}{5b}\right)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2}}{2b}}}}{}$
default	$B\left(\frac{2x\sqrt{b\,x^{3}+a}}{5b}+\frac{4ia\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/5*x*(b*x^3+a)^(1/2)/b+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
```

```
/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*A*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 42, normalized size = 0.18

$$\frac{2\left(\sqrt{bx^3+a}\,Bbx-(2\,Ba-5\,Ab)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(sqrt(b*x^3 + a)*B*b*x - (2*B*a - 5*A*b)*sqrt(b)*weierstrassPInverse(0,
 -4*a/b, x))/b^2
```

Sympy [A]
time = 0.99, size = 78, normalized size = 0.33

$$\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^(1/2),x)
[Out] int((A + B*x^3)/(a + b*x^3)^(1/2), x)
```

3.221 $\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=243

$$-\frac{A\sqrt{a+bx^3}}{2ax^2}-\frac{\sqrt{2+\sqrt{3}}\,(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.\right.}{2\sqrt[4]{3}\,a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] -1/2*A*(b*x^3+a)^(1/2)/a/x^2-1/6*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(1/3)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {464, 224}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(Ab-4aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{A\sqrt{a+bx^3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/2*(A*Sqrt[a + b*x^3])/(a*x^2) - (Sqrt[2 + Sqrt[3]]*(A*b - 4*a*B)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{2ax^2} - \frac{\left(\frac{Ab}{2}-2aB\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{2a}$$

$$= -\frac{A\sqrt{a+bx^3}}{2ax^2} - \frac{\sqrt{2+\sqrt{3}}\,(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}}}{2\sqrt[4]{3}\,a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 78, normalized size = 0.32

$$\frac{-2A(a+bx^3)+(-Ab+4aB)x^3\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{4ax^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*(a + b*x^3) + (-(A*b) + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(4*a*x^2*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(184) = 368.
time = 0.34, size = 587, normalized size = 2.42

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{2a\,x^2}-\frac{2i\left(B-\frac{Ab}{4a}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2}{2b}}}}{}$
risch	$-\frac{A\sqrt{b\,x^3+a}}{2a\,x^2}+\frac{i(Ab-4Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2}{2b}}}}{}$
default	$-\frac{2iB\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\right.}{}}}{3b\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3*I*B*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+A*

```
(-1/2*(b*x^3+a)^(1/2)/a/x^2+1/6*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 50, normalized size = 0.21

$$\frac{(4\,Ba - Ab)\sqrt{b}\,x^2 \text{weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right) - \sqrt{bx^3 + a}\,Ab}{2\,abx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 1/2*((4*B*a - A*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) - sqrt(b*x
^3 + a)*A*b)/(a*b*x^2)
```

Sympy [A]
time = 1.05, size = 82, normalized size = 0.34

$$\frac{A\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3}, \frac{1}{2} \\ \frac{1}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x^2\Gamma\left(\frac{1}{3}\right)} + \frac{Bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{1}{2} \\ \frac{4}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(1/2),x)
[Out] A*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^3\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/(x^3*(a + b*x^3)^(1/2)), x)
```

3.222 $\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=274

$$-\frac{A\sqrt{a+bx^3}}{5ax^5}+\frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2}+\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}{20\sqrt[4]{3}\,a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}$$

```
[Out] -1/5*A*(b*x^3+a)^(1/2)/a/x^5+1/20*(7*A*b-10*B*a)*(b*x^3+a)^(1/2)/a^2/x^2+1/
60*b^(2/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/
2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)*3^(3/4)/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 331, 224}

$$\frac{\sqrt{a+bx^3}\,(7Ab-10aB)}{20a^2x^2}+\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-10aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\,a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{A\sqrt{a+bx^3}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/5*(A*Sqrt[a + b*x^3])/(a*x^5) + ((7*A*b - 10*a*B)*Sqrt[a + b*x^3])/(20*a
^2*x^2) + (Sqrt[2 + Sqrt[3]]*b^(2/3)*(7*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(20*3^(1/4)*a^2*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{5ax^5} - \frac{\left(\frac{7Ab}{2}-5aB\right)\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{5a}$$

$$= -\frac{A\sqrt{a+bx^3}}{5ax^5} + \frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2} + \frac{(b(7Ab-10aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{40a^2}$$

$$= -\frac{A\sqrt{a+bx^3}}{5ax^5} + \frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2} + \frac{\sqrt{2+\sqrt{3}}\,b^{2/3}(7Ab-10aB)\left(\sqrt[3]{a}+\ldots\right.}{20\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 78, normalized size = 0.28

$$\frac{-4A(a+bx^3)+(7Ab-10aB)x^3\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{20ax^5\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
[Out] (-4*A*(a + b*x^3) + (7*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/(20*a*x^5*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(211) = 422.
time = 0.34, size = 625, normalized size = 2.28

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-7Ab\,x^3+10Ba\,x^3+4Aa)}{20a^2x^5}-\frac{i(7Ab-10Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\ \sqrt{\cdots}}{\cdots}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{5a\,x^5}+\frac{(7Ab-10Ba)\sqrt{b\,x^3+a}}{20a^2x^2}-\frac{i(7Ab-10Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}}{\cdots}$

default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{5a\,x^{5}}+\frac{7b\sqrt{b\,x^{3}+a}}{20a^{2}x^{2}}-\frac{7ib\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-}{-\frac{3\left(-a\,b^{2}\right)}{2b}}}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/5*(b*x^3+a)^(1/2)/a/x^5+7/20*b*(b*x^3+a)^(1/2)/a^2/x^2-7/60*I*b/a^2*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+B*(-1/2*(b*x^
3+a)^(1/2)/a/x^2+1/6*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 62, normalized size = 0.23

$$-\frac{(10\,Ba-7\,Ab)\sqrt{b}\,x^5\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left((10\,Ba-7\,Ab)x^3+4\,Aa\right)\sqrt{bx^3+a}}{20\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*((10*B*a - 7*A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) + ((1
0*B*a - 7*A*b)*x^3 + 4*A*a)*sqrt(b*x^3 + a))/(a^2*x^5)
```

Sympy [A]
time = 1.24, size = 90, normalized size = 0.33

$$\frac{A\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*x**2*gamma(1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^6\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/(x^6*(a + b*x^3)^(1/2)), x)
```

3.223 $\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=548

$$\frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}-\frac{8a(13Ab-10aB)\sqrt{a+bx^3}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}(}{}$$

[Out] 2/91*(13*A*b-10*B*a)*x^2*(b*x^3+a)^(1/2)/b^2+2/13*B*x^5*(b*x^3+a)^(1/2)/b-8/91*a*(13*A*b-10*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-8/273*a^(4/3)*(13*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/91*3^(1/4)*a^(4/3)*(13*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.18, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$,
Rules used = {470, 327, 309, 224, 1891}

$$-\frac{8\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(13Ab-10aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}\,b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(13Ab-10aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{8a\sqrt{a+bx^3}\,(13Ab-10aB)}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x^2\sqrt{a+bx^3}\,(13Ab-10aB)}{91b^2}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]

[Out] (2*(13*A*b - 10*a*B)*x^2*Sqrt[a + b*x^3])/(91*b^2) + (2*B*x^5*Sqrt[a + b*x^3])/(13*b) - (8*a*(13*A*b - 10*a*B)*Sqrt[a + b*x^3])/(91*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +

```
b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*a^(4/3)*(13*A*b - 10*a*B)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)
*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
```

```
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{\left(2\left(-\frac{13Ab}{2}+5aB\right)\right)\int \frac{x^4}{\sqrt{a+bx^3}}\,dx}{13b}$$

$$= \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{(4a(13Ab-10aB))\int \frac{x}{\sqrt{a+bx^3}}$$

$$= \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{(4a(13Ab-10aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt{a+}}}{91b^{7/3}}$$

$$= \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{8a(13Ab-10aB)\sqrt{a+bx^3}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 91, normalized size = 0.17

$$\frac{2x^2\left(-((a+bx^3)(-13Ab+10aB-7bBx^3))+a(-13Ab+10aB)\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{91b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x^2*(-((a + b*x^3)*(-13*A*b + 10*a*B - 7*b*B*x^3)) + a*(-13*A*b + 10*a*B
)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(91*
b^2*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 931 vs. 2(410) = 820.
time = 0.34, size = 932, normalized size = 1.70

method	result
risch	$\frac{2x^2(7bBx^3+13Ab-10Ba)\sqrt{bx^3+a}}{91b^2}+\frac{8i(13Ab-10Ba)a\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}$
elliptic	$\frac{2Bx^5\sqrt{bx^3+a}}{13b}+\frac{2\left(A-\frac{10aB}{13b}\right)x^2\sqrt{bx^3+a}}{7b}+\frac{8ia\left(A-\frac{10aB}{13b}\right)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\cdots}}{(-ab^2)^{\frac{1}{3}}}}}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/13*x^5*(b*x^3+a)^(1/2)/b-20/91*a*x^2*(b*x^3+a)^(1/2)/b^2-80/273*I*a^2/
b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
```

```
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(2/7*x^2*(b*x^3+a)^(1/2)/b
+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 78, normalized size = 0.14

$$-\frac{2\left(4\,(10\,Ba^2-13\,Aab)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-\left(7\,Bb^2x^5-(10\,Bab-13\,Ab^2)x^2\right)\sqrt{bx^3+a}\right)}{91\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] -2/91*(4*(10*B*a^2 - 13*A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstra
ssPInverse(0, -4*a/b, x)) - (7*B*b^2*x^5 - (10*B*a*b - 13*A*b^2)*x^2)*sqrt(
b*x^3 + a))/b^3
```

Sympy [A]
time = 1.40, size = 80, normalized size = 0.15

$$\frac{Ax^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)}+\frac{Bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(11/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(B\,x^3+A\right)}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] int((x^4*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

3.224 $\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=517

$$\frac{2Bx^2\sqrt{a+bx^3}}{7b}+\frac{2(7Ab-4aB)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a \cdots}{((\cdots}}}{7b^{5/3}\sqrt{\frac{\sqrt[3]{\cdots}}{((1+\cdots}}}$$

```
[Out] 2/7*B*x^2*(b*x^3+a)^(1/2)/b+2/7*(7*A*b-4*B*a)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))+2/21*a^(1/3)*(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*E
llipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/7*3^(1/4)*a^(1/3)*
(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {470, 309, 224, 1891}

$$\frac{2\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-4aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-4aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}\,(7Ab-4aB)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2Bx^2\sqrt{a+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*x^2*Sqrt[a + b*x^3])/(7*b) + (2*(7*A*b - 4*a*B)*Sqrt[a + b*x^3])/(7*b^
(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*(7*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(7*A*b - 4*
a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
```

```
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7
*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2Bx^2\sqrt{a+bx^3}}{7b} - \frac{\left(2\left(-\frac{7Ab}{2}+2aB\right)\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{7b}$$

$$= \frac{2Bx^2\sqrt{a+bx^3}}{7b} + \frac{(7Ab-4aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\,\sqrt[3]{a}\,(7A\right.}{7}$$

$$= \frac{2Bx^2\sqrt{a+bx^3}}{7b} + \frac{2(7Ab-4aB)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,(7Ab-4a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 75, normalized size = 0.15

$$\frac{x^2\left(4B(a+bx^3)+(7Ab-4aB)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{14b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (x^2*(4*B*(a + b*x^3) + (7*A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(14*b*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 891 vs. 2(383) = 766.
time = 0.34, size = 892, normalized size = 1.73

method	result

risch	$\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b} - \frac{2i(7Ab-4Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}}}}{}$
elliptic	$\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b} - \frac{2i\left(A-\frac{4aB}{7b}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b}{2b}}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/7*x^2*(b*x^3+a)^(1/2)/b+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
```

```
))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)))-2/3*I*A*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 51, normalized size = 0.10

$$\frac{2\left(\sqrt{bx^3+a}\,Bbx^2+(4\,Ba-7\,Ab)\sqrt{b}\ \mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)\right)}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/7*(sqrt(b*x^3 + a)*B*b*x^2 + (4*B*a - 7*A*b)*sqrt(b)*weierstrassZeta(0, -
4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b^2
```

Sympy [A]
time = 1.27, size = 80, normalized size = 0.15

$$\frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(B\,x^3+A\right)}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] int((x*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

3.225 $\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=509

$$-\frac{A\sqrt{a+bx^3}}{ax}+\frac{(Ab+2aB)\sqrt{a+bx^3}}{ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{}}{\left(\left(1+\sqrt{3}\right.\right.}}}{2a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}}{\left(\left(1+\sqrt{}\right.\right.}}}$$

```
[Out] -A*(b*x^3+a)^(1/2)/a/x+(A*b+2*B*a)*(b*x^3+a)^(1/2)/a/b^(2/3)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))+1/3*(A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*3^(1/4)*(A*b+2*B*a)*(a^(1/3)
+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/b^(2/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 309, 224, 1891}

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2aB+Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2aB+Ab)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{a+bx^3}\,(2aB+Ab)}{ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{A\sqrt{a+bx^3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] -((A*Sqrt[a + b*x^3])/(a*x)) + ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(a*b^(2/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(A*b + 2*a
*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*
a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (Sqrt[2]*(A*b + 2*a*B)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
```

```
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*b^(2/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{ax} - \frac{\left(-\frac{Ab}{2}-aB\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{a}$$

$$= -\frac{A\sqrt{a+bx^3}}{ax} + \frac{(Ab+2aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt[3]{b}} + \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,(Ab+2a\right.}{a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{A\sqrt{a+bx^3}}{ax} + \frac{(Ab+2aB)\sqrt{a+bx^3}}{ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(Ab+2aB)\left(\sqrt[3]{\right.}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 77, normalized size = 0.15

$$\frac{-4A(a+bx^3)+(Ab+2aB)x^3\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4ax\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*A*(a + b*x^3) + (A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1
[1/2, 2/3, 5/3, -((b*x^3)/a)])/(4*a*x*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 890 vs. 2(382) = 764.
time = 0.34, size = 891, normalized size = 1.75

method	result

elliptic	$-\frac{A\sqrt{b\,x^3+a}}{a x}-\frac{2i\left(B+\frac{Ab}{2a}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)}{2b}}}}{i(Ab+2Ba)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)}{2b}}}}$
risch	$-\frac{A\sqrt{b\,x^3+a}}{a x}-$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*B*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*El
```

```
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-(b*x^3+a)^(1/2)/a/
x-1/3*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 54, normalized size = 0.11

$$-\frac{(2\,Ba+Ab)\sqrt{b}\,x\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\sqrt{bx^3+a}\,Ab}{abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] -((2*B*a + A*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0,
 -4*a/b, x)) + sqrt(b*x^3 + a)*A*b)/(a*b*x)
```

Sympy [A]
time = 1.05, size = 82, normalized size = 0.16

$$\frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^2\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/(x^2*(a + b*x^3)^(1/2)), x)
```

3.226 $\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=550

$$-\frac{A\sqrt{a+bx^3}}{4ax^4}+\frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x}-\frac{\sqrt[3]{b}\,(5Ab-8aB)\sqrt{a+bx^3}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,(5Ab-8aB}{}$$

[Out] -1/4*A*(b*x^3+a)^(1/2)/a/x^4+1/8*(5*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^2/x-1/8*b^(1/3)*(5*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-1/24*b^(1/3)*(5*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/16*3^(1/4)*b^(1/3)*(5*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.17, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 331, 309, 224, 1891}

$$-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(5Ab-8aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(5Ab-8aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{\sqrt{a+bx^3}\,(5Ab-8aB)}{8a^2x}-\frac{\sqrt[3]{b}\,\sqrt{a+bx^3}\,(5Ab-8aB)}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{A\sqrt{a+bx^3}}{4ax^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]

[Out] -1/4*(A*Sqrt[a + b*x^3])/(a*x^4) + ((5*A*b - 8*a*B)*Sqrt[a + b*x^3])/(8*a^2*x) - (b^(1/3)*(5*A*b - 8*a*B)*Sqrt[a + b*x^3])/(8*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(5*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (b^(1/3)*(5*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^

```
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*a^(5/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
```

```
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{4ax^4} - \frac{\left(\frac{5Ab}{2}-4aB\right)\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{4a}$$

$$= -\frac{A\sqrt{a+bx^3}}{4ax^4} + \frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x} - \frac{(b(5Ab-8aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{16a^2}$$

$$= -\frac{A\sqrt{a+bx^3}}{4ax^4} + \frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x} - \frac{\left(b^{2/3}(5Ab-8aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}}{16a^2}$$

$$= -\frac{A\sqrt{a+bx^3}}{4ax^4} + \frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x} - \frac{\sqrt[3]{b}\,(5Ab-8aB)\sqrt{a+bx^3}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sqrt[4]{3}\,\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 78, normalized size = 0.14

$$\frac{-2A(a+bx^3)+(5Ab-8aB)x^3\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a}\right)}{8ax^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*(a + b*x^3) + (5*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(8*a*x^4*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 928 vs. 2(412) = 824.
time = 0.35, size = 929, normalized size = 1.69

method	result

risch	$-\frac{\sqrt{b\,x^3+a}\,(-5Ab\,x^3+8Ba\,x^3+2Aa)}{8a^2x^4}+\frac{i(5Ab-8Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\ldots}}{\ldots}$ … $i(5Ab-8Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{4a\,x^4}+\frac{(5Ab-8Ba)\sqrt{b\,x^3+a}}{8a^2x}+\ldots$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4*(b*x^3+a)^(1/2)/a/x^4+5/8*b*(b*x^3+a)^(1/2)/a^2/x+5/24*I*b/a^2*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
```

```
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+B*(-(b*x^3+a)^(1/2)/a/x-1/3*I/a*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 70, normalized size = 0.13

$$-\frac{(8\,Ba-5\,Ab)\sqrt{b}\,x^4\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+((8\,Ba-5\,Ab)x^3+2\,Aa)\sqrt{bx^3+a}}{8\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*((8*B*a - 5*A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPIn
verse(0, -4*a/b, x)) + ((8*B*a - 5*A*b)*x^3 + 2*A*a)*sqrt(b*x^3 + a))/(a^2*
x^4)
```

Sympy [A]
time = 1.17, size = 88, normalized size = 0.16

$$\frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^5\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/(x^5*(a + b*x^3)^(1/2)), x)
```

3.227 $\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=581

$$-\frac{A\sqrt{a+bx^3}}{7ax^7}+\frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4}-\frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x}+\frac{5b^{4/3}(11Ab-14aB)\sqrt{a+bx^3}}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

```
[Out] -1/7*A*(b*x^3+a)^(1/2)/a/x^7+1/56*(11*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^2/x^4-5
/112*b*(11*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^3/x+5/112*b^(4/3)*(11*A*b-14*B*a)*
(b*x^3+a)^(1/2)/a^3/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/336*b^(4/3)*(11*A*b-1
4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)*2^(1/2)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)-5/224*3^(1/4)*b^(4/3)*(11*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE
((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 331, 309, 224, 1891}

$$\frac{5b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(11Ab-14aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}\,a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(11Ab-14aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{5b^{4/3}\sqrt{a+bx^3}(11Ab-14aB)}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{5b\sqrt{a+bx^3}(11Ab-14aB)}{112a^3x}+\frac{\sqrt{a+bx^3}(11Ab-14aB)}{56a^2x^4}-\frac{A\sqrt{a+bx^3}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/7*(A*Sqrt[a + b*x^3])/(a*x^7) + ((11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(56*
a^2*x^4) - (5*b*(11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112*a^3*x) + (5*b^(4/3)
*(11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) - (5*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(11*A*b - 14*a*B)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(8/3)*Sqrt[
```

```
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (5*b^(4/3)*(11*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
```

```
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{7ax^7} - \frac{\left(\frac{11Ab}{2}-7aB\right)\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{7a}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} + \frac{(5b(11Ab-14aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{112a^2}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} - \frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x} + \frac{(5b^2(11\cdots}{\cdots}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} - \frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x} + \frac{(5b^{5/3}(\cdots}{\cdots}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} - \frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x} + \frac{5b^{4/3}(\cdots}{112a^3\,(\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 78, normalized size = 0.13

$$\frac{-8A(a+bx^3)+(11Ab-14aB)x^3\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)}{56ax^7\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

```
[Out] (-8*A*(a + b*x^3) + (11*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometri
c2F1[-4/3, 1/2, -1/3, -((b*x^3)/a)])/(56*a*x^7*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 969 vs. 2(439) = 878.
time = 0.33, size = 970, normalized size = 1.67

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(55A\,b^{2}x^{6}-70Bab\,x^{6}-22aAb\,x^{3}+28a^{2}B\,x^{3}+16a^{2}A\right)}{112a^{3}x^{7}}-\frac{5ib(11Ab-14Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{}}{}\right.}{(-a}}}{}$ ... $5ib(11Ab-14Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(\right.}{}}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{7a\,x^{7}}+\frac{(11Ab-14Ba)\sqrt{b\,x^{3}+a}}{56a^{2}x^{4}}-\frac{5b(11Ab-14Ba)\sqrt{b\,x^{3}+a}}{112a^{3}x}-\frac{}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/4*(b*x^3+a)^(1/2)/a/x^4+5/8*b*(b*x^3+a)^(1/2)/a^2/x+5/24*I*b/a^2*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
```

```
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-1/7/a*(b*x^3+a)^(1/2)/x^7+11/56*
b/a^2*(b*x^3+a)^(1/2)/x^4-55/112*b^2/a^3*(b*x^3+a)^(1/2)/x-55/336*I*b^2/a^3
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 97, normalized size = 0.17

$$\frac{5\,(14\,Bab-11\,Ab^2)\sqrt{b}\,x^7\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(5\,(14\,Bab-11\,Ab^2)x^6-2\,(14\,Ba^2-11\,Aab)x^3-16\,Aa^2\right)\sqrt{bx^3+a}}{112\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 1/112*(5*(14*B*a*b - 11*A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)) + (5*(14*B*a*b - 11*A*b^2)*x^6 - 2*(14*B*a^2
- 11*A*a*b)*x^3 - 16*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^7)
```

Sympy [A]
time = 1.37, size = 94, normalized size = 0.16

$$\frac{A\Gamma\left(-\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{3},\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{B\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-7/3)*hyper((-7/3, 1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*sqrt(a)*x**4*gamma(-1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^8\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^8*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/(x^8*(a + b*x^3)^(1/2)), x)
```

3.228 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}-\frac{2a(2Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}$$

```
[Out] 2/9*(A*b-3*B*a)*(b*x^3+a)^(3/2)/b^4+2/15*B*(b*x^3+a)^(5/2)/b^4-2/3*a^2*(A*b
-B*a)/b^4/(b*x^3+a)^(1/2)-2/3*a*(2*A*b-3*B*a)*(b*x^3+a)^(1/2)/b^4
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$-\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\left(a+bx^3\right)^{3/2}(Ab-3aB)}{9b^4}-\frac{2a\sqrt{a+bx^3}\,(2Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*a^2*(A*b - a*B))/(3*b^4*Sqrt[a + b*x^3]) - (2*a*(2*A*b - 3*a*B)*Sqrt[a
+ b*x^3])/(3*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(3/2))/(9*b^4) + (2*B*(a +
 b*x^3)^(5/2))/(15*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{3/2}} + \frac{a(-2Ab+3aB)}{b^3\sqrt{a+bx}} + \frac{(Ab-3aB)\sqrt{a+bx}}{b^3} + \frac{B(a+b}{b^3}\right.\right.$$
$$= -\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}} - \frac{2a(2Ab-3aB)\sqrt{a+bx^3}}{3b^4} + \frac{2(Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4} + \frac{2B(a+b}{15b}$$

Mathematica [A]
time = 0.05, size = 77, normalized size = 0.75

$$\frac{2(48a^3B - 8a^2b(5A-3Bx^3) + b^3x^6(5A+3Bx^3) - 2ab^2x^3(10A+3Bx^3))}{45b^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(48*a^3*B - 8*a^2*b*(5*A - 3*B*x^3) + b^3*x^6*(5*A + 3*B*x^3) - 2*a*b^2*
x^3*(10*A + 3*B*x^3)))/(45*b^4*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.35, size = 134, normalized size = 1.30

method	result
gosper	$-\frac{2\left(-3B\,x^9b^3-5A\,b^3x^6+6Ba\,b^2x^6+20Aa\,b^2x^3-24B\,a^2b\,x^3+40A\,a^2b-48B\,a^3\right)}{45\sqrt{b\,x^3+a}\;b^4}$
trager	$-\frac{2\left(-3B\,x^9b^3-5A\,b^3x^6+6Ba\,b^2x^6+20Aa\,b^2x^3-24B\,a^2b\,x^3+40A\,a^2b-48B\,a^3\right)}{45\sqrt{b\,x^3+a}\;b^4}$
risch	$-\frac{2\left(-3b^2B\,x^6-5A\,b^2x^3+9Bab\,x^3+25abA-33a^2B\right)\sqrt{b\,x^3+a}}{45b^4}-\frac{2a^2(Ab-Ba)}{3b^4\sqrt{b\,x^3+a}}$
default	$B\left(\frac{2a^3}{3b^4\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x^6\sqrt{b\,x^3+a}}{15b^2}-\frac{2a\,x^3\sqrt{b\,x^3+a}}{5b^3}+\frac{22a^2\sqrt{b\,x^3+a}}{15b^4}\right)+A\left(-\frac{2a^2}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)}}\right.$
elliptic	$-\frac{2a^2(Ab-Ba)}{3b^4\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B\,x^6\sqrt{b\,x^3+a}}{15b^2}+\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{4aB}{5b^2}\right)x^3\sqrt{b\,x^3+a}}{9b}+\frac{2\left(-\frac{a(Ab-Ba)}{b^3}-\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{4aB}{5b^2}\right)a}{3b}\right)\sqrt{}}{3b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/3/b^4*a^3/((x^3+a/b)*b)^(1/2)+2/15/b^2*x^6*(b*x^3+a)^(1/2)-2/5*a/b^3*x
^3*(b*x^3+a)^(1/2)+22/15*a^2*(b*x^3+a)^(1/2)/b^4)+A*(-2/3/b^3*a^2/((x^3+a/b
)*b)^(1/2)+2/9/b^2*x^3*(b*x^3+a)^(1/2)-10/9*a*(b*x^3+a)^(1/2)/b^3)
```

Maxima [A]
time = 0.28, size = 116, normalized size = 1.13

$$\frac{2}{15}\,B\left(\frac{(bx^3+a)^{\frac{5}{2}}}{b^4}-\frac{5\,(bx^3+a)^{\frac{3}{2}}a}{b^4}+\frac{15\,\sqrt{bx^3+a}\,a^2}{b^4}+\frac{5\,a^3}{\sqrt{bx^3+a}\,b^4}\right)+\frac{2}{9}\,A\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^3}-\frac{6\,\sqrt{bx^3+a}\,a}{b^3}-\frac{3\,a^2}{\sqrt{bx^3+a}\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*((b*x^3 + a)^(5/2)/b^4 - 5*(b*x^3 + a)^(3/2)*a/b^4 + 15*sqrt(b*x^3 +
 a)*a^2/b^4 + 5*a^3/(sqrt(b*x^3 + a)*b^4)) + 2/9*A*((b*x^3 + a)^(3/2)/b^3 -
 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3 + a)*b^3))
```

Fricas [A]
time = 1.91, size = 88, normalized size = 0.85

$$\frac{2\,(3\,Bb^3x^9-(6\,Bab^2-5\,Ab^3)x^6+48\,Ba^3-40\,Aa^2b+4\,(6\,Ba^2b-5\,Aab^2)x^3)\sqrt{bx^3+a}}{45\,(b^5x^3+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^3*x^9 - (6*B*a*b^2 - 5*A*b^3)*x^6 + 48*B*a^3 - 40*A*a^2*b + 4*(
6*B*a^2*b - 5*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^5*x^3 + a*b^4)
```

Sympy [A]
time = 0.60, size = 175, normalized size = 1.70

$$\begin{cases} -\frac{16Aa^2}{9b^3\sqrt{a+bx^3}}-\frac{8Aax^3}{9b^2\sqrt{a+bx^3}}+\frac{2Ax^6}{9b\sqrt{a+bx^3}}+\frac{32Ba^3}{15b^4\sqrt{a+bx^3}}+\frac{16Ba^2x^3}{15b^3\sqrt{a+bx^3}}-\frac{4Bax^6}{15b^2\sqrt{a+bx^3}}+\frac{2Bx^9}{15b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-16*A*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*A*a*x**3/(9*b**2*sqrt(a
 + b*x**3)) + 2*A*x**6/(9*b*sqrt(a + b*x**3)) + 32*B*a**3/(15*b**4*sqrt(a +
 b*x**3)) + 16*B*a**2*x**3/(15*b**3*sqrt(a + b*x**3)) - 4*B*a*x**6/(15*b**2
*sqrt(a + b*x**3)) + 2*B*x**9/(15*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**9/
9 + B*x**12/12)/a**(3/2), True))
```

Giac [A]
time = 0.78, size = 114, normalized size = 1.11

$$\frac{2\,(Ba^3-Aa^2b)}{3\,\sqrt{bx^3+a}\,b^4}+\frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}Bb^{16}-15\,(bx^3+a)^{\frac{3}{2}}Bab^{16}+45\,\sqrt{bx^3+a}\,Ba^2b^{16}+5\,(bx^3+a)^{\frac{3}{2}}Ab^{17}-30\,\sqrt{bx^3+a}\,Aab^{17}\right)}{45\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(B*a^3 - A*a^2*b)/(sqrt(b*x^3 + a)*b^4) + 2/45*(3*(b*x^3 + a)^(5/2)*B*b
^16 - 15*(b*x^3 + a)^(3/2)*B*a*b^16 + 45*sqrt(b*x^3 + a)*B*a^2*b^16 + 5*(b*
x^3 + a)^(3/2)*A*b^17 - 30*sqrt(b*x^3 + a)*A*a*b^17)/b^20
```

Mupad [B]
time = 2.77, size = 152, normalized size = 1.48

$$\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,\left(B\,a^2-A\,a\,b\right)}{b^3}-\frac{2\,a\,\left(\frac{2\,\left(A\,b^2-B\,a\,b\right)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{3\,b}\right)}{3\,b}+\frac{x^3\,\sqrt{b\,x^3+a}\,\left(\frac{2\,\left(A\,b^2-B\,a\,b\right)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{9\,b}-\frac{a^2\,\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)}{b^2\,\sqrt{b\,x^3+a}}+\frac{2\,B\,x^6\,\sqrt{b\,x^3+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] ((a + b*x^3)^(1/2)*((2*(B*a^2 - A*a*b))/b^3 - (2*a*((2*(A*b^2 - B*a*b))/b^3
 - (8*B*a)/(5*b^2)))/(3*b)))/(3*b) + (x^3*(a + b*x^3)^(1/2)*((2*(A*b^2 - B*
a*b))/b^3 - (8*B*a)/(5*b^2)))/(9*b) - (a^2*((2*A)/(3*b) - (2*B*a)/(3*b^2)))
/(b^2*(a + b*x^3)^(1/2)) + (2*B*x^6*(a + b*x^3)^(1/2))/(15*b^2)
```

3.229 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2(Ab-2aB)\sqrt{a+bx^3}}{3b^3}+\frac{2B(a+bx^3)^{3/2}}{9b^3}$$

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b^3+2/3*a*(A*b-B*a)/b^3/(b*x^3+a)^(1/2)+2/3*(A*b-2*B*
a)*(b*x^3+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\frac{2\sqrt{a+bx^3}\,(Ab-2aB)}{3b^3}+\frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B(a+bx^3)^{3/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*a*(A*b - a*B))/(3*b^3*Sqrt[a + b*x^3]) + (2*(A*b - 2*a*B)*Sqrt[a + b*x^3
])/(3*b^3) + (2*B*(a + b*x^3)^(3/2))/(9*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{3/2}} + \frac{Ab-2aB}{b^2\sqrt{a+bx}} + \frac{B\sqrt{a+bx}}{b^2}\right) dx, x, x^3\right)$$
$$= \frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}} + \frac{2(Ab-2aB)\sqrt{a+bx^3}}{3b^3} + \frac{2B(a+bx^3)^{3/2}}{9b^3}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.75

$$\frac{2(6aAb-8a^2B+3Ab^2x^3-4abBx^3+b^2Bx^6)}{9b^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(6*a*A*b - 8*a^2*B + 3*A*b^2*x^3 - 4*a*b*B*x^3 + b^2*B*x^6))/(9*b^3*Sqrt
[a + b*x^3])
```

Maple [A]
time = 0.34, size = 94, normalized size = 1.29

method	result
gosper	$\frac{\frac{2}{9}b^2 B x^6+\frac{2}{3}A b^2 x^3-\frac{8}{9}Bab x^3+\frac{4}{3}abA-\frac{16}{9}a^2 B}{\sqrt{b x^3+a}\, b^3}$
trager	$\frac{\frac{2}{9}b^2 B x^6+\frac{2}{3}A b^2 x^3-\frac{8}{9}Bab x^3+\frac{4}{3}abA-\frac{16}{9}a^2 B}{\sqrt{b x^3+a}\, b^3}$
risch	$\frac{2(bB x^3+3Ab-5Ba)\sqrt{b x^3+a}}{9b^3}+\frac{2a(Ab-Ba)}{3b^3\sqrt{b x^3+a}}$
elliptic	$\frac{2a(Ab-Ba)}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B x^3\sqrt{b x^3+a}}{9b^2}+\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{2aB}{3b^2}\right)\sqrt{b x^3+a}}{3b}$
default	$B\left(-\frac{2a^2}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x^3\sqrt{b x^3+a}}{9b^2}-\frac{10a\sqrt{b x^3+a}}{9b^3}\right)+A\left(\frac{2a}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2\sqrt{b x^3+a}}{3b^2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-2/3/b^3*a^2/((x^3+a/b)*b)^(1/2)+2/9/b^2*x^3*(b*x^3+a)^(1/2)-10/9*a*(b*x
^3+a)^(1/2)/b^3)+A*(2/3/b^2*a/((x^3+a/b)*b)^(1/2)+2/3*(b*x^3+a)^(1/2)/b^2)
```

Maxima [A]
time = 0.29, size = 81, normalized size = 1.11

$$\frac{2}{9}\,B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^3}-\frac{6\,\sqrt{bx^3+a}\,a}{b^3}-\frac{3\,a^2}{\sqrt{bx^3+a}\,b^3}\right)+\frac{2}{3}\,A\left(\frac{\sqrt{bx^3+a}}{b^2}+\frac{a}{\sqrt{bx^3+a}\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^3 - 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3
+ a)*b^3)) + 2/3*A*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2))
```

Fricas [A]
time = 2.86, size = 63, normalized size = 0.86

$$\frac{2\,(Bb^2x^6-(4\,Bab-3\,Ab^2)x^3-8\,Ba^2+6\,Aab)\sqrt{bx^3+a}}{9\,(b^4x^3+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b^2*x^6 - (4*B*a*b - 3*A*b^2)*x^3 - 8*B*a^2 + 6*A*a*b)*sqrt(b*x^3 +
a)/(b^4*x^3 + a*b^3)
```

Sympy [A]
time = 0.44, size = 124, normalized size = 1.70

$$\begin{cases}\frac{4Aa}{3b^2\sqrt{a+bx^3}}+\frac{2Ax^3}{3b\sqrt{a+bx^3}}-\frac{16Ba^2}{9b^3\sqrt{a+bx^3}}-\frac{8Bax^3}{9b^2\sqrt{a+bx^3}}+\frac{2Bx^6}{9b\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((4*A*a/(3*b**2*sqrt(a + b*x**3)) + 2*A*x**3/(3*b*sqrt(a + b*x**3)
) - 16*B*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*B*a*x**3/(9*b**2*sqrt(a + b*x**
3)) + 2*B*x**6/(9*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a*
*(3/2), True))
```

Giac [A]
time = 1.05, size = 77, normalized size = 1.05

$$-\frac{2\,(Ba^2-Aab)}{3\,\sqrt{bx^3+a}\,b^3}+\frac{2\left((bx^3+a)^{\frac{3}{2}}Bb^6-6\,\sqrt{bx^3+a}\,Bab^6+3\,\sqrt{bx^3+a}\,Ab^7\right)}{9\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(B*a^2 - A*a*b)/(sqrt(b*x^3 + a)*b^3) + 2/9*((b*x^3 + a)^(3/2)*B*b^6 -
 6*sqrt(b*x^3 + a)*B*a*b^6 + 3*sqrt(b*x^3 + a)*A*b^7)/b^9
```

Mupad [B]
time = 2.68, size = 60, normalized size = 0.82

$$\frac{2\,B\,{\left(b\,x^3+a\right)}^2-6\,B\,a^2+6\,A\,b\,\left(b\,x^3+a\right)-12\,B\,a\,\left(b\,x^3+a\right)+6\,A\,a\,b}{9\,b^3\,\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] (2*B*(a + b*x^3)^2 - 6*B*a^2 + 6*A*b*(a + b*x^3) - 12*B*a*(a + b*x^3) + 6*A
*a*b)/(9*b^3*(a + b*x^3)^(1/2))
```

3.230 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^2}$$

```
[Out] -2/3*(A*b-B*a)/b^2/(b*x^3+a)^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$\frac{2B\sqrt{a+bx^3}}{3b^2}-\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(A*b - a*B))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{3/2}}+\frac{B}{b\sqrt{a+bx}}\right)\,dx,x,x^3\right)\\ &= -\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.72

$$\frac{2(-Ab+2aB+bBx^3)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
[Out] (2*(-(A*b) + 2*a*B + b*B*x^3))/(3*b^2*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.34, size = 53, normalized size = 1.15

method	result	size
gosper	$-\frac{2(-bB\,x^3+Ab-2Ba)}{3\sqrt{b\,x^3+a}\;b^2}$	30
trager	$-\frac{2(-bB\,x^3+Ab-2Ba)}{3\sqrt{b\,x^3+a}\;b^2}$	30
risch	$-\frac{2(Ab-Ba)}{3b^2\sqrt{b\,x^3+a}}+\frac{2B\sqrt{b\,x^3+a}}{3b^2}$	39
elliptic	$-\frac{2(Ab-Ba)}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B\sqrt{b\,x^3+a}}{3b^2}$	43
default	$B\left(\frac{2a}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^3+a}}{3b^2}\right)-\frac{2A}{3b\sqrt{b\,x^3+a}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] B*(2/3/b^2*a/((x^3+a/b)*b)^(1/2)+2/3*(b*x^3+a)^(1/2)/b^2)-2/3*A/b/(b*x^3+a)
^(1/2)
```

Maxima [A]
time = 0.28, size = 47, normalized size = 1.02

$$\frac{2}{3}\,B\left(\frac{\sqrt{bx^3+a}}{b^2}+\frac{a}{\sqrt{bx^3+a}\,b^2}\right)-\frac{2\,A}{3\,\sqrt{bx^3+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] 2/3*B*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) - 2/3*A/(sqrt(b*x^3 +
 a)*b)
```

Fricas [A]
time = 2.83, size = 41, normalized size = 0.89

$$\frac{2\,(Bbx^3+2\,Ba-Ab)\sqrt{bx^3+a}}{3\,(b^3x^3+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(B*b*x^3 + 2*B*a - A*b)*sqrt(b*x^3 + a)/(b^3*x^3 + a*b^2)
```

Sympy [A]
time = 0.33, size = 75, normalized size = 1.63

$$\begin{cases} -\frac{2A}{3b\sqrt{a+bx^3}}+\frac{4Ba}{3b^2\sqrt{a+bx^3}}+\frac{2Bx^3}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-2*A/(3*b*sqrt(a + b*x**3)) + 4*B*a/(3*b**2*sqrt(a + b*x**3)) +
2*B*x**3/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a**(3/2)
, True))
```

Giac [A]
time = 1.28, size = 38, normalized size = 0.83

$$\frac{2\,\sqrt{bx^3+a}\,B}{3\,b^2}+\frac{2\,(Ba-Ab)}{3\,\sqrt{bx^3+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*B/b^2 + 2/3*(B*a - A*b)/(sqrt(b*x^3 + a)*b^2)
```

Mupad [B]
time = 2.61, size = 33, normalized size = 0.72

$$\frac{2\,B\,a-2\,A\,b+2\,B\,\left(b\,x^3+a\right)}{3\,b^2\,\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] (2*B*a - 2*A*b + 2*B*(a + b*x^3))/(3*b^2*(a + b*x^3)^(1/2))
```

3.231 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

```
[Out] -2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)+2/3*(A*b-B*a)/a/b/(b*x^3+a)
^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {457, 79, 65, 214}

$$\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*(A*b - a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(3*a^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\mathrm{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}+\frac{A\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a}\\
&= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}+\frac{(2A)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3ab}\\
&= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 58, normalized size = 1.00

$$\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*(A*b - a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(3*a^(3/2))
```

Maple [A]
time = 0.33, size = 57, normalized size = 0.98

method	result	size

elliptic	$\frac{\frac{2Ab}{3}-\frac{2Ba}{3}}{ba\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2A\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}$	51
default	$-\frac{2B}{3b\sqrt{b\,x^3+a}}+A\left(\frac{2}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*B/b/(b*x^3+a)^(1/2)+A*(2/3/a/((x^3+a/b)*b)^(1/2)-2/3*arctanh((b*x^3+a)
^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.50, size = 70, normalized size = 1.21

$$\frac{1}{3}\,A\left(\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx^3+a}\,a}\right)-\frac{2\,B}{3\,\sqrt{bx^3+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*A*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)
 + 2/(sqrt(b*x^3 + a)*a)) - 2/3*B/(sqrt(b*x^3 + a)*b)
```

Fricas [A]
time = 2.66, size = 170, normalized size = 2.93

$$\left[\frac{(Ab^2x^3+Aab)\sqrt{a}\,\log\left(\frac{bx^3-2\sqrt{bx^3+a}\,\sqrt{a}+2a}{x^3}\right)-2\sqrt{bx^3+a}\,(Ba^2-Aab)}{3\,(a^2b^2x^3+a^3b)},\ \frac{2\left((Ab^2x^3+Aab)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)-\sqrt{bx^3+a}\,(Ba^2-Aab)\right)}{3\,(a^2b^2x^3+a^3b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*((A*b^2*x^3 + A*a*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) +
2*a)/x^3) - 2*sqrt(b*x^3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b), 2/3*(
(A*b^2*x^3 + A*a*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^
3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b)]
```

Sympy [A]
time = 7.92, size = 56, normalized size = 0.97

$$\frac{2A\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a\sqrt{-a}}-\frac{2(-Ab+Ba)}{3ab\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*A*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a*sqrt(-a)) - 2*(-A*b + B*a)/(3*a*b*
sqrt(a + b*x**3))
```

Giac [A]
time = 1.22, size = 53, normalized size = 0.91

$$\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a}-\frac{2\,(Ba-Ab)}{3\,\sqrt{bx^3+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - 2/3*(B*a - A*b)/(sqrt
(b*x^3 + a)*a*b)
```

Mupad [B]
time = 2.77, size = 65, normalized size = 1.12

$$\frac{\frac{2\,A}{3\,a}-\frac{2\,B}{3\,b}}{\sqrt{b\,x^3+a}}+\frac{A\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(3/2)),x)
```

```
[Out] ((2*A)/(3*a) - (2*B)/(3*b))/(a + b*x^3)^(1/2) + (A*log((((a + b*x^3)^(1/2)
- a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(3/2))
```

3.232 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{-3Ab+2aB}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

```
[Out] 1/3*(3*A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)+1/3*(-3*A*b+2*B*
a)/a^2/(b*x^3+a)^(1/2)-1/3*A/a/x^3/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 53, 65, 214}

$$\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}-\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

```
[Out] -1/3*(3*A*b - 2*a*B)/(a^2*Sqrt[a + b*x^3]) - A/(3*a*x^3*Sqrt[a + b*x^3]) +
((3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx, x, x^3\right) \\
&= -\frac{A}{3ax^3\sqrt{a+bx^3}} + \frac{\left(-\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^3\right)}{3a} \\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}} - \frac{A}{3ax^3\sqrt{a+bx^3}} - \frac{(3Ab-2aB)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{6a^2} \\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}} - \frac{A}{3ax^3\sqrt{a+bx^3}} - \frac{(3Ab-2aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3a^2b} \\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}} - \frac{A}{3ax^3\sqrt{a+bx^3}} + \frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 77, normalized size = 0.90

$$\frac{-aA-3Abx^3+2aBx^3}{3a^2x^3\sqrt{a+bx^3}} + \frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-(a*A) - 3*A*b*x^3 + 2*a*B*x^3)/(3*a^2*x^3*Sqrt[a + b*x^3]) + ((3*A*b - 2*
a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Maple [A]
time = 0.36, size = 100, normalized size = 1.16

method	result
elliptic	$-\frac{2(Ab-Ba)}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}+\frac{(3Ab-2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}$
risch	$-\frac{A\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}-\frac{-\frac{2bA}{3\sqrt{b\,x^{3}+a}}+a(3Ab-2Ba)\left(\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)}{2a^{2}}$
default	$A\left(-\frac{2b}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)+B\left(\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\,\mathrm{arctanh}\left(\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-2/3*b/a^2/((x^3+a/b)*b)^(1/2)-1/3/a^2*(b*x^3+a)^(1/2)/x^3+b*arctanh((b*
x^3+a)^(1/2)/a^(1/2))/a^(5/2))+B*(2/3/a/((x^3+a/b)*b)^(1/2)-2/3*arctanh((b*
x^3+a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(70) = 140.
time = 0.50, size = 144, normalized size = 1.67

$$-\frac{1}{6}A\left(\frac{2\left(3\left(bx^{3}+a\right)b-2ab\right)}{\left(bx^{3}+a\right)^{\frac{3}{2}}a^{2}-\sqrt{bx^{3}+a}\,a^{3}}+\frac{3b\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)+\frac{1}{3}B\left(\frac{\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx^{3}+a}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*A*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a
)*a^3) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a
^(5/2)) + 1/3*B*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)
))/a^(3/2) + 2/(sqrt(b*x^3 + a)*a))
```

Fricas [A]
time = 1.91, size = 233, normalized size = 2.71

$$\left[-\frac{\left((2\,Bab-3\,Ab^{2})x^{6}+(2\,Ba^{2}-3\,Aab)x^{3}\right)\sqrt{a}\,\log\left(\frac{bx^{3}+2\,\sqrt{bx^{3}+a}\,\sqrt{a}+2\,a}{x^{3}}\right)-2\left((2\,Ba^{2}-3\,Aab)x^{3}-Aa^{2}\right)\sqrt{bx^{3}+a}}{6\left(a^{3}bx^{6}+a^{4}x^{3}\right)},\ \frac{\left((2\,Bab-3\,Ab^{2})x^{6}+(2\,Ba^{2}-3\,Aab)x^{3}\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^{3}+a}\,\sqrt{-a}}{a}\right)+\left((2\,Ba^{2}-3\,Aab)x^{3}-Aa^{2}\right)\sqrt{bx^{3}+a}}{3\left(a^{3}bx^{6}+a^{4}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(a)*log((b*x
^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2*((2*B*a^2 - 3*A*a*b)*x^3 - A
*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3), 1/3*(((2*B*a*b - 3*A*b^2)*x^6
 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (
(2*B*a^2 - 3*A*a*b)*x^3 - A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(73) = 146$.
time = 25.85, size = 264, normalized size = 3.07

$$A\left(-\frac{1}{3a\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{\sqrt{b}}{a^{2}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^{3}}+1}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right)+B\left(\frac{2a^{3}\sqrt{1+\frac{bx^{3}}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^{3}}+\frac{a^{3}\log\left(\frac{bx^{3}}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^{3}}-\frac{2a^{3}\log\left(\sqrt{1+\frac{bx^{3}}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^{3}}+\frac{a^{2}bx^{3}\log\left(\frac{bx^{3}}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^{3}}-\frac{2a^{2}bx^{3}\log\left(\sqrt{1+\frac{bx^{3}}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(3/2),x)
```

```
[Out] A*(-1/(3*a*sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*
sqrt(a/(b*x**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2)) + B*(
2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*x**
3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) + 1)/
(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2) +
3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2)
+ 3*a**(7/2)*b*x**3))
```

Giac [A]
time = 1.18, size = 99, normalized size = 1.15

$$\frac{(2\,Ba-3\,Ab)\arctan\left(\frac{\sqrt{bx^{3}+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a^{2}}+\frac{2\,(bx^{3}+a)Ba-2\,Ba^{2}-3\,(bx^{3}+a)Ab+2\,Aab}{3\left((bx^{3}+a)^{\frac{3}{2}}-\sqrt{bx^{3}+a}\,a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*B*a - 3*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(
2*(b*x^3 + a)*B*a - 2*B*a^2 - 3*(b*x^3 + a)*A*b + 2*A*a*b)/(((b*x^3 + a)^(3
/2) - sqrt(b*x^3 + a)*a)*a^2)
```

Mupad [B]
time = 2.93, size = 131, normalized size = 1.52

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}^3}{x^6}\right)\,\left(3\,A\,b-2\,B\,a\right)}{6\,a^{5/2}}-\frac{\frac{2\,B\,a^2-3\,A\,a\,b}{2\,a^3}-\frac{a\,\left(\frac{A\,b^2}{3\,a^3}+\frac{5\,b\,\left(2\,B\,a^2-3\,A\,a\,b\right)}{6\,a^4}\right)}{b}}{\sqrt{b\,x^3+a}}-\frac{A\,\sqrt{b\,x^3+a}}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
3*A*b - 2*B*a))/(6*a^(5/2)) - ((2*B*a^2 - 3*A*a*b)/(2*a^3) - (a*((A*b^2)/(3
*a^3) + (5*b*(2*B*a^2 - 3*A*a*b))/(6*a^4)))/b)/(a + b*x^3)^(1/2) - (A*(a +
b*x^3)^(1/2))/(3*a^2*x^3)
```

3.233 $\int \frac{A+Bx^3}{x^7(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{b(5Ab-4aB)}{4a^3\sqrt{a+bx^3}}-\frac{A}{6ax^6\sqrt{a+bx^3}}+\frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}}-\frac{b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}$$

```
[Out] -1/4*b*(5*A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2)+1/4*b*(5*A*b-
4*B*a)/a^3/(b*x^3+a)^(1/2)-1/6*A/a/x^6/(b*x^3+a)^(1/2)+1/12*(5*A*b-4*B*a)/a
^2/x^3/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 44, 53, 65, 214}

$$-\frac{b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{b(5Ab-4aB)}{4a^3\sqrt{a+bx^3}}+\frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}}-\frac{A}{6ax^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] (b*(5*A*b - 4*a*B))/(4*a^3*Sqrt[a + b*x^3]) - A/(6*a*x^6*Sqrt[a + b*x^3]) +
 (5*A*b - 4*a*B)/(12*a^2*x^3*Sqrt[a + b*x^3]) - (b*(5*A*b - 4*a*B)*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]])/(4*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^7 \left(a+bx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^3}} + \frac{\left(-\frac{5Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^3\right)}{6a}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^3}} - \frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}} - \frac{(5Ab-4aB)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^3\right.}{4a^2}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^3}} - \frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}} + \frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3} + \frac{(b(5Ab-4aB))\text{S}}{}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^3}} - \frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}} + \frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3} + \frac{(5Ab-4aB)\text{Sub}}{}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^3}} - \frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}} + \frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3} - \frac{b(5Ab-4aB)\,\text{ta}}{4}$$

Mathematica [A]
time = 0.15, size = 100, normalized size = 0.85

$$\frac{-2a^2A+5aAbx^3-4a^2Bx^3+15Ab^2x^6-12abBx^6}{12a^3x^6\sqrt{a+bx^3}} + \frac{b(-5Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*a^2*A + 5*a*A*b*x^3 - 4*a^2*B*x^3 + 15*A*b^2*x^6 - 12*a*b*B*x^6)/(12*a^
3*x^6*Sqrt[a + b*x^3]) + (b*(-5*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a
]])/(4*a^(7/2))
```

Maple [A]
time = 0.38, size = 141, normalized size = 1.19

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{6a^2x^6}+\frac{(7Ab-4Ba)\sqrt{b\,x^3+a}}{12a^3x^3}+\frac{2b(Ab-Ba)}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{b(5Ab-4Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}$

risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-7Ab\,x^3+4Ba\,x^3+2Aa\right)}{12a^3x^6}+\frac{b\left(-\frac{2(7Ab-4Ba)}{3\sqrt{b\,x^3+a}}+3a(5Ab-4Ba)\left(\frac{2}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+}}{\sqrt{a}}\right.}{3a^{\frac{3}{2}}}\right.\right.}{8a^3}$
default	$A\left(-\frac{\sqrt{b\,x^3+a}}{6a^2x^6}+\frac{7b\sqrt{b\,x^3+a}}{12a^3x^3}+\frac{2b^2}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{5b^2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}\right)+B\left(-\frac{2}{3a^2\sqrt{\left(x^3\right.}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/6/a^2*(b*x^3+a)^(1/2)/x^6+7/12/a^3*b*(b*x^3+a)^(1/2)/x^3+2/3*b^2/a^3/
((x^3+a/b)*b)^(1/2)-5/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2))+B*(-2
/3*b/a^2/((x^3+a/b)*b)^(1/2)-1/3/a^2*(b*x^3+a)^(1/2)/x^3+b*arctanh((b*x^3+a
)^(1/2)/a^(1/2))/a^(5/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(98) = 196$.

time = 0.49, size = 215, normalized size = 1.82

$$\frac{1}{24}A\left(\frac{2\left(15\,(bx^3+a)^2b^2-25\,(bx^3+a)ab^2+8\,a^2b^2\right)}{(bx^3+a)^{\frac{5}{2}}a^3-2\,(bx^3+a)^{\frac{3}{2}}a^4+\sqrt{bx^3+a}\,a^5}+\frac{15\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{1}{6}B\left(\frac{2\,(3\,(bx^3+a)b-2\,ab)}{(bx^3+a)^{\frac{3}{2}}a^2-\sqrt{bx^3+a}\,a^3}+\frac{3\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*A*(2*(15*(b*x^3 + a)^2*b^2 - 25*(b*x^3 + a)*a*b^2 + 8*a^2*b^2)/((b*x^3
 + a)^(5/2)*a^3 - 2*(b*x^3 + a)^(3/2)*a^4 + sqrt(b*x^3 + a)*a^5) + 15*b^2*l
og((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) - 1/6*
B*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a)*a^3
) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2
))
```

Fricas [A]

time = 3.22, size = 289, normalized size = 2.45

$$\left[-\frac{3\,((4\,Bab^2-5\,Ab^3)x^9+(4\,Ba^2b-5\,Aab^2)x^6)\sqrt{a}\,\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\,(3\,(4\,Ba^2b-5\,Aab^2)x^6+2\,Aa^3+(4\,Ba^3-5\,Aa^2b)x^3)\sqrt{bx^3+a}}{24\,(a^4bx^9+a^5x^6)},-\frac{3\,((4\,Bab^2-5\,Ab^3)x^9+(4\,Ba^2b-5\,Aab^2)x^6)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+(3\,(4\,Ba^2b-5\,Aab^2)x^6+2\,Aa^3+(4\,Ba^3-5\,Aa^2b)x^3)\sqrt{bx^3+a}}{12\,(a^4bx^9+a^5x^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^2)*x^6)*sqrt(a)
*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*(4*B*a^2*b - 5*A
```

```
*a*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*
x^9 + a^5*x^6), -1/12*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^
2)*x^6)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(4*B*a^2*b - 5*A*a
*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*x^
9 + a^5*x^6)]
```

Sympy [A]

time = 50.66, size = 192, normalized size = 1.63

$$A\left(-\frac{1}{6a\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5\sqrt{b}}{12a^2x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5b^{\frac{3}{2}}}{4a^3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{5b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{4a^{\frac{7}{2}}}\right)+B\left(-\frac{1}{3a\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**(3/2),x)
```

```
[Out] A*(-1/(6*a*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + 5*sqrt(b)/(12*a**2*x**
(9/2)*sqrt(a/(b*x**3) + 1)) + 5*b**(3/2)/(4*a**3*x**(3/2)*sqrt(a/(b*x**3) +
 1)) - 5*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(7/2))) + B*(-1/(3*a*
sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*sqrt(a/(b*x
**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2))
```

Giac [A]

time = 0.69, size = 137, normalized size = 1.16

$$-\frac{(4\,Bab-5\,Ab^2)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^3}-\frac{2\,(Bab-Ab^2)}{3\,\sqrt{bx^3+a}\,a^3}-\frac{4\,(bx^3+a)^{\frac{3}{2}}Bab-4\,\sqrt{bx^3+a}\,Ba^2b-7\,(bx^3+a)^{\frac{3}{2}}Ab^2+9\,\sqrt{bx^3+a}\,Aab^2}{12\,a^3b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) -
2/3*(B*a*b - A*b^2)/(sqrt(b*x^3 + a)*a^3) - 1/12*(4*(b*x^3 + a)^(3/2)*B*a*b
 - 4*sqrt(b*x^3 + a)*B*a^2*b - 7*(b*x^3 + a)^(3/2)*A*b^2 + 9*sqrt(b*x^3 + a
)*A*a*b^2)/(a^3*b^2*x^6)
```

Mupad [B]

time = 3.18, size = 167, normalized size = 1.42

$$\frac{b\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)(5\,A\,b-4\,B\,a)}{8\,a^{7/2}}-\frac{(4\,B\,a^2-7\,A\,a\,b)\,\sqrt{b\,x^3+a}}{12\,a^4\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,a^2\,x^6}-\frac{\frac{a\left(\frac{7\,A\,b^3-4\,B\,a\,b^2}{12\,a^4}-\frac{5\,b^2\,(5\,A\,b-4\,B\,a)}{8\,a^4}\right)}{b}+\frac{3\,b\,(5\,A\,b-4\,B\,a)}{8\,a^3}}{\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x)
```

```
[Out] (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)
*(5*A*b - 4*B*a))/(8*a^(7/2)) - ((4*B*a^2 - 7*A*a*b)*(a + b*x^3)^(1/2))/(12
*a^4*x^3) - (A*(a + b*x^3)^(1/2))/(6*a^2*x^6) - ((a*((7*A*b^3 - 4*B*a*b^2)/
(12*a^4) - (5*b^2*(5*A*b - 4*B*a))/(8*a^4)))/b + (3*b*(5*A*b - 4*B*a))/(8*a
^3))/(a + b*x^3)^(1/2)
```

3.234 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}}+\frac{2Bx^7}{11b\sqrt{a+bx^3}}+\frac{16(11Ab-14aB)x\sqrt{a+bx^3}}{165b^3}-\frac{32\sqrt{2+\sqrt{3}}\,a(11Ab-14aB)\left(\sqrt[3]{a}\right.}{16}$$

[Out] -2/33*(11*A*b-14*B*a)*x^4/b^2/(b*x^3+a)^(1/2)+2/11*B*x^7/b/(b*x^3+a)^(1/2)+16/165*(11*A*b-14*B*a)*x*(b*x^3+a)^(1/2)/b^3-32/495*a*(11*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(10/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {470, 294, 327, 224}

$$-\frac{32\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-14aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{165\sqrt[4]{3}\,b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{16x\sqrt{a+bx^3}(11Ab-14aB)}{165b^3}-\frac{2x^4(11Ab-14aB)}{33b^2\sqrt{a+bx^3}}+\frac{2Bx^7}{11b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]

[Out] (-2*(11*A*b - 14*a*B)*x^4)/(33*b^2*Sqrt[a + b*x^3]) + (2*B*x^7)/(11*b*Sqrt[a + b*x^3]) + (16*(11*A*b - 14*a*B)*x*Sqrt[a + b*x^3])/(165*b^3) - (32*Sqrt[2 + Sqrt[3]]*a*(11*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(165*3^(1/4)*b^(10/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2Bx^7}{11b\sqrt{a+bx^3}} - \frac{\left(2\left(-\frac{11Ab}{2}+7aB\right)\right)\int \frac{x^6}{(a+bx^3)^{3/2}}\,dx}{11b}$$

$$= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} + \frac{(8(11Ab-14aB))\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{33b^2}$$

$$= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} + \frac{16(11Ab-14aB)x\sqrt{a+bx^3}}{165b^3} - \frac{(16a(11Ab -}{}$$

$$32\sqrt{2+\sqrt{3}}$$

$$= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} + \frac{16(11Ab-14aB)x\sqrt{a+bx^3}}{165b^3} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 103, normalized size = 0.34

$$\frac{2x\left(-112a^2B+3b^2x^3(11A+5Bx^3)+a(88Ab-42bBx^3)+8a(-11Ab+14aB)\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{165b^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(-112*a^2*B + 3*b^2*x^3*(11*A + 5*B*x^3) + a*(88*A*b - 42*b*B*x^3) + 8
*a*(-11*A*b + 14*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)]))/(165*b^3*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(233) = 466.
time = 0.35, size = 666, normalized size = 2.22

method	result

elliptic	$\frac{2ax(Ab-Ba)}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2Bx^4\sqrt{bx^3+a}}{11b^2}+\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{8aB}{11b^2}\right)x\sqrt{bx^3+a}}{5b}-\frac{2i\left(-\frac{2a(Ab-Ba)}{3b^3}-\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{8aB}{11b^2}\right)a}{5b}\right)}{}$
default	$B\left(-\frac{2a^2x}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x^4\sqrt{bx^3+a}}{11b^2}-\frac{38ax\sqrt{bx^3+a}}{55b^3}-\frac{448ia^2\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}}{}\right)}{(-ab^2}}}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-2/3/b^3*a^2*x/((x^3+a/b)*b)^(1/2)+2/11/b^2*x^4*(b*x^3+a)^(1/2)-38/55*a/
b^3*x*(b*x^3+a)^(1/2)-448/495*I*a^2/b^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/3/b^2*a*x/((x^3+a/b)*b)^(1/2)+2/5*x*(b*x^3
+a)^(1/2)/b^2+32/45*I*a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*
(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
```

```
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 122, normalized size = 0.41

$$\frac{2\left(16\,(14\,Ba^3-11\,Aa^2b+(14\,Ba^2b-11\,Aab^2)x^3)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(15\,Bb^3x^7-3\,(14\,Bab^2-11\,Ab^3)x^4-8\,(14\,Ba^2b-11\,Aab^2)x)\sqrt{bx^3+a}\right)}{165\,(b^5x^3+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/165*(16*(14*B*a^3 - 11*A*a^2*b + (14*B*a^2*b - 11*A*a*b^2)*x^3)*sqrt(b)*w
eierstrassPInverse(0, -4*a/b, x) + (15*B*b^3*x^7 - 3*(14*B*a*b^2 - 11*A*b^3
)*x^4 - 8*(14*B*a^2*b - 11*A*a*b^2)*x)*sqrt(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

Sympy [A]
time = 11.86, size = 80, normalized size = 0.27

$$\frac{Ax^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}+\frac{Bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(3/2)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((3/2, 10/3), (13/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(13/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x)
[Out] int((x^6*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.235 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{2(5Ab-8aB)x}{15b^2\sqrt{a+bx^3}}+\frac{2Bx^4}{5b\sqrt{a+bx^3}}+\frac{4\sqrt{2+\sqrt{3}}\,(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}{15\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}$$

```
[Out] -2/15*(5*A*b-8*B*a)*x/b^2/(b*x^3+a)^(1/2)+2/5*B*x^4/b/(b*x^3+a)^(1/2)+4/45*
(5*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {470, 294, 224}

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(5Ab-8aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2x(5Ab-8aB)}{15b^2\sqrt{a+bx^3}}+\frac{2Bx^4}{5b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(5*A*b - 8*a*B)*x)/(15*b^2*Sqrt[a + b*x^3]) + (2*B*x^4)/(5*b*Sqrt[a + b
*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(5*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2Bx^4}{5b\sqrt{a+bx^3}} - \frac{\left(2\left(-\frac{5Ab}{2}+4aB\right)\right)\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx}{5b}$$

$$= -\frac{2(5Ab-8aB)x}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^4}{5b\sqrt{a+bx^3}} + \frac{(2(5Ab-8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{15b^2}$$

$$= -\frac{2(5Ab-8aB)x}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^4}{5b\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}\,(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2} \cdots}{((\cdots}}}{15\sqrt[4]{3}\,b^{7/3}\sqrt{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 78, normalized size = 0.29

$$\frac{2x\left(-5Ab+8aB+3bBx^3+(5Ab-8aB)\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{15b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(-5*A*b + 8*a*B + 3*b*B*x^3 + (5*A*b - 8*a*B)*Sqrt[1 + (b*x^3)/a]*Hype
rgeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(15*b^2*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(206) = 412.
time = 0.35, size = 627, normalized size = 2.33 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/3/b^2*a*x/((x^3+a/b)*b)^(1/2)+2/5*x*(b*x^3+a)^(1/2)/b^2+32/45*I*a/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-2/3/b*x/
((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 95, normalized size = 0.35

$$-\frac{2\left(2\left((8\,Bab-5\,Ab^2)x^3+8\,Ba^2-5\,Aab\right)\sqrt{b}\ \mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(3\,Bb^2x^4+(8\,Bab-5\,Ab^2)x\right)\sqrt{bx^3+a}\right)}{15\,(b^4x^3+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(2*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*sqrt(b)*weierstrassP
Inverse(0, -4*a/b, x) - (3*B*b^2*x^4 + (8*B*a*b - 5*A*b^2)*x)*sqrt(b*x^3 +
a))/(b^4*x^3 + a*b^3)
```

Sympy [A]
time = 5.17, size = 80, normalized size = 0.30

$$\frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] int((x^3*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.236 $\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=251

$$\frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}}+\frac{2\sqrt{2+\sqrt{3}}\,(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\ldots}{\left(1+\sqrt{3}\right)\ldots}\right.\right.}{3\sqrt[4]{3}\,ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] 2/3*(A*b-B*a)*x/a/b/(b*x^3+a)^(1/2)+2/9*(A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*Ell
ipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(4/3)/(b*x^3+a)^(1/
2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {393, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2aB+Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2x(Ab-aB)}{3ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*x)/(3*a*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(A*b + 2*a
*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*
3^(1/4)*a*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}} + \frac{\left(2\left(\frac{Ab}{2}+aB\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\,(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x \cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x \cdots\right.}}}{3\sqrt[4]{3}\,ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b} \cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.29

$$\frac{x\left(2Ab-2aB+(Ab+2aB)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{3ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^(3/2),x]
[Out] (x*(2*A*b - 2*a*B + (A*b + 2*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3
, 1/2, 4/3, -((b*x^3)/a)]))/(3*a*b*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(192) = 384.
time = 0.34, size = 613, normalized size = 2.44

method	result
elliptic	$\frac{2x(Ab-Ba)}{3ba\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2i\left(\frac{B}{b}+\frac{Ab-Ba}{3ab}\right)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}}}}{}$
default	$B\left(-\frac{2x}{3b\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{4i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})}{2b}}}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-2/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2
)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 83, normalized size = 0.33

$$-\frac{2\left(\sqrt{bx^3+a}\,(Bab-Ab^2)x-((2\,Bab+Ab^2)x^3+2\,Ba^2+Aab)\sqrt{b}\,\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{3\,(ab^3x^3+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(b*x^3 + a)*(B*a*b - A*b^2)*x - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2
+ A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x))/(a*b^3*x^3 + a^2*b^2)
```

Sympy [A]
time = 3.16, size = 78, normalized size = 0.31

$$\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^(3/2),x)
[Out] int((A + B*x^3)/(a + b*x^3)^(3/2), x)
```

3.237 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=272

$$-\frac{A}{2ax^2\sqrt{a+bx^3}}-\frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}}-\frac{\sqrt{2+\sqrt{3}}\,(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}{6\sqrt[4]{3}\,a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}$$

```
[Out] -1/2*A/a/x^2/(b*x^3+a)^(1/2)-1/6*(7*A*b-4*B*a)*x/a^2/(b*x^3+a)^(1/2)-1/18*(
7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)*3^(3/4)/a^2/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 205, 224}

$$-\frac{x(7Ab-4aB)}{6a^2\sqrt{a+bx^3}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-4aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\,a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{A}{2ax^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```

```
[Out] -1/2*A/(a*x^2*Sqrt[a + b*x^3]) - ((7*A*b - 4*a*B)*x)/(6*a^2*Sqrt[a + b*x^3]
) - (Sqrt[2 + Sqrt[3]]*(7*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(6*3^(1/4)*a^2*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
```

```
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx &= -\frac{A}{2ax^2\sqrt{a+bx^3}}-\frac{\left(\frac{7Ab}{2}-2aB\right)\int \frac{1}{(a+bx^3)^{3/2}}\,dx}{2a}\\ &= -\frac{A}{2ax^2\sqrt{a+bx^3}}-\frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}}-\frac{(7Ab-4aB)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{12a^2}\\ &= -\frac{A}{2ax^2\sqrt{a+bx^3}}-\frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}}-\frac{\sqrt{2+\sqrt{3}}\,(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a\ldots}{((\ldots}}}{6\sqrt[4]{3}\,a^2\sqrt[3]{b}\ldots}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 86, normalized size = 0.32

$$\frac{-6aA-14Abx^3+8aBx^3+(-7Ab+4aB)x^3\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{12a^2x^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-6*a*A - 14*A*b*x^3 + 8*a*B*x^3 + (-7*A*b + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(12*a^2*x^2*Sqrt[a + b*x^3
])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(209) = 418$.
time = 0.41, size = 631, normalized size = 2.32

method	result
elliptic	$-\frac{2x(Ab-Ba)}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{2a^2x^2}-\frac{2i\left(-\frac{Ab-Ba}{3a^2}-\frac{Ab}{4a^2}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}}{(-a\,b^2)^{\frac{1}{3}}}}}{}$
default	$B\left(\frac{2x}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
```

```
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-2/3*b*x/a^2/((x^3+a/b)*b)^(1/2)-1/2*(b*x^3
+a)^(1/2)/a^2/x^2+7/18*I/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 102, normalized size = 0.38

$$\frac{((4\,Bab-7\,Ab^2)x^5+(4\,Ba^2-7\,Aab)x^2)\sqrt{b}\ \mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+((4\,Bab-7\,Ab^2)x^3-3\,Aab)\sqrt{bx^3+a}}{6\,(a^2b^2x^5+a^3bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] 1/6*(((4*B*a*b - 7*A*b^2)*x^5 + (4*B*a^2 - 7*A*a*b)*x^2)*sqrt(b)*weierstras
sPInverse(0, -4*a/b, x) + ((4*B*a*b - 7*A*b^2)*x^3 - 3*A*a*b)*sqrt(b*x^3 +
a))/(a^2*b^2*x^5 + a^3*b*x^2)
```

Sympy [A]
time = 8.23, size = 82, normalized size = 0.30

$$\frac{A\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^3\,{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x)
```

```
[Out] int((A + B*x^3)/(x^3*(a + b*x^3)^(3/2)), x)
```

3.238 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=304

$$-\frac{A}{5ax^5\sqrt{a+bx^3}}-\frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}}+\frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2}+\frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}(13Ab-10aB)\left(\sqrt[3]{a}+\right.}{60}$$

[Out] -1/5*A/a/x^5/(b*x^3+a)^(1/2)+1/15*(-13*A*b+10*B*a)/a^2/x^2/(b*x^3+a)^(1/2)+7/60*(13*A*b-10*B*a)*(b*x^3+a)^(1/2)/a^3/x^2+7/180*b^(2/3)*(13*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 296, 331, 224}

$$\frac{7\sqrt{a+bx^3}(13Ab-10aB)}{60a^3x^2}-\frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}}+\frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(13Ab-10aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{60\sqrt[4]{3}\,a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{A}{5ax^5\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]

[Out] -1/5*A/(a*x^5*Sqrt[a + b*x^3]) - (13*A*b - 10*a*B)/(15*a^2*x^2*Sqrt[a + b*x^3]) + (7*(13*A*b - 10*a*B)*Sqrt[a + b*x^3])/(60*a^3*x^2) + (7*Sqrt[2 + Sqrt[3]]*b^(2/3)*(13*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(60*3^(1/4)*a^3*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6(a+bx^3)^{3/2}}\,dx = -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{\left(\frac{13Ab}{2}-5aB\right)\int \frac{1}{x^3(a+bx^3)^{3/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} - \frac{(7(13Ab-10aB))\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{30a^2}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} + \frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2} + \frac{(7b(13Ab-1}{}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} + \frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2} + \frac{7\sqrt{2+\sqrt{3}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 72, normalized size = 0.24

$$\frac{-4aA+(13Ab-10aB)x^3\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(-\frac{2}{3},\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{20a^2x^5\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-4*a*A + (13*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3,
 3/2, 1/3, -((b*x^3)/a)])/(20*a^2*x^5*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs. 2(237) = 474.
time = 0.44, size = 667, normalized size = 2.19

method	result

elliptic	$-\frac{A\sqrt{b\,x^3+a}}{5a^2x^5}+\frac{(17Ab-10Ba)\sqrt{b\,x^3+a}}{20a^3x^2}+\frac{2bx(Ab-Ba)}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\left(\frac{b(17Ab-10Ba)}{40a^3}+\frac{b(Ab-Ba)}{3a^3}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\cdots}{\cdots}$
default	$A\left(-\frac{\sqrt{b\,x^3+a}}{5a^2x^5}+\frac{17b\sqrt{b\,x^3+a}}{20a^3x^2}+\frac{2b^2x}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{91ib\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2\cdots}{2b}\cdots\right.}{(-a\,b^2)^{\frac{1}{3}}}}\cdots}{\cdots}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/5*(b*x^3+a)^(1/2)/a^2/x^5+17/20*b*(b*x^3+a)^(1/2)/a^3/x^2+2/3*b^2/a^3
*x/((x^3+a/b)*b)^(1/2)-91/180*I*b/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)))+B*(-2/3*b*x/a^2/((x^3+a/b)*b)^(1/2)-1/2*(b*x^3+a)
^(1/2)/a^2/x^2+7/18*I/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-
a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
```

```
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 119, normalized size = 0.39

$$-\frac{7\left((10\,Bab-13\,Ab^2)x^8+(10\,Ba^2-13\,Aab)x^5\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(7\,(10\,Bab-13\,Ab^2)x^6+3\,(10\,Ba^2-13\,Aab)x^3+12\,Aa^2\right)\sqrt{bx^3+a}}{60\,(a^3bx^8+a^4x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/60*(7*((10*B*a*b - 13*A*b^2)*x^8 + (10*B*a^2 - 13*A*a*b)*x^5)*sqrt(b)*we
ierstrassPInverse(0, -4*a/b, x) + (7*(10*B*a*b - 13*A*b^2)*x^6 + 3*(10*B*a^
2 - 13*A*a*b)*x^3 + 12*A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^8 + a^4*x^5)
```

Sympy [A]
time = 19.45, size = 90, normalized size = 0.30

$$\frac{A\Gamma\left(-\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{3},\frac{3}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-5/3)*hyper((-5/3, 3/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*x**2*gamma(1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^6\,{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/(x^6*(a + b*x^3)^(3/2)), x)
```

3.239 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=547

$$-\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}}+\frac{2Bx^5}{7b\sqrt{a+bx^3}}+\frac{8(7Ab-10aB)\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,(7Ab-10aB)\left(\sqrt[3]{a}\right.}{}$$

```
[Out] -2/21*(7*A*b-10*B*a)*x^2/b^2/(b*x^3+a)^(1/2)+2/7*B*x^5/b/(b*x^3+a)^(1/2)+8/
21*(7*A*b-10*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+8
/63*a^(1/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
*3^(3/4)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)-4/21*a^(1/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)
*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(8/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 294, 309, 224, 1891}

$$\frac{8\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-10aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}\,b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-10aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\,3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{8\sqrt{a+bx^3}\,(7Ab-10aB)}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{2x^2(7Ab-10aB)}{21b^2\sqrt{a+bx^3}}+\frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(7*A*b - 10*a*B)*x^2)/(21*b^2*Sqrt[a + b*x^3]) + (2*B*x^5)/(7*b*Sqrt[a
+ b*x^3]) + (8*(7*A*b - 10*a*B)*Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b - 10*a*B)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*
b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]) + (8*Sqrt[2]*a^(1/3)*(7*A*b - 10*a*B)*(a^(1/3) +
```

```
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(21*3^(1/4)*b^(8/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2Bx^5}{7b\sqrt{a+bx^3}} - \frac{\left(2\left(-\frac{7Ab}{2}+5aB\right)\right)\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx}{7b}$$

$$= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{(4(7Ab-10aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{21b^2}$$

$$= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{(4(7Ab-10aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{21b^{7/3}} + \ldots$$

$$= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{8(7Ab-10aB)\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{4\sqrt{2-\sqrt{3}}\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 79, normalized size = 0.14

$$\frac{2x^2\left(7Ab-10aB+bBx^3+(-7Ab+10aB)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{7b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2*(7*A*b - 10*a*B + b*B*x^3 + (-7*A*b + 10*a*B)*Sqrt[1 + (b*x^3)/a]*Hy
pergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(7*b^2*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 936 vs. 2(409) = 818.
time = 0.36, size = 937, normalized size = 1.71

method	result

elliptic	$-\frac{2x^2(Ab-Ba)}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b^2}-\frac{2i\left(\frac{\frac{4Ab}{3}-\frac{4Ba}{3}}{b^2}-\frac{4aB}{7b^2}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/3/b^2*a*x^2/((x^3+a/b)*b)^(1/2)+2/7*x^2*(b*x^3+a)^(1/2)/b^2+80/63*I*a/
b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-2/3/b*x^2/((x^3+a/b)*b)^
(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
```

```
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 104, normalized size = 0.19

$$\frac{2\left(4\left((10\,Bab-7\,Ab^2)x^3+10\,Ba^2-7\,Aab\right)\sqrt{b}\ \mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(3\,Bb^2x^5+(10\,Bab-7\,Ab^2)x^2\right)\sqrt{bx^3+a}\right)}{21\left(b^4x^3+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*((10*B*a*b - 7*A*b^2)*x^3 + 10*B*a^2 - 7*A*a*b)*sqrt(b)*weierstrass
Zeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (3*B*b^2*x^5 + (10*B*a
*b - 7*A*b^2)*x^2)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A]
time = 6.48, size = 80, normalized size = 0.15

$$\frac{Ax^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}+\frac{Bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(11/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x)
[Out] int((x^4*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.240 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=524

$$\frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}}-\frac{2(Ab-4aB)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt{2-\sqrt{3}}\,(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{\cdots}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\,(\cdots}{\left(\left(1+\sqrt{\cdots}\right.\right.}}}$$

```
[Out] 2/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)^(1/2)-2/3*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/b^(
5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-2/9*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*El
lipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/3*(A*b-4*B*
a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/
4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {468, 309, 224, 1891}

$$-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(Ab-4aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(Ab-4aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}\,(Ab-4aB)}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x^2(Ab-aB)}{3ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*x^2)/(3*a*b*Sqrt[a + b*x^3]) - (2*(A*b - 4*a*B)*Sqrt[a + b*x
^3])/(3*a*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]
*(A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(3/4)*a^(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*(A*b - 4*a*
B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
```

```
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3
^(1/4)*a^(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}} + \frac{\left(2\left(-\frac{Ab}{2}+2aB\right)\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}} - \frac{(Ab-4aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{3ab^{4/3}} - \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\,(Ab-4aB\right.}{3a^{2/3}b^{4/3}}$$

$$= \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}} - \frac{2(Ab-4aB)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sqrt{2-\sqrt{3}}\,(Ab-4aB)\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.14

$$\frac{x^2\left(4aB+(Ab-4aB)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (x^2*(4*a*B + (A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2,
 5/3, -((b*x^3)/a)]))/(2*a*b*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 920 vs. 2(393) = 786.
time = 0.33, size = 921, normalized size = 1.76

method	result

	$2i\left(\frac{B}{b}-\frac{Ab-Ba}{3ab}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}}}$
elliptic	$\frac{2x^2(Ab-Ba)}{3ba\sqrt{\left(x^3+\frac{a}{b}\right)b}}-$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-2/3/b*x^2/((x^3+a/b)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))))+A*(2/3/a*x^2/((x^3+a/b)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/
3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 94, normalized size = 0.18

$$-\frac{2\left(\sqrt{bx^3+a}\,(Bab-Ab^2)x^2+((4\,Bab-Ab^2)x^3+4\,Ba^2-Aab)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{3\,(ab^3x^3+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(b*x^3 + a)*(B*a*b - A*b^2)*x^2 + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^
2 - A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b
, x)))/(a*b^3*x^3 + a^2*b^2)
```

Sympy [A]
time = 3.19, size = 80, normalized size = 0.15

$$\frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp
_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(B\,x^3+A)}{{(b\,x^3+a)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^(3/2),x)
[Out] int((x*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.241 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=548

$$-\frac{A}{ax\sqrt{a+bx^3}}-\frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}}+\frac{(5Ab-2aB)\sqrt{a+bx^3}}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt{2-\sqrt{3}}\,(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{2\ 3^{3/}}$$

```
[Out] -A/a/x/(b*x^3+a)^(1/2)-1/3*(5*A*b-2*B*a)*x^2/a^2/(b*x^3+a)^(1/2)+1/3*(5*A*b
-2*B*a)*(b*x^3+a)^(1/2)/a^2/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/9*(5*
A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)
/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)-1/6*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 296, 309, 224, 1891}

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(5Ab-2aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(5Ab-2aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{a+bx^3}\,(5Ab-2aB)}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{x^2(5Ab-2aB)}{3a^2\sqrt{a+bx^3}}-\frac{A}{ax\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] -(A/(a*x*Sqrt[a + b*x^3])) - ((5*A*b - 2*a*B)*x^2)/(3*a^2*Sqrt[a + b*x^3])
+ ((5*A*b - 2*a*B)*Sqrt[a + b*x^3])/(3*a^2*b^(2/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (Sqrt[2 - Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (Sqrt[2]*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
```

```
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
```

```
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{ax\sqrt{a+bx^3}} - \frac{\left(\frac{5Ab}{2}-aB\right)\int \frac{x}{(a+bx^3)^{3/2}}\,dx}{a}$$

$$= -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{6a^2}$$

$$= -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{6a^2\sqrt[3]{b}} + \frac{\left(\sqrt{\frac{1}{2}\cdots\right.}{\cdots}$$

$$= -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\sqrt{a+bx^3}}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt{2-\sqrt{3}\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 72, normalized size = 0.13

$$\frac{-4aA+(-5Ab+2aB)x^3\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{4a^2x\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-4*a*A + (-5*A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3
/2, 5/3, -((b*x^3)/a)])/(4*a^2*x*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(412) = 824.
time = 0.35, size = 939, normalized size = 1.71

method	result

	$2i\left(\frac{Ab-Ba}{3a^{2}}+\frac{Ab}{2a^{2}}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}$
elliptic	$-\frac{2x^{2}(Ab-Ba)}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^{3}+a}}{a^{2}x}-$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/3/a*x^2/((x^3+a/b)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))))+A*(-2/3*b*x^2/a^2/((x^3+a/b)*b)^(1/2)-(b*x^3+a)^(1/2)/a^2/x-5/9*
I/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
```

```
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 106, normalized size = 0.19

$$\frac{((2\,Bab-5\,Ab^2)x^4+(2\,Ba^2-5\,Aab)x)\sqrt{b}\ \mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+((2\,Bab-5\,Ab^2)x^3-3\,Aab)\sqrt{bx^3+a}}{3\,(a^2b^2x^4+a^3bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((2*B*a*b - 5*A*b^2)*x^4 + (2*B*a^2 - 5*A*a*b)*x)*sqrt(b)*weierstrassZ
eta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + ((2*B*a*b - 5*A*b^2)*x^
3 - 3*A*a*b)*sqrt(b*x^3 + a))/(a^2*b^2*x^4 + a^3*b*x)
```

Sympy [A]
time = 6.64, size = 82, normalized size = 0.15

$$\frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^2\,{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/(x^2*(a + b*x^3)^(3/2)), x)
```

3.242 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=580

$$-\frac{A}{4ax^4\sqrt{a+bx^3}}-\frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}}+\frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x}-\frac{5\sqrt[3]{b}\,(11Ab-8aB)\sqrt{a+bx^3}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{5\sqrt{2-\cdots}}{\cdots}$$

[Out] -1/4*A/a/x^4/(b*x^3+a)^(1/2)+1/12*(-11*A*b+8*B*a)/a^2/x/(b*x^3+a)^(1/2)+5/24*(11*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^3/x-5/24*b^(1/3)*(11*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^3/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-5/72*b^(1/3)*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+5/48*b^(1/3)*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.22, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {464, 296, 331, 309, 224, 1891}

$$-\frac{5\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(11Ab-8aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}\,a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{5\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(11Ab-8aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16\,3^{3/4}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{5\sqrt{a+bx^3}(11Ab-8aB)}{24a^3x}-\frac{5\sqrt[3]{b}\sqrt{a+bx^3}(11Ab-8aB)}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}}-\frac{A}{4ax^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]

[Out] -1/4*A/(a*x^4*Sqrt[a + b*x^3]) - (11*A*b - 8*a*B)/(12*a^2*x*Sqrt[a + b*x^3]) + (5*(11*A*b - 8*a*B)*Sqrt[a + b*x^3])/(24*a^3*x) - (5*b^(1/3)*(11*A*b - 8*a*B)*Sqrt[a + b*x^3])/(24*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (5*Sqrt[2 - Sqrt[3]]*b^(1/3)*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*3^(3/4)*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/

```
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) -
(5*b^(1/3)*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{\left(\frac{11Ab}{2}-4aB\right)\int \frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx}{4a}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} - \frac{(5(11Ab-8aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{24a^2}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{(5b(11Ab-8a}{}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{(5b^{2/3}(11Ab-}{}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{5\sqrt[3]{b}\,(11Ab-}{24a^3\left(\left(1+\sqrt{3}\right.\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 72, normalized size = 0.12

$$\frac{-2aA+(11Ab-8aB)x^3\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a}\right)}{8a^2x^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*a*A + (11*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3,
3/2, 2/3, -((b*x^3)/a)])/(8*a^2*x^4*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 974 vs. 2(438) = 876.
time = 0.36, size = 975, normalized size = 1.68

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{4a^2x^4}+\frac{(13Ab-8Ba)\sqrt{b\,x^3+a}}{8a^3x}+\frac{2b\,x^2(Ab-Ba)}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\left(-\frac{b(13Ab-8Ba)}{16a^3}-\frac{b(Ab-Ba)}{3a^3}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{}}{}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4*(b*x^3+a)^(1/2)/a^2/x^4+13/8*b*(b*x^3+a)^(1/2)/a^3/x+2/3*b^2*x^2/a^
3/((x^3+a/b)*b)^(1/2)+55/72*I*b/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
```

```
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)))+B*(-2/3*b*x^2/a^2/((x^3+a/b)*b)^(1/2)-(b*x^3+a)^(1/2)/a^2/x-5/9*I/a^2*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 127, normalized size = 0.22

$$-\frac{5\left(\left(8\,Bab-11\,Ab^{2}\right)x^{7}+\left(8\,Ba^{2}-11\,Aab\right)x^{4}\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(5\left(8\,Bab-11\,Ab^{2}\right)x^{6}+3\left(8\,Ba^{2}-11\,Aab\right)x^{3}+6\,Aa^{2}\right)\sqrt{bx^{3}+a}}{24\left(a^{3}bx^{7}+a^{4}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(5*((8*B*a*b - 11*A*b^2)*x^7 + (8*B*a^2 - 11*A*a*b)*x^4)*sqrt(b)*weie
rstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*(8*B*a*b - 1
1*A*b^2)*x^6 + 3*(8*B*a^2 - 11*A*a*b)*x^3 + 6*A*a^2)*sqrt(b*x^3 + a))/(a^3*
b*x^7 + a^4*x^4)
```

Sympy [A]
time = 14.68, size = 88, normalized size = 0.15

$$\frac{A\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},\frac{3}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^{4}\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^5\,{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x)
```

```
[Out] int((A + B*x^3)/(x^5*(a + b*x^3)^(3/2)), x)
```

3.243 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=611

$$-\frac{A}{7ax^7\sqrt{a+bx^3}}-\frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}}+\frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4}-\frac{55b(17Ab-14aB)\sqrt{a+bx^3}}{336a^4x}+\frac{55b^{4/3}(}{336a^4\left(\right.}$$

[Out] -1/7*A/a/x^7/(b*x^3+a)^(1/2)+1/21*(-17*A*b+14*B*a)/a^2/x^4/(b*x^3+a)^(1/2)+11/168*(17*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^3/x^4-55/336*b*(17*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^4/x+55/336*b^(4/3)*(17*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^4/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+55/1008*b^(4/3)*(17*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(11/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-55/672*b^(4/3)*(17*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(11/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.26, antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {464, 296, 331, 309, 224, 1891}

$$\frac{55b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-14aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{168\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{55\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-14aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224\,3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{55b^{4/3}\sqrt{a+bx^3}(17Ab-14aB)}{336a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{55b\sqrt{a+bx^3}(17Ab-14aB)}{336a^4x}+\frac{11\sqrt{a+bx^3}(17Ab-14aB)}{168a^3x^4}-\frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]

[Out] -1/7*A/(a*x^7*Sqrt[a + b*x^3]) - (17*A*b - 14*a*B)/(21*a^2*x^4*Sqrt[a + b*x^3]) + (11*(17*A*b - 14*a*B)*Sqrt[a + b*x^3])/(168*a^3*x^4) - (55*b*(17*A*b - 14*a*B)*Sqrt[a + b*x^3])/(336*a^4*x) + (55*b^(4/3)*(17*A*b - 14*a*B)*Sqrt[a + b*x^3])/(336*a^4*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (55*Sqrt[2 - Sqrt[3]]*b^(4/3)*(17*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^

```
(1/3)*x)], -7 - 4*Sqrt[3]])/(224*3^(3/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (55*b
^(4/3)*(17*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(168*Sqrt[2]*3^(1/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{\left(\frac{17Ab}{2}-7aB\right)\int \frac{1}{x^5\left(a+bx^3\right)^{3/2}}\,dx}{7a}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} - \frac{(11(17Ab-14aB))\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{42a^2}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} + \frac{(55b(17Ab-}{}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} - \frac{55b(17Ab-}{3}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} - \frac{55b(17Ab-}{3}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} - \frac{55b(17Ab-}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 72, normalized size = 0.12

$$\frac{-8aA + (17Ab - 14aB)x^3\sqrt{1 + \frac{bx^3}{a}}\ {}_2F_1\left(-\frac{4}{3}, \frac{3}{2}; -\frac{1}{3}; -\frac{bx^3}{a}\right)}{56a^2x^7\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-8*a*A + (17*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-4/3,
 3/2, -1/3, -((b*x^3)/a)])/(56*a^2*x^7*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1017 vs. 2(465) = 930.
time = 0.35, size = 1018, normalized size = 1.67

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{7a^2x^7} + \frac{(25Ab-14Ba)\sqrt{b\,x^3+a}}{56a^3x^4} - \frac{(237Ab-182Ba)b\sqrt{b\,x^3+a}}{112a^4x} - \frac{2b^2x^2(Ab-Ba)}{3a^4\sqrt{\left(x^3+\frac{a}{b}\right)b}} - \frac{2i\left(\frac{b^2(237}{2}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/4*(b*x^3+a)^(1/2)/a^2/x^4+13/8*b*(b*x^3+a)^(1/2)/a^3/x+2/3*b^2*x^2/a^
3/((x^3+a/b)*b)^(1/2)+55/72*I*b/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
```

```
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)))+A*(-1/7/a^2*(b*x^3+a)^(1/2)/x^7+25/56/a^3*b*(b*x^3+a)^(1/2)/x^4-237/112
/a^4*b^2*(b*x^3+a)^(1/2)/x-2/3*b^3*x^2/a^4/((x^3+a/b)*b)^(1/2)-935/1008*I*b
^2/a^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(
b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 156, normalized size = 0.26

$$\frac{55\,((14\,Bab^2-17\,Ab^3)x^{10}+(14\,Ba^2b-17\,Aab^2)x^7)\sqrt{b}\ \text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(55\,(14\,Bab^2-17\,Ab^3)x^9+33\,(14\,Ba^2b-17\,Aab^2)x^6-48\,Aa^3-6\,(14\,Ba^3-17\,Aa^2b)x^3)\sqrt{bx^3+a}}{336\,(a^4bx^{10}+a^5x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/336*(55*((14*B*a*b^2 - 17*A*b^3)*x^10 + (14*B*a^2*b - 17*A*a*b^2)*x^7)*sq
rt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (55*(
14*B*a*b^2 - 17*A*b^3)*x^9 + 33*(14*B*a^2*b - 17*A*a*b^2)*x^6 - 48*A*a^3 -
6*(14*B*a^3 - 17*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*x^10 + a^5*x^7)
```

Sympy [A]
time = 30.16, size = 94, normalized size = 0.15

$$\frac{A\Gamma\left(-\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{3}, \frac{3}{2} \\ -\frac{4}{3} \end{matrix}\middle| {\frac{bx^{3}e^{i\pi}}{a}} \right)}{3a^{\frac{3}{2}}x^{7}\Gamma\left(-\frac{4}{3}\right)} + \frac{B\Gamma\left(-\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{3}{2} \\ -\frac{1}{3} \end{matrix}\middle| {\frac{bx^{3}e^{i\pi}}{a}} \right)}{3a^{\frac{3}{2}}x^{4}\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-7/3)*hyper((-7/3, 3/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(3/2)*x**4*gamma(-1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^8\,{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x)
```

```
[Out] int((A + B*x^3)/(x^8*(a + b*x^3)^(3/2)), x)
```

3.244 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2a^2(Ab-aB)}{9b^4(a+bx^3)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2(Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2B(a+bx^3)^{3/2}}{9b^4}$$

```
[Out] -2/9*a^2*(A*b-B*a)/b^4/(b*x^3+a)^(3/2)+2/9*B*(b*x^3+a)^(3/2)/b^4+2/3*a*(2*A
*b-3*B*a)/b^4/(b*x^3+a)^(1/2)+2/3*(A*b-3*B*a)*(b*x^3+a)^(1/2)/b^4
```

Rubi [A]

time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$-\frac{2a^2(Ab-aB)}{9b^4(a+bx^3)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}(Ab-3aB)}{3b^4}+\frac{2B(a+bx^3)^{3/2}}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*a^2*(A*b - a*B))/(9*b^4*(a + b*x^3)^(3/2)) + (2*a*(2*A*b - 3*a*B))/(3*b
^4*Sqrt[a + b*x^3]) + (2*(A*b - 3*a*B)*Sqrt[a + b*x^3])/(3*b^4) + (2*B*(a +
 b*x^3)^(3/2))/(9*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{5/2}} + \frac{a(-2Ab+3aB)}{b^3(a+bx)^{3/2}} + \frac{Ab-3aB}{b^3\sqrt{a+bx}} + \frac{B\sqrt{a+bx}}{b^3}\right) dx$$

$$= -\frac{2a^2(Ab-aB)}{9b^4(a+bx^3)^{3/2}} + \frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}} + \frac{2(Ab-3aB)\sqrt{a+bx^3}}{3b^4} + \frac{2B(a+bx^3)^{3/2}}{9b^4}$$

Mathematica [**A**]
time = 0.06, size = 73, normalized size = 0.71

$$\frac{2(-16a^3B+8a^2b(A-3Bx^3)-6ab^2x^3(-2A+Bx^3)+b^3x^6(3A+Bx^3))}{9b^4(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(-16*a^3*B + 8*a^2*b*(A - 3*B*x^3) - 6*a*b^2*x^3*(-2*A + B*x^3) + b^3*x^
6*(3*A + B*x^3)))/(9*b^4*(a + b*x^3)^(3/2))
```

Maple [**A**]
time = 0.32, size = 150, normalized size = 1.46

method	result
risch	$\frac{2(bBx^3+3Ab-8Ba)\sqrt{bx^3+a}}{9b^4} + \frac{2a(6Ab^2x^3-9Babx^3+5abA-8a^2B)}{9b^4(bx^3+a)^{\frac{3}{2}}}$
gosper	$\frac{\frac{2}{9}Bx^9b^3+\frac{2}{3}Ab^3x^6-\frac{4}{3}Bab^2x^6+\frac{8}{3}Aab^2x^3-\frac{16}{3}Ba^2bx^3+\frac{16}{9}Aa^2b-\frac{32}{9}Ba^3}{b^4(bx^3+a)^{\frac{3}{2}}}$
trager	$\frac{\frac{2}{9}Bx^9b^3+\frac{2}{3}Ab^3x^6-\frac{4}{3}Bab^2x^6+\frac{8}{3}Aab^2x^3-\frac{16}{3}Ba^2bx^3+\frac{16}{9}Aa^2b-\frac{32}{9}Ba^3}{b^4(bx^3+a)^{\frac{3}{2}}}$
elliptic	$-\frac{2a^2(Ab-Ba)\sqrt{bx^3+a}}{9b^6\left(x^3+\frac{a}{b}\right)^2} + \frac{2(2Ab-3Ba)a}{3b^4\sqrt{\left(x^3+\frac{a}{b}\right)b}} + \frac{2Bx^3\sqrt{bx^3+a}}{9b^3} + \frac{2\left(\frac{Ab-2Ba}{b^3}-\frac{2Ba}{3b^3}\right)\sqrt{bx^3+a}}{3b}$
default	$B\left(\frac{2a^3\sqrt{bx^3+a}}{9b^6\left(x^3+\frac{a}{b}\right)^2} - \frac{2a^2}{b^4\sqrt{\left(x^3+\frac{a}{b}\right)b}} + \frac{2x^3\sqrt{bx^3+a}}{9b^3} - \frac{16a\sqrt{bx^3+a}}{9b^4}\right) + A\left(-\frac{2a^2\sqrt{bx^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2} -\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/9*a^3/b^6*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/b^4*a^2/((x^3+a/b)*b)^(1/2)+2/
9/b^3*x^3*(b*x^3+a)^(1/2)-16/9*a/b^4*(b*x^3+a)^(1/2))+A*(-2/9*a^2/b^5*(b*x^
3+a)^(1/2)/(x^3+a/b)^2+4/3/b^3*a/((x^3+a/b)*b)^(1/2)+2/3/b^3*(b*x^3+a)^(1/2
))
```

Maxima [A]
time = 0.34, size = 116, normalized size = 1.13

$$\frac{2}{9}B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^4}-\frac{9\sqrt{bx^3+a}\,a}{b^4}-\frac{9\,a^2}{\sqrt{bx^3+a}\,b^4}+\frac{a^3}{(bx^3+a)^{\frac{3}{2}}b^4}\right)+\frac{2}{9}A\left(\frac{3\sqrt{bx^3+a}}{b^3}+\frac{6\,a}{\sqrt{bx^3+a}\,b^3}-\frac{a^2}{(bx^3+a)^{\frac{3}{2}}b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^4 - 9*sqrt(b*x^3 + a)*a/b^4 - 9*a^2/(sqrt(b*x^3
+ a)*b^4) + a^3/((b*x^3 + a)^(3/2)*b^4)) + 2/9*A*(3*sqrt(b*x^3 + a)/b^3 + 6
*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)^(3/2)*b^3))
```

Fricas [A]
time = 2.24, size = 98, normalized size = 0.95

$$\frac{2\,(Bb^3x^9-3\,(2\,Bab^2-Ab^3)x^6-16\,Ba^3+8\,Aa^2b-12\,(2\,Ba^2b-Aab^2)x^3)\sqrt{bx^3+a}}{9\,(b^6x^6+2\,ab^5x^3+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b^3*x^9 - 3*(2*B*a*b^2 - A*b^3)*x^6 - 16*B*a^3 + 8*A*a^2*b - 12*(2*B
*a^2*b - A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(99) = 198.
time = 0.68, size = 338, normalized size = 3.28

$$\begin{cases} \frac{16Aa^2b}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}}+\frac{24Aab^2x^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}}+\frac{6Ab^3x^6}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}}-\frac{32Ba^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}}-\frac{48Ba^2bx^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}}-\frac{12Bab^2x^6}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}}+\frac{2Bb^3x^9}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} & \text{for}\ b\neq 0 \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((16*A*a**2*b/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*
x**3)) + 24*A*a*b**2*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) + 6*A*b**3*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) - 32*B*a**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x
**3)) - 48*B*a**2*b*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
b*x**3)) - 12*B*a*b**2*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
```

```
+ b*x**3)) + 2*B*b**3*x**9/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
+ b*x**3)), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(5/2), True))
```

Giac [A]
time = 1.06, size = 104, normalized size = 1.01

$$-\frac{2\left(9\left(bx^3+a\right)Ba^2-Ba^3-6\left(bx^3+a\right)Aab+Aa^2b\right)}{9\left(bx^3+a\right)^{\frac{3}{2}}b^4}+\frac{2\left(\left(bx^3+a\right)^{\frac{3}{2}}Bb^8-9\sqrt{bx^3+a}\,Bab^8+3\sqrt{bx^3+a}\,Ab^9\right)}{9\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*(9*(b*x^3 + a)*B*a^2 - B*a^3 - 6*(b*x^3 + a)*A*a*b + A*a^2*b)/((b*x^3
+ a)^(3/2)*b^4) + 2/9*((b*x^3 + a)^(3/2)*B*b^8 - 9*sqrt(b*x^3 + a)*B*a*b^8
+ 3*sqrt(b*x^3 + a)*A*b^9)/b^12
```

Mupad [B]
time = 2.80, size = 145, normalized size = 1.41

$$\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,\left(A\,b-2\,B\,a\right)}{b^3}-\frac{4\,B\,a}{3\,b^3}\right)}{3\,b}-\frac{\frac{2\,B\,a^2-2\,A\,a\,b}{3\,b^4}-\frac{a\,\left(\frac{2\,A\,b^2-2\,B\,a\,b}{3\,b^4}-\frac{2\,B\,a}{3\,b^3}\right)}{b}}{\sqrt{b\,x^3+a}}-\frac{a^2\,\left(\frac{2\,A}{9\,b}-\frac{2\,B\,a}{9\,b^2}\right)}{b^2\,{\left(b\,x^3+a\right)}^{3/2}}+\frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] ((a + b*x^3)^(1/2)*((2*(A*b - 2*B*a))/b^3 - (4*B*a)/(3*b^3)))/(3*b) - ((2*B
*a^2 - 2*A*a*b)/(3*b^4) - (a*((2*A*b^2 - 2*B*a*b)/(3*b^4) - (2*B*a)/(3*b^3)
))/b)/(a + b*x^3)^(1/2) - (a^2*((2*A)/(9*b) - (2*B*a)/(9*b^2)))/(b^2*(a + b
*x^3)^(3/2)) + (2*B*x^3*(a + b*x^3)^(1/2))/(9*b^3)
```

3.245 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^3}$$

```
[Out] 2/9*a*(A*b-B*a)/b^3/(b*x^3+a)^(3/2)-2/3*(A*b-2*B*a)/b^3/(b*x^3+a)^(1/2)+2/3
*B*(b*x^3+a)^(1/2)/b^3
```

Rubi [A]

time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}+\frac{2B\sqrt{a+bx^3}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*a*(A*b - a*B))/(9*b^3*(a + b*x^3)^(3/2)) - (2*(A*b - 2*a*B))/(3*b^3*Sqrt
[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{5/2}} + \frac{Ab-2aB}{b^2(a+bx)^{3/2}} + \frac{B}{b^2\sqrt{a+bx}}\right)dx, x, x^3\right)$$
$$= \frac{2a(Ab-aB)}{9b^3(a+bx^3)^{3/2}} - \frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}} + \frac{2B\sqrt{a+bx^3}}{3b^3}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.77

$$\frac{2(-2aAb+8a^2B-3Ab^2x^3+12abBx^3+3b^2Bx^6)}{9b^3(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(-2*a*A*b + 8*a^2*B - 3*A*b^2*x^3 + 12*a*b*B*x^3 + 3*b^2*B*x^6))/(9*b^3*
(a + b*x^3)^(3/2))
```

Maple [A]
time = 0.33, size = 113, normalized size = 1.55

method	result
gosper	$-\frac{2(-3b^2Bx^6+3Ab^2x^3-12Babx^3+2abA-8a^2B)}{9(bx^3+a)^{\frac{3}{2}}b^3}$
trager	$-\frac{2(-3b^2Bx^6+3Ab^2x^3-12Babx^3+2abA-8a^2B)}{9(bx^3+a)^{\frac{3}{2}}b^3}$
risch	$\frac{2B\sqrt{bx^3+a}}{3b^3} - \frac{2(3Ab^2x^3-6Babx^3+2abA-5a^2B)}{9b^3(bx^3+a)^{\frac{3}{2}}}$
elliptic	$\frac{2(Ab-Ba)a\sqrt{bx^3+a}}{9b^5(x^3+\frac{a}{b})^2} - \frac{2(Ab-2Ba)}{3b^3\sqrt{(x^3+\frac{a}{b})b}} + \frac{2B\sqrt{bx^3+a}}{3b^3}$
default	$B\left(-\frac{2a^2\sqrt{bx^3+a}}{9b^5(x^3+\frac{a}{b})^2} + \frac{4a}{3b^3\sqrt{(x^3+\frac{a}{b})b}} + \frac{2\sqrt{bx^3+a}}{3b^3}\right) + A\left(\frac{2a\sqrt{bx^3+a}}{9b^4(x^3+\frac{a}{b})^2} - \frac{2}{3b^2\sqrt{(x^3+\frac{a}{b})b}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-2/9*a^2/b^5*(b*x^3+a)^(1/2)/(x^3+a/b)^2+4/3/b^3*a/((x^3+a/b)*b)^(1/2)+2
/3/b^3*(b*x^3+a)^(1/2))+A*(2/9/b^4*a*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/3/b^2/((
x^3+a/b)*b)^(1/2))
```

Maxima [A]
time = 0.28, size = 84, normalized size = 1.15

$$\frac{2}{9}\,B\left(\frac{3\,\sqrt{bx^3+a}}{b^3}+\frac{6\,a}{\sqrt{bx^3+a}\,b^3}-\frac{a^2}{(bx^3+a)^{\frac{3}{2}}b^3}\right)-\frac{2}{9}\,A\left(\frac{3}{\sqrt{bx^3+a}\,b^2}-\frac{a}{(bx^3+a)^{\frac{3}{2}}b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*(3*sqrt(b*x^3 + a)/b^3 + 6*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)
^(3/2)*b^3)) - 2/9*A*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2))
```

Fricas [A]
time = 1.83, size = 75, normalized size = 1.03

$$\frac{2\,(3\,Bb^2x^6+3\,(4\,Bab-Ab^2)x^3+8\,Ba^2-2\,Aab)\sqrt{bx^3+a}}{9\,(b^5x^6+2\,ab^4x^3+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*B*b^2*x^6 + 3*(4*B*a*b - A*b^2)*x^3 + 8*B*a^2 - 2*A*a*b)*sqrt(b*x^3
+ a)/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(70) = 140$.
time = 0.52, size = 240, normalized size = 3.29

$$\begin{cases} -\frac{4Aab}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}}-\frac{6Ab^2x^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}}+\frac{16Ba^2}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}}+\frac{24Babx^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}}+\frac{6Bb^2x^6}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((-4*A*a*b/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
3)) - 6*A*b**2*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
3)) + 16*B*a**2/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3))
+ 24*B*a*b*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3))
+ 6*B*b**2*x**6/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3)),
 Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a**(5/2), True))
```

Giac [A]
time = 1.38, size = 63, normalized size = 0.86

$$\frac{2\,\sqrt{bx^3+a}\,B}{3\,b^3}+\frac{2\,(6\,(bx^3+a)Ba-Ba^2-3\,(bx^3+a)Ab+Aab)}{9\,(bx^3+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*B/b^3 + 2/9*(6*(b*x^3 + a)*B*a - B*a^2 - 3*(b*x^3 + a)*
A*b + A*a*b)/((b*x^3 + a)^(3/2)*b^3)
```

Mupad [B]
time = 2.76, size = 60, normalized size = 0.82

$$\frac{6\,B\,{\left(b\,x^3+a\right)}^2-2\,B\,a^2-6\,A\,b\,\left(b\,x^3+a\right)+12\,B\,a\,\left(b\,x^3+a\right)+2\,A\,a\,b}{9\,b^3\,{\left(b\,x^3+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] (6*B*(a + b*x^3)^2 - 2*B*a^2 - 6*A*b*(a + b*x^3) + 12*B*a*(a + b*x^3) + 2*A
*a*b)/(9*b^3*(a + b*x^3)^(3/2))
```

3.246 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}$$

```
[Out] -2/9*(A*b-B*a)/b^2/(b*x^3+a)^(3/2)-2/3*B/b^2/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$-\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(A*b - a*B))/(9*b^2*(a + b*x^3)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*x^3])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{5/2}}+\frac{B}{b(a+bx)^{3/2}}\right)dx, x, x^3\right)\\
&= -\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.72

$$-\frac{2(Ab + 2aB + 3bBx^3)}{9b^2 (a + bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
[Out] (-2*(A*b + 2*a*B + 3*b*B*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

Maple [A]
time = 0.33, size = 64, normalized size = 1.39

method	result	size
gosper	$-\frac{2(3bB\,x^3+Ab+2Ba)}{9(b\,x^3+a)^{\frac{3}{2}}\,b^2}$	30
trager	$-\frac{2(3bB\,x^3+Ab+2Ba)}{9(b\,x^3+a)^{\frac{3}{2}}\,b^2}$	30
elliptic	$-\frac{2(Ab-Ba)\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}-\frac{2B}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}$	54
default	$B\left(\frac{2a\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}-\frac{2}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}\right)-\frac{2A}{9b(b\,x^3+a)^{\frac{3}{2}}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] B*(2/9/b^4*a*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/3/b^2/((x^3+a/b)*b)^(1/2))-2/9*A
/b/(b*x^3+a)^(3/2)
```

Maxima [A]
time = 0.29, size = 49, normalized size = 1.07

$$-\frac{2}{9}\,B\left(\frac{3}{\sqrt{bx^3+a}\,b^2}-\frac{a}{(bx^3+a)^{\frac{3}{2}}b^2}\right)-\frac{2\,A}{9\,(bx^3+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] -2/9*B*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2)) - 2/9*A/((b*x^
3 + a)^(3/2)*b)
```

Fricas [A]
time = 2.18, size = 52, normalized size = 1.13

$$-\frac{2\,{\left(3\,B b x^{3} + 2\,B a + A b\right)} \sqrt{b x^{3} + a}}{9\,{\left(b^{4} x^{6} + 2\,a b^{3} x^{3} + a^{2} b^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/9*(3*B*b*x^3 + 2*B*a + A*b)*sqrt(b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2
*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(44) = 88.
time = 0.40, size = 144, normalized size = 3.13

$$\begin{cases} - \frac{2 A b}{9 a b^{2} \sqrt{a + b x^{3}} + 9 b^{3} x^{3} \sqrt{a + b x^{3}}} - \frac{4 B a}{9 a b^{2} \sqrt{a + b x^{3}} + 9 b^{3} x^{3} \sqrt{a + b x^{3}}} - \frac{6 B b x^{3}}{9 a b^{2} \sqrt{a + b x^{3}} + 9 b^{3} x^{3} \sqrt{a + b x^{3}}} & \text{for}\: b \neq 0 \\\frac{\frac{A x^{3}}{3} + \frac{B x^{6}}{6}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((-2*A*b/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)
) - 4*B*a/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)) - 6*B*
b*x**3/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)), Ne(b, 0)
), ((A*x**3/3 + B*x**6/6)/a**(5/2), True))
```

Giac [A]
time = 1.48, size = 32, normalized size = 0.70

$$-\frac{2\,{\left(3\,{\left(b x^{3} + a\right)} B - B a + A b\right)}}{9\,{\left(b x^{3} + a\right)}^{\frac{3}{2}} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*(3*(b*x^3 + a)*B - B*a + A*b)/((b*x^3 + a)^(3/2)*b^2)
```

Mupad [B]
time = 2.68, size = 33, normalized size = 0.72

$$-\frac{2\,A\,b-2\,B\,a+6\,B\,\left(b\,x^3+a\right)}{9\,b^2\,{\left(b\,x^3+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] -(2*A*b - 2*B*a + 6*B*(a + b*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

3.247 $\int \frac{A+Bx^3}{x(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

```
[Out] 2/9*(A*b-B*a)/a/b/(b*x^3+a)^(3/2)-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^
(5/2)+2/3*A/a^2/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 53, 65, 214}

$$-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*(A*b - a*B))/(9*a*b*(a + b*x^3)^(3/2)) + (2*A)/(3*a^2*Sqrt[a + b*x^3]) -
 (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx,x,x^3\right)\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{A\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^3\right)}{3a}\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a^2}\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{(2A)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3a^2b}\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 70, normalized size = 0.91

$$-\frac{2(-4aAb+a^2B-3Ab^2x^3)}{9a^2b\left(a+bx^3\right)^{3/2}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*(-4*a*A*b + a^2*B - 3*A*b^2*x^3))/(9*a^2*b*(a + b*x^3)^(3/2)) - (2*A*Ar
cTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Maple [A]
time = 0.34, size = 85, normalized size = 1.10

method	result	size
elliptic	$\frac{2(Ab-Ba)\sqrt{b\,x^3+a}}{9b^3a\left(x^3+\frac{a}{b}\right)^2}+\frac{2A}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2A\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}$	77
default	$-\frac{2B}{9b(b\,x^3+a)^{\frac{3}{2}}}+A\left(\frac{2\sqrt{b\,x^3+a}}{9a\,b^2\left(x^3+\frac{a}{b}\right)^2}+\frac{2}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}\right)$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*B/b/(b*x^3+a)^(3/2)+A*(2/9/a/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/3/a^2/(
(x^3+a/b)*b)^(1/2)-2/3/a^(5/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 81, normalized size = 1.05

$$\frac{1}{9}A\left(\frac{3\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\,bx^3+4\,a\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2}\right)-\frac{2\,B}{9\left(bx^3+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*A*(3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/
2) + 2*(3*b*x^3 + 4*a)/((b*x^3 + a)^(3/2)*a^2)) - 2/9*B/((b*x^3 + a)^(3/2)*
b)
```

Fricas [A]
time = 3.18, size = 243, normalized size = 3.16

$$\left[\frac{3\,(Ab^3x^6+2\,Aab^2x^3+Aa^2b)\sqrt{a}\,\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\,(3\,Aab^2x^3-Ba^3+4\,Aa^2b)\sqrt{bx^3+a}}{9\,(a^3b^3x^6+2\,a^4b^2x^3+a^5b)},\ \frac{2\left(3\,(Ab^3x^6+2\,Aab^2x^3+Aa^2b)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+(3\,Aab^2x^3-Ba^3+4\,Aa^2b)\sqrt{bx^3+a}\right)}{9\,(a^3b^3x^6+2\,a^4b^2x^3+a^5b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/9*(3*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b
*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*A*a*b^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(
b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b), 2/9*(3*(A*b^3*x^6 + 2*A*
a*b^2*x^3 + A*a^2*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*A*a*b
^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 +
 a^5*b)]
```

Sympy [A]
time = 12.19, size = 76, normalized size = 0.99

$$\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{2A\,\mathrm{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a^2\sqrt{-a}}-\frac{2(-Ab+Ba)}{9ab\,(a+bx^3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(5/2),x)
```

```
[Out] 2*A/(3*a**2*sqrt(a + b*x**3)) + 2*A*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a**2
*sqrt(-a)) - 2*(-A*b + B*a)/(9*a*b*(a + b*x**3)**(3/2))
```

Giac [A]
time = 0.99, size = 67, normalized size = 0.87

$$\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a^2}-\frac{2\,(Ba^2-3\,(bx^3+a)Ab-Aab)}{9\,(bx^3+a)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 2/9*(B*a^2 - 3*(b*x
^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a^2*b)
```

Mupad [B]
time = 2.78, size = 80, normalized size = 1.04

$$\frac{\frac{2\,A}{9\,a}-\frac{2\,B}{9\,b}}{{\left(b\,x^3+a\right)}^{3/2}}+\frac{2\,A}{3\,a^2\,\sqrt{b\,x^3+a}}+\frac{A\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(5/2)),x)
```

```
[Out] ((2*A)/(9*a) - (2*B)/(9*b))/(a + b*x^3)^(3/2) + (2*A)/(3*a^2*(a + b*x^3)^(1
/2)) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)
))/x^6))/(3*a^(5/2))
```

3.248 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{-5Ab+2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}$$

```
[Out] 1/9*(-5*A*b+2*B*a)/a^2/(b*x^3+a)^(3/2)-1/3*A/a/x^3/(b*x^3+a)^(3/2)+1/3*(5*A
*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2)+1/3*(-5*A*b+2*B*a)/a^3/(
b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 53, 65, 214}

$$\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}-\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

```
[Out] -1/9*(5*A*b - 2*a*B)/(a^2*(a + b*x^3)^(3/2)) - A/(3*a*x^3*(a + b*x^3)^(3/2)
) - (5*A*b - 2*a*B)/(3*a^3*Sqrt[a + b*x^3]) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt
[a + b*x^3]/Sqrt[a]])/(3*a^(7/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx,x,x^3\right)$$

$$= -\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} + \frac{\left(-\frac{5Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx,x,x^3\right)}{3a}$$

$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} - \frac{(5Ab-2aB)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^3\right)}{6a^2}$$

$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} - \frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}} - \frac{(5Ab-2aB)\text{Subst}\left(\int \ldots\right.}{6a^3}$$

$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} - \frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}} - \frac{(5Ab-2aB)\text{Subst}\left(\int \ldots\right.}{3a\ldots}$$

$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} - \frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\tanh^{-1}\left(\ldots\right.}{3a^{7/2}}$$

Mathematica [A]
time = 0.12, size = 99, normalized size = 0.88

$$\frac{-3a^2A-20aAbx^3+8a^2Bx^3-15Ab^2x^6+6abBx^6}{9a^3x^3(a+bx^3)^{3/2}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-3*a^2*A - 20*a*A*b*x^3 + 8*a^2*B*x^3 - 15*A*b^2*x^6 + 6*a*b*B*x^6)/(9*a^3
*x^3*(a + b*x^3)^(3/2)) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]
)/(3*a^(7/2))
```

Maple [A]
time = 0.36, size = 157, normalized size = 1.39

method	result
risch	$-\frac{A\sqrt{b\,x^3+a}}{3a^3x^3}-\frac{\frac{4a(Ab-Ba)}{9(b\,x^3+a)^{\frac{3}{2}}}-\frac{2(5Ab-2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{\frac{8Ab}{3}-\frac{4Ba}{3}}{\sqrt{b\,x^3+a}}}{2a^3}$
elliptic	$-\frac{2(Ab-Ba)\sqrt{b\,x^3+a}}{9a^2b^2\left(x^3+\frac{a}{b}\right)^2}-\frac{2(2Ab-Ba)}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{3a^3x^3}+\frac{(5Ab-2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{7}{2}}}$
default	$A\left(-\frac{2\sqrt{b\,x^3+a}}{9a^2b\left(x^3+\frac{a}{b}\right)^2}-\frac{4b}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^3+a}}{3a^3x^3}+\frac{5b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{7}{2}}}\right)+B\left(\frac{2\sqrt{b\,x^3+a}}{9a\,b^2\left(x^3+\frac{a}{b}\right)^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-2/9/a^2/b*(b*x^3+a)^(1/2)/(x^3+a/b)^2-4/3*b/a^3/((x^3+a/b)*b)^(1/2)-1/3
/a^3*(b*x^3+a)^(1/2)/x^3+5/3/a^(7/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+B*
(2/9/a/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/3/a^2/((x^3+a/b)*b)^(1/2)-2/3/a^(5
/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 170, normalized size = 1.50

$$-\frac{1}{18}A\left(\frac{2\left(15\,(bx^3+a)^2b-10\,(bx^3+a)ab-2\,a^2b\right)}{(bx^3+a)^{\frac{5}{2}}a^3-(bx^3+a)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)+\frac{1}{9}B\left(\frac{3\,\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx^3+4\,a)}{(bx^3+a)^{\frac{3}{2}}a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/18*A*(2*(15*(b*x^3 + a)^2*b - 10*(b*x^3 + a)*a*b - 2*a^2*b)/((b*x^3 + a)
^(5/2)*a^3 - (b*x^3 + a)^(3/2)*a^4) + 15*b*log((sqrt(b*x^3 + a) - sqrt(a))/
(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) + 1/9*B*(3*log((sqrt(b*x^3 + a) - sqr
t(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*b*x^3 + 4*a)/((b*x^3 + a)
^(3/2)*a^2))
```

Fricas [A]
time = 2.91, size = 351, normalized size = 3.11

$$\left[-\frac{3\left((2\,Bab^2-5\,Ab^3)x^9+2\,(2\,Ba^2b-5\,Aab^2)x^6+(2\,Ba^3-5\,Aa^2b)x^3\right)\sqrt{a}\,\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)-2\left(3\,(2\,Ba^2b-5\,Aab^2)x^6-3\,Aa^3+4\,(2\,Ba^3-5\,Aa^2b)x^3\right)\sqrt{bx^3+a}}{18\,(a^4b^2x^9+2\,a^5bx^6+a^6x^3)},\ \frac{3\left((2\,Bab^2-5\,Ab^3)x^9+2\,(2\,Ba^2b-5\,Aab^2)x^6+(2\,Ba^3-5\,Aa^2b)x^3\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\left(3\,(2\,Ba^2b-5\,Aab^2)x^6-3\,Aa^3+4\,(2\,Ba^3-5\,Aa^2b)x^3\right)\sqrt{bx^3+a}}{9\,(a^4b^2x^9+2\,a^5bx^6+a^6x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/18*(3*((2*B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B
*a^3 - 5*A*a^2*b)*x^3)*sqrt(a)*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a
)/x^3) - 2*(3*(2*B*a^2*b - 5*A*a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*
b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3), 1/9*(3*((2*
B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B*a^3 - 5*A*a^2
*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(2*B*a^2*b - 5*A*
a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b
^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(107) = 214$.
time = 87.16, size = 1608, normalized size = 14.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 5
4*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 46*a**16*b*x**3*sqrt(1 +
 b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**
9 + 18*a**(33/2)*b**3*x**12) - 15*a**16*b*x**3*log(b*x**3/a)/(18*a**(39/2)*
x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**
12) + 30*a**16*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(18*a**(39/2)*x**3 + 54*a
**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 70*a*
*15*b**2*x**6*sqrt(1 + b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 +
 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 45*a**15*b**2*x**6*log
(b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**
9 + 18*a**(33/2)*b**3*x**12) + 90*a**15*b**2*x**6*log(sqrt(1 + b*x**3/a) +
1)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a
```

```
**(33/2)*b**3*x**12) - 30*a**14*b**3*x**9*sqrt(1 + b*x**3/a)/(18*a**(39/2)*
x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**
12) - 45*a**14*b**3*x**9*log(b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*
x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) + 90*a**14*b**3*x*
*9*log(sqrt(1 + b*x**3/a) + 1)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 5
4*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 15*a**13*b**4*x**12*log(
b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9
 + 18*a**(33/2)*b**3*x**12) + 30*a**13*b**4*x**12*log(sqrt(1 + b*x**3/a) +
1)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a
**(33/2)*b**3*x**12)) + B*(8*a**7*sqrt(1 + b*x**3/a)/(9*a**(19/2) + 27*a**(
17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9) + 3*a**7*log
(b*x**3/a)/(9*a**(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*
a**(13/2)*b**3*x**9) - 6*a**7*log(sqrt(1 + b*x**3/a) + 1)/(9*a**(19/2) + 27
*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9) + 14*a*
*6*b*x**3*sqrt(1 + b*x**3/a)/(9*a**(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15
/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9) + 9*a**6*b*x**3*log(b*x**3/a)/(9*a**
(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x*
*9) - 18*a**6*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(9*a**(19/2) + 27*a**(17/2
)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9) + 6*a**5*b**2*x*
*6*sqrt(1 + b*x**3/a)/(9*a**(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**
2*x**6 + 9*a**(13/2)*b**3*x**9) + 9*a**5*b**2*x**6*log(b*x**3/a)/(9*a**(19/
2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9)
- 18*a**5*b**2*x**6*log(sqrt(1 + b*x**3/a) + 1)/(9*a**(19/2) + 27*a**(17/2)
*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9) + 3*a**4*b**3*x**
9*log(b*x**3/a)/(9*a**(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6
 + 9*a**(13/2)*b**3*x**9) - 6*a**4*b**3*x**9*log(sqrt(1 + b*x**3/a) + 1)/(9
*a**(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**
3*x**9))
```

Giac [A]
time = 1.46, size = 101, normalized size = 0.89

$$\frac{(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a^3}+\frac{2\,(3\,(bx^3+a)Ba+Ba^2-6\,(bx^3+a)Ab-Aab)}{9\,(bx^3+a)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx^3+a}\,A}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*B*a - 5*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2/9*(
3*(b*x^3 + a)*B*a + B*a^2 - 6*(b*x^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a
^3) - 1/3*sqrt(b*x^3 + a)*A/(a^3*x^3)
```

Mupad [B]

time = 2.97, size = 198, normalized size = 1.75

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(5\,A\,b-2\,B\,a)}{6\,a^{7/2}}-\frac{\frac{2\,B\,a^2-5\,A\,a\,b}{2\,a^4}-\frac{a\left(\frac{A\,b^2}{3\,a^4}+\frac{5\,b\,(2\,B\,a^2-5\,A\,a\,b)}{6\,a^5}\right)}{b}}{\sqrt{b\,x^3+a}}-\frac{\frac{2\,B\,a^3-5\,A\,a^2\,b}{4\,a^4}-\frac{a\left(\frac{13\,b\,(2\,B\,a^3-5\,A\,a^2\,b)}{36\,a^5}+\frac{A\,b^2}{3\,a^3}\right)}{b}}{(b\,x^3+a)^{3/2}}-\frac{A\,\sqrt{b\,x^3+a}}{3\,a^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
5*A*b - 2*B*a))/(6*a^(7/2)) - ((2*B*a^2 - 5*A*a*b)/(2*a^4) - (a*((A*b^2)/(3
*a^4) + (5*b*(2*B*a^2 - 5*A*a*b))/(6*a^5)))/b)/(a + b*x^3)^(1/2) - ((2*B*a^
3 - 5*A*a^2*b)/(4*a^4) - (a*((13*b*(2*B*a^3 - 5*A*a^2*b))/(36*a^5) + (A*b^2
)/(3*a^3)))/b)/(a + b*x^3)^(3/2) - (A*(a + b*x^3)^(1/2))/(3*a^3*x^3)
```

3.249 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=299

$$-\frac{2(5Ab-14aB)x^4}{45b^2(a+bx^3)^{3/2}}+\frac{2Bx^7}{5b(a+bx^3)^{3/2}}-\frac{16(5Ab-14aB)x}{135b^3\sqrt{a+bx^3}}+\frac{32\sqrt{2+\sqrt{3}}\,(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a\cdots}{((\cdots}}}{135\sqrt[4]{3}\,b^{10/3}\sqrt{\cdots}}$$

```
[Out] -2/45*(5*A*b-14*B*a)*x^4/b^2/(b*x^3+a)^(3/2)+2/5*B*x^7/b/(b*x^3+a)^(3/2)-16
/135*(5*A*b-14*B*a)*x/b^3/(b*x^3+a)^(1/2)+32/405*(5*A*b-14*B*a)*(a^(1/3)+b^
(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(10/3)/(b
*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {470, 294, 224}

$$\frac{32\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(5Ab-14aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{135\sqrt[4]{3}\,b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{16x(5Ab-14aB)}{135b^3\sqrt{a+bx^3}}-\frac{2x^4(5Ab-14aB)}{45b^2(a+bx^3)^{3/2}}+\frac{2Bx^7}{5b(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(5*A*b - 14*a*B)*x^4)/(45*b^2*(a + b*x^3)^(3/2)) + (2*B*x^7)/(5*b*(a +
b*x^3)^(3/2)) - (16*(5*A*b - 14*a*B)*x)/(135*b^3*Sqrt[a + b*x^3]) + (32*Sqr
t[2 + Sqrt[3]]*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(135*3^(1/4)*b^(10/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2Bx^7}{5b\,(a+bx^3)^{3/2}} - \frac{\left(2\left(-\frac{5Ab}{2}+7aB\right)\right)\int \frac{x^6}{(a+bx^3)^{5/2}}\,dx}{5b}$$

$$= -\frac{2(5Ab-14aB)x^4}{45b^2\,(a+bx^3)^{3/2}} + \frac{2Bx^7}{5b\,(a+bx^3)^{3/2}} + \frac{(8(5Ab-14aB))\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx}{45b^2}$$

$$= -\frac{2(5Ab-14aB)x^4}{45b^2\,(a+bx^3)^{3/2}} + \frac{2Bx^7}{5b\,(a+bx^3)^{3/2}} - \frac{16(5Ab-14aB)x}{135b^3\sqrt{a+bx^3}} + \frac{(16(5Ab-14aB))\int -}{135b^3}$$

$$= -\frac{2(5Ab-14aB)x^4}{45b^2\,(a+bx^3)^{3/2}} + \frac{2Bx^7}{5b\,(a+bx^3)^{3/2}} - \frac{16(5Ab-14aB)x}{135b^3\sqrt{a+bx^3}} + \frac{32\sqrt{2+\sqrt{3}}\,(5Ab-1}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.10, size = 108, normalized size = 0.36

$$\frac{2x\left(112a^2B+b^2x^3(-55A+27Bx^3)+a(-40Ab+154bBx^3)+8(5Ab-14aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{135b^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(112*a^2*B + b^2*x^3*(-55*A + 27*B*x^3) + a*(-40*A*b + 154*b*B*x^3) +
8*(5*A*b - 14*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1
/2, 4/3, -((b*x^3)/a)]))/(135*b^3*(a + b*x^3)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 682 vs. 2(232) = 464.
time = 0.37, size = 683, normalized size = 2.28

method	result
elliptic	$\frac{2ax(Ab-Ba)\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}-\frac{2x(11Ab-20Ba)}{27b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2Bx\sqrt{b\,x^3+a}}{5b^3}-\frac{2i\left(\frac{Ab-2Ba}{b^3}-\frac{11Ab-20Ba}{27b^3}-\frac{2Ba}{5b^3}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{}$
default	$B\left(-\frac{2a^2x\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}+\frac{40ax}{27b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x\sqrt{b\,x^3+a}}{5b^3}+\frac{448ia\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(}{2}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] B*(-2/9*a^2*x/b^5*(b*x^3+a)^(1/2)/(x^3+a/b)^2+40/27/b^3*a*x/((x^3+a/b)*b)^(1/2)+2/5/b^3*x*(b*x^3+a)^(1/2)+448/405*I*a/b^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/9*a*x/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^2-22/27/b^2*x/((x^3+a/b)*b)^(1/2)-32/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 153, normalized size = 0.51

$$-\frac{2\left(16\left(\left(14\,Bab^{2}-5\,Ab^{3}\right)x^{6}+14\,Ba^{3}-5\,Aa^{2}b+2\left(14\,Ba^{2}b-5\,Aab^{2}\right)x^{3}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\left(27\,Bb^{3}x^{7}+11\left(14\,Bab^{2}-5\,Ab^{3}\right)x^{4}+8\left(14\,Ba^{2}b-5\,Aab^{2}\right)x\right)\sqrt{bx^{3}+a}\right)}{135\left(b^{6}x^{6}+2\,ab^{5}x^{3}+a^{2}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")

[Out] -2/135*(16*((14*B*a*b^2 - 5*A*b^3)*x^6 + 14*B*a^3 - 5*A*a^2*b + 2*(14*B*a^2*b - 5*A*a*b^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - (27*B*b^3*x^7 + 11*(14*B*a*b^2 - 5*A*b^3)*x^4 + 8*(14*B*a^2*b - 5*A*a*b^2)*x)*sqrt(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)

Sympy [A]

time = 43.16, size = 80, normalized size = 0.27

$$\frac{Ax^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{7}{3},\frac{5}{2}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}+\frac{Bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**7*gamma(7/3)*hyper((7/3, 5/2), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(5/2)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((5/2, 10/3), (13/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(13/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] int((x^6*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.250 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=283

$$\frac{2(Ab-aB)x^4}{9ab\left(a+bx^3\right)^{3/2}}-\frac{2(Ab+8aB)x}{27ab^2\sqrt{a+bx^3}}+\frac{4\sqrt{2+\sqrt{3}}\,(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}{27\sqrt[4]{3}\,ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}$$

```
[Out] 2/9*(A*b-B*a)*x^4/a/b/(b*x^3+a)^(3/2)-2/27*(A*b+8*B*a)*x/a/b^2/(b*x^3+a)^(1
/2)+4/81*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/a/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {468, 294, 224}

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(8aB+Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2x(8aB+Ab)}{27ab^2\sqrt{a+bx^3}}+\frac{2x^4(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x^4)/(9*a*b*(a + b*x^3)^(3/2)) - (2*(A*b + 8*a*B)*x)/(27*a*b
^2*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(A*b + 8*a*B)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a*b^(7/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx &= \frac{2(Ab-aB)x^4}{9ab\,(a+bx^3)^{3/2}} + \frac{\left(2\left(\frac{Ab}{2}+4aB\right)\right)\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx}{9ab} \\
&= \frac{2(Ab-aB)x^4}{9ab\,(a+bx^3)^{3/2}} - \frac{2(Ab+8aB)x}{27ab^2\sqrt{a+bx^3}} + \frac{(2(Ab+8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{27ab^2} \\
&= \frac{2(Ab-aB)x^4}{9ab\,(a+bx^3)^{3/2}} - \frac{2(Ab+8aB)x}{27ab^2\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}\,(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/}}{((1}}}{27\sqrt[4]{3}\,ab^{7/3}\sqrt{}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.08, size = 99, normalized size = 0.35

$$\frac{2x\left(-8a^2B+2Ab^2x^3-ab(A+11Bx^3)+(Ab+8aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{27ab^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(-8*a^2*B + 2*A*b^2*x^3 - a*b*(A + 11*B*x^3) + (A*b + 8*a*B)*(a + b*x^
3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(27
*a*b^2*(a + b*x^3)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(220) = 440.
time = 0.31, size = 669, normalized size = 2.36

method	result
elliptic	$-\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}+\frac{2x(2Ab-11Ba)}{27b^2a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\left(\frac{B}{b^2}+\frac{2Ab-11Ba}{27b^2a}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\right.}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{}$
default	$B\left(\frac{2ax\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}-\frac{22x}{27b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{32i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3}{}}}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/9*a*x/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^2-22/27/b^2*x/((x^3+a/b)*b)^(1/2)-
32/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+
A*(-2/9*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+4/27/b/a*x/((x^3+a/b)*b)^(1/2)-4/
81*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 141, normalized size = 0.50

$$\frac{2\left(2\left((8\,Bab^2+Ab^3)x^6+8\,Ba^3+Aa^2b+2\,(8\,Ba^2b+Aab^2)x^3\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\left((11\,Bab^2-2\,Ab^3)x^4+(8\,Ba^2b+Aab^2)x\right)\sqrt{bx^3+a}\right)}{27\,(ab^5x^6+2\,a^2b^4x^3+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*(2*((8*B*a*b^2 + A*b^3)*x^6 + 8*B*a^3 + A*a^2*b + 2*(8*B*a^2*b + A*a*b
^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - ((11*B*a*b^2 - 2*A*b^3
)*x^4 + (8*B*a^2*b + A*a*b^2)*x)*sqrt(b*x^3 + a))/(a*b^5*x^6 + 2*a^2*b^4*x^
3 + a^3*b^3)
```

Sympy [A]
time = 29.41, size = 80, normalized size = 0.28

$$\frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{7}{3},\frac{5}{2}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(5/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((7/3, 5/2), (10/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*gamma(10/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] int((x^3*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.251 $\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=283

$$\frac{2(Ab-aB)x}{9ab\,(a+bx^3)^{3/2}}+\frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}}+\frac{2\sqrt{2+\sqrt{3}}\,(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}{27\sqrt[4]{3}\,a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}$$

```
[Out] 2/9*(A*b-B*a)*x/a/b/(b*x^3+a)^(3/2)+2/27*(7*A*b+2*B*a)*x/a^2/b/(b*x^3+a)^(1
/2)+2/81*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(3/4)/a^2/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {393, 205, 224}

$$\frac{2x(2aB+7Ab)}{27a^2b\sqrt{a+bx^3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2aB+7Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(Ab-aB)}{9ab\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(7*A*b + 2*a*B)*x)/(27*a^2
*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*A*b + 2*a*B)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(4/3)
*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)x}{9ab\,(a+bx^3)^{3/2}} + \frac{\left(2\left(\frac{7Ab}{2}+aB\right)\right)\int \frac{1}{(a+bx^3)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x}{9ab\,(a+bx^3)^{3/2}} + \frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}} + \frac{(7Ab+2aB)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)x}{9ab\,(a+bx^3)^{3/2}} + \frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\,(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a\ldots}{((\ldots}}}{27\sqrt[4]{3}\,a^2b^{4/3}\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 103, normalized size = 0.36

$$\frac{-2a^2Bx+14Ab^2x^4+4abx(5A+Bx^3)+(7Ab+2aB)x(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{27a^2b\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*a^2*B*x + 14*A*b^2*x^4 + 4*a*b*x*(5*A + B*x^3) + (7*A*b + 2*a*B)*x*(a +
 b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])
/(27*a^2*b*(a + b*x^3)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(220) = 440$.
time = 0.32, size = 674, normalized size = 2.38

method	result
elliptic	$\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9a\,b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{2x(7Ab+2Ba)}{27b\,a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i(7Ab+2Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{}}{(-a\,b^2)^{\frac{1}{3}}}}}{}$
default	$B\left(-\frac{2x\sqrt{b\,x^3+a}}{9b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{4x}{27ba\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{4i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{-\frac{3(}{}}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-2/9*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+4/27/b/a*x/((x^3+a/b)*b)^(1/2)-4/
81*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
```

```
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A
*(2/9/a*x/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+14/27/a^2*x/((x^3+a/b)*b)^(1/2)-1
4/81*I/a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 145, normalized size = 0.51

$$\frac{2\left(\left((2\,Bab^2+7\,Ab^3)x^6+2\,Ba^3+7\,Aa^2b+2\,(2\,Ba^2b+7\,Aab^2)x^3\right)\sqrt{b}\ \mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\left((2\,Bab^2+7\,Ab^3)x^4-(Ba^2b-10\,Aab^2)x\right)\sqrt{bx^3+a}\right)}{27\,(a^2b^4x^6+2\,a^3b^3x^3+a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*(((2*B*a*b^2 + 7*A*b^3)*x^6 + 2*B*a^3 + 7*A*a^2*b + 2*(2*B*a^2*b + 7*A
*a*b^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + ((2*B*a*b^2 + 7*A*
b^3)*x^4 - (B*a^2*b - 10*A*a*b^2)*x)*sqrt(b*x^3 + a))/(a^2*b^4*x^6 + 2*a^3*
b^3*x^3 + a^4*b^2)
```

Sympy [A]
time = 22.08, size = 78, normalized size = 0.28

$$\frac{Ax\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(5/2)*gamma(7/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)/(a + b*x^3)^(5/2), x)
```

3.252 $\int \frac{A+Bx^3}{x^3(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}-\frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}}-\frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}}-\frac{7\sqrt{2+\sqrt{3}}\,(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\cdots}}{54\sqrt[4]{3}\,a^3\sqrt[3]{b}\,\cdots}$$

[Out] -1/2*A/a/x^2/(b*x^3+a)^(3/2)-1/18*(13*A*b-4*B*a)*x/a^2/(b*x^3+a)^(3/2)-7/54*(13*A*b-4*B*a)*x/a^3/(b*x^3+a)^(1/2)-7/162*(13*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 205, 224}

$$-\frac{7x(13Ab-4aB)}{54a^3\sqrt{a+bx^3}}-\frac{x(13Ab-4aB)}{18a^2\left(a+bx^3\right)^{3/2}}-\frac{7\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(13Ab-4aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{54\sqrt[4]{3}\,a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]

[Out] -1/2*A/(a*x^2*(a + b*x^3)^(3/2)) - ((13*A*b - 4*a*B)*x)/(18*a^2*(a + b*x^3)^(3/2)) - (7*(13*A*b - 4*a*B)*x)/(54*a^3*Sqrt[a + b*x^3]) - (7*Sqrt[2 + Sqrt[3]]*(13*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(54*3^(1/4)*a^3*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}-\frac{\left(\frac{13Ab}{2}-2aB\right)\int \frac{1}{\left(a+bx^3\right)^{5/2}}\,dx}{2a}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}-\frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}}-\frac{(7(13Ab-4aB))\int \frac{1}{\left(a+bx^3\right)^{3/2}}\,dx}{36a^2}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}-\frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}}-\frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}}-\frac{(7(13Ab-4aB))\int \frac{}{\sqrt{}}}{108a^3}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}-\frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}}-\frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}}-\frac{7\sqrt{2+\sqrt{3}}\,(13Ab-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 116, normalized size = 0.39

$$\frac{-182Ab^2x^6+a^2(-54A+80Bx^3)+a(-260Abx^3+56bBx^6)+7(-13Ab+4aB)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{108a^3x^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-182*A*b^2*x^6 + a^2*(-54*A + 80*B*x^3) + a*(-260*A*b*x^3 + 56*b*B*x^6) +
7*(-13*A*b + 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1
/3, 1/2, 4/3, -((b*x^3)/a)])/(108*a^3*x^2*(a + b*x^3)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(233) = 466.
time = 0.36, size = 689, normalized size = 2.30

method	result
elliptic	$-\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9a^2b^2\left(x^3+\frac{a}{b}\right)^2}-\frac{2x(16Ab-7Ba)}{27a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{2a^3x^2}-\frac{2i\left(-\frac{16Ab-7Ba}{27a^3}-\frac{Ab}{4a^3}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\cdots\right.}{\cdots}}\cdots}{\cdots}$
default	$B\left(\frac{2x\sqrt{b\,x^3+a}}{9a\,b^2\left(x^3+\frac{a}{b}\right)^2}+\frac{14x}{27a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{14i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-\frac{3(\cdots}{}}}\cdots}{\cdots}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/9/a*x/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+14/27/a^2*x/((x^3+a/b)*b)^(1/2)-
14/81*I/a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
)+A*(-2/9/a^2/b*x*(b*x^3+a)^(1/2)/(x^3+a/b)^2-32/27*b*x/a^3/((x^3+a/b)*b)^(
1/2)-1/2/a^3*(b*x^3+a)^(1/2)/x^2+91/162*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 163, normalized size = 0.54

$$\frac{7\,((4\,Bab^2-13\,Ab^3)x^8+2\,(4\,Ba^2b-13\,Aab^2)x^5+(4\,Ba^3-13\,Aa^2b)x^2)\sqrt{b}\ \mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(7\,(4\,Bab^2-13\,Ab^3)x^6-27\,Aa^2b+10\,(4\,Ba^2b-13\,Aab^2)x^3)\sqrt{bx^3+a}}{54\,(a^3b^3x^8+2\,a^4b^2x^5+a^5bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/54*(7*((4*B*a*b^2 - 13*A*b^3)*x^8 + 2*(4*B*a^2*b - 13*A*a*b^2)*x^5 + (4*B
*a^3 - 13*A*a^2*b)*x^2)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (7*(4*B
*a*b^2 - 13*A*b^3)*x^6 - 27*A*a^2*b + 10*(4*B*a^2*b - 13*A*a*b^2)*x^3)*sqrt
(b*x^3 + a))/(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)
```

Sympy [A]
time = 55.63, size = 82, normalized size = 0.27

$$\frac{A\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(5/2),x)
```

```
[Out] A*gamma(-2/3)*hyper((-2/3, 5/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/2)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(5/2)*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^3\,{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/(x^3*(a + b*x^3)^(5/2)), x)
```

3.253 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=334

$$-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}-\frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}}-\frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}}+\frac{91(19Ab-10aB)\sqrt{a+bx^3}}{540a^4x^2}+\frac{91\sqrt{2+\sqrt{3}}\,b \cdots}{\cdots}$$

```
[Out] -1/5*A/a/x^5/(b*x^3+a)^(3/2)+1/45*(-19*A*b+10*B*a)/a^2/x^2/(b*x^3+a)^(3/2)-
13/135*(19*A*b-10*B*a)/a^3/x^2/(b*x^3+a)^(1/2)+91/540*(19*A*b-10*B*a)*(b*x^
3+a)^(1/2)/a^4/x^2+91/1620*b^(2/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*Elli
pticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^4/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 296, 331, 224}

$$\frac{91\sqrt{a+bx^3}\,(19Ab-10aB)}{540a^4x^2}-\frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}}-\frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}}+\frac{91\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(19Ab-10aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{540\sqrt[4]{3}\,a^4\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

```
[Out] -1/5*A/(a*x^5*(a + b*x^3)^(3/2)) - (19*A*b - 10*a*B)/(45*a^2*x^2*(a + b*x^3
)^(3/2)) - (13*(19*A*b - 10*a*B))/(135*a^3*x^2*Sqrt[a + b*x^3]) + (91*(19*A
*b - 10*a*B)*Sqrt[a + b*x^3])/(540*a^4*x^2) + (91*Sqrt[2 + Sqrt[3]]*b^(2/3)
*(19*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(540*3^(1/4)*a^4*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{5/2}}\,dx = -\frac{A}{5ax^5 (a+bx^3)^{3/2}} - \frac{\left(\frac{19Ab}{2}-5aB\right)\int \frac{1}{x^3(a+bx^3)^{5/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5 (a+bx^3)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2 (a+bx^3)^{3/2}} - \frac{(13(19Ab-10aB))\int \frac{1}{x^3(a+bx^3)^{3/2}}\,dx}{90a^2}$$

$$= -\frac{A}{5ax^5 (a+bx^3)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2 (a+bx^3)^{3/2}} - \frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}} - \frac{(91(19Ab-10aB}{27}$$

$$= -\frac{A}{5ax^5 (a+bx^3)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2 (a+bx^3)^{3/2}} - \frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}} + \frac{91(19Ab-10aB)}{540a^4x^2}$$

$$= -\frac{A}{5ax^5 (a+bx^3)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2 (a+bx^3)^{3/2}} - \frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}} + \frac{91(19Ab-10aB)}{540a^4x^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 83, normalized size = 0.25

$$\frac{-2a^2A+\left(\frac{19Ab}{2}-5aB\right)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(-\frac{2}{3},\frac{5}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{10a^3x^5 (a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*a^2*A + ((19*A*b)/2 - 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[-2/3, 5/2, 1/3, -((b*x^3)/a)])/(10*a^3*x^5*(a + b*x^3)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 721 vs. 2(263) = 526.
time = 0.39, size = 722, normalized size = 2.16

method	result

elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{5a^{3}x^{5}}+\frac{(27Ab-10Ba)\sqrt{b\,x^{3}+a}}{20a^{4}x^{2}}+\frac{2x(Ab-Ba)\sqrt{b\,x^{3}+a}}{9a^{3}b\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2bx(25Ab-16Ba)}{27a^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2i\left(\frac{b(27Ab-10B}{40a^{4}}\right.}{}$
default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{5a^{3}x^{5}}+\frac{27b\sqrt{b\,x^{3}+a}}{20a^{4}x^{2}}+\frac{2x\sqrt{b\,x^{3}+a}}{9a^{3}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{50b^{2}x}{27a^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{1729ib\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\right.}{}}}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/5/a^3*(b*x^3+a)^(1/2)/x^5+27/20/a^4*b*(b*x^3+a)^(1/2)/x^2+2/9*x/a^3*(
b*x^3+a)^(1/2)/(x^3+a/b)^2+50/27*b^2*x/a^4/((x^3+a/b)*b)^(1/2)-1729/1620*I/
a^4*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+B*(-2/
9/a^2/b*x*(b*x^3+a)^(1/2)/(x^3+a/b)^2-32/27*b*x/a^3/((x^3+a/b)*b)^(1/2)-1/2
/a^3*(b*x^3+a)^(1/2)/x^2+91/162*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
```

```
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 178, normalized size = 0.53

$$-\frac{91\left(\left(10\,Bab^2-19\,Ab^3\right)x^{11}+2\left(10\,Ba^2b-19\,Aab^2\right)x^8+\left(10\,Ba^3-19\,Aa^2b\right)x^5\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\left(91\left(10\,Bab^2-19\,Ab^3\right)x^9+130\left(10\,Ba^2b-19\,Aab^2\right)x^6+108\,Aa^3+27\left(10\,Ba^3-19\,Aa^2b\right)x^3\right)\sqrt{bx^3+a}}{540\left(a^4b^2x^{11}+2\,a^5bx^8+a^6x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/540*(91*((10*B*a*b^2 - 19*A*b^3)*x^11 + 2*(10*B*a^2*b - 19*A*a*b^2)*x^8
+ (10*B*a^3 - 19*A*a^2*b)*x^5)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) +
(91*(10*B*a*b^2 - 19*A*b^3)*x^9 + 130*(10*B*a^2*b - 19*A*a*b^2)*x^6 + 108*A
*a^3 + 27*(10*B*a^3 - 19*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^11 + 2*a
^5*b*x^8 + a^6*x^5)
```

Sympy [A]
time = 127.63, size = 90, normalized size = 0.27

$$\frac{A\Gamma\left(-\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{3},\frac{5}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(5/2),x)
```

```
[Out] A*gamma(-5/3)*hyper((-5/3, 5/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/2)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 5/2), (1/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*x**2*gamma(1/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^6\,{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/(x^6*(a + b*x^3)^(5/2)), x)
```

3.254 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=577

$$-\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}}+\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}-\frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}}+\frac{80(7Ab-16aB)\sqrt{a+bx^3}}{189b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{40\sqrt{2-\sqrt{\vphantom{3}}}}{}$$

```
[Out] -2/63*(7*A*b-16*B*a)*x^5/b^2/(b*x^3+a)^(3/2)+2/7*B*x^8/b/(b*x^3+a)^(3/2)-20
/189*(7*A*b-16*B*a)*x^2/b^3/(b*x^3+a)^(1/2)+80/189*(7*A*b-16*B*a)*(b*x^3+a)
^(1/2)/b^(11/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+80/567*a^(1/3)*(7*A*b-16*B*
a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(11/3)/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)-40/189*a^(1/3)*(7*A*b-16*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6
^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(11/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 294, 309, 224, 1891}

$$\frac{80\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-16aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{189\sqrt[4]{3}\,b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{40\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-16aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{63\ 3^{3/4}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{80\sqrt{a+bx^3}\,(7Ab-16aB)}{189b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{20x^2(7Ab-16aB)}{189b^3\sqrt{a+bx^3}}-\frac{2x^5(7Ab-16aB)}{63b^2\left(a+bx^3\right)^{3/2}}+\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(7*A*b - 16*a*B)*x^5)/(63*b^2*(a + b*x^3)^(3/2)) + (2*B*x^8)/(7*b*(a +
b*x^3)^(3/2)) - (20*(7*A*b - 16*a*B)*x^2)/(189*b^3*Sqrt[a + b*x^3]) + (80*(
7*A*b - 16*a*B)*Sqrt[a + b*x^3])/(189*b^(11/3)*((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)) - (40*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(63*3^(3/4)*b^(11/3)*Sqr
```

```
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3]) + (80*Sqrt[2]*a^(1/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(189*3^(1/4)*b^(11/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
```

```
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2Bx^8}{7b\,(a+bx^3)^{3/2}} - \frac{\left(2\left(-\frac{7Ab}{2}+8aB\right)\right)\int \frac{x^7}{(a+bx^3)^{5/2}}\,dx}{7b}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\,(a+bx^3)^{3/2}} + \frac{2Bx^8}{7b\,(a+bx^3)^{3/2}} + \frac{(10(7Ab-16aB))\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx}{63b^2}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\,(a+bx^3)^{3/2}} + \frac{2Bx^8}{7b\,(a+bx^3)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{(40(7Ab-16aB))\int -\sqrt{}}{189b^3}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\,(a+bx^3)^{3/2}} + \frac{2Bx^8}{7b\,(a+bx^3)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{(40(7Ab-16aB))\int \frac{(1}{}}{189b^1}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\,(a+bx^3)^{3/2}} + \frac{2Bx^8}{7b\,(a+bx^3)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{80(7Ab-16aB)\sqrt{}}{189b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 109, normalized size = 0.19

$$\frac{2x^2\left(-32a^2B+2ab(7A-8Bx^3)+b^2x^3(7A+Bx^3)+2(-7Ab+16aB)\,(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{7b^3\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x^2*(-32*a^2*B + 2*a*b*(7*A - 8*B*x^3) + b^2*x^3*(7*A + B*x^3) + 2*(-7*A
*b + 16*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/
3, -((b*x^3)/a)]))/(7*b^3*(a + b*x^3)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 996 vs. 2(435) = 870.
time = 0.38, size = 997, normalized size = 1.73

method	result
	$2i\left(\frac{Ab-2Ba}{b^3}+\frac{13Ab-22Ba}{27b^3}-\frac{4Ba}{7b^3}\right)\sqrt{3}\,(-ab$
elliptic	$\frac{2a\,x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}-\frac{2x^2(13Ab-22Ba)}{27b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b^3}-$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-2/9*a^2*x^2/b^5*(b*x^3+a)^(1/2)/(x^3+a/b)^2+44/27/b^3*a*x^2/((x^3+a/b)*
b)^(1/2)+2/7/b^3*x^2*(b*x^3+a)^(1/2)+1280/567*I*a/b^4*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))))+A*(2/9*a*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^2-26/27/b^
2*x^2/((x^3+a/b)*b)^(1/2)-80/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
```

```
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 162, normalized size = 0.28

$$\frac{2\left(40\left((16\,Bab^2-7\,Ab^3)x^6+16\,Ba^3-7\,Aa^2b+2\,(16\,Ba^2b-7\,Aab^2)x^3\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(27\,Bb^3x^8+13\,(16\,Bab^2-7\,Ab^3)x^5+10\,(16\,Ba^2b-7\,Aab^2)x^2\right)\sqrt{bx^3+a}\right)}{189\,(b^6x^6+2\,ab^5x^3+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/189*(40*((16*B*a*b^2 - 7*A*b^3)*x^6 + 16*B*a^3 - 7*A*a^2*b + 2*(16*B*a^2*
b - 7*A*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(
0, -4*a/b, x)) + (27*B*b^3*x^8 + 13*(16*B*a*b^2 - 7*A*b^3)*x^5 + 10*(16*B*a
^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A]
time = 53.89, size = 80, normalized size = 0.14

$$\frac{Ax^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{11}{3}\right)}+\frac{Bx^{11}\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{11}{3}\\ \frac{14}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{14}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**8*gamma(8/3)*hyper((5/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(5/2)*gamma(11/3)) + B*x**11*gamma(11/3)*hyper((5/2, 11/3), (14/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(14/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x)
[Out] int((x^7*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.255 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=559

$$\frac{2(Ab-aB)x^5}{9ab\,(a+bx^3)^{3/2}}+\frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}}-\frac{8(Ab-10aB)\sqrt{a+bx^3}}{27ab^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{4\sqrt{2-\sqrt{3}}\,(Ab-10aB)\left(\sqrt[3]{a}+\right.}{9}$$

[Out] 2/9*(A*b-B*a)*x^5/a/b/(b*x^3+a)^(3/2)+2/27*(A*b-10*B*a)*x^2/a/b^2/(b*x^3+a)^(1/2)-8/27*(A*b-10*B*a)*(b*x^3+a)^(1/2)/a/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-8/81*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/27*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.18, antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {468, 294, 309, 224, 1891}

$$-\frac{8\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(Ab-10aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(Ab-10aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{8\sqrt{a+bx^3}\,(Ab-10aB)}{27ab^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x^2(Ab-10aB)}{27ab^2\sqrt{a+bx^3}}+\frac{2x^5(Ab-aB)}{9ab\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]

[Out] (2*(A*b - a*B)*x^5)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(A*b - 10*a*B)*x^2)/(27*a*b^2*Sqrt[a + b*x^3]) - (8*(A*b - 10*a*B)*Sqrt[a + b*x^3])/(27*a*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*Sqrt[2 - Sqrt[3]]*(A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(2/3)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1

```
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*(A*b - 10*a*B)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^(2/
3)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
```

```
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx &= \frac{2(Ab-aB)x^5}{9ab\,(a+bx^3)^{3/2}}+\frac{\left(2\left(-\frac{Ab}{2}+5aB\right)\right)\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx}{9ab}\\
&= \frac{2(Ab-aB)x^5}{9ab\,(a+bx^3)^{3/2}}+\frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}}-\frac{(4(Ab-10aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{27ab^2}\\
&= \frac{2(Ab-aB)x^5}{9ab\,(a+bx^3)^{3/2}}+\frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}}-\frac{(4(Ab-10aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{27ab^{7/3}}-\cdots\\
&= \frac{2(Ab-aB)x^5}{9ab\,(a+bx^3)^{3/2}}+\frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}}-\frac{8(Ab-10aB)\sqrt{a+bx^3}}{27ab^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\cdots
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 92, normalized size = 0.16

$$\frac{2x^2\left(-aAb+5aB(2a+bx^3)+(Ab-10aB)\,(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\!\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{5ab^2\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x^2*(-(a*A*b) + 5*a*B*(2*a + b*x^3) + (A*b - 10*a*B)*(a + b*x^3)*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(5*a*b^2*(a +
 b*x^3)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 980 vs. 2(421) = 842.
time = 0.31, size = 981, normalized size = 1.75

method	result
	$2i\left(\frac{B}{b^2}-\frac{4Ab-13Ba}{27a\,b^2}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-\cdots}{2b}\cdots\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$
elliptic	$-\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}+\frac{2x^2(4Ab-13Ba)}{27b^2a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\cdots$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/9*a*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^2-26/27/b^2*x^2/((x^3+a/b)*b)^(1
/2)-80/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-2/9*x^2/b^3*
(b*x^3+a)^(1/2)/(x^3+a/b)^2+8/27/b/a*x^2/((x^3+a/b)*b)^(1/2)+8/81*I/b^2/a*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*
```

```
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 154, normalized size = 0.28

$$-\frac{2\left(4\left(\left(10\,Bab^{2}-Ab^{3}\right)x^{6}+10\,Ba^{3}-Aa^{2}b+2\left(10\,Ba^{2}b-Aab^{2}\right)x^{3}\right)\sqrt{b}\ \mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(\left(13\,Bab^{2}-4\,Ab^{3}\right)x^{5}+\left(10\,Ba^{2}b-Aab^{2}\right)x^{2}\right)\sqrt{bx^{3}+a}\right)}{27\left(ab^{5}x^{6}+2\,a^{2}b^{4}x^{3}+a^{3}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(4*((10*B*a*b^2 - A*b^3)*x^6 + 10*B*a^3 - A*a^2*b + 2*(10*B*a^2*b - A
*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a
/b, x)) + ((13*B*a*b^2 - 4*A*b^3)*x^5 + (10*B*a^2*b - A*a*b^2)*x^2)*sqrt(b*
x^3 + a))/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)
```

Sympy [A]
time = 29.61, size = 80, normalized size = 0.14

$$\frac{Ax^{5}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}+\frac{Bx^{8}\Gamma\left(\frac{8}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(5/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((5/2, 8/3), (11/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*gamma(11/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] int((x^4*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.256 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=563

$$\frac{2(Ab-aB)x^2}{9ab\,(a+bx^3)^{3/2}}+\frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}}-\frac{2(5Ab+4aB)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt{2-\sqrt{3}}\,(5Ab+4aB)\left(\sqrt[3]{a}+\right.}{9}$$

```
[Out] 2/9*(A*b-B*a)*x^2/a/b/(b*x^3+a)^(3/2)+2/27*(5*A*b+4*B*a)*x^2/a^2/b/(b*x^3+a
)^(1/2)-2/27*(5*A*b+4*B*a)*(b*x^3+a)^(1/2)/a^2/b^(5/3)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))-2/81*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(
1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)*3^(3/4)/a^(5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/27*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)
*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/b^(5/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {468, 296, 309, 224, 1891}

$$-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(4aB+5Ab)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(4aB+5Ab)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}\,(4aB+5Ab)}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{2x^2(4aB+5Ab)}{27a^2b\sqrt{a+bx^3}}+\frac{2x^2(Ab-aB)}{9ab\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x^2)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(5*A*b + 4*a*B)*x^2)/(27
*a^2*b*Sqrt[a + b*x^3]) - (2*(5*A*b + 4*a*B)*Sqrt[a + b*x^3])/(27*a^2*b^(5/
3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*(5*A*b + 4*a*B
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^
(3/4)*a^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
```

```
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*(5*A*b + 4*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a
^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
```

```
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx &= \frac{2(Ab-aB)x^2}{9ab\,(a+bx^3)^{3/2}} + \frac{\left(2\left(\frac{5Ab}{2}+2aB\right)\right)\int \frac{x}{(a+bx^3)^{3/2}}\,dx}{9ab} \\
&= \frac{2(Ab-aB)x^2}{9ab\,(a+bx^3)^{3/2}} + \frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}} - \frac{(5Ab+4aB)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{27a^2b} \\
&= \frac{2(Ab-aB)x^2}{9ab\,(a+bx^3)^{3/2}} + \frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}} - \frac{(5Ab+4aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{27a^2b^{4/3}} - \frac{\left(\sqrt{2\left(\cdots\right.}\right.}{\cdots} \\
&= \frac{2(Ab-aB)x^2}{9ab\,(a+bx^3)^{3/2}} + \frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}} - \frac{2(5Ab+4aB)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sqrt{2-\sqrt{3}\cdots}}{\cdots}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 81, normalized size = 0.14

$$\frac{x^2\left(-4a^2B+(5Ab+4aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{10a^2b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (x^2*(-4*a^2*B + (5*A*b + 4*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 985 vs. 2(425) = 850.
time = 0.32, size = 986, normalized size = 1.75

method	result
	$2i(5Ab+4Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)}{(-a\,b^2)^{\frac{1}{3}}}}$
elliptic	$\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9a\,b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{2x^2(5Ab+4Ba)}{27b\,a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-2/9*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+8/27/b/a*x^2/((x^3+a/b)*b)^(1/2
)+8/81*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(2/9/a*x^2/b^2*
(b*x^3+a)^(1/2)/(x^3+a/b)^2+10/27/a^2*x^2/((x^3+a/b)*b)^(1/2)+10/81*I/a^2*3
^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
```

```
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 154, normalized size = 0.27

$$\frac{2\left(\left(\left(4\,Bab^2+5\,Ab^3\right)x^6+4\,Ba^3+5\,Aa^2b+2\left(4\,Ba^2b+5\,Aab^2\right)x^3\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\left(\left(4\,Bab^2+5\,Ab^3\right)x^5+\left(Ba^2b+8\,Aab^2\right)x^2\right)\sqrt{bx^3+a}\right)}{27\left(a^2b^4x^6+2\,a^3b^3x^3+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*(((4*B*a*b^2 + 5*A*b^3)*x^6 + 4*B*a^3 + 5*A*a^2*b + 2*(4*B*a^2*b + 5*A
*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a
/b, x)) + ((4*B*a*b^2 + 5*A*b^3)*x^5 + (B*a^2*b + 8*A*a*b^2)*x^2)*sqrt(b*x^
3 + a))/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)
```

Sympy [A]
time = 20.51, size = 80, normalized size = 0.14

$$\frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(5/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp
_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] int((x*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.257 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=578

$$-\frac{A}{ax\left(a+bx^3\right)^{3/2}}-\frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}}+\frac{5(11Ab-2aB)\sqrt{a+bx^3}}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{5\sqrt{2-\sqrt{3}}}{}$$

```
[Out] -A/a/x/(b*x^3+a)^(3/2)-1/9*(11*A*b-2*B*a)*x^2/a^2/(b*x^3+a)^(3/2)-5/27*(11*
A*b-2*B*a)*x^2/a^3/(b*x^3+a)^(1/2)+5/27*(11*A*b-2*B*a)*(b*x^3+a)^(1/2)/a^3/
b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/81*(11*A*b-2*B*a)*(a^(1/3)+b^(1/3
)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b^(2/3)/(b*x^3+a)^(1/2)/(
a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/54*(
11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(1/4)/a^(8/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {464, 296, 309, 224, 1891}

$$\frac{5\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(11Ab-2aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}(11Ab-2aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{18\,3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{5\sqrt{a+bx^3}(11Ab-2aB)}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{5x^2(11Ab-2aB)}{27a^3\sqrt{a+bx^3}}-\frac{x^2(11Ab-2aB)}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{ax\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^3)^(3/2))) - ((11*A*b - 2*a*B)*x^2)/(9*a^2*(a + b*x^3)^(3
/2)) - (5*(11*A*b - 2*a*B)*x^2)/(27*a^3*Sqrt[a + b*x^3]) + (5*(11*A*b - 2*a
*B)*Sqrt[a + b*x^3])/(27*a^3*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
 (5*Sqrt[2 - Sqrt[3]]*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(18*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(
```

```
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
]) + (5*Sqrt[2]*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
```

```
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{\left(\frac{11Ab}{2}-aB\right)\int \frac{x}{\left(a+bx^3\right)^{5/2}}\,dx}{a}$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{(5(11Ab-2aB))\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx}{18a^2}$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{(5(11Ab-2aB))\int \frac{\cdots}{\sqrt{a \cdots}}}{54a^3}$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{(5(11Ab-2aB))\int \frac{\left(1-\cdots\right.}{\cdots}}{54a^3\sqrt[3]{\cdots}}$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{5(11Ab-2aB)\sqrt{a\cdots}}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 86, normalized size = 0.15

$$-\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{\left(\frac{11Ab}{2}-aB\right)x^2\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]
[Out] -(A/(a*x*(a + b*x^3)^(3/2))) - ((((11*A*b)/2 - a*B)*x^2*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(2*a^3*Sqrt[a + b*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1000 vs. 2(438) = 876.
time = 0.35, size = 1001, normalized size = 1.73

method	result
	$2i\left(\frac{14Ab-5Ba}{27a^3}+\frac{Ab}{2a^3}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\right.}{}}$
elliptic	$-\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9a^2b^2\left(x^3+\frac{a}{b}\right)^2}-\frac{2x^2(14Ab-5Ba)}{27a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{a^3x}-$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(2/9/a*x^2/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+10/27/a^2*x^2/((x^3+a/b)*b)^(1
/2)+10/81*I/a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-2/9/a^2*x^
2/b*(b*x^3+a)^(1/2)/(x^3+a/b)^2-28/27*b*x^2/a^3/((x^3+a/b)*b)^(1/2)-1/a^3*(
b*x^3+a)^(1/2)/x-55/81*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
```

```
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b
*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 167, normalized size = 0.29

$$\frac{5\,\left(\left(2\,Bab^{2}-11\,Ab^{3}\right)x^{7}+2\,\left(2\,Ba^{2}b-11\,Aab^{2}\right)x^{4}+\left(2\,Ba^{3}-11\,Aa^{2}b\right)x\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(5\,\left(2\,Bab^{2}-11\,Ab^{3}\right)x^{6}-27\,Aa^{2}b+8\,\left(2\,Ba^{2}b-11\,Aab^{2}\right)x^{3}\right)\sqrt{bx^{3}+a}}{27\,\left(a^{3}b^{3}x^{7}+2\,a^{4}b^{2}x^{4}+a^{5}bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*(5*((2*B*a*b^2 - 11*A*b^3)*x^7 + 2*(2*B*a^2*b - 11*A*a*b^2)*x^4 + (2*B
*a^3 - 11*A*a^2*b)*x)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInvers
e(0, -4*a/b, x)) + (5*(2*B*a*b^2 - 11*A*b^3)*x^6 - 27*A*a^2*b + 8*(2*B*a^2*
b - 11*A*a*b^2)*x^3)*sqrt(b*x^3 + a))/(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*
x)
```

Sympy [A]
time = 41.40, size = 82, normalized size = 0.14

$$\frac{A\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{5}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(5/2),x)
```

```
[Out] A*gamma(-1/3)*hyper((-1/3, 5/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/2)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(5/2)*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^2\,{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/(x^2*(a + b*x^3)^(5/2)), x)
```

3.258 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=610

$$-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}-\frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}}-\frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}}+\frac{55(17Ab-8aB)\sqrt{a+bx^3}}{216a^4x}-\frac{55\sqrt[3]{b}\,(17Ab-8}{216a^4\left(\left(1+\sqrt{3}\right.\right.}$$

```
[Out] -1/4*A/a/x^4/(b*x^3+a)^(3/2)+1/36*(-17*A*b+8*B*a)/a^2/x/(b*x^3+a)^(3/2)-11/
108*(17*A*b-8*B*a)/a^3/x/(b*x^3+a)^(1/2)+55/216*(17*A*b-8*B*a)*(b*x^3+a)^(1
/2)/a^4/x-55/216*b^(1/3)*(17*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^4/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))-55/648*b^(1/3)*(17*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*Elliptic
F((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/a^(11/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+55/432*b^(1/3)*(17*A*b-8*B
*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1
/4)/a^(11/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$,
Rules used = {464, 296, 331, 309, 224, 1891}

$$-\frac{55\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{55\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{144\,3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{55\sqrt{a+bx^3}(17Ab-8aB)}{216a^4x}-\frac{55\sqrt[3]{b}\sqrt{a+bx^3}(17Ab-8aB)}{216a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}}-\frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}}-\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
```

```
[Out] -1/4*A/(a*x^4*(a + b*x^3)^(3/2)) - (17*A*b - 8*a*B)/(36*a^2*x*(a + b*x^3)^(
3/2)) - (11*(17*A*b - 8*a*B))/(108*a^3*x*Sqrt[a + b*x^3]) + (55*(17*A*b - 8
*a*B)*Sqrt[a + b*x^3])/(216*a^4*x) - (55*b^(1/3)*(17*A*b - 8*a*B)*Sqrt[a +
b*x^3])/(216*a^4*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (55*Sqrt[2 - Sqrt[3
]]*b^(1/3)*(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
```

```
)], -7 - 4*Sqrt[3]])/(144*3^(3/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (55*b^(1/3)*
(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(108*Sqrt[2]*3^(1/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{\left(\frac{17Ab}{2}-4aB\right)\int \frac{1}{x^2\left(a+bx^3\right)^{5/2}}\,dx}{4a}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{(11(17Ab-8aB))\int \frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx}{72a^2}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} - \frac{(55(17Ab-8aB))\int}{216a}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a}}{216a^4x}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a}}{216a^4x}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a}}{216a^4x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 83, normalized size = 0.14

$$\frac{-a^2A + \left(\frac{17Ab}{2} - 4aB\right)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{2}{3};-\frac{bx^3}{a}\right)}{4a^3x^4\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-(a^2*A) + ((17*A*b)/2 - 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[-1/3, 5/2, 2/3, -((b*x^3)/a)])/(4*a^3*x^4*(a + b*x^3)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1033 vs. 2(464) = 928.
time = 0.37, size = 1034, normalized size = 1.70

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{4a^3x^4}+\frac{(21Ab-8Ba)\sqrt{b\,x^3+a}}{8a^4x}+\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9a^3b\left(x^3+\frac{a}{b}\right)^2}+\frac{2b\,x^2(23Ab-14Ba)}{27a^4\sqrt{\left(x^3+\frac{a}{b}\right)b}}-$ … $2i\left(-\frac{b(21Ab-8\ldots}{16a^4}\right.$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4/a^3*(b*x^3+a)^(1/2)/x^4+21/8/a^4*b*(b*x^3+a)^(1/2)/x+2/9*x^2/a^3*(b
*x^3+a)^(1/2)/(x^3+a/b)^2+46/27*b^2*x^2/a^4/((x^3+a/b)*b)^(1/2)+935/648*I/a
^4*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
```

```
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+B*(-2/9/a^2*x^2/b*(b*x^3+a)
^(1/2)/(x^3+a/b)^2-28/27*b*x^2/a^3/((x^3+a/b)*b)^(1/2)-1/a^3*(b*x^3+a)^(1/2
)/x-55/81*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 186, normalized size = 0.30

$$-\frac{55\left(\left(8\,Bab^{2}-17\,Ab^{3}\right)x^{10}+2\left(8\,Ba^{2}b-17\,Aab^{2}\right)x^{7}+\left(8\,Ba^{3}-17\,Aa^{2}b\right)x^{4}\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(55\left(8\,Bab^{2}-17\,Ab^{3}\right)x^{9}+88\left(8\,Ba^{2}b-17\,Aab^{2}\right)x^{6}+54\,Aa^{3}+27\left(8\,Ba^{3}-17\,Aa^{2}b\right)x^{3}\right)\sqrt{bx^{3}+a}}{216\left(a^{4}b^{2}x^{10}+2\,a^{5}bx^{7}+a^{6}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/216*(55*((8*B*a*b^2 - 17*A*b^3)*x^10 + 2*(8*B*a^2*b - 17*A*a*b^2)*x^7 +
(8*B*a^3 - 17*A*a^2*b)*x^4)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassP
Inverse(0, -4*a/b, x)) + (55*(8*B*a*b^2 - 17*A*b^3)*x^9 + 88*(8*B*a^2*b - 1
7*A*a*b^2)*x^6 + 54*A*a^3 + 27*(8*B*a^3 - 17*A*a^2*b)*x^3)*sqrt(b*x^3 + a))
/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x^4)
```

Sympy [A]
time = 97.77, size = 88, normalized size = 0.14

$$\frac{A\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},\frac{5}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x^{4}\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{5}{2}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(5/2),x)
```

```
[Out] A*gamma(-4/3)*hyper((-4/3, 5/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/2)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 5/2), (2/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*x*gamma(2/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^5\,{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/(x^5*(a + b*x^3)^(5/2)), x)
```

3.259 $\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=97

$$\frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c(c+dx^3)^{3/2}}{9d^3}+\frac{2(c+dx^3)^{5/2}}{15d^3}-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^3}$$

```
[Out] -10/9*c*(d*x^3+c)^(3/2)/d^3+2/15*(d*x^3+c)^(5/2)/d^3-32/3*c^(5/2)*arctan(1/
3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))/d^3*3^(1/2)+32/3*c^2*(d*x^3+c)^(1/2)/d^3
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {457, 90, 52, 65, 209}

$$-\frac{32c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^3}+\frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c(c+dx^3)^{3/2}}{9d^3}+\frac{2(c+dx^3)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (32*c^2*Sqrt[c + d*x^3])/(3*d^3) - (10*c*(c + d*x^3)^(3/2))/(9*d^3) + (2*(c
 + d*x^3)^(5/2))/(15*d^3) - (32*c^(5/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqr
t[c])])/(Sqrt[3]*d^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{4c+dx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{5c\sqrt{c+dx}}{d^2}+\frac{(c+dx)^{3/2}}{d^2}+\frac{16c^2\sqrt{c+dx}}{d^2(4c+dx)}\right)\,dx,x,x^3\right)\\
&= -\frac{10c(c+dx^3)^{3/2}}{9d^3}+\frac{2(c+dx^3)^{5/2}}{15d^3}+\frac{(16c^2)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx,x,x^3\right)}{3d^2}\\
&= \frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c(c+dx^3)^{3/2}}{9d^3}+\frac{2(c+dx^3)^{5/2}}{15d^3}-\frac{(16c^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\right.}{d^2}\\
&= \frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c(c+dx^3)^{3/2}}{9d^3}+\frac{2(c+dx^3)^{5/2}}{15d^3}-\frac{(32c^3)\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx,x,\sqrt{c\ldots}\right.}{d^3}\\
&= \frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c(c+dx^3)^{3/2}}{9d^3}+\frac{2(c+dx^3)^{5/2}}{15d^3}-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^3}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 78, normalized size = 0.80

$$\frac{2\sqrt{c+dx^3}\,(218c^2-19cdx^3+3d^2x^6)}{45d^3}-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(218*c^2 - 19*c*d*x^3 + 3*d^2*x^6))/(45*d^3) - (32*c^(5/
2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.22, size = 503, normalized size = 5.19 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/15*x^6*(d*x^3+c)^(1/2)+2/45*c/d*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(d*x^3+
c)^(1/2)/d^2)-8/9*c*(d*x^3+c)^(3/2)/d^3+16*c^2/d^2*(2/3*(d*x^3+c)^(1/2)/d+1
/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_al
pha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=R
ootOf(_Z^3*d+4*c)))
```

Maxima [A]
time = 0.49, size = 69, normalized size = 0.71

$$-\frac{2\left(240\sqrt{3}\,c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\,(dx^3+c)^{\frac{5}{2}}+25\,(dx^3+c)^{\frac{3}{2}}c-240\,\sqrt{dx^3+c}\,c^2\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] -2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*
(d*x^3 + c)^(5/2) + 25*(d*x^3 + c)^(3/2)*c - 240*sqrt(d*x^3 + c)*c^2)/d^3
```

Fricas [A]
time = 2.24, size = 156, normalized size = 1.61

$$\left[\frac{2\left(120\sqrt{3}\,\sqrt{-c}\,c^2\log\left(\frac{dx^3-2\sqrt{3}\,\sqrt{dx^3+c}\,\sqrt{-c}-2c}{dx^3+4c}\right)+(3\,d^2x^6-19\,cdx^3+218\,c^2)\sqrt{dx^3+c}\right)}{45\,d^3},\,-\frac{2\left(240\sqrt{3}\,c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\sqrt{c}}\right)-(3\,d^2x^6-19\,cdx^3+218\,c^2)\sqrt{dx^3+c}\right)}{45\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [2/45*(120*sqrt(3)*sqrt(-c)*c^2*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt
(-c) - 2*c)/(d*x^3 + 4*c)) + (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3
+ c))/d^3, -2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sq
rt(c)) - (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3 + c))/d^3]
```

Sympy [A]
time = 11.85, size = 85, normalized size = 0.88

$$\frac{2\left(-\frac{16\sqrt{3}\,c^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{16c^2\sqrt{c+dx^3}}{3}-\frac{5c\left(c+dx^3\right)^{\frac{3}{2}}}{9}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] 2*(-16*sqrt(3)*c**(5/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 + 16*c
**2*sqrt(c + d*x**3)/3 - 5*c*(c + d*x**3)**(3/2)/9 + (c + d*x**3)**(5/2)/15
)/d**3
```

Giac [A]
time = 1.07, size = 82, normalized size = 0.85

$$-\frac{32\,\sqrt{3}\,c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{3\,d^3}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^{12}-25\left(dx^3+c\right)^{\frac{3}{2}}cd^{12}+240\,\sqrt{dx^3+c}\,c^2d^{12}\right)}{45\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -32/3*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/4
5*(3*(d*x^3 + c)^(5/2)*d^12 - 25*(d*x^3 + c)^(3/2)*c*d^12 + 240*sqrt(d*x^3
+ c)*c^2*d^12)/d^15
```

Mupad [B]
time = 4.54, size = 109, normalized size = 1.12

$$\frac{436\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d}-\frac{38\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}+\frac{\sqrt{3}\,c^{5/2}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,16{}\mathrm{i}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (436*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2))/(15*d) - (
38*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (3^(1/2)*c^(5/2)*log((2*3^(1/2)*c +
c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*16i)/(3*d^3)
```

3.260 $\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=76

$$-\frac{8c\sqrt{c+dx^3}}{3d^2}+\frac{2(c+dx^3)^{3/2}}{9d^2}+\frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^2}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^2+8/3*c^(3/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1
/2))/d^2*3^(1/2)-8/3*c*(d*x^3+c)^(1/2)/d^2
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {457, 81, 52, 65, 209}

$$\frac{8c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^2}-\frac{8c\sqrt{c+dx^3}}{3d^2}+\frac{2(c+dx^3)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (-8*c*Sqrt[c + d*x^3])/(3*d^2) + (2*(c + d*x^3)^(3/2))/(9*d^2) + (8*c^(3/2)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)\\
&= \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{(4c)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)}{3d}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{\left(4c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{d}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{\left(8c^2\right)\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^2}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^2}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 66, normalized size = 0.87

$$
\frac{2\left(-11c+dx^3\right)\sqrt{c+dx^3}}{9d^2} + \frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*(-11*c + d*x^3)*Sqrt[c + d*x^3])/(9*d^2) + (8*c^(3/2)*ArcTan[Sqrt[c + d*
x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 446, normalized size = 5.87 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^2-4*c/d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
)/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [A]
time = 0.52, size = 53, normalized size = 0.70

$$\frac{2\left(12\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}-12\sqrt{dx^3+c}\,c\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] 2/9*(12*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x^
3 + c)^(3/2) - 12*sqrt(d*x^3 + c)*c)/d^2
```

Fricas [A]
time = 2.41, size = 129, normalized size = 1.70

$$\left[\frac{2\left(6\sqrt{3}\sqrt{-c}\,c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}\,(dx^3-11c)\right)}{9\,d^2},\ \frac{2\left(12\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{dx^3+c}\,(dx^3-11c)\right)}{9\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [2/9*(6*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c)
- 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 11*c))/d^2, 2/9*(12*sqrt(3
)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*(d*
x^3 - 11*c))/d^2]
```

Sympy [A]
time = 6.45, size = 68, normalized size = 0.89

$$\frac{2\cdot\left(\frac{4\sqrt{3}\,c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}-\frac{4c\sqrt{c+dx^3}}{3}+\frac{(c+dx^3)^{\frac{3}{2}}}{9}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] 2*(4*sqrt(3)*c**(3/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 - 4*c*sq
rt(c + d*x**3)/3 + (c + d*x**3)**(3/2)/9)/d**2
```

Giac [A]
time = 1.52, size = 64, normalized size = 0.84

$$\frac{8\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3\,d^2}+\frac{2\left((dx^3+c)^{\frac{3}{2}}d^4-12\sqrt{dx^3+c}\,cd^4\right)}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] 8/3*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^2 + 2/9*(
(d*x^3 + c)^(3/2)*d^4 - 12*sqrt(d*x^3 + c)*c*d^4)/d^6
```

Mupad [B]
time = 4.28, size = 88, normalized size = 1.16

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}-\frac{22\,c\,\sqrt{d\,x^3+c}}{9\,d^2}+\frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,4{}\mathrm{i}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*d) - (22*c*(c + d*x^3)^(1/2))/(9*d^2) + (3^(1/
2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)
/(4*c + d*x^3))*4i)/(3*d^2)
```

3.261 $\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{\sqrt{3}\,d}$$

```
[Out] -2/3*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*c^(1/2)/d*3^(1/2)+2/3*(d*x
^3+c)^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {455, 52, 65, 209}

$$\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{\sqrt{3}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d) - (2*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(Sqrt[3]*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx,x,x^3\right)\\ &= \frac{2\sqrt{c+dx^3}}{3d}-c\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx,x,x^3\right)\\ &= \frac{2\sqrt{c+dx^3}}{3d}-\frac{(2c)\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx,x,\sqrt{c+dx^3}\right)}{d}\\ &= \frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{\sqrt{3}\,d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 0.95

$$\frac{2\left(\sqrt{c+dx^3}-\sqrt{3}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*(Sqrt[c + d*x^3] - Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[
c])]))/(3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 425, normalized size = 7.46 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1
/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^
(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [A]
time = 0.50, size = 42, normalized size = 0.74

$$-\frac{2\left(\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] -2/3*(sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d*
x^3 + c))/d
```

Fricas [A]
time = 2.87, size = 110, normalized size = 1.93

$$\left[\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+2\sqrt{dx^3+c}}{3\,d},-\frac{2\left(\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*
c)/(d*x^3 + 4*c)) + 2*sqrt(d*x^3 + c))/d, -2/3*(sqrt(3)*sqrt(c)*arctan(1/3*
sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d*x^3 + c))/d]
```

Sympy [A]
time = 1.77, size = 51, normalized size = 0.89

$$\frac{2\left(-\frac{\sqrt{3}\sqrt{c}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{\sqrt{c+dx^3}}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] 2*(-sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 + sqrt(c +
 d*x**3)/3)/d
```

Giac [A]
time = 1.60, size = 44, normalized size = 0.77

$$-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3d}+\frac{2\sqrt{dx^3+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d + 2/3*sq
rt(d*x^3 + c)/d
```

Mupad [B]
time = 3.87, size = 71, normalized size = 1.25

$$\frac{2\sqrt{d\,x^3+c}}{3\,d}+\frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(
c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(3*d)
```

3.262 $\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

```
[Out] -1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/6*arctan(1/3*(d*x^3+c)^(1/2
)*3^(1/2)/c^(1/2))*3^(1/2)/c^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {457, 85, 65, 214, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

```
[Out] ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(2*Sqrt[3]*Sqrt[c]) - ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]]/(6*Sqrt[c])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x\,(4c+dx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(4c+dx)}\,dx, x, x^3\right)\\ &= \frac{1}{12}\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)+\frac{1}{4}d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{6d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.91

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)-\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]])/(6*Sqrt[c])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.52, size = 468, normalized size = 7.20

method	result
default	$-\frac{d\left(\frac{2\sqrt{d\,x^{3}+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\,\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\cdots\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+\cdots}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d/c*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)
*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
```

```
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-
3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))+1/4/c*(2/3*(d*x^3+c)^(
1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x), x)
```

Fricas [A]
time = 2.40, size = 147, normalized size = 2.26

$$\left[\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{12c}, -\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)-2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)}{12c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="fricas")
[Out] [1/12*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt
(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, -1/12*(sqrt(3)*s
qrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c
)) - 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c]
```

Sympy [A]
time = 3.00, size = 66, normalized size = 1.02

$$\frac{2\left(\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}}+\frac{\sqrt{3}\,d\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12\sqrt{c}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(d*x**3+4*c),x)
[Out] 2*(d*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*sqrt(-c)) + sqrt(3)*d*atan(sqrt(3)
*sqrt(c + d*x**3)/(3*sqrt(c)))/(12*sqrt(c)))/d
```

Giac [A]
time = 0.71, size = 50, normalized size = 0.77

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/sqrt(c) + 1/6*arcta
n(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c)
```

Mupad [B]
time = 4.66, size = 93, normalized size = 1.43

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\,\sqrt{c}}+\frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,1{}\mathrm{i}}{12\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(4*c + d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(1
2*c^(1/2)) + (3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(
1/2)*d*x^3)/(4*c + d*x^3))*1i)/(12*c^(1/2))
```

3.263 $\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx$

Optimal. Leaf size=88

$$-\frac{\sqrt{c+dx^3}}{12cx^3}-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}$$

```
[Out] -1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/24*d*arctan(1/3*(d*x^3+c
)^(1/2)*3^(1/2)/c^(1/2))/c^(3/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {457, 101, 162, 65, 214, 209}

$$-\frac{d\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}-\frac{\sqrt{c+dx^3}}{12cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(4c+dx)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{\text{Subst}\left(\int \frac{cd-\frac{d^2x}{2}}{x\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{12c}\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{48c} - \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x\right.}{16c}\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{24c} - \frac{d\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right.}{8c}\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3} - \frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 88, normalized size = 1.00

$$-\frac{\sqrt{c+dx^3}}{12cx^3}-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.46, size = 511, normalized size = 5.81

method	result
risch	$-\frac{\sqrt{d\,x^3+c}}{12c\,x^3}+\frac{d\left(i\sqrt{2}\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x \cdots\right.}{-3\left(-c\,d^2\right)\cdots}}\cdots}{\cdots}\right)}{\cdots}$

default	$\frac{d^2\left(\frac{2\sqrt{d x^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{(-}{}\right)}{-3(-c d^2)^{\frac{1}{3}}+i}}}{}\right)}{}\right)}{}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/c^2*d^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3
)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_al
pha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
```

```
2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))+1/4/c*(-1/3*(d*x^3
+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))-1/16*d/c^2*(2
/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^4), x)
```

Fricas [A]
time = 2.92, size = 194, normalized size = 2.20

$$\left[-\frac{2\,\sqrt{3}\,\sqrt{c}\,dx^{3}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^{3}+c}}{3\,\sqrt{c}}\right)-\sqrt{c}\,dx^{3}\log\left(\frac{dx^{3}-2\,\sqrt{dx^{3}+c}\,\sqrt{c}+2\,c}{x^{3}}\right)+4\,\sqrt{dx^{3}+c}\,c}{48\,c^{2}x^{3}},\,-\frac{\sqrt{3}\,\sqrt{-c}\,dx^{3}\log\left(\frac{dx^{3}+2\,\sqrt{3}\,\sqrt{dx^{3}+c}\,\sqrt{-c}-2\,c}{dx^{3}+4\,c}\right)-2\,\sqrt{-c}\,dx^{3}\arctan\left(\frac{\sqrt{dx^{3}+c}\,\sqrt{-c}}{c}\right)+4\,\sqrt{dx^{3}+c}\,c}{48\,c^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [-1/48*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))
 - sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 4*sqr
t(d*x^3 + c)*c)/(c^2*x^3), -1/48*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 + 2*sqr
t(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) - 2*sqrt(-c)*d*x^3*arct
an(sqrt(d*x^3 + c)*sqrt(-c)/c) + 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^{3}}}{x^{4}\cdot\left(4c + dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**4*(4*c + d*x**3)), x)
```

Giac [A]
time = 0.72, size = 72, normalized size = 0.82

$$-\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^{3}+c}}{3\,\sqrt{c}}\right)}{24\,c^{\frac{3}{2}}}+\frac{d\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{24\,\sqrt{-c}\,c}-\frac{\sqrt{dx^{3}+c}}{12\,cx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/24*
d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/12*sqrt(d*x^3 + c)/(c*x
^3)
```

Mupad [B]
time = 4.86, size = 113, normalized size = 1.28

$$\frac{d\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{48\,c^{3/2}}-\frac{\sqrt{d\,x^3+c}}{12\,c\,x^3}+\frac{\sqrt{3}\,d\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{48\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(4*c + d*x^3)),x)
```

```
[Out] (d*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)
)/(48*c^(3/2)) - (c + d*x^3)^(1/2)/(12*c*x^3) + (3^(1/2)*d*log((2*3^(1/2)*c
 + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(48*c^(
3/2))
```

3.264 $\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=689

$$\frac{2x^2\sqrt{c+dx^3}}{7d}-\frac{50c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{2\sqrt[3]{2}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}\,d^{5/3}}+\frac{2\sqrt[3]{2}\,c^{7/6}}{}$$

```
[Out] -2*2^(1/3)*c^(7/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1
/2))/d^(5/3)+2/3*2^(1/3)*c^(7/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-2
/3*2^(1/3)*c^(7/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^
3+c)^(1/2))/d^(5/3)*3^(1/2)+2/3*2^(1/3)*c^(7/6)*arctan(1/3*(d*x^3+c)^(1/2)*
3^(1/2)/c^(1/2))/d^(5/3)*3^(1/2)+2/7*x^2*(d*x^3+c)^(1/2)/d-50/7*c*(d*x^3+c)
^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-50/21*c^(4/3)*(c^(1/3)+d^(1/
3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)
*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+25/7*3^(1/4)*
c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/
d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {489, 598, 309, 224, 1891, 497}

$$-\frac{50\sqrt{2}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{25\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{2\sqrt[3]{2}\,c^{7/6}\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}\,d^{5/3}}+\frac{2\sqrt[3]{2}\,c^{7/6}\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^{5/3}}-\frac{2\sqrt[3]{2}\,c^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{2\sqrt[3]{2}\,c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3d^{5/3}}-\frac{50c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{2x^2\sqrt{c+dx^3}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*x^2*Sqrt[c + d*x^3])/(7*d) - (50*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) - (2*2^(1/3)*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c
^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(5/3)) + (2*2^(1/
3)*c^(7/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^(5/3)) - (
2*2^(1/3)*c^(7/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c +
```

```
d*x^3]])/d^(5/3) + (2*2^(1/3)*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*
d^(5/3)) + (25*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5
0*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
```

```
0] && PosQ[c]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\int \frac{x\left(8c^2+\frac{25}{2}cdx^3\right)}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx}{7d}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\int \left(\frac{25cx}{2\sqrt{c+dx^3}} - \frac{42c^2x}{\sqrt{c+dx^3}\,(4c+dx^3)}\right)dx}{7d}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(25c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d} + \frac{(12c^2)\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx}{d}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\sqrt[3]{2}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}\,d^{5/3}} + \frac{2\sqrt[3]{2}\,c^{7/6}\tan^{-1}\left(\sqrt{\ldots}\right.}{\sqrt{3}\,d^{5/3}}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{50c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{2\sqrt[3]{2}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\ldots\right.}{\sqrt{c+d\ldots}}\right.}{\sqrt{3}\,d^{5/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 4.10, size = 133, normalized size = 0.19

$$\frac{8x^2(c+dx^3)-8cx^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-5dx^5\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{28d\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (8*x^2*(c + d*x^3) - 8*c*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), -1/4*(d*x^3)/c] - 5*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(28*d*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 1309, normalized size = 1.90

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1309

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/7*x^2*(d*x^3+c)^(1/2)-2/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2))))-4*c/d*(-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d
^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-
I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d
```

```
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*
2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_al
pha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=R
ootOf(_Z^3*d+4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 19.98, size = 3827, normalized size = 5.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] -1/42*(28*sqrt(3)*(4/27)^(1/6)*d^2*(-c^7/d^10)^(1/6)*arctan(-1/3*(108*4^(2/
3)*sqrt(3)*(c^7*d^12*x^16 - 39*c^8*d^11*x^13 - 72*c^9*d^10*x^10 - 32*c^10*d
^9*x^7)*(-c^7/d^10)^(2/3) + 12*4^(1/3)*sqrt(3)*(c^9*d^9*x^17 - 271*c^10*d^8
*x^14 + 112*c^11*d^7*x^11 + 1216*c^12*d^6*x^8 + 1088*c^13*d^5*x^5 + 256*c^1
4*d^4*x^2)*(-c^7/d^10)^(1/3) + 3*sqrt(1/3)*(324*sqrt(3)*(4/27)^(5/6)*(d^14*
x^16 + 229*c*d^13*x^13 + 492*c^2*d^12*x^10 + 328*c^3*d^11*x^7 + 64*c^4*d^10
*x^4)*(-c^7/d^10)^(5/6) + 12*sqrt(3)*sqrt(1/3)*(c^2*d^11*x^17 + 737*c^3*d^1
0*x^14 + 2704*c^4*d^9*x^11 + 3376*c^5*d^8*x^8 + 1664*c^6*d^7*x^5 + 256*c^7*
d^6*x^2)*sqrt(-c^7/d^10) + sqrt(3)*(4/27)^(1/6)*(c^4*d^8*x^18 + 1098*c^5*d^
7*x^15 - 24720*c^6*d^6*x^12 - 56704*c^7*d^5*x^9 - 44928*c^8*d^4*x^6 - 15360
*c^9*d^3*x^3 - 2048*c^10*d^2)*(-c^7/d^10)^(1/6) - sqrt(d*x^3 + c)*(4^(2/3)*
sqrt(3)*(5*c*d^12*x^15 - 3272*c^2*d^11*x^12 - 12544*c^3*d^10*x^9 - 14656*c^
4*d^9*x^6 - 6656*c^5*d^8*x^3 - 1024*c^6*d^7)*(-c^7/d^10)^(2/3) - 1728*4^(1/
```

3)*sqrt(3)*(c^4*d^8*x^13 + 2*c^5*d^7*x^10 + c^6*d^6*x^7)*(-c^7/d^10)^(1/3)
- 12*sqrt(3)*(17*c^6*d^5*x^14 - 1456*c^7*d^4*x^11 - 2544*c^8*d^3*x^8 - 1408
*c^9*d^2*x^5 - 256*c^10*d*x^2)))*sqrt((24*c^12*d^2*x^8 - 168*c^13*d*x^5 - 1
92*c^14*x^2 - 4^(2/3)*(c^7*d^9*x^9 + 60*c^8*d^8*x^6 - 32*c^10*d^6)*(-c^7/d^
10)^(2/3) - 24*4^(1/3)*(c^10*d^5*x^7 + 5*c^11*d^4*x^4 + 4*c^12*d^3*x)*(-c^7
/d^10)^(1/3) + 6*(36*sqrt(1/3)*c^9*d^6*x^5*sqrt(-c^7/d^10) + 9*(4/27)^(5/6)
*(c^6*d^10*x^7 + 2*c^7*d^9*x^4 - 8*c^8*d^8*x)*(-c^7/d^10)^(5/6) - 4*(4/27)^
(1/6)*(c^11*d^3*x^6 - 16*c^12*d^2*x^3 - 8*c^13*d)*(-c^7/d^10)^(1/6))*sqrt(d
*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + sqrt(3)*(c^1
1*d^6*x^18 - 1416*c^12*d^5*x^15 + 14352*c^13*d^4*x^12 + 44480*c^14*d^3*x^9
+ 49920*c^15*d^2*x^6 + 24576*c^16*d*x^3 + 4096*c^17) - 6*sqrt(d*x^3 + c)*(2
7*sqrt(3)*(4/27)^(5/6)*(31*c^6*d^13*x^14 + 1744*c^7*d^12*x^11 + 2976*c^8*d^
11*x^8 + 1600*c^9*d^10*x^5 + 256*c^10*d^9*x^2)*(-c^7/d^10)^(5/6) + 24*sqrt(
3)*sqrt(1/3)*(c^8*d^10*x^15 + 157*c^9*d^9*x^12 + 348*c^10*d^8*x^9 + 256*c^1
1*d^7*x^6 + 64*c^12*d^6*x^3)*sqrt(-c^7/d^10) + 2*sqrt(3)*(4/27)^(1/6)*(c^10
*d^7*x^16 + 686*c^11*d^6*x^13 + 7072*c^12*d^5*x^10 + 11008*c^13*d^4*x^7 + 5
888*c^14*d^3*x^4 + 1024*c^15*d^2*x)*(-c^7/d^10)^(1/6)))/(c^11*d^6*x^18 + 21
84*c^12*d^5*x^15 + 57696*c^13*d^4*x^12 + 125696*c^14*d^3*x^9 + 100608*c^15*
d^2*x^6 + 33792*c^16*d*x^3 + 4096*c^17)) - 28*sqrt(3)*(4/27)^(1/6)*d^2*(-c^
7/d^10)^(1/6)*arctan(-1/3*(108*4^(2/3)*sqrt(3)*(c^7*d^12*x^16 - 39*c^8*d^11
*x^13 - 72*c^9*d^10*x^10 - 32*c^10*d^9*x^7)*(-c^7/d^10)^(2/3) + 12*4^(1/3)*
sqrt(3)*(c^9*d^9*x^17 - 271*c^10*d^8*x^14 + 112*c^11*d^7*x^11 + 1216*c^12*d
^6*x^8 + 1088*c^13*d^5*x^5 + 256*c^14*d^4*x^2)*(-c^7/d^10)^(1/3) - 3*sqrt(1
/3)*(324*sqrt(3)*(4/27)^(5/6)*(d^14*x^16 + 229*c*d^13*x^13 + 492*c^2*d^12*x
^10 + 328*c^3*d^11*x^7 + 64*c^4*d^10*x^4)*(-c^7/d^10)^(5/6) + 12*sqrt(3)*sq
rt(1/3)*(c^2*d^11*x^17 + 737*c^3*d^10*x^14 + 2704*c^4*d^9*x^11 + 3376*c^5*d
^8*x^8 + 1664*c^6*d^7*x^5 + 256*c^7*d^6*x^2)*sqrt(-c^7/d^10) + sqrt(3)*(4/2
7)^(1/6)*(c^4*d^8*x^18 + 1098*c^5*d^7*x^15 - 24720*c^6*d^6*x^12 - 56704*c^7
*d^5*x^9 - 44928*c^8*d^4*x^6 - 15360*c^9*d^3*x^3 - 2048*c^10*d^2)*(-c^7/d^1
0)^(1/6) + sqrt(d*x^3 + c)*(4^(2/3)*sqrt(3)*(5*c*d^12*x^15 - 3272*c^2*d^11*
x^12 - 12544*c^3*d^10*x^9 - 14656*c^4*d^9*x^6 - 6656*c^5*d^8*x^3 - 1024*c^6
d^7)(-c^7/d^10)^(2/3) - 1728*4^(1/3)*sqrt(3)*(c^4*d^8*x^13 + 2*c^5*d^7*x^
10 + c^6*d^6*x^7)*(-c^7/d^10)^(1/3) - 12*sqrt(3)*(17*c^6*d^5*x^14 - 1456*c^
7*d^4*x^11 - 2544*c^8*d^3*x^8 - 1408*c^9*d^2*x^5 - 256*c^10*d*x^2)))*sqrt((
24*c^12*d^2*x^8 - 168*c^13*d*x^5 - 192*c^14*x^2 - 4^(2/3)*(c^7*d^9*x^9 + 60
*c^8*d^8*x^6 - 32*c^10*d^6)*(-c^7/d^10)^(2/3) - 24*4^(1/3)*(c^10*d^5*x^7 +
5*c^11*d^4*x^4 + 4*c^12*d^3*x)*(-c^7/d^10)^(1/3) - 6*(36*sqrt(1/3)*c^9*d^6*
x^5*sqrt(-c^7/d^10) + 9*(4/27)^(5/6)*(c^6*d^10*x^7 + 2*c^7*d^9*x^4 - 8*c^8*
d^8*x)*(-c^7/d^10)^(5/6) - 4*(4/27)^(1/6)*(c^11*d^3*x^6 - 16*c^12*d^2*x^3 -
8*c^13*d)*(-c^7/d^10)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48
*c^2*d*x^3 + 64*c^3)) + sqrt(3)*(c^11*d^6*x^18 - 1416*c^12*d^5*x^15 + 14352
*c^13*d^4*x^12 + 44480*c^14*d^3*x^9 + 49920*c^15*d^2*x^6 + 24576*c^16*d*x^3
+ 4096*c^17) + 6*sqrt(d*x^3 + c)*(27*sqrt(3)*(4/27)^(5/6)*(31*c^6*d^13*x^1
4 + 1744*c^7*d^12*x^11 + 2976*c^8*d^11*x^8 + 1600*c^9*d^10*x^5 + 256*c^10*d
^9*x^2)*(-c^7/d^10)^(5/6) + 24*sqrt(3)*sqrt(1/3)*(c^8*d^10*x^15 + 157*c^9*d

```
^9*x^12 + 348*c^10*d^8*x^9 + 256*c^11*d^7*x^6 + 64*c^12*d^6*x^3)*sqrt(-c^7/
d^10) + 2*sqrt(3)*(4/27)^(1/6)*(c^10*d^7*x^16 + 686*c^11*d^6*x^13 + 7072*c^
12*d^5*x^10 + 11008*c^13*d^4*x^7 + 5888*c^14*d^3*x^4 + 1024*c^15*d^2*x)*(-c
^7/d^10)^(1/6)))/(c^11*d^6*x^18 + 2184*c^12*d^5*x^15 + 57696*c^13*d^4*x^12
+ 125696*c^14*d^3*x^9 + 100608*c^15*d^2*x^6 + 33792*c^16*d*x^3 + 4096*c^17)
) - 12*sqrt(d*x^3 + c)*d*x^2 + 7*(4/27)^(1/6)*d^2*(-c^7/d^10)^(1/6)*log(163
84/3*(24*c^12*d^2*x^8 - 168*c^13*d*x^5 - 192*c^14*x^2 - 4^(2/3)*(c^7*d^9*x^
9 + 60*c^8*d^8*x^6 - 32*c^10*d^6)*(-c^7/d^10)^(2/3) - 24*4^(1/3)*(c^10*d^5*
x^7 + 5*c^11*d^4*x^4 + 4*c^12*d^3*x)*(-c^7/d^10...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] int((x^4*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

3.265 $\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=659

$$\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}\,d^{2/3}}-\frac{\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{2^{2/3}\sqrt{3}\,d^{2/3}}+\frac{\sqrt[6]{c}\,\mathrm{ta}}{}$$

```
[Out] 1/2*c^(1/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2^
(1/3)/d^(2/3)-1/6*c^(1/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/d^(2/3)+
1/6*c^(1/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1
/2))*2^(1/3)/d^(2/3)*3^(1/2)-1/6*c^(1/6)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)
/c^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)+2*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))+2/3*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/
3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)*3^(3/4)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE
((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(
1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {495, 309, 224, 1891, 497}

$$\frac{2\sqrt{2}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[6]{c}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}\,d^{2/3}}-\frac{\sqrt[6]{c}\,\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{2^{2/3}\sqrt{3}\,d^{2/3}}+\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}d^{2/3}}-\frac{\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3\;2^{2/3}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6
)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/
(2^(2/3)*Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
])])/(2^(2/3)*Sqrt[3]*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/
3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(2^(2/3)*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(3*2^(2/3)*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/
```

```
3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/
3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(
2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\left((3c)\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx\right) + \int \frac{x}{\sqrt{c+dx^3}}\,dx$$

$$= \frac{\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}\,d^{2/3}} - \frac{\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{2^{2/3}\sqrt{3}\,d^{2/3}} + \frac{\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt[6]{}\ldots}{\ldots}\right)}{2\ldots}$$

$$= \frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}\,d^{2/3}} - \frac{\sqrt[6]{c}\,\tan^{-1}\ldots}{2^{2/3}\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.23, size = 63, normalized size = 0.10

$$\frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d*
x^3)/c])/(8*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 848, normalized size = 1.29

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/
_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4
*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 12.06, size = 3547, normalized size = 5.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*(1/432)^(1/6)*d*(-c/d^4)^(1/6)*arctan(-1/3*(432*sqrt(3)*(1/
2)^(2/3)*(c*d^8*x^16 - 39*c^2*d^7*x^13 - 72*c^3*d^6*x^10 - 32*c^4*d^5*x^7)*
(-c/d^4)^(2/3) + 24*sqrt(3)*(1/2)^(1/3)*(c*d^7*x^17 - 271*c^2*d^6*x^14 + 11
2*c^3*d^5*x^11 + 1216*c^4*d^4*x^8 + 1088*c^5*d^3*x^5 + 256*c^6*d^2*x^2)*(-c
/d^4)^(1/3) + 12*sqrt(1/3)*(5184*sqrt(3)*(1/432)^(5/6)*(d^9*x^16 + 229*c*d^
8*x^13 + 492*c^2*d^7*x^10 + 328*c^3*d^6*x^7 + 64*c^4*d^5*x^4)*(-c/d^4)^(5/6
) + 6*sqrt(3)*sqrt(1/3)*(d^8*x^17 + 737*c*d^7*x^14 + 2704*c^2*d^6*x^11 + 33
76*c^3*d^5*x^8 + 1664*c^4*d^4*x^5 + 256*c^5*d^3*x^2)*sqrt(-c/d^4) + sqrt(3)
*(1/432)^(1/6)*(d^7*x^18 + 1098*c*d^6*x^15 - 24720*c^2*d^5*x^12 - 56704*c^3
*d^4*x^9 - 44928*c^4*d^3*x^6 - 15360*c^5*d^2*x^3 - 2048*c^6*d)*(-c/d^4)^(1/
6) - 2*sqrt(d*x^3 + c)*(sqrt(3)*(1/2)^(2/3)*(5*d^8*x^15 - 3272*c*d^7*x^12 -
 12544*c^2*d^6*x^9 - 14656*c^3*d^5*x^6 - 6656*c^4*d^4*x^3 - 1024*c^5*d^3)*(
-c/d^4)^(2/3) - 864*sqrt(3)*(1/2)^(1/3)*(c*d^6*x^13 + 2*c^2*d^5*x^10 + c^3*
d^4*x^7)*(-c/d^4)^(1/3) - 3*sqrt(3)*(17*c*d^5*x^14 - 1456*c^2*d^4*x^11 - 25
44*c^3*d^3*x^8 - 1408*c^4*d^2*x^5 - 256*c^5*d*x^2)))*sqrt((6*c^2*d^2*x^8 -
42*c^3*d*x^5 - 48*c^4*x^2 - (1/2)^(2/3)*(c*d^5*x^9 + 60*c^2*d^4*x^6 - 32*c^
4*d^2)*(-c/d^4)^(2/3) - 12*(1/2)^(1/3)*(c^2*d^3*x^7 + 5*c^3*d^2*x^4 + 4*c^4
*d*x)*(-c/d^4)^(1/3) + 6*(9*sqrt(1/3)*c^2*d^3*x^5*sqrt(-c/d^4) + 72*(1/432)
^(5/6)*(c*d^5*x^7 + 2*c^2*d^4*x^4 - 8*c^3*d^3*x)*(-c/d^4)^(5/6) - 2*(1/432)
^(1/6)*(c^2*d^2*x^6 - 16*c^3*d*x^3 - 8*c^4)*(-c/d^4)^(1/6))*sqrt(d*x^3 + c)
)/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + sqrt(3)*(c*d^6*x^18 -
 1416*c^2*d^5*x^15 + 14352*c^3*d^4*x^12 + 44480*c^4*d^3*x^9 + 49920*c^5*d^2
*x^6 + 24576*c^6*d*x^3 + 4096*c^7) - 24*sqrt(d*x^3 + c)*(216*sqrt(3)*(1/432
)^(5/6)*(31*c*d^8*x^14 + 1744*c^2*d^7*x^11 + 2976*c^3*d^6*x^8 + 1600*c^4*d^
5*x^5 + 256*c^5*d^4*x^2)*(-c/d^4)^(5/6) + 6*sqrt(3)*sqrt(1/3)*(c*d^7*x^15 +
 157*c^2*d^6*x^12 + 348*c^3*d^5*x^9 + 256*c^4*d^4*x^6 + 64*c^5*d^3*x^3)*sqr
t(-c/d^4) + sqrt(3)*(1/432)^(1/6)*(c*d^6*x^16 + 686*c^2*d^5*x^13 + 7072*c^3
*d^4*x^10 + 11008*c^4*d^3*x^7 + 5888*c^5*d^2*x^4 + 1024*c^6*d*x)*(-c/d^4)^(
1/6)))/(c*d^6*x^18 + 2184*c^2*d^5*x^15 + 57696*c^3*d^4*x^12 + 125696*c^4*d^
3*x^9 + 100608*c^5*d^2*x^6 + 33792*c^6*d*x^3 + 4096*c^7)) - 4*sqrt(3)*(1/43
2)^(1/6)*d*(-c/d^4)^(1/6)*arctan(-1/3*(432*sqrt(3)*(1/2)^(2/3)*(c*d^8*x^16
- 39*c^2*d^7*x^13 - 72*c^3*d^6*x^10 - 32*c^4*d^5*x^7)*(-c/d^4)^(2/3) + 24*s
qrt(3)*(1/2)^(1/3)*(c*d^7*x^17 - 271*c^2*d^6*x^14 + 112*c^3*d^5*x^11 + 1216
*c^4*d^4*x^8 + 1088*c^5*d^3*x^5 + 256*c^6*d^2*x^2)*(-c/d^4)^(1/3) - 12*sqrt
(1/3)*(5184*sqrt(3)*(1/432)^(5/6)*(d^9*x^16 + 229*c*d^8*x^13 + 492*c^2*d^7*
x^10 + 328*c^3*d^6*x^7 + 64*c^4*d^5*x^4)*(-c/d^4)^(5/6) + 6*sqrt(3)*sqrt(1/
3)*(d^8*x^17 + 737*c*d^7*x^14 + 2704*c^2*d^6*x^11 + 3376*c^3*d^5*x^8 + 1664
*c^4*d^4*x^5 + 256*c^5*d^3*x^2)*sqrt(-c/d^4) + sqrt(3)*(1/432)^(1/6)*(d^7*x
^18 + 1098*c*d^6*x^15 - 24720*c^2*d^5*x^12 - 56704*c^3*d^4*x^9 - 44928*c^4*
d^3*x^6 - 15360*c^5*d^2*x^3 - 2048*c^6*d)*(-c/d^4)^(1/6) + 2*sqrt(d*x^3 + c
)*(sqrt(3)*(1/2)^(2/3)*(5*d^8*x^15 - 3272*c*d^7*x^12 - 12544*c^2*d^6*x^9 -
```

```
14656*c^3*d^5*x^6 - 6656*c^4*d^4*x^3 - 1024*c^5*d^3)*(-c/d^4)^(2/3) - 864*s
qrt(3)*(1/2)^(1/3)*(c*d^6*x^13 + 2*c^2*d^5*x^10 + c^3*d^4*x^7)*(-c/d^4)^(1/
3) - 3*sqrt(3)*(17*c*d^5*x^14 - 1456*c^2*d^4*x^11 - 2544*c^3*d^3*x^8 - 1408
*c^4*d^2*x^5 - 256*c^5*d*x^2)))*sqrt((6*c^2*d^2*x^8 - 42*c^3*d*x^5 - 48*c^4
*x^2 - (1/2)^(2/3)*(c*d^5*x^9 + 60*c^2*d^4*x^6 - 32*c^4*d^2)*(-c/d^4)^(2/3)
 - 12*(1/2)^(1/3)*(c^2*d^3*x^7 + 5*c^3*d^2*x^4 + 4*c^4*d*x)*(-c/d^4)^(1/3)
- 6*(9*sqrt(1/3)*c^2*d^3*x^5*sqrt(-c/d^4) + 72*(1/432)^(5/6)*(c*d^5*x^7 + 2
*c^2*d^4*x^4 - 8*c^3*d^3*x)*(-c/d^4)^(5/6) - 2*(1/432)^(1/6)*(c^2*d^2*x^6 -
 16*c^3*d*x^3 - 8*c^4)*(-c/d^4)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2
*x^6 + 48*c^2*d*x^3 + 64*c^3)) + sqrt(3)*(c*d^6*x^18 - 1416*c^2*d^5*x^15 +
14352*c^3*d^4*x^12 + 44480*c^4*d^3*x^9 + 49920*c^5*d^2*x^6 + 24576*c^6*d*x^
3 + 4096*c^7) + 24*sqrt(d*x^3 + c)*(216*sqrt(3)*(1/432)^(5/6)*(31*c*d^8*x^1
4 + 1744*c^2*d^7*x^11 + 2976*c^3*d^6*x^8 + 1600*c^4*d^5*x^5 + 256*c^5*d^4*x
^2)*(-c/d^4)^(5/6) + 6*sqrt(3)*sqrt(1/3)*(c*d^7*x^15 + 157*c^2*d^6*x^12 + 3
48*c^3*d^5*x^9 + 256*c^4*d^4*x^6 + 64*c^5*d^3*x^3)*sqrt(-c/d^4) + sqrt(3)*(
1/432)^(1/6)*(c*d^6*x^16 + 686*c^2*d^5*x^13 + 7072*c^3*d^4*x^10 + 11008*c^4
*d^3*x^7 + 5888*c^5*d^2*x^4 + 1024*c^6*d*x)*(-c/d^4)^(1/6)))/(c*d^6*x^18 +
2184*c^2*d^5*x^15 + 57696*c^3*d^4*x^12 + 125696*c^4*d^3*x^9 + 100608*c^5*d^
2*x^6 + 33792*c^6*d*x^3 + 4096*c^7)) + (1/432)^(1/6)*d*(-c/d^4)^(1/6)*log(1
/12*(6*c^2*d^2*x^8 - 42*c^3*d*x^5 - 48*c^4*x^2 - (1/2)^(2/3)*(c*d^5*x^9 + 6
0*c^2*d^4*x^6 - 32*c^4*d^2)*(-c/d^4)^(2/3) - 12*(1/2)^(1/3)*(c^2*d^3*x^7 +
5*c^3*d^2*x^4 + 4*c^4*d*x)*(-c/d^4)^(1/3) + 6*(9*sqrt(1/3)*c^2*d^3*x^5*sqrt
(-c/d^4) + 72*(1/432)^(5/6)*(c*d^5*x^7 + 2*c^2*d^4*x^4 - 8*c^3*d^3*x)*(-c/d
^4)^(5/6) - 2*(1/432)^(1/6)*(c^2*d^2*x^6 - 16*c^3*d*x^3 - 8*c^4)*(-c/d^4)^(
1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) -
(1/432)^(1/6)*d*(-c/d^4)^(1/6)*log(1/12*(6*c^2*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
[Out] int((x*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

3.266 $\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=697

$$-\frac{\sqrt{c+dx^3}}{4cx}+\frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{4c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{4\,2^{2/3}\sqrt{3}\,c^{5/6}}+\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{c+}}{\sqrt{3}}\right.}{4\,2^{2/3}\sqrt{3}\,c^{5/}}$$

```
[Out] -1/8*d^(1/3)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2
^(1/3)/c^(5/6)+1/24*d^(1/3)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(5/6
)-1/24*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+c)
^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)+1/24*d^(1/3)*arctan(1/3*(d*x^3+c)^(1/2)*3^(
1/2)/c^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)-1/4*(d*x^3+c)^(1/2)/c/x+1/4*d^(1/3)*(
d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/12*d^(1/3)*(c^(1/3)+d^(1
/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3
)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/8*3^(1/4)*
d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/
c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {486, 598, 309, 224, 1891, 497}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{2}\,\sqrt[4]{3}\,c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{8c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{\sqrt[3]{d}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{4\,2^{2/3}\sqrt{3}\,c^{5/6}}+\frac{\sqrt[3]{d}\,\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{4\,2^{2/3}\sqrt{3}\,c^{5/6}}-\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{4\,2^{2/3}c^{5/6}}+\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\,2^{2/3}c^{5/6}}-\frac{\sqrt{c+dx^3}}{4cx}+\frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{4c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
```

```
[Out] -1/4*Sqrt[c + d*x^3]/(c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(4*c*((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)
*d^(1/3)*x))/Sqrt[c + d*x^3]])/(4*2^(2/3)*Sqrt[3]*c^(5/6)) + (d^(1/3)*ArcTa
n[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(4*2^(2/3)*Sqrt[3]*c^(5/6)) - (d^(1/3
```

```
)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(4*2^(2
/3)*c^(5/6)) + (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(12*2^(2/3)*c^(5/
6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[
2]*3^(1/4)*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
```

```
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{4cx} + \frac{\int \frac{x\left(5cd+\frac{d^2x^3}{2}\right)}{\sqrt{c+dx^3}\,(4c+dx^3)}dx}{4c}$$

$$= -\frac{\sqrt{c+dx^3}}{4cx} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{3cdx}{\sqrt{c+dx^3}\,(4c+dx^3)}\right)dx}{4c}$$

$$= -\frac{\sqrt{c+dx^3}}{4cx} + \frac{1}{4}(3d)\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx + \frac{d\int \frac{x}{\sqrt{c+dx^3}}dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{4cx} - \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\,\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{4\,2^{2/3}\sqrt{3}\,c^{5/6}} + \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{4\,2^{2/3}\sqrt{3}\,c^{5/6}} -$$

$$= -\frac{\sqrt{c+dx^3}}{4cx} + \frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{4c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\,\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{4\,2^{2/3}\sqrt{3}\,c^{5/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 20.07, size = 136, normalized size = 0.20

$$\frac{-40c(c+dx^3)+25cdx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+d^2x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{160c^2x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
```

```
[Out] (-40*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(160*c^2*x*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 1306, normalized size = 1.87

method	result	size

elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	1306

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d/c*(-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^
(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x
+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^
(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1
/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c
*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^
2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/
6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*d+4*c))+1/4/c*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.71, size = 2319, normalized size = 3.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(3)*(1/432)^(1/6)*c*x*(-d^2/c^5)^(1/6)*arctan(-1/3*(3*(sqrt(3)*
sqrt(1/3)*c^3*d^2*x*sqrt(-d^2/c^5) + 2*sqrt(3)*(1/432)^(1/6)*c*d^3*x^2*(-d^
2/c^5)^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^2*x^3 + 4*c^5*d)*(-d^2/c^5)^
(5/6))*sqrt(d*x^3 + c) + (2*sqrt(3)*(1/2)^(1/3)*(c^2*d^2*x^3 + c^3*d)*(-d^2
/c^5)^(1/3) + sqrt(3)*(d^3*x^4 + c*d^2*x) + 3*(sqrt(3)*sqrt(1/3)*c^3*d*x*sq
rt(-d^2/c^5) + 2*sqrt(3)*(1/432)^(1/6)*c*d^2*x^2*(-d^2/c^5)^(1/6) - 24*sqrt
(3)*(1/432)^(5/6)*(c^4*d*x^3 - 2*c^5)*(-d^2/c^5)^(5/6))*sqrt(d*x^3 + c))*sq
rt((d^5*x^9 + 60*c*d^4*x^6 - 32*c^3*d^2 - 24*(1/2)^(2/3)*(c^4*d^3*x^7 + 5*c
^5*d^2*x^4 + 4*c^6*d*x)*(-d^2/c^5)^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*
c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-d^2/c^5)^(1/3) + 12*(648*(1/432)^(5/6)*c^5*d
^2*x^5*(-d^2/c^5)^(5/6) - sqrt(1/3)*(c^3*d^3*x^6 - 16*c^4*d^2*x^3 - 8*c^5*d
)*sqrt(-d^2/c^5) - (1/432)^(1/6)*(c*d^4*x^7 + 2*c^2*d^3*x^4 - 8*c^3*d^2*x)*
(-d^2/c^5)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 +
 64*c^3)))/(d^4*x^4 + c*d^3*x)) + 4*sqrt(3)*(1/432)^(1/6)*c*x*(-d^2/c^5)^(1
/6)*arctan(-1/3*(3*(sqrt(3)*sqrt(1/3)*c^3*d^2*x*sqrt(-d^2/c^5) + 2*sqrt(3)*
(1/432)^(1/6)*c*d^3*x^2*(-d^2/c^5)^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^
2*x^3 + 4*c^5*d)*(-d^2/c^5)^(5/6))*sqrt(d*x^3 + c) - (2*sqrt(3)*(1/2)^(1/3)
*(c^2*d^2*x^3 + c^3*d)*(-d^2/c^5)^(1/3) + sqrt(3)*(d^3*x^4 + c*d^2*x) - 3*(
sqrt(3)*sqrt(1/3)*c^3*d*x*sqrt(-d^2/c^5) + 2*sqrt(3)*(1/432)^(1/6)*c*d^2*x^
2*(-d^2/c^5)^(1/6) - 24*sqrt(3)*(1/432)^(5/6)*(c^4*d*x^3 - 2*c^5)*(-d^2/c^5
)^(5/6))*sqrt(d*x^3 + c))*sqrt((d^5*x^9 + 60*c*d^4*x^6 - 32*c^3*d^2 - 24*(1
/2)^(2/3)*(c^4*d^3*x^7 + 5*c^5*d^2*x^4 + 4*c^6*d*x)*(-d^2/c^5)^(2/3) + 12*(
1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-d^2/c^5)^(1/3) -
 12*(648*(1/432)^(5/6)*c^5*d^2*x^5*(-d^2/c^5)^(5/6) - sqrt(1/3)*(c^3*d^3*x^
6 - 16*c^4*d^2*x^3 - 8*c^5*d)*sqrt(-d^2/c^5) - (1/432)^(1/6)*(c*d^4*x^7 + 2
*c^2*d^3*x^4 - 8*c^3*d^2*x)*(-d^2/c^5)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 1
2*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d^4*x^4 + c*d^3*x)) - (1/432)^(1/6)
*c*x*(-d^2/c^5)^(1/6)*log((d^5*x^9 + 60*c*d^4*x^6 - 32*c^3*d^2 - 24*(1/2)^(
```

```
2/3)*(c^4*d^3*x^7 + 5*c^5*d^2*x^4 + 4*c^6*d*x)*(-d^2/c^5)^(2/3) + 12*(1/2)^
(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-d^2/c^5)^(1/3) + 12*(
648*(1/432)^(5/6)*c^5*d^2*x^5*(-d^2/c^5)^(5/6) - sqrt(1/3)*(c^3*d^3*x^6 - 1
6*c^4*d^2*x^3 - 8*c^5*d)*sqrt(-d^2/c^5) - (1/432)^(1/6)*(c*d^4*x^7 + 2*c^2*
d^3*x^4 - 8*c^3*d^2*x)*(-d^2/c^5)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d
^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/432)^(1/6)*c*x*(-d^2/c^5)^(1/6)*log((
d^5*x^9 + 60*c*d^4*x^6 - 32*c^3*d^2 - 24*(1/2)^(2/3)*(c^4*d^3*x^7 + 5*c^5*d
^2*x^4 + 4*c^6*d*x)*(-d^2/c^5)^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*
d^3*x^5 - 8*c^4*d^2*x^2)*(-d^2/c^5)^(1/3) - 12*(648*(1/432)^(5/6)*c^5*d^2*x
^5*(-d^2/c^5)^(5/6) - sqrt(1/3)*(c^3*d^3*x^6 - 16*c^4*d^2*x^3 - 8*c^5*d)*sq
rt(-d^2/c^5) - (1/432)^(1/6)*(c*d^4*x^7 + 2*c^2*d^3*x^4 - 8*c^3*d^2*x)*(-d^
2/c^5)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*
c^3)) + 2*(1/432)^(1/6)*c*x*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 -
72*c^2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6
*x)*(-d^2/c^5)^(2/3) + 12*(1/2)^(1/3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*
d*x^2)*(-d^2/c^5)^(1/3) + 6*(1296*(1/432)^(5/6)*c^5*d*x^5*(-d^2/c^5)^(5/6)
+ sqrt(1/3)*(5*c^3*d^2*x^6 - 20*c^4*d*x^3 - 16*c^5)*sqrt(-d^2/c^5) + 2*(1/4
32)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-d^2/c^5)^(1/6))*sqrt(d
*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(1/432)^(1
/6)*c*x*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c^2*d^2*x^3 - 32*
c^3*d + 48*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x)*(-d^2/c^5)^(2/3)
 + 12*(1/2)^(1/3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*d*x^2)*(-d^2/c^5)^(1
/3) - 6*(1296*(1/432)^(5/6)*c^5*d*x^5*(-d^2/c^5)^(5/6) + sqrt(1/3)*(5*c^3*d
^2*x^6 - 20*c^4*d*x^3 - 16*c^5)*sqrt(-d^2/c^5) + 2*(1/432)^(1/6)*(c*d^3*x^7
 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-d^2/c^5)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9
+ 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 12*sqrt(d)*x*weierstrassZeta(0,
-4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 12*sqrt(d*x^3 + c))/(c*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^2 \cdot (4c + dx^3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**2*(4*c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^2*(4*c + d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^2*(4*c + d*x^3)), x)
```

3.267 $\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/16*x^4*AppellF1(4/3,-1/2,1,7/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/(
1+d*x^3/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -1/4*(d*x^3)/c, -((d*x^3)/
c)])/(16*c*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{4c+dx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 236 vs. 2(66) = 132.
time = 3.88, size = 236, normalized size = 3.58

$$\frac{x\left(-17x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+32\left(\frac{c}{d}+x^3+\frac{64c^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{d(4c+dx^3)\left(-16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)\right)}\right)\right)}{80\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (x*(-17*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1
/4*(d*x^3)/c] + 32*(c/d + x^3 + (64*c^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3
)/c), -1/4*(d*x^3)/c])/(d*(4*c + d*x^3)*(-16*c*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), -1/4*(d*x^3)/c] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3
)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*
x^3)/c]))))))/(80*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.40, size = 1003, normalized size = 15.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/5*x*(d*x^3+c)^(1/2)-2/5*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)))-4*c/d*(-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
```

```
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*
3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*
_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3821 vs. $2(52) = 104$.
time = 18.90, size = 3821, normalized size = 57.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] -1/60*(20*sqrt(3)*(16/27)^(1/6)*d^2*(-c^5/d^8)^(1/6)*arctan(1/24*(96*sqrt(3
)*2^(2/3)*(c^5*d^11*x^17 - 271*c^6*d^10*x^14 + 112*c^7*d^9*x^11 + 1216*c^8*
d^8*x^8 + 1088*c^9*d^7*x^5 + 256*c^10*d^6*x^2)*(-c^5/d^8)^(2/3) + 1728*sqrt
(3)*2^(1/3)*(c^7*d^8*x^16 - 39*c^8*d^7*x^13 - 72*c^9*d^6*x^10 - 32*c^10*d^5
*x^7)*(-c^5/d^8)^(1/3) - 3*sqrt(1/3)*(9*sqrt(3)*(16/27)^(5/6)*(d^13*x^18 +
1098*c*d^12*x^15 - 24720*c^2*d^11*x^12 - 56704*c^3*d^10*x^9 - 44928*c^4*d^9
*x^6 - 15360*c^5*d^8*x^3 - 2048*c^6*d^7)*(-c^5/d^8)^(5/6) + 96*sqrt(3)*sqrt
(1/3)*(c^2*d^10*x^17 + 737*c^3*d^9*x^14 + 2704*c^4*d^8*x^11 + 3376*c^5*d^7*
x^8 + 1664*c^6*d^6*x^5 + 256*c^7*d^5*x^2)*sqrt(-c^5/d^8) + 576*sqrt(3)*(16/
27)^(1/6)*(c^4*d^7*x^16 + 229*c^5*d^6*x^13 + 492*c^6*d^5*x^10 + 328*c^7*d^4
*x^7 + 64*c^8*d^3*x^4)*(-c^5/d^8)^(1/6) - 16*sqrt(d*x^3 + c)*(864*sqrt(3)*2
```

^(2/3)*(c^2*d^10*x^13 + 2*c^3*d^9*x^10 + c^4*d^8*x^7)*(-c^5/d^8)^(2/3) - sqrt(3)*2^(1/3)*(5*c^3*d^8*x^15 - 3272*c^4*d^7*x^12 - 12544*c^5*d^6*x^9 - 14656*c^6*d^5*x^6 - 6656*c^7*d^4*x^3 - 1024*c^8*d^3)*(-c^5/d^8)^(1/3) - 6*sqrt(3)*(17*c^5*d^5*x^14 - 1456*c^6*d^4*x^11 - 2544*c^7*d^3*x^8 - 1408*c^8*d^2*x^5 - 256*c^9*d*x^2)))*sqrt((24*c^8*d^2*x^8 - 168*c^9*d*x^5 - 192*c^10*x^2 + 24*2^(2/3)*(c^5*d^7*x^7 + 5*c^6*d^6*x^4 + 4*c^7*d^5*x)*(-c^5/d^8)^(2/3) + 2*2^(1/3)*(c^6*d^5*x^9 + 60*c^7*d^4*x^6 - 32*c^9*d^2)*(-c^5/d^8)^(1/3) + 3*(72*sqrt(1/3)*c^6*d^5*x^5*sqrt(-c^5/d^8) - 9*(16/27)^(5/6)*(c^4*d^8*x^6 - 16*c^5*d^7*x^3 - 8*c^6*d^6)*(-c^5/d^8)^(5/6) + 4*(16/27)^(1/6)*(c^7*d^3*x^7 + 2*c^8*d^2*x^4 - 8*c^9*d*x)*(-c^5/d^8)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 8*sqrt(3)*(c^8*d^6*x^18 - 1416*c^9*d^5*x^15 + 14352*c^10*d^4*x^12 + 44480*c^11*d^3*x^9 + 49920*c^12*d^2*x^6 + 24576*c^13*d*x^3 + 4096*c^14) + 36*sqrt(d*x^3 + c)*(3*sqrt(3)*(16/27)^(5/6)*(c^4*d^12*x^16 + 686*c^5*d^11*x^13 + 7072*c^6*d^10*x^10 + 11008*c^7*d^9*x^7 + 5888*c^8*d^8*x^4 + 1024*c^9*d^7*x)*(-c^5/d^8)^(5/6) + 32*sqrt(3)*sqrt(1/3)*(c^6*d^9*x^15 + 157*c^7*d^8*x^12 + 348*c^8*d^7*x^9 + 256*c^9*d^6*x^6 + 64*c^10*d^5*x^3)*sqrt(-c^5/d^8) + 8*sqrt(3)*(16/27)^(1/6)*(31*c^8*d^6*x^14 + 1744*c^9*d^5*x^11 + 2976*c^10*d^4*x^8 + 1600*c^11*d^3*x^5 + 256*c^12*d^2*x^2)*(-c^5/d^8)^(1/6)))/(c^8*d^6*x^18 + 2184*c^9*d^5*x^15 + 57696*c^10*d^4*x^12 + 125696*c^11*d^3*x^9 + 100608*c^12*d^2*x^6 + 33792*c^13*d*x^3 + 4096*c^14)) - 20*sqrt(3)*(16/27)^(1/6)*d^2*(-c^5/d^8)^(1/6)*arctan(1/24*(96*sqrt(3)*2^(2/3)*(c^5*d^11*x^17 - 271*c^6*d^10*x^14 + 112*c^7*d^9*x^11 + 1216*c^8*d^8*x^8 + 1088*c^9*d^7*x^5 + 256*c^10*d^6*x^2)*(-c^5/d^8)^(2/3) + 1728*sqrt(3)*2^(1/3)*(c^7*d^8*x^16 - 39*c^8*d^7*x^13 - 72*c^9*d^6*x^10 - 32*c^10*d^5*x^7)*(-c^5/d^8)^(1/3) + 3*sqrt(1/3)*(9*sqrt(3)*(16/27)^(5/6)*(d^13*x^18 + 1098*c*d^12*x^15 - 24720*c^2*d^11*x^12 - 56704*c^3*d^10*x^9 - 44928*c^4*d^9*x^6 - 15360*c^5*d^8*x^3 - 2048*c^6*d^7)*(-c^5/d^8)^(5/6) + 96*sqrt(3)*sqrt(1/3)*(c^2*d^10*x^17 + 737*c^3*d^9*x^14 + 2704*c^4*d^8*x^11 + 3376*c^5*d^7*x^8 + 1664*c^6*d^6*x^5 + 256*c^7*d^5*x^2)*sqrt(-c^5/d^8) + 576*sqrt(3)*(16/27)^(1/6)*(c^4*d^7*x^16 + 229*c^5*d^6*x^13 + 492*c^6*d^5*x^10 + 328*c^7*d^4*x^7 + 64*c^8*d^3*x^4)*(-c^5/d^8)^(1/6) + 16*sqrt(d*x^3 + c)*(864*sqrt(3)*2^(2/3)*(c^2*d^10*x^13 + 2*c^3*d^9*x^10 + c^4*d^8*x^7)*(-c^5/d^8)^(2/3) - sqrt(3)*2^(1/3)*(5*c^3*d^8*x^15 - 3272*c^4*d^7*x^12 - 12544*c^5*d^6*x^9 - 14656*c^6*d^5*x^6 - 6656*c^7*d^4*x^3 - 1024*c^8*d^3)*(-c^5/d^8)^(1/3) - 6*sqrt(3)*(17*c^5*d^5*x^14 - 1456*c^6*d^4*x^11 - 2544*c^7*d^3*x^8 - 1408*c^8*d^2*x^5 - 256*c^9*d*x^2)))*sqrt((24*c^8*d^2*x^8 - 168*c^9*d*x^5 - 192*c^10*x^2 + 24*2^(2/3)*(c^5*d^7*x^7 + 5*c^6*d^6*x^4 + 4*c^7*d^5*x)*(-c^5/d^8)^(2/3) + 2*2^(1/3)*(c^6*d^5*x^9 + 60*c^7*d^4*x^6 - 32*c^9*d^2)*(-c^5/d^8)^(1/3) - 3*(72*sqrt(1/3)*c^6*d^5*x^5*sqrt(-c^5/d^8) - 9*(16/27)^(5/6)*(c^4*d^8*x^6 - 16*c^5*d^7*x^3 - 8*c^6*d^6)*(-c^5/d^8)^(5/6) + 4*(16/27)^(1/6)*(c^7*d^3*x^7 + 2*c^8*d^2*x^4 - 8*c^9*d*x)*(-c^5/d^8)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 8*sqrt(3)*(c^8*d^6*x^18 - 1416*c^9*d^5*x^15 + 14352*c^10*d^4*x^12 + 44480*c^11*d^3*x^9 + 49920*c^12*d^2*x^6 + 24576*c^13*d*x^3 + 4096*c^14) - 36*sqrt(d*x^3 + c)*(3*sqrt(3)*(16/27)^(5/6)*(c^4*d^12*x^16 + 686*c^5*d^11*x^13 + 7072*c^6*d^10*x^10 + 11008*c^7*

```
d^9*x^7 + 5888*c^8*d^8*x^4 + 1024*c^9*d^7*x)*(-c^5/d^8)^(5/6) + 32*sqrt(3)*
sqrt(1/3)*(c^6*d^9*x^15 + 157*c^7*d^8*x^12 + 348*c^8*d^7*x^9 + 256*c^9*d^6*
x^6 + 64*c^10*d^5*x^3)*sqrt(-c^5/d^8) + 8*sqrt(3)*(16/27)^(1/6)*(31*c^8*d^6
*x^14 + 1744*c^9*d^5*x^11 + 2976*c^10*d^4*x^8 + 1600*c^11*d^3*x^5 + 256*c^1
2*d^2*x^2)*(-c^5/d^8)^(1/6)))/(c^8*d^6*x^18 + 2184*c^9*d^5*x^15 + 57696*c^1
0*d^4*x^12 + 125696*c^11*d^3*x^9 + 100608*c^12*d^2*x^6 + 33792*c^13*d*x^3 +
 4096*c^14)) - 5*(16/27)^(1/6)*d^2*(-c^5/d^8)^(1/6)*log(16/3*(24*c^8*d^2*x^
8 - 168*c^9*d*x^5 - 192*c^10*x^2 + 24*2^(2/3)*(c^5*d^7*x^7 + 5*c^6*d^6*x^4
+ 4*c^7*d^5*x)*(-c^5/d^8)^(2/3) + 2*2^(1/3)*(c^6*d^5*x^9 + 60*c^7*d^4*x^6 -
 32*c^9*d^2)*(-c^5/d^8)^(1/3) + 3*(72*sqrt(1/3)*c^6*d^5*x^5*sqrt(-c^5/d^8)
- 9*(16/27)^(5/6)*(c^4*d^8*x^6 - 16*c^5*d^7*x^3...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
[Out] Integral(x**3*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
[Out] int((x^3*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

3.268 $\int \frac{\sqrt{c+dx^3}}{4c+dx^3} \, dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/4*x*AppellF1(1/3,-1/2,1,4/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/(1+d
*x^3/c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {441, 440}

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -1/4*(d*x^3)/c, -((d*x^3)/c)
])/(4*c*Sqrt[1 + (d*x^3)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{4c+dx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(64) = 128.
time = 8.83, size = 165, normalized size = 2.58

$$\frac{16cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{(4c+dx^3)\left(16cF_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(F_1\left(\frac{4}{3};-\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-2F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

```
[Out] (16*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/((4*c + d*x^3)*(16*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -1/4
*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -1/4*(d*x^
3)/c] - 2*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.35, size = 696, normalized size = 10.88 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*
I/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/
3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^
```

```
2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2187 vs. 2(50) = 100.
time = 4.25, size = 2187, normalized size = 34.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] 1/24*(4*sqrt(3)*(1/108)^(1/6)*d*(-1/(c*d^2))^(1/6)*arctan(1/3*((108*sqrt(3)
*(1/108)^(5/6)*c*d^2*x^2*(-1/(c*d^2))^(5/6) + 3*sqrt(3)*sqrt(1/3)*c*d*x*sqr
t(-1/(c*d^2)) + sqrt(3)*(1/108)^(1/6)*(d*x^3 + 4*c)*(-1/(c*d^2))^(1/6))*sqr
t(d*x^3 + c) - (4*sqrt(3)*(1/4)^(2/3)*(c*d^2*x^3 + c^2*d)*(-1/(c*d^2))^(2/3
) - sqrt(3)*(d*x^4 + c*x) - (108*sqrt(3)*(1/108)^(5/6)*c*d^2*x^2*(-1/(c*d^2
))^(5/6) + 3*sqrt(3)*sqrt(1/3)*c*d*x*sqrt(-1/(c*d^2)) - sqrt(3)*(1/108)^(1/
6)*(d*x^3 - 2*c)*(-1/(c*d^2))^(1/6))*sqrt(d*x^3 + c))*sqrt((d^3*x^9 + 60*c*
d^2*x^6 - 32*c^3 - 24*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^
2)*(-1/(c*d^2))^(2/3) + 24*(1/4)^(1/3)*(c*d^3*x^7 + 5*c^2*d^2*x^4 + 4*c^3*d
*x)*(-1/(c*d^2))^(1/3) + 12*(9*(1/108)^(1/6)*c*d^2*x^5*(-1/(c*d^2))^(1/6) -
 18*(1/108)^(5/6)*(c*d^4*x^7 + 2*c^2*d^3*x^4 - 8*c^3*d^2*x)*(-1/(c*d^2))^(5
/6) - sqrt(1/3)*(c*d^3*x^6 - 16*c^2*d^2*x^3 - 8*c^3*d)*sqrt(-1/(c*d^2)))*sq
rt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 +
c*x)) + 4*sqrt(3)*(1/108)^(1/6)*d*(-1/(c*d^2))^(1/6)*arctan(1/3*((108*sqrt(
3)*(1/108)^(5/6)*c*d^2*x^2*(-1/(c*d^2))^(5/6) + 3*sqrt(3)*sqrt(1/3)*c*d*x*s
qrt(-1/(c*d^2)) + sqrt(3)*(1/108)^(1/6)*(d*x^3 + 4*c)*(-1/(c*d^2))^(1/6))*s
qrt(d*x^3 + c) + (4*sqrt(3)*(1/4)^(2/3)*(c*d^2*x^3 + c^2*d)*(-1/(c*d^2))^(2
/3) - sqrt(3)*(d*x^4 + c*x) + (108*sqrt(3)*(1/108)^(5/6)*c*d^2*x^2*(-1/(c*d
^2))^(5/6) + 3*sqrt(3)*sqrt(1/3)*c*d*x*sqrt(-1/(c*d^2)) - sqrt(3)*(1/108)^(
1/6)*(d*x^3 - 2*c)*(-1/(c*d^2))^(1/6))*sqrt(d*x^3 + c))*sqrt((d^3*x^9 + 60*
```

```
c*d^2*x^6 - 32*c^3 - 24*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*
x^2)*(-1/(c*d^2))^(2/3) + 24*(1/4)^(1/3)*(c*d^3*x^7 + 5*c^2*d^2*x^4 + 4*c^3
*d*x)*(-1/(c*d^2))^(1/3) - 12*(9*(1/108)^(1/6)*c*d^2*x^5*(-1/(c*d^2))^(1/6)
 - 18*(1/108)^(5/6)*(c*d^4*x^7 + 2*c^2*d^3*x^4 - 8*c^3*d^2*x)*(-1/(c*d^2))^
(5/6) - sqrt(1/3)*(c*d^3*x^6 - 16*c^2*d^2*x^3 - 8*c^3*d)*sqrt(-1/(c*d^2)))*
sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4
+ c*x)) - (1/108)^(1/6)*d*(-1/(c*d^2))^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 -
32*c^3 - 24*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)*(-1/(c*
d^2))^(2/3) + 24*(1/4)^(1/3)*(c*d^3*x^7 + 5*c^2*d^2*x^4 + 4*c^3*d*x)*(-1/(c
*d^2))^(1/3) + 12*(9*(1/108)^(1/6)*c*d^2*x^5*(-1/(c*d^2))^(1/6) - 18*(1/108
)^(5/6)*(c*d^4*x^7 + 2*c^2*d^3*x^4 - 8*c^3*d^2*x)*(-1/(c*d^2))^(5/6) - sqrt
(1/3)*(c*d^3*x^6 - 16*c^2*d^2*x^3 - 8*c^3*d)*sqrt(-1/(c*d^2)))*sqrt(d*x^3 +
 c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/108)^(1/6)*d*(-
1/(c*d^2))^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/4)^(2/3)*(c*d
^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)*(-1/(c*d^2))^(2/3) + 24*(1/4)^(1/3)
*(c*d^3*x^7 + 5*c^2*d^2*x^4 + 4*c^3*d*x)*(-1/(c*d^2))^(1/3) - 12*(9*(1/108)
^(1/6)*c*d^2*x^5*(-1/(c*d^2))^(1/6) - 18*(1/108)^(5/6)*(c*d^4*x^7 + 2*c^2*d
^3*x^4 - 8*c^3*d^2*x)*(-1/(c*d^2))^(5/6) - sqrt(1/3)*(c*d^3*x^6 - 16*c^2*d^
2*x^3 - 8*c^3*d)*sqrt(-1/(c*d^2)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6
 + 48*c^2*d*x^3 + 64*c^3)) + 2*(1/108)^(1/6)*d*(-1/(c*d^2))^(1/6)*log((d^3*
x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/4)^(2/3)*(c*d^4*x^8 - 7*
c^2*d^3*x^5 - 8*c^3*d^2*x^2)*(-1/(c*d^2))^(2/3) - 48*(1/4)^(1/3)*(c*d^3*x^7
 - c^2*d^2*x^4 - 2*c^3*d*x)*(-1/(c*d^2))^(1/3) + 6*(18*(1/108)^(1/6)*c*d^2*
x^5*(-1/(c*d^2))^(1/6) + 36*(1/108)^(5/6)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c
^3*d^2*x)*(-1/(c*d^2))^(5/6) + sqrt(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16
*c^3*d)*sqrt(-1/(c*d^2)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2
*d*x^3 + 64*c^3)) - 2*(1/108)^(1/6)*d*(-1/(c*d^2))^(1/6)*log((d^3*x^9 - 66*
c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*x
^5 - 8*c^3*d^2*x^2)*(-1/(c*d^2))^(2/3) - 48*(1/4)^(1/3)*(c*d^3*x^7 - c^2*d^
2*x^4 - 2*c^3*d*x)*(-1/(c*d^2))^(1/3) - 6*(18*(1/108)^(1/6)*c*d^2*x^5*(-1/(
c*d^2))^(1/6) + 36*(1/108)^(5/6)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x)
*(-1/(c*d^2))^(5/6) + sqrt(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d)*s
qrt(-1/(c*d^2)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 +
64*c^3)) + 24*sqrt(d)*weierstrassPInverse(0, -4*c/d, x))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{4c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(4*c + d*x^3),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(4*c + d*x^3), x)
```

3.269 $\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -1/8*AppellF1(-2/3,-1/2,1,1/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/x^2/
(1+d*x^3/c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

```
[Out] -1/8*(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -1/4*(d*x^3)/c, -((d*x^3
)/c)])/(c*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3(4c+dx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= -\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(66) = 132.
time = 20.10, size = 244, normalized size = 3.70

$$\frac{-32c(c+dx^3)-d^2x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+\frac{2048c^3dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{(4c+dx^3)\left(16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)\right)}}{256c^2x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

```
[Out] (-32*c*(c + d*x^3) - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3,
 -((d*x^3)/c), -1/4*(d*x^3)/c] + (2048*c^3*d*x^3*AppellF1[1/3, 1/2, 1, 4/3,
 -((d*x^3)/c), -1/4*(d*x^3)/c])/((4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2, 1,
4/3, -((d*x^3)/c), -1/4*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -(
(d*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1
/4*(d*x^3)/c]))))/(256*c^2*x^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.40, size = 1002, normalized size = 15.18 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c*(-1/2*(d*x^3+c)^(1/2)/x^2-1/2*I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)))-1/4*d/c*(-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
```

```
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1
/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d
-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/
3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2372 vs. $2(52) = 104$.
time = 4.66, size = 2372, normalized size = 35.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] -1/96*(4*sqrt(3)*(1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*arctan(1/3*((108*sqrt
(3)*(1/108)^(5/6)*c^6*d^3*x^2*(-d^4/c^7)^(5/6) + 3*sqrt(3)*sqrt(1/3)*c^4*d^
4*x*sqrt(-d^4/c^7) + sqrt(3)*(1/108)^(1/6)*(c*d^6*x^3 + 4*c^2*d^5)*(-d^4/c^
7)^(1/6))*sqrt(d*x^3 + c) - (4*sqrt(3)*(1/4)^(2/3)*(c^5*d*x^3 + c^6)*(-d^4/
c^7)^(2/3) - sqrt(3)*(d^4*x^4 + c*d^3*x) - (108*sqrt(3)*(1/108)^(5/6)*c^6*x
^2*(-d^4/c^7)^(5/6) + 3*sqrt(3)*sqrt(1/3)*c^4*d*x*sqrt(-d^4/c^7) - sqrt(3)*
(1/108)^(1/6)*(c*d^3*x^3 - 2*c^2*d^2)*(-d^4/c^7)^(1/6))*sqrt(d*x^3 + c))*sq
rt((d^9*x^9 + 60*c*d^8*x^6 - 32*c^3*d^6 - 24*(1/4)^(2/3)*(c^5*d^6*x^8 - 7*c
^6*d^5*x^5 - 8*c^7*d^4*x^2)*(-d^4/c^7)^(2/3) + 24*(1/4)^(1/3)*(c^3*d^7*x^7
+ 5*c^4*d^6*x^4 + 4*c^5*d^5*x)*(-d^4/c^7)^(1/3) + 12*(9*(1/108)^(1/6)*c^2*d
^7*x^5*(-d^4/c^7)^(1/6) - 18*(1/108)^(5/6)*(c^6*d^5*x^7 + 2*c^7*d^4*x^4 - 8
```

*c^8*d^3*x)*(-d^4/c^7)^(5/6) - sqrt(1/3)*(c^4*d^6*x^6 - 16*c^5*d^5*x^3 - 8*c^6*d^4)*sqrt(-d^4/c^7))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d^7*x^4 + c*d^6*x)) + 4*sqrt(3)*(1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*arctan(1/3*((108*sqrt(3)*(1/108)^(5/6)*c^6*d^3*x^2*(-d^4/c^7)^(5/6) + 3*sqrt(3)*sqrt(1/3)*c^4*d^4*x*sqrt(-d^4/c^7) + sqrt(3)*(1/108)^(1/6)*(c*d^6*x^3 + 4*c^2*d^5)*(-d^4/c^7)^(1/6))*sqrt(d*x^3 + c) + (4*sqrt(3)*(1/4)^(2/3)*(c^5*d*x^3 + c^6)*(-d^4/c^7)^(2/3) - sqrt(3)*(d^4*x^4 + c*d^3*x) + (108*sqrt(3)*(1/108)^(5/6)*c^6*x^2*(-d^4/c^7)^(5/6) + 3*sqrt(3)*sqrt(1/3)*c^4*d*x*sqrt(-d^4/c^7) - sqrt(3)*(1/108)^(1/6)*(c*d^3*x^3 - 2*c^2*d^2)*(-d^4/c^7)^(1/6))*sqrt(d*x^3 + c))*sqrt((d^9*x^9 + 60*c*d^8*x^6 - 32*c^3*d^6 - 24*(1/4)^(2/3)*(c^5*d^6*x^8 - 7*c^6*d^5*x^5 - 8*c^7*d^4*x^2)*(-d^4/c^7)^(2/3) + 24*(1/4)^(1/3)*(c^3*d^7*x^7 + 5*c^4*d^6*x^4 + 4*c^5*d^5*x)*(-d^4/c^7)^(1/3) - 12*(9*(1/108)^(1/6)*c^2*d^7*x^5*(-d^4/c^7)^(1/6) - 18*(1/108)^(5/6)*(c^6*d^5*x^7 + 2*c^7*d^4*x^4 - 8*c^8*d^3*x)*(-d^4/c^7)^(5/6) - sqrt(1/3)*(c^4*d^6*x^6 - 16*c^5*d^5*x^3 - 8*c^6*d^4)*sqrt(-d^4/c^7))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d^7*x^4 + c*d^6*x)) - (1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*log((d^9*x^9 + 60*c*d^8*x^6 - 32*c^3*d^6 - 24*(1/4)^(2/3)*(c^5*d^6*x^8 - 7*c^6*d^5*x^5 - 8*c^7*d^4*x^2)*(-d^4/c^7)^(2/3) + 24*(1/4)^(1/3)*(c^3*d^7*x^7 + 5*c^4*d^6*x^4 + 4*c^5*d^5*x)*(-d^4/c^7)^(1/3) + 12*(9*(1/108)^(1/6)*c^2*d^7*x^5*(-d^4/c^7)^(1/6) - 18*(1/108)^(5/6)*(c^6*d^5*x^7 + 2*c^7*d^4*x^4 - 8*c^8*d^3*x)*(-d^4/c^7)^(5/6) - sqrt(1/3)*(c^4*d^6*x^6 - 16*c^5*d^5*x^3 - 8*c^6*d^4)*sqrt(-d^4/c^7))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*log((d^9*x^9 + 60*c*d^8*x^6 - 32*c^3*d^6 - 24*(1/4)^(2/3)*(c^5*d^6*x^8 - 7*c^6*d^5*x^5 - 8*c^7*d^4*x^2)*(-d^4/c^7)^(2/3) + 24*(1/4)^(1/3)*(c^3*d^7*x^7 + 5*c^4*d^6*x^4 + 4*c^5*d^5*x)*(-d^4/c^7)^(1/3) - 12*(9*(1/108)^(1/6)*c^2*d^7*x^5*(-d^4/c^7)^(1/6) - 18*(1/108)^(5/6)*(c^6*d^5*x^7 + 2*c^7*d^4*x^4 - 8*c^8*d^3*x)*(-d^4/c^7)^(5/6) - sqrt(1/3)*(c^4*d^6*x^6 - 16*c^5*d^5*x^3 - 8*c^6*d^4)*sqrt(-d^4/c^7))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 2*(1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*c^7*d*x^2)*(-d^4/c^7)^(2/3) - 48*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*x^4 - 2*c^5*d^2*x)*(-d^4/c^7)^(1/3) + 6*(18*(1/108)^(1/6)*c^2*d^4*x^5*(-d^4/c^7)^(1/6) + 36*(1/108)^(5/6)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x)*(-d^4/c^7)^(5/6) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt(-d^4/c^7))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*c^7*d*x^2)*(-d^4/c^7)^(2/3) - 48*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*x^4 - 2*c^5*d^2*x)*(-d^4/c^7)^(1/3) - 6*(18*(1/108)^(1/6)*c^2*d^4*x^5*(-d^4/c^7)^(1/6) + 36*(1/108)^(5/6)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x)*(-d^4/c^7)^(5/6) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt(-d^4/c^7))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 12*sqrt(d)*x^2*weierstrassPInverse(0, -4*c/d, x) + 12*sqrt(d*x^3 + c))/(c*x^2)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^3\cdot(4c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(d*x**3+4*c),x)
[Out] Integral(sqrt(c + d*x**3)/(x**3*(4*c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{x^3\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^3*(4*c + d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^3*(4*c + d*x^3)), x)
```

3.270 $\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=78

$$-\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\,d^3}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^3+32/9*c^(3/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(
1/2))/d^3*3^(1/2)-10/3*c*(d*x^3+c)^(1/2)/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {457, 90, 65, 209}

$$\frac{32c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\,d^3}-\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (-10*c*Sqrt[c + d*x^3])/(3*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (32*c^(3/
2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{5c}{d^2\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{d^2}+\frac{16c^2}{d^2\sqrt{c+dx}\,(4c+dx)}\right) dx, x, x^3\right)\\
&= -\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{(16c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{3d^2}\\
&= -\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{(32c^2)\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^3}\\
&= -\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\,d^3}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 65, normalized size = 0.83

$$
\frac{2(-14c+dx^3)\sqrt{c+dx^3}+32\sqrt{3}\,c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{9d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*(-14*c + d*x^3)*Sqrt[c + d*x^3] + 32*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*x
^3]/(Sqrt[3]*Sqrt[c])])/(9*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 464, normalized size = 5.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2)/d^2)-8/3*c*(d*x^3+c)^(
1/2)/d^3-16/9*I*c/d^5*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2
)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_a
lpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [A]
time = 0.50, size = 53, normalized size = 0.68

$$\frac{2\left(16\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}-15\sqrt{dx^3+c}\,c\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(16*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x^
3 + c)^(3/2) - 15*sqrt(d*x^3 + c)*c)/d^3
```

Fricas [A]
time = 2.81, size = 129, normalized size = 1.65

$$\left[\frac{2\left(8\sqrt{3}\,\sqrt{-c}\,c\log\left(\frac{dx^3+2\sqrt{3}\,\sqrt{dx^3+c}\,\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}\,(dx^3-14\,c)\right)}{9\,d^3},\ \frac{2\left(16\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{dx^3+c}\,(dx^3-14\,c)\right)}{9\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(8*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c)
- 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 14*c))/d^3, 2/9*(16*sqrt(3
)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*(d*
x^3 - 14*c))/d^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^8}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [A]
time = 1.12, size = 64, normalized size = 0.82

$$\frac{32\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{9\,d^3}+\frac{2\left((dx^3+c)^{\frac{3}{2}}d^6-15\,\sqrt{dx^3+c}\,cd^6\right)}{9\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 32/9*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/9*
((d*x^3 + c)^(3/2)*d^6 - 15*sqrt(d*x^3 + c)*c*d^6)/d^9
```

Mupad [B]
time = 5.38, size = 88, normalized size = 1.13

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}-\frac{28\,c\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,16\mathrm{i}}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*d^2) - (28*c*(c + d*x^3)^(1/2))/(9*d^3) + (3^(
1/2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^
3)/(4*c + d*x^3))*16i)/(9*d^3)
```

3.271 $\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=59

$$\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\sqrt{3}\,d^2}$$

```
[Out] -8/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*c^(1/2)/d^2*3^(1/2)+2/3*(d
*x^3+c)^(1/2)/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {457, 81, 65, 209}

$$\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\sqrt{3}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^2) - (8*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sq
rt[c])])/(3*Sqrt[3]*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)\\ &= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{(4c)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{3d}\\ &= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{(8c)\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^2}\\ &= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{8\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\sqrt{3}\,d^2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.95

$$\frac{6\sqrt{c+dx^3} - 8\sqrt{3}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (6*Sqrt[c + d*x^3] - 8*Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(9*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 425, normalized size = 7.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d^2+4/9*I/d^4*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x
-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2
)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [A]
time = 0.50, size = 43, normalized size = 0.73

$$-\frac{2\left(4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{dx^3+c}\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqr
t(d*x^3 + c))/d^2
```

Fricas [A]
time = 2.55, size = 112, normalized size = 1.90

$$\left[\frac{2\left(2\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+3\sqrt{dx^3+c}\right)}{9\,d^2},-\frac{2\left(4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(2*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) -
2*c)/(d*x^3 + 4*c)) + 3*sqrt(d*x^3 + c))/d^2, -2/9*(4*sqrt(3)*sqrt(c)*arcta
n(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqrt(d*x^3 + c))/d^2]
```

Sympy [A]
time = 7.00, size = 65, normalized size = 1.10

$$\begin{cases} \frac{2\left(-\frac{4\sqrt{3}\sqrt{c}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d}+\frac{\sqrt{c+dx^3}}{3d}\right)}{d} & \text{for}\: d \neq 0 \\ \frac{x^6}{24c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-4*sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))
/(9*d) + sqrt(c + d*x**3)/(3*d))/d, Ne(d, 0)), (x**6/(24*c**(3/2)), True))
```

Giac [A]
time = 1.74, size = 49, normalized size = 0.83

$$-\frac{2\left(\frac{4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{d}-\frac{3\sqrt{dx^3+c}}{d}\right)}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d - 3*s
qrt(d*x^3 + c)/d)/d
```

Mupad [B]
time = 4.86, size = 71, normalized size = 1.20

$$\frac{2\sqrt{d\,x^3+c}}{3\,d^2}+\frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,4\mathrm{i}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^2) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2)
*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*4i)/(9*d^2)
```

3.272 $\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}$$

[Out] 2/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))/d*3^(1/2)/c^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {455, 65, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.34, size = 413, normalized size = 10.32

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d+4*c))
```

Maxima [A]
time = 0.50, size = 29, normalized size = 0.72

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

Fricas [A]

time = 2.60, size = 87, normalized size = 2.18

$$\left[-\frac{\sqrt{3}\,\sqrt{-c}\,\log\left(\frac{dx^3-2\sqrt{3}\,\sqrt{dx^3+c}\,\sqrt{-c}-2c}{dx^3+4c}\right)}{9cd},\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/9*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*
c)/(d*x^3 + 4*c))/(c*d), 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqr
t(c))/(sqrt(c)*d)]
```

Sympy [A]
time = 5.18, size = 37, normalized size = 0.92

$$\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] 2*sqrt(3)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(9*sqrt(c)*d)
```

Giac [A]
time = 1.68, size = 29, normalized size = 0.72

$$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

Mupad [B]
time = 5.21, size = 56, normalized size = 1.40

$$\frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{2\,d\,x^3+8\,c}\right)\,1\mathrm{i}}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(
8*c + 2*d*x^3))*1i)/(9*c^(1/2)*d)
```

3.273 $\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=65

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{6\sqrt{3}\,c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

```
[Out] -1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/18*arctan(1/3*(d*x^3+c)^(1/
2)*3^(1/2)/c^(1/2))/c^(3/2)*3^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {457, 88, 65, 214, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{6\sqrt{3}\,c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/6*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(Sqrt[3]*c^(3/2)) - ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]]/(6*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\,(4c+dx)}\,dx,x,x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{12c} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx,x,x^3\right)}{12c}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx,x,\sqrt{c+dx^3}\right)}{6c} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{6cd}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}\,c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.91

$$-\frac{\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)+3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
[Out] -1/18*(Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] + 3*ArcTanh[Sqrt[c
 + d*x^3]/Sqrt[c]])/c^(3/2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 433, normalized size = 6.66

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-}}{}\right.}{}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*I/d^2/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*d+4*c))-1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x), x)
```

Fricas [A]
time = 3.35, size = 148, normalized size = 2.28

$$\left[-\frac{2\,\sqrt{3}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)-3\,\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)}{36\,c^2},-\frac{\sqrt{3}\,\sqrt{-c}\,\log\left(\frac{dx^3+2\,\sqrt{3}\,\sqrt{dx^3+c}\,\sqrt{-c}-2\,c}{dx^3+4\,c}\right)-6\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)}{36\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/36*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*s
qrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, -1/36*(sqrt
(3)*sqrt(-c)*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3
+ 4*c)) - 6*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c^2]
```

Sympy [A]
time = 3.84, size = 63, normalized size = 0.97

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{6c\sqrt{-c}}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**3)/sqrt(-c))/(6*c*sqrt(-c)) - sqrt(3)*atan(sqrt(3)*sqrt(
c + d*x**3)/(3*sqrt(c)))/(18*c**(3/2))
```

Giac [A]
time = 1.43, size = 53, normalized size = 0.82

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{18\,c^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\,\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/6*arc
tan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

Mupad [B]
time = 5.51, size = 94, normalized size = 1.45

$$\frac{\ln\left(\frac{{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\,c^{3/2}}+\frac{\sqrt{3}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,1{}\mathrm{i}}{36\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(1
2*c^(3/2)) + (3^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(
1/2)*d*x^3)/(4*c + d*x^3))*1i)/(36*c^(3/2))
```

3.274 $\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=88

$$-\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{24\sqrt{3}\,c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

```
[Out] 1/8*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/72*d*arctan(1/3*(d*x^3+c)^
(1/2)*3^(1/2)/c^(1/2))/c^(5/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c^2/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {457, 105, 162, 65, 214, 209}

$$\frac{d\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{24\sqrt{3}\,c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}-\frac{\sqrt{c+dx^3}}{12c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
]])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2)
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{\text{Subst}\left(\int \frac{3cd+\frac{d^2x}{2}}{x\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{12c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{16c^2} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+\cdots}\right)}{48c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{8c^2} + \frac{d\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \cdots\right)}{24c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} + \frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}\,c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 88, normalized size = 1.00

$$-\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}\,c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2)
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.45, size = 477, normalized size = 5.42

method	result

risch	$-\frac{\sqrt{d\,x^{3}+c}}{12c^{2}x^{3}}-\frac{d\left(i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\cdots\right.}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}\cdots}}}{\cdots}\right)}{\cdots}\right)}{\cdots}$
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right)}{\cdots}$

elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/144*I/c^3/d*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*d+4*c))+1/4/c*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d
*x^3+c)^(1/2)/c^(1/2))/c^(3/2))+1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(
5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^4), x)
```

Fricas [A]
time = 3.69, size = 194, normalized size = 2.20

$$\left[\frac{2\sqrt{3}\sqrt{c}\,dx^3\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+9\sqrt{c}\,dx^3\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-12\sqrt{dx^3+c}\,c}{144\,c^3x^3},-\frac{\sqrt{3}\sqrt{-c}\,dx^3\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+18\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+12\sqrt{dx^3+c}\,c}{144\,c^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/144*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))
 + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 12*
sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/144*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 - 2
*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + 18*sqrt(-c)*d*x^3
*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [A]
time = 1.27, size = 72, normalized size = 0.82

$$\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{72\,c^{\frac{5}{2}}}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}\,c^2}-\frac{\sqrt{dx^3+c}}{12\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(5/2) - 1/8*d*
arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/12*sqrt(d*x^3 + c)/(c^2
*x^3)
```

Mupad [B]
time = 5.72, size = 112, normalized size = 1.27

$$\frac{d\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\,{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}^3}{x^6}\right)}{16\,c^{5/2}}-\frac{\sqrt{d\,x^3+c}}{12\,c^2\,x^3}+\frac{\sqrt{3}\,d\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,1{}\mathrm{i}}{144\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (d*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)
)/(16*c^(5/2)) - (c + d*x^3)^(1/2)/(12*c^2*x^3) + (3^(1/2)*d*log((c^(1/2)*(
c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(144*
c^(5/2))
```

3.275 $\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=667

$$\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{2\sqrt[3]{2}\,\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}}-\frac{2\sqrt[3]{2}\,\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{3}\,\sqrt{c}}\right.}{3\sqrt{3}\,d^{5/3}}$$

```
[Out] 2/3*2^(1/3)*c^(1/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(
1/2))/d^(5/3)-2/9*2^(1/3)*c^(1/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)+
2/9*2^(1/3)*c^(1/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x
^3+c)^(1/2))/d^(5/3)*3^(1/2)-2/9*2^(1/3)*c^(1/6)*arctan(1/3*(d*x^3+c)^(1/2)
*3^(1/2)/c^(1/2))/d^(5/3)*3^(1/2)+2*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))+2/3*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/
3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE
((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(
1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 667, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {494, 309, 224, 1891, 497}

$$\frac{2\sqrt{2}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}+\frac{2\sqrt[3]{2}\,\sqrt[6]{c}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}}-\frac{2\sqrt[3]{2}\,\sqrt[6]{c}\,\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\sqrt{3}\,d^{5/3}}+\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{2\sqrt[3]{2}\,\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3d^{5/3}}-\frac{2\sqrt[3]{2}\,\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (2*2^(1
/3)*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c +
 d*x^3]])/(3*Sqrt[3]*d^(5/3)) - (2*2^(1/3)*c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(
Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^(5/3)) + (2*2^(1/3)*c^(1/6)*ArcTanh[(c^(1/6
)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*d^(5/3)) - (2*2^(1/3)
*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(9*d^(5/3)) - (3^(1/4)*Sqrt[2 -
```

```
Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3
)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(5/3)*Sqrt[(c
^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
 + d*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{d} - \frac{(4c)\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx}{d}$$

$$= \frac{2\sqrt[3]{2}\,\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}} - \frac{2\sqrt[3]{2}\,\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right.}{3\sqrt{3}\,d^{5/3}}$$

$$= \frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{2\sqrt[3]{2}\,\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right.}{\sqrt{c+dx^3}}\right.}{3\sqrt{3}\,d^{5/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.03, size = 67, normalized size = 0.10

$$\frac{x^5\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{20c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/(20*c*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 848, normalized size = 1.27

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+4/9*I/d^4*2^(1/2)*sum(
1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)
*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d
+4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 15.49, size = 3600, normalized size = 5.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(4*sqrt(3)*(4/27)^(1/6)*d^2*(-c/d^10)^(1/6)*arctan(-1/3*(108*4^(2/3)*s
qrt(3)*(c*d^12*x^16 - 39*c^2*d^11*x^13 - 72*c^3*d^10*x^10 - 32*c^4*d^9*x^7)
*(-c/d^10)^(2/3) + 12*4^(1/3)*sqrt(3)*(c*d^9*x^17 - 271*c^2*d^8*x^14 + 112*
c^3*d^7*x^11 + 1216*c^4*d^6*x^8 + 1088*c^5*d^5*x^5 + 256*c^6*d^4*x^2)*(-c/d
^10)^(1/3) + 3*sqrt(1/3)*(324*sqrt(3)*(4/27)^(5/6)*(d^14*x^16 + 229*c*d^13*
x^13 + 492*c^2*d^12*x^10 + 328*c^3*d^11*x^7 + 64*c^4*d^10*x^4)*(-c/d^10)^(5
/6) + 12*sqrt(3)*sqrt(1/3)*(d^11*x^17 + 737*c*d^10*x^14 + 2704*c^2*d^9*x^11
 + 3376*c^3*d^8*x^8 + 1664*c^4*d^7*x^5 + 256*c^5*d^6*x^2)*sqrt(-c/d^10) + s
qrt(3)*(4/27)^(1/6)*(d^8*x^18 + 1098*c*d^7*x^15 - 24720*c^2*d^6*x^12 - 5670
4*c^3*d^5*x^9 - 44928*c^4*d^4*x^6 - 15360*c^5*d^3*x^3 - 2048*c^6*d^2)*(-c/d
^10)^(1/6) - sqrt(d*x^3 + c)*(4^(2/3)*sqrt(3)*(5*d^12*x^15 - 3272*c*d^11*x^
12 - 12544*c^2*d^10*x^9 - 14656*c^3*d^9*x^6 - 6656*c^4*d^8*x^3 - 1024*c^5*d
^7)*(-c/d^10)^(2/3) - 1728*4^(1/3)*sqrt(3)*(c*d^8*x^13 + 2*c^2*d^7*x^10 + c
^3*d^6*x^7)*(-c/d^10)^(1/3) - 12*sqrt(3)*(17*c*d^5*x^14 - 1456*c^2*d^4*x^11
 - 2544*c^3*d^3*x^8 - 1408*c^4*d^2*x^5 - 256*c^5*d*x^2)))*sqrt((24*c^2*d^2*
x^8 - 168*c^3*d*x^5 - 192*c^4*x^2 - 4^(2/3)*(c*d^9*x^9 + 60*c^2*d^8*x^6 - 3
2*c^4*d^6)*(-c/d^10)^(2/3) - 24*4^(1/3)*(c^2*d^5*x^7 + 5*c^3*d^4*x^4 + 4*c^
4*d^3*x)*(-c/d^10)^(1/3) + 6*(36*sqrt(1/3)*c^2*d^6*x^5*sqrt(-c/d^10) + 9*(4
/27)^(5/6)*(c*d^10*x^7 + 2*c^2*d^9*x^4 - 8*c^3*d^8*x)*(-c/d^10)^(5/6) - 4*(
4/27)^(1/6)*(c^2*d^3*x^6 - 16*c^3*d^2*x^3 - 8*c^4*d)*(-c/d^10)^(1/6))*sqrt(
d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + sqrt(3)*(c*
d^6*x^18 - 1416*c^2*d^5*x^15 + 14352*c^3*d^4*x^12 + 44480*c^4*d^3*x^9 + 499
20*c^5*d^2*x^6 + 24576*c^6*d*x^3 + 4096*c^7) - 6*sqrt(d*x^3 + c)*(27*sqrt(3
)*(4/27)^(5/6)*(31*c*d^13*x^14 + 1744*c^2*d^12*x^11 + 2976*c^3*d^11*x^8 + 1
600*c^4*d^10*x^5 + 256*c^5*d^9*x^2)*(-c/d^10)^(5/6) + 24*sqrt(3)*sqrt(1/3)*
(c*d^10*x^15 + 157*c^2*d^9*x^12 + 348*c^3*d^8*x^9 + 256*c^4*d^7*x^6 + 64*c^
5*d^6*x^3)*sqrt(-c/d^10) + 2*sqrt(3)*(4/27)^(1/6)*(c*d^7*x^16 + 686*c^2*d^6
*x^13 + 7072*c^3*d^5*x^10 + 11008*c^4*d^4*x^7 + 5888*c^5*d^3*x^4 + 1024*c^6
*d^2*x)*(-c/d^10)^(1/6)))/(c*d^6*x^18 + 2184*c^2*d^5*x^15 + 57696*c^3*d^4*x
^12 + 125696*c^4*d^3*x^9 + 100608*c^5*d^2*x^6 + 33792*c^6*d*x^3 + 4096*c^7)
) - 4*sqrt(3)*(4/27)^(1/6)*d^2*(-c/d^10)^(1/6)*arctan(-1/3*(108*4^(2/3)*sqr
t(3)*(c*d^12*x^16 - 39*c^2*d^11*x^13 - 72*c^3*d^10*x^10 - 32*c^4*d^9*x^7)*(
-c/d^10)^(2/3) + 12*4^(1/3)*sqrt(3)*(c*d^9*x^17 - 271*c^2*d^8*x^14 + 112*c^
3*d^7*x^11 + 1216*c^4*d^6*x^8 + 1088*c^5*d^5*x^5 + 256*c^6*d^4*x^2)*(-c/d^1
0)^(1/3) - 3*sqrt(1/3)*(324*sqrt(3)*(4/27)^(5/6)*(d^14*x^16 + 229*c*d^13*x^
13 + 492*c^2*d^12*x^10 + 328*c^3*d^11*x^7 + 64*c^4*d^10*x^4)*(-c/d^10)^(5/6
) + 12*sqrt(3)*sqrt(1/3)*(d^11*x^17 + 737*c*d^10*x^14 + 2704*c^2*d^9*x^11 +
 3376*c^3*d^8*x^8 + 1664*c^4*d^7*x^5 + 256*c^5*d^6*x^2)*sqrt(-c/d^10) + sqr
t(3)*(4/27)^(1/6)*(d^8*x^18 + 1098*c*d^7*x^15 - 24720*c^2*d^6*x^12 - 56704*
c^3*d^5*x^9 - 44928*c^4*d^4*x^6 - 15360*c^5*d^3*x^3 - 2048*c^6*d^2)*(-c/d^1
0)^(1/6) + sqrt(d*x^3 + c)*(4^(2/3)*sqrt(3)*(5*d^12*x^15 - 3272*c*d^11*x^12
```

```
 - 12544*c^2*d^10*x^9 - 14656*c^3*d^9*x^6 - 6656*c^4*d^8*x^3 - 1024*c^5*d^7
)*(-c/d^10)^(2/3) - 1728*4^(1/3)*sqrt(3)*(c*d^8*x^13 + 2*c^2*d^7*x^10 + c^3
*d^6*x^7)*(-c/d^10)^(1/3) - 12*sqrt(3)*(17*c*d^5*x^14 - 1456*c^2*d^4*x^11 -
 2544*c^3*d^3*x^8 - 1408*c^4*d^2*x^5 - 256*c^5*d*x^2)))*sqrt((24*c^2*d^2*x^
8 - 168*c^3*d*x^5 - 192*c^4*x^2 - 4^(2/3)*(c*d^9*x^9 + 60*c^2*d^8*x^6 - 32*
c^4*d^6)*(-c/d^10)^(2/3) - 24*4^(1/3)*(c^2*d^5*x^7 + 5*c^3*d^4*x^4 + 4*c^4*
d^3*x)*(-c/d^10)^(1/3) - 6*(36*sqrt(1/3)*c^2*d^6*x^5*sqrt(-c/d^10) + 9*(4/2
7)^(5/6)*(c*d^10*x^7 + 2*c^2*d^9*x^4 - 8*c^3*d^8*x)*(-c/d^10)^(5/6) - 4*(4/
27)^(1/6)*(c^2*d^3*x^6 - 16*c^3*d^2*x^3 - 8*c^4*d)*(-c/d^10)^(1/6))*sqrt(d*
x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + sqrt(3)*(c*d^
6*x^18 - 1416*c^2*d^5*x^15 + 14352*c^3*d^4*x^12 + 44480*c^4*d^3*x^9 + 49920
*c^5*d^2*x^6 + 24576*c^6*d*x^3 + 4096*c^7) + 6*sqrt(d*x^3 + c)*(27*sqrt(3)*
(4/27)^(5/6)*(31*c*d^13*x^14 + 1744*c^2*d^12*x^11 + 2976*c^3*d^11*x^8 + 160
0*c^4*d^10*x^5 + 256*c^5*d^9*x^2)*(-c/d^10)^(5/6) + 24*sqrt(3)*sqrt(1/3)*(c
*d^10*x^15 + 157*c^2*d^9*x^12 + 348*c^3*d^8*x^9 + 256*c^4*d^7*x^6 + 64*c^5*
d^6*x^3)*sqrt(-c/d^10) + 2*sqrt(3)*(4/27)^(1/6)*(c*d^7*x^16 + 686*c^2*d^6*x
^13 + 7072*c^3*d^5*x^10 + 11008*c^4*d^4*x^7 + 5888*c^5*d^3*x^4 + 1024*c^6*d
^2*x)*(-c/d^10)^(1/6)))/(c*d^6*x^18 + 2184*c^2*d^5*x^15 + 57696*c^3*d^4*x^1
2 + 125696*c^4*d^3*x^9 + 100608*c^5*d^2*x^6 + 33792*c^6*d*x^3 + 4096*c^7))
+ (4/27)^(1/6)*d^2*(-c/d^10)^(1/6)*log(16384/3*(24*c^2*d^2*x^8 - 168*c^3*d*
x^5 - 192*c^4*x^2 - 4^(2/3)*(c*d^9*x^9 + 60*c^2*d^8*x^6 - 32*c^4*d^6)*(-c/d
^10)^(2/3) - 24*4^(1/3)*(c^2*d^5*x^7 + 5*c^3*d^4*x^4 + 4*c^4*d^3*x)*(-c/d^1
0)^(1/3) + 6*(36*sqrt(1/3)*c^2*d^6*x^5*sqrt(-c/d^10) + 9*(4/27)^(5/6)*(c*d^
10*x^7 + 2*c^2*d^9*x^4 - 8*c^3*d^8*x)*(-c/d^10)^(5/6) - 4*(4/27)^(1/6)*(c^2
*d^3*x^6 - 16*c^3*d^2*x^3 - 8*c^4*d)*(-c/d^10)^(1/6))*sqrt(d*x^3 + c))/(d^3
*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) -...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
[Out] int(x^4/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.276 $\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=206

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}\,c^{5/6}d^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}\,c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{\ldots}\right.}{9\ 2^{2/3}c\ldots}$$

```
[Out] -1/6*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2^(1/3)/c
^(5/6)/d^(2/3)+1/18*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)/d^(2/3
)-1/18*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*
2^(1/3)/c^(5/6)/d^(2/3)*3^(1/2)+1/18*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(
1/2))*2^(1/3)/c^(5/6)/d^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {497}

$$-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}\,c^{5/6}d^{2/3}}+\frac{\mathrm{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}\,c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]
]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[
c])]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1
/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\,\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}\,c^{5/6}d^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}\,c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\right.}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.03, size = 67, normalized size = 0.33

$$\frac{x^2\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/(8*c*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.33, size = 416, normalized size = 2.02

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right)}{}$

elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{}}{}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2
)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_a
lpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2274 vs. $2(141) = 282$.
time = 4.03, size = 2274, normalized size = 11.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] 1/9*sqrt(3)*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*arctan(-1/3*(3*(sqrt(3)*sqrt(1/3)*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 + 4*c^5*d^3)*(-1/(c^5*d^4))^(5/6))*sqrt(d*x^3 + c) + (2*sqrt(3)*(1/2)^(1/3)*(c^2*d^2*x^3 + c^3*d)*(-1/(c^5*d^4))^(1/3) + sqrt(3)*(d*x^4 + c*x) + 3*(sqrt(3)*sqrt(1/3)*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(1/6) - 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 - 2*c^5*d^3)*(-1/(c^5*d^4))^(5/6))*sqrt(d*x^3 + c))*sqrt((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(2/3)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3) + 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 + c*x)) + 1/9*sqrt(3)*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*arctan(-1/3*(3*(sqrt(3)*sqrt(1/3)*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 + 4*c^5*d^3)*(-1/(c^5*d^4))^(5/6))*sqrt(d*x^3 + c) - (2*sqrt(3)*(1/2)^(1/3)*(c^2*d^2*x^3 + c^3*d)*(-1/(c^5*d^4))^(1/3) + sqrt(3)*(d*x^4 + c*x) - 3*(sqrt(3)*sqrt(1/3)*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(1/6) - 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 - 2*c^5*d^3)*(-1/(c^5*d^4))^(5/6))*sqrt(d*x^3 + c))*sqrt((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(2/3)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3) - 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 + c*x)) - 1/36*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(2/3)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3) + 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/36*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(2/3)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3) - 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 +

```
 c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/18*(1/432)^(1/6)
*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 +
 48*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x)*(-1/(c^5*d^4))^(2
/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5
*d^4))^(1/3) + 6*(1296*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) + sqr
t(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) + 2
*(1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6
))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 1/1
8*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d
*x^3 - 32*c^3 + 48*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x)*(-
1/(c^5*d^4))^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^
2*x^2)*(-1/(c^5*d^4))^(1/3) - 6*(1296*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^
4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1
/(c^5*d^4)) + 2*(1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/
(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 +
 64*c^3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c + dx^3} \cdot (4c + dx^3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [B]
time = 25.80, size = 453, normalized size = 2.20

$$\frac{\sqrt{3}\,314928^{1/3}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{3}\,\sqrt{-c}-2^{1/3}\,\sqrt{3}\,{(-c)}^{1/6}\,d^{1/3}\,x\right)^3\,\left(54\,\sqrt{d\,x^3+c}-54\,\sqrt{3}\,\sqrt{-c}+54\,2^{1/3}\,\sqrt{3}\,{(-c)}^{1/6}\,d^{1/3}\,x\right)}{{\left(d^{1/3}\,x-2^{2/3}\,{(-c)}^{1/3}\right)}^6}\right)}{2916\,{(-c)}^{5/6}\,d^{2/3}}+\frac{\sqrt{3}\,314928^{1/3}\,\ln\left(\frac{\left(2\,\sqrt{3}\,\sqrt{-c}-2\,\sqrt{d\,x^3+c}+2^{1/3}\,\sqrt{3}\,{(-c)}^{1/6}\,d^{1/3}\,x+2^{1/3}\,{(-c)}^{1/6}\,d^{1/3}\,x\,3{}\mathrm{i}\right)^3\,\left(108\,\sqrt{d\,x^3+c}+108\,\sqrt{3}\,\sqrt{-c}+54\,2^{1/3}\,\sqrt{3}\,{(-c)}^{1/6}\,d^{1/3}\,x+2^{1/3}\,{(-c)}^{1/6}\,d^{1/3}\,x\,162{}\mathrm{i}\right)}{{\left(2\,d^{1/3}\,x+2^{2/3}\,{(-c)}^{1/3}-2^{2/3}\,\sqrt{3}\,{(-c)}^{1/3}\,1{}\mathrm{i}\right)}^6}\right)\,\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}{2916\,{(-c)}^{5/6}\,d^{2/3}}+\frac{\sqrt{3}\,314928^{1/3}\,\ln\left(\frac{\left(2\,\sqrt{d\,x^3+c}+2\,\sqrt{3}\,\sqrt{-c}+2^{1/3}\,\sqrt{3}\,{(-c)}^{1/6}\,d^{1/3}\,x-2^{1/3}\,{(-c)}^{1/6}\,d^{1/3}\,x\,3{}\mathrm{i}\right)^3\,\left(108\,\sqrt{d\,x^3+c}-108\,\sqrt{3}\,\sqrt{-c}-54\,2^{1/3}\,\sqrt{3}\,{(-c)}^{1/6}\,d^{1/3}\,x+2^{1/3}\,{(-c)}^{1/6}\,d^{1/3}\,x\,162{}\mathrm{i}\right)}{{\left(2\,d^{1/3}\,x+2^{2/3}\,{(-c)}^{1/3}+2^{2/3}\,\sqrt{3}\,{(-c)}^{1/3}\,1{}\mathrm{i}\right)}^6}\right)\,\sqrt{\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\,1{}\mathrm{i}}{2916\,{(-c)}^{5/6}\,d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (3^(1/2)*314928^(1/3)*log((((c + d*x^3)^(1/2) + 3^(1/2)*(-c)^(1/2) - 2^(1/3
)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(54*(c + d*x^3)^(1/2) - 54*3^(1/2)*(-c)^(
1/2) + 54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(d^(1/3)*x - 2^(2/3)*(-c)^
(1/3))^6))/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)*314928^(1/3)*log(((2*3^(1/2
)*(-c)^(1/2) - 2*(c + d*x^3)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*3i + 2^(1
/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d*x^3)^(1/2) + 108*3^(1/2)*(-
c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i + 54*2^(1/3)*3^(1/2)*(-c)^(1/6
)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3) - 2^(2/3)*3^(1/2)*(-c)^(1/3
)*1i)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)
*314928^(1/3)*log(((2*(c + d*x^3)^(1/2) + 2*3^(1/2)*(-c)^(1/2) - 2^(1/3)*(-
c)^(1/6)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d
*x^3)^(1/2) - 108*3^(1/2)*(-c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i -
54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3)
 + 2^(2/3)*3^(1/2)*(-c)^(1/3)*1i)^6)*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/(2916
*(-c)^(5/6)*d^(2/3))
```

3.277 $\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=697

$$-\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{4c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{12\;2^{2/3}\sqrt{3}\,c^{11/6}}-\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{c+\ldots}}{\sqrt{3}\ldots}\right)}{12\;2^{2/3}\sqrt{3}\,c^{11/\ldots}}$$

```
[Out] 1/24*d^(1/3)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2
^(1/3)/c^(11/6)-1/72*d^(1/3)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(11
/6)+1/72*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+
c)^(1/2))*2^(1/3)/c^(11/6)*3^(1/2)-1/72*d^(1/3)*arctan(1/3*(d*x^3+c)^(1/2)*
3^(1/2)/c^(1/2))*2^(1/3)/c^(11/6)*3^(1/2)-1/4*(d*x^3+c)^(1/2)/c^2/x+1/4*d^(
1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/12*d^(1/3)*(c^(1
/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)
/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/8*
3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)
))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))
*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {491, 598, 309, 224, 1891, 497}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{2}\,\sqrt[4]{3}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{8c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[3]{d}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{12\;2^{2/3}\sqrt{3}\,c^{11/6}}-\frac{\sqrt[3]{d}\,\text{ArcTan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{12\;2^{2/3}\sqrt{3}\,c^{11/6}}+\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{12\;2^{2/3}c^{11/6}}-\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36\;2^{2/3}c^{11/6}}-\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{4c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/4*Sqrt[c + d*x^3]/(c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(4*c^2*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) + (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(
1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(12*2^(2/3)*Sqrt[3]*c^(11/6)) - (d^(1/3)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(12*2^(2/3)*Sqrt[3]*c^(11/6)) +
 (d^(1/3)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])
```

```
/(12*2^(2/3)*c^(11/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(36*2^(
2/3)*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*c^(5/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) +
 (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]
])/(2*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
```

```
0] && PosQ[c]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\int \frac{x\left(cd+\frac{d^2x^3}{2}\right)}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx}{4c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}}-\frac{cdx}{\sqrt{c+dx^3}\,(4c+dx^3)}\right)dx}{4c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{8c^2}-\frac{d\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx}{4c}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{12\,2^{2/3}\sqrt{3}\,c^{11/6}}-\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{c}\ldots}{\sqrt{3}\ldots}\right.}{12\,2^{2/3}\sqrt{3}\,c\ldots}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{4c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}\ldots\right.}{\sqrt{c+\ldots}}\right.}{12\,2^{2/3}\sqrt{3}\,c^{11\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 20.06, size = 136, normalized size = 0.20

$$\frac{-40c(c+dx^3)+5cdx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+d^2x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{160c^3x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (-40*c*(c + d*x^3) + 5*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), -1/4*(d*x^3)/c] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(160*c^3*x*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 874, normalized size = 1.25

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	874

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*I/d^2/c^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*
_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*d+4*c))+1/4/c*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/
2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1
```

/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.16, size = 2351, normalized size = 3.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/144*(4*sqrt(3)*(1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6)*arctan(-1/3*(3*(sqrt(3)*sqrt(1/3)*c^6*d^2*x*sqrt(-d^2/c^11) + 2*sqrt(3)*(1/432)^(1/6)*c^2*d^3*x^2*(-d^2/c^11)^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^9*d^2*x^3 + 4*c^10*d)*(-d^2/c^11)^(5/6))*sqrt(d*x^3 + c) + (2*sqrt(3)*(1/2)^(1/3)*(c^4*d^2*x^3 + c^5*d)*(-d^2/c^11)^(1/3) + sqrt(3)*(d^3*x^4 + c*d^2*x) + 3*(sqrt(3)*sqrt(1/3)*c^6*d*x*sqrt(-d^2/c^11) + 2*sqrt(3)*(1/432)^(1/6)*c^2*d^2*x^2*(-d^2/c^11)^(1/6) - 24*sqrt(3)*(1/432)^(5/6)*(c^9*d*x^3 - 2*c^10)*(-d^2/c^11)^(5/6))*sqrt(d*x^3 + c))*sqrt((d^5*x^9 + 60*c*d^4*x^6 - 32*c^3*d^2 - 24*(1/2)^(2/3)*(c^8*d^3*x^7 + 5*c^9*d^2*x^4 + 4*c^10*d*x)*(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^4*x^8 - 7*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(-d^2/c^11)^(1/3) + 12*(648*(1/432)^(5/6)*c^10*d^2*x^5*(-d^2/c^11)^(5/6) - sqrt(1/3)*(c^6*d^3*x^6 - 16*c^7*d^2*x^3 - 8*c^8*d)*sqrt(-d^2/c^11) - (1/432)^(1/6)*(c^2*d^4*x^7 + 2*c^3*d^3*x^4 - 8*c^4*d^2*x)*(-d^2/c^11)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d^4*x^4 + c*d^3*x)) + 4*sqrt(3)*(1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6)*arctan(-1/3*(3*(sqrt(3)*sqrt(1/3)*c^6*d^2*x*sqrt(-d^2/c^11) + 2*sqrt(3)*(1/432)^(1/6)*c^2*d^3*x^2*(-d^2/c^11)^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^9*d^2*x^3 + 4*c^10*d)*(-d^2/c^11)^(5/6))*sqrt(d*x^3 + c) - (2*sqrt(3)*(1/2)^(1/3)*(c^4*d^2*x^3 + c^5*d)*(-d^2/c^11)^(1/3) + sqrt(3)*(d^3*x^4 + c*d^2*x) - 3*(sqrt(3)*sqrt(1/3)*c^6*d*x*sqrt(-d^2/c^11) + 2*sqrt(3)*(1/432)^(1/6)*c^2*d^2*x^2*(-d^2/c^11)^(1/6) - 24*sqrt(3)*(1/432)^(5/6)*(c^9*d*x^3 - 2*c^10)*(-d^2/c^11)^(5/6))*sqrt(d*x^3 + c))*sqrt((d^5*x^9 + 60*c*d^4*x^6 - 32*c^3*d^2 - 24*(1/2)^(2/3)*(c^8*d^3*x^7 + 5*c^9*d^2*x^4 + 4*c^10*d*x)*(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^4*x^8 - 7*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(-d^2/c^11)^(1/3) - 12*(648*(1/432)^(5/6)*c^10*d
```

```
^2*x^5*(-d^2/c^11)^(5/6) - sqrt(1/3)*(c^6*d^3*x^6 - 16*c^7*d^2*x^3 - 8*c^8*
d)*sqrt(-d^2/c^11) - (1/432)^(1/6)*(c^2*d^4*x^7 + 2*c^3*d^3*x^4 - 8*c^4*d^2
*x)*(-d^2/c^11)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*
x^3 + 64*c^3)))/(d^4*x^4 + c*d^3*x)) - (1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6
)*log((d^5*x^9 + 60*c*d^4*x^6 - 32*c^3*d^2 - 24*(1/2)^(2/3)*(c^8*d^3*x^7 +
5*c^9*d^2*x^4 + 4*c^10*d*x)*(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^4*x^8
 - 7*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(-d^2/c^11)^(1/3) + 12*(648*(1/432)^(5/6)
*c^10*d^2*x^5*(-d^2/c^11)^(5/6) - sqrt(1/3)*(c^6*d^3*x^6 - 16*c^7*d^2*x^3 -
 8*c^8*d)*sqrt(-d^2/c^11) - (1/432)^(1/6)*(c^2*d^4*x^7 + 2*c^3*d^3*x^4 - 8*
c^4*d^2*x)*(-d^2/c^11)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48
*c^2*d*x^3 + 64*c^3)) + (1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6)*log((d^5*x^9
+ 60*c*d^4*x^6 - 32*c^3*d^2 - 24*(1/2)^(2/3)*(c^8*d^3*x^7 + 5*c^9*d^2*x^4 +
 4*c^10*d*x)*(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^4*x^8 - 7*c^5*d^3*x^
5 - 8*c^6*d^2*x^2)*(-d^2/c^11)^(1/3) - 12*(648*(1/432)^(5/6)*c^10*d^2*x^5*(
-d^2/c^11)^(5/6) - sqrt(1/3)*(c^6*d^3*x^6 - 16*c^7*d^2*x^3 - 8*c^8*d)*sqrt(
-d^2/c^11) - (1/432)^(1/6)*(c^2*d^4*x^7 + 2*c^3*d^3*x^4 - 8*c^4*d^2*x)*(-d^
2/c^11)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64
*c^3)) + 2*(1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^
6 - 72*c^2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2
*c^10*x)*(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 -
8*c^6*d*x^2)*(-d^2/c^11)^(1/3) + 6*(1296*(1/432)^(5/6)*c^10*d*x^5*(-d^2/c^1
1)^(5/6) + sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d*x^3 - 16*c^8)*sqrt(-d^2/c^11
) + 2*(1/432)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x)*(-d^2/c^11)^
(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) -
 2*(1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c
^2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2*c^10*x)
*(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d*
x^2)*(-d^2/c^11)^(1/3) - 6*(1296*(1/432)^(5/6)*c^10*d*x^5*(-d^2/c^11)^(5/6)
 + sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d*x^3 - 16*c^8)*sqrt(-d^2/c^11) + 2*(1
/432)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x)*(-d^2/c^11)^(1/6))*s
qrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 36*sqrt
(d)*x*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 36*sq
rt(d*x^3 + c))/(c^2*x)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] int(1/(x^2*(c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.278 $\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

```
[Out] 1/16*x^4*AppellF1(4/3,1/2,1,7/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/
(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -1/4*(d*x^3)/c, -((d*x^
3)/c)])/(16*c*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(4c+dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}} = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

Mathematica [A]
time = 10.03, size = 67, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{16c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/(16*c*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.36, size = 696, normalized size = 10.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+4/
9*I/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3
```

```
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2261 vs. $2(52) = 104$.
time = 3.80, size = 2261, normalized size = 34.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(16/27)^(1/6)*d^2*(-1/(c*d^8))^(1/6)*arctan(1/48*(2*(27*sqr
t(3)*(16/27)^(5/6)*c*d^7*x^2*(-1/(c*d^8))^(5/6) + 24*sqrt(3)*sqrt(1/3)*c*d^
4*x*sqrt(-1/(c*d^8)) + 4*sqrt(3)*(16/27)^(1/6)*(d^2*x^3 + 4*c*d)*(-1/(c*d^8
))^(1/6))*sqrt(d*x^3 + c) - (8*sqrt(3)*2^(2/3)*(c*d^6*x^3 + c^2*d^5)*(-1/(c
*d^8))^(2/3) - 8*sqrt(3)*(d*x^4 + c*x) - (27*sqrt(3)*(16/27)^(5/6)*c*d^7*x^
2*(-1/(c*d^8))^(5/6) + 24*sqrt(3)*sqrt(1/3)*c*d^4*x*sqrt(-1/(c*d^8)) - 4*sq
rt(3)*(16/27)^(1/6)*(d^2*x^3 - 2*c*d)*(-1/(c*d^8))^(1/6))*sqrt(d*x^3 + c))*
sqrt((4*d^3*x^9 + 240*c*d^2*x^6 - 128*c^3 - 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d
^7*x^5 - 8*c^3*d^6*x^2)*(-1/(c*d^8))^(2/3) + 48*2^(1/3)*(c*d^5*x^7 + 5*c^2*
d^4*x^4 + 4*c^3*d^3*x)*(-1/(c*d^8))^(1/3) + 3*(72*(16/27)^(1/6)*c*d^3*x^5*(
-1/(c*d^8))^(1/6) - 9*(16/27)^(5/6)*(c*d^9*x^7 + 2*c^2*d^8*x^4 - 8*c^3*d^7*
x)*(-1/(c*d^8))^(5/6) - 16*sqrt(1/3)*(c*d^6*x^6 - 16*c^2*d^5*x^3 - 8*c^3*d^
4)*sqrt(-1/(c*d^8)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^
3 + 64*c^3)))/(d*x^4 + c*x)) + 4*sqrt(3)*(16/27)^(1/6)*d^2*(-1/(c*d^8))^(1/
6)*arctan(1/48*(2*(27*sqrt(3)*(16/27)^(5/6)*c*d^7*x^2*(-1/(c*d^8))^(5/6) +
24*sqrt(3)*sqrt(1/3)*c*d^4*x*sqrt(-1/(c*d^8)) + 4*sqrt(3)*(16/27)^(1/6)*(d^
2*x^3 + 4*c*d)*(-1/(c*d^8))^(1/6))*sqrt(d*x^3 + c) + (8*sqrt(3)*2^(2/3)*(c*
d^6*x^3 + c^2*d^5)*(-1/(c*d^8))^(2/3) - 8*sqrt(3)*(d*x^4 + c*x) + (27*sqrt(
3)*(16/27)^(5/6)*c*d^7*x^2*(-1/(c*d^8))^(5/6) + 24*sqrt(3)*sqrt(1/3)*c*d^4*
x*sqrt(-1/(c*d^8)) - 4*sqrt(3)*(16/27)^(1/6)*(d^2*x^3 - 2*c*d)*(-1/(c*d^8))
^(1/6))*sqrt(d*x^3 + c))*sqrt((4*d^3*x^9 + 240*c*d^2*x^6 - 128*c^3 - 24*2^(
2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^3*d^6*x^2)*(-1/(c*d^8))^(2/3) + 48*2^
(1/3)*(c*d^5*x^7 + 5*c^2*d^4*x^4 + 4*c^3*d^3*x)*(-1/(c*d^8))^(1/3) - 3*(72*
```

```
(16/27)^(1/6)*c*d^3*x^5*(-1/(c*d^8))^(1/6) - 9*(16/27)^(5/6)*(c*d^9*x^7 + 2
*c^2*d^8*x^4 - 8*c^3*d^7*x)*(-1/(c*d^8))^(5/6) - 16*sqrt(1/3)*(c*d^6*x^6 -
16*c^2*d^5*x^3 - 8*c^3*d^4)*sqrt(-1/(c*d^8)))*sqrt(d*x^3 + c))/(d^3*x^9 + 1
2*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 + c*x)) - (16/27)^(1/6)*d^2*(
-1/(c*d^8))^(1/6)*log(16*(4*d^3*x^9 + 240*c*d^2*x^6 - 128*c^3 - 24*2^(2/3)*
(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^3*d^6*x^2)*(-1/(c*d^8))^(2/3) + 48*2^(1/3)
*(c*d^5*x^7 + 5*c^2*d^4*x^4 + 4*c^3*d^3*x)*(-1/(c*d^8))^(1/3) + 3*(72*(16/2
7)^(1/6)*c*d^3*x^5*(-1/(c*d^8))^(1/6) - 9*(16/27)^(5/6)*(c*d^9*x^7 + 2*c^2*
d^8*x^4 - 8*c^3*d^7*x)*(-1/(c*d^8))^(5/6) - 16*sqrt(1/3)*(c*d^6*x^6 - 16*c^
2*d^5*x^3 - 8*c^3*d^4)*sqrt(-1/(c*d^8)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d
^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (16/27)^(1/6)*d^2*(-1/(c*d^8))^(1/6)*log
(16*(4*d^3*x^9 + 240*c*d^2*x^6 - 128*c^3 - 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^
7*x^5 - 8*c^3*d^6*x^2)*(-1/(c*d^8))^(2/3) + 48*2^(1/3)*(c*d^5*x^7 + 5*c^2*d
^4*x^4 + 4*c^3*d^3*x)*(-1/(c*d^8))^(1/3) - 3*(72*(16/27)^(1/6)*c*d^3*x^5*(-
1/(c*d^8))^(1/6) - 9*(16/27)^(5/6)*(c*d^9*x^7 + 2*c^2*d^8*x^4 - 8*c^3*d^7*x
)*(-1/(c*d^8))^(5/6) - 16*sqrt(1/3)*(c*d^6*x^6 - 16*c^2*d^5*x^3 - 8*c^3*d^4
)*sqrt(-1/(c*d^8)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3
 + 64*c^3)) + 2*(16/27)^(1/6)*d^2*(-1/(c*d^8))^(1/6)*log((4*d^3*x^9 - 264*c
*d^2*x^6 - 288*c^2*d*x^3 - 128*c^3 - 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5
- 8*c^3*d^6*x^2)*(-1/(c*d^8))^(2/3) - 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 -
 2*c^3*d^3*x)*(-1/(c*d^8))^(1/3) + 3*(72*(16/27)^(1/6)*c*d^3*x^5*(-1/(c*d^8
))^(1/6) + 9*(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x)*(-1/(
c*d^8))^(5/6) + 8*sqrt(1/3)*(5*c*d^6*x^6 - 20*c^2*d^5*x^3 - 16*c^3*d^4)*sqr
t(-1/(c*d^8)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64
*c^3)) - 2*(16/27)^(1/6)*d^2*(-1/(c*d^8))^(1/6)*log((4*d^3*x^9 - 264*c*d^2*
x^6 - 288*c^2*d*x^3 - 128*c^3 - 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c
^3*d^6*x^2)*(-1/(c*d^8))^(2/3) - 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^
3*d^3*x)*(-1/(c*d^8))^(1/3) - 3*(72*(16/27)^(1/6)*c*d^3*x^5*(-1/(c*d^8))^(1
/6) + 9*(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x)*(-1/(c*d^8
))^(5/6) + 8*sqrt(1/3)*(5*c*d^6*x^6 - 20*c^2*d^5*x^3 - 16*c^3*d^4)*sqrt(-1/
(c*d^8)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)
) + 24*sqrt(d)*weierstrassPInverse(0, -4*c/d, x))/d^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] int(x^3/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.279 $\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

```
[Out] 1/4*x*AppellF1(1/3,1/2,1,4/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/(d*
x^3+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -1/4*(d*x^3)/c, -((d*x^3)
/c)])/(4*c*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(4c+dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(64) = 128.
time = 10.04, size = 165, normalized size = 2.58

$$\frac{16cxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{\sqrt{c+dx^3}\,(4c+dx^3)\left(16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (16*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(Sqrt[c +
 d*x^3]*(4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -1/4*(
d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/
c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.34, size = 416, normalized size = 6.50

method	result

default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*
_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2
```

```
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2288 vs. $2(50) = 100$.
time = 3.76, size = 2288, normalized size = 35.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] -1/72*(4*sqrt(3)*(1/108)^(1/6)*c*d*(-1/(c^7*d^2))^(1/6)*arctan(1/3*((108*sq
rt(3)*(1/108)^(5/6)*c^6*d^2*x^2*(-1/(c^7*d^2))^(5/6) + 3*sqrt(3)*sqrt(1/3)*
c^4*d*x*sqrt(-1/(c^7*d^2)) + sqrt(3)*(1/108)^(1/6)*(c*d*x^3 + 4*c^2)*(-1/(c
^7*d^2))^(1/6))*sqrt(d*x^3 + c) - (4*sqrt(3)*(1/4)^(2/3)*(c^5*d^2*x^3 + c^6
*d)*(-1/(c^7*d^2))^(2/3) - sqrt(3)*(d*x^4 + c*x) - (108*sqrt(3)*(1/108)^(5/
6)*c^6*d^2*x^2*(-1/(c^7*d^2))^(5/6) + 3*sqrt(3)*sqrt(1/3)*c^4*d*x*sqrt(-1/(
c^7*d^2)) - sqrt(3)*(1/108)^(1/6)*(c*d*x^3 - 2*c^2)*(-1/(c^7*d^2))^(1/6))*s
qrt(d*x^3 + c))*sqrt((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/4)^(2/3)*(c^5
*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-1/(c^7*d^2))^(2/3) + 24*(1/4)^(
1/3)*(c^3*d^3*x^7 + 5*c^4*d^2*x^4 + 4*c^5*d*x)*(-1/(c^7*d^2))^(1/3) + 12*(9
*(1/108)^(1/6)*c^2*d^2*x^5*(-1/(c^7*d^2))^(1/6) - 18*(1/108)^(5/6)*(c^6*d^4
*x^7 + 2*c^7*d^3*x^4 - 8*c^8*d^2*x)*(-1/(c^7*d^2))^(5/6) - sqrt(1/3)*(c^4*d
^3*x^6 - 16*c^5*d^2*x^3 - 8*c^6*d)*sqrt(-1/(c^7*d^2)))*sqrt(d*x^3 + c))/(d^
3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 + c*x)) + 4*sqrt(3)*
(1/108)^(1/6)*c*d*(-1/(c^7*d^2))^(1/6)*arctan(1/3*((108*sqrt(3)*(1/108)^(5/
6)*c^6*d^2*x^2*(-1/(c^7*d^2))^(5/6) + 3*sqrt(3)*sqrt(1/3)*c^4*d*x*sqrt(-1/(
c^7*d^2)) + sqrt(3)*(1/108)^(1/6)*(c*d*x^3 + 4*c^2)*(-1/(c^7*d^2))^(1/6))*s
qrt(d*x^3 + c) + (4*sqrt(3)*(1/4)^(2/3)*(c^5*d^2*x^3 + c^6*d)*(-1/(c^7*d^2)
)^(2/3) - sqrt(3)*(d*x^4 + c*x) + (108*sqrt(3)*(1/108)^(5/6)*c^6*d^2*x^2*(-
1/(c^7*d^2))^(5/6) + 3*sqrt(3)*sqrt(1/3)*c^4*d*x*sqrt(-1/(c^7*d^2)) - sqrt(
3)*(1/108)^(1/6)*(c*d*x^3 - 2*c^2)*(-1/(c^7*d^2))^(1/6))*sqrt(d*x^3 + c))*s
qrt((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/4)^(2/3)*(c^5*d^4*x^8 - 7*c^6*
d^3*x^5 - 8*c^7*d^2*x^2)*(-1/(c^7*d^2))^(2/3) + 24*(1/4)^(1/3)*(c^3*d^3*x^7
 + 5*c^4*d^2*x^4 + 4*c^5*d*x)*(-1/(c^7*d^2))^(1/3) - 12*(9*(1/108)^(1/6)*c^
```

```
2*d^2*x^5*(-1/(c^7*d^2))^(1/6) - 18*(1/108)^(5/6)*(c^6*d^4*x^7 + 2*c^7*d^3*
x^4 - 8*c^8*d^2*x)*(-1/(c^7*d^2))^(5/6) - sqrt(1/3)*(c^4*d^3*x^6 - 16*c^5*d
^2*x^3 - 8*c^6*d)*sqrt(-1/(c^7*d^2)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*
x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 + c*x)) - (1/108)^(1/6)*c*d*(-1/(c^7*
d^2))^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/4)^(2/3)*(c^5*d^4*
x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-1/(c^7*d^2))^(2/3) + 24*(1/4)^(1/3)*
(c^3*d^3*x^7 + 5*c^4*d^2*x^4 + 4*c^5*d*x)*(-1/(c^7*d^2))^(1/3) + 12*(9*(1/1
08)^(1/6)*c^2*d^2*x^5*(-1/(c^7*d^2))^(1/6) - 18*(1/108)^(5/6)*(c^6*d^4*x^7
+ 2*c^7*d^3*x^4 - 8*c^8*d^2*x)*(-1/(c^7*d^2))^(5/6) - sqrt(1/3)*(c^4*d^3*x^
6 - 16*c^5*d^2*x^3 - 8*c^6*d)*sqrt(-1/(c^7*d^2)))*sqrt(d*x^3 + c))/(d^3*x^9
 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/108)^(1/6)*c*d*(-1/(c^7*d^2)
)^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/4)^(2/3)*(c^5*d^4*x^8
- 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-1/(c^7*d^2))^(2/3) + 24*(1/4)^(1/3)*(c^3
*d^3*x^7 + 5*c^4*d^2*x^4 + 4*c^5*d*x)*(-1/(c^7*d^2))^(1/3) - 12*(9*(1/108)^
(1/6)*c^2*d^2*x^5*(-1/(c^7*d^2))^(1/6) - 18*(1/108)^(5/6)*(c^6*d^4*x^7 + 2*
c^7*d^3*x^4 - 8*c^8*d^2*x)*(-1/(c^7*d^2))^(5/6) - sqrt(1/3)*(c^4*d^3*x^6 -
16*c^5*d^2*x^3 - 8*c^6*d)*sqrt(-1/(c^7*d^2)))*sqrt(d*x^3 + c))/(d^3*x^9 + 1
2*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 2*(1/108)^(1/6)*c*d*(-1/(c^7*d^2))^
(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/4)^(2/3)*
(c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-1/(c^7*d^2))^(2/3) - 48*(1/
4)^(1/3)*(c^3*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d*x)*(-1/(c^7*d^2))^(1/3) + 6*(
18*(1/108)^(1/6)*c^2*d^2*x^5*(-1/(c^7*d^2))^(1/6) + 36*(1/108)^(5/6)*(c^6*d
^4*x^7 - 16*c^7*d^3*x^4 - 8*c^8*d^2*x)*(-1/(c^7*d^2))^(5/6) + sqrt(1/3)*(5*
c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt(-1/(c^7*d^2)))*sqrt(d*x^3 + c
))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(1/108)^(1/6)*c*d*
(-1/(c^7*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 -
24*(1/4)^(2/3)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-1/(c^7*d^2))
^(2/3) - 48*(1/4)^(1/3)*(c^3*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d*x)*(-1/(c^7*d^
2))^(1/3) - 6*(18*(1/108)^(1/6)*c^2*d^2*x^5*(-1/(c^7*d^2))^(1/6) + 36*(1/10
8)^(5/6)*(c^6*d^4*x^7 - 16*c^7*d^3*x^4 - 8*c^8*d^2*x)*(-1/(c^7*d^2))^(5/6)
+ sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt(-1/(c^7*d^2)))
*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 24*sq
rt(d)*weierstrassPInverse(0, -4*c/d, x))/(c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c + dx^3} \cdot (4c + dx^3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] int(1/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.280 $\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

```
[Out] -1/8*AppellF1(-2/3,1/2,1,1/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x^2
/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/8*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -1/4*(d*x^3)/c, -((d*
x^3)/c)])/(c*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(4c+dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 243 vs. 2(66) = 132.
time = 20.13, size = 243, normalized size = 3.68

$$\frac{-\frac{32(c+dx^3)}{c^2}-\frac{d^2x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{c^3}+\frac{2048dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{(4c+dx^3)\left(-16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)\right)}}{256x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] ((-32*(c + d*x^3))/c^2 - (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/c^3 + (2048*d*x^3*AppellF1[1/3, 1/2, 1
, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/((4*c + d*x^3)*(-16*c*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + 3*d*x^3*(AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -1/4*(d*x^3)/c]))))/(256*x^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.41, size = 722, normalized size = 10.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c*(-1/2/c*(d*x^3+c)^(1/2)/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/36*I/d^2/c^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2
)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c
```

```
*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/
2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^
2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)
^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2392 vs. $2(52) = 104$.
time = 6.00, size = 2392, normalized size = 36.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/288*(4*sqrt(3)*(1/108)^(1/6)*c^2*x^2*(-d^4/c^13)^(1/6)*arctan(1/3*((108*s
qrt(3)*(1/108)^(5/6)*c^11*d^3*x^2*(-d^4/c^13)^(5/6) + 3*sqrt(3)*sqrt(1/3)*c
^7*d^4*x*sqrt(-d^4/c^13) + sqrt(3)*(1/108)^(1/6)*(c^2*d^6*x^3 + 4*c^3*d^5)*
(-d^4/c^13)^(1/6))*sqrt(d*x^3 + c) - (4*sqrt(3)*(1/4)^(2/3)*(c^9*d*x^3 + c^
10)*(-d^4/c^13)^(2/3) - sqrt(3)*(d^4*x^4 + c*d^3*x) - (108*sqrt(3)*(1/108)^
(5/6)*c^11*x^2*(-d^4/c^13)^(5/6) + 3*sqrt(3)*sqrt(1/3)*c^7*d*x*sqrt(-d^4/c^
13) - sqrt(3)*(1/108)^(1/6)*(c^2*d^3*x^3 - 2*c^3*d^2)*(-d^4/c^13)^(1/6))*sq
rt(d*x^3 + c))*sqrt((d^9*x^9 + 60*c*d^8*x^6 - 32*c^3*d^6 - 24*(1/4)^(2/3)*(
c^9*d^6*x^8 - 7*c^10*d^5*x^5 - 8*c^11*d^4*x^2)*(-d^4/c^13)^(2/3) + 24*(1/4)
^(1/3)*(c^5*d^7*x^7 + 5*c^6*d^6*x^4 + 4*c^7*d^5*x)*(-d^4/c^13)^(1/3) + 12*(
9*(1/108)^(1/6)*c^3*d^7*x^5*(-d^4/c^13)^(1/6) - 18*(1/108)^(5/6)*(c^11*d^5*
x^7 + 2*c^12*d^4*x^4 - 8*c^13*d^3*x)*(-d^4/c^13)^(5/6) - sqrt(1/3)*(c^7*d^6
*x^6 - 16*c^8*d^5*x^3 - 8*c^9*d^4)*sqrt(-d^4/c^13))*sqrt(d*x^3 + c))/(d^3*x
^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d^7*x^4 + c*d^6*x)) + 4*sqrt(
3)*(1/108)^(1/6)*c^2*x^2*(-d^4/c^13)^(1/6)*arctan(1/3*((108*sqrt(3)*(1/108)
^(5/6)*c^11*d^3*x^2*(-d^4/c^13)^(5/6) + 3*sqrt(3)*sqrt(1/3)*c^7*d^4*x*sqrt(
-d^4/c^13) + sqrt(3)*(1/108)^(1/6)*(c^2*d^6*x^3 + 4*c^3*d^5)*(-d^4/c^13)^(1
/6))*sqrt(d*x^3 + c) + (4*sqrt(3)*(1/4)^(2/3)*(c^9*d*x^3 + c^10)*(-d^4/c^13
```

```
)^(2/3) - sqrt(3)*(d^4*x^4 + c*d^3*x) + (108*sqrt(3)*(1/108)^(5/6)*c^11*x^2
*(-d^4/c^13)^(5/6) + 3*sqrt(3)*sqrt(1/3)*c^7*d*x*sqrt(-d^4/c^13) - sqrt(3)*
(1/108)^(1/6)*(c^2*d^3*x^3 - 2*c^3*d^2)*(-d^4/c^13)^(1/6))*sqrt(d*x^3 + c))
*sqrt((d^9*x^9 + 60*c*d^8*x^6 - 32*c^3*d^6 - 24*(1/4)^(2/3)*(c^9*d^6*x^8 -
7*c^10*d^5*x^5 - 8*c^11*d^4*x^2)*(-d^4/c^13)^(2/3) + 24*(1/4)^(1/3)*(c^5*d^
7*x^7 + 5*c^6*d^6*x^4 + 4*c^7*d^5*x)*(-d^4/c^13)^(1/3) - 12*(9*(1/108)^(1/6
)*c^3*d^7*x^5*(-d^4/c^13)^(1/6) - 18*(1/108)^(5/6)*(c^11*d^5*x^7 + 2*c^12*d
^4*x^4 - 8*c^13*d^3*x)*(-d^4/c^13)^(5/6) - sqrt(1/3)*(c^7*d^6*x^6 - 16*c^8*
d^5*x^3 - 8*c^9*d^4)*sqrt(-d^4/c^13))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*
x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d^7*x^4 + c*d^6*x)) - (1/108)^(1/6)*c^2*x^2
*(-d^4/c^13)^(1/6)*log((d^9*x^9 + 60*c*d^8*x^6 - 32*c^3*d^6 - 24*(1/4)^(2/3
)*(c^9*d^6*x^8 - 7*c^10*d^5*x^5 - 8*c^11*d^4*x^2)*(-d^4/c^13)^(2/3) + 24*(1
/4)^(1/3)*(c^5*d^7*x^7 + 5*c^6*d^6*x^4 + 4*c^7*d^5*x)*(-d^4/c^13)^(1/3) + 1
2*(9*(1/108)^(1/6)*c^3*d^7*x^5*(-d^4/c^13)^(1/6) - 18*(1/108)^(5/6)*(c^11*d
^5*x^7 + 2*c^12*d^4*x^4 - 8*c^13*d^3*x)*(-d^4/c^13)^(5/6) - sqrt(1/3)*(c^7*
d^6*x^6 - 16*c^8*d^5*x^3 - 8*c^9*d^4)*sqrt(-d^4/c^13))*sqrt(d*x^3 + c))/(d^
3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/108)^(1/6)*c^2*x^2*(-d^
4/c^13)^(1/6)*log((d^9*x^9 + 60*c*d^8*x^6 - 32*c^3*d^6 - 24*(1/4)^(2/3)*(c^
9*d^6*x^8 - 7*c^10*d^5*x^5 - 8*c^11*d^4*x^2)*(-d^4/c^13)^(2/3) + 24*(1/4)^(
1/3)*(c^5*d^7*x^7 + 5*c^6*d^6*x^4 + 4*c^7*d^5*x)*(-d^4/c^13)^(1/3) - 12*(9*
(1/108)^(1/6)*c^3*d^7*x^5*(-d^4/c^13)^(1/6) - 18*(1/108)^(5/6)*(c^11*d^5*x^
7 + 2*c^12*d^4*x^4 - 8*c^13*d^3*x)*(-d^4/c^13)^(5/6) - sqrt(1/3)*(c^7*d^6*x
^6 - 16*c^8*d^5*x^3 - 8*c^9*d^4)*sqrt(-d^4/c^13))*sqrt(d*x^3 + c))/(d^3*x^9
 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 2*(1/108)^(1/6)*c^2*x^2*(-d^4/c
^13)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(
1/4)^(2/3)*(c^9*d^3*x^8 - 7*c^10*d^2*x^5 - 8*c^11*d*x^2)*(-d^4/c^13)^(2/3)
- 48*(1/4)^(1/3)*(c^5*d^4*x^7 - c^6*d^3*x^4 - 2*c^7*d^2*x)*(-d^4/c^13)^(1/3
) + 6*(18*(1/108)^(1/6)*c^3*d^4*x^5*(-d^4/c^13)^(1/6) + 36*(1/108)^(5/6)*(c
^11*d^2*x^7 - 16*c^12*d*x^4 - 8*c^13*x)*(-d^4/c^13)^(5/6) + sqrt(1/3)*(5*c^
7*d^3*x^6 - 20*c^8*d^2*x^3 - 16*c^9*d)*sqrt(-d^4/c^13))*sqrt(d*x^3 + c))/(d
^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(1/108)^(1/6)*c^2*x^2*(
-d^4/c^13)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3
- 24*(1/4)^(2/3)*(c^9*d^3*x^8 - 7*c^10*d^2*x^5 - 8*c^11*d*x^2)*(-d^4/c^13)^
(2/3) - 48*(1/4)^(1/3)*(c^5*d^4*x^7 - c^6*d^3*x^4 - 2*c^7*d^2*x)*(-d^4/c^13
)^(1/3) - 6*(18*(1/108)^(1/6)*c^3*d^4*x^5*(-d^4/c^13)^(1/6) + 36*(1/108)^(5
/6)*(c^11*d^2*x^7 - 16*c^12*d*x^4 - 8*c^13*x)*(-d^4/c^13)^(5/6) + sqrt(1/3)
*(5*c^7*d^3*x^6 - 20*c^8*d^2*x^3 - 16*c^9*d)*sqrt(-d^4/c^13))*sqrt(d*x^3 +
c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 60*sqrt(d)*x^2*weie
rstrassPInverse(0, -4*c/d, x) - 36*sqrt(d*x^3 + c))/(c^2*x^2)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] int(1/(x^3*(c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.281 $\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx$

Optimal. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[Out] -1/6*arctanh((1+2^(1/3)*x)/(-x^3+1)^(1/2))*2^(1/3)+1/18*arctanh((-x^3+1)^(1
/2))*2^(1/3)-1/18*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*2^(1/3)*3^(1
/2)+1/18*arctan(1/3*(-x^3+1)^(1/2)*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{497\}$

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(2^(2/3)*Sqrt[3]) + Ar
cTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/S
qrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tan}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.03, size = 28, normalized size = 0.22

$$\frac{1}{8}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 11.08, size = 164, normalized size = 1.29

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3-4\right)}\frac{\_\alpha^2\sqrt{2}\sqrt{i\left(2x+1-i\sqrt{3}\right)}\sqrt{\frac{x-1}{i\sqrt{3}-3}}\sqrt{-\frac{i\left(2x+1+i\sqrt{3}\right)}{2}}\left(-2\_\alpha^2+\_\alpha+1+i\sqrt{3}\right.}{}\right.}{36}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3-4\right)}\frac{\_\alpha^2\sqrt{2}\sqrt{i\left(2x+1-i\sqrt{3}\right)}\sqrt{\frac{x-1}{i\sqrt{3}-3}}\sqrt{-\frac{i\left(2x+1+i\sqrt{3}\right)}{2}}\left(-2\_\alpha^2+\_\alpha+1+i\sqrt{3}\right.}{}\right.}{36}$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(2*x+1-I*3^(1/2)))^(1/2)*((x-1)/(I*3^(1/
2)-3))^(1/2)*(-1/2*I*(2*x+1+I*3^(1/2)))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2+_
```

```
alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1/2
)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^3-
4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. 2(92) = 184.
time = 3.14, size = 1191, normalized size = 9.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/31104*432^(5/6)*sqrt(3)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2
/3)*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 -
 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3
888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/31104*432
^(5/6)*sqrt(3)*log(36*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*
x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*
x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(
x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3
)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2)
 - (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432
^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) -
 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3)*log(36*(36
*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6
- 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3
)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
 12*x^6 + 48*x^3 - 64)) - 1/1944*432^(5/6)*arctan(1/216*sqrt(-x^3 + 1)*(72*
432^(1/6)*x^2 + 432^(5/6)*x + 72*sqrt(3))/(2*x^3 - 1)) + 1/3888*432^(5/6)*a
rctan(-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4
) + 18*432^(1/6)*(x^5 + 8*x^2)) + (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sq
rt(3)*2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) - sqrt(-x^3 + 1)*(432^(5/
6)*(2*x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqr
t((36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) + (2592
```

```
*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*s
qrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(
x^9 - 12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4)) + 1/3888*432^(5/6)*arctan(
-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4) + 18
*432^(1/6)*(x^5 + 8*x^2)) - (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sqrt(3)*
2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) + sqrt(-x^3 + 1)*(432^(5/6)*(2*
x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqrt((36*
x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6 -
 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)
*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Mupad [B]
time = 3.42, size = 653, normalized size = 5.14

$$-\frac{2^{1/3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{2^{2/3}-1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\sqrt{1-x^3}\,\left(2^{2/3}-1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2^{1/3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{1-x^3}\,\left(2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2^{1/3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{2^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{1-x^3}\,\left(2^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)
```

```
[Out] - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1
/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(2^
```

```
(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(2/
3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 + 1
/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(
-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/2)*1i)/2 - 1/2) -
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(1 - x^3)^(1/2)*(2^(2/3)
*((3^(1/2)*1i)/2 - 1/2) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.282 $\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=111

$$-\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2(c+dx^3)^{3/2}}{3d^4}-\frac{4c(c+dx^3)^{5/2}}{5d^4}-\frac{2(c+dx^3)^{7/2}}{21d^4}+\frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

```
[Out] -38/3*c^2*(d*x^3+c)^(3/2)/d^4-4/5*c*(d*x^3+c)^(5/2)/d^4-2/21*(d*x^3+c)^(7/2
)/d^4+1024*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4-1024/3*c^3*(d*x
^3+c)^(1/2)/d^4
```

Rubi [A]
time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 90, 52, 65, 212}

$$\frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2(c+dx^3)^{3/2}}{3d^4}-\frac{4c(c+dx^3)^{5/2}}{5d^4}-\frac{2(c+dx^3)^{7/2}}{21d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-1024*c^3*Sqrt[c + d*x^3])/(3*d^4) - (38*c^2*(c + d*x^3)^(3/2))/(3*d^4) -
(4*c*(c + d*x^3)^(5/2))/(5*d^4) - (2*(c + d*x^3)^(7/2))/(21*d^4) + (1024*c^
(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{57c^2\sqrt{c+dx}}{d^3} + \frac{512c^3\sqrt{c+dx}}{d^3(8c-dx)} - \frac{6c(c+dx)^{3/2}}{d^3} - \frac{(c+dx)^{5/2}}{d^3}\right) dx$$

$$= -\frac{38c^2(c+dx^3)^{3/2}}{3d^4} - \frac{4c(c+dx^3)^{5/2}}{5d^4} - \frac{2(c+dx^3)^{7/2}}{21d^4} + \frac{(512c^3)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,d\right.}{3d^3}$$

$$= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4} - \frac{38c^2(c+dx^3)^{3/2}}{3d^4} - \frac{4c(c+dx^3)^{5/2}}{5d^4} - \frac{2(c+dx^3)^{7/2}}{21d^4} + \frac{(1536c^4)}{}$$

$$= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4} - \frac{38c^2(c+dx^3)^{3/2}}{3d^4} - \frac{4c(c+dx^3)^{5/2}}{5d^4} - \frac{2(c+dx^3)^{7/2}}{21d^4} + \frac{(3072c^4)}{}$$

$$= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4} - \frac{38c^2(c+dx^3)^{3/2}}{3d^4} - \frac{4c(c+dx^3)^{5/2}}{5d^4} - \frac{2(c+dx^3)^{7/2}}{21d^4} + \frac{1024c^{7/2}}{}$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 0.74

$$-\frac{2\sqrt{c+dx^3}\,(18632c^3+764c^2dx^3+57cd^2x^6+5d^3x^9)}{105d^4} + \frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(18632*c^3 + 764*c^2*d*x^3 + 57*c*d^2*x^6 + 5*d^3*x^9))
/(105*d^4) + (1024*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.41, size = 582, normalized size = 5.24 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/21*x^9*(d*x^3+c)^(1/2)+2/105*c/d*x^6*(d*x^3+c)^(1/2)-8/315*c^2/d^2*
x^3*(d*x^3+c)^(1/2)+16/315*c^3*(d*x^3+c)^(1/2)/d^3)-8/d^2*c*(2/15*x^6*(d*x^
3+c)^(1/2)+2/45*c/d*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(d*x^3+c)^(1/2)/d^2)-128/9
*c^2*(d*x^3+c)^(3/2)/d^4-512*c^3/d^3*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/
2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))
```

Maxima [A]
time = 0.53, size = 96, normalized size = 0.86

$$-\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+5\left(dx^3+c\right)^{\frac{7}{2}}+42\left(dx^3+c\right)^{\frac{5}{2}}c+665\left(dx^3+c\right)^{\frac{3}{2}}c^2+17920\sqrt{dx^3+c}\,c^3\right)}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/105*(26880*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
3*sqrt(c))) + 5*(d*x^3 + c)^(7/2) + 42*(d*x^3 + c)^(5/2)*c + 665*(d*x^3 + c
)^(3/2)*c^2 + 17920*sqrt(d*x^3 + c)*c^3)/d^4
```

Fricas [A]
time = 2.74, size = 169, normalized size = 1.52

$$\left[\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3\right)\sqrt{dx^3+c}\right)}{105\,d^4},\,-\frac{2\left(53760\,\sqrt{-c}\,c^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+\left(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3\right)\sqrt{dx^3+c}\right)}{105\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/105*(26880*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) - (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^
3 + c))/d^4, -2/105*(53760*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)
/c) + (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^3 + c
))/d^4]
```

Sympy [A]
time = 25.80, size = 99, normalized size = 0.89

$$\frac{2\left(-\frac{512c^4\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{512c^3\sqrt{c+dx^3}}{3}-\frac{19c^2(c+dx^3)^{\frac{3}{2}}}{3}-\frac{2c(c+dx^3)^{\frac{5}{2}}}{5}-\frac{(c+dx^3)^{\frac{7}{2}}}{21}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-512*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 512*c**3*sqrt(c
 + d*x**3)/3 - 19*c**2*(c + d*x**3)**(3/2)/3 - 2*c*(c + d*x**3)**(5/2)/5 -
(c + d*x**3)**(7/2)/21)/d**4
```

Giac [A]
time = 0.78, size = 100, normalized size = 0.90

$$-\frac{1024\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^4}-\frac{2\left(5\,(dx^3+c)^{\frac{7}{2}}d^{24}+42\,(dx^3+c)^{\frac{5}{2}}cd^{24}+665\,(dx^3+c)^{\frac{3}{2}}c^2d^{24}+17920\,\sqrt{dx^3+c}\,c^3d^{24}\right)}{105\,d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1024*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/105*(5*(d
*x^3 + c)^(7/2)*d^24 + 42*(d*x^3 + c)^(5/2)*c*d^24 + 665*(d*x^3 + c)^(3/2)*
c^2*d^24 + 17920*sqrt(d*x^3 + c)*c^3*d^24)/d^28
```

Mupad [B]
time = 3.51, size = 118, normalized size = 1.06

$$\frac{512\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4}-\frac{37264\,c^3\,\sqrt{d\,x^3+c}}{105\,d^4}-\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d}-\frac{38\,c\,x^6\,\sqrt{d\,x^3+c}}{35\,d^2}-\frac{1528\,c^2\,x^3\,\sqrt{d\,x^3+c}}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (512*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^4 - (37264*c^3*(c + d*x^3)^(1/2))/(105*d^4) - (2*x^9*(c + d*x^3)^(1/2)
)/(21*d) - (38*c*x^6*(c + d*x^3)^(1/2))/(35*d^2) - (1528*c^2*x^3*(c + d*x^3
)^(1/2))/(105*d^3)
```

3.283 $\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=90

$$-\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c(c+dx^3)^{3/2}}{9d^3}-\frac{2(c+dx^3)^{5/2}}{15d^3}+\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

```
[Out] -14/9*c*(d*x^3+c)^(3/2)/d^3-2/15*(d*x^3+c)^(5/2)/d^3+128*c^(5/2)*arctanh(1/
3*(d*x^3+c)^(1/2)/c^(1/2))/d^3-128/3*c^2*(d*x^3+c)^(1/2)/d^3
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 90, 52, 65, 212}

$$\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c(c+dx^3)^{3/2}}{9d^3}-\frac{2(c+dx^3)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-128*c^2*Sqrt[c + d*x^3])/(3*d^3) - (14*c*(c + d*x^3)^(3/2))/(9*d^3) - (2*
(c + d*x^3)^(5/2))/(15*d^3) + (128*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/d^3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{7c\sqrt{c+dx}}{d^2} + \frac{64c^2\sqrt{c+dx}}{d^2(8c-dx)} - \frac{(c+dx)^{3/2}}{d^2}\right)dx, x, x^3\right)$$

$$= -\frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{(64c^2)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{128c^2\sqrt{c+dx^3}}{3d^3} - \frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{(192c^3)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+ \cdots}}\right.}{d^2}$$

$$= -\frac{128c^2\sqrt{c+dx^3}}{3d^3} - \frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{(384c^3)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \cdots\right.}{d^3}$$

$$= -\frac{128c^2\sqrt{c+dx^3}}{3d^3} - \frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

Mathematica [A]
time = 0.06, size = 71, normalized size = 0.79

$$-\frac{2\sqrt{c+dx^3}\,(998c^2+41cdx^3+3d^2x^6)}{45d^3} + \frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(998*c^2 + 41*c*d*x^3 + 3*d^2*x^6))/(45*d^3) + (128*c^(
5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 504, normalized size = 5.60 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/15*x^6*(d*x^3+c)^(1/2)+2/45*c/d*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(d*x^3
+c)^(1/2)/d^2)-16/9*c*(d*x^3+c)^(3/2)/d^3-64*c^2/d^2*(2/3*(d*x^3+c)^(1/2)/d
+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-
(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.48, size = 82, normalized size = 0.91

$$-\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+35\,(dx^3+c)^{\frac{3}{2}}c+960\,\sqrt{dx^3+c}\,c^2\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/45*(1440*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 35*(d*x^3 + c)^(3/2)*c + 960*sqrt(d*x^3 +
 c)*c^2)/d^3
```

Fricas [A]
time = 2.35, size = 147, normalized size = 1.63

$$\left[\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(3\,d^2x^6+41\,cdx^3+998\,c^2)\sqrt{dx^3+c}\right)}{45\,d^3},-\frac{2\left(2880\,\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+(3\,d^2x^6+41\,cdx^3+998\,c^2)\sqrt{dx^3+c}\right)}{45\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/45*(1440*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - (3*d^2*x^6 + 41*c*d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3, -2/45*(28
80*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 41*c*
d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3]
```

Sympy [A]
time = 12.26, size = 82, normalized size = 0.91

$$\frac{2\left(-\frac{64c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{64c^2\sqrt{c+dx^3}}{3}-\frac{7c(c+dx^3)^{\frac{3}{2}}}{9}-\frac{(c+dx^3)^{\frac{5}{2}}}{15}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-64*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 64*c**2*sqrt(c +
 d*x**3)/3 - 7*c*(c + d*x**3)**(3/2)/9 - (c + d*x**3)**(5/2)/15)/d**3
```

Giac [A]
time = 0.86, size = 83, normalized size = 0.92

$$-\frac{128\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^3}-\frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}d^{12}+35\,(dx^3+c)^{\frac{3}{2}}cd^{12}+960\,\sqrt{dx^3+c}\,c^2d^{12}\right)}{45\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -128*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/45*(3*(d*x
^3 + c)^(5/2)*d^12 + 35*(d*x^3 + c)^(3/2)*c*d^12 + 960*sqrt(d*x^3 + c)*c^2*
d^12)/d^15
```

Mupad [B]
time = 3.40, size = 98, normalized size = 1.09

$$\frac{64\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3}-\frac{1996\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3}-\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d}-\frac{82\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (64*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d^3 - (1996*c^2*(c + d*x^3)^(1/2))/(45*d^3) - (2*x^6*(c + d*x^3)^(1/2))/(
15*d) - (82*c*x^3*(c + d*x^3)^(1/2))/(45*d^2)
```

3.284 $\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=69

$$-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2(c+dx^3)^{3/2}}{9d^2}+\frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^2+16*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^
2-16/3*c*(d*x^3+c)^(1/2)/d^2
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.185$, Rules used = {457, 81, 52, 65, 212}

$$\frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2(c+dx^3)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-16*c*Sqrt[c + d*x^3])/(3*d^2) - (2*(c + d*x^3)^(3/2))/(9*d^2) + (16*c^(3/
2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right) \\
&= -\frac{2(c+dx^3)^{3/2}}{9d^2} + \frac{(8c)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{3d} \\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2} - \frac{2(c+dx^3)^{3/2}}{9d^2} + \frac{(24c^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d} \\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2} - \frac{2(c+dx^3)^{3/2}}{9d^2} + \frac{(48c^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^2} \\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2} - \frac{2(c+dx^3)^{3/2}}{9d^2} + \frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.86

$$-\frac{2\sqrt{c+dx^3}\,(25c+dx^3)}{9d^2} + \frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(25*c + d*x^3))/(9*d^2) + (16*c^(3/2)*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])])/d^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 446, normalized size = 6.46 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^2-8*c/d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum
((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*
d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)
```

Maxima [A]
time = 0.51, size = 66, normalized size = 0.96

$$-\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}+24\sqrt{dx^3+c}\,c\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/9*(36*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + (d*x^3 + c)^(3/2) + 24*sqrt(d*x^3 + c)*c)/d^2
```

Fricas [A]
time = 2.69, size = 121, normalized size = 1.75

$$\left[\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(dx^3+25\,c)\sqrt{dx^3+c}\right)}{9\,d^2},-\frac{2\left(72\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+(dx^3+25\,c)\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/9*(36*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2, -2/9*(72*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2]
```

Sympy [A]
time = 7.27, size = 65, normalized size = 0.94

$$\frac{2\left(-\frac{8c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{8c\sqrt{c+dx^3}}{3}-\frac{(c+dx^3)^{\frac{3}{2}}}{9}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-8*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 8*c*sqrt(c + d*x*
*3)/3 - (c + d*x**3)**(3/2)/9)/d**2
```

Giac [A]
time = 1.29, size = 65, normalized size = 0.94

$$-\frac{16\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d^2}-\frac{2\left((dx^3+c)^{\frac{3}{2}}d^4+24\,\sqrt{dx^3+c}\,cd^4\right)}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -16*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/9*((d*x^3 +
 c)^(3/2)*d^4 + 24*sqrt(d*x^3 + c)*c*d^4)/d^6
```

Mupad [B]
time = 3.51, size = 78, normalized size = 1.13

$$\frac{8\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2}-\frac{50\,c\,\sqrt{d\,x^3+c}}{9\,d^2}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (8*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/d^2 - (50*c*(c + d*x^3)^(1/2))/(9*d^2) - (2*x^3*(c + d*x^3)^(1/2))/(9*d)
```

3.285 $\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=50

$$-\frac{2\sqrt{c+dx^3}}{3d}+\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

```
[Out] 2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d-2/3*(d*x^3+c)^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {455, 52, 65, 212}

$$\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d) + (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\ &= -\frac{2\sqrt{c+dx^3}}{3d} + (3c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= -\frac{2\sqrt{c+dx^3}}{3d} + \frac{(6c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}\\ &= -\frac{2\sqrt{c+dx^3}}{3d} + \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.94

$$-\frac{2\left(\sqrt{c+dx^3}-3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
[Out] (-2*(Sqrt[c + d*x^3] - 3*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(3*
d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 425, normalized size = 8.50 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
[Out] -2/3*(d*x^3+c)^(1/2)/d-1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-
```

```
1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)
^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/
c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.57, size = 56, normalized size = 1.12

$$-\frac{3\,\sqrt{c}\,\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\,\sqrt{dx^3+c}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 2*sqrt(d*x^3 + c))/d
```

Fricas [A]
time = 2.62, size = 101, normalized size = 2.02

$$\left[\frac{3\,\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-2\,\sqrt{dx^3+c}}{3\,d},\,-\frac{2\left(3\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+\sqrt{dx^3+c}\right)}{3\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) - 2*sqrt(d*x^3 + c))/d, -2/3*(3*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt
(-c)/c) + sqrt(d*x^3 + c))/d]
```

Sympy [A]
time = 2.06, size = 46, normalized size = 0.92

$$\frac{2\left(-\frac{c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
[Out] 2*(-c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - sqrt(c + d*x**3)/3)/d
```

Giac [A]
time = 1.55, size = 43, normalized size = 0.86

$$-\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d}-\frac{2\,\sqrt{dx^3+c}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
[Out] -2*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/3*sqrt(d*x^3 + c
)/d
```

Mupad [B]
time = 3.50, size = 59, normalized size = 1.18

$$\frac{\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d}-\frac{2\,\sqrt{d\,x^3+c}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
[Out] (c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d
 - (2*(c + d*x^3)^(1/2))/(3*d)
```

3.286 $\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)}\,dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

```
[Out] 1/4*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/12*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 85, 65, 214, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(4*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*Sqrt[c])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(8c-dx)}\,dx,x,x^3\right)\\
&= \frac{1}{24}\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)+\frac{1}{8}(3d)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{3}{4}\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{12d}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}
\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 53, normalized size = 0.91

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

```
[Out] (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(12*Sqrt[c])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.41, size = 468, normalized size = 8.07

method	result
default	$-\frac{d\left(\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\right.}{-3\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right)}{}\right)}{}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d/c*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)
*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alp
```

```
ha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alph
a-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/8/c*(2/3*(d*x^3+c)
^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x), x)
```

Fricas [A]
time = 2.58, size = 138, normalized size = 2.38

$$\left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{24c}, \frac{\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)}{12c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, 1/12*(
sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*arctan(1/3*sqrt(d*
x^3 + c)*sqrt(-c)/c))/c]
```

Sympy [A]
time = 3.13, size = 60, normalized size = 1.03

$$\frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24\sqrt{-c}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c),x)
```

```
[Out] 2*(-d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(8*sqrt(-c)) + d*atan(sqrt(c + d*
x**3)/sqrt(-c))/(24*sqrt(-c)))/d
```

Giac [A]
time = 0.92, size = 48, normalized size = 0.83

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/4*arctan(1/3*sqrt(d*x^3
+ c)/sqrt(-c))/sqrt(-c)
```

Mupad [B]
time = 4.69, size = 125, normalized size = 2.16

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\,\left(6\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^3\,\left(24\,c^2-24\,c^{3/2}\,\sqrt{d\,x^3+c}+d^2\,x^6-20\,c\,d\,x^3\right)^3}{x^{15}\,{\left(8\,c-d\,x^3\right)}^3\,{\left(24\,c-d\,x^3\right)}^3}\right)}{24\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))*(6*c + d
*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^3*(24*c^2 - 24*c^(3/2)*(c + d*x^3)^(1/2
) + d^2*x^6 - 20*c*d*x^3)^3)/(x^15*(8*c - d*x^3)^3*(24*c - d*x^3)^3))/(24*c
^(1/2))
```

3.287 $\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx$

Optimal. Leaf size=81

$$-\frac{\sqrt{c+dx^3}}{24cx^3}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

```
[Out] 1/32*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-5/96*d*arctanh((d*x^3+c
)^(1/2)/c^(1/2))/c^(3/2)-1/24*(d*x^3+c)^(1/2)/c/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {457, 101, 162, 65, 214, 212}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}-\frac{\sqrt{c+dx^3}}{24cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3
2*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx = \frac{1}{3}\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(8c-dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{\mathrm{Subst}\left(\int \frac{5cd+\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}$$

$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{(5d)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c} + \frac{\left(3d^2\right)\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\right.}{64c}$$

$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{5\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96c} + \frac{(3d)\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c-}\right.}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

Mathematica [**A**]

time = 0.08, size = 81, normalized size = 1.00

$$-\frac{\sqrt{c+dx^3}}{24cx^3}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3
2*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.42, size = 511, normalized size = 6.31

method	result
risch	$-\frac{\sqrt{d\,x^3+c}}{24c\,x^3}+\frac{d\left(-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{6\sqrt{c}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\cdots\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}\cdots}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$

default	$-\frac{d^{2}\left(\frac{2\sqrt{d x^{3}+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x\cdots\right.}{-3\left(-c\,d^{2}\right)\cdots}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/c^2*d^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/
3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^
(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*
_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
```

```
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/8/c*(-1/3*(d*
x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/64*d/c^2
*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^4), x)
```

Fricas [A]
time = 2.80, size = 186, normalized size = 2.30

$$\left[\frac{3\sqrt{c}\,dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\,\sqrt{c}+10c}{dx^3-8c}\right)+5\sqrt{c}\,dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\,\sqrt{c}+2c}{x^3}\right)-8\sqrt{dx^3+c}\,c}{192\,c^2x^3},\ \frac{5\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-3\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3c}\right)-4\sqrt{dx^3+c}\,c}{96\,c^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
[Out] [1/192*(3*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x
^3 - 8*c)) + 5*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/
x^3) - 8*sqrt(d*x^3 + c)*c)/(c^2*x^3), 1/96*(5*sqrt(-c)*d*x^3*arctan(sqrt(d
*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c
)/c) - 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c),x)
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

Giac [A]
time = 1.06, size = 73, normalized size = 0.90

$$\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}\,c}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}\,c}-\frac{\sqrt{dx^3+c}}{24\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 5/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/32*d*arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/(c*x^3)
```

Mupad [B]
time = 3.75, size = 69, normalized size = 0.85

$$\frac{d\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{32\,\sqrt{c^3}}-\frac{5\,d\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\,\sqrt{c^3}}-\frac{\sqrt{d\,x^3+c}}{24\,c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)),x)
```

```
[Out] (d*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(32*(c^3)^(1/2)) - (5*d*at
anh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)))/(96*(c^3)^(1/2)) - (c + d*x^3)^(1/2
)/(24*c*x^3)
```

3.288 $\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{c+dx^3}}{48cx^6}-\frac{d\sqrt{c+dx^3}}{64c^2x^3}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

```
[Out] 1/256*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/256*d^2*arctanh((d
*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/48*(d*x^3+c)^(1/2)/c/x^6-1/64*d*(d*x^3+c)^
(1/2)/c^2/x^3
```

Rubi [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {457, 101, 156, 162, 65, 214, 212}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}-\frac{d\sqrt{c+dx^3}}{64c^2x^3}-\frac{\sqrt{c+dx^3}}{48cx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

```
[Out] -1/48*Sqrt[c + d*x^3]/(c*x^6) - (d*Sqrt[c + d*x^3])/(64*c^2*x^3) + (d^2*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*
x^3]/Sqrt[c]])/(256*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} + \frac{\text{Subst}\left(\int \frac{6cd+\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{\text{Subst}\left(\int \frac{6c^2d^2-3cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{512c^2} + \frac{(3d^3)\,\text{Subst}\left(\int \frac{}{(8c-}\right.}{5}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{256c^2} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{}{9}\right.}{}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

Mathematica [A]
time = 0.11, size = 95, normalized size = 0.89

$$\frac{\left(-4c-3dx^3\right)\sqrt{c+dx^3}}{192c^2x^6} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

```
[Out] ((-4*c - 3*d*x^3)*Sqrt[c + d*x^3])/(192*c^2*x^6) + (d^2*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(
256*c^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.42, size = 574, normalized size = 5.36

method	result

risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(3d\,x^{3}+4c\right)}{192c^{2}x^{6}}-\frac{3d^{2}\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i}{}\right.}{}}}{}\right.}{}\right.}{}$

default	$\frac{-\frac{\sqrt{d\,x^3+c}}{6x^6}-\frac{d\sqrt{d\,x^3+c}}{12c\,x^3}+\frac{d^2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{12c^{\frac{3}{2}}}}{8c}-\frac{d^3\left(\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{(-c\,d^2)^{\frac{1}{3}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/6*(d*x^3+c)^(1/2)/x^6-1/12*d*(d*x^3+c)^(1/2)/c/x^3+1/12*d^2*arcta
nh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-1/512/c^3*d^3*(2/3*(d*x^3+c)^(1/2)/d+1
/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_
```

```
alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha
=RootOf(_Z^3*d-8*c))+1/64*d/c^2*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d
*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/512/c^3*d^2*(2/3*(d*x^3+c)^(1/2)-2/3*arct
anh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^7), x)
```

Fricas [A]
time = 2.48, size = 188, normalized size = 1.76

$$\left[\frac{3\,\sqrt{c}\,d^{2}x^{6}\log\left(\frac{d^{2}x^{6}+24\,cdx^{3}+8\,(dx^{3}+4\,c)\sqrt{dx^{3}+c}\,\sqrt{c}+32\,c^{2}}{dx^{6}-8\,cx^{3}}\right)-8\,(3\,cdx^{3}+4\,c^{2})\sqrt{dx^{3}+c}}{1536\,c^{3}x^{6}},\ -\frac{3\,\sqrt{-c}\,d^{2}x^{6}\arctan\left(\frac{(dx^{3}+4\,c)\sqrt{dx^{3}+c}\,\sqrt{-c}}{4\,(cdx^{3}+c^{2})}\right)+4\,(3\,cdx^{3}+4\,c^{2})\sqrt{dx^{3}+c}}{768\,c^{3}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*sqrt(c)*d^2*x^6*log((d^2*x^6 + 24*c*d*x^3 + 8*(d*x^3 + 4*c)*sqrt
(d*x^3 + c)*sqrt(c) + 32*c^2)/(d*x^6 - 8*c*x^3)) - 8*(3*c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^3*x^6), -1/768*(3*sqrt(-c)*d^2*x^6*arctan(1/4*(d*x^3 + 4*
c)*sqrt(d*x^3 + c)*sqrt(-c)/(c*d*x^3 + c^2)) + 4*(3*c*d*x^3 + 4*c^2)*sqrt(d
*x^3 + c))/(c^3*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{c+dx^{3}}}{-8cx^{7}+dx^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

Giac [A]
time = 1.09, size = 100, normalized size = 0.93

$$-\frac{d^{2}\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{256\,\sqrt{-c}\,c^{2}}-\frac{d^{2}\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\,\sqrt{-c}}\right)}{256\,\sqrt{-c}\,c^{2}}-\frac{3\,(dx^{3}+c)^{\frac{3}{2}}d^{2}+\sqrt{dx^{3}+c}\,cd^{2}}{192\,c^{2}d^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/256*d^2*arct
an(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/192*(3*(d*x^3 + c)^(3/2
)*d^2 + sqrt(d*x^3 + c)*c*d^2)/(c^2*d^2*x^6)
```

Mupad [B]
time = 3.91, size = 83, normalized size = 0.78

$$\frac{d^2\,\mathrm{atanh}\left(\frac{d^4\,\sqrt{d\,x^3+c}}{2048\,c^{7/2}\,\left(\frac{d^4}{2048\,c^3}+\frac{d^5\,x^3}{8192\,c^4}\right)}\right)}{256\,c^{5/2}}-\frac{\sqrt{d\,x^3+c}}{192\,c\,x^6}-\frac{{\left(d\,x^3+c\right)}^{3/2}}{64\,c^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)),x)
```

```
[Out] (d^2*atanh((d^4*(c + d*x^3)^(1/2))/(2048*c^(7/2)*(d^4/(2048*c^3) + (d^5*x^3
)/(8192*c^4)))))/(256*c^(5/2)) - (c + d*x^3)^(1/2)/(192*c*x^6) - (c + d*x^3
)^(3/2)/(64*c^2*x^6)
```

3.289 $\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=648

$$-\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}-\frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{32\sqrt{3}\,c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\right.}{\sqrt{c+dx}}\right.}{d^{8/3}}$$

```
[Out] 32*c^(13/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8
/3)-32*c^(13/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32*c^(13/6)*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3)-2
14/91*c*x^2*(d*x^3+c)^(1/2)/d^2-2/13*x^5*(d*x^3+c)^(1/2)/d-12248/91*c^2*(d*
x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-12248/273*c^(7/3)*(c^(
1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2
)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+612
4/91*3^(1/4)*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^
(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(
1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2
)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.68, antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {489, 596, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{12248\sqrt{2}\,c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}\,d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{6124\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{91d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{32\sqrt{3}\,c^{13/6}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{32c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{32c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}-\frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-214*c*x^2*Sqrt[c + d*x^3])/(91*d^2) - (2*x^5*Sqrt[c + d*x^3])/(13*d) - (1
2248*c^2*Sqrt[c + d*x^3])/(91*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
- (32*Sqrt[3]*c^(13/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[
c + d*x^3]])/d^(8/3) + (32*c^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1
/6)*Sqrt[c + d*x^3])])/d^(8/3) - (32*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/d^(8/3) + (6124*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/
```

```
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(8/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d
*x^3]) - (12248*Sqrt[2]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2x^5\sqrt{c+dx^3}}{13d} + \frac{2\int \frac{x^4\left(40c^2+\frac{107}{2}cdx^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{13d}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} + \frac{4\int \frac{x\left(856c^3d+1531c^2d^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{91d^3}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} + \frac{4\int \left(-\frac{1531c^2dx}{\sqrt{c+dx^3}} + \frac{13104c^3dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{91d^3}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{(6124c^2)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{91d^2} + \frac{(576c^3)\int \frac{}{(8c-dx^3)}}{d^2}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{(48c^2)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^3} - \frac{(6}{}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{6124\sqrt[4]{3}}{}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{32\sqrt{3}\,c^{1}}{}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{32\sqrt{3}\,c^{1}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order

4 in optimal.
time = 4.20, size = 150, normalized size = 0.23

$$\frac{-20(107c^2x^2+114cdx^5+7d^2x^8)+2140c^2x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+1531cdx^5\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{910d^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-20*(107*c^2*x^2 + 114*c*d*x^5 + 7*d^2*x^8) + 2140*c^2*x^2*Sqrt[1 + (d*x^3
)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 1531*c*d*x^5
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(910*d^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 1788, normalized size = 2.76

method	result	size
risch	Expression too large to display	884
elliptic	Expression too large to display	889
default	Expression too large to display	1788

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/13*x^5*(d*x^3+c)^(1/2)+6/91*c*x^2*(d*x^3+c)^(1/2)/d+8/91*I*c^2/d^2*
3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-8/d^2*c*(2/7*x^2*(d*x^3+c)^(1/2)
-2/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
```

```
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-64*c^2/d^2*(-2/3*I*3^
(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-
c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*
3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 34.86, size = 3782, normalized size = 5.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
[Out] 2/273*(1456*sqrt(3)*d^3*(c^13/d^16)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c^13*
d^16*x^16 + 784*c^14*d^15*x^13 + 7680*c^15*d^14*x^10 + 10752*c^16*d^13*x^7
```

+ 4096*c^17*d^12*x^4)*(c^13/d^16)^(2/3) + 36*sqrt(3)*(c^17*d^11*x^17 + 1772
*c^18*d^10*x^14 + 42592*c^19*d^9*x^11 + 96256*c^20*d^8*x^8 + 69632*c^21*d^7
*x^5 + 16384*c^22*d^6*x^2)*(c^13/d^16)^(1/3) + sqrt(3)*(c^21*d^6*x^18 + 945
6*c^22*d^5*x^15 + 749184*c^23*d^4*x^12 + 3017216*c^24*d^3*x^9 + 3489792*c^2
5*d^2*x^6 + 1572864*c^26*d*x^3 + 262144*c^27) + 12*sqrt(d*x^3 + c)*(12*sqrt
(3)*(35*c^11*d^18*x^14 - 14440*c^12*d^17*x^11 - 24576*c^13*d^16*x^8 - 16384
*c^14*d^15*x^5 - 4096*c^15*d^14*x^2)*(c^13/d^16)^(5/6) + 18*sqrt(3)*(c^15*d
^13*x^15 - 1112*c^16*d^12*x^12 + 7296*c^17*d^11*x^9 + 11776*c^18*d^10*x^6 +
 4096*c^19*d^9*x^3)*sqrt(c^13/d^16) + sqrt(3)*(c^19*d^8*x^16 - 4768*c^20*d^
7*x^13 + 362752*c^21*d^6*x^10 + 709120*c^22*d^5*x^7 + 413696*c^23*d^4*x^4 +
 65536*c^24*d^3*x)*(c^13/d^16)^(1/6)) - 2*(324*sqrt(3)*(d^19*x^16 - 1858*c*
d^18*x^13 - 4176*c^2*d^17*x^10 - 3584*c^3*d^16*x^7 - 1024*c^4*d^15*x^4)*(c^
13/d^16)^(5/6) + 18*sqrt(3)*(c^4*d^14*x^17 - 5290*c^5*d^13*x^14 - 21152*c^6
*d^12*x^11 - 47744*c^7*d^11*x^8 - 37888*c^8*d^10*x^5 - 8192*c^9*d^9*x^2)*sq
rt(c^13/d^16) + sqrt(3)*(c^8*d^9*x^18 - 7698*c^9*d^8*x^15 - 1664688*c^10*d^
7*x^12 - 5524864*c^11*d^6*x^9 - 6223872*c^12*d^5*x^6 - 2703360*c^13*d^4*x^3
 - 327680*c^14*d^3)*(c^13/d^16)^(1/6) + 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c^2*d
^16*x^15 + 37352*c^3*d^15*x^12 - 230336*c^4*d^14*x^9 - 515072*c^5*d^13*x^6
 - 286720*c^6*d^12*x^3 - 32768*c^7*d^11)*(c^13/d^16)^(2/3) + 108*sqrt(3)*(53
*c^7*d^10*x^13 + 1320*c^8*d^9*x^10 + 1536*c^9*d^8*x^7 + 512*c^10*d^7*x^4)*(
c^13/d^16)^(1/3) + 6*sqrt(3)*(37*c^11*d^5*x^14 + 28912*c^12*d^4*x^11 + 4358
4*c^13*d^3*x^8 + 20992*c^14*d^2*x^5 + 4096*c^15*d*x^2)))*sqrt((18*c^22*d^2*
x^8 + 360*c^23*d*x^5 - 144*c^24*x^2 + (c^13*d^13*x^9 - 276*c^14*d^12*x^6 -
1608*c^15*d^11*x^3 - 1088*c^16*d^10)*(c^13/d^16)^(2/3) + 6*sqrt(d*x^3 + c)*
((c^11*d^15*x^7 - 28*c^12*d^14*x^4 - 272*c^13*d^13*x)*(c^13/d^16)^(5/6) - 2
4*(c^16*d^9*x^5 + c^17*d^8*x^2)*sqrt(c^13/d^16) + 4*(c^20*d^4*x^6 + 41*c^21
*d^3*x^3 + 40*c^22*d^2)*(c^13/d^16)^(1/6)) - 18*(c^18*d^7*x^7 - 52*c^19*d^6
*x^4 - 80*c^20*d^5*x)*(c^13/d^16)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)))/(c^21*d^6*x^18 - 14952*c^22*d^5*x^15 + 2872896*c^23*d^4*
x^12 + 7330304*c^24*d^3*x^9 + 6696960*c^25*d^2*x^6 + 2457600*c^26*d*x^3 + 2
62144*c^27)) - 1456*sqrt(3)*d^3*(c^13/d^16)^(1/6)*arctan(-1/3*(324*sqrt(3)*
(3*c^13*d^16*x^16 + 784*c^14*d^15*x^13 + 7680*c^15*d^14*x^10 + 10752*c^16*d
^13*x^7 + 4096*c^17*d^12*x^4)*(c^13/d^16)^(2/3) + 36*sqrt(3)*(c^17*d^11*x^1
7 + 1772*c^18*d^10*x^14 + 42592*c^19*d^9*x^11 + 96256*c^20*d^8*x^8 + 69632*
c^21*d^7*x^5 + 16384*c^22*d^6*x^2)*(c^13/d^16)^(1/3) + sqrt(3)*(c^21*d^6*x^
18 + 9456*c^22*d^5*x^15 + 749184*c^23*d^4*x^12 + 3017216*c^24*d^3*x^9 + 348
9792*c^25*d^2*x^6 + 1572864*c^26*d*x^3 + 262144*c^27) - 12*sqrt(d*x^3 + c)*
(12*sqrt(3)*(35*c^11*d^18*x^14 - 14440*c^12*d^17*x^11 - 24576*c^13*d^16*x^8
 - 16384*c^14*d^15*x^5 - 4096*c^15*d^14*x^2)*(c^13/d^16)^(5/6) + 18*sqrt(3)
*(c^15*d^13*x^15 - 1112*c^16*d^12*x^12 + 7296*c^17*d^11*x^9 + 11776*c^18*d^
10*x^6 + 4096*c^19*d^9*x^3)*sqrt(c^13/d^16) + sqrt(3)*(c^19*d^8*x^16 - 4768
*c^20*d^7*x^13 + 362752*c^21*d^6*x^10 + 709120*c^22*d^5*x^7 + 413696*c^23*d
^4*x^4 + 65536*c^24*d^3*x)*(c^13/d^16)^(1/6)) + 2*(324*sqrt(3)*(d^19*x^16 -
 1858*c*d^18*x^13 - 4176*c^2*d^17*x^10 - 3584*c^3*d^16*x^7 - 1024*c^4*d^15*
x^4)*(c^13/d^16)^(5/6) + 18*sqrt(3)*(c^4*d^14*x^17 - 5290*c^5*d^13*x^14 - 2

```
1152*c^6*d^12*x^11 - 47744*c^7*d^11*x^8 - 37888*c^8*d^10*x^5 - 8192*c^9*d^9
*x^2)*sqrt(c^13/d^16) + sqrt(3)*(c^8*d^9*x^18 - 7698*c^9*d^8*x^15 - 1664688
*c^10*d^7*x^12 - 5524864*c^11*d^6*x^9 - 6223872*c^12*d^5*x^6 - 2703360*c^13
*d^4*x^3 - 327680*c^14*d^3)*(c^13/d^16)^(1/6) - 6*sqrt(d*x^3 + c)*(sqrt(3)*
(7*c^2*d^16*x^15 + 37352*c^3*d^15*x^12 - 230336*c^4*d^14*x^9 - 515072*c^5*d
^13*x^6 - 286720*c^6*d^12*x^3 - 32768*c^7*d^11)*(c^13/d^16)^(2/3) + 108*sqr
t(3)*(53*c^7*d^10*x^13 + 1320*c^8*d^9*x^10 + 1536*c^9*d^8*x^7 + 512*c^10*d^
7*x^4)*(c^13/d^16)^(1/3) + 6*sqrt(3)*(37*c^11*d^5*x^14 + 28912*c^12*d^4*x^1
1 + 43584*c^13*d^3*x^8 + 20992*c^14*d^2*x^5 + 4096*c^15*d*x^2)))*sqrt((18*c
^22*d^2*x^8 + 360*c^23*d*x^5 - 144*c^24*x^2 + (c^13*d^13*x^9 - 276*c^14*d^1
2*x^6 - 1608*c^15*d^11*x^3 - 1088*c^16*d^10)*(c^13/d^16)^(2/3) - 6*sqrt(d*x
^3 + c)*((c^11*d^15*x^7 - 28*c^12*d^14*x^4 - 272*c^13*d^13*x)*(c^13/d^16)^(
5/6) - 24*(c^16*d^9*x^5 + c^17*d^8*x^2)*sqrt(c^13/d^16) + 4*(c^20*d^4*x^6 +
 41*c^21*d^3*x^3 + 40*c^22*d^2)*(c^13/d^16)^(1/6)) - 18*(c^18*d^7*x^7 - 52*
c^19*d^6*x^4 - 80*c^20*d^5*x)*(c^13/d^16)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)))/(c^21*d^6*x^18 - 14952*c^22*d^5*x^15 + 2872896*c
^23*d^4*x^12 + 7330304*c^24*d^3*x^9 + 6696960*c^25*d^2*x^6 + 2457600*c^26*d
*x^3 + 262144*c^27)) + 364*d^3*(c^13/d^16)^(1/6)*log(4503599627370496*(18*c
^22*d^2*x^8 + 360*c^23*d*x^5 - 144*c^24*x^2 + (c^13*d^13*x^9 - 276*c^14*d^1
2*x^6 - 1608*c^15*d^11*x^3 - 1088*c^16*d^10)*(c...
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
[Out] -Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

3.290 $\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=624

$$-\frac{2x^2\sqrt{c+dx^3}}{7d}-\frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{4\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{4c^{7/6}\tanh^{-}}{}$$

```
[Out] 4*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5/3
)-4*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-4*c^(7/6)*arctan(c
^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)-2/7*x^2
*(d*x^3+c)^(1/2)/d-118/7*c*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2)))-118/21*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-
3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-
c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(
3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3
)*(1+3^(1/2)))^2)^(1/2)+59/7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE(
(d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {489, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{118\sqrt{2}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{59\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{4\sqrt{3}\,c^{7/6}\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{4c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{4c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}-\frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{2x^2\sqrt{c+dx^3}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/(7*d) - (118*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (4*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*
(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(5/3) + (4*c^(7/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(5/3) - (4*c^(7/6)*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/d^(5/3) + (59*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^
(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^
```

```
(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
7*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3]) - (118*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3
)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*
x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2x^2\sqrt{c+dx^3}}{7d} + \frac{2\int \frac{x\left(16c^2+\frac{59}{2}cdx^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{7d}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} + \frac{2\int \left(-\frac{59cx}{2\sqrt{c+dx^3}} + \frac{252c^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{7d}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(59c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d} + \frac{(72c^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(6c)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^2} - \frac{(59c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}}{\sqrt{c+dx^3}}}{7d^{4/3}}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\right.}{}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{4\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}\right.}{\sqrt{c+d}}\right.}{d^{5/3}}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{4\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}\right.}{\sqrt{c+d}}\right.}{d^{5/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 4.07, size = 130, normalized size = 0.21

$$\frac{x^2\left(-80(c+dx^3)+80c\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+59dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{280d\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (x^2*(-80*(c + d*x^3) + 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), (d*x^3)/(8*c)] + 59*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(280*d*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 1310, normalized size = 2.10

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1310

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/7*x^2*(d*x^3+c)^(1/2)-2/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2))))-8*c/d*(-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*
d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(
-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*
d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
```

```
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3
*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.90, size = 3756, normalized size = 6.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] 1/21*(28*sqrt(3)*d^2*(c^7/d^10)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c^7*d^12*
x^16 + 784*c^8*d^11*x^13 + 7680*c^9*d^10*x^10 + 10752*c^10*d^9*x^7 + 4096*c
^11*d^8*x^4)*(c^7/d^10)^(2/3) + 36*sqrt(3)*(c^9*d^9*x^17 + 1772*c^10*d^8*x^
14 + 42592*c^11*d^7*x^11 + 96256*c^12*d^6*x^8 + 69632*c^13*d^5*x^5 + 16384*
c^14*d^4*x^2)*(c^7/d^10)^(1/3) + sqrt(3)*(c^11*d^6*x^18 + 9456*c^12*d^5*x^1
5 + 749184*c^13*d^4*x^12 + 3017216*c^14*d^3*x^9 + 3489792*c^15*d^2*x^6 + 15
72864*c^16*d*x^3 + 262144*c^17) + 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c^6*d^
13*x^14 - 14440*c^7*d^12*x^11 - 24576*c^8*d^11*x^8 - 16384*c^9*d^10*x^5 - 4
096*c^10*d^9*x^2)*(c^7/d^10)^(5/6) + 18*sqrt(3)*(c^8*d^10*x^15 - 1112*c^9*d
^9*x^12 + 7296*c^10*d^8*x^9 + 11776*c^11*d^7*x^6 + 4096*c^12*d^6*x^3)*sqrt(
c^7/d^10) + sqrt(3)*(c^10*d^7*x^16 - 4768*c^11*d^6*x^13 + 362752*c^12*d^5*x
^10 + 709120*c^13*d^4*x^7 + 413696*c^14*d^3*x^4 + 65536*c^15*d^2*x)*(c^7/d^
10)^(1/6)) - 2*(324*sqrt(3)*(d^14*x^16 - 1858*c*d^13*x^13 - 4176*c^2*d^12*x
^10 - 3584*c^3*d^11*x^7 - 1024*c^4*d^10*x^4)*(c^7/d^10)^(5/6) + 18*sqrt(3)*
```

(c^2*d^11*x^17 - 5290*c^3*d^10*x^14 - 21152*c^4*d^9*x^11 - 47744*c^5*d^8*x^
8 - 37888*c^6*d^7*x^5 - 8192*c^7*d^6*x^2)*sqrt(c^7/d^10) + sqrt(3)*(c^4*d^8
*x^18 - 7698*c^5*d^7*x^15 - 1664688*c^6*d^6*x^12 - 5524864*c^7*d^5*x^9 - 62
23872*c^8*d^4*x^6 - 2703360*c^9*d^3*x^3 - 327680*c^10*d^2)*(c^7/d^10)^(1/6)
 + 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c*d^12*x^15 + 37352*c^2*d^11*x^12 - 230336
*c^3*d^10*x^9 - 515072*c^4*d^9*x^6 - 286720*c^5*d^8*x^3 - 32768*c^6*d^7)*(c
^7/d^10)^(2/3) + 108*sqrt(3)*(53*c^4*d^8*x^13 + 1320*c^5*d^7*x^10 + 1536*c^
6*d^6*x^7 + 512*c^7*d^5*x^4)*(c^7/d^10)^(1/3) + 6*sqrt(3)*(37*c^6*d^5*x^14
+ 28912*c^7*d^4*x^11 + 43584*c^8*d^3*x^8 + 20992*c^9*d^2*x^5 + 4096*c^10*d*
x^2)))*sqrt((18*c^12*d^2*x^8 + 360*c^13*d*x^5 - 144*c^14*x^2 + (c^7*d^9*x^9
 - 276*c^8*d^8*x^6 - 1608*c^9*d^7*x^3 - 1088*c^10*d^6)*(c^7/d^10)^(2/3) + 6
*sqrt(d*x^3 + c)*((c^6*d^10*x^7 - 28*c^7*d^9*x^4 - 272*c^8*d^8*x)*(c^7/d^10
)^(5/6) - 24*(c^9*d^6*x^5 + c^10*d^5*x^2)*sqrt(c^7/d^10) + 4*(c^11*d^3*x^6
+ 41*c^12*d^2*x^3 + 40*c^13*d)*(c^7/d^10)^(1/6)) - 18*(c^10*d^5*x^7 - 52*c^
11*d^4*x^4 - 80*c^12*d^3*x)*(c^7/d^10)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192
*c^2*d*x^3 - 512*c^3)))/(c^11*d^6*x^18 - 14952*c^12*d^5*x^15 + 2872896*c^13
*d^4*x^12 + 7330304*c^14*d^3*x^9 + 6696960*c^15*d^2*x^6 + 2457600*c^16*d*x^
3 + 262144*c^17)) - 28*sqrt(3)*d^2*(c^7/d^10)^(1/6)*arctan(-1/3*(324*sqrt(3
)*(3*c^7*d^12*x^16 + 784*c^8*d^11*x^13 + 7680*c^9*d^10*x^10 + 10752*c^10*d^
9*x^7 + 4096*c^11*d^8*x^4)*(c^7/d^10)^(2/3) + 36*sqrt(3)*(c^9*d^9*x^17 + 17
72*c^10*d^8*x^14 + 42592*c^11*d^7*x^11 + 96256*c^12*d^6*x^8 + 69632*c^13*d^
5*x^5 + 16384*c^14*d^4*x^2)*(c^7/d^10)^(1/3) + sqrt(3)*(c^11*d^6*x^18 + 945
6*c^12*d^5*x^15 + 749184*c^13*d^4*x^12 + 3017216*c^14*d^3*x^9 + 3489792*c^1
5*d^2*x^6 + 1572864*c^16*d*x^3 + 262144*c^17) - 12*sqrt(d*x^3 + c)*(12*sqrt
(3)*(35*c^6*d^13*x^14 - 14440*c^7*d^12*x^11 - 24576*c^8*d^11*x^8 - 16384*c^
9*d^10*x^5 - 4096*c^10*d^9*x^2)*(c^7/d^10)^(5/6) + 18*sqrt(3)*(c^8*d^10*x^1
5 - 1112*c^9*d^9*x^12 + 7296*c^10*d^8*x^9 + 11776*c^11*d^7*x^6 + 4096*c^12*
d^6*x^3)*sqrt(c^7/d^10) + sqrt(3)*(c^10*d^7*x^16 - 4768*c^11*d^6*x^13 + 362
752*c^12*d^5*x^10 + 709120*c^13*d^4*x^7 + 413696*c^14*d^3*x^4 + 65536*c^15*
d^2*x)*(c^7/d^10)^(1/6)) + 2*(324*sqrt(3)*(d^14*x^16 - 1858*c*d^13*x^13 - 4
176*c^2*d^12*x^10 - 3584*c^3*d^11*x^7 - 1024*c^4*d^10*x^4)*(c^7/d^10)^(5/6)
 + 18*sqrt(3)*(c^2*d^11*x^17 - 5290*c^3*d^10*x^14 - 21152*c^4*d^9*x^11 - 47
744*c^5*d^8*x^8 - 37888*c^6*d^7*x^5 - 8192*c^7*d^6*x^2)*sqrt(c^7/d^10) + sq
rt(3)*(c^4*d^8*x^18 - 7698*c^5*d^7*x^15 - 1664688*c^6*d^6*x^12 - 5524864*c^
7*d^5*x^9 - 6223872*c^8*d^4*x^6 - 2703360*c^9*d^3*x^3 - 327680*c^10*d^2)*(c
^7/d^10)^(1/6) - 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c*d^12*x^15 + 37352*c^2*d^11
*x^12 - 230336*c^3*d^10*x^9 - 515072*c^4*d^9*x^6 - 286720*c^5*d^8*x^3 - 327
68*c^6*d^7)*(c^7/d^10)^(2/3) + 108*sqrt(3)*(53*c^4*d^8*x^13 + 1320*c^5*d^7*
x^10 + 1536*c^6*d^6*x^7 + 512*c^7*d^5*x^4)*(c^7/d^10)^(1/3) + 6*sqrt(3)*(37
*c^6*d^5*x^14 + 28912*c^7*d^4*x^11 + 43584*c^8*d^3*x^8 + 20992*c^9*d^2*x^5
+ 4096*c^10*d*x^2)))*sqrt((18*c^12*d^2*x^8 + 360*c^13*d*x^5 - 144*c^14*x^2
+ (c^7*d^9*x^9 - 276*c^8*d^8*x^6 - 1608*c^9*d^7*x^3 - 1088*c^10*d^6)*(c^7/d
^10)^(2/3) - 6*sqrt(d*x^3 + c)*((c^6*d^10*x^7 - 28*c^7*d^9*x^4 - 272*c^8*d^
8*x)*(c^7/d^10)^(5/6) - 24*(c^9*d^6*x^5 + c^10*d^5*x^2)*sqrt(c^7/d^10) + 4*
(c^11*d^3*x^6 + 41*c^12*d^2*x^3 + 40*c^13*d)*(c^7/d^10)^(1/6)) - 18*(c^10*d

```
^5*x^7 - 52*c^11*d^4*x^4 - 80*c^12*d^3*x)*(c^7/d^10)^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^11*d^6*x^18 - 14952*c^12*d^5*x^15
+ 2872896*c^13*d^4*x^12 + 7330304*c^14*d^3*x^9 + 6696960*c^15*d^2*x^6 + 245
7600*c^16*d*x^3 + 262144*c^17)) - 6*sqrt(d*x^3 + c)*d*x^2 + 7*d^2*(c^7/d^10
)^(1/6)*log(4194304*(18*c^12*d^2*x^8 + 360*c^13*d*x^5 - 144*c^14*x^2 + (c^7
*d^9*x^9 - 276*c^8*d^8*x^6 - 1608*c^9*d^7*x^3 - 1088*c^10*d^6)*(c^7/d^10)^(
2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^10*x^7 - 28*c^7*d^9*x^4 - 272*c^8*d^8*x)*(
c^7/d^10)^(5/6) - 24*(c^9*d^6*x^5 + c^10*d^5*x^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
[Out] -Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
[Out] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

3.291 $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=601

$$-\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\sqrt{3}\,\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}+\frac{\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{2d^{2/3}}$$

```
[Out] 1/2*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(2
/3)-1/2*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)-1/2*c^(1/6)*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(2/3)-2
*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-2/3*c^(1/3)*(c^(1/
3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(2/3)/(d*x^3+c)^(1/2)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+3^(1/
4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {495, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{2\sqrt{2}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{\sqrt{3}\,\sqrt[6]{c}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}-\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{2d^{2/3}}-\frac{\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[
3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(2*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
 d*x^3])])/(2*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*
d^(2/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
```

```
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^
(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt
[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = (9c)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx - \int \frac{x}{\sqrt{c+dx^3}}\,dx$$

$$= -\frac{3\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{4d} - \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{\sqrt[3]{d}} + \frac{\left(3\sqrt[3]{c}\right)\int \frac{1\cdots}{\left(2-\frac{\sqrt[3]{d}}{\sqrt[3]{c}}\cdots\right.}}{4\sqrt[3]{\cdots}}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\cdots}{\left(\left(1+\sqrt{3}\right.\right.\cdots}}}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\cdots}{\left(\left(1+\cdots\right.\right.}}}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{3}\,\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{\sqrt[6]{c}\,\mathrm{ta}\cdots}{\cdots}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{3}\,\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{\sqrt[6]{c}\,\mathrm{ta}\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 5.92, size = 63, normalized size = 0.10

$$\frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/
(8*c)])/(16*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 848, normalized size = 1.41

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/3*I/d^3*2^(1/2)*sum(1/_
alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_a
lpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-
8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")

[Out] -integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.62, size = 3481, normalized size = 5.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")

[Out] 1/24*(4*sqrt(3)*d*(c/d^4)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c*d^8*x^16 + 784*c^2*d^7*x^13 + 7680*c^3*d^6*x^10 + 10752*c^4*d^5*x^7 + 4096*c^5*d^4*x^4)*(c/d^4)^(2/3) + 36*sqrt(3)*(c*d^7*x^17 + 1772*c^2*d^6*x^14 + 42592*c^3*d^5*x^11 + 96256*c^4*d^4*x^8 + 69632*c^5*d^3*x^5 + 16384*c^6*d^2*x^2)*(c/d^4)^(1/3) + sqrt(3)*(c*d^6*x^18 + 9456*c^2*d^5*x^15 + 749184*c^3*d^4*x^12 + 3017216*c^4*d^3*x^9 + 3489792*c^5*d^2*x^6 + 1572864*c^6*d*x^3 + 262144*c^7) + 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c*d^8*x^14 - 14440*c^2*d^7*x^11 - 24576*c^3*d^6*x^8 - 16384*c^4*d^5*x^5 - 4096*c^5*d^4*x^2)*(c/d^4)^(5/6) + 18*sqrt(3)*(c*d^7*x^15 - 1112*c^2*d^6*x^12 + 7296*c^3*d^5*x^9 + 11776*c^4*d^4*x^6 + 4096*c^5*d^3*x^3)*sqrt(c/d^4) + sqrt(3)*(c*d^6*x^16 - 4768*c^2*d^5*x^13 + 362752*c^3*d^4*x^10 + 709120*c^4*d^3*x^7 + 413696*c^5*d^2*x^4 + 65536*c^6*d*x)*(c/d^4)^(1/6)) - 2*(324*sqrt(3)*(d^9*x^16 - 1858*c*d^8*x^13 - 4176*c^2*d^7*x^10 - 3584*c^3*d^6*x^7 - 1024*c^4*d^5*x^4)*(c/d^4)^(5/6) + 18*sqrt(3)*(d^8*x^17 - 5290*c*d^7*x^14 - 21152*c^2*d^6*x^11 - 47744*c^3*d^5*x^8 - 37888*c^4*d^4*x^5 - 8192*c^5*d^3*x^2)*sqrt(c/d^4) + sqrt(3)*(d^7*x^18 - 7698*c*d^6*x^15 - 1664688*c^2*d^5*x^12 - 5524864*c^3*d^4*x^9 - 6223872*c^4*d^3*x^6 - 2703360*c^5*d^2*x^3 - 327680*c^6*d)*(c/d^4)^(1/6) + 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*d^8*x^15 + 37352*c*d^7*x^12 - 230336*c^2*d^6*x^9 - 515072*c^3*d^5*x^6 - 286720*c^4*d^4*x^3 - 32768*c^5*d^3)*(c/d^4)^(2/3) + 108*sqrt(3)*(53*c*d^6*x^13 + 1320*c^2*d^5*x^10 + 1536*c^3*d^4*x^7 + 512*c^4*d^3*x^4)*(c/d^4)^(1/3) + 6*sqrt(3)*(37*c*d^5*x^14 + 28912*c^2*d^4*x^11 + 43584*c^3*d^3*x^8 + 20992*c^4*d^2*x^5 + 4096*c^5*d*x^2)))*sqrt((18*c^2*d^2*x^8 + 360*c^3*d*x^5 - 144*c^4*x^2 + (c*d^5*x^9 - 276*c^2*d^4*x^6 - 1608*c^3*d^3*x^3 - 1088*c^4*d^2)*(c/d^4)^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^5*x^7 - 28*c^2*d^4*x^4 - 272*c^3*d^3*x)*(c/d^4)^(5/6) - 24*(c^2*d^3*x^5 + c^3*d^2*x^2)*sqrt(c/d^4) + 4*(c^2*d^2*x^6 + 41*c^3*d*x^3 + 40*c^4)*(c/d^4)^(1/6)) - 18*(c^2*d^3*x^7 - 52*c^3*d^2*x^4 - 80*c^4*d*x)*(c/d^4)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^6*x^18 - 14952*c^2*d^5*x^15 + 2872896*c^3*d^4*x^12 + 7330304*c^4*d^3*x^9 + 6696960*c^5*d^2*x^6 + 2457600*c^6*d*x^3 + 262144*c^7)) - 4*sqrt(3)*d*(c/d^4)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c*d^8*x^16 + 784*c^2*d^7*x^13 + 7680*c^3*d^6*x^10 + 10752*c^4*d^5*x^7 + 4096*c^5*d^4*x^4)*(c/d^4)^(2/3) + 36*sqrt(3)*(c*d^7*x^17 + 1772*c^2*d^6*x^14 + 42592*c^3*d^5*x^11 + 96256*c^4*d^4*x^8 + 69632*c^5*d^3*x^5 + 16384*c^6*d^2*x^2)*(c/d^4)^(1/3) + sqrt(3)*(c*d^6*x^18 + 9456*c^2*d^5*x^15 + 749184*c^3*d^4*x^12 + 30

```
17216*c^4*d^3*x^9 + 3489792*c^5*d^2*x^6 + 1572864*c^6*d*x^3 + 262144*c^7) -
 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c*d^8*x^14 - 14440*c^2*d^7*x^11 - 24576
*c^3*d^6*x^8 - 16384*c^4*d^5*x^5 - 4096*c^5*d^4*x^2)*(c/d^4)^(5/6) + 18*sqr
t(3)*(c*d^7*x^15 - 1112*c^2*d^6*x^12 + 7296*c^3*d^5*x^9 + 11776*c^4*d^4*x^6
 + 4096*c^5*d^3*x^3)*sqrt(c/d^4) + sqrt(3)*(c*d^6*x^16 - 4768*c^2*d^5*x^13
+ 362752*c^3*d^4*x^10 + 709120*c^4*d^3*x^7 + 413696*c^5*d^2*x^4 + 65536*c^6
*d*x)*(c/d^4)^(1/6)) + 2*(324*sqrt(3)*(d^9*x^16 - 1858*c*d^8*x^13 - 4176*c^
2*d^7*x^10 - 3584*c^3*d^6*x^7 - 1024*c^4*d^5*x^4)*(c/d^4)^(5/6) + 18*sqrt(3
)*(d^8*x^17 - 5290*c*d^7*x^14 - 21152*c^2*d^6*x^11 - 47744*c^3*d^5*x^8 - 37
888*c^4*d^4*x^5 - 8192*c^5*d^3*x^2)*sqrt(c/d^4) + sqrt(3)*(d^7*x^18 - 7698*
c*d^6*x^15 - 1664688*c^2*d^5*x^12 - 5524864*c^3*d^4*x^9 - 6223872*c^4*d^3*x
^6 - 2703360*c^5*d^2*x^3 - 327680*c^6*d)*(c/d^4)^(1/6) - 6*sqrt(d*x^3 + c)*
(sqrt(3)*(7*d^8*x^15 + 37352*c*d^7*x^12 - 230336*c^2*d^6*x^9 - 515072*c^3*d
^5*x^6 - 286720*c^4*d^4*x^3 - 32768*c^5*d^3)*(c/d^4)^(2/3) + 108*sqrt(3)*(5
3*c*d^6*x^13 + 1320*c^2*d^5*x^10 + 1536*c^3*d^4*x^7 + 512*c^4*d^3*x^4)*(c/d
^4)^(1/3) + 6*sqrt(3)*(37*c*d^5*x^14 + 28912*c^2*d^4*x^11 + 43584*c^3*d^3*x
^8 + 20992*c^4*d^2*x^5 + 4096*c^5*d*x^2)))*sqrt((18*c^2*d^2*x^8 + 360*c^3*d
*x^5 - 144*c^4*x^2 + (c*d^5*x^9 - 276*c^2*d^4*x^6 - 1608*c^3*d^3*x^3 - 1088
*c^4*d^2)*(c/d^4)^(2/3) - 6*sqrt(d*x^3 + c)*((c*d^5*x^7 - 28*c^2*d^4*x^4 -
272*c^3*d^3*x)*(c/d^4)^(5/6) - 24*(c^2*d^3*x^5 + c^3*d^2*x^2)*sqrt(c/d^4) +
 4*(c^2*d^2*x^6 + 41*c^3*d*x^3 + 40*c^4)*(c/d^4)^(1/6)) - 18*(c^2*d^3*x^7 -
 52*c^3*d^2*x^4 - 80*c^4*d*x)*(c/d^4)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*
c^2*d*x^3 - 512*c^3)))/(c*d^6*x^18 - 14952*c^2*d^5*x^15 + 2872896*c^3*d^4*x
^12 + 7330304*c^4*d^3*x^9 + 6696960*c^5*d^2*x^6 + 2457600*c^6*d*x^3 + 26214
4*c^7)) + d*(c/d^4)^(1/6)*log(1/4*(18*c^2*d^2*x^8 + 360*c^3*d*x^5 - 144*c^4
*x^2 + (c*d^5*x^9 - 276*c^2*d^4*x^6 - 1608*c^3*d^3*x^3 - 1088*c^4*d^2)*(c/d
^4)^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^5*x^7 - 28*c^2*d^4*x^4 - 272*c^3*d^3*x)
*(c/d^4)^(5/6) - 24*(c^2*d^3*x^5 + c^3*d^2*x^2)*sqrt(c/d^4) + 4*(c^2*d^2*x^
6 + 41*c^3*d*x^3 + 40*c^4)*(c/d^4)^(1/6)) - 18*(c^2*d^3*x^7 - 52*c^3*d^2*x^
4 - 80*c^4*d*x)*(c/d^4)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 51
2*c^3)) - d*(c/d^4)^(1/6)*log(1/4*(18*c^2*d^2*x^8 + 360*c^3*d*x^5 - 144*c^4
*x^2 + (c*d^5*x^9 - 276*c^2*d^4*x^6 - 1608*c^3*d^3*x^3 - 1088*c^4*d^2)*(c/d
^4)^(2/3) - 6*sqrt(d*x^3 + c)*((c*d^5*x^7 - 28*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
[Out] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

3.292 $\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx$

Optimal. Leaf size=632

$$-\frac{\sqrt{c+dx^3}}{8cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}}+\frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c} \ldots\right.}{3\sqrt[6]{c}\ldots}\right.}{16c^{5/6}}$$

```
[Out] 1/16*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(
5/6)-1/16*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)-1/16*d^(1/3)
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c^(5/6
)-1/8*(d*x^3+c)^(1/2)/c/x+1/8*d^(1/3)*(d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))+1/24*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*
(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/
3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2)))^2)^(1/2)-1/16*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*Ellipti
cE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c
^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {486, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\,c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{16c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{3}\sqrt[3]{d}\,\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}}+\frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16c^{5/6}}-\frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16c^{5/6}}-\frac{\sqrt{c+dx^3}}{8cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

```
[Out] -1/8*Sqrt[c + d*x^3]/(c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(8*c*((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) +
 d^(1/3)*x))/Sqrt[c + d*x^3]])/(16*c^(5/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d
^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(16*c^(5/6)) - (d^(1/3)*ArcTanh[S
qrt[c + d*x^3]/(3*Sqrt[c])])/(16*c^(5/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1
/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
```

```
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16
*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) -
 c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*c^(2/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}dx = -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\int \frac{x\left(13cd-\frac{d^2x^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{9cdx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{1}{8}(9d)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}dx + \frac{d\int \frac{x}{\sqrt{c+dx^3}}dx}{16c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} - \frac{3\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{32c} + \frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}dx}{16c} +$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\ldots}{16c^{2}\ldots}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\ldots\right.}{16c^{5/6}}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\ldots\right.}{16c^{5/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 20.07, size = 137, normalized size = 0.22

$$\frac{-80c(c+dx^3)+65cdx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-d^2x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{640c^2x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

```
[Out] (-80*c*(c + d*x^3) + 65*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^2*x*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 1306, normalized size = 2.07

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	1306

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d/c*(-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^
(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x
+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^
(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1
/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c
*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^
2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1
/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Roo
```

```
tOf(_Z^3*d-8*c))+1/8/c*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.62, size = 2490, normalized size = 3.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] -1/192*(4*sqrt(3)*c*x*(d^2/c^5)^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^4*x^5*(d^2
/c^5)^(1/6) - sqrt(3)*(c^4*d^3*x^6 - 40*c^5*d^2*x^3 - 32*c^6*d)*(d^2/c^5)^(
5/6) + 3*sqrt(3)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(d^2/c^5))*sqrt(d*x^3 +
c) + (18*sqrt(3)*(c^4*d^2*x^5 + c^5*d*x^2)*(d^2/c^5)^(2/3) + 12*sqrt(3)*(c^
2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(d^2/c^5)^(1/3) + 3*sqrt(3)*(d^4*x^7 + 5
*c*d^3*x^4 + 4*c^2*d^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^2*x^6 + 32*c^5*
d*x^3 + 40*c^6)*(d^2/c^5)^(5/6) + 3*sqrt(3)*(7*c^3*d^2*x^4 + 4*c^4*d*x)*sqr
t(d^2/c^5) + 9*sqrt(3)*(c*d^3*x^5 + 2*c^2*d^2*x^2)*(d^2/c^5)^(1/6)))*sqrt((
d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^4*d^3*x^7
 - 52*c^5*d^2*x^4 - 80*c^6*d*x)*(d^2/c^5)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^
5*d^2*x^5 + c^6*d*x^2)*(d^2/c^5)^(5/6) - 4*(c^3*d^3*x^6 + 41*c^4*d^2*x^3 +
40*c^5*d)*sqrt(d^2/c^5) - (c*d^4*x^7 - 28*c^2*d^3*x^4 - 272*c^3*d^2*x)*(d^2
/c^5)^(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(d^2/c^5)^
(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^5*x^7 - 7*c*
d^4*x^4 - 8*c^2*d^3*x)) + 4*sqrt(3)*c*x*(d^2/c^5)^(1/6)*arctan(1/9*((9*sqrt
```

```
(3)*c*d^4*x^5*(d^2/c^5)^(1/6) - sqrt(3)*(c^4*d^3*x^6 - 40*c^5*d^2*x^3 - 32*
c^6*d)*(d^2/c^5)^(5/6) + 3*sqrt(3)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(d^2/c
^5))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^4*d^2*x^5 + c^5*d*x^2)*(d^2/c^5)^(2/3
) + 12*sqrt(3)*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(d^2/c^5)^(1/3) + 3*sq
rt(3)*(d^4*x^7 + 5*c*d^3*x^4 + 4*c^2*d^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^4
*d^2*x^6 + 32*c^5*d*x^3 + 40*c^6)*(d^2/c^5)^(5/6) + 3*sqrt(3)*(7*c^3*d^2*x^
4 + 4*c^4*d*x)*sqrt(d^2/c^5) + 9*sqrt(3)*(c*d^3*x^5 + 2*c^2*d^2*x^2)*(d^2/c
^5)^(1/6)))*sqrt((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2
 - 18*(c^4*d^3*x^7 - 52*c^5*d^2*x^4 - 80*c^6*d*x)*(d^2/c^5)^(2/3) - 6*sqrt(
d*x^3 + c)*(24*(c^5*d^2*x^5 + c^6*d*x^2)*(d^2/c^5)^(5/6) - 4*(c^3*d^3*x^6 +
 41*c^4*d^2*x^3 + 40*c^5*d)*sqrt(d^2/c^5) - (c*d^4*x^7 - 28*c^2*d^3*x^4 - 2
72*c^3*d^2*x)*(d^2/c^5)^(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d
^2*x^2)*(d^2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)
))/(d^5*x^7 - 7*c*d^4*x^4 - 8*c^2*d^3*x)) + c*x*(d^2/c^5)^(1/6)*log((d^5*x^
9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^4*d^3*x^7 - 52*
c^5*d^2*x^4 - 80*c^6*d*x)*(d^2/c^5)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^2*
x^5 + c^6*d*x^2)*(d^2/c^5)^(5/6) - 4*(c^3*d^3*x^6 + 41*c^4*d^2*x^3 + 40*c^5
*d)*sqrt(d^2/c^5) - (c*d^4*x^7 - 28*c^2*d^3*x^4 - 272*c^3*d^2*x)*(d^2/c^5)^
(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(d^2/c^5)^(1/3))
/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - c*x*(d^2/c^5)^(1/6)*
log((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^4*d^
3*x^7 - 52*c^5*d^2*x^4 - 80*c^6*d*x)*(d^2/c^5)^(2/3) - 6*sqrt(d*x^3 + c)*(2
4*(c^5*d^2*x^5 + c^6*d*x^2)*(d^2/c^5)^(5/6) - 4*(c^3*d^3*x^6 + 41*c^4*d^2*x
^3 + 40*c^5*d)*sqrt(d^2/c^5) - (c*d^4*x^7 - 28*c^2*d^3*x^4 - 272*c^3*d^2*x)
*(d^2/c^5)^(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(d^2/
c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c*x*(d^
2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d +
18*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x)*(d^2/c^5)^(2/3) + 6*sqrt(d*x^3
 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^2/c^5)^(5/6) + (7*c^3*d^2*x^6 + 152*
c^4*d*x^3 + 64*c^5)*sqrt(d^2/c^5) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d
*x)*(d^2/c^5)^(1/6)) + 18*(c^2*d^3*x^8 + 38*c^3*d^2*x^5 + 64*c^4*d*x^2)*(d^
2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*c*x*(
d^2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d
+ 18*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x)*(d^2/c^5)^(2/3) - 6*sqrt(d*x
^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^2/c^5)^(5/6) + (7*c^3*d^2*x^6 + 15
2*c^4*d*x^3 + 64*c^5)*sqrt(d^2/c^5) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3
*d*x)*(d^2/c^5)^(1/6)) + 18*(c^2*d^3*x^8 + 38*c^3*d^2*x^5 + 64*c^4*d*x^2)*(
d^2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 24*sq
rt(d)*x*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 24*
sqrt(d*x^3 + c))/(c*x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c),x)
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)), x)
```

3.293 $\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)}\,dx$

Optimal. Leaf size=654

$$-\frac{\sqrt{c+dx^3}}{32cx^4}-\frac{d\sqrt{c+dx^3}}{16c^2x}+\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}}+\frac{d^{4}}{}$$

```
[Out] 1/128*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(11/6)-1/128*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/128*d^
(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c
^(11/6)-1/32*(d*x^3+c)^(1/2)/c/x^4-1/16*d*(d*x^3+c)^(1/2)/c^2/x+1/16*d^(4/3
)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/48*d^(4/3)*(c^(1/3)
+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/32*3^
(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 654, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {486, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{32c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{3}\,d^{4/3}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{11/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{11/6}}+\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{d\sqrt{c+dx^3}}{16c^2x}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

```
[Out] -1/32*Sqrt[c + d*x^3]/(c*x^4) - (d*Sqrt[c + d*x^3])/(16*c^2*x) + (d^(4/3)*S
qrt[c + d*x^3])/(16*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(
4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(128*
c^(11/6)) + (d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*
x^3])])/(128*c^(11/6)) - (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(12
```

```
8*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (
d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x
^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])
/(8*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{32cx^4} + \frac{\int \frac{16cd+\frac{5d^2x^3}{2}}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} - \frac{\int \frac{x\left(-100c^2d^2+8cd^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} - \frac{\int \left(-\frac{8cd^2x}{\sqrt{c+dx^3}} - \frac{36c^2d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c^2} + \frac{(9d^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{64c}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} - \frac{(3d)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{256c^2} + \cdots$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \cdots$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \cdots$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 20.07, size = 153, normalized size = 0.23

$$\frac{125cd^2x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\left(40c(c^2+3cdx^3+2d^2x^6)+d^3x^9\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{5120c^3x^4\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

```
[Out] (125*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 3*c*d*x^3 + 2*d^2*x^6) + d^3*x^9*Sqrt[1 +
(d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(5120*
c^3*x^4*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 1782, normalized size = 2.72

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	887
default	Expression too large to display	1782

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*d*(d*x^3+c)^(1/2)/c/x-1/8*I/c*d*3^(1/2)
*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/64/c^2*d^2*(-2/3*I*3^(1/2)/d*(-c*d^2)
^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
```

```
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/
2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a
lpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64*d/c^2*(-(d*x
^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/
3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.85, size = 2612, normalized size = 3.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

[Out] -1/1536*(4*sqrt(3)*c^2*x^4*(d^8/c^11)^(1/6)*arctan(1/9*((9*sqrt(3)*c^2*d^13
x^5(d^8/c^11)^(1/6) - sqrt(3)*(c^9*d^8*x^6 - 40*c^10*d^7*x^3 - 32*c^11*d^
6)*(d^8/c^11)^(5/6) + 3*sqrt(3)*(5*c^6*d^10*x^4 + 8*c^7*d^9*x)*sqrt(d^8/c^1
1))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^8*d^3*x^5 + c^9*d^2*x^2)*(d^8/c^11)^(2
/3) + 12*sqrt(3)*(c^4*d^6*x^6 - c^5*d^5*x^3 - 2*c^6*d^4)*(d^8/c^11)^(1/3) +
 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*x^4 + 4*c^2*d^7*x) + sqrt(d*x^3 + c)*(sqrt(3)
*(c^9*d^2*x^6 + 32*c^10*d*x^3 + 40*c^11)*(d^8/c^11)^(5/6) + 3*sqrt(3)*(7*c^
6*d^4*x^4 + 4*c^7*d^3*x)*sqrt(d^8/c^11) + 9*sqrt(3)*(c^2*d^7*x^5 + 2*c^3*d^
6*x^2)*(d^8/c^11)^(1/6)))*sqrt((d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x
^3 - 1088*c^3*d^12 - 18*(c^8*d^9*x^7 - 52*c^9*d^8*x^4 - 80*c^10*d^7*x)*(d^8
/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^7*x^5 + c^11*d^6*x^2)*(d^8/c^1
1)^(5/6) - 4*(c^6*d^10*x^6 + 41*c^7*d^9*x^3 + 40*c^8*d^8)*sqrt(d^8/c^11) -
(c^2*d^13*x^7 - 28*c^3*d^12*x^4 - 272*c^4*d^11*x)*(d^8/c^11)^(1/6)) + 18*(c
^4*d^12*x^8 + 20*c^5*d^11*x^5 - 8*c^6*d^10*x^2)*(d^8/c^11)^(1/3))/(d^3*x^9
- 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^15*x^7 - 7*c*d^14*x^4 - 8*c^
2*d^13*x)) + 4*sqrt(3)*c^2*x^4*(d^8/c^11)^(1/6)*arctan(1/9*((9*sqrt(3)*c^2*
d^13*x^5*(d^8/c^11)^(1/6) - sqrt(3)*(c^9*d^8*x^6 - 40*c^10*d^7*x^3 - 32*c^1
1*d^6)*(d^8/c^11)^(5/6) + 3*sqrt(3)*(5*c^6*d^10*x^4 + 8*c^7*d^9*x)*sqrt(d^8
/c^11))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^8*d^3*x^5 + c^9*d^2*x^2)*(d^8/c^11
)^(2/3) + 12*sqrt(3)*(c^4*d^6*x^6 - c^5*d^5*x^3 - 2*c^6*d^4)*(d^8/c^11)^(1/
3) + 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*x^4 + 4*c^2*d^7*x) - sqrt(d*x^3 + c)*(sqr
t(3)*(c^9*d^2*x^6 + 32*c^10*d*x^3 + 40*c^11)*(d^8/c^11)^(5/6) + 3*sqrt(3)*(
7*c^6*d^4*x^4 + 4*c^7*d^3*x)*sqrt(d^8/c^11) + 9*sqrt(3)*(c^2*d^7*x^5 + 2*c^
3*d^6*x^2)*(d^8/c^11)^(1/6)))*sqrt((d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^
13*x^3 - 1088*c^3*d^12 - 18*(c^8*d^9*x^7 - 52*c^9*d^8*x^4 - 80*c^10*d^7*x)*
(d^8/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^10*d^7*x^5 + c^11*d^6*x^2)*(d^8
/c^11)^(5/6) - 4*(c^6*d^10*x^6 + 41*c^7*d^9*x^3 + 40*c^8*d^8)*sqrt(d^8/c^11
) - (c^2*d^13*x^7 - 28*c^3*d^12*x^4 - 272*c^4*d^11*x)*(d^8/c^11)^(1/6)) + 1
8*(c^4*d^12*x^8 + 20*c^5*d^11*x^5 - 8*c^6*d^10*x^2)*(d^8/c^11)^(1/3))/(d^3*
x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^15*x^7 - 7*c*d^14*x^4 -
8*c^2*d^13*x)) + c^2*x^4*(d^8/c^11)^(1/6)*log((d^15*x^9 - 276*c*d^14*x^6 -
1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^8*d^9*x^7 - 52*c^9*d^8*x^4 - 80*c
^10*d^7*x)*(d^8/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^7*x^5 + c^11*d^
6*x^2)*(d^8/c^11)^(5/6) - 4*(c^6*d^10*x^6 + 41*c^7*d^9*x^3 + 40*c^8*d^8)*sq
rt(d^8/c^11) - (c^2*d^13*x^7 - 28*c^3*d^12*x^4 - 272*c^4*d^11*x)*(d^8/c^11)
^(1/6)) + 18*(c^4*d^12*x^8 + 20*c^5*d^11*x^5 - 8*c^6*d^10*x^2)*(d^8/c^11)^(
1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - c^2*x^4*(d^8/c^
11)^(1/6)*log((d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^1
2 - 18*(c^8*d^9*x^7 - 52*c^9*d^8*x^4 - 80*c^10*d^7*x)*(d^8/c^11)^(2/3) - 6*
sqrt(d*x^3 + c)*(24*(c^10*d^7*x^5 + c^11*d^6*x^2)*(d^8/c^11)^(5/6) - 4*(c^6
*d^10*x^6 + 41*c^7*d^9*x^3 + 40*c^8*d^8)*sqrt(d^8/c^11) - (c^2*d^13*x^7 - 2
8*c^3*d^12*x^4 - 272*c^4*d^11*x)*(d^8/c^11)^(1/6)) + 18*(c^4*d^12*x^8 + 20*
c^5*d^11*x^5 - 8*c^6*d^10*x^2)*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) - 2*c^2*x^4*(d^8/c^11)^(1/6)*log((d^9*x^9 + 318*c
*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^8*d^3*x^7 + 64*c^9*d^2*

```
x^4 + 32*c^10*d*x)*(d^8/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 +
32*c^11*x^2)*(d^8/c^11)^(5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d
^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x)*(d^8/c^
11)^(1/6)) + 18*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2)*(d^8/c^11)^
(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*c^2*x^4*(d^8
/c^11)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6
+ 18*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x)*(d^8/c^11)^(2/3) - 6*sq
rt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^8/c^11)^(5/6) + (7*c^6*d^4
*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3
*d^6*x^4 + 160*c^4*d^5*x)*(d^8/c^11)^(1/6)) + 18*(c^4*d^6*x^8 + 38*c^5*d^5*
x^5 + 64*c^6*d^4*x^2)*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d
*x^3 - 512*c^3)) + 96*d^(3/2)*x^4*weierstrassZeta(0, -4*c/d, weierstrassPIn
verse(0, -4*c/d, x)) + 48*(2*d*x^3 + c)*sqrt(d*x^3 + c))/(c^2*x^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c),x)
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{x^5\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)), x)
```

3.294 $\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx$

Optimal. Leaf size=678

$$-\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}-\frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\sqrt{3}\,d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\,\left(\sqrt[3]{\right.}}{\sqrt{c+}}\right.}{1024c^{17/6}}$$

```
[Out] 1/1024*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-1/1024*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/1024
*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)/c^(17/6)-1/56*(d*x^3+c)^(1/2)/c/x^7-19/1792*d*(d*x^3+c)^(1/2)/c^2/x^4+1/1
12*d^2*(d*x^3+c)^(1/2)/c^3/x-1/112*d^(7/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))-1/336*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+
c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/224*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x
)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 1.07, antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {486, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\,\sqrt[4]{3}\,c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{224c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{3}\,d^{7/3}\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{17/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{1024c^{17/6}}-\frac{d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{17/6}}-\frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}-\frac{\sqrt{c+dx^3}}{56cx^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

```
[Out] -1/56*Sqrt[c + d*x^3]/(c*x^7) - (19*d*Sqrt[c + d*x^3])/(1792*c^2*x^4) + (d^
2*Sqrt[c + d*x^3])/(112*c^3*x) - (d^(7/3)*Sqrt[c + d*x^3])/(112*c^3*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c
^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(17/6)) + (d^(7/3)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(17/6)) - (d^(
```

```
7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(17/6)) + (3^(1/4)*Sqrt[
2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
- 4*Sqrt[3]])/(224*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (d^(7/3)*(c^(1/3) + d^(1/3)*
x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*3^(1/4)*c^(8/3
)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{56cx^7} + \frac{\int \frac{19cd+\frac{11d^2x^3}{2}}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{56c}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} - \frac{\int \frac{128c^2d^2-\frac{95}{2}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1792c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} + \frac{\int \frac{x\left(-260c^3d^3+64c^2d^4x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} + \frac{\int \left(-\frac{64c^2d^3x}{\sqrt{c+dx^3}} + \frac{252c^3d^3x}{(8c-dx^3)\sqrt{c+dx^3}}\right.}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{d^3\int \frac{x}{\sqrt{c+dx^3}}\,dx}{224c^3} + \frac{(9d^3)\int \frac{}{(8c-dx^3)}}{512}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{(3d^2)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}}{2048c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} +$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} -$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.06, size = 164, normalized size = 0.24

$$\frac{-160c(32c^3+51c^2dx^3+3cd^2x^6-16d^3x^9)-325cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+32d^4x^{12}\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{286720c^4x^7\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

```
[Out] (-160*c*(32*c^3 + 51*c^2*d*x^3 + 3*c*d^2*x^6 - 16*d^3*x^9) - 325*c*d^3*x^9*
Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)]
 + 32*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/(286720*c^4*x^7*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 2280, normalized size = 3.36

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	2280

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*d/c^2*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*d*(d*x^3+c)^(1/2)/c/x-1/8*I/c*d*3^
(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))+1/8/c*(-1/7*(d*x^3+c)^(1/2)/x^7-3/
56*d*(d*x^3+c)^(1/2)/x^4/c+15/112*d^2*(d*x^3+c)^(1/2)/c^2/x+5/112*I/c^2*d^2
*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(
```

```
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-1/512/c^3*d^3*(-2/3*I*3^(1/2)/d
*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^
(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d
^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)
^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/512/c^
3*d^2*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*
(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*
(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")

[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.25, size = 2630, normalized size = 3.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")

[Out] -1/86016*(28*sqrt(3)*c^3*x^7*(d^14/c^17)^(1/6)*arctan(1/9*((9*sqrt(3)*c^3*d^22*x^5*(d^14/c^17)^(1/6) - sqrt(3)*(c^14*d^13*x^6 - 40*c^15*d^12*x^3 - 32*c^16*d^11)*(d^14/c^17)^(5/6) + 3*sqrt(3)*(5*c^9*d^17*x^4 + 8*c^10*d^16*x)*sqrt(d^14/c^17))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^12*d^4*x^5 + c^13*d^3*x^2)*(d^14/c^17)^(2/3) + 12*sqrt(3)*(c^6*d^9*x^6 - c^7*d^8*x^3 - 2*c^8*d^7)*(d^14/c^17)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 + 4*c^2*d^12*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^14*d^2*x^6 + 32*c^15*d*x^3 + 40*c^16)*(d^14/c^17)^(5/6) + 3*sqrt(3)*(7*c^9*d^6*x^4 + 4*c^10*d^5*x)*sqrt(d^14/c^17) + 9*sqrt(3)*(c^3*d^11*x^5 + 2*c^4*d^10*x^2)*(d^14/c^17)^(1/6)))*sqrt((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^12*d^15*x^7 - 52*c^13*d^14*x^4 - 80*c^14*d^13*x)*(d^14/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^15*d^12*x^5 + c^16*d^11*x^2)*(d^14/c^17)^(5/6) - 4*(c^9*d^17*x^6 + 41*c^10*d^16*x^3 + 40*c^11*d^15)*sqrt(d^14/c^17) - (c^3*d^22*x^7 - 28*c^4*d^21*x^4 - 272*c^5*d^20*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^20*x^8 + 20*c^7*d^19*x^5 - 8*c^8*d^18*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*x)) + 28*sqrt(3)*c^3*x^7*(d^14/c^17)^(1/6)*arctan(1/9*((9*sqrt(3)*c^3*d^22*x^5*(d^14/c^17)^(1/6) - sqrt(3)*(c^14*d^13*x^6 - 40*c^15*d^12*x^3 - 32*c^16*d^11)*(d^14/c^17)^(5/6) + 3*sqrt(3)*(5*c^9*d^17*x^4 + 8*c^10*d^16*x)*sqrt(d^14/c^17))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^12*d^4*x^5 + c^13*d^3*x^2)*(d^14/c^17)^(2/3) + 12*sqrt(3)*(c^6*d^9*x^6 - c^7*d^8*x^3 - 2*c^8*d^7)*(d^14/c^17)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 + 4*c^2*d^12*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^14*d^2*x^6 + 32*c^15*d*x^3 + 40*c^16)*(d^14/c^17)^(5/6) + 3*sqrt(3)*(7*c^9*d^6*x^4 + 4*c^10*d^5*x)*sqrt(d^14/c^17) + 9*sqrt(3)*(c^3*d^11*x^5 + 2*c^4*d^10*x^2)*(d^14/c^17)^(1/6)))*sqrt((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^12*d^15*x^7 - 52*c^13*d^14*x^4 - 80*c^14*d^13*x)*(d^14/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^15*d^12*x^5 + c^16*d^11*x^2)*(d^14/c^17)^(5/6) - 4*(c^9*d^17*x^6 + 41*c^10*d^16*x^3 + 40*c^11*d^15)*sqrt(d^14/c^17) - (c^3*d^22*x^7 - 28*c^4*d^21*x^4 - 272*c^5*d^20*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^20*x^8 + 20*c^7*d^19*x^5 - 8*c^8*d^18*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*x)) + 7*c^3*x^7*(d^14/c^17)^(1/6)*log((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^12*d^15*x^7 - 52*c^13*d^14*x^4 - 80*c^14*d^13*x)*(d^14/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^15*

```
d^12*x^5 + c^16*d^11*x^2)*(d^14/c^17)^(5/6) - 4*(c^9*d^17*x^6 + 41*c^10*d^1
6*x^3 + 40*c^11*d^15)*sqrt(d^14/c^17) - (c^3*d^22*x^7 - 28*c^4*d^21*x^4 - 2
72*c^5*d^20*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^20*x^8 + 20*c^7*d^19*x^5 - 8*
c^8*d^18*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) - 7*c^3*x^7*(d^14/c^17)^(1/6)*log((d^25*x^9 - 276*c*d^24*x^6 - 16
08*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^12*d^15*x^7 - 52*c^13*d^14*x^4 - 80
*c^14*d^13*x)*(d^14/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^15*d^12*x^5 + c^
16*d^11*x^2)*(d^14/c^17)^(5/6) - 4*(c^9*d^17*x^6 + 41*c^10*d^16*x^3 + 40*c^
11*d^15)*sqrt(d^14/c^17) - (c^3*d^22*x^7 - 28*c^4*d^21*x^4 - 272*c^5*d^20*x
)*(d^14/c^17)^(1/6)) + 18*(c^6*d^20*x^8 + 20*c^7*d^19*x^5 - 8*c^8*d^18*x^2)
*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1
4*c^3*x^7*(d^14/c^17)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*
x^3 + 640*c^3*d^11 + 18*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x)*
(d^14/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^14
/c^17)^(5/6) + (7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c
^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x)*(d^14/c^17)^(1/6))
+ 18*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2)*(d^14/c^17)^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 14*c^3*x^7*(d^14/c^17)^(
1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18
*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x)*(d^14/c^17)^(2/3) - 6*s
qrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^14/c^17)^(5/6) + (7*c^9*d
^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*d^11*x^7 +
80*c^4*d^10*x^4 + 160*c^5*d^9*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^9*x^8 + 38*
c^7*d^8*x^5 + 64*c^8*d^7*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) - 768*d^(5/2)*x^7*weierstrassZeta(0, -4*c/d, weie
rstrassPInverse(0, -4*c/d, x)) - 48*(16*d^2*x^6 - 19*c*d*x^3 - 32*c^2)*sqrt
(d*x^3 + c))/(c^3*x^7)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**8 + d*x**11), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{x^8\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)), x)
```

3.295 $\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=130

$$-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3(c+dx^3)^{3/2}}{9d^4}-\frac{38c^2(c+dx^3)^{5/2}}{5d^4}-\frac{4c(c+dx^3)^{7/2}}{7d^4}-\frac{2(c+dx^3)^{9/2}}{27d^4}+\frac{9216c^{9/2}\tanh^{-1}}{d^4}$$

```
[Out] -1024/9*c^3*(d*x^3+c)^(3/2)/d^4-38/5*c^2*(d*x^3+c)^(5/2)/d^4-4/7*c*(d*x^3+c
)^(7/2)/d^4-2/27*(d*x^3+c)^(9/2)/d^4+9216*c^(9/2)*arctanh(1/3*(d*x^3+c)^(1/
2)/c^(1/2))/d^4-3072*c^4*(d*x^3+c)^(1/2)/d^4
```

Rubi [A]
time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 90, 52, 65, 212}

$$\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3(c+dx^3)^{3/2}}{9d^4}-\frac{38c^2(c+dx^3)^{5/2}}{5d^4}-\frac{4c(c+dx^3)^{7/2}}{7d^4}-\frac{2(c+dx^3)^{9/2}}{27d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-3072*c^4*Sqrt[c + d*x^3])/d^4 - (1024*c^3*(c + d*x^3)^(3/2))/(9*d^4) - (3
8*c^2*(c + d*x^3)^(5/2))/(5*d^4) - (4*c*(c + d*x^3)^(7/2))/(7*d^4) - (2*(c
+ d*x^3)^(9/2))/(27*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/d^4
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{57c^2(c+dx)^{3/2}}{d^3} + \frac{512c^3(c+dx)^{3/2}}{d^3(8c-dx)} - \frac{6c(c+dx)^{5/2}}{d^3} - \frac{(c+dx)^{7/2}}{d^3}\right.\right.$$

$$= -\frac{38c^2(c+dx^3)^{5/2}}{5d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - \frac{2(c+dx^3)^{9/2}}{27d^4} + \frac{(512c^3)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,d\right.}{3d^3}$$

$$= -\frac{1024c^3(c+dx^3)^{3/2}}{9d^4} - \frac{38c^2(c+dx^3)^{5/2}}{5d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - \frac{2(c+dx^3)^{9/2}}{27d^4} + \frac{(1536}{}$$

$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3(c+dx^3)^{3/2}}{9d^4} - \frac{38c^2(c+dx^3)^{5/2}}{5d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - 2$$

$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3(c+dx^3)^{3/2}}{9d^4} - \frac{38c^2(c+dx^3)^{5/2}}{5d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - 2$$

$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3(c+dx^3)^{3/2}}{9d^4} - \frac{38c^2(c+dx^3)^{5/2}}{5d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - 2$$

Mathematica [A]

time = 0.08, size = 93, normalized size = 0.72

$$-\frac{2\sqrt{c+dx^3}\,(1509176c^4+61892c^3dx^3+4611c^2d^2x^6+410cd^3x^9+35d^4x^{12})}{945d^4}+\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1509176*c^4 + 61892*c^3*d*x^3 + 4611*c^2*d^2*x^6 + 410
*c*d^3*x^9 + 35*d^4*x^12))/(945*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3
]/(3*Sqrt[c])])/d^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 634, normalized size = 4.88 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/27*d*x^12*(d*x^3+c)^(1/2)+20/189*c*x^9*(d*x^3+c)^(1/2)+2/315*c^2/d*
x^6*(d*x^3+c)^(1/2)-8/945*c^3/d^2*x^3*(d*x^3+c)^(1/2)+16/945*c^4/d^3*(d*x^3
+c)^(1/2))-8/d^2*c*(2/21*d*x^9*(d*x^3+c)^(1/2)+16/105*c*x^6*(d*x^3+c)^(1/2)
+2/105*c^2/d*x^3*(d*x^3+c)^(1/2)-4/105*c^3/d^2*(d*x^3+c)^(1/2))-128/15*c^2*
(d*x^3+c)^(5/2)/d^4-512*c^3/d^3*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(
1/2)/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/
3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^
2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [A]
time = 0.55, size = 110, normalized size = 0.85

$$-\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+35\,(dx^3+c)^{\frac{9}{2}}+270\,(dx^3+c)^{\frac{7}{2}}c+3591\,(dx^3+c)^{\frac{5}{2}}c^2+53760\,(dx^3+c)^{\frac{3}{2}}c^3+1451520\,\sqrt{dx^3+c}\,c^4\right)}{945\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/945*(2177280*c^(9/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c)
+ 3*sqrt(c))) + 35*(d*x^3 + c)^(9/2) + 270*(d*x^3 + c)^(7/2)*c + 3591*(d*x^
```

```
3 + c)^(5/2)*c^2 + 53760*(d*x^3 + c)^(3/2)*c^3 + 1451520*sqrt(d*x^3 + c)*c^
4)/d^4
```

Fricas [A]
time = 2.28, size = 191, normalized size = 1.47

$$\left[\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(35\,d^4x^{12}+410\,cd^3x^9+4611\,c^2d^2x^6+61892\,c^3dx^3+1509176\,c^4)\sqrt{dx^3+c}\right)}{945\,d^4},\ -\frac{2\left(4354560\,\sqrt{-c}\,c^4\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+(35\,d^4x^{12}+410\,cd^3x^9+4611\,c^2d^2x^6+61892\,c^3dx^3+1509176\,c^4)\sqrt{dx^3+c}\right)}{945\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/945*(2177280*c^(9/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x
^3 - 8*c)) - (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2*d^2*x^6 + 61892*c^3*d*
x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4, -2/945*(4354560*sqrt(-c)*c^4*arcta
n(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2
*d^2*x^6 + 61892*c^3*d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4]
```

Sympy [A]
time = 78.44, size = 131, normalized size = 1.01

$$-\frac{9216c^5\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^4\sqrt{-c}}-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3(c+dx^3)^{\frac{3}{2}}}{9d^4}-\frac{38c^2(c+dx^3)^{\frac{5}{2}}}{5d^4}-\frac{4c(c+dx^3)^{\frac{7}{2}}}{7d^4}-\frac{2(c+dx^3)^{\frac{9}{2}}}{27d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -9216*c**5*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**4*sqrt(-c)) - 3072*c**4*
sqrt(c + d*x**3)/d**4 - 1024*c**3*(c + d*x**3)**(3/2)/(9*d**4) - 38*c**2*(c
 + d*x**3)**(5/2)/(5*d**4) - 4*c*(c + d*x**3)**(7/2)/(7*d**4) - 2*(c + d*x*
*3)**(9/2)/(27*d**4)
```

Giac [A]
time = 0.94, size = 117, normalized size = 0.90

$$-\frac{9216\,c^5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^4}-\frac{2\left(35\,(dx^3+c)^{\frac{9}{2}}d^{32}+270\,(dx^3+c)^{\frac{7}{2}}cd^{32}+3591\,(dx^3+c)^{\frac{5}{2}}c^2d^{32}+53760\,(dx^3+c)^{\frac{3}{2}}c^3d^{32}+1451520\,\sqrt{dx^3+c}\,c^4d^{32}\right)}{945\,d^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -9216*c^5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/945*(35*(
d*x^3 + c)^(9/2)*d^32 + 270*(d*x^3 + c)^(7/2)*c*d^32 + 3591*(d*x^3 + c)^(5/
2)*c^2*d^32 + 53760*(d*x^3 + c)^(3/2)*c^3*d^32 + 1451520*sqrt(d*x^3 + c)*c^
4*d^32)/d^36
```

Mupad [B]
time = 3.53, size = 135, normalized size = 1.04

$$\frac{4608\,c^{9/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4}-\frac{2\,x^{12}\,\sqrt{d\,x^3+c}}{27}-\frac{3018352\,c^4\,\sqrt{d\,x^3+c}}{945\,d^4}-\frac{164\,c\,x^9\,\sqrt{d\,x^3+c}}{189\,d}-\frac{123784\,c^3\,x^3\,\sqrt{d\,x^3+c}}{945\,d^3}-\frac{3074\,c^2\,x^6\,\sqrt{d\,x^3+c}}{315\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (4608*c^(9/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^4 - (2*x^12*(c + d*x^3)^(1/2))/27 - (3018352*c^4*(c + d*x^3)^(1/2))/(
945*d^4) - (164*c*x^9*(c + d*x^3)^(1/2))/(189*d) - (123784*c^3*x^3*(c + d*x
^3)^(1/2))/(945*d^3) - (3074*c^2*x^6*(c + d*x^3)^(1/2))/(315*d^2)
```

3.296 $\int \frac{x^8 (c+dx^3)^{3/2}}{8c-dx^3} \, dx$

Optimal. Leaf size=109

$$-\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2(c+dx^3)^{3/2}}{9d^3}-\frac{14c(c+dx^3)^{5/2}}{15d^3}-\frac{2(c+dx^3)^{7/2}}{21d^3}+\frac{1152c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

```
[Out] -128/9*c^2*(d*x^3+c)^(3/2)/d^3-14/15*c*(d*x^3+c)^(5/2)/d^3-2/21*(d*x^3+c)^(
7/2)/d^3+1152*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3-384*c^3*(d*x
^3+c)^(1/2)/d^3
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 90, 52, 65, 212}

$$\frac{1152c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2(c+dx^3)^{3/2}}{9d^3}-\frac{14c(c+dx^3)^{5/2}}{15d^3}-\frac{2(c+dx^3)^{7/2}}{21d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-384*c^3*Sqrt[c + d*x^3])/d^3 - (128*c^2*(c + d*x^3)^(3/2))/(9*d^3) - (14*
c*(c + d*x^3)^(5/2))/(15*d^3) - (2*(c + d*x^3)^(7/2))/(21*d^3) + (1152*c^(7
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{8c-dx}\,dx,x,x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{7c(c+dx)^{3/2}}{d^2}+\frac{64c^2(c+dx)^{3/2}}{d^2(8c-dx)}-\frac{(c+dx)^{5/2}}{d^2}\right)dx,x,x^3\right) \\
&= -\frac{14c(c+dx^3)^{5/2}}{15d^3}-\frac{2(c+dx^3)^{7/2}}{21d^3}+\frac{(64c^2)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx,x,x^3\right)}{3d^2} \\
&= -\frac{128c^2(c+dx^3)^{3/2}}{9d^3}-\frac{14c(c+dx^3)^{5/2}}{15d^3}-\frac{2(c+dx^3)^{7/2}}{21d^3}+\frac{(192c^3)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\right.}{d^2} \\
&= -\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2(c+dx^3)^{3/2}}{9d^3}-\frac{14c(c+dx^3)^{5/2}}{15d^3}-\frac{2(c+dx^3)^{7/2}}{21d^3}+\frac{(1728c^4}{} \\
&= -\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2(c+dx^3)^{3/2}}{9d^3}-\frac{14c(c+dx^3)^{5/2}}{15d^3}-\frac{2(c+dx^3)^{7/2}}{21d^3}+\frac{(3456c^4}{} \\
&= -\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2(c+dx^3)^{3/2}}{9d^3}-\frac{14c(c+dx^3)^{5/2}}{15d^3}-\frac{2(c+dx^3)^{7/2}}{21d^3}+\frac{1152c^{7/}}{}
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 82, normalized size = 0.75

$$-\frac{2\sqrt{c+dx^3}\,(62882c^3+2579c^2dx^3+192cd^2x^6+15d^3x^9)}{315d^3}+\frac{1152c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(62882*c^3 + 2579*c^2*d*x^3 + 192*c*d^2*x^6 + 15*d^3*x^
9))/(315*d^3) + (1152*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 538, normalized size = 4.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/21*d*x^9*(d*x^3+c)^(1/2)+16/105*c*x^6*(d*x^3+c)^(1/2)+2/105*c^2/d*x
^3*(d*x^3+c)^(1/2)-4/105*c^3/d^2*(d*x^3+c)^(1/2))-16/15*c*(d*x^3+c)^(5/2)/d
^3-64*c^2/d^2*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I*c/d^3*2
^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/
3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*
(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^
(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*d-8*c)))
```

Maxima [A]
time = 0.52, size = 96, normalized size = 0.88

$$-\frac{2\left(90720\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+15\,(dx^3+c)^{\frac{7}{2}}+147\,(dx^3+c)^{\frac{5}{2}}c+2240\,(dx^3+c)^{\frac{3}{2}}c^2+60480\,\sqrt{dx^3+c}\,c^3\right)}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/315*(90720*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
3*sqrt(c))) + 15*(d*x^3 + c)^(7/2) + 147*(d*x^3 + c)^(5/2)*c + 2240*(d*x^3
+ c)^(3/2)*c^2 + 60480*sqrt(d*x^3 + c)*c^3)/d^3
```

Fricas [A]
time = 2.25, size = 169, normalized size = 1.55

$$\left[\frac{2\,\left(90720\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+62882\,c^3\right)\sqrt{dx^3+c}\right)}{315\,d^3},\,-\frac{2\,\left(181440\,\sqrt{-c}\,c^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+\left(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+62882\,c^3\right)\sqrt{dx^3+c}\right)}{315\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/315*(90720*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) - (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d
*x^3 + c))/d^3, -2/315*(181440*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt
(-c)/c) + (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d*
x^3 + c))/d^3]
```

Sympy [A]
time = 48.89, size = 110, normalized size = 1.01

$$-\frac{1152c^4\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^3\sqrt{-c}}-\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2(c+dx^3)^{\frac{3}{2}}}{9d^3}-\frac{14c(c+dx^3)^{\frac{5}{2}}}{15d^3}-\frac{2(c+dx^3)^{\frac{7}{2}}}{21d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -1152*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**3*sqrt(-c)) - 384*c**3*s
qrt(c + d*x**3)/d**3 - 128*c**2*(c + d*x**3)**(3/2)/(9*d**3) - 14*c*(c + d*
x**3)**(5/2)/(15*d**3) - 2*(c + d*x**3)**(7/2)/(21*d**3)
```

Giac [A]
time = 1.74, size = 100, normalized size = 0.92

$$-\frac{1152\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d^3}-\frac{2\,\left(15\,(dx^3+c)^{\frac{7}{2}}d^{18}+147\,(dx^3+c)^{\frac{5}{2}}cd^{18}+2240\,(dx^3+c)^{\frac{3}{2}}c^2d^{18}+60480\,\sqrt{dx^3+c}\,c^3d^{18}\right)}{315\,d^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1152*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/315*(15*(
d*x^3 + c)^(7/2)*d^18 + 147*(d*x^3 + c)^(5/2)*c*d^18 + 2240*(d*x^3 + c)^(3/
2)*c^2*d^18 + 60480*sqrt(d*x^3 + c)*c^3*d^18)/d^21
```

Mupad [B]
time = 3.50, size = 115, normalized size = 1.06

$$\frac{576\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3}-\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21}-\frac{125764\,c^3\,\sqrt{d\,x^3+c}}{315\,d^3}-\frac{128\,c\,x^6\,\sqrt{d\,x^3+c}}{105\,d}-\frac{5158\,c^2\,x^3\,\sqrt{d\,x^3+c}}{315\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (576*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^3 - (2*x^9*(c + d*x^3)^(1/2))/21 - (125764*c^3*(c + d*x^3)^(1/2))/(315
*d^3) - (128*c*x^6*(c + d*x^3)^(1/2))/(105*d) - (5158*c^2*x^3*(c + d*x^3)^(
1/2))/(315*d^2)
```

3.297 $\int \frac{x^5 (c+dx^3)^{3/2}}{8c-dx^3} \, dx$

Optimal. Leaf size=88

$$-\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c(c+dx^3)^{3/2}}{9d^2}-\frac{2(c+dx^3)^{5/2}}{15d^2}+\frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

[Out] -16/9*c*(d*x^3+c)^(3/2)/d^2-2/15*(d*x^3+c)^(5/2)/d^2+144*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2-48*c^2*(d*x^3+c)^(1/2)/d^2

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 81, 52, 65, 212}

$$\frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c(c+dx^3)^{3/2}}{9d^2}-\frac{2(c+dx^3)^{5/2}}{15d^2}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]

[Out] (-48*c^2*Sqrt[c + d*x^3])/d^2 - (16*c*(c + d*x^3)^(3/2))/(9*d^2) - (2*(c + d*x^3)^(5/2))/(15*d^2) + (144*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)\\
&= -\frac{2(c+dx^3)^{5/2}}{15d^2} + \frac{(8c)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{3d}\\
&= -\frac{16c(c+dx^3)^{3/2}}{9d^2} - \frac{2(c+dx^3)^{5/2}}{15d^2} + \frac{(24c^2)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d}\\
&= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c(c+dx^3)^{3/2}}{9d^2} - \frac{2(c+dx^3)^{5/2}}{15d^2} + \frac{(216c^3)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c-}}\right.}{d}\\
&= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c(c+dx^3)^{3/2}}{9d^2} - \frac{2(c+dx^3)^{5/2}}{15d^2} + \frac{(432c^3)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x,\right.}{d^2}\\
&= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c(c+dx^3)^{3/2}}{9d^2} - \frac{2(c+dx^3)^{5/2}}{15d^2} + \frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right.}{d^2}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 71, normalized size = 0.81

$$-\frac{2\sqrt{c+dx^3}\,(1123c^2+46cdx^3+3d^2x^6)}{45d^2}+\frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1123*c^2 + 46*c*d*x^3 + 3*d^2*x^6))/(45*d^2) + (144*c^
(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 462, normalized size = 5.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(d*x^3+c)^(5/2)/d^2-8*c/d*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(
1/2)/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/
3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^
2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [A]
time = 0.52, size = 82, normalized size = 0.93

$$-\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+40\,(dx^3+c)^{\frac{3}{2}}c+1080\sqrt{dx^3+c}\,c^2\right)}{45\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/45*(1620*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 40*(d*x^3 + c)^(3/2)*c + 1080*sqrt(d*x^3
+ c)*c^2)/d^2
```

Fricas [A]
time = 1.56, size = 147, normalized size = 1.67

$$\left[\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(3\,d^2x^6+46\,cdx^3+1123\,c^2)\sqrt{dx^3+c}\right)}{45\,d^2},-\frac{2\left(3240\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(3\,d^2x^6+46\,cdx^3+1123\,c^2)\sqrt{dx^3+c}\right)}{45\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/45*(1620*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - (3*d^2*x^6 + 46*c*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2, -2/45*(3
240*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 46*c
*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2]
```

Sympy [A]
time = 28.87, size = 90, normalized size = 1.02

$$-\frac{144c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^2\sqrt{-c}}-\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c(c+dx^3)^{\frac{3}{2}}}{9d^2}-\frac{2(c+dx^3)^{\frac{5}{2}}}{15d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -144*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**2*sqrt(-c)) - 48*c**2*sqr
t(c + d*x**3)/d**2 - 16*c*(c + d*x**3)**(3/2)/(9*d**2) - 2*(c + d*x**3)**(5
/2)/(15*d**2)
```

Giac [A]
time = 1.26, size = 83, normalized size = 0.94

$$-\frac{144\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d^2}-\frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}d^8+40\,(dx^3+c)^{\frac{3}{2}}cd^8+1080\,\sqrt{dx^3+c}\,c^2d^8\right)}{45\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -144*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/45*(3*(d*x
^3 + c)^(5/2)*d^8 + 40*(d*x^3 + c)^(3/2)*c*d^8 + 1080*sqrt(d*x^3 + c)*c^2*d
^8)/d^10
```

Mupad [B]
time = 3.52, size = 95, normalized size = 1.08

$$\frac{72\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2}-\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15}-\frac{2246\,c^2\,\sqrt{d\,x^3+c}}{45\,d^2}-\frac{92\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (72*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d^2 - (2*x^6*(c + d*x^3)^(1/2))/15 - (2246*c^2*(c + d*x^3)^(1/2))/(45*d^2
) - (92*c*x^3*(c + d*x^3)^(1/2))/(45*d)
```

3.298 $\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=67

$$-\frac{6c\sqrt{c+dx^3}}{d}-\frac{2(c+dx^3)^{3/2}}{9d}+\frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d+18*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d-6*
c*(d*x^3+c)^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {455, 52, 65, 212}

$$\frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{6c\sqrt{c+dx^3}}{d}-\frac{2(c+dx^3)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-6*c*Sqrt[c + d*x^3])/d - (2*(c + d*x^3)^(3/2))/(9*d) + (18*c^(3/2)*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx &= \frac{1}{3}\mathrm{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)\\ &= -\frac{2(c+dx^3)^{3/2}}{9d} + (3c)\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\ &= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2(c+dx^3)^{3/2}}{9d} + (27c^2)\,\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2(c+dx^3)^{3/2}}{9d} + \frac{(54c^2)\,\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}\\ &= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2(c+dx^3)^{3/2}}{9d} + \frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.88

$$-\frac{2\sqrt{c+dx^3}\,(28c+dx^3)}{9d}+\frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(28*c + d*x^3))/(9*d) + (18*c^(3/2)*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.34, size = 441, normalized size = 6.58 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*x^3*(d*x^3+c)^(1/2)-56/9*c*(d*x^3+c)^(1/2)/d-3*I*c/d^3*2^(1/2)*sum((-c
*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)
*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-
c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.50, size = 68, normalized size = 1.01

$$-\frac{81\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\,(dx^3+c)^{\frac{3}{2}}+54\,\sqrt{dx^3+c}\,c}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -1/9*(81*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 2*(d*x^3 + c)^(3/2) + 54*sqrt(d*x^3 + c)*c)/d
```

Fricas [A]
time = 1.51, size = 121, normalized size = 1.81

$$\left[\frac{81\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-2\,(dx^3+28\,c)\sqrt{dx^3+c}}{9\,d},\,-\frac{2\left(81\,\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+(dx^3+28\,c)\sqrt{dx^3+c}\right)}{9\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/9*(81*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - 2*(d*x^3 + 28*c)*sqrt(d*x^3 + c))/d, -2/9*(81*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 28*c)*sqrt(d*x^3 + c))/d]
```

Sympy [A]
time = 12.28, size = 65, normalized size = 0.97

$$-\frac{18c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d\sqrt{-c}}-\frac{6c\sqrt{c+dx^3}}{d}-\frac{2(c+dx^3)^{\frac{3}{2}}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -18*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d*sqrt(-c)) - 6*c*sqrt(c + d*
x**3)/d - 2*(c + d*x**3)**(3/2)/(9*d)
```

Giac [A]
time = 0.96, size = 65, normalized size = 0.97

$$-\frac{18\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d}-\frac{2\left((dx^3+c)^{\frac{3}{2}}d^2+27\sqrt{dx^3+c}\,cd^2\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -18*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/9*((d*x^3 + c
)^(3/2)*d^2 + 27*sqrt(d*x^3 + c)*c*d^2)/d^3
```

Mupad [B]
time = 3.45, size = 75, normalized size = 1.12

$$\frac{9\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d}-\frac{56\,c\,\sqrt{d\,x^3+c}}{9\,d}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (9*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/d - (56*c*(c + d*x^3)^(1/2))/(9*d) - (2*x^3*(c + d*x^3)^(1/2))/9
```

3.299 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx$

Optimal. Leaf size=73

$$-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

```
[Out] 9/4*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)-1/12*arctanh((d*x^3+c)^(1/
2)/c^(1/2))*c^(1/2)-2/3*(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {457, 86, 162, 65, 214, 212}

$$-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(8c-dx)}\,dx, x, x^3\right)\\
&= -\frac{2}{3}\sqrt{c+dx^3} - \frac{\text{Subst}\left(\int \frac{-c^2d-10cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}\\
&= -\frac{2}{3}\sqrt{c+dx^3} + \frac{1}{24}c\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) + \frac{1}{8}(27cd)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{\cdots}}\right.\\
&= -\frac{2}{3}\sqrt{c+dx^3} + \frac{1}{4}(27c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{c\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x \cdots\right.}{12d}\\
&= -\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 73, normalized size = 1.00

$$
-\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 500, normalized size = 6.85

method	result
default	$-\frac{d\left(\frac{2x^3\sqrt{d\,x^3+c}}{9}+\frac{56c\sqrt{d\,x^3+c}}{9d}+\frac{3ic\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right.}{d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}\right.}{}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
[Out] -1/8*d/c*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I*c/d^3*2^(1/2
)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
```

```
(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_a
lpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-
8*c))+1/8/c*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*a
rctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x), x)
```

Fricas [A]
time = 1.60, size = 152, normalized size = 2.08

$$\left[\frac{9}{8}\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\frac{1}{24}\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-\frac{2}{3}\sqrt{dx^3+c},\frac{1}{12}\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\frac{9}{4}\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-\frac{2}{3}\sqrt{dx^3+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [9/8*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))
+ 1/24*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2/3*sqr
t(d*x^3 + c), 1/12*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 9/4*sqrt(-
c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 2/3*sqrt(d*x^3 + c)]
```

Sympy [A]
time = 8.06, size = 73, normalized size = 1.00

$$-\frac{9c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}}-\frac{2\sqrt{c+dx^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c),x)
```

```
[Out] -9*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(4*sqrt(-c)) + c*atan(sqrt(c + d*x
**3)/sqrt(-c))/(12*sqrt(-c)) - 2*sqrt(c + d*x**3)/3
```

Giac [A]
time = 1.56, size = 61, normalized size = 0.84

$$\frac{c \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}} - \frac{9\,c \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}} - \frac{2}{3}\sqrt{dx^3+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 1/12*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/4*c*arctan(1/3*sqrt(d*
x^3 + c)/sqrt(-c))/sqrt(-c) - 2/3*sqrt(d*x^3 + c)
```

Mupad [B]
time = 5.89, size = 89, normalized size = 1.22

$$\frac{\sqrt{c}\ \ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(10\,c+d\,x^3+6\,\sqrt{c}\ \sqrt{d\,x^3+c}\right)^{27}}{x^6\,\left(8\,c-d\,x^3\right)^{27}}\right)}{24} - \frac{2\,\sqrt{d\,x^3+c}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
*(10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^27)/(x^6*(8*c - d*x^3)^27)))/
24 - (2*(c + d*x^3)^(1/2))/3
```

3.300 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)} dx$

Optimal. Leaf size=78

$$-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

```
[Out] 9/32*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-13/96*d*arctanh((d*x^3+
c)^(1/2)/c^(1/2))/c^(1/2)-1/24*(d*x^3+c)^(1/2)/x^3
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {457, 100, 162, 65, 214, 212}

$$-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/x^3 + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(8c-dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} - \frac{\text{Subst}\left(\int \frac{-13c^2d-\frac{17}{2}cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{1}{192}(13d)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) + \frac{1}{64}(27d^2)\,\text{Subst}\left(\int \frac{}{(8c-}\right.$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{13}{96}\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right) + \frac{1}{32}(27d)\text{Subst}\left(\int \frac{1}{9c-x^2}\right.$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Mathematica [A]

time = 0.07, size = 78, normalized size = 1.00

$$-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/x^3 + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.42, size = 556, normalized size = 7.13

method	result
risch	$-\frac{\sqrt{d\,x^3+c}}{24x^3}+\frac{d\left(-\frac{13\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{6\sqrt{c}}-\frac{3i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d\ldots\right.}{\ldots}\right)}{\left(-c\,d^2\ldots\right.}}\ldots}{\ldots}\right)}{\ldots}\right)}{\ldots}$

default	$-\frac{d^{2}\left(\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{56c\sqrt{d\,x^{3}+c}}{9d}+\frac{3ic\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d\ldots\right.}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}\ldots}}{\ldots}\right.}{\ldots}\right.}{\ldots}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/c^2*d^2*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I*c/d^3*2
^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/
3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*
(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^
(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
```

```
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*d-8*c)))+1/8/c*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)
*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))+1/64*d/c^2*(2/9*d*x^3*(d*x^3+c)^(1/2)+
8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^4), x)
```

Fricas [A]
time = 1.72, size = 186, normalized size = 2.38

$$\left[\frac{27\sqrt{c}\,dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\,\sqrt{c}+10c}{dx^3-8c}\right)+13\sqrt{c}\,dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\,\sqrt{c}+2c}{x^3}\right)-8\sqrt{dx^3+c}\,c}{192\,cx^3},\ \frac{13\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-27\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3c}\right)-4\sqrt{dx^3+c}\,c}{96\,cx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
[Out] [1/192*(27*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) + 13*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c
)/x^3) - 8*sqrt(d*x^3 + c)*c)/(c*x^3), 1/96*(13*sqrt(-c)*d*x^3*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(
-c)/c) - 4*sqrt(d*x^3 + c)*c)/(c*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{c\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx-\int\frac{dx^3\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c),x)
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

Giac [A]
time = 0.83, size = 64, normalized size = 0.82

$$\frac{13\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}}-\frac{9\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}}-\frac{\sqrt{dx^3+c}}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 13/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/32*d*arctan(1/3*sqrt(
d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/24*sqrt(d*x^3 + c)/x^3
```

Mupad [B]
time = 3.52, size = 56, normalized size = 0.72

$$\frac{9\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{3\,\sqrt{c}}\right)}{32\,\sqrt{c}}-\frac{13\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96\,\sqrt{c}}-\frac{\sqrt{d\,x^3+c}}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x)
```

```
[Out] (9*d*atanh((c + d*x^3)^(1/2)/(3*c^(1/2))))/(32*c^(1/2)) - (13*d*atanh((c +
d*x^3)^(1/2)/c^(1/2)))/(96*c^(1/2)) - (c + d*x^3)^(1/2)/(24*x^3)
```

3.301 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{c+dx^3}}{48x^6}-\frac{11d\sqrt{c+dx^3}}{192cx^3}+\frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}$$

```
[Out] 9/256*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-37/768*d^2*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/48*(d*x^3+c)^(1/2)/x^6-11/192*d*(d*x^3+c)
^(1/2)/c/x^3
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {457, 100, 156, 162, 65, 214, 212}

$$\frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}-\frac{11d\sqrt{c+dx^3}}{192cx^3}-\frac{\sqrt{c+dx^3}}{48x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

```
[Out] -1/48*Sqrt[c + d*x^3]/x^6 - (11*d*Sqrt[c + d*x^3])/(192*c*x^3) + (9*d^2*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(768*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(8c-dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{\text{Subst}\left(\int \frac{-22c^2d-\frac{35}{2}cd^2x}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{\text{Subst}\left(\int \frac{74c^3d^2+11c^2d^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{(37d^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{1536c} + \frac{(27d^3)\,\text{Sub}}{}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{(37d)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{768c} + \frac{(27d^2)\,\text{S}}{}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}} - \frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right.}{768c^{3/2}}$$

Mathematica [A]
time = 0.11, size = 95, normalized size = 0.91

$$\frac{(-4c-11dx^3)\sqrt{c+dx^3}}{192cx^6} + \frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}} - \frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

```
[Out] ((-4*c - 11*d*x^3)*Sqrt[c + d*x^3])/(192*c*x^6) + (9*d^2*ArcTanh[Sqrt[c + d
*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]
])/(768*c^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.43, size = 617, normalized size = 5.93

method	result

risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(11d\,x^{3}+4c\right)}{192x^{6}c}+\frac{d^{2}\left(-\frac{37\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}-\frac{3i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\,\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\cdots\right.}{\cdots}}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$

default	$\frac{-\frac{c\sqrt{d\,x^3+c}}{6x^6}-\frac{5d\sqrt{d\,x^3+c}}{12x^3}-\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{4\sqrt{c}}}{8c}-\frac{d^3\left(\frac{2x^3\sqrt{d\,x^3+c}}{9}+\frac{56c\sqrt{d\,x^3+c}}{9d}+\frac{3ic\sqrt{2}\,\left(\_\alpha\ldots\right.}{\ldots}\right.}{\ldots}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/6*c*(d*x^3+c)^(1/2)/x^6-5/12*d*(d*x^3+c)^(1/2)/x^3-1/4*d^2*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))-1/512/c^3*d^3*(2/9*x^3*(d*x^3+c)^(1/2)+
56/9*c*(d*x^3+c)^(1/2)/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^
2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-
```

```
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64*d/c^2*(-1/3*c*(d*x^3+c
)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)
))+1/512/c^3*d^2*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/
2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^7), x)
```

Fricas [A]
time = 1.71, size = 218, normalized size = 2.10

$$\left[\frac{27\,\sqrt{c}\,d^2x^6\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+37\,\sqrt{c}\,d^2x^6\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-8\,(11\,cdx^3+4\,c^2)\sqrt{dx^3+c}}{1536\,c^2x^6},\ \frac{37\,\sqrt{-c}\,d^2x^6\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-27\,\sqrt{-c}\,d^2x^6\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-4\,(11\,cdx^3+4\,c^2)\sqrt{dx^3+c}}{768\,c^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/1536*(27*sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/
(d*x^3 - 8*c)) + 37*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c)
+ 2*c)/x^3) - 8*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*x^6), 1/768*(37*
sqrt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d^2*x^6*a
rctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 +
 c))/(c^2*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{c\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx-\int\frac{dx^3\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x) - Integral(d*x**3*sq
rt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

Giac [A]
time = 1.00, size = 101, normalized size = 0.97

$$\frac{37\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\,\sqrt{-c}\,c}-\frac{9\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{256\,\sqrt{-c}\,c}-\frac{11\,(dx^3+c)^{\frac{3}{2}}d^2-7\,\sqrt{dx^3+c}\,cd^2}{192\,cd^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 37/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 9/256*d^2*arctan
(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/192*(11*(d*x^3 + c)^(3/2)*d
^2 - 7*sqrt(d*x^3 + c)*c*d^2)/(c*d^2*x^6)
```

Mupad [B]
time = 3.74, size = 87, normalized size = 0.84

$$\frac{7\,\sqrt{d\,x^3+c}}{192\,x^6}-\frac{37\,d^2\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{768\,\sqrt{c^3}}+\frac{9\,d^2\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{256\,\sqrt{c^3}}-\frac{11\,{\left(d\,x^3+c\right)}^{3/2}}{192\,c\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x)
```

```
[Out] (7*(c + d*x^3)^(1/2))/(192*x^6) - (37*d^2*atanh((c*(c + d*x^3)^(1/2))/(c^3)
^(1/2)))/(768*(c^3)^(1/2)) + (9*d^2*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1
/2))))/(256*(c^3)^(1/2)) - (11*(c + d*x^3)^(3/2))/(192*c*x^6)
```

3.302 $\int \frac{x^7 \left(c+dx^3\right)^{3/2}}{8c-dx^3} \, dx$

Optimal. Leaf size=669

$$-\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2}-\frac{348cx^5\sqrt{c+dx^3}}{247d}-\frac{2}{19}x^8\sqrt{c+dx^3}-\frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{288\sqrt{3}\,c}{}$$

```
[Out] 288*c^(19/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
8/3)-288*c^(19/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-288*c^(19/6)
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3
)-36534/1729*c^2*x^2*(d*x^3+c)^(1/2)/d^2-348/247*c*x^5*(d*x^3+c)^(1/2)/d-2/
19*x^8*(d*x^3+c)^(1/2)-2094648/1729*c^3*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))-698216/1729*3^(3/4)*c^(10/3)*(c^(1/3)+d^(1/3)*x)*Ellip
ticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1
/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3
)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1047324/1729*3^(1/4)*c^(10/3)*(c
^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)
*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d
*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A]

time = 0.76, antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {488, 596, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{698216\sqrt{2}\,3^{3/4}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{1047324\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{288\sqrt{3}\,c^{19/6}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{288c^{19/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{288c^{19/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}-\frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2}-\frac{2}{19}x^8\sqrt{c+dx^3}-\frac{348cx^5\sqrt{c+dx^3}}{247d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-36534*c^2*x^2*Sqrt[c + d*x^3])/(1729*d^2) - (348*c*x^5*Sqrt[c + d*x^3])/(
247*d) - (2*x^8*Sqrt[c + d*x^3])/19 - (2094648*c^3*Sqrt[c + d*x^3])/(1729*d
^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (288*Sqrt[3]*c^(19/6)*ArcTan[
(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(8/3) + (288*c^
(19/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(8/3
```

```
) - (288*c^(19/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(8/3) + (1047324*
3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(10/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (698216*Sqrt[
2]*3^(3/4)*c^(10/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
 4*Sqrt[3]])/(1729*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2\int \frac{x^7\left(-\frac{147c^2d}{2}-87cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{19d}$$

$$= -\frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{4\int \frac{x^4\left(-3480c^3d^2-\frac{18267}{4}c^2d^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{247d^3}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{8\int \frac{x\left(-73068c^4d^3-\frac{26183}{2}\right.}{(8c-dx^3)\sqrt{c+}}}{1729d^5}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{8\int\left(\frac{261831c^3d^3x}{2\sqrt{c+dx^3}} - \right.}{17}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{(1047324c^3)\int \frac{}{\sqrt{c+}}}{1729d^2}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{(432c^3)\int \frac{2\sqrt[3]{c}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right.}}{}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2094648c^3\sqrt{}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right.\right.}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2094648c^3\sqrt{}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right.\right.}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2094648c^3\sqrt{}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right.\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 5.83, size = 163, normalized size = 0.24

$$\frac{-20x^2(18267c^3+19485c^2dx^3+1309cd^2x^6+91d^3x^9)+365340c^3x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+261831c^2dx^5\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{17290d^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-20*x^2*(18267*c^3 + 19485*c^2*d*x^3 + 1309*c*d^2*x^6 + 91*d^3*x^9) + 3653
40*c^3*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*
x^3)/(8*c)] + 261831*c^2*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/
3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17290*d^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 1840, normalized size = 2.75

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	1840

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/19*d*x^8*(d*x^3+c)^(1/2)+44/247*c*x^5*(d*x^3+c)^(1/2)+54/1729*c^2/d
*x^2*(d*x^3+c)^(1/2)+72/1729*I*c^3/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2))))-8/d^2*c*(2/13*d*x^5*(d*x^3+c)^(1/2)+32/91*c*x^2*(d*x^3+c)^(1/2)-18/9
1*I*c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
```

```
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-64*c^2/d^2*(2/7*x^2*(d
*x^3+c)^(1/2)-44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c
*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*
(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c
*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3*I*c/d^
3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-
(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 51.21, size = 3793, normalized size = 5.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

[Out] 2/1729*(82992*sqrt(3)*(c^19/d^16)^(1/6)*d^3*arctan(-1/3*(324*sqrt(3)*(3*c^1
9*d^16*x^16 + 784*c^20*d^15*x^13 + 7680*c^21*d^14*x^10 + 10752*c^22*d^13*x^
7 + 4096*c^23*d^12*x^4)*(c^19/d^16)^(2/3) + 36*sqrt(3)*(c^25*d^11*x^17 + 17
72*c^26*d^10*x^14 + 42592*c^27*d^9*x^11 + 96256*c^28*d^8*x^8 + 69632*c^29*d
^7*x^5 + 16384*c^30*d^6*x^2)*(c^19/d^16)^(1/3) + sqrt(3)*(c^31*d^6*x^18 + 9
456*c^32*d^5*x^15 + 749184*c^33*d^4*x^12 + 3017216*c^34*d^3*x^9 + 3489792*c
^35*d^2*x^6 + 1572864*c^36*d*x^3 + 262144*c^37) + 12*sqrt(d*x^3 + c)*(12*sq
rt(3)*(35*c^16*d^18*x^14 - 14440*c^17*d^17*x^11 - 24576*c^18*d^16*x^8 - 163
84*c^19*d^15*x^5 - 4096*c^20*d^14*x^2)*(c^19/d^16)^(5/6) + 18*sqrt(3)*(c^22
*d^13*x^15 - 1112*c^23*d^12*x^12 + 7296*c^24*d^11*x^9 + 11776*c^25*d^10*x^6
 + 4096*c^26*d^9*x^3)*sqrt(c^19/d^16) + sqrt(3)*(c^28*d^8*x^16 - 4768*c^29*
d^7*x^13 + 362752*c^30*d^6*x^10 + 709120*c^31*d^5*x^7 + 413696*c^32*d^4*x^4
 + 65536*c^33*d^3*x)*(c^19/d^16)^(1/6)) - 2*(324*sqrt(3)*(d^19*x^16 - 1858*
c*d^18*x^13 - 4176*c^2*d^17*x^10 - 3584*c^3*d^16*x^7 - 1024*c^4*d^15*x^4)*(
c^19/d^16)^(5/6) + 18*sqrt(3)*(c^6*d^14*x^17 - 5290*c^7*d^13*x^14 - 21152*c
^8*d^12*x^11 - 47744*c^9*d^11*x^8 - 37888*c^10*d^10*x^5 - 8192*c^11*d^9*x^2
)*sqrt(c^19/d^16) + sqrt(3)*(c^12*d^9*x^18 - 7698*c^13*d^8*x^15 - 1664688*c
^14*d^7*x^12 - 5524864*c^15*d^6*x^9 - 6223872*c^16*d^5*x^6 - 2703360*c^17*d
^4*x^3 - 327680*c^18*d^3)*(c^19/d^16)^(1/6) + 6*sqrt(d*x^3 + c)*(sqrt(3)*(7
*c^3*d^16*x^15 + 37352*c^4*d^15*x^12 - 230336*c^5*d^14*x^9 - 515072*c^6*d^1
3*x^6 - 286720*c^7*d^12*x^3 - 32768*c^8*d^11)*(c^19/d^16)^(2/3) + 108*sqrt(
3)*(53*c^10*d^10*x^13 + 1320*c^11*d^9*x^10 + 1536*c^12*d^8*x^7 + 512*c^13*d
^7*x^4)*(c^19/d^16)^(1/3) + 6*sqrt(3)*(37*c^16*d^5*x^14 + 28912*c^17*d^4*x^
11 + 43584*c^18*d^3*x^8 + 20992*c^19*d^2*x^5 + 4096*c^20*d*x^2)))*sqrt((18*
c^32*d^2*x^8 + 360*c^33*d*x^5 - 144*c^34*x^2 + (c^19*d^13*x^9 - 276*c^20*d^
12*x^6 - 1608*c^21*d^11*x^3 - 1088*c^22*d^10)*(c^19/d^16)^(2/3) + 6*sqrt(d*
x^3 + c)*((c^16*d^15*x^7 - 28*c^17*d^14*x^4 - 272*c^18*d^13*x)*(c^19/d^16)^
(5/6) - 24*(c^23*d^9*x^5 + c^24*d^8*x^2)*sqrt(c^19/d^16) + 4*(c^29*d^4*x^6
+ 41*c^30*d^3*x^3 + 40*c^31*d^2)*(c^19/d^16)^(1/6)) - 18*(c^26*d^7*x^7 - 52
*c^27*d^6*x^4 - 80*c^28*d^5*x)*(c^19/d^16)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
 192*c^2*d*x^3 - 512*c^3)))/(c^31*d^6*x^18 - 14952*c^32*d^5*x^15 + 2872896*
c^33*d^4*x^12 + 7330304*c^34*d^3*x^9 + 6696960*c^35*d^2*x^6 + 2457600*c^36*
d*x^3 + 262144*c^37)) - 82992*sqrt(3)*(c^19/d^16)^(1/6)*d^3*arctan(-1/3*(32
4*sqrt(3)*(3*c^19*d^16*x^16 + 784*c^20*d^15*x^13 + 7680*c^21*d^14*x^10 + 10
752*c^22*d^13*x^7 + 4096*c^23*d^12*x^4)*(c^19/d^16)^(2/3) + 36*sqrt(3)*(c^2
5*d^11*x^17 + 1772*c^26*d^10*x^14 + 42592*c^27*d^9*x^11 + 96256*c^28*d^8*x^
8 + 69632*c^29*d^7*x^5 + 16384*c^30*d^6*x^2)*(c^19/d^16)^(1/3) + sqrt(3)*(c
^31*d^6*x^18 + 9456*c^32*d^5*x^15 + 749184*c^33*d^4*x^12 + 3017216*c^34*d^3
*x^9 + 3489792*c^35*d^2*x^6 + 1572864*c^36*d*x^3 + 262144*c^37) - 12*sqrt(d
x^3 + c)(12*sqrt(3)*(35*c^16*d^18*x^14 - 14440*c^17*d^17*x^11 - 24576*c^1
8*d^16*x^8 - 16384*c^19*d^15*x^5 - 4096*c^20*d^14*x^2)*(c^19/d^16)^(5/6) +
18*sqrt(3)*(c^22*d^13*x^15 - 1112*c^23*d^12*x^12 + 7296*c^24*d^11*x^9 + 117
76*c^25*d^10*x^6 + 4096*c^26*d^9*x^3)*sqrt(c^19/d^16) + sqrt(3)*(c^28*d^8*x
^16 - 4768*c^29*d^7*x^13 + 362752*c^30*d^6*x^10 + 709120*c^31*d^5*x^7 + 413
696*c^32*d^4*x^4 + 65536*c^33*d^3*x)*(c^19/d^16)^(1/6)) + 2*(324*sqrt(3)*(d

```
^19*x^16 - 1858*c*d^18*x^13 - 4176*c^2*d^17*x^10 - 3584*c^3*d^16*x^7 - 1024
*c^4*d^15*x^4)*(c^19/d^16)^(5/6) + 18*sqrt(3)*(c^6*d^14*x^17 - 5290*c^7*d^1
3*x^14 - 21152*c^8*d^12*x^11 - 47744*c^9*d^11*x^8 - 37888*c^10*d^10*x^5 - 8
192*c^11*d^9*x^2)*sqrt(c^19/d^16) + sqrt(3)*(c^12*d^9*x^18 - 7698*c^13*d^8*
x^15 - 1664688*c^14*d^7*x^12 - 5524864*c^15*d^6*x^9 - 6223872*c^16*d^5*x^6
- 2703360*c^17*d^4*x^3 - 327680*c^18*d^3)*(c^19/d^16)^(1/6) - 6*sqrt(d*x^3
+ c)*(sqrt(3)*(7*c^3*d^16*x^15 + 37352*c^4*d^15*x^12 - 230336*c^5*d^14*x^9
- 515072*c^6*d^13*x^6 - 286720*c^7*d^12*x^3 - 32768*c^8*d^11)*(c^19/d^16)^(
2/3) + 108*sqrt(3)*(53*c^10*d^10*x^13 + 1320*c^11*d^9*x^10 + 1536*c^12*d^8*
x^7 + 512*c^13*d^7*x^4)*(c^19/d^16)^(1/3) + 6*sqrt(3)*(37*c^16*d^5*x^14 + 2
8912*c^17*d^4*x^11 + 43584*c^18*d^3*x^8 + 20992*c^19*d^2*x^5 + 4096*c^20*d*
x^2))*sqrt((18*c^32*d^2*x^8 + 360*c^33*d*x^5 - 144*c^34*x^2 + (c^19*d^13*x
^9 - 276*c^20*d^12*x^6 - 1608*c^21*d^11*x^3 - 1088*c^22*d^10)*(c^19/d^16)^(
2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^15*x^7 - 28*c^17*d^14*x^4 - 272*c^18*d^13
*x)*(c^19/d^16)^(5/6) - 24*(c^23*d^9*x^5 + c^24*d^8*x^2)*sqrt(c^19/d^16) +
4*(c^29*d^4*x^6 + 41*c^30*d^3*x^3 + 40*c^31*d^2)*(c^19/d^16)^(1/6)) - 18*(c
^26*d^7*x^7 - 52*c^27*d^6*x^4 - 80*c^28*d^5*x)*(c^19/d^16)^(1/3))/(d^3*x^9
- 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^31*d^6*x^18 - 14952*c^32*d^5
*x^15 + 2872896*c^33*d^4*x^12 + 7330304*c^34*d^3*x^9 + 6696960*c^35*d^2*x^6
 + 2457600*c^36*d*x^3 + 262144*c^37)) + 1047324*c^3*sqrt(d)*weierstrassZeta
(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 20748*(c^19/d^16)^(1/6)*d^
3*log(157030809290648581004328896*(18*c^32*d^2*x...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx-\int \frac{dx^{10}\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*x**7*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**10*sq
rt(c + d*x**3)/(-8*c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,{\left(d\,x^3+c\right)}^{3/2}}{8\,c-d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
[Out] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

3.303 $\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=645

$$-\frac{240cx^2\sqrt{c+dx^3}}{91d}-\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{36\sqrt{3}\,c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\right.}{\sqrt{c+dx}}\right.}{d^{5/3}}$$

```
[Out] 36*c^(13/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5
/3)-36*c^(13/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-36*c^(13/6)*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)-2
40/91*c*x^2*(d*x^3+c)^(1/2)/d-2/13*x^5*(d*x^3+c)^(1/2)-13782/91*c^2*(d*x^3+
c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-4594/91*3^(3/4)*c^(7/3)*(c
^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)
*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(
1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+6891/91*3
^(1/4)*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2))
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 645, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {488, 596, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{4594\sqrt{2}\,3^{3/4}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{6891\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{36\sqrt{3}\,c^{13/6}\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{36c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{36c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}-\frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{240cx^2\sqrt{c+dx^3}}{91d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-240*c*x^2*Sqrt[c + d*x^3])/(91*d) - (2*x^5*Sqrt[c + d*x^3])/13 - (13782*c
^2*Sqrt[c + d*x^3])/(91*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (36*
Sqrt[3]*c^(13/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/d^(5/3) + (36*c^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sq
rt[c + d*x^3])])/d^(5/3) - (36*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d^(5/3) + (6891*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*
```

```
Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(5/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (4594*Sqrt[2]*3^(3/4)*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2}{13}x^5\sqrt{c+dx^3} - \frac{2\int \frac{x^4\left(-\frac{93c^2d}{2}-60cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{13d}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{4\int \frac{x\left(-960c^3d^2-\frac{6891}{4}c^2d^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{91d^3}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{4\int \left(\frac{6891c^2d^2x}{4\sqrt{c+dx^3}} - \frac{14742c^3d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{91d^3}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{(6891c^2)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{91d} + \frac{(648c^3)\int \frac{}{(8c-dx^3}}{d}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{(54c^2)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^2} - $$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{6891\sqrt[4]{}}{}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{36\sqrt{3}}{}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{36\sqrt{3}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 5.62, size = 150, normalized size = 0.23

$$\frac{-80(120c^2x^2+127cdx^5+7d^2x^8)+9600c^2x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+6891cdx^5\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3640d\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-80*(120*c^2*x^2 + 127*c*d*x^5 + 7*d^2*x^8) + 9600*c^2*x^2*Sqrt[1 + (d*x^3
)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 6891*c*d*x^5
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(3640*d*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 1344, normalized size = 2.08

method	result	size
risch	Expression too large to display	884
elliptic	Expression too large to display	886
default	Expression too large to display	1344

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/13*d*x^5*(d*x^3+c)^(1/2)+32/91*c*x^2*(d*x^3+c)^(1/2)-18/91*I*c^2*3^
(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-8*c/d*(2/7*x^2*(d*x^3+c)^(1/2)-4
4/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
```

```
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1
/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3
)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/
3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*
_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/
2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*
d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 22.57, size = 3774, normalized size = 5.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
[Out] 1/91*(1092*sqrt(3)*(c^13/d^10)^(1/6)*d^2*arctan(-1/3*(324*sqrt(3)*(3*c^13*d
^12*x^16 + 784*c^14*d^11*x^13 + 7680*c^15*d^10*x^10 + 10752*c^16*d^9*x^7 +
4096*c^17*d^8*x^4)*(c^13/d^10)^(2/3) + 36*sqrt(3)*(c^17*d^9*x^17 + 1772*c^1
8*d^8*x^14 + 42592*c^19*d^7*x^11 + 96256*c^20*d^6*x^8 + 69632*c^21*d^5*x^5
+ 16384*c^22*d^4*x^2)*(c^13/d^10)^(1/3) + sqrt(3)*(c^21*d^6*x^18 + 9456*c^2
2*d^5*x^15 + 749184*c^23*d^4*x^12 + 3017216*c^24*d^3*x^9 + 3489792*c^25*d^2
*x^6 + 1572864*c^26*d*x^3 + 262144*c^27) + 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(
35*c^11*d^13*x^14 - 14440*c^12*d^12*x^11 - 24576*c^13*d^11*x^8 - 16384*c^14
*d^10*x^5 - 4096*c^15*d^9*x^2)*(c^13/d^10)^(5/6) + 18*sqrt(3)*(c^15*d^10*x^
15 - 1112*c^16*d^9*x^12 + 7296*c^17*d^8*x^9 + 11776*c^18*d^7*x^6 + 4096*c^1
9*d^6*x^3)*sqrt(c^13/d^10) + sqrt(3)*(c^19*d^7*x^16 - 4768*c^20*d^6*x^13 +
```

362752*c^21*d^5*x^10 + 709120*c^22*d^4*x^7 + 413696*c^23*d^3*x^4 + 65536*c^24*d^2*x)*(c^13/d^10)^(1/6)) - 2*(324*sqrt(3)*(d^14*x^16 - 1858*c*d^13*x^13 - 4176*c^2*d^12*x^10 - 3584*c^3*d^11*x^7 - 1024*c^4*d^10*x^4)*(c^13/d^10)^(5/6) + 18*sqrt(3)*(c^4*d^11*x^17 - 5290*c^5*d^10*x^14 - 21152*c^6*d^9*x^11 - 47744*c^7*d^8*x^8 - 37888*c^8*d^7*x^5 - 8192*c^9*d^6*x^2)*sqrt(c^13/d^10) + sqrt(3)*(c^8*d^8*x^18 - 7698*c^9*d^7*x^15 - 1664688*c^10*d^6*x^12 - 5524864*c^11*d^5*x^9 - 6223872*c^12*d^4*x^6 - 2703360*c^13*d^3*x^3 - 327680*c^14*d^2)*(c^13/d^10)^(1/6) + 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c^2*d^12*x^15 + 37352*c^3*d^11*x^12 - 230336*c^4*d^10*x^9 - 515072*c^5*d^9*x^6 - 286720*c^6*d^8*x^3 - 32768*c^7*d^7)*(c^13/d^10)^(2/3) + 108*sqrt(3)*(53*c^7*d^8*x^13 + 1320*c^8*d^7*x^10 + 1536*c^9*d^6*x^7 + 512*c^10*d^5*x^4)*(c^13/d^10)^(1/3) + 6*sqrt(3)*(37*c^11*d^5*x^14 + 28912*c^12*d^4*x^11 + 43584*c^13*d^3*x^8 + 20992*c^14*d^2*x^5 + 4096*c^15*d*x^2)))*sqrt((18*c^22*d^2*x^8 + 360*c^23*d*x^5 - 144*c^24*x^2 + (c^13*d^9*x^9 - 276*c^14*d^8*x^6 - 1608*c^15*d^7*x^3 - 1088*c^16*d^6)*(c^13/d^10)^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^10*x^7 - 28*c^12*d^9*x^4 - 272*c^13*d^8*x)*(c^13/d^10)^(5/6) - 24*(c^16*d^6*x^5 + c^17*d^5*x^2)*sqrt(c^13/d^10) + 4*(c^20*d^3*x^6 + 41*c^21*d^2*x^3 + 40*c^22*d)*(c^13/d^10)^(1/6)) - 18*(c^18*d^5*x^7 - 52*c^19*d^4*x^4 - 80*c^20*d^3*x)*(c^13/d^10)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^21*d^6*x^18 - 14952*c^22*d^5*x^15 + 2872896*c^23*d^4*x^12 + 7330304*c^24*d^3*x^9 + 6696960*c^25*d^2*x^6 + 2457600*c^26*d*x^3 + 262144*c^27)) - 1092*sqrt(3)*(c^13/d^10)^(1/6)*d^2*arctan(-1/3*(324*sqrt(3)*(3*c^13*d^12*x^16 + 784*c^14*d^11*x^13 + 7680*c^15*d^10*x^10 + 10752*c^16*d^9*x^7 + 4096*c^17*d^8*x^4)*(c^13/d^10)^(2/3) + 36*sqrt(3)*(c^17*d^9*x^17 + 1772*c^18*d^8*x^14 + 42592*c^19*d^7*x^11 + 96256*c^20*d^6*x^8 + 69632*c^21*d^5*x^5 + 16384*c^22*d^4*x^2)*(c^13/d^10)^(1/3) + sqrt(3)*(c^21*d^6*x^18 + 9456*c^22*d^5*x^15 + 749184*c^23*d^4*x^12 + 3017216*c^24*d^3*x^9 + 3489792*c^25*d^2*x^6 + 1572864*c^26*d*x^3 + 262144*c^27) - 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c^11*d^13*x^14 - 14440*c^12*d^12*x^11 - 24576*c^13*d^11*x^8 - 16384*c^14*d^10*x^5 - 4096*c^15*d^9*x^2)*(c^13/d^10)^(5/6) + 18*sqrt(3)*(c^15*d^10*x^15 - 1112*c^16*d^9*x^12 + 7296*c^17*d^8*x^9 + 11776*c^18*d^7*x^6 + 4096*c^19*d^6*x^3)*sqrt(c^13/d^10) + sqrt(3)*(c^19*d^7*x^16 - 4768*c^20*d^6*x^13 + 362752*c^21*d^5*x^10 + 709120*c^22*d^4*x^7 + 413696*c^23*d^3*x^4 + 65536*c^24*d^2*x)*(c^13/d^10)^(1/6)) + 2*(324*sqrt(3)*(d^14*x^16 - 1858*c*d^13*x^13 - 4176*c^2*d^12*x^10 - 3584*c^3*d^11*x^7 - 1024*c^4*d^10*x^4)*(c^13/d^10)^(5/6) + 18*sqrt(3)*(c^4*d^11*x^17 - 5290*c^5*d^10*x^14 - 21152*c^6*d^9*x^11 - 47744*c^7*d^8*x^8 - 37888*c^8*d^7*x^5 - 8192*c^9*d^6*x^2)*sqrt(c^13/d^10) + sqrt(3)*(c^8*d^8*x^18 - 7698*c^9*d^7*x^15 - 1664688*c^10*d^6*x^12 - 5524864*c^11*d^5*x^9 - 6223872*c^12*d^4*x^6 - 2703360*c^13*d^3*x^3 - 327680*c^14*d^2)*(c^13/d^10)^(1/6) - 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c^2*d^12*x^15 + 37352*c^3*d^11*x^12 - 230336*c^4*d^10*x^9 - 515072*c^5*d^9*x^6 - 286720*c^6*d^8*x^3 - 32768*c^7*d^7)*(c^13/d^10)^(2/3) + 108*sqrt(3)*(53*c^7*d^8*x^13 + 1320*c^8*d^7*x^10 + 1536*c^9*d^6*x^7 + 512*c^10*d^5*x^4)*(c^13/d^10)^(1/3) + 6*sqrt(3)*(37*c^11*d^5*x^14 + 28912*c^12*d^4*x^11 + 43584*c^13*d^3*x^8 + 20992*c^14*d^2*x^5 + 4096*c^15*d*x^2)))*sqrt((18*c^22*d^2*x^8 + 360*c^23*d*x^5 - 144*c^24

```
*x^2 + (c^13*d^9*x^9 - 276*c^14*d^8*x^6 - 1608*c^15*d^7*x^3 - 1088*c^16*d^6
)*(c^13/d^10)^(2/3) - 6*sqrt(d*x^3 + c)*((c^11*d^10*x^7 - 28*c^12*d^9*x^4 -
 272*c^13*d^8*x)*(c^13/d^10)^(5/6) - 24*(c^16*d^6*x^5 + c^17*d^5*x^2)*sqrt(
c^13/d^10) + 4*(c^20*d^3*x^6 + 41*c^21*d^2*x^3 + 40*c^22*d)*(c^13/d^10)^(1/
6)) - 18*(c^18*d^5*x^7 - 52*c^19*d^4*x^4 - 80*c^20*d^3*x)*(c^13/d^10)^(1/3)
)/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^21*d^6*x^18 - 149
52*c^22*d^5*x^15 + 2872896*c^23*d^4*x^12 + 7330304*c^24*d^3*x^9 + 6696960*c
^25*d^2*x^6 + 2457600*c^26*d*x^3 + 262144*c^27)) + 13782*c^2*sqrt(d)*weiers
trassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 273*(c^13/d^10)^(
1/6)*d^2*log(14624633760251904*(18*c^22*d^2*x^8 + 360*c^23*d*x^5 - 144*c^24
*x^2 + (c^13*d^9*x^9 - 276*c^14*d^8*x^6 - 1608*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx-\int \frac{dx^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**7*sqr
t(c + d*x**3)/(-8*c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d\,x^3+c\right)}^{3/2}}{8\,c-d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

3.304 $\int \frac{x\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=627

$$-\frac{2}{7}x^2\sqrt{c+dx^3}-\frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}+\frac{9c^{7/6}\tanh^{-}}{}$$

```
[Out] 9/2*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(2
/3)-9/2*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)-9/2*c^(7/6)*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(2/3)-2
/7*x^2*(d*x^3+c)^(1/2)-132/7*c*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))-44/7*3^(3/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c
^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))^2)^(1/2)+66/7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*Elliptic
E((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^
(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 627, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {488, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{44\sqrt{2}\,3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{66\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{9\sqrt{3}\,c^{7/6}\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}+\frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}}-\frac{9c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}}-\frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{2}{7}x^2\sqrt{c+dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/7 - (132*c*Sqrt[c + d*x^3])/(7*d^(2/3)*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) + (9*c^(7/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) - (9*c^(7/6)*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) + (66*3^(1/4)*Sqrt[2 - Sqr
t[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
```

```
t[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqr
t[3]])/(7*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (44*Sqrt[2]*3^(3/4)*c^(4/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{2\int \frac{x\left(-\frac{39c^2d}{2}-33cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{7d}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{2\int \left(\frac{33cdx}{\sqrt{c+dx^3}} - \frac{567c^2dx}{2(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{7d}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{1}{7}(66c)\int \frac{x}{\sqrt{c+dx^3}}\,dx + \left(81c^2\right)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{(27c)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{4d} - \frac{(66c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{}}{\sqrt{c+dx^3}}}{7\sqrt[3]{d}}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{66\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\right.}{}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{}\right.}{\sqrt{c+}}\right.}{2d^{2/3}}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{}\right.}{\sqrt{c+}}\right.}{2d^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 7.54, size = 127, normalized size = 0.20

$$\frac{x^2\left(-160(c+dx^3)+195c\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+132dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{560\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (x^2*(-160*(c + d*x^3) + 195*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), (d*x^3)/(8*c)] + 132*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(560*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 864, normalized size = 1.38

method	result	size
default	Expression too large to display	864
elliptic	Expression too large to display	864
risch	Expression too large to display	866

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*x^2*(d*x^3+c)^(1/2)+44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
))-3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2
)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_a
lpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2
```

```
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 19.20, size = 3708, normalized size = 5.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] -1/56*(16*sqrt(d*x^3 + c)*d*x^2 - 84*sqrt(3)*(c^7/d^4)^(1/6)*d*arctan(-1/3*
(324*sqrt(3)*(3*c^7*d^8*x^16 + 784*c^8*d^7*x^13 + 7680*c^9*d^6*x^10 + 10752
*c^10*d^5*x^7 + 4096*c^11*d^4*x^4)*(c^7/d^4)^(2/3) + 36*sqrt(3)*(c^9*d^7*x^
17 + 1772*c^10*d^6*x^14 + 42592*c^11*d^5*x^11 + 96256*c^12*d^4*x^8 + 69632*
c^13*d^3*x^5 + 16384*c^14*d^2*x^2)*(c^7/d^4)^(1/3) + sqrt(3)*(c^11*d^6*x^18
 + 9456*c^12*d^5*x^15 + 749184*c^13*d^4*x^12 + 3017216*c^14*d^3*x^9 + 34897
92*c^15*d^2*x^6 + 1572864*c^16*d*x^3 + 262144*c^17) + 12*sqrt(d*x^3 + c)*(1
2*sqrt(3)*(35*c^6*d^8*x^14 - 14440*c^7*d^7*x^11 - 24576*c^8*d^6*x^8 - 16384
*c^9*d^5*x^5 - 4096*c^10*d^4*x^2)*(c^7/d^4)^(5/6) + 18*sqrt(3)*(c^8*d^7*x^1
5 - 1112*c^9*d^6*x^12 + 7296*c^10*d^5*x^9 + 11776*c^11*d^4*x^6 + 4096*c^12*
d^3*x^3)*sqrt(c^7/d^4) + sqrt(3)*(c^10*d^6*x^16 - 4768*c^11*d^5*x^13 + 3627
52*c^12*d^4*x^10 + 709120*c^13*d^3*x^7 + 413696*c^14*d^2*x^4 + 65536*c^15*d
*x)*(c^7/d^4)^(1/6)) - 2*(324*sqrt(3)*(d^9*x^16 - 1858*c*d^8*x^13 - 4176*c^
2*d^7*x^10 - 3584*c^3*d^6*x^7 - 1024*c^4*d^5*x^4)*(c^7/d^4)^(5/6) + 18*sqrt
(3)*(c^2*d^8*x^17 - 5290*c^3*d^7*x^14 - 21152*c^4*d^6*x^11 - 47744*c^5*d^5*
x^8 - 37888*c^6*d^4*x^5 - 8192*c^7*d^3*x^2)*sqrt(c^7/d^4) + sqrt(3)*(c^4*d^
7*x^18 - 7698*c^5*d^6*x^15 - 1664688*c^6*d^5*x^12 - 5524864*c^7*d^4*x^9 - 6
223872*c^8*d^3*x^6 - 2703360*c^9*d^2*x^3 - 327680*c^10*d)*(c^7/d^4)^(1/6) +
 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c*d^8*x^15 + 37352*c^2*d^7*x^12 - 230336*c^3
*d^6*x^9 - 515072*c^4*d^5*x^6 - 286720*c^5*d^4*x^3 - 32768*c^6*d^3)*(c^7/d^
4)^(2/3) + 108*sqrt(3)*(53*c^4*d^6*x^13 + 1320*c^5*d^5*x^10 + 1536*c^6*d^4*
x^7 + 512*c^7*d^3*x^4)*(c^7/d^4)^(1/3) + 6*sqrt(3)*(37*c^6*d^5*x^14 + 28912
*c^7*d^4*x^11 + 43584*c^8*d^3*x^8 + 20992*c^9*d^2*x^5 + 4096*c^10*d*x^2)))*
sqrt((18*c^12*d^2*x^8 + 360*c^13*d*x^5 - 144*c^14*x^2 + (c^7*d^5*x^9 - 276*
c^8*d^4*x^6 - 1608*c^9*d^3*x^3 - 1088*c^10*d^2)*(c^7/d^4)^(2/3) + 6*sqrt(d*
```

```
x^3 + c)*((c^6*d^5*x^7 - 28*c^7*d^4*x^4 - 272*c^8*d^3*x)*(c^7/d^4)^(5/6) -
24*(c^9*d^3*x^5 + c^10*d^2*x^2)*sqrt(c^7/d^4) + 4*(c^11*d^2*x^6 + 41*c^12*d
*x^3 + 40*c^13)*(c^7/d^4)^(1/6)) - 18*(c^10*d^3*x^7 - 52*c^11*d^2*x^4 - 80*
c^12*d*x)*(c^7/d^4)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^
3)))/(c^11*d^6*x^18 - 14952*c^12*d^5*x^15 + 2872896*c^13*d^4*x^12 + 7330304
*c^14*d^3*x^9 + 6696960*c^15*d^2*x^6 + 2457600*c^16*d*x^3 + 262144*c^17)) +
 84*sqrt(3)*(c^7/d^4)^(1/6)*d*arctan(-1/3*(324*sqrt(3)*(3*c^7*d^8*x^16 + 78
4*c^8*d^7*x^13 + 7680*c^9*d^6*x^10 + 10752*c^10*d^5*x^7 + 4096*c^11*d^4*x^4
)*(c^7/d^4)^(2/3) + 36*sqrt(3)*(c^9*d^7*x^17 + 1772*c^10*d^6*x^14 + 42592*c
^11*d^5*x^11 + 96256*c^12*d^4*x^8 + 69632*c^13*d^3*x^5 + 16384*c^14*d^2*x^2
)*(c^7/d^4)^(1/3) + sqrt(3)*(c^11*d^6*x^18 + 9456*c^12*d^5*x^15 + 749184*c^
13*d^4*x^12 + 3017216*c^14*d^3*x^9 + 3489792*c^15*d^2*x^6 + 1572864*c^16*d*
x^3 + 262144*c^17) - 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c^6*d^8*x^14 - 1444
0*c^7*d^7*x^11 - 24576*c^8*d^6*x^8 - 16384*c^9*d^5*x^5 - 4096*c^10*d^4*x^2)
*(c^7/d^4)^(5/6) + 18*sqrt(3)*(c^8*d^7*x^15 - 1112*c^9*d^6*x^12 + 7296*c^10
*d^5*x^9 + 11776*c^11*d^4*x^6 + 4096*c^12*d^3*x^3)*sqrt(c^7/d^4) + sqrt(3)*
(c^10*d^6*x^16 - 4768*c^11*d^5*x^13 + 362752*c^12*d^4*x^10 + 709120*c^13*d^
3*x^7 + 413696*c^14*d^2*x^4 + 65536*c^15*d*x)*(c^7/d^4)^(1/6)) + 2*(324*sqr
t(3)*(d^9*x^16 - 1858*c*d^8*x^13 - 4176*c^2*d^7*x^10 - 3584*c^3*d^6*x^7 - 1
024*c^4*d^5*x^4)*(c^7/d^4)^(5/6) + 18*sqrt(3)*(c^2*d^8*x^17 - 5290*c^3*d^7*
x^14 - 21152*c^4*d^6*x^11 - 47744*c^5*d^5*x^8 - 37888*c^6*d^4*x^5 - 8192*c^
7*d^3*x^2)*sqrt(c^7/d^4) + sqrt(3)*(c^4*d^7*x^18 - 7698*c^5*d^6*x^15 - 1664
688*c^6*d^5*x^12 - 5524864*c^7*d^4*x^9 - 6223872*c^8*d^3*x^6 - 2703360*c^9*
d^2*x^3 - 327680*c^10*d)*(c^7/d^4)^(1/6) - 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c*
d^8*x^15 + 37352*c^2*d^7*x^12 - 230336*c^3*d^6*x^9 - 515072*c^4*d^5*x^6 - 2
86720*c^5*d^4*x^3 - 32768*c^6*d^3)*(c^7/d^4)^(2/3) + 108*sqrt(3)*(53*c^4*d^
6*x^13 + 1320*c^5*d^5*x^10 + 1536*c^6*d^4*x^7 + 512*c^7*d^3*x^4)*(c^7/d^4)^
(1/3) + 6*sqrt(3)*(37*c^6*d^5*x^14 + 28912*c^7*d^4*x^11 + 43584*c^8*d^3*x^8
 + 20992*c^9*d^2*x^5 + 4096*c^10*d*x^2)))*sqrt((18*c^12*d^2*x^8 + 360*c^13*
d*x^5 - 144*c^14*x^2 + (c^7*d^5*x^9 - 276*c^8*d^4*x^6 - 1608*c^9*d^3*x^3 -
1088*c^10*d^2)*(c^7/d^4)^(2/3) - 6*sqrt(d*x^3 + c)*((c^6*d^5*x^7 - 28*c^7*d
^4*x^4 - 272*c^8*d^3*x)*(c^7/d^4)^(5/6) - 24*(c^9*d^3*x^5 + c^10*d^2*x^2)*s
qrt(c^7/d^4) + 4*(c^11*d^2*x^6 + 41*c^12*d*x^3 + 40*c^13)*(c^7/d^4)^(1/6))
- 18*(c^10*d^3*x^7 - 52*c^11*d^2*x^4 - 80*c^12*d*x)*(c^7/d^4)^(1/3))/(d^3*x
^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^11*d^6*x^18 - 14952*c^12*
d^5*x^15 + 2872896*c^13*d^4*x^12 + 7330304*c^14*d^3*x^9 + 6696960*c^15*d^2*
x^6 + 2457600*c^16*d*x^3 + 262144*c^17)) - 1056*c*sqrt(d)*weierstrassZeta(0
, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 21*(c^7/d^4)^(1/6)*d*log(348
6784401/4*(18*c^12*d^2*x^8 + 360*c^13*d*x^5 - 144*c^14*x^2 + (c^7*d^5*x^9 -
 276*c^8*d^4*x^6 - 1608*c^9*d^3*x^3 - 1088*c^10*d^2)*(c^7/d^4)^(2/3) + 6*sq
rt(d*x^3 + c)*((c^6*d^5*x^7 - 28*c^7*d^4*x^4 - 272*c^8*d^3*x)*(c^7/d^4)^(5/
6) - 24*(c^9*d^3*x^5 + c^10*d^2*x^2)*sqrt(c^7/d...
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx\sqrt{c+dx^3}}{-8c+dx^3}\,dx-\int \frac{dx^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*x*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**4*sqrt(c
 + d*x**3)/(-8*c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(d\,x^3+c\right)}^{3/2}}{8\,c-d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

3.305 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=626

$$-\frac{\sqrt{c+dx^3}}{8x}-\frac{15\sqrt[3]{d}\,\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{9}{16}\sqrt{3}\,\sqrt[6]{c}\,\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)+\frac{9}{16}\sqrt[6]{c}\,\sqrt[3]{d}\,\mathrm{t}$$

```
[Out] 9/16*c^(1/6)*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1
/2))-9/16*c^(1/6)*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-9/16*c^(1/6)
*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)-1/8*(d*x^3+c)^(1/2)/x-15/8*d^(1/3)*(d*x^3+c)^(1/2)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))-5/8*3^(3/4)*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3
)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c
^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2)))^2)^(1/2)+15/16*3^(1/4)*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*El
lipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.51, antiderivative size = 626, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {485, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{5\,3^{3/4}\sqrt[3]{c}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\middle|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\middle|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{9}{16}\sqrt{3}\,\sqrt[6]{c}\,\sqrt[3]{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)-\frac{\sqrt{c+dx^3}}{8x}-\frac{15\sqrt[3]{d}\,\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{9}{16}\sqrt[6]{c}\,\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)-\frac{9}{16}\sqrt[6]{c}\,\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

```
[Out] -1/8*Sqrt[c + d*x^3]/x - (15*d^(1/3)*Sqrt[c + d*x^3])/(8*((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(1/6)*d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/16 + (9*c^(1/6)*d^(1/3)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/16 - (9*c^(1/6)*d^(1/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/16 + (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1
/3)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
```

```
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(16*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) - (5*3^(3/4)*c^(1/3)*d^(1/3)*(c^(1/3) + d^(1/3)*
x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*Sqrt[(c^(1/3)*(
c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3
])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{8x} + \frac{\int \frac{x\left(21c^2d+\frac{15}{2}cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8x} + \frac{\int \left(-\frac{15cdx}{2\sqrt{c+dx^3}} + \frac{81c^2dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{1}{16}(15d)\int \frac{x}{\sqrt{c+dx^3}}\,dx + \frac{1}{8}(81cd)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{27}{32}\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx - \frac{1}{16}\left(15d^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right.}{\sqrt{c}}$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{}$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{9}{16}\sqrt{3}\,\sqrt[6]{c}\,\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}\right.}{\sqrt{c+dx}}\right.$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{9}{16}\sqrt{3}\,\sqrt[6]{c}\,\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}\right.}{\sqrt{c+dx}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.06, size = 137, normalized size = 0.22

$$\frac{-16c(c+dx^3)+21cdx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3d^2x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{128cx\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

```
[Out] (-16*c*(c + d*x^3) + 21*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5
/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(128*c*x*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 1339, normalized size = 2.14

method	result	size
elliptic	Expression too large to display	859
risch	Expression too large to display	866
default	Expression too large to display	1339

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d/c*(2/7*x^2*(d*x^3+c)^(1/2)-44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/
d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2
*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
```

```
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/8/c*(-c*(d*x^3+c)^(1/2)/x+2/7
*(d*x^3+c)^(1/2)*d*x^2-9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1
/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1
/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx-\int \frac{dx^3\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c),x)
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^3+c\right)}^{3/2}}{x^2\,\left(8\,c-d\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)), x)
```

3.306 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=651

$$-\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}+\frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{9\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}}+\frac{9d}{}$$

```
[Out] 9/128*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(5/6)-9/128*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)-9/128*d^(4
/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c^(
5/6)-1/32*(d*x^3+c)^(1/2)/x^4-3/16*d*(d*x^3+c)^(1/2)/c/x+3/16*d^(4/3)*(d*x^
3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/16*3^(3/4)*d^(4/3)*(c^(1/3)+
d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c
^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/32*3^(1/4)*d^(
4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-
c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(
2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {485, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\middle|-7-4\sqrt{3}\right)}{8\sqrt{2}\,c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\middle|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{9\sqrt{3}\,d^{4/3}\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}}+\frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{5/6}}-\frac{9d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{5/6}}+\frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{3d\sqrt{c+dx^3}}{16cx}-\frac{\sqrt{c+dx^3}}{32x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]
```

```
[Out] -1/32*Sqrt[c + d*x^3]/x^4 - (3*d*Sqrt[c + d*x^3])/(16*c*x) + (3*d^(4/3)*Sqr
t[c + d*x^3])/(16*c*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*d^(4/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(128*c^
(5/6)) + (9*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x
^3])])/(128*c^(5/6)) - (9*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(12
```

```
8*c^(5/6)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) +
(3^(3/4)*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(8*Sqrt[2]*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{32x^4}+\frac{\int \frac{48c^2d+\frac{69}{2}cd^2x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}-\frac{\int \frac{x\left(-516c^3d^2+24c^2d^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}-\frac{\int \left(-\frac{24c^2d^2x}{\sqrt{c+dx^3}}-\frac{324c^3d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}+\frac{1}{64}\left(81d^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx+\frac{\left(3d^2\right)\int \frac{x}{\sqrt{c+\cdots}}\cdots}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}-\frac{(27d)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{256c}+\frac{\left(3d^{5/3}\right)\int \cdots}{\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}+\frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d^{4/\cdots}\cdots}{\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}+\frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{9\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right)\cdots}{128\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}+\frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{9\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right)\cdots}{128\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.06, size = 154, normalized size = 0.24

$$\frac{645cd^2x^6\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\left(40c(c^2+7cdx^3+6d^2x^6)+3d^3x^9\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{5120c^2x^4\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]
```

```
[Out] (645*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 7*c*d*x^3 + 6*d^2*x^6) + 3*d^3*x^9*Sqrt[1
+ (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(512
0*c^2*x^4*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 1810, normalized size = 2.78

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	884
default	Expression too large to display	1810

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/4*c*(d*x^3+c)^(1/2)/x^4-11/8*d*(d*x^3+c)^(1/2)/x-9/8*I*d*3^(1/2)*
(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-1/64/c^2*d^2*(2/7*x^2*(d*x^3+c)^(1/2)-44
/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
```

```
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1/
_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))+1/64*d/c^2*(-c*(d*x^3+c)^(1/2)/x+2/7*(d*x^3+c)^(1/2)*d*x^2-9/7*I*c*
3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.79, size = 2585, normalized size = 3.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] -1/512*(12*sqrt(3)*(d^8/c^5)^(1/6)*c*x^4*arctan(1/9*((9*sqrt(3)*(d^8/c^5)^(
1/6)*c*d^13*x^5 - sqrt(3)*(c^4*d^8*x^6 - 40*c^5*d^7*x^3 - 32*c^6*d^6)*(d^8/
c^5)^(5/6) + 3*sqrt(3)*(5*c^3*d^10*x^4 + 8*c^4*d^9*x)*sqrt(d^8/c^5))*sqrt(d
*x^3 + c) + (18*sqrt(3)*(c^4*d^3*x^5 + c^5*d^2*x^2)*(d^8/c^5)^(2/3) + 12*sq
rt(3)*(c^2*d^6*x^6 - c^3*d^5*x^3 - 2*c^4*d^4)*(d^8/c^5)^(1/3) + 3*sqrt(3)*(
d^9*x^7 + 5*c*d^8*x^4 + 4*c^2*d^7*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^2*x^
6 + 32*c^5*d*x^3 + 40*c^6)*(d^8/c^5)^(5/6) + 3*sqrt(3)*(7*c^3*d^4*x^4 + 4*c
^4*d^3*x)*sqrt(d^8/c^5) + 9*sqrt(3)*(c*d^7*x^5 + 2*c^2*d^6*x^2)*(d^8/c^5)^(
1/6)))*sqrt((d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12
- 18*(c^4*d^9*x^7 - 52*c^5*d^8*x^4 - 80*c^6*d^7*x)*(d^8/c^5)^(2/3) + 6*sqrt
(d*x^3 + c)*(24*(c^5*d^7*x^5 + c^6*d^6*x^2)*(d^8/c^5)^(5/6) - 4*(c^3*d^10*x
^6 + 41*c^4*d^9*x^3 + 40*c^5*d^8)*sqrt(d^8/c^5) - (c*d^13*x^7 - 28*c^2*d^12
*x^4 - 272*c^3*d^11*x)*(d^8/c^5)^(1/6)) + 18*(c^2*d^12*x^8 + 20*c^3*d^11*x^
5 - 8*c^4*d^10*x^2)*(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^
3 - 512*c^3)))/(d^15*x^7 - 7*c*d^14*x^4 - 8*c^2*d^13*x)) + 12*sqrt(3)*(d^8/
c^5)^(1/6)*c*x^4*arctan(1/9*((9*sqrt(3)*(d^8/c^5)^(1/6)*c*d^13*x^5 - sqrt(3
)*(c^4*d^8*x^6 - 40*c^5*d^7*x^3 - 32*c^6*d^6)*(d^8/c^5)^(5/6) + 3*sqrt(3)*(
5*c^3*d^10*x^4 + 8*c^4*d^9*x)*sqrt(d^8/c^5))*sqrt(d*x^3 + c) - (18*sqrt(3)*
(c^4*d^3*x^5 + c^5*d^2*x^2)*(d^8/c^5)^(2/3) + 12*sqrt(3)*(c^2*d^6*x^6 - c^3
*d^5*x^3 - 2*c^4*d^4)*(d^8/c^5)^(1/3) + 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*x^4 +
4*c^2*d^7*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^2*x^6 + 32*c^5*d*x^3 + 40*c^
6)*(d^8/c^5)^(5/6) + 3*sqrt(3)*(7*c^3*d^4*x^4 + 4*c^4*d^3*x)*sqrt(d^8/c^5)
+ 9*sqrt(3)*(c*d^7*x^5 + 2*c^2*d^6*x^2)*(d^8/c^5)^(1/6)))*sqrt((d^15*x^9 -
276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^4*d^9*x^7 - 52*c
^5*d^8*x^4 - 80*c^6*d^7*x)*(d^8/c^5)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^7
*x^5 + c^6*d^6*x^2)*(d^8/c^5)^(5/6) - 4*(c^3*d^10*x^6 + 41*c^4*d^9*x^3 + 40
*c^5*d^8)*sqrt(d^8/c^5) - (c*d^13*x^7 - 28*c^2*d^12*x^4 - 272*c^3*d^11*x)*(
d^8/c^5)^(1/6)) + 18*(c^2*d^12*x^8 + 20*c^3*d^11*x^5 - 8*c^4*d^10*x^2)*(d^8
/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^15*x^7
 - 7*c*d^14*x^4 - 8*c^2*d^13*x)) + 96*d^(3/2)*x^4*weierstrassZeta(0, -4*c/d
, weierstrassPInverse(0, -4*c/d, x)) + 3*(d^8/c^5)^(1/6)*c*x^4*log(43046721
*(d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^4*d
^9*x^7 - 52*c^5*d^8*x^4 - 80*c^6*d^7*x)*(d^8/c^5)^(2/3) + 6*sqrt(d*x^3 + c)
*(24*(c^5*d^7*x^5 + c^6*d^6*x^2)*(d^8/c^5)^(5/6) - 4*(c^3*d^10*x^6 + 41*c^4
*d^9*x^3 + 40*c^5*d^8)*sqrt(d^8/c^5) - (c*d^13*x^7 - 28*c^2*d^12*x^4 - 272*
c^3*d^11*x)*(d^8/c^5)^(1/6)) + 18*(c^2*d^12*x^8 + 20*c^3*d^11*x^5 - 8*c^4*d
^10*x^2)*(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) - 3*(d^8/c^5)^(1/6)*c*x^4*log(43046721*(d^15*x^9 - 276*c*d^14*x^6 - 1608
*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^4*d^9*x^7 - 52*c^5*d^8*x^4 - 80*c^6*d
^7*x)*(d^8/c^5)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^7*x^5 + c^6*d^6*x^2)*(
d^8/c^5)^(5/6) - 4*(c^3*d^10*x^6 + 41*c^4*d^9*x^3 + 40*c^5*d^8)*sqrt(d^8/c^
5) - (c*d^13*x^7 - 28*c^2*d^12*x^4 - 272*c^3*d^11*x)*(d^8/c^5)^(1/6)) + 18*
(c^2*d^12*x^8 + 20*c^3*d^11*x^5 - 8*c^4*d^10*x^2)*(d^8/c^5)^(1/3))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 6*(d^8/c^5)^(1/6)*c*x^4*log(6
561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^4*d
```

```
^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x)*(d^8/c^5)^(2/3) + 6*sqrt(d*x^3 + c)*(
6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^8/c^5)^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3
*x^3 + 64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^
5*x)*(d^8/c^5)^(1/6)) + 18*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*
(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 6*(d
^8/c^5)^(1/6)*c*x^4*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 +
640*c^3*d^6 + 18*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x)*(d^8/c^5)^(2
/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^8/c^5)^(5/6) + (7*
c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80
*c^2*d^6*x^4 + 160*c^3*d^5*x)*(d^8/c^5)^(1/6)) + 18*(c^2*d^6*x^8 + 38*c^3*d
^5*x^5 + 64*c^4*d^4*x^2)*(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2
*d*x^3 - 512*c^3)) + 16*(6*d*x^3 + c)*sqrt(d*x^3 + c))/(c*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx-\int \frac{dx^3\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^3+c\right)}^{3/2}}{x^5\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)), x)
```

3.307 $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx$

Optimal. Leaf size=675

$$-\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{9\sqrt{3}\,d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}}{\sqrt{c}}\right.}{1024c^{11/6}}$$

```
[Out] 9/1024*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(11/6)-9/1024*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-9/1024
*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)/c^(11/6)-1/56*(d*x^3+c)^(1/2)/x^7-75/1792*d*(d*x^3+c)^(1/2)/c/x^4-3/56*d^
2*(d*x^3+c)^(1/2)/c^2/x+3/56*d^(7/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2)))+1/56*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x
+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2
/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)-3/112*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*Ellip
ticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1
/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*
(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$,
Rules used = {485, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{112c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{9\sqrt{3}\,d^{7/3}\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}-\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]
```

```
[Out] -1/56*Sqrt[c + d*x^3]/x^7 - (75*d*Sqrt[c + d*x^3])/(1792*c*x^4) - (3*d^2*Sq
rt[c + d*x^3])/(56*c^2*x) + (3*d^(7/3)*Sqrt[c + d*x^3])/(56*c^2*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(11/6)) + (9*d^(7/3)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(11/6)) - (9*d
```

```
^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(11/6)) - (3*3^(1/4)*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(112*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(3/4)*d^(7/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*c^
(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{56x^7}+\frac{\int \frac{75c^2d+\frac{123}{2}cd^2x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{56c}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{\int \frac{-768c^3d^2-\frac{375}{2}c^2d^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1792c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{\int \frac{x(5340c^4d^3-384c^3d^4x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{\int \left(\frac{384c^3d^3x}{\sqrt{c+dx^3}}+\frac{2268c^4d^3x}{(8c-dx^3)\sqrt{c+dx^3}}\right.}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{(3d^3)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{112c^2}+\frac{(81d^3)\int}{}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}-\frac{(27d^2)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+d}}}{2048c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.07, size = 167, normalized size = 0.25

$$\frac{6675cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\left(5c(32c^3+107c^2dx^3+171cd^2x^6+96d^3x^9)+6d^4x^{12}\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{286720c^3x^7\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]
```

```
[Out] (6675*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 + 107*c^2*d*x^3 + 171*c*d^2*x^6 + 96*d^3
*x^9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)
/c), (d*x^3)/(8*c)]))/(286720*c^3*x^7*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 2306, normalized size = 3.42

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	903
default	Expression too large to display	2306

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*d/c^2*(-1/4*c*(d*x^3+c)^(1/2)/x^4-11/8*d*(d*x^3+c)^(1/2)/x-9/8*I*d*3^(
1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))+1/8/c*(-1/7*c*(d*x^3+c)^(1/2)/x^7-1
7/56*d*(d*x^3+c)^(1/2)/x^4-27/112/c*d^2*(d*x^3+c)^(1/2)/x-9/112*I*d^2/c*3^(
1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*
```

```
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-1/512/c^3*d^3*(2/7*x^2*(d*x^3+c)^(1
/2)-44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3
))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*
sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/
d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))+1/512/c^3*d^2*(-c*(d*x^3+c)^(1/2)/x+2/7*(d*x^3+c)^(1/2)*d*x^2
-9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.91, size = 2634, normalized size = 3.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] -1/28672*(84*sqrt(3)*(d^14/c^11)^(1/6)*c^2*x^7*arctan(1/9*((9*sqrt(3)*(d^14
/c^11)^(1/6)*c^2*d^22*x^5 - sqrt(3)*(c^9*d^13*x^6 - 40*c^10*d^12*x^3 - 32*c
^11*d^11)*(d^14/c^11)^(5/6) + 3*sqrt(3)*(5*c^6*d^17*x^4 + 8*c^7*d^16*x)*sqr
t(d^14/c^11))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^8*d^4*x^5 + c^9*d^3*x^2)*(d^
14/c^11)^(2/3) + 12*sqrt(3)*(c^4*d^9*x^6 - c^5*d^8*x^3 - 2*c^6*d^7)*(d^14/c
^11)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 + 4*c^2*d^12*x) + sqrt(d*x^
3 + c)*(sqrt(3)*(c^9*d^2*x^6 + 32*c^10*d*x^3 + 40*c^11)*(d^14/c^11)^(5/6) +
 3*sqrt(3)*(7*c^6*d^6*x^4 + 4*c^7*d^5*x)*sqrt(d^14/c^11) + 9*sqrt(3)*(c^2*d
^11*x^5 + 2*c^3*d^10*x^2)*(d^14/c^11)^(1/6)))*sqrt((d^25*x^9 - 276*c*d^24*x
^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^8*d^15*x^7 - 52*c^9*d^14*x^4
 - 80*c^10*d^13*x)*(d^14/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^12*x^5
 + c^11*d^11*x^2)*(d^14/c^11)^(5/6) - 4*(c^6*d^17*x^6 + 41*c^7*d^16*x^3 + 4
0*c^8*d^15)*sqrt(d^14/c^11) - (c^2*d^22*x^7 - 28*c^3*d^21*x^4 - 272*c^4*d^2
0*x)*(d^14/c^11)^(1/6)) + 18*(c^4*d^20*x^8 + 20*c^5*d^19*x^5 - 8*c^6*d^18*x
^2)*(d^14/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))
/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*x)) + 84*sqrt(3)*(d^14/c^11)^(1/6)*c
^2*x^7*arctan(1/9*((9*sqrt(3)*(d^14/c^11)^(1/6)*c^2*d^22*x^5 - sqrt(3)*(c^9
*d^13*x^6 - 40*c^10*d^12*x^3 - 32*c^11*d^11)*(d^14/c^11)^(5/6) + 3*sqrt(3)*
(5*c^6*d^17*x^4 + 8*c^7*d^16*x)*sqrt(d^14/c^11))*sqrt(d*x^3 + c) - (18*sqrt
(3)*(c^8*d^4*x^5 + c^9*d^3*x^2)*(d^14/c^11)^(2/3) + 12*sqrt(3)*(c^4*d^9*x^6
 - c^5*d^8*x^3 - 2*c^6*d^7)*(d^14/c^11)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d
^13*x^4 + 4*c^2*d^12*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^9*d^2*x^6 + 32*c^10*d
*x^3 + 40*c^11)*(d^14/c^11)^(5/6) + 3*sqrt(3)*(7*c^6*d^6*x^4 + 4*c^7*d^5*x)
*sqrt(d^14/c^11) + 9*sqrt(3)*(c^2*d^11*x^5 + 2*c^3*d^10*x^2)*(d^14/c^11)^(1
/6)))*sqrt((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 -
 18*(c^8*d^15*x^7 - 52*c^9*d^14*x^4 - 80*c^10*d^13*x)*(d^14/c^11)^(2/3) - 6
*sqrt(d*x^3 + c)*(24*(c^10*d^12*x^5 + c^11*d^11*x^2)*(d^14/c^11)^(5/6) - 4*
(c^6*d^17*x^6 + 41*c^7*d^16*x^3 + 40*c^8*d^15)*sqrt(d^14/c^11) - (c^2*d^22*
x^7 - 28*c^3*d^21*x^4 - 272*c^4*d^20*x)*(d^14/c^11)^(1/6)) + 18*(c^4*d^20*x
^8 + 20*c^5*d^19*x^5 - 8*c^6*d^18*x^2)*(d^14/c^11)^(1/3))/(d^3*x^9 - 24*c*d
^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*x
)) + 1536*d^(5/2)*x^7*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*
c/d, x)) + 21*(d^14/c^11)^(1/6)*c^2*x^7*log(43046721*(d^25*x^9 - 276*c*d^24
*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^8*d^15*x^7 - 52*c^9*d^14*x
```

```
^4 - 80*c^10*d^13*x)*(d^14/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^12*x
^5 + c^11*d^11*x^2)*(d^14/c^11)^(5/6) - 4*(c^6*d^17*x^6 + 41*c^7*d^16*x^3 +
 40*c^8*d^15)*sqrt(d^14/c^11) - (c^2*d^22*x^7 - 28*c^3*d^21*x^4 - 272*c^4*d
^20*x)*(d^14/c^11)^(1/6)) + 18*(c^4*d^20*x^8 + 20*c^5*d^19*x^5 - 8*c^6*d^18
*x^2)*(d^14/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)
) - 21*(d^14/c^11)^(1/6)*c^2*x^7*log(43046721*(d^25*x^9 - 276*c*d^24*x^6 -
1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^8*d^15*x^7 - 52*c^9*d^14*x^4 - 80
*c^10*d^13*x)*(d^14/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^10*d^12*x^5 + c^
11*d^11*x^2)*(d^14/c^11)^(5/6) - 4*(c^6*d^17*x^6 + 41*c^7*d^16*x^3 + 40*c^8
*d^15)*sqrt(d^14/c^11) - (c^2*d^22*x^7 - 28*c^3*d^21*x^4 - 272*c^4*d^20*x)*
(d^14/c^11)^(1/6)) + 18*(c^4*d^20*x^8 + 20*c^5*d^19*x^5 - 8*c^6*d^18*x^2)*(
d^14/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 42*
(d^14/c^11)^(1/6)*c^2*x^7*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^
12*x^3 + 640*c^3*d^11 + 18*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x)
*(d^14/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^1
4/c^11)^(5/6) + (7*c^6*d^6*x^6 + 152*c^7*d^5*x^3 + 64*c^8*d^4)*sqrt(d^14/c^
11) + (c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x)*(d^14/c^11)^(1/6)) +
 18*(c^4*d^9*x^8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^2)*(d^14/c^11)^(1/3))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 42*(d^14/c^11)^(1/6)*c^2*
x^7*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11
+ 18*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x)*(d^14/c^11)^(2/3) - 6
*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^14/c^11)^(5/6) + (7*c^6
*d^6*x^6 + 152*c^7*d^5*x^3 + 64*c^8*d^4)*sqrt(d^14/c^11) + (c^2*d^11*x^7 +
80*c^3*d^10*x^4 + 160*c^4*d^9*x)*(d^14/c^11)^(1/6)) + 18*(c^4*d^9*x^8 + 38*
c^5*d^8*x^5 + 64*c^6*d^7*x^2)*(d^14/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) + 16*(96*d^2*x^6 + 75*c*d*x^3 + 32*c^2)*sqrt(d*x^
3 + c))/(c^2*x^7)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx-\int \frac{dx^3\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**8 + d*x**11), x) - Integral(d*x**3*sq
rt(c + d*x**3)/(-8*c*x**8 + d*x**11), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{x^8\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)), x)
```

3.308 $\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=90

$$-\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c(c+dx^3)^{3/2}}{3d^4}-\frac{2(c+dx^3)^{5/2}}{15d^4}+\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

```
[Out] -4/3*c*(d*x^3+c)^(3/2)/d^4-2/15*(d*x^3+c)^(5/2)/d^4+1024/9*c^(5/2)*arctanh(
1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4-38*c^2*(d*x^3+c)^(1/2)/d^4
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {457, 90, 65, 212}

$$\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}-\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c(c+dx^3)^{3/2}}{3d^4}-\frac{2(c+dx^3)^{5/2}}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-38*c^2*Sqrt[c + d*x^3])/d^4 - (4*c*(c + d*x^3)^(3/2))/(3*d^4) - (2*(c + d
*x^3)^(5/2))/(15*d^4) + (1024*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/(9*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{57c^2}{d^3\sqrt{c+dx}} + \frac{512c^3}{d^3(8c-dx)\sqrt{c+dx}} - \frac{6c\sqrt{c+dx}}{d^3} - \frac{(c+dx}{d^3}\right.\right.$$

$$= -\frac{38c^2\sqrt{c+dx^3}}{d^4} - \frac{4c(c+dx^3)^{3/2}}{3d^4} - \frac{2(c+dx^3)^{5/2}}{15d^4} + \frac{(512c^3)\,\text{Subst}\left(\int \frac{}{(8c-dx)}\right.}{3d^3}$$

$$= -\frac{38c^2\sqrt{c+dx^3}}{d^4} - \frac{4c(c+dx^3)^{3/2}}{3d^4} - \frac{2(c+dx^3)^{5/2}}{15d^4} + \frac{(1024c^3)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\right.}{3d^4}$$

$$= -\frac{38c^2\sqrt{c+dx^3}}{d^4} - \frac{4c(c+dx^3)^{3/2}}{3d^4} - \frac{2(c+dx^3)^{5/2}}{15d^4} + \frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+}}{3\sqrt{}}\right.}{9d^4}$$

Mathematica [A]
time = 0.06, size = 69, normalized size = 0.77

$$\frac{-6\sqrt{c+dx^3}\,(296c^2+12cdx^3+d^2x^6)+5120c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{45d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-6*Sqrt[c + d*x^3]*(296*c^2 + 12*c*d*x^3 + d^2*x^6) + 5120*c^(5/2)*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 528, normalized size = 5.87 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/15/d*x^6*(d*x^3+c)^(1/2)-8/45*c/d^2*x^3*(d*x^3+c)^(1/2)+16/45*c^2*(
d*x^3+c)^(1/2)/d^3)-8/d^2*c*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2
)/d^2)-128/3*c^2*(d*x^3+c)^(1/2)/d^4-512/27*I*c^2/d^6*2^(1/2)*sum((-c*d^2)^
(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d
^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)
^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.50, size = 82, normalized size = 0.91

$$-\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+30\,(dx^3+c)^{\frac{3}{2}}c+855\,\sqrt{dx^3+c}\,c^2\right)}{45\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/45*(1280*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 30*(d*x^3 + c)^(3/2)*c + 855*sqrt(d*x^3 +
 c)*c^2)/d^4
```

Fricas [A]
time = 3.16, size = 146, normalized size = 1.62

$$\left[\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(d^2x^6+12\,cdx^3+296\,c^2)\sqrt{dx^3+c}\right)}{45\,d^4},-\frac{2\left(2560\,\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3c}\right)+3\,(d^2x^6+12\,cdx^3+296\,c^2)\sqrt{dx^3+c}\right)}{45\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/45*(1280*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - 3*(d^2*x^6 + 12*c*d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4, -2/45*(25
60*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^2*x^6 + 12*c*
d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^{11}}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**11/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.20, size = 82, normalized size = 0.91

$$-\frac{1024\,c^{3}\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d^{4}}-\frac{2\left((dx^{3}+c)^{\frac{5}{2}}d^{16}+10\,(dx^{3}+c)^{\frac{3}{2}}cd^{16}+285\sqrt{dx^{3}+c}\,c^{2}d^{16}\right)}{15\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1024/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/15*((d*
x^3 + c)^(5/2)*d^16 + 10*(d*x^3 + c)^(3/2)*c*d^16 + 285*sqrt(d*x^3 + c)*c^2
*d^16)/d^20
```

Mupad [B]
time = 3.22, size = 98, normalized size = 1.09

$$\frac{512\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^{3}+6\,\sqrt{c}\,\sqrt{d\,x^{3}+c}}{8\,c-d\,x^{3}}\right)}{9\,d^{4}}-\frac{592\,c^{2}\,\sqrt{d\,x^{3}+c}}{15\,d^{4}}-\frac{2\,x^{6}\,\sqrt{d\,x^{3}+c}}{15\,d^{2}}-\frac{8\,c\,x^{3}\,\sqrt{d\,x^{3}+c}}{5\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (512*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(9*d^4) - (592*c^2*(c + d*x^3)^(1/2))/(15*d^4) - (2*x^6*(c + d*x^3)^(1/2
))/(15*d^2) - (8*c*x^3*(c + d*x^3)^(1/2))/(5*d^3)
```

3.309 $\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=71

$$-\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^3+128/9*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))
/d^3-14/3*c*(d*x^3+c)^(1/2)/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {457, 90, 65, 212}

$$\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}-\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2(c+dx^3)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-14*c*Sqrt[c + d*x^3])/(3*d^3) - (2*(c + d*x^3)^(3/2))/(9*d^3) + (128*c^(3
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{7c}{d^2\sqrt{c+dx}}+\frac{64c^2}{d^2(8c-dx)\sqrt{c+dx}}-\frac{\sqrt{c+dx}}{d^2}\right)dx,x,x^3\right)\\
&= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{(64c^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3d^2}\\
&= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{(128c^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3d^3}\\
&= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 0.82

$$\frac{-2\sqrt{c+dx^3}\,(22c+dx^3)+128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(22*c + d*x^3) + 128*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(9*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 465, normalized size = 6.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2)/d^2)-16/3*c*(d*x^3+c)
^(1/2)/d^3-64/27*I*c/d^5*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2
*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.51, size = 66, normalized size = 0.93

$$-\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}+21\sqrt{dx^3+c}\,c\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(32*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + (d*x^3 + c)^(3/2) + 21*sqrt(d*x^3 + c)*c)/d^3
```

Fricas [A]
time = 2.66, size = 121, normalized size = 1.70

$$\left[\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(dx^3+22\,c)\sqrt{dx^3+c}\right)}{9\,d^3},\,-\frac{2\left(64\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+(dx^3+22\,c)\sqrt{dx^3+c}\right)}{9\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(32*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3, -2/9*(64*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^8}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**8/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.88, size = 65, normalized size = 0.92

$$-\frac{128\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d^3}-\frac{2\left((dx^3+c)^{\frac{3}{2}}d^6+21\sqrt{dx^3+c}\,cd^6\right)}{9\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -128/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/9*((d*x^
3 + c)^(3/2)*d^6 + 21*sqrt(d*x^3 + c)*c*d^6)/d^9
```

Mupad [B]
time = 3.39, size = 78, normalized size = 1.10

$$\frac{64\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3}-\frac{44\,c\,\sqrt{d\,x^3+c}}{9\,d^3}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (64*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/(9*d^3) - (44*c*(c + d*x^3)^(1/2))/(9*d^3) - (2*x^3*(c + d*x^3)^(1/2))/(9
*d^2)
```

3.310 $\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=52

$$-\frac{2\sqrt{c+dx^3}}{3d^2}+\frac{16\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

```
[Out] 16/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^2-2/3*(d*x^3+c)^(1/2)/d
^2
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {457, 81, 65, 212}

$$\frac{16\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}-\frac{2\sqrt{c+dx^3}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d^2) + (16*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(9*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= -\frac{2\sqrt{c+dx^3}}{3d^2}+\frac{(8c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}\\ &= -\frac{2\sqrt{c+dx^3}}{3d^2}+\frac{(16c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^2}\\ &= -\frac{2\sqrt{c+dx^3}}{3d^2}+\frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.94

$$-\frac{2\left(3\sqrt{c+dx^3}-8\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
[Out] (-2*(3*Sqrt[c + d*x^3] - 8*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(
9*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.35, size = 425, normalized size = 8.17 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(d*x^3+c)^(1/2)/d^2-8/27*I/d^4*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*
x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
```

```
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d
^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*
d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.49, size = 56, normalized size = 1.08

$$-\frac{2\left(4\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\sqrt{dx^3+c}\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(4*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 3*sqrt(d*x^3 + c))/d^2
```

Fricas [A]
time = 2.17, size = 103, normalized size = 1.98

$$\left[\frac{2\left(4\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-3\sqrt{dx^3+c}\right)}{9\,d^2},-\frac{2\left(8\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(4*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) - 3*sqrt(d*x^3 + c))/d^2, -2/9*(8*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sq
rt(-c)/c) + 3*sqrt(d*x^3 + c))/d^2]
```

Sympy [A]
time = 6.65, size = 61, normalized size = 1.17

$$\begin{cases} \frac{2\left(-\frac{8c\,\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3d}\right)}{d} & \text{for}\ d\neq 0\\ \frac{x^6}{48c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-8*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c)) - sqr
t(c + d*x**3)/(3*d))/d, Ne(d, 0)), (x**6/(48*c**(3/2)), True))
```

Giac [A]
time = 0.90, size = 48, normalized size = 0.92

$$-\frac{2\left(\frac{8\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d}+\frac{3\sqrt{dx^3+c}}{d}\right)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*(8*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 3*sqrt(d*x^3
+ c)/d)/d
```

Mupad [B]
time = 3.27, size = 60, normalized size = 1.15

$$\frac{8\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2}-\frac{2\,\sqrt{d\,x^3+c}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (8*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/(9*d^2) - (2*(c + d*x^3)^(1/2))/(3*d^2)
```

3.311 $\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=33

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

[Out] 2/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d/c^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {455, 65, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Antiderivative was successfully verified.

[In] Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 413, normalized size = 12.52

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{}}{}\right.}{}$

elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d
^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.51, size = 42, normalized size = 1.27

$$-\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/(sqrt
(c)*d)
```

Fricas [A]
time = 2.95, size = 78, normalized size = 2.36

$$\left[\frac{\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)}{9\sqrt{c}\,d}, -\frac{2\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)}{9\,cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/9*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))/(sqrt(c)
*d), -2/9*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)/(c*d)]
```

Sympy [A]
time = 3.92, size = 32, normalized size = 0.97

$$-\frac{2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c))
```

Giac [A]
time = 0.80, size = 27, normalized size = 0.82

$$-\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d)
```

Mupad [B]
time = 3.23, size = 45, normalized size = 1.36

$$\frac{\ln\left(\frac{10\,c+d\,x^3+6\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(9*c^(1/2)*
d)
```

3.312 $\int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

```
[Out] 1/36*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/12*arctanh((d*x^3+c)^(1
/2)/c^(1/2))/c^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 88, 65, 214, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(36*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]
/Sqrt[c]]/(12*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}+\frac{d\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}\\
&= \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{12c}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{12cd}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.88

$$
\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(36*c^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 433, normalized size = 7.47

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right)}{}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/216*I/d^2/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/
3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^
2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*d-8*c))-1/12*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x), x)
```

Fricas [A]
time = 3.59, size = 139, normalized size = 2.40

$$\left[\frac{\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{72c^2}, \frac{3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)}{36c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/72*(sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)
) + 3*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, 1/36
*(3*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*arctan(1/3*sqrt(
d*x^3 + c)*sqrt(-c)/c))/c^2]
```

Sympy [A]
time = 3.95, size = 58, normalized size = 1.00

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{36c\sqrt{-c}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(36*c*sqrt(-c)) + atan(sqrt(c + d*x**3
)/sqrt(-c))/(12*c*sqrt(-c))
```

Giac [A]
time = 0.96, size = 54, normalized size = 0.93

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}\,c}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{36\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/36*arctan(1/3*sqrt(d
*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

Mupad [B]
time = 3.28, size = 47, normalized size = 0.81

$$-\frac{3\operatorname{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)-\operatorname{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{3\sqrt{c^3}}\right)}{36\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] -(3*atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)) - atanh((c*(c + d*x^3)^(1/2))/
(3*(c^3)^(1/2))))/(36*(c^3)^(1/2))
```

3.313 $\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=81

$$-\frac{\sqrt{c+dx^3}}{24c^2x^3}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

```
[Out] 1/288*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/32*d*arctanh((d*x^3+
c)^(1/2)/c^(1/2))/c^(5/2)-1/24*(d*x^3+c)^(1/2)/c^2/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 105, 162, 65, 214, 212}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}-\frac{\sqrt{c+dx^3}}{24c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4 (8c - dx^3)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{\text{Subst}\left(\int \frac{3cd-\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{64c^2} + \frac{d^2\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c-}}\right.}{192c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{32c^2} + \frac{d\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,\right.}{96c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

Mathematica [A]

time = 0.07, size = 81, normalized size = 1.00

$$-\frac{\sqrt{c+dx^3}}{24c^2x^3}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.41, size = 477, normalized size = 5.89

method	result
risch	$-\frac{\sqrt{d\,x^3+c}}{24c^2x^3}-\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+}{d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}\right.}{}$

default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right)}{}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1728*I/c^3/d*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3
))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2
*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*d-8*c))+1/8/c*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh
((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-1/96*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/
c^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^4), x)
```

Fricas [A]
time = 3.16, size = 184, normalized size = 2.27

$$\left[\frac{\sqrt{c}\,dx^3\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\,\sqrt{c}\,dx^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-24\,\sqrt{dx^3+c}\,c}{576\,c^3x^3},-\frac{9\,\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)+\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+12\,\sqrt{dx^3+c}\,c}{288\,c^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/576*(sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^
3) - 24*sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/288*(9*sqrt(-c)*d*x^3*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) + sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)
/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^4\sqrt{c+dx^3}+dx^7\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)

[Out] -Integral(1/(-8*c*x**4*sqrt(c + d*x**3) + d*x**7*sqrt(c + d*x**3)), x)

Giac [A]
time = 0.75, size = 73, normalized size = 0.90

$$-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{32\,\sqrt{-c}\,c^2}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{288\,\sqrt{-c}\,c^2}-\frac{\sqrt{dx^3+c}}{24\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -1/32*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/288*d*arctan(1/
3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/24*sqrt(d*x^3 + c)/(c^2*x^3)

Mupad [B]
time = 3.42, size = 73, normalized size = 0.90

$$\frac{d\,\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{32\,\sqrt{c^5}}+\frac{d\,\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)}{288\,\sqrt{c^5}}-\frac{\sqrt{d\,x^3+c}}{24\,c^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (d*atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2)))/(32*(c^5)^(1/2)) + (d*atanh(
(c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(288*(c^5)^(1/2)) - (c + d*x^3)^(
1/2)/(24*c^2*x^3)
```

3.314 $\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{c+dx^3}}{48c^2x^6}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}$$

```
[Out] 1/2304*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-7/256*d^2*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/48*(d*x^3+c)^(1/2)/c^2/x^6+5/192*d*(d*x^3
+c)^(1/2)/c^3/x^3
```

Rubi [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {457, 105, 156, 162, 65, 214, 212}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}-\frac{\sqrt{c+dx^3}}{48c^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/48*Sqrt[c + d*x^3]/(c^2*x^6) + (5*d*Sqrt[c + d*x^3])/(192*c^3*x^3) + (d^
2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]])/(256*c^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} - \frac{\text{Subst}\left(\int \frac{10cd-\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{48c^2}$$
$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{\text{Subst}\left(\int \frac{42c^2d^2-5cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^4}$$
$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{(7d^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{512c^3} + \frac{d^3\text{S}}{}$$
$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{(7d)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{256c^3} + \frac{d}{}$$
$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}} - \frac{7d^2\tanh^{-1}\left(\sqrt{\right.}{256c^{7/}}$$

Mathematica [**A**]
time = 0.12, size = 95, normalized size = 0.89

$$\frac{\sqrt{c+dx^3}\,(-4c+5dx^3)}{192c^3x^6} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}} - \frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[c + d*x^3]*(-4*c + 5*d*x^3))/(192*c^3*x^6) + (d^2*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]
)/(256*c^(7/2))
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 540, normalized size = 5.05

method	result

risch

$$-\frac{\sqrt{d\,x^{3}+c}\,\left(-5d\,x^{3}+4c\right)}{192c^{3}x^{6}}+\frac{d^{2}\left(-\frac{7\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\cdots\right.}{\cdots}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

default

$$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6c\,x^{6}}+\frac{d\sqrt{d\,x^{3}+c}}{4c^{2}x^{3}}-\frac{d^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4c^{\frac{5}{2}}}}{8c}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\cdots\right.}{\cdots}}\cdots}{\cdots}\right)}{\cdots}$$

elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/6*(d*x^3+c)^(1/2)/c/x^6+1/4*d*(d*x^3+c)^(1/2)/c^2/x^3-1/4*d^2*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-1/13824*I/c^4*2^(1/2)*sum((-c*d^2)^(
1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^
2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*
d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^
(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64*d/c^2
*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
)-1/768*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^7), x)
```

Fricas [A]
time = 3.84, size = 217, normalized size = 2.03

$$\left[\frac{\sqrt{c}\,d^2x^6\log\left(\frac{dx^3+6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+63\,\sqrt{c}\,d^2x^6\log\left(\frac{dx^3-2\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)+24\,(5\,cdx^3-4\,c^2)\sqrt{dx^3+c}}{4608\,c^4x^6}, \frac{63\,\sqrt{-c}\,d^2x^6\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-\sqrt{-c}\,d^2x^6\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+12\,(5\,cdx^3-4\,c^2)\sqrt{dx^3+c}}{2304\,c^4x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4608*(sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) + 63*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
*c)/x^3) + 24*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^4*x^6), 1/2304*(63*sq
rt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*d^2*x^6*arctan
(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/
(c^4*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^7\sqrt{c+dx^3}+dx^{10}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**7*sqrt(c + d*x**3) + d*x**10*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.07, size = 101, normalized size = 0.94

$$\frac{7\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\,\sqrt{-c}\,c^3}-\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2304\,\sqrt{-c}\,c^3}+\frac{5\,(dx^3+c)^{\frac{3}{2}}d^2-9\,\sqrt{dx^3+c}\,cd^2}{192\,c^3d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 7/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2304*d^2*arct
an(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/192*(5*(d*x^3 + c)^(3/2
)*d^2 - 9*sqrt(d*x^3 + c)*c*d^2)/(c^3*d^2*x^6)
```

Mupad [B]
time = 3.50, size = 94, normalized size = 0.88

$$\frac{d^2\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)}{2304\,\sqrt{c^7}}-\frac{7\,d^2\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)}{256\,\sqrt{c^7}}-\frac{3\,\sqrt{d\,x^3+c}}{64\,c^2\,x^6}+\frac{5\,{\left(d\,x^3+c\right)}^{3/2}}{192\,c^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (d^2*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2))))/(2304*(c^7)^(1/2)) - (
7*d^2*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(256*(c^7)^(1/2)) - (3*(c
 + d*x^3)^(1/2))/(64*c^2*x^6) + (5*(c + d*x^3)^(3/2))/(192*c^3*x^6)
```

3.315 $\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=630

$$-\frac{2x^2\sqrt{c+dx^3}}{7d^2}-\frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{8/3}}+\frac{32c^{7/6}\tanh^{-1}\Bigg(}{9\ldots}$$

```
[Out] 32/9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
8/3)-32/9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32/9*c^(7/6)
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^(1/2
)-2/7*x^2*(d*x^3+c)^(1/2)/d^2-104/7*c*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))-104/21*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+
c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)
*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+52/7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)
*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3
)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^
(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {490, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{104\sqrt{2}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{52\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{32c^{7/6}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{8/3}}+\frac{32c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}-\frac{32c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}-\frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{2x^2\sqrt{c+dx^3}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/(7*d^2) - (104*c*Sqrt[c + d*x^3])/(7*d^(8/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (32*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1
/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(8/3)) + (32*c^(7/6)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(8/3)) - (32*
c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^(8/3)) + (52*3^(1/4)*Sqr
t[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
```

```
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(7*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (104*Sqrt[2]*c^(4/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(
8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2x^2\sqrt{c+dx^3}}{7d^2} + \frac{2\int \frac{x(16c^2+26cdx^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{7d^2}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} + \frac{2\int \left(-\frac{26cx}{\sqrt{c+dx^3}} + \frac{224c^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{7d^2}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{(52c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d^2} + \frac{(64c^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^2}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{(16c)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^3} - \frac{(52c)\int \frac{\left(1-\sqrt{3}\right)\ldots}{\sqrt{c+\ldots}}}{7d^{7/\ldots}}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{52\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{4/3}\big(\ldots}{\ldots}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\ldots}{\sqrt{c\ldots}}\right.}{3\sqrt{3}\,d^{8/3}\ldots}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\ldots}{\sqrt{c\ldots}}\right.}{3\sqrt{3}\,d^{8/3}\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.07, size = 130, normalized size = 0.21

$$\frac{x^2\left(-20(c+dx^3)+20c\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+13dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{70d^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*(-20*(c + d*x^3) + 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), (d*x^3)/(8*c)] + 13*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(70*d^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 1308, normalized size = 2.08

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1308

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/7*x^2*(d*x^3+c)^(1/2)/d+8/21*I*c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2))))+16/3*I/d^3*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-
c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-
c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
```

```
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-64/27*I
*c/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3
))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2
*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 28.19, size = 3764, normalized size = 5.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/189*(112*sqrt(3)*d^3*(c^7/d^16)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c^7*d^1
6*x^16 + 784*c^8*d^15*x^13 + 7680*c^9*d^14*x^10 + 10752*c^10*d^13*x^7 + 409
6*c^11*d^12*x^4)*(c^7/d^16)^(2/3) + 36*sqrt(3)*(c^9*d^11*x^17 + 1772*c^10*d
^10*x^14 + 42592*c^11*d^9*x^11 + 96256*c^12*d^8*x^8 + 69632*c^13*d^7*x^5 +
16384*c^14*d^6*x^2)*(c^7/d^16)^(1/3) + sqrt(3)*(c^11*d^6*x^18 + 9456*c^12*d
^5*x^15 + 749184*c^13*d^4*x^12 + 3017216*c^14*d^3*x^9 + 3489792*c^15*d^2*x^
6 + 1572864*c^16*d*x^3 + 262144*c^17) + 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*
c^6*d^18*x^14 - 14440*c^7*d^17*x^11 - 24576*c^8*d^16*x^8 - 16384*c^9*d^15*x
^5 - 4096*c^10*d^14*x^2)*(c^7/d^16)^(5/6) + 18*sqrt(3)*(c^8*d^13*x^15 - 111
2*c^9*d^12*x^12 + 7296*c^10*d^11*x^9 + 11776*c^11*d^10*x^6 + 4096*c^12*d^9*
x^3)*sqrt(c^7/d^16) + sqrt(3)*(c^10*d^8*x^16 - 4768*c^11*d^7*x^13 + 362752*
c^12*d^6*x^10 + 709120*c^13*d^5*x^7 + 413696*c^14*d^4*x^4 + 65536*c^15*d^3*
x)*(c^7/d^16)^(1/6)) - 2*(324*sqrt(3)*(d^19*x^16 - 1858*c*d^18*x^13 - 4176*
c^2*d^17*x^10 - 3584*c^3*d^16*x^7 - 1024*c^4*d^15*x^4)*(c^7/d^16)^(5/6) + 1
```

8*sqrt(3)*(c^2*d^14*x^17 - 5290*c^3*d^13*x^14 - 21152*c^4*d^12*x^11 - 47744 *c^5*d^11*x^8 - 37888*c^6*d^10*x^5 - 8192*c^7*d^9*x^2)*sqrt(c^7/d^16) + sqr t(3)*(c^4*d^9*x^18 - 7698*c^5*d^8*x^15 - 1664688*c^6*d^7*x^12 - 5524864*c^7 *d^6*x^9 - 6223872*c^8*d^5*x^6 - 2703360*c^9*d^4*x^3 - 327680*c^10*d^3)*(c^ 7/d^16)^(1/6) + 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c*d^16*x^15 + 37352*c^2*d^15* x^12 - 230336*c^3*d^14*x^9 - 515072*c^4*d^13*x^6 - 286720*c^5*d^12*x^3 - 32 768*c^6*d^11)*(c^7/d^16)^(2/3) + 108*sqrt(3)*(53*c^4*d^10*x^13 + 1320*c^5*d ^9*x^10 + 1536*c^6*d^8*x^7 + 512*c^7*d^7*x^4)*(c^7/d^16)^(1/3) + 6*sqrt(3)* (37*c^6*d^5*x^14 + 28912*c^7*d^4*x^11 + 43584*c^8*d^3*x^8 + 20992*c^9*d^2*x ^5 + 4096*c^10*d*x^2)))*sqrt((18*c^12*d^2*x^8 + 360*c^13*d*x^5 - 144*c^14*x ^2 + (c^7*d^13*x^9 - 276*c^8*d^12*x^6 - 1608*c^9*d^11*x^3 - 1088*c^10*d^10) *(c^7/d^16)^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^15*x^7 - 28*c^7*d^14*x^4 - 27 2*c^8*d^13*x)*(c^7/d^16)^(5/6) - 24*(c^9*d^9*x^5 + c^10*d^8*x^2)*sqrt(c^7/d ^16) + 4*(c^11*d^4*x^6 + 41*c^12*d^3*x^3 + 40*c^13*d^2)*(c^7/d^16)^(1/6)) - 18*(c^10*d^7*x^7 - 52*c^11*d^6*x^4 - 80*c^12*d^5*x)*(c^7/d^16)^(1/3))/(d^3 *x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^11*d^6*x^18 - 14952*c^1 2*d^5*x^15 + 2872896*c^13*d^4*x^12 + 7330304*c^14*d^3*x^9 + 6696960*c^15*d^ 2*x^6 + 2457600*c^16*d*x^3 + 262144*c^17)) - 112*sqrt(3)*d^3*(c^7/d^16)^(1/ 6)*arctan(-1/3*(324*sqrt(3)*(3*c^7*d^16*x^16 + 784*c^8*d^15*x^13 + 7680*c^9 *d^14*x^10 + 10752*c^10*d^13*x^7 + 4096*c^11*d^12*x^4)*(c^7/d^16)^(2/3) + 3 6*sqrt(3)*(c^9*d^11*x^17 + 1772*c^10*d^10*x^14 + 42592*c^11*d^9*x^11 + 9625 6*c^12*d^8*x^8 + 69632*c^13*d^7*x^5 + 16384*c^14*d^6*x^2)*(c^7/d^16)^(1/3) + sqrt(3)*(c^11*d^6*x^18 + 9456*c^12*d^5*x^15 + 749184*c^13*d^4*x^12 + 3017 216*c^14*d^3*x^9 + 3489792*c^15*d^2*x^6 + 1572864*c^16*d*x^3 + 262144*c^17) - 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c^6*d^18*x^14 - 14440*c^7*d^17*x^11 - 24576*c^8*d^16*x^8 - 16384*c^9*d^15*x^5 - 4096*c^10*d^14*x^2)*(c^7/d^16)^(5/6) + 18*sqrt(3)*(c^8*d^13*x^15 - 1112*c^9*d^12*x^12 + 7296*c^10*d^11*x^9 + 11776*c^11*d^10*x^6 + 4096*c^12*d^9*x^3)*sqrt(c^7/d^16) + sqrt(3)*(c^10*d ^8*x^16 - 4768*c^11*d^7*x^13 + 362752*c^12*d^6*x^10 + 709120*c^13*d^5*x^7 + 413696*c^14*d^4*x^4 + 65536*c^15*d^3*x)*(c^7/d^16)^(1/6)) + 2*(324*sqrt(3) *(d^19*x^16 - 1858*c*d^18*x^13 - 4176*c^2*d^17*x^10 - 3584*c^3*d^16*x^7 - 1 024*c^4*d^15*x^4)*(c^7/d^16)^(5/6) + 18*sqrt(3)*(c^2*d^14*x^17 - 5290*c^3*d ^13*x^14 - 21152*c^4*d^12*x^11 - 47744*c^5*d^11*x^8 - 37888*c^6*d^10*x^5 - 8192*c^7*d^9*x^2)*sqrt(c^7/d^16) + sqrt(3)*(c^4*d^9*x^18 - 7698*c^5*d^8*x^1 5 - 1664688*c^6*d^7*x^12 - 5524864*c^7*d^6*x^9 - 6223872*c^8*d^5*x^6 - 2703 360*c^9*d^4*x^3 - 327680*c^10*d^3)*(c^7/d^16)^(1/6) - 6*sqrt(d*x^3 + c)*(sq rt(3)*(7*c*d^16*x^15 + 37352*c^2*d^15*x^12 - 230336*c^3*d^14*x^9 - 515072*c ^4*d^13*x^6 - 286720*c^5*d^12*x^3 - 32768*c^6*d^11)*(c^7/d^16)^(2/3) + 108* sqrt(3)*(53*c^4*d^10*x^13 + 1320*c^5*d^9*x^10 + 1536*c^6*d^8*x^7 + 512*c^7* d^7*x^4)*(c^7/d^16)^(1/3) + 6*sqrt(3)*(37*c^6*d^5*x^14 + 28912*c^7*d^4*x^11 + 43584*c^8*d^3*x^8 + 20992*c^9*d^2*x^5 + 4096*c^10*d*x^2)))*sqrt((18*c^12 *d^2*x^8 + 360*c^13*d*x^5 - 144*c^14*x^2 + (c^7*d^13*x^9 - 276*c^8*d^12*x^6 - 1608*c^9*d^11*x^3 - 1088*c^10*d^10)*(c^7/d^16)^(2/3) - 6*sqrt(d*x^3 + c) *((c^6*d^15*x^7 - 28*c^7*d^14*x^4 - 272*c^8*d^13*x)*(c^7/d^16)^(5/6) - 24*(c^9*d^9*x^5 + c^10*d^8*x^2)*sqrt(c^7/d^16) + 4*(c^11*d^4*x^6 + 41*c^12*d^3*

```
x^3 + 40*c^13*d^2)*(c^7/d^16)^(1/6)) - 18*(c^10*d^7*x^7 - 52*c^11*d^6*x^4 -
 80*c^12*d^5*x)*(c^7/d^16)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
 512*c^3)))/(c^11*d^6*x^18 - 14952*c^12*d^5*x^15 + 2872896*c^13*d^4*x^12 +
7330304*c^14*d^3*x^9 + 6696960*c^15*d^2*x^6 + 2457600*c^16*d*x^3 + 262144*c
^17)) + 28*d^3*(c^7/d^16)^(1/6)*log(4503599627370496/9*(18*c^12*d^2*x^8 + 3
60*c^13*d*x^5 - 144*c^14*x^2 + (c^7*d^13*x^9 - 276*c^8*d^12*x^6 - 1608*c^9*
d^11*x^3 - 1088*c^10*d^10)*(c^7/d^16)^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^15*
x^7 - 28*c^7*d^14*x^4 - 272*c^8*d^13*x)*(c^7/d^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^7}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(x**7/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.316 $\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=601

$$-\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{4\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}}+\frac{4\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{9d^{5/3}}$$

```
[Out] 4/9*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5
/3)-4/9*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-4/9*c^(1/6)*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(5/3)*3^(1/2)-2
*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-2/3*c^(1/3)*(c^(1/
3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+3^(1/
4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {494, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{2\sqrt{2}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{4\sqrt[6]{c}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}}-\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{4\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{9d^{5/3}}-\frac{4\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (4*c^(
1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sq
rt[3]*d^(5/3)) + (4*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt
[c + d*x^3])]])/(9*d^(5/3)) - (4*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
]])/(9*d^(5/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
```

```
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (
2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{d} + \frac{(8c)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d}$$

$$= -\frac{2\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^2} - \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{d^{4/3}} + \frac{\left(2\sqrt[3]{c}\right)\int}{}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^2}{((}}}{d^{5/3}\sqrt{}}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{4\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}} + \frac{4}{}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{4\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}} + \frac{4}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.04, size = 67, normalized size = 0.11

$$\frac{x^5\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{40c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(40*c*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 848, normalized size = 1.41

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-8/27*I/d^4*2^(1/2)*sum(
1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)
*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.67, size = 3529, normalized size = 5.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(4*sqrt(3)*d^2*(c/d^10)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c*d^12*x^16
+ 784*c^2*d^11*x^13 + 7680*c^3*d^10*x^10 + 10752*c^4*d^9*x^7 + 4096*c^5*d^8
*x^4)*(c/d^10)^(2/3) + 36*sqrt(3)*(c*d^9*x^17 + 1772*c^2*d^8*x^14 + 42592*c
^3*d^7*x^11 + 96256*c^4*d^6*x^8 + 69632*c^5*d^5*x^5 + 16384*c^6*d^4*x^2)*(c
/d^10)^(1/3) + sqrt(3)*(c*d^6*x^18 + 9456*c^2*d^5*x^15 + 749184*c^3*d^4*x^1
2 + 3017216*c^4*d^3*x^9 + 3489792*c^5*d^2*x^6 + 1572864*c^6*d*x^3 + 262144*
c^7) + 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c*d^13*x^14 - 14440*c^2*d^12*x^11
 - 24576*c^3*d^11*x^8 - 16384*c^4*d^10*x^5 - 4096*c^5*d^9*x^2)*(c/d^10)^(5/
6) + 18*sqrt(3)*(c*d^10*x^15 - 1112*c^2*d^9*x^12 + 7296*c^3*d^8*x^9 + 11776
*c^4*d^7*x^6 + 4096*c^5*d^6*x^3)*sqrt(c/d^10) + sqrt(3)*(c*d^7*x^16 - 4768*
c^2*d^6*x^13 + 362752*c^3*d^5*x^10 + 709120*c^4*d^4*x^7 + 413696*c^5*d^3*x^
4 + 65536*c^6*d^2*x)*(c/d^10)^(1/6)) - 2*(324*sqrt(3)*(d^14*x^16 - 1858*c*d
^13*x^13 - 4176*c^2*d^12*x^10 - 3584*c^3*d^11*x^7 - 1024*c^4*d^10*x^4)*(c/d
^10)^(5/6) + 18*sqrt(3)*(d^11*x^17 - 5290*c*d^10*x^14 - 21152*c^2*d^9*x^11
- 47744*c^3*d^8*x^8 - 37888*c^4*d^7*x^5 - 8192*c^5*d^6*x^2)*sqrt(c/d^10) +
sqrt(3)*(d^8*x^18 - 7698*c*d^7*x^15 - 1664688*c^2*d^6*x^12 - 5524864*c^3*d^
5*x^9 - 6223872*c^4*d^4*x^6 - 2703360*c^5*d^3*x^3 - 327680*c^6*d^2)*(c/d^10
)^(1/6) + 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*d^12*x^15 + 37352*c*d^11*x^12 - 230
336*c^2*d^10*x^9 - 515072*c^3*d^9*x^6 - 286720*c^4*d^8*x^3 - 32768*c^5*d^7)
*(c/d^10)^(2/3) + 108*sqrt(3)*(53*c*d^8*x^13 + 1320*c^2*d^7*x^10 + 1536*c^3
*d^6*x^7 + 512*c^4*d^5*x^4)*(c/d^10)^(1/3) + 6*sqrt(3)*(37*c*d^5*x^14 + 289
12*c^2*d^4*x^11 + 43584*c^3*d^3*x^8 + 20992*c^4*d^2*x^5 + 4096*c^5*d*x^2)))
*sqrt((18*c^2*d^2*x^8 + 360*c^3*d*x^5 - 144*c^4*x^2 + (c*d^9*x^9 - 276*c^2*
d^8*x^6 - 1608*c^3*d^7*x^3 - 1088*c^4*d^6)*(c/d^10)^(2/3) + 6*sqrt(d*x^3 +
c)*((c*d^10*x^7 - 28*c^2*d^9*x^4 - 272*c^3*d^8*x)*(c/d^10)^(5/6) - 24*(c^2*
d^6*x^5 + c^3*d^5*x^2)*sqrt(c/d^10) + 4*(c^2*d^3*x^6 + 41*c^3*d^2*x^3 + 40*
c^4*d)*(c/d^10)^(1/6)) - 18*(c^2*d^5*x^7 - 52*c^3*d^4*x^4 - 80*c^4*d^3*x)*(
c/d^10)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^6*
x^18 - 14952*c^2*d^5*x^15 + 2872896*c^3*d^4*x^12 + 7330304*c^4*d^3*x^9 + 66
96960*c^5*d^2*x^6 + 2457600*c^6*d*x^3 + 262144*c^7)) - 4*sqrt(3)*d^2*(c/d^1
0)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c*d^12*x^16 + 784*c^2*d^11*x^13 + 7680
*c^3*d^10*x^10 + 10752*c^4*d^9*x^7 + 4096*c^5*d^8*x^4)*(c/d^10)^(2/3) + 36*
sqrt(3)*(c*d^9*x^17 + 1772*c^2*d^8*x^14 + 42592*c^3*d^7*x^11 + 96256*c^4*d^
6*x^8 + 69632*c^5*d^5*x^5 + 16384*c^6*d^4*x^2)*(c/d^10)^(1/3) + sqrt(3)*(c*
```

```
d^6*x^18 + 9456*c^2*d^5*x^15 + 749184*c^3*d^4*x^12 + 3017216*c^4*d^3*x^9 +
3489792*c^5*d^2*x^6 + 1572864*c^6*d*x^3 + 262144*c^7) - 12*sqrt(d*x^3 + c)*
(12*sqrt(3)*(35*c*d^13*x^14 - 14440*c^2*d^12*x^11 - 24576*c^3*d^11*x^8 - 16
384*c^4*d^10*x^5 - 4096*c^5*d^9*x^2)*(c/d^10)^(5/6) + 18*sqrt(3)*(c*d^10*x^
15 - 1112*c^2*d^9*x^12 + 7296*c^3*d^8*x^9 + 11776*c^4*d^7*x^6 + 4096*c^5*d^
6*x^3)*sqrt(c/d^10) + sqrt(3)*(c*d^7*x^16 - 4768*c^2*d^6*x^13 + 362752*c^3*
d^5*x^10 + 709120*c^4*d^4*x^7 + 413696*c^5*d^3*x^4 + 65536*c^6*d^2*x)*(c/d^
10)^(1/6)) + 2*(324*sqrt(3)*(d^14*x^16 - 1858*c*d^13*x^13 - 4176*c^2*d^12*x
^10 - 3584*c^3*d^11*x^7 - 1024*c^4*d^10*x^4)*(c/d^10)^(5/6) + 18*sqrt(3)*(d
^11*x^17 - 5290*c*d^10*x^14 - 21152*c^2*d^9*x^11 - 47744*c^3*d^8*x^8 - 3788
8*c^4*d^7*x^5 - 8192*c^5*d^6*x^2)*sqrt(c/d^10) + sqrt(3)*(d^8*x^18 - 7698*c
*d^7*x^15 - 1664688*c^2*d^6*x^12 - 5524864*c^3*d^5*x^9 - 6223872*c^4*d^4*x^
6 - 2703360*c^5*d^3*x^3 - 327680*c^6*d^2)*(c/d^10)^(1/6) - 6*sqrt(d*x^3 + c
)*(sqrt(3)*(7*d^12*x^15 + 37352*c*d^11*x^12 - 230336*c^2*d^10*x^9 - 515072*
c^3*d^9*x^6 - 286720*c^4*d^8*x^3 - 32768*c^5*d^7)*(c/d^10)^(2/3) + 108*sqrt
(3)*(53*c*d^8*x^13 + 1320*c^2*d^7*x^10 + 1536*c^3*d^6*x^7 + 512*c^4*d^5*x^4
)*(c/d^10)^(1/3) + 6*sqrt(3)*(37*c*d^5*x^14 + 28912*c^2*d^4*x^11 + 43584*c^
3*d^3*x^8 + 20992*c^4*d^2*x^5 + 4096*c^5*d*x^2)))*sqrt((18*c^2*d^2*x^8 + 36
0*c^3*d*x^5 - 144*c^4*x^2 + (c*d^9*x^9 - 276*c^2*d^8*x^6 - 1608*c^3*d^7*x^3
 - 1088*c^4*d^6)*(c/d^10)^(2/3) - 6*sqrt(d*x^3 + c)*((c*d^10*x^7 - 28*c^2*d
^9*x^4 - 272*c^3*d^8*x)*(c/d^10)^(5/6) - 24*(c^2*d^6*x^5 + c^3*d^5*x^2)*sqr
t(c/d^10) + 4*(c^2*d^3*x^6 + 41*c^3*d^2*x^3 + 40*c^4*d)*(c/d^10)^(1/6)) - 1
8*(c^2*d^5*x^7 - 52*c^3*d^4*x^4 - 80*c^4*d^3*x)*(c/d^10)^(1/3))/(d^3*x^9 -
24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^6*x^18 - 14952*c^2*d^5*x^15
+ 2872896*c^3*d^4*x^12 + 7330304*c^4*d^3*x^9 + 6696960*c^5*d^2*x^6 + 245760
0*c^6*d*x^3 + 262144*c^7)) + d^2*(c/d^10)^(1/6)*log(4194304/9*(18*c^2*d^2*x
^8 + 360*c^3*d*x^5 - 144*c^4*x^2 + (c*d^9*x^9 - 276*c^2*d^8*x^6 - 1608*c^3*
d^7*x^3 - 1088*c^4*d^6)*(c/d^10)^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^10*x^7 - 2
8*c^2*d^9*x^4 - 272*c^3*d^8*x)*(c/d^10)^(5/6) - 24*(c^2*d^6*x^5 + c^3*d^5*x
^2)*sqrt(c/d^10) + 4*(c^2*d^3*x^6 + 41*c^3*d^2*x^3 + 40*c^4*d)*(c/d^10)^(1/
6)) - 18*(c^2*d^5*x^7 - 52*c^3*d^4*x^4 - 80*c^4*d^3*x)*(c/d^10)^(1/3))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - d^2*(c/d^10)^(1/6)*log(41
94304/9*(18*c^2*d^2*x^8 + 360*c^3*d*x^5 - 144*c...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**4/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.317 $\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=141

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}\,c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

```
[Out] 1/18*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^(
2/3)-1/18*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(2/3)-1/18*arctan(
c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(2/3)*3^(1/2
)
```

Rubi [A]
time = 0.28, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}\,c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/6*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(Sqrt[
3]*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d
*x^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(18*c^(
5/6)*d^(2/3))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{12cd} + \frac{\int \frac{1+\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{12c^{2/3}\sqrt[3]{d}} - \frac{\sqrt[3]{d}\int \cdots}{\cdots}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{6\sqrt[3]{c}\,d^{2/3}} - \frac{\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^{\cdots}\right)}{12\sqrt[3]{c}} \cdots$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}\,c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\text{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}\,c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\cdots\right)}{18c\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.03, size = 67, normalized size = 0.48

$$\frac{x^2\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(16*c*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 416, normalized size = 2.95

method	result

default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{}}{}\right)}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{}}{}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^
2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_
alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/
```

```
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2459 vs. $2(95) = 190$.
time = 6.49, size = 2459, normalized size = 17.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/54*sqrt(3)*(1/(c^5*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^2*x^5*(1/(c^5*
d^4))^(1/6) - sqrt(3)*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d
^4))^(5/6) + 3*sqrt(3)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(1/(c^5*d^4)))*sqr
t(d*x^3 + c) + (18*sqrt(3)*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^(2/3)
+ 12*sqrt(3)*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^(1/3) + 3*
sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^5
*x^6 + 32*c^5*d^4*x^3 + 40*c^6*d^3)*(1/(c^5*d^4))^(5/6) + 3*sqrt(3)*(7*c^3*
d^3*x^4 + 4*c^4*d^2*x)*sqrt(1/(c^5*d^4)) + 9*sqrt(3)*(c*d^2*x^5 + 2*c^2*d*x
^2)*(1/(c^5*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 -
1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(
2/3) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6
) - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^
3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^
8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) - 1/54*
sqrt(3)*(1/(c^5*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^2*x^5*(1/(c^5*d^4))^
(1/6) - sqrt(3)*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d^4))^(
5/6) + 3*sqrt(3)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(1/(c^5*d^4)))*sqrt(d*x^
3 + c) - (18*sqrt(3)*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^(2/3) + 12*s
qrt(3)*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^(1/3) + 3*sqrt(3
)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^5*x^6 +
 32*c^5*d^4*x^3 + 40*c^6*d^3)*(1/(c^5*d^4))^(5/6) + 3*sqrt(3)*(7*c^3*d^3*x^
4 + 4*c^4*d^2*x)*sqrt(1/(c^5*d^4)) + 9*sqrt(3)*(c*d^2*x^5 + 2*c^2*d*x^2)*(1
/(c^5*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c
```

```
^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) -
 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*
(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7
- 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20
*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
+ 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 1/108*(1/(c
^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18
*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) + 6*sq
rt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) + (7*
c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7
+ 80*c^2*d^2*x^4 + 160*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 38
*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) - 1/108*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318
*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4
+ 32*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 +
 32*c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 6
4*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(
1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1
/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1/
216*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 108
8*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3
) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) -
 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x
^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 +
 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x
^6 + 192*c^2*d*x^3 - 512*c^3)) + 1/216*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 2
76*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4
 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 +
 c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^
5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c
^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5
*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(x/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [B]
time = 40.22, size = 272, normalized size = 1.93

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\,{\left(\sqrt{d\,x^3+c}-\sqrt{c}+2\,c^{1/6}\,d^{1/3}\,x\right)}^3}{x^3\,{\left(d^{1/3}\,x-2\,c^{1/3}\right)}^3}\right)}{54\,c^{5/6}\,d^{2/3}}+\frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\,{\left(-\sqrt{3}\,c^{1/6}\,d^{1/3}\,x+\sqrt{d\,x^3+c}\,1\mathrm{i}+\sqrt{c}\,1\mathrm{i}+c^{1/6}\,d^{1/3}\,x\,1\mathrm{i}\right)}^3}{x^3\,{\left(d^{1/3}\,x+c^{1/3}-\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)}^3}\right)\,\sqrt{-1+\sqrt{3}\,1\mathrm{i}}}{108\,c^{5/6}\,d^{2/3}}+\frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\,{\left(\sqrt{3}\,c^{1/6}\,d^{1/3}\,x-\sqrt{d\,x^3+c}\,1\mathrm{i}+\sqrt{c}\,1\mathrm{i}+c^{1/6}\,d^{1/3}\,x\,1\mathrm{i}\right)}^3}{x^3\,{\left(d^{1/3}\,x+c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)}^3}\right)\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{108\,c^{5/6}\,d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) + c^(1/2))*((c + d*x^3)^(1/2) - c^(1/2) + 2*c^(1/6)
*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 2*c^(1/3))^3))/(54*c^(5/6)*d^(2/3)) + (2^(
1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2)*1i + c^(1/2)*1i
+ c^(1/6)*d^(1/3)*x*1i - 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 3^
(1/2)*c^(1/3)*1i + c^(1/3))^3))*(3^(1/2)*1i - 1)^(1/2))/(108*c^(5/6)*d^(2/3
)) + (2^(1/2)*log((((c + d*x^3)^(1/2) + c^(1/2))*(c^(1/2)*1i - (c + d*x^3)^
(1/2)*1i + c^(1/6)*d^(1/3)*x*1i + 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(3^(1/
2)*c^(1/3)*1i + d^(1/3)*x + c^(1/3))^3))*(3^(1/2)*1i + 1)^(1/2)*1i)/(108*c^
(5/6)*d^(2/3))
```

3.318 $\int \frac{1}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=632

$$-\frac{\sqrt{c+dx^3}}{8c^2x}+\frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}\,c^{11/6}}+\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{\ }\right.}{3\sqrt[6]{c}\,\sqrt{c}}\right.}{144c^{11/6}}$$

```
[Out] 1/144*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(11/6)-1/144*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/144*d^
(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)*
3^(1/2)-1/8*(d*x^3+c)^(1/2)/c^2/x+1/8*d^(1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))+1/24*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(
2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2
)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/16*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*
x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$,
Rules used = {491, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{16c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{\sqrt[3]{d}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}\,c^{11/6}}+\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{144c^{11/6}}-\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}}-\frac{\sqrt{c+dx^3}}{8c^2x}+\frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/8*Sqrt[c + d*x^3]/(c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(8*c^2*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(48*Sqrt[3]*c^(11/6)) + (d^(1/3)*ArcTanh[(c^(1/3
) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(144*c^(11/6)) - (d^(1/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(144*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt
[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt
```

```
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(16*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^
(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
+ d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\int \frac{x\left(5cd-\frac{d^2x^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{8c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{cdx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{8c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{16c^2} + \frac{d\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} - \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{96c^2} + \frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{}}{\sqrt{c+dx^3}}}{16c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}\right.}{}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}\right.}{\sqrt{c+d}}\right.}{48\sqrt{3}\,c^{11/6}}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}\right.}{\sqrt{c+d}}\right.}{48\sqrt{3}\,c^{11/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 20.07, size = 137, normalized size = 0.22

$$\frac{-80c(c+dx^3)+25cdx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-d^2x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{640c^3x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-80*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^3*x*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 874, normalized size = 1.38

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	874

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/216*I/d^2/c^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+
2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2
)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/8/c*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3
^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
```

```
d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.14, size = 2522, normalized size = 3.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1728*(4*sqrt(3)*c^2*x*(d^2/c^11)^(1/6)*arctan(1/9*((9*sqrt(3)*c^2*d^4*x^
5*(d^2/c^11)^(1/6) - sqrt(3)*(c^9*d^3*x^6 - 40*c^10*d^2*x^3 - 32*c^11*d)*(d
^2/c^11)^(5/6) + 3*sqrt(3)*(5*c^6*d^3*x^4 + 8*c^7*d^2*x)*sqrt(d^2/c^11))*sq
rt(d*x^3 + c) + (18*sqrt(3)*(c^8*d^2*x^5 + c^9*d*x^2)*(d^2/c^11)^(2/3) + 12
*sqrt(3)*(c^4*d^3*x^6 - c^5*d^2*x^3 - 2*c^6*d)*(d^2/c^11)^(1/3) + 3*sqrt(3)
*(d^4*x^7 + 5*c*d^3*x^4 + 4*c^2*d^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^9*d^2*
x^6 + 32*c^10*d*x^3 + 40*c^11)*(d^2/c^11)^(5/6) + 3*sqrt(3)*(7*c^6*d^2*x^4
+ 4*c^7*d*x)*sqrt(d^2/c^11) + 9*sqrt(3)*(c^2*d^3*x^5 + 2*c^3*d^2*x^2)*(d^2/
c^11)^(1/6)))*sqrt((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d
^2 - 18*(c^8*d^3*x^7 - 52*c^9*d^2*x^4 - 80*c^10*d*x)*(d^2/c^11)^(2/3) + 6*s
qrt(d*x^3 + c)*(24*(c^10*d^2*x^5 + c^11*d*x^2)*(d^2/c^11)^(5/6) - 4*(c^6*d^
3*x^6 + 41*c^7*d^2*x^3 + 40*c^8*d)*sqrt(d^2/c^11) - (c^2*d^4*x^7 - 28*c^3*d
^3*x^4 - 272*c^4*d^2*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d^4*x^8 + 20*c^5*d^3*x^
5 - 8*c^6*d^2*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^
3 - 512*c^3)))/(d^5*x^7 - 7*c*d^4*x^4 - 8*c^2*d^3*x)) + 4*sqrt(3)*c^2*x*(d^
2/c^11)^(1/6)*arctan(1/9*((9*sqrt(3)*c^2*d^4*x^5*(d^2/c^11)^(1/6) - sqrt(3)
*(c^9*d^3*x^6 - 40*c^10*d^2*x^3 - 32*c^11*d)*(d^2/c^11)^(5/6) + 3*sqrt(3)*(
5*c^6*d^3*x^4 + 8*c^7*d^2*x)*sqrt(d^2/c^11))*sqrt(d*x^3 + c) - (18*sqrt(3)*
(c^8*d^2*x^5 + c^9*d*x^2)*(d^2/c^11)^(2/3) + 12*sqrt(3)*(c^4*d^3*x^6 - c^5*
d^2*x^3 - 2*c^6*d)*(d^2/c^11)^(1/3) + 3*sqrt(3)*(d^4*x^7 + 5*c*d^3*x^4 + 4*
c^2*d^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^9*d^2*x^6 + 32*c^10*d*x^3 + 40*c^1
1)*(d^2/c^11)^(5/6) + 3*sqrt(3)*(7*c^6*d^2*x^4 + 4*c^7*d*x)*sqrt(d^2/c^11)
+ 9*sqrt(3)*(c^2*d^3*x^5 + 2*c^3*d^2*x^2)*(d^2/c^11)^(1/6)))*sqrt((d^5*x^9
- 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^8*d^3*x^7 - 52*c^
9*d^2*x^4 - 80*c^10*d*x)*(d^2/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^10*d^2
```

```
*x^5 + c^11*d*x^2)*(d^2/c^11)^(5/6) - 4*(c^6*d^3*x^6 + 41*c^7*d^2*x^3 + 40*
c^8*d)*sqrt(d^2/c^11) - (c^2*d^4*x^7 - 28*c^3*d^3*x^4 - 272*c^4*d^2*x)*(d^2
/c^11)^(1/6)) + 18*(c^4*d^4*x^8 + 20*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(d^2/c^11
)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^5*x^7 - 7*
c*d^4*x^4 - 8*c^2*d^3*x)) + c^2*x*(d^2/c^11)^(1/6)*log((d^5*x^9 - 276*c*d^4
*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^8*d^3*x^7 - 52*c^9*d^2*x^4 -
 80*c^10*d*x)*(d^2/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^2*x^5 + c^11
*d*x^2)*(d^2/c^11)^(5/6) - 4*(c^6*d^3*x^6 + 41*c^7*d^2*x^3 + 40*c^8*d)*sqrt
(d^2/c^11) - (c^2*d^4*x^7 - 28*c^3*d^3*x^4 - 272*c^4*d^2*x)*(d^2/c^11)^(1/6
)) + 18*(c^4*d^4*x^8 + 20*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(d^2/c^11)^(1/3))/(d
^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - c^2*x*(d^2/c^11)^(1/6)*
log((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^8*d^
3*x^7 - 52*c^9*d^2*x^4 - 80*c^10*d*x)*(d^2/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*
(24*(c^10*d^2*x^5 + c^11*d*x^2)*(d^2/c^11)^(5/6) - 4*(c^6*d^3*x^6 + 41*c^7*
d^2*x^3 + 40*c^8*d)*sqrt(d^2/c^11) - (c^2*d^4*x^7 - 28*c^3*d^3*x^4 - 272*c^
4*d^2*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d^4*x^8 + 20*c^5*d^3*x^5 - 8*c^6*d^2*x
^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
 2*c^2*x*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 +
 640*c^3*d + 18*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x)*(d^2/c^11)^(2/3)
 + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^2/c^11)^(5/6) + (7*
c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7 + 80*c^
3*d^2*x^4 + 160*c^4*d*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d^3*x^8 + 38*c^5*d^2*x
^5 + 64*c^6*d*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^
3 - 512*c^3)) + 2*c^2*x*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 120
0*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x)*(
d^2/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^2/c^
11)^(5/6) + (7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*
d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d^3*x^8
 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) + 216*sqrt(d)*x*weierstrassZeta(0, -4*c/d, wei
erstrassPInverse(0, -4*c/d, x)) + 216*sqrt(d*x^3 + c))/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^2\sqrt{c+dx^3}+dx^5\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**2*sqrt(c + d*x**3) + d*x**5*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.319 $\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=654

$$-\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}\,c^{17/6}}+\frac{d^{4/3}\,\mathrm{ta}}{}$$

[Out] 1/1152*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/6)-1/1152*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/1152*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/6)*3^(1/2)-1/32*(d*x^3+c)^(1/2)/c^2/x^4+1/16*d*(d*x^3+c)^(1/2)/c^3/x-1/16*d^(4/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-1/48*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/32*3^(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.58, antiderivative size = 654, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {491, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\,\sqrt[4]{3}\,c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{32c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{d^{4/3}\mathrm{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}\,c^{17/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1152c^{17/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{17/6}}-\frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{\sqrt{c+dx^3}}{32c^2x^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] -1/32*Sqrt[c + d*x^3]/(c^2*x^4) + (d*Sqrt[c + d*x^3])/(16*c^3*x) - (d^(4/3)*Sqrt[c + d*x^3])/(16*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(384*Sqrt[3]*c^(17/6)) + (d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1152*c^(17/6)) - (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1152*c^(17/6)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*

```
Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(8/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(8*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{\int \frac{-16cd+\frac{5d^2x^3}{2}}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{32c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{\int \frac{x(60c^2d^2-8cd^3x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{256c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{\int \left(\frac{8cd^2x}{\sqrt{c+dx^3}} - \frac{4c^2d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{256c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c^3} + \frac{d^2\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}}{64c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{768c^3} - \frac{d^{5/3}}{}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\,\sqrt{2}}{}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{d^{4/3}\tan}{}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{d^{4/3}\tan}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 20.08, size = 152, normalized size = 0.23

$$\frac{160c(-c^2+cdx^3+2d^2x^6)-75cd^2x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+4d^3x^9\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{5120c^4x^4\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (160*c*(-c^2 + c*d*x^3 + 2*d^2*x^6) - 75*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*Appe
llF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 4*d^3*x^9*Sqrt[1 + (d
*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(5120*c^4
*x^4*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 1351, normalized size = 2.07

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	887
default	Expression too large to display	1351

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/4*(d*x^3+c)^(1/2)/x^4/c+5/8*d*(d*x^3+c)^(1/2)/c^2/x+5/24*I/c^2*d*
3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/1728*I/c^3/d*2^(1/2)*sum(1/_al
pha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_
alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alp
ha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
```

```
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-
c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c
*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*
c))+1/64*d/c^2*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.28, size = 2614, normalized size = 4.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/13824*(4*sqrt(3)*c^3*x^4*(d^8/c^17)^(1/6)*arctan(1/9*((9*sqrt(3)*c^3*d^1
3*x^5*(d^8/c^17)^(1/6) - sqrt(3)*(c^14*d^8*x^6 - 40*c^15*d^7*x^3 - 32*c^16*
d^6)*(d^8/c^17)^(5/6) + 3*sqrt(3)*(5*c^9*d^10*x^4 + 8*c^10*d^9*x)*sqrt(d^8/
c^17))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^12*d^3*x^5 + c^13*d^2*x^2)*(d^8/c^1
7)^(2/3) + 12*sqrt(3)*(c^6*d^6*x^6 - c^7*d^5*x^3 - 2*c^8*d^4)*(d^8/c^17)^(1
/3) + 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*x^4 + 4*c^2*d^7*x) + sqrt(d*x^3 + c)*(sq
rt(3)*(c^14*d^2*x^6 + 32*c^15*d*x^3 + 40*c^16)*(d^8/c^17)^(5/6) + 3*sqrt(3)
*(7*c^9*d^4*x^4 + 4*c^10*d^3*x)*sqrt(d^8/c^17) + 9*sqrt(3)*(c^3*d^7*x^5 + 2
*c^4*d^6*x^2)*(d^8/c^17)^(1/6)))*sqrt((d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2
*d^13*x^3 - 1088*c^3*d^12 - 18*(c^12*d^9*x^7 - 52*c^13*d^8*x^4 - 80*c^14*d^
7*x)*(d^8/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^15*d^7*x^5 + c^16*d^6*x^2)
```

(d^8/c^17)^(5/6) - 4(c^9*d^10*x^6 + 41*c^10*d^9*x^3 + 40*c^11*d^8)*sqrt(d
^8/c^17) - (c^3*d^13*x^7 - 28*c^4*d^12*x^4 - 272*c^5*d^11*x)*(d^8/c^17)^(1/
6)) + 18*(c^6*d^12*x^8 + 20*c^7*d^11*x^5 - 8*c^8*d^10*x^2)*(d^8/c^17)^(1/3)
)/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^15*x^7 - 7*c*d^14
*x^4 - 8*c^2*d^13*x)) + 4*sqrt(3)*c^3*x^4*(d^8/c^17)^(1/6)*arctan(1/9*((9*s
qrt(3)*c^3*d^13*x^5*(d^8/c^17)^(1/6) - sqrt(3)*(c^14*d^8*x^6 - 40*c^15*d^7*
x^3 - 32*c^16*d^6)*(d^8/c^17)^(5/6) + 3*sqrt(3)*(5*c^9*d^10*x^4 + 8*c^10*d^
9*x)*sqrt(d^8/c^17))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^12*d^3*x^5 + c^13*d^2
x^2)(d^8/c^17)^(2/3) + 12*sqrt(3)*(c^6*d^6*x^6 - c^7*d^5*x^3 - 2*c^8*d^4)
*(d^8/c^17)^(1/3) + 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*x^4 + 4*c^2*d^7*x) - sqrt(
d*x^3 + c)*(sqrt(3)*(c^14*d^2*x^6 + 32*c^15*d*x^3 + 40*c^16)*(d^8/c^17)^(5/
6) + 3*sqrt(3)*(7*c^9*d^4*x^4 + 4*c^10*d^3*x)*sqrt(d^8/c^17) + 9*sqrt(3)*(c
^3*d^7*x^5 + 2*c^4*d^6*x^2)*(d^8/c^17)^(1/6)))*sqrt((d^15*x^9 - 276*c*d^14*
x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^12*d^9*x^7 - 52*c^13*d^8*x^
4 - 80*c^14*d^7*x)*(d^8/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^15*d^7*x^5 +
 c^16*d^6*x^2)*(d^8/c^17)^(5/6) - 4*(c^9*d^10*x^6 + 41*c^10*d^9*x^3 + 40*c^
11*d^8)*sqrt(d^8/c^17) - (c^3*d^13*x^7 - 28*c^4*d^12*x^4 - 272*c^5*d^11*x)*
(d^8/c^17)^(1/6)) + 18*(c^6*d^12*x^8 + 20*c^7*d^11*x^5 - 8*c^8*d^10*x^2)*(d
^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^15*
x^7 - 7*c*d^14*x^4 - 8*c^2*d^13*x)) + c^3*x^4*(d^8/c^17)^(1/6)*log((d^15*x^
9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^12*d^9*x^7 -
 52*c^13*d^8*x^4 - 80*c^14*d^7*x)*(d^8/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(24*
(c^15*d^7*x^5 + c^16*d^6*x^2)*(d^8/c^17)^(5/6) - 4*(c^9*d^10*x^6 + 41*c^10*
d^9*x^3 + 40*c^11*d^8)*sqrt(d^8/c^17) - (c^3*d^13*x^7 - 28*c^4*d^12*x^4 - 2
72*c^5*d^11*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^12*x^8 + 20*c^7*d^11*x^5 - 8*c
^8*d^10*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 51
2*c^3)) - c^3*x^4*(d^8/c^17)^(1/6)*log((d^15*x^9 - 276*c*d^14*x^6 - 1608*c^
2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^12*d^9*x^7 - 52*c^13*d^8*x^4 - 80*c^14*d
^7*x)*(d^8/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^15*d^7*x^5 + c^16*d^6*x^2
)*(d^8/c^17)^(5/6) - 4*(c^9*d^10*x^6 + 41*c^10*d^9*x^3 + 40*c^11*d^8)*sqrt(
d^8/c^17) - (c^3*d^13*x^7 - 28*c^4*d^12*x^4 - 272*c^5*d^11*x)*(d^8/c^17)^(1
/6)) + 18*(c^6*d^12*x^8 + 20*c^7*d^11*x^5 - 8*c^8*d^10*x^2)*(d^8/c^17)^(1/3
))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c^3*x^4*(d^8/c^1
7)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18
*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x)*(d^8/c^17)^(2/3) + 6*sqrt
(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^8/c^17)^(5/6) + (7*c^9*d^4*x
^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4
*d^6*x^4 + 160*c^5*d^5*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^6*x^8 + 38*c^7*d^5*
x^5 + 64*c^8*d^4*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d
*x^3 - 512*c^3)) + 2*c^3*x^4*(d^8/c^17)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6
+ 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 3
2*c^14*d*x)*(d^8/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16
x^2)(d^8/c^17)^(5/6) + (7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*s
qrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x)*(d^8/c^17)^(
1/6)) + 18*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2)*(d^8/c^17)^(1/3)

```
)/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 864*d^(3/2)*x^4*wei
erstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 432*(2*d*x^3 -
 c)*sqrt(d*x^3 + c))/(c^3*x^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^5\sqrt{c+dx^3}+dx^8\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(1/(-8*c*x**5*sqrt(c + d*x**3) + d*x**8*sqrt(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.320 $\int \frac{1}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=678

$$-\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\right.}{\sqrt{c+dx^3}}\right.}{3072\sqrt{3}\,c^{23/6}}$$

```
[Out] 1/9216*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(23/6)-1/9216*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/9216
*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(23/
6)*3^(1/2)-1/56*(d*x^3+c)^(1/2)/c^2/x^7+37/1792*d*(d*x^3+c)^(1/2)/c^3/x^4-3
/56*d^2*(d*x^3+c)^(1/2)/c^4/x+3/56*d^(7/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))+1/56*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(
1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/112*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x
)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(11/3)/(d*x^3+c)^(1/2)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.66, antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {491, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}\,c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{112c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{d^{7/3}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}\,c^{23/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{23/6}}-\frac{d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{23/6}}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{\sqrt{c+dx^3}}{56c^2x^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/56*Sqrt[c + d*x^3]/(c^2*x^7) + (37*d*Sqrt[c + d*x^3])/(1792*c^3*x^4) - (
3*d^2*Sqrt[c + d*x^3])/(56*c^4*x) + (3*d^(7/3)*Sqrt[c + d*x^3])/(56*c^4*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/
3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3072*Sqrt[3]*c^(23/6)) + (d^(7/3)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9216*c^(23/6)) -
(d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9216*c^(23/6)) - (3*3^(1/4)
```

```
*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[Arc
Sin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
], -7 - 4*Sqrt[3]])/(112*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(3/4)*d^(7/3)*(c^(
1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^
(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]
*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^8 (8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{56c^2x^7} + \frac{\int \frac{-37cd+\frac{11d^2x^3}{2}}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{56c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7} + \frac{37d\sqrt{c+dx^3}}{1792c^3x^4} - \frac{\int \frac{-768c^2d^2+\frac{185}{2}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1792c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7} + \frac{37d\sqrt{c+dx^3}}{1792c^3x^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^4x} + \frac{\int \frac{x(3100c^3d^3-384c^2d^4x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{14336c^6}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7} + \frac{37d\sqrt{c+dx^3}}{1792c^3x^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^4x} + \frac{\int \left(\frac{384c^2d^3x}{\sqrt{c+dx^3}} + \frac{28}{(8c-dx^3)}\right.}{14336c^6}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7} + \frac{37d\sqrt{c+dx^3}}{1792c^3x^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^4x} + \frac{(3d^3)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{112c^4} +$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7} + \frac{37d\sqrt{c+dx^3}}{1792c^3x^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^4x} - \frac{d^2\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^4}{}}{\left(4+\frac{2\sqrt[3]{d}}{\sqrt[3]{c}}x+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{}}}{6144c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7} + \frac{37d\sqrt{c+dx^3}}{1792c^3x^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^4x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7} + \frac{37d\sqrt{c+dx^3}}{1792c^3x^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^4x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7} + \frac{37d\sqrt{c+dx^3}}{1792c^3x^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^4x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.08, size = 167, normalized size = 0.25

$$\frac{3875cd^3x^9\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\left(5c(32c^3-5c^2dx^3+59cd^2x^6+96d^3x^9)+6d^4x^{12}\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{286720c^5x^7\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (3875*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 - 5*c^2*d*x^3 + 59*c*d^2*x^6 + 96*d^3*x^
9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c)
, (d*x^3)/(8*c)]))/(286720*c^5*x^7*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 1849, normalized size = 2.73

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	1849

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*d/c^2*(-1/4*(d*x^3+c)^(1/2)/x^4/c+5/8*d*(d*x^3+c)^(1/2)/c^2/x+5/24*I/c
^2*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))+1/8/c*(-1/7*(d*x^3+c)^(1/2)
/c/x^7+11/56*d*(d*x^3+c)^(1/2)/c^2/x^4-55/112*d^2*(d*x^3+c)^(1/2)/c^3/x-55/
336*I/c^3*d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
```

```
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/13824*I/c^4*2^(1
/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c
*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^
2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1
/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*d-8*c))+1/512/c^3*d^2*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^
2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.58, size = 2630, normalized size = 3.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/774144*(28*sqrt(3)*c^4*x^7*(d^14/c^23)^(1/6)*arctan(1/9*((9*sqrt(3)*c^4*
d^22*x^5*(d^14/c^23)^(1/6) - sqrt(3)*(c^19*d^13*x^6 - 40*c^20*d^12*x^3 - 32
*c^21*d^11)*(d^14/c^23)^(5/6) + 3*sqrt(3)*(5*c^12*d^17*x^4 + 8*c^13*d^16*x)
*sqrt(d^14/c^23))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^16*d^4*x^5 + c^17*d^3*x^
2)*(d^14/c^23)^(2/3) + 12*sqrt(3)*(c^8*d^9*x^6 - c^9*d^8*x^3 - 2*c^10*d^7)*
(d^14/c^23)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 + 4*c^2*d^12*x) + sq
rt(d*x^3 + c)*(sqrt(3)*(c^19*d^2*x^6 + 32*c^20*d*x^3 + 40*c^21)*(d^14/c^23)
^(5/6) + 3*sqrt(3)*(7*c^12*d^6*x^4 + 4*c^13*d^5*x)*sqrt(d^14/c^23) + 9*sqrt
(3)*(c^4*d^11*x^5 + 2*c^5*d^10*x^2)*(d^14/c^23)^(1/6)))*sqrt((d^25*x^9 - 27
6*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^16*d^15*x^7 - 52*c
^17*d^14*x^4 - 80*c^18*d^13*x)*(d^14/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c
^20*d^12*x^5 + c^21*d^11*x^2)*(d^14/c^23)^(5/6) - 4*(c^12*d^17*x^6 + 41*c^1
3*d^16*x^3 + 40*c^14*d^15)*sqrt(d^14/c^23) - (c^4*d^22*x^7 - 28*c^5*d^21*x^
4 - 272*c^6*d^20*x)*(d^14/c^23)^(1/6)) + 18*(c^8*d^20*x^8 + 20*c^9*d^19*x^5
 - 8*c^10*d^18*x^2)*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*
x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*x)) + 28*sqrt(3)*c^4
*x^7*(d^14/c^23)^(1/6)*arctan(1/9*((9*sqrt(3)*c^4*d^22*x^5*(d^14/c^23)^(1/6
) - sqrt(3)*(c^19*d^13*x^6 - 40*c^20*d^12*x^3 - 32*c^21*d^11)*(d^14/c^23)^(
5/6) + 3*sqrt(3)*(5*c^12*d^17*x^4 + 8*c^13*d^16*x)*sqrt(d^14/c^23))*sqrt(d*
x^3 + c) - (18*sqrt(3)*(c^16*d^4*x^5 + c^17*d^3*x^2)*(d^14/c^23)^(2/3) + 12
*sqrt(3)*(c^8*d^9*x^6 - c^9*d^8*x^3 - 2*c^10*d^7)*(d^14/c^23)^(1/3) + 3*sqr
t(3)*(d^14*x^7 + 5*c*d^13*x^4 + 4*c^2*d^12*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c
^19*d^2*x^6 + 32*c^20*d*x^3 + 40*c^21)*(d^14/c^23)^(5/6) + 3*sqrt(3)*(7*c^1
2*d^6*x^4 + 4*c^13*d^5*x)*sqrt(d^14/c^23) + 9*sqrt(3)*(c^4*d^11*x^5 + 2*c^5
*d^10*x^2)*(d^14/c^23)^(1/6)))*sqrt((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d
^23*x^3 - 1088*c^3*d^22 - 18*(c^16*d^15*x^7 - 52*c^17*d^14*x^4 - 80*c^18*d^
13*x)*(d^14/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^20*d^12*x^5 + c^21*d^11*
x^2)*(d^14/c^23)^(5/6) - 4*(c^12*d^17*x^6 + 41*c^13*d^16*x^3 + 40*c^14*d^15
)*sqrt(d^14/c^23) - (c^4*d^22*x^7 - 28*c^5*d^21*x^4 - 272*c^6*d^20*x)*(d^14
/c^23)^(1/6)) + 18*(c^8*d^20*x^8 + 20*c^9*d^19*x^5 - 8*c^10*d^18*x^2)*(d^14
/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^
7 - 7*c*d^24*x^4 - 8*c^2*d^23*x)) + 7*c^4*x^7*(d^14/c^23)^(1/6)*log((d^25*x
^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^16*d^15*x^7
 - 52*c^17*d^14*x^4 - 80*c^18*d^13*x)*(d^14/c^23)^(2/3) + 6*sqrt(d*x^3 + c)
*(24*(c^20*d^12*x^5 + c^21*d^11*x^2)*(d^14/c^23)^(5/6) - 4*(c^12*d^17*x^6 +
 41*c^13*d^16*x^3 + 40*c^14*d^15)*sqrt(d^14/c^23) - (c^4*d^22*x^7 - 28*c^5*
d^21*x^4 - 272*c^6*d^20*x)*(d^14/c^23)^(1/6)) + 18*(c^8*d^20*x^8 + 20*c^9*d
^19*x^5 - 8*c^10*d^18*x^2)*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192
*c^2*d*x^3 - 512*c^3)) - 7*c^4*x^7*(d^14/c^23)^(1/6)*log((d^25*x^9 - 276*c*
d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^16*d^15*x^7 - 52*c^17*
d^14*x^4 - 80*c^18*d^13*x)*(d^14/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^20*
d^12*x^5 + c^21*d^11*x^2)*(d^14/c^23)^(5/6) - 4*(c^12*d^17*x^6 + 41*c^13*d^
16*x^3 + 40*c^14*d^15)*sqrt(d^14/c^23) - (c^4*d^22*x^7 - 28*c^5*d^21*x^4 -
272*c^6*d^20*x)*(d^14/c^23)^(1/6)) + 18*(c^8*d^20*x^8 + 20*c^9*d^19*x^5 - 8
*c^10*d^18*x^2)*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
```

```
- 512*c^3)) - 14*c^4*x^7*(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 +
 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 +
32*c^18*d^2*x)*(d^14/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*
c^21*x^2)*(d^14/c^23)^(5/6) + (7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*
d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)*(d^
14/c^23)^(1/6)) + 18*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(d^14
/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 14*c^4*
x^7*(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 +
640*c^3*d^11 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x)*(d^14/
c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^14/c^23)
^(5/6) + (7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)*sqrt(d^14/c^23)
+ (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)*(d^14/c^23)^(1/6)) + 18*
(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(d^14/c^23)^(1/3))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 41472*d^(5/2)*x^7*weierstras
sZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 432*(96*d^2*x^6 - 37*
c*d*x^3 + 32*c^2)*sqrt(d*x^3 + c))/(c^4*x^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^8\sqrt{c+dx^3}+dx^{11}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(1/(-8*c*x**8*sqrt(c + d*x**3) + d*x**11*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^8\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.321 $\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

```
[Out] 1/32*x^4*AppellF1(4/3,1/2,1,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/(
d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(32*c*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

Mathematica [A]
time = 10.03, size = 67, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(32*c*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.32, size = 696, normalized size = 10.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-8/2
7*I/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)
```

```
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2419 vs. $2(52) = 104$.
time = 4.75, size = 2419, normalized size = 36.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(4*sqrt(3)*d^2*(1/(c*d^8))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^8*x^5*(1/(
c*d^8))^(5/6) + 3*sqrt(3)*(5*c*d^5*x^4 + 8*c^2*d^4*x)*sqrt(1/(c*d^8)) - sqr
t(3)*(d^3*x^6 - 40*c*d^2*x^3 - 32*c^2*d)*(1/(c*d^8))^(1/6))*sqrt(d*x^3 + c)
 - (12*sqrt(3)*(c*d^7*x^6 - c^2*d^6*x^3 - 2*c^3*d^5)*(1/(c*d^8))^(2/3) + 18
*sqrt(3)*(c*d^4*x^5 + c^2*d^3*x^2)*(1/(c*d^8))^(1/3) + 3*sqrt(3)*(d^2*x^7 +
 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(9*sqrt(3)*(c*d^8*x^5 + 2*c^2*d^7*x
^2)*(1/(c*d^8))^(5/6) + 3*sqrt(3)*(7*c*d^5*x^4 + 4*c^2*d^4*x)*sqrt(1/(c*d^8
)) + sqrt(3)*(d^3*x^6 + 32*c*d^2*x^3 + 40*c^2*d)*(1/(c*d^8))^(1/6)))*sqrt((
d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c*d^8*x^8 + 20*c^
2*d^7*x^5 - 8*c^3*d^6*x^2)*(1/(c*d^8))^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^9*x^
7 - 28*c^2*d^8*x^4 - 272*c^3*d^7*x)*(1/(c*d^8))^(5/6) + 4*(c*d^6*x^6 + 41*c
^2*d^5*x^3 + 40*c^3*d^4)*sqrt(1/(c*d^8)) - 24*(c*d^3*x^5 + c^2*d^2*x^2)*(1/
(c*d^8))^(1/6)) - 18*(c*d^5*x^7 - 52*c^2*d^4*x^4 - 80*c^3*d^3*x)*(1/(c*d^8)
)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*
c*d*x^4 - 8*c^2*x)) + 4*sqrt(3)*d^2*(1/(c*d^8))^(1/6)*arctan(1/9*((9*sqrt(3
)*c*d^8*x^5*(1/(c*d^8))^(5/6) + 3*sqrt(3)*(5*c*d^5*x^4 + 8*c^2*d^4*x)*sqrt(
1/(c*d^8)) - sqrt(3)*(d^3*x^6 - 40*c*d^2*x^3 - 32*c^2*d)*(1/(c*d^8))^(1/6))
*sqrt(d*x^3 + c) + (12*sqrt(3)*(c*d^7*x^6 - c^2*d^6*x^3 - 2*c^3*d^5)*(1/(c*
d^8))^(2/3) + 18*sqrt(3)*(c*d^4*x^5 + c^2*d^3*x^2)*(1/(c*d^8))^(1/3) + 3*sq
rt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c*d^8*x
^5 + 2*c^2*d^7*x^2)*(1/(c*d^8))^(5/6) + 3*sqrt(3)*(7*c*d^5*x^4 + 4*c^2*d^4*
x)*sqrt(1/(c*d^8)) + sqrt(3)*(d^3*x^6 + 32*c*d^2*x^3 + 40*c^2*d)*(1/(c*d^8)
)^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c
```

```
*d^8*x^8 + 20*c^2*d^7*x^5 - 8*c^3*d^6*x^2)*(1/(c*d^8))^(2/3) - 6*sqrt(d*x^3
 + c)*((c*d^9*x^7 - 28*c^2*d^8*x^4 - 272*c^3*d^7*x)*(1/(c*d^8))^(5/6) + 4*(
c*d^6*x^6 + 41*c^2*d^5*x^3 + 40*c^3*d^4)*sqrt(1/(c*d^8)) - 24*(c*d^3*x^5 +
c^2*d^2*x^2)*(1/(c*d^8))^(1/6)) - 18*(c*d^5*x^7 - 52*c^2*d^4*x^4 - 80*c^3*d
^3*x)*(1/(c*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)
))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 2*d^2*(1/(c*d^8))^(1/6)*log((d^3*x^9
+ 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c*d^8*x^8 + 38*c^2*d^7*x^5
 + 64*c^3*d^6*x^2)*(1/(c*d^8))^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^9*x^7 + 80*c
^2*d^8*x^4 + 160*c^3*d^7*x)*(1/(c*d^8))^(5/6) + (7*c*d^6*x^6 + 152*c^2*d^5*
x^3 + 64*c^3*d^4)*sqrt(1/(c*d^8)) + 6*(5*c*d^3*x^5 + 32*c^2*d^2*x^2)*(1/(c*
d^8))^(1/6)) + 18*(5*c*d^5*x^7 + 64*c^2*d^4*x^4 + 32*c^3*d^3*x)*(1/(c*d^8))
^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*d^2*(1/(c*d
^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c*
d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2)*(1/(c*d^8))^(2/3) - 6*sqrt(d*x^3
 + c)*((c*d^9*x^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x)*(1/(c*d^8))^(5/6) + (7*
c*d^6*x^6 + 152*c^2*d^5*x^3 + 64*c^3*d^4)*sqrt(1/(c*d^8)) + 6*(5*c*d^3*x^5
+ 32*c^2*d^2*x^2)*(1/(c*d^8))^(1/6)) + 18*(5*c*d^5*x^7 + 64*c^2*d^4*x^4 + 3
2*c^3*d^3*x)*(1/(c*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) + d^2*(1/(c*d^8))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*
x^3 - 1088*c^3 + 18*(c*d^8*x^8 + 20*c^2*d^7*x^5 - 8*c^3*d^6*x^2)*(1/(c*d^8)
)^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^9*x^7 - 28*c^2*d^8*x^4 - 272*c^3*d^7*x)*(
1/(c*d^8))^(5/6) + 4*(c*d^6*x^6 + 41*c^2*d^5*x^3 + 40*c^3*d^4)*sqrt(1/(c*d^
8)) - 24*(c*d^3*x^5 + c^2*d^2*x^2)*(1/(c*d^8))^(1/6)) - 18*(c*d^5*x^7 - 52*
c^2*d^4*x^4 - 80*c^3*d^3*x)*(1/(c*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 19
2*c^2*d*x^3 - 512*c^3)) - d^2*(1/(c*d^8))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^
6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c*d^8*x^8 + 20*c^2*d^7*x^5 - 8*c^3*d^6*
x^2)*(1/(c*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c*d^9*x^7 - 28*c^2*d^8*x^4 - 2
72*c^3*d^7*x)*(1/(c*d^8))^(5/6) + 4*(c*d^6*x^6 + 41*c^2*d^5*x^3 + 40*c^3*d^
4)*sqrt(1/(c*d^8)) - 24*(c*d^3*x^5 + c^2*d^2*x^2)*(1/(c*d^8))^(1/6)) - 18*(
c*d^5*x^7 - 52*c^2*d^4*x^4 - 80*c^3*d^3*x)*(1/(c*d^8))^(1/3))/(d^3*x^9 - 24
*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 36*sqrt(d)*weierstrassPInverse(0,
-4*c/d, x))/d^2
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**3/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.322 $\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

```
[Out] 1/8*x*AppellF1(1/3,1/2,1,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/(d*x
^3+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(8*c*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}} = \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 166 vs. 2(64) = 128.
time = 10.10, size = 166, normalized size = 2.59

$$\frac{32cxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\sqrt{c+dx^3}\left(32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (32*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*
x^3)*Sqrt[c + d*x^3]*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)
/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]
- 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.30, size = 416, normalized size = 6.50

method	result

default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{}}{}\right)}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{}}{}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2
*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(
```

1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2448 vs. $2(50) = 100$.
time = 6.36, size = 2448, normalized size = 38.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*(4*sqrt(3)*c*d*(1/(c^7*d^2))^(1/6)*arctan(1/9*((9*sqrt(3)*c^6*d^3*x^5
*(1/(c^7*d^2))^(5/6) + 3*sqrt(3)*(5*c^4*d^2*x^4 + 8*c^5*d*x)*sqrt(1/(c^7*d^
2)) - sqrt(3)*(c*d^2*x^6 - 40*c^2*d*x^3 - 32*c^3)*(1/(c^7*d^2))^(1/6))*sqrt
(d*x^3 + c) - (12*sqrt(3)*(c^5*d^3*x^6 - c^6*d^2*x^3 - 2*c^7*d)*(1/(c^7*d^2
))^(2/3) + 18*sqrt(3)*(c^3*d^2*x^5 + c^4*d*x^2)*(1/(c^7*d^2))^(1/3) + 3*sqr
t(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(9*sqrt(3)*(c^6*d^3*
x^5 + 2*c^7*d^2*x^2)*(1/(c^7*d^2))^(5/6) + 3*sqrt(3)*(7*c^4*d^2*x^4 + 4*c^5
*d*x)*sqrt(1/(c^7*d^2)) + sqrt(3)*(c*d^2*x^6 + 32*c^2*d*x^3 + 40*c^3)*(1/(c
^7*d^2))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3
+ 18*(c^5*d^4*x^8 + 20*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(1/(c^7*d^2))^(2/3) + 6
*sqrt(d*x^3 + c)*((c^6*d^4*x^7 - 28*c^7*d^3*x^4 - 272*c^8*d^2*x)*(1/(c^7*d^
2))^(5/6) + 4*(c^4*d^3*x^6 + 41*c^5*d^2*x^3 + 40*c^6*d)*sqrt(1/(c^7*d^2)) -
 24*(c^2*d^2*x^5 + c^3*d*x^2)*(1/(c^7*d^2))^(1/6)) - 18*(c^3*d^3*x^7 - 52*c
^4*d^2*x^4 - 80*c^5*d*x)*(1/(c^7*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192
*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 4*sqrt(3)*c*d*(1
/(c^7*d^2))^(1/6)*arctan(1/9*((9*sqrt(3)*c^6*d^3*x^5*(1/(c^7*d^2))^(5/6) +
3*sqrt(3)*(5*c^4*d^2*x^4 + 8*c^5*d*x)*sqrt(1/(c^7*d^2)) - sqrt(3)*(c*d^2*x^
6 - 40*c^2*d*x^3 - 32*c^3)*(1/(c^7*d^2))^(1/6))*sqrt(d*x^3 + c) + (12*sqrt(
3)*(c^5*d^3*x^6 - c^6*d^2*x^3 - 2*c^7*d)*(1/(c^7*d^2))^(2/3) + 18*sqrt(3)*(
c^3*d^2*x^5 + c^4*d*x^2)*(1/(c^7*d^2))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x
^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^6*d^3*x^5 + 2*c^7*d^2*x^2)*(1
/(c^7*d^2))^(5/6) + 3*sqrt(3)*(7*c^4*d^2*x^4 + 4*c^5*d*x)*sqrt(1/(c^7*d^2))
 + sqrt(3)*(c*d^2*x^6 + 32*c^2*d*x^3 + 40*c^3)*(1/(c^7*d^2))^(1/6)))*sqrt((
d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^5*d^4*x^8 + 20*
```

```
c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(1/(c^7*d^2))^(2/3) - 6*sqrt(d*x^3 + c)*((c^6*
d^4*x^7 - 28*c^7*d^3*x^4 - 272*c^8*d^2*x)*(1/(c^7*d^2))^(5/6) + 4*(c^4*d^3*
x^6 + 41*c^5*d^2*x^3 + 40*c^6*d)*sqrt(1/(c^7*d^2)) - 24*(c^2*d^2*x^5 + c^3*
d*x^2)*(1/(c^7*d^2))^(1/6)) - 18*(c^3*d^3*x^7 - 52*c^4*d^2*x^4 - 80*c^5*d*x
)*(1/(c^7*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))
/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 2*c*d*(1/(c^7*d^2))^(1/6)*log((d^3*x^9
+ 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x
^5 + 64*c^7*d^2*x^2)*(1/(c^7*d^2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^4*x^7
+ 80*c^7*d^3*x^4 + 160*c^8*d^2*x)*(1/(c^7*d^2))^(5/6) + (7*c^4*d^3*x^6 + 15
2*c^5*d^2*x^3 + 64*c^6*d)*sqrt(1/(c^7*d^2)) + 6*(5*c^2*d^2*x^5 + 32*c^3*d*x
^2)*(1/(c^7*d^2))^(1/6)) + 18*(5*c^3*d^3*x^7 + 64*c^4*d^2*x^4 + 32*c^5*d*x)
*(1/(c^7*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
 2*c*d*(1/(c^7*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 +
640*c^3 + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(1/(c^7*d^2))^
(2/3) - 6*sqrt(d*x^3 + c)*((c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8*d^2*x)*(
1/(c^7*d^2))^(5/6) + (7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d)*sqrt(1/(c
^7*d^2)) + 6*(5*c^2*d^2*x^5 + 32*c^3*d*x^2)*(1/(c^7*d^2))^(1/6)) + 18*(5*c^
3*d^3*x^7 + 64*c^4*d^2*x^4 + 32*c^5*d*x)*(1/(c^7*d^2))^(1/3))/(d^3*x^9 - 24
*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + c*d*(1/(c^7*d^2))^(1/6)*log((d^3*x
^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^5*d^4*x^8 + 20*c^6*d
^3*x^5 - 8*c^7*d^2*x^2)*(1/(c^7*d^2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^4*x
^7 - 28*c^7*d^3*x^4 - 272*c^8*d^2*x)*(1/(c^7*d^2))^(5/6) + 4*(c^4*d^3*x^6 +
 41*c^5*d^2*x^3 + 40*c^6*d)*sqrt(1/(c^7*d^2)) - 24*(c^2*d^2*x^5 + c^3*d*x^2
)*(1/(c^7*d^2))^(1/6)) - 18*(c^3*d^3*x^7 - 52*c^4*d^2*x^4 - 80*c^5*d*x)*(1/
(c^7*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - c*d
*(1/(c^7*d^2))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c
^3 + 18*(c^5*d^4*x^8 + 20*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(1/(c^7*d^2))^(2/3)
- 6*sqrt(d*x^3 + c)*((c^6*d^4*x^7 - 28*c^7*d^3*x^4 - 272*c^8*d^2*x)*(1/(c^7
*d^2))^(5/6) + 4*(c^4*d^3*x^6 + 41*c^5*d^2*x^3 + 40*c^6*d)*sqrt(1/(c^7*d^2)
) - 24*(c^2*d^2*x^5 + c^3*d*x^2)*(1/(c^7*d^2))^(1/6)) - 18*(c^3*d^3*x^7 - 5
2*c^4*d^2*x^4 - 80*c^5*d*x)*(1/(c^7*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) + 72*sqrt(d)*weierstrassPInverse(0, -4*c/d, x))/(
c*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.323 $\int \frac{1}{x^3(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

```
[Out] -1/16*AppellF1(-2/3,1/2,1,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x^2
/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/16*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, (d*x^3)/(8*c), -((d*
x^3)/c)])/(c*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 (8c - dx^3) \sqrt{c + dx^3}} dx = \frac{\sqrt{1 + \frac{dx^3}{c}} \int \frac{1}{x^3(8c - dx^3)\sqrt{1+\frac{dx^3}{c}}} dx}{\sqrt{c + dx^3}}$$
$$= -\frac{\sqrt{1 + \frac{dx^3}{c}} F_1\left(-\frac{2}{3}; 1, \frac{1}{2}; \frac{1}{3}; \frac{dx^3}{8c}, -\frac{dx^3}{c}\right)}{16cx^2\sqrt{c + dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 242 vs. 2(66) = 132.
time = 20.12, size = 242, normalized size = 3.67

$$\frac{-\frac{64(c+dx^3)}{c^2} + \frac{d^2x^6\sqrt{1+\frac{dx^3}{c}} F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3} + \frac{4096dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(-8c+dx^3)\left(32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)\right)}}{1024x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] ((-64*(c + d*x^3))/c^2 + (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^3 + (4096*d*x^3*AppellF1[1/3, 1/2, 1,
 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((-8*c + d*x^3)*(32*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3
, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)]))))/(1024*x^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.40, size = 722, normalized size = 10.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/2/c*(d*x^3+c)^(1/2)/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/216*I/d^2/c^2*2^(1/2)*sum(1/_alpha^2*(-c*d^
2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
```

```
c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1
/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d
^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2574 vs. $2(52) = 104$.
time = 6.63, size = 2574, normalized size = 39.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3456*(4*sqrt(3)*c^2*x^2*(d^4/c^13)^(1/6)*arctan(1/9*((9*sqrt(3)*c^11*d^4*
x^5*(d^4/c^13)^(5/6) + 3*sqrt(3)*(5*c^7*d^5*x^4 + 8*c^8*d^4*x)*sqrt(d^4/c^1
3) - sqrt(3)*(c^2*d^7*x^6 - 40*c^3*d^6*x^3 - 32*c^4*d^5)*(d^4/c^13)^(1/6))*
sqrt(d*x^3 + c) - (12*sqrt(3)*(c^9*d^2*x^6 - c^10*d*x^3 - 2*c^11)*(d^4/c^13
)^(2/3) + 18*sqrt(3)*(c^5*d^3*x^5 + c^6*d^2*x^2)*(d^4/c^13)^(1/3) + 3*sqrt(
3)*(d^5*x^7 + 5*c*d^4*x^4 + 4*c^2*d^3*x) - sqrt(d*x^3 + c)*(9*sqrt(3)*(c^11
*d*x^5 + 2*c^12*x^2)*(d^4/c^13)^(5/6) + 3*sqrt(3)*(7*c^7*d^2*x^4 + 4*c^8*d*
x)*sqrt(d^4/c^13) + sqrt(3)*(c^2*d^4*x^6 + 32*c^3*d^3*x^3 + 40*c^4*d^2)*(d^
4/c^13)^(1/6)))*sqrt((d^9*x^9 - 276*c*d^8*x^6 - 1608*c^2*d^7*x^3 - 1088*c^3
*d^6 + 18*(c^9*d^6*x^8 + 20*c^10*d^5*x^5 - 8*c^11*d^4*x^2)*(d^4/c^13)^(2/3)
 + 6*sqrt(d*x^3 + c)*((c^11*d^5*x^7 - 28*c^12*d^4*x^4 - 272*c^13*d^3*x)*(d^
4/c^13)^(5/6) + 4*(c^7*d^6*x^6 + 41*c^8*d^5*x^3 + 40*c^9*d^4)*sqrt(d^4/c^13
) - 24*(c^3*d^7*x^5 + c^4*d^6*x^2)*(d^4/c^13)^(1/6)) - 18*(c^5*d^7*x^7 - 52
*c^6*d^6*x^4 - 80*c^7*d^5*x)*(d^4/c^13)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 19
2*c^2*d*x^3 - 512*c^3)))/(d^8*x^7 - 7*c*d^7*x^4 - 8*c^2*d^6*x)) + 4*sqrt(3)
*c^2*x^2*(d^4/c^13)^(1/6)*arctan(1/9*((9*sqrt(3)*c^11*d^4*x^5*(d^4/c^13)^(5
/6) + 3*sqrt(3)*(5*c^7*d^5*x^4 + 8*c^8*d^4*x)*sqrt(d^4/c^13) - sqrt(3)*(c^2
*d^7*x^6 - 40*c^3*d^6*x^3 - 32*c^4*d^5)*(d^4/c^13)^(1/6))*sqrt(d*x^3 + c) +
```

```
(12*sqrt(3)*(c^9*d^2*x^6 - c^10*d*x^3 - 2*c^11)*(d^4/c^13)^(2/3) + 18*sqrt
(3)*(c^5*d^3*x^5 + c^6*d^2*x^2)*(d^4/c^13)^(1/3) + 3*sqrt(3)*(d^5*x^7 + 5*c
*d^4*x^4 + 4*c^2*d^3*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^11*d*x^5 + 2*c^12*x
^2)*(d^4/c^13)^(5/6) + 3*sqrt(3)*(7*c^7*d^2*x^4 + 4*c^8*d*x)*sqrt(d^4/c^13)
 + sqrt(3)*(c^2*d^4*x^6 + 32*c^3*d^3*x^3 + 40*c^4*d^2)*(d^4/c^13)^(1/6)))*s
qrt((d^9*x^9 - 276*c*d^8*x^6 - 1608*c^2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^9*d^
6*x^8 + 20*c^10*d^5*x^5 - 8*c^11*d^4*x^2)*(d^4/c^13)^(2/3) - 6*sqrt(d*x^3 +
 c)*((c^11*d^5*x^7 - 28*c^12*d^4*x^4 - 272*c^13*d^3*x)*(d^4/c^13)^(5/6) + 4
*(c^7*d^6*x^6 + 41*c^8*d^5*x^3 + 40*c^9*d^4)*sqrt(d^4/c^13) - 24*(c^3*d^7*x
^5 + c^4*d^6*x^2)*(d^4/c^13)^(1/6)) - 18*(c^5*d^7*x^7 - 52*c^6*d^6*x^4 - 80
*c^7*d^5*x)*(d^4/c^13)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512
*c^3)))/(d^8*x^7 - 7*c*d^7*x^4 - 8*c^2*d^6*x)) + c^2*x^2*(d^4/c^13)^(1/6)*l
og((d^9*x^9 - 276*c*d^8*x^6 - 1608*c^2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^9*d^6
*x^8 + 20*c^10*d^5*x^5 - 8*c^11*d^4*x^2)*(d^4/c^13)^(2/3) + 6*sqrt(d*x^3 +
c)*((c^11*d^5*x^7 - 28*c^12*d^4*x^4 - 272*c^13*d^3*x)*(d^4/c^13)^(5/6) + 4*
(c^7*d^6*x^6 + 41*c^8*d^5*x^3 + 40*c^9*d^4)*sqrt(d^4/c^13) - 24*(c^3*d^7*x^
5 + c^4*d^6*x^2)*(d^4/c^13)^(1/6)) - 18*(c^5*d^7*x^7 - 52*c^6*d^6*x^4 - 80*
c^7*d^5*x)*(d^4/c^13)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)) - c^2*x^2*(d^4/c^13)^(1/6)*log((d^9*x^9 - 276*c*d^8*x^6 - 1608*c^2*d^
7*x^3 - 1088*c^3*d^6 + 18*(c^9*d^6*x^8 + 20*c^10*d^5*x^5 - 8*c^11*d^4*x^2)*
(d^4/c^13)^(2/3) - 6*sqrt(d*x^3 + c)*((c^11*d^5*x^7 - 28*c^12*d^4*x^4 - 272
*c^13*d^3*x)*(d^4/c^13)^(5/6) + 4*(c^7*d^6*x^6 + 41*c^8*d^5*x^3 + 40*c^9*d^
4)*sqrt(d^4/c^13) - 24*(c^3*d^7*x^5 + c^4*d^6*x^2)*(d^4/c^13)^(1/6)) - 18*(
c^5*d^7*x^7 - 52*c^6*d^6*x^4 - 80*c^7*d^5*x)*(d^4/c^13)^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*c^2*x^2*(d^4/c^13)^(1/6)*log((d
^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + 18*(c^9*d^3*x^8 +
 38*c^10*d^2*x^5 + 64*c^11*d*x^2)*(d^4/c^13)^(2/3) + 6*sqrt(d*x^3 + c)*((c^
11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x)*(d^4/c^13)^(5/6) + (7*c^7*d^3*x^6
+ 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(d^4/c^13) + 6*(5*c^3*d^4*x^5 + 32*c^4*d^
3*x^2)*(d^4/c^13)^(1/6)) + 18*(5*c^5*d^4*x^7 + 64*c^6*d^3*x^4 + 32*c^7*d^2*
x)*(d^4/c^13)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
2*c^2*x^2*(d^4/c^13)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3
+ 640*c^3*d^3 + 18*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2)*(d^4/c^1
3)^(2/3) - 6*sqrt(d*x^3 + c)*((c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x)*(
d^4/c^13)^(5/6) + (7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(d^4/c^1
3) + 6*(5*c^3*d^4*x^5 + 32*c^4*d^3*x^2)*(d^4/c^13)^(1/6)) + 18*(5*c^5*d^4*x
^7 + 64*c^6*d^3*x^4 + 32*c^7*d^2*x)*(d^4/c^13)^(1/3))/(d^3*x^9 - 24*c*d^2*x
^6 + 192*c^2*d*x^3 - 512*c^3)) - 144*sqrt(d)*x^2*weierstrassPInverse(0, -4*
c/d, x) - 216*sqrt(d*x^3 + c))/(c^2*x^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^3\sqrt{c+dx^3}+dx^6\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(1/(-8*c*x**3*sqrt(c + d*x**3) + d*x**6*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.324 $\int \frac{1}{x^6(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{5}{3};1,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

```
[Out] -1/40*AppellF1(-5/3,1/2,1,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x^
5/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};1,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/40*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 1/2, -2/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c*x^5*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^6 (8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{5}{3};1,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(66) = 132.
time = 20.14, size = 261, normalized size = 3.95

$$\frac{-23d^3x^9\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+64c\left(-16c^2+7cdx^3+23d^2x^6+\frac{3264c^2d^2x^6F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)\right)}\right)}{40960c^4x^5\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-23*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + 64*c*(-16*c^2 + 7*c*d*x^3 + 23*d^2*x^6 + (3264*c^2*d^2*x^6*
AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32
*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(Appel
lF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1
, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/(40960*c^4*x^5*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.36, size = 1047, normalized size = 15.86 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/5/c*(d*x^3+c)^(1/2)/x^5+7/20*d/c^2*(d*x^3+c)^(1/2)/x^2-7/60*I/c^2
*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/64*d/c^
2*(-1/2/c*(d*x^3+c)^(1/2)/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
```

```
1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2))-1/1728*I/c^3/d*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1
/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d
-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(
2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2603 vs. $2(52) = 104$.
time = 9.51, size = 2603, normalized size = 39.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/138240*(20*sqrt(3)*c^3*x^5*(d^10/c^19)^(1/6)*arctan(1/9*((9*sqrt(3)*c^16*
d^9*x^5*(d^10/c^19)^(5/6) + 3*sqrt(3)*(5*c^10*d^12*x^4 + 8*c^11*d^11*x)*sqr
t(d^10/c^19) - sqrt(3)*(c^3*d^16*x^6 - 40*c^4*d^15*x^3 - 32*c^5*d^14)*(d^10
/c^19)^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c^13*d^3*x^6 - c^14*d^2*x^3 -
2*c^15*d)*(d^10/c^19)^(2/3) + 18*sqrt(3)*(c^7*d^6*x^5 + c^8*d^5*x^2)*(d^10/
c^19)^(1/3) + 3*sqrt(3)*(d^10*x^7 + 5*c*d^9*x^4 + 4*c^2*d^8*x) - sqrt(d*x^3
 + c)*(9*sqrt(3)*(c^16*d*x^5 + 2*c^17*x^2)*(d^10/c^19)^(5/6) + 3*sqrt(3)*(7
*c^10*d^4*x^4 + 4*c^11*d^3*x)*sqrt(d^10/c^19) + sqrt(3)*(c^3*d^8*x^6 + 32*c
^4*d^7*x^3 + 40*c^5*d^6)*(d^10/c^19)^(1/6)))*sqrt((d^19*x^9 - 276*c*d^18*x^
6 - 1608*c^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^13*d^12*x^8 + 20*c^14*d^11*x^
5 - 8*c^15*d^10*x^2)*(d^10/c^19)^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^10*x^7
```

```
- 28*c^17*d^9*x^4 - 272*c^18*d^8*x)*(d^10/c^19)^(5/6) + 4*(c^10*d^13*x^6 +
41*c^11*d^12*x^3 + 40*c^12*d^11)*sqrt(d^10/c^19) - 24*(c^4*d^16*x^5 + c^5*d
^15*x^2)*(d^10/c^19)^(1/6)) - 18*(c^7*d^15*x^7 - 52*c^8*d^14*x^4 - 80*c^9*d
^13*x)*(d^10/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)))/(d^18*x^7 - 7*c*d^17*x^4 - 8*c^2*d^16*x)) + 20*sqrt(3)*c^3*x^5*(d^10/c^
19)^(1/6)*arctan(1/9*((9*sqrt(3)*c^16*d^9*x^5*(d^10/c^19)^(5/6) + 3*sqrt(3)
*(5*c^10*d^12*x^4 + 8*c^11*d^11*x)*sqrt(d^10/c^19) - sqrt(3)*(c^3*d^16*x^6
- 40*c^4*d^15*x^3 - 32*c^5*d^14)*(d^10/c^19)^(1/6))*sqrt(d*x^3 + c) + (12*s
qrt(3)*(c^13*d^3*x^6 - c^14*d^2*x^3 - 2*c^15*d)*(d^10/c^19)^(2/3) + 18*sqrt
(3)*(c^7*d^6*x^5 + c^8*d^5*x^2)*(d^10/c^19)^(1/3) + 3*sqrt(3)*(d^10*x^7 + 5
*c*d^9*x^4 + 4*c^2*d^8*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^16*d*x^5 + 2*c^17
*x^2)*(d^10/c^19)^(5/6) + 3*sqrt(3)*(7*c^10*d^4*x^4 + 4*c^11*d^3*x)*sqrt(d^
10/c^19) + sqrt(3)*(c^3*d^8*x^6 + 32*c^4*d^7*x^3 + 40*c^5*d^6)*(d^10/c^19)^
(1/6)))*sqrt((d^19*x^9 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^3 - 1088*c^3*d^16
 + 18*(c^13*d^12*x^8 + 20*c^14*d^11*x^5 - 8*c^15*d^10*x^2)*(d^10/c^19)^(2/3
) - 6*sqrt(d*x^3 + c)*((c^16*d^10*x^7 - 28*c^17*d^9*x^4 - 272*c^18*d^8*x)*(
d^10/c^19)^(5/6) + 4*(c^10*d^13*x^6 + 41*c^11*d^12*x^3 + 40*c^12*d^11)*sqrt
(d^10/c^19) - 24*(c^4*d^16*x^5 + c^5*d^15*x^2)*(d^10/c^19)^(1/6)) - 18*(c^7
*d^15*x^7 - 52*c^8*d^14*x^4 - 80*c^9*d^13*x)*(d^10/c^19)^(1/3))/(d^3*x^9 -
24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^18*x^7 - 7*c*d^17*x^4 - 8*c^2*
d^16*x)) + 5*c^3*x^5*(d^10/c^19)^(1/6)*log((d^19*x^9 - 276*c*d^18*x^6 - 160
8*c^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^13*d^12*x^8 + 20*c^14*d^11*x^5 - 8*c
^15*d^10*x^2)*(d^10/c^19)^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^10*x^7 - 28*c^
17*d^9*x^4 - 272*c^18*d^8*x)*(d^10/c^19)^(5/6) + 4*(c^10*d^13*x^6 + 41*c^11
*d^12*x^3 + 40*c^12*d^11)*sqrt(d^10/c^19) - 24*(c^4*d^16*x^5 + c^5*d^15*x^2
)*(d^10/c^19)^(1/6)) - 18*(c^7*d^15*x^7 - 52*c^8*d^14*x^4 - 80*c^9*d^13*x)*
(d^10/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*
c^3*x^5*(d^10/c^19)^(1/6)*log((d^19*x^9 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^
3 - 1088*c^3*d^16 + 18*(c^13*d^12*x^8 + 20*c^14*d^11*x^5 - 8*c^15*d^10*x^2)
*(d^10/c^19)^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^10*x^7 - 28*c^17*d^9*x^4 -
272*c^18*d^8*x)*(d^10/c^19)^(5/6) + 4*(c^10*d^13*x^6 + 41*c^11*d^12*x^3 + 4
0*c^12*d^11)*sqrt(d^10/c^19) - 24*(c^4*d^16*x^5 + c^5*d^15*x^2)*(d^10/c^19)
^(1/6)) - 18*(c^7*d^15*x^7 - 52*c^8*d^14*x^4 - 80*c^9*d^13*x)*(d^10/c^19)^(
1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 10*c^3*x^5*(d^1
0/c^19)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d
^8 + 18*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2)*(d^10/c^19)^(2/3
) + 6*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x)*(d^10/c^
19)^(5/6) + (7*c^10*d^5*x^6 + 152*c^11*d^4*x^3 + 64*c^12*d^3)*sqrt(d^10/c^1
9) + 6*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2)*(d^10/c^19)^(1/6)) + 18*(5*c^7*d^7*
x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x)*(d^10/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2
*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 10*c^3*x^5*(d^10/c^19)^(1/6)*log((d^11*x
^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + 18*(c^13*d^4*x^8 + 3
8*c^14*d^3*x^5 + 64*c^15*d^2*x^2)*(d^10/c^19)^(2/3) - 6*sqrt(d*x^3 + c)*((c
^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x)*(d^10/c^19)^(5/6) + (7*c^10*d^5*x
^6 + 152*c^11*d^4*x^3 + 64*c^12*d^3)*sqrt(d^10/c^19) + 6*(5*c^4*d^8*x^5 + 3
```

```
2*c^5*d^7*x^2)*(d^10/c^19)^(1/6)) + 18*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32
*c^9*d^5*x)*(d^10/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 51
2*c^3)) + 5328*d^(3/2)*x^5*weierstrassPInverse(0, -4*c/d, x) + 216*(23*d*x^
3 - 16*c)*sqrt(d*x^3 + c))/(c^3*x^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^6\sqrt{c+dx^3}+dx^9\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(1/(-8*c*x**6*sqrt(c + d*x**3) + d*x**9*sqrt(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^6\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.325 $\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2c^2}{27d^4\sqrt{c+dx^3}}-\frac{4c\sqrt{c+dx^3}}{d^4}-\frac{2(c+dx^3)^{3/2}}{9d^4}+\frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^4+1024/81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2
))/d^4+2/27*c^2/d^4/(d*x^3+c)^(1/2)-4*c*(d*x^3+c)^(1/2)/d^4
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 89, 45, 65, 212}

$$\frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}+\frac{2c^2}{27d^4\sqrt{c+dx^3}}-\frac{4c\sqrt{c+dx^3}}{d^4}-\frac{2(c+dx^3)^{3/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*c^2)/(27*d^4*Sqrt[c + d*x^3]) - (4*c*Sqrt[c + d*x^3])/d^4 - (2*(c + d*x^
3)^(3/2))/(9*d^4) + (1024*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81
*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
```

```
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{c^2}{9d^3(c+dx)^{3/2}} - \frac{7c}{d^3\sqrt{c+dx}} - \frac{x}{d^2\sqrt{c+dx}} + \frac{51\cdots}{9d^3(8c-d\cdots}\right.\right.$$

$$= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{14c\sqrt{c+dx^3}}{3d^4} + \frac{(512c^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, \cdots\right.}{27d^3}$$

$$= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{14c\sqrt{c+dx^3}}{3d^4} + \frac{(1024c^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\cdots\right.}{27d^4}$$

$$= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{4c\sqrt{c+dx^3}}{d^4} - \frac{2(c+dx^3)^{3/2}}{9d^4} + \frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\cdots}}{3\sqrt{\cdots}}\right.}{81d^4}$$

Mathematica [A]
time = 0.08, size = 70, normalized size = 0.78

$$\frac{2\left(-\frac{3(56c^2+60cdx^3+3d^2x^6)}{\sqrt{c+dx^3}} + 512c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*((-3*(56*c^2 + 60*c*d*x^3 + 3*d^2*x^6))/Sqrt[c + d*x^3] + 512*c^(3/2)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 560, normalized size = 6.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-2/3/d^3*c^2/((x^3+c/d)*d)^(1/2)+2/9/d^2*x^3*(d*x^3+c)^(1/2)-10/9*c*(
d*x^3+c)^(1/2)/d^3)-8/d^2*c*(2/3/d^2*c/((x^3+c/d)*d)^(1/2)+2/3*(d*x^3+c)^(1
/2)/d^2)+128/3*c^2/d^4/(d*x^3+c)^(1/2)-512*c^3/d^3*(2/27/d/c/((x^3+c/d)*d)^
(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2
)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_a
lpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [A]
time = 0.49, size = 82, normalized size = 0.91

$$-\frac{2\left(256\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+9\,(dx^3+c)^{\frac{3}{2}}+162\,\sqrt{dx^3+c}\,c-\frac{3\,c^2}{\sqrt{dx^3+c}}\right)}{81\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/81*(256*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 9*(d*x^3 + c)^(3/2) + 162*sqrt(d*x^3 + c)*c - 3*c^2/sqrt(d*x^3 +
 c))/d^4
```

Fricas [A]
time = 3.49, size = 189, normalized size = 2.10

$$\left[\frac{2\left(256\,(cdx^3+c^2)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(3\,d^2x^6+60\,cdx^3+56\,c^2)\sqrt{dx^3+c}\right)}{81\,(d^5x^3+cd^4)},\ -\frac{2\left(512\,(cdx^3+c^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\,(3\,d^2x^6+60\,cdx^3+56\,c^2)\sqrt{dx^3+c}\right)}{81\,(d^5x^3+cd^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(256*(c*d*x^3 + c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) - 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 + c)
)/(d^5*x^3 + c*d^4), -2/81*(512*(c*d*x^3 + c^2)*sqrt(-c)*arctan(1/3*sqrt(d*
x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 + c))
/(d^5*x^3 + c*d^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.63, size = 82, normalized size = 0.91

$$-\frac{1024\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\,d^4}+\frac{2\,c^2}{27\sqrt{dx^3+c}\,d^4}-\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^8+18\sqrt{dx^3+c}\,cd^8\right)}{9\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1024/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/27*c^2
/(sqrt(d*x^3 + c)*d^4) - 2/9*((d*x^3 + c)^(3/2)*d^8 + 18*sqrt(d*x^3 + c)*c*
d^8)/d^12
```

Mupad [**B**]
time = 3.78, size = 95, normalized size = 1.06

$$\frac{512\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^4}-\frac{38\,c\,\sqrt{d\,x^3+c}}{9\,d^4}+\frac{2\,c^2}{27\,d^4\,\sqrt{d\,x^3+c}}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (512*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(81*d^4) - (38*c*(c + d*x^3)^(1/2))/(9*d^4) + (2*c^2)/(27*d^4*(c + d*x^3
)^(1/2)) - (2*x^3*(c + d*x^3)^(1/2))/(9*d^3)
```

3.326 $\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

```
[Out] 128/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^3-2/27*c/d^3/(d*x^3+c
)^(1/2)-2/3*(d*x^3+c)^(1/2)/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {457, 89, 65, 212}

$$-\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*c)/(27*d^3*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(3*d^3) + (128*Sqrt[c
]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{9d^2(c+dx)^{3/2}} - \frac{1}{d^2\sqrt{c+dx}} + \frac{64c}{9d^2(8c-dx)\sqrt{c+dx}}\right) dx\right.\\
&= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{(64c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right.}{27d^2}\\
&= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{(128c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^3}\\
&= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{128\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 59, normalized size = 0.83

$$\frac{2\left(-\frac{3(10c+9dx^3)}{\sqrt{c+dx^3}} + 64\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*((-3*(10*c + 9*d*x^3))/Sqrt[c + d*x^3] + 64*Sqrt[c]*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])]))/(81*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.42, size = 498, normalized size = 7.01 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/3/d^2*c/((x^3+c/d)*d)^(1/2)+2/3*(d*x^3+c)^(1/2)/d^2)+16/3*c/d^3/(d*
x^3+c)^(1/2)-64*c^2/d^2*(2/27/d/c/((x^3+c/d)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/
2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))
```

Maxima [A]
time = 0.49, size = 68, normalized size = 0.96

$$-\frac{2\left(32\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+27\sqrt{dx^3+c}+\frac{3c}{\sqrt{dx^3+c}}\right)}{81d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/81*(32*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 27*sqrt(d*x^3 + c) + 3*c/sqrt(d*x^3 + c))/d^3
```

Fricas [A]
time = 2.48, size = 161, normalized size = 2.27

$$\left[\frac{2\left(32(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-3(9dx^3+10c)\sqrt{dx^3+c}\right)}{81(d^4x^3+cd^3)}, -\frac{2\left(64(dx^3+c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3(9dx^3+10c)\sqrt{dx^3+c}\right)}{81(d^4x^3+cd^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(32*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 3*(9*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3), -
2/81*(64*(d*x^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(9
*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.75, size = 58, normalized size = 0.82

$$-\frac{128\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{81\,\sqrt{-c}\,d^3}-\frac{2\,\sqrt{dx^3+c}}{3\,d^3}-\frac{2\,c}{27\,\sqrt{dx^3+c}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -128/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/3*sqrt(d*
x^3 + c)/d^3 - 2/27*c/(sqrt(d*x^3 + c)*d^3)
```

Mupad [B]
time = 3.71, size = 75, normalized size = 1.06

$$\frac{64\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^3}-\frac{2\,c}{27\,d^3\,\sqrt{d\,x^3+c}}-\frac{2\,\sqrt{d\,x^3+c}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (64*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/(81*d^3) - (2*c)/(27*d^3*(c + d*x^3)^(1/2)) - (2*(c + d*x^3)^(1/2))/(3*d^
3)
```

3.327 $\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}} dx$

Optimal. Leaf size=52

$$\frac{2}{27d^2\sqrt{c+dx^3}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

```
[Out] 16/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2/c^(1/2)+2/27/d^2/(d*x^3+c)^(
1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {457, 79, 65, 212}

$$\frac{2}{27d^2\sqrt{c+dx^3}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*d^2*Sqrt[c + d*x^3]) + (16*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
Sqrt[c]*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx &= \frac{1}{3}\mathrm{Subst}\left(\int \frac{x}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{8\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d}\\
&= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^2}\\
&= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.94

$$\frac{2\left(\frac{3}{\sqrt{c+dx^3}}+\frac{8\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{81d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*(3/Sqrt[c + d*x^3] + (8*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/
(81*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.35, size = 456, normalized size = 8.77

method	result
elliptic	$\frac{2}{27d^2\sqrt{\left(x^3+\frac{c}{d}\right)d}} - \frac{8i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)} \frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d}{-3(-c}}}{}\right.}{}$

default	$\frac{2}{3d^2\sqrt{d\,x^3+c}}-\frac{8c\left(\frac{2}{27dc\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+}{d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d^2/(d*x^3+c)^(1/2)-8*c/d*(2/27/d/c/((x^3+c/d)*d)^(1/2)+1/243*I/d^3/c^2
*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/
d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
```

```
_Z^3*d-8*c)))
```

Maxima [A]
time = 0.49, size = 56, normalized size = 1.08

$$-\frac{2\left(\frac{4\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}}-\frac{3}{\sqrt{dx^3+c}}\right)}{81\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -2/81*(4*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/s
qrt(c) - 3/sqrt(d*x^3 + c))/d^2
```

Fricas [A]
time = 3.06, size = 149, normalized size = 2.87

$$\left[\frac{2\left(4\,(dx^3+c)\sqrt{c}\,\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\sqrt{dx^3+c}\,c\right)}{81\,(cd^3x^3+c^2d^2)},-\frac{2\left(8\,(dx^3+c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)-3\sqrt{dx^3+c}\,c\right)}{81\,(cd^3x^3+c^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] [2/81*(4*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) + 3*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 + c^2*d^2), -2/81*(8*(d*x
^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c)
*c)/(c*d^3*x^3 + c^2*d^2)]
```

Sympy [A]
time = 10.20, size = 58, normalized size = 1.12

$$\begin{cases}\frac{2\cdot\left(\frac{1}{27d\sqrt{c+dx^3}}-\frac{8\,\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81d\sqrt{-c}}\right)}{d} & \text{for } d\neq 0\\ \frac{x^6}{48c^{\frac{5}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] Piecewise((2*(1/(27*d*sqrt(c + d*x**3)) - 8*atan(sqrt(c + d*x**3)/(3*sqrt(-
c)))/(81*d*sqrt(-c)))/d, Ne(d, 0)), (x**6/(48*c**(5/2)), True))
```

Giac [A]
time = 1.30, size = 47, normalized size = 0.90

$$-\frac{2\left(\frac{8\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d}-\frac{3}{\sqrt{dx^3+c}\,d}\right)}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/81*(8*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 3/(sqrt(d*x^3
+ c)*d))/d
```

Mupad [B]
time = 3.68, size = 60, normalized size = 1.15

$$\frac{2}{27\,d^2\,\sqrt{d\,x^3+c}}+\frac{8\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] 2/(27*d^2*(c + d*x^3)^(1/2)) + (8*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)
^(1/2))/(8*c - d*x^3)))/(81*c^(1/2)*d^2)
```

3.328 $\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{2}{27cd\sqrt{c+dx^3}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

```
[Out] 2/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d-2/27/c/d/(d*x^3+c)^(1/2
)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {455, 53, 65, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}-\frac{2}{27cd\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(27*c*d*Sqrt[c + d*x^3]) + (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
c^(3/2)*d)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)\\
&= -\frac{2}{27cd\sqrt{c+dx^3}}+\frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27c}\\
&= -\frac{2}{27cd\sqrt{c+dx^3}}+\frac{2\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27cd}\\
&= -\frac{2}{27cd\sqrt{c+dx^3}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}
\end{aligned}
$$

Mathematica [**A**]
time = 0.06, size = 52, normalized size = 0.95

$$
\frac{2\left(-\frac{3\sqrt{c}}{\sqrt{c+dx^3}}+\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81c^{3/2}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*((-3*Sqrt[c])/Sqrt[c + d*x^3] + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(
81*c^(3/2)*d)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.31, size = 435, normalized size = 7.91

method	result
default	$-\frac{2}{27dc\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-3(-c\cdots}}\cdots}{\cdots}\right)}{\cdots}$
elliptic	$-\frac{2}{27dc\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-3(-c\cdots}}\cdots}{\cdots}\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/27/d/c/((x^3+c/d)*d)^(1/2)-1/243*I/d^3/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3
)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*
```

```
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_
alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.49, size = 58, normalized size = 1.05

$$-\frac{\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{6}{\sqrt{dx^3+c}\,c}}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/81*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(
3/2) + 6/(sqrt(d*x^3 + c)*c))/d
```

Fricas [A]
time = 2.73, size = 147, normalized size = 2.67

$$\left[\frac{(dx^3+c)\sqrt{c}\,\log\left(\frac{dx^3+6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\sqrt{dx^3+c}\,c}{81\,(c^2d^2x^3+c^3d)},\,-\frac{2\left((dx^3+c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\sqrt{dx^3+c}\,c\right)}{81\,(c^2d^2x^3+c^3d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/81*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(
d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 + c^3*d), -2/81*((d*x^3 +
 c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 + c)*c)/
(c^2*d^2*x^3 + c^3*d)]
```

Sympy [A]
time = 8.37, size = 51, normalized size = 0.93

$$-\frac{2}{27cd\sqrt{c+dx^3}}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81cd\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -2/(27*c*d*sqrt(c + d*x**3)) - 2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(81*c*
d*sqrt(-c))
```

Giac [A]
time = 1.50, size = 48, normalized size = 0.87

$$-\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\,cd}-\frac{2}{27\sqrt{dx^3+c}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 2/27/(sqrt(d*x^
3 + c)*c*d)
```

Mupad [B]
time = 3.63, size = 63, normalized size = 1.15

$$\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,c^{3/2}\,d}-\frac{2}{27\,c\,d\,\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(81*c^(3/2)
*d) - 2/(27*c*d*(c + d*x^3)^(1/2))
```

3.329 $\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{2}{27c^2\sqrt{c+dx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

```
[Out] 1/324*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/12*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(5/2)+2/27/c^2/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {457, 87, 162, 65, 214, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}+\frac{2}{27c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(324*c^(5
/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right) \\
&= \frac{2}{27c^2\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-9cd+d^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27c^2d} \\
&= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2} + \frac{d\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+\cdots}}\right.}{216c^2} \\
&= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{108c^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, \cdots\right.}{12c^2d} \\
&= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.07, size = 69, normalized size = 0.91

$$\frac{\frac{24\sqrt{c}}{\sqrt{c+dx^3}}+\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-27\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{324c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((24*Sqrt[c])/Sqrt[c + d*x^3] + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 27*A
rcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(324*c^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 485, normalized size = 6.38

method	result
default	$-\frac{d\left(\frac{2}{27dc\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-3(\cdots}}\cdots}{\cdots}\right.}{\cdots}\right)}{\cdots}$

elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d/c*(2/27/d/c/((x^3+c/d)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2
)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2
)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/8/c*(
2/3/c/((x^3+c/d)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x), x)
```

Fricas [A]
time = 3.19, size = 213, normalized size = 2.80

$$\left[\frac{(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+27(dx^3+c)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+48\sqrt{dx^3+c}\,c}{648(c^3dx^3+c^4)}, \frac{27(dx^3+c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-(dx^3+c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+24\sqrt{dx^3+c}\,c}{324(c^3dx^3+c^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/648*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/
(d*x^3 - 8*c)) + 27*(d*x^3 + c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt
(c) + 2*c)/x^3) + 48*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c^4), 1/324*(27*(d*x^3
 + c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d*x^3 + c)*sqrt(-c)*ar
ctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c
^4)]
```

Sympy [A]
time = 6.20, size = 78, normalized size = 1.03

$$\frac{2}{27c^2\sqrt{c+dx^3}} - \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{324c^2\sqrt{-c}} + \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] 2/(27*c**2*sqrt(c + d*x**3)) - atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(324*c**
2*sqrt(-c)) + atan(sqrt(c + d*x**3)/sqrt(-c))/(12*c**2*sqrt(-c))
```

Giac [A]
time = 0.90, size = 68, normalized size = 0.89

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}\,c^2} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{324\sqrt{-c}\,c^2} + \frac{2}{27\sqrt{dx^3+c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/324*arctan(1/3*sqr
t(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + 2/27/(sqrt(d*x^3 + c)*c^2)
```

Mupad [B]
time = 3.66, size = 68, normalized size = 0.89

$$\frac{2}{27\,c^2\,\sqrt{d\,x^3+c}} - \frac{\operatorname{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{12\,\sqrt{c^5}} + \frac{\operatorname{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)}{324\,\sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] 2/(27*c^2*(c + d*x^3)^(1/2)) - atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/(
12*(c^5)^(1/2)) + atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))/(324*(c^5)
^(1/2))
```

3.330 $\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{25d}{216c^3\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\sqrt{c+dx^3}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

```
[Out] 1/2592*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+11/96*d*arctanh((d*x^
3+c)^(1/2)/c^(1/2))/c^(7/2)-25/216*d/c^3/(d*x^3+c)^(1/2)-1/24/c^2/x^3/(d*x^
3+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {457, 105, 157, 162, 65, 214, 212}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}-\frac{25d}{216c^3\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-25*d)/(216*c^3*Sqrt[c + d*x^3]) - 1/(24*c^2*x^3*Sqrt[c + d*x^3]) + (d*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2592*c^(7/2)) + (11*d*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(96*c^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(8c-dx)(c+dx)^{3/2}} \, dx, x, x^3\right)$$

$$= -\frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{11cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}} \, dx, x, x^3\right)}{24c^2}$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{99c^2d^2}{2}-\frac{25}{4}cd^3x}{x(8c-dx)\sqrt{c+dx}} \, dx, x\right.}{108c^4d}$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{(11d)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} \, dx, x,\right.}{192c^3}$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{11\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} \, dx, x, \sqrt{c+d}\right.}{96c^3}$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}} + \frac{11d\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.13, size = 85, normalized size = 0.85

$$\frac{-\frac{12\sqrt{c}\,(9c+25dx^3)}{x^3\sqrt{c+dx^3}} + d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 297d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2592c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((-12*Sqrt[c]*(9*c + 25*d*x^3))/(x^3*Sqrt[c + d*x^3]) + d*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])] + 297*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(7/2)
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 549, normalized size = 5.49

method	result

risch	$-\frac{\sqrt{d x^3+c}}{24 c^3 x^3} - \frac{d \left(-\frac{11 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{6\sqrt{c}} + \frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)} \frac{\left(-c d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c d^2 \ldots\right.}{\ldots}\right.}{\left(-c d^2\right)\ldots}}}{\ldots}\right.}{\ldots}\right.}{\ldots}$

default	$-\frac{d^{2}\left(\frac{2}{27dc\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-3(-}}}{}\right)}{}\right)}{}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/c^2*d^2*(2/27/d/c/((x^3+c/d)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-c
*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)
*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-
c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(
```

```
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/
8/c*(-2/3*d/c^2/((x^3+c/d)*d)^(1/2)-1/3*(d*x^3+c)^(1/2)/c^2/x^3+d*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+1/64*d/c^2*(2/3/c/((x^3+c/d)*d)^(1/2)-2/3*
arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^4), x)
```

Fricas [A]
time = 2.45, size = 272, normalized size = 2.72

$$\left[\frac{\left(d^{2} x^{6}+c d x^{3}\right) \sqrt{c} \log \left(\frac{d x^{3}+6 \sqrt{d x^{3}+c} \sqrt{c}+10 c}{d x^{3}-8 c}\right)+297\left(d^{2} x^{6}+c d x^{3}\right) \sqrt{c} \log \left(\frac{d x^{3}+2 \sqrt{d x^{3}+c} \sqrt{c}+2 c}{x^{3}}\right)-24\left(25 c d x^{3}+9 c^{2}\right) \sqrt{d x^{3}+c}}{5184\left(c^{4} d x^{6}+c^{5} x^{3}\right)},-\frac{297\left(d^{2} x^{6}+c d x^{3}\right) \sqrt{-c} \arctan \left(\frac{\sqrt{d x^{3}+c} \sqrt{-c}}{c}\right)+\left(d^{2} x^{6}+c d x^{3}\right) \sqrt{-c} \arctan \left(\frac{\sqrt{d x^{3}+c} \sqrt{-c}}{3 c}\right)+12\left(25 c d x^{3}+9 c^{2}\right) \sqrt{d x^{3}+c}}{2592\left(c^{4} d x^{6}+c^{5} x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5184*((d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 297*(d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 2*sq
rt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c)
)/(c^4*d*x^6 + c^5*x^3), -1/2592*(297*(d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d
*x^3 + c)*sqrt(-c)/c) + 12*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^6
 + c^5*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^{2}x^{4}\sqrt{c+dx^{3}}-7cdx^{7}\sqrt{c+dx^{3}}+d^{2}x^{10}\sqrt{c+dx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**4*sqrt(c + d*x**3) - 7*c*d*x**7*sqrt(c + d*x**3) +
d**2*x**10*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 0.79, size = 100, normalized size = 1.00

$$-\frac{11\,d\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}\,c^{3}}-\frac{d\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\sqrt{-c}}\right)}{2592\sqrt{-c}\,c^{3}}-\frac{25\,(dx^{3}+c)d-16\,cd}{216\left((dx^{3}+c)^{\frac{3}{2}}-\sqrt{dx^{3}+c}\,c\right)c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -11/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2592*d*arctan(
1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/216*(25*(d*x^3 + c)*d - 16
*c*d)/(((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)*c^3)
```

Mupad [B]
time = 3.80, size = 88, normalized size = 0.88

$$\frac{11\,d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)}{96\,\sqrt{c^7}}-\frac{25\,d}{216\,c^3\,\sqrt{d\,x^3+c}}+\frac{d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)}{2592\,\sqrt{c^7}}-\frac{1}{24\,c^2\,x^3\,\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (11*d*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(96*(c^7)^(1/2)) - (25*d)
/(216*c^3*(c + d*x^3)^(1/2)) + (d*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1
/2))))/(2592*(c^7)^(1/2)) - 1/(24*c^2*x^3*(c + d*x^3)^(1/2))
```

3.331 $\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{245d^2}{1728c^4\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}}-\frac{109d^2\tanh^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c}}\right.}{768c^{9/2}}$$

```
[Out] 1/20736*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-109/768*d^2*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)+245/1728*d^2/c^4/(d*x^3+c)^(1/2)-1/48/c^
2/x^6/(d*x^3+c)^(1/2)+3/64*d/c^3/x^3/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {457, 105, 156, 157, 162, 65, 214, 212}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}}-\frac{109d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}+\frac{245d^2}{1728c^4\sqrt{c+dx^3}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (245*d^2)/(1728*c^4*Sqrt[c + d*x^3]) - 1/(48*c^2*x^6*Sqrt[c + d*x^3]) + (3*
d)/(64*c^3*x^3*Sqrt[c + d*x^3]) + (d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/(20736*c^(9/2)) - (109*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(9/2)
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7 (8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{48c^2x^6\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{18cd-\frac{5d^2x}{2}}{x^2(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{48c^2}$$

$$= -\frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{218c^2d^2-27cd^3x}{x(8c-dx)(c+dx)^{3/2}}\,dx, x\right.}{384c^4}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int -\right.}{}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{(109d^2)\,\text{Su}}{}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{(109d)\text{Sub}}{}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{d^2\tanh^{-1}\left(\right.}{207}$$

Mathematica [A]
time = 0.16, size = 100, normalized size = 0.78

$$\frac{\frac{12\sqrt{c}\,(-36c^2+81cdx^3+245d^2x^6)}{x^6\sqrt{c+dx^3}} + d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2943d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{20736c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((12*Sqrt[c]*(-36*c^2 + 81*c*d*x^3 + 245*d^2*x^6))/(x^6*Sqrt[c + d*x^3]) +
d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 2943*d^2*ArcTanh[Sqrt[c + d*x^3]
/Sqrt[c]])/(20736*c^(9/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 636, normalized size = 4.97

method	result

risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(-13 d\,x^{3}+4 c\right)}{192 c^{4} x^{6}}+\frac{d^{2}\left(-\frac{109\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\cdots\right.}{\cdots}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$

default	$\frac{-\frac{\sqrt{d\,x^3+c}}{6c^2x^6}+\frac{7d\sqrt{d\,x^3+c}}{12c^3x^3}+\frac{2d^2}{3c^3\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{5d^2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{4c^{\frac{7}{2}}}}{8c} - \frac{d^3\left(\frac{2}{27dc\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{i\sqrt{2}\ldots}{\ldots}\right.}{\ldots}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/6*(d*x^3+c)^(1/2)/c^2/x^6+7/12*d*(d*x^3+c)^(1/2)/c^3/x^3+2/3*d^2/
c^3/((x^3+c/d)*d)^(1/2)-5/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2))-1
/512/c^3*d^3*(2/27/d/c/((x^3+c/d)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-c*
d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(
```

```
1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c
*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/6
4*d/c^2*(-2/3*d/c^2/((x^3+c/d)*d)^(1/2)-1/3*(d*x^3+c)^(1/2)/c^2/x^3+d*arcta
nh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+1/512/c^3*d^2*(2/3/c/((x^3+c/d)*d)^(1/
2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^7), x)
```

Fricas [A]
time = 2.99, size = 303, normalized size = 2.37

$$\left[\frac{(d^3x^9 + cd^2x^6)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 2943\,(d^3x^9 + cd^2x^6)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) + 24\,(245\,cd^2x^6 + 81\,c^2dx^3 - 36\,c^3)\sqrt{dx^3+c}}{41472\,(c^5dx^9 + c^6x^6)}, \frac{2943\,(d^3x^9 + cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - (d^3x^9 + cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 12\,(245\,cd^2x^6 + 81\,c^2dx^3 - 36\,c^3)\sqrt{dx^3+c}}{20736\,(c^5dx^9 + c^6x^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/41472*((d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 2943*(d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3
- 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(245*c*d^2*x^6 + 81*c^2*d*x^3
- 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6), 1/20736*(2943*(d^3*x^9 +
c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d^3*x^9 + c*d^2*x
^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(245*c*d^2*x^6 + 8
1*c^2*d*x^3 - 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^7\sqrt{c + dx^3} - 7cdx^{10}\sqrt{c + dx^3} + d^2x^{13}\sqrt{c + dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**7*sqrt(c + d*x**3) - 7*c*d*x**10*sqrt(c + d*x**3) +
 d**2*x**13*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 0.78, size = 118, normalized size = 0.92

$$\frac{109\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\,\sqrt{-c}\,c^4}-\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{20736\,\sqrt{-c}\,c^4}+\frac{2\,d^2}{27\,\sqrt{dx^3+c}\,c^4}+\frac{13\,(dx^3+c)^{\frac{3}{2}}d^2-17\,\sqrt{dx^3+c}\,cd^2}{192\,c^4d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 109/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/20736*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) + 2/27*d^2/(sqrt(d*x^3 +
 c)*c^4) + 1/192*(13*(d*x^3 + c)^(3/2)*d^2 - 17*sqrt(d*x^3 + c)*c*d^2)/(c^4
*d^2*x^6)
```

Mupad [B]
time = 4.03, size = 112, normalized size = 0.88

$$\frac{245\,d^2}{1728\,c^4\,\sqrt{d\,x^3+c}}-\frac{109\,d^2\,\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)}{768\,\sqrt{c^9}}+\frac{d^2\,\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)}{20736\,\sqrt{c^9}}-\frac{1}{48\,c^2\,x^6\,\sqrt{d\,x^3+c}}+\frac{3\,d}{64\,c^3\,x^3\,\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (245*d^2)/(1728*c^4*(c + d*x^3)^(1/2)) - (109*d^2*atanh((c^4*(c + d*x^3)^(1
/2))/(c^9)^(1/2)))/(768*(c^9)^(1/2)) + (d^2*atanh((c^4*(c + d*x^3)^(1/2))/(
3*(c^9)^(1/2))))/(20736*(c^9)^(1/2)) - 1/(48*c^2*x^6*(c + d*x^3)^(1/2)) + (
3*d)/(64*c^3*x^3*(c + d*x^3)^(1/2))
```

3.332 $\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=629

$$\frac{2x^2}{27d^2\sqrt{c+dx^3}}-\frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{32\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,d^{8/3}}+\frac{32\sqrt[6]{c}\,\tanh^{-1}\left(\cdots\right.}{8\cdots}$$

```
[Out] 32/81*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(8/3)-32/81*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32/81*c^(1
/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^(
1/2)+2/27*x^2/d^2/(d*x^3+c)^(1/2)-56/27*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))-56/81*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x
+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2
)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+28/27*c^(1/3)*(c^(1/3)+d^(1/3)*x)*Ellip
ticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1
/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/d^(8/3)/(d*x^3+c)^(1/2)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {481, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{9\;3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{32\sqrt[6]{c}\,\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,d^{8/3}}-\frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{32\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}}-\frac{32\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}}+\frac{2x^2}{27d^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*x^2)/(27*d^2*Sqrt[c + d*x^3]) - (56*Sqrt[c + d*x^3])/(27*d^(8/3)*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (32*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3
) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*d^(8/3)) + (32*c^(1/6)*ArcTan
h[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*d^(8/3)) - (32*
c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^(8/3)) + (28*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
```

```
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*S
qrt[3]])/(9*3^(3/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (56*Sqrt[2]*c^(1/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*d^
(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{2\int \frac{x(16c^2-14cdx^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27cd^2}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{2\int \left(\frac{14cx}{\sqrt{c+dx^3}} - \frac{96c^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27cd^2}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{28\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27d^2} + \frac{(64c)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9d^2}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{16\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{27d^3} - \frac{28\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+}{\sqrt{c+dx^3}}}{27d^{7/3}}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{28\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{}\right.}{}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{32\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{}}{}\right.}{27\sqrt{3}\,}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{32\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{}}{}\right.}{27\sqrt{3}\,}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 6.86, size = 127, normalized size = 0.20

$$\frac{x^2\left(20c-20c\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{270cd^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(20*c - 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), (d*x^3)/(8*c)] + 7*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))/(270*c*d^2*Sqrt[c + d*x^3])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 1810, normalized size = 2.88

method	result	size
elliptic	Expression too large to display	869
default	Expression too large to display	1810

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-2/3/d*x^2/((x^3+c/d)*d)^(1/2)-8/9*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2))))-8/d^2*c*(2/3*x^2/c/((x^3+c/d)*d)^(1/2)+2/9*I/c*3^(1/2)/d*(-c
*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
```

```
^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2))))-64*c^2/d^2*(-2/27*x^2/c^2/((x^3+c/d)*d)^(1/
2)-2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/
3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/243*I/c^2/d^3*2
^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*
(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c
*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_a
lpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=
RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 26.54, size = 3632, normalized size = 5.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/243*(9*sqrt(d*x^3 + c)*d*x^2 + 16*sqrt(3)*(d^4*x^3 + c*d^3)*(c/d^16)^(1/6
)*arctan(-1/3*(324*sqrt(3)*(3*c*d^16*x^16 + 784*c^2*d^15*x^13 + 7680*c^3*d^
14*x^10 + 10752*c^4*d^13*x^7 + 4096*c^5*d^12*x^4)*(c/d^16)^(2/3) + 36*sqrt(
```

3)*(c*d^11*x^17 + 1772*c^2*d^10*x^14 + 42592*c^3*d^9*x^11 + 96256*c^4*d^8*x^8 + 69632*c^5*d^7*x^5 + 16384*c^6*d^6*x^2)*(c/d^16)^(1/3) + sqrt(3)*(c*d^6*x^18 + 9456*c^2*d^5*x^15 + 749184*c^3*d^4*x^12 + 3017216*c^4*d^3*x^9 + 3489792*c^5*d^2*x^6 + 1572864*c^6*d*x^3 + 262144*c^7) + 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c*d^18*x^14 - 14440*c^2*d^17*x^11 - 24576*c^3*d^16*x^8 - 16384*c^4*d^15*x^5 - 4096*c^5*d^14*x^2)*(c/d^16)^(5/6) + 18*sqrt(3)*(c*d^13*x^15 - 1112*c^2*d^12*x^12 + 7296*c^3*d^11*x^9 + 11776*c^4*d^10*x^6 + 4096*c^5*d^9*x^3)*sqrt(c/d^16) + sqrt(3)*(c*d^8*x^16 - 4768*c^2*d^7*x^13 + 362752*c^3*d^6*x^10 + 709120*c^4*d^5*x^7 + 413696*c^5*d^4*x^4 + 65536*c^6*d^3*x)*(c/d^16)^(1/6)) - 2*(324*sqrt(3)*(d^19*x^16 - 1858*c*d^18*x^13 - 4176*c^2*d^17*x^10 - 3584*c^3*d^16*x^7 - 1024*c^4*d^15*x^4)*(c/d^16)^(5/6) + 18*sqrt(3)*(d^14*x^17 - 5290*c*d^13*x^14 - 21152*c^2*d^12*x^11 - 47744*c^3*d^11*x^8 - 37888*c^4*d^10*x^5 - 8192*c^5*d^9*x^2)*sqrt(c/d^16) + sqrt(3)*(d^9*x^18 - 7698*c*d^8*x^15 - 1664688*c^2*d^7*x^12 - 5524864*c^3*d^6*x^9 - 6223872*c^4*d^5*x^6 - 2703360*c^5*d^4*x^3 - 327680*c^6*d^3)*(c/d^16)^(1/6) + 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*d^16*x^15 + 37352*c*d^15*x^12 - 230336*c^2*d^14*x^9 - 515072*c^3*d^13*x^6 - 286720*c^4*d^12*x^3 - 32768*c^5*d^11)*(c/d^16)^(2/3) + 108*sqrt(3)*(53*c*d^10*x^13 + 1320*c^2*d^9*x^10 + 1536*c^3*d^8*x^7 + 512*c^4*d^7*x^4)*(c/d^16)^(1/3) + 6*sqrt(3)*(37*c*d^5*x^14 + 28912*c^2*d^4*x^11 + 43584*c^3*d^3*x^8 + 20992*c^4*d^2*x^5 + 4096*c^5*d*x^2)))*sqrt((18*c^2*d^2*x^8 + 360*c^3*d*x^5 - 144*c^4*x^2 + (c*d^13*x^9 - 276*c^2*d^12*x^6 - 1608*c^3*d^11*x^3 - 1088*c^4*d^10)*(c/d^16)^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^15*x^7 - 28*c^2*d^14*x^4 - 272*c^3*d^13*x)*(c/d^16)^(5/6) - 24*(c^2*d^9*x^5 + c^3*d^8*x^2)*sqrt(c/d^16) + 4*(c^2*d^4*x^6 + 41*c^3*d^3*x^3 + 40*c^4*d^2)*(c/d^16)^(1/6)) - 18*(c^2*d^7*x^7 - 52*c^3*d^6*x^4 - 80*c^4*d^5*x)*(c/d^16)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^6*x^18 - 14952*c^2*d^5*x^15 + 2872896*c^3*d^4*x^12 + 7330304*c^4*d^3*x^9 + 6696960*c^5*d^2*x^6 + 2457600*c^6*d*x^3 + 262144*c^7)) - 16*sqrt(3)*(d^4*x^3 + c*d^3)*(c/d^16)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c*d^16*x^16 + 784*c^2*d^15*x^13 + 7680*c^3*d^14*x^10 + 10752*c^4*d^13*x^7 + 4096*c^5*d^12*x^4)*(c/d^16)^(2/3) + 36*sqrt(3)*(c*d^11*x^17 + 1772*c^2*d^10*x^14 + 42592*c^3*d^9*x^11 + 96256*c^4*d^8*x^8 + 69632*c^5*d^7*x^5 + 16384*c^6*d^6*x^2)*(c/d^16)^(1/3) + sqrt(3)*(c*d^6*x^18 + 9456*c^2*d^5*x^15 + 749184*c^3*d^4*x^12 + 3017216*c^4*d^3*x^9 + 3489792*c^5*d^2*x^6 + 1572864*c^6*d*x^3 + 262144*c^7) - 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c*d^18*x^14 - 14440*c^2*d^17*x^11 - 24576*c^3*d^16*x^8 - 16384*c^4*d^15*x^5 - 4096*c^5*d^14*x^2)*(c/d^16)^(5/6) + 18*sqrt(3)*(c*d^13*x^15 - 1112*c^2*d^12*x^12 + 7296*c^3*d^11*x^9 + 11776*c^4*d^10*x^6 + 4096*c^5*d^9*x^3)*sqrt(c/d^16) + sqrt(3)*(c*d^8*x^16 - 4768*c^2*d^7*x^13 + 362752*c^3*d^6*x^10 + 709120*c^4*d^5*x^7 + 413696*c^5*d^4*x^4 + 65536*c^6*d^3*x)*(c/d^16)^(1/6)) + 2*(324*sqrt(3)*(d^19*x^16 - 1858*c*d^18*x^13 - 4176*c^2*d^17*x^10 - 3584*c^3*d^16*x^7 - 1024*c^4*d^15*x^4)*(c/d^16)^(5/6) + 18*sqrt(3)*(d^14*x^17 - 5290*c*d^13*x^14 - 21152*c^2*d^12*x^11 - 47744*c^3*d^11*x^8 - 37888*c^4*d^10*x^5 - 8192*c^5*d^9*x^2)*sqrt(c/d^16) + sqrt(3)*(d^9*x^18 - 7698*c*d^8*x^15 - 1664688*c^2*d^7*x^12 - 5524864*c^3*d^6*x^9 - 6223872*c^4*d^5*x^6 - 2703360*c^5*d^4*x^3 - 327680*c^6*d^3)*(c/d^16)^(1/6) -

```
6*sqrt(d*x^3 + c)*(sqrt(3)*(7*d^16*x^15 + 37352*c*d^15*x^12 - 230336*c^2*d^
14*x^9 - 515072*c^3*d^13*x^6 - 286720*c^4*d^12*x^3 - 32768*c^5*d^11)*(c/d^1
6)^(2/3) + 108*sqrt(3)*(53*c*d^10*x^13 + 1320*c^2*d^9*x^10 + 1536*c^3*d^8*x
^7 + 512*c^4*d^7*x^4)*(c/d^16)^(1/3) + 6*sqrt(3)*(37*c*d^5*x^14 + 28912*c^2
*d^4*x^11 + 43584*c^3*d^3*x^8 + 20992*c^4*d^2*x^5 + 4096*c^5*d*x^2)))*sqrt(
(18*c^2*d^2*x^8 + 360*c^3*d*x^5 - 144*c^4*x^2 + (c*d^13*x^9 - 276*c^2*d^12*
x^6 - 1608*c^3*d^11*x^3 - 1088*c^4*d^10)*(c/d^16)^(2/3) - 6*sqrt(d*x^3 + c)
*((c*d^15*x^7 - 28*c^2*d^14*x^4 - 272*c^3*d^13*x)*(c/d^16)^(5/6) - 24*(c^2*
d^9*x^5 + c^3*d^8*x^2)*sqrt(c/d^16) + 4*(c^2*d^4*x^6 + 41*c^3*d^3*x^3 + 40*
c^4*d^2)*(c/d^16)^(1/6)) - 18*(c^2*d^7*x^7 - 52*c^3*d^6*x^4 - 80*c^4*d^5*x)
*(c/d^16)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^
6*x^18 - 14952*c^2*d^5*x^15 + 2872896*c^3*d^4*x^12 + 7330304*c^4*d^3*x^9 +
6696960*c^5*d^2*x^6 + 2457600*c^6*d*x^3 + 262144*c^7)) + 252*(d*x^3 + c)*sq
rt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 4*(d^
4*x^3 + c*d^3)*(c/d^16)^(1/6)*log(4503599627370496/9*(18*c^2*d^2*x^8 + 360*
c^3*d*x^5 - 144*c^4*x^2 + (c*d^13*x^9 - 276*c^2*d^12*x^6 - 1608*c^3*d^11*x^
3 - 1088*c^4*d^10)*(c/d^16)^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^15*x^7 - 28*c^2
*d^14*x^4 - 272*c^3*d^13*x)*(c/d^16)^(5/6) - 24*(c^2*d^9*x^5 + c^3*d^8*x^2)
*sqrt(c/d^16) + 4*(c^2*d^4*x^6 + 41*c^3*d^3*x^3...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.333 $\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=635

$$-\frac{2x^2}{27cd\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{4\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,c^{5/6}d^{5/3}}+\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}\right.}{3\sqrt[6]{c}}\right.}{81c^{5/6}}$$

```
[Out] 4/81*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^(
5/3)-4/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(5/3)-4/81*arctan(
c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(5/3)*3^(1/2
)-2/27*x^2/c/d/(d*x^3+c)^(1/2)+2/27*(d*x^3+c)^(1/2)/c/d^(5/3)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))+2/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1
-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)
-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/c^(2/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/27*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3
)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1
/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.51, antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {482, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{4\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,c^{5/6}d^{5/3}}+\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}-\frac{4\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}+\frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{2x^2}{27cd\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*x^2)/(27*c*d*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(27*c*d^(5/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (4*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*c^(5/6)*d^(5/3)) + (4*ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*c^(5/6)*d^(5/3)) - (4*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*c^(5/6)*d^(5/3)) - (Sqrt[2 - Sqrt[
3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
```

```
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*
3^(3/4)*c^(2/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x
)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(2/3)*d^(5/3)
*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(16c-\frac{dx^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\int \left(\frac{x}{2\sqrt{c+dx^3}} + \frac{12cx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{8\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9d} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} - \frac{2\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{27cd^2} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+}{\sqrt{c+dx^3}}}{27cd^{4/3}}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}\right.}{}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt{}}\right.}{27\sqrt{3}\,c}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt{}}\right.}{27\sqrt{3}\,c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 6.56, size = 126, normalized size = 0.20

$$-\frac{x^2\left(80c-80c\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{1080c^2d\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/1080*(x^2*(80*c - 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -(
(d*x^3)/c), (d*x^3)/(8*c)] + d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1
, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(c^2*d*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 1346, normalized size = 2.12

method	result	size
elliptic	Expression too large to display	878
default	Expression too large to display	1346

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/3*x^2/c/((x^3+c/d)*d)^(1/2)+2/9*I/c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2))))-8*c/d*(-2/27*x^2/c^2/((x^3+c/d)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d
*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
```

```
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c
*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)
*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-
c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.11, size = 2638, normalized size = 4.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/243*(18*sqrt(d*x^3 + c)*d*x^2 + 4*sqrt(3)*(c*d^3*x^3 + c^2*d^2)*(1/(c^5*
d^10))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^3*x^5*(1/(c^5*d^10))^(1/6) - sqrt(3
)*(c^4*d^10*x^6 - 40*c^5*d^9*x^3 - 32*c^6*d^8)*(1/(c^5*d^10))^(5/6) + 3*sqr
t(3)*(5*c^3*d^6*x^4 + 8*c^4*d^5*x)*sqrt(1/(c^5*d^10)))*sqrt(d*x^3 + c) + (1
8*sqrt(3)*(c^4*d^8*x^5 + c^5*d^7*x^2)*(1/(c^5*d^10))^(2/3) + 12*sqrt(3)*(c^
2*d^5*x^6 - c^3*d^4*x^3 - 2*c^4*d^3)*(1/(c^5*d^10))^(1/3) + 3*sqrt(3)*(d^2*
x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^10*x^6 + 32*c^
5*d^9*x^3 + 40*c^6*d^8)*(1/(c^5*d^10))^(5/6) + 3*sqrt(3)*(7*c^3*d^6*x^4 + 4
*c^4*d^5*x)*sqrt(1/(c^5*d^10)) + 9*sqrt(3)*(c*d^3*x^5 + 2*c^2*d^2*x^2)*(1/(
c^5*d^10))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^
3 - 18*(c^4*d^9*x^7 - 52*c^5*d^8*x^4 - 80*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) +
 6*sqrt(d*x^3 + c)*(24*(c^5*d^10*x^5 + c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6) -
4*(c^3*d^7*x^6 + 41*c^4*d^6*x^3 + 40*c^5*d^5)*sqrt(1/(c^5*d^10)) - (c*d^4*x
^7 - 28*c^2*d^3*x^4 - 272*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^6*x^
8 + 20*c^3*d^5*x^5 - 8*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d
```

^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 4*sqrt(3)*(c*d^3*x^3 + c^2*d^2)*(1/(c^5*d^10))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^3*x^5*(1/(c^5*d^10))^(1/6) - sqrt(3)*(c^4*d^10*x^6 - 40*c^5*d^9*x^3 - 32*c^6*d^8)*(1/(c^5*d^10))^(5/6) + 3*sqrt(3)*(5*c^3*d^6*x^4 + 8*c^4*d^5*x)*sqrt(1/(c^5*d^10)))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^4*d^8*x^5 + c^5*d^7*x^2)*(1/(c^5*d^10))^(2/3) + 12*sqrt(3)*(c^2*d^5*x^6 - c^3*d^4*x^3 - 2*c^4*d^3)*(1/(c^5*d^10))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^10*x^6 + 32*c^5*d^9*x^3 + 40*c^6*d^8)*(1/(c^5*d^10))^(5/6) + 3*sqrt(3)*(7*c^3*d^6*x^4 + 4*c^4*d^5*x)*sqrt(1/(c^5*d^10)) + 9*sqrt(3)*(c*d^3*x^5 + 2*c^2*d^2*x^2)*(1/(c^5*d^10))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^9*x^7 - 52*c^5*d^8*x^4 - 80*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^10*x^5 + c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6) - 4*(c^3*d^7*x^6 + 41*c^4*d^6*x^3 + 40*c^5*d^5)*sqrt(1/(c^5*d^10)) - (c*d^4*x^7 - 28*c^2*d^3*x^4 - 272*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^6*x^8 + 20*c^3*d^5*x^5 - 8*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 18*(d*x^3 + c)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 2*(c*d^3*x^3 + c^2*d^2)*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6) + (7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c*d^3*x^3 + c^2*d^2)*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6) + (7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 + c^2*d^2)*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^9*x^7 - 52*c^5*d^8*x^4 - 80*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^10*x^5 + c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6) - 4*(c^3*d^7*x^6 + 41*c^4*d^6*x^3 + 40*c^5*d^5)*sqrt(1/(c^5*d^10)) - (c*d^4*x^7 - 28*c^2*d^3*x^4 - 272*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^6*x^8 + 20*c^3*d^5*x^5 - 8*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^3*x^3 + c^2*d^2)*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^9*x^7 - 52*c^5*d^8*x^4 - 80*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^10*x^5 + c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6) - 4*(c^3*d^7*x^6 + 41*c^4*d^6*x^3 + 40*c^5*d^5)*sqrt(1/(c^5*d^10)) - (c*d^4*x^7 - 28*c^2*d^3*x^4 - 272*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^6*x^8 + 20*c^3*d^5*x^5 - 8*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))

```
/(c*d^3*x^3 + c^2*d^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.334 $\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=632

$$\frac{2x^2}{27c^2\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}\,c^{11/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{3\sqrt[6]{c}\,\sqrt{c+}}\right.}{162c^{11/6}d^{2/3}}$$

```
[Out] 1/162*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/d
^(2/3)-1/162*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(2/3)-1/162*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^(2/3)*
3^(1/2)+2/27*x^2/c^2/(d*x^3+c)^(1/2)-2/27*(d*x^3+c)^(1/2)/c^2/d^(2/3)/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2)))-2/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^
(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*(
(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/27*(c^(1/3)+d^(1/3)*x)*EllipticE
((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/
2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = $\{483, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211\}$

$$-\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}\,c^{11/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{162c^{11/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{162c^{11/6}d^{2/3}}-\frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{2x^2}{27c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*x^2)/(27*c^2*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(27*c^2*d^(2/3)*((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)) - ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/
3)*x))/Sqrt[c + d*x^3]]/(54*Sqrt[3]*c^(11/6)*d^(2/3)) + ArcTanh[(c^(1/3) +
d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(162*c^(11/6)*d^(2/3)) - ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])]/(162*c^(11/6)*d^(2/3)) + (Sqrt[2 - Sqrt[3]]*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
```

```
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4
)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(5/3)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\int \frac{x\left(\frac{5cd}{2}-\frac{d^2x^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27c^2d}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\int \left(\frac{dx}{2\sqrt{c+dx^3}} - \frac{3cdx}{2(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27c^2d}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27c^2} + \frac{\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9c}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{108c^2d} - \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}}{27c^2\sqrt[3]{d}}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\right.}{}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{}\right.}{\sqrt{c+}}\right.}{54\sqrt{3}\,c^{11/6}}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{}\right.}{\sqrt{c+}}\right.}{54\sqrt{3}\,c^{11/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 8.69, size = 124, normalized size = 0.20

$$\frac{x^2\left(160c-25c\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+2dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{2160c^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(160*c - 25*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)
/c), (d*x^3)/(8*c)] + 2*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3
, -((d*x^3)/c), (d*x^3)/(8*c)]))/(2160*c^3*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 875, normalized size = 1.38

method	result	size
default	Expression too large to display	875
elliptic	Expression too large to display	875

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*x^2/c^2/((x^3+c/d)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(
2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(
d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2
)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c
*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*
c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.55, size = 2638, normalized size = 4.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1944*(144*sqrt(d*x^3 + c)*d*x^2 - 4*sqrt(3)*(c^2*d^2*x^3 + c^3*d)*(1/(c^1
1*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c^2*d^2*x^5*(1/(c^11*d^4))^(1/6) - sqr
t(3)*(c^9*d^5*x^6 - 40*c^10*d^4*x^3 - 32*c^11*d^3)*(1/(c^11*d^4))^(5/6) + 3
*sqrt(3)*(5*c^6*d^3*x^4 + 8*c^7*d^2*x)*sqrt(1/(c^11*d^4)))*sqrt(d*x^3 + c)
+ (18*sqrt(3)*(c^8*d^4*x^5 + c^9*d^3*x^2)*(1/(c^11*d^4))^(2/3) + 12*sqrt(3)
*(c^4*d^3*x^6 - c^5*d^2*x^3 - 2*c^6*d)*(1/(c^11*d^4))^(1/3) + 3*sqrt(3)*(d^
2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^9*d^5*x^6 + 32*c
^10*d^4*x^3 + 40*c^11*d^3)*(1/(c^11*d^4))^(5/6) + 3*sqrt(3)*(7*c^6*d^3*x^4
+ 4*c^7*d^2*x)*sqrt(1/(c^11*d^4)) + 9*sqrt(3)*(c^2*d^2*x^5 + 2*c^3*d*x^2)*(
1/(c^11*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088
*c^3 - 18*(c^8*d^5*x^7 - 52*c^9*d^4*x^4 - 80*c^10*d^3*x)*(1/(c^11*d^4))^(2/
3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^5*x^5 + c^11*d^4*x^2)*(1/(c^11*d^4))^(5/
6) - 4*(c^6*d^4*x^6 + 41*c^7*d^3*x^3 + 40*c^8*d^2)*sqrt(1/(c^11*d^4)) - (c^
2*d^3*x^7 - 28*c^3*d^2*x^4 - 272*c^4*d*x)*(1/(c^11*d^4))^(1/6)) + 18*(c^4*d
^4*x^8 + 20*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) -
 4*sqrt(3)*(c^2*d^2*x^3 + c^3*d)*(1/(c^11*d^4))^(1/6)*arctan(1/9*((9*sqrt(3
)*c^2*d^2*x^5*(1/(c^11*d^4))^(1/6) - sqrt(3)*(c^9*d^5*x^6 - 40*c^10*d^4*x^3
 - 32*c^11*d^3)*(1/(c^11*d^4))^(5/6) + 3*sqrt(3)*(5*c^6*d^3*x^4 + 8*c^7*d^2
*x)*sqrt(1/(c^11*d^4)))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^8*d^4*x^5 + c^9*d^
3*x^2)*(1/(c^11*d^4))^(2/3) + 12*sqrt(3)*(c^4*d^3*x^6 - c^5*d^2*x^3 - 2*c^6
*d)*(1/(c^11*d^4))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt
(d*x^3 + c)*(sqrt(3)*(c^9*d^5*x^6 + 32*c^10*d^4*x^3 + 40*c^11*d^3)*(1/(c^11
*d^4))^(5/6) + 3*sqrt(3)*(7*c^6*d^3*x^4 + 4*c^7*d^2*x)*sqrt(1/(c^11*d^4)) +
 9*sqrt(3)*(c^2*d^2*x^5 + 2*c^3*d*x^2)*(1/(c^11*d^4))^(1/6)))*sqrt((d^3*x^9
 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^8*d^5*x^7 - 52*c^9*d^4
*x^4 - 80*c^10*d^3*x)*(1/(c^11*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^10*d^
```

```
5*x^5 + c^11*d^4*x^2)*(1/(c^11*d^4))^(5/6) - 4*(c^6*d^4*x^6 + 41*c^7*d^3*x^
3 + 40*c^8*d^2)*sqrt(1/(c^11*d^4)) - (c^2*d^3*x^7 - 28*c^3*d^2*x^4 - 272*c^
4*d*x)*(1/(c^11*d^4))^(1/6)) + 18*(c^4*d^4*x^8 + 20*c^5*d^3*x^5 - 8*c^6*d^2
*x^2)*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c
^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 144*(d*x^3 + c)*sqrt(d)*weierstras
sZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 2*(c^2*d^2*x^3 + c^3*
d)*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640
*c^3 + 18*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x)*(1/(c^11*d^4))^(
2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2)*(1/(c^11*d^4
))^(5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)
) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(1/(c^11*d^4))^(1/6)) + 18
*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2)*(1/(c^11*d^4))^(1/3))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^2*d^2*x^3 + c^3*d)*(
1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3
 + 18*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x)*(1/(c^11*d^4))^(2/3)
 - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2)*(1/(c^11*d^4))^(
5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)) +
(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(1/(c^11*d^4))^(1/6)) + 18*(c^
4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2)*(1/(c^11*d^4))^(1/3))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^2*d^2*x^3 + c^3*d)*(1/(c^1
1*d^4))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18
*(c^8*d^5*x^7 - 52*c^9*d^4*x^4 - 80*c^10*d^3*x)*(1/(c^11*d^4))^(2/3) + 6*sq
rt(d*x^3 + c)*(24*(c^10*d^5*x^5 + c^11*d^4*x^2)*(1/(c^11*d^4))^(5/6) - 4*(c
^6*d^4*x^6 + 41*c^7*d^3*x^3 + 40*c^8*d^2)*sqrt(1/(c^11*d^4)) - (c^2*d^3*x^7
 - 28*c^3*d^2*x^4 - 272*c^4*d*x)*(1/(c^11*d^4))^(1/6)) + 18*(c^4*d^4*x^8 +
20*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x
^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^2*d^2*x^3 + c^3*d)*(1/(c^11*d^4))^(1/6)
*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^8*d^5*x^7
 - 52*c^9*d^4*x^4 - 80*c^10*d^3*x)*(1/(c^11*d^4))^(2/3) - 6*sqrt(d*x^3 + c)
*(24*(c^10*d^5*x^5 + c^11*d^4*x^2)*(1/(c^11*d^4))^(5/6) - 4*(c^6*d^4*x^6 +
41*c^7*d^3*x^3 + 40*c^8*d^2)*sqrt(1/(c^11*d^4)) - (c^2*d^3*x^7 - 28*c^3*d^2
*x^4 - 272*c^4*d*x)*(1/(c^11*d^4))^(1/6)) + 18*(c^4*d^4*x^8 + 20*c^5*d^3*x^
5 - 8*c^6*d^2*x^2)*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)))/(c^2*d^2*x^3 + c^3*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-8c^2\sqrt{c+dx^3}-7cdx^3\sqrt{c+dx^3}+d^2x^6\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(x/(-8*c**2*sqrt(c + d*x**3) - 7*c*d*x**3*sqrt(c + d*x**3) + d**2*
x**6*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.335 $\int \frac{1}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=653

$$\frac{2}{27c^2x\sqrt{c+dx^3}}-\frac{43\sqrt{c+dx^3}}{216c^3x}+\frac{43\sqrt[3]{d}\,\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}\,c^{17/6}}+$$

[Out] 1/1296*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/6)-1/1296*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/1296*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/6)*3^(1/2)+2/27/c^2/x/(d*x^3+c)^(1/2)-43/216*(d*x^3+c)^(1/2)/c^3/x+43/216*d^(1/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+43/648*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-43/432*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.60, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\,\sqrt[4]{3}\,c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{43\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{144\,3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[3]{d}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}\,c^{17/6}}+\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{1296c^{17/6}}-\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{17/6}}-\frac{43\sqrt{c+dx^3}}{216c^3x}+\frac{43\sqrt[3]{d}\,\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{2}{27c^2x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] 2/(27*c^2*x*Sqrt[c + d*x^3]) - (43*Sqrt[c + d*x^3])/(216*c^3*x) + (43*d^(1/3)*Sqrt[c + d*x^3])/(216*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(432*Sqrt[3]*c^(17/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1296*c^(17/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1296*c^(17/6)) - (43*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq

```
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(144*3^(3/4)*c^(8/3)*Sqrt[(c^(1
/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3]) + (43*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
 - 4*Sqrt[3]])/(108*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3
)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{43cd}{2}+\frac{5d^2x^3}{2}}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27c^2d}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{\int \frac{x\left(\frac{175c^2d^2}{2}-\frac{43}{4}cd^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{108c^4d}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{\int \left(\frac{43cd^2x}{4\sqrt{c+dx^3}} + \frac{3c^2d^2x}{2(8c-dx^3)\sqrt{c+dx^3}} \cdots\right.}{108c^4d}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{(43d)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{432c^3} + \frac{d\int \frac{\cdots}{(8c-dx^3)\cdots}}{72\cdots}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} - \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{864c^3} + \cdots$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\,\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \cdots$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\,\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \cdots$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\,\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 20.08, size = 140, normalized size = 0.21

$$\frac{-80c(27c+43dx^3)+875cdx^3\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-43d^2x^6\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{17280c^4x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-80*c*(27*c + 43*d*x^3) + 875*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/
2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 43*d^2*x^6*Sqrt[1 + (d*x^3)/c]*Ap
pellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17280*c^4*x*Sqrt[c
+ d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 1361, normalized size = 2.08

method	result	size
elliptic	Expression too large to display	890
risch	Expression too large to display	1334
default	Expression too large to display	1361

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d/c*(-2/27*x^2/c^2/((x^3+c/d)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(
1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*
3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
```

```
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3
)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/8/c*(-2/3*d
*x^2/c^2/((x^3+c/d)*d)^(1/2)-(d*x^3+c)^(1/2)/c^2/x-5/9*I/c^2*3^(1/2)*(-c*d^
2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.11, size = 2610, normalized size = 4.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15552*(4*sqrt(3)*(c^3*d*x^4 + c^4*x)*(d^2/c^17)^(1/6)*arctan(1/9*((9*sqr
t(3)*c^3*d^4*x^5*(d^2/c^17)^(1/6) - sqrt(3)*(c^14*d^3*x^6 - 40*c^15*d^2*x^3
 - 32*c^16*d)*(d^2/c^17)^(5/6) + 3*sqrt(3)*(5*c^9*d^3*x^4 + 8*c^10*d^2*x)*s
qrt(d^2/c^17))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^12*d^2*x^5 + c^13*d*x^2)*(d
^2/c^17)^(2/3) + 12*sqrt(3)*(c^6*d^3*x^6 - c^7*d^2*x^3 - 2*c^8*d)*(d^2/c^17
)^(1/3) + 3*sqrt(3)*(d^4*x^7 + 5*c*d^3*x^4 + 4*c^2*d^2*x) + sqrt(d*x^3 + c)
*(sqrt(3)*(c^14*d^2*x^6 + 32*c^15*d*x^3 + 40*c^16)*(d^2/c^17)^(5/6) + 3*sqr
t(3)*(7*c^9*d^2*x^4 + 4*c^10*d*x)*sqrt(d^2/c^17) + 9*sqrt(3)*(c^3*d^3*x^5 +
 2*c^4*d^2*x^2)*(d^2/c^17)^(1/6)))*sqrt((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2
*d^3*x^3 - 1088*c^3*d^2 - 18*(c^12*d^3*x^7 - 52*c^13*d^2*x^4 - 80*c^14*d*x)
*(d^2/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^15*d^2*x^5 + c^16*d*x^2)*(d^2/
```

c^17)^(5/6) - 4*(c^9*d^3*x^6 + 41*c^10*d^2*x^3 + 40*c^11*d)*sqrt(d^2/c^17) - (c^3*d^4*x^7 - 28*c^4*d^3*x^4 - 272*c^5*d^2*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^4*x^8 + 20*c^7*d^3*x^5 - 8*c^8*d^2*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^5*x^7 - 7*c*d^4*x^4 - 8*c^2*d^3*x)) + 4*sqrt(3)*(c^3*d*x^4 + c^4*x)*(d^2/c^17)^(1/6)*arctan(1/9*((9*sqrt(3)*c^3*d^4*x^5*(d^2/c^17)^(1/6) - sqrt(3)*(c^14*d^3*x^6 - 40*c^15*d^2*x^3 - 32*c^16*d)*(d^2/c^17)^(5/6) + 3*sqrt(3)*(5*c^9*d^3*x^4 + 8*c^10*d^2*x)*sqrt(d^2/c^17))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^12*d^2*x^5 + c^13*d*x^2)*(d^2/c^17)^(2/3) + 12*sqrt(3)*(c^6*d^3*x^6 - c^7*d^2*x^3 - 2*c^8*d)*(d^2/c^17)^(1/3) + 3*sqrt(3)*(d^4*x^7 + 5*c*d^3*x^4 + 4*c^2*d^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^14*d^2*x^6 + 32*c^15*d*x^3 + 40*c^16)*(d^2/c^17)^(5/6) + 3*sqrt(3)*(7*c^9*d^2*x^4 + 4*c^10*d*x)*sqrt(d^2/c^17) + 9*sqrt(3)*(c^3*d^3*x^5 + 2*c^4*d^2*x^2)*(d^2/c^17)^(1/6)))*sqrt((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^12*d^3*x^7 - 52*c^13*d^2*x^4 - 80*c^14*d*x)*(d^2/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^15*d^2*x^5 + c^16*d*x^2)*(d^2/c^17)^(5/6) - 4*(c^9*d^3*x^6 + 41*c^10*d^2*x^3 + 40*c^11*d)*sqrt(d^2/c^17) - (c^3*d^4*x^7 - 28*c^4*d^3*x^4 - 272*c^5*d^2*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^4*x^8 + 20*c^7*d^3*x^5 - 8*c^8*d^2*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^5*x^7 - 7*c*d^4*x^4 - 8*c^2*d^3*x)) + 3096*(d*x^4 + c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (c^3*d*x^4 + c^4*x)*(d^2/c^17)^(1/6)*log((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^12*d^3*x^7 - 52*c^13*d^2*x^4 - 80*c^14*d*x)*(d^2/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^15*d^2*x^5 + c^16*d*x^2)*(d^2/c^17)^(5/6) - 4*(c^9*d^3*x^6 + 41*c^10*d^2*x^3 + 40*c^11*d)*sqrt(d^2/c^17) - (c^3*d^4*x^7 - 28*c^4*d^3*x^4 - 272*c^5*d^2*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^4*x^8 + 20*c^7*d^3*x^5 - 8*c^8*d^2*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^3*d*x^4 + c^4*x)*(d^2/c^17)^(1/6)*log((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^12*d^3*x^7 - 52*c^13*d^2*x^4 - 80*c^14*d*x)*(d^2/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^15*d^2*x^5 + c^16*d*x^2)*(d^2/c^17)^(5/6) - 4*(c^9*d^3*x^6 + 41*c^10*d^2*x^3 + 40*c^11*d)*sqrt(d^2/c^17) - (c^3*d^4*x^7 - 28*c^4*d^3*x^4 - 272*c^5*d^2*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^4*x^8 + 20*c^7*d^3*x^5 - 8*c^8*d^2*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^3*d*x^4 + c^4*x)*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x)*(d^2/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^2/c^17)^(5/6) + (7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^3*d*x^4 + c^4*x)*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x)*(d^2/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^2/c^17)^(5/6) + (7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^3*x^8 + 38*c^7*d^2

```
*x^5 + 64*c^8*d*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*
x^3 - 512*c^3)) + 72*(43*d*x^3 + 27*c)*sqrt(d*x^3 + c))/(c^3*d*x^4 + c^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^2\sqrt{c+dx^3} - 7cdx^5\sqrt{c+dx^3} + d^2x^8\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] -Integral(1/(-8*c**2*x**2*sqrt(c + d*x**3) - 7*c*d*x**5*sqrt(c + d*x**3) +
d**2*x**8*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.336 $\int \frac{1}{x^5(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=675

$$\frac{2}{27c^2x^4\sqrt{c+dx^3}}-\frac{91\sqrt{c+dx^3}}{864c^3x^4}+\frac{113d\sqrt{c+dx^3}}{432c^4x}-\frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\,(}{\sqrt{c}}\right.}{3456\sqrt{3}\,c}$$

```
[Out] 1/10368*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(23/6)-1/10368*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/10
368*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(
23/6)*3^(1/2)+2/27/c^2/x^4/(d*x^3+c)^(1/2)-91/864*(d*x^3+c)^(1/2)/c^3/x^4+1
13/432*d*(d*x^3+c)^(1/2)/c^4/x-113/432*d^(4/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))-113/1296*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(
1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+113/864*d^(4/3)*(c^(1/3)+d^(1/3)
*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(11/3)/(d*x^3+
c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A]
time = 0.66, antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{113d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{216\sqrt{2}\,\sqrt[4]{3}\,c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{113\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{288\,3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{d^{4/3}\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3456\sqrt{3}\,c^{23/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{10368c^{23/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{23/6}}-\frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{113d\sqrt{c+dx^3}}{432c^4x}-\frac{91\sqrt{c+dx^3}}{864c^3x^4}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*x^4*Sqrt[c + d*x^3]) - (91*Sqrt[c + d*x^3])/(864*c^3*x^4) + (113*
d*Sqrt[c + d*x^3])/(432*c^4*x) - (113*d^(4/3)*Sqrt[c + d*x^3])/(432*c^4*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/
3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3456*Sqrt[3]*c^(23/6)) + (d^(4/3)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(10368*c^(23/6)) -
```

```
(d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(23/6)) + (113*Sqr
t[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(288*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (113*d^(4/3)*(c^(
1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^
(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(216*Sqrt[2
]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^5 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{91cd}{2}+\frac{11d^2x^3}{2}}{x^5(8c-dx^3)\sqrt{c+dx^3}} \, dx}{27c^2d}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{\int \frac{-904c^2d^2+\frac{455}{4}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}} \, dx}{432c^4d}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{\int \frac{x(3610c^3d^3-452c^2d^4x}{(8c-dx^3)\sqrt{c+dx}}}{3456c^6d}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{\int \left(\frac{452c^2d^3x}{\sqrt{c+dx^3}} - \right.}{34}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{(113d^2)\int \frac{x}{\sqrt{c+dx}}}{864c^4}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{d\int \frac{2\sqrt[3]{c}\,d^{2/3}-2}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}}{c^{2/3}}\right.}}{6912}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c}}{432c^4\left(\left(1+\sqrt{3}\right)\right.}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c}}{432c^4\left(\left(1+\sqrt{3}\right)\right.}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c}}{432c^4\left(\left(1+\sqrt{3}\right)\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 20.07, size = 153, normalized size = 0.23

$$\frac{160c(-27c^2+135cdx^3+226d^2x^6)-9025cd^2x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+452d^3x^9\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{138240c^5x^4\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (160*c*(-27*c^2 + 135*c*d*x^3 + 226*d^2*x^6) - 9025*c*d^2*x^6*Sqrt[1 + (d*x
^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 452*d^3*x^
9*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c
)])/(138240*c^5*x^4*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 1864, normalized size = 2.76

method	result	size
elliptic	Expression too large to display	911
risch	Expression too large to display	1344
default	Expression too large to display	1864

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/4*(d*x^3+c)^(1/2)/c^2/x^4+13/8*d*(d*x^3+c)^(1/2)/c^3/x+2/3*d^2/c^
3*x^2/((x^3+c/d)*d)^(1/2)+55/72*I/c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2))))-1/64/c^2*d^2*(-2/27*x^2/c^2/((x^3+c/d)*d)^(1/2)-2/81*I/c^2*3^(1/2)/
d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(
```

```
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-
c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*
3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1
/64*d/c^2*(-2/3*d*x^2/c^2/((x^3+c/d)*d)^(1/2)-(d*x^3+c)^(1/2)/c^2/x-5/9*I/c
^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.93, size = 2708, normalized size = 4.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/124416*(4*sqrt(3)*(c^4*d*x^7 + c^5*x^4)*(d^8/c^23)^(1/6)*arctan(1/9*((9*
sqrt(3)*c^4*d^13*x^5*(d^8/c^23)^(1/6) - sqrt(3)*(c^19*d^8*x^6 - 40*c^20*d^7
*x^3 - 32*c^21*d^6)*(d^8/c^23)^(5/6) + 3*sqrt(3)*(5*c^12*d^10*x^4 + 8*c^13*
d^9*x)*sqrt(d^8/c^23))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^16*d^3*x^5 + c^17*d
^2*x^2)*(d^8/c^23)^(2/3) + 12*sqrt(3)*(c^8*d^6*x^6 - c^9*d^5*x^3 - 2*c^10*d
^4)*(d^8/c^23)^(1/3) + 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*x^4 + 4*c^2*d^7*x) + sq
rt(d*x^3 + c)*(sqrt(3)*(c^19*d^2*x^6 + 32*c^20*d*x^3 + 40*c^21)*(d^8/c^23)^
(5/6) + 3*sqrt(3)*(7*c^12*d^4*x^4 + 4*c^13*d^3*x)*sqrt(d^8/c^23) + 9*sqrt(3
)*(c^4*d^7*x^5 + 2*c^5*d^6*x^2)*(d^8/c^23)^(1/6)))*sqrt((d^15*x^9 - 276*c*d
^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^16*d^9*x^7 - 52*c^17*d^
8*x^4 - 80*c^18*d^7*x)*(d^8/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^20*d^7*x
^5 + c^21*d^6*x^2)*(d^8/c^23)^(5/6) - 4*(c^12*d^10*x^6 + 41*c^13*d^9*x^3 +
40*c^14*d^8)*sqrt(d^8/c^23) - (c^4*d^13*x^7 - 28*c^5*d^12*x^4 - 272*c^6*d^1
1*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^12*x^8 + 20*c^9*d^11*x^5 - 8*c^10*d^10*x
^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/
(d^15*x^7 - 7*c*d^14*x^4 - 8*c^2*d^13*x)) + 4*sqrt(3)*(c^4*d*x^7 + c^5*x^4)
*(d^8/c^23)^(1/6)*arctan(1/9*((9*sqrt(3)*c^4*d^13*x^5*(d^8/c^23)^(1/6) - sq
rt(3)*(c^19*d^8*x^6 - 40*c^20*d^7*x^3 - 32*c^21*d^6)*(d^8/c^23)^(5/6) + 3*s
qrt(3)*(5*c^12*d^10*x^4 + 8*c^13*d^9*x)*sqrt(d^8/c^23))*sqrt(d*x^3 + c) - (
18*sqrt(3)*(c^16*d^3*x^5 + c^17*d^2*x^2)*(d^8/c^23)^(2/3) + 12*sqrt(3)*(c^8
*d^6*x^6 - c^9*d^5*x^3 - 2*c^10*d^4)*(d^8/c^23)^(1/3) + 3*sqrt(3)*(d^9*x^7
+ 5*c*d^8*x^4 + 4*c^2*d^7*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^19*d^2*x^6 + 32*
c^20*d*x^3 + 40*c^21)*(d^8/c^23)^(5/6) + 3*sqrt(3)*(7*c^12*d^4*x^4 + 4*c^13
*d^3*x)*sqrt(d^8/c^23) + 9*sqrt(3)*(c^4*d^7*x^5 + 2*c^5*d^6*x^2)*(d^8/c^23)
^(1/6)))*sqrt((d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^1
2 - 18*(c^16*d^9*x^7 - 52*c^17*d^8*x^4 - 80*c^18*d^7*x)*(d^8/c^23)^(2/3) -
6*sqrt(d*x^3 + c)*(24*(c^20*d^7*x^5 + c^21*d^6*x^2)*(d^8/c^23)^(5/6) - 4*(c
^12*d^10*x^6 + 41*c^13*d^9*x^3 + 40*c^14*d^8)*sqrt(d^8/c^23) - (c^4*d^13*x^
7 - 28*c^5*d^12*x^4 - 272*c^6*d^11*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^12*x^8
+ 20*c^9*d^11*x^5 - 8*c^10*d^10*x^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*
x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^15*x^7 - 7*c*d^14*x^4 - 8*c^2*d^13*x))
- 32544*(d^2*x^7 + c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassP
Inverse(0, -4*c/d, x)) + (c^4*d*x^7 + c^5*x^4)*(d^8/c^23)^(1/6)*log((d^15*x
^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^16*d^9*x^7
- 52*c^17*d^8*x^4 - 80*c^18*d^7*x)*(d^8/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(24
*(c^20*d^7*x^5 + c^21*d^6*x^2)*(d^8/c^23)^(5/6) - 4*(c^12*d^10*x^6 + 41*c^1
3*d^9*x^3 + 40*c^14*d^8)*sqrt(d^8/c^23) - (c^4*d^13*x^7 - 28*c^5*d^12*x^4 -
 272*c^6*d^11*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^12*x^8 + 20*c^9*d^11*x^5 - 8
*c^10*d^10*x^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
 512*c^3)) - (c^4*d*x^7 + c^5*x^4)*(d^8/c^23)^(1/6)*log((d^15*x^9 - 276*c*d
^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^16*d^9*x^7 - 52*c^17*d^
8*x^4 - 80*c^18*d^7*x)*(d^8/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^20*d^7*x
^5 + c^21*d^6*x^2)*(d^8/c^23)^(5/6) - 4*(c^12*d^10*x^6 + 41*c^13*d^9*x^3 +
40*c^14*d^8)*sqrt(d^8/c^23) - (c^4*d^13*x^7 - 28*c^5*d^12*x^4 - 272*c^6*d^1
1*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^12*x^8 + 20*c^9*d^11*x^5 - 8*c^10*d^10*x
```

```
^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
 2*(c^4*d*x^7 + c^5*x^4)*(d^8/c^23)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 12
00*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^
18*d*x)*(d^8/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2
)*(d^8/c^23)^(5/6) + (7*c^12*d^4*x^6 + 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt
(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x)*(d^8/c^23)^(1/6
)) + 18*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2)*(d^8/c^23)^(1/3))/
(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^4*d*x^7 + c^5*x^
4)*(d^8/c^23)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c
^3*d^6 + 18*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x)*(d^8/c^23)^(2/
3) - 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^8/c^23)^(5/6) + (
7*c^12*d^4*x^6 + 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*
x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^6*x^8 +
 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^
6 + 192*c^2*d*x^3 - 512*c^3)) - 144*(226*d^2*x^6 + 135*c*d*x^3 - 27*c^2)*sq
rt(d*x^3 + c))/(c^4*d*x^7 + c^5*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^5\sqrt{c+dx^3}-7cdx^8\sqrt{c+dx^3}+d^2x^{11}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] -Integral(1/(-8*c**2*x**5*sqrt(c + d*x**3) - 7*c*d*x**8*sqrt(c + d*x**3) +
d**2*x**11*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.337 $\int \frac{1}{x^8(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=699

$$\frac{2}{27c^2x^7\sqrt{c+dx^3}}-\frac{139\sqrt{c+dx^3}}{1512c^3x^7}+\frac{6095d\sqrt{c+dx^3}}{48384c^4x^4}-\frac{953d^2\sqrt{c+dx^3}}{3024c^5x}+\frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

```
[Out] 1/82944*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(29/6)-1/82944*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6)-1/82
944*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(
29/6)*3^(1/2)+2/27/c^2/x^7/(d*x^3+c)^(1/2)-139/1512*(d*x^3+c)^(1/2)/c^3/x^7
+6095/48384*d*(d*x^3+c)^(1/2)/c^4/x^4-953/3024*d^2*(d*x^3+c)^(1/2)/c^5/x+95
3/3024*d^(7/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+953/9072
*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(14/3)*2^(1/2)/
(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)-953/6048*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3
)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-
1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1
/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 699, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{953d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{1512\sqrt{2}\sqrt[4]{3}\,c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{953\sqrt{2-\sqrt{3}}\,d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2016\,3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{d^{7/3}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}\,c^{29/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{29/6}}-\frac{d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{29/6}}+\frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{953d^2\sqrt{c+dx^3}}{3024c^5x}+\frac{6095d\sqrt{c+dx^3}}{48384c^4x^4}-\frac{139\sqrt{c+dx^3}}{1512c^3x^7}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*x^7*Sqrt[c + d*x^3]) - (139*Sqrt[c + d*x^3])/(1512*c^3*x^7) + (60
95*d*Sqrt[c + d*x^3])/(48384*c^4*x^4) - (953*d^2*Sqrt[c + d*x^3])/(3024*c^5
*x) + (953*d^(7/3)*Sqrt[c + d*x^3])/(3024*c^5*((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)) - (d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c +
d*x^3]])/(27648*Sqrt[3]*c^(29/6)) + (d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^
```

```
2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(82944*c^(29/6)) - (d^(7/3)*ArcTanh[Sqrt[c
+ d*x^3]/(3*Sqrt[c])])/(82944*c^(29/6)) - (953*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2016*3^(
3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (953*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(1512*Sqrt[2]*3^(1/4)*c^(14/3)*Sqrt
[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqr
t[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^8 (8c-dx^3) (c+dx^3)^{3/2}} dx = \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{139cd}{2}+\frac{17d^2x^3}{2}}{x^8(8c-dx^3)\sqrt{c+dx^3}} dx}{27c^2d}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{\int \frac{-\frac{6095}{2}c^2d^2+\frac{1529}{4}cd^3x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}} dx}{756c^4d}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{\int \frac{-60992c^3d^3+30}{x^2(8c-dx^3)\sqrt{c}}}{24192c}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx}}{3024c^5x}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx}}{3024c^5x}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx}}{3024c^5x}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx}}{3024c^5x}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx}}{3024c^5x}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx}}{3024c^5x}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx}}{3024c^5x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.08, size = 167, normalized size = 0.24

$$\frac{610025cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\left(5c(864c^3-1647c^2dx^3+9153cd^2x^6+15248d^3x^9)+953d^4x^{12}\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{7741440c^6x^7\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (610025*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), (d*x^3)/(8*c)] - 32*(5*c*(864*c^3 - 1647*c^2*d*x^3 + 9153*c*d^2*x^6 + 1
5248*d^3*x^9) + 953*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))/(7741440*c^6*x^7*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 2389, normalized size = 3.42

method	result	size
elliptic	Expression too large to display	930
risch	Expression too large to display	1357
default	Expression too large to display	2389

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*d/c^2*(-1/4*(d*x^3+c)^(1/2)/c^2/x^4+13/8*d*(d*x^3+c)^(1/2)/c^3/x+2/3*d
^2/c^3*x^2/((x^3+c/d)*d)^(1/2)+55/72*I/c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2))))+1/8/c*(-1/7*(d*x^3+c)^(1/2)/c^2/x^7+25/56*d*(d*x^3+c)^(1/2)/c^
3/x^4-237/112*d^2*(d*x^3+c)^(1/2)/c^4/x-2/3*d^3/c^4*x^2/((x^3+c/d)*d)^(1/2)
-935/1008*I*d^2/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)
^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(
x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
```

```
1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-1/512/c^3*d^
3*(-2/27*x^2/c^2/((x^3+c/d)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)
*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alph
a-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/512/c^3*d^2*(-2/3*
d*x^2/c^2/((x^3+c/d)*d)^(1/2)-(d*x^3+c)^(1/2)/c^2/x-5/9*I/c^2*3^(1/2)*(-c*d
^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 16.06, size = 2726, normalized size = 3.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6967296*(28*sqrt(3)*(c^5*d*x^10 + c^6*x^7)*(d^14/c^29)^(1/6)*arctan(1/9*
((9*sqrt(3)*c^5*d^22*x^5*(d^14/c^29)^(1/6) - sqrt(3)*(c^24*d^13*x^6 - 40*c^
25*d^12*x^3 - 32*c^26*d^11)*(d^14/c^29)^(5/6) + 3*sqrt(3)*(5*c^15*d^17*x^4
+ 8*c^16*d^16*x)*sqrt(d^14/c^29))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^20*d^4*x
^5 + c^21*d^3*x^2)*(d^14/c^29)^(2/3) + 12*sqrt(3)*(c^10*d^9*x^6 - c^11*d^8*
x^3 - 2*c^12*d^7)*(d^14/c^29)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 +
4*c^2*d^12*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^24*d^2*x^6 + 32*c^25*d*x^3 + 40
*c^26)*(d^14/c^29)^(5/6) + 3*sqrt(3)*(7*c^15*d^6*x^4 + 4*c^16*d^5*x)*sqrt(d
^14/c^29) + 9*sqrt(3)*(c^5*d^11*x^5 + 2*c^6*d^10*x^2)*(d^14/c^29)^(1/6)))*s
qrt((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^
20*d^15*x^7 - 52*c^21*d^14*x^4 - 80*c^22*d^13*x)*(d^14/c^29)^(2/3) + 6*sqrt
(d*x^3 + c)*(24*(c^25*d^12*x^5 + c^26*d^11*x^2)*(d^14/c^29)^(5/6) - 4*(c^15
*d^17*x^6 + 41*c^16*d^16*x^3 + 40*c^17*d^15)*sqrt(d^14/c^29) - (c^5*d^22*x^
7 - 28*c^6*d^21*x^4 - 272*c^7*d^20*x)*(d^14/c^29)^(1/6)) + 18*(c^10*d^20*x^
8 + 20*c^11*d^19*x^5 - 8*c^12*d^18*x^2)*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*
d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*
x)) + 28*sqrt(3)*(c^5*d*x^10 + c^6*x^7)*(d^14/c^29)^(1/6)*arctan(1/9*((9*sq
rt(3)*c^5*d^22*x^5*(d^14/c^29)^(1/6) - sqrt(3)*(c^24*d^13*x^6 - 40*c^25*d^1
2*x^3 - 32*c^26*d^11)*(d^14/c^29)^(5/6) + 3*sqrt(3)*(5*c^15*d^17*x^4 + 8*c^
16*d^16*x)*sqrt(d^14/c^29))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^20*d^4*x^5 + c
^21*d^3*x^2)*(d^14/c^29)^(2/3) + 12*sqrt(3)*(c^10*d^9*x^6 - c^11*d^8*x^3 -
2*c^12*d^7)*(d^14/c^29)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 + 4*c^2*
d^12*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^24*d^2*x^6 + 32*c^25*d*x^3 + 40*c^26)
*(d^14/c^29)^(5/6) + 3*sqrt(3)*(7*c^15*d^6*x^4 + 4*c^16*d^5*x)*sqrt(d^14/c^
29) + 9*sqrt(3)*(c^5*d^11*x^5 + 2*c^6*d^10*x^2)*(d^14/c^29)^(1/6)))*sqrt((d
^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^20*d^1
5*x^7 - 52*c^21*d^14*x^4 - 80*c^22*d^13*x)*(d^14/c^29)^(2/3) - 6*sqrt(d*x^3
 + c)*(24*(c^25*d^12*x^5 + c^26*d^11*x^2)*(d^14/c^29)^(5/6) - 4*(c^15*d^17*
x^6 + 41*c^16*d^16*x^3 + 40*c^17*d^15)*sqrt(d^14/c^29) - (c^5*d^22*x^7 - 28
*c^6*d^21*x^4 - 272*c^7*d^20*x)*(d^14/c^29)^(1/6)) + 18*(c^10*d^20*x^8 + 20
*c^11*d^19*x^5 - 8*c^12*d^18*x^2)*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^
6 + 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*x)) +
```

```
2195712*(d^3*x^10 + c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstra
ssPInverse(0, -4*c/d, x)) + 7*(c^5*d*x^10 + c^6*x^7)*(d^14/c^29)^(1/6)*log(
(d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^20*d
^15*x^7 - 52*c^21*d^14*x^4 - 80*c^22*d^13*x)*(d^14/c^29)^(2/3) + 6*sqrt(d*x
^3 + c)*(24*(c^25*d^12*x^5 + c^26*d^11*x^2)*(d^14/c^29)^(5/6) - 4*(c^15*d^1
7*x^6 + 41*c^16*d^16*x^3 + 40*c^17*d^15)*sqrt(d^14/c^29) - (c^5*d^22*x^7 -
28*c^6*d^21*x^4 - 272*c^7*d^20*x)*(d^14/c^29)^(1/6)) + 18*(c^10*d^20*x^8 +
20*c^11*d^19*x^5 - 8*c^12*d^18*x^2)*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*
x^6 + 192*c^2*d*x^3 - 512*c^3)) - 7*(c^5*d*x^10 + c^6*x^7)*(d^14/c^29)^(1/6
)*log((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(
c^20*d^15*x^7 - 52*c^21*d^14*x^4 - 80*c^22*d^13*x)*(d^14/c^29)^(2/3) - 6*sq
rt(d*x^3 + c)*(24*(c^25*d^12*x^5 + c^26*d^11*x^2)*(d^14/c^29)^(5/6) - 4*(c^
15*d^17*x^6 + 41*c^16*d^16*x^3 + 40*c^17*d^15)*sqrt(d^14/c^29) - (c^5*d^22*
x^7 - 28*c^6*d^21*x^4 - 272*c^7*d^20*x)*(d^14/c^29)^(1/6)) + 18*(c^10*d^20*
x^8 + 20*c^11*d^19*x^5 - 8*c^12*d^18*x^2)*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 14*(c^5*d*x^10 + c^6*x^7)*(d^14/c^2
9)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11
+ 18*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x)*(d^14/c^29)^(2/3) +
 6*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2)*(d^14/c^29)^(5/6) + (7*c
^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x
^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x)*(d^14/c^29)^(1/6)) + 18*(c^10*d^9*x^8
 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2)*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 14*(c^5*d*x^10 + c^6*x^7)*(d^14/c^29)^(
1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18
*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x)*(d^14/c^29)^(2/3) - 6*s
qrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2)*(d^14/c^29)^(5/6) + (7*c^15*
d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 +
 80*c^6*d^10*x^4 + 160*c^7*d^9*x)*(d^14/c^29)^(1/6)) + 18*(c^10*d^9*x^8 + 3
8*c^11*d^8*x^5 + 64*c^12*d^7*x^2)*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^
6 + 192*c^2*d*x^3 - 512*c^3)) + 144*(15248*d^3*x^9 + 9153*c*d^2*x^6 - 1647*
c^2*d*x^3 + 864*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^10 + c^6*x^7)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^8\sqrt{c+dx^3} - 7cdx^{11}\sqrt{c+dx^3} + d^2x^{14}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**8*sqrt(c + d*x**3) - 7*c*d*x**11*sqrt(c + d*x**3) +
 d**2*x**14*sqrt(c + d*x**3)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^8\,{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.338 $\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

```
[Out] 1/32*x^4*AppellF1(4/3,3/2,1,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2
/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(32*c^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(66) = 132.
time = 6.35, size = 233, normalized size = 3.53

$$\frac{x\left(x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+\frac{64c\left(-1+\frac{256c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)\right)}\right)}{d}\right)}{864c^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3
)/(8*c)] + (64*c*(-1 + (256*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d
*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/d)
)/(864*c^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.33, size = 1038, normalized size = 15.73

method	result

elliptic	$-\frac{2x}{27dc\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{2i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/3*x/c/((x^3+c/d)*d)^(1/2)-2/9*I/c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-8*c/d*(-2/27*x/c^2/((x^3+c/d)*d)^(1/2)+2/81
*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/24
3*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^
2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_
alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
```

```
2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2628 vs. 2(52) = 104.
time = 6.87, size = 2628, normalized size = 39.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] 1/486*(4*sqrt(3)*(c*d^3*x^3 + c^2*d^2)*(1/(c^7*d^8))^(1/6)*arctan(1/9*((9*s
qrt(3)*c^6*d^8*x^5*(1/(c^7*d^8))^(5/6) + 3*sqrt(3)*(5*c^4*d^5*x^4 + 8*c^5*d
^4*x)*sqrt(1/(c^7*d^8)) - sqrt(3)*(c*d^3*x^6 - 40*c^2*d^2*x^3 - 32*c^3*d)*(
1/(c^7*d^8))^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c^5*d^7*x^6 - c^6*d^6*x^
3 - 2*c^7*d^5)*(1/(c^7*d^8))^(2/3) + 18*sqrt(3)*(c^3*d^4*x^5 + c^4*d^3*x^2)
*(1/(c^7*d^8))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x
^3 + c)*(9*sqrt(3)*(c^6*d^8*x^5 + 2*c^7*d^7*x^2)*(1/(c^7*d^8))^(5/6) + 3*sq
rt(3)*(7*c^4*d^5*x^4 + 4*c^5*d^4*x)*sqrt(1/(c^7*d^8)) + sqrt(3)*(c*d^3*x^6
+ 32*c^2*d^2*x^3 + 40*c^3*d)*(1/(c^7*d^8))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^
2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^5*d^8*x^8 + 20*c^6*d^7*x^5 - 8*c^
7*d^6*x^2)*(1/(c^7*d^8))^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^9*x^7 - 28*c^7*d
^8*x^4 - 272*c^8*d^7*x)*(1/(c^7*d^8))^(5/6) + 4*(c^4*d^6*x^6 + 41*c^5*d^5*x
^3 + 40*c^6*d^4)*sqrt(1/(c^7*d^8)) - 24*(c^2*d^3*x^5 + c^3*d^2*x^2)*(1/(c^7
*d^8))^(1/6)) - 18*(c^3*d^5*x^7 - 52*c^4*d^4*x^4 - 80*c^5*d^3*x)*(1/(c^7*d^
8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 -
7*c*d*x^4 - 8*c^2*x)) + 4*sqrt(3)*(c*d^3*x^3 + c^2*d^2)*(1/(c^7*d^8))^(1/6)
*arctan(1/9*((9*sqrt(3)*c^6*d^8*x^5*(1/(c^7*d^8))^(5/6) + 3*sqrt(3)*(5*c^4*
d^5*x^4 + 8*c^5*d^4*x)*sqrt(1/(c^7*d^8)) - sqrt(3)*(c*d^3*x^6 - 40*c^2*d^2*
x^3 - 32*c^3*d)*(1/(c^7*d^8))^(1/6))*sqrt(d*x^3 + c) + (12*sqrt(3)*(c^5*d^7
*x^6 - c^6*d^6*x^3 - 2*c^7*d^5)*(1/(c^7*d^8))^(2/3) + 18*sqrt(3)*(c^3*d^4*x
^5 + c^4*d^3*x^2)*(1/(c^7*d^8))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*
c^2*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^6*d^8*x^5 + 2*c^7*d^7*x^2)*(1/(c^7*d
```

```
^8))^(5/6) + 3*sqrt(3)*(7*c^4*d^5*x^4 + 4*c^5*d^4*x)*sqrt(1/(c^7*d^8)) + sq
rt(3)*(c*d^3*x^6 + 32*c^2*d^2*x^3 + 40*c^3*d)*(1/(c^7*d^8))^(1/6)))*sqrt((d
^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^5*d^8*x^8 + 20*c
^6*d^7*x^5 - 8*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c^6*d
^9*x^7 - 28*c^7*d^8*x^4 - 272*c^8*d^7*x)*(1/(c^7*d^8))^(5/6) + 4*(c^4*d^6*x
^6 + 41*c^5*d^5*x^3 + 40*c^6*d^4)*sqrt(1/(c^7*d^8)) - 24*(c^2*d^3*x^5 + c^3
*d^2*x^2)*(1/(c^7*d^8))^(1/6)) - 18*(c^3*d^5*x^7 - 52*c^4*d^4*x^4 - 80*c^5*
d^3*x)*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c
^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) - 36*sqrt(d*x^3 + c)*d*x + 36*(d*x^3
 + c)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) + 2*(c*d^3*x^3 + c^2*d^2)*(
1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3
+ 18*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) +
6*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x)*(1/(c^7*d
^8))^(5/6) + (7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8
)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2)*(1/(c^7*d^8))^(1/6)) + 18*(5*c^3*d^
5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x)*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c*d^3*x^3 + c^2*d^2)*(1/(c^7*d^8)
)^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^5*d
^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) - 6*sqrt(d*x^
3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x)*(1/(c^7*d^8))^(5/6)
+ (7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c
^2*d^3*x^5 + 32*c^3*d^2*x^2)*(1/(c^7*d^8))^(1/6)) + 18*(5*c^3*d^5*x^7 + 64*
c^4*d^4*x^4 + 32*c^5*d^3*x)*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 + c^2*d^2)*(1/(c^7*d^8))^(1/6)*log((
d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^5*d^8*x^8 + 20*
c^6*d^7*x^5 - 8*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*
d^9*x^7 - 28*c^7*d^8*x^4 - 272*c^8*d^7*x)*(1/(c^7*d^8))^(5/6) + 4*(c^4*d^6*
x^6 + 41*c^5*d^5*x^3 + 40*c^6*d^4)*sqrt(1/(c^7*d^8)) - 24*(c^2*d^3*x^5 + c^
3*d^2*x^2)*(1/(c^7*d^8))^(1/6)) - 18*(c^3*d^5*x^7 - 52*c^4*d^4*x^4 - 80*c^5
*d^3*x)*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)) - (c*d^3*x^3 + c^2*d^2)*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 - 276*c*d^2*
x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^5*d^8*x^8 + 20*c^6*d^7*x^5 - 8*c^7*
d^6*x^2)*(1/(c^7*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c^6*d^9*x^7 - 28*c^7*d^8
*x^4 - 272*c^8*d^7*x)*(1/(c^7*d^8))^(5/6) + 4*(c^4*d^6*x^6 + 41*c^5*d^5*x^3
 + 40*c^6*d^4)*sqrt(1/(c^7*d^8)) - 24*(c^2*d^3*x^5 + c^3*d^2*x^2)*(1/(c^7*d
^8))^(1/6)) - 18*(c^3*d^5*x^7 - 52*c^4*d^4*x^4 - 80*c^5*d^3*x)*(1/(c^7*d^8)
)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^3*x^3 +
c^2*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.339 $\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

```
[Out] 1/8*x*AppellF1(1/3,3/2,1,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/(d
*x^3+c)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(8*c^2*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 230 vs. 2(64) = 128.
time = 10.08, size = 230, normalized size = 3.59

$$\frac{x\left(-\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+64\left(\frac{1}{c^2}+\frac{176F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)\right)}\right)\right)}{864\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)])/c^3) + 64*(c^(-2) + (176*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3
)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*
x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]
))))))/(864*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.30, size = 721, normalized size = 11.27 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*x/c^2/((x^3+c/d)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d
^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
```

```
-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(
1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*
d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2572 vs. $2(50) = 100$.
time = 6.58, size = 2572, normalized size = 40.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3888*(4*sqrt(3)*(c^2*d^2*x^3 + c^3*d)*(1/(c^13*d^2))^(1/6)*arctan(1/9*((9
*sqrt(3)*c^11*d^3*x^5*(1/(c^13*d^2))^(5/6) + 3*sqrt(3)*(5*c^7*d^2*x^4 + 8*c
^8*d*x)*sqrt(1/(c^13*d^2)) - sqrt(3)*(c^2*d^2*x^6 - 40*c^3*d*x^3 - 32*c^4)*
(1/(c^13*d^2))^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c^9*d^3*x^6 - c^10*d^2
*x^3 - 2*c^11*d)*(1/(c^13*d^2))^(2/3) + 18*sqrt(3)*(c^5*d^2*x^5 + c^6*d*x^2
)*(1/(c^13*d^2))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d
*x^3 + c)*(9*sqrt(3)*(c^11*d^3*x^5 + 2*c^12*d^2*x^2)*(1/(c^13*d^2))^(5/6) +
 3*sqrt(3)*(7*c^7*d^2*x^4 + 4*c^8*d*x)*sqrt(1/(c^13*d^2)) + sqrt(3)*(c^2*d^
2*x^6 + 32*c^3*d*x^3 + 40*c^4)*(1/(c^13*d^2))^(1/6)))*sqrt((d^3*x^9 - 276*c
*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^9*d^4*x^8 + 20*c^10*d^3*x^5 -
8*c^11*d^2*x^2)*(1/(c^13*d^2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^4*x^7 - 2
8*c^12*d^3*x^4 - 272*c^13*d^2*x)*(1/(c^13*d^2))^(5/6) + 4*(c^7*d^3*x^6 + 41
*c^8*d^2*x^3 + 40*c^9*d)*sqrt(1/(c^13*d^2)) - 24*(c^3*d^2*x^5 + c^4*d*x^2)*
(1/(c^13*d^2))^(1/6)) - 18*(c^5*d^3*x^7 - 52*c^6*d^2*x^4 - 80*c^7*d*x)*(1/(
c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2
*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 4*sqrt(3)*(c^2*d^2*x^3 + c^3*d)*(1/(c^13*d^2
))^(1/6)*arctan(1/9*((9*sqrt(3)*c^11*d^3*x^5*(1/(c^13*d^2))^(5/6) + 3*sqrt(
3)*(5*c^7*d^2*x^4 + 8*c^8*d*x)*sqrt(1/(c^13*d^2)) - sqrt(3)*(c^2*d^2*x^6 -
```

```
40*c^3*d*x^3 - 32*c^4)*(1/(c^13*d^2))^(1/6))*sqrt(d*x^3 + c) + (12*sqrt(3)*
(c^9*d^3*x^6 - c^10*d^2*x^3 - 2*c^11*d)*(1/(c^13*d^2))^(2/3) + 18*sqrt(3)*(
c^5*d^2*x^5 + c^6*d*x^2)*(1/(c^13*d^2))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*
x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^11*d^3*x^5 + 2*c^12*d^2*x^2)
*(1/(c^13*d^2))^(5/6) + 3*sqrt(3)*(7*c^7*d^2*x^4 + 4*c^8*d*x)*sqrt(1/(c^13*
d^2)) + sqrt(3)*(c^2*d^2*x^6 + 32*c^3*d*x^3 + 40*c^4)*(1/(c^13*d^2))^(1/6))
)*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^9*d^4*x
^8 + 20*c^10*d^3*x^5 - 8*c^11*d^2*x^2)*(1/(c^13*d^2))^(2/3) - 6*sqrt(d*x^3
+ c)*((c^11*d^4*x^7 - 28*c^12*d^3*x^4 - 272*c^13*d^2*x)*(1/(c^13*d^2))^(5/6
) + 4*(c^7*d^3*x^6 + 41*c^8*d^2*x^3 + 40*c^9*d)*sqrt(1/(c^13*d^2)) - 24*(c^
3*d^2*x^5 + c^4*d*x^2)*(1/(c^13*d^2))^(1/6)) - 18*(c^5*d^3*x^7 - 52*c^6*d^2
*x^4 - 80*c^7*d*x)*(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 288*sqrt(d*x^3 + c)*d
*x + 360*(d*x^3 + c)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) + 2*(c^2*d^2
*x^3 + c^3*d)*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*
d*x^3 + 640*c^3 + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)*(1/(
c^13*d^2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160
*c^13*d^2*x)*(1/(c^13*d^2))^(5/6) + (7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c
^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2)*(1/(c^13*d^2))^
(1/6)) + 18*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x)*(1/(c^13*d^2))^(1
/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^2*d^2*x^3 +
 c^3*d)*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3
+ 640*c^3 + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)*(1/(c^13*d
^2))^(2/3) - 6*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*
d^2*x)*(1/(c^13*d^2))^(5/6) + (7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*
sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2)*(1/(c^13*d^2))^(1/6))
 + 18*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x)*(1/(c^13*d^2))^(1/3))/(
d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^2*d^2*x^3 + c^3*d)*
(1/(c^13*d^2))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c
^3 + 18*(c^9*d^4*x^8 + 20*c^10*d^3*x^5 - 8*c^11*d^2*x^2)*(1/(c^13*d^2))^(2/
3) + 6*sqrt(d*x^3 + c)*((c^11*d^4*x^7 - 28*c^12*d^3*x^4 - 272*c^13*d^2*x)*(
1/(c^13*d^2))^(5/6) + 4*(c^7*d^3*x^6 + 41*c^8*d^2*x^3 + 40*c^9*d)*sqrt(1/(c
^13*d^2)) - 24*(c^3*d^2*x^5 + c^4*d*x^2)*(1/(c^13*d^2))^(1/6)) - 18*(c^5*d^
3*x^7 - 52*c^6*d^2*x^4 - 80*c^7*d*x)*(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*
d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^2*d^2*x^3 + c^3*d)*(1/(c^13*d^2))^
(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^9*d^
4*x^8 + 20*c^10*d^3*x^5 - 8*c^11*d^2*x^2)*(1/(c^13*d^2))^(2/3) - 6*sqrt(d*x
^3 + c)*((c^11*d^4*x^7 - 28*c^12*d^3*x^4 - 272*c^13*d^2*x)*(1/(c^13*d^2))^(
5/6) + 4*(c^7*d^3*x^6 + 41*c^8*d^2*x^3 + 40*c^9*d)*sqrt(1/(c^13*d^2)) - 24*
(c^3*d^2*x^5 + c^4*d*x^2)*(1/(c^13*d^2))^(1/6)) - 18*(c^5*d^3*x^7 - 52*c^6*
d^2*x^4 - 80*c^7*d*x)*(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c
^2*d*x^3 - 512*c^3)))/(c^2*d^2*x^3 + c^3*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2\sqrt{c+dx^3}-7cdx^3\sqrt{c+dx^3}+d^2x^6\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*sqrt(c + d*x**3) - 7*c*d*x**3*sqrt(c + d*x**3) + d**2*
x**6*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.340 $\int \frac{1}{x^3(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

```
[Out] -1/16*AppellF1(-2/3,3/2,1,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/x
^2/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/16*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, (d*x^3)/(8*c), -((d*
x^3)/c)])/(c^2*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \frac{\sqrt{1 + \frac{dx^3}{c}} \int \frac{1}{x^3 (8c - dx^3) \left(1 + \frac{dx^3}{c}\right)^{3/2}} \, dx}{c\sqrt{c + dx^3}}$$
$$= -\frac{\sqrt{1 + \frac{dx^3}{c}} \, F_1\left(-\frac{2}{3}; 1, \frac{3}{2}; \frac{1}{3}; \frac{dx^3}{8c}, -\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c + dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(66) = 132.
time = 20.12, size = 248, normalized size = 3.76

$$\frac{59d^2x^6\sqrt{1 + \frac{dx^3}{c}} \, F_1\left(\frac{4}{3}; \frac{1}{2}, 1; \frac{7}{3}; -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) + 64c\left(-27c - 59dx^3 - \frac{7360c^2dx^3F_1\left(\frac{1}{3}; \frac{1}{2}, 1; \frac{4}{3}; -\frac{dx^3}{c}, \frac{dx^3}{8c}\right)}{(8c - dx^3)\left(32cF_1\left(\frac{1}{3}; \frac{1}{2}, 1; \frac{4}{3}; -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) + 3dx^3\left(F_1\left(\frac{4}{3}; \frac{1}{2}, 2; \frac{7}{3}; -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) - 4F_1\left(\frac{4}{3}; \frac{3}{2}, 1; \frac{7}{3}; -\frac{dx^3}{c}, \frac{dx^3}{8c}\right)\right)\right)}\right)}{27648c^4x^2\sqrt{c + dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (59*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d
*x^3)/(8*c)] + 64*c*(-27*c - 59*d*x^3 - (7360*c^2*d*x^3*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/
2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/
3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c)
, (d*x^3)/(8*c)])))))/(27648*c^4*x^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.40, size = 1053, normalized size = 15.95

method	result

elliptic	$-\frac{\sqrt{d\,x^3+c}}{16c^3x^2}-\frac{2dx}{27c^3\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{59i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\dots}{-\frac{3(-c\,d^2)^{\frac{1}{3}}}{2d}\dots}}\dots}{\dots}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/2/c^2*(d*x^3+c)^(1/2)/x^2-2/3*d*x/c^2/((x^3+c/d)*d)^(1/2)+7/18*I/
c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/8*d/c*
(-2/27*x/c^2/((x^3+c/d)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c
*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)
*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
```

```
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-
c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2664 vs. 2(52) = 104.
time = 11.03, size = 2664, normalized size = 40.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] 1/31104*(4*sqrt(3)*(c^3*d*x^5 + c^4*x^2)*(d^4/c^19)^(1/6)*arctan(1/9*((9*sq
rt(3)*c^16*d^4*x^5*(d^4/c^19)^(5/6) + 3*sqrt(3)*(5*c^10*d^5*x^4 + 8*c^11*d^
4*x)*sqrt(d^4/c^19) - sqrt(3)*(c^3*d^7*x^6 - 40*c^4*d^6*x^3 - 32*c^5*d^5)*(
d^4/c^19)^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c^13*d^2*x^6 - c^14*d*x^3 -
 2*c^15)*(d^4/c^19)^(2/3) + 18*sqrt(3)*(c^7*d^3*x^5 + c^8*d^2*x^2)*(d^4/c^1
9)^(1/3) + 3*sqrt(3)*(d^5*x^7 + 5*c*d^4*x^4 + 4*c^2*d^3*x) - sqrt(d*x^3 + c
)*(9*sqrt(3)*(c^16*d*x^5 + 2*c^17*x^2)*(d^4/c^19)^(5/6) + 3*sqrt(3)*(7*c^10
*d^2*x^4 + 4*c^11*d*x)*sqrt(d^4/c^19) + sqrt(3)*(c^3*d^4*x^6 + 32*c^4*d^3*x
^3 + 40*c^5*d^2)*(d^4/c^19)^(1/6)))*sqrt((d^9*x^9 - 276*c*d^8*x^6 - 1608*c^
2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^13*d^6*x^8 + 20*c^14*d^5*x^5 - 8*c^15*d^4*
x^2)*(d^4/c^19)^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^5*x^7 - 28*c^17*d^4*x^4
- 272*c^18*d^3*x)*(d^4/c^19)^(5/6) + 4*(c^10*d^6*x^6 + 41*c^11*d^5*x^3 + 40
*c^12*d^4)*sqrt(d^4/c^19) - 24*(c^4*d^7*x^5 + c^5*d^6*x^2)*(d^4/c^19)^(1/6)
) - 18*(c^7*d^7*x^7 - 52*c^8*d^6*x^4 - 80*c^9*d^5*x)*(d^4/c^19)^(1/3))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^8*x^7 - 7*c*d^7*x^4 - 8
*c^2*d^6*x)) + 4*sqrt(3)*(c^3*d*x^5 + c^4*x^2)*(d^4/c^19)^(1/6)*arctan(1/9*
((9*sqrt(3)*c^16*d^4*x^5*(d^4/c^19)^(5/6) + 3*sqrt(3)*(5*c^10*d^5*x^4 + 8*c
^11*d^4*x)*sqrt(d^4/c^19) - sqrt(3)*(c^3*d^7*x^6 - 40*c^4*d^6*x^3 - 32*c^5*
d^5)*(d^4/c^19)^(1/6))*sqrt(d*x^3 + c) + (12*sqrt(3)*(c^13*d^2*x^6 - c^14*d
*x^3 - 2*c^15)*(d^4/c^19)^(2/3) + 18*sqrt(3)*(c^7*d^3*x^5 + c^8*d^2*x^2)*(d
^4/c^19)^(1/3) + 3*sqrt(3)*(d^5*x^7 + 5*c*d^4*x^4 + 4*c^2*d^3*x) + sqrt(d*x
```

```
^3 + c)*(9*sqrt(3)*(c^16*d*x^5 + 2*c^17*x^2)*(d^4/c^19)^(5/6) + 3*sqrt(3)*(
7*c^10*d^2*x^4 + 4*c^11*d*x)*sqrt(d^4/c^19) + sqrt(3)*(c^3*d^4*x^6 + 32*c^4
*d^3*x^3 + 40*c^5*d^2)*(d^4/c^19)^(1/6)))*sqrt((d^9*x^9 - 276*c*d^8*x^6 - 1
608*c^2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^13*d^6*x^8 + 20*c^14*d^5*x^5 - 8*c^1
5*d^4*x^2)*(d^4/c^19)^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^5*x^7 - 28*c^17*d^
4*x^4 - 272*c^18*d^3*x)*(d^4/c^19)^(5/6) + 4*(c^10*d^6*x^6 + 41*c^11*d^5*x^
3 + 40*c^12*d^4)*sqrt(d^4/c^19) - 24*(c^4*d^7*x^5 + c^5*d^6*x^2)*(d^4/c^19)
^(1/6)) - 18*(c^7*d^7*x^7 - 52*c^8*d^6*x^4 - 80*c^9*d^5*x)*(d^4/c^19)^(1/3)
)/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^8*x^7 - 7*c*d^7*x
^4 - 8*c^2*d^6*x)) - 4176*(d*x^5 + c*x^2)*sqrt(d)*weierstrassPInverse(0, -4
*c/d, x) + (c^3*d*x^5 + c^4*x^2)*(d^4/c^19)^(1/6)*log((d^9*x^9 - 276*c*d^8*
x^6 - 1608*c^2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^13*d^6*x^8 + 20*c^14*d^5*x^5
 - 8*c^15*d^4*x^2)*(d^4/c^19)^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^5*x^7 - 28*
c^17*d^4*x^4 - 272*c^18*d^3*x)*(d^4/c^19)^(5/6) + 4*(c^10*d^6*x^6 + 41*c^11
*d^5*x^3 + 40*c^12*d^4)*sqrt(d^4/c^19) - 24*(c^4*d^7*x^5 + c^5*d^6*x^2)*(d^
4/c^19)^(1/6)) - 18*(c^7*d^7*x^7 - 52*c^8*d^6*x^4 - 80*c^9*d^5*x)*(d^4/c^19
)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^3*d*x^5 +
 c^4*x^2)*(d^4/c^19)^(1/6)*log((d^9*x^9 - 276*c*d^8*x^6 - 1608*c^2*d^7*x^3
 - 1088*c^3*d^6 + 18*(c^13*d^6*x^8 + 20*c^14*d^5*x^5 - 8*c^15*d^4*x^2)*(d^4/
c^19)^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^5*x^7 - 28*c^17*d^4*x^4 - 272*c^18
*d^3*x)*(d^4/c^19)^(5/6) + 4*(c^10*d^6*x^6 + 41*c^11*d^5*x^3 + 40*c^12*d^4)
*sqrt(d^4/c^19) - 24*(c^4*d^7*x^5 + c^5*d^6*x^2)*(d^4/c^19)^(1/6)) - 18*(c^
7*d^7*x^7 - 52*c^8*d^6*x^4 - 80*c^9*d^5*x)*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^3*d*x^5 + c^4*x^2)*(d^4/c^19)^
(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + 18*(c
^13*d^3*x^8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2)*(d^4/c^19)^(2/3) + 6*sqrt(d*
x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x)*(d^4/c^19)^(5/6) + (7
*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4
*x^5 + 32*c^5*d^3*x^2)*(d^4/c^19)^(1/6)) + 18*(5*c^7*d^4*x^7 + 64*c^8*d^3*x
^4 + 32*c^9*d^2*x)*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^
3 - 512*c^3)) - 2*(c^3*d*x^5 + c^4*x^2)*(d^4/c^19)^(1/6)*log((d^6*x^9 + 318
*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + 18*(c^13*d^3*x^8 + 38*c^14*d^
2*x^5 + 64*c^15*d*x^2)*(d^4/c^19)^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^2*x^7
 + 80*c^17*d*x^4 + 160*c^18*x)*(d^4/c^19)^(5/6) + (7*c^10*d^3*x^6 + 152*c^11
*d^2*x^3 + 64*c^12*d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2)*(
d^4/c^19)^(1/6)) + 18*(5*c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x)*(d^4/
c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 72*(59*d
*x^3 + 27*c)*sqrt(d*x^3 + c))/(c^3*d*x^5 + c^4*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^3\sqrt{c+dx^3} - 7cdx^6\sqrt{c+dx^3} + d^2x^9\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**3*sqrt(c + d*x**3) - 7*c*d*x**6*sqrt(c + d*x**3) +
d**2*x**9*sqrt(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.341 $\int \frac{1}{x^6(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{5}{3};1,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

```
[Out] -1/40*AppellF1(-5/3,3/2,1,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/
x^5/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};1,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/40*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 3/2, -2/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c^2*x^5*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^6 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{1}{x^6(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}} \, dx}{c\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\, F_1\left(-\frac{5}{3}; 1, \frac{3}{2}; -\frac{2}{3}; \frac{dx^3}{8c}, -\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(66) = 132.
time = 20.15, size = 261, normalized size = 3.95

$$\frac{-2981d^3x^9\sqrt{1+\frac{dx^3}{c}}\, F_1\left(\frac{4}{3}; \frac{1}{2}, 1; \frac{7}{3}; -\frac{dx^3}{c}, \frac{dx^3}{8c}\right) + 64c\left(-432c^2 + 1269cdx^3 + 2981d^2x^6 + \frac{382528c^2d^2x^6F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)\right)}\right)}{1105920c^5x^5\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2981*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 64*c*(-432*c^2 + 1269*c*d*x^3 + 2981*d^2*x^6 + (382528*c^
2*d^2*x^6*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c -
d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*
x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4
/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/(1105920*c^5*x^5*Sqrt[c
+ d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.40, size = 1402, normalized size = 21.24

method	result

elliptic	$-\frac{\sqrt{d\,x^3+c}}{40c^3x^5}+\frac{63d\sqrt{d\,x^3+c}}{640c^4x^2}+\frac{2d^2x}{27c^4\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{2981id\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right.}{(-c\,d^2)^{\frac{1}{3}}}}}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c*(-1/5/c^2*(d*x^3+c)^(1/2)/x^5+17/20/c^3*d*(d*x^3+c)^(1/2)/x^2+2/3*d^2
/c^3*x/((x^3+c/d)*d)^(1/2)-91/180*I/c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/64*d/c^2*(-1/2/c^2*(d*x^3+c)^(1/2)/x^2-2/3*
d*x/c^2/((x^3+c/d)*d)^(1/2)+7/18*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)))-1/64/c^2*d^2*(-2/27*x/c^2/((x^3+c/d)*d)^(1/2)+2/
81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
```

```
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/
243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2
*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2697 vs. $2(52) = 104$.
time = 16.56, size = 2697, normalized size = 40.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1244160*(20*sqrt(3)*(c^4*d*x^8 + c^5*x^5)*(d^10/c^25)^(1/6)*arctan(1/9*((
9*sqrt(3)*c^21*d^9*x^5*(d^10/c^25)^(5/6) + 3*sqrt(3)*(5*c^13*d^12*x^4 + 8*c
^14*d^11*x)*sqrt(d^10/c^25) - sqrt(3)*(c^4*d^16*x^6 - 40*c^5*d^15*x^3 - 32*
c^6*d^14)*(d^10/c^25)^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c^17*d^3*x^6 -
c^18*d^2*x^3 - 2*c^19*d)*(d^10/c^25)^(2/3) + 18*sqrt(3)*(c^9*d^6*x^5 + c^10
*d^5*x^2)*(d^10/c^25)^(1/3) + 3*sqrt(3)*(d^10*x^7 + 5*c*d^9*x^4 + 4*c^2*d^8
*x) - sqrt(d*x^3 + c)*(9*sqrt(3)*(c^21*d*x^5 + 2*c^22*x^2)*(d^10/c^25)^(5/6
) + 3*sqrt(3)*(7*c^13*d^4*x^4 + 4*c^14*d^3*x)*sqrt(d^10/c^25) + sqrt(3)*(c^
4*d^8*x^6 + 32*c^5*d^7*x^3 + 40*c^6*d^6)*(d^10/c^25)^(1/6)))*sqrt((d^19*x^9
 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^17*d^12*x^8 +
 20*c^18*d^11*x^5 - 8*c^19*d^10*x^2)*(d^10/c^25)^(2/3) + 6*sqrt(d*x^3 + c)*
((c^21*d^10*x^7 - 28*c^22*d^9*x^4 - 272*c^23*d^8*x)*(d^10/c^25)^(5/6) + 4*(
```

c^13*d^13*x^6 + 41*c^14*d^12*x^3 + 40*c^15*d^11)*sqrt(d^10/c^25) - 24*(c^5*
d^16*x^5 + c^6*d^15*x^2)*(d^10/c^25)^(1/6)) - 18*(c^9*d^15*x^7 - 52*c^10*d^
14*x^4 - 80*c^11*d^13*x)*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c
^2*d*x^3 - 512*c^3)))/(d^18*x^7 - 7*c*d^17*x^4 - 8*c^2*d^16*x)) + 20*sqrt(3
)*(c^4*d*x^8 + c^5*x^5)*(d^10/c^25)^(1/6)*arctan(1/9*((9*sqrt(3)*c^21*d^9*x
^5*(d^10/c^25)^(5/6) + 3*sqrt(3)*(5*c^13*d^12*x^4 + 8*c^14*d^11*x)*sqrt(d^1
0/c^25) - sqrt(3)*(c^4*d^16*x^6 - 40*c^5*d^15*x^3 - 32*c^6*d^14)*(d^10/c^25
)^(1/6))*sqrt(d*x^3 + c) + (12*sqrt(3)*(c^17*d^3*x^6 - c^18*d^2*x^3 - 2*c^1
9*d)*(d^10/c^25)^(2/3) + 18*sqrt(3)*(c^9*d^6*x^5 + c^10*d^5*x^2)*(d^10/c^25
)^(1/3) + 3*sqrt(3)*(d^10*x^7 + 5*c*d^9*x^4 + 4*c^2*d^8*x) + sqrt(d*x^3 + c
)*(9*sqrt(3)*(c^21*d*x^5 + 2*c^22*x^2)*(d^10/c^25)^(5/6) + 3*sqrt(3)*(7*c^1
3*d^4*x^4 + 4*c^14*d^3*x)*sqrt(d^10/c^25) + sqrt(3)*(c^4*d^8*x^6 + 32*c^5*d
^7*x^3 + 40*c^6*d^6)*(d^10/c^25)^(1/6)))*sqrt((d^19*x^9 - 276*c*d^18*x^6 -
1608*c^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^17*d^12*x^8 + 20*c^18*d^11*x^5 -
8*c^19*d^10*x^2)*(d^10/c^25)^(2/3) - 6*sqrt(d*x^3 + c)*((c^21*d^10*x^7 - 28
*c^22*d^9*x^4 - 272*c^23*d^8*x)*(d^10/c^25)^(5/6) + 4*(c^13*d^13*x^6 + 41*c
^14*d^12*x^3 + 40*c^15*d^11)*sqrt(d^10/c^25) - 24*(c^5*d^16*x^5 + c^6*d^15*
x^2)*(d^10/c^25)^(1/6)) - 18*(c^9*d^15*x^7 - 52*c^10*d^14*x^4 - 80*c^11*d^1
3*x)*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
)/(d^18*x^7 - 7*c*d^17*x^4 - 8*c^2*d^16*x)) + 214992*(d^2*x^8 + c*d*x^5)*sq
rt(d)*weierstrassPInverse(0, -4*c/d, x) + 5*(c^4*d*x^8 + c^5*x^5)*(d^10/c^2
5)^(1/6)*log((d^19*x^9 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^3 - 1088*c^3*d^16
 + 18*(c^17*d^12*x^8 + 20*c^18*d^11*x^5 - 8*c^19*d^10*x^2)*(d^10/c^25)^(2/3
) + 6*sqrt(d*x^3 + c)*((c^21*d^10*x^7 - 28*c^22*d^9*x^4 - 272*c^23*d^8*x)*(
d^10/c^25)^(5/6) + 4*(c^13*d^13*x^6 + 41*c^14*d^12*x^3 + 40*c^15*d^11)*sqrt
(d^10/c^25) - 24*(c^5*d^16*x^5 + c^6*d^15*x^2)*(d^10/c^25)^(1/6)) - 18*(c^9
*d^15*x^7 - 52*c^10*d^14*x^4 - 80*c^11*d^13*x)*(d^10/c^25)^(1/3))/(d^3*x^9
- 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(c^4*d*x^8 + c^5*x^5)*(d^10/
c^25)^(1/6)*log((d^19*x^9 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^3 - 1088*c^3*d
^16 + 18*(c^17*d^12*x^8 + 20*c^18*d^11*x^5 - 8*c^19*d^10*x^2)*(d^10/c^25)^(
2/3) - 6*sqrt(d*x^3 + c)*((c^21*d^10*x^7 - 28*c^22*d^9*x^4 - 272*c^23*d^8*x
)*(d^10/c^25)^(5/6) + 4*(c^13*d^13*x^6 + 41*c^14*d^12*x^3 + 40*c^15*d^11)*s
qrt(d^10/c^25) - 24*(c^5*d^16*x^5 + c^6*d^15*x^2)*(d^10/c^25)^(1/6)) - 18*(
c^9*d^15*x^7 - 52*c^10*d^14*x^4 - 80*c^11*d^13*x)*(d^10/c^25)^(1/3))/(d^3*x
^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 10*(c^4*d*x^8 + c^5*x^5)*(d
^10/c^25)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3
d^8 + 18(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2)*(d^10/c^25)^(2
/3) + 6*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x)*(d^10/
c^25)^(5/6) + (7*c^13*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c
^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2)*(d^10/c^25)^(1/6)) + 18*(5*c^9*d^
7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x)*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 10*(c^4*d*x^8 + c^5*x^5)*(d^10/c^25)
^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + 18
*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2)*(d^10/c^25)^(2/3) - 6*s
qrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x)*(d^10/c^25)^(5/

```
6) + (7*c^13*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*
(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2)*(d^10/c^25)^(1/6)) + 18*(5*c^9*d^7*x^7 + 6
4*c^10*d^6*x^4 + 32*c^11*d^5*x)*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
+ 192*c^2*d*x^3 - 512*c^3)) + 72*(2981*d^2*x^6 + 1269*c*d*x^3 - 432*c^2)*sq
rt(d*x^3 + c))/(c^4*d*x^8 + c^5*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^6\sqrt{c+dx^3} - 7cdx^9\sqrt{c+dx^3} + d^2x^{12}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**6*sqrt(c + d*x**3) - 7*c*d*x**9*sqrt(c + d*x**3) +
d**2*x**12*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^6\,{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.342 $\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

Optimal. Leaf size=737

$$\frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{\ldots}}{\sqrt{2}\,3^{3/4}\ldots}\right.}{\sqrt{2}\sqrt[4]{3}\,b^{2/3}}$$

```
[Out] 1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/
2))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1-3^(1
/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/4*3
^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*
2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctanh(1/2*3^
(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3+a)^(1/2))/b
^(2/3)*2^(1/2)+2*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+2/
3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {495, 309, 224, 1891, 500}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{3^{3/4}\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}\,b^{2/3}}+\frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}\,b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(3/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 - Sqrt[3]
```

```
)*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) +
(3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3
)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcT
anh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a +
 b*x^3])]])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
```

```
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = -\left(\left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a+bx^3}\,\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a+\cdots}}\cdots$$

$$= \frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a}\cdots}{\sqrt{2}\,3^{3/4}\cdots}\right.}{\sqrt{2}\,\sqrt[4]{3}\,b^{2/3}}\cdots$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{\sqrt{2}\,\sqrt{a+bx^3}}\cdots\right.}{2\sqrt{2}\,b^{2/3}}\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 8.82, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}\,a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a + 6*Sqrt[3]*a))])/((20 + 12*Sqrt[3])*Sqrt[a + b*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 995, normalized size = 1.35

method	result	size
elliptic	Expression too large to display	977
default	Expression too large to display	995

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*I/(-5+3*3^(1/2))/(5+3*3^(1/2))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))+1/9*I/b^3*2^(1/2)*sum(1/_alpha*(3+2*3^(1/2))*(-a*b^2)^(1/3)*(1/2*I*b*(
2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b
*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)
*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alph
a^2*3^(1/2)+3*I*(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*b^
2)^(1/3)*_alpha*3^(1/2)*b-6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*(-a*b^2)^(2/3
)*3^(1/2)+3*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),-1/6/b*(2*I*(-a*b^2)^(1/3)*_alpha^2*3^(1/2)*b-I*(-a*b^2)^
(2/3)*_alpha*3^(1/2)-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*I*(-a*b^2)^(2/3)*_alph
a+2*(-a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I
*a*b+2*3^(1/2)*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*
a+10*a))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="maxima"
)
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+bx^3}}{10a+6\sqrt{3}\,a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/2)/(b*x**3+2*a*(5+3*3**(1/2))),x)
[Out] Integral(x*sqrt(a + b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{b\,x^3+a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)),x)
[Out] int((x*(a + b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)), x)
```

3.343 $\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=757

$$\frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}+\frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{\cdots}}{\sqrt{2}\,3^{3/\cdots}}\right)}{\sqrt{2}\,\sqrt[4]{3}\,b^{2/3\cdots}}$$

```
[Out] 1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/
2))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1-3^(
1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/4
*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2)
)*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctanh(1/2
*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3+a)^(1/2)
)/b^(2/3)*2^(1/2)+2*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))+2/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-
b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 757, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {495, 309, 224, 1891, 500}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{3^{3/4}\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}\,b^{2/3}}+\frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (2*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(3/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 - Sqrt[3]
```

```
)*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) +
(3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*
x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcT
anh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a
 - b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt
[a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
```

```
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\left(\left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx\right)+\int \frac{}{\sqrt{a}}$$

$$= \frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{}}{\sqrt{2}\,3^{3/4}}\right)}{\sqrt{2}\sqrt[4]{3}\,b^{2/3}}$$

$$= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}+\frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{}\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 9.40, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}\,a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a + 6*Sqrt[3]*a)])/((20 + 12*Sqrt[3])*Sqrt[a - b*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.08, size = 942, normalized size = 1.24

method	result	size
elliptic	Expression too large to display	924
default	Expression too large to display	942

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*I/(-5+3*3^(1/2))/(5+3*3^(1/2))*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((
x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^
(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a
*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1
/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2
)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3
/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2
)*sum(1/_alpha*(3+2*3^(1/2))*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a
*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(
-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*(3*
I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2/3)*3
^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b-6*b^2*_a
lpha^2+6*I*(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)+3*(a*b^2)^(1/3)*_alpha*b+3
*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*b^2)^(1
/3)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)+4*I*(a*b^2)^(1/3)*_al
pha^2*b-2*I*(a*b^2)^(2/3)*_alpha+2*(a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*a
*b-3*(a*b^2)^(2/3)*_alpha-2*I*a*b-2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)/b*(a*b
^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alp
ha=RootOf(b*_Z^3-6*3^(1/2)*a-10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{a-bx^3}}{-6\sqrt{3}\,a-10a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3+a)**(1/2)/(-b*x**3+2*a*(5+3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(a - b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="giac"
)
```

```
[Out] integrate(-sqrt(-b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x\,\sqrt{a-b\,x^3}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)),x)
```

```
[Out] -int((x*(a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)), x)
```

3.344 $\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

Optimal. Leaf size=774

$$-\frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\ \cdots}{\cdots}\right)}{\cdots}$$

```
[Out] 1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/
2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctan(1/2
*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/2))
/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b
^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)-1/6*a^(1/6)*
arctanh(1/6*(1-3^(1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^
(2/3)*2^(1/2)-2*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-2/
3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-
b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/
3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/
3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/
3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {495, 310, 225, 1893, 501}

$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{\sqrt{2}b^{2/3}}-\frac{2\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{3^{3/4}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}b^{2/3}}-\frac{\sqrt[6]{a}\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (-2*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1
/4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(S
qrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcTan[(3
```

```
^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*
x^3])])/(Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])
*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(
2/3)) - (a^(1/6)*ArcTanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*
Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)
*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
```

```
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx + \int \frac{3}{\sqrt{-a}}$$

$$= \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}} + \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{}}{}\right)}{\sqrt{}}$$

$$= -\frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}\right.}{\sqrt{2}\sqrt{-a+bx^3}}\right.}{2\sqrt{2}\,b^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 9.41, size = 87, normalized size = 0.11

$$-\frac{x^2\sqrt{-a+bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{1-\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)])/((5 + 3*Sqrt[3])*a*Sqrt[1 - (b*x^3)/a])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 944, normalized size = 1.22

method	result	size
elliptic	Expression too large to display	926
default	Expression too large to display	944

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*I/(-5+3*3^(1/2))/(5+3*3^(1/2))*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((
x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^
(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a
*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/
3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/
2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)
*sum(1/_alpha*(3+2*3^(1/2))*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*
b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-
3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/
2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(3*I*
(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2/3)*3^(
1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b-6*b^2*_alp
ha^2+6*I*(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)+3*(a*b^2)^(1/3)*_alpha*b+3*(
a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*b^2)^(1/3
)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)+4*I*(a*b^2)^(1/3)*_alph
a^2*b-2*I*(a*b^2)^(2/3)*_alpha+2*(a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*a*b
-3*(a*b^2)^(2/3)*_alpha-2*I*a*b-2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)/b*(a*b^2
)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alpha
=RootOf(b*_Z^3-6*3^(1/2)*a-10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-a+bx^3}}{-6\sqrt{3}\,a-10a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3-a)**(1/2)/(b*x**3-2*a*(5+3*3**(1/2))),x)
```

```
[Out] Integral(x*sqrt(-a + b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{b\,x^3-a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b*x^3 - a)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)),x)
```

```
[Out] int((x*(b*x^3 - a)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)), x)
```

3.345 $\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=768

$$-\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{\sqrt{2}\,b^{2/3}}+\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\right.}{}$$

```
[Out] 1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/
2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctan(1/
2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1/
2))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3
)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)-1/6*a^(1
/6)*arctanh(1/6*(1-3^(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/
4)/b^(2/3)*2^(1/2)-2*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))-2/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^(
2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*
(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 768, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {495, 310, 225, 1893, 501}

$$-\frac{2\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{\sqrt{2}\,b^{2/3}}+\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}-\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{3^{3/4}\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}-\frac{\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\,\sqrt[4]{3}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*a^(1/6)*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/
(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcTan[(3
```

```
^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^
3])])/(2*Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])
*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*b^(
2/3)) - (a^(1/6)*ArcTanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*
Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)
*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2)]*Sqrt[-a - b*x^3]) - (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
```

```
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx+\int \frac{}{\sqrt{-\cdots}}$$

$$= \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}\,b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\cdots\right.}{2\cdots}$$

$$= -\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}{\sqrt{2}\sqrt{-a-b\cdots}}\right.}{\sqrt{2}\,b^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 9.34, size = 90, normalized size = 0.12

$$-\frac{x^2\sqrt{-a-bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{\frac{a+bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x
^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*a*Sqrt[(a + b*x^3)/a])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 1001, normalized size = 1.30

method	result	size
elliptic	Expression too large to display	983
default	Expression too large to display	1001

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*I/(-5+3*3^(1/2))/(5+3*3^(1/2))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)))+1/9*I/b^3*2^(1/2)*sum(1/_alpha*(3+2*3^(1/2))*(-a*b^2)^(1/3)*(1/2*I*b*
(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(
b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2
)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1
/3))^(1/2)/(-b*x^3-a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_al
pha^2*3^(1/2)+3*I*(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*
b^2)^(1/3)*_alpha*3^(1/2)*b-6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*(-a*b^2)^(2
/3)*3^(1/2)+3*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),-1/6/b*(2*I*(-a*b^2)^(1/3)*_alpha^2*3^(1/2)*b-I*(-a*b^2
)^(2/3)*_alpha*3^(1/2)-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*I*(-a*b^2)^(2/3)*_al
pha+2*(-a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2
*I*a*b+2*3^(1/2)*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2
)*a+10*a))
```

Maple [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{-a-bx^3}}{10a+6\sqrt{3}\,a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3-a)**(1/2)/(-b*x**3-2*a*(5+3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(-a - b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="giac"
)
```

```
[Out] integrate(-sqrt(-b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,\sqrt{-b\,x^3-a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(- a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)),x)
[Out] int(-(x*(- a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)), x)
```

3.346 $\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

Optimal. Leaf size=738

$$\frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a+bx^3}}\right)}{\sqrt{2}\,b^{2/3}}-\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\cdots}{\cdots}\right)}{2\cdots}$$

```
[Out] -1/2*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)-1/4*3^(1/4)*a^(1/6)*arctan
(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2
))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)
+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6
)*arctanh(1/6*(1+3^(1/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/
b^(2/3)*2^(1/2)+2*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+2
/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)
/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-
1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 738, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {495, 309, 224, 1891, 500}

$$\frac{2\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a+bx^3}}\right)}{\sqrt{2}\,b^{2/3}}-\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{3^{3/4}\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\,\sqrt[4]{3}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*a^(1/6)*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(S
qrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1
```

```
/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]
)/(2*Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(
1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*b^(2/3))
 + (a^(1/6)*ArcTanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])])/(Sqrt[2]*3^(1/4)*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
```

```
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = -\left(\left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx\right)+\int \frac{}{\sqrt{a}}$$

$$= -\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a+bx^3}}\right)}{\sqrt{2}\,b^{2/3}}-\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}}{}\right.}{2}$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\right.}{\sqrt{2}\,\sqrt{a+bx^3}}\right.}{\sqrt{2}\,b^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 8.78, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}\,a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a - 6*Sqrt[3]*a))])/((20 - 12*Sqrt[3])*Sqrt[a + b*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 995, normalized size = 1.35

method	result	size
elliptic	Expression too large to display	977
default	Expression too large to display	995

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*I/(-5+3*3^(1/2))/(5+3*3^(1/2))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))+1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)-3)*(-a*b^2)^(1/3)*(1/2*I*b*(
2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b
*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)
*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha
^2*3^(1/2)-3*I*(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*b^2
)^(1/3)*_alpha*3^(1/2)*b+6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*(-a*b^2)^(2/3)
*3^(1/2)-3*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),-1/6/b*(2*I*(-a*b^2)^(1/3)*_alpha^2*3^(1/2)*b-I*(-a*b^2)^(
2/3)*_alpha*3^(1/2)+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*I*(-a*b^2)^(2/3)*_alpha
-2*(-a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*
a*b-2*3^(1/2)*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a
+10*a))
```

Maple [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+bx^3}}{-6\sqrt{3}\,a+10a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/2)/(b*x**3+2*a*(5-3*3**(1/2))),x)
```

```
[Out] Integral(x*sqrt(a + b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{b\,x^3+a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)),x)
```

```
[Out] int((x*(a + b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)), x)
```

3.347 $\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=758

$$\frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}-\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\cdots}{\cdots}\right)}{\sqrt{2}\cdots}$$

[Out] -1/2*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)-1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctanh(1/6*(1+3^(1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+2*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+2/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.18, antiderivative size = 758, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {495, 309, 224, 1891, 500}

$$\frac{2\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}-\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a-bx^3}}\right)}{\sqrt{2}\,b^{2/3}}+\frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}+\frac{3^{3/4}\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\,\sqrt[4]{3}\,b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]

[Out] (2*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1

```
/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3]
)])/(Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(
1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3))
 + (a^(1/6)*ArcTanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])])/(Sqrt[2]*3^(1/4)*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt
[a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
```

```
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = -\left(\left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a-}}$$

$$= -\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}} - \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{}}{}\right.}{\sqrt{}}$$

$$= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{}\right.}{\sqrt{2}\sqrt{a-bx^3}}\right.}{2\sqrt{2}\,b^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 9.30, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a - 6*Sqrt[3]*a)])/((20 - 12*Sqrt[3])*Sqrt[a - b*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 942, normalized size = 1.24

method	result	size
elliptic	Expression too large to display	924
default	Expression too large to display	942

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*I/(-5+3*3^(1/2))/(5+3*3^(1/2))*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((
x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^
(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a
*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1
/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2
)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3
/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2
)*sum(1/_alpha*(2*3^(1/2)-3)*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a
*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(
-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*(-3
*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(2/3)*
3^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+6*b^2*_
alpha^2+6*I*(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)-3*(a*b^2)^(1/3)*_alpha*b-
3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*b^2)^(
1/3)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)-4*I*(a*b^2)^(1/3)*_a
lpha^2*b+2*I*(a*b^2)^(2/3)*_alpha-2*(a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*
a*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)/b*(a*
b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_al
pha=RootOf(b*_Z^3+6*3^(1/2)*a-10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{a-bx^3}}{-10a+6\sqrt{3}\,a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3+a)**(1/2)/(-b*x**3+2*a*(5-3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(a - b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac"
)
```

```
[Out] integrate(-sqrt(-b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,\sqrt{a-b\,x^3}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)),x)
```

```
[Out] int(-(x*(a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)), x)
```

3.348 $\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=774

$$\frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}-\frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{\ldots}}{\sqrt{2}\,3\ldots}\right)}{\sqrt{2}\sqrt[4]{3}\,b^{2/\ldots}}$$

```
[Out] -1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1
/2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1+3^(
1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/2*
3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/4*3^(1/4)*a^(1/6)*arctanh(1/2
*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1/2))/b
^(2/3)*2^(1/2)+2*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+2
/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2
/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1
/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {495, 310, 225, 1893, 501}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{bx^3-a}}-\frac{3^{3/4}\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}\,b^{2/3}}+\frac{2\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}+\frac{\sqrt[4]{3}\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{\sqrt{2}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(3/
4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sq
rt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 + Sqrt[
```

```
3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3))
 + (3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/
3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])]])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*
ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sq
rt[-a + b*x^3])]])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2
/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
```

```
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx - \int \frac{x}{\sqrt{-a+\cdots}}$$

$$= -\frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}} + \frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\cdots}{\sqrt{2}\cdots}\right)}{\sqrt{2}\sqrt[4]{3}\,b^{\cdots}}$$

$$= \frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\cdots\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 9.41, size = 89, normalized size = 0.11

$$-\frac{x^2\sqrt{-a+bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},-\frac{bx^3}{-10a+6\sqrt{3}\,a}\right)}{4\left(-5+3\sqrt{3}\right)a\sqrt{\frac{a-bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, -((b*x^3)
/(-10*a + 6*Sqrt[3]*a))])/((-5 + 3*Sqrt[3])*a*Sqrt[(a - b*x^3)/a])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 944, normalized size = 1.22

method	result	size
elliptic	Expression too large to display	926
default	Expression too large to display	944

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*I/(-5+3*3^(1/2))/(5+3*3^(1/2))*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*(
(x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))
^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(
a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1
/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2
)^(1/3)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3
/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(1/2
)*sum(1/_alpha*(2*3^(1/2)-3)*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a
*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(
-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(-3*
I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(2/3)*3
^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+6*b^2*_a
lpha^2+6*I*(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)-3*(a*b^2)^(1/3)*_alpha*b-3
*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*b^2)^(1
/3)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)-4*I*(a*b^2)^(1/3)*_al
pha^2*b+2*I*(a*b^2)^(2/3)*_alpha-2*(a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*a
*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)/b*(a*b
^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alp
ha=RootOf(b*_Z^3+6*3^(1/2)*a-10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima
")
```

```
[Out] -integrate(sqrt(b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{-a+bx^3}}{-10a+6\sqrt{3}\,a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3-a)**(1/2)/(-b*x**3+2*a*(5-3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(-a + b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,\sqrt{b\,x^3-a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(b*x^3 - a)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)),x)
```

```
[Out] int(-(x*(b*x^3 - a)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)), x)
```

3.349 $\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

Optimal. Leaf size=768

$$\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{\cdots}}{\sqrt{2}\,3^{3/\cdots}}\right.}{\sqrt{2}\,\sqrt[4]{3}\,b^{2/3}\cdots}$$

```
[Out] -1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1
/2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1+3^
(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/
2*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/4*3^(1/4)*a^(1/6)*arctanh
(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-a)^(1/
2))/b^(2/3)*2^(1/2)+2*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))+2/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^
(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 768, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {495, 310, 225, 1893, 501}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{3^{3/4}\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\text{ArcTan}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}\,b^{2/3}}+\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}\,b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (2*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(3/
4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sq
rt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 + Sqrt[
```

```
3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3))
 + (3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(
1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*
ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt
[-a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2)]*Sqrt[-a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
```

```
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx - \int \frac{x}{\sqrt{-a-b\ldots}}$$

$$= -\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}} + \frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\ldots}{\sqrt{2}\,3^{\ldots}}\right)}{\sqrt{2}\sqrt[4]{3}\,b^{2/\ldots}}$$

$$= \frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\ldots\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 9.37, size = 89, normalized size = 0.12

$$-\frac{x^2\sqrt{-a-bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{4\left(-5+3\sqrt{3}\right)a\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x
^3)/(10*a - 6*Sqrt[3]*a))])/((-5 + 3*Sqrt[3])*a*Sqrt[1 + (b*x^3)/a])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 1001, normalized size = 1.30

method	result	size
elliptic	Expression too large to display	983
default	Expression too large to display	1001

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*I/(-5+3*3^(1/2))/(5+3*3^(1/2))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)-3)*(-a*b^2)^(1/3)*(1/2*I*b*(
2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b
*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)
*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/
3))^(1/2)/(-b*x^3-a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alph
a^2*3^(1/2)-3*I*(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*b^
2)^(1/3)*_alpha*3^(1/2)*b+6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*(-a*b^2)^(2/3
)*3^(1/2)-3*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),-1/6/b*(2*I*(-a*b^2)^(1/3)*_alpha^2*3^(1/2)*b-I*(-a*b^2)^
(2/3)*_alpha*3^(1/2)+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*I*(-a*b^2)^(2/3)*_alph
a-2*(-a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I
*a*b-2*3^(1/2)*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*
a+10*a))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima
")
[Out] integrate(sqrt(-b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas
")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-a-bx^3}}{-6\sqrt{3}\,a+10a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3-a)**(1/2)/(b*x**3+2*a*(5-3*3**(1/2))),x)
[Out] Integral(x*sqrt(-a - b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{-b\,x^3-a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(- a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)),x)
[Out] int((x*(- a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)), x)
```

3.350 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=318

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{\right.}}{}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b
*x^3+a)^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6*
(1-3^(1/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/a^
(5/6)/b^(2/3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*
(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^
(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2
^(1/2)/(b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {500}

$$-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3
)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 -
 Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]
)/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(
1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqr
t[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
```

```
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.07, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}\,a}\right)}{\left(20a+12\sqrt{3}\,a\right)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 538, normalized size = 1.69 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^
2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2
)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x
+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^
3+a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I
*(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*b^2)^(1/3)*_alpha
*3^(1/2)*b-6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*(-a*b^2)^(2/3)*3^(1/2)+3*(-a
*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
```

```
2),-1/6/b*(2*I*(-a*b^2)^(1/3)*_alpha^2*3^(1/2)*b-I*(-a*b^2)^(2/3)*_alpha*3^
(1/2)-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*I*(-a*b^2)^(2/3)*_alpha+2*(-a*b^2)^(2
/3)*_alpha*3^(1/2)+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I*a*b+2*3^(1/2)*
a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a+10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 + a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5012 vs. $2(211) = 422$.
time = 25.81, size = 5012, normalized size = 15.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/6)*arctan(
-1/3*(3*sqrt(b*x^3 + a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1978*a^5*b^3
 + sqrt(3)*(153*a^4*b^4*x^3 + 1142*a^5*b^3))*(-(1351*sqrt(3) - 2340)/(a^5*b
^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x + 71*a^3*b^2*x)*sqrt(-(1351*sq
rt(3) - 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 + 9*a*b*x^2)*(
-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/6)) + (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 +
7*a^3*b + 4*sqrt(3)*(a^2*b^2*x^3 + a^3*b))*(-(1351*sqrt(3) - 2340)/(a^5*b^4
))^(1/3) + sqrt(3)*(b*x^4 + a*x) - 3*sqrt(b*x^3 + a)*(108*(1/1944)^(5/6)*(2
65*a^4*b^4*x^3 - 1448*a^5*b^3 + sqrt(3)*(153*a^4*b^4*x^3 - 836*a^5*b^3))*(-
(1351*sqrt(3) - 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x +
71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) - 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sq
rt(3)*a*b*x^2 + 9*a*b*x^2)*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/6)))*sqrt(
(b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(
1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 211
2*a^7*b^3*x + 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4
*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(
1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2
+ 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*
x^2))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 - 6*
```

a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) + 2*sqrt(b*x^3 + a)*(1944*(1/1944)^(5/6
)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 + sqrt(3)*(2131*a^
5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) - 2340)/(a
^5*b^4))^(5/6) - 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5
*b^3*x^3 - 768*a^6*b^2 + sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*
a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) - 2340)/(a^5*b^4)) - 3*(1/1
944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x +
3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(135
1*sqrt(3) - 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2
*x^6 - 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 + a*x)) + 1/6*sqrt(3)*(1/1944)^(1/6
)*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/6)*arctan(-1/3*(3*sqrt(b*x^3 + a)*(
108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1978*a^5*b^3 + sqrt(3)*(153*a^4*b^4*x
^3 + 1142*a^5*b^3))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*(4
1*sqrt(3)*a^3*b^2*x + 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) - 2340)/(a^5*b^4))
+ (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 + 9*a*b*x^2)*(-(1351*sqrt(3) - 2340)/(a
^5*b^4))^(1/6)) - (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 + 7*a^3*b + 4*sqrt(3)*(a^2*
b^2*x^3 + a^3*b))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/3) + sqrt(3)*(b*x^4
 + a*x) + 3*sqrt(b*x^3 + a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1448*a^5
b^3 + sqrt(3)(153*a^4*b^4*x^3 - 836*a^5*b^3))*(-(1351*sqrt(3) - 2340)/(a^
5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x + 71*a^3*b^2*x)*sqrt(-(1351
*sqrt(3) - 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 + 9*a*b*x^2
)*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 + 100*a*b^3*x^9
 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*
x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x + 4*sqrt(3)*(
223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-
(1351*sqrt(3) - 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 4
98*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 + 3*sqrt(3)*(5*a^2*b^5*x^
11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) - 2
340)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^
3 - 8*a^4) - 2*sqrt(b*x^3 + a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 289
6*a^6*b^5*x^5 + 568*a^7*b^4*x^2 + sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*
x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1
/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 +
sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2
))*sqrt(-(1351*sqrt(3) - 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10
+ 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x + 3*sqrt(3)*(a*b^4*x^10 + 4
*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) - 2340)/(a^5*b^
4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64
*a^4)))/(b*x^4 + a*x)) - 1/24*(1/1944)^(1/6)*(-(1351*sqrt(3) - 2340)/(a^5*b
^4))^(1/6)*log((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3
+ 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^
6*b^4*x^4 + 2112*a^7*b^3*x + 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7
 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(2
/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 -
64*a^5*b^2*x^2 + 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^

```
5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/3) + 32*sqrt(3)*
(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) + 2*sqrt(b*x^3 + a)*(194
4*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^3}\cdot\left(10a+6\sqrt{3}\,a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5+3*3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(10*a + 6*sqrt(3)*a + b*x**3)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3+a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

3.351 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal. Leaf size=324

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right.}{}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-
b*x^3+a)^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6
*(1-3^(1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/
a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x
)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)
*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
*2^(1/2)/(-b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {500}

$$-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3
)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 -
 Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]
)/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1
- Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(6*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((
1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]
*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
```

```
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.05, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 509, normalized size = 1.57

method	result

default	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3-6\sqrt{3}\,a-10a\right)} \frac{\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{2}\,\sqrt{\cdots}}{\cdots}\right.$
elliptic	$i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3-6\sqrt{3}\,a-10a\right)} \frac{\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{2}\,\sqrt{\cdots}}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/
2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/
3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I
*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2
)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2
/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b-6*b
^2*_alpha^2+6*I*(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)+3*(a*b^2)^(1/3)*_alph
a*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*b^
2)^(1/3)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)+4*I*(a*b^2)^(1/3
```

```
)*_alpha^2*b-2*I*(a*b^2)^(2/3)*_alpha+2*(a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1
/2)*a*b-3*(a*b^2)^(2/3)*_alpha-2*I*a*b-2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)/b
*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
,_alpha=RootOf(b*_Z^3-6*3^(1/2)*a-10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 + a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5036 vs. $2(218) = 436$.
time = 24.99, size = 5036, normalized size = 15.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/6)*arctan(
1/3*(3*sqrt(-b*x^3 + a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1978*a^5*b^3
 + sqrt(3)*(153*a^4*b^4*x^3 - 1142*a^5*b^3))*(-(1351*sqrt(3) - 2340)/(a^5*b
^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x + 71*a^3*b^2*x)*sqrt(-(1351*sq
rt(3) - 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 + 9*a*b*x^2)*(
-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/6)) - (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 -
7*a^3*b + 4*sqrt(3)*(a^2*b^2*x^3 - a^3*b))*(-(1351*sqrt(3) - 2340)/(a^5*b^4
))^(1/3) - sqrt(3)*(b*x^4 - a*x) - 3*sqrt(-b*x^3 + a)*(108*(1/1944)^(5/6)*(
265*a^4*b^4*x^3 + 1448*a^5*b^3 + sqrt(3)*(153*a^4*b^4*x^3 + 836*a^5*b^3))*(
-(1351*sqrt(3) - 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x +
 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) - 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*s
qrt(3)*a*b*x^2 + 9*a*b*x^2)*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/6)))*sqrt
((b^4*x^12 - 100*a*b^3*x^9 + 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*
(1/9)^(2/3)*(1545*a^4*b^6*x^10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 21
12*a^7*b^3*x + 4*sqrt(3)*(223*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^
4*x^4 - 304*a^7*b^3*x))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^
(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2
 + 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2
*x^2))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 + 6
*a^2*b^2*x^6 - 15*a^3*b*x^3 + 8*a^4) + 2*sqrt(-b*x^3 + a)*(1944*(1/1944)^(5
```

/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 + sqrt(3)*(2131*
a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) - 2340)/
(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a
^5*b^3*x^3 + 768*a^6*b^2 + sqrt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 228
0*a^5*b^3*x^3 + 448*a^6*b^2))*sqrt(-(1351*sqrt(3) - 2340)/(a^5*b^4)) - 3*(1
/1944)^(1/6)*(5*a*b^4*x^10 - 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x
+ 3*sqrt(3)*(a*b^4*x^10 - 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(1
351*sqrt(3) - 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b
^2*x^6 + 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 - a*x)) + 1/6*sqrt(3)*(1/1944)^(1
/6)*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/6)*arctan(1/3*(3*sqrt(-b*x^3 + a)
(108(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1978*a^5*b^3 + sqrt(3)*(153*a^4*b^4
*x^3 - 1142*a^5*b^3))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*
(41*sqrt(3)*a^3*b^2*x + 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) - 2340)/(a^5*b^4)
) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 + 9*a*b*x^2)*(-(1351*sqrt(3) - 2340)/
(a^5*b^4))^(1/6)) + (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 - 7*a^3*b + 4*sqrt(3)*(a^
2*b^2*x^3 - a^3*b))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/3) - sqrt(3)*(b*x
^4 - a*x) + 3*sqrt(-b*x^3 + a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1448*
a^5*b^3 + sqrt(3)*(153*a^4*b^4*x^3 + 836*a^5*b^3))*(-(1351*sqrt(3) - 2340)/
(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x + 71*a^3*b^2*x)*sqrt(-(1
351*sqrt(3) - 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 + 9*a*b*
x^2)*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 - 100*a*b^3*
x^9 + 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b
^6*x^10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x + 4*sqrt(3
)*(223*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))
*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11
+ 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2 + 3*sqrt(3)*(5*a^2*b^5
*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3)
- 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b
*x^3 + 8*a^4) - 2*sqrt(-b*x^3 + a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 +
2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 + sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*
b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(5/6) + 2*sq
rt(1/6)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^
2 + sqrt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6
*b^2))*sqrt(-(1351*sqrt(3) - 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x
^10 - 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x + 3*sqrt(3)*(a*b^4*x^10
- 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(1351*sqrt(3) - 2340)/(a^
5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3
+ 64*a^4)))/(b*x^4 - a*x)) + 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) - 2340)/(a
^5*b^4))^(1/6)*log(-(b^4*x^12 + 68*a*b^3*x^9 + 168*a^2*b^2*x^6 - 544*a^3*b*
x^3 + 64*a^4 + 6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 + 11556*a^5*b^5*x^7 + 7776*
a^6*b^4*x^4 + 1440*a^7*b^3*x + 8*sqrt(3)*(202*a^4*b^6*x^10 + 834*a^5*b^5*x^
7 + 561*a^6*b^4*x^4 + 104*a^7*b^3*x))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(2
/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 +
64*a^5*b^2*x^2 + 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^
5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) - 2340)/(a^5*b^4))^(1/3) + 64*sqrt(3)*

```
(a*b^3*x^9 - 3*a^2*b^2*x^6 + 3*a^3*b*x^3 - a^4) + 2*sqrt(-b*x^3 + a)*(1944*
(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-6\sqrt{3}\,a\sqrt{a-bx^3}-10a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5+3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-6*sqrt(3)*a*sqrt(a - b*x**3) - 10*a*sqrt(a - b*x**3) + b*x**3
*sqrt(a - b*x**3)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x}{\sqrt{a-b\,x^3}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] -int(x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

3.352 $\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=328

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+$$

```
[Out] 1/36*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*
x^3-a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctan(1/2*3
^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/2))*(
2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1/6)
*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1/2))*(2-3^(1/2))*3^(1/
4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/6*(1-3^(1/2))*(b*x^3-a)^(1/2)*3^(
1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {501}

$$\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - S
qrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*S
qrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
```

```
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}} + \frac{(2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.06, size = 85, normalized size = 0.26

$$-\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}\,a}\right)}{\left(20a+12\sqrt{3}\,a\right)\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a + b*x^3]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 510, normalized size = 1.55

method	result

default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{2}\ \cdots}{\cdots}\right)}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{2}\ \cdots}{\cdots}\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1
/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-
I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2
)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2
/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b-6*b
^2*_alpha^2+6*I*(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)+3*(a*b^2)^(1/3)*_alph
a*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*b^
2)^(1/3)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)+4*I*(a*b^2)^(1/3
```

```
)*_alpha^2*b-2*I*(a*b^2)^(2/3)*_alpha+2*(a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(1
/2)*a*b-3*(a*b^2)^(2/3)*_alpha-2*I*a*b-2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)/b
*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
,_alpha=RootOf(b*_Z^3-6*3^(1/2)*a-10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima"
)
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 - a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-6\sqrt{3}\,a-10a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3-2*a*(5+3*3**(1/2)))/(b*x**3-a)**(1/2),x)
[Out] Integral(x/(sqrt(-a + b*x**3)*(-6*sqrt(3)*a - 10*a + b*x**3)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3-a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
[Out] int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

3.353 $\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal. Leaf size=330

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\,\sqrt[4]{3}\,a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\,\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

```
[Out] 1/36*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b
*x^3-a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctan(1/2*
3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1/2)
)*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1
/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-a)^(1/2))*(2-3^(1/2))*3
^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/6*(1-3^(1/2))*(-b*x^3-a)^(1/2
)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {501}

$$\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\,\sqrt[4]{3}\,a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\,\sqrt[4]{3}\,a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x
))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 -
 Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*S
qrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
```

```
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\,\sqrt[4]{3}\,a^{5/6}b^{2/3}} + \frac{\left(2\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.05, size = 87, normalized size = 0.26

$$-\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}\,a}\right)}{\left(20a+12\sqrt{3}\,a\right)\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^
3)/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a - b*x^3]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 541, normalized size = 1.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)
^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+
1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b*x^
3-a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I
*(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*b^2)^(1/3)*_alpha
*3^(1/2)*b-6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*(-a*b^2)^(2/3)*3^(1/2)+3*(-a
*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
```

```
2),-1/6/b*(2*I*(-a*b^2)^(1/3)*_alpha^2*3^(1/2)*b-I*(-a*b^2)^(2/3)*_alpha*3^
(1/2)-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*I*(-a*b^2)^(2/3)*_alpha+2*(-a*b^2)^(2
/3)*_alpha*3^(1/2)+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I*a*b+2*3^(1/2)*
a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a+10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxim
a")
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 - a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{10a\sqrt{-a-bx^3}+6\sqrt{3}\,a\sqrt{-a-bx^3}+bx^3\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3-2*a*(5+3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
[Out] -Integral(x/(10*a*sqrt(-a - b*x**3) + 6*sqrt(3)*a*sqrt(-a - b*x**3) + b*x**
3*sqrt(-a - b*x**3)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

3.354 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=310

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

```
[Out] -1/18*arctan(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)))*2^(1/2)
/(b*x^3+a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctan(1
/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))
*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1/
6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))*(2+3^(1/2))*3^(
1/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctanh(1/6*(1+3^(1/2))*(b*x^3+a)^(1/2)*3
^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {500}

$$-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -1/3*((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1
/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2
 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(S
qrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[
3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*Arc
Tanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]
*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
```

```
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}} - \frac{\left(2+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.08, size = 83, normalized size = 0.27

$$\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}\,a}\right)}{\left(20a-12\sqrt{3}\,a\right)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 538, normalized size = 1.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)
^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+
1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(
-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*b^2)^(1/3)*_alpha*3
^(1/2)*b+6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*(-a*b^2)^(2/3)*3^(1/2)-3*(-a*b
^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
```

```
,-1/6/b*(2*I*(-a*b^2)^(1/3)*_alpha^2*3^(1/2)*b-I*(-a*b^2)^(2/3)*_alpha*3^(1
/2)+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*I*(-a*b^2)^(2/3)*_alpha-2*(-a*b^2)^(2/3
)*_alpha*3^(1/2)+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a*b-2*3^(1/2)*a*
b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a+10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^3}\,\left(-6\sqrt{3}\,a+10a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3+a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
[Out] int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

3.355 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal. Leaf size=316

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

```
[Out] -1/18*arctan(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)))*2^(1/2)/
(-b*x^3+a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctan(1
/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2)
)*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1
/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))*(2+3^(1/2))*3
^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctanh(1/6*(1+3^(1/2))*(-b*x^3+a)^(1/2
)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {500}

$$-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -1/6*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3
)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 +
 Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(S
qrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[
3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*
Sqrt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*Arc
Tanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]
*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
```

```
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}} - \frac{\left(2+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.05, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}\,a}\right)}{\left(20a-12\sqrt{3}\,a\right)\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 509, normalized size = 1.61

method	result

default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{2}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{2}\cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1
/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-
I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/
2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^
(2/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+6
*b^2*_alpha^2+6*I*(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)-3*(a*b^2)^(1/3)*_al
pha*b-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*
b^2)^(1/3)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)-4*I*(a*b^2)^(1
```

```
/3)*_alpha^2*b+2*I*(a*b^2)^(2/3)*_alpha-2*(a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^
(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)
/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2
)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a-10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-10a\sqrt{a-bx^3}+6\sqrt{3}\,a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
[Out] -Integral(x/(-10*a*sqrt(a - b*x**3) + 6*sqrt(3)*a*sqrt(a - b*x**3) + b*x**3
*sqrt(a - b*x**3)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{a-b\,x^3}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
[Out] int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

3.356 $\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=320

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right.}{}$$

```
[Out] 1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*
x^3-a)^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6*(
1+3^(1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a^(
5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2
^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(
1/2)/(b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {501}

$$\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 +
 Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((
1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
```

```
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx=\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.06, size = 84, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}\,a}\right)}{\left(20a-12\sqrt{3}\,a\right)\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a + b*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 510, normalized size = 1.59

method	result

default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{2}\ \cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{2}\ \cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1
/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-
I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2
)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(
2/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+6*
b^2*_alpha^2+6*I*(a*b^2)^(2/3)-2*(a*b^2)^(2/3)*3^(1/2)-3*(a*b^2)^(1/3)*_alp
ha*b-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*(a*b
^2)^(1/3)*_alpha^2*3^(1/2)*b+I*(a*b^2)^(2/3)*_alpha*3^(1/2)-4*I*(a*b^2)^(1/
```

```
3)*_alpha^2*b+2*I*(a*b^2)^(2/3)*_alpha-2*(a*b^2)^(2/3)*_alpha*3^(1/2)+I*3^(
1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)/
b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)
),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a-10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima
")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 - a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5060 vs. 2(223) = 446.
time = 30.83, size = 5060, normalized size = 15.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan
(1/3*(3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1978*a^5*b^3
 - sqrt(3)*(153*a^4*b^4*x^3 - 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b
^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sq
rt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(
-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) - (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 -
7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 - a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4
))^(1/3) - sqrt(3)*(b*x^4 - a*x) + 3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(2
65*a^4*b^4*x^3 + 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 + 836*a^5*b^3))*(-
(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x -
71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sq
rt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt(
(b^4*x^12 - 100*a*b^3*x^9 + 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(
1/9)^(2/3)*(1545*a^4*b^6*x^10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 211
2*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4
*x^4 - 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(
1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2
 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*
x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 + 6*
a^2*b^2*x^6 - 15*a^3*b*x^3 + 8*a^4) + 2*sqrt(b*x^3 - a)*(1944*(1/1944)^(5/6
```

)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/6)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 - 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 - a*x)) - 1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan(1/3*(3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1978*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 - 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) + (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 - 7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 - a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - sqrt(3)*(b*x^4 - a*x) - 3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 + 836*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 - 100*a*b^3*x^9 + 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3 + 8*a^4) - 2*sqrt(b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/6)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 - 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 - a*x)) + 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log((b^4*x^12 + 68*a*b^3*x^9 + 168*a^2*b^2*x^6 - 544*a^3*b*x^3 + 64*a^4 + 6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 + 11556*a^5*b^5*x^7 + 7776*a^6*b^4*x^4 + 1440*a^7*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 + 834*a^5*b^5*x^7 + 561*a^6*b^4*x^4 + 104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 64*sqrt(3)*(a*b^3

```
*x^9 - 3*a^2*b^2*x^6 + 3*a^3*b*x^3 - a^4) + 2*sqrt(b*x^3 - a)*(1944*(1/1944
)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-10a\sqrt{-a+bx^3}+6\sqrt{3}\,a\sqrt{-a+bx^3}+bx^3\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-10*a*sqrt(-a + b*x**3) + 6*sqrt(3)*a*sqrt(-a + b*x**3) + b*x*
*3*sqrt(-a + b*x**3)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{b\,x^3-a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

3.357 $\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=322

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right.}{}$$

```
[Out] 1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b
*x^3-a)^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6*
(1+3^(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a
^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/
3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))
*2^(1/2)/(-b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {501}

$$\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1
/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqr
t[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
```

```
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}} - \frac{(2+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.05, size = 86, normalized size = 0.27

$$\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}\,a}\right)}{\left(20a-12\sqrt{3}\,a\right)\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a - b*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 541, normalized size = 1.68 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)
^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+
1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b*x^
3-a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*
(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*(-a*b^2)^(1/3)*_alpha*
3^(1/2)*b+6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*(-a*b^2)^(2/3)*3^(1/2)-3*(-a*
b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
```

```
),-1/6/b*(2*I*(-a*b^2)^(1/3)*_alpha^2*3^(1/2)*b-I*(-a*b^2)^(2/3)*_alpha*3^(
1/2)+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*I*(-a*b^2)^(2/3)*_alpha-2*(-a*b^2)^(2/
3)*_alpha*3^(1/2)+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a*b-2*3^(1/2)*a
*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a+10*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 - a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5060 vs. 2(222) = 444.
time = 34.72, size = 5060, normalized size = 15.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan
(-1/3*(3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1978*a^5*b
^3 - sqrt(3)*(153*a^4*b^4*x^3 + 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5
*b^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*
sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)
*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) + (6*(1/9)^(1/3)*(7*a^2*b^2*x^3
+ 7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 + a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b
^4))^(1/3) + sqrt(3)*(b*x^4 + a*x) + 3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)
*(265*a^4*b^4*x^3 - 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 - 836*a^5*b^3))
*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x
 - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5
*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sq
rt((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 -
6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 +
2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*
b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9
)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x
^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b
^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 -
```

6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) + 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 + a*x)) - 1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan(-1/3*(3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1978*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 + 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) - (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 + 7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 + a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + sqrt(3)*(b*x^4 + a*x) - 3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 - 836*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) - 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 + a*x)) - 1/24*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4

```
*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*s
qrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) + 2*sqrt(-b*x^3 -
 a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 28...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-6\sqrt{3}\,a+10a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a - b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

3.358 $\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=125

$$\frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}-\frac{2a^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

```
[Out] -2/9*(a*d+b*c)*(d*x^3+c)^(3/2)/b^2/d^2+2/15*(d*x^3+c)^(5/2)/b/d^2-2/3*a^2*a
rctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)+2
/3*a^2*(d*x^3+c)^(1/2)/b^3
```

Rubi [A]

time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 90, 52, 65, 214}

$$-\frac{2a^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2\left(c+dx^3\right)^{3/2}(ad+bc)}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*a^2*Sqrt[c + d*x^3])/(3*b^3) - (2*(b*c + a*d)*(c + d*x^3)^(3/2))/(9*b^2*
d^2) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(-bc-ad)\sqrt{c+dx}}{b^2d} + \frac{a^2\sqrt{c+dx}}{b^2(a+bx)} + \frac{(c+dx)^{3/2}}{bd}\right)dx, x, x^3\right)$$

$$= -\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd^2} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b^2}$$

$$= \frac{2a^2\sqrt{c+dx^3}}{3b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd^2} + \frac{\left(a^2(bc-ad)\right)\text{Subst}\left(\int \frac{}{(a+}\right.}{3b^3}$$

$$= \frac{2a^2\sqrt{c+dx^3}}{3b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd^2} + \frac{\left(2a^2(bc-ad)\right)\text{Subst}\left(\int \frac{}{a-}\right.}{3b^3}$$

$$= \frac{2a^2\sqrt{c+dx^3}}{3b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd^2} - \frac{2a^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{3b^{7/2}}$$

Mathematica [A]
time = 0.22, size = 121, normalized size = 0.97

$$\frac{2\sqrt{c+dx^3}\left(15a^2d^2-5abd\left(c+dx^3\right)+b^2\left(-2c^2+cdx^3+3d^2x^6\right)\right)}{45b^3d^2} - \frac{2a^2\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(15*a^2*d^2 - 5*a*b*d*(c + d*x^3) + b^2*(-2*c^2 + c*d*x^
3 + 3*d^2*x^6)))/(45*b^3*d^2) - (2*a^2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*S
qrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 514, normalized size = 4.11 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2/15*x^6*(d*x^3+c)^(1/2)+2/45*c/d*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(d*x^3+
c)^(1/2)/d^2)-2/9*a/b^2*(d*x^3+c)^(3/2)/d+a^2/b^2*(2/3*(d*x^3+c)^(1/2)/b+1/
3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.23, size = 280, normalized size = 2.24

$$\left[\frac{15\,a^{2}d^{2}\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx^{3}+2\,bc-ad-2\,\sqrt{dx^{3}+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^{3}+a}\right)+2\,(3\,b^{2}d^{2}x^{6}-2\,b^{2}c^{2}-5\,abcd+15\,a^{2}d^{2}+(b^{2}cd-5\,abd^{2})x^{3})\sqrt{dx^{3}+c}}{45\,b^{3}d^{2}},\ -\frac{2\left(15\,a^{2}d^{2}\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx^{3}+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(3\,b^{2}d^{2}x^{6}-2\,b^{2}c^{2}-5\,abcd+15\,a^{2}d^{2}+(b^{2}cd-5\,abd^{2})x^{3})\sqrt{dx^{3}+c}\right)}{45\,b^{3}d^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/45*(15*a^2*d^2*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d
*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(3*b^2*d^2*x^6 - 2*b^2*c^
2 - 5*a*b*c*d + 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b
^3*d^2), -2/45*(15*a^2*d^2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*s
qrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^2*d^2*x^6 - 2*b^2*c^2 - 5*a*b*c*d +
 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^3*d^2)]
```

Sympy [A]
time = 12.26, size = 128, normalized size = 1.02

$$\frac{2\left(\frac{a^2d^3\sqrt{c+dx^3}}{3b^3}-\frac{a^2d^3(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{d(c+dx^3)^{\frac{5}{2}}}{15b}+\frac{(c+dx^3)^{\frac{3}{2}}(-ad^2-bcd)}{9b^2}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(a**2*d**3*sqrt(c + d*x**3)/(3*b**3) - a**2*d**3*(a*d - b*c)*atan(sqrt(c
+ d*x**3)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + d*(c + d*x**3
)**(5/2)/(15*b) + (c + d*x**3)**(3/2)*(-a*d**2 - b*c*d)/(9*b**2))/d**3
```

Giac [A]
time = 0.52, size = 139, normalized size = 1.11

$$\frac{2\left(a^2bc-a^3d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^3}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^4d^8-5\left(dx^3+c\right)^{\frac{3}{2}}b^4cd^8-5\left(dx^3+c\right)^{\frac{3}{2}}ab^3d^9+15\sqrt{dx^3+c}\,a^2b^2d^{10}\right)}{45\,b^5d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*b*c - a^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(
-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^8 - 5*(d*x^3 + c)^(3
/2)*b^4*c*d^8 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^9 + 15*sqrt(d*x^3 + c)*a^2*b^2*
d^10)/(b^5*d^10)
```

Mupad [B]
time = 6.17, size = 176, normalized size = 1.41

$$\frac{2\,a^2\,\sqrt{d\,x^3+c}}{3\,b^3}+\frac{2\,{\left(d\,x^3+c\right)}^{5/2}}{15\,b\,d^2}-\frac{2\,a\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b^2\,d}-\frac{2\,c\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b\,d^2}+\frac{a^2\,\ln\left(\frac{a^2\,d^2\,1{}\mathrm{i}+b^2\,c^2\,2{}\mathrm{i}-2\,\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}-a\,b\,d^2\,x^3\,1{}\mathrm{i}+b^2\,c\,d\,x^3\,1{}\mathrm{i}-a\,b\,c\,d\,3{}\mathrm{i}}{2\,b\,x^3+2\,a}\right)\,\sqrt{a\,d-b\,c}\,1{}\mathrm{i}}{3\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*a^2*(c + d*x^3)^(1/2))/(3*b^3) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*a
*(c + d*x^3)^(3/2))/(9*b^2*d) - (2*c*(c + d*x^3)^(3/2))/(9*b*d^2) + (a^2*lo
g((a^2*d^2*1i + b^2*c^2*2i - 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)
- a*b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c
)^(1/2)*1i)/(3*b^(7/2))
```

3.359 $\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=93

$$-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9bd}+\frac{2a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b/d+2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1
/2))*(-a*d+b*c)^(1/2)/b^(5/2)-2/3*a*(d*x^3+c)^(1/2)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 81, 52, 65, 214}

$$\frac{2a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (-2*a*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b*d) + (2*a*Sqrt[
b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx,x,x^3\right)\\
&= \frac{2(c+dx^3)^{3/2}}{9bd}-\frac{a\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^3\right)}{3b}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9bd}-\frac{(a(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b^2}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9bd}-\frac{(2a(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b^2d}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9bd}+\frac{2a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 88, normalized size = 0.95

$$
\frac{2\sqrt{c+dx^3}\,(-3ad+b(c+dx^3))}{9b^2d}+\frac{2a\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-3*a*d + b*(c + d*x^3)))/(9*b^2*d) + (2*a*Sqrt[-(b*c) +
 a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 458, normalized size = 4.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/b/d-a/b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
)/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3
*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*
b+a))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 4.38, size = 195, normalized size = 2.10

$$\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,(bdx^3+bc-3\,ad)\sqrt{dx^3+c}}{9\,b^2d},\ \frac{2\left(3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(bdx^3+bc-3\,ad)\sqrt{dx^3+c}\right)}{9\,b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/9*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
 c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(b*d*x^3 + b*c - 3*a*d)*sqrt(d*
x^3 + c))/(b^2*d), 2/9*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*
b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^3 + b*c - 3*a*d)*sqrt(d*x^3 +
c))/(b^2*d)]
```

Sympy [A]
time = 7.33, size = 95, normalized size = 1.02

$$\frac{2\left(-\frac{ad^2\sqrt{c+dx^3}}{3b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d(c+dx^3)^{\frac{3}{2}}}{9b}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**3)/(3*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x
**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**3)**(3
/2)/(9*b))/d**2
```

Giac [A]
time = 0.59, size = 96, normalized size = 1.03

$$-\frac{2\left(abc-a^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^2}+\frac{2\left((dx^3+c)^{\frac{3}{2}}b^2d^2-3\sqrt{dx^3+c}\,abd^3\right)}{9\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^3 + c)*a*
b*d^3)/(b^3*d^3)
```

Mupad [B]
time = 6.06, size = 136, normalized size = 1.46

$$\frac{2\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b\,d}-\frac{2\,a\,\sqrt{d\,x^3+c}}{3\,b^2}+\frac{a\,\ln\left(\frac{a^2\,d^2\,1{}\mathrm{i}+b^2\,c^2\,2{}\mathrm{i}+2\,\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}-a\,b\,d^2\,x^3\,1{}\mathrm{i}+b^2\,c\,d\,x^3\,1{}\mathrm{i}-a\,b\,c\,d\,3{}\mathrm{i}}{2\,b\,x^3+2\,a}\right)\,\sqrt{a\,d-b\,c}\,1{}\mathrm{i}}{3\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(3/2))/(9*b*d) - (2*a*(c + d*x^3)^(1/2))/(3*b^2) + (a*log((a
^2*d^2*1i + b^2*c^2*2i + 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2) - a*
b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c)^(1
/2)*1i)/(3*b^(5/2))
```

3.360 $\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=70

$$\frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

```
[Out] -2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(
3/2)+2/3*(d*x^3+c)^(1/2)/b
```

Rubi [A]

time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {455, 52, 65, 214}

$$\frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^3\right)\\ &= \frac{2\sqrt{c+dx^3}}{3b}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b}\\ &= \frac{2\sqrt{c+dx^3}}{3b}+\frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3bd}\\ &= \frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 70, normalized size = 1.00

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] ((2*Sqrt[c + d*x^3])/b - (2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x
^3])/Sqrt[-(b*c) + a*d]])/b^(3/2))/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.34, size = 434, normalized size = 6.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x
-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2
)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)
/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.55, size = 156, normalized size = 2.23

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\sqrt{dx^3+c}}{3b},\ -\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^3+c}\right)}{3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c))/b, -2/3*(sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqr
t(d*x^3 + c))/b]
```

Sympy [A]

time = 2.63, size = 68, normalized size = 0.97

$$\frac{2\left(\frac{d\sqrt{c+dx^3}}{3b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**3)/(3*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**3)/sqrt((a*d
 - b*c)/b))/(3*b**2*sqrt((a*d - b*c)/b)))/d
```

Giac [A]
time = 0.59, size = 66, normalized size = 0.94

$$\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b}+\frac{2\,\sqrt{dx^3+c}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b) + 2/3*sqrt(d*x^3 + c)/b
```

Mupad [B]
time = 6.16, size = 82, normalized size = 1.17

$$\frac{2\,\sqrt{d\,x^3+c}}{3\,b}+\frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)
*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*b^(3
/2))
```

3.361 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx$

Optimal. Leaf size=85

$$-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a+2/3*arctanh(b^(1/2)*(d*x^3+
c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 85, 65, 214}

$$\frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

```
[Out] (-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a) + (2*Sqrt[b*c - a*d]*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx, x, x^3\right)\\ &= \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}\\ &= \frac{(2c)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad} - \frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad}\\ &= -\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 81, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}} - \sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

```
[Out] (2*((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]
])/Sqrt[b] - Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]))/(3*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.43, size = 476, normalized size = 5.60

method	result

default	$-\frac{b\left(\frac{2\sqrt{dx^3+c}}{3b}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\cdots\right.}{-3\left(-cd^2\right)^{\frac{1}{3}}\cdots}}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-
3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
```

```
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a*(2/3*(d*x^3+
c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x), x)
```

Fricas [A]
time = 4.26, size = 383, normalized size = 4.51

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)}{3\,a}, \frac{2\,\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)}{3\,a}, \frac{2\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)+\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)}{3\,a}, \frac{2\left(\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)\right)}{3\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="fricas")
[Out] [1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*
sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
c)*sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/
c) + sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*s
qrt((b*c - a*d)/b))/(b*x^3 + a)))/a, 2/3*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt
(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqrt(d*x^
3 + c)*sqrt(-c)/c))/a]
```

Sympy [A]
time = 4.03, size = 85, normalized size = 1.00

$$\frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a),x)
```

```
[Out] 2*(c*d*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)) + d*(a*d - b*c)*atan(
sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*b*sqrt((a*d - b*c)/b)))/d
```

Giac [A]
time = 0.57, size = 79, normalized size = 0.93

$$-\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a}+\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*a) + 2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B]
time = 7.94, size = 114, normalized size = 1.34

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,a\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(a + b*x^3)),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(
1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*a*b^(1/2))
```

3.362 $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{c+dx^3}}{3ax^3}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}-\frac{2\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}$$

```
[Out] 1/3*(-a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(1/2)-2/3*arctanh(b
^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)*(-a*d+b*c)^(1/2)/a^2-1/3*(
d*x^3+c)^(1/2)/a/x^3
```

Rubi [A]
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 101, 162, 65, 214}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}-\frac{2\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}-\frac{\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(a*x^3) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a^2*Sqrt[c]) - (2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[b*c - a*d]])/(3*a^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc-ad)\text{Subst}\left(\int\right.}{6}$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(2b(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc-ad)\text{Subst}}{}$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+}}{\sqrt{bc-}}\right.}{3a^2}$$

Mathematica [A]
time = 0.28, size = 107, normalized size = 0.93

$$\frac{-\frac{a\sqrt{c+dx^3}}{x^3} - 2\sqrt{b}\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right) + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]

[Out] (-((a*Sqrt[c + d*x^3])/x^3) - 2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]] + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/(3*a^2)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.41, size = 518, normalized size = 4.50

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}}{3a\,x^{3}}-\frac{-\frac{2(-ad+2bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}-\frac{2ib(ad-bc)\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i}{\cdots}\right.}{\cdots}}}{\cdots}\right.}{\cdots}}{\cdots}$

default	$b^2 \left(\frac{2\sqrt{d x^3+c}}{3b} + \frac{i\sqrt{2} \left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(-c\,d^2\right)^{\frac{1}{3}} \sqrt{2} \sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}} \sqrt{\frac{d\left(x-\frac{(}{}\right.}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+}}}{} \right.}{} \right.$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*
I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2
)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alp
ha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
```

```
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a*(-1/3*(d*
x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))-b/a^2*(2/3
*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^4), x)
```

Fricas [A]
time = 3.34, size = 513, normalized size = 4.46

$$\left[\frac{2\,\sqrt{b^2c-abd}\,cx^3\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-(2\,bc-ad)\sqrt{c}\,x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-2\,\sqrt{dx^3+c}\,ac}{6\,a^2cx^3}, \frac{4\,\sqrt{-b^2c+abd}\,cx^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(2\,bc-ad)\sqrt{c}\,x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-2\,\sqrt{dx^3+c}\,ac}{6\,a^2cx^3}, -\frac{(2\,bc-ad)\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-\sqrt{b^2c-abd}\,cx^3\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+\sqrt{dx^3+c}\,ac}{3\,a^2cx^3}, \frac{2\,\sqrt{-b^2c+abd}\,cx^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(2\,bc-ad)\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-\sqrt{dx^3+c}\,ac}{3\,a^2cx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(b^2*c - a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3
 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*
x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c
*x^3), 1/6*(4*sqrt(-b^2*c + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^3 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*x^3 - 2*sqrt(
d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), -1/3*(
(2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(b^2*c
- a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c -
a*b*d))/(b*x^3 + a)) + sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), 1/3*(2*sqrt(-b^2*c
 + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)
) - (2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(d*
x^3 + c)*a*c)/(a^2*c*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^4\left(a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)), x)
```

Giac [A]
time = 0.62, size = 107, normalized size = 0.93

$$\frac{2\,(b^2c-abd)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}-\frac{\sqrt{dx^3+c}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b^2*c - a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a^2) - 1/3*(2*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^
2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)/(a*x^3)
```

Mupad [B]
time = 5.13, size = 137, normalized size = 1.19

$$\frac{\ln\left(\frac{a\,d-2\,b\,c+2\,\sqrt{d\,x^3+c}\,\sqrt{b^2\,c-a\,b\,d}-b\,d\,x^3}{b\,x^3+a}\right)\,\sqrt{b^2\,c-a\,b\,d}}{3\,a^2}-\frac{\sqrt{d\,x^3+c}}{3\,a\,x^3}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)\,(a\,d-2\,b\,c)}{6\,a^2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)),x)
```

```
[Out] (log((a*d - 2*b*c + 2*(c + d*x^3)^(1/2)*(b^2*c - a*b*d)^(1/2) - b*d*x^3)/(a
 + b*x^3))*(b^2*c - a*b*d)^(1/2))/(3*a^2) - (c + d*x^3)^(1/2)/(3*a*x^3) + (
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(a
*d - 2*b*c))/(6*a^2*c^(1/2))
```

3.363 $\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1,-1/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x
^3/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(64) = 128$.
time = 4.79, size = 241, normalized size = 3.77

$$\frac{x\left(\frac{(3bc-5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+8\left(c+dx^3+\frac{8a^2c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}\right)\right)}{20b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x*(((3*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)])/a + 8*(c + d*x^3 + (8*a^2*c^2*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)]))))))/(20*b*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.39, size = 1012, normalized size = 15.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2/5*x*(d*x^3+c)^(1/2)-2/5*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2))-a/b*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
```

```
2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2))+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*
3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3
)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3),x)
[Out] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3), x)
```

3.364 $\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1,-1/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x
^3/c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{\sqrt{c+dx^3}\int \frac{x\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}} = \frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [A]
time = 7.31, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{\frac{c+dx^3}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)
])/(2*a*Sqrt[(c + d*x^3)/c])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.31, size = 857, normalized size = 13.39

method	result	size
default	Expression too large to display	857
elliptic	Expression too large to display	857

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
```

```
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/b/d^2*2^(1/2)*sum(
1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)
*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c + dx^3}}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(1/2))/(a + b*x^3),x)
[Out] int((x*(c + d*x^3)^(1/2))/(a + b*x^3), x)
```

3.365 $\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] x*AppellF1(1/3,1,-1/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x^3/c)^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])
/(a*Sqrt[1 + (d*x^3)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}} = \frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 161 vs. $2(59) = 118$.
time = 9.99, size = 161, normalized size = 2.73

$$\frac{8acx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(8acF_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(-2bcF_1\left(\frac{4}{3};-\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

```
[Out] (8*a*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3
)/a)])/((a + b*x^3)*(8*a*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(-2*b*c*AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)
/a)] + a*d*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.30, size = 705, normalized size = 11.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*
I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
```

```
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(a + b*x^3),x)
[Out] int((c + d*x^3)^(1/2)/(a + b*x^3), x)
```

3.366 $\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -AppellF1(-1/3,1,-1/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x/(1+d*x^3/c
)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

```
[Out] -((Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx=\frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^2(a+bx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$=-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 139 vs. 2(62) = 124.
time = 10.08, size = 139, normalized size = 2.24

$$\frac{-20a(c+dx^3)+5(-2bc+3ad)x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{20a^2x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

```
[Out] (-20*a*(c + d*x^3) + 5*(-2*b*c + 3*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/c]
*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*x*Sqrt[c +
 d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.41, size = 1314, normalized size = 21.19

method	result	size
elliptic	Expression too large to display	892
risch	Expression too large to display	893
default	Expression too large to display	1314

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3
))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
```

```
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/b/d^2*2^(1/2
)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^
2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d
^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)
^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*
b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*b+a)))+1/a*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(a+bx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**2*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(b\,x^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)), x)
```

3.367 $\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -1/2*AppellF1(-2/3,1,-1/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x^2/(1+d
*x^3/c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3(a+bx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 335 vs. $2(64) = 128$.
time = 10.19, size = 335, normalized size = 5.23

$$\frac{-bdx^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(32ac(2ac+6bcx^3-adx^3+2bdx^6)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3(a+bx^3)(c+dx^3)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}}{16a^2x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

```
[Out] (-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)]) + (a*(32*a*c*(2*a*c + 6*b*c*x^3 - a*d*x^3 + 2*b*d*x^6)*AppellF1
[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(a + b*x^3)*(c + d*
x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Ap
pellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*
c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*App
ellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2
, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*x^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.39, size = 1010, normalized size = 15.78 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3
))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
)+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
```

```
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2
*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a*(-1/2*(d*x^3+c)^(1/2)/x^2-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^3 \left(a + bx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a),x)
[Out] Integral(sqrt(c + d*x**3)/(x**3*(a + b*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{x^3\,\left(b\,x^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)), x)
```

3.368 $\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal. Leaf size=154

$$\frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4}+\frac{2a^2(c+dx^3)^{3/2}}{9b^3}-\frac{2(bc+ad)\,(c+dx^3)^{5/2}}{15b^2d^2}+\frac{2(c+dx^3)^{7/2}}{21bd^2}-\frac{2a^2(bc-ad)^{3/2}\tanh^{-1}\Big(}{3b^{9/2}}$$

```
[Out] 2/9*a^2*(d*x^3+c)^(3/2)/b^3-2/15*(a*d+b*c)*(d*x^3+c)^(5/2)/b^2/d^2+2/21*(d*
x^3+c)^(7/2)/b/d^2-2/3*a^2*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)
/(-a*d+b*c)^(1/2))/b^(9/2)+2/3*a^2*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^4
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$,
Rules used = {457, 90, 52, 65, 214}

$$-\frac{2a^2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}+\frac{2a^2\sqrt{c+dx^3}\,(bc-ad)}{3b^4}+\frac{2a^2(c+dx^3)^{3/2}}{9b^3}-\frac{2(c+dx^3)^{5/2}\,(ad+bc)}{15b^2d^2}+\frac{2(c+dx^3)^{7/2}}{21bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*a^2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^4) + (2*a^2*(c + d*x^3)^(3/2))/(9*
b^3) - (2*(b*c + a*d)*(c + d*x^3)^(5/2))/(15*b^2*d^2) + (2*(c + d*x^3)^(7/2
))/(21*b*d^2) - (2*a^2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[b*c - a*d]])/(3*b^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(-bc-ad)(c+dx)^{3/2}}{b^2d} + \frac{a^2(c+dx)^{3/2}}{b^2(a+bx)} + \frac{(c+dx)^{5/2}}{bd}\right)dx, x, x^3\right)$$

$$= -\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2} + \frac{2\left(c+dx^3\right)^{7/2}}{21bd^2} + \frac{a^2\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{3b^2}$$

$$= \frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2} + \frac{2\left(c+dx^3\right)^{7/2}}{21bd^2} + \frac{\left(a^2(bc-ad)\right)\text{Subst}\left(\int \cdots\right.}{3b\cdots}$$

$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4} + \frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2} + \frac{2\left(c+dx^3\right)^{7/\cdots}}{21bd^2}\cdots$$

$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4} + \frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2} + \frac{2\left(c+dx^3\right)^{7/\cdots}}{21bd^2}\cdots$$

$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4} + \frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2} + \frac{2\left(c+dx^3\right)^{7/\cdots}}{21bd^2}\cdots$$

Mathematica [A]
time = 0.34, size = 143, normalized size = 0.93

$$\frac{2\sqrt{c+dx^3}\left(-105a^3d^3-21ab^2d(c+dx^3)^2-3b^3(2c-5dx^3)\left(c+dx^3\right)^2+35a^2bd^2(4c+dx^3)\right)}{315b^4d^2}+\frac{2a^2(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-105*a^3*d^3 - 21*a*b^2*d*(c + d*x^3)^2 - 3*b^3*(2*c -
5*d*x^3)*(c + d*x^3)^2 + 35*a^2*b*d^2*(4*c + d*x^3)))/(315*b^4*d^2) + (2*a^
2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]]
)/(3*b^(9/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 605, normalized size = 3.93 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2/21*d*x^9*(d*x^3+c)^(1/2)+16/105*c*x^6*(d*x^3+c)^(1/2)+2/105*c^2/d*x^
3*(d*x^3+c)^(1/2)-4/105*c^3/d^2*(d*x^3+c)^(1/2))-2/15*a/b^2/d*(d*x^3+c)^(5/
2)+a^2/b^2*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*c*d/b)/
d*(d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a
*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3
)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/
3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*
_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/
2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Root
Of(_Z^3*b+a))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.28, size = 410, normalized size = 2.66

$$\left[-\frac{105\,(a^2bcd^2-a^3d^3)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\,(15\,b^3d^3x^9+3\,(8\,b^3cd^2-7\,ab^2d^3)x^6-6\,b^3c^3-21\,ab^2c^2d+140\,a^2bcd^2-105\,a^3d^3+(3\,b^3c^2d-42\,ab^2cd^2+35\,a^2bd^3)x^3)\sqrt{dx^3+c}}{315\,b^4d^2},\ -\frac{2\left(105\,(a^2bcd^2-a^3d^3)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(15\,b^3d^3x^9+3\,(8\,b^3cd^2-7\,ab^2d^3)x^6-6\,b^3c^3-21\,ab^2c^2d+140\,a^2bcd^2-105\,a^3d^3+(3\,b^3c^2d-42\,ab^2cd^2+35\,a^2bd^3)x^3)\sqrt{dx^3+c}\right)}{315\,b^4d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/315*(105*(a^2*b*c*d^2 - a^3*d^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b
*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(15*b^
3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21*a*b^2*c^2*d
+ 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^2 + 35*a^2*b*
d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2), -2/315*(105*(a^2*b*c*d^2 - a^3*d^3)*s
qrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a
*d)) - (15*b^3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21
*a*b^2*c^2*d + 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^
2 + 35*a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2)]
```

Sympy [A]
time = 53.17, size = 153, normalized size = 0.99

$$\frac{2a^2(c+dx^3)^{\frac{3}{2}}}{9b^3}+\frac{2a^2(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^5\sqrt{\frac{ad-bc}{b}}}+\frac{2(c+dx^3)^{\frac{7}{2}}}{21bd^2}+\frac{(c+dx^3)^{\frac{5}{2}}(-2ad-2bc)}{15b^2d^2}+\frac{\sqrt{c+dx^3}\,(-2a^3d+2a^2bc)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] 2*a**2*(c + d*x**3)**(3/2)/(9*b**3) + 2*a**2*(a*d - b*c)**2*atan(sqrt(c + d
*x**3)/sqrt((a*d - b*c)/b))/(3*b**5*sqrt((a*d - b*c)/b)) + 2*(c + d*x**3)**
(7/2)/(21*b*d**2) + (c + d*x**3)**(5/2)*(-2*a*d - 2*b*c)/(15*b**2*d**2) + s
qrt(c + d*x**3)*(-2*a**3*d + 2*a**2*b*c)/(3*b**4)
```

Giac [A]
time = 1.44, size = 193, normalized size = 1.25

$$\frac{2\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^4}+\frac{2\left(15\,(dx^3+c)^{\frac{7}{2}}b^6d^{12}-21\,(dx^3+c)^{\frac{5}{2}}b^6cd^{12}-21\,(dx^3+c)^{\frac{5}{2}}ab^5d^{13}+35\,(dx^3+c)^{\frac{3}{2}}a^2b^4d^{14}+105\,\sqrt{dx^3+c}\,a^2b^4cd^{14}-105\,\sqrt{dx^3+c}\,a^3b^3d^{15}\right)}{315\,b^7d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^
2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 2/315*(15*(d*x^3 + c)^(7/2)*b^6*
d^12 - 21*(d*x^3 + c)^(5/2)*b^6*c*d^12 - 21*(d*x^3 + c)^(5/2)*a*b^5*d^13 +
35*(d*x^3 + c)^(3/2)*a^2*b^4*d^14 + 105*sqrt(d*x^3 + c)*a^2*b^4*c*d^14 - 10
5*sqrt(d*x^3 + c)*a^3*b^3*d^15)/(b^7*d^14)
```

Mupad [B]
time = 6.12, size = 330, normalized size = 2.14

$$\frac{2\,d\,x^9\,\sqrt{d\,x^3+c}}{21\,b}-\frac{\left(\frac{2\,a\,\left(\frac{c^2}{b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)}{b}+\frac{2\,c\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{4\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{5\,d}\right)}{3\,d}\right)\,\sqrt{d\,x^3+c}}{3\,d}+\frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{4\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{5\,d}\right)}{9\,d}-\frac{x^6\,\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{15\,d}+\frac{a^2\,\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] (2*d*x^9*(c + d*x^3)^(1/2))/(21*b) - (((2*a*(c^2/b + (a*((a*d^2)/b^2 - (2*c
*d)/b))/b))/b + (2*c*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (4*c*
((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(3*d))*(c + d*x^3)^(1/2))/(3*d) +
 (x^3*(c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (4
*c*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(9*d) - (x^6*(c + d*x^3)^(1/2)
*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(15*d) + (a^2*log((a^2*d^2 + 2*b^2*c^2 -
 b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3
 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*b^(9/2))
```

3.369 $\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal. Leaf size=120

$$-\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3}-\frac{2a(c+dx^3)^{3/2}}{9b^2}+\frac{2(c+dx^3)^{5/2}}{15bd}+\frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

```
[Out] -2/9*a*(d*x^3+c)^(3/2)/b^2+2/15*(d*x^3+c)^(5/2)/b/d+2/3*a*(-a*d+b*c)^(3/2)*
arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)-2/3*a*(-a*d+b*c)*
(d*x^3+c)^(1/2)/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 81, 52, 65, 214}

$$\frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}-\frac{2a\sqrt{c+dx^3}\,(bc-ad)}{3b^3}-\frac{2a(c+dx^3)^{3/2}}{9b^2}+\frac{2(c+dx^3)^{5/2}}{15bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (-2*a*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^3) - (2*a*(c + d*x^3)^(3/2))/(9*b^2
) + (2*(c + d*x^3)^(5/2))/(15*b*d) + (2*a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)$$

$$= \frac{2(c+dx^3)^{5/2}}{15bd} - \frac{a\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{3b}$$

$$= -\frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd} - \frac{(a(bc-ad))\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b^2}$$

$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd} - \frac{(a(bc-ad)^2)\,\text{Subst}\Big(\int}{3}$$

$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd} - \frac{(2a(bc-ad)^2)\,\text{Subst}\Big(\int}{}$$

$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd} + \frac{2a(bc-ad)^{3/2}\tanh^{-1}\Big(}{3b^{7/2}}$$

Mathematica [A]
time = 0.22, size = 111, normalized size = 0.92

$$\frac{2\sqrt{c+dx^3}\left(15a^2d^2+3b^2(c+dx^3)^2-5abd(4c+dx^3)\right)}{45b^3d} - \frac{2a(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(15*a^2*d^2 + 3*b^2*(c + d*x^3)^2 - 5*a*b*d*(4*c + d*x^3
)))/(45*b^3*d) - (2*a*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 531, normalized size = 4.42 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/b/d-a/b*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*
c)/b^2-2/3*c*d/b)/d*(d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a
*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d
^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.91, size = 297, normalized size = 2.48

$$\left[-\frac{15\,(abcd - a^2d^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right) - 2\,(3\,b^2d^2x^6 + 3\,b^2c^2 - 20\,abcd + 15\,a^2d^2 + (6\,b^2cd - 5\,abd^2)x^3)\sqrt{dx^3+c}}{45\,b^3d}, \frac{2\left(15\,(abcd - a^2d^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right) + (3\,b^2d^2x^6 + 3\,b^2c^2 - 20\,abcd + 15\,a^2d^2 + (6\,b^2cd - 5\,abd^2)x^3)\sqrt{dx^3+c}\right)}{45\,b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/45*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a
*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(3*b^2*d^2*x
^6 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqr
t(d*x^3 + c))/(b^3*d), 2/45*(15*(a*b*c*d - a^2*d^2)*sqrt(-(b*c - a*d)/b)*ar
ctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (3*b^2*d^2*x^6
+ 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d
*x^3 + c))/(b^3*d)]
```

Sympy [A]
time = 28.86, size = 116, normalized size = 0.97

$$-\frac{2a(c+dx^3)^{\frac{3}{2}}}{9b^2}-\frac{2a(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{2(c+dx^3)^{\frac{5}{2}}}{15bd}+\frac{\sqrt{c+dx^3}\cdot(2a^2d-2abc)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] -2*a*(c + d*x**3)**(3/2)/(9*b**2) - 2*a*(a*d - b*c)**2*atan(sqrt(c + d*x**3
)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + 2*(c + d*x**3)**(5/2)
/(15*b*d) + sqrt(c + d*x**3)*(2*a**2*d - 2*a*b*c)/(3*b**3)
```

Giac [A]
time = 1.47, size = 151, normalized size = 1.26

$$-\frac{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^3}+\frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}b^4d^4-5\,(dx^3+c)^{\frac{3}{2}}ab^3d^5-15\,\sqrt{dx^3+c}\,ab^3cd^5+15\,\sqrt{dx^3+c}\,a^2b^2d^6\right)}{45\,b^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2
*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^4
 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^3 + c)*a*b^3*c*d^5 + 15*sqrt
(d*x^3 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

Mupad [B]
time = 6.13, size = 215, normalized size = 1.79

$$\frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{2\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{3\,d}\right)}{3\,d}+\frac{2\,d\,x^6\,\sqrt{d\,x^3+c}}{15\,b}-\frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{9\,d}+\frac{a\,\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d+\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] ((c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (2*c*((
2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(3*d)))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2))
/(15*b) - (x^3*(c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(9*d) +
(a*log((a^2*d^2 + 2*b^2*c^2 + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2
i - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1
i)/(3*b^(7/2))
```

3.370 $\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal. Leaf size=96

$$\frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9b}-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b-2/3*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/
(-a*d+b*c)^(1/2))/b^(5/2)+2/3*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {455, 52, 65, 214}

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}+\frac{2\sqrt{c+dx^3}\,(bc-ad)}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b) - (2*
(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^
(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)$$
$$= \frac{2(c+dx^3)^{3/2}}{9b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b}$$
$$= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9b} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x$$
$$= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9b} + \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c-}$$
$$= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9b} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

Mathematica [A]
time = 0.18, size = 85, normalized size = 0.89

$$\frac{2\sqrt{c+dx^3}\,(4bc-3ad+bdx^3)}{9b^2} + \frac{2(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(4*b*c - 3*a*d + b*d*x^3))/(9*b^2) + (2*(-(b*c) + a*d)^(
3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.33, size = 507, normalized size = 5.28 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*c*d/b)/d*(d*x^3+c)^
(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*
d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c
*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.03, size = 204, normalized size = 2.12

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\,(bdx^3+4\,bc-3\,ad)\sqrt{dx^3+c}}{9\,b^2},\;-\frac{2\left(3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(bdx^3+4\,bc-3\,ad)\sqrt{dx^3+c}\right)}{9\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqr
t(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(b*d*x^3 + 4*b*c - 3*a
*d)*sqrt(d*x^3 + c))/b^2, -2/9*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-
sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x^3 + 4*b*c - 3*
a*d)*sqrt(d*x^3 + c))/b^2]
```

Sympy [A]
time = 14.37, size = 90, normalized size = 0.94

$$\frac{2(c+dx^3)^{\frac{3}{2}}}{9b}+\frac{\sqrt{c+dx^3}\,(-2ad+2bc)}{3b^2}+\frac{2(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] 2*(c + d*x**3)**(3/2)/(9*b) + sqrt(c + d*x**3)*(-2*a*d + 2*b*c)/(3*b**2) +
2*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a
*d - b*c)/b))
```

Giac [A]
time = 1.33, size = 113, normalized size = 1.18

$$\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^2}+\frac{2\left((dx^3+c)^{\frac{3}{2}}b^2+3\sqrt{dx^3+c}\,b^2c-3\sqrt{dx^3+c}\,abd\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2 + 3*sqrt(d*
x^3 + c)*b^2*c - 3*sqrt(d*x^3 + c)*a*b*d)/b^3
```

Mupad [B]
time = 5.91, size = 143, normalized size = 1.49

$$\frac{2\,d\,x^3\,\sqrt{d\,x^3+c}}{9\,b}-\frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{8\,c\,d}{3\,b}\right)}{3\,d}+\frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] (log((a^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i
- a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)
/(3*b^(5/2)) - ((c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (8*c*d)/(3*b)))/(3*d) +
(2*d*x^3*(c + d*x^3)^(1/2))/(9*b)
```

3.371 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx$

Optimal. Leaf size=104

$$\frac{2d\sqrt{c+dx^3}}{3b}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}$$

```
[Out] -2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a+2/3*(-a*d+b*c)^(3/2)*arctan
h(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(3/2)+2/3*d*(d*x^3+c)^(1/2)
/b
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 86, 162, 65, 214}

$$\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2d\sqrt{c+dx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

```
[Out] (2*d*Sqrt[c + d*x^3])/(3*b) - (2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/
(3*a) + (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a
*d]])/(3*a*b^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)}\,dx,x,x^3\right)\\
&= \frac{2d\sqrt{c+dx^3}}{3b}+\frac{\text{Subst}\left(\int \frac{bc^2+d(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b}\\
&= \frac{2d\sqrt{c+dx^3}}{3b}+\frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{3a}-\frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}} \cdots\right.}{3ab}\\
&= \frac{2d\sqrt{c+dx^3}}{3b}+\frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3ad}-\frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{\cdots}{a-\cdots} \cdots\right.}{3ab \cdots}\\
&= \frac{2d\sqrt{c+dx^3}}{3b}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 106, normalized size = 1.02

$$
\frac{2\left(a\sqrt{b}\,d\sqrt{c+dx^3}-(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)-b^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3ab^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

```
[Out] (2*(a*Sqrt[b]*d*Sqrt[c + d*x^3] - (-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt
[c + d*x^3])/Sqrt[-(b*c) + a*d]] - b^(3/2)*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]]))/(3*a*b^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 565, normalized size = 5.43

method	result
default	$-\frac{b\left(\frac{2dx^3\sqrt{dx^3+c}}{9b}+\frac{2\left(-\frac{d(ad-2bc)}{b^2}-\frac{2cd}{3b}\right)\sqrt{dx^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*c*d/b)/d*(d*x
^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c
)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
```

```
)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha
*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d
-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*b+a)))+1/a*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*a
rctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x), x)
```

Fricas [A]
time = 2.52, size = 486, normalized size = 4.67

$$\left[\frac{bc^{\frac{3}{2}}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}\,ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)}{3ab}, \frac{bc^{\frac{3}{2}}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}\,ad+2(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)}{3ab}, \frac{2b\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+2\sqrt{dx^3+c}\,ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)}{3ab}, \frac{2\left(b\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\sqrt{dx^3+c}\,ad+(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)\right)}{3ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(b*c^(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt
(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*
d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)))/(a*b), 1/3*(b*c^
(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c
)*a*d + 2*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-
(b*c - a*d)/b)/(b*c - a*d)))/(a*b), 1/3*(2*b*sqrt(-c)*c*arctan(sqrt(d*x^3 +
 c)*sqrt(-c)/c) + 2*sqrt(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*l
og((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3
 + a)))/(a*b), 2/3*(b*sqrt(-c)*c*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(
d*x^3 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b
*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

Sympy [A]

time = 11.64, size = 102, normalized size = 0.98

$$\frac{2d\sqrt{c+dx^3}}{3b}+\frac{2c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}-\frac{2(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab^2\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a),x)
```

```
[Out] 2*d*sqrt(c + d*x**3)/(3*b) + 2*c**2*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sq
rt(-c)) - 2*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*
b**2*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 1.58, size = 112, normalized size = 1.08

$$\frac{2\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}+\frac{2\,\sqrt{dx^3+c}\,d}{3\,b}-\frac{2\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)) + 2/3*sqrt(d*x^3 + c)
*d/b - 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b
^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

Mupad [B]
time = 7.88, size = 155, normalized size = 1.49

$$\frac{c^{3/2}\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a}+\frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b}+\frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d+\sqrt{b}\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d-b\,c\right)^{3/2}\,1{}\mathrm{i}}{3\,a\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(a + b*x^3)),x)
```

```
[Out] (c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a) + (2*d*(c + d*x^3)^(1/2))/(3*b) + (log((a^2*d^2 + 2*b^2*c^2 +
 b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3
 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a*b^(3/2))
```

3.372 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)} dx$

Optimal. Leaf size=116

$$-\frac{c\sqrt{c+dx^3}}{3ax^3}+\frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}$$

```
[Out] -2/3*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2
/b^(1/2)+1/3*(-3*a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^2-1/
3*c*(d*x^3+c)^(1/2)/a/x^3
```

Rubi [A]
time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 100, 162, 65, 214}

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}+\frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}-\frac{c\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*(c*Sqrt[c + d*x^3])/(a*x^3) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c
+ d*x^3]/Sqrt[c]])/(3*a^2) - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)}\,dx, x, x^3\right)$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(2bc-3ad)+\frac{1}{2}d(bc-2ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{(c(2bc-3ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{}{(a}\right.}{3}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{(c(2bc-3ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} + \frac{(2(bc-ad)^2)\,\text{Subs}}{}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} + \frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{}}{\sqrt{bc}}\right.}{3a^2\sqrt{b}}$$

Mathematica [A]

time = 0.25, size = 108, normalized size = 0.93

$$\frac{-\frac{ac\sqrt{c+dx^3}}{x^3}+\frac{2(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}}+\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

```
[Out] (-((a*c*Sqrt[c + d*x^3])/x^3) + (2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqr
t[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[b] + Sqrt[c]*(2*b*c - 3*a*d)*ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 620, normalized size = 5.34

method	result
risch	$-\frac{c\sqrt{d\,x^3+c}}{3a\,x^3}+\frac{-\frac{2\sqrt{c}\,(3ad-2bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a}-\frac{2i\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{}}{}\right.}{}}{}$

default	$$\frac{b^2\left(\frac{2d\,x^3\sqrt{d\,x^3+c}}{9b}+\frac{2\left(-\frac{d(ad-2bc)}{b^2}-\frac{2cd}{3b}\right)\sqrt{d\,x^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{}}{}\right.}{}\right.}{}$$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^2*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*c*d/b)/d*(
d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-
b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*
_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_al
pha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
```

```
c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*b+a)))+1/a*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d
*arctanh((d*x^3+c)^(1/2)/c^(1/2)))-b/a^2*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(
d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^4), x)
```

Fricas [A]
time = 2.88, size = 538, normalized size = 4.64

$$\left[-\frac{2\,(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+(2\,bc-3\,ad)\sqrt{c}\,x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2c}{x^3}\right)+2\,\sqrt{dx^3+c}\,ac}{6\,a^2x^3}, -\frac{4\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(2\,bc-3\,ad)\sqrt{c}\,x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2c}{x^3}\right)+2\,\sqrt{dx^3+c}\,ac}{6\,a^2x^3}, -\frac{(2\,bc-3\,ad)\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)+(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{dx^3+c}\,ac}{3\,a^2x^3}, -\frac{2\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(2\,bc-3\,ad)\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)+\sqrt{dx^3+c}\,ac}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2
*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + (2*b*c - 3*a*d)*sqrt
(c)*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 +
 c)*a*c)/(a^2*x^3), -1/6*(4*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan(-sq
rt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sqrt(c)
*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c)
*a*c)/(a^2*x^3), -1/3*((2*b*c - 3*a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*
sqrt(-c)/c) + (b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*
d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(d*x^3 + c)
*a*c)/(a^2*x^3), -1/3*(2*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(
d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sqrt(-c)*x
^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^3 + c)*a*c)/(a^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^{\frac{3}{2}}}{x^4\left(a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a),x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x**4*(a + b*x**3)), x)
```

Giac [A]
time = 1.10, size = 121, normalized size = 1.04

$$\frac{2\,\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^2}-\frac{\left(2\,bc^2-3\,acd\right)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}-\frac{\sqrt{dx^3+c}\,c}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/3*(2*b*c^2 - 3*a*c*d)*arctan(sqrt(d*
x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)*c/(a*x^3)
```

Mupad [B]
time = 9.52, size = 167, normalized size = 1.44

$$\frac{\sqrt{c}\,\ln\left(\frac{{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)\,\left(3\,a\,d-2\,b\,c\right)}{6\,a^2}-\frac{c\,\sqrt{d\,x^3+c}}{3\,a\,x^3}+\frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,a^2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6)*(3*a*d - 2*b*c))/(6*a^2) - (c*(c + d*x^3)^(1/2))/(3*a*x^3) + (log((a
^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d
^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a^2
*b^(1/2))
```

3.373 $\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal. Leaf size=65

$$\frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/4*c*x^4*AppellF1(4/3,1,-3/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d
*x^3/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x^3\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(65) = 130.
time = 6.61, size = 280, normalized size = 4.31

$$\frac{x\left(8(c+dx^3)\left(14bc-11ad+5bdx^3\right)+\frac{(27b^2c^2-88abcd+55a^2d^2)x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}-\frac{64a^2c^2(-14bc+11ad)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}\right)}{220b^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (x*(8*(c + d*x^3)*(14*b*c - 11*a*d + 5*b*d*x^3) + ((27*b^2*c^2 - 88*a*b*c*d
 + 55*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)])/a - (64*a^2*c^2*(-14*b*c + 11*a*d)*AppellF1[1/3, 1/2, 1
, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2,
 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7
/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/
c), -((b*x^3)/a)])))))/(220*b^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.38, size = 1101, normalized size = 16.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2/11*d*x^4*(d*x^3+c)^(1/2)+28/55*c*x*(d*x^3+c)^(1/2)-18/55*I*c^2*3^(1/
2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-a/b*(2/5*d/b*x*
(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-2/5*c*d/b)*3^(1/2)/d*(-c*d^2)^(1/
```

```
3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d
^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(
-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*
(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/
3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*
d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d
-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(c+dx^3)^{\frac{3}{2}}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] Integral(x**3*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,{\left(d\,x^3+c\right)}^{3/2}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3), x)
```

3.374 $\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal. Leaf size=65

$$\frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

[Out] 1/2*c*x^2*AppellF1(2/3,1,-3/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x^3/c)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]

[Out] (c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)])/(2*a*Sqrt[1 + (d*x^3)/c])

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}} = \frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 149 vs. 2(65) = 130.
time = 8.94, size = 149, normalized size = 2.29

$$\frac{x^2\left(20ad(c+dx^3)+5c(7bc-4ad)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2d(10bc-7ad)x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{70ab\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (x^2*(20*a*d*(c + d*x^3) + 5*c*(7*b*c - 4*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1
[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(10*b*c - 7*a*d)*x^3*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)]))
/(70*a*b*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.38, size = 930, normalized size = 14.31

method	result	size
risch	Expression too large to display	921
default	Expression too large to display	930
elliptic	Expression too large to display	930

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*d/b*x^2*(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-4/7*c*d/b)*3^(1/2)/d*
(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/
```

```
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b
*c*d-b^2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c+dx^3)^{\frac{3}{2}}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] Integral(x*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,{\left(d\,x^3+c\right)}^{3/2}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(3/2))/(a + b*x^3),x)
[Out] int((x*(c + d*x^3)^(3/2))/(a + b*x^3), x)
```

3.375 $\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal. Leaf size=60

$$\frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] c*x*AppellF1(1/3,1,-3/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x^3/c
)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

```
[Out] (c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a*Sqrt[1 + (d*x^3)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(60) = 120.
time = 10.22, size = 351, normalized size = 5.85

$$\frac{x\left(\frac{d(8bc-5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(-4ac(2ad^2x^3+b(5c^2+2cdx^3+2d^2x^6))F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3dx^3(a+bx^3)(c+dx^3)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}\right)}{20b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

```
[Out] (x*((d*(8*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-4*a*c*(2*a*d^2*x^3 + b*(5*c^2 + 2*c*d*
x^3 + 2*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] +
3*d*x^3*(a + b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)
/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)
/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*
x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(20*b
*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.35, size = 776, normalized size = 12.93 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*d/b*x*(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-2/5*c*d/b)*3^(1/2)/d*(-
c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
```

```
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I/b^2/d^2*2^(1/2)*s
um((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*
(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*
(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/
2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^
2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3
*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^3)^{\frac{3}{2}}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/(b*x**3+a),x)
[Out] Integral((c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x^3+c)^{3/2}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(a + b*x^3),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(a + b*x^3), x)
```

3.376 $\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=63

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -c*AppellF1(-1/3,1,-3/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x/(1+d*x^3
/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^2(a+bx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= -\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(63) = 126.
time = 10.09, size = 148, normalized size = 2.35

$$\frac{-20ac(c+dx^3)+5c(-2bc+5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2d(bc+2ad)x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{20a^2x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-20*a*c*(c + d*x^3) + 5*c*(-2*b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(b*c + 2*a*d)*x^6*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(2
0*a^2*x*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.40, size = 1404, normalized size = 22.29

method	result	size
risch	Expression too large to display	920
elliptic	Expression too large to display	924
default	Expression too large to display	1404

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a*(2/7*d/b*x^2*(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-4/7*c*d/b)*3^(1
/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3
```

```
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2
+2*a*b*c*d-b^2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2
*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a*(-c*(d*x^3+c)^(1/2)/x+2/7*(
d*x^3+c)^(1/2)*d*x^2-9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d
*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d
*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{3}\right)^{\frac{3}{2}}}{x^{2} \left(a + bx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a),x)
[Out] Integral((c + d*x**3)**(3/2)/(x**2*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(d\,x^3+c\right)}^{3/2}}{x^2\,\left(b\,x^3+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x)
[Out] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)), x)
```

3.377 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=65

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -1/2*c*AppellF1(-2/3,1,-3/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -3/2, 1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^3(a+bx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= -\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(65) = 130.

time = 10.23, size = 343, normalized size = 5.28

$$-\frac{d(bc-4ad)x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8ac\left(-4ac(2ac+6bcx^3-5adx^3+2bdx^6)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3(a+bx^3)(c+dx^3)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}}{16a^2x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/16*(d*(b*c - 4*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)] + (8*a*c*(-4*a*c*(2*a*c + 6*b*c*x^3 - 5*a*d*x^3
+ 2*b*d*x^6)*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3
*(a + b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))
/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a
*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a^2*x^2*Sqrt
[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.40, size = 1096, normalized size = 16.86 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a*(2/5*d/b*x*(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-2/5*c*d/b)*3^(1/2
)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I/b^2/d^2*2^(
```

```
1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/
2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a
lpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a*(-1/2*c
*(d*x^3+c)^(1/2)/x^2+2/5*d*x*(d*x^3+c)^(1/2)-9/10*I*c*3^(1/2)*(-c*d^2)^(1/3
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^3)^{\frac{3}{2}}}{x^3\,(a+bx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a),x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x**3*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(d\,x^3+c\right)}^{3/2}}{x^3\,\left(b\,x^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)), x)
```

3.378 $\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=104

$$-\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2(c+dx^3)^{3/2}}{9bd^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b/d^2-2/3*a^2*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c
)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-2/3*(a*d+b*c)*(d*x^3+c)^(1/2)/b^2/d^2
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 90, 65, 214}

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^3}\,(ad+bc)}{3b^2d^2}+\frac{2(c+dx^3)^{3/2}}{9bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*(b*c + a*d)*Sqrt[c + d*x^3])/(3*b^2*d^2) + (2*(c + d*x^3)^(3/2))/(9*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)
*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx, x, x^3\right)\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2(c+dx^3)^{3/2}}{9bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b^2}\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2(c+dx^3)^{3/2}}{9bd^2}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+\ldots}\right)}{3b^2d}\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2(c+dx^3)^{3/2}}{9bd^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 91, normalized size = 0.88

$$\frac{2\sqrt{c+dx^3}\,(-2bc-3ad+bdx^3)}{9b^2d^2}+\frac{2a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-2*b*c - 3*a*d + b*d*x^3))/(9*b^2*d^2) + (2*a^2*ArcTan[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2)*Sqrt[-(b*c) + a*d
])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.35, size = 488, normalized size = 4.69 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2)/d^2)-2/3*a/b^2*(d*x^3+
c)^(1/2)/d-1/3*I*a^2/b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(
1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha
-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.49, size = 289, normalized size = 2.78

$$\left[\frac{3\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^3+2\,bc-ad-2\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2d^2)x^3\right)\sqrt{dx^3+c}}{9\left(b^4cd^2-ab^3d^3\right)}, \frac{2\left(3\sqrt{-b^2c+abd}\,a^2d^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2d^2)x^3\right)\sqrt{dx^3+c}\right)}{9\left(b^4cd^2-ab^3d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/9*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x
^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a^
2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^
3), 2/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c
+ a*b*d)/(b*d*x^3 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d
 - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.08, size = 106, normalized size = 1.02

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^2}+\frac{2\left((dx^3+c)^{\frac{3}{2}}b^2d^4-3\,\sqrt{dx^3+c}\,b^2cd^4-3\,\sqrt{dx^3+c}\,abd^5\right)}{9\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^3 + c)*b^2*c*d^4 - 3*s
qrt(d*x^3 + c)*a*b*d^5)/(b^3*d^6)
```

Mupad [B]
time = 5.43, size = 121, normalized size = 1.16

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b\,d}-\frac{\left(\frac{2\,a}{b^2}+\frac{4\,c}{3\,b\,d}\right)\,\sqrt{d\,x^3+c}}{3\,d}+\frac{a^2\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*b*d) - (((2*a)/b^2 + (4*c)/(3*b*d))*(c + d*x^3
)^(1/2))/(3*d) + (a^2*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b
*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

3.379 $\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=74

$$\frac{2\sqrt{c+dx^3}}{3bd}+\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

```
[Out] 2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+2/3*(d*x^3+c)^(1/2)/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 81, 65, 214}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b
*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{2\sqrt{c+dx^3}}{3bd} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}$$
$$= \frac{2\sqrt{c+dx^3}}{3bd} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd}$$
$$= \frac{2\sqrt{c+dx^3}}{3bd} + \frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 1.01

$$\frac{2\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{d} - \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*((Sqrt[b]*Sqrt[c + d*x^3])/d - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[
-(b*c) + a*d]])/Sqrt[-(b*c) + a*d]))/(3*b^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 448, normalized size = 6.05 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b/d+1/3*I*a/b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/
3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.92, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}\,(b^2c-abd)}{3\,(b^3cd-ab^2d^2)},\ -\frac{2\left(\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-\sqrt{dx^3+c}\,(b^2c-abd)\right)}{3\,(b^3cd-ab^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c
)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -2/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^3 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^3)\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.41, size = 64, normalized size = 0.86

$$-\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^3+c}}{b}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^3 + c)/b)/d
```

Mupad [B]
time = 5.10, size = 86, normalized size = 1.16

$$\frac{2\sqrt{d\,x^3+c}}{3\,b\,d}+\frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b*d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(
1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b*c
)^(1/2))
```

3.380 $\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {455, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*c
 - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [**A**]
time = 0.04, size = 51, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3\sqrt{b}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*Sqrt[b]*Sqrt[-(
b*c) + a*d])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 426, normalized size = 8.35

method	result

default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^
2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_
alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),
```

```
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.47, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)}{3\,\sqrt{b^2c-abd}},\ \frac{2\,\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)}{3\,(b^2c-abd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b
*x^3 + a))/sqrt(b^2*c - a*b*d), 2/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A]
time = 5.23, size = 39, normalized size = 0.76

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] 2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 1.64, size = 40, normalized size = 0.78

$$\frac{2\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B]
time = 5.89, size = 70, normalized size = 1.37

$$\frac{\ln\left(\frac{a\,d\,1\mathrm{i}-b\,c\,2\mathrm{i}+2\,\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}-b\,d\,x^3\,1\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (log((a*d*1i - b*c*2i + 2*(c + d*x^3)^(1/2)*(a*b*d - b^2*c)^(1/2) - b*d*x^3
*1i)/(a + b*x^3))*1i)/(3*(a*b*d - b^2*c)^(1/2))
```

3.381 $\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=85

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c^(1/2)+2/3*arctanh(b^(1/2)*(d*x^3+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 88, 65, 214}

$$\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{3a}-\frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3a}\\ &= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3ad}-\frac{(2b)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3ad}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 82, normalized size = 0.96

$$-\frac{\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*((2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt
[-(b*c) + a*d] + (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/a
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 453, normalized size = 5.33

method	result

default	$\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{-}}{}\right.}{}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*b/a/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+
2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2
)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*
c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-2/3*arctanh((d*x^3+c)^(1/2)/c^(
1/2))/a/c^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x), x)
```

Fricas [A]
time = 3.68, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)}{3\,ac}, \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)}{3\,ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+2\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)}{3\,ac}, \frac{2\left(c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)\right)}{3\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(c*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt
(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + sqrt
(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-
c)/c))/(a*c), 2/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^3 + c)*
sqrt(-c)/c))/(a*c)]
```

Sympy [A]
time = 5.42, size = 70, normalized size = 0.82

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] -2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)) + 2
*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c))
```

Giac [A]
time = 1.19, size = 71, normalized size = 0.84

$$-\frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a}+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

[Out] -2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))

Mupad [B]
time = 7.31, size = 114, normalized size = 1.34

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a\,\sqrt{c}}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,a\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^3)*(c + d*x^3)^(1/2)),x)

[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3*a*c^(1/2)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*a*(a*d - b*c)^(1/2))

3.382 $\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{c+dx^3}}{3acx^3}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

```
[Out] 1/3*(a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-2/3*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/3*(d
*x^3+c)^(1/2)/a/c/x^3
```

Rubi [A]
time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 105, 162, 65, 214}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}-\frac{\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(a*c*x^3) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqr
t[c]])/(3*a^2*c^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[
b*c - a*d]])/(3*a^2*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc+ad)\text{Subst}\left(\int \right.}{6a}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc+ad)\text{Subs}}{}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+}}{\sqrt{bc-a}}\right.}{3a^2\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.27, size = 109, normalized size = 0.93

$$\frac{-\frac{a\sqrt{c+dx^3}}{cx^3} + \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{3/2}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-((a*Sqrt[c + d*x^3])/(c*x^3)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[c
+ d*x^3]/Sqrt[c]])/c^(3/2))/(3*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.41, size = 498, normalized size = 4.26

method	result
risch	$-\frac{\sqrt{d x^3+c}}{3ac x^3}-\frac{-\frac{2(ad+2bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{2ib^2c\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\ldots}{\ldots}\right)}{(\ldots}}\ldots}{\ldots}\right)}{\ldots}}{\ldots}$
default	$-\frac{ib^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}\ldots}}\ldots}{\ldots}\right)}{\ldots}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*b^2/a^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/
d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2
*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a
*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a*(-1/3*(d*x^3+c)^(1/2)/
c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))+2/3*b/a^2*arctanh((d*
x^3+c)^(1/2)/c^(1/2))/c^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^4), x)
```

Fricas [A]
time = 4.08, size = 565, normalized size = 4.83

$$\left[\frac{2\,bc^2x^3\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+(2\,bc+ad)\sqrt{c}\,x^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-2\,\sqrt{dx^3+c}\,ac}{6\,a^2c^2x^3},\ -\frac{4\,bc^2x^3\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-(2\,bc+ad)\sqrt{c}\,x^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)+2\,\sqrt{dx^3+c}\,ac}{6\,a^2c^2x^3},\ \frac{bc^2x^3\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-(2\,bc+ad)\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-\sqrt{dx^3+c}\,ac}{3\,a^2c^2x^3},\ -\frac{2\,bc^2x^3\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+(2\,bc+ad)\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)+\sqrt{dx^3+c}\,ac}{3\,a^2c^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d
*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + (2*b*c + a*d)*sqr
t(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3
+ c)*a*c)/(a^2*c^2*x^3), -1/6*(4*b*c^2*x^3*sqrt(-b/(b*c - a*d))*arctan(-sqr
t(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (2*b*c + a
*d)*sqrt(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt
(d*x^3 + c)*a*c)/(a^2*c^2*x^3), 1/3*(b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d
*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*
x^3 + a)) - (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) -
 sqrt(d*x^3 + c)*a*c)/(a^2*c^2*x^3), -1/3*(2*b*c^2*x^3*sqrt(-b/(b*c - a*d))
*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
```

```
+ (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^
3 + c)*a*c)/(a^2*c^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 0.94, size = 104, normalized size = 0.89

$$\frac{2\,b^2\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}\,c}-\frac{\sqrt{dx^3+c}}{3\,acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] 2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/3*(2*b*c + a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/3*sqrt(d*x^3 + c)/(a*c*x^3)
```

Mupad [B]
time = 8.42, size = 142, normalized size = 1.21

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\,{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}^3}{x^6}\right)\,\left(a\,d+2\,b\,c\right)}{6\,a^2\,c^{3/2}}-\frac{\sqrt{d\,x^3+c}}{3\,a\,c\,x^3}+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,a^2\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
[Out] (log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(
a*d + 2*b*c))/(6*a^2*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a*c*x^3) + (b^(3/2)*lo
g((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/
(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(1/2))
```

3.383 $\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x^
3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

Mathematica [A]
time = 10.04, size = 65, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{4a\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)
/a)])/(4*a*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.34, size = 719, normalized size = 11.23 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/b*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/
3*I*a/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-
1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)
^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*
```

```
(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/
(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{(b\,x^3+a)\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
[Out] int(x^3/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.384 $\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x^
3+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

Mathematica [A]
time = 10.03, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)
/a)])/(2*a*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.31, size = 429, normalized size = 6.70

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\ \sqrt{}}{}\right.}{}$

elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d
*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*
d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

<u>Maxima</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

<u>Fricas</u> [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(b\,x^3+a\right)\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(x/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.385 $\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,1,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x^3+c)^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}} = \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.
time = 10.04, size = 161, normalized size = 2.73

$$-\frac{8acxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\sqrt{c+dx^3}\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x
^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a
)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.31, size = 429, normalized size = 7.27

method	result

default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1
/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^
(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(
```

```
a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\,x^3+a)\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.386 $\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,1,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/x/(d*x^3+c)
^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*x*Sqrt[c + d*x^3]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.
time = 10.08, size = 141, normalized size = 2.27

$$\frac{-20a(c+dx^3)+5(-2bc+ad)x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{20a^2cx\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-20*a*(c + d*x^3) + 5*(-2*b*c + a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3,
 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/c]*A
ppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*c*x*Sqrt[c +
 d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.40, size = 890, normalized size = 14.35

method	result	size
default	Expression too large to display	890
elliptic	Expression too large to display	891
risch	Expression too large to display	892

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*b/a/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-
1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)
^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/
```

```
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/
(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a*(-(d*x^3+c)^(1/2)/c/
x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + bx^3) \sqrt{c + dx^3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,\left(b\,x^3+a\right)\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.387 $\int \frac{1}{x^3(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,1,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/x^2/(d*
x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 339 vs. 2(64) = 128.
time = 10.18, size = 339, normalized size = 5.30

$$\frac{-bdx^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a\left(-4ac(2ac+6bcx^3+3adx^3+2bdx^6)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3(a+bx^3)(c+dx^3)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}}{16a^2cx^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)]) + (8*a*(-4*a*c*(2*a*c + 6*b*c*x^3 + 3*a*d*x^3 + 2*b*d*x^6)*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(a + b*x^3)*(c +
 d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d
*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*
a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*A
ppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3
/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*c*x^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.39, size = 738, normalized size = 11.53 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*b/a/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^
2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
```

```
)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a*(-1/2/c*(d*x^3+c)^
(1/2)/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + bx^3) \sqrt{c + dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\left(b\,x^3+a\right)\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.388 $\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=107

$$\frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -2/3*a^2*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*
c)^(3/2)+2/3*c^2/d^2/(-a*d+b*c)/(d*x^3+c)^(1/2)+2/3*(d*x^3+c)^(1/2)/b/d^2
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 89, 65, 214}

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}+\frac{2c^2}{3d^2\sqrt{c+dx^3}\,(bc-ad)}+\frac{2\sqrt{c+dx^3}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*c^2)/(3*d^2*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(3*b*d^2)
 - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b
*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c^2}{d(-bc+ad)(c+dx)^{3/2}} + \frac{1}{bd\sqrt{c+dx}} + \frac{a^2}{b(bc-ad)(a+bx)\sqrt{c}}\right.\right.$$

$$= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{3bd^2} + \frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right.}{3b(bc-ad)}$$

$$= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{3bd^2} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+}\right.}{3bd(bc-ad)}$$

$$= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{3bd^2} - \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

Mathematica [A]
time = 0.31, size = 111, normalized size = 1.04

$$\frac{2\left(\frac{\sqrt{b}\,(ad(c+dx^3)-bc(2c+dx^3))}{d^2(-bc+ad)\sqrt{c+dx^3}} - \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*((Sqrt[b]*(a*d*(c + d*x^3) - b*c*(2*c + d*x^3)))/(d^2*(-(b*c) + a*d)*Sqr
t[c + d*x^3]) - (a^2*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/
(-(b*c) + a*d)^(3/2)))/(3*b^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 527, normalized size = 4.93 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2/3/d^2*c/((x^3+c/d)*d)^(1/2)+2/3*(d*x^3+c)^(1/2)/d^2)+2/3*a/b^2/d/(d*
x^3+c)^(1/2)+a^2/b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2
)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/
3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^
2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.
time = 3.74, size = 440, normalized size = 4.11

$$\left[-\frac{(a^2d^3x^3+a^2cd^2)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(2\,b^3c^3-3\,ab^2c^2d+a^2bcd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^3)\sqrt{dx^3+c}}{3\,(b^4c^3d^2-2\,ab^3c^2d^3+a^2b^2cd^4+(b^4c^2d^3-2\,ab^3cd^4+a^2b^2d^5)x^3)},\ \frac{2\left((a^2d^3x^3+a^2cd^2)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+(2\,b^3c^3-3\,ab^2c^2d+a^2bcd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^3)\sqrt{dx^3+c}\right)}{3\,(b^4c^3d^2-2\,ab^3c^2d^3+a^2b^2cd^4+(b^4c^2d^3-2\,ab^3cd^4+a^2b^2d^5)x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((a^2*d^3*x^3 + a^2*c*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c -
 a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^3 -
 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^3)
*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b^2*c*d^4 + (b^4*c^2
*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3), 2/3*((a^2*d^3*x^3 + a^2*c*d^2)*sq
rt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b
```

```
*c)) + (2*b^3*c^3 - 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^
2 + a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b
^2*c*d^4 + (b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 1.50, size = 103, normalized size = 0.96

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,(b^2c-abd)\sqrt{-b^2c+abd}}+\frac{2\,c^2}{3\,(bcd^2-ad^3)\sqrt{dx^3+c}}+\frac{2\,\sqrt{dx^3+c}}{3\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqr
t(-b^2*c + a*b*d)) + 2/3*c^2/((b*c*d^2 - a*d^3)*sqrt(d*x^3 + c)) + 2/3*sqrt
(d*x^3 + c)/(b*d^2)
```

Mupad [B]
time = 6.46, size = 115, normalized size = 1.07

$$\frac{2\,\sqrt{d\,x^3+c}}{3\,b\,d^2}-\frac{2\,c^2}{3\,d^2\,\sqrt{d\,x^3+c}\,\,(a\,d-b\,c)}+\frac{a^2\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,b^{3/2}\,{(a\,d-b\,c)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b*d^2) - (2*c^2)/(3*d^2*(c + d*x^3)^(1/2)*(a*d - b
*c)) + (a^2*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*
2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b*c)^(3/2))
```

3.389 $\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=82

$$-\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}}+\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}$$

```
[Out] 2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-2/3*c/d/(-a*d+b*c)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 79, 65, 214}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}-\frac{2c}{3d\sqrt{c+dx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*c)/(3*d*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\ &= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3(bc-ad)}\\ &= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d(bc-ad)}\\ &= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} + \frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 80, normalized size = 0.98

$$\frac{2}{3}\left(\frac{c}{d(-bc+ad)\sqrt{c+dx^3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
[Out] (2*(c/(d*(-(b*c) + a*d)*Sqrt[c + d*x^3]) + (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))))/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 487, normalized size = 5.94

method	result
elliptic	$\frac{2c}{3d(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\ldots}}{\ldots}\right.}{\ldots}$

default	$-\frac{2}{3bd\sqrt{d\,x^3+c}}-\frac{a\left(-\frac{2}{3(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d\ldots\right.}{\ldots}\right)}{\left(-c\,d^2\right)\ldots}}}{\ldots}\right)}{\ldots}\right)}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/3/b/d/(d*x^3+c)^(1/2)-a/b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^

```
2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(66) = 132$.
time = 2.57, size = 326, normalized size = 3.98

$$\left[-\frac{(ad^2x^3 + acd)\sqrt{b^2c - abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right) + 2\,(b^2c^2 - abcd)\sqrt{dx^3 + c}}{3\,(b^3c^3d - 2\,ab^2c^2d^2 + a^2bcd^3 + (b^3c^2d^2 - 2\,ab^2cd^3 + a^2bd^4)x^3)}, -\frac{2\left((ad^2x^3 + acd)\sqrt{-b^2c + abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right) + (b^2c^2 - abcd)\sqrt{dx^3 + c}\right)}{3\,(b^3c^3d - 2\,ab^2c^2d^2 + a^2bcd^3 + (b^3c^2d^2 - 2\,ab^2cd^3 + a^2bd^4)x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((a*d^2*x^3 + a*c*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d -
 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(b^2*c^2 - a*b*c*d
)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3 + (b^3*c^2*d^
2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3), -2/3*((a*d^2*x^3 + a*c*d)*sqrt(-b^2*c
+ a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (b^
2*c^2 - a*b*c*d)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^
3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + bx^3)\,(c + dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 1.78, size = 78, normalized size = 0.95

$$-\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{c}{\sqrt{dx^3+c}\,(bc-ad)}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*(b*c - a*d)) + c/(sqrt(d*x^3 + c)*(b*c - a*d)))/d
```

Mupad [B]
time = 5.99, size = 94, normalized size = 1.15

$$\frac{2\,c}{3\,d\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)}+\frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (2*c)/(3*d*(c + d*x^3)^(1/2)*(a*d - b*c)) + (a*log((2*b*c - a*d + b^(1/2)*(
c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(1/2
)*(a*d - b*c)^(3/2))
```

3.390 $\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2}{3(bc-ad)\sqrt{c+dx^3}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

```
[Out] -2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(
3/2)+2/3/(-a*d+b*c)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {455, 53, 65, 214}

$$\frac{2}{3\sqrt{c+dx^3}\,(bc-ad)}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(3*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*(b*c - a*d)^(3/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right) \\
&= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} + \frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3(bc-ad)} \\
&= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d(bc-ad)} \\
&= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} - \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 76, normalized size = 0.99

$$
\frac{2}{(3bc-3ad)\sqrt{c+dx^3}} - \frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3(-bc+ad)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/((3*b*c - 3*a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[-(b*c) + a*d]])/(3*(-(b*c) + a*d)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.31, size = 463, normalized size = 6.01

method	result
default	$-\frac{2}{3(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{}}{}\right.}{}$
elliptic	$-\frac{2}{3(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*
d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
```

```
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*b+a))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**A**]
time = 4.39, size = 236, normalized size = 3.06

$$\left[-\frac{(dx^3+c)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}}{3\,((bcd-ad^2)x^3+bc^2-acd)},-\frac{2\left((dx^3+c)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-\sqrt{dx^3+c}\right)}{3\,((bcd-ad^2)x^3+bc^2-acd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((d*x^3 + c)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c
))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d), -2/3*((d*x^3 + c)*sqrt(-b/(b*c -
a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b
*c)) - sqrt(d*x^3 + c))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)]
```

Sympy [**A**]
time = 8.88, size = 66, normalized size = 0.86

$$-\frac{2}{3\sqrt{c+dx^3}\,(ad-bc)}-\frac{2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] -2/(3*sqrt(c + d*x**3)*(a*d - b*c)) - 2*atan(sqrt(c + d*x**3)/sqrt((a*d - b
*c)/b))/(3*sqrt((a*d - b*c)/b)*(a*d - b*c))
```

Giac [A]
time = 0.97, size = 73, normalized size = 0.95

$$\frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{2}{3\,\sqrt{dx^3+c}\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
(b*c - a*d)) + 2/3/(sqrt(d*x^3 + c)*(b*c - a*d))
```

Mupad [B]
time = 5.85, size = 89, normalized size = 1.16

$$-\frac{2}{3\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i
- b*d*x^3)/(a + b*x^3))*1i)/(3*(a*d - b*c)^(3/2)) - 2/(3*(c + d*x^3)^(1/2)*
(a*d - b*c))
```

3.391 $\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c^(3/2)+2/3*b^(3/2)*arctanh(b^(1/2)
*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c)^(3/2)-2/3*d/c/(-a*d+b*c)/(d
*x^3+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 87, 162, 65, 214}

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}-\frac{2d}{3c\sqrt{c+dx^3}\,(bc-ad)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*d)/(3*c*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a*c^(3/2)) + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*a*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{bc-ad-bdx}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3c(bc-ad)}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{3ac} - \frac{b^2\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3acd} - \frac{(2b^2)\,\text{Su}\cdots}{\cdots}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\cdots}}{\sqrt{bc\cdots}}\right)}{3a(bc-ad)^{3\cdots}}$$

Mathematica [A]
time = 0.39, size = 110, normalized size = 0.96

$$\frac{2}{3}\left(\frac{d}{c(-bc+ad)\sqrt{c+dx^3}} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{a(-bc+ad)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{ac^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*(d/(c*(-(b*c) + a*d)*Sqrt[c + d*x^3]) + (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^3])/Sqrt[-(b*c) + a*d]])/(a*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]]/(a*c^(3/2))))/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 512, normalized size = 4.49

method	result
default	$-\frac{b\left(-\frac{2}{3(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}\right)}{\cdots}\right)}{\cdots}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*
c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/
d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha
=RootOf(_Z^3*b+a))+1/a*(2/3/c/((x^3+c/d)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1
/2)/c^(1/2))/c^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(90) = 180$.
time = 4.85, size = 790, normalized size = 6.93

$$\left[-\frac{2\sqrt{dx^3+c}\,acd + (bc^2dx^3+bc^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right) - ((bcd-ad^2)x^3+bc^2-acd)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{3(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^3)}, -\frac{2\sqrt{dx^3+c}\,acd - 2(bc^2dx^3+bc^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right) - ((bcd-ad^2)x^3+bc^2-acd)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{3(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^3)}, -\frac{2\sqrt{dx^3+c}\,acd - 2((bcd-ad^2)x^3+bc^2-acd)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + (bc^2dx^3+bc^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)}{3(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^3)}, -\frac{2\left(\sqrt{dx^3+c}\,acd - (bc^2dx^3+bc^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right) - ((bcd-ad^2)x^3+bc^2-acd)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)\right)}{3(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*sqrt(d*x^3 + c)*a*c*d + (b*c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))*
log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*
d)))/(b*x^3 + a)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log((d*x^
3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*
d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*(b*c^2*d*x^3 + b*c
^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c -
 a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log
((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*
b*c^3*d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*((b*c*d - a*
d^2)*x^3 + b*c^2 - a*c*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (b*
c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(a*b*c^4 - a^2*c^
```

```
3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3), -2/3*(sqrt(d*x^3 + c)*a*c*d - (b*c^2*
d*x^3 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqr
t(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*
sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/(a*b*c^4 - a^2*c^3*d + (a*b*c^
3*d - a^2*c^2*d^2)*x^3)]
```

Sympy [A]
time = 7.31, size = 104, normalized size = 0.91

$$\frac{2d}{3c\sqrt{c+dx^3}\,(ad-bc)}+\frac{2b\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] 2*d/(3*c*sqrt(c + d*x**3)*(a*d - b*c)) + 2*b*atan(sqrt(c + d*x**3)/sqrt((a*
d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + 2*atan(sqrt(c + d*x**3
)/sqrt(-c))/(3*a*c*sqrt(-c))
```

Giac [A]
time = 1.04, size = 111, normalized size = 0.97

$$-\frac{2\,b^2\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,(abc-a^2d)\sqrt{-b^2c+abd}}-\frac{2\,d}{3\,\sqrt{dx^3+c}\,(bc^2-acd)}+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c - a^2*d)*sq
rt(-b^2*c + a*b*d)) - 2/3*d/(sqrt(d*x^3 + c)*(b*c^2 - a*c*d)) + 2/3*arctan(
sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

Mupad [B]
time = 8.44, size = 139, normalized size = 1.22

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a\,c^{3/2}}+\frac{2\,d}{3\,c\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)}+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,a\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a*c^(3/2)) + (2*d)/(3*c*(c + d*x^3)^(1/2)*(a*d - b*c)) + (b^(3/2)*log((2*b
*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b
*x^3))*1i)/(3*a*(a*d - b*c)^(3/2))
```

3.392 $\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=158

$$-\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}}-\frac{1}{3acx^3\sqrt{c+dx^3}}+\frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

```
[Out] 1/3*(3*a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(5/2)-2/3*b^(5/2)*
arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(3/2)-1/3*
d*(-3*a*d+b*c)/a/c^2/(-a*d+b*c)/(d*x^3+c)^(1/2)-1/3/a/c/x^3/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 105, 157, 162, 65, 214}

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}}-\frac{d(bc-3ad)}{3ac^2\sqrt{c+dx^3}(bc-ad)}-\frac{1}{3acx^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/3*(d*(b*c - 3*a*d))/(a*c^2*(b*c - a*d)*Sqrt[c + d*x^3]) - 1/(3*a*c*x^3*S
qrt[c + d*x^3]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2
*c^(5/2)) - (2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/
(3*a^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}} dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^{3/2}} dx, x, x^3\right)$$
$$= -\frac{1}{3acx^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+3ad)+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}} dx, x, x^3\right)}{3ac}$$
$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{4}(bc-ad)(2bc+3ad}{x(a+bx)\sqrt{c}}\right.}{3ac^2(bc}$$
$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{b^3\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{3a^2(bc-ad)}$$
$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{(2b^3)\text{ Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}} dx\right.}{3a^2d(bc-a}$$
$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{}}\right.}{3a^2c^{5/2}}$$

Mathematica [A]
time = 0.43, size = 142, normalized size = 0.90

$$\frac{\frac{a(-bc(c+dx^3)+ad(c+3dx^3))}{c^2(bc-ad)x^3\sqrt{c+dx^3}} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{5/2}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((a*(-(b*c*(c + d*x^3)) + a*d*(c + 3*d*x^3)))/(c^2*(b*c - a*d)*x^3*Sqrt[c +
 d*x^3]) - (2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])
/(-(b*c) + a*d)^(3/2) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/
c^(5/2))/(3*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.45, size = 575, normalized size = 3.64

method	result

risch	$-\frac{\sqrt{d\,x^3+c}}{3c^2a\,x^3}-\frac{-\frac{2(3ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}-\frac{2ib^3c^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{}}{}\right.}{}}}{}\right.}{}}{}$

default	$b^{2}\left(-\frac{2}{3(ad-bc)\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d
+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d
^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*
d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2
)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2
*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^
```

```
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_al
pha=RootOf(_Z^3*b+a)))+1/a*(-2/3*d/c^2/((x^3+c/d)*d)^(1/2)-1/3*(d*x^3+c)^(1
/2)/c^2/x^3+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-b/a^2*(2/3/c/((x^3+
c/d)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(130) = 260.
time = 3.88, size = 1120, normalized size = 7.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2
*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)
) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3
*a^2*c*d^2)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3)
 + 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))
/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/6*(4*(
b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(
b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((2*b^2*c^2*d + a*b*c*d^
2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(c)*log
((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a*b*c^3 - a^2*c^2*d +
(a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2
)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/3*(((2*b^2*c^2*d + a*b*c*d^2 - 3*a
^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(-c)*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) + (b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d
))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c -
 a*d)))/(b*x^3 + a)) + (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3
)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*
d)*x^3), -1/3*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(
-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + ((2*b^
2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2
)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a*b*c^3 - a^2*c^2*d +
```

```
(a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^
2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\,(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 1.35, size = 173, normalized size = 1.09

$$\frac{2\,b^3\arctan\left(\frac{\sqrt{dx^3+c}\;b}{\sqrt{-b^2c+abd}}\right)}{3\,(a^2bc-a^3d)\sqrt{-b^2c+abd}}-\frac{(dx^3+c)bcd-3\,(dx^3+c)ad^2+2\,acd^2}{3\,(abc^3-a^2c^2d)\left((dx^3+c)^{\frac{3}{2}}-\sqrt{dx^3+c}\;c\right)}-\frac{(2\,bc+3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}\;c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b^3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b*c - a^3*d)*s
qrt(-b^2*c + a*b*d)) - 1/3*((d*x^3 + c)*b*c*d - 3*(d*x^3 + c)*a*d^2 + 2*a*c
*d^2)/((a*b*c^3 - a^2*c^2*d)*((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)) - 1/3
*(2*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^2)
```

Mupad [B]
time = 10.47, size = 597, normalized size = 3.78

$$\frac{\ln\left(\frac{\left(\sqrt{dx^3+c}-\sqrt{c}\right)\left(\sqrt{dx^3+c}+\sqrt{c}\right)^3}{x^6}\right)(3\,a\,d+2\,b\,c)}{6\,a^2\,c^{5/2}}-\frac{\sqrt{dx^3+c}}{3\,a\,c^2\,x^3}-\frac{\frac{c\left(\frac{c\left(\frac{c\left(\frac{3\,a^2d^4+24\,a\,b\,c\,d^3+15\,b^2c^2d^2}{8\,a^3c^5}+\frac{c\left(\frac{c\left(\frac{3\,b^2d^4}{8\,a^3c^5}+\frac{b^2d^4(5\,a\,d-3\,b\,c)}{8\,a^3c^4(b\,c^2-a\,c\,d)}-\frac{b\,d^4(a\,d+2\,b\,c)(5\,a\,d-3\,b\,c)}{4\,a^3c^5(b\,c^2-a\,c\,d)}\right)}{d}-\frac{3\,b\,d^3(a\,d+2\,b\,c)}{4\,a^3c^5}+\frac{d(5\,a\,d-3\,b\,c)(3\,a^2d^4+24\,a\,b\,c\,d^3+15\,b^2c^2d^2)}{24\,a^3c^5(b\,c^2-a\,c\,d)}\right)}{d}-\frac{d^2(5\,a\,d-3\,b\,c)(6\,a^2d^2+14\,a\,b\,c\,d+3\,b^2c^2)}{12\,a^3c^4(b\,c^2-a\,c\,d)}\right)}{d}-\frac{d(6\,a^2d^2+14\,a\,b\,c\,d+3\,b^2c^2)}{4\,a^3c^4}+\frac{d^2(5\,a\,d-3\,b\,c)(13\,a\,d+18\,b\,c)}{24\,a^2c^3(b\,c^2-a\,c\,d)}\right)}{d}+\frac{d(13\,a\,d+18\,b\,c)}{8\,a^2c^3}-\frac{d(3\,a\,d+2\,b\,c)(5\,a\,d-3\,b\,c)}{6\,a^2c^2(b\,c^2-a\,c\,d)}\right)}{d}}{\sqrt{dx^3+c}}-\frac{3\,a\,d+2\,b\,c}{3\,a^2c^2}+\frac{b^{3/2}\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{dx^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)1{}\mathrm{i}}{3\,a^2\,(a\,d-b\,c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(
3*a*d + 2*b*c))/(6*a^2*c^(5/2)) - (c + d*x^3)^(1/2)/(3*a*c^2*x^3) - ((c*((c
*((c*((3*a^2*d^4 + 15*b^2*c^2*d^2 + 24*a*b*c*d^3)/(8*a^3*c^5) + (c*((c*((3*
b^2*d^4)/(8*a^3*c^5) + (b^2*d^4*(5*a*d - 3*b*c))/(8*a^3*c^4*(b*c^2 - a*c*d)
```

```
) - (b*d^4*(a*d + 2*b*c)*(5*a*d - 3*b*c))/(4*a^3*c^5*(b*c^2 - a*c*d))))/d -
 (3*b*d^3*(a*d + 2*b*c))/(4*a^3*c^5) + (d*(5*a*d - 3*b*c)*(3*a^2*d^4 + 15*b
^2*c^2*d^2 + 24*a*b*c*d^3))/(24*a^3*c^5*(b*c^2 - a*c*d))))/d - (d^2*(5*a*d
- 3*b*c)*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(12*a^3*c^4*(b*c^2 - a*c*d))
))/d - (d*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(4*a^3*c^4) + (d^2*(5*a*d -
 3*b*c)*(13*a*d + 18*b*c))/(24*a^2*c^3*(b*c^2 - a*c*d))))/d + (d*(13*a*d +
18*b*c))/(8*a^2*c^3) - (d*(3*a*d + 2*b*c)*(5*a*d - 3*b*c))/(6*a^2*c^2*(b*c^
2 - a*c*d))))/d - (3*a*d + 2*b*c)/(2*a^2*c^2))/(c + d*x^3)^(1/2) + (b^(5/2)
*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^
3)/(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(3/2))
```

3.393 $\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1,3/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d*
x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*c*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 231 vs. $2(67) = 134$.
time = 7.74, size = 231, normalized size = 3.45

$$\frac{x\left(-8-\frac{bx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}-\frac{64a^2cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}\right)}{12(-bc+ad)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-8 - (b*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c)
, -((b*x^3)/a)])/a - (64*a^2*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((
b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x
^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(
12*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.35, size = 1069, normalized size = 15.96

method	result

elliptic	$-\frac{2x}{3(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-c\,d^2)^{\frac{1}{3}}}{d}}{-\frac{3(-c\,d^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,(-c\,d^2}{2d}}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2/3*x/c/((x^3+c/d)*d)^(1/2)-2/9*I/c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)))-a/b*(2/3*d*x/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)
-2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^
(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x
+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(
1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
```

```
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha
-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(b\,x^3+a\right)\,{\left(d\,x^3+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
[Out] int(x^3/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.394 $\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};1,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1,3/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d*
x^3+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};1,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*c*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};1,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 142 vs. 2(67) = 134.
time = 10.08, size = 142, normalized size = 2.12

$$\frac{x^2\left(-20ad+5(3bc+ad)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{30ac(bc-ad)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(-20*a*d + 5*(3*b*c + a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a*c*(b*c - a*d)*Sqrt[c +
d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.33, size = 907, normalized size = 13.54

method	result	size
default	Expression too large to display	907
elliptic	Expression too large to display	907

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*d*x^2/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)+2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^2
)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
```

```
1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha*(-c
*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)
*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-
c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a
))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(b\,x^3+a)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
[Out] int(x/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.395 $\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}} dx$

Optimal. Leaf size=62

$$\frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,1,3/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d*x^3+c)
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*c*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(62) = 124.
time = 10.21, size = 338, normalized size = 5.45

$$\frac{x\left(\frac{bdx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a(-bc+ad)}+\frac{32ac(-3bc+3ad+2bdx^3)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24dx^3(a+bx^3)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{(bc-ad)(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}\right)}{12c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*((b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
((b*x^3)/a)])/(a*(-(b*c) + a*d)) + (32*a*c*(-3*b*c + 3*a*d + 2*b*d*x^3)*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*d*x^3*(a + b*x^3)*
(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF
1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/((b*c - a*d)*(a + b*x^3)*
(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b
*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/
3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(12*c*Sqrt[c + d*x^3])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.31, size = 753, normalized size = 12.15 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*d*x/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d
```

```
-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c
*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^
2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/
2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**(3/2),x)
[Out] Integral(1/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\,x^3+a)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.396 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};1,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,1,3/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/x/(d*x^3+
c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};1,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*c*x*Sqrt[c + d*x^3]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};1,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(65) = 130.
time = 10.13, size = 193, normalized size = 2.97

$$\frac{20a(-3bc(c+dx^3)+ad(3c+5dx^3))-5(6b^2c^2-3abcd+5a^2d^2)x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bd(3bc-5ad)x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{60a^2c^2(bc-ad)x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (20*a*(-3*b*c*(c + d*x^3) + a*d*(3*c + 5*d*x^3)) - 5*(6*b^2*c^2 - 3*a*b*c*d
 + 5*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), -((b*x^3)/a)] + 2*b*d*(3*b*c - 5*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^2*c^2*(b*c - a*d)*x*Sq
rt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.42, size = 1392, normalized size = 21.42

method	result	size
elliptic	Expression too large to display	952
risch	Expression too large to display	1382
default	Expression too large to display	1392

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a*(2/3*d*x^2/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)+2/9*I/c/(a*d-b*c)*3^(1/2)*(
-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
```

```
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_alp
ha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3
^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_a
lpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alph
a*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*
d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*b+a)))+1/a*(-2/3*d*x^2/c^2/((x^3+c/d)*d)^(1/2)-(d*x^3+c)^(1/2)/c^2/x-5/9
*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + bx^3) (c + dx^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 \left(b x^3 + a\right) \left(d x^3 + c\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.397 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,1,3/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/x^2/(
d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a*c*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 425 vs. 2(67) = 134.
time = 10.36, size = 425, normalized size = 6.34

$$\frac{bd(3bc-7ad)x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a\left(-4ac(-6b^2cx^3(3c+dx^3)+3a^2d(2c+7dx^3)+ab(-6c^2-3cdx^3+14d^2x^6))F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3(-3b^2cx^3(c+dx^3)+a^2d(3c+7dx^3)+ab(-3c^2+7d^2x^6))\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}}{48a^2c^2(-bc+ad)x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b*d*(3*b*c - 7*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)] + (8*a*(-4*a*c*(-6*b^2*c*x^3*(3*c + d*x^3) + 3*a^2
*d*(2*c + 7*d*x^3) + a*b*(-6*c^2 - 3*c*d*x^3 + 14*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(-3*b^2*c*x^3*(c + d*x^3) +
 a^2*d*(3*c + 7*d*x^3) + a*b*(-3*c^2 + 7*d^2*x^6))*(2*b*c*AppellF1[4/3, 1/2
, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d
*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x
^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x
^3)/a)]))))/(48*a^2*c^2*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.45, size = 1084, normalized size = 16.18

method	result

elliptic	$-\frac{2d^2x}{3c^2(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{\sqrt{d\,x^3+c}}{2c^2a\,x^2}-\frac{2i\left(-\frac{d^2}{3(ad-bc)c^2}-\frac{d}{4a\,c^2}\right)\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a*(2/3*d*x/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-c
*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I*b/d^2*2^(1/2)*sum(
1/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-
(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)),_alpha=RootOf(_Z^3*b+a)))+1/a*(-1/2/c^2*(d*x^3+c)^(1/2)/x^2-2/3*d*x/c^2
/((x^3+c/d)*d)^(1/2)+7/18*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-
```

```
1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + bx^3) (c + dx^3)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\left(b\,x^3+a\right)\,{\left(d\,x^3+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
[Out] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.398 $\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=117

$$\frac{7x^6\sqrt{c+dx^3}}{15d^2}+\frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)}+\frac{2c\sqrt{c+dx^3}\,(1146c+47dx^3)}{15d^4}-\frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

```
[Out] -3968/9*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4+7/15*x^6*(d*x^3+c)
^(1/2)/d^2+1/3*x^9*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+2/15*c*(47*d*x^3+1146*c)*
(d*x^3+c)^(1/2)/d^4
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {457, 99, 158, 152, 65, 212}

$$-\frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}+\frac{2c\sqrt{c+dx^3}\,(1146c+47dx^3)}{15d^4}+\frac{7x^6\sqrt{c+dx^3}}{15d^2}+\frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (7*x^6*Sqrt[c + d*x^3])/(15*d^2) + (x^9*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)
) + (2*c*Sqrt[c + d*x^3]*(1146*c + 47*d*x^3))/(15*d^4) - (3968*c^(5/2)*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3c+\frac{7dx}{2}\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)} + \frac{2\text{Subst}\left(\int \frac{x\left(-56c^2d-\frac{141}{2}cd^2x\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{15d^3}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} - \frac{(1984c^3)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} - \frac{(3968c^3)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d(8c-dx^3)} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} - \frac{3968c^{5/2}\tanh^{-1}\left(\cdots\right.}{9d^4}$$

Mathematica [A]
time = 0.16, size = 91, normalized size = 0.78

$$\frac{2\left(\frac{3\sqrt{c+dx^3}\,(-9168c^3+770c^2dx^3+19cd^2x^6+d^3x^9)}{-8c+dx^3} - 9920c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{45d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (2*((3*Sqrt[c + d*x^3]*(-9168*c^3 + 770*c^2*d*x^3 + 19*c*d^2*x^6 + d^3*x^9)
)/(-8*c + d*x^3) - 9920*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(45*
d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.42, size = 953, normalized size = 8.15

method	result

elliptic	$\frac{512c^3\sqrt{d\,x^3+c}}{3d^4(-d\,x^3+8c)}+\frac{2x^6\sqrt{d\,x^3+c}}{15d^2}+\frac{18c\,x^3\sqrt{d\,x^3+c}}{5d^3}+\frac{1972c^2\sqrt{d\,x^3+c}}{15d^4}+\frac{1984ic^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_\right.}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d*(2/15*x^6*(d*x^3+c)^(1/2)+2/45*c/d*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(d
*x^3+c)^(1/2)/d^2)+32/9*c*(d*x^3+c)^(3/2)/d)+512*c^3/d^3*(1/3*(d*x^3+c)^(1/
2)/d/(-d*x^3+8*c)+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/
d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2
*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+192/d^3*c^2*(2/3*(d*x^3+c)^(1/2
)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d
^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*
```

```
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [A]
time = 0.48, size = 107, normalized size = 0.91

$$\frac{2\left(4960\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+75\left(dx^3+c\right)^{\frac{3}{2}}c+2880\sqrt{dx^3+c}\,c^2-\frac{3840\sqrt{dx^3+c}\,c^3}{dx^3-8\,c}\right)}{45\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/45*(4960*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 3*(d*x^3 + c)^(5/2) + 75*(d*x^3 + c)^(3/2)*c + 2880*sqrt(d*x^3 +
 c)*c^2 - 3840*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^4
```

Fricas [A]
time = 2.11, size = 219, normalized size = 1.87

$$\left[\frac{2\left(4960\left(c^2dx^3-8\,c^3\right)\sqrt{c}\,\log\left(\frac{dx^3-6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\left(d^3x^9+19\,cd^2x^6+770\,c^2dx^3-9168\,c^3\right)\sqrt{dx^3+c}\right)}{45\left(d^5x^3-8\,cd^4\right)},\ \frac{2\left(9920\left(c^2dx^3-8\,c^3\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\left(d^3x^9+19\,cd^2x^6+770\,c^2dx^3-9168\,c^3\right)\sqrt{dx^3+c}\right)}{45\left(d^5x^3-8\,cd^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/45*(4960*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 3*(d^3*x^9 + 19*c*d^2*x^6 + 770*c^2*d*x^3 - 91
68*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/45*(9920*(c^2*d*x^3 - 8*c^3
)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^3*x^9 + 19*c*d^2*x
^6 + 770*c^2*d*x^3 - 9168*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**11*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [A]
time = 1.37, size = 110, normalized size = 0.94

$$\frac{3968\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d^4}-\frac{512\sqrt{dx^3+c}\,c^3}{3\left(dx^3-8\,c\right)d^4}+\frac{2\left(\left(dx^3+c\right)^{\frac{5}{2}}d^{16}+25\left(dx^3+c\right)^{\frac{3}{2}}cd^{16}+960\sqrt{dx^3+c}\,c^2d^{16}\right)}{15\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 3968/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/3*sqrt
(d*x^3 + c)*c^3/((d*x^3 - 8*c)*d^4) + 2/15*((d*x^3 + c)^(5/2)*d^16 + 25*(d*
x^3 + c)^(3/2)*c*d^16 + 960*sqrt(d*x^3 + c)*c^2*d^16)/d^20
```

Mupad [B]
time = 4.09, size = 127, normalized size = 1.09

$$\frac{1984\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^4}+\frac{1972\,c^2\,\sqrt{d\,x^3+c}}{15\,d^4}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d^2}+\frac{18\,c\,x^3\,\sqrt{d\,x^3+c}}{5\,d^3}+\frac{512\,c^3\,\sqrt{d\,x^3+c}}{3\,d^4\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (1984*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/(9*d^4) + (1972*c^2*(c + d*x^3)^(1/2))/(15*d^4) + (2*x^6*(c + d*x^3)^(1
/2))/(15*d^2) + (18*c*x^3*(c + d*x^3)^(1/2))/(5*d^3) + (512*c^3*(c + d*x^3)
^(1/2))/(3*d^4*(8*c - d*x^3))
```

3.399 $\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=102

$$\frac{352c\sqrt{c+dx^3}}{27d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{64c(c+dx^3)^{3/2}}{27d^3\,(8c-dx^3)}-\frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^3+64/27*c*(d*x^3+c)^(3/2)/d^3/(-d*x^3+8*c)-352/9*c^(3
/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3+352/27*c*(d*x^3+c)^(1/2)/d^3
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 91, 81, 52, 65, 212}

$$-\frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}+\frac{64c(c+dx^3)^{3/2}}{27d^3\,(8c-dx^3)}+\frac{352c\sqrt{c+dx^3}}{27d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (352*c*Sqrt[c + d*x^3])/(27*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (64*c*(c
 + d*x^3)^(3/2))/(27*d^3*(8*c - d*x^3)) - (352*c^(3/2)*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])])/(9*d^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)$$
$$= \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\,(104c^2d+9cd^2x)}{8c-dx}\,dx, x, x^3\right)}{27cd^3}$$
$$= \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} - \frac{(176c)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{27d^2}$$
$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} - \frac{(176c^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\right.}{3d^2}$$
$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} - \frac{(352c^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c\cdot}\right.}{3d^3}$$
$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} - \frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

Mathematica [A]
time = 0.11, size = 79, normalized size = 0.77

$$\frac{2\left(\frac{\sqrt{c+dx^3}\,(-488c^2+41cdx^3+d^2x^6)}{-8c+dx^3} - 176c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (2*((Sqrt[c + d*x^3]*(-488*c^2 + 41*c*d*x^3 + d^2*x^6))/(-8*c + d*x^3) - 17
6*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(9*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 893, normalized size = 8.75

method	result

elliptic	$\frac{64c^2\sqrt{d\,x^3+c}}{3d^3(-d\,x^3+8c)}+\frac{2x^3\sqrt{d\,x^3+c}}{9d^2}+\frac{98c\sqrt{d\,x^3+c}}{9d^3}+\frac{176ic\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x\cdots\right.}{\cdots}}\cdots}{\cdots}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^3+64*c^2/d^2*(1/3*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+1/54
*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_
alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha
=RootOf(_Z^3*d-8*c)))+16/d^2*c*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum
((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*
d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

)

Maxima [A]
time = 0.49, size = 91, normalized size = 0.89

$$\frac{2 \left(88 c^{\frac{3}{2}} \log \left(\frac{\sqrt{d x^3+c}-3 \sqrt{c}}{\sqrt{d x^3+c}+3 \sqrt{c}}\right)+\left(d x^3+c\right)^{\frac{3}{2}}+48 \sqrt{d x^3+c}\, c-\frac{96 \sqrt{d x^3+c}\, c^2}{d x^3-8 c}\right)}{9 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/9*(88*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + (d*x^3 + c)^(3/2) + 48*sqrt(d*x^3 + c)*c - 96*sqrt(d*x^3 + c)*c^2/(
d*x^3 - 8*c))/d^3
```

Fricas [A]
time = 2.62, size = 191, normalized size = 1.87

$$\left[\frac{2 \left(88 \left(c d x^3-8 c^2\right) \sqrt{c} \log \left(\frac{d x^3-6 \sqrt{d x^3+c} \sqrt{c}+10 c}{d x^3-8 c}\right)+\left(d^2 x^6+41 c d x^3-488 c^2\right) \sqrt{d x^3+c}\right)}{9 \left(d^4 x^3-8 c d^3\right)}, \frac{2 \left(176 \left(c d x^3-8 c^2\right) \sqrt{-c} \arctan \left(\frac{\sqrt{d x^3+c} \sqrt{-c}}{3 c}\right)+\left(d^2 x^6+41 c d x^3-488 c^2\right) \sqrt{d x^3+c}\right)}{9 \left(d^4 x^3-8 c d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/9*(88*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(
d^4*x^3 - 8*c*d^3), 2/9*(176*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x
^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(d^
4*x^3 - 8*c*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 \sqrt{c+d x^3}}{\left(-8 c+d x^3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**8*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [A]
time = 1.15, size = 93, normalized size = 0.91

$$\frac{352 c^2 \arctan \left(\frac{\sqrt{d x^3+c}}{3 \sqrt{-c}}\right)}{9 \sqrt{-c}\, d^3}-\frac{64 \sqrt{d x^3+c}\, c^2}{3 \left(d x^3-8 c\right) d^3}+\frac{2 \left(\left(d x^3+c\right)^{\frac{3}{2}} d^6+48 \sqrt{d x^3+c}\, c d^6\right)}{9 d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 352/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 64/3*sqrt(d
*x^3 + c)*c^2/((d*x^3 - 8*c)*d^3) + 2/9*((d*x^3 + c)^(3/2)*d^6 + 48*sqrt(d*
x^3 + c)*c*d^6)/d^9
```

Mupad [B]
time = 4.01, size = 107, normalized size = 1.05

$$\frac{98\,c\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{176\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3}+\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}+\frac{64\,c^2\,\sqrt{d\,x^3+c}}{3\,d^3\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (98*c*(c + d*x^3)^(1/2))/(9*d^3) + (176*c^(3/2)*log((10*c + d*x^3 - 6*c^(1/
2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^3) + (2*x^3*(c + d*x^3)^(1/2))/(
9*d^2) + (64*c^2*(c + d*x^3)^(1/2))/(3*d^3*(8*c - d*x^3))
```

3.400 $\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=82

$$\frac{26\sqrt{c+dx^3}}{27d^2}+\frac{8(c+dx^3)^{3/2}}{27d^2\,(8c-dx^3)}-\frac{26\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

```
[Out] 8/27*(d*x^3+c)^(3/2)/d^2/(-d*x^3+8*c)-26/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1
/2))*c^(1/2)/d^2+26/27*(d*x^3+c)^(1/2)/d^2
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 79, 52, 65, 212}

$$\frac{8(c+dx^3)^{3/2}}{27d^2\,(8c-dx^3)}+\frac{26\sqrt{c+dx^3}}{27d^2}-\frac{26\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (26*Sqrt[c + d*x^3])/(27*d^2) + (8*(c + d*x^3)^(3/2))/(27*d^2*(8*c - d*x^3)
) - (26*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)\\
&= \frac{8(c+dx^3)^{3/2}}{27d^2(8c-dx^3)} - \frac{13\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{27d}\\
&= \frac{26\sqrt{c+dx^3}}{27d^2} + \frac{8(c+dx^3)^{3/2}}{27d^2(8c-dx^3)} - \frac{(13c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}\\
&= \frac{26\sqrt{c+dx^3}}{27d^2} + \frac{8(c+dx^3)^{3/2}}{27d^2(8c-dx^3)} - \frac{(26c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^2}\\
&= \frac{26\sqrt{c+dx^3}}{27d^2} + \frac{8(c+dx^3)^{3/2}}{27d^2(8c-dx^3)} - \frac{26\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}
\end{aligned}
$$

Mathematica [A]

time = 0.09, size = 69, normalized size = 0.84

$$\frac{2\left(\frac{3(-12c+dx^3)\sqrt{c+dx^3}}{-8c+dx^3}-13\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (2*((3*(-12*c + d*x^3)*Sqrt[c + d*x^3])/(-8*c + d*x^3) - 13*Sqrt[c]*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(9*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 875, normalized size = 10.67

method	result
elliptic	$\frac{8c\sqrt{d\,x^3+c}}{3d^2(-d\,x^3+8c)}+\frac{2\sqrt{d\,x^3+c}}{3d^2}+\frac{13i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2
*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d
*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*
d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*
```

```
d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c
*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+8*c/d*(1/3*(d*x^3+c)^(1/2
)/d/(-d*x^3+8*c)+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d
*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [A]
time = 0.50, size = 79, normalized size = 0.96

$$\frac{13\,\sqrt{c}\,\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+6\,\sqrt{dx^3+c}-\frac{24\,\sqrt{dx^3+c}\,c}{dx^3-8\,c}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] 1/9*(13*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 6*sqrt(d*x^3 + c) - 24*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^2
```

Fricas [A]
time = 3.13, size = 165, normalized size = 2.01

$$\left[\frac{13\,(dx^3-8\,c)\sqrt{c}\,\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+6\,\sqrt{dx^3+c}\,(dx^3-12\,c)}{9\,(d^3x^3-8\,cd^2)},\ \frac{2\left(13\,(dx^3-8\,c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\,\sqrt{dx^3+c}\,(dx^3-12\,c)\right)}{9\,(d^3x^3-8\,cd^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
[Out] [1/9*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) + 6*sqrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2),
2/9*(13*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*s
qrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [A]
time = 0.67, size = 69, normalized size = 0.84

$$\frac{26\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}\,d^2}+\frac{2\,\sqrt{dx^3+c}}{3\,d^2}-\frac{8\,\sqrt{dx^3+c}\,c}{3\,(dx^3-8\,c)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 26/9*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) + 2/3*sqrt(d*x^3
 + c)/d^2 - 8/3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^2)
```

Mupad [B]
time = 3.99, size = 87, normalized size = 1.06

$$\frac{2\,\sqrt{d\,x^3+c}}{3\,d^2}+\frac{13\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2}+\frac{8\,c\,\sqrt{d\,x^3+c}}{3\,d^2\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^2) + (13*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*(
c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^2) + (8*c*(c + d*x^3)^(1/2))/(3*d^2*
(8*c - d*x^3))
```

3.401 $\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{c+dx^3}}{3d\,(8c-dx^3)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

```
[Out] -1/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d/c^(1/2)+1/3*(d*x^3+c)^(1/2)/d/(
-d*x^3+8*c)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {455, 43, 65, 212}

$$\frac{\sqrt{c+dx^3}}{3d\,(8c-dx^3)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/(3*d*(8*c - d*x^3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/
(9*Sqrt[c]*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\
&= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 0.95

$$\frac{\frac{3\sqrt{c+dx^3}}{8c-dx^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] ((3*Sqrt[c + d*x^3])/(8*c - d*x^3) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/S
qrt[c])/(9*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 440, normalized size = 6.88

method	result

default	$\frac{\sqrt{d\,x^3+c}}{3d(-d\,x^3+8c)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{(-\cdots}{\cdots}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\cdots}}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{d\,x^3+c}}{3d(-d\,x^3+8c)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{(-\cdots}{\cdots}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\cdots}}\cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2
)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)
```

```
*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.48, size = 66, normalized size = 1.03

$$\frac{\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{6\sqrt{dx^3+c}}{dx^3-8\,c}}{18\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sqrt
(c) - 6*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d
```

Fricas [A]
time = 3.47, size = 149, normalized size = 2.33

$$\left[\frac{(dx^3-8\,c)\sqrt{c}\,\log\left(\frac{dx^3-6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\sqrt{dx^3+c}\,c}{18\,(cd^2x^3-8\,c^2d)}, \frac{(dx^3-8\,c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-3\sqrt{dx^3+c}\,c}{9\,(cd^2x^3-8\,c^2d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/18*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c*d^2*x^3 - 8*c^2*d), 1/9*((d*x^3 -
 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c)*c
)/(c*d^2*x^3 - 8*c^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [A]
time = 0.58, size = 53, normalized size = 0.83

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d} - \frac{\sqrt{dx^3+c}}{3\,(dx^3-8\,c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 1/3*sqrt(d*x^3 + c)
/((d*x^3 - 8*c)*d)
```

Mupad [B]
time = 3.93, size = 72, normalized size = 1.12

$$\frac{\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{18\,\sqrt{c}\,d}+\frac{\sqrt{d\,x^3+c}}{3\,d\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(18*c^(1/2)
*d) + (c + d*x^3)^(1/2)/(3*d*(8*c - d*x^3))
```

3.402 $\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

```
[Out] 5/288*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/96*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(3/2)+1/24*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 101, 162, 65, 214, 212}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}+\frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(24*c*(8*c - d*x^3)) + (5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/(288*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(8c-dx)^2}\,dx,x,x^3\right)$$

$$= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}$$

$$= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{192c} + \frac{(5d)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx\right.}{192c}$$

$$= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{5\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{96c} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+d\right.}{96cd}$$

$$= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

Mathematica [A]

time = 0.09, size = 83, normalized size = 0.94

$$\frac{\frac{12\sqrt{c}\,\sqrt{c+dx^3}}{8c-dx^3}+5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{288c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 5*ArcTanh[Sqrt[c + d*x^3]/(3*
Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(288*c^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 913, normalized size = 10.38

method	result	size
default	Expression too large to display	913
elliptic	Expression too large to display	1534

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*d/c^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/
2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a
lpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(2/3*(d*
x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))+1/8*d/c*(1/3*(d*
x^3+c)^(1/2)/d/(-d*x^3+8*c)+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(
1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha
-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x), x)
```

Fricas [A]
time = 4.21, size = 226, normalized size = 2.57

$$\left[\frac{5\,(dx^3-8\,c)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(dx^3-8\,c)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-24\,\sqrt{dx^3+c}\,c}{576\,(c^2dx^3-8\,c^3)}, \frac{3\,(dx^3-8\,c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-5\,(dx^3-8\,c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-12\,\sqrt{dx^3+c}\,c}{288\,(c^2dx^3-8\,c^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
[Out] [1/576*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 3*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)
*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^2*d*x^3 - 8*c^3), 1/288*(3*
(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 5*(d*x^3 - 8*c)
*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)*c)/(c
^2*d*x^3 - 8*c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x*(-8*c + d*x**3)**2), x)
```

Giac [A]
time = 0.52, size = 79, normalized size = 0.90

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}\,c}-\frac{5\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{288\,\sqrt{-c}\,c}-\frac{\sqrt{dx^3+c}}{24\,(dx^3-8\,c)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 5/288*arctan(1/3*sqrt(
d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/((d*x^3 - 8*c)*c)
```

Mupad [B]
time = 3.98, size = 76, normalized size = 0.86

$$\frac{5\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{288\,\sqrt{c^3}}-\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\,\sqrt{c^3}}+\frac{\sqrt{d\,x^3+c}}{8\,c\,\left(24\,c-3\,d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)^2),x)
```

```
[Out] (5*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(288*(c^3)^(1/2)) - atanh(
(c*(c + d*x^3)^(1/2))/(c^3)^(1/2))/(96*(c^3)^(1/2)) + (c + d*x^3)^(1/2)/(8*
c*(24*c - 3*d*x^3))
```

3.403 $\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2} \, dx$

Optimal. Leaf size=124

$$\frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}+\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}$$

```
[Out] 7/1152*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/128*d*arctanh((d*x^
3+c)^(1/2)/c^(1/2))/c^(5/2)+1/96*d*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/24*(d
*x^3+c)^(1/2)/c/x^3/(-d*x^3+8*c)
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {457, 101, 156, 162, 65, 214, 212}

$$\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}+\frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] (d*Sqrt[c + d*x^3])/(96*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c*x^3*(8*c
 - d*x^3)) + (7*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1152*c^(5/2)) - (d
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(128*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(8c-dx)^2}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{6cd+\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-54c^2d^2-9cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{1728c^3d}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{256c^2} + \frac{(7d^2)\,\text{Sub}}{}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{128c^2} + \frac{(7d)\text{Su}}{}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c+}}{\sqrt{c}}\right)}{128c^{5/2}}$$

Mathematica [A]
time = 0.17, size = 97, normalized size = 0.78

$$\frac{\frac{12\sqrt{c}\left(4c-dx^3\right)\sqrt{c+dx^3}}{-8cx^3+dx^6} + 7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{1152c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] ((12*Sqrt[c]*(4*c - d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 7*d*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(115
2*c^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 958, normalized size = 7.73

method	result	size
risch	Expression too large to display	901
default	Expression too large to display	958
elliptic	Expression too large to display	1550

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*d^2*(1/3*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+1/54*I/d^3/c*2^(1/2)*sum((
-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha
*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*
(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))-
1/256/c^3*d^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/
3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^
(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*
_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/3*
(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/256/c
^3*d*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^4), x)
```

Fricas [A]
time = 2.08, size = 278, normalized size = 2.24

$$\left[\frac{7\,(d^2x^6 - 8\,cdx^3)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right) + 9\,(d^2x^6 - 8\,cdx^3)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right) - 24\,(cdx^3 - 4\,c^2)\sqrt{dx^3+c}}{2304\,(c^3dx^6 - 8\,c^4x^3)}, \frac{9\,(d^2x^6 - 8\,cdx^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right) - 7\,(d^2x^6 - 8\,cdx^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right) - 12\,(cdx^3 - 4\,c^2)\sqrt{dx^3+c}}{1152\,(c^3dx^6 - 8\,c^4x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/2304*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 -
2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c
))/(c^3*d*x^6 - 8*c^4*x^3), 1/1152*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan
(sqrt(d*x^3 + c)*sqrt(-c)/c) - 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*
sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^3*d*
x^6 - 8*c^4*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^4 \left(-8c+dx^3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**4*(-8*c + d*x**3)**2), x)
```

Giac [A]
time = 0.52, size = 113, normalized size = 0.91

$$\frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{128\sqrt{-c}\,c^2} - \frac{7\,d \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{1152\sqrt{-c}\,c^2} - \frac{(dx^3+c)^{\frac{3}{2}}d - 5\sqrt{dx^3+c}\,cd}{96\left((dx^3+c)^2 - 10\,(dx^3+c)c + 9\,c^2\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/128*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 7/1152*d*arctan(1
/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/96*((d*x^3 + c)^(3/2)*d - 5
*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^2)
```

Mupad [B]
time = 4.21, size = 117, normalized size = 0.94

$$\frac{\frac{5\,d\,\sqrt{d\,x^3+c}}{32\,c} - \frac{d\,{\left(d\,x^3+c\right)}^{3/2}}{32\,c^2}}{3\,{\left(d\,x^3+c\right)}^2 - 30\,c\,\left(d\,x^3+c\right) + 27\,c^2} + \frac{d\,\left(\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)\,1{}\mathrm{i} - \frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)\,7{}\mathrm{i}}{9}\right)\,1{}\mathrm{i}}{128\,\sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)^2),x)
```

```
[Out] ((5*d*(c + d*x^3)^(1/2))/(32*c) - (d*(c + d*x^3)^(3/2))/(32*c^2))/(3*(c + d
*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c^2*(c + d*x^3)^(1/2))/(c
^5)^(1/2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*7i)/9)*1i)/
(128*(c^5)^(1/2))
```

3.404 $\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)^2}\,dx$

Optimal. Leaf size=164

$$\frac{5d^2\sqrt{c+dx^3}}{1536c^3(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{48cx^6(8c-dx^3)}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3(8c-dx^3)}+\frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}}-\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}$$

```
[Out] 23/18432*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/2048*d^2*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+5/1536*d^2*(d*x^3+c)^(1/2)/c^3/(-d*x^3+8
*c)-1/48*(d*x^3+c)^(1/2)/c/x^6/(-d*x^3+8*c)-7/384*d*(d*x^3+c)^(1/2)/c^2/x^3
/(-d*x^3+8*c)
```

Rubi [A]
time = 0.09, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {457, 101, 156, 162, 65, 214, 212}

$$\frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}}-\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}+\frac{5d^2\sqrt{c+dx^3}}{1536c^3(8c-dx^3)}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{48cx^6(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] (5*d^2*Sqrt[c + d*x^3])/(1536*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c*x^
6*(8*c - d*x^3)) - (7*d*Sqrt[c + d*x^3])/(384*c^2*x^3*(8*c - d*x^3)) + (23*
d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(18432*c^(7/2)) - (d^2*ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]])/(2048*c^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{7cd+\frac{5d^2x}{2}}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-6c^2d^2-\frac{21}{2}cd^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{54c^3d^3+45c^2 \cdots}{x(8c-dx)\sqrt{c+ \cdots}} \cdots\right.}{27648c^5 \cdots}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{d^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} \cdots\right.}{4096c^3}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{d\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, \cdots\right.}{2048c^3}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+d \cdots}}{3\sqrt{c}}\right.}{18432c^{7/2}}$$

Mathematica [A]
time = 0.23, size = 112, normalized size = 0.68

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}\left(32c^2+28cdx^3-5d^2x^6\right)}{-8cx^6+dx^9} + 23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18432c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 28*c*d*x^3 - 5*d^2*x^6))/(-8*c*x^6 +
 d*x^9) + 23*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d^2*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(18432*c^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.45, size = 1021, normalized size = 6.23

method	result	size

risch	Expression too large to display	904
default	Expression too large to display	1021
elliptic	Expression too large to display	1580

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/x^6-1/12*d*(d*x^3+c)^(1/2)/c/x^3+1/12*d^2*ar
ctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-3/4096*d^3/c^4*(2/3*(d*x^3+c)^(1/2)
/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c)))+1/512/c^3*d^3*(1/3*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)
+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d
^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arct
anh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+3/4096*d^2/c^4*(2/3*(d*x^3+c)^(1/2)-2
/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^7), x)
```

Fricas [A]
time = 2.51, size = 310, normalized size = 1.89

$$\left[\frac{23\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\,(5\,cd^2x^6-28\,c^2dx^3-32\,c^3)\sqrt{dx^3+c}}{36864\,(c^4dx^9-8\,c^5x^6)},\ \frac{9\,(d^3x^9-8\,cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-23\,(d^3x^9-8\,cd^2x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-12\,(5\,cd^2x^6-28\,c^2dx^3-32\,c^3)\sqrt{dx^3+c}}{18432\,(c^4dx^9-8\,c^5x^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/36864*(23*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)
*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*
x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d^2*x^6 - 28*c^2*d*x^
3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6), 1/18432*(9*(d^3*x^9 -
 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 23*(d^3*x^9 - 8
*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(5*c*d^2*x
^6 - 28*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**7*(-8*c + d*x**3)**2), x)
```

Giac [A]
time = 0.52, size = 105, normalized size = 0.64

$$\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}\,c^3}-\frac{23\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{18432\,\sqrt{-c}\,c^3}-\frac{\sqrt{dx^3+c}\,d^2}{1536\,(dx^3-8\,c)c^3}-\frac{(dx^3+c)^{\frac{3}{2}}}{384\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 23/18432*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/1536*sqrt(d*x^3 + c)
*d^2/((d*x^3 - 8*c)*c^3) - 1/384*(d*x^3 + c)^(3/2)/(c^3*x^6)
```

Mupad [B]
time = 4.45, size = 154, normalized size = 0.94

$$\frac{\frac{d^2\sqrt{d\,x^3+c}}{512\,c}-\frac{19\,d^2\,{\left(d\,x^3+c\right)}^{3/2}}{256\,c^2}+\frac{5\,d^2\,{\left(d\,x^3+c\right)}^{5/2}}{512\,c^3}}{33\,c\,{\left(d\,x^3+c\right)}^2-57\,c^2\,\left(d\,x^3+c\right)-3\,{\left(d\,x^3+c\right)}^3+27\,c^3}+\frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,23{}\mathrm{i}}{9}\right)\,1{}\mathrm{i}}{2048\,\sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)^2),x)
```

```
[Out] ((d^2*(c + d*x^3)^(1/2))/(512*c) - (19*d^2*(c + d*x^3)^(3/2))/(256*c^2) + (
5*d^2*(c + d*x^3)^(5/2))/(512*c^3))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3
) - 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(
1/2))*1i - (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*23i)/9)*1i)/(204
8*(c^7)^(1/2))
```

3.405 $\int \frac{x^7 \sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=663

$$\frac{13x^2\sqrt{c+dx^3}}{21d^2}+\frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)}+\frac{76c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{8/3}}-$$

```
[Out] -76/9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(8/3)+76/9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)+76/9*c^(7/6
)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^(1/
2)+13/21*x^2*(d*x^3+c)^(1/2)/d^2+1/3*x^5*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+746
/21*c*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+746/63*c^(4/3
)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c
)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)-373/21*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*
(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/
2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.81, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {478, 596, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{746\sqrt{2}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}\,d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{373\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7\,3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{76c^{7/6}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{8/3}}-\frac{76c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}+\frac{76c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}+\frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{13x^2\sqrt{c+dx^3}}{21d^2}+\frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (13*x^2*Sqrt[c + d*x^3])/(21*d^2) + (746*c*Sqrt[c + d*x^3])/(21*d^(8/3)*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^5*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3
)) + (76*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/(3*Sqrt[3]*d^(8/3)) - (76*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3
*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(8/3)) + (76*c^(7/6)*ArcTanh[Sqrt[c + d*x^
```

```
3]/(3*Sqrt[c])])/(9*d^(8/3)) - (373*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^
(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*d^(8/3)*S
qrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) + (746*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(21*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{\int \frac{x^4\left(5c+\frac{13dx^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{3d}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{2\int \frac{x\left(104c^2d+\frac{373}{2}cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{21d^3}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{2\int \left(-\frac{373cdx}{2\sqrt{c+dx^3}} + \frac{1596c^2dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{21d^3}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} + \frac{(373c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{21d^2} - \frac{(152c^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx}}}{d^2}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} + \frac{(38c)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^3} + \frac{(373c)\int}{}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{373\sqrt{2-\sqrt{3}}}{}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} + \frac{76c^{7/6}\tan^{-1}\left(\right.}{}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} + \frac{76c^{7/6}\tan^{-1}\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 5.37, size = 176, normalized size = 0.27

$$-\frac{80(52c^2x^2+49cdx^5-3d^2x^8)+520cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+373dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{840d^2\left(-8c+dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] -1/840*(80*(52*c^2*x^2 + 49*c*d*x^5 - 3*d^2*x^8) + 520*c*x^2*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)
] + 373*d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)])/(d^2*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 2199, normalized size = 3.32

method	result	size
elliptic	Expression too large to display	897
risch	Expression too large to display	1758
default	Expression too large to display	2199

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(2/7*x^2*(d*x^3+c)^(1/2)-2/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1
/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
)^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2))))+64*c^2/d^2*(1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/72*I/c*3^(1/
2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
```

```
d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(
-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha
*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*
(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+
16/d^2*c*(-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^
(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x
+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^
(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1
/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c
*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^
2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1
/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 34.31, size = 3866, normalized size = 5.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/189*(532*sqrt(3)*(d^4*x^3 - 8*c*d^3)*(c^7/d^16)^(1/6)*arctan(-1/3*(324*s
qrt(3)*(3*c^7*d^16*x^16 + 784*c^8*d^15*x^13 + 7680*c^9*d^14*x^10 + 10752*c^
10*d^13*x^7 + 4096*c^11*d^12*x^4)*(c^7/d^16)^(2/3) + 36*sqrt(3)*(c^9*d^11*x
^17 + 1772*c^10*d^10*x^14 + 42592*c^11*d^9*x^11 + 96256*c^12*d^8*x^8 + 6963
2*c^13*d^7*x^5 + 16384*c^14*d^6*x^2)*(c^7/d^16)^(1/3) + sqrt(3)*(c^11*d^6*x
^18 + 9456*c^12*d^5*x^15 + 749184*c^13*d^4*x^12 + 3017216*c^14*d^3*x^9 + 34
89792*c^15*d^2*x^6 + 1572864*c^16*d*x^3 + 262144*c^17) + 12*sqrt(d*x^3 + c)
*(12*sqrt(3)*(35*c^6*d^18*x^14 - 14440*c^7*d^17*x^11 - 24576*c^8*d^16*x^8 -
 16384*c^9*d^15*x^5 - 4096*c^10*d^14*x^2)*(c^7/d^16)^(5/6) + 18*sqrt(3)*(c^
8*d^13*x^15 - 1112*c^9*d^12*x^12 + 7296*c^10*d^11*x^9 + 11776*c^11*d^10*x^6
 + 4096*c^12*d^9*x^3)*sqrt(c^7/d^16) + sqrt(3)*(c^10*d^8*x^16 - 4768*c^11*d
^7*x^13 + 362752*c^12*d^6*x^10 + 709120*c^13*d^5*x^7 + 413696*c^14*d^4*x^4
+ 65536*c^15*d^3*x)*(c^7/d^16)^(1/6)) - 2*(324*sqrt(3)*(d^19*x^16 - 1858*c*
d^18*x^13 - 4176*c^2*d^17*x^10 - 3584*c^3*d^16*x^7 - 1024*c^4*d^15*x^4)*(c^
7/d^16)^(5/6) + 18*sqrt(3)*(c^2*d^14*x^17 - 5290*c^3*d^13*x^14 - 21152*c^4*
d^12*x^11 - 47744*c^5*d^11*x^8 - 37888*c^6*d^10*x^5 - 8192*c^7*d^9*x^2)*sqr
t(c^7/d^16) + sqrt(3)*(c^4*d^9*x^18 - 7698*c^5*d^8*x^15 - 1664688*c^6*d^7*x
^12 - 5524864*c^7*d^6*x^9 - 6223872*c^8*d^5*x^6 - 2703360*c^9*d^4*x^3 - 327
680*c^10*d^3)*(c^7/d^16)^(1/6) + 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c*d^16*x^15
+ 37352*c^2*d^15*x^12 - 230336*c^3*d^14*x^9 - 515072*c^4*d^13*x^6 - 286720*
c^5*d^12*x^3 - 32768*c^6*d^11)*(c^7/d^16)^(2/3) + 108*sqrt(3)*(53*c^4*d^10*
x^13 + 1320*c^5*d^9*x^10 + 1536*c^6*d^8*x^7 + 512*c^7*d^7*x^4)*(c^7/d^16)^(
1/3) + 6*sqrt(3)*(37*c^6*d^5*x^14 + 28912*c^7*d^4*x^11 + 43584*c^8*d^3*x^8
+ 20992*c^9*d^2*x^5 + 4096*c^10*d*x^2)))*sqrt((18*c^12*d^2*x^8 + 360*c^13*d
*x^5 - 144*c^14*x^2 + (c^7*d^13*x^9 - 276*c^8*d^12*x^6 - 1608*c^9*d^11*x^3
- 1088*c^10*d^10)*(c^7/d^16)^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^15*x^7 - 28*
c^7*d^14*x^4 - 272*c^8*d^13*x)*(c^7/d^16)^(5/6) - 24*(c^9*d^9*x^5 + c^10*d^
8*x^2)*sqrt(c^7/d^16) + 4*(c^11*d^4*x^6 + 41*c^12*d^3*x^3 + 40*c^13*d^2)*(c
^7/d^16)^(1/6)) - 18*(c^10*d^7*x^7 - 52*c^11*d^6*x^4 - 80*c^12*d^5*x)*(c^7/
d^16)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^11*d^6
*x^18 - 14952*c^12*d^5*x^15 + 2872896*c^13*d^4*x^12 + 7330304*c^14*d^3*x^9
+ 6696960*c^15*d^2*x^6 + 2457600*c^16*d*x^3 + 262144*c^17)) - 532*sqrt(3)*(
d^4*x^3 - 8*c*d^3)*(c^7/d^16)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c^7*d^16*x^
16 + 784*c^8*d^15*x^13 + 7680*c^9*d^14*x^10 + 10752*c^10*d^13*x^7 + 4096*c^
11*d^12*x^4)*(c^7/d^16)^(2/3) + 36*sqrt(3)*(c^9*d^11*x^17 + 1772*c^10*d^10*
```

```
x^14 + 42592*c^11*d^9*x^11 + 96256*c^12*d^8*x^8 + 69632*c^13*d^7*x^5 + 1638
4*c^14*d^6*x^2)*(c^7/d^16)^(1/3) + sqrt(3)*(c^11*d^6*x^18 + 9456*c^12*d^5*x
^15 + 749184*c^13*d^4*x^12 + 3017216*c^14*d^3*x^9 + 3489792*c^15*d^2*x^6 +
1572864*c^16*d*x^3 + 262144*c^17) - 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c^6*
d^18*x^14 - 14440*c^7*d^17*x^11 - 24576*c^8*d^16*x^8 - 16384*c^9*d^15*x^5 -
 4096*c^10*d^14*x^2)*(c^7/d^16)^(5/6) + 18*sqrt(3)*(c^8*d^13*x^15 - 1112*c^
9*d^12*x^12 + 7296*c^10*d^11*x^9 + 11776*c^11*d^10*x^6 + 4096*c^12*d^9*x^3)
*sqrt(c^7/d^16) + sqrt(3)*(c^10*d^8*x^16 - 4768*c^11*d^7*x^13 + 362752*c^12
*d^6*x^10 + 709120*c^13*d^5*x^7 + 413696*c^14*d^4*x^4 + 65536*c^15*d^3*x)*(
c^7/d^16)^(1/6)) + 2*(324*sqrt(3)*(d^19*x^16 - 1858*c*d^18*x^13 - 4176*c^2*
d^17*x^10 - 3584*c^3*d^16*x^7 - 1024*c^4*d^15*x^4)*(c^7/d^16)^(5/6) + 18*sq
rt(3)*(c^2*d^14*x^17 - 5290*c^3*d^13*x^14 - 21152*c^4*d^12*x^11 - 47744*c^5
*d^11*x^8 - 37888*c^6*d^10*x^5 - 8192*c^7*d^9*x^2)*sqrt(c^7/d^16) + sqrt(3)
*(c^4*d^9*x^18 - 7698*c^5*d^8*x^15 - 1664688*c^6*d^7*x^12 - 5524864*c^7*d^6
*x^9 - 6223872*c^8*d^5*x^6 - 2703360*c^9*d^4*x^3 - 327680*c^10*d^3)*(c^7/d^
16)^(1/6) - 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c*d^16*x^15 + 37352*c^2*d^15*x^12
 - 230336*c^3*d^14*x^9 - 515072*c^4*d^13*x^6 - 286720*c^5*d^12*x^3 - 32768*
c^6*d^11)*(c^7/d^16)^(2/3) + 108*sqrt(3)*(53*c^4*d^10*x^13 + 1320*c^5*d^9*x
^10 + 1536*c^6*d^8*x^7 + 512*c^7*d^7*x^4)*(c^7/d^16)^(1/3) + 6*sqrt(3)*(37*
c^6*d^5*x^14 + 28912*c^7*d^4*x^11 + 43584*c^8*d^3*x^8 + 20992*c^9*d^2*x^5 +
 4096*c^10*d*x^2)))*sqrt((18*c^12*d^2*x^8 + 360*c^13*d*x^5 - 144*c^14*x^2 +
 (c^7*d^13*x^9 - 276*c^8*d^12*x^6 - 1608*c^9*d^11*x^3 - 1088*c^10*d^10)*(c^
7/d^16)^(2/3) - 6*sqrt(d*x^3 + c)*((c^6*d^15*x^7 - 28*c^7*d^14*x^4 - 272*c^
8*d^13*x)*(c^7/d^16)^(5/6) - 24*(c^9*d^9*x^5 + c^10*d^8*x^2)*sqrt(c^7/d^16)
 + 4*(c^11*d^4*x^6 + 41*c^12*d^3*x^3 + 40*c^13*d^2)*(c^7/d^16)^(1/6)) - 18*
(c^10*d^7*x^7 - 52*c^11*d^6*x^4 - 80*c^12*d^5*x)*(c^7/d^16)^(1/3))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^11*d^6*x^18 - 14952*c^12*d^
5*x^15 + 2872896*c^13*d^4*x^12 + 7330304*c^14*d^3*x^9 + 6696960*c^15*d^2*x^
6 + 2457600*c^16*d*x^3 + 262144*c^17)) + 6714*(c*d*x^3 - 8*c^2)*sqrt(d)*wei
erstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 133*(d^4*x^3 -
 8*c*d^3)*(c^7/d^16)^(1/6)*log(2571555572935976550 4/9*(18*c^12*d^2*x^8 + 36
0*c^13*d*x^5 - 144*c^14*x^2 + (c^7*d^13*x^9 - 2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\sqrt{d\,x^3+c}}{{\left(8\,c-d\,x^3\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

3.406 $\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=641

$$\frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{x^2\sqrt{c+dx^3}}{3d\,(8c-dx^3)}+\frac{5\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}}-\frac{5\sqrt[6]{c}\,\tanh^{-1}\left(\frac{(\sqrt[3]{\ldots}}{3\sqrt[6]{c}\ldots}\right.}{9d^{5/3}}$$

```
[Out] -5/9*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
5/3)+5/9*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)+5/9*c^(1/6)*a
rctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(5/3)*3^(1/2)+
1/3*x^2*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+7/3*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))+7/9*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/
2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-7/6*3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x
)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {478, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{5\sqrt[6]{c}\,\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}}+\frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{5\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}}+\frac{5\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}}+\frac{x^2\sqrt{c+dx^3}}{3d\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (7*Sqrt[c + d*x^3])/(3*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*
Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) + (5*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(5/3)) - (5*c^(1/6)*Ar
cTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(5/3)) + (
5*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^(5/3)) - (7*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
```

```
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*S
qrt[3]])/(2*3^(3/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (7*Sqrt[2]*c^(1/3)*(c^(1/3)
 + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3
)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^(5
/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\int \frac{x\left(2c+\frac{7dx^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{3d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\int \left(-\frac{7x}{2\sqrt{c+dx^3}} + \frac{30cx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{3d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{7\int \frac{x}{\sqrt{c+dx^3}}\,dx}{6d} - \frac{(10c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{5\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{6d^2} + \frac{7\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{6d^{4/3}} - \cdots (5 \cdots$$

$$= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{7\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\cdots}}{2\ 3^{3\cdots}\cdots}$$

$$= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{5\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\cdots}{\sqrt{c+dx^3}}\cdots\right)}{3\sqrt{3}\,d^{5/3}}\cdots$$

$$= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{5\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\cdots}{\sqrt{c+dx^3}}\cdots\right)}{3\sqrt{3}\,d^{5/3}}\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 8.29, size = 167, normalized size = 0.26

$$\frac{80cx^2(c+dx^3)+10cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{240cd\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7*d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(240*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 1741, normalized size = 2.72

method	result	size
elliptic	Expression too large to display	877
default	Expression too large to display	1741

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8*c/d*(1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/72*I/c*3^(1/2)/d*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2
)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)
*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
```

```
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/d*(-2/3*I*3^
(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-
c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*
3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 21.50, size = 3633, normalized size = 5.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/108*(36*sqrt(d*x^3 + c)*d*x^2 + 20*sqrt(3)*(d^3*x^3 - 8*c*d^2)*(c/d^10)^
(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c*d^12*x^16 + 784*c^2*d^11*x^13 + 7680*c^
3*d^10*x^10 + 10752*c^4*d^9*x^7 + 4096*c^5*d^8*x^4)*(c/d^10)^(2/3) + 36*sqr
t(3)*(c*d^9*x^17 + 1772*c^2*d^8*x^14 + 42592*c^3*d^7*x^11 + 96256*c^4*d^6*x
^8 + 69632*c^5*d^5*x^5 + 16384*c^6*d^4*x^2)*(c/d^10)^(1/3) + sqrt(3)*(c*d^6
```

*x^18 + 9456*c^2*d^5*x^15 + 749184*c^3*d^4*x^12 + 3017216*c^4*d^3*x^9 + 348
9792*c^5*d^2*x^6 + 1572864*c^6*d*x^3 + 262144*c^7) + 12*sqrt(d*x^3 + c)*(12
sqrt(3)(35*c*d^13*x^14 - 14440*c^2*d^12*x^11 - 24576*c^3*d^11*x^8 - 16384
*c^4*d^10*x^5 - 4096*c^5*d^9*x^2)*(c/d^10)^(5/6) + 18*sqrt(3)*(c*d^10*x^15
- 1112*c^2*d^9*x^12 + 7296*c^3*d^8*x^9 + 11776*c^4*d^7*x^6 + 4096*c^5*d^6*x
^3)*sqrt(c/d^10) + sqrt(3)*(c*d^7*x^16 - 4768*c^2*d^6*x^13 + 362752*c^3*d^5
*x^10 + 709120*c^4*d^4*x^7 + 413696*c^5*d^3*x^4 + 65536*c^6*d^2*x)*(c/d^10)
^(1/6)) - 2*(324*sqrt(3)*(d^14*x^16 - 1858*c*d^13*x^13 - 4176*c^2*d^12*x^10
 - 3584*c^3*d^11*x^7 - 1024*c^4*d^10*x^4)*(c/d^10)^(5/6) + 18*sqrt(3)*(d^11
*x^17 - 5290*c*d^10*x^14 - 21152*c^2*d^9*x^11 - 47744*c^3*d^8*x^8 - 37888*c
^4*d^7*x^5 - 8192*c^5*d^6*x^2)*sqrt(c/d^10) + sqrt(3)*(d^8*x^18 - 7698*c*d^
7*x^15 - 1664688*c^2*d^6*x^12 - 5524864*c^3*d^5*x^9 - 6223872*c^4*d^4*x^6 -
 2703360*c^5*d^3*x^3 - 327680*c^6*d^2)*(c/d^10)^(1/6) + 6*sqrt(d*x^3 + c)*(
sqrt(3)*(7*d^12*x^15 + 37352*c*d^11*x^12 - 230336*c^2*d^10*x^9 - 515072*c^3
*d^9*x^6 - 286720*c^4*d^8*x^3 - 32768*c^5*d^7)*(c/d^10)^(2/3) + 108*sqrt(3)
*(53*c*d^8*x^13 + 1320*c^2*d^7*x^10 + 1536*c^3*d^6*x^7 + 512*c^4*d^5*x^4)*(
c/d^10)^(1/3) + 6*sqrt(3)*(37*c*d^5*x^14 + 28912*c^2*d^4*x^11 + 43584*c^3*d
^3*x^8 + 20992*c^4*d^2*x^5 + 4096*c^5*d*x^2)))*sqrt((18*c^2*d^2*x^8 + 360*c
^3*d*x^5 - 144*c^4*x^2 + (c*d^9*x^9 - 276*c^2*d^8*x^6 - 1608*c^3*d^7*x^3 -
1088*c^4*d^6)*(c/d^10)^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^10*x^7 - 28*c^2*d^9*
x^4 - 272*c^3*d^8*x)*(c/d^10)^(5/6) - 24*(c^2*d^6*x^5 + c^3*d^5*x^2)*sqrt(c
/d^10) + 4*(c^2*d^3*x^6 + 41*c^3*d^2*x^3 + 40*c^4*d)*(c/d^10)^(1/6)) - 18*(
c^2*d^5*x^7 - 52*c^3*d^4*x^4 - 80*c^4*d^3*x)*(c/d^10)^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^6*x^18 - 14952*c^2*d^5*x^15 + 2
872896*c^3*d^4*x^12 + 7330304*c^4*d^3*x^9 + 6696960*c^5*d^2*x^6 + 2457600*c
^6*d*x^3 + 262144*c^7)) - 20*sqrt(3)*(d^3*x^3 - 8*c*d^2)*(c/d^10)^(1/6)*arc
tan(-1/3*(324*sqrt(3)*(3*c*d^12*x^16 + 784*c^2*d^11*x^13 + 7680*c^3*d^10*x^
10 + 10752*c^4*d^9*x^7 + 4096*c^5*d^8*x^4)*(c/d^10)^(2/3) + 36*sqrt(3)*(c*d
^9*x^17 + 1772*c^2*d^8*x^14 + 42592*c^3*d^7*x^11 + 96256*c^4*d^6*x^8 + 6963
2*c^5*d^5*x^5 + 16384*c^6*d^4*x^2)*(c/d^10)^(1/3) + sqrt(3)*(c*d^6*x^18 + 9
456*c^2*d^5*x^15 + 749184*c^3*d^4*x^12 + 3017216*c^4*d^3*x^9 + 3489792*c^5*
d^2*x^6 + 1572864*c^6*d*x^3 + 262144*c^7) - 12*sqrt(d*x^3 + c)*(12*sqrt(3)*
(35*c*d^13*x^14 - 14440*c^2*d^12*x^11 - 24576*c^3*d^11*x^8 - 16384*c^4*d^10
*x^5 - 4096*c^5*d^9*x^2)*(c/d^10)^(5/6) + 18*sqrt(3)*(c*d^10*x^15 - 1112*c^
2*d^9*x^12 + 7296*c^3*d^8*x^9 + 11776*c^4*d^7*x^6 + 4096*c^5*d^6*x^3)*sqrt(
c/d^10) + sqrt(3)*(c*d^7*x^16 - 4768*c^2*d^6*x^13 + 362752*c^3*d^5*x^10 + 7
09120*c^4*d^4*x^7 + 413696*c^5*d^3*x^4 + 65536*c^6*d^2*x)*(c/d^10)^(1/6)) +
 2*(324*sqrt(3)*(d^14*x^16 - 1858*c*d^13*x^13 - 4176*c^2*d^12*x^10 - 3584*c
^3*d^11*x^7 - 1024*c^4*d^10*x^4)*(c/d^10)^(5/6) + 18*sqrt(3)*(d^11*x^17 - 5
290*c*d^10*x^14 - 21152*c^2*d^9*x^11 - 47744*c^3*d^8*x^8 - 37888*c^4*d^7*x^
5 - 8192*c^5*d^6*x^2)*sqrt(c/d^10) + sqrt(3)*(d^8*x^18 - 7698*c*d^7*x^15 -
1664688*c^2*d^6*x^12 - 5524864*c^3*d^5*x^9 - 6223872*c^4*d^4*x^6 - 2703360*
c^5*d^3*x^3 - 327680*c^6*d^2)*(c/d^10)^(1/6) - 6*sqrt(d*x^3 + c)*(sqrt(3)*(
7*d^12*x^15 + 37352*c*d^11*x^12 - 230336*c^2*d^10*x^9 - 515072*c^3*d^9*x^6
- 286720*c^4*d^8*x^3 - 32768*c^5*d^7)*(c/d^10)^(2/3) + 108*sqrt(3)*(53*c*d^

```
8*x^13 + 1320*c^2*d^7*x^10 + 1536*c^3*d^6*x^7 + 512*c^4*d^5*x^4)*(c/d^10)^(
1/3) + 6*sqrt(3)*(37*c*d^5*x^14 + 28912*c^2*d^4*x^11 + 43584*c^3*d^3*x^8 +
20992*c^4*d^2*x^5 + 4096*c^5*d*x^2)))*sqrt((18*c^2*d^2*x^8 + 360*c^3*d*x^5
- 144*c^4*x^2 + (c*d^9*x^9 - 276*c^2*d^8*x^6 - 1608*c^3*d^7*x^3 - 1088*c^4*
d^6)*(c/d^10)^(2/3) - 6*sqrt(d*x^3 + c)*((c*d^10*x^7 - 28*c^2*d^9*x^4 - 272
*c^3*d^8*x)*(c/d^10)^(5/6) - 24*(c^2*d^6*x^5 + c^3*d^5*x^2)*sqrt(c/d^10) +
4*(c^2*d^3*x^6 + 41*c^3*d^2*x^3 + 40*c^4*d)*(c/d^10)^(1/6)) - 18*(c^2*d^5*x
^7 - 52*c^3*d^4*x^4 - 80*c^4*d^3*x)*(c/d^10)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^6*x^18 - 14952*c^2*d^5*x^15 + 2872896*c^
3*d^4*x^12 + 7330304*c^4*d^3*x^9 + 6696960*c^5*d^2*x^6 + 2457600*c^6*d*x^3
+ 262144*c^7)) + 252*(d*x^3 - 8*c)*sqrt(d)*weierstrassZeta(0, -4*c/d, weier
strassPInverse(0, -4*c/d, x)) + 5*(d^3*x^3 - 8*c*d^2)*(c/d^10)^(1/6)*log(39
062500/9*(18*c^2*d^2*x^8 + 360*c^3*d*x^5 - 144*c^4*x^2 + (c*d^9*x^9 - 276*c
^2*d^8*x^6 - 1608*c^3*d^7*x^3 - 1088*c^4*d^6)*(c/d^10)^(2/3) + 6*sqrt(d*x^3
 + c)*((c*d^10*x^7 - 28*c^2*d^9*x^4 - 272*c^3*d^8*x)*(c/d^10)^(5/6) - 24*(c
^2*d^6*x^5 + c^3*d^5*x^2)*sqrt(c/d^10) + 4*(c^2*d^3*x^6 + 41*c^3*d^2*x^3 +
40*c^4*d)*(c/d^10)^(1/6)) - 18*(c^2*d^5*x^7 - 5...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
[Out] Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{d\,x^3+c}}{{\left(8\,c-d\,x^3\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
[Out] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

3.407 $\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=644

$$\frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{x^2\sqrt{c+dx^3}}{24c\,(8c-dx^3)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}\,c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{3\sqrt[6]{c}\,\sqrt{c+}}\right.}{144c^{5/6}d^{2/3}}$$

```
[Out] -1/144*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d
^(2/3)+1/144*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(2/3)+1/144*arc
tan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(2/3)*3^
(1/2)+1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)+1/24*(d*x^3+c)^(1/2)/c/d^(2/3
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/72*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/
3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((
c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)*3^(3/4)/c^(2/3)/d^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1
/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/48*(c^(1/3)+d^(1/3)*x)*El
lipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(2/3)/(d*x^3
+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {480, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\,\sqrt[4]{3}\,c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{16\,3^{3/4}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}\,c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{5/6}d^{2/3}}+\frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{x^2\sqrt{c+dx^3}}{24c\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/(24*c*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*S
qrt[c + d*x^3])/(24*c*(8*c - d*x^3)) + ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d
^(1/3)*x))/Sqrt[c + d*x^3]]/(48*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/3)
 + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(144*c^(5/6)*d^(2/3)) + ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(144*c^(5/6)*d^(2/3)) - (Sqrt[2 - Sqrt[3]]*(
```

```
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*3^(3
/4)*c^(2/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + ((c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*c^(2/3)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)} - \frac{\int \frac{x\left(-c+\frac{dx^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)} - \frac{\int \left(-\frac{x}{2\sqrt{c+dx^3}} + \frac{3cx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{24c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)} - \frac{1}{8}\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{48c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)} + \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{96cd} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{48c\sqrt[3]{d}} - \frac{\int \frac{}{(}}{}$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{}}{16\ 3^{3}}$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}\,c^{5/6}d^{2/3}}$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}\,c^{5/6}d^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.07, size = 164, normalized size = 0.25

$$\frac{80cx^2(c+dx^3)+5cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{1920c^2\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 5*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[
2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqrt[
1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(19
20*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 883, normalized size = 1.37

method	result	size
default	Expression too large to display	883
elliptic	Expression too large to display	883

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/72*I/c*3^(1/2)/d*(-c*d^2)^(1/3)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(
1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha
-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.95, size = 2613, normalized size = 4.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/1728*(72*sqrt(d*x^3 + c)*d*x^2 - 4*sqrt(3)*(c*d^2*x^3 - 8*c^2*d)*(1/(c^5
*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^2*x^5*(1/(c^5*d^4))^(1/6) - sqrt(3)
*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d^4))^(5/6) + 3*sqrt(3
)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(1/(c^5*d^4)))*sqrt(d*x^3 + c) + (18*sq
rt(3)*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^(2/3) + 12*sqrt(3)*(c^2*d^3
*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*
c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^5*x^6 + 32*c^5*d^4*x^3
 + 40*c^6*d^3)*(1/(c^5*d^4))^(5/6) + 3*sqrt(3)*(7*c^3*d^3*x^4 + 4*c^4*d^2*x
)*sqrt(1/(c^5*d^4)) + 9*sqrt(3)*(c*d^2*x^5 + 2*c^2*d*x^2)*(1/(c^5*d^4))^(1/
6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^
5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) + 6*sqrt(d*x^3 +
 c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*(c^3*d^4*x^6 +
41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7 - 28*c^2*d^2*x^
4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 -
8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) - 4*sqrt(3)*(c*d^2*x^3 - 8*c
^2*d)*(1/(c^5*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^2*x^5*(1/(c^5*d^4))^(1
/6) - sqrt(3)*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d^4))^(5/
6) + 3*sqrt(3)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(1/(c^5*d^4)))*sqrt(d*x^3
+ c) - (18*sqrt(3)*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^(2/3) + 12*sqr
t(3)*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^(1/3) + 3*sqrt(3)*
(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^5*x^6 + 3
2*c^5*d^4*x^3 + 40*c^6*d^3)*(1/(c^5*d^4))^(5/6) + 3*sqrt(3)*(7*c^3*d^3*x^4
+ 4*c^4*d^2*x)*sqrt(1/(c^5*d^4)) + 9*sqrt(3)*(c*d^2*x^5 + 2*c^2*d*x^2)*(1/(
c^5*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3
 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6
*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*(c
```

```
^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7 -
28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20*c
^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 72*(d*x^3 - 8
*c)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) +
 2*(c*d^2*x^3 - 8*c^2*d)*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 +
 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3
*x)*(1/(c^5*d^4))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*
x^2)*(1/(c^5*d^4))^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*s
qrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(1/(c^5*d^4))
^(1/6)) + 18*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1/(c^5*d^4))^
(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c*d^2*x^3 -
 8*c^2*d)*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3
 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x)*(1/(c^5*d^4
))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2)*(1/(c^5*d^
4))^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)
) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c
^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^2*x^3 - 8*c^2*d)*(1/(c^5
*d^4))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*
(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) + 6*sqrt(
d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*(c^3*d^4
*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7 - 28*c^2
*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3
*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^
2*d*x^3 - 512*c^3)) + (c*d^2*x^3 - 8*c^2*d)*(1/(c^5*d^4))^(1/6)*log((d^3*x^
9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^
4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*
x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 +
40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*
(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1
/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*
d^2*x^3 - 8*c^2*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{d\,x^3+c}}{{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
[Out] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

3.408 $\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=665

$$-\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}-\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}\,c^{11/6}}+\frac{\sqrt[3]{d}}{}$$

```
[Out] 1/144*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(11/6)-1/144*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/144*d^
(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)*
3^(1/2)-1/48*(d*x^3+c)^(1/2)/c^2/x+1/24*(d*x^3+c)^(1/2)/c/x/(-d*x^3+8*c)+1/
48*d^(1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/144*d^(1/3
)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c
)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)-1/96*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1
-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*
2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {480, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{24\sqrt{2}\,\sqrt[4]{3}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{32\,3^{3/4}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{\sqrt[3]{d}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}\,c^{11/6}}+\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{144c^{11/6}}-\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}}-\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] -1/48*Sqrt[c + d*x^3]/(c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(48*c^2*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x*(8*c - d*x^3)) - (d^(
1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(48*S
qrt[3]*c^(11/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt
[c + d*x^3])])/(144*c^(11/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
```

```
)])/(144*c^(11/6)) - (Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[
(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*3^(3/4)*c^(5/3)*Sqrt[(c^(1/3)*
(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^
3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^
(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sq
rt[3]])/(24*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} - \frac{\int \frac{-4c-\frac{5dx^3}{2}}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} + \frac{\int \frac{x\left(40c^2d-2cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{192c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} + \frac{\int \left(\frac{2cdx}{\sqrt{c+dx^3}} + \frac{24c^2dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{192c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{96c^2} + \frac{d\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} - \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{96c^2} + \frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\cdots\right.}{\sqrt{c\cdots}}}{96\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} - \frac{\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\cdots}{\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} - \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right.}{48\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} - \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right.}{48\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 20.08, size = 179, normalized size = 0.27

$$\frac{-80c(6c^2+5cdx^3-d^2x^6)+50cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+d^2x^6(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3840c^3\sqrt{c+dx^3}\,(8cx-dx^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] (-80*c*(6*c^2 + 5*c*d*x^3 - d^2*x^6) + 50*c*d*x^3*(8*c - d*x^3)*Sqrt[1 + (d
*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d^2*x^6*
(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/(3840*c^3*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 2194, normalized size = 3.30

method	result	size
elliptic	Expression too large to display	898
risch	Expression too large to display	1758
default	Expression too large to display	2194

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d/c*(1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/72*I/c*3^(1/2)/d*(-c*d^2
)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
```

```
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/
3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))-1/64*d/c^2*(
-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/
_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d
-8*c))+1/64/c^2*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.27, size = 2617, normalized size = 3.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/1728*(4*sqrt(3)*(c^2*d*x^4 - 8*c^3*x)*(d^2/c^11)^(1/6)*arctan(1/9*((9*sq
rt(3)*c^2*d^4*x^5*(d^2/c^11)^(1/6) - sqrt(3)*(c^9*d^3*x^6 - 40*c^10*d^2*x^3
 - 32*c^11*d)*(d^2/c^11)^(5/6) + 3*sqrt(3)*(5*c^6*d^3*x^4 + 8*c^7*d^2*x)*sq
rt(d^2/c^11))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^8*d^2*x^5 + c^9*d*x^2)*(d^2/
c^11)^(2/3) + 12*sqrt(3)*(c^4*d^3*x^6 - c^5*d^2*x^3 - 2*c^6*d)*(d^2/c^11)^(
1/3) + 3*sqrt(3)*(d^4*x^7 + 5*c*d^3*x^4 + 4*c^2*d^2*x) + sqrt(d*x^3 + c)*(s
qrt(3)*(c^9*d^2*x^6 + 32*c^10*d*x^3 + 40*c^11)*(d^2/c^11)^(5/6) + 3*sqrt(3)
*(7*c^6*d^2*x^4 + 4*c^7*d*x)*sqrt(d^2/c^11) + 9*sqrt(3)*(c^2*d^3*x^5 + 2*c^
3*d^2*x^2)*(d^2/c^11)^(1/6)))*sqrt((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*
x^3 - 1088*c^3*d^2 - 18*(c^8*d^3*x^7 - 52*c^9*d^2*x^4 - 80*c^10*d*x)*(d^2/c
^11)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^2*x^5 + c^11*d*x^2)*(d^2/c^11)^(
5/6) - 4*(c^6*d^3*x^6 + 41*c^7*d^2*x^3 + 40*c^8*d)*sqrt(d^2/c^11) - (c^2*d^
4*x^7 - 28*c^3*d^3*x^4 - 272*c^4*d^2*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d^4*x^8
 + 20*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^
6 + 192*c^2*d*x^3 - 512*c^3)))/(d^5*x^7 - 7*c*d^4*x^4 - 8*c^2*d^3*x)) + 4*s
qrt(3)*(c^2*d*x^4 - 8*c^3*x)*(d^2/c^11)^(1/6)*arctan(1/9*((9*sqrt(3)*c^2*d^
4*x^5*(d^2/c^11)^(1/6) - sqrt(3)*(c^9*d^3*x^6 - 40*c^10*d^2*x^3 - 32*c^11*d
)*(d^2/c^11)^(5/6) + 3*sqrt(3)*(5*c^6*d^3*x^4 + 8*c^7*d^2*x)*sqrt(d^2/c^11)
)*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^8*d^2*x^5 + c^9*d*x^2)*(d^2/c^11)^(2/3)
+ 12*sqrt(3)*(c^4*d^3*x^6 - c^5*d^2*x^3 - 2*c^6*d)*(d^2/c^11)^(1/3) + 3*sqr
t(3)*(d^4*x^7 + 5*c*d^3*x^4 + 4*c^2*d^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^9*
d^2*x^6 + 32*c^10*d*x^3 + 40*c^11)*(d^2/c^11)^(5/6) + 3*sqrt(3)*(7*c^6*d^2*
x^4 + 4*c^7*d*x)*sqrt(d^2/c^11) + 9*sqrt(3)*(c^2*d^3*x^5 + 2*c^3*d^2*x^2)*(
d^2/c^11)^(1/6)))*sqrt((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c
^3*d^2 - 18*(c^8*d^3*x^7 - 52*c^9*d^2*x^4 - 80*c^10*d*x)*(d^2/c^11)^(2/3) -
 6*sqrt(d*x^3 + c)*(24*(c^10*d^2*x^5 + c^11*d*x^2)*(d^2/c^11)^(5/6) - 4*(c^
6*d^3*x^6 + 41*c^7*d^2*x^3 + 40*c^8*d)*sqrt(d^2/c^11) - (c^2*d^4*x^7 - 28*c
^3*d^3*x^4 - 272*c^4*d^2*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d^4*x^8 + 20*c^5*d^
3*x^5 - 8*c^6*d^2*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)))/(d^5*x^7 - 7*c*d^4*x^4 - 8*c^2*d^3*x)) + 36*(d*x^4 - 8*c
*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) +
 (c^2*d*x^4 - 8*c^3*x)*(d^2/c^11)^(1/6)*log((d^5*x^9 - 276*c*d^4*x^6 - 1608
*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^8*d^3*x^7 - 52*c^9*d^2*x^4 - 80*c^10*d*
x)*(d^2/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^2*x^5 + c^11*d*x^2)*(d^
2/c^11)^(5/6) - 4*(c^6*d^3*x^6 + 41*c^7*d^2*x^3 + 40*c^8*d)*sqrt(d^2/c^11)
```

```
- (c^2*d^4*x^7 - 28*c^3*d^3*x^4 - 272*c^4*d^2*x)*(d^2/c^11)^(1/6)) + 18*(c^
4*d^4*x^8 + 20*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24
*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^2*d*x^4 - 8*c^3*x)*(d^2/c^11)^(
1/6)*log((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c
^8*d^3*x^7 - 52*c^9*d^2*x^4 - 80*c^10*d*x)*(d^2/c^11)^(2/3) - 6*sqrt(d*x^3
+ c)*(24*(c^10*d^2*x^5 + c^11*d*x^2)*(d^2/c^11)^(5/6) - 4*(c^6*d^3*x^6 + 41
*c^7*d^2*x^3 + 40*c^8*d)*sqrt(d^2/c^11) - (c^2*d^4*x^7 - 28*c^3*d^3*x^4 - 2
72*c^4*d^2*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d^4*x^8 + 20*c^5*d^3*x^5 - 8*c^6*
d^2*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^
3)) - 2*(c^2*d*x^4 - 8*c^3*x)*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6
 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^1
0*x)*(d^2/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(
d^2/c^11)^(5/6) + (7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) +
 (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d
^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d
^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^2*d*x^4 - 8*c^3*x)*(d^2/c^11)^(1/
6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^8*
d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x)*(d^2/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(6
*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^2/c^11)^(5/6) + (7*c^6*d^2*x^6 + 152*c^7*d
*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x
)*(d^2/c^11)^(1/6)) + 18*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2)*(d^2
/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 36*sqrt
(d*x^3 + c)*(d*x^3 - 6*c))/(c^2*d*x^4 - 8*c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**2*(-8*c + d*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{x^2\,{\left(8\,c-d\,x^3\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)^2), x)
```

3.409 $\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)^2}\,dx$

Optimal. Leaf size=687

$$-\frac{7\sqrt{c+dx^3}}{768c^2x^4}-\frac{d\sqrt{c+dx^3}}{96c^3x}+\frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}-\frac{17d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{\ \ }\right.}{\sqrt{c+\ }}\right.}{3072\sqrt{3}\,c^{17/6}}$$

[Out] 17/9216*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/6)-17/9216*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-17/9216*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/6)*3^(1/2)-7/768*(d*x^3+c)^(1/2)/c^2/x^4-1/96*d*(d*x^3+c)^(1/2)/c^3/x+1/24*(d*x^3+c)^(1/2)/c/x^4/(-d*x^3+8*c)+1/96*d^(4/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/288*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/192*3^(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.66, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {480, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{48\sqrt{2}\,\sqrt[4]{3}\,c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{64\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{17d^{4/3}\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}\,c^{17/6}}+\frac{17d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{17/6}}-\frac{17d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{17/6}}+\frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{d\sqrt{c+dx^3}}{96c^3x}-\frac{7\sqrt{c+dx^3}}{768c^2x^4}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]

[Out] (-7*Sqrt[c + d*x^3])/(768*c^2*x^4) - (d*Sqrt[c + d*x^3])/(96*c^3*x) + (d^(4/3)*Sqrt[c + d*x^3])/(96*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x^4*(8*c - d*x^3)) - (17*d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3072*Sqrt[3]*c^(17/6)) + (17*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9216*c^(17/

```
6)) - (17*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9216*c^(17/6)) - (
Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(64*3^(3/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(4/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(48*Sqrt[2]*3^
(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{\int \frac{-7c-\frac{11dx^3}{2}}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} + \frac{\int \frac{64c^2d+\frac{35}{2}cd^2x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{768c^3}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{\int \frac{x\left(-460c^3d^2+32c^2d^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{6144c^5}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{\int \left(-\frac{32c^2d^2x}{\sqrt{c+dx^3}} - \frac{204c^3d^2x}{(8c-dx^3)\sqrt{c+d}}\right.}{6144c^5}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} + \frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{192c^3} + \frac{(17d^2)\int \frac{}{(8c}}{}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{(17d)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+d}}}{6144c^3}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.09, size = 199, normalized size = 0.29

$$\sqrt{c+dx^3}\left(-\frac{1}{256c^2x^4}-\frac{5d}{512c^3x}-\frac{d^2x^2}{1536c^3(-8c+dx^3)}\right)+\frac{115d^2x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{24576c^3\sqrt{c+dx^3}}-\frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{7680c^4\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/256*1/(c^2*x^4) - (5*d)/(512*c^3*x) - (d^2*x^2)/(1536*c
^3*(-8*c + d*x^3))) + (115*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1
, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(24576*c^3*Sqrt[c + d*x^3]) - (d^3*x^5
*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(7680*c^4*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 2672, normalized size = 3.89

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2672

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*d*(d*x^3+c)^(1/2)/c/x-1/8*I/c*d*3^(1
/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))+1/64/c^2*d^2*(1/24*x^2*(d*x^3+c)^(1/
2)/c/(-d*x^3+8*c)-1/72*I/c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d
*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
```

(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))-1/256/c^3*d^2*(-2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/256/c^3*d*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)

```
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.99, size = 2721, normalized size = 3.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
[Out] -1/110592*(68*sqrt(3)*(c^3*d*x^7 - 8*c^4*x^4)*(d^8/c^17)^(1/6)*arctan(1/9*(
(9*sqrt(3)*c^3*d^13*x^5*(d^8/c^17)^(1/6) - sqrt(3)*(c^14*d^8*x^6 - 40*c^15*
d^7*x^3 - 32*c^16*d^6)*(d^8/c^17)^(5/6) + 3*sqrt(3)*(5*c^9*d^10*x^4 + 8*c^1
0*d^9*x)*sqrt(d^8/c^17))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^12*d^3*x^5 + c^13
*d^2*x^2)*(d^8/c^17)^(2/3) + 12*sqrt(3)*(c^6*d^6*x^6 - c^7*d^5*x^3 - 2*c^8*
d^4)*(d^8/c^17)^(1/3) + 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*x^4 + 4*c^2*d^7*x) + s
qrt(d*x^3 + c)*(sqrt(3)*(c^14*d^2*x^6 + 32*c^15*d*x^3 + 40*c^16)*(d^8/c^17)
^(5/6) + 3*sqrt(3)*(7*c^9*d^4*x^4 + 4*c^10*d^3*x)*sqrt(d^8/c^17) + 9*sqrt(3
)*(c^3*d^7*x^5 + 2*c^4*d^6*x^2)*(d^8/c^17)^(1/6)))*sqrt((d^15*x^9 - 276*c*d
^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^12*d^9*x^7 - 52*c^13*d^
8*x^4 - 80*c^14*d^7*x)*(d^8/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^15*d^7*x
^5 + c^16*d^6*x^2)*(d^8/c^17)^(5/6) - 4*(c^9*d^10*x^6 + 41*c^10*d^9*x^3 + 4
0*c^11*d^8)*sqrt(d^8/c^17) - (c^3*d^13*x^7 - 28*c^4*d^12*x^4 - 272*c^5*d^11
*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^12*x^8 + 20*c^7*d^11*x^5 - 8*c^8*d^10*x^2
)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d
^15*x^7 - 7*c*d^14*x^4 - 8*c^2*d^13*x)) + 68*sqrt(3)*(c^3*d*x^7 - 8*c^4*x^4
)*(d^8/c^17)^(1/6)*arctan(1/9*((9*sqrt(3)*c^3*d^13*x^5*(d^8/c^17)^(1/6) - s
qrt(3)*(c^14*d^8*x^6 - 40*c^15*d^7*x^3 - 32*c^16*d^6)*(d^8/c^17)^(5/6) + 3*
sqrt(3)*(5*c^9*d^10*x^4 + 8*c^10*d^9*x)*sqrt(d^8/c^17))*sqrt(d*x^3 + c) - (
18*sqrt(3)*(c^12*d^3*x^5 + c^13*d^2*x^2)*(d^8/c^17)^(2/3) + 12*sqrt(3)*(c^6
*d^6*x^6 - c^7*d^5*x^3 - 2*c^8*d^4)*(d^8/c^17)^(1/3) + 3*sqrt(3)*(d^9*x^7 +
 5*c*d^8*x^4 + 4*c^2*d^7*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^14*d^2*x^6 + 32*c
^15*d*x^3 + 40*c^16)*(d^8/c^17)^(5/6) + 3*sqrt(3)*(7*c^9*d^4*x^4 + 4*c^10*d
^3*x)*sqrt(d^8/c^17) + 9*sqrt(3)*(c^3*d^7*x^5 + 2*c^4*d^6*x^2)*(d^8/c^17)^(
```

```
1/6)))*sqrt((d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12
- 18*(c^12*d^9*x^7 - 52*c^13*d^8*x^4 - 80*c^14*d^7*x)*(d^8/c^17)^(2/3) - 6*
sqrt(d*x^3 + c)*(24*(c^15*d^7*x^5 + c^16*d^6*x^2)*(d^8/c^17)^(5/6) - 4*(c^9
*d^10*x^6 + 41*c^10*d^9*x^3 + 40*c^11*d^8)*sqrt(d^8/c^17) - (c^3*d^13*x^7 -
 28*c^4*d^12*x^4 - 272*c^5*d^11*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^12*x^8 + 2
0*c^7*d^11*x^5 - 8*c^8*d^10*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
+ 192*c^2*d*x^3 - 512*c^3)))/(d^15*x^7 - 7*c*d^14*x^4 - 8*c^2*d^13*x)) + 11
52*(d^2*x^7 - 8*c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInv
erse(0, -4*c/d, x)) + 17*(c^3*d*x^7 - 8*c^4*x^4)*(d^8/c^17)^(1/6)*log(20159
93900449*(d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 1
8*(c^12*d^9*x^7 - 52*c^13*d^8*x^4 - 80*c^14*d^7*x)*(d^8/c^17)^(2/3) + 6*sqr
t(d*x^3 + c)*(24*(c^15*d^7*x^5 + c^16*d^6*x^2)*(d^8/c^17)^(5/6) - 4*(c^9*d^
10*x^6 + 41*c^10*d^9*x^3 + 40*c^11*d^8)*sqrt(d^8/c^17) - (c^3*d^13*x^7 - 28
*c^4*d^12*x^4 - 272*c^5*d^11*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^12*x^8 + 20*c
^7*d^11*x^5 - 8*c^8*d^10*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)) - 17*(c^3*d*x^7 - 8*c^4*x^4)*(d^8/c^17)^(1/6)*log(
2015993900449*(d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^1
2 - 18*(c^12*d^9*x^7 - 52*c^13*d^8*x^4 - 80*c^14*d^7*x)*(d^8/c^17)^(2/3) -
6*sqrt(d*x^3 + c)*(24*(c^15*d^7*x^5 + c^16*d^6*x^2)*(d^8/c^17)^(5/6) - 4*(c
^9*d^10*x^6 + 41*c^10*d^9*x^3 + 40*c^11*d^8)*sqrt(d^8/c^17) - (c^3*d^13*x^7
 - 28*c^4*d^12*x^4 - 272*c^5*d^11*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^12*x^8 +
 20*c^7*d^11*x^5 - 8*c^8*d^10*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^
6 + 192*c^2*d*x^3 - 512*c^3)) - 34*(c^3*d*x^7 - 8*c^4*x^4)*(d^8/c^17)^(1/6)
*log(1419857*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18
*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x)*(d^8/c^17)^(2/3) + 6*sqrt
(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^8/c^17)^(5/6) + (7*c^9*d^4*x
^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4
*d^6*x^4 + 160*c^5*d^5*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^6*x^8 + 38*c^7*d^5*
x^5 + 64*c^8*d^4*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d
*x^3 - 512*c^3)) + 34*(c^3*d*x^7 - 8*c^4*x^4)*(d^8/c^17)^(1/6)*log(1419857*
(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^12*d^3*
x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x)*(d^8/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(
6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^8/c^17)^(5/6) + (7*c^9*d^4*x^6 + 152*c^10
*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 16
0*c^5*d^5*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*
d^4*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^
3)) + 144*(8*d^2*x^6 - 57*c*d*x^3 - 24*c^2)*sqrt(d*x^3 + c))/(c^3*d*x^7 - 8
*c^4*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x**5*(-8*c + d*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{x^5\,{\left(8\,c-d\,x^3\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)^2), x)
```

3.410 $\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=711

$$-\frac{5\sqrt{c+dx^3}}{672c^2x^7}-\frac{53d\sqrt{c+dx^3}}{21504c^3x^4}-\frac{d^2\sqrt{c+dx^3}}{5376c^4x}+\frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)}-\frac{13d^{7/3}}{}$$

```
[Out] 13/36864*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(23/6)-13/36864*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-13
/36864*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(23/6)*3^(1/2)-5/672*(d*x^3+c)^(1/2)/c^2/x^7-53/21504*d*(d*x^3+c)^(1/2)/c
^3/x^4-1/5376*d^2*(d*x^3+c)^(1/2)/c^4/x+1/24*(d*x^3+c)^(1/2)/c/x^7/(-d*x^3+
8*c)+1/5376*d^(7/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/1
6128*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^
(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(11/3)*2^(1
/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)-1/10752*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/
3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x
+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(11/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d
^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {480, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2688\sqrt{2}\,\sqrt[4]{3}\,c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\,d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{3584\,3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{13d^{7/3}\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{12288\sqrt{3}\,c^{23/6}}+\frac{13d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{36864c^{23/6}}-\frac{13d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36864c^{23/6}}+\frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{d^2\sqrt{c+dx^3}}{5376c^4x}-\frac{53d\sqrt{c+dx^3}}{21504c^3x^4}-\frac{5\sqrt{c+dx^3}}{672c^2x^7}+\frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (-5*Sqrt[c + d*x^3])/(672*c^2*x^7) - (53*d*Sqrt[c + d*x^3])/(21504*c^3*x^4)
 - (d^2*Sqrt[c + d*x^3])/(5376*c^4*x) + (d^(7/3)*Sqrt[c + d*x^3])/(5376*c^4
*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x^7*(8*c - d*
x^3)) - (13*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c +
```

```
d*x^3]])/(12288*Sqrt[3]*c^(23/6)) + (13*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)
*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(36864*c^(23/6)) - (13*d^(7/3)*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(36864*c^(23/6)) - (Sqrt[2 - Sqrt[3]]*d^(7/3)
*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3584*
3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2688*Sqrt[2]*3^(1/4)*c^(11/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{\int \frac{-10c-\frac{17dx^3}{2}}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{\int \frac{106c^2d+55cd^2x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1344c^3}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{\int \frac{-64c^3d^2-265c^2d^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{43008c^5}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{\int \frac{x(2440c^4d^3-32c^3}{(8c-dx^3)\sqrt{c+}}}{344064c}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{\int \left(\frac{32c^3d^3x}{\sqrt{c+dx^3}}\right.}{}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{d^3\int \frac{x}{\sqrt{c+dx^3}}}{10752c^4}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{(13d^2)\int \frac{2}{\left(4+\frac{2\sqrt{}}{}\right.}}{}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.12, size = 209, normalized size = 0.29

$$\frac{1525cd^3x^9(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-8\left(20c(384c^4+648c^3dx^3+243c^2d^2x^6-25cd^3x^9-4d^4x^{12})+d^4x^{12}(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{3440640c^5x^7\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (1525*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(384*c^4 + 648*c^3*d*x^3 + 243*c^2
*d^2*x^6 - 25*c*d^3*x^9 - 4*d^4*x^12) + d^4*x^12*(8*c - d*x^3)*Sqrt[1 + (d*
x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3440640*
c^5*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 3170, normalized size = 4.46

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	3170

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/256/c^3*d*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*d*(d*x^3+c)^(1/2)/c/x-1/8*I/c*d*3
^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))+1/64/c^2*(-1/7*(d*x^3+c)^(1/2)/x^
7-3/56*d*(d*x^3+c)^(1/2)/x^4/c+15/112*d^2*(d*x^3+c)^(1/2)/c^2/x+5/112*I/c^2
*d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ellipt
```

```
icE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-3/4096*d^3/c^4*(-2/3*I*3^(1
/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*
d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c
*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+3/4
096*d^2/c^4*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((
x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
)+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
)+1/512/c^3*d^3*(1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/72*I/c*3^(1/2)/d
*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
```

```
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.37, size = 2738, normalized size = 3.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/3096576*(364*sqrt(3)*(c^4*d*x^10 - 8*c^5*x^7)*(d^14/c^23)^(1/6)*arctan(1
/9*((9*sqrt(3)*c^4*d^22*x^5*(d^14/c^23)^(1/6) - sqrt(3)*(c^19*d^13*x^6 - 40
*c^20*d^12*x^3 - 32*c^21*d^11)*(d^14/c^23)^(5/6) + 3*sqrt(3)*(5*c^12*d^17*x
^4 + 8*c^13*d^16*x)*sqrt(d^14/c^23))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^16*d^
4*x^5 + c^17*d^3*x^2)*(d^14/c^23)^(2/3) + 12*sqrt(3)*(c^8*d^9*x^6 - c^9*d^8
*x^3 - 2*c^10*d^7)*(d^14/c^23)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 +
 4*c^2*d^12*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^19*d^2*x^6 + 32*c^20*d*x^3 + 4
0*c^21)*(d^14/c^23)^(5/6) + 3*sqrt(3)*(7*c^12*d^6*x^4 + 4*c^13*d^5*x)*sqrt(
d^14/c^23) + 9*sqrt(3)*(c^4*d^11*x^5 + 2*c^5*d^10*x^2)*(d^14/c^23)^(1/6)))*
sqrt((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c
^16*d^15*x^7 - 52*c^17*d^14*x^4 - 80*c^18*d^13*x)*(d^14/c^23)^(2/3) + 6*sqr
t(d*x^3 + c)*(24*(c^20*d^12*x^5 + c^21*d^11*x^2)*(d^14/c^23)^(5/6) - 4*(c^1
2*d^17*x^6 + 41*c^13*d^16*x^3 + 40*c^14*d^15)*sqrt(d^14/c^23) - (c^4*d^22*x
^7 - 28*c^5*d^21*x^4 - 272*c^6*d^20*x)*(d^14/c^23)^(1/6)) + 18*(c^8*d^20*x^
8 + 20*c^9*d^19*x^5 - 8*c^10*d^18*x^2)*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d
^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*x
)) + 364*sqrt(3)*(c^4*d*x^10 - 8*c^5*x^7)*(d^14/c^23)^(1/6)*arctan(1/9*((9*
sqrt(3)*c^4*d^22*x^5*(d^14/c^23)^(1/6) - sqrt(3)*(c^19*d^13*x^6 - 40*c^20*d
^12*x^3 - 32*c^21*d^11)*(d^14/c^23)^(5/6) + 3*sqrt(3)*(5*c^12*d^17*x^4 + 8*
c^13*d^16*x)*sqrt(d^14/c^23))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^16*d^4*x^5 +
 c^17*d^3*x^2)*(d^14/c^23)^(2/3) + 12*sqrt(3)*(c^8*d^9*x^6 - c^9*d^8*x^3 -
2*c^10*d^7)*(d^14/c^23)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 + 4*c^2*
d^12*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^19*d^2*x^6 + 32*c^20*d*x^3 + 40*c^21)
```

```
*(d^14/c^23)^(5/6) + 3*sqrt(3)*(7*c^12*d^6*x^4 + 4*c^13*d^5*x)*sqrt(d^14/c^
23) + 9*sqrt(3)*(c^4*d^11*x^5 + 2*c^5*d^10*x^2)*(d^14/c^23)^(1/6)))*sqrt((d
^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^16*d^1
5*x^7 - 52*c^17*d^14*x^4 - 80*c^18*d^13*x)*(d^14/c^23)^(2/3) - 6*sqrt(d*x^3
 + c)*(24*(c^20*d^12*x^5 + c^21*d^11*x^2)*(d^14/c^23)^(5/6) - 4*(c^12*d^17*
x^6 + 41*c^13*d^16*x^3 + 40*c^14*d^15)*sqrt(d^14/c^23) - (c^4*d^22*x^7 - 28
*c^5*d^21*x^4 - 272*c^6*d^20*x)*(d^14/c^23)^(1/6)) + 18*(c^8*d^20*x^8 + 20*
c^9*d^19*x^5 - 8*c^10*d^18*x^2)*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
+ 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*x)) + 57
6*(d^3*x^10 - 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPI
nverse(0, -4*c/d, x)) + 91*(c^4*d*x^10 - 8*c^5*x^7)*(d^14/c^23)^(1/6)*log(1
37858491849*(d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22
- 18*(c^16*d^15*x^7 - 52*c^17*d^14*x^4 - 80*c^18*d^13*x)*(d^14/c^23)^(2/3)
+ 6*sqrt(d*x^3 + c)*(24*(c^20*d^12*x^5 + c^21*d^11*x^2)*(d^14/c^23)^(5/6) -
 4*(c^12*d^17*x^6 + 41*c^13*d^16*x^3 + 40*c^14*d^15)*sqrt(d^14/c^23) - (c^4
*d^22*x^7 - 28*c^5*d^21*x^4 - 272*c^6*d^20*x)*(d^14/c^23)^(1/6)) + 18*(c^8*
d^20*x^8 + 20*c^9*d^19*x^5 - 8*c^10*d^18*x^2)*(d^14/c^23)^(1/3))/(d^3*x^9 -
 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 91*(c^4*d*x^10 - 8*c^5*x^7)*(d^
14/c^23)^(1/6)*log(137858491849*(d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*
x^3 - 1088*c^3*d^22 - 18*(c^16*d^15*x^7 - 52*c^17*d^14*x^4 - 80*c^18*d^13*x
)*(d^14/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^20*d^12*x^5 + c^21*d^11*x^2)
*(d^14/c^23)^(5/6) - 4*(c^12*d^17*x^6 + 41*c^13*d^16*x^3 + 40*c^14*d^15)*sq
rt(d^14/c^23) - (c^4*d^22*x^7 - 28*c^5*d^21*x^4 - 272*c^6*d^20*x)*(d^14/c^2
3)^(1/6)) + 18*(c^8*d^20*x^8 + 20*c^9*d^19*x^5 - 8*c^10*d^18*x^2)*(d^14/c^2
3)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 182*(c^4*d*
x^10 - 8*c^5*x^7)*(d^14/c^23)^(1/6)*log(371293*(d^14*x^9 + 318*c*d^13*x^6 +
 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 +
32*c^18*d^2*x)*(d^14/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*
c^21*x^2)*(d^14/c^23)^(5/6) + (7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*
d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)*(d^
14/c^23)^(1/6)) + 18*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(d^14
/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 182*(c^
4*d*x^10 - 8*c^5*x^7)*(d^14/c^23)^(1/6)*log(371293*(d^14*x^9 + 318*c*d^13*x
^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^
4 + 32*c^18*d^2*x)*(d^14/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 +
 32*c^21*x^2)*(d^14/c^23)^(5/6) + (7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c
^14*d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)
*(d^14/c^23)^(1/6)) + 18*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(
d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 144
*(4*d^3*x^9 + 21*c*d^2*x^6 - 264*c^2*d*x^3 - 384*c^3)*sqrt(d*x^3 + c))/(c^4
*d*x^10 - 8*c^5*x^7)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**8*(-8*c + d*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{x^8\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)^2),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)^2), x)
```

3.411 $\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=134

$$\frac{1664c^3\sqrt{c+dx^3}}{d^4}+\frac{3x^6(c+dx^3)^{3/2}}{7d^2}+\frac{x^9(c+dx^3)^{3/2}}{3d(8c-dx^3)}+\frac{2c(c+dx^3)^{3/2}(694c+51dx^3)}{21d^4}-\frac{4992c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

```
[Out] 3/7*x^6*(d*x^3+c)^(3/2)/d^2+1/3*x^9*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)+2/21*c*(
d*x^3+c)^(3/2)*(51*d*x^3+694*c)/d^4-4992*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2
)/c^(1/2))/d^4+1664*c^3*(d*x^3+c)^(1/2)/d^4
```

Rubi [A]

time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {457, 99, 158, 152, 52, 65, 212}

$$-\frac{4992c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}+\frac{1664c^3\sqrt{c+dx^3}}{d^4}+\frac{2c(c+dx^3)^{3/2}(694c+51dx^3)}{21d^4}+\frac{3x^6(c+dx^3)^{3/2}}{7d^2}+\frac{x^9(c+dx^3)^{3/2}}{3d(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (1664*c^3*Sqrt[c + d*x^3])/d^4 + (3*x^6*(c + d*x^3)^(3/2))/(7*d^2) + (x^9*(
c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (2*c*(c + d*x^3)^(3/2)*(694*c + 51*
d*x^3))/(21*d^4) - (4992*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}\,\left(3c+\frac{9dx}{2}\right)}{8c-dx}\,dx, x, x^3\right)}{3d}$$

$$= \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2\text{Subst}\left(\int \frac{x\sqrt{c+dx}\,\left(-72c^2d-\frac{255}{2}cd^2x\right)}{8c-dx}\,dx, x, x^3\right)}{21d^3}$$

$$= \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2c(c+dx^3)^{3/2}\,(694c+51dx^3)}{21d^4} - \frac{(832c^3)\,\text{Subst}\left(\right.}{}$$

$$= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2c(c+dx^3)^{3/2}\,(694c+51dx}{21d^4}$$

$$= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2c(c+dx^3)^{3/2}\,(694c+51dx}{21d^4}$$

$$= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2c(c+dx^3)^{3/2}\,(694c+51dx}{21d^4}$$

Mathematica [A]
time = 0.15, size = 103, normalized size = 0.77

$$\frac{2\sqrt{c+dx^3}\,(-145328c^4+12206c^3dx^3+301c^2d^2x^6+16cd^3x^9+d^4x^{12})}{21d^4\,(-8c+dx^3)} - \frac{4992c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-145328*c^4 + 12206*c^3*d*x^3 + 301*c^2*d^2*x^6 + 16*c*
d^3*x^9 + d^4*x^12))/(21*d^4*(-8*c + d*x^3)) - (4992*c^(7/2)*ArcTanh[Sqrt[c
 + d*x^3]/(3*Sqrt[c])])/d^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 999, normalized size = 7.46

method	result

	8
elliptic	$\frac{1536c^4\sqrt{d\,x^3+c}}{d^4(-d\,x^3+8c)}+\frac{2x^9\sqrt{d\,x^3+c}}{21d}+\frac{16c\,x^6\sqrt{d\,x^3+c}}{7d^2}+\frac{986c^2x^3\sqrt{d\,x^3+c}}{21d^3}+\frac{32300c^3\sqrt{d\,x^3+c}}{21d^4}+-$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d*(2/21*d*x^9*(d*x^3+c)^(1/2)+16/105*c*x^6*(d*x^3+c)^(1/2)+2/105*c^2
/d*x^3*(d*x^3+c)^(1/2)-4/105*c^3/d^2*(d*x^3+c)^(1/2))+32/15*c/d*(d*x^3+c)^(
5/2))+192/d^3*c^2*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I*c/d
^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^
2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d
^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)
^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/1
8/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*d-8*c))+512*c^3/d^3*(3*c/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/3*(d*x^3+
c)^(1/2)/d+1/2*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)
^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+
1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_al
pha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/
```

```
3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [A]
time = 0.48, size = 119, normalized size = 0.89

$$\frac{2\left(26208\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+(dx^3+c)^{\frac{7}{2}}+21\,(dx^3+c)^{\frac{5}{2}}c+448\,(dx^3+c)^{\frac{3}{2}}c^2+15680\sqrt{dx^3+c}\,c^3-\frac{16128\sqrt{dx^3+c}\,c^4}{dx^3-8\,c}\right)}{21\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/21*(26208*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + (d*x^3 + c)^(7/2) + 21*(d*x^3 + c)^(5/2)*c + 448*(d*x^3 + c)^(3
/2)*c^2 + 15680*sqrt(d*x^3 + c)*c^3 - 16128*sqrt(d*x^3 + c)*c^4/(d*x^3 - 8*
c))/d^4
```

Fricas [A]
time = 3.08, size = 239, normalized size = 1.78

$$\left[\frac{2\left(26208\,(c^3dx^3-8\,c^4)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(d^4x^{12}+16\,cd^3x^9+301\,c^2d^2x^6+12206\,c^3dx^3-145328\,c^4)\sqrt{dx^3+c}\right)}{21\,(d^5x^3-8\,cd^4)},\frac{2\left(52416\,(c^3dx^3-8\,c^4)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(d^4x^{12}+16\,cd^3x^9+301\,c^2d^2x^6+12206\,c^3dx^3-145328\,c^4)\sqrt{dx^3+c}\right)}{21\,(d^5x^3-8\,cd^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/21*(26208*(c^3*d*x^3 - 8*c^4)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + (d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 +
12206*c^3*d*x^3 - 145328*c^4)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/21*(5
2416*(c^3*d*x^3 - 8*c^4)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) +
(d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 + 12206*c^3*d*x^3 - 145328*c^4)*
sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 127, normalized size = 0.95

$$\frac{4992\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^4}-\frac{1536\sqrt{dx^3+c}\,c^4}{(dx^3-8\,c)d^4}+\frac{2\left((dx^3+c)^{\frac{7}{2}}d^{24}+21\,(dx^3+c)^{\frac{5}{2}}cd^{24}+448\,(dx^3+c)^{\frac{3}{2}}c^2d^{24}+15680\sqrt{dx^3+c}\,c^3d^{24}\right)}{21\,d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] 4992*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 1536*sqrt(d*x^3 + c)*c^4/((d*x^3 - 8*c)*d^4) + 2/21*((d*x^3 + c)^(7/2)*d^24 + 21*(d*x^3 + c)^(5/2)*c*d^24 + 448*(d*x^3 + c)^(3/2)*c^2*d^24 + 15680*sqrt(d*x^3 + c)*c^3*d^24)/d^28

Mupad [B]
time = 4.10, size = 147, normalized size = 1.10

$$\frac{2496\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4}+\frac{32300\,c^3\,\sqrt{d\,x^3+c}}{21\,d^4}+\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d}+\frac{16\,c\,x^6\,\sqrt{d\,x^3+c}}{7\,d^2}+\frac{986\,c^2\,x^3\,\sqrt{d\,x^3+c}}{21\,d^3}+\frac{1536\,c^4\,\sqrt{d\,x^3+c}}{d^4\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)

[Out] (2496*c^(7/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d^4 + (32300*c^3*(c + d*x^3)^(1/2))/(21*d^4) + (2*x^9*(c + d*x^3)^(1/2))/(21*d) + (16*c*x^6*(c + d*x^3)^(1/2))/(7*d^2) + (986*c^2*x^3*(c + d*x^3)^(1/2))/(21*d^3) + (1536*c^4*(c + d*x^3)^(1/2))/(d^4*(8*c - d*x^3))

3.412 $\int \frac{x^8\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=119

$$\frac{160c^2\sqrt{c+dx^3}}{d^3}+\frac{160c(c+dx^3)^{3/2}}{27d^3}+\frac{2(c+dx^3)^{5/2}}{15d^3}+\frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)}-\frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

```
[Out] 160/27*c*(d*x^3+c)^(3/2)/d^3+2/15*(d*x^3+c)^(5/2)/d^3+64/27*c*(d*x^3+c)^(5/
2)/d^3/(-d*x^3+8*c)-480*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3+16
0*c^2*(d*x^3+c)^(1/2)/d^3
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 91, 81, 52, 65, 212}

$$-\frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}+\frac{160c^2\sqrt{c+dx^3}}{d^3}+\frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)}+\frac{160c(c+dx^3)^{3/2}}{27d^3}+\frac{2(c+dx^3)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (160*c^2*Sqrt[c + d*x^3])/d^3 + (160*c*(c + d*x^3)^(3/2))/(27*d^3) + (2*(c
+ d*x^3)^(5/2))/(15*d^3) + (64*c*(c + d*x^3)^(5/2))/(27*d^3*(8*c - d*x^3))
- (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{(c+dx)^{3/2}(168c^2d+9cd^2x)}{8c-dx}\,dx, x, x^3\right)}{27cd^3}$$

$$= \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{(80c)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{9d^2}$$

$$= \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{(80c^2)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,\right.}{d^2}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{(720c^3)\,\text{Su}}{}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{(1440c^3)\,\text{S}}{}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{480c^{5/2}\tan}{}$$

Mathematica [A]
time = 0.13, size = 93, normalized size = 0.78

$$\frac{2\sqrt{c+dx^3}\,(-29944c^3+2515c^2dx^3+62cd^2x^6+3d^3x^9)}{45d^3(-8c+dx^3)} - \frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-29944*c^3 + 2515*c^2*d*x^3 + 62*c*d^2*x^6 + 3*d^3*x^9)
)/(45*d^3*(-8*c + d*x^3)) - (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/d^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.41, size = 921, normalized size = 7.74

method	result

elliptic	$\frac{192c^3\sqrt{d\,x^3+c}}{d^3(-d\,x^3+8c)}+\frac{2x^6\sqrt{d\,x^3+c}}{15d}+\frac{172c\,x^3\sqrt{d\,x^3+c}}{45d^2}+\frac{6406c^2\sqrt{d\,x^3+c}}{45d^3}+\frac{80ic^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z\right.}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/d^3+64*c^2/d^2*(3*c/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/3
*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(
-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*
(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/
3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*
d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+16/d^2*c*(2/9*x^3*(d*x^3+c)^(1/2)
+56/9*c*(d*x^3+c)^(1/2)/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*
x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d
^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*
d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
```

```
*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [A]
time = 0.49, size = 107, normalized size = 0.90

$$\frac{2\left(5400\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+80\,(dx^3+c)^{\frac{3}{2}}c+3120\,\sqrt{dx^3+c}\,c^2-\frac{4320\,\sqrt{dx^3+c}\,c^3}{dx^3-8\,c}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/45*(5400*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 3*(d*x^3 + c)^(5/2) + 80*(d*x^3 + c)^(3/2)*c + 3120*sqrt(d*x^3 +
 c)*c^2 - 4320*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^3
```

Fricas [A]
time = 2.39, size = 219, normalized size = 1.84

$$\left[\frac{2\left(5400\,(c^2dx^3-8\,c^3)\sqrt{c}\,\log\left(\frac{dx^3-6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(3\,d^3x^9+62\,cd^2x^6+2515\,c^2dx^3-29944\,c^3)\sqrt{dx^3+c}\right)}{45\,(d^4x^3-8\,cd^3)},\ \frac{2\left(10800\,(c^2dx^3-8\,c^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+(3\,d^3x^9+62\,cd^2x^6+2515\,c^2dx^3-29944\,c^3)\sqrt{dx^3+c}\right)}{45\,(d^4x^3-8\,cd^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/45*(5400*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + (3*d^3*x^9 + 62*c*d^2*x^6 + 2515*c^2*d*x^3 - 2
9944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3), 2/45*(10800*(c^2*d*x^3 - 8*
c^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^3*x^9 + 62*c*d^
2*x^6 + 2515*c^2*d*x^3 - 29944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**8*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Giac [A]
time = 0.61, size = 111, normalized size = 0.93

$$\frac{480\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^3}-\frac{192\,\sqrt{dx^3+c}\,c^3}{(dx^3-8\,c)d^3}+\frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}d^{12}+80\,(dx^3+c)^{\frac{3}{2}}cd^{12}+3120\,\sqrt{dx^3+c}\,c^2d^{12}\right)}{45\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 480*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 192*sqrt(d*x^
3 + c)*c^3/((d*x^3 - 8*c)*d^3) + 2/45*(3*(d*x^3 + c)^(5/2)*d^12 + 80*(d*x^3
 + c)^(3/2)*c*d^12 + 3120*sqrt(d*x^3 + c)*c^2*d^12)/d^15
```

Mupad [B]
time = 4.05, size = 127, normalized size = 1.07

$$\frac{240\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3}+\frac{6406\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d}+\frac{172\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}+\frac{192\,c^3\,\sqrt{d\,x^3+c}}{d^3\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (240*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^3 + (6406*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2))/
(15*d) + (172*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (192*c^3*(c + d*x^3)^(1/2
))/(d^3*(8*c - d*x^3))
```

3.413 $\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=97

$$\frac{14c\sqrt{c+dx^3}}{d^2}+\frac{14(c+dx^3)^{3/2}}{27d^2}+\frac{8(c+dx^3)^{5/2}}{27d^2\,(8c-dx^3)}-\frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

```
[Out] 14/27*(d*x^3+c)^(3/2)/d^2+8/27*(d*x^3+c)^(5/2)/d^2/(-d*x^3+8*c)-42*c^(3/2)*
arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2+14*c*(d*x^3+c)^(1/2)/d^2
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 79, 52, 65, 212}

$$-\frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}+\frac{8(c+dx^3)^{5/2}}{27d^2\,(8c-dx^3)}+\frac{14(c+dx^3)^{3/2}}{27d^2}+\frac{14c\sqrt{c+dx^3}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (14*c*Sqrt[c + d*x^3])/d^2 + (14*(c + d*x^3)^(3/2))/(27*d^2) + (8*(c + d*x^
3)^(5/2))/(27*d^2*(8*c - d*x^3)) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*S
qrt[c])])/d^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)} - \frac{7\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{9d}$$

$$= \frac{14(c+dx^3)^{3/2}}{27d^2} + \frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)} - \frac{(7c)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d}$$

$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14(c+dx^3)^{3/2}}{27d^2} + \frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)} - \frac{(63c^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+d}}\right.}{d}$$

$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14(c+dx^3)^{3/2}}{27d^2} + \frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)} - \frac{(126c^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{}\right.}{d^2}$$

$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14(c+dx^3)^{3/2}}{27d^2} + \frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)} - \frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

Mathematica [A]

time = 0.11, size = 81, normalized size = 0.84

$$\frac{2\sqrt{c+dx^3}\,(-524c^2+44cdx^3+d^2x^6)}{9d^2\,(-8c+dx^3)}-\frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] (2*Sqrt[c + d*x^3]*(-524*c^2 + 44*c*d*x^3 + d^2*x^6))/(9*d^2*(-8*c + d*x^3)) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 903, normalized size = 9.31

method	result
elliptic	$\frac{24c^2\sqrt{d\,x^3+c}}{d^2(-d\,x^3+8c)}+\frac{2x^3\sqrt{d\,x^3+c}}{9d}+\frac{104c\sqrt{d\,x^3+c}}{9d^2}+\frac{7ic\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}(d\_Z^3-8c)}\frac{(-c\,d^2)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\right.}{}}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3

```
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)+8*c/d*(3*c/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3
*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/
d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))
```

Maxima [A]
time = 0.50, size = 93, normalized size = 0.96

$$\frac{189\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\,(dx^3+c)^{\frac{3}{2}}+102\,\sqrt{dx^3+c}\,c-\frac{216\,\sqrt{dx^3+c}\,c^2}{dx^3-8\,c}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(189*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 2*(d*x^3 + c)^(3/2) + 102*sqrt(d*x^3 + c)*c - 216*sqrt(d*x^3 + c)*
c^2/(d*x^3 - 8*c))/d^2
```

Fricas [A]
time = 4.15, size = 192, normalized size = 1.98

$$\left[\frac{189\,(cdx^3-8\,c^2)\sqrt{c}\,\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+2\,(d^2x^6+44\,cdx^3-524\,c^2)\sqrt{dx^3+c}}{9\,(d^3x^3-8\,cd^2)},\frac{2\left(189\,(cdx^3-8\,c^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+(d^2x^6+44\,cdx^3-524\,c^2)\sqrt{dx^3+c}\right)}{9\,(d^3x^3-8\,cd^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(189*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
+ 10*c)/(d*x^3 - 8*c)) + 2*(d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 + c)
)/(d^3*x^3 - 8*c*d^2), 2/9*(189*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(
d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 + c))/
(d^3*x^3 - 8*c*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (c + dx^3)^{\frac{3}{2}}}{(-8c + dx^3)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**5*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Giac [A]
time = 0.55, size = 93, normalized size = 0.96

$$\frac{42\, c^2 \arctan\left(\frac{\sqrt{dx^3 + c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\, d^2} - \frac{24\sqrt{dx^3 + c}\, c^2}{(dx^3 - 8\, c) d^2} + \frac{2\left((dx^3 + c)^{\frac{3}{2}} d^4 + 51\sqrt{dx^3 + c}\, cd^4\right)}{9\, d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 42*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 24*sqrt(d*x^3
+ c)*c^2/((d*x^3 - 8*c)*d^2) + 2/9*((d*x^3 + c)^(3/2)*d^4 + 51*sqrt(d*x^3 +
 c)*c*d^4)/d^6
```

Mupad [B]
time = 4.04, size = 107, normalized size = 1.10

$$\frac{104\, c\, \sqrt{d\, x^3 + c}}{9\, d^2} + \frac{21\, c^{3/2}\, \ln\left(\frac{10\, c + d\, x^3 - 6\, \sqrt{c}\, \sqrt{d\, x^3 + c}}{8\, c - d\, x^3}\right)}{d^2} + \frac{2\, x^3\, \sqrt{d\, x^3 + c}}{9\, d} + \frac{24\, c^2\, \sqrt{d\, x^3 + c}}{d^2\, (8\, c - d\, x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (104*c*(c + d*x^3)^(1/2))/(9*d^2) + (21*c^(3/2)*log((10*c + d*x^3 - 6*c^(1/
2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d^2 + (2*x^3*(c + d*x^3)^(1/2))/(9*d)
 + (24*c^2*(c + d*x^3)^(1/2))/(d^2*(8*c - d*x^3))
```

3.414 $\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{c+dx^3}}{d}+\frac{(c+dx^3)^{3/2}}{3d\,(8c-dx^3)}-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

```
[Out] 1/3*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c
^(1/2)/d+(d*x^3+c)^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {455, 43, 52, 65, 212}

$$\frac{(c+dx^3)^{3/2}}{3d\,(8c-dx^3)}+\frac{\sqrt{c+dx^3}}{d}-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/d + (c + d*x^3)^(3/2)/(3*d*(8*c - d*x^3)) - (3*Sqrt[c]*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)\\
&= \frac{(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{d} + \frac{(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} - \frac{1}{2}(9c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{d} + \frac{(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} - \frac{(9c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}\\
&= \frac{\sqrt{c+dx^3}}{d} + \frac{(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} - \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 0.94

$$\frac{(25c-2dx^3)\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] ((25*c - 2*d*x^3)*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) - (3*Sqrt[c]*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 452, normalized size = 5.87

method	result
default	$\frac{3c\sqrt{d\,x^{3}+c}}{d\left(-d\,x^{3}+8c\right)}+\frac{2\sqrt{d\,x^{3}+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$
elliptic	$\frac{3c\sqrt{d\,x^{3}+c}}{d\left(-d\,x^{3}+8c\right)}+\frac{2\sqrt{d\,x^{3}+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)

[Out] 3*c/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*
sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1

```
/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_
alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alp
ha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-
c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c
*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*
c))
```

Maxima [A]
time = 0.49, size = 79, normalized size = 1.03

$$\frac{9\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+4\sqrt{dx^3+c}-\frac{18\sqrt{dx^3+c}\,c}{dx^3-8c}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(9*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(
c))) + 4*sqrt(d*x^3 + c) - 18*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d
```

Fricas [A]
time = 3.95, size = 162, normalized size = 2.10

$$\left[\frac{9\,(dx^3-8\,c)\sqrt{c}\,\log\left(\frac{dx^3-6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+2\,(2\,dx^3-25\,c)\sqrt{dx^3+c}}{6\,(d^2x^3-8\,cd)},\ \frac{9\,(dx^3-8\,c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+(2\,dx^3-25\,c)\sqrt{dx^3+c}}{3\,(d^2x^3-8\,cd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) + 2*(2*d*x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d), 1
/3*(9*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*d*
x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**2*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Giac [A]
time = 0.63, size = 69, normalized size = 0.90

$$\frac{3\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d}+\frac{2\,\sqrt{dx^3+c}}{3\,d}-\frac{3\,\sqrt{dx^3+c}\,c}{(dx^3-8\,c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 3*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 2/3*sqrt(d*x^3 + c)
/d - 3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d)
```

Mupad [B]
time = 3.99, size = 87, normalized size = 1.13

$$\frac{2\,\sqrt{d\,x^3+c}}{3\,d}+\frac{3\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{2\,d}+\frac{3\,c\,\sqrt{d\,x^3+c}}{d\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d) + (3*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c +
 d*x^3)^(1/2))/(8*c - d*x^3)))/(2*d) + (3*c*(c + d*x^3)^(1/2))/(d*(8*c - d*
x^3))
```

3.415 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2} dx$

Optimal. Leaf size=85

$$\frac{3\sqrt{c+dx^3}}{8(8c-dx^3)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

```
[Out] -3/32*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/96*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(1/2)+3/8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {457, 100, 162, 65, 214, 212}

$$\frac{3\sqrt{c+dx^3}}{8(8c-dx^3)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (3*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(32*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*Sqrt[c])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x\,(8c-dx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{3\sqrt{c+dx^3}}{8\,(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{-c^2d+\frac{7}{2}cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24cd}$$

$$= \frac{3\sqrt{c+dx^3}}{8\,(8c-dx^3)} + \frac{1}{192}\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) - \frac{1}{64}(9d)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c}}\right.$$

$$= \frac{3\sqrt{c+dx^3}}{8\,(8c-dx^3)} - \frac{9}{32}\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c}\right.}{96d}$$

$$= \frac{3\sqrt{c+dx^3}}{8\,(8c-dx^3)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Mathematica [A]
time = 0.08, size = 85, normalized size = 1.00

$$\frac{3\sqrt{c+dx^3}}{8(8c-dx^3)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (3*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(32*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*Sqrt[c])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 957, normalized size = 11.26

method	result	size
default	Expression too large to display	957
elliptic	Expression too large to display	1525

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*d/c^2*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I*c/d^3*2^(
1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)
*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*d-8*c)))+1/64/c^2*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(
3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))+1/8*d/c*(3*c/d*(d*x^3+c)^(1/2)/(-d*x
^3+8*c)+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-
```

```
3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x), x)
```

Fricas [A]
time = 3.35, size = 220, normalized size = 2.59

$$\left[\frac{9\,(dx^3-8\,c)\sqrt{c}\,\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(dx^3-8\,c)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-72\,\sqrt{dx^3+c}\,c}{192\,(cdx^3-8\,c^2)},\ \frac{(dx^3-8\,c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)+9\,(dx^3-8\,c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-36\,\sqrt{dx^3+c}\,c}{96\,(cdx^3-8\,c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
[Out] [1/192*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + (d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*s
qrt(c) + 2*c)/x^3) - 72*sqrt(d*x^3 + c)*c)/(c*d*x^3 - 8*c^2), 1/96*((d*x^3
- 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 9*(d*x^3 - 8*c)*sqrt(-
c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 36*sqrt(d*x^3 + c)*c)/(c*d*x^3
- 8*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^3)^{\frac{3}{2}}}{x\,(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c)**2,x)
[Out] Integral((c + d*x**3)**(3/2)/(x*(-8*c + d*x**3)**2), x)
```

Giac [A]
time = 0.59, size = 70, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}}+\frac{3\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{32\,\sqrt{-c}}-\frac{3\,\sqrt{dx^3+c}}{8\,(dx^3-8\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) + 3/32*arctan(1/3*sqrt(d*x^3
 + c)/sqrt(-c))/sqrt(-c) - 3/8*sqrt(d*x^3 + c)/(d*x^3 - 8*c)
```

Mupad [B]
time = 4.72, size = 101, normalized size = 1.19

$$\frac{3\sqrt{dx^3+c}}{8(8c-dx^3)}+\frac{\ln\left(\frac{\left(\sqrt{dx^3+c}-\sqrt{c}\right)^3\left(\sqrt{dx^3+c}+\sqrt{c}\right)\left(10c+dx^3-6\sqrt{c}\sqrt{dx^3+c}\right)^9}{x^6(8c-dx^3)^9}\right)}{192\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x)
```

```
[Out] (3*(c + d*x^3)^(1/2))/(8*(8*c - d*x^3)) + log((((c + d*x^3)^(1/2) - c^(1/2)
)^3*((c + d*x^3)^(1/2) + c^(1/2))*(10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/
2))^9)/(x^6*(8*c - d*x^3)^9))/(192*c^(1/2))
```

3.416 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx$

Optimal. Leaf size=121

$$\frac{5d\sqrt{c+dx^3}}{96c\,(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{24x^3\,(8c-dx^3)}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}}-\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}$$

```
[Out] 3/128*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-7/384*d*arctanh((d*x^3
+c)^(1/2)/c^(1/2))/c^(3/2)+5/96*d*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/24*(d*x^
3+c)^(1/2)/x^3/(-d*x^3+8*c)
```

Rubi [A]
time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {457, 100, 156, 162, 65, 214, 212}

$$\frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}}-\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}+\frac{5d\sqrt{c+dx^3}}{96c\,(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{24x^3\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] (5*d*Sqrt[c + d*x^3])/(96*c*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*x^3*(8*c -
 d*x^3)) + (3*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*c^(3/2)) - (7*d*
ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(8c-dx)^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{-14c^2d-\frac{19}{2}cd^2x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}$$

$$= \frac{5d\sqrt{c+dx^3}}{96c(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24x^3(8c-dx^3)} + \frac{\text{Subst}\left(\int \frac{126c^3d^2+45c^2d^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^3d}$$

$$= \frac{5d\sqrt{c+dx^3}}{96c(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24x^3(8c-dx^3)} + \frac{(7d)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{768c} + \frac{(9d^2)\,\text{Su}}{}$$

$$= \frac{5d\sqrt{c+dx^3}}{96c(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24x^3(8c-dx^3)} + \frac{7\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{384c} + \frac{(9d)\text{Sub}}{}$$

$$= \frac{5d\sqrt{c+dx^3}}{96c(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24x^3(8c-dx^3)} + \frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}} - \frac{7d\tanh^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c}}\right.}{384c^{3/2}}$$

Mathematica [A]
time = 0.17, size = 97, normalized size = 0.80

$$\frac{\frac{4\sqrt{c}\,(4c-5dx^3)\sqrt{c+dx^3}}{-8cx^3+dx^6} + 9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] ((4*Sqrt[c]*(4*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 9*d*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 7*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(38
4*c^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.43, size = 1015, normalized size = 8.39

method	result	size
risch	Expression too large to display	902
default	Expression too large to display	1015
elliptic	Expression too large to display	1550

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/256/c^3*d^2*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I*c/d^3*
2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1
/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d
*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3
^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*d-8*c))+1/64/c^2*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(
1/2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))+1/256/c^3*d*(2/9*d*x^3*(d*x^3+c)^(
1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))+1/
64/c^2*d^2*(3*c/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/3*(d*x^3+c)^(1/2)/d+1/2*I/
d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d
^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*
d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2
)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/
18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Root
Of(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^4), x)
```

Fricas [A]
time = 3.00, size = 280, normalized size = 2.31

$$\left[\frac{9\,(d^2x^6 - 8\,cdx^3)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right) + 7\,(d^2x^6 - 8\,cdx^3)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right) - 8\,(5\,cdx^3 - 4\,c^2)\sqrt{dx^3+c}}{768\,(c^2dx^6 - 8\,c^3x^3)}, \frac{7\,(d^2x^6 - 8\,cdx^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right) - 9\,(d^2x^6 - 8\,cdx^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right) - 4\,(5\,cdx^3 - 4\,c^2)\sqrt{dx^3+c}}{384\,(c^2dx^6 - 8\,c^3x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/768*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 - 2
*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c
))/(c^2*d*x^6 - 8*c^3*x^3), 1/384*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(
sqrt(d*x^3 + c)*sqrt(-c)/c) - 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) - 4*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^2*d*
x^6 - 8*c^3*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^3)^{\frac{3}{2}}}{x^4\,(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x**4*(-8*c + d*x**3)**2), x)
```

Giac [A]
time = 0.53, size = 114, normalized size = 0.94

$$\frac{7\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}\,c}-\frac{3\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{128\,\sqrt{-c}\,c}-\frac{5\,(dx^3+c)^{\frac{3}{2}}d-9\,\sqrt{dx^3+c}\,cd}{96\,\left((dx^3+c)^2-10\,(dx^3+c)c+9\,c^2\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 7/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 3/128*d*arctan(1/3*
sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/96*(5*(d*x^3 + c)^(3/2)*d - 9*sq
rt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c)
```

Mupad [B]
time = 4.24, size = 110, normalized size = 0.91

$$\frac{\frac{9\,d\,\sqrt{d\,x^3+c}}{32}-\frac{5\,d\,{\left(d\,x^3+c\right)}^{3/2}}{32\,c}}{3\,{\left(d\,x^3+c\right)}^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2}+\frac{d\,\left(\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)\,9{}\mathrm{i}}{7}\right)\,7{}\mathrm{i}}{384\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x)
```

```
[Out] ((9*d*(c + d*x^3)^(1/2))/32 - (5*d*(c + d*x^3)^(3/2))/(32*c))/(3*(c + d*x^3
)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1
/2))*1i - (atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2)))*9i)/7)*7i)/(384*(c^
3)^(1/2))
```

3.417 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx$

Optimal. Leaf size=161

$$\frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)}-\frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)}+\frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}}-\frac{17d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}$$

```
[Out] 15/2048*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-17/2048*d^2*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+7/512*d^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*
c)-1/48*(d*x^3+c)^(1/2)/x^6/(-d*x^3+8*c)-23/384*d*(d*x^3+c)^(1/2)/c/x^3/(-d
*x^3+8*c)
```

Rubi [A]
time = 0.09, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {457, 100, 156, 162, 65, 214, 212}

$$\frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}}-\frac{17d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}+\frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)}-\frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] (7*d^2*Sqrt[c + d*x^3])/(512*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*x^6*(
8*c - d*x^3)) - (23*d*Sqrt[c + d*x^3])/(384*c*x^3*(8*c - d*x^3)) + (15*d^2*
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2048*c^(5/2)) - (17*d^2*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(2048*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(8c-dx)^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{-23c^2d-\frac{37}{2}cd^2x}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)} - \frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)} + \frac{\text{Subst}\left(\int \frac{102c^3d^2+\frac{69}{2}c^2d^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)} - \frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{-918c^4d^3-189c^3d}{x(8c-dx)\sqrt{c+d}}\right.}{27648c^5d}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)} - \frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)} + \frac{(17d^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+}}\right.}{4096c^2}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)} - \frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)} + \frac{(17d)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,d\right.}{2048c^2}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)} - \frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)} + \frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right.}{2048c^{5/2}}$$

Mathematica [A]
time = 0.22, size = 112, normalized size = 0.70

$$\frac{\frac{4\sqrt{c}\sqrt{c+dx^3}(32c^2+92cdx^3-21d^2x^6)}{-8cx^6+dx^9} + 45d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 51d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] ((4*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 92*c*d*x^3 - 21*d^2*x^6))/(-8*c*x^6 +
 d*x^9) + 45*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 51*d^2*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(6144*c^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 1076, normalized size = 6.68

method	result	size

risch	Expression too large to display	912
default	Expression too large to display	1076
elliptic	Expression too large to display	1580

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*(-1/6*c*(d*x^3+c)^(1/2)/x^6-5/12*d*(d*x^3+c)^(1/2)/x^3-1/4*d^2*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))-3/4096*d^3/c^4*(2/9*x^3*(d*x^3+c)^(1
/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*
(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*
(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/
2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^
2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3
*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(-1/3*c*(d*
x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d*arctanh((d*x^3+c)^(1/2)/c^
(1/2)))+3/4096*d^2/c^4*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3
*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))+1/512/c^3*d^3*(3*c/d*(d*x^3+c)^(
1/2)/(-d*x^3+8*c)+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/
3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^7), x)
```

Fricas [A]
time = 2.65, size = 310, normalized size = 1.93

$$\left[\frac{45\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+51\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-8\,(21\,cd^2x^6-92\,c^2dx^3-32\,c^3)\sqrt{dx^3+c}}{12288\,(c^3dx^9-8\,c^4x^6)},\ \frac{51\,(d^3x^9-8\,cd^2x^6)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-45\,(d^3x^9-8\,cd^2x^6)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-4\,(21\,cd^2x^6-92\,c^2dx^3-32\,c^3)\sqrt{dx^3+c}}{6144\,(c^3dx^9-8\,c^4x^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/12288*(45*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)
*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 51*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d
*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(21*c*d^2*x^6 - 92*c^2*d*x
^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6), 1/6144*(51*(d^3*x^9
- 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 45*(d^3*x^9 -
8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(21*c*d^2*
x^6 - 92*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.83, size = 129, normalized size = 0.80

$$\frac{17\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}\,c^2}-\frac{15\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2048\,\sqrt{-c}\,c^2}-\frac{3\,\sqrt{dx^3+c}\,d^2}{512\,(dx^3-8\,c)c^2}-\frac{3\,(dx^3+c)^{\frac{3}{2}}d^2-2\,\sqrt{dx^3+c}\,cd^2}{384\,c^2d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 17/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 15/2048*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 3/512*sqrt(d*x^3 + c)*
d^2/((d*x^3 - 8*c)*c^2) - 1/384*(3*(d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3 + c
)*c*d^2)/(c^2*d^2*x^6)
```

Mupad [B]
time = 4.62, size = 151, normalized size = 0.94

$$\frac{\frac{81\,d^2\,\sqrt{d\,x^3+c}}{512}-\frac{67\,d^2\,{\left(d\,x^3+c\right)}^{3/2}}{256\,c}+\frac{21\,d^2\,{\left(d\,x^3+c\right)}^{5/2}}{512\,c^2}}{33\,c\,{\left(d\,x^3+c\right)}^2-57\,c^2\,\left(d\,x^3+c\right)-3\,{\left(d\,x^3+c\right)}^3+27\,c^3}+\frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)\,15{}\mathrm{i}}{17}\right)\,17{}\mathrm{i}}{2048\,\sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x)

[Out] ((81*d^2*(c + d*x^3)^(1/2))/512 - (67*d^2*(c + d*x^3)^(3/2))/(256*c) + (21*d^2*(c + d*x^3)^(5/2))/(512*c^2))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3) - 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*15i)/17)*17i)/(2048*(c^5)^(1/2))

3.418 $\int \frac{x^7 (c+dx^3)^{3/2}}{(8c-dx^3)^2} \, dx$

Optimal. Leaf size=681

$$\frac{103cx^2\sqrt{c+dx^3}}{13d^2}+\frac{19x^5\sqrt{c+dx^3}}{39d}+\frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{x^5(c+dx^3)^{3/2}}{3d\,(8c-dx^3)}+\frac{108\sqrt{3}\,c^{13/6}\tan^{-1}}{}$$

```
[Out] 1/3*x^5*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-108*c^(13/6)*arctanh(1/3*(c^(1/3)+d^
(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8/3)+108*c^(13/6)*arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/d^(8/3)+108*c^(13/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*
3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3)+103/13*c*x^2*(d*x^3+c)^(1/2)/d^2+1
9/39*x^5*(d*x^3+c)^(1/2)/d+5906/13*c^2*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))+5906/39*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/
2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-2953/13*3^(1/4)*c^(7/3)*(c^(1/3)+d^(1/
3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d
^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2
)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$, Rules used = {478, 595, 596, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{5906\sqrt{2}\,c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\,d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{2953\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{13d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{108\sqrt{3}\,c^{13/6}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{108c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{108c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}+\frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{103cx^2\sqrt{c+dx^3}}{13d^2}+\frac{x^5(c+dx^3)^{3/2}}{3d\,(8c-dx^3)}+\frac{19x^5\sqrt{c+dx^3}}{39d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (103*c*x^2*Sqrt[c + d*x^3])/(13*d^2) + (19*x^5*Sqrt[c + d*x^3])/(39*d) + (5
906*c^2*Sqrt[c + d*x^3])/(13*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) +
 (x^5*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (108*Sqrt[3]*c^(13/6)*ArcTan
[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(8/3) - (108*c
^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(8/
```

```
3) + (108*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(8/3) - (2953*3^
(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(13*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (5906*Sqrt[2]*c^(7
/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(13
*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{x^5(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} - \frac{\int \frac{x^4\sqrt{c+dx^3}\left(5c+\frac{19dx^3}{2}\right)}{8c-dx^3}\,dx}{3d}$$

$$= \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2\int \frac{x^4\left(-\frac{825c^2d}{2}-\frac{2163}{4}cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{39d^2}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{4\int \frac{x\left(-8652c^3d^2-\frac{62013}{4}c^2d^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,d}{273d^4}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{4\int \left(\frac{62013c^2d^2x}{4\sqrt{c+dx^3}} - \frac{132}{(8c-dx^3)}\right.}{273d^4}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{(2953c^2)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{13d^2} -$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{(162c^2)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)}}{d^3}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5(c+}{3d\,(8c}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5(c+}{3d\,(8c}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5(c+}{3d\,(8c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.85, size = 191, normalized size = 0.28

$$\frac{80x^2(-412c^3-388c^2dx^3+25cd^2x^6+d^3x^9)+4120c^2x^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+2953cdx^5(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{520d^2\left(-8c+dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (80*x^2*(-412*c^3 - 388*c^2*d*x^3 + 25*c*d^2*x^6 + d^3*x^9) + 4120*c^2*x^2*
(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
(d*x^3)/(8*c)] + 2953*c*d*x^5*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(520*d^2*(-8*c + d*x^3)*Sqrt[
c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 2224, normalized size = 3.27

method	result	size
elliptic	Expression too large to display	921
risch	Expression too large to display	1769
default	Expression too large to display	2224

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(2/13*d*x^5*(d*x^3+c)^(1/2)+32/91*c*x^2*(d*x^3+c)^(1/2)-18/91*I*c^2*3
^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))+16/d^2*c*(2/7*x^2*(d*x^3+c)^(1/
2)-44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
```

```
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*s
um(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1
/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d
*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3
^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*d-8*c)))+64*c^2/d^2*(3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-19/24*I*3^(1/
2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d
^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(
1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*
d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 58.72, size = 3882, normalized size = 5.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/13*(468*sqrt(3)*(d^4*x^3 - 8*c*d^3)*(c^13/d^16)^(1/6)*arctan(-1/3*(324*s
qrt(3)*(3*c^13*d^16*x^16 + 784*c^14*d^15*x^13 + 7680*c^15*d^14*x^10 + 10752
*c^16*d^13*x^7 + 4096*c^17*d^12*x^4)*(c^13/d^16)^(2/3) + 36*sqrt(3)*(c^17*d
^11*x^17 + 1772*c^18*d^10*x^14 + 42592*c^19*d^9*x^11 + 96256*c^20*d^8*x^8 +
 69632*c^21*d^7*x^5 + 16384*c^22*d^6*x^2)*(c^13/d^16)^(1/3) + sqrt(3)*(c^21
*d^6*x^18 + 9456*c^22*d^5*x^15 + 749184*c^23*d^4*x^12 + 3017216*c^24*d^3*x^
9 + 3489792*c^25*d^2*x^6 + 1572864*c^26*d*x^3 + 262144*c^27) + 12*sqrt(d*x^
3 + c)*(12*sqrt(3)*(35*c^11*d^18*x^14 - 14440*c^12*d^17*x^11 - 24576*c^13*d
^16*x^8 - 16384*c^14*d^15*x^5 - 4096*c^15*d^14*x^2)*(c^13/d^16)^(5/6) + 18*
sqrt(3)*(c^15*d^13*x^15 - 1112*c^16*d^12*x^12 + 7296*c^17*d^11*x^9 + 11776*
c^18*d^10*x^6 + 4096*c^19*d^9*x^3)*sqrt(c^13/d^16) + sqrt(3)*(c^19*d^8*x^16
 - 4768*c^20*d^7*x^13 + 362752*c^21*d^6*x^10 + 709120*c^22*d^5*x^7 + 413696
*c^23*d^4*x^4 + 65536*c^24*d^3*x)*(c^13/d^16)^(1/6)) - 2*(324*sqrt(3)*(d^19
*x^16 - 1858*c*d^18*x^13 - 4176*c^2*d^17*x^10 - 3584*c^3*d^16*x^7 - 1024*c^
4*d^15*x^4)*(c^13/d^16)^(5/6) + 18*sqrt(3)*(c^4*d^14*x^17 - 5290*c^5*d^13*x
^14 - 21152*c^6*d^12*x^11 - 47744*c^7*d^11*x^8 - 37888*c^8*d^10*x^5 - 8192*
c^9*d^9*x^2)*sqrt(c^13/d^16) + sqrt(3)*(c^8*d^9*x^18 - 7698*c^9*d^8*x^15 -
1664688*c^10*d^7*x^12 - 5524864*c^11*d^6*x^9 - 6223872*c^12*d^5*x^6 - 27033
60*c^13*d^4*x^3 - 327680*c^14*d^3)*(c^13/d^16)^(1/6) + 6*sqrt(d*x^3 + c)*(s
qrt(3)*(7*c^2*d^16*x^15 + 37352*c^3*d^15*x^12 - 230336*c^4*d^14*x^9 - 51507
2*c^5*d^13*x^6 - 286720*c^6*d^12*x^3 - 32768*c^7*d^11)*(c^13/d^16)^(2/3) +
108*sqrt(3)*(53*c^7*d^10*x^13 + 1320*c^8*d^9*x^10 + 1536*c^9*d^8*x^7 + 512*
c^10*d^7*x^4)*(c^13/d^16)^(1/3) + 6*sqrt(3)*(37*c^11*d^5*x^14 + 28912*c^12*
d^4*x^11 + 43584*c^13*d^3*x^8 + 20992*c^14*d^2*x^5 + 4096*c^15*d*x^2)))*sqr
t((18*c^22*d^2*x^8 + 360*c^23*d*x^5 - 144*c^24*x^2 + (c^13*d^13*x^9 - 276*c
^14*d^12*x^6 - 1608*c^15*d^11*x^3 - 1088*c^16*d^10)*(c^13/d^16)^(2/3) + 6*s
qrt(d*x^3 + c)*((c^11*d^15*x^7 - 28*c^12*d^14*x^4 - 272*c^13*d^13*x)*(c^13/
d^16)^(5/6) - 24*(c^16*d^9*x^5 + c^17*d^8*x^2)*sqrt(c^13/d^16) + 4*(c^20*d^
4*x^6 + 41*c^21*d^3*x^3 + 40*c^22*d^2)*(c^13/d^16)^(1/6)) - 18*(c^18*d^7*x^
7 - 52*c^19*d^6*x^4 - 80*c^20*d^5*x)*(c^13/d^16)^(1/3))/(d^3*x^9 - 24*c*d^2
*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^21*d^6*x^18 - 14952*c^22*d^5*x^15 + 28
72896*c^23*d^4*x^12 + 7330304*c^24*d^3*x^9 + 6696960*c^25*d^2*x^6 + 2457600
*c^26*d*x^3 + 262144*c^27)) - 468*sqrt(3)*(d^4*x^3 - 8*c*d^3)*(c^13/d^16)^(
1/6)*arctan(-1/3*(324*sqrt(3)*(3*c^13*d^16*x^16 + 784*c^14*d^15*x^13 + 7680
*c^15*d^14*x^10 + 10752*c^16*d^13*x^7 + 4096*c^17*d^12*x^4)*(c^13/d^16)^(2/
```

```
3) + 36*sqrt(3)*(c^17*d^11*x^17 + 1772*c^18*d^10*x^14 + 42592*c^19*d^9*x^11
 + 96256*c^20*d^8*x^8 + 69632*c^21*d^7*x^5 + 16384*c^22*d^6*x^2)*(c^13/d^16
)^(1/3) + sqrt(3)*(c^21*d^6*x^18 + 9456*c^22*d^5*x^15 + 749184*c^23*d^4*x^1
2 + 3017216*c^24*d^3*x^9 + 3489792*c^25*d^2*x^6 + 1572864*c^26*d*x^3 + 2621
44*c^27) - 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c^11*d^18*x^14 - 14440*c^12*d
^17*x^11 - 24576*c^13*d^16*x^8 - 16384*c^14*d^15*x^5 - 4096*c^15*d^14*x^2)*
(c^13/d^16)^(5/6) + 18*sqrt(3)*(c^15*d^13*x^15 - 1112*c^16*d^12*x^12 + 7296
*c^17*d^11*x^9 + 11776*c^18*d^10*x^6 + 4096*c^19*d^9*x^3)*sqrt(c^13/d^16) +
 sqrt(3)*(c^19*d^8*x^16 - 4768*c^20*d^7*x^13 + 362752*c^21*d^6*x^10 + 70912
0*c^22*d^5*x^7 + 413696*c^23*d^4*x^4 + 65536*c^24*d^3*x)*(c^13/d^16)^(1/6))
 + 2*(324*sqrt(3)*(d^19*x^16 - 1858*c*d^18*x^13 - 4176*c^2*d^17*x^10 - 3584
*c^3*d^16*x^7 - 1024*c^4*d^15*x^4)*(c^13/d^16)^(5/6) + 18*sqrt(3)*(c^4*d^14
*x^17 - 5290*c^5*d^13*x^14 - 21152*c^6*d^12*x^11 - 47744*c^7*d^11*x^8 - 378
88*c^8*d^10*x^5 - 8192*c^9*d^9*x^2)*sqrt(c^13/d^16) + sqrt(3)*(c^8*d^9*x^18
 - 7698*c^9*d^8*x^15 - 1664688*c^10*d^7*x^12 - 5524864*c^11*d^6*x^9 - 62238
72*c^12*d^5*x^6 - 2703360*c^13*d^4*x^3 - 327680*c^14*d^3)*(c^13/d^16)^(1/6)
 - 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c^2*d^16*x^15 + 37352*c^3*d^15*x^12 - 2303
36*c^4*d^14*x^9 - 515072*c^5*d^13*x^6 - 286720*c^6*d^12*x^3 - 32768*c^7*d^1
1)*(c^13/d^16)^(2/3) + 108*sqrt(3)*(53*c^7*d^10*x^13 + 1320*c^8*d^9*x^10 +
1536*c^9*d^8*x^7 + 512*c^10*d^7*x^4)*(c^13/d^16)^(1/3) + 6*sqrt(3)*(37*c^11
*d^5*x^14 + 28912*c^12*d^4*x^11 + 43584*c^13*d^3*x^8 + 20992*c^14*d^2*x^5 +
 4096*c^15*d*x^2)))*sqrt((18*c^22*d^2*x^8 + 360*c^23*d*x^5 - 144*c^24*x^2 +
 (c^13*d^13*x^9 - 276*c^14*d^12*x^6 - 1608*c^15*d^11*x^3 - 1088*c^16*d^10)*
(c^13/d^16)^(2/3) - 6*sqrt(d*x^3 + c)*((c^11*d^15*x^7 - 28*c^12*d^14*x^4 -
272*c^13*d^13*x)*(c^13/d^16)^(5/6) - 24*(c^16*d^9*x^5 + c^17*d^8*x^2)*sqrt(
c^13/d^16) + 4*(c^20*d^4*x^6 + 41*c^21*d^3*x^3 + 40*c^22*d^2)*(c^13/d^16)^(
1/6)) - 18*(c^18*d^7*x^7 - 52*c^19*d^6*x^4 - 80*c^20*d^5*x)*(c^13/d^16)^(1/
3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^21*d^6*x^18 - 1
4952*c^22*d^5*x^15 + 2872896*c^23*d^4*x^12 + 7330304*c^24*d^3*x^9 + 6696960
*c^25*d^2*x^6 + 2457600*c^26*d*x^3 + 262144*c^27)) + 5906*(c^2*d*x^3 - 8*c^
3)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) +
117*(d^4*x^3 - 8*c*d^3)*(c^13/d^16)^(1/6)*log(8...
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**7*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,{\left(d\,x^3+c\right)}^{3/2}}{{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
[Out] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

3.419 $\int \frac{x^4\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=657

$$\frac{13x^2\sqrt{c+dx^3}}{21d}+\frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

```
[Out] 1/3*x^2*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/
3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5/3)+9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1
/2)/c^(1/2))/d^(5/3)+9*c^(7/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(
d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)+13/21*x^2*(d*x^3+c)^(1/2)/d+265/7*c*(d*x^3+
c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+265/21*c^(4/3)*(c^(1/3)+d^
(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1
/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-265/14*3^(
1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((
c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {478, 595, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{265\sqrt{2}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{14d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{9\sqrt{3}\,c^{7/6}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{9c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}+\frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{13x^2\sqrt{c+dx^3}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (13*x^2*Sqrt[c + d*x^3])/(21*d) + (265*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)
) + (9*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[
c + d*x^3]])/d^(5/3) - (9*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6
)*Sqrt[c + d*x^3])])/d^(5/3) + (9*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
```

```
])])/d^(5/3) - (265*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)
*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*d^(5/3)*Sqrt[(c^(1/3)*(c
^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (265*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 224</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

<u>Rule 309</u>

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

<u>Rule 455</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} - \frac{\int \frac{x\sqrt{c+dx^3}\left(2c+\frac{13dx^3}{2}\right)}{8c-dx^3}\,dx}{3d}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2\int \frac{x\left(-111c^2d-\frac{795}{4}cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{21d^2}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2\int\left(\frac{795cdx}{4\sqrt{c+dx^3}} - \frac{1701c^2dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{21d^2}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{(265c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{14d} - \frac{(162c^2)\int \frac{x}{(8c-dx^3)\sqrt{c \cdots}}}{d}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{(27c)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2d^2} + \frac{(265c \cdots}{\cdots}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} - \frac{265\sqrt[4]{3}\sqrt{2 \cdots}}{\cdots}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{9\sqrt{3}\,c^{7/6}\,\mathrm{ta}\cdots}{\cdots}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{9\sqrt{3}\,c^{7/6}\,\mathrm{ta}\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 9.41, size = 176, normalized size = 0.27

$$-\frac{16x^2(37c^2+35cdx^3-2d^2x^6)+74cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+53dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{112d\,(-8c+dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] -1/112*(16*x^2*(37*c^2 + 35*c*d*x^3 - 2*d^2*x^6) + 74*c*x^2*(-8*c + d*x^3)*
Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)]
 + 53*d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -
((d*x^3)/c), (d*x^3)/(8*c)])/(d*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 1748, normalized size = 2.66

method	result	size
elliptic	Expression too large to display	897
default	Expression too large to display	1748
risch	Expression too large to display	1758

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8*c/d*(3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-19/24*I*3^(1/2)/d*(-c*d^2)^(1/3
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
```

```
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-
3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/d*(2/7*x^2*(d*x^3+c)
^(1/2)-44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/
2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d
^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*
d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2
)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/
18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Root
Of(_Z^3*d-8*c)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 23.97, size = 3858, normalized size = 5.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

[Out] -1/28*(84*sqrt(3)*(d^3*x^3 - 8*c*d^2)*(c^7/d^10)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c^7*d^12*x^16 + 784*c^8*d^11*x^13 + 7680*c^9*d^10*x^10 + 10752*c^10*d^9*x^7 + 4096*c^11*d^8*x^4)*(c^7/d^10)^(2/3) + 36*sqrt(3)*(c^9*d^9*x^17 + 1772*c^10*d^8*x^14 + 42592*c^11*d^7*x^11 + 96256*c^12*d^6*x^8 + 69632*c^13*d^5*x^5 + 16384*c^14*d^4*x^2)*(c^7/d^10)^(1/3) + sqrt(3)*(c^11*d^6*x^18 + 9456*c^12*d^5*x^15 + 749184*c^13*d^4*x^12 + 3017216*c^14*d^3*x^9 + 3489792*c^15*d^2*x^6 + 1572864*c^16*d*x^3 + 262144*c^17) + 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c^6*d^13*x^14 - 14440*c^7*d^12*x^11 - 24576*c^8*d^11*x^8 - 16384*c^9*d^10*x^5 - 4096*c^10*d^9*x^2)*(c^7/d^10)^(5/6) + 18*sqrt(3)*(c^8*d^10*x^15 - 1112*c^9*d^9*x^12 + 7296*c^10*d^8*x^9 + 11776*c^11*d^7*x^6 + 4096*c^12*d^6*x^3)*sqrt(c^7/d^10) + sqrt(3)*(c^10*d^7*x^16 - 4768*c^11*d^6*x^13 + 362752*c^12*d^5*x^10 + 709120*c^13*d^4*x^7 + 413696*c^14*d^3*x^4 + 65536*c^15*d^2*x)*(c^7/d^10)^(1/6)) - 2*(324*sqrt(3)*(d^14*x^16 - 1858*c*d^13*x^13 - 4176*c^2*d^12*x^10 - 3584*c^3*d^11*x^7 - 1024*c^4*d^10*x^4)*(c^7/d^10)^(5/6) + 18*sqrt(3)*(c^2*d^11*x^17 - 5290*c^3*d^10*x^14 - 21152*c^4*d^9*x^11 - 47744*c^5*d^8*x^8 - 37888*c^6*d^7*x^5 - 8192*c^7*d^6*x^2)*sqrt(c^7/d^10) + sqrt(3)*(c^4*d^8*x^18 - 7698*c^5*d^7*x^15 - 1664688*c^6*d^6*x^12 - 5524864*c^7*d^5*x^9 - 6223872*c^8*d^4*x^6 - 2703360*c^9*d^3*x^3 - 327680*c^10*d^2)*(c^7/d^10)^(1/6) + 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*c*d^12*x^15 + 37352*c^2*d^11*x^12 - 230336*c^3*d^10*x^9 - 515072*c^4*d^9*x^6 - 286720*c^5*d^8*x^3 - 32768*c^6*d^7)*(c^7/d^10)^(2/3) + 108*sqrt(3)*(53*c^4*d^8*x^13 + 1320*c^5*d^7*x^10 + 1536*c^6*d^6*x^7 + 512*c^7*d^5*x^4)*(c^7/d^10)^(1/3) + 6*sqrt(3)*(37*c^6*d^5*x^14 + 28912*c^7*d^4*x^11 + 43584*c^8*d^3*x^8 + 20992*c^9*d^2*x^5 + 4096*c^10*d*x^2)))*sqrt((18*c^12*d^2*x^8 + 360*c^13*d*x^5 - 144*c^14*x^2 + (c^7*d^9*x^9 - 276*c^8*d^8*x^6 - 1608*c^9*d^7*x^3 - 1088*c^10*d^6)*(c^7/d^10)^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^10*x^7 - 28*c^7*d^9*x^4 - 272*c^8*d^8*x)*(c^7/d^10)^(5/6) - 24*(c^9*d^6*x^5 + c^10*d^5*x^2)*sqrt(c^7/d^10) + 4*(c^11*d^3*x^6 + 41*c^12*d^2*x^3 + 40*c^13*d)*(c^7/d^10)^(1/6)) - 18*(c^10*d^5*x^7 - 52*c^11*d^4*x^4 - 80*c^12*d^3*x)*(c^7/d^10)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^11*d^6*x^18 - 14952*c^12*d^5*x^15 + 2872896*c^13*d^4*x^12 + 7330304*c^14*d^3*x^9 + 6696960*c^15*d^2*x^6 + 2457600*c^16*d*x^3 + 262144*c^17)) - 84*sqrt(3)*(d^3*x^3 - 8*c*d^2)*(c^7/d^10)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c^7*d^12*x^16 + 784*c^8*d^11*x^13 + 7680*c^9*d^10*x^10 + 10752*c^10*d^9*x^7 + 4096*c^11*d^8*x^4)*(c^7/d^10)^(2/3) + 36*sqrt(3)*(c^9*d^9*x^17 + 1772*c^10*d^8*x^14 + 42592*c^11*d^7*x^11 + 96256*c^12*d^6*x^8 + 69632*c^13*d^5*x^5 + 16384*c^14*d^4*x^2)*(c^7/d^10)^(1/3) + sqrt(3)*(c^11*d^6*x^18 + 9456*c^12*d^5*x^15 + 749184*c^13*d^4*x^12 + 3017216*c^14*d^3*x^9 + 3489792*c^15*d^2*x^6 + 1572864*c^16*d*x^3 + 262144*c^17) - 12*sqrt(d*x^3 + c)*(12*sqrt(3)*(35*c^6*d^13*x^14 - 14440*c^7*d^12*x^11 - 24576*c^8*d^11*x^8 - 16384*c^9*d^10*x^5 - 4096*c^10*d^9*x^2)*(c^7/d^10)^(5/6) + 18*sqrt(3)*(c^8*d^10*x^15 - 1112*c^9*d^9*x^12 + 7296*c^10*d^8*x^9 + 11776*c^11*d^7*x^6 + 4096*c^12*d^6*x^3)*sqrt(c^7/d^10) + sqrt(3)*(c^10*d^7*x^16 - 4768*c^11*d^6*x^13 + 362752*c^12*d^5*x^10 + 709120*c^13*d^4*x^7 + 413696*c^14*d^3*x^4 + 65536*c^15*d^2*x)*(c^7/d^10)^(1/6)) + 2*(324*sqrt(3)*(d^14*x^16 - 1858*c*d^13*x^13 - 4176*c^2*d^12*x^10 - 3584*c^3*d^11*x^7 - 10

```
24*c^4*d^10*x^4)*(c^7/d^10)^(5/6) + 18*sqrt(3)*(c^2*d^11*x^17 - 5290*c^3*d^
10*x^14 - 21152*c^4*d^9*x^11 - 47744*c^5*d^8*x^8 - 37888*c^6*d^7*x^5 - 8192
*c^7*d^6*x^2)*sqrt(c^7/d^10) + sqrt(3)*(c^4*d^8*x^18 - 7698*c^5*d^7*x^15 -
1664688*c^6*d^6*x^12 - 5524864*c^7*d^5*x^9 - 6223872*c^8*d^4*x^6 - 2703360*
c^9*d^3*x^3 - 327680*c^10*d^2)*(c^7/d^10)^(1/6) - 6*sqrt(d*x^3 + c)*(sqrt(3
)*(7*c*d^12*x^15 + 37352*c^2*d^11*x^12 - 230336*c^3*d^10*x^9 - 515072*c^4*d
^9*x^6 - 286720*c^5*d^8*x^3 - 32768*c^6*d^7)*(c^7/d^10)^(2/3) + 108*sqrt(3)
*(53*c^4*d^8*x^13 + 1320*c^5*d^7*x^10 + 1536*c^6*d^6*x^7 + 512*c^7*d^5*x^4)
*(c^7/d^10)^(1/3) + 6*sqrt(3)*(37*c^6*d^5*x^14 + 28912*c^7*d^4*x^11 + 43584
*c^8*d^3*x^8 + 20992*c^9*d^2*x^5 + 4096*c^10*d*x^2)))*sqrt((18*c^12*d^2*x^8
 + 360*c^13*d*x^5 - 144*c^14*x^2 + (c^7*d^9*x^9 - 276*c^8*d^8*x^6 - 1608*c^
9*d^7*x^3 - 1088*c^10*d^6)*(c^7/d^10)^(2/3) - 6*sqrt(d*x^3 + c)*((c^6*d^10*
x^7 - 28*c^7*d^9*x^4 - 272*c^8*d^8*x)*(c^7/d^10)^(5/6) - 24*(c^9*d^6*x^5 +
c^10*d^5*x^2)*sqrt(c^7/d^10) + 4*(c^11*d^3*x^6 + 41*c^12*d^2*x^3 + 40*c^13*
d)*(c^7/d^10)^(1/6)) - 18*(c^10*d^5*x^7 - 52*c^11*d^4*x^4 - 80*c^12*d^3*x)*
(c^7/d^10)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^1
1*d^6*x^18 - 14952*c^12*d^5*x^15 + 2872896*c^13*d^4*x^12 + 7330304*c^14*d^3
*x^9 + 6696960*c^15*d^2*x^6 + 2457600*c^16*d*x^3 + 262144*c^17)) + 1060*(c*
d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4
*c/d, x)) + 21*(d^3*x^3 - 8*c*d^2)*(c^7/d^10)^(1/6)*log(13947137604*(18*c^1
2*d^2*x^8 + 360*c^13*d*x^5 - 144*c^14*x^2 + (c^7*d^9*x^9 - 276*c^8*d^8*x^6
- 1608*c^9*d^7*x^3 - 1088*c^10*d^6)*(c^7/d^10)^...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**4*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d\,x^3+c\right)}^{3/2}}{{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
[Out] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

3.420 $\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=638

$$\frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{9\sqrt{3}\,\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}}-\frac{9\sqrt[6]{c}\,\tanh^{-1}}{}$$

```
[Out] -9/16*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(2/3)+9/16*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)+9/16*c^(1/6
)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(2/
3)+3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+19/8*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))+19/24*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1
/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2
^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-19/16*3^(1/4)*c^(1/3)*(c^(1/3)+d^(
1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x
+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1
/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.51, antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {479, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\,d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{16d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{9\sqrt{3}\,\sqrt[6]{c}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}}+\frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{9\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16d^{2/3}}+\frac{9\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16d^{2/3}}+\frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (19*Sqrt[c + d*x^3])/(8*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*x
^2*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) + (9*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(16*d^(2/3)) - (9*c^(1/6)*
ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(16*d^(2/3))
+ (9*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(16*d^(2/3)) - (19*3^(1/
4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*
```

```
d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)], -7 - 4*Sqrt[3]])/(16*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (19*c^(1/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/
4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)} + \frac{\int \frac{x\left(-15c^2d-\frac{57}{2}cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24cd}$$

$$= \frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)} + \frac{\int \left(\frac{57cdx}{2\sqrt{c+dx^3}} - \frac{243c^2dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{24cd}$$

$$= \frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)} + \frac{19}{16}\int \frac{x}{\sqrt{c+dx^3}}\,dx - \frac{1}{8}(81c)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= \frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)} + \frac{27\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{32d} + \frac{19\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{16\sqrt[3]{d}}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)} - \frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)} + \frac{9\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}\right.}{\sqrt{c+dx}}\right.}{16d^{2/3}}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)} + \frac{9\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}\right.}{\sqrt{c+dx}}\right.}{16d^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.13, size = 141, normalized size = 0.22

$$\frac{x^2\left(\frac{240(c+dx^3)}{8c-dx^3}-25\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-\frac{19dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c}\right)}{640\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (x^2*((240*(c + d*x^3))/(8*c - d*x^3) - 25*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3
, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - (19*d*x^3*Sqrt[1 + (d*x^3)/c]
*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c))/(640*Sqrt[c +
 d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 874, normalized size = 1.37

method	result	size
default	Expression too large to display	874
elliptic	Expression too large to display	874

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-19/24*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-
1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)
^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*
```

```
(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/
c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 17.34, size = 3583, normalized size = 5.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/64*(24*sqrt(d*x^3 + c)*d*x^2 + 12*sqrt(3)*(d^2*x^3 - 8*c*d)*(c/d^4)^(1/6
)*arctan(-1/3*(324*sqrt(3)*(3*c*d^8*x^16 + 784*c^2*d^7*x^13 + 7680*c^3*d^6*
x^10 + 10752*c^4*d^5*x^7 + 4096*c^5*d^4*x^4)*(c/d^4)^(2/3) + 36*sqrt(3)*(c*
d^7*x^17 + 1772*c^2*d^6*x^14 + 42592*c^3*d^5*x^11 + 96256*c^4*d^4*x^8 + 696
32*c^5*d^3*x^5 + 16384*c^6*d^2*x^2)*(c/d^4)^(1/3) + sqrt(3)*(c*d^6*x^18 + 9
456*c^2*d^5*x^15 + 749184*c^3*d^4*x^12 + 3017216*c^4*d^3*x^9 + 3489792*c^5*
d^2*x^6 + 1572864*c^6*d*x^3 + 262144*c^7) + 12*sqrt(d*x^3 + c)*(12*sqrt(3)*
(35*c*d^8*x^14 - 14440*c^2*d^7*x^11 - 24576*c^3*d^6*x^8 - 16384*c^4*d^5*x^5
 - 4096*c^5*d^4*x^2)*(c/d^4)^(5/6) + 18*sqrt(3)*(c*d^7*x^15 - 1112*c^2*d^6*
x^12 + 7296*c^3*d^5*x^9 + 11776*c^4*d^4*x^6 + 4096*c^5*d^3*x^3)*sqrt(c/d^4)
 + sqrt(3)*(c*d^6*x^16 - 4768*c^2*d^5*x^13 + 362752*c^3*d^4*x^10 + 709120*c
^4*d^3*x^7 + 413696*c^5*d^2*x^4 + 65536*c^6*d*x)*(c/d^4)^(1/6)) - 2*(324*sq
rt(3)*(d^9*x^16 - 1858*c*d^8*x^13 - 4176*c^2*d^7*x^10 - 3584*c^3*d^6*x^7 -
1024*c^4*d^5*x^4)*(c/d^4)^(5/6) + 18*sqrt(3)*(d^8*x^17 - 5290*c*d^7*x^14 -
21152*c^2*d^6*x^11 - 47744*c^3*d^5*x^8 - 37888*c^4*d^4*x^5 - 8192*c^5*d^3*x
^2)*sqrt(c/d^4) + sqrt(3)*(d^7*x^18 - 7698*c*d^6*x^15 - 1664688*c^2*d^5*x^1
2 - 5524864*c^3*d^4*x^9 - 6223872*c^4*d^3*x^6 - 2703360*c^5*d^2*x^3 - 32768
0*c^6*d)*(c/d^4)^(1/6) + 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*d^8*x^15 + 37352*c*d
^7*x^12 - 230336*c^2*d^6*x^9 - 515072*c^3*d^5*x^6 - 286720*c^4*d^4*x^3 - 32
768*c^5*d^3)*(c/d^4)^(2/3) + 108*sqrt(3)*(53*c*d^6*x^13 + 1320*c^2*d^5*x^10
 + 1536*c^3*d^4*x^7 + 512*c^4*d^3*x^4)*(c/d^4)^(1/3) + 6*sqrt(3)*(37*c*d^5*
x^14 + 28912*c^2*d^4*x^11 + 43584*c^3*d^3*x^8 + 20992*c^4*d^2*x^5 + 4096*c^
5*d*x^2)))*sqrt((18*c^2*d^2*x^8 + 360*c^3*d*x^5 - 144*c^4*x^2 + (c*d^5*x^9
- 276*c^2*d^4*x^6 - 1608*c^3*d^3*x^3 - 1088*c^4*d^2)*(c/d^4)^(2/3) + 6*sqrt
```

```
(d*x^3 + c)*((c*d^5*x^7 - 28*c^2*d^4*x^4 - 272*c^3*d^3*x)*(c/d^4)^(5/6) - 2
4*(c^2*d^3*x^5 + c^3*d^2*x^2)*sqrt(c/d^4) + 4*(c^2*d^2*x^6 + 41*c^3*d*x^3 +
 40*c^4)*(c/d^4)^(1/6)) - 18*(c^2*d^3*x^7 - 52*c^3*d^2*x^4 - 80*c^4*d*x)*(c
/d^4)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^6*x^
18 - 14952*c^2*d^5*x^15 + 2872896*c^3*d^4*x^12 + 7330304*c^4*d^3*x^9 + 6696
960*c^5*d^2*x^6 + 2457600*c^6*d*x^3 + 262144*c^7)) - 12*sqrt(3)*(d^2*x^3 -
8*c*d)*(c/d^4)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c*d^8*x^16 + 784*c^2*d^7*x
^13 + 7680*c^3*d^6*x^10 + 10752*c^4*d^5*x^7 + 4096*c^5*d^4*x^4)*(c/d^4)^(2/
3) + 36*sqrt(3)*(c*d^7*x^17 + 1772*c^2*d^6*x^14 + 42592*c^3*d^5*x^11 + 9625
6*c^4*d^4*x^8 + 69632*c^5*d^3*x^5 + 16384*c^6*d^2*x^2)*(c/d^4)^(1/3) + sqrt
(3)*(c*d^6*x^18 + 9456*c^2*d^5*x^15 + 749184*c^3*d^4*x^12 + 3017216*c^4*d^3
*x^9 + 3489792*c^5*d^2*x^6 + 1572864*c^6*d*x^3 + 262144*c^7) - 12*sqrt(d*x^
3 + c)*(12*sqrt(3)*(35*c*d^8*x^14 - 14440*c^2*d^7*x^11 - 24576*c^3*d^6*x^8
- 16384*c^4*d^5*x^5 - 4096*c^5*d^4*x^2)*(c/d^4)^(5/6) + 18*sqrt(3)*(c*d^7*x
^15 - 1112*c^2*d^6*x^12 + 7296*c^3*d^5*x^9 + 11776*c^4*d^4*x^6 + 4096*c^5*d
^3*x^3)*sqrt(c/d^4) + sqrt(3)*(c*d^6*x^16 - 4768*c^2*d^5*x^13 + 362752*c^3*
d^4*x^10 + 709120*c^4*d^3*x^7 + 413696*c^5*d^2*x^4 + 65536*c^6*d*x)*(c/d^4)
^(1/6)) + 2*(324*sqrt(3)*(d^9*x^16 - 1858*c*d^8*x^13 - 4176*c^2*d^7*x^10 -
3584*c^3*d^6*x^7 - 1024*c^4*d^5*x^4)*(c/d^4)^(5/6) + 18*sqrt(3)*(d^8*x^17 -
 5290*c*d^7*x^14 - 21152*c^2*d^6*x^11 - 47744*c^3*d^5*x^8 - 37888*c^4*d^4*x
^5 - 8192*c^5*d^3*x^2)*sqrt(c/d^4) + sqrt(3)*(d^7*x^18 - 7698*c*d^6*x^15 -
1664688*c^2*d^5*x^12 - 5524864*c^3*d^4*x^9 - 6223872*c^4*d^3*x^6 - 2703360*
c^5*d^2*x^3 - 327680*c^6*d)*(c/d^4)^(1/6) - 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*d
^8*x^15 + 37352*c*d^7*x^12 - 230336*c^2*d^6*x^9 - 515072*c^3*d^5*x^6 - 2867
20*c^4*d^4*x^3 - 32768*c^5*d^3)*(c/d^4)^(2/3) + 108*sqrt(3)*(53*c*d^6*x^13
+ 1320*c^2*d^5*x^10 + 1536*c^3*d^4*x^7 + 512*c^4*d^3*x^4)*(c/d^4)^(1/3) + 6
*sqrt(3)*(37*c*d^5*x^14 + 28912*c^2*d^4*x^11 + 43584*c^3*d^3*x^8 + 20992*c^
4*d^2*x^5 + 4096*c^5*d*x^2)))*sqrt((18*c^2*d^2*x^8 + 360*c^3*d*x^5 - 144*c^
4*x^2 + (c*d^5*x^9 - 276*c^2*d^4*x^6 - 1608*c^3*d^3*x^3 - 1088*c^4*d^2)*(c/
d^4)^(2/3) - 6*sqrt(d*x^3 + c)*((c*d^5*x^7 - 28*c^2*d^4*x^4 - 272*c^3*d^3*x
)*(c/d^4)^(5/6) - 24*(c^2*d^3*x^5 + c^3*d^2*x^2)*sqrt(c/d^4) + 4*(c^2*d^2*x
^6 + 41*c^3*d*x^3 + 40*c^4)*(c/d^4)^(1/6)) - 18*(c^2*d^3*x^7 - 52*c^3*d^2*x
^4 - 80*c^4*d*x)*(c/d^4)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)))/(c*d^6*x^18 - 14952*c^2*d^5*x^15 + 2872896*c^3*d^4*x^12 + 7330304
*c^4*d^3*x^9 + 6696960*c^5*d^2*x^6 + 2457600*c^6*d*x^3 + 262144*c^7)) + 152
*(d*x^3 - 8*c)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4
*c/d, x)) + 3*(d^2*x^3 - 8*c*d)*(c/d^4)^(1/6)*log(3486784401/4*(18*c^2*d^2*
x^8 + 360*c^3*d*x^5 - 144*c^4*x^2 + (c*d^5*x^9 - 276*c^2*d^4*x^6 - 1608*c^3
*d^3*x^3 - 1088*c^4*d^2)*(c/d^4)^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^5*x^7 - 28
*c^2*d^4*x^4 - 272*c^3*d^3*x)*(c/d^4)^(5/6) - 24*(c^2*d^3*x^5 + c^3*d^2*x^2
)*sqrt(c/d^4) + 4*(c^2*d^2*x^6 + 41*c^3*d*x^3 + 40*c^4)*(c/d^4)^(1/6)) - 18
*(c^2*d^3*x^7 - 52*c^3*d^2*x^4 - 80*c^4*d*x)*(c/d^4)^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 3*(d^2*x...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
[Out] Integral(x*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(d\,x^3+c\right)}^{3/2}}{{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
[Out] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

3.421 $\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx$

Optimal. Leaf size=522

$$-\frac{\sqrt{c+dx^3}}{16cx}+\frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{3\sqrt{c+dx^3}}{8x\,(8c-dx^3)}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\,\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\cdots}{\left(\left(1+\cdots\right.\right.}}}{32c^{2/3}\sqrt{\frac{\cdots}{\left(\left(\cdots\right.\right.}}}$$

```
[Out] -1/16*(d*x^3+c)^(1/2)/c/x+3/8*(d*x^3+c)^(1/2)/x/(-d*x^3+8*c)+1/16*d^(1/3)*(
d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/48*d^(1/3)*(c^(1/3)+d^(1
/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3
)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/32*3^(1/4)
*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2
/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
/c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$,
Rules used = {479, 21, 331, 309, 224, 1891}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\,\sqrt[4]{3}\,c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{c+dx^3}}{16cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{3\sqrt{c+dx^3}}{8x\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] -1/16*Sqrt[c + d*x^3]/(c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(16*c*((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[c + d*x^3])/(8*x*(8*c - d*x^3)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)], -7 - 4*Sqrt[3]])/(32*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^
```

```
(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*Sqrt[2]*3^(1/4)*c
^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
```

```
x]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx &= \frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)} + \frac{\int \frac{12c^2d-\frac{3}{2}cd^2x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24cd} \\ &= \frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)} + \frac{1}{16}\int \frac{1}{x^2\sqrt{c+dx^3}}\,dx \\ &= -\frac{\sqrt{c+dx^3}}{16cx} + \frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)} + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c} \\ &= -\frac{\sqrt{c+dx^3}}{16cx} + \frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)} + \frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{32c} + \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,d^2 \cdots\right.}{16c \cdots} \\ &= -\frac{\sqrt{c+dx^3}}{16cx} + \frac{\sqrt[3]{d}\,\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)} - \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{c} \cdots}{\cdots}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.51, size = 242, normalized size = 0.46

$$\frac{(2c-dx^3)\sqrt{c+dx^3}}{16cx(-8c+dx^3)} - \frac{\sqrt[6]{-1}\,\sqrt[3]{-d}\,\sqrt{(-1)^{5/6}\left(-1+\frac{\sqrt[3]{-d}\,x}{\sqrt[3]{c}}\right)}\,\sqrt{1+\frac{\sqrt[3]{-d}\,x}{\sqrt[3]{c}}+\frac{(-d)^{2/3}x^2}{c^{2/3}}}\left(-i\sqrt{3}\,E\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}-\frac{i\sqrt[3]{-d}\,x}{\sqrt[3]{c}}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+\sqrt[3]{-1}\,F\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}-\frac{i\sqrt[3]{-d}\,x}{\sqrt[3]{c}}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)\right)}{16\sqrt[4]{3}\,\sqrt[3]{c}\,\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] ((2*c - d*x^3)*Sqrt[c + d*x^3])/(16*c*x*(-8*c + d*x^3)) - ((-1)^(1/6)*(-d)^
(1/3)*Sqrt[(-1)^(5/6)*(-1 + ((-d)^(1/3)*x)/c^(1/3))]*Sqrt[1 + ((-d)^(1/3)*x
)/c^(1/3) + ((-d)^(2/3)*x^2)/c^(2/3)]*((-I)*Sqrt[3]*EllipticE[ArcSin[Sqrt[-
(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)] + (-1)^(1/3)*E
llipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)
^(1/3)]))/(16*3^(1/4)*c^(1/3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 2218, normalized size = 4.25

method	result
elliptic	$\frac{3d\,x^2\sqrt{d\,x^3+c}}{64c(-d\,x^3+8c)}-\frac{\sqrt{d\,x^3+c}}{64cx}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-c\,d}{d}}{-\frac{3(-c\,d^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{}}{}}}}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d/c*(3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-19/24*I*3^(1/2)/d*(-c*d^2)^(1
/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
```

d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I
d(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)
*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alph
a-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))-1/64*d/c^2*(2/7*x^2*
(d*x^3+c)^(1/2)-44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(
-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(
-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3*I*c/
d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-c*(d*x^3+c)^(1/2)/x+2/7*(d*x^3+c)^(1/2
)*d*x^2-9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/
3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/

```
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.64, size = 68, normalized size = 0.13

$$-\frac{(dx^4 - 8\,cx)\sqrt{d}\,\text{weierstrassZeta}\left(0, -\frac{4\,c}{d}, \text{weierstrassPInverse}\left(0, -\frac{4\,c}{d}, x\right)\right) + \sqrt{dx^3 + c}\,(dx^3 - 2\,c)}{16\,(cdx^4 - 8\,c^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
[Out] -1/16*((d*x^4 - 8*c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInver
se(0, -4*c/d, x)) + sqrt(d*x^3 + c)*(d*x^3 - 2*c))/(c*d*x^4 - 8*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^{\frac{3}{2}}}{x^2\,(-8c + dx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c)**2,x)
[Out] Integral((c + d*x**3)**(3/2)/(x**2*(-8*c + d*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x^3+c)^{3/2}}{x^2\,(8\,c-d\,x^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2), x)
```

3.422 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=684

$$-\frac{13\sqrt{c+dx^3}}{256cx^4}-\frac{d\sqrt{c+dx^3}}{32c^2x}+\frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}-\frac{9\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}}{\sqrt{}}\right.}{1024c^{11/6}}$$

[Out] 9/1024*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)-9/1024*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-9/1024*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c^(11/6)-13/256*(d*x^3+c)^(1/2)/c/x^4-1/32*d*(d*x^3+c)^(1/2)/c^2/x+3/8*(d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)+1/32*d^(4/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/96*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/64*3^(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.66, antiderivative size = 684, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {479, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{16\sqrt{2}\sqrt[4]{3}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{64c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{9\sqrt{3}\,d^{4/3}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}+\frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{d\sqrt{c+dx^3}}{32c^2x}+\frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}-\frac{13\sqrt{c+dx^3}}{256cx^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]

[Out] (-13*Sqrt[c + d*x^3])/(256*c*x^4) - (d*Sqrt[c + d*x^3])/(32*c^2*x) + (d^(4/3)*Sqrt[c + d*x^3])/(32*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[c + d*x^3])/(8*x^4*(8*c - d*x^3)) - (9*Sqrt[3]*d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(11/6)) + (9*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(11/6)

```
) - (9*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(11/6)) - (3^(
1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(64*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(4/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*Sqrt[2]*3^(1/
4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)} + \frac{\int \frac{39c^2d+\frac{51}{2}cd^2x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24cd}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} + \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)} - \frac{\int \frac{-192c^3d^2-\frac{195}{2}c^2d^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{768c^3d}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)} + \frac{\int \frac{x(1740c^4d^3-96c^3d^4x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{6144c^5d}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)} + \frac{\int \left(\frac{96c^3d^3x}{\sqrt{c+dx^3}} + \frac{972c^4d^3x}{(8c-dx^3)\sqrt{c+dx}}\right.}{6144c^5d}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)} + \frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{64c^2} + \frac{(81d^2)\int \frac{}{(8c}}{}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)} - \frac{(27d)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+d}}}{2048c^2}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.10, size = 199, normalized size = 0.29

$$\sqrt{c+dx^3}\left(-\frac{1}{256cx^4}-\frac{13d}{512c^2x}-\frac{3d^2x^2}{512c^2(-8c+dx^3)}\right)+\frac{145d^2x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{8192c^2\sqrt{c+dx^3}}-\frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{2560c^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/256*1/(c*x^4) - (13*d)/(512*c^2*x) - (3*d^2*x^2)/(512*c
^2*(-8*c + d*x^3))) + (145*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1
, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(8192*c^2*Sqrt[c + d*x^3]) - (d^3*x^5*
Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]
)/(2560*c^3*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 2691, normalized size = 3.93

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2691

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*(-1/4*c*(d*x^3+c)^(1/2)/x^4-11/8*d*(d*x^3+c)^(1/2)/x-9/8*I*d*3^(1/
2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))+1/64/c^2*d^2*(3/8*x^2*(d*x^3+c)^(1/2)
/(-d*x^3+8*c)-19/24*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*
d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(
-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
```

/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))-1/256/c^3*d^2*(2/7*x^2*(d*x^3+c)^(1/2)-44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/256/c^3*d*(-c*(d*x^3+c)^(1/2)/x+2/7*(d*x^3+c)^(1/2)*d*x^2-9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3

```
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.89, size = 2721, normalized size = 3.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
[Out] -1/4096*(12*sqrt(3)*(c^2*d*x^7 - 8*c^3*x^4)*(d^8/c^11)^(1/6)*arctan(1/9*((9
*sqrt(3)*c^2*d^13*x^5*(d^8/c^11)^(1/6) - sqrt(3)*(c^9*d^8*x^6 - 40*c^10*d^7
*x^3 - 32*c^11*d^6)*(d^8/c^11)^(5/6) + 3*sqrt(3)*(5*c^6*d^10*x^4 + 8*c^7*d^
9*x)*sqrt(d^8/c^11))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^8*d^3*x^5 + c^9*d^2*x
^2)*(d^8/c^11)^(2/3) + 12*sqrt(3)*(c^4*d^6*x^6 - c^5*d^5*x^3 - 2*c^6*d^4)*(
d^8/c^11)^(1/3) + 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*x^4 + 4*c^2*d^7*x) + sqrt(d*
x^3 + c)*(sqrt(3)*(c^9*d^2*x^6 + 32*c^10*d*x^3 + 40*c^11)*(d^8/c^11)^(5/6)
+ 3*sqrt(3)*(7*c^6*d^4*x^4 + 4*c^7*d^3*x)*sqrt(d^8/c^11) + 9*sqrt(3)*(c^2*d
^7*x^5 + 2*c^3*d^6*x^2)*(d^8/c^11)^(1/6)))*sqrt((d^15*x^9 - 276*c*d^14*x^6
- 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^8*d^9*x^7 - 52*c^9*d^8*x^4 - 80
*c^10*d^7*x)*(d^8/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^7*x^5 + c^11*
d^6*x^2)*(d^8/c^11)^(5/6) - 4*(c^6*d^10*x^6 + 41*c^7*d^9*x^3 + 40*c^8*d^8)*
sqrt(d^8/c^11) - (c^2*d^13*x^7 - 28*c^3*d^12*x^4 - 272*c^4*d^11*x)*(d^8/c^1
1)^(1/6)) + 18*(c^4*d^12*x^8 + 20*c^5*d^11*x^5 - 8*c^6*d^10*x^2)*(d^8/c^11)
^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^15*x^7 - 7*
c*d^14*x^4 - 8*c^2*d^13*x)) + 12*sqrt(3)*(c^2*d*x^7 - 8*c^3*x^4)*(d^8/c^11)
^(1/6)*arctan(1/9*((9*sqrt(3)*c^2*d^13*x^5*(d^8/c^11)^(1/6) - sqrt(3)*(c^9*
d^8*x^6 - 40*c^10*d^7*x^3 - 32*c^11*d^6)*(d^8/c^11)^(5/6) + 3*sqrt(3)*(5*c^
6*d^10*x^4 + 8*c^7*d^9*x)*sqrt(d^8/c^11))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^
8*d^3*x^5 + c^9*d^2*x^2)*(d^8/c^11)^(2/3) + 12*sqrt(3)*(c^4*d^6*x^6 - c^5*d
^5*x^3 - 2*c^6*d^4)*(d^8/c^11)^(1/3) + 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*x^4 + 4
*c^2*d^7*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^9*d^2*x^6 + 32*c^10*d*x^3 + 40*c^
11)*(d^8/c^11)^(5/6) + 3*sqrt(3)*(7*c^6*d^4*x^4 + 4*c^7*d^3*x)*sqrt(d^8/c^1
```

```
1) + 9*sqrt(3)*(c^2*d^7*x^5 + 2*c^3*d^6*x^2)*(d^8/c^11)^(1/6)))*sqrt((d^15*
x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^8*d^9*x^7
- 52*c^9*d^8*x^4 - 80*c^10*d^7*x)*(d^8/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(24*
(c^10*d^7*x^5 + c^11*d^6*x^2)*(d^8/c^11)^(5/6) - 4*(c^6*d^10*x^6 + 41*c^7*d
^9*x^3 + 40*c^8*d^8)*sqrt(d^8/c^11) - (c^2*d^13*x^7 - 28*c^3*d^12*x^4 - 272
*c^4*d^11*x)*(d^8/c^11)^(1/6)) + 18*(c^4*d^12*x^8 + 20*c^5*d^11*x^5 - 8*c^6
*d^10*x^2)*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)))/(d^15*x^7 - 7*c*d^14*x^4 - 8*c^2*d^13*x)) + 128*(d^2*x^7 - 8*c*d*x^4
)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 3
*(c^2*d*x^7 - 8*c^3*x^4)*(d^8/c^11)^(1/6)*log(43046721*(d^15*x^9 - 276*c*d^
14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^8*d^9*x^7 - 52*c^9*d^8*x
^4 - 80*c^10*d^7*x)*(d^8/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^7*x^5
+ c^11*d^6*x^2)*(d^8/c^11)^(5/6) - 4*(c^6*d^10*x^6 + 41*c^7*d^9*x^3 + 40*c^
8*d^8)*sqrt(d^8/c^11) - (c^2*d^13*x^7 - 28*c^3*d^12*x^4 - 272*c^4*d^11*x)*(
d^8/c^11)^(1/6)) + 18*(c^4*d^12*x^8 + 20*c^5*d^11*x^5 - 8*c^6*d^10*x^2)*(d^
8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 3*(c^2
*d*x^7 - 8*c^3*x^4)*(d^8/c^11)^(1/6)*log(43046721*(d^15*x^9 - 276*c*d^14*x^
6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^8*d^9*x^7 - 52*c^9*d^8*x^4 -
80*c^10*d^7*x)*(d^8/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^10*d^7*x^5 + c^1
1*d^6*x^2)*(d^8/c^11)^(5/6) - 4*(c^6*d^10*x^6 + 41*c^7*d^9*x^3 + 40*c^8*d^8
)*sqrt(d^8/c^11) - (c^2*d^13*x^7 - 28*c^3*d^12*x^4 - 272*c^4*d^11*x)*(d^8/c
^11)^(1/6)) + 18*(c^4*d^12*x^8 + 20*c^5*d^11*x^5 - 8*c^6*d^10*x^2)*(d^8/c^1
1)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 6*(c^2*d*x^
7 - 8*c^3*x^4)*(d^8/c^11)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^
2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x)
*(d^8/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^8/
c^11)^(5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11)
 + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x)*(d^8/c^11)^(1/6)) + 18*(c
^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2)*(d^8/c^11)^(1/3))/(d^3*x^9 -
24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 6*(c^2*d*x^7 - 8*c^3*x^4)*(d^8/c
^11)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d
^6 + 18*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x)*(d^8/c^11)^(2/3) - 6
*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^8/c^11)^(5/6) + (7*c^6*
d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*
c^3*d^6*x^4 + 160*c^4*d^5*x)*(d^8/c^11)^(1/6)) + 18*(c^4*d^6*x^8 + 38*c^5*d
^5*x^5 + 64*c^6*d^4*x^2)*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^
2*d*x^3 - 512*c^3)) + 16*(8*d^2*x^6 - 51*c*d*x^3 - 8*c^2)*sqrt(d*x^3 + c))/
(c^2*d*x^7 - 8*c^3*x^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^3+c\right)}^{3/2}}{x^5\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2), x)
```

3.423 $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2} dx$

Optimal. Leaf size=708

$$-\frac{11\sqrt{c+dx^3}}{224cx^7}-\frac{83d\sqrt{c+dx^3}}{7168c^2x^4}-\frac{19d^2\sqrt{c+dx^3}}{1792c^3x}+\frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{3\sqrt{c+dx^3}}{8x^7(8c-dx^3)}-\frac{9\sqrt{}}{}$$

```
[Out] 9/4096*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-9/4096*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-9/4096
*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)/c^(17/6)-11/224*(d*x^3+c)^(1/2)/c/x^7-83/7168*d*(d*x^3+c)^(1/2)/c^2/x^4-1
9/1792*d^2*(d*x^3+c)^(1/2)/c^3/x+3/8*(d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)+19/17
92*d^(7/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+19/5376*d^(7
/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3
+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)-19/3584*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/
3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)
-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {479, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{19d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{896\sqrt{2}\sqrt[4]{3}\,c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{3584c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{9\sqrt{3}\,d^{7/3}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{4096c^{17/6}}+\frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{4096c^{17/6}}-\frac{9d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4096c^{17/6}}+\frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{19d^2\sqrt{c+dx^3}}{1792c^3x}-\frac{83d\sqrt{c+dx^3}}{7168c^2x^4}+\frac{3\sqrt{c+dx^3}}{8x^7(8c-dx^3)}-\frac{11\sqrt{c+dx^3}}{224cx^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (-11*Sqrt[c + d*x^3])/(224*c*x^7) - (83*d*Sqrt[c + d*x^3])/(7168*c^2*x^4) -
 (19*d^2*Sqrt[c + d*x^3])/(1792*c^3*x) + (19*d^(7/3)*Sqrt[c + d*x^3])/(1792
*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[c + d*x^3])/(8*x^7*(8*c
 - d*x^3)) - (9*Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*
```

```
x))/Sqrt[c + d*x^3]])/(4096*c^(17/6)) + (9*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/
3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(4096*c^(17/6)) - (9*d^(7/3)*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(4096*c^(17/6)) - (19*3^(1/4)*Sqrt[2 - Sqrt[3
]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(3584*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (19*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*Sqrt[2]*3^(1/4)*c^(8/3)*Sqr
t[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sq
rt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)^2}\,dx = \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} + \frac{\int \frac{66c^2d+\frac{105}{2}cd^2x^3}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24cd}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\int \frac{-498c^3d^2-363c^2d^3x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1344c^3d}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} + \frac{\int \frac{3648c^4d^3+1245c^3d^4x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{43008c^5d}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\int \frac{x(-28200c^5d\ldots}{(8c-dx^3)\ldots}}{344\ldots}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\int \left(-\frac{1824c^4\ldots}{\sqrt{c+\ldots}}\right.}{\ldots}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} + \frac{\left(19d^3\right)\int \frac{\ldots}{\sqrt{c\ldots}}}{358\ldots}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\left(27d^2\right)\int \frac{\ldots}{\left(4\ldots\right.}}{\ldots}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\ldots\right.}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\ldots\right.}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\ldots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 212, normalized size = 0.30

$$\sqrt{c+dx^3}\left(-\frac{1}{448cx^7}-\frac{41d}{7168c^2x^4}-\frac{283d^2}{28672c^3x}-\frac{3d^3x^2}{4096c^3(-8c+dx^3)}\right)+\frac{1175d^3x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{229376c^3\sqrt{c+dx^3}}-\frac{19d^4x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{143360c^4\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/448*1/(c*x^7) - (41*d)/(7168*c^2*x^4) - (283*d^2)/(2867
2*c^3*x) - (3*d^3*x^2)/(4096*c^3*(-8*c + d*x^3))) + (1175*d^3*x^2*Sqrt[(c +
 d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(229376
*c^3*Sqrt[c + d*x^3]) - (19*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2,
1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(143360*c^4*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 3187, normalized size = 4.50

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	3187

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/256/c^3*d*(-1/4*c*(d*x^3+c)^(1/2)/x^4-11/8*d*(d*x^3+c)^(1/2)/x-9/8*I*d*3^
(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/64/c^2*(-1/7*c*(d*x^3+c)^(1/2)/x
^7-17/56*d*(d*x^3+c)^(1/2)/x^4-27/112/c*d^2*(d*x^3+c)^(1/2)/x-9/112*I*d^2/c
*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(
```

```
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-3/4096*d^3/c^4*(2/7*x^2*(d*x^3+
c)^(1/2)-44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)
^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(
x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3*I*c/d^3*2^(
1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d
^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-
1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Ro
otOf(_Z^3*d-8*c)))+1/512/c^3*d^3*(3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-19/2
4*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_al
pha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_
alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alp
ha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
```

```
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-
c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c
*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.36, size = 2738, normalized size = 3.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/114688*(84*sqrt(3)*(c^3*d*x^10 - 8*c^4*x^7)*(d^14/c^17)^(1/6)*arctan(1/9
*((9*sqrt(3)*c^3*d^22*x^5*(d^14/c^17)^(1/6) - sqrt(3)*(c^14*d^13*x^6 - 40*c
^15*d^12*x^3 - 32*c^16*d^11)*(d^14/c^17)^(5/6) + 3*sqrt(3)*(5*c^9*d^17*x^4
+ 8*c^10*d^16*x)*sqrt(d^14/c^17))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^12*d^4*x
^5 + c^13*d^3*x^2)*(d^14/c^17)^(2/3) + 12*sqrt(3)*(c^6*d^9*x^6 - c^7*d^8*x^
3 - 2*c^8*d^7)*(d^14/c^17)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 + 4*c
^2*d^12*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^14*d^2*x^6 + 32*c^15*d*x^3 + 40*c^
16)*(d^14/c^17)^(5/6) + 3*sqrt(3)*(7*c^9*d^6*x^4 + 4*c^10*d^5*x)*sqrt(d^14/
c^17) + 9*sqrt(3)*(c^3*d^11*x^5 + 2*c^4*d^10*x^2)*(d^14/c^17)^(1/6)))*sqrt(
(d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^12*d
^15*x^7 - 52*c^13*d^14*x^4 - 80*c^14*d^13*x)*(d^14/c^17)^(2/3) + 6*sqrt(d*x
^3 + c)*(24*(c^15*d^12*x^5 + c^16*d^11*x^2)*(d^14/c^17)^(5/6) - 4*(c^9*d^17
*x^6 + 41*c^10*d^16*x^3 + 40*c^11*d^15)*sqrt(d^14/c^17) - (c^3*d^22*x^7 - 2
8*c^4*d^21*x^4 - 272*c^5*d^20*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^20*x^8 + 20
*c^7*d^19*x^5 - 8*c^8*d^18*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
+ 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*x)) + 84
*sqrt(3)*(c^3*d*x^10 - 8*c^4*x^7)*(d^14/c^17)^(1/6)*arctan(1/9*((9*sqrt(3)*
c^3*d^22*x^5*(d^14/c^17)^(1/6) - sqrt(3)*(c^14*d^13*x^6 - 40*c^15*d^12*x^3
- 32*c^16*d^11)*(d^14/c^17)^(5/6) + 3*sqrt(3)*(5*c^9*d^17*x^4 + 8*c^10*d^16
*x)*sqrt(d^14/c^17))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^12*d^4*x^5 + c^13*d^3
*x^2)*(d^14/c^17)^(2/3) + 12*sqrt(3)*(c^6*d^9*x^6 - c^7*d^8*x^3 - 2*c^8*d^7
)*(d^14/c^17)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 + 4*c^2*d^12*x) -
sqrt(d*x^3 + c)*(sqrt(3)*(c^14*d^2*x^6 + 32*c^15*d*x^3 + 40*c^16)*(d^14/c^1
```

```
7)^(5/6) + 3*sqrt(3)*(7*c^9*d^6*x^4 + 4*c^10*d^5*x)*sqrt(d^14/c^17) + 9*sqr
t(3)*(c^3*d^11*x^5 + 2*c^4*d^10*x^2)*(d^14/c^17)^(1/6)))*sqrt((d^25*x^9 - 2
76*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^12*d^15*x^7 - 52*
c^13*d^14*x^4 - 80*c^14*d^13*x)*(d^14/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(
c^15*d^12*x^5 + c^16*d^11*x^2)*(d^14/c^17)^(5/6) - 4*(c^9*d^17*x^6 + 41*c^1
0*d^16*x^3 + 40*c^11*d^15)*sqrt(d^14/c^17) - (c^3*d^22*x^7 - 28*c^4*d^21*x^
4 - 272*c^5*d^20*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^20*x^8 + 20*c^7*d^19*x^5
 - 8*c^8*d^18*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x
^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*x)) + 1216*(d^3*x^10
- 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4
*c/d, x)) + 21*(c^3*d*x^10 - 8*c^4*x^7)*(d^14/c^17)^(1/6)*log(43046721*(d^2
5*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^12*d^15*
x^7 - 52*c^13*d^14*x^4 - 80*c^14*d^13*x)*(d^14/c^17)^(2/3) + 6*sqrt(d*x^3 +
 c)*(24*(c^15*d^12*x^5 + c^16*d^11*x^2)*(d^14/c^17)^(5/6) - 4*(c^9*d^17*x^6
 + 41*c^10*d^16*x^3 + 40*c^11*d^15)*sqrt(d^14/c^17) - (c^3*d^22*x^7 - 28*c^
4*d^21*x^4 - 272*c^5*d^20*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^20*x^8 + 20*c^7
*d^19*x^5 - 8*c^8*d^18*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 19
2*c^2*d*x^3 - 512*c^3)) - 21*(c^3*d*x^10 - 8*c^4*x^7)*(d^14/c^17)^(1/6)*log
(43046721*(d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 -
18*(c^12*d^15*x^7 - 52*c^13*d^14*x^4 - 80*c^14*d^13*x)*(d^14/c^17)^(2/3) -
6*sqrt(d*x^3 + c)*(24*(c^15*d^12*x^5 + c^16*d^11*x^2)*(d^14/c^17)^(5/6) - 4
*(c^9*d^17*x^6 + 41*c^10*d^16*x^3 + 40*c^11*d^15)*sqrt(d^14/c^17) - (c^3*d^
22*x^7 - 28*c^4*d^21*x^4 - 272*c^5*d^20*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^2
0*x^8 + 20*c^7*d^19*x^5 - 8*c^8*d^18*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 42*(c^3*d*x^10 - 8*c^4*x^7)*(d^14/c
^17)^(1/6)*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^
3*d^11 + 18*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x)*(d^14/c^17)^
(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^14/c^17)^(5/6)
 + (7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*
d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^
9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 42*(c^3*d*x^10 - 8*c^4*x^7)*(d^14/c^
17)^(1/6)*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3
*d^11 + 18*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x)*(d^14/c^17)^(
2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^14/c^17)^(5/6)
+ (7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*d
^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^9
*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*
d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 16*(76*d^3*x^9 - 525*c*d^2*x^6 - 312*
c^2*d*x^3 - 128*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^10 - 8*c^4*x^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x^3+c\right)}^{3/2}}{x^8\,{\left(8\,c-d\,x^3\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2), x)
```

3.424 $\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=95

$$\frac{8x^6\sqrt{c+dx^3}}{27d^2\,(8c-dx^3)}+\frac{2\sqrt{c+dx^3}\,(170c+7dx^3)}{27d^4}-\frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

```
[Out] -2944/81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4+8/27*x^6*(d*x^3+c
)^(1/2)/d^2/(-d*x^3+8*c)+2/27*(7*d*x^3+170*c)*(d*x^3+c)^(1/2)/d^4
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 100, 152, 65, 212}

$$-\frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}+\frac{2\sqrt{c+dx^3}\,(170c+7dx^3)}{27d^4}+\frac{8x^6\sqrt{c+dx^3}}{27d^2\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (8*x^6*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (2*Sqrt[c + d*x^3]*(170*c
+ 7*d*x^3))/(27*d^4) - (2944*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(81*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right) \\
&= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{x\left(16c^2+21cdx\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27cd^2} \\
&= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{\left(1472c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c-}}\right.}{27d^3} \\
&= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{\left(2944c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x,\right.}{27d^4} \\
&= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 81, normalized size = 0.85

$$\frac{2\left(\frac{3\sqrt{c+dx^3}\,(-1360c^2+114cdx^3+3d^2x^6)}{-8c+dx^3}-1472c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*((3*Sqrt[c + d*x^3]*(-1360*c^2 + 114*c*d*x^3 + 3*d^2*x^6))/(-8*c + d*x^3
) - 1472*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 917, normalized size = 9.65

method	result
elliptic	$\frac{512c^2\sqrt{d\,x^3+c}}{27d^4(-d\,x^3+8c)}+\frac{2x^3\sqrt{d\,x^3+c}}{9d^3}+\frac{92c\sqrt{d\,x^3+c}}{9d^4}+\frac{1472ic\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id(}{}}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2)/d^2)+32/3*c*(d*x^
3+c)^(1/2)/d)+512*c^3/d^3*(1/27*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)-1/486*I/d^
3/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d
```

```
^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-
1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Ro
otOf(_Z^3*d-8*c))+64/9*I/d^6*c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1
/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^
(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.48, size = 93, normalized size = 0.98

$$\frac{2\left(736\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+9\,(dx^3+c)^{\frac{3}{2}}+405\,\sqrt{dx^3+c}\,c-\frac{768\,\sqrt{dx^3+c}\,c^2}{dx^3-8\,c}\right)}{81\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/81*(736*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 9*(d*x^3 + c)^(3/2) + 405*sqrt(d*x^3 + c)*c - 768*sqrt(d*x^3 + c)
*c^2/(d*x^3 - 8*c))/d^4
```

Fricas [A]
time = 3.21, size = 195, normalized size = 2.05

$$\left[\frac{2\left(736\,(cdx^3-8\,c^2)\sqrt{c}\,\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(3\,d^2x^6+114\,cdx^3-1360\,c^2)\sqrt{dx^3+c}\right)}{81\,(d^5x^3-8\,cd^4)},\frac{2\left(1472\,(cdx^3-8\,c^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\,(3\,d^2x^6+114\,cdx^3-1360\,c^2)\sqrt{dx^3+c}\right)}{81\,(d^5x^3-8\,cd^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/81*(736*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt(d*x^3
 + c))/(d^5*x^3 - 8*c*d^4), 2/81*(1472*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/
3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt
(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**11/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.35, size = 93, normalized size = 0.98

$$\frac{2944\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{81\,\sqrt{-c}\,d^4}-\frac{512\,\sqrt{dx^3+c}\,c^2}{27\,(dx^3-8\,c)d^4}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^8+45\,\sqrt{dx^3+c}\,cd^8\right)}{9\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2944/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/27*sq
rt(d*x^3 + c)*c^2/((d*x^3 - 8*c)*d^4) + 2/9*((d*x^3 + c)^(3/2)*d^8 + 45*sqr
t(d*x^3 + c)*c*d^8)/d^12
```

Mupad [B]
time = 4.06, size = 107, normalized size = 1.13

$$\frac{92\,c\,\sqrt{d\,x^3+c}}{9\,d^4}+\frac{1472\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^4}+\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{512\,c^2\,\sqrt{d\,x^3+c}}{27\,d^4\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (92*c*(c + d*x^3)^(1/2))/(9*d^4) + (1472*c^(3/2)*log((10*c + d*x^3 - 6*c^(1
/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^4) + (2*x^3*(c + d*x^3)^(1/2))
/(9*d^3) + (512*c^2*(c + d*x^3)^(1/2))/(27*d^4*(8*c - d*x^3))
```

3.425 $\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{64c\sqrt{c+dx^3}}{27d^3\,(8c-dx^3)}-\frac{224\sqrt{c}\ \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

```
[Out] -224/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^3+2/3*(d*x^3+c)^(1/2
)/d^3+64/27*c*(d*x^3+c)^(1/2)/d^3/(-d*x^3+8*c)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 91, 81, 65, 212}

$$\frac{64c\sqrt{c+dx^3}}{27d^3\,(8c-dx^3)}+\frac{2\sqrt{c+dx^3}}{3d^3}-\frac{224\sqrt{c}\ \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^3) + (64*c*Sqrt[c + d*x^3])/(27*d^3*(8*c - d*x^3))
 - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
```

```
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{40c^2d+9cd^2x}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27cd^3}\\
&= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)} - \frac{(112c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27d^2}\\
&= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)} - \frac{(224c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27d^3}\\
&= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)} - \frac{224\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 70, normalized size = 0.84

$$
\frac{2\left(\frac{3\sqrt{c+dx^3}(-104c+9dx^3)}{-8c+dx^3} - 112\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*((3*Sqrt[c + d*x^3]*(-104*c + 9*d*x^3))/(-8*c + d*x^3) - 112*Sqrt[c]*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.42, size = 875, normalized size = 10.54

method	result
elliptic	$\frac{64c\sqrt{d\,x^3+c}}{27d^3(-d\,x^3+8c)}+\frac{2\sqrt{d\,x^3+c}}{3d^3}+\frac{112i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)\cdots}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right)}{\cdots}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d^3+64*c^2/d^2*(1/27*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)-1
/486*I/d^3/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*d-8*c)))+16/27*I/d^5*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
```

```
*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-
3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.49, size = 79, normalized size = 0.95

$$\frac{2\left(56\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+27\sqrt{dx^3+c}-\frac{96\sqrt{dx^3+c}\,c}{dx^3-8c}\right)}{81\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/81*(56*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 27*sqrt(d*x^3 + c) - 96*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^3
```

Fricas [A]
time = 3.23, size = 167, normalized size = 2.01

$$\left[\frac{2\left(56\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(9dx^3-104c\right)\sqrt{dx^3+c}\right)}{81\left(d^4x^3-8cd^3\right)},\frac{2\left(112\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\left(9dx^3-104c\right)\sqrt{dx^3+c}\right)}{81\left(d^4x^3-8cd^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/81*(56*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^
3), 2/81*(112*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)
 + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 0.87, size = 69, normalized size = 0.83

$$\frac{224\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\,d^3}+\frac{2\sqrt{dx^3+c}}{3\,d^3}-\frac{64\sqrt{dx^3+c}\,c}{27\,(dx^3-8\,c)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 224/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) + 2/3*sqrt(d*x
^3 + c)/d^3 - 64/27*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^3)
```

Mupad [B]
time = 4.00, size = 87, normalized size = 1.05

$$\frac{2\sqrt{d\,x^3+c}}{3\,d^3}+\frac{112\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^3}+\frac{64\,c\,\sqrt{d\,x^3+c}}{27\,d^3\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^3) + (112*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*
(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^3) + (64*c*(c + d*x^3)^(1/2))/(27*
d^3*(8*c - d*x^3))
```

3.426 $\int \frac{x^5}{(8c-dx^3)^2 \sqrt{c+dx^3}}\, dx$

Optimal. Leaf size=64

$$\frac{8\sqrt{c+dx^3}}{27d^2\,(8c-dx^3)} - \frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

```
[Out] -10/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2/c^(1/2)+8/27*(d*x^3+c)^(1/2
)/d^2/(-d*x^3+8*c)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {457, 79, 65, 212}

$$\frac{8\sqrt{c+dx^3}}{27d^2\,(8c-dx^3)} - \frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (8*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{8\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{5\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d}\\
&= \frac{8\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{10\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^2}\\
&= \frac{8\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.98

$$-\frac{8\sqrt{c+dx^3}}{27d^2(-8c+dx^3)} - \frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*Sqrt[c + d*x^3])/(27*d^2*(-8*c + d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 862, normalized size = 13.47

method	result
elliptic	$\frac{8\sqrt{d x^3+c}}{27 d^2(-d x^3+8c)} + \frac{5i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)} \frac{(-c d^2)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,(-c d^2)^{\frac{1}{3}}+(-c d^2)^{\frac{1}{3}}}{d}\right)}{(-c d^2)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x- \cdots\right.}{-3(-c d^2)^{\frac{1}{3}} \cdots}} \cdots}{\cdots}\right. \cdots}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8*c/d*(1/27*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)-1/486*I/d^3/c^2*2^(1/2)*sum((-
c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*
3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1
/27*I/d^4/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2
-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2
)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_al
pha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.49, size = 67, normalized size = 1.05

$$\frac{\frac{5 \log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{24\sqrt{dx^3+c}}{dx^3-8c}}{81d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/81*(5*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sq
rt(c) - 24*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d^2
```

Fricas [A]
time = 3.16, size = 155, normalized size = 2.42

$$\left[\frac{5\,(dx^3-8\,c)\sqrt{c}\,\log\left(\frac{dx^3-6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-24\sqrt{dx^3+c}\,c}{81\,(cd^3x^3-8\,c^2d^2)}, \frac{2\left(5\,(dx^3-8\,c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-12\sqrt{dx^3+c}\,c\right)}{81\,(cd^3x^3-8\,c^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/81*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) - 24*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2), 2/81*(5*
(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x
^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.46, size = 58, normalized size = 0.91

$$\frac{2\left(\frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d} - \frac{12\sqrt{dx^3+c}}{(dx^3-8\,c)d}\right)}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/81*(5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 12*sqrt(d*x^3 +
 c)/((d*x^3 - 8*c)*d))/d
```

Mupad [B]
time = 4.01, size = 72, normalized size = 1.12

$$\frac{5\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2}+\frac{8\,\sqrt{d\,x^3+c}}{27\,d^2\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (5*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*c^(
1/2)*d^2) + (8*(c + d*x^3)^(1/2))/(27*d^2*(8*c - d*x^3))
```

3.427 $\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

```
[Out] 1/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d+1/27*(d*x^3+c)^(1/2)/c/
d/(-d*x^3+8*c)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {455, 44, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}+\frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(27*c*d*(8*c - d*x^3)) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)]/(81*c^(3/2)*d)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{54c}\\ &= \frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27cd}\\ &= \frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 64, normalized size = 0.96

$$\frac{\frac{3\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((3*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqr
t[c])])/(81*c^(3/2)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 443, normalized size = 6.61

method	result

default	$\frac{\sqrt{d\,x^3+c}}{27cd(-d\,x^3+8c)} - \frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)} \frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{(-c\,d^2)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{(-c}{}\right.}{-3(-c\,d^2)^{\frac{1}{3}}+i}}}{}\right.}{}$
elliptic	$\frac{\sqrt{d\,x^3+c}}{27cd(-d\,x^3+8c)} - \frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)} \frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{(-c\,d^2)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{(-c}{}\right.}{-3(-c\,d^2)^{\frac{1}{3}}+i}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)-1/486*I/d^3/c^2*2^(1/2)*sum((-c*d^2)^
(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d
^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)
```

```
^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.49, size = 72, normalized size = 1.07

$$-\frac{\frac{6\sqrt{dx^3+c}}{(dx^3+c)c-9\,c^2}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{162\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -1/162*(6*sqrt(d*x^3 + c)/((d*x^3 + c)*c - 9*c^2) + log((sqrt(d*x^3 + c) -
3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d
```

Fricas [A]
time = 2.69, size = 153, normalized size = 2.28

$$\left[\frac{(dx^3-8\,c)\sqrt{c}\,\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\sqrt{dx^3+c}\,c}{162\,(c^2d^2x^3-8\,c^3d)},-\frac{(dx^3-8\,c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\sqrt{dx^3+c}\,c}{81\,(c^2d^2x^3-8\,c^3d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] [1/162*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 - 8*c^3*d), -1/81*((d*
x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 +
 c)*c)/(c^2*d^2*x^3 - 8*c^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**2/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.17, size = 59, normalized size = 0.88

$$-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\,cd}-\frac{\sqrt{dx^3+c}}{27\,(dx^3-8\,c)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 1/27*sqrt(d*x^3
 + c)/((d*x^3 - 8*c)*c*d)
```

Mupad [B]
time = 3.98, size = 75, normalized size = 1.12

$$\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{162\,c^{3/2}\,d}+\frac{\sqrt{d\,x^3+c}}{27\,c\,d\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(162*c^(3/2
)*d) + (c + d*x^3)^(1/2)/(27*c*d*(8*c - d*x^3))
```

3.428 $\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}+\frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

```
[Out] 13/2592*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/96*arctanh((d*x^3+c)
^(1/2)/c^(1/2))/c^(5/2)+1/216*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 105, 162, 65, 214, 212}

$$\frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}+\frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(216*c^2*(8*c - d*x^3)) + (13*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(2592*c^(5/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-9cd-\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{216c^2d}\\
&= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^2} + \frac{(13d)\text{Subst}\left(\int \frac{\cdots}{(8c-dx)\cdots}\cdots\right)}{1728\cdots}\\
&= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{13\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{864c^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\cdots\right)}{96\cdots}\\
&= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 83, normalized size = 0.94

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3}+13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-27\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2592c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 13*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])] - 27*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.41, size = 881, normalized size = 10.01

method	result	size
default	Expression too large to display	881
elliptic	Expression too large to display	1534

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d/c*(1/27*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)-1/486*I/d^3/c^2*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
)/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3
*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
-1/1728*I/d^2/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1
/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d
*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha
^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
)),_alpha=RootOf(_Z^3*d-8*c))-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x), x)
```

Fricas [A]
time = 1.82, size = 226, normalized size = 2.57

$$\left[\frac{13\,(dx^3-8\,c)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+27\,(dx^3-8\,c)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-24\,\sqrt{dx^3+c}\,c}{5184\,(c^3dx^3-8\,c^4)}, \frac{27\,(dx^3-8\,c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-13\,(dx^3-8\,c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-12\,\sqrt{dx^3+c}\,c}{2592\,(c^3dx^3-8\,c^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/5184*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
10*c)/(d*x^3 - 8*c)) + 27*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
 c)*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 - 8*c^4), 1/2592
*(27*(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 13*(d*x^3
- 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)
*c)/(c^3*d*x^3 - 8*c^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.01, size = 79, normalized size = 0.90

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}\,c^2}-\frac{13\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2592\,\sqrt{-c}\,c^2}-\frac{\sqrt{dx^3+c}}{216\,(dx^3-8\,c)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 13/2592*arctan(1/3*s
qrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/216*sqrt(d*x^3 + c)/((d*x^3 - 8
*c)*c^2)
```

Mupad [B]
time = 4.01, size = 80, normalized size = 0.91

$$\frac{13\,\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)}{2592\,\sqrt{c^5}}-\frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{96\,\sqrt{c^5}}+\frac{\sqrt{d\,x^3+c}}{72\,c^2\,\left(24\,c-3\,d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (13*atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(2592*(c^5)^(1/2)) - at
anh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/(96*(c^5)^(1/2)) + (c + d*x^3)^(1/
2)/(72*c^2*(24*c - 3*d*x^3))
```

3.429 $\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=124

$$\frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}$$

```
[Out] 11/10368*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+1/384*d*arctanh((d*
x^3+c)^(1/2)/c^(1/2))/c^(7/2)+5/864*d*(d*x^3+c)^(1/2)/c^3/(-d*x^3+8*c)-1/24
*(d*x^3+c)^(1/2)/c^2/x^3/(-d*x^3+8*c)
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 105, 156, 162, 65, 214, 212}

$$\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}+\frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (5*d*Sqrt[c + d*x^3])/(864*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c^2*x^3
*(8*c - d*x^3)) + (11*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(7/2
)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{2cd-\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} + \frac{\text{Subst}\left(\int \frac{-18c^2d^2+5cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x\right.}{1728c^4d}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} - \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{768c^3}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{384c^3}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} + \frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}} + \frac{d\tan}{}$$

Mathematica [A]
time = 0.17, size = 97, normalized size = 0.78

$$\frac{\frac{12\sqrt{c}(36c-5dx^3)\sqrt{c+dx^3}}{-8cx^3+dx^6} + 11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 27d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{10368c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*(36*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 11*d*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 27*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(10368*c^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.48, size = 927, normalized size = 7.48

method	result	size
risch	Expression too large to display	902
default	Expression too large to display	927
elliptic	Expression too large to display	1550

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/64/c^2*d^2*(1/27*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)-1/486*I/d^3/c^2*2^(1/2)
*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I
3^(1/2)(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*
_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_al
pha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8
*c))-1/6912*I/c^4/d*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)
^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+
1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_al
pha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
(x+1/2/d(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/
3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d
*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-1/384*d*arctanh((d*x^3+c)^(1/2)/
c^(1/2))/c^(7/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^4), x)

Fricas [A]
time = 3.80, size = 280, normalized size = 2.26

$$\left[\frac{11\,(d^2x^6 - 8\,cdx^3)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right) + 27\,(d^2x^6 - 8\,cdx^3)\sqrt{c}\,\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right) - 24\,(5\,cdx^3 - 36\,c^2)\sqrt{dx^3+c}}{20736\,(c^4dx^6 - 8\,c^5x^3)}, -\frac{27\,(d^2x^6 - 8\,cdx^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right) + 11\,(d^2x^6 - 8\,cdx^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right) + 12\,(5\,cdx^3 - 36\,c^2)\sqrt{dx^3+c}}{10368\,(c^4dx^6 - 8\,c^5x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20736*(11*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*s
qrt(c) + 10*c)/(d*x^3 - 8*c)) + 27*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3
 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d*x^3 - 36*c^2)*sqrt(d*x
^3 + c))/(c^4*d*x^6 - 8*c^5*x^3), -1/10368*(27*(d^2*x^6 - 8*c*d*x^3)*sqrt(-
c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 11*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*a
rctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 36*c^2)*sqrt(d*x^3
+ c))/(c^4*d*x^6 - 8*c^5*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.34, size = 114, normalized size = 0.92

$$-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\sqrt{-c}\,c^3}-\frac{11\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{10368\sqrt{-c}\,c^3}-\frac{5\left(dx^3+c\right)^{\frac{3}{2}}d-41\sqrt{dx^3+c}\,cd}{864\left(\left(dx^3+c\right)^2-10\left(dx^3+c\right)c+9\,c^2\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 11/10368*d*arcta
n(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/864*(5*(d*x^3 + c)^(3/2)
*d - 41*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^
3)
```

Mupad [B]
time = 4.11, size = 117, normalized size = 0.94

$$\frac{\frac{41\,d\,\sqrt{d\,x^3+c}}{288\,c^2}-\frac{5\,d\,{\left(d\,x^3+c\right)}^{3/2}}{288\,c^3}}{3\,{\left(d\,x^3+c\right)}^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2}-\frac{d\,\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1{}\mathrm{i}+\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,11{}\mathrm{i}}{27}\right)\,1{}\mathrm{i}}{384\,\sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((41*d*(c + d*x^3)^(1/2))/(288*c^2) - (5*d*(c + d*x^3)^(3/2))/(288*c^3))/(3
*(c + d*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) - (d*(atanh((c^3*(c + d*x^3)^(1
/2))/(c^7)^(1/2))*1i + (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*11i)
/27)*1i)/(384*(c^7)^(1/2))
```

3.430 $\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=164

$$-\frac{35d^2\sqrt{c+dx^3}}{13824c^4(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{48c^2x^6(8c-dx^3)}+\frac{3d\sqrt{c+dx^3}}{128c^3x^3(8c-dx^3)}+\frac{31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}}-\frac{19d^2\tanh^{-1}\left(\sqrt{\cdots}\right)}{6144c^{9}\cdots}$$

```
[Out] 31/165888*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-19/6144*d^2*arct
anh((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-35/13824*d^2*(d*x^3+c)^(1/2)/c^4/(-d*x
^3+8*c)-1/48*(d*x^3+c)^(1/2)/c^2/x^6/(-d*x^3+8*c)+3/128*d*(d*x^3+c)^(1/2)/c
^3/x^3/(-d*x^3+8*c)
```

Rubi [A]
time = 0.10, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {457, 105, 156, 162, 65, 214, 212}

$$\frac{31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}}-\frac{19d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}-\frac{35d^2\sqrt{c+dx^3}}{13824c^4(8c-dx^3)}+\frac{3d\sqrt{c+dx^3}}{128c^3x^3(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{48c^2x^6(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-35*d^2*Sqrt[c + d*x^3])/(13824*c^4*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c
^2*x^6*(8*c - d*x^3)) + (3*d*Sqrt[c + d*x^3])/(128*c^3*x^3*(8*c - d*x^3)) +
 (31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(165888*c^(9/2)) - (19*d^2*A
rcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(6144*c^(9/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}} \, dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)^2\sqrt{c+dx}} \, dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6 (8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{9cd-\frac{5d^2x}{2}}{x^2(8c-dx)^2\sqrt{c+dx}} \, dx, x, x^3\right)}{48c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6 (8c-dx^3)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3 (8c-dx^3)} + \frac{\text{Subst}\left(\int \frac{38c^2d^2-\frac{27}{2}cd^3x}{x(8c-dx)^2\sqrt{c+dx}} \, d\right.}{384c^4}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4 (8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48c^2x^6 (8c-dx^3)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3 (8c-dx^3)} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4 (8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48c^2x^6 (8c-dx^3)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3 (8c-dx^3)} + \frac{(19d^2)\,\text{S}}{}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4 (8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48c^2x^6 (8c-dx^3)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3 (8c-dx^3)} + \frac{(19d)\text{Su}}{}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4 (8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48c^2x^6 (8c-dx^3)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3 (8c-dx^3)} + \frac{31d^2\,\text{ta}}{}$$

Mathematica [A]
time = 0.24, size = 112, normalized size = 0.68

$$\frac{\frac{12\sqrt{c}\,\sqrt{c+dx^3}\,(288c^2-324cdx^3+35d^2x^6)}{-8cx^6+dx^9} + 31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 513d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{165888c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(288*c^2 - 324*c*d*x^3 + 35*d^2*x^6))/(-8*c*x^
6 + d*x^9) + 31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 513*d^2*ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]])/(165888*c^(9/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.51, size = 990, normalized size = 6.04

method	result	size

risch	Expression too large to display	912
default	Expression too large to display	990
elliptic	Expression too large to display	1580

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/c/x^6+1/4*d*(d*x^3+c)^(1/2)/c^2/x^3-1/4*d^2*
arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-1/36864*I/c^5*2^(1/2)*sum((-c*d^2
)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c
*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/
2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^
2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^
2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/256/c
^3*d*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(
3/2))-1/2048*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)+1/512/c^3*d^3*(1/
27*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)-1/486*I/d^3/c^2*2^(1/2)*sum((-c*d^2)^(1
/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d
-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(
2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^7), x)
```

Fricas [A]
time = 3.97, size = 310, normalized size = 1.89

$$\left[\frac{31\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+513\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)+24\,(35\,cd^2x^6-324\,c^2dx^3+288\,c^3)\sqrt{dx^3+c}}{331776\,(c^5dx^9-8\,c^6x^6)},\ \frac{513\,(d^3x^9-8\,cd^2x^6)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-31\,(d^3x^9-8\,cd^2x^6)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+12\,(35\,cd^2x^6-324\,c^2dx^3+288\,c^3)\sqrt{dx^3+c}}{165888\,(c^5dx^9-8\,c^6x^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/331776*(31*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 513*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log(
(d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(35*c*d^2*x^6 - 324*c^2
*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6*x^6), 1/165888*(513*(
d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 31*(d^
3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(
35*c*d^2*x^6 - 324*c^2*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6
*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.70, size = 128, normalized size = 0.78

$$\frac{19\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6144\,\sqrt{-c}\,c^4}-\frac{31\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{165888\,\sqrt{-c}\,c^4}-\frac{\sqrt{dx^3+c}\,d^2}{13824\,(dx^3-8\,c)c^4}+\frac{(dx^3+c)^{\frac{3}{2}}d^2-2\,\sqrt{dx^3+c}\,cd^2}{384\,c^4d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 19/6144*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 31/165888*d^2
*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/13824*sqrt(d*x^3 +
 c)*d^2/((d*x^3 - 8*c)*c^4) + 1/384*((d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3 +
 c)*c*d^2)/(c^4*d^2*x^6)
```

Mupad [B]
time = 4.37, size = 155, normalized size = 0.95

$$-\frac{\frac{647\,d^2\,\sqrt{d\,x^3+c}}{4608\,c^2}-\frac{197\,d^2\,{\left(d\,x^3+c\right)}^{3/2}}{2304\,c^3}+\frac{35\,d^2\,{\left(d\,x^3+c\right)}^{5/2}}{4608\,c^4}}{33\,c\,{\left(d\,x^3+c\right)}^2-57\,c^2\,\left(d\,x^3+c\right)-3\,{\left(d\,x^3+c\right)}^3+27\,c^3}+\frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,31{}\mathrm{i}}{513}\right)\,19{}\mathrm{i}}{6144\,\sqrt{c^9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)

[Out] (d^2*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i - (atanh((c^4*(c + d*x^3)^(1/2))/(3*(c^9)^(1/2)))*31i)/513)*19i)/(6144*(c^9)^(1/2)) - ((647*d^2*(c + d*x^3)^(1/2))/(4608*c^2) - (197*d^2*(c + d*x^3)^(3/2))/(2304*c^3) + (35*d^2*(c + d*x^3)^(5/2))/(4608*c^4))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3) - 3*(c + d*x^3)^3 + 27*c^3)

3.431 $\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=641

$$\frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}+\frac{44\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,d^{8/3}}-\frac{44\sqrt[6]{c}\,\tanh^{-1}}{8\ldots}$$

```
[Out] -44/81*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d
^(8/3)+44/81*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)+44/81*c^(
1/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^
(1/2)+8/27*x^2*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)+62/27*(d*x^3+c)^(1/2)/d^(8/
3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+62/81*c^(1/3)*(c^(1/3)+d^(1/3)*x)*Ellipt
icF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/
2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^
(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-31/27*c^(1/3)*(c^(1/3)+d^
(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*
x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/d^(8/3)/(d*
x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {481, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{62\sqrt{2}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\middle|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{31\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\middle|-7-4\sqrt{3}\right)}{9\,3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{44\sqrt[6]{c}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,d^{8/3}}+\frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{44\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{81d^{8/3}}+\frac{44\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}}+\frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (62*Sqrt[c + d*x^3])/(27*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (8*
x^2*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (44*c^(1/6)*ArcTan[(Sqrt[3]*c
^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*d^(8/3)) - (44*
c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*d
^(8/3)) + (44*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^(8/3)) -
```

```
(31*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (62*Sqrt[2]*c
^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c
^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/
(27*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{\int \frac{x(16c^2+31cdx^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27cd^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{\int \left(-\frac{31cx}{\sqrt{c+dx^3}}+\frac{264c^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27cd^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)} + \frac{31\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27d^2} - \frac{(88c)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9d^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)} + \frac{22\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{27d^3} + \frac{31\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+ \cdots}{\sqrt{c+dx^3}}}{27d^{7/3}}$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{31\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\,\left(\sqrt[3]{\cdots}\right.}{\cdots}$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)} + \frac{44\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{\cdots}}{\cdots}\right.}{27\sqrt{3}\,\cdots}$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)} + \frac{44\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{\cdots}}{\cdots}\right.}{27\sqrt{3}\,\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.11, size = 167, normalized size = 0.26

$$\frac{320cx^2(c+dx^3)+40cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+31dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{1080cd^2\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (320*c*x^2*(c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 31*d*x^5*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(1080*c*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 1738, normalized size = 2.71

method	result	size
elliptic	Expression too large to display	877
default	Expression too large to display	1738

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/d^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+64*c^2/d^2*(1/216*x^2*
(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/648*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
```

```
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-
3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+16/27*I/d^5*2^(1/2)*su
m(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/
3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*
(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^
(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 29.97, size = 3641, normalized size = 5.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/243*(72*sqrt(d*x^3 + c)*d*x^2 + 44*sqrt(3)*(d^4*x^3 - 8*c*d^3)*(c/d^16)^
(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c*d^16*x^16 + 784*c^2*d^15*x^13 + 7680*c^
3*d^14*x^10 + 10752*c^4*d^13*x^7 + 4096*c^5*d^12*x^4)*(c/d^16)^(2/3) + 36*s
qrt(3)*(c*d^11*x^17 + 1772*c^2*d^10*x^14 + 42592*c^3*d^9*x^11 + 96256*c^4*d
```

^8*x^8 + 69632*c^5*d^7*x^5 + 16384*c^6*d^6*x^2)*(c/d^16)^(1/3) + sqrt(3)*(c
*d^6*x^18 + 9456*c^2*d^5*x^15 + 749184*c^3*d^4*x^12 + 3017216*c^4*d^3*x^9 +
3489792*c^5*d^2*x^6 + 1572864*c^6*d*x^3 + 262144*c^7) + 12*sqrt(d*x^3 + c)
*(12*sqrt(3)*(35*c*d^18*x^14 - 14440*c^2*d^17*x^11 - 24576*c^3*d^16*x^8 - 1
6384*c^4*d^15*x^5 - 4096*c^5*d^14*x^2)*(c/d^16)^(5/6) + 18*sqrt(3)*(c*d^13*
x^15 - 1112*c^2*d^12*x^12 + 7296*c^3*d^11*x^9 + 11776*c^4*d^10*x^6 + 4096*c
^5*d^9*x^3)*sqrt(c/d^16) + sqrt(3)*(c*d^8*x^16 - 4768*c^2*d^7*x^13 + 362752
*c^3*d^6*x^10 + 709120*c^4*d^5*x^7 + 413696*c^5*d^4*x^4 + 65536*c^6*d^3*x)*
(c/d^16)^(1/6)) - 2*(324*sqrt(3)*(d^19*x^16 - 1858*c*d^18*x^13 - 4176*c^2*d
^17*x^10 - 3584*c^3*d^16*x^7 - 1024*c^4*d^15*x^4)*(c/d^16)^(5/6) + 18*sqrt(
3)*(d^14*x^17 - 5290*c*d^13*x^14 - 21152*c^2*d^12*x^11 - 47744*c^3*d^11*x^8
- 37888*c^4*d^10*x^5 - 8192*c^5*d^9*x^2)*sqrt(c/d^16) + sqrt(3)*(d^9*x^18
- 7698*c*d^8*x^15 - 1664688*c^2*d^7*x^12 - 5524864*c^3*d^6*x^9 - 6223872*c^
4*d^5*x^6 - 2703360*c^5*d^4*x^3 - 327680*c^6*d^3)*(c/d^16)^(1/6) + 6*sqrt(d
x^3 + c)(sqrt(3)*(7*d^16*x^15 + 37352*c*d^15*x^12 - 230336*c^2*d^14*x^9 -
515072*c^3*d^13*x^6 - 286720*c^4*d^12*x^3 - 32768*c^5*d^11)*(c/d^16)^(2/3)
+ 108*sqrt(3)*(53*c*d^10*x^13 + 1320*c^2*d^9*x^10 + 1536*c^3*d^8*x^7 + 512
*c^4*d^7*x^4)*(c/d^16)^(1/3) + 6*sqrt(3)*(37*c*d^5*x^14 + 28912*c^2*d^4*x^1
1 + 43584*c^3*d^3*x^8 + 20992*c^4*d^2*x^5 + 4096*c^5*d*x^2))*sqrt((18*c^2*
d^2*x^8 + 360*c^3*d*x^5 - 144*c^4*x^2 + (c*d^13*x^9 - 276*c^2*d^12*x^6 - 16
08*c^3*d^11*x^3 - 1088*c^4*d^10)*(c/d^16)^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^1
5*x^7 - 28*c^2*d^14*x^4 - 272*c^3*d^13*x)*(c/d^16)^(5/6) - 24*(c^2*d^9*x^5
+ c^3*d^8*x^2)*sqrt(c/d^16) + 4*(c^2*d^4*x^6 + 41*c^3*d^3*x^3 + 40*c^4*d^2)
(c/d^16)^(1/6)) - 18(c^2*d^7*x^7 - 52*c^3*d^6*x^4 - 80*c^4*d^5*x)*(c/d^16
)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^6*x^18 -
14952*c^2*d^5*x^15 + 2872896*c^3*d^4*x^12 + 7330304*c^4*d^3*x^9 + 6696960*
c^5*d^2*x^6 + 2457600*c^6*d*x^3 + 262144*c^7)) - 44*sqrt(3)*(d^4*x^3 - 8*c*
d^3)*(c/d^16)^(1/6)*arctan(-1/3*(324*sqrt(3)*(3*c*d^16*x^16 + 784*c^2*d^15*
x^13 + 7680*c^3*d^14*x^10 + 10752*c^4*d^13*x^7 + 4096*c^5*d^12*x^4)*(c/d^16
)^(2/3) + 36*sqrt(3)*(c*d^11*x^17 + 1772*c^2*d^10*x^14 + 42592*c^3*d^9*x^11
+ 96256*c^4*d^8*x^8 + 69632*c^5*d^7*x^5 + 16384*c^6*d^6*x^2)*(c/d^16)^(1/3
) + sqrt(3)*(c*d^6*x^18 + 9456*c^2*d^5*x^15 + 749184*c^3*d^4*x^12 + 3017216
*c^4*d^3*x^9 + 3489792*c^5*d^2*x^6 + 1572864*c^6*d*x^3 + 262144*c^7) - 12*s
qrt(d*x^3 + c)*(12*sqrt(3)*(35*c*d^18*x^14 - 14440*c^2*d^17*x^11 - 24576*c^
3*d^16*x^8 - 16384*c^4*d^15*x^5 - 4096*c^5*d^14*x^2)*(c/d^16)^(5/6) + 18*sq
rt(3)*(c*d^13*x^15 - 1112*c^2*d^12*x^12 + 7296*c^3*d^11*x^9 + 11776*c^4*d^1
0*x^6 + 4096*c^5*d^9*x^3)*sqrt(c/d^16) + sqrt(3)*(c*d^8*x^16 - 4768*c^2*d^7
*x^13 + 362752*c^3*d^6*x^10 + 709120*c^4*d^5*x^7 + 413696*c^5*d^4*x^4 + 655
36*c^6*d^3*x)*(c/d^16)^(1/6)) + 2*(324*sqrt(3)*(d^19*x^16 - 1858*c*d^18*x^1
3 - 4176*c^2*d^17*x^10 - 3584*c^3*d^16*x^7 - 1024*c^4*d^15*x^4)*(c/d^16)^(5
/6) + 18*sqrt(3)*(d^14*x^17 - 5290*c*d^13*x^14 - 21152*c^2*d^12*x^11 - 4774
4*c^3*d^11*x^8 - 37888*c^4*d^10*x^5 - 8192*c^5*d^9*x^2)*sqrt(c/d^16) + sqrt
(3)*(d^9*x^18 - 7698*c*d^8*x^15 - 1664688*c^2*d^7*x^12 - 5524864*c^3*d^6*x^
9 - 6223872*c^4*d^5*x^6 - 2703360*c^5*d^4*x^3 - 327680*c^6*d^3)*(c/d^16)^(1
/6) - 6*sqrt(d*x^3 + c)*(sqrt(3)*(7*d^16*x^15 + 37352*c*d^15*x^12 - 230336*

```
c^2*d^14*x^9 - 515072*c^3*d^13*x^6 - 286720*c^4*d^12*x^3 - 32768*c^5*d^11)*
(c/d^16)^(2/3) + 108*sqrt(3)*(53*c*d^10*x^13 + 1320*c^2*d^9*x^10 + 1536*c^3
*d^8*x^7 + 512*c^4*d^7*x^4)*(c/d^16)^(1/3) + 6*sqrt(3)*(37*c*d^5*x^14 + 289
12*c^2*d^4*x^11 + 43584*c^3*d^3*x^8 + 20992*c^4*d^2*x^5 + 4096*c^5*d*x^2)))
*sqrt((18*c^2*d^2*x^8 + 360*c^3*d*x^5 - 144*c^4*x^2 + (c*d^13*x^9 - 276*c^2
*d^12*x^6 - 1608*c^3*d^11*x^3 - 1088*c^4*d^10)*(c/d^16)^(2/3) - 6*sqrt(d*x^
3 + c)*((c*d^15*x^7 - 28*c^2*d^14*x^4 - 272*c^3*d^13*x)*(c/d^16)^(5/6) - 24
*(c^2*d^9*x^5 + c^3*d^8*x^2)*sqrt(c/d^16) + 4*(c^2*d^4*x^6 + 41*c^3*d^3*x^3
 + 40*c^4*d^2)*(c/d^16)^(1/6)) - 18*(c^2*d^7*x^7 - 52*c^3*d^6*x^4 - 80*c^4*
d^5*x)*(c/d^16)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))
/(c*d^6*x^18 - 14952*c^2*d^5*x^15 + 2872896*c^3*d^4*x^12 + 7330304*c^4*d^3*
x^9 + 6696960*c^5*d^2*x^6 + 2457600*c^6*d*x^3 + 262144*c^7)) + 558*(d*x^3 -
 8*c)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x))
 + 11*(d^4*x^3 - 8*c*d^3)*(c/d^16)^(1/6)*log(108789443753672704/9*(18*c^2*d
^2*x^8 + 360*c^3*d*x^5 - 144*c^4*x^2 + (c*d^13*x^9 - 276*c^2*d^12*x^6 - 160
8*c^3*d^11*x^3 - 1088*c^4*d^10)*(c/d^16)^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^15
*x^7 - 28*c^2*d^14*x^4 - 272*c^3*d^13*x)*(c/d^16)^(5/6) - 24*(c^2*d^9*x^5 +
 c^3*d^8*x^2)*sqrt(c/d^16) + 4*(c^2*d^4*x^6 + 4...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**7/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.432 $\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=647

$$\frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,c^{5/6}d^{5/3}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{}\right.}{3\sqrt[6]{c}\sqrt{c}}\right.}{81c^{5/6}d^{5/3}}$$

```
[Out] -1/81*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^
(5/3)+1/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(5/3)+1/81*arctan
(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(5/3)*3^(1/
2)+1/27*x^2*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)+1/27*(d*x^3+c)^(1/2)/c/d^(5/3)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3
)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(
1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
))^2)^(1/2)*3^(3/4)/c^(2/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/
3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/54*(c^(1/3)+d^(1/3)*x)*Ell
ipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^
2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(5/3)/(d*x^3+
c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A]

time = 0.52, antiderivative size = 647, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {482, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,c^{5/6}d^{5/3}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}+\frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(27*c*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*S
qrt[c + d*x^3])/(27*c*d*(8*c - d*x^3)) + ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) +
 d^(1/3)*x))/Sqrt[c + d*x^3]]/(27*Sqrt[3]*c^(5/6)*d^(5/3)) - ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(81*c^(5/6)*d^(5/3)) + ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(81*c^(5/6)*d^(5/3)) - (Sqrt[2 - Sqrt[3]]*(
```

```
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(18*3^(3
/4)*c^(2/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(2/3)*d^(5/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} - \frac{\int \frac{x\left(2c+\frac{dx^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27cd}$$

$$= \frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} - \frac{\int \left(-\frac{x}{2\sqrt{c+dx^3}} + \frac{6cx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27cd}$$

$$= \frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} - \frac{2\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9d} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{54cd}$$

$$= \frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} + \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{54cd^2} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}}{\sqrt{c+dx^3}}}{54cd^{4/3}}$$

$$= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\right.}{}$$

$$= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\,(}{\sqrt{c+}}\right.}{27\sqrt{3}\,c^{5/6}}$$

$$= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\,(}{\sqrt{c+}}\right.}{27\sqrt{3}\,c^{5/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.08, size = 166, normalized size = 0.26

$$\frac{80cx^2(c+dx^3)+10cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{2160c^2d\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(
2160*c^2*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 1305, normalized size = 2.02

method	result	size
elliptic	Expression too large to display	886
default	Expression too large to display	1305

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*I/d^4/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2
)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_a
lpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*d-8*c))+8*c/d*(1/216*x^2*(d*x^3+c)^(1/2)/c^2/(-d*
x^3+8*c)-1/648*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d
^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-
I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)
```

```
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7/1944*I/c
^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3
))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2
*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.50, size = 2647, normalized size = 4.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/972*(36*sqrt(d*x^3 + c)*d*x^2 - 4*sqrt(3)*(c*d^3*x^3 - 8*c^2*d^2)*(1/(c^
5*d^10))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^3*x^5*(1/(c^5*d^10))^(1/6) - sqrt
(3)*(c^4*d^10*x^6 - 40*c^5*d^9*x^3 - 32*c^6*d^8)*(1/(c^5*d^10))^(5/6) + 3*s
qrt(3)*(5*c^3*d^6*x^4 + 8*c^4*d^5*x)*sqrt(1/(c^5*d^10)))*sqrt(d*x^3 + c) +
(18*sqrt(3)*(c^4*d^8*x^5 + c^5*d^7*x^2)*(1/(c^5*d^10))^(2/3) + 12*sqrt(3)*(
c^2*d^5*x^6 - c^3*d^4*x^3 - 2*c^4*d^3)*(1/(c^5*d^10))^(1/3) + 3*sqrt(3)*(d^
2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^10*x^6 + 32*
c^5*d^9*x^3 + 40*c^6*d^8)*(1/(c^5*d^10))^(5/6) + 3*sqrt(3)*(7*c^3*d^6*x^4 +
 4*c^4*d^5*x)*sqrt(1/(c^5*d^10)) + 9*sqrt(3)*(c*d^3*x^5 + 2*c^2*d^2*x^2)*(1
/(c^5*d^10))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*
c^3 - 18*(c^4*d^9*x^7 - 52*c^5*d^8*x^4 - 80*c^6*d^7*x)*(1/(c^5*d^10))^(2/3)
 + 6*sqrt(d*x^3 + c)*(24*(c^5*d^10*x^5 + c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6)
- 4*(c^3*d^7*x^6 + 41*c^4*d^6*x^3 + 40*c^5*d^5)*sqrt(1/(c^5*d^10)) - (c*d^4
*x^7 - 28*c^2*d^3*x^4 - 272*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^6*
x^8 + 20*c^3*d^5*x^5 - 8*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c
```

```
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) - 4*
sqrt(3)*(c*d^3*x^3 - 8*c^2*d^2)*(1/(c^5*d^10))^(1/6)*arctan(1/9*((9*sqrt(3)
*c*d^3*x^5*(1/(c^5*d^10))^(1/6) - sqrt(3)*(c^4*d^10*x^6 - 40*c^5*d^9*x^3 -
32*c^6*d^8)*(1/(c^5*d^10))^(5/6) + 3*sqrt(3)*(5*c^3*d^6*x^4 + 8*c^4*d^5*x)*
sqrt(1/(c^5*d^10)))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^4*d^8*x^5 + c^5*d^7*x^
2)*(1/(c^5*d^10))^(2/3) + 12*sqrt(3)*(c^2*d^5*x^6 - c^3*d^4*x^3 - 2*c^4*d^3
)*(1/(c^5*d^10))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d
*x^3 + c)*(sqrt(3)*(c^4*d^10*x^6 + 32*c^5*d^9*x^3 + 40*c^6*d^8)*(1/(c^5*d^1
0))^(5/6) + 3*sqrt(3)*(7*c^3*d^6*x^4 + 4*c^4*d^5*x)*sqrt(1/(c^5*d^10)) + 9*
sqrt(3)*(c*d^3*x^5 + 2*c^2*d^2*x^2)*(1/(c^5*d^10))^(1/6)))*sqrt((d^3*x^9 -
276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^9*x^7 - 52*c^5*d^8*x^
4 - 80*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^10*x^
5 + c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6) - 4*(c^3*d^7*x^6 + 41*c^4*d^6*x^3 + 4
0*c^5*d^5)*sqrt(1/(c^5*d^10)) - (c*d^4*x^7 - 28*c^2*d^3*x^4 - 272*c^3*d^2*x
)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^6*x^8 + 20*c^3*d^5*x^5 - 8*c^4*d^4*x^2)
*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))
/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 36*(d*x^3 - 8*c)*sqrt(d)*weierstrassZet
a(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 2*(c*d^3*x^3 - 8*c^2*d^2)
*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c
^3 + 18*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x)*(1/(c^5*d^10))^(2/3
) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2)*(1/(c^5*d^10))^(
5/6) + (7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) +
(c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^
2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c*d^3*x^3 - 8*c^2*d^2)*(1/
(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 +
 18*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) -
6*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6)
 + (7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d
^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^
6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^3*x^3 - 8*c^2*d^2)*(1/(c^5*d
^10))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(
c^4*d^9*x^7 - 52*c^5*d^8*x^4 - 80*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) + 6*sqrt(
d*x^3 + c)*(24*(c^5*d^10*x^5 + c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6) - 4*(c^3*d
^7*x^6 + 41*c^4*d^6*x^3 + 40*c^5*d^5)*sqrt(1/(c^5*d^10)) - (c*d^4*x^7 - 28*
c^2*d^3*x^4 - 272*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^6*x^8 + 20*c
^3*d^5*x^5 - 8*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 - 8*c^2*d^2)*(1/(c^5*d^10))^(1/6)*l
og((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^9*x^7 -
 52*c^5*d^8*x^4 - 80*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) - 6*sqrt(d*x^3 + c)*(2
4*(c^5*d^10*x^5 + c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6) - 4*(c^3*d^7*x^6 + 41*c
^4*d^6*x^3 + 40*c^5*d^5)*sqrt(1/(c^5*d^10)) - (c*d^4*x^7 - 28*c^2*d^3*x^4 -
 272*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^6*x^8 + 20*c^3*d^5*x^5 -
8*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^
```

```
3 - 512*c^3)))/(c*d^3*x^3 - 8*c^2*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**4/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.433 $\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=644

$$\frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}-\frac{7\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}\,c^{11/6}d^{2/3}}+\frac{7\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}\right.}{3\sqrt[6]{c}}\right.}{1296c^{11/}}$$

```
[Out] 7/1296*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/
d^(2/3)-7/1296*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(2/3)-7/1296
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^(2/
3)*3^(1/2)+1/216*x^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)+1/216*(d*x^3+c)^(1/2)
/c^2/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/648*(c^(1/3)+d^(1/3)*x)*Elli
pticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(
c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/432*(c^(1/3)+
d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(5/3)/d
^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {483, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\,\sqrt[4]{3}\,c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{144\,3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{7\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}\,c^{11/6}d^{2/3}}+\frac{7\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{1296c^{11/6}d^{2/3}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{11/6}d^{2/3}}+\frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(216*c^2*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^
2*Sqrt[c + d*x^3])/(216*c^2*(8*c - d*x^3)) - (7*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(432*Sqrt[3]*c^(11/6)*d^(2/3)) + (7*A
rcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1296*c^(11/6)
*d^(2/3)) - (7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1296*c^(11/6)*d^(2/3)
) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
```

)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(144*3^(3/4)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + ((c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(108*Sqrt[2]*3^(1/4)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)} + \frac{\int \frac{x\left(25cd-\frac{d^2x^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{21cdx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{216c^2d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{432c^2} + \frac{7\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{72c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)} - \frac{7\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{864c^2d} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{c}}{\sqrt{c+dx^3}}}{432c^2\sqrt[3]{d}}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}\right.}{}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt{}}\right.}{432\sqrt{3}\,c}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}}{\sqrt{}}\right.}{432\sqrt{3}\,c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.07, size = 164, normalized size = 0.25

$$\frac{80cx^2(c+dx^3)+125cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{17280c^3\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 125*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(
17280*c^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 883, normalized size = 1.37

method	result	size
default	Expression too large to display	883
elliptic	Expression too large to display	883

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*x^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/648*I/c^2*3^(1/2)/d*(-c*d^2)^(
1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/
3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.10, size = 2648, normalized size = 4.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15552*(72*sqrt(d*x^3 + c)*d*x^2 + 28*sqrt(3)*(c^2*d^2*x^3 - 8*c^3*d)*(1/
(c^11*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c^2*d^2*x^5*(1/(c^11*d^4))^(1/6) -
 sqrt(3)*(c^9*d^5*x^6 - 40*c^10*d^4*x^3 - 32*c^11*d^3)*(1/(c^11*d^4))^(5/6)
 + 3*sqrt(3)*(5*c^6*d^3*x^4 + 8*c^7*d^2*x)*sqrt(1/(c^11*d^4)))*sqrt(d*x^3 +
 c) + (18*sqrt(3)*(c^8*d^4*x^5 + c^9*d^3*x^2)*(1/(c^11*d^4))^(2/3) + 12*sqr
t(3)*(c^4*d^3*x^6 - c^5*d^2*x^3 - 2*c^6*d)*(1/(c^11*d^4))^(1/3) + 3*sqrt(3)
*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^9*d^5*x^6 +
32*c^10*d^4*x^3 + 40*c^11*d^3)*(1/(c^11*d^4))^(5/6) + 3*sqrt(3)*(7*c^6*d^3*
x^4 + 4*c^7*d^2*x)*sqrt(1/(c^11*d^4)) + 9*sqrt(3)*(c^2*d^2*x^5 + 2*c^3*d*x^
2)*(1/(c^11*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 -
1088*c^3 - 18*(c^8*d^5*x^7 - 52*c^9*d^4*x^4 - 80*c^10*d^3*x)*(1/(c^11*d^4))
^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^5*x^5 + c^11*d^4*x^2)*(1/(c^11*d^4))
^(5/6) - 4*(c^6*d^4*x^6 + 41*c^7*d^3*x^3 + 40*c^8*d^2)*sqrt(1/(c^11*d^4)) -
 (c^2*d^3*x^7 - 28*c^3*d^2*x^4 - 272*c^4*d*x)*(1/(c^11*d^4))^(1/6)) + 18*(c
^4*d^4*x^8 + 20*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(1/(c^11*d^4))^(1/3))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x
)) + 28*sqrt(3)*(c^2*d^2*x^3 - 8*c^3*d)*(1/(c^11*d^4))^(1/6)*arctan(1/9*((9
*sqrt(3)*c^2*d^2*x^5*(1/(c^11*d^4))^(1/6) - sqrt(3)*(c^9*d^5*x^6 - 40*c^10*
d^4*x^3 - 32*c^11*d^3)*(1/(c^11*d^4))^(5/6) + 3*sqrt(3)*(5*c^6*d^3*x^4 + 8*
c^7*d^2*x)*sqrt(1/(c^11*d^4)))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^8*d^4*x^5 +
 c^9*d^3*x^2)*(1/(c^11*d^4))^(2/3) + 12*sqrt(3)*(c^4*d^3*x^6 - c^5*d^2*x^3
- 2*c^6*d)*(1/(c^11*d^4))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x)
 - sqrt(d*x^3 + c)*(sqrt(3)*(c^9*d^5*x^6 + 32*c^10*d^4*x^3 + 40*c^11*d^3)*(
1/(c^11*d^4))^(5/6) + 3*sqrt(3)*(7*c^6*d^3*x^4 + 4*c^7*d^2*x)*sqrt(1/(c^11*
d^4)) + 9*sqrt(3)*(c^2*d^2*x^5 + 2*c^3*d*x^2)*(1/(c^11*d^4))^(1/6)))*sqrt((
d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^8*d^5*x^7 - 52*
c^9*d^4*x^4 - 80*c^10*d^3*x)*(1/(c^11*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(
```

```
c^10*d^5*x^5 + c^11*d^4*x^2)*(1/(c^11*d^4))^(5/6) - 4*(c^6*d^4*x^6 + 41*c^7
*d^3*x^3 + 40*c^8*d^2)*sqrt(1/(c^11*d^4)) - (c^2*d^3*x^7 - 28*c^3*d^2*x^4 -
 272*c^4*d*x)*(1/(c^11*d^4))^(1/6)) + 18*(c^4*d^4*x^8 + 20*c^5*d^3*x^5 - 8*
c^6*d^2*x^2)*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
- 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 72*(d*x^3 - 8*c)*sqrt(d)*we
ierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 14*(c^2*d^2*x
^3 - 8*c^3*d)*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*
d*x^3 + 640*c^3 + 18*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x)*(1/(c
^11*d^4))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2)*(
1/(c^11*d^4))^(5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1
/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(1/(c^11*d^4))^
(1/6)) + 18*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2)*(1/(c^11*d^4))^
(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 14*(c^2*d^2*x^
3 - 8*c^3*d)*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d
*x^3 + 640*c^3 + 18*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x)*(1/(c^
11*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2)*(1
/(c^11*d^4))^(5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/
(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(1/(c^11*d^4))^(
1/6)) + 18*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2)*(1/(c^11*d^4))^(
1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 7*(c^2*d^2*x^3
- 8*c^3*d)*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x
^3 - 1088*c^3 - 18*(c^8*d^5*x^7 - 52*c^9*d^4*x^4 - 80*c^10*d^3*x)*(1/(c^11*
d^4))^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^5*x^5 + c^11*d^4*x^2)*(1/(c^11*
d^4))^(5/6) - 4*(c^6*d^4*x^6 + 41*c^7*d^3*x^3 + 40*c^8*d^2)*sqrt(1/(c^11*d^
4)) - (c^2*d^3*x^7 - 28*c^3*d^2*x^4 - 272*c^4*d*x)*(1/(c^11*d^4))^(1/6)) +
18*(c^4*d^4*x^8 + 20*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(1/(c^11*d^4))^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 7*(c^2*d^2*x^3 - 8*c^3*d
)*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088
*c^3 - 18*(c^8*d^5*x^7 - 52*c^9*d^4*x^4 - 80*c^10*d^3*x)*(1/(c^11*d^4))^(2/
3) - 6*sqrt(d*x^3 + c)*(24*(c^10*d^5*x^5 + c^11*d^4*x^2)*(1/(c^11*d^4))^(5/
6) - 4*(c^6*d^4*x^6 + 41*c^7*d^3*x^3 + 40*c^8*d^2)*sqrt(1/(c^11*d^4)) - (c^
2*d^3*x^7 - 28*c^3*d^2*x^4 - 272*c^4*d*x)*(1/(c^11*d^4))^(1/6)) + 18*(c^4*d
^4*x^8 + 20*c^5*d^3*x^5 - 8*c^6*d^2*x^2)*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^2*d^2*x^3 - 8*c^3*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.434 $\int \frac{1}{x^2(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=665

$$-\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{7\sqrt[3]{d}\,\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x\,(8c-dx^3)}-\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\,\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{216\sqrt{3}\,c^{17/6}}$$

```
[Out] 1/648*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(17/6)-1/648*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/648*d^
(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/6)*
3^(1/2)-7/432*(d*x^3+c)^(1/2)/c^3/x+1/216*(d*x^3+c)^(1/2)/c^2/x/(-d*x^3+8*c
)+7/432*d^(1/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+7/1296*
d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d
*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)-7/864*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-
3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(1/4)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{216\sqrt{2}\,\sqrt[4]{3}\,c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{288\;3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[3]{d}\,\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{216\sqrt{3}\,c^{17/6}}+\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{648c^{17/6}}-\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{648c^{17/6}}-\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{7\sqrt[3]{d}\,\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-7*Sqrt[c + d*x^3])/(432*c^3*x) + (7*d^(1/3)*Sqrt[c + d*x^3])/(432*c^3*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(216*c^2*x*(8*c - d*x^3
)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3
]])/(216*Sqrt[3]*c^(17/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(648*c^(17/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(648*c^(17/6)) - (7*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1
```

```
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(288*3^(3/4)*c^(8/3)*S
qrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) + (7*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(216*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} + \frac{\int \frac{28cd+\frac{5d^2x^3}{2}}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} - \frac{\int \frac{x(-160c^2d^2+14cd^3x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1728c^4d}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} - \frac{\int \left(-\frac{14cd^2x}{\sqrt{c+dx^3}} - \frac{48c^2d^2x}{(8c-dx^3)\sqrt{c+dx}}\right.}{1728c^4d}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} + \frac{(7d)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{864c^3} + \frac{d\int \frac{}{(8c-dx^3)}}{3}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} - \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{432c^3}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{7\sqrt[3]{d}\,\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{7\sqrt[3]{d}\,\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{7\sqrt[3]{d}\,\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 20.10, size = 180, normalized size = 0.27

$$\frac{-80c(54c^2+47cdx^3-7d^2x^6)+200cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7d^2x^6(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{34560c^4\sqrt{c+dx^3}\,(8cx-dx^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-80*c*(54*c^2 + 47*c*d*x^3 - 7*d^2*x^6) + 200*c*d*x^3*(8*c - d*x^3)*Sqrt[1
 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7*d
^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x
^3)/c), (d*x^3)/(8*c)])/(34560*c^4*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 1762, normalized size = 2.65

method	result	size
elliptic	Expression too large to display	898
risch	Expression too large to display	1758
default	Expression too large to display	1762

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1728*I/d^2/c^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-
c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/8*d/c*(1/216*x^2*(d*x^3+c)^(1/2)/c
^2/(-d*x^3+8*c)-1/648*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/
d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
```

```
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/
d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7/1
944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^
2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_
alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*
3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.68, size = 2618, normalized size = 3.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/7776*(4*sqrt(3)*(c^3*d*x^4 - 8*c^4*x)*(d^2/c^17)^(1/6)*arctan(1/9*((9*sq
rt(3)*c^3*d^4*x^5*(d^2/c^17)^(1/6) - sqrt(3)*(c^14*d^3*x^6 - 40*c^15*d^2*x^
3 - 32*c^16*d)*(d^2/c^17)^(5/6) + 3*sqrt(3)*(5*c^9*d^3*x^4 + 8*c^10*d^2*x)*
sqrt(d^2/c^17))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^12*d^2*x^5 + c^13*d*x^2)*(
d^2/c^17)^(2/3) + 12*sqrt(3)*(c^6*d^3*x^6 - c^7*d^2*x^3 - 2*c^8*d)*(d^2/c^1
7)^(1/3) + 3*sqrt(3)*(d^4*x^7 + 5*c*d^3*x^4 + 4*c^2*d^2*x) + sqrt(d*x^3 + c
)*(sqrt(3)*(c^14*d^2*x^6 + 32*c^15*d*x^3 + 40*c^16)*(d^2/c^17)^(5/6) + 3*sq
rt(3)*(7*c^9*d^2*x^4 + 4*c^10*d*x)*sqrt(d^2/c^17) + 9*sqrt(3)*(c^3*d^3*x^5
+ 2*c^4*d^2*x^2)*(d^2/c^17)^(1/6)))*sqrt((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^
2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^12*d^3*x^7 - 52*c^13*d^2*x^4 - 80*c^14*d*x
)*(d^2/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^15*d^2*x^5 + c^16*d*x^2)*(d^2
/c^17)^(5/6) - 4*(c^9*d^3*x^6 + 41*c^10*d^2*x^3 + 40*c^11*d)*sqrt(d^2/c^17)
 - (c^3*d^4*x^7 - 28*c^4*d^3*x^4 - 272*c^5*d^2*x)*(d^2/c^17)^(1/6)) + 18*(c
^6*d^4*x^8 + 20*c^7*d^3*x^5 - 8*c^8*d^2*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^5*x^7 - 7*c*d^4*x^4 - 8*c^2*d^3
*x)) + 4*sqrt(3)*(c^3*d*x^4 - 8*c^4*x)*(d^2/c^17)^(1/6)*arctan(1/9*((9*sqrt
(3)*c^3*d^4*x^5*(d^2/c^17)^(1/6) - sqrt(3)*(c^14*d^3*x^6 - 40*c^15*d^2*x^3
- 32*c^16*d)*(d^2/c^17)^(5/6) + 3*sqrt(3)*(5*c^9*d^3*x^4 + 8*c^10*d^2*x)*sq
rt(d^2/c^17))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^12*d^2*x^5 + c^13*d*x^2)*(d^
2/c^17)^(2/3) + 12*sqrt(3)*(c^6*d^3*x^6 - c^7*d^2*x^3 - 2*c^8*d)*(d^2/c^17)
^(1/3) + 3*sqrt(3)*(d^4*x^7 + 5*c*d^3*x^4 + 4*c^2*d^2*x) - sqrt(d*x^3 + c)*
(sqrt(3)*(c^14*d^2*x^6 + 32*c^15*d*x^3 + 40*c^16)*(d^2/c^17)^(5/6) + 3*sqrt
(3)*(7*c^9*d^2*x^4 + 4*c^10*d*x)*sqrt(d^2/c^17) + 9*sqrt(3)*(c^3*d^3*x^5 +
2*c^4*d^2*x^2)*(d^2/c^17)^(1/6)))*sqrt((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*
d^3*x^3 - 1088*c^3*d^2 - 18*(c^12*d^3*x^7 - 52*c^13*d^2*x^4 - 80*c^14*d*x)*
(d^2/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^15*d^2*x^5 + c^16*d*x^2)*(d^2/c
^17)^(5/6) - 4*(c^9*d^3*x^6 + 41*c^10*d^2*x^3 + 40*c^11*d)*sqrt(d^2/c^17) -
 (c^3*d^4*x^7 - 28*c^4*d^3*x^4 - 272*c^5*d^2*x)*(d^2/c^17)^(1/6)) + 18*(c^6
*d^4*x^8 + 20*c^7*d^3*x^5 - 8*c^8*d^2*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^5*x^7 - 7*c*d^4*x^4 - 8*c^2*d^3*x
)) + 126*(d*x^4 - 8*c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInv
erse(0, -4*c/d, x)) + (c^3*d*x^4 - 8*c^4*x)*(d^2/c^17)^(1/6)*log((d^5*x^9 -
 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^12*d^3*x^7 - 52*c^
13*d^2*x^4 - 80*c^14*d*x)*(d^2/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^15*d^
2*x^5 + c^16*d*x^2)*(d^2/c^17)^(5/6) - 4*(c^9*d^3*x^6 + 41*c^10*d^2*x^3 + 4
0*c^11*d)*sqrt(d^2/c^17) - (c^3*d^4*x^7 - 28*c^4*d^3*x^4 - 272*c^5*d^2*x)*(
d^2/c^17)^(1/6)) + 18*(c^6*d^4*x^8 + 20*c^7*d^3*x^5 - 8*c^8*d^2*x^2)*(d^2/c
^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^3*d*x^
4 - 8*c^4*x)*(d^2/c^17)^(1/6)*log((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x
^3 - 1088*c^3*d^2 - 18*(c^12*d^3*x^7 - 52*c^13*d^2*x^4 - 80*c^14*d*x)*(d^2/
c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^15*d^2*x^5 + c^16*d*x^2)*(d^2/c^17)^
(5/6) - 4*(c^9*d^3*x^6 + 41*c^10*d^2*x^3 + 40*c^11*d)*sqrt(d^2/c^17) - (c^3
*d^4*x^7 - 28*c^4*d^3*x^4 - 272*c^5*d^2*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^4*
x^8 + 20*c^7*d^3*x^5 - 8*c^8*d^2*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2
*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^3*d*x^4 - 8*c^4*x)*(d^2/c^17)^(1/6)
```

```
*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^12*d
^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x)*(d^2/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6
*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^2/c^17)^(5/6) + (7*c^9*d^2*x^6 + 152*c^10*
d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d
*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2)*(d
^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^
3*d*x^4 - 8*c^4*x)*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2
*d^2*x^3 + 640*c^3*d + 18*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x)*(d^2
/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^2/c^17)
^(5/6) + (7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*d
^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^3*x^8
+ 38*c^7*d^2*x^5 + 64*c^8*d*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
+ 192*c^2*d*x^3 - 512*c^3)) + 18*(7*d*x^3 - 54*c)*sqrt(d*x^3 + c))/(c^3*d*x
^4 - 8*c^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**2*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.435 $\int \frac{1}{x^5 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=687

$$-\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{5d\sqrt{c+dx^3}}{864c^4x}-\frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}-\frac{25d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{\ldots}}{\ldots}\right.}{27648\sqrt{3}\ldots}$$

```
[Out] 25/82944*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(23/6)-25/82944*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-25
/82944*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(23/6)*3^(1/2)-31/6912*(d*x^3+c)^(1/2)/c^3/x^4+5/864*d*(d*x^3+c)^(1/2)/c^
4/x+1/216*(d*x^3+c)^(1/2)/c^2/x^4/(-d*x^3+8*c)-5/864*d^(4/3)*(d*x^3+c)^(1/2
)/c^4/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-5/2592*d^(4/3)*(c^(1/3)+d^(1/3)*x)*El
lipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+5/1728*d^(4/3)*(c^(1
/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(11
/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$,
Rules used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{5d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{432\sqrt{2}\sqrt[4]{3}\,c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{5\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{576\,3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{25d^{4/3}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}\,c^{23/6}}+\frac{25d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{23/6}}-\frac{25d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{23/6}}-\frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{5d\sqrt{c+dx^3}}{864c^4x}-\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-31*Sqrt[c + d*x^3])/(6912*c^3*x^4) + (5*d*Sqrt[c + d*x^3])/(864*c^4*x) -
(5*d^(4/3)*Sqrt[c + d*x^3])/(864*c^4*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) +
 Sqrt[c + d*x^3]/(216*c^2*x^4*(8*c - d*x^3)) - (25*d^(4/3)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27648*Sqrt[3]*c^(23/6)) +
 (25*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/
```

```
(82944*c^(23/6)) - (25*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(82944
*c^(23/6)) + (5*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(576*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(
1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
- (5*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(432*Sqrt[2]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^5 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} + \frac{\int \frac{31cd+\frac{11d^2x^3}{2}}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} - \frac{\int \frac{320c^2d^2-\frac{155}{2}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{6912c^4d}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} + \frac{\int \frac{x(-980c^3d^3+160c^2d^{\cdots}}{(8c-dx^3)\sqrt{c+dx^{\cdots}}}}{55296c^6d}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} + \frac{\int \left(-\frac{160c^2d^3x}{\sqrt{c+dx^3}} \cdots\right.}{5\cdots}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} - \frac{(5d^2)\int \frac{x}{\sqrt{c+dx^3}}\cdots}{1728c^4}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} - \frac{(25d)\int \frac{2\sqrt[3]{c}\cdots}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}\cdots\right.}}{55\cdots}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} - \frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\cdots}{216\cdots}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} - \frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\cdots}{216\cdots}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} - \frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\cdots}{216\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 196, normalized size = 0.29

$$\frac{245cd^2x^6(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-16\left(2c(216c^3-135c^2dx^3-311cd^2x^6+40d^3x^9)+d^3x^9(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{221184c^5x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (245*c*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(2*c*(216*c^3 - 135*c^2*d*x^3 - 311*c*d
^2*x^6 + 40*d^3*x^9) + d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(221184*c^5*x^4*(8*c - d*x
^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 2241, normalized size = 3.26

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2241

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*d^2*(1/216*x^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/648*I/c^2*3^(1/2
)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha
*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*
```

```
d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)+1/64/c^2*(-1/4*(d*x^3+c)^(1/2)/x^4/c+5/8*d*(d*x^3+c)^(1/2)/c^2/x+5/24*I/c
^2*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-1/6912*I/c^4/d*2^(1/2)*sum(
1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)
*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*d-8*c))+1/256/c^3*d*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 12.47, size = 2721, normalized size = 3.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/995328*(100*sqrt(3)*(c^4*d*x^7 - 8*c^5*x^4)*(d^8/c^23)^(1/6)*arctan(1/9*
((9*sqrt(3)*c^4*d^13*x^5*(d^8/c^23)^(1/6) - sqrt(3)*(c^19*d^8*x^6 - 40*c^20
*d^7*x^3 - 32*c^21*d^6)*(d^8/c^23)^(5/6) + 3*sqrt(3)*(5*c^12*d^10*x^4 + 8*c
^13*d^9*x)*sqrt(d^8/c^23))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^16*d^3*x^5 + c^
17*d^2*x^2)*(d^8/c^23)^(2/3) + 12*sqrt(3)*(c^8*d^6*x^6 - c^9*d^5*x^3 - 2*c^
10*d^4)*(d^8/c^23)^(1/3) + 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*x^4 + 4*c^2*d^7*x)
+ sqrt(d*x^3 + c)*(sqrt(3)*(c^19*d^2*x^6 + 32*c^20*d*x^3 + 40*c^21)*(d^8/c^
23)^(5/6) + 3*sqrt(3)*(7*c^12*d^4*x^4 + 4*c^13*d^3*x)*sqrt(d^8/c^23) + 9*sq
rt(3)*(c^4*d^7*x^5 + 2*c^5*d^6*x^2)*(d^8/c^23)^(1/6)))*sqrt((d^15*x^9 - 276
*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^16*d^9*x^7 - 52*c^1
7*d^8*x^4 - 80*c^18*d^7*x)*(d^8/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^20*d
^7*x^5 + c^21*d^6*x^2)*(d^8/c^23)^(5/6) - 4*(c^12*d^10*x^6 + 41*c^13*d^9*x^
3 + 40*c^14*d^8)*sqrt(d^8/c^23) - (c^4*d^13*x^7 - 28*c^5*d^12*x^4 - 272*c^6
*d^11*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^12*x^8 + 20*c^9*d^11*x^5 - 8*c^10*d^
10*x^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)))/(d^15*x^7 - 7*c*d^14*x^4 - 8*c^2*d^13*x)) + 100*sqrt(3)*(c^4*d*x^7 - 8*
c^5*x^4)*(d^8/c^23)^(1/6)*arctan(1/9*((9*sqrt(3)*c^4*d^13*x^5*(d^8/c^23)^(1
/6) - sqrt(3)*(c^19*d^8*x^6 - 40*c^20*d^7*x^3 - 32*c^21*d^6)*(d^8/c^23)^(5/
6) + 3*sqrt(3)*(5*c^12*d^10*x^4 + 8*c^13*d^9*x)*sqrt(d^8/c^23))*sqrt(d*x^3
+ c) - (18*sqrt(3)*(c^16*d^3*x^5 + c^17*d^2*x^2)*(d^8/c^23)^(2/3) + 12*sqrt
(3)*(c^8*d^6*x^6 - c^9*d^5*x^3 - 2*c^10*d^4)*(d^8/c^23)^(1/3) + 3*sqrt(3)*(
d^9*x^7 + 5*c*d^8*x^4 + 4*c^2*d^7*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^19*d^2*x
^6 + 32*c^20*d*x^3 + 40*c^21)*(d^8/c^23)^(5/6) + 3*sqrt(3)*(7*c^12*d^4*x^4
+ 4*c^13*d^3*x)*sqrt(d^8/c^23) + 9*sqrt(3)*(c^4*d^7*x^5 + 2*c^5*d^6*x^2)*(d
^8/c^23)^(1/6)))*sqrt((d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088
*c^3*d^12 - 18*(c^16*d^9*x^7 - 52*c^17*d^8*x^4 - 80*c^18*d^7*x)*(d^8/c^23)^
(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^20*d^7*x^5 + c^21*d^6*x^2)*(d^8/c^23)^(5/6
) - 4*(c^12*d^10*x^6 + 41*c^13*d^9*x^3 + 40*c^14*d^8)*sqrt(d^8/c^23) - (c^4
*d^13*x^7 - 28*c^5*d^12*x^4 - 272*c^6*d^11*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d
^12*x^8 + 20*c^9*d^11*x^5 - 8*c^10*d^10*x^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^15*x^7 - 7*c*d^14*x^4 - 8*c^2*d
^13*x)) - 5760*(d^2*x^7 - 8*c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, wei
erstrassPInverse(0, -4*c/d, x)) + 25*(c^4*d*x^7 - 8*c^5*x^4)*(d^8/c^23)^(1/
6)*log(95367431640625*(d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088
```

```
*c^3*d^12 - 18*(c^16*d^9*x^7 - 52*c^17*d^8*x^4 - 80*c^18*d^7*x)*(d^8/c^23)^
(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^20*d^7*x^5 + c^21*d^6*x^2)*(d^8/c^23)^(5/6
) - 4*(c^12*d^10*x^6 + 41*c^13*d^9*x^3 + 40*c^14*d^8)*sqrt(d^8/c^23) - (c^4
*d^13*x^7 - 28*c^5*d^12*x^4 - 272*c^6*d^11*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d
^12*x^8 + 20*c^9*d^11*x^5 - 8*c^10*d^10*x^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 25*(c^4*d*x^7 - 8*c^5*x^4)*(d^8/c
^23)^(1/6)*log(95367431640625*(d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^
3 - 1088*c^3*d^12 - 18*(c^16*d^9*x^7 - 52*c^17*d^8*x^4 - 80*c^18*d^7*x)*(d^
8/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^20*d^7*x^5 + c^21*d^6*x^2)*(d^8/c^
23)^(5/6) - 4*(c^12*d^10*x^6 + 41*c^13*d^9*x^3 + 40*c^14*d^8)*sqrt(d^8/c^23
) - (c^4*d^13*x^7 - 28*c^5*d^12*x^4 - 272*c^6*d^11*x)*(d^8/c^23)^(1/6)) + 1
8*(c^8*d^12*x^8 + 20*c^9*d^11*x^5 - 8*c^10*d^10*x^2)*(d^8/c^23)^(1/3))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 50*(c^4*d*x^7 - 8*c^5*x^4
)*(d^8/c^23)^(1/6)*log(9765625*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3
+ 640*c^3*d^6 + 18*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x)*(d^8/c^
23)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^8/c^23)^(5
/6) + (7*c^12*d^4*x^6 + 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c
^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^
6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 50*(c^4*d*x^7 - 8*c^5*x^4)*(d^8/c^23
)^(1/6)*log(9765625*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d
^6 + 18*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x)*(d^8/c^23)^(2/3) -
 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^8/c^23)^(5/6) + (7*c^
12*d^4*x^6 + 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7
+ 80*c^5*d^6*x^4 + 160*c^6*d^5*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^6*x^8 + 38*
c^9*d^5*x^5 + 64*c^10*d^4*x^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) - 144*(40*d^2*x^6 - 351*c*d*x^3 + 216*c^2)*sqrt(d
*x^3 + c))/(c^4*d*x^7 - 8*c^5*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (-8c + dx^3)^2 \sqrt{c + dx^3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.436 $\int \frac{1}{x^8(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=711

$$-\frac{17\sqrt{c+dx^3}}{6048c^3x^7}+\frac{391d\sqrt{c+dx^3}}{193536c^4x^4}-\frac{289d^2\sqrt{c+dx^3}}{48384c^5x}+\frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right.}$$

```
[Out] 17/331776*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2)
)/c^(29/6)-17/331776*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6)-
17/331776*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2
))/c^(29/6)*3^(1/2)-17/6048*(d*x^3+c)^(1/2)/c^3/x^7+391/193536*d*(d*x^3+c)^
(1/2)/c^4/x^4-289/48384*d^2*(d*x^3+c)^(1/2)/c^5/x+1/216*(d*x^3+c)^(1/2)/c^2
/x^7/(-d*x^3+8*c)+289/48384*d^(7/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))+289/145152*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^
(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)
-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/c^(14/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-289/96768*d^(7/3)*(c^(1/3)+d^(1/3)*x)*Ell
ipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^
2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/(d*x^3+c)^(1/2
)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{289d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\middle|-7-4\sqrt{3}\right)}{24192\sqrt{2}\sqrt[4]{3}\,c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{289\sqrt{2-\sqrt{3}}\,d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\middle|-7-4\sqrt{3}\right)}{32256\,3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{17d^{7/3}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{110592\sqrt{3}\,c^{29/6}}+\frac{17d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{331776c^{29/6}}-\frac{17d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{331776c^{29/6}}+\frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{289d^2\sqrt{c+dx^3}}{48384c^5x}+\frac{391d\sqrt{c+dx^3}}{193536c^4x^4}-\frac{17\sqrt{c+dx^3}}{6048c^3x^7}+\frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-17*Sqrt[c + d*x^3])/(6048*c^3*x^7) + (391*d*Sqrt[c + d*x^3])/(193536*c^4*
x^4) - (289*d^2*Sqrt[c + d*x^3])/(48384*c^5*x) + (289*d^(7/3)*Sqrt[c + d*x^
3])/(48384*c^5*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(216*
c^2*x^7*(8*c - d*x^3)) - (17*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(110592*Sqrt[3]*c^(29/6)) + (17*d^(7/3)*ArcTanh[
```

```
(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(331776*c^(29/6)) - (
17*d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(331776*c^(29/6)) - (289*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(32256*3^(3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (289*d^(7/3)*
(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3)
+ d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(24192*
Sqrt[2]*3^(1/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)} + \frac{\int \frac{34cd+\frac{17d^2x^3}{2}}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)} - \frac{\int \frac{782c^2d^2-187cd^3x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{12096c^4d}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)} + \frac{\int \frac{18496c^3d^3-1955}{x^2(8c-dx^3)\sqrt{c-}}}{387072c}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{}}{48384c^5\left(\left(1+\sqrt{3}\right.\right.}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{}}{48384c^5\left(\left(1+\sqrt{3}\right.\right.}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{}}{48384c^5\left(\left(1+\sqrt{3}\right.\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 212, normalized size = 0.30

$$\sqrt{c+dx^3}\left(-\frac{1}{448c^3x^7}+\frac{15d}{7168c^4x^4}-\frac{171d^2}{28672c^5x}-\frac{d^3x^2}{110592c^5(-8c+dx^3)}\right)+\frac{9605d^3x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{6193152c^5\sqrt{c+dx^3}}-\frac{289d^4x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3870720c^6\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/448*1/(c^3*x^7) + (15*d)/(7168*c^4*x^4) - (171*d^2)/(28
672*c^5*x) - (d^3*x^2)/(110592*c^5*(-8*c + d*x^3))) + (9605*d^3*x^2*Sqrt[(c
 + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(6193
152*c^5*Sqrt[c + d*x^3]) - (289*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1
/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(3870720*c^6*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 2739, normalized size = 3.85

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	2739

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/256/c^3*d*(-1/4*(d*x^3+c)^(1/2)/x^4/c+5/8*d*(d*x^3+c)^(1/2)/c^2/x+5/24*I/
c^2*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))+1/64/c^2*(-1/7*(d*x^3+c)^(
1/2)/c/x^7+11/56*d*(d*x^3+c)^(1/2)/c^2/x^4-55/112*d^2*(d*x^3+c)^(1/2)/c^3/x
-55/336*I/c^3*d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
```

(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-1/36864*I/c^5*
2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_
alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha
=RootOf(_Z^3*d-8*c))+1/512/c^3*d^3*(1/216*x^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8
*c)-1/648*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7/1944*I/c^2/d^
3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-
(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c)))+3/4096*d^2/c^4*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2
)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(

```
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 12.62, size = 2738, normalized size = 3.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] -1/27869184*(476*sqrt(3)*(c^5*d*x^10 - 8*c^6*x^7)*(d^14/c^29)^(1/6)*arctan(
1/9*((9*sqrt(3)*c^5*d^22*x^5*(d^14/c^29)^(1/6) - sqrt(3)*(c^24*d^13*x^6 - 4
0*c^25*d^12*x^3 - 32*c^26*d^11)*(d^14/c^29)^(5/6) + 3*sqrt(3)*(5*c^15*d^17*
x^4 + 8*c^16*d^16*x)*sqrt(d^14/c^29))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^20*d
^4*x^5 + c^21*d^3*x^2)*(d^14/c^29)^(2/3) + 12*sqrt(3)*(c^10*d^9*x^6 - c^11*
d^8*x^3 - 2*c^12*d^7)*(d^14/c^29)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^
4 + 4*c^2*d^12*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^24*d^2*x^6 + 32*c^25*d*x^3
+ 40*c^26)*(d^14/c^29)^(5/6) + 3*sqrt(3)*(7*c^15*d^6*x^4 + 4*c^16*d^5*x)*sq
rt(d^14/c^29) + 9*sqrt(3)*(c^5*d^11*x^5 + 2*c^6*d^10*x^2)*(d^14/c^29)^(1/6)
))*sqrt((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18
*(c^20*d^15*x^7 - 52*c^21*d^14*x^4 - 80*c^22*d^13*x)*(d^14/c^29)^(2/3) + 6*
sqrt(d*x^3 + c)*(24*(c^25*d^12*x^5 + c^26*d^11*x^2)*(d^14/c^29)^(5/6) - 4*(
c^15*d^17*x^6 + 41*c^16*d^16*x^3 + 40*c^17*d^15)*sqrt(d^14/c^29) - (c^5*d^2
2*x^7 - 28*c^6*d^21*x^4 - 272*c^7*d^20*x)*(d^14/c^29)^(1/6)) + 18*(c^10*d^2
0*x^8 + 20*c^11*d^19*x^5 - 8*c^12*d^18*x^2)*(d^14/c^29)^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d
^23*x)) + 476*sqrt(3)*(c^5*d*x^10 - 8*c^6*x^7)*(d^14/c^29)^(1/6)*arctan(1/9
*((9*sqrt(3)*c^5*d^22*x^5*(d^14/c^29)^(1/6) - sqrt(3)*(c^24*d^13*x^6 - 40*c
^25*d^12*x^3 - 32*c^26*d^11)*(d^14/c^29)^(5/6) + 3*sqrt(3)*(5*c^15*d^17*x^4
 + 8*c^16*d^16*x)*sqrt(d^14/c^29))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^20*d^4*
x^5 + c^21*d^3*x^2)*(d^14/c^29)^(2/3) + 12*sqrt(3)*(c^10*d^9*x^6 - c^11*d^8
*x^3 - 2*c^12*d^7)*(d^14/c^29)^(1/3) + 3*sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 +
 4*c^2*d^12*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^24*d^2*x^6 + 32*c^25*d*x^3 + 4
```

```
0*c^26)*(d^14/c^29)^(5/6) + 3*sqrt(3)*(7*c^15*d^6*x^4 + 4*c^16*d^5*x)*sqrt(
d^14/c^29) + 9*sqrt(3)*(c^5*d^11*x^5 + 2*c^6*d^10*x^2)*(d^14/c^29)^(1/6)))*
sqrt((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c
^20*d^15*x^7 - 52*c^21*d^14*x^4 - 80*c^22*d^13*x)*(d^14/c^29)^(2/3) - 6*sqr
t(d*x^3 + c)*(24*(c^25*d^12*x^5 + c^26*d^11*x^2)*(d^14/c^29)^(5/6) - 4*(c^1
5*d^17*x^6 + 41*c^16*d^16*x^3 + 40*c^17*d^15)*sqrt(d^14/c^29) - (c^5*d^22*x
^7 - 28*c^6*d^21*x^4 - 272*c^7*d^20*x)*(d^14/c^29)^(1/6)) + 18*(c^10*d^20*x
^8 + 20*c^11*d^19*x^5 - 8*c^12*d^18*x^2)*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23
*x)) + 166464*(d^3*x^10 - 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/d, w
eierstrassPInverse(0, -4*c/d, x)) + 119*(c^5*d*x^10 - 8*c^6*x^7)*(d^14/c^29
)^(1/6)*log(2015993900449*(d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 -
1088*c^3*d^22 - 18*(c^20*d^15*x^7 - 52*c^21*d^14*x^4 - 80*c^22*d^13*x)*(d^1
4/c^29)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^25*d^12*x^5 + c^26*d^11*x^2)*(d^14
/c^29)^(5/6) - 4*(c^15*d^17*x^6 + 41*c^16*d^16*x^3 + 40*c^17*d^15)*sqrt(d^1
4/c^29) - (c^5*d^22*x^7 - 28*c^6*d^21*x^4 - 272*c^7*d^20*x)*(d^14/c^29)^(1/
6)) + 18*(c^10*d^20*x^8 + 20*c^11*d^19*x^5 - 8*c^12*d^18*x^2)*(d^14/c^29)^(
1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 119*(c^5*d*x^10
 - 8*c^6*x^7)*(d^14/c^29)^(1/6)*log(2015993900449*(d^25*x^9 - 276*c*d^24*x^
6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^20*d^15*x^7 - 52*c^21*d^14*x^
4 - 80*c^22*d^13*x)*(d^14/c^29)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^25*d^12*x^
5 + c^26*d^11*x^2)*(d^14/c^29)^(5/6) - 4*(c^15*d^17*x^6 + 41*c^16*d^16*x^3
+ 40*c^17*d^15)*sqrt(d^14/c^29) - (c^5*d^22*x^7 - 28*c^6*d^21*x^4 - 272*c^7
*d^20*x)*(d^14/c^29)^(1/6)) + 18*(c^10*d^20*x^8 + 20*c^11*d^19*x^5 - 8*c^12
*d^18*x^2)*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512
*c^3)) - 238*(c^5*d*x^10 - 8*c^6*x^7)*(d^14/c^29)^(1/6)*log(1419857*(d^14*x
^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^20*d^4*x^7
 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x)*(d^14/c^29)^(2/3) + 6*sqrt(d*x^3 + c)*(
6*(5*c^25*d*x^5 + 32*c^26*x^2)*(d^14/c^29)^(5/6) + (7*c^15*d^6*x^6 + 152*c^
16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d^10*x^4
 + 160*c^7*d^9*x)*(d^14/c^29)^(1/6)) + 18*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 +
 64*c^12*d^7*x^2)*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^
3 - 512*c^3)) + 238*(c^5*d*x^10 - 8*c^6*x^7)*(d^14/c^29)^(1/6)*log(1419857*
(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^20*
d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x)*(d^14/c^29)^(2/3) - 6*sqrt(d*x^3
 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2)*(d^14/c^29)^(5/6) + (7*c^15*d^6*x^6 +
 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d
^10*x^4 + 160*c^7*d^9*x)*(d^14/c^29)^(1/6)) + 18*(c^10*d^9*x^8 + 38*c^11*d^
8*x^5 + 64*c^12*d^7*x^2)*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c
^2*d*x^3 - 512*c^3)) + 144*(1156*d^3*x^9 - 9639*c*d^2*x^6 + 3672*c^2*d*x^3
- 3456*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^10 - 8*c^6*x^7)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 \left(-8c+dx^3\right)^2 \sqrt{c+dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**8*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^8\,\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.437 $\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{x^7\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{7}{3};2,\frac{1}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

```
[Out] 1/448*x^7*AppellF1(7/3,1/2,2,10/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c
^2/(d*x^3+c)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{7}{3};2,\frac{1}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 1/2, 10/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(448*c^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^6}{(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}} = \frac{x^7\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{7}{3};2,\frac{1}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 239 vs. $2(66) = 132$.
time = 10.21, size = 239, normalized size = 3.62

$$\frac{x\left(-\frac{23dx^3\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c}+\frac{256\left(c+dx^3-\frac{32c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}\right)}{8c-dx^3}\right)}{864d^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((-23*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/c + (256*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(8*
c - d*x^3)))/(864*d^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.37, size = 1432, normalized size = 21.70

method	result

elliptic	$\frac{8x\sqrt{d\,x^{3}+c}}{27d^{2}\left(-d\,x^{3}+8c\right)}-\frac{46i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\cdots}}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/d^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+64
*c^2/d^2*(1/216*x/c^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+1/648*I/c^2*3^(1/2)/d*(-
c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-5/972*I/c^2/d^3*2^(1/2)
*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d
^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*
d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2
)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/
18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
```

```
+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Root
Of(_Z^3*d-8*c)))+16/27*I/d^5*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-
3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2529 vs. $2(52) = 104$.
time = 6.86, size = 2529, normalized size = 38.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/243*(8*sqrt(3)*(d^4*x^3 - 8*c*d^3)*(1/(c*d^14))^(1/6)*arctan(1/9*((9*sqr
t(3)*c*d^13*x^5*(1/(c*d^14))^(5/6) + 3*sqrt(3)*(5*c*d^8*x^4 + 8*c^2*d^7*x)*
sqrt(1/(c*d^14)) - sqrt(3)*(d^4*x^6 - 40*c*d^3*x^3 - 32*c^2*d^2)*(1/(c*d^14
))^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c*d^11*x^6 - c^2*d^10*x^3 - 2*c^3*
d^9)*(1/(c*d^14))^(2/3) + 18*sqrt(3)*(c*d^6*x^5 + c^2*d^5*x^2)*(1/(c*d^14))
^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(9*sqr
t(3)*(c*d^13*x^5 + 2*c^2*d^12*x^2)*(1/(c*d^14))^(5/6) + 3*sqrt(3)*(7*c*d^8*
x^4 + 4*c^2*d^7*x)*sqrt(1/(c*d^14)) + sqrt(3)*(d^4*x^6 + 32*c*d^3*x^3 + 40*
c^2*d^2)*(1/(c*d^14))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^
3 - 1088*c^3 + 18*(c*d^12*x^8 + 20*c^2*d^11*x^5 - 8*c^3*d^10*x^2)*(1/(c*d^1
4))^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^14*x^7 - 28*c^2*d^13*x^4 - 272*c^3*d^12
*x)*(1/(c*d^14))^(5/6) + 4*(c*d^9*x^6 + 41*c^2*d^8*x^3 + 40*c^3*d^7)*sqrt(1
/(c*d^14)) - 24*(c*d^4*x^5 + c^2*d^3*x^2)*(1/(c*d^14))^(1/6)) - 18*(c*d^7*x
```

^7 - 52*c^2*d^6*x^4 - 80*c^3*d^5*x)*(1/(c*d^14))^(1/3))/(d^3*x^9 - 24*c*d^2
*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 8*sqrt
(3)*(d^4*x^3 - 8*c*d^3)*(1/(c*d^14))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^13*x^
5*(1/(c*d^14))^(5/6) + 3*sqrt(3)*(5*c*d^8*x^4 + 8*c^2*d^7*x)*sqrt(1/(c*d^14
)) - sqrt(3)*(d^4*x^6 - 40*c*d^3*x^3 - 32*c^2*d^2)*(1/(c*d^14))^(1/6))*sqrt
(d*x^3 + c) + (12*sqrt(3)*(c*d^11*x^6 - c^2*d^10*x^3 - 2*c^3*d^9)*(1/(c*d^1
4))^(2/3) + 18*sqrt(3)*(c*d^6*x^5 + c^2*d^5*x^2)*(1/(c*d^14))^(1/3) + 3*sqr
t(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c*d^13*x
^5 + 2*c^2*d^12*x^2)*(1/(c*d^14))^(5/6) + 3*sqrt(3)*(7*c*d^8*x^4 + 4*c^2*d^
7*x)*sqrt(1/(c*d^14)) + sqrt(3)*(d^4*x^6 + 32*c*d^3*x^3 + 40*c^2*d^2)*(1/(c
*d^14))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 +
 18*(c*d^12*x^8 + 20*c^2*d^11*x^5 - 8*c^3*d^10*x^2)*(1/(c*d^14))^(2/3) - 6*
sqrt(d*x^3 + c)*((c*d^14*x^7 - 28*c^2*d^13*x^4 - 272*c^3*d^12*x)*(1/(c*d^14
))^(5/6) + 4*(c*d^9*x^6 + 41*c^2*d^8*x^3 + 40*c^3*d^7)*sqrt(1/(c*d^14)) - 2
4*(c*d^4*x^5 + c^2*d^3*x^2)*(1/(c*d^14))^(1/6)) - 18*(c*d^7*x^7 - 52*c^2*d^
6*x^4 - 80*c^3*d^5*x)*(1/(c*d^14))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2
*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 36*sqrt(d*x^3 + c)*d
x - 63(d*x^3 - 8*c)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) + 4*(d^4*x^
3 - 8*c*d^3)*(1/(c*d^14))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x
^3 + 640*c^3 + 18*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2)*(1/(c*d^
14))^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^1
2*x)*(1/(c*d^14))^(5/6) + (7*c*d^9*x^6 + 152*c^2*d^8*x^3 + 64*c^3*d^7)*sqrt
(1/(c*d^14)) + 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2)*(1/(c*d^14))^(1/6)) + 18*(5
*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*(1/(c*d^14))^(1/3))/(d^3*x^9 -
24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 4*(d^4*x^3 - 8*c*d^3)*(1/(c*d^14
))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c*d^
12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2)*(1/(c*d^14))^(2/3) - 6*sqrt(d*x
^3 + c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x)*(1/(c*d^14))^(5/6)
 + (7*c*d^9*x^6 + 152*c^2*d^8*x^3 + 64*c^3*d^7)*sqrt(1/(c*d^14)) + 6*(5*c*d
^4*x^5 + 32*c^2*d^3*x^2)*(1/(c*d^14))^(1/6)) + 18*(5*c*d^7*x^7 + 64*c^2*d^6
*x^4 + 32*c^3*d^5*x)*(1/(c*d^14))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)) + 2*(d^4*x^3 - 8*c*d^3)*(1/(c*d^14))^(1/6)*log((d^3*x^9 -
 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c*d^12*x^8 + 20*c^2*d^11*x
^5 - 8*c^3*d^10*x^2)*(1/(c*d^14))^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^14*x^7 -
28*c^2*d^13*x^4 - 272*c^3*d^12*x)*(1/(c*d^14))^(5/6) + 4*(c*d^9*x^6 + 41*c^
2*d^8*x^3 + 40*c^3*d^7)*sqrt(1/(c*d^14)) - 24*(c*d^4*x^5 + c^2*d^3*x^2)*(1/
(c*d^14))^(1/6)) - 18*(c*d^7*x^7 - 52*c^2*d^6*x^4 - 80*c^3*d^5*x)*(1/(c*d^1
4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(d^4*x^3
 - 8*c*d^3)*(1/(c*d^14))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^
3 - 1088*c^3 + 18*(c*d^12*x^8 + 20*c^2*d^11*x^5 - 8*c^3*d^10*x^2)*(1/(c*d^1
4))^(2/3) - 6*sqrt(d*x^3 + c)*((c*d^14*x^7 - 28*c^2*d^13*x^4 - 272*c^3*d^12
x)(1/(c*d^14))^(5/6) + 4*(c*d^9*x^6 + 41*c^2*d^8*x^3 + 40*c^3*d^7)*sqrt(1
/(c*d^14)) - 24*(c*d^4*x^5 + c^2*d^3*x^2)*(1/(c*d^14))^(1/6)) - 18*(c*d^7*x
^7 - 52*c^2*d^6*x^4 - 80*c^3*d^5*x)*(1/(c*d^14))^(1/3))/(d^3*x^9 - 24*c*d^2
*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^4*x^3 - 8*c*d^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**6/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6}{\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x^6/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.438 $\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

```
[Out] 1/256*x^4*AppellF1(4/3,1/2,2,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^
2/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(256*c^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}} = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(66) = 132.
time = 10.13, size = 237, normalized size = 3.59

$$\frac{x\left(x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-\frac{64c\left(c+dx^3-\frac{32c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}\right)}{d(-8c+dx^3)}\right)}{1728c^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3
)/(8*c)] - (64*c*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/
c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/
(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] -
 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(d*(-8*c + d
*x^3))))/(1728*c^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.38, size = 1151, normalized size = 17.44

method	result

elliptic	$\frac{x\sqrt{d\,x^3+c}}{27cd(-d\,x^3+8c)}+\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-c\,d^2)^{\frac{1}{3}}}{d}}{-\frac{3(-c\,d^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x\right.}{}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8*c/d*(1/216*x/c^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+1/648*I/c^2*3^(1/2)/d*(-c*d
^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-5/972*I/c^2/d^3*2^(1/2)*su
m(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/
d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))+1/27*I/d^4/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(
2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(
d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2
)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c
```

```
*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*
c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2637 vs. $2(52) = 104$.
time = 7.54, size = 2637, normalized size = 39.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3888*(4*sqrt(3)*(c*d^3*x^3 - 8*c^2*d^2)*(1/(c^7*d^8))^(1/6)*arctan(1/9*((
9*sqrt(3)*c^6*d^8*x^5*(1/(c^7*d^8))^(5/6) + 3*sqrt(3)*(5*c^4*d^5*x^4 + 8*c^
5*d^4*x)*sqrt(1/(c^7*d^8)) - sqrt(3)*(c*d^3*x^6 - 40*c^2*d^2*x^3 - 32*c^3*d
)*(1/(c^7*d^8))^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c^5*d^7*x^6 - c^6*d^6
*x^3 - 2*c^7*d^5)*(1/(c^7*d^8))^(2/3) + 18*sqrt(3)*(c^3*d^4*x^5 + c^4*d^3*x
^2)*(1/(c^7*d^8))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(
d*x^3 + c)*(9*sqrt(3)*(c^6*d^8*x^5 + 2*c^7*d^7*x^2)*(1/(c^7*d^8))^(5/6) + 3
*sqrt(3)*(7*c^4*d^5*x^4 + 4*c^5*d^4*x)*sqrt(1/(c^7*d^8)) + sqrt(3)*(c*d^3*x
^6 + 32*c^2*d^2*x^3 + 40*c^3*d)*(1/(c^7*d^8))^(1/6)))*sqrt((d^3*x^9 - 276*c
*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^5*d^8*x^8 + 20*c^6*d^7*x^5 - 8
*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^9*x^7 - 28*c^
7*d^8*x^4 - 272*c^8*d^7*x)*(1/(c^7*d^8))^(5/6) + 4*(c^4*d^6*x^6 + 41*c^5*d^
5*x^3 + 40*c^6*d^4)*sqrt(1/(c^7*d^8)) - 24*(c^2*d^3*x^5 + c^3*d^2*x^2)*(1/(
c^7*d^8))^(1/6)) - 18*(c^3*d^5*x^7 - 52*c^4*d^4*x^4 - 80*c^5*d^3*x)*(1/(c^7
*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7
 - 7*c*d*x^4 - 8*c^2*x)) + 4*sqrt(3)*(c*d^3*x^3 - 8*c^2*d^2)*(1/(c^7*d^8))^
(1/6)*arctan(1/9*((9*sqrt(3)*c^6*d^8*x^5*(1/(c^7*d^8))^(5/6) + 3*sqrt(3)*(5
*c^4*d^5*x^4 + 8*c^5*d^4*x)*sqrt(1/(c^7*d^8)) - sqrt(3)*(c*d^3*x^6 - 40*c^2
*d^2*x^3 - 32*c^3*d)*(1/(c^7*d^8))^(1/6))*sqrt(d*x^3 + c) + (12*sqrt(3)*(c^
5*d^7*x^6 - c^6*d^6*x^3 - 2*c^7*d^5)*(1/(c^7*d^8))^(2/3) + 18*sqrt(3)*(c^3*
```

```
d^4*x^5 + c^4*d^3*x^2)*(1/(c^7*d^8))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4
 + 4*c^2*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^6*d^8*x^5 + 2*c^7*d^7*x^2)*(1/(
c^7*d^8))^(5/6) + 3*sqrt(3)*(7*c^4*d^5*x^4 + 4*c^5*d^4*x)*sqrt(1/(c^7*d^8))
 + sqrt(3)*(c*d^3*x^6 + 32*c^2*d^2*x^3 + 40*c^3*d)*(1/(c^7*d^8))^(1/6)))*sq
rt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^5*d^8*x^8 +
 20*c^6*d^7*x^5 - 8*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((
c^6*d^9*x^7 - 28*c^7*d^8*x^4 - 272*c^8*d^7*x)*(1/(c^7*d^8))^(5/6) + 4*(c^4*
d^6*x^6 + 41*c^5*d^5*x^3 + 40*c^6*d^4)*sqrt(1/(c^7*d^8)) - 24*(c^2*d^3*x^5
+ c^3*d^2*x^2)*(1/(c^7*d^8))^(1/6)) - 18*(c^3*d^5*x^7 - 52*c^4*d^4*x^4 - 80
*c^5*d^3*x)*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) - 144*sqrt(d*x^3 + c)*d*x - 72*
(d*x^3 - 8*c)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) + 2*(c*d^3*x^3 - 8*
c^2*d^2)*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3
+ 640*c^3 + 18*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2)*(1/(c^7*d^8)
)^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x)
*(1/(c^7*d^8))^(5/6) + (7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(
1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2)*(1/(c^7*d^8))^(1/6)) + 18
*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x)*(1/(c^7*d^8))^(1/3))/(d^3*
x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c*d^3*x^3 - 8*c^2*d^2)*
(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3
 + 18*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) -
 6*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x)*(1/(c^7*
d^8))^(5/6) + (7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^
8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2)*(1/(c^7*d^8))^(1/6)) + 18*(5*c^3*d
^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x)*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 - 8*c^2*d^2)*(1/(c^7*d^8
))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^5
*d^8*x^8 + 20*c^6*d^7*x^5 - 8*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) + 6*sqrt(d*x
^3 + c)*((c^6*d^9*x^7 - 28*c^7*d^8*x^4 - 272*c^8*d^7*x)*(1/(c^7*d^8))^(5/6)
 + 4*(c^4*d^6*x^6 + 41*c^5*d^5*x^3 + 40*c^6*d^4)*sqrt(1/(c^7*d^8)) - 24*(c^
2*d^3*x^5 + c^3*d^2*x^2)*(1/(c^7*d^8))^(1/6)) - 18*(c^3*d^5*x^7 - 52*c^4*d^
4*x^4 - 80*c^5*d^3*x)*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^
2*d*x^3 - 512*c^3)) - (c*d^3*x^3 - 8*c^2*d^2)*(1/(c^7*d^8))^(1/6)*log((d^3*
x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^5*d^8*x^8 + 20*c^6*
d^7*x^5 - 8*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c^6*d^9*
x^7 - 28*c^7*d^8*x^4 - 272*c^8*d^7*x)*(1/(c^7*d^8))^(5/6) + 4*(c^4*d^6*x^6
+ 41*c^5*d^5*x^3 + 40*c^6*d^4)*sqrt(1/(c^7*d^8)) - 24*(c^2*d^3*x^5 + c^3*d^
2*x^2)*(1/(c^7*d^8))^(1/6)) - 18*(c^3*d^5*x^7 - 52*c^4*d^4*x^4 - 80*c^5*d^3
*x)*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)
))/(c*d^3*x^3 - 8*c^2*d^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(-8c+dx^3)^2\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**3/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.439 $\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

```
[Out] 1/64*x*AppellF1(1/3,1/2,2,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/(
d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(64*c^2*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}} = \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(64) = 128.
time = 10.11, size = 237, normalized size = 3.70

$$\frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{64\left(\frac{c+dx^3}{c^2}+\frac{832F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}\right)}{8c-dx^3}\right)}{13824\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)])/c^3 + (64*((c + d*x^3)/c^2 + (832*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(8*
c - d*x^3)))/(13824*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.34, size = 729, normalized size = 11.39 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*x/c^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+1/648*I/c^2*3^(1/2)/d*(-c*d^2)^(1/
3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-5/972*I/c^2/d^3*2^(1/2)*sum(1/_al
pha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
```

```
(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_a
lpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-
8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2582 vs. 2(50) = 100.
time = 7.46, size = 2582, normalized size = 40.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15552*(20*sqrt(3)*(c^2*d^2*x^3 - 8*c^3*d)*(1/(c^13*d^2))^(1/6)*arctan(1/9
*((9*sqrt(3)*c^11*d^3*x^5*(1/(c^13*d^2))^(5/6) + 3*sqrt(3)*(5*c^7*d^2*x^4 +
 8*c^8*d*x)*sqrt(1/(c^13*d^2)) - sqrt(3)*(c^2*d^2*x^6 - 40*c^3*d*x^3 - 32*c
^4)*(1/(c^13*d^2))^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c^9*d^3*x^6 - c^10
*d^2*x^3 - 2*c^11*d)*(1/(c^13*d^2))^(2/3) + 18*sqrt(3)*(c^5*d^2*x^5 + c^6*d
*x^2)*(1/(c^13*d^2))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sq
rt(d*x^3 + c)*(9*sqrt(3)*(c^11*d^3*x^5 + 2*c^12*d^2*x^2)*(1/(c^13*d^2))^(5/
6) + 3*sqrt(3)*(7*c^7*d^2*x^4 + 4*c^8*d*x)*sqrt(1/(c^13*d^2)) + sqrt(3)*(c^
2*d^2*x^6 + 32*c^3*d*x^3 + 40*c^4)*(1/(c^13*d^2))^(1/6)))*sqrt((d^3*x^9 - 2
76*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^9*d^4*x^8 + 20*c^10*d^3*x^
5 - 8*c^11*d^2*x^2)*(1/(c^13*d^2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^4*x^7
 - 28*c^12*d^3*x^4 - 272*c^13*d^2*x)*(1/(c^13*d^2))^(5/6) + 4*(c^7*d^3*x^6
+ 41*c^8*d^2*x^3 + 40*c^9*d)*sqrt(1/(c^13*d^2)) - 24*(c^3*d^2*x^5 + c^4*d*x
^2)*(1/(c^13*d^2))^(1/6)) - 18*(c^5*d^3*x^7 - 52*c^6*d^2*x^4 - 80*c^7*d*x)*
(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/
(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 20*sqrt(3)*(c^2*d^2*x^3 - 8*c^3*d)*(1/(c
```

```
^13*d^2))^(1/6)*arctan(1/9*((9*sqrt(3)*c^11*d^3*x^5*(1/(c^13*d^2))^(5/6) +
3*sqrt(3)*(5*c^7*d^2*x^4 + 8*c^8*d*x)*sqrt(1/(c^13*d^2)) - sqrt(3)*(c^2*d^2
*x^6 - 40*c^3*d*x^3 - 32*c^4)*(1/(c^13*d^2))^(1/6))*sqrt(d*x^3 + c) + (12*s
qrt(3)*(c^9*d^3*x^6 - c^10*d^2*x^3 - 2*c^11*d)*(1/(c^13*d^2))^(2/3) + 18*sq
rt(3)*(c^5*d^2*x^5 + c^6*d*x^2)*(1/(c^13*d^2))^(1/3) + 3*sqrt(3)*(d^2*x^7 +
 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^11*d^3*x^5 + 2*c^12*d
^2*x^2)*(1/(c^13*d^2))^(5/6) + 3*sqrt(3)*(7*c^7*d^2*x^4 + 4*c^8*d*x)*sqrt(1
/(c^13*d^2)) + sqrt(3)*(c^2*d^2*x^6 + 32*c^3*d*x^3 + 40*c^4)*(1/(c^13*d^2))
^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^
9*d^4*x^8 + 20*c^10*d^3*x^5 - 8*c^11*d^2*x^2)*(1/(c^13*d^2))^(2/3) - 6*sqrt
(d*x^3 + c)*((c^11*d^4*x^7 - 28*c^12*d^3*x^4 - 272*c^13*d^2*x)*(1/(c^13*d^2
))^(5/6) + 4*(c^7*d^3*x^6 + 41*c^8*d^2*x^3 + 40*c^9*d)*sqrt(1/(c^13*d^2)) -
 24*(c^3*d^2*x^5 + c^4*d*x^2)*(1/(c^13*d^2))^(1/6)) - 18*(c^5*d^3*x^7 - 52*
c^6*d^2*x^4 - 80*c^7*d*x)*(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) - 72*sqrt(d*x^3
+ c)*d*x + 288*(d*x^3 - 8*c)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) + 10
*(c^2*d^2*x^3 - 8*c^3*d)*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6
+ 1200*c^2*d*x^3 + 640*c^3 + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^
2*x^2)*(1/(c^13*d^2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^
3*x^4 + 160*c^13*d^2*x)*(1/(c^13*d^2))^(5/6) + (7*c^7*d^3*x^6 + 152*c^8*d^2
*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2)*(1/(
c^13*d^2))^(1/6)) + 18*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x)*(1/(c^
13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 10*(c
^2*d^2*x^3 - 8*c^3*d)*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1
200*c^2*d*x^3 + 640*c^3 + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x
^2)*(1/(c^13*d^2))^(2/3) - 6*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x
^4 + 160*c^13*d^2*x)*(1/(c^13*d^2))^(5/6) + (7*c^7*d^3*x^6 + 152*c^8*d^2*x^
3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2)*(1/(c^1
3*d^2))^(1/6)) + 18*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x)*(1/(c^13*
d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 5*(c^2*d
^2*x^3 - 8*c^3*d)*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*
c^2*d*x^3 - 1088*c^3 + 18*(c^9*d^4*x^8 + 20*c^10*d^3*x^5 - 8*c^11*d^2*x^2)*
(1/(c^13*d^2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^4*x^7 - 28*c^12*d^3*x^4 -
 272*c^13*d^2*x)*(1/(c^13*d^2))^(5/6) + 4*(c^7*d^3*x^6 + 41*c^8*d^2*x^3 + 4
0*c^9*d)*sqrt(1/(c^13*d^2)) - 24*(c^3*d^2*x^5 + c^4*d*x^2)*(1/(c^13*d^2))^(
1/6)) - 18*(c^5*d^3*x^7 - 52*c^6*d^2*x^4 - 80*c^7*d*x)*(1/(c^13*d^2))^(1/3)
)/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(c^2*d^2*x^3 - 8*
c^3*d)*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 -
 1088*c^3 + 18*(c^9*d^4*x^8 + 20*c^10*d^3*x^5 - 8*c^11*d^2*x^2)*(1/(c^13*d^
2))^(2/3) - 6*sqrt(d*x^3 + c)*((c^11*d^4*x^7 - 28*c^12*d^3*x^4 - 272*c^13*d
^2*x)*(1/(c^13*d^2))^(5/6) + 4*(c^7*d^3*x^6 + 41*c^8*d^2*x^3 + 40*c^9*d)*sq
rt(1/(c^13*d^2)) - 24*(c^3*d^2*x^5 + c^4*d*x^2)*(1/(c^13*d^2))^(1/6)) - 18*
(c^5*d^3*x^7 - 52*c^6*d^2*x^4 - 80*c^7*d*x)*(1/(c^13*d^2))^(1/3))/(d^3*x^9
- 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^2*d^2*x^3 - 8*c^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.440 $\int \frac{1}{x^3(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

```
[Out] -1/128*AppellF1(-2/3,1/2,2,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/
x^2/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/128*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c^2*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{1}{x^3(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\, F_1\left(-\frac{2}{3}; 2, \frac{1}{2}; \frac{1}{3}; \frac{dx^3}{8c}, -\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 266 vs. 2(66) = 132.
time = 10.12, size = 266, normalized size = 4.03

$$\frac{-\frac{64(c+dx^3)(-216c+29dx^3)}{c^3x^2(-8c+dx^3)} + \frac{29d^2x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^4} - \frac{4096dxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c(8c-dx^3)\left(32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)\right)}}{221184\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((-64*(c + d*x^3)*(-216*c + 29*d*x^3))/(c^3*x^2*(-8*c + d*x^3)) + (29*d^2*x
^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*
c)])/c^4 - (4096*d*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(c*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/
(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] -
 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(221184*Sqrt
[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.45, size = 1456, normalized size = 22.06

method	result

elliptic	$-\frac{\sqrt{d\,x^{3}+c}}{128 c^{3} x^{2}}+\frac{d x \sqrt{d\,x^{3}+c}}{1728 c^{3}\left(-d\,x^{3}+8 c\right)}+\frac{29 i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}-\frac{i \sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\ \sqrt{\frac{x-\frac{(}{}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}+}}}{}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*(-1/2/c*(d*x^3+c)^(1/2)/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/1728*I/d^2/c^3*2^(1/2)*sum(1/_alpha^2*(-
c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*
3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/
8*d/c*(1/216*x/c^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+1/648*I/c^2*3^(1/2)/d*(-c*d
^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
```

```
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-5/972*I/c^2/d^3*2^(1/2)*su
m(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/
d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2677 vs. $2(52) = 104$.
time = 11.24, size = 2677, normalized size = 40.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/248832*(76*sqrt(3)*(c^3*d*x^5 - 8*c^4*x^2)*(d^4/c^19)^(1/6)*arctan(1/9*((
9*sqrt(3)*c^16*d^4*x^5*(d^4/c^19)^(5/6) + 3*sqrt(3)*(5*c^10*d^5*x^4 + 8*c^1
1*d^4*x)*sqrt(d^4/c^19) - sqrt(3)*(c^3*d^7*x^6 - 40*c^4*d^6*x^3 - 32*c^5*d^
5)*(d^4/c^19)^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c^13*d^2*x^6 - c^14*d*x
^3 - 2*c^15)*(d^4/c^19)^(2/3) + 18*sqrt(3)*(c^7*d^3*x^5 + c^8*d^2*x^2)*(d^4
/c^19)^(1/3) + 3*sqrt(3)*(d^5*x^7 + 5*c*d^4*x^4 + 4*c^2*d^3*x) - sqrt(d*x^3
 + c)*(9*sqrt(3)*(c^16*d*x^5 + 2*c^17*x^2)*(d^4/c^19)^(5/6) + 3*sqrt(3)*(7*
c^10*d^2*x^4 + 4*c^11*d*x)*sqrt(d^4/c^19) + sqrt(3)*(c^3*d^4*x^6 + 32*c^4*d
^3*x^3 + 40*c^5*d^2)*(d^4/c^19)^(1/6)))*sqrt((d^9*x^9 - 276*c*d^8*x^6 - 160
8*c^2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^13*d^6*x^8 + 20*c^14*d^5*x^5 - 8*c^15*
d^4*x^2)*(d^4/c^19)^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^5*x^7 - 28*c^17*d^4*
```

x^4 - 272*c^18*d^3*x)*(d^4/c^19)^(5/6) + 4*(c^10*d^6*x^6 + 41*c^11*d^5*x^3 + 40*c^12*d^4)*sqrt(d^4/c^19) - 24*(c^4*d^7*x^5 + c^5*d^6*x^2)*(d^4/c^19)^(1/6)) - 18*(c^7*d^7*x^7 - 52*c^8*d^6*x^4 - 80*c^9*d^5*x)*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^8*x^7 - 7*c*d^7*x^4 - 8*c^2*d^6*x)) + 76*sqrt(3)*(c^3*d*x^5 - 8*c^4*x^2)*(d^4/c^19)^(1/6)*arctan(1/9*((9*sqrt(3)*c^16*d^4*x^5*(d^4/c^19)^(5/6) + 3*sqrt(3)*(5*c^10*d^5*x^4 + 8*c^11*d^4*x)*sqrt(d^4/c^19) - sqrt(3)*(c^3*d^7*x^6 - 40*c^4*d^6*x^3 - 32*c^5*d^5)*(d^4/c^19)^(1/6))*sqrt(d*x^3 + c) + (12*sqrt(3)*(c^13*d^2*x^6 - c^14*d*x^3 - 2*c^15)*(d^4/c^19)^(2/3) + 18*sqrt(3)*(c^7*d^3*x^5 + c^8*d^2*x^2)*(d^4/c^19)^(1/3) + 3*sqrt(3)*(d^5*x^7 + 5*c*d^4*x^4 + 4*c^2*d^3*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^16*d*x^5 + 2*c^17*x^2)*(d^4/c^19)^(5/6) + 3*sqrt(3)*(7*c^10*d^2*x^4 + 4*c^11*d*x)*sqrt(d^4/c^19) + sqrt(3)*(c^3*d^4*x^6 + 32*c^4*d^3*x^3 + 40*c^5*d^2)*(d^4/c^19)^(1/6)))*sqrt((d^9*x^9 - 276*c*d^8*x^6 - 1608*c^2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^13*d^6*x^8 + 20*c^14*d^5*x^5 - 8*c^15*d^4*x^2)*(d^4/c^19)^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^5*x^7 - 28*c^17*d^4*x^4 - 272*c^18*d^3*x)*(d^4/c^19)^(5/6) + 4*(c^10*d^6*x^6 + 41*c^11*d^5*x^3 + 40*c^12*d^4)*sqrt(d^4/c^19) - 24*(c^4*d^7*x^5 + c^5*d^6*x^2)*(d^4/c^19)^(1/6)) - 18*(c^7*d^7*x^7 - 52*c^8*d^6*x^4 - 80*c^9*d^5*x)*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^8*x^7 - 7*c*d^7*x^4 - 8*c^2*d^6*x)) - 720*(d*x^5 - 8*c*x^2)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) + 19*(c^3*d*x^5 - 8*c^4*x^2)*(d^4/c^19)^(1/6)*log(6131066257801*(d^9*x^9 - 276*c*d^8*x^6 - 1608*c^2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^13*d^6*x^8 + 20*c^14*d^5*x^5 - 8*c^15*d^4*x^2)*(d^4/c^19)^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^5*x^7 - 28*c^17*d^4*x^4 - 272*c^18*d^3*x)*(d^4/c^19)^(5/6) + 4*(c^10*d^6*x^6 + 41*c^11*d^5*x^3 + 40*c^12*d^4)*sqrt(d^4/c^19) - 24*(c^4*d^7*x^5 + c^5*d^6*x^2)*(d^4/c^19)^(1/6)) - 18*(c^7*d^7*x^7 - 52*c^8*d^6*x^4 - 80*c^9*d^5*x)*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 19*(c^3*d*x^5 - 8*c^4*x^2)*(d^4/c^19)^(1/6)*log(6131066257801*(d^9*x^9 - 276*c*d^8*x^6 - 1608*c^2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^13*d^6*x^8 + 20*c^14*d^5*x^5 - 8*c^15*d^4*x^2)*(d^4/c^19)^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^5*x^7 - 28*c^17*d^4*x^4 - 272*c^18*d^3*x)*(d^4/c^19)^(5/6) + 4*(c^10*d^6*x^6 + 41*c^11*d^5*x^3 + 40*c^12*d^4)*sqrt(d^4/c^19) - 24*(c^4*d^7*x^5 + c^5*d^6*x^2)*(d^4/c^19)^(1/6)) - 18*(c^7*d^7*x^7 - 52*c^8*d^6*x^4 - 80*c^9*d^5*x)*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 38*(c^3*d*x^5 - 8*c^4*x^2)*(d^4/c^19)^(1/6)*log(2476099*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + 18*(c^13*d^3*x^8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2)*(d^4/c^19)^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x)*(d^4/c^19)^(5/6) + (7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2)*(d^4/c^19)^(1/6)) + 18*(5*c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x)*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 38*(c^3*d*x^5 - 8*c^4*x^2)*(d^4/c^19)^(1/6)*log(2476099*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + 18*(c^13*d^3*x^8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2)*(d^4/c^19)^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x)*(d^4/c^19)^(5/6) + (7*c^10*d^3*x^6 + 152*c^11*d^2*x^

```
3 + 64*c^12*d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2)*(d^4/c^1
9)^(1/6)) + 18*(5*c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x)*(d^4/c^19)^(
1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 72*(29*d*x^3 -
216*c)*sqrt(d*x^3 + c))/(c^3*d*x^5 - 8*c^4*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.441 $\int \frac{1}{x^6(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{5}{3};2,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

```
[Out] -1/320*AppellF1(-5/3,1/2,2,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2
/x^5/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};2,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/320*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 1/2, -2/3, (d*x^3)/(8*c), -((
d*x^3)/c)])/(c^2*x^5*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^6 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\, dx = \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{1}{x^6(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\, dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\, F_1\left(-\frac{5}{3};2,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 279 vs. 2(66) = 132.
time = 10.15, size = 279, normalized size = 4.23

$$\frac{\frac{64(c+dx^3)(864c^2-1080cdx^3+119d^2x^6)}{c^4x^5(-8c+dx^3)} - \frac{119d^3x^4\sqrt{1+\frac{dx^3}{c}}\, F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^5} + \frac{1404928d^2xF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^2(8c-dx^3)\left(32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)\right)}}{2211840\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((64*(c + d*x^3)*(864*c^2 - 1080*c*d*x^3 + 119*d^2*x^6))/(c^4*x^5*(-8*c + d
*x^3)) - (119*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x
^3)/c), (d*x^3)/(8*c)])/c^5 + (1404928*d^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((
d*x^3)/c), (d*x^3)/(8*c)])/(c^2*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4
/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d
*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^
3)/(8*c)]))))/(2211840*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.44, size = 1783, normalized size = 27.02

method	result

elliptic	$-\frac{\sqrt{d\,x^{3}+c}}{320c^{3}x^{5}}+\frac{9d\sqrt{d\,x^{3}+c}}{2560c^{4}x^{2}}+\frac{d^{2}x\sqrt{d\,x^{3}+c}}{13824c^{4}(-d\,x^{3}+8c)}-\frac{119id\sqrt{3}\,(-c\,d^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^{2})^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^{2})^{\frac{1}{3}}}{2d}\right)\sqrt{}}{(-c\,d^{2})^{\frac{1}{3}}}}}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*(-1/5/c*(d*x^3+c)^(1/2)/x^5+7/20*d/c^2*(d*x^3+c)^(1/2)/x^2-7/60*I/
c^2*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/64/c
^2*d^2*(1/216*x/c^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+1/648*I/c^2*3^(1/2)/d*(-c*
d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-5/972*I/c^2/d^3*2^(1/2)*s
um(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
```

```
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18
/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf
(_Z^3*d-8*c))+1/256/c^3*d*(-1/2/c*(d*x^3+c)^(1/2)/x^2+1/6*I/c*3^(1/2)*(-c*
d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/6912*I/c^4/d*2^(1/2)*s
um(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18
/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf
(_Z^3*d-8*c))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2709 vs. $2(52) = 104$.
time = 12.39, size = 2709, normalized size = 41.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2488320*(140*sqrt(3)*(c^4*d*x^8 - 8*c^5*x^5)*(d^10/c^25)^(1/6)*arctan(1/9
*((9*sqrt(3)*c^21*d^9*x^5*(d^10/c^25)^(5/6) + 3*sqrt(3)*(5*c^13*d^12*x^4 +
8*c^14*d^11*x)*sqrt(d^10/c^25) - sqrt(3)*(c^4*d^16*x^6 - 40*c^5*d^15*x^3 -
```

32*c^6*d^14)*(d^10/c^25)^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c^17*d^3*x^6 - c^18*d^2*x^3 - 2*c^19*d)*(d^10/c^25)^(2/3) + 18*sqrt(3)*(c^9*d^6*x^5 + c^10*d^5*x^2)*(d^10/c^25)^(1/3) + 3*sqrt(3)*(d^10*x^7 + 5*c*d^9*x^4 + 4*c^2*d^8*x) - sqrt(d*x^3 + c)*(9*sqrt(3)*(c^21*d*x^5 + 2*c^22*x^2)*(d^10/c^25)^(5/6) + 3*sqrt(3)*(7*c^13*d^4*x^4 + 4*c^14*d^3*x)*sqrt(d^10/c^25) + sqrt(3)*(c^4*d^8*x^6 + 32*c^5*d^7*x^3 + 40*c^6*d^6)*(d^10/c^25)^(1/6)))*sqrt((d^19*x^9 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^17*d^12*x^8 + 20*c^18*d^11*x^5 - 8*c^19*d^10*x^2)*(d^10/c^25)^(2/3) + 6*sqrt(d*x^3 + c)*((c^21*d^10*x^7 - 28*c^22*d^9*x^4 - 272*c^23*d^8*x)*(d^10/c^25)^(5/6) + 4*(c^13*d^13*x^6 + 41*c^14*d^12*x^3 + 40*c^15*d^11)*sqrt(d^10/c^25) - 24*(c^5*d^16*x^5 + c^6*d^15*x^2)*(d^10/c^25)^(1/6)) - 18*(c^9*d^15*x^7 - 52*c^10*d^14*x^4 - 80*c^11*d^13*x)*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^18*x^7 - 7*c*d^17*x^4 - 8*c^2*d^16*x)) + 140*sqrt(3)*(c^4*d*x^8 - 8*c^5*x^5)*(d^10/c^25)^(1/6)*arctan(1/9*((9*sqrt(3)*c^21*d^9*x^5*(d^10/c^25)^(5/6) + 3*sqrt(3)*(5*c^13*d^12*x^4 + 8*c^14*d^11*x)*sqrt(d^10/c^25) - sqrt(3)*(c^4*d^16*x^6 - 40*c^5*d^15*x^3 - 32*c^6*d^14)*(d^10/c^25)^(1/6))*sqrt(d*x^3 + c) + (12*sqrt(3)*(c^17*d^3*x^6 - c^18*d^2*x^3 - 2*c^19*d)*(d^10/c^25)^(2/3) + 18*sqrt(3)*(c^9*d^6*x^5 + c^10*d^5*x^2)*(d^10/c^25)^(1/3) + 3*sqrt(3)*(d^10*x^7 + 5*c*d^9*x^4 + 4*c^2*d^8*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^21*d*x^5 + 2*c^22*x^2)*(d^10/c^25)^(5/6) + 3*sqrt(3)*(7*c^13*d^4*x^4 + 4*c^14*d^3*x)*sqrt(d^10/c^25) + sqrt(3)*(c^4*d^8*x^6 + 32*c^5*d^7*x^3 + 40*c^6*d^6)*(d^10/c^25)^(1/6)))*sqrt((d^19*x^9 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^17*d^12*x^8 + 20*c^18*d^11*x^5 - 8*c^19*d^10*x^2)*(d^10/c^25)^(2/3) - 6*sqrt(d*x^3 + c)*((c^21*d^10*x^7 - 28*c^22*d^9*x^4 - 272*c^23*d^8*x)*(d^10/c^25)^(5/6) + 4*(c^13*d^13*x^6 + 41*c^14*d^12*x^3 + 40*c^15*d^11)*sqrt(d^10/c^25) - 24*(c^5*d^16*x^5 + c^6*d^15*x^2)*(d^10/c^25)^(1/6)) - 18*(c^9*d^15*x^7 - 52*c^10*d^14*x^4 - 80*c^11*d^13*x)*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^18*x^7 - 7*c*d^17*x^4 - 8*c^2*d^16*x)) + 11088*(d^2*x^8 - 8*c*d*x^5)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) + 35*(c^4*d*x^8 - 8*c^5*x^5)*(d^10/c^25)^(1/6)*log(282475249*(d^19*x^9 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^17*d^12*x^8 + 20*c^18*d^11*x^5 - 8*c^19*d^10*x^2)*(d^10/c^25)^(2/3) + 6*sqrt(d*x^3 + c)*((c^21*d^10*x^7 - 28*c^22*d^9*x^4 - 272*c^23*d^8*x)*(d^10/c^25)^(5/6) + 4*(c^13*d^13*x^6 + 41*c^14*d^12*x^3 + 40*c^15*d^11)*sqrt(d^10/c^25) - 24*(c^5*d^16*x^5 + c^6*d^15*x^2)*(d^10/c^25)^(1/6)) - 18*(c^9*d^15*x^7 - 52*c^10*d^14*x^4 - 80*c^11*d^13*x)*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 35*(c^4*d*x^8 - 8*c^5*x^5)*(d^10/c^25)^(1/6)*log(282475249*(d^19*x^9 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^17*d^12*x^8 + 20*c^18*d^11*x^5 - 8*c^19*d^10*x^2)*(d^10/c^25)^(2/3) - 6*sqrt(d*x^3 + c)*((c^21*d^10*x^7 - 28*c^22*d^9*x^4 - 272*c^23*d^8*x)*(d^10/c^25)^(5/6) + 4*(c^13*d^13*x^6 + 41*c^14*d^12*x^3 + 40*c^15*d^11)*sqrt(d^10/c^25) - 24*(c^5*d^16*x^5 + c^6*d^15*x^2)*(d^10/c^25)^(1/6)) - 18*(c^9*d^15*x^7 - 52*c^10*d^14*x^4 - 80*c^11*d^13*x)*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 70*(c^4*d*x^8 - 8*c^5*x^5)*(d^10/c^25)^(1/6)*log(16807*(d^11*x^9 + 31

```
8*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + 18*(c^17*d^4*x^8 + 38*c^18*
d^3*x^5 + 64*c^19*d^2*x^2)*(d^10/c^25)^(2/3) + 6*sqrt(d*x^3 + c)*((c^21*d^2
*x^7 + 80*c^22*d*x^4 + 160*c^23*x)*(d^10/c^25)^(5/6) + (7*c^13*d^5*x^6 + 15
2*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d
^7*x^2)*(d^10/c^25)^(1/6)) + 18*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*
d^5*x)*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) - 70*(c^4*d*x^8 - 8*c^5*x^5)*(d^10/c^25)^(1/6)*log(16807*(d^11*x^9 + 318
*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + 18*(c^17*d^4*x^8 + 38*c^18*d
^3*x^5 + 64*c^19*d^2*x^2)*(d^10/c^25)^(2/3) - 6*sqrt(d*x^3 + c)*((c^21*d^2*
x^7 + 80*c^22*d*x^4 + 160*c^23*x)*(d^10/c^25)^(5/6) + (7*c^13*d^5*x^6 + 152
*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^
7*x^2)*(d^10/c^25)^(1/6)) + 18*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d
^5*x)*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)
) + 72*(119*d^2*x^6 - 1080*c*d*x^3 + 864*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^8 -
 8*c^5*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**6*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^6\,\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.442 $\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2(38c+39dx^3)}{81d^4\sqrt{c+dx^3}}-\frac{640\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

```
[Out] -640/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^4+8/27*x^6/d^2/(-d*
x^3+8*c)/(d*x^3+c)^(1/2)+2/81*(39*d*x^3+38*c)/d^4/(d*x^3+c)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 100, 151, 65, 212}

$$\frac{2(38c+39dx^3)}{81d^4\sqrt{c+dx^3}}-\frac{640\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}+\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (8*x^6)/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (2*(38*c + 39*d*x^3))/(81*
d^4*Sqrt[c + d*x^3]) - (640*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
243*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(8c-dx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{8x^6}{27d^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{x(16c^2+13cdx)}{(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{27cd^2}$$

$$= \frac{8x^6}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2(38c+39dx^3)}{81d^4\sqrt{c+dx^3}} - \frac{(320c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+}}\right.}{81d^3}$$

$$= \frac{8x^6}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2(38c+39dx^3)}{81d^4\sqrt{c+dx^3}} - \frac{(640c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\right.}{81d^4}$$

$$= \frac{8x^6}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2(38c+39dx^3)}{81d^4\sqrt{c+dx^3}} - \frac{640\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right.}{243d^4}$$

Mathematica [A]
time = 0.09, size = 99, normalized size = 1.04

$$-\frac{2\left(912c^2+822cdx^3-81d^2x^6-320\sqrt{c}\,(8c-dx^3)\sqrt{c+dx^3}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{243d^4\,(-8c+dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(912*c^2 + 822*c*d*x^3 - 81*d^2*x^6 - 320*Sqrt[c]*(8*c - d*x^3)*Sqrt[c
+ d*x^3]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(243*d^4*(-8*c + d*x^3)*Sqr
t[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.42, size = 971, normalized size = 10.22

method	result
elliptic	$\frac{512c\sqrt{d\,x^3+c}}{243d^4(-d\,x^3+8c)}+\frac{2c}{243d^4\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2\sqrt{d\,x^3+c}}{3d^4}+\frac{320i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id}{\ldots}}\ldots}{\ldots}\right.}{\ldots}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d*(2/3/d^2*c/((x^3+c/d)*d)^(1/2)+2/3*(d*x^3+c)^(1/2)/d^2)-32/3*c/d/(
d*x^3+c)^(1/2))+512*c^3/d^3*(1/243/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243
/d/c^2/((x^3+c/d)*d)^(1/2)-1/1458*I/c^3/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2
*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
```

```
1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
)^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/
2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_al
pha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+192/d^3*c^2*(2/27/
d/c/((x^3+c/d)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-
3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [A]
time = 0.49, size = 98, normalized size = 1.03

$$\frac{2\left(160\sqrt{c}\,\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+81\sqrt{dx^3+c}-\frac{3\left(85\,(dx^3+c)c+3\,c^2\right)}{(dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c}\right)}{243\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/243*(160*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 81*sqrt(d*x^3 + c) - 3*(85*(d*x^3 + c)*c + 3*c^2)/((d*x^3 + c)^(
3/2) - 9*sqrt(d*x^3 + c)*c))/d^4
```

Fricas [A]
time = 2.39, size = 233, normalized size = 2.45

$$\left[\frac{2\left(160\,(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\,\log\left(\frac{dx^3-6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(27\,d^2x^6-274\,cdx^3-304\,c^2)\sqrt{dx^3+c}\right)}{243\,(d^6x^6-7\,cd^5x^3-8\,c^2d^4)},\ \frac{2\left(320\,(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\,(27\,d^2x^6-274\,cdx^3-304\,c^2)\sqrt{dx^3+c}\right)}{243\,(d^6x^6-7\,cd^5x^3-8\,c^2d^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/243*(160*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3
 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 3*(27*d^2*x^6 - 274*c*d*x^3 - 304*c^
```

```
2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^3 - 8*c^2*d^4), 2/243*(320*(d^2*x^
6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*
(27*d^2*x^6 - 274*c*d*x^3 - 304*c^2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^
3 - 8*c^2*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(-8c + dx^3)^2 (c + dx^3)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**11/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 0.99, size = 88, normalized size = 0.93

$$\frac{640\, c \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}\, d^4} + \frac{2\sqrt{dx^3+c}}{3\, d^4} - \frac{2\,(85\,(dx^3+c)c + 3\,c^2)}{81\left((dx^3+c)^{\frac{3}{2}} - 9\sqrt{dx^3+c}\, c\right) d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 640/243*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/3*sqrt(d*
x^3 + c)/d^4 - 2/81*(85*(d*x^3 + c)*c + 3*c^2)/(((d*x^3 + c)^(3/2) - 9*sqrt
(d*x^3 + c)*c)*d^4)
```

Mupad [B]
time = 4.38, size = 111, normalized size = 1.17

$$\frac{2\sqrt{d\,x^3+c}}{3\,d^4} + \frac{320\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,d^4} + \frac{\sqrt{d\,x^3+c}\,\left(\frac{176\,c^2}{81\,d^4} + \frac{170\,c\,x^3}{81\,d^3}\right)}{8\,c^2 + 7\,c\,d\,x^3 - d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^4) + (320*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*
(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*d^4) + ((c + d*x^3)^(1/2)*((176*c^2
)/(81*d^4) + (170*c*x^3)/(81*d^3)))/(8*c^2 - d^2*x^6 + 7*c*d*x^3)
```

3.443 $\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{22}{81d^3\sqrt{c+dx^3}}+\frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}\,d^3}$$

```
[Out] -32/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3/c^(1/2)-22/81/d^3/(d*x^3+c
)^(1/2)+64/27*c/d^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 91, 79, 65, 212}

$$\frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{22}{81d^3\sqrt{c+dx^3}}-\frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}\,d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -22/(81*d^3*Sqrt[c + d*x^3]) + (64*c)/(27*d^3*(8*c - d*x^3)*Sqrt[c + d*x^3]
) - (32*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*Sqrt[c]*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(8c-dx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-24c^2d+9cd^2x}{(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{27cd^3}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{16\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+\cdots}}\right.}{81d^2}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{32\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\cdots\right.}{81d^3}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}\,d^3}$$

Mathematica [A]

time = 0.11, size = 71, normalized size = 0.86

$$\frac{2\left(\frac{3(8c+11dx^3)}{(8c-dx^3)\sqrt{c+dx^3}}-\frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{243d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*((3*(8*c + 11*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) - (16*ArcTanh[Sqrt
[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/(243*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 927, normalized size = 11.17

method	result
elliptic	$-\frac{2}{243d^3\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{64\sqrt{d\,x^3+c}}{243d^3(-d\,x^3+8c)}+\frac{16i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{(-c\,d^2)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{d}\cdots\right.}{(-c\,d^2)^{\frac{1}{3}}}}\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d^3/(d*x^3+c)^(1/2)+64*c^2/d^2*(1/243/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*
c)-2/243/d/c^2/((x^3+c/d)*d)^(1/2)-1/1458*I/c^3/d^3*2^(1/2)*sum((-c*d^2)^(1
/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d
```

```
-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(
2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+16/d^2*c*(
2/27/d/c/((x^3+c/d)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/
2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a
lpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [A]
time = 0.48, size = 81, normalized size = 0.98

$$\frac{2\left(\frac{8\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}}-\frac{3\left(11\,dx^3+8\,c\right)}{\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c}\right)}{243\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/243*(8*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/s
qrt(c) - 3*(11*d*x^3 + 8*c)/((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c))/d^3
```

Fricas [A]
time = 2.78, size = 223, normalized size = 2.69

$$\left[\frac{2\left(8\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\,\log\left(\frac{dx^3-6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\left(11\,cdx^3+8\,c^2\right)\sqrt{dx^3+c}\right)}{243\left(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3\right)},\ \frac{2\left(16\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-3\left(11\,cdx^3+8\,c^2\right)\sqrt{dx^3+c}\right)}{243\left(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/243*(8*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 +
 c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 3*(11*c*d*x^3 + 8*c^2)*sqrt(d*x^3 + c)
)/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3), 2/243*(16*(d^2*x^6 - 7*c*d*x^3 -
 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*(11*c*d*x^3 + 8
*c^2)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**8/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 1.41, size = 67, normalized size = 0.81

$$\frac{32\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{243\,\sqrt{-c}\,d^3}-\frac{2\,(11\,dx^3+8\,c)}{81\left((dx^3+c)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}\,c\right)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 32/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/81*(11*d*x^3
 + 8*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*d^3)
```

Mupad [B]
time = 4.30, size = 94, normalized size = 1.13

$$\frac{16\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,\sqrt{c}\,d^3}+\frac{\sqrt{d\,x^3+c}\,\left(\frac{16\,c}{81\,d^3}+\frac{22\,x^3}{81\,d^2}\right)}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (16*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*c
^(1/2)*d^3) + ((c + d*x^3)^(1/2)*((16*c)/(81*d^3) + (22*x^3)/(81*d^2)))/(8*
c^2 - d^2*x^6 + 7*c*d*x^3)
```

3.444 $\int \frac{x^5}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2}{81cd^2\sqrt{c+dx^3}}+\frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}$$

```
[Out] 2/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d^2-2/81/c/d^2/(d*x^3+c)
^(1/2)+8/27/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {457, 79, 53, 65, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}+\frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{2}{81cd^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(81*c*d^2*Sqrt[c + d*x^3]) + 8/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
(2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*c^(3/2)*d^2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{9d}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\right.}{81cd}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{}\right.}{81cd^2}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}$$

Mathematica [A]

time = 0.09, size = 73, normalized size = 0.86

$$\frac{2\left(\frac{3\sqrt{c}\,(4c+dx^3)}{(8c-dx^3)\sqrt{c+dx^3}}+\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{243c^{3/2}d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (2*((3*Sqrt[c]*(4*c + d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(243*c^(3/2)*d^2)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 909, normalized size = 10.69

method	result
elliptic	$\frac{8\sqrt{d\,x^3+c}}{243d^2c(-d\,x^3+8c)}+\frac{2}{243d^2c\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 8*c/d*(1/243/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243/d/c^2/((x^3+c/d)*d)^(1/2)-1/1458*I/c^3/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2

```
)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/d*(2/27/d/c/((x^3+c/d)*d)^(1/2)+1/243*
I/d^3/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*d-8*c)))
```

Maxima [A]
time = 0.50, size = 83, normalized size = 0.98

$$-\frac{\frac{6\,(dx^3+4\,c)}{(dx^3+c)^{\frac{3}{2}}c-9\,\sqrt{dx^3+c}\,c^2}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{243\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/243*(6*(d*x^3 + 4*c)/((d*x^3 + c)^(3/2)*c - 9*sqrt(d*x^3 + c)*c^2) + log
((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d^2
```

Fricas [A]
time = 3.15, size = 223, normalized size = 2.62

$$\left[\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,(cdx^3+4\,c^2)\sqrt{dx^3+c}}{243\,(c^2d^4x^6-7\,c^3d^3x^3-8\,c^4d^2)},\,-\frac{2\left((d^2x^6-7\,cdx^3-8\,c^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\,(cdx^3+4\,c^2)\sqrt{dx^3+c}\right)}{243\,(c^2d^4x^6-7\,c^3d^3x^3-8\,c^4d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^
2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2), -2/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^
2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**5/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 1.32, size = 76, normalized size = 0.89

$$-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,cd}+\frac{3\,(dx^3+4\,c)}{\left((dx^3+c)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}\,c\right)cd}\right)}{243\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/243*(arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) + 3*(d*x^3 + 4*
c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c*d))/d
```

Mupad [B]
time = 4.26, size = 96, normalized size = 1.13

$$\frac{\left(\frac{8}{81\,d^2}+\frac{2\,x^3}{81\,c\,d}\right)\,\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}+\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,c^{3/2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((8/(81*d^2) + (2*x^3)/(81*c*d))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 + 7*c*
d*x^3) + log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(2
43*c^(3/2)*d^2)
```

3.445 $\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{1}{81c^2d\sqrt{c+dx^3}}+\frac{1}{27cd\,(8c-dx^3)\sqrt{c+dx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}$$

```
[Out] 1/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)/d-1/81/c^2/d/(d*x^3+c)^(
1/2)+1/27/c/d/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {455, 44, 53, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}-\frac{1}{81c^2d\sqrt{c+dx^3}}+\frac{1}{27cd\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/81*1/(c^2*d*Sqrt[c + d*x^3]) + 1/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
+ ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c^(5/2)*d)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx &= \frac{1}{3}\mathrm{Subst}\left(\int \frac{1}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{1}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}} + \frac{\mathrm{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{18c}\\
&= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}} + \frac{\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+d}}\right.}{162c^2}\\
&= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}} + \frac{\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{}\right.}{81c^2d}\\
&= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 73, normalized size = 0.83

$$
\frac{\frac{3\sqrt{c}\,(-5c+dx^3)}{(8c-dx^3)\sqrt{c+dx^3}} + \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((3*Sqrt[c]*(-5*c + d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(243*c^(5/2)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.34, size = 464, normalized size = 5.27

method	result
default	$\frac{\sqrt{d\,x^3+c}}{243c^2 d(-d\,x^3+8c)} - \frac{2}{243d\,c^2\sqrt{\left(x^3+\frac{c}{d}\right)d}} - \frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)} \frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+}{d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$
elliptic	$\frac{\sqrt{d\,x^3+c}}{243c^2 d(-d\,x^3+8c)} - \frac{2}{243d\,c^2\sqrt{\left(x^3+\frac{c}{d}\right)d}} - \frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)} \frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+}{d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/243/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243/d/c^2/((x^3+c/d)*d)^(1/2)-1/
1458*I/c^3/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [A]
time = 0.49, size = 85, normalized size = 0.97

$$-\frac{\frac{6\,(dx^3-5\,c)}{(dx^3+c)^{\frac{3}{2}}c^2-9\,\sqrt{dx^3+c}\,c^3}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{486\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/486*(6*(d*x^3 - 5*c)/((d*x^3 + c)^(3/2)*c^2 - 9*sqrt(d*x^3 + c)*c^3) + l
og((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(5/2))/d
```

Fricas [A]
time = 2.82, size = 219, normalized size = 2.49

$$\left[\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,(cdx^3-5\,c^2)\sqrt{dx^3+c}}{486\,(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d)},\,-\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\,(cdx^3-5\,c^2)\sqrt{dx^3+c}}{243\,(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/486*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 - 5*c^2)*sqrt(d*x^3 + c))/(c^
3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d), -1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)
*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 - 5*c^2)*sqrt
(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**2/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 0.93, size = 72, normalized size = 0.82

$$-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}\,c^2d}-\frac{dx^3-5\,c}{81\left((dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2*d) - 1/81*(d*x^3
- 5*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^2*d)
```

Mupad [B]
time = 4.26, size = 97, normalized size = 1.10

$$\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{486\,c^{5/2}\,d}-\frac{\left(\frac{5}{81\,c\,d}-\frac{x^3}{81\,c^2}\right)\,\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(486*c^(5/2
)*d) - ((5/(81*c*d) - x^3/(81*c^2))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 + 7
*c*d*x^3)
```

3.446 $\int \frac{1}{x(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{5}{648c^3\sqrt{c+dx^3}}+\frac{1}{216c^2(8c-dx^3)\sqrt{c+dx^3}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

```
[Out] 7/7776*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/96*arctanh((d*x^3+c)^
(1/2)/c^(1/2))/c^(7/2)+5/648/c^3/(d*x^3+c)^(1/2)+1/216/c^2/(-d*x^3+8*c)/(d*
x^3+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {457, 105, 157, 162, 65, 214, 212}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}+\frac{5}{648c^3\sqrt{c+dx^3}}+\frac{1}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*Sqrt[c + d*x^3]) + 1/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (
7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(7776*c^(7/2)) - ArcTanh[Sqrt[c + d
*x^3]/Sqrt[c]]/(96*c^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-9cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{216c^2d}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-\frac{81}{2}c^2d^2+\frac{15}{4}c}{x(8c-dx)\sqrt{c+}}\right.}{972c^4d^2}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,d\right.}{192c^3}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{7\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\right.}{2592c^3}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}}$$

Mathematica [A]
time = 0.14, size = 93, normalized size = 0.88

$$\frac{\frac{12\sqrt{c}\left(43c-5dx^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}} + 7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 81\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{7776c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((12*Sqrt[c]*(43*c - 5*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + 7*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])] - 81*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(7776*c
^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.46, size = 954, normalized size = 9.00

method	result	size
default	Expression too large to display	954
elliptic	Expression too large to display	1552

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d/c*(1/243/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243/d/c^2/((x^3+c/d)*d)
^(1/2)-1/1458*I/c^3/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*
_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^
(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1
/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))-1/64*d/c^2*(2/27/d/c/((x^3+c/d)*d)^(1/
2)+1/243*I/d^3/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(2/3/c/((x^3+c/d)*d)^(1/2)-2/3*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x), x)
```

Fricas [A]
time = 2.65, size = 316, normalized size = 2.98

$$\left[\frac{7\,(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+81\,(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)+24\,(5\,cdx^3-43\,c^2)\sqrt{dx^3+c}}{15552\,(c^4d^2x^6-7\,c^5dx^3-8\,c^6)}, \frac{81\,(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-7\,(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+12\,(5\,cdx^3-43\,c^2)\sqrt{dx^3+c}}{7776\,(c^4d^2x^6-7\,c^5dx^3-8\,c^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/15552*(7*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3
 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 81*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqr
t(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(5*c*d*x^3 - 4
3*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 - 8*c^6), 1/7776*(81*(d^
2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 7*
(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/
c) + 12*(5*c*d*x^3 - 43*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 -
8*c^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 0.94, size = 93, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}\,c^3}-\frac{7\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{7776\sqrt{-c}\,c^3}+\frac{5\,dx^3-43\,c}{648\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 7/7776*arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/648*(5*d*x^3 - 43*c)/(((d*x^3 +
c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^3)
```

Mupad [B]
time = 4.33, size = 101, normalized size = 0.95

$$-\frac{\frac{5\,(d\,x^3+c)}{216\,c^3}-\frac{2}{9\,c^2}}{27\,c\,\sqrt{d\,x^3+c}\,-3\,{(d\,x^3+c)}^{3/2}}+\frac{\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,7\mathrm{i}}{81}\right)\,1\mathrm{i}}{96\,\sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2))*1i - (atanh((c^3*(c + d*x^3)^(
1/2))/(3*(c^7)^(1/2)))*7i)/81)*1i)/(96*(c^7)^(1/2)) - ((5*(c + d*x^3))/(216
*c^3) - 2/(9*c^2))/(27*c*(c + d*x^3)^(1/2) - 3*(c + d*x^3)^(3/2))
```

3.447 $\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{35d}{2592c^4\sqrt{c+dx^3}}+\frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+$$

```
[Out] 5/31104*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)+5/384*d*arctanh((d*x
^3+c)^(1/2)/c^(1/2))/c^(9/2)-35/2592*d/c^4/(d*x^3+c)^(1/2)+5/864*d/c^3/(-d*
x^3+8*c)/(d*x^3+c)^(1/2)-1/24/c^2/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {457, 105, 156, 157, 162, 65, 214, 212}

$$\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}-\frac{35d}{2592c^4\sqrt{c+dx^3}}+\frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-35*d)/(2592*c^4*Sqrt[c + d*x^3]) + (5*d)/(864*c^3*(8*c - d*x^3)*Sqrt[c +
d*x^3]) - 1/(24*c^2*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (5*d*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(31104*c^(9/2)) + (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqr
t[c]])/(384*c^(9/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}} dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(8c-dx)^2(c+dx)^{3/2}} dx, x, x^3\right)$$
$$= -\frac{1}{24c^2x^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{10cd-\frac{5d^2x}{2}}{x(8c-dx)^2(c+dx)^{3/2}} dx, x, x^3\right)}{24c^2}$$
$$= \frac{5d}{864c^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3(8c-dx^3)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3(8c-dx^3)}$$
$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3(8c-dx^3)}$$
$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3(8c-dx^3)}$$
$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3(8c-dx^3)}$$

Mathematica [**A**]
time = 0.23, size = 109, normalized size = 0.76

$$\frac{-\frac{12\sqrt{c}(108c^2+265cdx^3-35d^2x^6)}{x^3(8c-dx^3)\sqrt{c+dx^3}} + 5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 405d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{31104c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((-12*Sqrt[c]*(108*c^2 + 265*c*d*x^3 - 35*d^2*x^6))/(x^3*(8*c - d*x^3)*Sqrt
[c + d*x^3]) + 5*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 405*d*ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]])/(31104*c^(9/2))
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.50, size = 1020, normalized size = 7.13

method	result	size
risch	Expression too large to display	913
default	Expression too large to display	1020
elliptic	Expression too large to display	1569

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*d^2*(1/243/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243/d/c^2/((x^3+c/
d)*d)^(1/2)-1/1458*I/c^3/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(
-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*
(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/
3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*
d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))-1/256/c^3*d^2*(2/27/d/c/((x^3+c/d
)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-
c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(-2/3*d/c^2/((x^3+c/d)*d)^
(1/2)-1/3*(d*x^3+c)^(1/2)/c^2/x^3+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2
))+1/256/c^3*d*(2/3/c/((x^3+c/d)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/
2))/c^(3/2))
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^4), x)
```

Fricas [A]
time = 3.33, size = 368, normalized size = 2.57

$$\left[\frac{5\,(d^3x^9-7\,cd^2x^6-8\,c^2dx^3)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+405\,(d^3x^9-7\,cd^2x^6-8\,c^2dx^3)\sqrt{c}\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\,(35\,cd^2x^6-265\,c^2dx^3-108\,c^3)\sqrt{dx^3+c}}{62208\,(c^5d^2x^9-7\,c^6dx^6-8\,c^7x^3)},\ -\frac{405\,(d^3x^9-7\,cd^2x^6-8\,c^2dx^3)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+5\,(d^3x^9-7\,cd^2x^6-8\,c^2dx^3)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+12\,(35\,cd^2x^6-265\,c^2dx^3-108\,c^3)\sqrt{dx^3+c}}{31104\,(c^5d^2x^9-7\,c^6dx^6-8\,c^7x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] [1/62208*(5*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 405*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(35*c*d^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 - 7*c^6*d*x^6 - 8*c^7*x^3), -1/31104*(405*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 5*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(35*c*d^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 - 7*c^6*d*x^6 - 8*c^7*x^3)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)

[Out] Integral(1/(x**4*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)

Giac [A]
time = 1.32, size = 129, normalized size = 0.90

$$-\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}\,c^4}-\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{31104\,\sqrt{-c}\,c^4}-\frac{35\,(dx^3+c)^2d-335\,(dx^3+c)cd+192\,c^2d}{2592\left((dx^3+c)^{\frac{5}{2}}-10\,(dx^3+c)^{\frac{3}{2}}c+9\,\sqrt{dx^3+c}\,c^2\right)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] -5/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 5/31104*d*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/2592*(35*(d*x^3 + c)^2*d - 335*(d*x^3 + c)*c*d + 192*c^2*d)/(((d*x^3 + c)^(5/2) - 10*(d*x^3 + c)^(3/2)*c + 9*sqrt(d*x^3 + c)*c^2)*c^4)

Mupad [B]
time = 4.56, size = 133, normalized size = 0.93

$$-\frac{\frac{2\,d}{9\,c^2}+\frac{35\,d\,{\left(d\,x^3+c\right)}^2}{864\,c^4}-\frac{335\,d\,\left(d\,x^3+c\right)}{864\,c^3}}{3\,{\left(d\,x^3+c\right)}^{5/2}-30\,c\,{\left(d\,x^3+c\right)}^{3/2}+27\,c^2\,\sqrt{d\,x^3+c}}-\frac{d\,\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1\mathrm{i}+\frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,1\mathrm{i}}{81}\right)\,5\mathrm{i}}{384\,\sqrt{c^9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] - ((2*d)/(9*c^2) + (35*d*(c + d*x^3)^2)/(864*c^4) - (335*d*(c + d*x^3))/(86
4*c^3))/(3*(c + d*x^3)^(5/2) - 30*c*(c + d*x^3)^(3/2) + 27*c^2*(c + d*x^3)^
(1/2)) - (d*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i + (atanh((c^4*(c
 + d*x^3)^(1/2))/(3*(c^9)^(1/2)))*1i)/81)*5i)/(384*(c^9)^(1/2))
```

3.448 $\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{665d^2}{41472c^5\sqrt{c+dx^3}}-\frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}}-\frac{1}{48c^2x^6(8c-dx^3)\sqrt{c+dx^3}}+\frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+}}$$

```
[Out] 13/497664*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/2)-33/2048*d^2*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))/c^(11/2)+665/41472*d^2/c^5/(d*x^3+c)^(1/2)-71
/13824*d^2/c^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-1/48/c^2/x^6/(-d*x^3+8*c)/(d*x^
3+c)^(1/2)+17/384*d/c^3/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {457, 105, 156, 157, 162, 65, 214, 212}

$$\frac{13d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}}-\frac{33d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}}+\frac{665d^2}{41472c^5\sqrt{c+dx^3}}-\frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}}+\frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+dx^3}}-\frac{1}{48c^2x^6(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (665*d^2)/(41472*c^5*Sqrt[c + d*x^3]) - (71*d^2)/(13824*c^4*(8*c - d*x^3)*S
qrt[c + d*x^3]) - 1/(48*c^2*x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (17*d)/(38
4*c^3*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (13*d^2*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(497664*c^(11/2)) - (33*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(2048*c^(11/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= -\frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{17cd-\frac{7d^2x}{2}}{x^2(8c-dx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)}{48c^2}$$

$$= -\frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + \text{Subs}$$

$$= -\frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{}{384c}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right.}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right.}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right.}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right.}$$

Mathematica [A]
time = 0.28, size = 123, normalized size = 0.66

$$\frac{\frac{12\sqrt{c}\left(864c^3-1836c^2dx^3-5107cd^2x^6+665d^3x^9\right)}{x^6\left(-8c+dx^3\right)\sqrt{c+dx^3}} + 13d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 8019d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{497664c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
[Out] ((12*Sqrt[c]*(864*c^3 - 1836*c^2*d*x^3 - 5107*c*d^2*x^6 + 665*d^3*x^9))/(x^
6*(-8*c + d*x^3)*Sqrt[c + d*x^3]) + 13*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])] - 8019*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(497664*c^(11/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.54, size = 1107, normalized size = 5.98

method	result	size
risch	Expression too large to display	923
default	Expression too large to display	1107
elliptic	Expression too large to display	1601

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/c^2/x^6+7/12*d*(d*x^3+c)^(1/2)/c^3/x^3+2/3*d
^2/c^3/((x^3+c/d)*d)^(1/2)-5/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)
)+1/512/c^3*d^3*(1/243/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243/d/c^2/((x^3
+c/d)*d)^(1/2)-1/1458*I/c^3/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1
/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^
(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))-3/4096*d^3/c^4*(2/27/d/c/((x^3
+c/d)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/
d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2
*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(-2/3*d/c^2/((x^3+c
/d)*d)^(1/2)-1/3*(d*x^3+c)^(1/2)/c^2/x^3+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))
/c^(5/2))+3/4096*d^2/c^4*(2/3/c/((x^3+c/d)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^7), x)
```

Fricas [A]
time = 2.67, size = 398, normalized size = 2.15

$$\left[\frac{13\left(d^{4}x^{12}-7\,cd^{3}x^{9}-8\,c^{2}d^{2}x^{6}\right)\sqrt{c}\,\log\left(\frac{dx^{3}+6\,\sqrt{dx^{3}+c}\,\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+8019\left(d^{4}x^{12}-7\,cd^{3}x^{9}-8\,c^{2}d^{2}x^{6}\right)\sqrt{c}\,\log\left(\frac{dx^{3}-2\,\sqrt{dx^{3}+c}\,\sqrt{c}+2\,c}{x^{3}}\right)+24\left(665\,cd^{3}x^{9}-5107\,c^{2}d^{2}x^{6}-1836\,c^{3}dx^{3}+864\,c^{4}\right)\sqrt{dx^{3}+c}}{995328\left(c^{6}d^{2}x^{12}-7\,c^{7}dx^{9}-8\,c^{8}x^{6}\right)},\ \frac{8019\left(d^{4}x^{12}-7\,cd^{3}x^{9}-8\,c^{2}d^{2}x^{6}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^{3}+c}\,\sqrt{-c}}{c}\right)-13\left(d^{4}x^{12}-7\,cd^{3}x^{9}-8\,c^{2}d^{2}x^{6}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^{3}+c}\,\sqrt{-c}}{3\,c}\right)+12\left(665\,cd^{3}x^{9}-5107\,c^{2}d^{2}x^{6}-1836\,c^{3}dx^{3}+864\,c^{4}\right)\sqrt{dx^{3}+c}}{497664\left(c^{6}d^{2}x^{12}-7\,c^{7}dx^{9}-8\,c^{8}x^{6}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/995328*(13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 +
 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 8019*(d^4*x^12 - 7*c*d^
3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c
)/x^3) + 24*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^3*d*x^3 + 864*c^4)*s
qrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6), 1/497664*(8019*(d
^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt
(-c)/c) - 13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + 12*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^
3*d*x^3 + 864*c^4)*sqrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7}\left(-8c+dx^{3}\right)^{2}\left(c+dx^{3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**7*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 1.84, size = 149, normalized size = 0.81

$$\frac{33\,d^{2}\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}\,c^{5}}-\frac{13\,d^{2}\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\,\sqrt{-c}}\right)}{497664\,\sqrt{-c}\,c^{5}}+\frac{341\,(dx^{3}+c)d^{2}-3072\,cd^{2}}{41472\left(\left(dx^{3}+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^{3}+c}\,c\right)c^{5}}+\frac{3\,(dx^{3}+c)^{\frac{3}{2}}d^{2}-4\,\sqrt{dx^{3}+c}\,cd^{2}}{384\,c^{5}d^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 33/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) - 13/497664*d^2
*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) + 1/41472*(341*(d*x^3
```

```
+ c)*d^2 - 3072*c*d^2)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^5) + 1/
384*(3*(d*x^3 + c)^(3/2)*d^2 - 4*sqrt(d*x^3 + c)*c*d^2)/(c^5*d^2*x^6)
```

Mupad [B]
time = 4.76, size = 171, normalized size = 0.92

$$\frac{\frac{2d^2}{9c^2}-\frac{10373\,d^2\,(d\,x^3+c)}{13824\,c^3}+\frac{3551\,d^2\,(d\,x^3+c)^2}{6912\,c^4}-\frac{665\,d^2\,(d\,x^3+c)^3}{13824\,c^5}}{33\,c\,(d\,x^3+c)^{5/2}-3\,(d\,x^3+c)^{7/2}+27\,c^3\,\sqrt{d\,x^3+c}-57\,c^2\,(d\,x^3+c)^{3/2}}+\frac{d^2\left(\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{\sqrt{c^{11}}}\right)\,1\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^{11}}}\right)\,13\mathrm{i}}{8019}\right)\,33\mathrm{i}}{2048\,\sqrt{c^{11}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((2*d^2)/(9*c^2) - (10373*d^2*(c + d*x^3))/(13824*c^3) + (3551*d^2*(c + d*x
^3)^2)/(6912*c^4) - (665*d^2*(c + d*x^3)^3)/(13824*c^5))/(33*c*(c + d*x^3)^
(5/2) - 3*(c + d*x^3)^(7/2) + 27*c^3*(c + d*x^3)^(1/2) - 57*c^2*(c + d*x^3)
^(3/2)) + (d^2*(atanh((c^5*(c + d*x^3)^(1/2))/(c^11)^(1/2))*1i - (atanh((c^
5*(c + d*x^3)^(1/2))/(3*(c^11)^(1/2)))*13i)/8019)*33i)/(2048*(c^11)^(1/2))
```

3.449 $\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=668

$$-\frac{2x^2}{81cd^2\sqrt{c+dx^3}}+\frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{4\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{\cdots}\right.}{\sqrt{c+\cdots}}\right.}{81\sqrt{3}\,c^{5/6}d^{8\cdots}}$$

[Out] -4/243*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^(8/3)+4/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(8/3)+4/243*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(8/3)*3^(1/2)-2/81*x^2/c/d^2/(d*x^3+c)^(1/2)+8/27*x^2/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)+2/81*(d*x^3+c)^(1/2)/c/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+2/243*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/81*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.57, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {481, 593, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{81\sqrt[4]{3}\,c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\,3^{3/4}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{4\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}\,c^{5/6}d^{8/3}}-\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{5/6}d^{8/3}}+\frac{4\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/6}d^{8/3}}+\frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{2x^2}{81cd^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (-2*x^2)/(81*c*d^2*Sqrt[c + d*x^3]) + (8*x^2)/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(81*c*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (4*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(81*Sqrt[3]*c^(5/6)*d^(8/3)) - (4*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(243*c^(5/6)*d^(8/3)) + (4*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*c^(5/6)*d^(8/3)) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)

```
*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(3/4)*c^(2/3)*d^(8/
3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(81*3^(1/4)*c^(2/3)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{x\left(16c^2+7cdx^3\right)}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{27cd^2}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(-72c^3d-\frac{9}{2}c^2d^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,d}{729c^3d^3}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \left(\frac{9c^2dx}{2\sqrt{c+dx^3}} - \frac{}{(8c-}\right.}{729c^3}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{8\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,d}{27d^2}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d}{}}{\left(4+\frac{2\sqrt[3]{d}}{\sqrt[3]{c}}x+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{}}}{81cd^3}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\right.}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\right.}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 8.91, size = 168, normalized size = 0.25

$$\frac{80cx^2(4c+dx^3)+40cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3240c^2d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (80*c*x^2*(4*c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(3240*c^2*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 2256, normalized size = 3.38

method	result	size
elliptic	Expression too large to display	910
default	Expression too large to display	2256

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(2/3*x^2/c/((x^3+c/d)*d)^(1/2)+2/9*I/c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2))))+16/d^2*c*(-2/27*x^2/c^2/((x^3+c/d)*d)^(1/2)-2/81*I/c^2*3^(1/
2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
```

```
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha
*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*
d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)+64*c^2/d^2*(2/243*x^2/c^3/((x^3+c/d)*d)^(1/2)+1/1944/c^3*x^2*(d*x^3+c)^(1
/2)/(-d*x^3+8*c)+5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-
1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+
1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-5
/5832*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2
*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

[Out] integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.61, size = 2746, normalized size = 4.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] 1/729*(4*sqrt(3)*(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)*(1/(c^5*d^16))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^4*x^5*(1/(c^5*d^16))^(1/6) - sqrt(3)*(c^4*d^15*x^6 - 40*c^5*d^14*x^3 - 32*c^6*d^13)*(1/(c^5*d^16))^(5/6) + 3*sqrt(3)*(5*c^3*d^9*x^4 + 8*c^4*d^8*x)*sqrt(1/(c^5*d^16)))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^4*d^12*x^5 + c^5*d^11*x^2)*(1/(c^5*d^16))^(2/3) + 12*sqrt(3)*(c^2*d^7*x^6 - c^3*d^6*x^3 - 2*c^4*d^5)*(1/(c^5*d^16))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^15*x^6 + 32*c^5*d^14*x^3 + 40*c^6*d^13)*(1/(c^5*d^16))^(5/6) + 3*sqrt(3)*(7*c^3*d^9*x^4 + 4*c^4*d^8*x)*sqrt(1/(c^5*d^16)) + 9*sqrt(3)*(c*d^4*x^5 + 2*c^2*d^3*x^2)*(1/(c^5*d^16))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^13*x^7 - 52*c^5*d^12*x^4 - 80*c^6*d^11*x)*(1/(c^5*d^16))^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^15*x^5 + c^6*d^14*x^2)*(1/(c^5*d^16))^(5/6) - 4*(c^3*d^10*x^6 + 41*c^4*d^9*x^3 + 40*c^5*d^8)*sqrt(1/(c^5*d^16)) - (c*d^5*x^7 - 28*c^2*d^4*x^4 - 272*c^3*d^3*x)*(1/(c^5*d^16))^(1/6)) + 18*(c^2*d^8*x^8 + 20*c^3*d^7*x^5 - 8*c^4*d^6*x^2)*(1/(c^5*d^16))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 4*sqrt(3)*(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)*(1/(c^5*d^16))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^4*x^5*(1/(c^5*d^16))^(1/6) - sqrt(3)*(c^4*d^15*x^6 - 40*c^5*d^14*x^3 - 32*c^6*d^13)*(1/(c^5*d^16))^(5/6) + 3*sqrt(3)*(5*c^3*d^9*x^4 + 8*c^4*d^8*x)*sqrt(1/(c^5*d^16)))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^4*d^12*x^5 + c^5*d^11*x^2)*(1/(c^5*d^16))^(2/3) + 12*sqrt(3)*(c^2*d^7*x^6 - c^3*d^6*x^3 - 2*c^4*d^5)*(1/(c^5*d^16))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^15*x^6 + 32*c^5*d^14*x^3 + 40*c^6*d^13)*(1/(c^5*d^16))^(5/6) + 3*sqrt(3)*(7*c^3*d^9*x^4 + 4*c^4*d^8*x)*sqrt(1/(c^5*d^16)) + 9*sqrt(3)*(c*d^4*x^5 + 2*c^2*d^3*x^2)*(1/(c^5*d^16))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^13*x^7 - 52*c^5*d^12*x^4 - 80*c^6*d^11*x)*(1/(c^5*d^16))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^15*x^5 + c^6*d^14*x^2)*(1/(c^5*d^16))^(5/6) - 4*(c^3*d^10*x^6 + 41*c^4*d^9*x^3 + 40*c^5*d^8)*sqrt(1/(c^5*d^16)) - (c*d^5*x^7 - 28*c^2*d^4*x^4 - 272*c^3*d^3*x)*(1/(c^5*d^16))^(1/6)) + 18*(c^2*d^8*x^8 + 20*c^3*d^7*x^5 - 8*c^4*d^6*x^2)*(1/(c^5*d^16))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) - 18*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 2*(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)*(1/(c^5*d^16))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^

```
13*x^7 + 64*c^5*d^12*x^4 + 32*c^6*d^11*x)*(1/(c^5*d^16))^(2/3) + 6*sqrt(d*x
^3 + c)*(6*(5*c^5*d^15*x^5 + 32*c^6*d^14*x^2)*(1/(c^5*d^16))^(5/6) + (7*c^3
*d^10*x^6 + 152*c^4*d^9*x^3 + 64*c^5*d^8)*sqrt(1/(c^5*d^16)) + (c*d^5*x^7 +
 80*c^2*d^4*x^4 + 160*c^3*d^3*x)*(1/(c^5*d^16))^(1/6)) + 18*(c^2*d^8*x^8 +
38*c^3*d^7*x^5 + 64*c^4*d^6*x^2)*(1/(c^5*d^16))^(1/3))/(d^3*x^9 - 24*c*d^2*
x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)
*(1/(c^5*d^16))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c
^3 + 18*(5*c^4*d^13*x^7 + 64*c^5*d^12*x^4 + 32*c^6*d^11*x)*(1/(c^5*d^16))^(
2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^15*x^5 + 32*c^6*d^14*x^2)*(1/(c^5*d^16
))^(5/6) + (7*c^3*d^10*x^6 + 152*c^4*d^9*x^3 + 64*c^5*d^8)*sqrt(1/(c^5*d^16
)) + (c*d^5*x^7 + 80*c^2*d^4*x^4 + 160*c^3*d^3*x)*(1/(c^5*d^16))^(1/6)) + 1
8*(c^2*d^8*x^8 + 38*c^3*d^7*x^5 + 64*c^4*d^6*x^2)*(1/(c^5*d^16))^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^5*x^6 - 7*c^2*d^4*x
^3 - 8*c^3*d^3)*(1/(c^5*d^16))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^
2*d*x^3 - 1088*c^3 - 18*(c^4*d^13*x^7 - 52*c^5*d^12*x^4 - 80*c^6*d^11*x)*(1
/(c^5*d^16))^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^15*x^5 + c^6*d^14*x^2)*(1
/(c^5*d^16))^(5/6) - 4*(c^3*d^10*x^6 + 41*c^4*d^9*x^3 + 40*c^5*d^8)*sqrt(1/
(c^5*d^16)) - (c*d^5*x^7 - 28*c^2*d^4*x^4 - 272*c^3*d^3*x)*(1/(c^5*d^16))^(
1/6)) + 18*(c^2*d^8*x^8 + 20*c^3*d^7*x^5 - 8*c^4*d^6*x^2)*(1/(c^5*d^16))^(1
/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^5*x^6 - 7*c
^2*d^4*x^3 - 8*c^3*d^3)*(1/(c^5*d^16))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 -
 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^13*x^7 - 52*c^5*d^12*x^4 - 80*c^6*d^
11*x)*(1/(c^5*d^16))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^15*x^5 + c^6*d^14
*x^2)*(1/(c^5*d^16))^(5/6) - 4*(c^3*d^10*x^6 + 41*c^4*d^9*x^3 + 40*c^5*d^8)
*sqrt(1/(c^5*d^16)) - (c*d^5*x^7 - 28*c^2*d^4*x^4 - 272*c^3*d^3*x)*(1/(c^5*
d^16))^(1/6)) + 18*(c^2*d^8*x^8 + 20*c^3*d^7*x^5 - 8*c^4*d^6*x^2)*(1/(c^5*d
^16))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 18*(d^2*
x^5 + 4*c*d*x^2)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**7/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.450 $\int \frac{x^4}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=671

$$-\frac{x^2}{81c^2d\sqrt{c+dx^3}}+\frac{x^2}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}}+\frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\,\left(\sqrt[3]{c}\right.}{\sqrt{c+d}}\right.}{81\sqrt{3}\,c^{11/6}d}$$

[Out] 1/243*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/d^(5/3)-1/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(5/3)-1/243*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^(5/3)*3^(1/2)-1/81*x^2/c^2/d/(d*x^3+c)^(1/2)+1/27*x^2/c/d/(-d*x^3+8*c)/(d*x^3+c)^(1/2)+1/81*(d*x^3+c)^(1/2)/c^2/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/243*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/162*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(5/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.59, antiderivative size = 671, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {482, 593, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{81\sqrt[4]{3}\,c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{54\,3^{3/4}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}\,c^{11/6}d^{5/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{243c^{11/6}d^{5/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{11/6}d^{5/3}}+\frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{x^2}{81c^2d\sqrt{c+dx^3}}+\frac{x^2}{27cd\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] -1/81*x^2/(c^2*d*Sqrt[c + d*x^3]) + x^2/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) + Sqrt[c + d*x^3]/(81*c^2*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(81*Sqrt[3]*c^(11/6)*d^(5/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(243*c^(11/6)*d^(5/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c^(11/6)*d^(5/3)) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)

```
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(54*3^(3/4)*c^(5/3)*d^(5/3)*Sqrt[(c^(1/3)*
(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^
3]) + (Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^
(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sq
rt[3]])/(81*3^(1/4)*c^(5/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{x\left(2c-\frac{5dx^3}{2}\right)}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{27cd}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(45c^2d-\frac{9}{4}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,d\cdots}{729c^3d^2}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int\left(\frac{9cdx}{4\sqrt{c+dx^3}} + \cdots\right.}{729c^3\cdots}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{162c^2d} + \frac{2\int\cdots}{\cdots}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}\cdots}{\sqrt[3]{\cdots}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c\cdots}}}{162c^2d^2}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}\cdots}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}\cdots}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}\cdots}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.07, size = 169, normalized size = 0.25

$$\frac{80cx^2(-5c+dx^3)+50cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{6480c^3d\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (80*c*x^2*(-5*c + d*x^3) + 50*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(6480*c^3*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 1789, normalized size = 2.67

method	result	size
elliptic	Expression too large to display	910
default	Expression too large to display	1789

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8*c/d*(2/243*x^2/c^3/((x^3+c/d)*d)^(1/2)+1/1944/c^3*x^2*(d*x^3+c)^(1/2)/(-d
*x^3+8*c)+5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c
*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*
(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c
*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-5/5832*I
/c^3/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1
/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d
*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha
^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
```

```
/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
)),_alpha=RootOf(_Z^3*d-8*c))+1/d*(-2/27*x^2/c^2/((x^3+c/d)*d)^(1/2)-2/81*
I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*s
um(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1
/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d
*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3
^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*d-8*c)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.76, size = 2780, normalized size = 4.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(4*sqrt(3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^11*d^10)
)^(1/6)*arctan(1/9*((9*sqrt(3)*c^2*d^3*x^5*(1/(c^11*d^10))^(1/6) - sqrt(3)*
(c^9*d^10*x^6 - 40*c^10*d^9*x^3 - 32*c^11*d^8)*(1/(c^11*d^10))^(5/6) + 3*sq
rt(3)*(5*c^6*d^6*x^4 + 8*c^7*d^5*x)*sqrt(1/(c^11*d^10)))*sqrt(d*x^3 + c) +
(18*sqrt(3)*(c^8*d^8*x^5 + c^9*d^7*x^2)*(1/(c^11*d^10))^(2/3) + 12*sqrt(3)*
(c^4*d^5*x^6 - c^5*d^4*x^3 - 2*c^6*d^3)*(1/(c^11*d^10))^(1/3) + 3*sqrt(3)*(
d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^9*d^10*x^6 + 3
2*c^10*d^9*x^3 + 40*c^11*d^8)*(1/(c^11*d^10))^(5/6) + 3*sqrt(3)*(7*c^6*d^6*
x^4 + 4*c^7*d^5*x)*sqrt(1/(c^11*d^10)) + 9*sqrt(3)*(c^2*d^3*x^5 + 2*c^3*d^2
*x^2)*(1/(c^11*d^10))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^
3 - 1088*c^3 - 18*(c^8*d^9*x^7 - 52*c^9*d^8*x^4 - 80*c^10*d^7*x)*(1/(c^11*d
^10))^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^10*d^10*x^5 + c^11*d^9*x^2)*(1/(c^11
*d^10))^(5/6) - 4*(c^6*d^7*x^6 + 41*c^7*d^6*x^3 + 40*c^8*d^5)*sqrt(1/(c^11*
d^10)) - (c^2*d^4*x^7 - 28*c^3*d^3*x^4 - 272*c^4*d^2*x)*(1/(c^11*d^10))^(1/
6)) + 18*(c^4*d^6*x^8 + 20*c^5*d^5*x^5 - 8*c^6*d^4*x^2)*(1/(c^11*d^10))^(1/
3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x
^4 - 8*c^2*x)) + 4*sqrt(3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^
11*d^10))^(1/6)*arctan(1/9*((9*sqrt(3)*c^2*d^3*x^5*(1/(c^11*d^10))^(1/6) -
sqrt(3)*(c^9*d^10*x^6 - 40*c^10*d^9*x^3 - 32*c^11*d^8)*(1/(c^11*d^10))^(5/6
) + 3*sqrt(3)*(5*c^6*d^6*x^4 + 8*c^7*d^5*x)*sqrt(1/(c^11*d^10)))*sqrt(d*x^3
 + c) - (18*sqrt(3)*(c^8*d^8*x^5 + c^9*d^7*x^2)*(1/(c^11*d^10))^(2/3) + 12*
sqrt(3)*(c^4*d^5*x^6 - c^5*d^4*x^3 - 2*c^6*d^3)*(1/(c^11*d^10))^(1/3) + 3*s
qrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^9*d^10
*x^6 + 32*c^10*d^9*x^3 + 40*c^11*d^8)*(1/(c^11*d^10))^(5/6) + 3*sqrt(3)*(7*
c^6*d^6*x^4 + 4*c^7*d^5*x)*sqrt(1/(c^11*d^10)) + 9*sqrt(3)*(c^2*d^3*x^5 + 2
*c^3*d^2*x^2)*(1/(c^11*d^10))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*
c^2*d*x^3 - 1088*c^3 - 18*(c^8*d^9*x^7 - 52*c^9*d^8*x^4 - 80*c^10*d^7*x)*(1
/(c^11*d^10))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^10*d^10*x^5 + c^11*d^9*x^2)*
(1/(c^11*d^10))^(5/6) - 4*(c^6*d^7*x^6 + 41*c^7*d^6*x^3 + 40*c^8*d^5)*sqrt(
1/(c^11*d^10)) - (c^2*d^4*x^7 - 28*c^3*d^3*x^4 - 272*c^4*d^2*x)*(1/(c^11*d^
10))^(1/6)) + 18*(c^4*d^6*x^8 + 20*c^5*d^5*x^5 - 8*c^6*d^4*x^2)*(1/(c^11*d^
10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 -
 7*c*d*x^4 - 8*c^2*x)) + 36*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstra
ssZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 2*(c^2*d^4*x^6 - 7*c
^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^11*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6
+ 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^8*d^9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d
^7*x)*(1/(c^11*d^10))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d^10*x^5 + 32*c^
11*d^9*x^2)*(1/(c^11*d^10))^(5/6) + (7*c^6*d^7*x^6 + 152*c^7*d^6*x^3 + 64*c
^8*d^5)*sqrt(1/(c^11*d^10)) + (c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x
)*(1/(c^11*d^10))^(1/6)) + 18*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^
2)*(1/(c^11*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) + 2*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^11*d^10))^(1/6)*log(
(d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^8*d^9*x^7 + 6
4*c^9*d^8*x^4 + 32*c^10*d^7*x)*(1/(c^11*d^10))^(2/3) - 6*sqrt(d*x^3 + c)*(6
*(5*c^10*d^10*x^5 + 32*c^11*d^9*x^2)*(1/(c^11*d^10))^(5/6) + (7*c^6*d^7*x^6
```

```
 + 152*c^7*d^6*x^3 + 64*c^8*d^5)*sqrt(1/(c^11*d^10)) + (c^2*d^4*x^7 + 80*c^
3*d^3*x^4 + 160*c^4*d^2*x)*(1/(c^11*d^10))^(1/6)) + 18*(c^4*d^6*x^8 + 38*c^
5*d^5*x^5 + 64*c^6*d^4*x^2)*(1/(c^11*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
+ 192*c^2*d*x^3 - 512*c^3)) + (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/
(c^11*d^10))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3
 - 18*(c^8*d^9*x^7 - 52*c^9*d^8*x^4 - 80*c^10*d^7*x)*(1/(c^11*d^10))^(2/3)
+ 6*sqrt(d*x^3 + c)*(24*(c^10*d^10*x^5 + c^11*d^9*x^2)*(1/(c^11*d^10))^(5/6
) - 4*(c^6*d^7*x^6 + 41*c^7*d^6*x^3 + 40*c^8*d^5)*sqrt(1/(c^11*d^10)) - (c^
2*d^4*x^7 - 28*c^3*d^3*x^4 - 272*c^4*d^2*x)*(1/(c^11*d^10))^(1/6)) + 18*(c^
4*d^6*x^8 + 20*c^5*d^5*x^5 - 8*c^6*d^4*x^2)*(1/(c^11*d^10))^(1/3))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^2*d^4*x^6 - 7*c^3*d^3*x^3
- 8*c^4*d^2)*(1/(c^11*d^10))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*
d*x^3 - 1088*c^3 - 18*(c^8*d^9*x^7 - 52*c^9*d^8*x^4 - 80*c^10*d^7*x)*(1/(c^
11*d^10))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^10*d^10*x^5 + c^11*d^9*x^2)*(1/(
c^11*d^10))^(5/6) - 4*(c^6*d^7*x^6 + 41*c^7*d^6*x^3 + 40*c^8*d^5)*sqrt(1/(c
^11*d^10)) - (c^2*d^4*x^7 - 28*c^3*d^3*x^4 - 272*c^4*d^2*x)*(1/(c^11*d^10))
^(1/6)) + 18*(c^4*d^6*x^8 + 20*c^5*d^5*x^5 - 8*c^6*d^4*x^2)*(1/(c^11*d^10))
^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 36*(d^2*x^5 -
 5*c*d*x^2)*sqrt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**4/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.451 $\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=665

$$\frac{5x^2}{648c^3\sqrt{c+dx^3}}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}-\frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{5\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\,\left(\sqrt[3]{\right.}}{\sqrt{c+}}\right)}{1296\sqrt{3}\,c^{17/}}$$

```
[Out] 5/3888*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/6)/
d^(2/3)-5/3888*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)/d^(2/3)-5/3888
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/6)/d^(2/
3)*3^(1/2)+5/648*x^2/c^3/(d*x^3+c)^(1/2)+1/216*x^2/c^2/(-d*x^3+8*c)/(d*x^3+
c)^(1/2)-5/648*(d*x^3+c)^(1/2)/c^3/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-
5/1944*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3
)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)/d^(2/3)*2^(
1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)+5/1296*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-
3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(1/4)/c^(8/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^
(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.57, antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {483, 593, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{5\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{324\sqrt{2}\,\sqrt[4]{3}\,c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}+\frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{432\,3^{3/4}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{5\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}\,c^{17/6}d^{2/3}}+\frac{5\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{17/6}d^{2/3}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{17/6}d^{2/3}}-\frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{5x^2}{648c^3\sqrt{c+dx^3}}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (5*x^2)/(648*c^3*Sqrt[c + d*x^3]) + x^2/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (5*Sqrt[c + d*x^3])/(648*c^3*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)) - (5*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(1296*Sqrt[3]*c^(17/6)*d^(2/3)) + (5*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(3888*c^(17/6)*d^(2/3)) - (5*ArcTanh[Sqrt[c + d*x^
3]/(3*Sqrt[c])])/(3888*c^(17/6)*d^(2/3)) + (5*Sqrt[2 - Sqrt[3]]*(c^(1/3) +
```

```
d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(432*3^(3/4)*c^(8/
3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(324*Sqrt[2]*3^(1/4)*c^(8/3)*d^(2/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d
*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{x\left(25cd+\frac{5d^2x^3}{2}\right)}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{x\left(\frac{45c^2d^2}{2}-\frac{45}{4}cd^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{2916c^4d^2}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \left(\frac{45cd^2x}{4\sqrt{c+dx^3}} - \frac{\cdots}{2(8c-d\cdots}\right.}{2916c^4\cdots}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\int \frac{x}{\sqrt{c+dx^3}}\,dx}{1296c^3} + \frac{5\int \cdots}{\cdots}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/\cdots}}{\sqrt[3]{\cdots}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c\cdots}}}{2592c^3d}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.09, size = 167, normalized size = 0.25

$$\frac{16cx^2(43c-5dx^3)+5cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{10368c^4\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (16*c*x^2*(43*c - 5*d*x^3) + 5*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*App
ellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(8*c - d*x^3)*
Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]
)/(10368*c^4*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 904, normalized size = 1.36

method	result	size
default	Expression too large to display	904
elliptic	Expression too large to display	904

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/243*x^2/c^3/((x^3+c/d)*d)^(1/2)+1/1944/c^3*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*
c)+5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-5/5832*I/c^3/d^
3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-
(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
```

```
/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.34, size = 2748, normalized size = 4.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/46656*(20*sqrt(3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)*(1/(c^17*d^4))
^(1/6)*arctan(1/9*((9*sqrt(3)*c^3*d^2*x^5*(1/(c^17*d^4))^(1/6) - sqrt(3)*(c
^14*d^5*x^6 - 40*c^15*d^4*x^3 - 32*c^16*d^3)*(1/(c^17*d^4))^(5/6) + 3*sqrt(
3)*(5*c^9*d^3*x^4 + 8*c^10*d^2*x)*sqrt(1/(c^17*d^4)))*sqrt(d*x^3 + c) + (18
*sqrt(3)*(c^12*d^4*x^5 + c^13*d^3*x^2)*(1/(c^17*d^4))^(2/3) + 12*sqrt(3)*(c
^6*d^3*x^6 - c^7*d^2*x^3 - 2*c^8*d)*(1/(c^17*d^4))^(1/3) + 3*sqrt(3)*(d^2*x
^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^14*d^5*x^6 + 32*c^1
5*d^4*x^3 + 40*c^16*d^3)*(1/(c^17*d^4))^(5/6) + 3*sqrt(3)*(7*c^9*d^3*x^4 +
4*c^10*d^2*x)*sqrt(1/(c^17*d^4)) + 9*sqrt(3)*(c^3*d^2*x^5 + 2*c^4*d*x^2)*(1
/(c^17*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*
c^3 - 18*(c^12*d^5*x^7 - 52*c^13*d^4*x^4 - 80*c^14*d^3*x)*(1/(c^17*d^4))^(2
/3) + 6*sqrt(d*x^3 + c)*(24*(c^15*d^5*x^5 + c^16*d^4*x^2)*(1/(c^17*d^4))^(5
/6) - 4*(c^9*d^4*x^6 + 41*c^10*d^3*x^3 + 40*c^11*d^2)*sqrt(1/(c^17*d^4)) -
(c^3*d^3*x^7 - 28*c^4*d^2*x^4 - 272*c^5*d*x)*(1/(c^17*d^4))^(1/6)) + 18*(c^
6*d^4*x^8 + 20*c^7*d^3*x^5 - 8*c^8*d^2*x^2)*(1/(c^17*d^4))^(1/3))/(d^3*x^9
- 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)
) + 20*sqrt(3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)*(1/(c^17*d^4))^(1/6)
*arctan(1/9*((9*sqrt(3)*c^3*d^2*x^5*(1/(c^17*d^4))^(1/6) - sqrt(3)*(c^14*d^
5*x^6 - 40*c^15*d^4*x^3 - 32*c^16*d^3)*(1/(c^17*d^4))^(5/6) + 3*sqrt(3)*(5*
c^9*d^3*x^4 + 8*c^10*d^2*x)*sqrt(1/(c^17*d^4)))*sqrt(d*x^3 + c) - (18*sqrt(
3)*(c^12*d^4*x^5 + c^13*d^3*x^2)*(1/(c^17*d^4))^(2/3) + 12*sqrt(3)*(c^6*d^3
*x^6 - c^7*d^2*x^3 - 2*c^8*d)*(1/(c^17*d^4))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5
*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^14*d^5*x^6 + 32*c^15*d^4*
```

```
x^3 + 40*c^16*d^3)*(1/(c^17*d^4))^(5/6) + 3*sqrt(3)*(7*c^9*d^3*x^4 + 4*c^10
*d^2*x)*sqrt(1/(c^17*d^4)) + 9*sqrt(3)*(c^3*d^2*x^5 + 2*c^4*d*x^2)*(1/(c^17
*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 -
18*(c^12*d^5*x^7 - 52*c^13*d^4*x^4 - 80*c^14*d^3*x)*(1/(c^17*d^4))^(2/3) -
6*sqrt(d*x^3 + c)*(24*(c^15*d^5*x^5 + c^16*d^4*x^2)*(1/(c^17*d^4))^(5/6) -
4*(c^9*d^4*x^6 + 41*c^10*d^3*x^3 + 40*c^11*d^2)*sqrt(1/(c^17*d^4)) - (c^3*d
^3*x^7 - 28*c^4*d^2*x^4 - 272*c^5*d*x)*(1/(c^17*d^4))^(1/6)) + 18*(c^6*d^4*
x^8 + 20*c^7*d^3*x^5 - 8*c^8*d^2*x^2)*(1/(c^17*d^4))^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) - 36
0*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstr
assPInverse(0, -4*c/d, x)) - 10*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)*(1/
(c^17*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 +
 18*(5*c^12*d^5*x^7 + 64*c^13*d^4*x^4 + 32*c^14*d^3*x)*(1/(c^17*d^4))^(2/3)
 + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2)*(1/(c^17*d^4))^(
5/6) + (7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(1/(c^17*d^4))
+ (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x)*(1/(c^17*d^4))^(1/6)) + 18*(
c^6*d^4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2)*(1/(c^17*d^4))^(1/3))/(d^3*x
^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 10*(c^3*d^3*x^6 - 7*c^4*d^2
*x^3 - 8*c^5*d)*(1/(c^17*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^
2*d*x^3 + 640*c^3 + 18*(5*c^12*d^5*x^7 + 64*c^13*d^4*x^4 + 32*c^14*d^3*x)*(
1/(c^17*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^
2)*(1/(c^17*d^4))^(5/6) + (7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*
sqrt(1/(c^17*d^4)) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x)*(1/(c^17*
d^4))^(1/6)) + 18*(c^6*d^4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2)*(1/(c^17*
d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 5*(c^3*d
^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)*(1/(c^17*d^4))^(1/6)*log((d^3*x^9 - 276*c
*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^12*d^5*x^7 - 52*c^13*d^4*x^4 -
 80*c^14*d^3*x)*(1/(c^17*d^4))^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^15*d^5*x^5
+ c^16*d^4*x^2)*(1/(c^17*d^4))^(5/6) - 4*(c^9*d^4*x^6 + 41*c^10*d^3*x^3 + 4
0*c^11*d^2)*sqrt(1/(c^17*d^4)) - (c^3*d^3*x^7 - 28*c^4*d^2*x^4 - 272*c^5*d*
x)*(1/(c^17*d^4))^(1/6)) + 18*(c^6*d^4*x^8 + 20*c^7*d^3*x^5 - 8*c^8*d^2*x^2
)*(1/(c^17*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
 - 5*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)*(1/(c^17*d^4))^(1/6)*log((d^3*
x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^12*d^5*x^7 - 52*c^1
3*d^4*x^4 - 80*c^14*d^3*x)*(1/(c^17*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^
15*d^5*x^5 + c^16*d^4*x^2)*(1/(c^17*d^4))^(5/6) - 4*(c^9*d^4*x^6 + 41*c^10*
d^3*x^3 + 40*c^11*d^2)*sqrt(1/(c^17*d^4)) - (c^3*d^3*x^7 - 28*c^4*d^2*x^4 -
 272*c^5*d*x)*(1/(c^17*d^4))^(1/6)) + 18*(c^6*d^4*x^8 + 20*c^7*d^3*x^5 - 8*
c^8*d^2*x^2)*(1/(c^17*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
- 512*c^3)) - 72*(5*d^2*x^5 - 43*c*d*x^2)*sqrt(d*x^3 + c))/(c^3*d^3*x^6 - 7
*c^4*d^2*x^3 - 8*c^5*d)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.452 $\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=686

$$\frac{5}{648c^3x\sqrt{c+dx^3}}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{31\sqrt{c+dx^3}}{1296c^4x}+\frac{31\sqrt[3]{d}\,\sqrt{c+dx^3}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\sqrt[3]{d}\ \mathrm{ta}}{}$$

[Out] 1/3888*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(23/6)-1/3888*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/3888*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(23/6)*3^(1/2)+5/648/c^3/x/(d*x^3+c)^(1/2)+1/216/c^2/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-31/1296*(d*x^3+c)^(1/2)/c^4/x+31/1296*d^(1/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+31/3888*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-31/2592*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(11/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.67, antiderivative size = 686, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$, Rules used = {483, 593, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{648\sqrt{2}\,\sqrt[4]{3}\,c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{31\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{864\,3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}-\frac{\sqrt[3]{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}\,c^{23/6}}+\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{3888c^{23/6}}-\frac{\sqrt[3]{d}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{23/6}}-\frac{31\sqrt{c+dx^3}}{1296c^4x}+\frac{31\sqrt[3]{d}\,\sqrt{c+dx^3}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{5}{648c^3x\sqrt{c+dx^3}}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] 5/(648*c^3*x*Sqrt[c + d*x^3]) + 1/(216*c^2*x*(8*c - d*x^3)*Sqrt[c + d*x^3]) - (31*Sqrt[c + d*x^3])/(1296*c^4*x) + (31*d^(1/3)*Sqrt[c + d*x^3])/(1296*c^4*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1296*Sqrt[3]*c^(23/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3888*c^(23/

```
6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3888*c^(23/6)) - (31*
Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(864*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x
))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (31*d^(1/3)*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(648*Sqrt
[2]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^2 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{28cd+\frac{11d^2x^3}{2}}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{-558c^2d^2+\frac{225}{4}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}}{2916c^4d^2}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{\int \frac{x(}{(8}}{}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{\int \Big(}{}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{(31c}{}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} - \frac{\int -}{}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{}{129}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{}{129}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{}{129}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 20.09, size = 180, normalized size = 0.26

$$\frac{-80c(162c^2+227cdx^3-31d^2x^6)+650cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+31d^2x^6(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{103680c^5\sqrt{c+dx^3}\,(8cx-dx^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-80*c*(162*c^2 + 227*c*d*x^3 - 31*d^2*x^6) + 650*c*d*x^3*(8*c - d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] +
31*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -(
(d*x^3)/c), (d*x^3)/(8*c)])/(103680*c^5*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 2270, normalized size = 3.31

method	result	size
elliptic	Expression too large to display	920
risch	Expression too large to display	2220
default	Expression too large to display	2270

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d/c*(2/243*x^2/c^3/((x^3+c/d)*d)^(1/2)+1/1944/c^3*x^2*(d*x^3+c)^(1/2)/(
-d*x^3+8*c)+5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(
-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(
-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-5/5832
*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^
(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1
/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alp
ha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
```

```
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2)),_alpha=RootOf(_Z^3*d-8*c)))-1/64*d/c^2*(-2/27*x^2/c^2/((x^3+c/d)*d)^(1
/2)-2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1
/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/243*I/c^2/d^3*
2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_
alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha
=RootOf(_Z^3*d-8*c))+1/64/c^2*(-2/3*d*x^2/c^2/((x^3+c/d)*d)^(1/2)-(d*x^3+c
)^(1/2)/c^2/x-5/9*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*
d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(
-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*
d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 4.06, size = 2717, normalized size = 3.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/46656*(4*sqrt(3)*(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x)*(d^2/c^23)^(1/6)*
arctan(1/9*((9*sqrt(3)*c^4*d^4*x^5*(d^2/c^23)^(1/6) - sqrt(3)*(c^19*d^3*x^6
 - 40*c^20*d^2*x^3 - 32*c^21*d)*(d^2/c^23)^(5/6) + 3*sqrt(3)*(5*c^12*d^3*x^
4 + 8*c^13*d^2*x)*sqrt(d^2/c^23))*sqrt(d*x^3 + c) + (18*sqrt(3)*(c^16*d^2*x
^5 + c^17*d*x^2)*(d^2/c^23)^(2/3) + 12*sqrt(3)*(c^8*d^3*x^6 - c^9*d^2*x^3 -
 2*c^10*d)*(d^2/c^23)^(1/3) + 3*sqrt(3)*(d^4*x^7 + 5*c*d^3*x^4 + 4*c^2*d^2*
x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^19*d^2*x^6 + 32*c^20*d*x^3 + 40*c^21)*(d^2
/c^23)^(5/6) + 3*sqrt(3)*(7*c^12*d^2*x^4 + 4*c^13*d*x)*sqrt(d^2/c^23) + 9*s
qrt(3)*(c^4*d^3*x^5 + 2*c^5*d^2*x^2)*(d^2/c^23)^(1/6)))*sqrt((d^5*x^9 - 276
*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^16*d^3*x^7 - 52*c^17*d
^2*x^4 - 80*c^18*d*x)*(d^2/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^20*d^2*x^
5 + c^21*d*x^2)*(d^2/c^23)^(5/6) - 4*(c^12*d^3*x^6 + 41*c^13*d^2*x^3 + 40*c
^14*d)*sqrt(d^2/c^23) - (c^4*d^4*x^7 - 28*c^5*d^3*x^4 - 272*c^6*d^2*x)*(d^2
/c^23)^(1/6)) + 18*(c^8*d^4*x^8 + 20*c^9*d^3*x^5 - 8*c^10*d^2*x^2)*(d^2/c^2
3)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^5*x^7 - 7
*c*d^4*x^4 - 8*c^2*d^3*x)) + 4*sqrt(3)*(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x
)*(d^2/c^23)^(1/6)*arctan(1/9*((9*sqrt(3)*c^4*d^4*x^5*(d^2/c^23)^(1/6) - sq
rt(3)*(c^19*d^3*x^6 - 40*c^20*d^2*x^3 - 32*c^21*d)*(d^2/c^23)^(5/6) + 3*sqr
t(3)*(5*c^12*d^3*x^4 + 8*c^13*d^2*x)*sqrt(d^2/c^23))*sqrt(d*x^3 + c) - (18*
sqrt(3)*(c^16*d^2*x^5 + c^17*d*x^2)*(d^2/c^23)^(2/3) + 12*sqrt(3)*(c^8*d^3*
x^6 - c^9*d^2*x^3 - 2*c^10*d)*(d^2/c^23)^(1/3) + 3*sqrt(3)*(d^4*x^7 + 5*c*d
^3*x^4 + 4*c^2*d^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^19*d^2*x^6 + 32*c^20*d*
x^3 + 40*c^21)*(d^2/c^23)^(5/6) + 3*sqrt(3)*(7*c^12*d^2*x^4 + 4*c^13*d*x)*s
qrt(d^2/c^23) + 9*sqrt(3)*(c^4*d^3*x^5 + 2*c^5*d^2*x^2)*(d^2/c^23)^(1/6)))*
sqrt((d^5*x^9 - 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^16*
d^3*x^7 - 52*c^17*d^2*x^4 - 80*c^18*d*x)*(d^2/c^23)^(2/3) - 6*sqrt(d*x^3 +
c)*(24*(c^20*d^2*x^5 + c^21*d*x^2)*(d^2/c^23)^(5/6) - 4*(c^12*d^3*x^6 + 41*
c^13*d^2*x^3 + 40*c^14*d)*sqrt(d^2/c^23) - (c^4*d^4*x^7 - 28*c^5*d^3*x^4 -
272*c^6*d^2*x)*(d^2/c^23)^(1/6)) + 18*(c^8*d^4*x^8 + 20*c^9*d^3*x^5 - 8*c^1
0*d^2*x^2)*(d^2/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)))/(d^5*x^7 - 7*c*d^4*x^4 - 8*c^2*d^3*x)) + 1116*(d^2*x^7 - 7*c*d*x^4 -
 8*c^2*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d,
 x)) + (c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x)*(d^2/c^23)^(1/6)*log((d^5*x^9
- 276*c*d^4*x^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^16*d^3*x^7 - 52*c
```

```
^17*d^2*x^4 - 80*c^18*d*x)*(d^2/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^20*d
^2*x^5 + c^21*d*x^2)*(d^2/c^23)^(5/6) - 4*(c^12*d^3*x^6 + 41*c^13*d^2*x^3 +
 40*c^14*d)*sqrt(d^2/c^23) - (c^4*d^4*x^7 - 28*c^5*d^3*x^4 - 272*c^6*d^2*x)
*(d^2/c^23)^(1/6)) + 18*(c^8*d^4*x^8 + 20*c^9*d^3*x^5 - 8*c^10*d^2*x^2)*(d^
2/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^4*d
^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x)*(d^2/c^23)^(1/6)*log((d^5*x^9 - 276*c*d^4*x
^6 - 1608*c^2*d^3*x^3 - 1088*c^3*d^2 - 18*(c^16*d^3*x^7 - 52*c^17*d^2*x^4 -
 80*c^18*d*x)*(d^2/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^20*d^2*x^5 + c^21
*d*x^2)*(d^2/c^23)^(5/6) - 4*(c^12*d^3*x^6 + 41*c^13*d^2*x^3 + 40*c^14*d)*s
qrt(d^2/c^23) - (c^4*d^4*x^7 - 28*c^5*d^3*x^4 - 272*c^6*d^2*x)*(d^2/c^23)^(
1/6)) + 18*(c^8*d^4*x^8 + 20*c^9*d^3*x^5 - 8*c^10*d^2*x^2)*(d^2/c^23)^(1/3)
)/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^4*d^2*x^7 - 7*
c^5*d*x^4 - 8*c^6*x)*(d^2/c^23)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c
^2*d^2*x^3 + 640*c^3*d + 18*(5*c^16*d^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x)*(d
^2/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^2/c^2
3)^(5/6) + (7*c^12*d^2*x^6 + 152*c^13*d*x^3 + 64*c^14)*sqrt(d^2/c^23) + (c^
4*d^3*x^7 + 80*c^5*d^2*x^4 + 160*c^6*d*x)*(d^2/c^23)^(1/6)) + 18*(c^8*d^3*x
^8 + 38*c^9*d^2*x^5 + 64*c^10*d*x^2)*(d^2/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*
x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x)*(
d^2/c^23)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d
 + 18*(5*c^16*d^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x)*(d^2/c^23)^(2/3) - 6*sqr
t(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^2/c^23)^(5/6) + (7*c^12*d^2
*x^6 + 152*c^13*d*x^3 + 64*c^14)*sqrt(d^2/c^23) + (c^4*d^3*x^7 + 80*c^5*d^2
*x^4 + 160*c^6*d*x)*(d^2/c^23)^(1/6)) + 18*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 +
64*c^10*d*x^2)*(d^2/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) + 36*(31*d^2*x^6 - 227*c*d*x^3 - 162*c^2)*sqrt(d*x^3 + c))/(c^4*d
^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(-8c + dx^3\right)^2 \left(c + dx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.453 $\int \frac{1}{x^5(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=708

$$\frac{5}{648c^3x^4\sqrt{c+dx^3}}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{253\sqrt{c+dx^3}}{20736c^4x^4}+\frac{77d\sqrt{c+dx^3}}{2592c^5x}-\frac{77d^{4/3}\sqrt{c+dx}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\right.}$$

```
[Out] 11/248832*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2)
)/c^(29/6)-11/248832*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6)-
11/248832*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2
))/c^(29/6)*3^(1/2)+5/648/c^3/x^4/(d*x^3+c)^(1/2)+1/216/c^2/x^4/(-d*x^3+8*c
)/(d*x^3+c)^(1/2)-253/20736*(d*x^3+c)^(1/2)/c^4/x^4+77/2592*d*(d*x^3+c)^(1/
2)/c^5/x-77/2592*d^(4/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)-77/7776*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/
2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/
3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(14/3)
*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))^2)^(1/2)+77/5184*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.73, antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$, Rules used = {483, 593, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{77d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{1296\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{77\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{1728\,3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{11d^{4/3}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{82944\sqrt{3}c^{29/6}}+\frac{11d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{248832c^{29/6}}-\frac{11d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{248832c^{29/6}}-\frac{77d^{4/3}\sqrt{c+dx^3}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{77d\sqrt{c+dx^3}}{2592c^5x}-\frac{253\sqrt{c+dx^3}}{20736c^4x^4}+\frac{5}{648c^3x^4\sqrt{c+dx^3}}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*x^4*Sqrt[c + d*x^3]) + 1/(216*c^2*x^4*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (253*Sqrt[c + d*x^3])/(20736*c^4*x^4) + (77*d*Sqrt[c + d*x^3])/(2592
*c^5*x) - (77*d^(4/3)*Sqrt[c + d*x^3])/(2592*c^5*((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)) - (11*d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqr
t[c + d*x^3]])/(82944*Sqrt[3]*c^(29/6)) + (11*d^(4/3)*ArcTanh[(c^(1/3) + d^
```

```
(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(248832*c^(29/6)) - (11*d^(4/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(248832*c^(29/6)) + (77*Sqrt[2 - Sqrt[3
]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(1728*3^(3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (77*d^(4/3)*(c^(1/3) + d^(1/
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(1296*Sqrt[2]*3^(1/4)*c
^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{31cd+\frac{17d^2x^3}{2}}{x^5(8c-dx^3)(c+dx^3)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{-\frac{2277}{2}c^2d^2+\frac{495}{4}cd^3}{x^5(8c-dx^3)\sqrt{c+d}}}{2916c^4d^2}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} +$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} +$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} +$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} +$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} +$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} +$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} +$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 198, normalized size = 0.28

$$\frac{-24475cd^2x^6(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-16\left(10c(648c^3-2997c^2dx^3-4565cd^2x^6+616d^3x^9)+77d^3x^9(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{3317760c^6x^4\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-24475*c*d^2*x^6*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(10*c*(648*c^3 - 2997*c^2*d*x^3 - 456
5*c*d^2*x^6 + 616*d^3*x^9) + 77*d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*
AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3317760*c^6*x^4*
(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 2775, normalized size = 3.92

method	result	size
elliptic	Expression too large to display	943
risch	Expression too large to display	2232
default	Expression too large to display	2775

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*d^2*(2/243*x^2/c^3/((x^3+c/d)*d)^(1/2)+1/1944/c^3*x^2*(d*x^3+c)^(1
/2)/(-d*x^3+8*c)+5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-
1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+
1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-5
/5832*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2
*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
```

)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/4*(d*x^3+c)^(1/2)/c^2/x^
4+13/8*d*(d*x^3+c)^(1/2)/c^3/x+2/3*d^2/c^3*x^2/((x^3+c/d)*d)^(1/2)+55/72*I/
c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d
x^3+c)^(1/2)((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))-1/256/c^3*d^2*(-2/27*x^2/c
^2/((x^3+c/d)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3
)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d
^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/256/c^3*d*(-2/3*d*x^2/c^2/((x^3+
c/d)*d)^(1/2)-(d*x^3+c)^(1/2)/c^2/x-5/9*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^

```
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.97, size = 2820, normalized size = 3.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] -1/2985984*(44*sqrt(3)*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4)*(d^8/c^29)^
(1/6)*arctan(1/9*((9*sqrt(3)*c^5*d^13*x^5*(d^8/c^29)^(1/6) - sqrt(3)*(c^24*
d^8*x^6 - 40*c^25*d^7*x^3 - 32*c^26*d^6)*(d^8/c^29)^(5/6) + 3*sqrt(3)*(5*c^
15*d^10*x^4 + 8*c^16*d^9*x)*sqrt(d^8/c^29))*sqrt(d*x^3 + c) + (18*sqrt(3)*(
c^20*d^3*x^5 + c^21*d^2*x^2)*(d^8/c^29)^(2/3) + 12*sqrt(3)*(c^10*d^6*x^6 -
c^11*d^5*x^3 - 2*c^12*d^4)*(d^8/c^29)^(1/3) + 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*
x^4 + 4*c^2*d^7*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^24*d^2*x^6 + 32*c^25*d*x^3
 + 40*c^26)*(d^8/c^29)^(5/6) + 3*sqrt(3)*(7*c^15*d^4*x^4 + 4*c^16*d^3*x)*sq
rt(d^8/c^29) + 9*sqrt(3)*(c^5*d^7*x^5 + 2*c^6*d^6*x^2)*(d^8/c^29)^(1/6)))*s
qrt((d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^
20*d^9*x^7 - 52*c^21*d^8*x^4 - 80*c^22*d^7*x)*(d^8/c^29)^(2/3) + 6*sqrt(d*x
^3 + c)*(24*(c^25*d^7*x^5 + c^26*d^6*x^2)*(d^8/c^29)^(5/6) - 4*(c^15*d^10*x
^6 + 41*c^16*d^9*x^3 + 40*c^17*d^8)*sqrt(d^8/c^29) - (c^5*d^13*x^7 - 28*c^6
*d^12*x^4 - 272*c^7*d^11*x)*(d^8/c^29)^(1/6)) + 18*(c^10*d^12*x^8 + 20*c^11
*d^11*x^5 - 8*c^12*d^10*x^2)*(d^8/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 19
2*c^2*d*x^3 - 512*c^3)))/(d^15*x^7 - 7*c*d^14*x^4 - 8*c^2*d^13*x)) + 44*sqr
t(3)*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4)*(d^8/c^29)^(1/6)*arctan(1/9*(
(9*sqrt(3)*c^5*d^13*x^5*(d^8/c^29)^(1/6) - sqrt(3)*(c^24*d^8*x^6 - 40*c^25*
d^7*x^3 - 32*c^26*d^6)*(d^8/c^29)^(5/6) + 3*sqrt(3)*(5*c^15*d^10*x^4 + 8*c^
16*d^9*x)*sqrt(d^8/c^29))*sqrt(d*x^3 + c) - (18*sqrt(3)*(c^20*d^3*x^5 + c^2
1*d^2*x^2)*(d^8/c^29)^(2/3) + 12*sqrt(3)*(c^10*d^6*x^6 - c^11*d^5*x^3 - 2*c
^12*d^4)*(d^8/c^29)^(1/3) + 3*sqrt(3)*(d^9*x^7 + 5*c*d^8*x^4 + 4*c^2*d^7*x)
 - sqrt(d*x^3 + c)*(sqrt(3)*(c^24*d^2*x^6 + 32*c^25*d*x^3 + 40*c^26)*(d^8/c
```

```
^29)^(5/6) + 3*sqrt(3)*(7*c^15*d^4*x^4 + 4*c^16*d^3*x)*sqrt(d^8/c^29) + 9*s
qrt(3)*(c^5*d^7*x^5 + 2*c^6*d^6*x^2)*(d^8/c^29)^(1/6)))*sqrt((d^15*x^9 - 27
6*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^20*d^9*x^7 - 52*c^
21*d^8*x^4 - 80*c^22*d^7*x)*(d^8/c^29)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^25*
d^7*x^5 + c^26*d^6*x^2)*(d^8/c^29)^(5/6) - 4*(c^15*d^10*x^6 + 41*c^16*d^9*x
^3 + 40*c^17*d^8)*sqrt(d^8/c^29) - (c^5*d^13*x^7 - 28*c^6*d^12*x^4 - 272*c^
7*d^11*x)*(d^8/c^29)^(1/6)) + 18*(c^10*d^12*x^8 + 20*c^11*d^11*x^5 - 8*c^12
*d^10*x^2)*(d^8/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)))/(d^15*x^7 - 7*c*d^14*x^4 - 8*c^2*d^13*x)) - 88704*(d^3*x^10 - 7*c*d^
2*x^7 - 8*c^2*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse
(0, -4*c/d, x)) + 11*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4)*(d^8/c^29)^(1
/6)*log(25937424601*(d^15*x^9 - 276*c*d^14*x^6 - 1608*c^2*d^13*x^3 - 1088*c
^3*d^12 - 18*(c^20*d^9*x^7 - 52*c^21*d^8*x^4 - 80*c^22*d^7*x)*(d^8/c^29)^(2
/3) + 6*sqrt(d*x^3 + c)*(24*(c^25*d^7*x^5 + c^26*d^6*x^2)*(d^8/c^29)^(5/6)
- 4*(c^15*d^10*x^6 + 41*c^16*d^9*x^3 + 40*c^17*d^8)*sqrt(d^8/c^29) - (c^5*d
^13*x^7 - 28*c^6*d^12*x^4 - 272*c^7*d^11*x)*(d^8/c^29)^(1/6)) + 18*(c^10*d^
12*x^8 + 20*c^11*d^11*x^5 - 8*c^12*d^10*x^2)*(d^8/c^29)^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 11*(c^5*d^2*x^10 - 7*c^6*d*x^7 -
8*c^7*x^4)*(d^8/c^29)^(1/6)*log(25937424601*(d^15*x^9 - 276*c*d^14*x^6 - 16
08*c^2*d^13*x^3 - 1088*c^3*d^12 - 18*(c^20*d^9*x^7 - 52*c^21*d^8*x^4 - 80*c
^22*d^7*x)*(d^8/c^29)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^25*d^7*x^5 + c^26*d^
6*x^2)*(d^8/c^29)^(5/6) - 4*(c^15*d^10*x^6 + 41*c^16*d^9*x^3 + 40*c^17*d^8)
*sqrt(d^8/c^29) - (c^5*d^13*x^7 - 28*c^6*d^12*x^4 - 272*c^7*d^11*x)*(d^8/c^
29)^(1/6)) + 18*(c^10*d^12*x^8 + 20*c^11*d^11*x^5 - 8*c^12*d^10*x^2)*(d^8/c
^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 22*(c^5*d
^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4)*(d^8/c^29)^(1/6)*log(161051*(d^9*x^9 + 3
18*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^20*d^3*x^7 + 64*c^2
1*d^2*x^4 + 32*c^22*d*x)*(d^8/c^29)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^25*d*
x^5 + 32*c^26*x^2)*(d^8/c^29)^(5/6) + (7*c^15*d^4*x^6 + 152*c^16*d^3*x^3 +
64*c^17*d^2)*sqrt(d^8/c^29) + (c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x
)*(d^8/c^29)^(1/6)) + 18*(c^10*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4*x^2)
*(d^8/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 22
*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4)*(d^8/c^29)^(1/6)*log(161051*(d^9*
x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^20*d^3*x^7 +
 64*c^21*d^2*x^4 + 32*c^22*d*x)*(d^8/c^29)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*
c^25*d*x^5 + 32*c^26*x^2)*(d^8/c^29)^(5/6) + (7*c^15*d^4*x^6 + 152*c^16*d^3
*x^3 + 64*c^17*d^2)*sqrt(d^8/c^29) + (c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^
7*d^5*x)*(d^8/c^29)^(1/6)) + 18*(c^10*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d
^4*x^2)*(d^8/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) - 144*(616*d^3*x^9 - 4565*c*d^2*x^6 - 2997*c^2*d*x^3 + 648*c^3)*sqrt(d*x
^3 + c))/(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (-8c + dx^3)^2 (c + dx^3)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \left(d\,x^3+c\right)^{3/2} \left(8\,c-d\,x^3\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.454 $\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}} \, dx$

Optimal. Leaf size=732

$$\frac{5}{648c^3x^7\sqrt{c+dx^3}}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{191\sqrt{c+dx^3}}{18144c^4x^7}+\frac{8257d\sqrt{c+dx^3}}{580608c^5x^4}-\frac{5179d^2\sqrt{c+dx^3}}{145152c^6x}+\frac{}{1}$$

```
[Out] 7/995328*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(35/6)-7/995328*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(35/6)-7/
995328*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(35/6)*3^(1/2)+5/648/c^3/x^7/(d*x^3+c)^(1/2)+1/216/c^2/x^7/(-d*x^3+8*c)/(
d*x^3+c)^(1/2)-191/18144*(d*x^3+c)^(1/2)/c^4/x^7+8257/580608*d*(d*x^3+c)^(1
/2)/c^5/x^4-5179/145152*d^2*(d*x^3+c)^(1/2)/c^6/x+5179/145152*d^(7/3)*(d*x^
3+c)^(1/2)/c^6/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+5179/435456*d^(7/3)*(c^(1/3)
+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(17/3)*2^(1/2)/(d*x^3+c)^(1/2)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-5179/2
90304*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(1/4)/c^(17/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.82, antiderivative size = 732, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$, Rules used = {483, 593, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\frac{5179d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{72576\sqrt{2}\sqrt[4]{3}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{5179\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{96768\,3^{3/4}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7d^{7/3}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{331776\sqrt{3}c^{35/6}}+\frac{7d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{995328c^{35/6}}-\frac{7d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{995328c^{35/6}}+\frac{5179d^{7/3}\sqrt{c+dx^3}}{145152c^6\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{5179d^2\sqrt{c+dx^3}}{145152c^6x}+\frac{8257d\sqrt{c+dx^3}}{580608c^5x^4}-\frac{191\sqrt{c+dx^3}}{18144c^4x^7}+\frac{5}{648c^3x^7\sqrt{c+dx^3}}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*x^7*Sqrt[c + d*x^3]) + 1/(216*c^2*x^7*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (191*Sqrt[c + d*x^3])/(18144*c^4*x^7) + (8257*d*Sqrt[c + d*x^3])/(58
0608*c^5*x^4) - (5179*d^2*Sqrt[c + d*x^3])/(145152*c^6*x) + (5179*d^(7/3)*S
qrt[c + d*x^3])/(145152*c^6*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (7*d^(7/
```

```
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(331776
*Sqrt[3]*c^(35/6)) + (7*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*
Sqrt[c + d*x^3])]])/(995328*c^(35/6)) - (7*d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])]])/(995328*c^(35/6)) - (5179*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(96768*3^(3/4)*c^
(17/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3]) + (5179*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(72576*Sqrt[2]*3^(1/4)*c^(17/3)*Sqrt[(c^(
1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
 d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^8\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{34cd+\frac{23d^2x^3}{2}}{x^8(8c-dx^3)(c+dx^3)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{-1719c^2d^2+\frac{765}{4}cd^3 \cdots}{x^8(8c-dx^3)\sqrt{c+d \cdots}}}{2916c^4d^2}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \cdots$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8 \cdots}{\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8 \cdots}{\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8 \cdots}{\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8 \cdots}{\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8 \cdots}{\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8 \cdots}{\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8 \cdots}{\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8 \cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.13, size = 210, normalized size = 0.29

$$\frac{829375cd^3x^9(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-8\left(20c(10368c^4-18792c^3dx^3+101817c^2d^2x^6+153269cd^3x^9-20716d^4x^{12})+5179d^4x^{12}(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{92897280c^7x^7\,(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (829375*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(10368*c^4 - 18792*c^3*d*x^3 + 1
01817*c^2*d^2*x^6 + 153269*c*d^3*x^9 - 20716*d^4*x^12) + 5179*d^4*x^12*(8*c
 - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x
^3)/(8*c)]))/(92897280*c^7*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 3300, normalized size = 4.51

method	result	size
elliptic	Expression too large to display	962
risch	Expression too large to display	2243
default	Expression too large to display	3300

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/512/c^3*d^3*(2/243*x^2/c^3/((x^3+c/d)*d)^(1/2)+1/1944/c^3*x^2*(d*x^3+c)^(
1/2)/(-d*x^3+8*c)+5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x
-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
)^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-
5/5832*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+
2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
```

2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
d^2)^(1/3))^(1/2),-1/18/d(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2
)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(-1/4*(d*x^3+c)^(1/2)/c^
2/x^4+13/8*d*(d*x^3+c)^(1/2)/c^3/x+2/3*d^2/c^3*x^2/((x^3+c/d)*d)^(1/2)+55/7
2*I/c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))))+1/64/c^2*(-1/7*(d*x^3+
c)^(1/2)/c^2/x^7+25/56*d*(d*x^3+c)^(1/2)/c^3/x^4-237/112*d^2*(d*x^3+c)^(1/2
)/c^4/x-2/3*d^3/c^4*x^2/((x^3+c/d)*d)^(1/2)-935/1008*I*d^2/c^4*3^(1/2)*(-c*
d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2))))-3/4096*d^3/c^4*(-2/27*x^2/c^2/((x^3+c/d)*d)^
(1/2)-2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^
(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x
+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^
(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/243*I/c^2/d^
3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*

```
(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-
(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 20.19, size = 2837, normalized size = 3.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/83607552*(196*sqrt(3)*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8*x^7)*(d^14/c^
35)^(1/6)*arctan(1/9*((9*sqrt(3)*c^6*d^22*x^5*(d^14/c^35)^(1/6) - sqrt(3)*(
c^29*d^13*x^6 - 40*c^30*d^12*x^3 - 32*c^31*d^11)*(d^14/c^35)^(5/6) + 3*sqrt
(3)*(5*c^18*d^17*x^4 + 8*c^19*d^16*x)*sqrt(d^14/c^35))*sqrt(d*x^3 + c) + (1
8*sqrt(3)*(c^24*d^4*x^5 + c^25*d^3*x^2)*(d^14/c^35)^(2/3) + 12*sqrt(3)*(c^1
2*d^9*x^6 - c^13*d^8*x^3 - 2*c^14*d^7)*(d^14/c^35)^(1/3) + 3*sqrt(3)*(d^14*
x^7 + 5*c*d^13*x^4 + 4*c^2*d^12*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^29*d^2*x^6
 + 32*c^30*d*x^3 + 40*c^31)*(d^14/c^35)^(5/6) + 3*sqrt(3)*(7*c^18*d^6*x^4 +
 4*c^19*d^5*x)*sqrt(d^14/c^35) + 9*sqrt(3)*(c^6*d^11*x^5 + 2*c^7*d^10*x^2)*
(d^14/c^35)^(1/6)))*sqrt((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1
088*c^3*d^22 - 18*(c^24*d^15*x^7 - 52*c^25*d^14*x^4 - 80*c^26*d^13*x)*(d^14
/c^35)^(2/3) + 6*sqrt(d*x^3 + c)*(24*(c^30*d^12*x^5 + c^31*d^11*x^2)*(d^14/
c^35)^(5/6) - 4*(c^18*d^17*x^6 + 41*c^19*d^16*x^3 + 40*c^20*d^15)*sqrt(d^14
/c^35) - (c^6*d^22*x^7 - 28*c^7*d^21*x^4 - 272*c^8*d^20*x)*(d^14/c^35)^(1/6
)) + 18*(c^12*d^20*x^8 + 20*c^13*d^19*x^5 - 8*c^14*d^18*x^2)*(d^14/c^35)^(1
/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^25*x^7 - 7*c*d
^24*x^4 - 8*c^2*d^23*x)) + 196*sqrt(3)*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8
*x^7)*(d^14/c^35)^(1/6)*arctan(1/9*((9*sqrt(3)*c^6*d^22*x^5*(d^14/c^35)^(1/
6) - sqrt(3)*(c^29*d^13*x^6 - 40*c^30*d^12*x^3 - 32*c^31*d^11)*(d^14/c^35)^
(5/6) + 3*sqrt(3)*(5*c^18*d^17*x^4 + 8*c^19*d^16*x)*sqrt(d^14/c^35))*sqrt(d
*x^3 + c) - (18*sqrt(3)*(c^24*d^4*x^5 + c^25*d^3*x^2)*(d^14/c^35)^(2/3) + 1
2*sqrt(3)*(c^12*d^9*x^6 - c^13*d^8*x^3 - 2*c^14*d^7)*(d^14/c^35)^(1/3) + 3*
sqrt(3)*(d^14*x^7 + 5*c*d^13*x^4 + 4*c^2*d^12*x) - sqrt(d*x^3 + c)*(sqrt(3)
```

```
*(c^29*d^2*x^6 + 32*c^30*d*x^3 + 40*c^31)*(d^14/c^35)^(5/6) + 3*sqrt(3)*(7*
c^18*d^6*x^4 + 4*c^19*d^5*x)*sqrt(d^14/c^35) + 9*sqrt(3)*(c^6*d^11*x^5 + 2*
c^7*d^10*x^2)*(d^14/c^35)^(1/6)))*sqrt((d^25*x^9 - 276*c*d^24*x^6 - 1608*c^
2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^24*d^15*x^7 - 52*c^25*d^14*x^4 - 80*c^26
*d^13*x)*(d^14/c^35)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^30*d^12*x^5 + c^31*d^
11*x^2)*(d^14/c^35)^(5/6) - 4*(c^18*d^17*x^6 + 41*c^19*d^16*x^3 + 40*c^20*d
^15)*sqrt(d^14/c^35) - (c^6*d^22*x^7 - 28*c^7*d^21*x^4 - 272*c^8*d^20*x)*(d
^14/c^35)^(1/6)) + 18*(c^12*d^20*x^8 + 20*c^13*d^19*x^5 - 8*c^14*d^18*x^2)*
(d^14/c^35)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^
25*x^7 - 7*c*d^24*x^4 - 8*c^2*d^23*x)) + 2983104*(d^4*x^13 - 7*c*d^3*x^10 -
 8*c^2*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -
4*c/d, x)) + 49*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8*x^7)*(d^14/c^35)^(1/6)
*log(282475249*(d^25*x^9 - 276*c*d^24*x^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^
22 - 18*(c^24*d^15*x^7 - 52*c^25*d^14*x^4 - 80*c^26*d^13*x)*(d^14/c^35)^(2/
3) + 6*sqrt(d*x^3 + c)*(24*(c^30*d^12*x^5 + c^31*d^11*x^2)*(d^14/c^35)^(5/6
) - 4*(c^18*d^17*x^6 + 41*c^19*d^16*x^3 + 40*c^20*d^15)*sqrt(d^14/c^35) - (
c^6*d^22*x^7 - 28*c^7*d^21*x^4 - 272*c^8*d^20*x)*(d^14/c^35)^(1/6)) + 18*(c
^12*d^20*x^8 + 20*c^13*d^19*x^5 - 8*c^14*d^18*x^2)*(d^14/c^35)^(1/3))/(d^3*
x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 49*(c^6*d^2*x^13 - 7*c^7*d
*x^10 - 8*c^8*x^7)*(d^14/c^35)^(1/6)*log(282475249*(d^25*x^9 - 276*c*d^24*x
^6 - 1608*c^2*d^23*x^3 - 1088*c^3*d^22 - 18*(c^24*d^15*x^7 - 52*c^25*d^14*x
^4 - 80*c^26*d^13*x)*(d^14/c^35)^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^30*d^12*x
^5 + c^31*d^11*x^2)*(d^14/c^35)^(5/6) - 4*(c^18*d^17*x^6 + 41*c^19*d^16*x^3
 + 40*c^20*d^15)*sqrt(d^14/c^35) - (c^6*d^22*x^7 - 28*c^7*d^21*x^4 - 272*c^
8*d^20*x)*(d^14/c^35)^(1/6)) + 18*(c^12*d^20*x^8 + 20*c^13*d^19*x^5 - 8*c^1
4*d^18*x^2)*(d^14/c^35)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 51
2*c^3)) - 98*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8*x^7)*(d^14/c^35)^(1/6)*lo
g(16807*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*
(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x)*(d^14/c^35)^(2/3) + 6*sq
rt(d*x^3 + c)*(6*(5*c^30*d*x^5 + 32*c^31*x^2)*(d^14/c^35)^(5/6) + (7*c^18*d
^6*x^6 + 152*c^19*d^5*x^3 + 64*c^20*d^4)*sqrt(d^14/c^35) + (c^6*d^11*x^7 +
80*c^7*d^10*x^4 + 160*c^8*d^9*x)*(d^14/c^35)^(1/6)) + 18*(c^12*d^9*x^8 + 38
*c^13*d^8*x^5 + 64*c^14*d^7*x^2)*(d^14/c^35)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) + 98*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8*x^7)
*(d^14/c^35)^(1/6)*log(16807*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3
 + 640*c^3*d^11 + 18*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x)*(d^
14/c^35)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^30*d*x^5 + 32*c^31*x^2)*(d^14/c^
35)^(5/6) + (7*c^18*d^6*x^6 + 152*c^19*d^5*x^3 + 64*c^20*d^4)*sqrt(d^14/c^3
5) + (c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 160*c^8*d^9*x)*(d^14/c^35)^(1/6)) +
18*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2)*(d^14/c^35)^(1/3))/(d
^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 144*(20716*d^4*x^12 - 1
53269*c*d^3*x^9 - 101817*c^2*d^2*x^6 + 18792*c^3*d*x^3 - 10368*c^4)*sqrt(d*
x^3 + c))/(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8*x^7)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 \left(-8c + dx^3\right)^2 \left(c + dx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**8*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^8 \left(d\,x^3 + c\right)^{3/2} \left(8\,c - d\,x^3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.455 $\int \frac{x^6}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=256

$$\frac{2x(4c+dx^3)}{81cd^2(8c-dx^3)\sqrt{c+dx^3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.\right.}{81\sqrt[4]{3}\,cd^{7/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx}}$$

```
[Out] 2/81*x*(d*x^3+4*c)/c/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-2/243*(c^(1/3)+d^(1/3
)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^
(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c/d^(7/3)/(d*x^
3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 0.26, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{7}{3};2,\frac{3}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 3/2, 10/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(448*c^3*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^6}{(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x^7\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{7}{3};2,\frac{3}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^3\sqrt{c+dx^3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.33, size = 189, normalized size = 0.74

$$-\frac{6\sqrt[3]{-d}\,x(4c+dx^3)+2i3^{3/4}\sqrt[3]{c}\sqrt{\frac{(-1)^{5/6}\left(-\sqrt[3]{c}+\sqrt[3]{-d}\,x\right)}{\sqrt[3]{c}}}\sqrt{1+\frac{\sqrt[3]{-d}\,x}{\sqrt[3]{c}}+\frac{(-d)^{2/3}x^2}{c^{2/3}}}\,(-8c+dx^3)\,F\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}-\frac{i\sqrt[3]{-d}\,x}{\sqrt[3]{c}}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)}{243c(-d)^{7/3}(-8c+dx^3)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/243*(6*(-d)^(1/3)*x*(4*c + d*x^3) + (2*I)*3^(3/4)*c^(1/3)*Sqrt[((-1)^(5/
6)*(-c^(1/3) + (-d)^(1/3)*x))/c^(1/3)]*Sqrt[1 + ((-d)^(1/3)*x)/c^(1/3) + ((
-d)^(2/3)*x^2)/c^(2/3)]*(-8*c + d*x^3)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) -
(I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)])/(c*(-d)^(7/3)*(-8*c + d*x^
3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 1792, normalized size = 7.00

method	result

elliptic	$\frac{2x}{243 d^{2} c \sqrt{\left(x^{3}+\frac{c}{d}\right) d}}+\frac{8x\sqrt{d\,x^{3}+c}}{243 c\,d^{2}\left(-d\,x^{3}+8c\right)}+\frac{2i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-\frac{3\left(-c\,d^{2}\cdots\right.}{2d}\cdots}}\cdots}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/d^2*(2/3*x/c/((x^3+c/d)*d)^(1/2)-2/9*I/c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
d^2)^(1/3)))^(1/2)(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+16/d^2*c*(-2/27*x/c^2/((x^3+c/d)*d)^(1/2)+
2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
3^(1/2)/d(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c
d^2)^(1/3)/(-3/2/d(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+
1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
3^(1/2)(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-
c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+64*c^2/d^2*(2/243*x/c^3/((x^3+c/d)*
d)^(1/2)+1/1944*x/c^3*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-5/1944*I/c^3*3^(1/2)/d*(
-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3

```
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/972*I/c^3/d^3*2^(1/2
)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c
*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^
2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1
/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.69, size = 89, normalized size = 0.35

$$-\frac{2\left(\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{d}\ \mathrm{weierstrassPInverse}\left(0,-\frac{4\,c}{d},x\right)+\left(d^2x^4+4\,cdx\right)\sqrt{dx^3+c}\right)}{81\left(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] -2/81*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassPInverse(0, -4*c/d,
 x) + (d^2*x^4 + 4*c*d*x)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c
^3*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^6}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**6/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(x^6/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.456 $\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

```
[Out] 1/256*x^4*AppellF1(4/3,3/2,2,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^
3/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(256*c^3*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 242 vs. 2(66) = 132.
time = 10.19, size = 242, normalized size = 3.67

$$\frac{x\left(\frac{3x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{192\left(\frac{-5c+dx^3}{c^2}+\frac{160F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}\right)}{d(8c-dx^3)}\right)}{15552\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*((3*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)])/c^3 + (192*((-5*c + d*x^3)/c^2 + (160*AppellF1[1/3, 1/2, 1, 4/
3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)
/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))
/(d*(8*c - d*x^3))))/(15552*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.37, size = 1479, normalized size = 22.41

method	result

elliptic	$-\frac{2x}{243 d\, c^{2} \sqrt{\left(x^{3}+\frac{c}{d}\right) d}}+\frac{x \sqrt{d\, x^{3}+c}}{243 c^{2} d \left(-d\, x^{3}+8 c\right)}+\frac{i \sqrt{3}\, \left(-c\, d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}-\frac{i \sqrt{3}\, \left(-c\, d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\, d}{\left(-c\, d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{\cdots}{-\frac{3(\cdots}{}}}}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/27*x/c^2/((x^3+c/d)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2
*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*
d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)+8*c/d*(2/243*x/c^3/((x^3+c/d)*d)^(1/2)+1/1944*x/c^3*(d*x^3+c)^(1/2)/(-d*x
^3+8*c)-5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d
^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-
```

```
I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2
*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d
*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*
d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*
d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c
*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2758 vs. $2(52) = 104$.
time = 6.46, size = 2758, normalized size = 41.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3888*(4*sqrt(3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^13*d^8))^
(1/6)*arctan(1/9*((9*sqrt(3)*c^11*d^8*x^5*(1/(c^13*d^8))^(5/6) + 3*sqrt(3)*
(5*c^7*d^5*x^4 + 8*c^8*d^4*x)*sqrt(1/(c^13*d^8)) - sqrt(3)*(c^2*d^3*x^6 - 4
0*c^3*d^2*x^3 - 32*c^4*d)*(1/(c^13*d^8))^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(
3)*(c^9*d^7*x^6 - c^10*d^6*x^3 - 2*c^11*d^5)*(1/(c^13*d^8))^(2/3) + 18*sqrt
(3)*(c^5*d^4*x^5 + c^6*d^3*x^2)*(1/(c^13*d^8))^(1/3) + 3*sqrt(3)*(d^2*x^7 +
 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(9*sqrt(3)*(c^11*d^8*x^5 + 2*c^12*d
^7*x^2)*(1/(c^13*d^8))^(5/6) + 3*sqrt(3)*(7*c^7*d^5*x^4 + 4*c^8*d^4*x)*sqrt
(1/(c^13*d^8)) + sqrt(3)*(c^2*d^3*x^6 + 32*c^3*d^2*x^3 + 40*c^4*d)*(1/(c^13
*d^8))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 +
18*(c^9*d^8*x^8 + 20*c^10*d^7*x^5 - 8*c^11*d^6*x^2)*(1/(c^13*d^8))^(2/3) +
```

6*sqrt(d*x^3 + c)*((c^11*d^9*x^7 - 28*c^12*d^8*x^4 - 272*c^13*d^7*x)*(1/(c^13*d^8))^(5/6) + 4*(c^7*d^6*x^6 + 41*c^8*d^5*x^3 + 40*c^9*d^4)*sqrt(1/(c^13*d^8)) - 24*(c^3*d^3*x^5 + c^4*d^2*x^2)*(1/(c^13*d^8))^(1/6)) - 18*(c^5*d^5*x^7 - 52*c^6*d^4*x^4 - 80*c^7*d^3*x)*(1/(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 4*sqrt(3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^13*d^8))^(1/6)*arctan(1/9*((9*sqrt(3)*c^11*d^8*x^5*(1/(c^13*d^8))^(5/6) + 3*sqrt(3)*(5*c^7*d^5*x^4 + 8*c^8*d^4*x)*sqrt(1/(c^13*d^8)) - sqrt(3)*(c^2*d^3*x^6 - 40*c^3*d^2*x^3 - 32*c^4*d)*(1/(c^13*d^8))^(1/6))*sqrt(d*x^3 + c) + (12*sqrt(3)*(c^9*d^7*x^6 - c^10*d^6*x^3 - 2*c^11*d^5)*(1/(c^13*d^8))^(2/3) + 18*sqrt(3)*(c^5*d^4*x^5 + c^6*d^3*x^2)*(1/(c^13*d^8))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^11*d^8*x^5 + 2*c^12*d^7*x^2)*(1/(c^13*d^8))^(5/6) + 3*sqrt(3)*(7*c^7*d^5*x^4 + 4*c^8*d^4*x)*sqrt(1/(c^13*d^8)) + sqrt(3)*(c^2*d^3*x^6 + 32*c^3*d^2*x^3 + 40*c^4*d)*(1/(c^13*d^8))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^9*d^8*x^8 + 20*c^10*d^7*x^5 - 8*c^11*d^6*x^2)*(1/(c^13*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c^11*d^9*x^7 - 28*c^12*d^8*x^4 - 272*c^13*d^7*x)*(1/(c^13*d^8))^(5/6) + 4*(c^7*d^6*x^6 + 41*c^8*d^5*x^3 + 40*c^9*d^4)*sqrt(1/(c^13*d^8)) - 24*(c^3*d^3*x^5 + c^4*d^2*x^2)*(1/(c^13*d^8))^(1/6)) - 18*(c^5*d^5*x^7 - 52*c^6*d^4*x^4 - 80*c^7*d^3*x)*(1/(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 24*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) + 2*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d^6*x^2)*(1/(c^13*d^8))^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^13*d^7*x)*(1/(c^13*d^8))^(5/6) + (7*c^7*d^6*x^6 + 152*c^8*d^5*x^3 + 64*c^9*d^4)*sqrt(1/(c^13*d^8)) + 6*(5*c^3*d^3*x^5 + 32*c^4*d^2*x^2)*(1/(c^13*d^8))^(1/6)) + 18*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4 + 32*c^7*d^3*x)*(1/(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d^6*x^2)*(1/(c^13*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^13*d^7*x)*(1/(c^13*d^8))^(5/6) + (7*c^7*d^6*x^6 + 152*c^8*d^5*x^3 + 64*c^9*d^4)*sqrt(1/(c^13*d^8)) + 6*(5*c^3*d^3*x^5 + 32*c^4*d^2*x^2)*(1/(c^13*d^8))^(1/6)) + 18*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4 + 32*c^7*d^3*x)*(1/(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^9*d^8*x^8 + 20*c^10*d^7*x^5 - 8*c^11*d^6*x^2)*(1/(c^13*d^8))^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^9*x^7 - 28*c^12*d^8*x^4 - 272*c^13*d^7*x)*(1/(c^13*d^8))^(5/6) + 4*(c^7*d^6*x^6 + 41*c^8*d^5*x^3 + 40*c^9*d^4)*sqrt(1/(c^13*d^8)) - 24*(c^3*d^3*x^5 + c^4*d^2*x^2)*(1/(c^13*d^8))^(1/6)) - 18*(c^5*d^5*x^7 - 52*c^6*d^4*x^4 - 80*c^7*d^3*x)*(1/(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3

```
- 1088*c^3 + 18*(c^9*d^8*x^8 + 20*c^10*d^7*x^5 - 8*c^11*d^6*x^2)*(1/(c^13*d
^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c^11*d^9*x^7 - 28*c^12*d^8*x^4 - 272*c^13*
d^7*x)*(1/(c^13*d^8))^(5/6) + 4*(c^7*d^6*x^6 + 41*c^8*d^5*x^3 + 40*c^9*d^4)
*sqrt(1/(c^13*d^8)) - 24*(c^3*d^3*x^5 + c^4*d^2*x^2)*(1/(c^13*d^8))^(1/6))
- 18*(c^5*d^5*x^7 - 52*c^6*d^4*x^4 - 80*c^7*d^3*x)*(1/(c^13*d^8))^(1/3))/(d
^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 48*(d^2*x^4 - 5*c*d*x)*
sqrt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**3/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.457 $\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

```
[Out] 1/64*x*AppellF1(1/3,3/2,2,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^3/(
d*x^3+c)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(64*c^3*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 253 vs. 2(64) = 128.
time = 10.17, size = 253, normalized size = 3.95

$$\frac{x\left(-15dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+192c\left(\frac{-43c+5dx^3}{-8c+dx^3}+\frac{1216c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)\right)}\right)\right)}{124416c^4\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-15*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
(d*x^3)/(8*c)] + 192*c*((-43*c + 5*d*x^3)/(-8*c + d*x^3) + (1216*c^2*Appell
F1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/
3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))))))/(124416*c^4*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.32, size = 748, normalized size = 11.69 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/243*x/c^3/((x^3+c/d)*d)^(1/2)+1/1944*x/c^3*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-5
/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*
```

```
d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3
)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d
^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2681 vs. $2(50) = 100$.
time = 6.99, size = 2681, normalized size = 41.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15552*(4*sqrt(3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)*(1/(c^19*d^2))^(
1/6)*arctan(1/9*((9*sqrt(3)*c^16*d^3*x^5*(1/(c^19*d^2))^(5/6) + 3*sqrt(3)*(
5*c^10*d^2*x^4 + 8*c^11*d*x)*sqrt(1/(c^19*d^2)) - sqrt(3)*(c^3*d^2*x^6 - 40
*c^4*d*x^3 - 32*c^5)*(1/(c^19*d^2))^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c
^13*d^3*x^6 - c^14*d^2*x^3 - 2*c^15*d)*(1/(c^19*d^2))^(2/3) + 18*sqrt(3)*(c
^7*d^2*x^5 + c^8*d*x^2)*(1/(c^19*d^2))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x
^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(9*sqrt(3)*(c^16*d^3*x^5 + 2*c^17*d^2*x^2)*
(1/(c^19*d^2))^(5/6) + 3*sqrt(3)*(7*c^10*d^2*x^4 + 4*c^11*d*x)*sqrt(1/(c^19
*d^2)) + sqrt(3)*(c^3*d^2*x^6 + 32*c^4*d*x^3 + 40*c^5)*(1/(c^19*d^2))^(1/6)
))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^13*d^4
*x^8 + 20*c^14*d^3*x^5 - 8*c^15*d^2*x^2)*(1/(c^19*d^2))^(2/3) + 6*sqrt(d*x^
3 + c)*((c^16*d^4*x^7 - 28*c^17*d^3*x^4 - 272*c^18*d^2*x)*(1/(c^19*d^2))^(5
/6) + 4*(c^10*d^3*x^6 + 41*c^11*d^2*x^3 + 40*c^12*d)*sqrt(1/(c^19*d^2)) - 2
4*(c^4*d^2*x^5 + c^5*d*x^2)*(1/(c^19*d^2))^(1/6)) - 18*(c^7*d^3*x^7 - 52*c^
8*d^2*x^4 - 80*c^9*d*x)*(1/(c^19*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192
*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 4*sqrt(3)*(c^3*d
^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)*(1/(c^19*d^2))^(1/6)*arctan(1/9*((9*sqrt(
3)*c^16*d^3*x^5*(1/(c^19*d^2))^(5/6) + 3*sqrt(3)*(5*c^10*d^2*x^4 + 8*c^11*d
```

```
*x)*sqrt(1/(c^19*d^2)) - sqrt(3)*(c^3*d^2*x^6 - 40*c^4*d*x^3 - 32*c^5)*(1/(
c^19*d^2))^(1/6))*sqrt(d*x^3 + c) + (12*sqrt(3)*(c^13*d^3*x^6 - c^14*d^2*x^
3 - 2*c^15*d)*(1/(c^19*d^2))^(2/3) + 18*sqrt(3)*(c^7*d^2*x^5 + c^8*d*x^2)*(
1/(c^19*d^2))^(1/3) + 3*sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^
3 + c)*(9*sqrt(3)*(c^16*d^3*x^5 + 2*c^17*d^2*x^2)*(1/(c^19*d^2))^(5/6) + 3*
sqrt(3)*(7*c^10*d^2*x^4 + 4*c^11*d*x)*sqrt(1/(c^19*d^2)) + sqrt(3)*(c^3*d^2
*x^6 + 32*c^4*d*x^3 + 40*c^5)*(1/(c^19*d^2))^(1/6)))*sqrt((d^3*x^9 - 276*c*
d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^13*d^4*x^8 + 20*c^14*d^3*x^5 -
8*c^15*d^2*x^2)*(1/(c^19*d^2))^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^4*x^7 - 2
8*c^17*d^3*x^4 - 272*c^18*d^2*x)*(1/(c^19*d^2))^(5/6) + 4*(c^10*d^3*x^6 + 4
1*c^11*d^2*x^3 + 40*c^12*d)*sqrt(1/(c^19*d^2)) - 24*(c^4*d^2*x^5 + c^5*d*x^
2)*(1/(c^19*d^2))^(1/6)) - 18*(c^7*d^3*x^7 - 52*c^8*d^2*x^4 - 80*c^9*d*x)*(
1/(c^19*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(
d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 192*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)
*weierstrassPInverse(0, -4*c/d, x) + 2*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5
*d)*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 64
0*c^3 + 18*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2)*(1/(c^19*d^2)
)^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d^2
*x)*(1/(c^19*d^2))^(5/6) + (7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*d)*
sqrt(1/(c^19*d^2)) + 6*(5*c^4*d^2*x^5 + 32*c^5*d*x^2)*(1/(c^19*d^2))^(1/6))
 + 18*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c^9*d*x)*(1/(c^19*d^2))^(1/3))/(
d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^3*d^3*x^6 - 7*c^4
*d^2*x^3 - 8*c^5*d)*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 120
0*c^2*d*x^3 + 640*c^3 + 18*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^
2)*(1/(c^19*d^2))^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^4*x^7 + 80*c^17*d^3*x^
4 + 160*c^18*d^2*x)*(1/(c^19*d^2))^(5/6) + (7*c^10*d^3*x^6 + 152*c^11*d^2*x
^3 + 64*c^12*d)*sqrt(1/(c^19*d^2)) + 6*(5*c^4*d^2*x^5 + 32*c^5*d*x^2)*(1/(c
^19*d^2))^(1/6)) + 18*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c^9*d*x)*(1/(c^1
9*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^3*d
^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 - 276*c
*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^13*d^4*x^8 + 20*c^14*d^3*x^5 -
 8*c^15*d^2*x^2)*(1/(c^19*d^2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^4*x^7 -
28*c^17*d^3*x^4 - 272*c^18*d^2*x)*(1/(c^19*d^2))^(5/6) + 4*(c^10*d^3*x^6 +
41*c^11*d^2*x^3 + 40*c^12*d)*sqrt(1/(c^19*d^2)) - 24*(c^4*d^2*x^5 + c^5*d*x
^2)*(1/(c^19*d^2))^(1/6)) - 18*(c^7*d^3*x^7 - 52*c^8*d^2*x^4 - 80*c^9*d*x)*
(1/(c^19*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
 (c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)*(1/(c^19*d^2))^(1/6)*log((d^3*x^9
- 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 + 18*(c^13*d^4*x^8 + 20*c^14*d^
3*x^5 - 8*c^15*d^2*x^2)*(1/(c^19*d^2))^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^4
*x^7 - 28*c^17*d^3*x^4 - 272*c^18*d^2*x)*(1/(c^19*d^2))^(5/6) + 4*(c^10*d^3
*x^6 + 41*c^11*d^2*x^3 + 40*c^12*d)*sqrt(1/(c^19*d^2)) - 24*(c^4*d^2*x^5 +
c^5*d*x^2)*(1/(c^19*d^2))^(1/6)) - 18*(c^7*d^3*x^7 - 52*c^8*d^2*x^4 - 80*c^
9*d*x)*(1/(c^19*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)) + 24*(5*d^2*x^4 - 43*c*d*x)*sqrt(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2
*x^3 - 8*c^5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-8c + dx^3\right)^2 \left(c + dx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.458 $\int \frac{1}{x^3(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

```
[Out] -1/128*AppellF1(-2/3,3/2,2,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^3/
x^2/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/128*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c^3*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{1}{x^3(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\; F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(66) = 132.
time = 10.15, size = 259, normalized size = 3.92

$$\frac{167d^2x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+\frac{64c\left(-648c^2-1249cdx^3+167d^2x^6-\frac{19648c^2dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}\right)}{8c-dx^3}}{663552c^5x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (167*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + (64*c*(-648*c^2 - 1249*c*d*x^3 + 167*d^2*x^6 - (19648*c^2*d
*x^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF
1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1
/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((
d*x^3)/c), (d*x^3)/(8*c)]))))/(8*c - d*x^3))/(663552*c^5*x^2*Sqrt[c + d*x^3
])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.45, size = 1806, normalized size = 27.36

method	result

elliptic	$\frac{dx\sqrt{d\,x^3+c}}{15552c^4(-d\,x^3+8c)}-\frac{\sqrt{d\,x^3+c}}{128c^4x^2}-\frac{2dx}{243c^4\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{167i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)}{(-c\,d^2)^{\frac{1}{3}}}}}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d/c*(2/243*x/c^3/((x^3+c/d)*d)^(1/2)+1/1944*x/c^3*(d*x^3+c)^(1/2)/(-d*x
^3+8*c)-5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d
^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-
I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2
*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d
*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*
d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*
d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c
*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(-1/2/c^2*(d*x^3
+c)^(1/2)/x^2-2/3*d*x/c^2/((x^3+c/d)*d)^(1/2)+7/18*I/c^2*3^(1/2)*(-c*d^2)^(
1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
```

```
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/64*d/c^2*(-2/27*x/c^2/((x^3+
c/d)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(
-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(
1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha
-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2771 vs. $2(52) = 104$.
time = 10.37, size = 2771, normalized size = 41.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] 1/82944*(4*sqrt(3)*(c^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2)*(d^4/c^25)^(1/6)
*arctan(1/9*((9*sqrt(3)*c^21*d^4*x^5*(d^4/c^25)^(5/6) + 3*sqrt(3)*(5*c^13*d
```

^5*x^4 + 8*c^14*d^4*x)*sqrt(d^4/c^25) - sqrt(3)*(c^4*d^7*x^6 - 40*c^5*d^6*x
^3 - 32*c^6*d^5)*(d^4/c^25)^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c^17*d^2*
x^6 - c^18*d*x^3 - 2*c^19)*(d^4/c^25)^(2/3) + 18*sqrt(3)*(c^9*d^3*x^5 + c^1
0*d^2*x^2)*(d^4/c^25)^(1/3) + 3*sqrt(3)*(d^5*x^7 + 5*c*d^4*x^4 + 4*c^2*d^3*
x) - sqrt(d*x^3 + c)*(9*sqrt(3)*(c^21*d*x^5 + 2*c^22*x^2)*(d^4/c^25)^(5/6)
+ 3*sqrt(3)*(7*c^13*d^2*x^4 + 4*c^14*d*x)*sqrt(d^4/c^25) + sqrt(3)*(c^4*d^4
*x^6 + 32*c^5*d^3*x^3 + 40*c^6*d^2)*(d^4/c^25)^(1/6)))*sqrt((d^9*x^9 - 276*
c*d^8*x^6 - 1608*c^2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^17*d^6*x^8 + 20*c^18*d^
5*x^5 - 8*c^19*d^4*x^2)*(d^4/c^25)^(2/3) + 6*sqrt(d*x^3 + c)*((c^21*d^5*x^7
 - 28*c^22*d^4*x^4 - 272*c^23*d^3*x)*(d^4/c^25)^(5/6) + 4*(c^13*d^6*x^6 + 4
1*c^14*d^5*x^3 + 40*c^15*d^4)*sqrt(d^4/c^25) - 24*(c^5*d^7*x^5 + c^6*d^6*x^
2)*(d^4/c^25)^(1/6)) - 18*(c^9*d^7*x^7 - 52*c^10*d^6*x^4 - 80*c^11*d^5*x)*(
d^4/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^8*
x^7 - 7*c*d^7*x^4 - 8*c^2*d^6*x)) + 4*sqrt(3)*(c^4*d^2*x^8 - 7*c^5*d*x^5 -
8*c^6*x^2)*(d^4/c^25)^(1/6)*arctan(1/9*((9*sqrt(3)*c^21*d^4*x^5*(d^4/c^25)^
(5/6) + 3*sqrt(3)*(5*c^13*d^5*x^4 + 8*c^14*d^4*x)*sqrt(d^4/c^25) - sqrt(3)*
(c^4*d^7*x^6 - 40*c^5*d^6*x^3 - 32*c^6*d^5)*(d^4/c^25)^(1/6))*sqrt(d*x^3 +
c) + (12*sqrt(3)*(c^17*d^2*x^6 - c^18*d*x^3 - 2*c^19)*(d^4/c^25)^(2/3) + 18
sqrt(3)(c^9*d^3*x^5 + c^10*d^2*x^2)*(d^4/c^25)^(1/3) + 3*sqrt(3)*(d^5*x^7
 + 5*c*d^4*x^4 + 4*c^2*d^3*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^21*d*x^5 + 2*
c^22*x^2)*(d^4/c^25)^(5/6) + 3*sqrt(3)*(7*c^13*d^2*x^4 + 4*c^14*d*x)*sqrt(d
^4/c^25) + sqrt(3)*(c^4*d^4*x^6 + 32*c^5*d^3*x^3 + 40*c^6*d^2)*(d^4/c^25)^(
1/6)))*sqrt((d^9*x^9 - 276*c*d^8*x^6 - 1608*c^2*d^7*x^3 - 1088*c^3*d^6 + 18
*(c^17*d^6*x^8 + 20*c^18*d^5*x^5 - 8*c^19*d^4*x^2)*(d^4/c^25)^(2/3) - 6*sqr
t(d*x^3 + c)*((c^21*d^5*x^7 - 28*c^22*d^4*x^4 - 272*c^23*d^3*x)*(d^4/c^25)^
(5/6) + 4*(c^13*d^6*x^6 + 41*c^14*d^5*x^3 + 40*c^15*d^4)*sqrt(d^4/c^25) - 2
4*(c^5*d^7*x^5 + c^6*d^6*x^2)*(d^4/c^25)^(1/6)) - 18*(c^9*d^7*x^7 - 52*c^10
*d^6*x^4 - 80*c^11*d^5*x)*(d^4/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c
^2*d*x^3 - 512*c^3)))/(d^8*x^7 - 7*c*d^7*x^4 - 8*c^2*d^6*x)) - 1264*(d^2*x^
8 - 7*c*d*x^5 - 8*c^2*x^2)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) + (c^4
*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2)*(d^4/c^25)^(1/6)*log((d^9*x^9 - 276*c*d
^8*x^6 - 1608*c^2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^17*d^6*x^8 + 20*c^18*d^5*x
^5 - 8*c^19*d^4*x^2)*(d^4/c^25)^(2/3) + 6*sqrt(d*x^3 + c)*((c^21*d^5*x^7 -
28*c^22*d^4*x^4 - 272*c^23*d^3*x)*(d^4/c^25)^(5/6) + 4*(c^13*d^6*x^6 + 41*c
^14*d^5*x^3 + 40*c^15*d^4)*sqrt(d^4/c^25) - 24*(c^5*d^7*x^5 + c^6*d^6*x^2)*
(d^4/c^25)^(1/6)) - 18*(c^9*d^7*x^7 - 52*c^10*d^6*x^4 - 80*c^11*d^5*x)*(d^4
/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^4*d^
2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2)*(d^4/c^25)^(1/6)*log((d^9*x^9 - 276*c*d^8*
x^6 - 1608*c^2*d^7*x^3 - 1088*c^3*d^6 + 18*(c^17*d^6*x^8 + 20*c^18*d^5*x^5
- 8*c^19*d^4*x^2)*(d^4/c^25)^(2/3) - 6*sqrt(d*x^3 + c)*((c^21*d^5*x^7 - 28*
c^22*d^4*x^4 - 272*c^23*d^3*x)*(d^4/c^25)^(5/6) + 4*(c^13*d^6*x^6 + 41*c^14
*d^5*x^3 + 40*c^15*d^4)*sqrt(d^4/c^25) - 24*(c^5*d^7*x^5 + c^6*d^6*x^2)*(d^
4/c^25)^(1/6)) - 18*(c^9*d^7*x^7 - 52*c^10*d^6*x^4 - 80*c^11*d^5*x)*(d^4/c^
25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^4*d^2
*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2)*(d^4/c^25)^(1/6)*log((d^6*x^9 + 318*c*d^5*x

```
^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + 18*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 +
64*c^19*d*x^2)*(d^4/c^25)^(2/3) + 6*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^2
2*d*x^4 + 160*c^23*x)*(d^4/c^25)^(5/6) + (7*c^13*d^3*x^6 + 152*c^14*d^2*x^3
 + 64*c^15*d)*sqrt(d^4/c^25) + 6*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2)*(d^4/c^25
)^(1/6)) + 18*(5*c^9*d^4*x^7 + 64*c^10*d^3*x^4 + 32*c^11*d^2*x)*(d^4/c^25)^
(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^4*d^2*x^8
 - 7*c^5*d*x^5 - 8*c^6*x^2)*(d^4/c^25)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 +
 1200*c^2*d^4*x^3 + 640*c^3*d^3 + 18*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c
^19*d*x^2)*(d^4/c^25)^(2/3) - 6*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*
x^4 + 160*c^23*x)*(d^4/c^25)^(5/6) + (7*c^13*d^3*x^6 + 152*c^14*d^2*x^3 + 6
4*c^15*d)*sqrt(d^4/c^25) + 6*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2)*(d^4/c^25)^(1
/6)) + 18*(5*c^9*d^4*x^7 + 64*c^10*d^3*x^4 + 32*c^11*d^2*x)*(d^4/c^25)^(1/3
))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 8*(167*d^2*x^6 - 1
249*c*d*x^3 - 648*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*
x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(-8c + dx^3\right)^2 \left(c + dx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**3*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.459 $\int \frac{1}{x^6(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{5}{3};2,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

```
[Out] -1/320*AppellF1(-5/3,3/2,2,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^3
/x^5/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};2,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/320*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 3/2, -2/3, (d*x^3)/(8*c), -((
d*x^3)/c)])/(c^3*x^5*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{5}{3};2,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 283 vs. 2(66) = 132.
time = 10.14, size = 283, normalized size = 4.29

$$\frac{\frac{64(2592c^3-7128c^2dx^3-15373cd^2x^6+2027d^3x^9)}{c^5x^5(-8c+dx^3)}-\frac{2027d^3x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^6}+\frac{16789504d^2xF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3(8c-dx^3)\left(32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)\right)}}{6635520\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((64*(2592*c^3 - 7128*c^2*d*x^3 - 15373*c*d^2*x^6 + 2027*d^3*x^9))/(c^5*x^5
*(-8*c + d*x^3)) - (2027*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^6 + (16789504*d^2*x*AppellF1[1/3, 1/2,
 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(c^3*(8*c - d*x^3)*(32*c*AppellF1[1/
3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2,
2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^
3)/c), (d*x^3)/(8*c)]))))/(6635520*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.46, size = 2157, normalized size = 32.68

method	result

	$2027 i d \sqrt{3}\, (-c\, d^2)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{(}{}\right.}{}}$
elliptic	$-\frac{\sqrt{d\, x^3+c}}{320 c^4 x^5}+\frac{29 d \sqrt{d\, x^3+c}}{2560 c^5 x^2}+\frac{2 d^2 x}{243 c^5 \sqrt{\left(x^3+\frac{c}{d}\right) d}}+\frac{d^2 x \sqrt{d\, x^3+c}}{124416 c^5 (-d\, x^3+8 c)}-$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/c^2*(-1/5/c^2*(d*x^3+c)^(1/2)/x^5+17/20/c^3*d*(d*x^3+c)^(1/2)/x^2+2/3*
d^2/c^3*x/((x^3+c/d)*d)^(1/2)-91/180*I/c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/256/c^3*d*(-1/2/c^2*(d*x^3+c)^(1/2)/x^2-
2/3*d*x/c^2/((x^3+c/d)*d)^(1/2)+7/18*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/256/c^3*d^2*(-2/27*x/c^2/((x^3+c/d)*d)^(1/
2)+2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/
3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
```

```
)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d
*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*d^2*(2/243*x/c^3/((x^3+
c/d)*d)^(1/2)+1/1944*x/c^3*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-5/1944*I/c^3*3^(1/2
)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/972*I/c^3/d^3*2
^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*d-8*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2804 vs. $2(52) = 104$.

time = 17.17, size = 2804, normalized size = 42.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] 1/2488320*(20*sqrt(3)*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5)*(d^10/c^31)^(1/6)*arctan(1/9*((9*sqrt(3)*c^26*d^9*x^5*(d^10/c^31)^(5/6) + 3*sqrt(3)*(5*c^16*d^12*x^4 + 8*c^17*d^11*x)*sqrt(d^10/c^31) - sqrt(3)*(c^5*d^16*x^6 - 40*c^6*d^15*x^3 - 32*c^7*d^14)*(d^10/c^31)^(1/6))*sqrt(d*x^3 + c) - (12*sqrt(3)*(c^21*d^3*x^6 - c^22*d^2*x^3 - 2*c^23*d)*(d^10/c^31)^(2/3) + 18*sqrt(3)*(c^11*d^6*x^5 + c^12*d^5*x^2)*(d^10/c^31)^(1/3) + 3*sqrt(3)*(d^10*x^7 + 5*c*d^9*x^4 + 4*c^2*d^8*x) - sqrt(d*x^3 + c)*(9*sqrt(3)*(c^26*d*x^5 + 2*c^27*x^2)*(d^10/c^31)^(5/6) + 3*sqrt(3)*(7*c^16*d^4*x^4 + 4*c^17*d^3*x)*sqrt(d^10/c^31) + sqrt(3)*(c^5*d^8*x^6 + 32*c^6*d^7*x^3 + 40*c^7*d^6)*(d^10/c^31)^(1/6)))*sqrt((d^19*x^9 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^21*d^12*x^8 + 20*c^22*d^11*x^5 - 8*c^23*d^10*x^2)*(d^10/c^31)^(2/3) + 6*sqrt(d*x^3 + c)*((c^26*d^10*x^7 - 28*c^27*d^9*x^4 - 272*c^28*d^8*x)*(d^10/c^31)^(5/6) + 4*(c^16*d^13*x^6 + 41*c^17*d^12*x^3 + 40*c^18*d^11)*sqrt(d^10/c^31) - 24*(c^6*d^16*x^5 + c^7*d^15*x^2)*(d^10/c^31)^(1/6)) - 18*(c^11*d^15*x^7 - 52*c^12*d^14*x^4 - 80*c^13*d^13*x)*(d^10/c^31)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^18*x^7 - 7*c*d^17*x^4 - 8*c^2*d^16*x)) + 20*sqrt(3)*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5)*(d^10/c^31)^(1/6)*arctan(1/9*((9*sqrt(3)*c^26*d^9*x^5*(d^10/c^31)^(5/6) + 3*sqrt(3)*(5*c^16*d^12*x^4 + 8*c^17*d^11*x)*sqrt(d^10/c^31) - sqrt(3)*(c^5*d^16*x^6 - 40*c^6*d^15*x^3 - 32*c^7*d^14)*(d^10/c^31)^(1/6))*sqrt(d*x^3 + c) + (12*sqrt(3)*(c^21*d^3*x^6 - c^22*d^2*x^3 - 2*c^23*d)*(d^10/c^31)^(2/3) + 18*sqrt(3)*(c^11*d^6*x^5 + c^12*d^5*x^2)*(d^10/c^31)^(1/3) + 3*sqrt(3)*(d^10*x^7 + 5*c*d^9*x^4 + 4*c^2*d^8*x) + sqrt(d*x^3 + c)*(9*sqrt(3)*(c^26*d*x^5 + 2*c^27*x^2)*(d^10/c^31)^(5/6) + 3*sqrt(3)*(7*c^16*d^4*x^4 + 4*c^17*d^3*x)*sqrt(d^10/c^31) + sqrt(3)*(c^5*d^8*x^6 + 32*c^6*d^7*x^3 + 40*c^7*d^6)*(d^10/c^31)^(1/6)))*sqrt((d^19*x^9 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^21*d^12*x^8 + 20*c^22*d^11*x^5 - 8*c^23*d^10*x^2)*(d^10/c^31)^(2/3) - 6*sqrt(d*x^3 + c)*((c^26*d^10*x^7 - 28*c^27*d^9*x^4 - 272*c^28*d^8*x)*(d^10/c^31)^(5/6) + 4*(c^16*d^13*x^6 + 41*c^17*d^12*x^3 + 40*c^18*d^11)*sqrt(d^10/c^31) - 24*(c^6*d^16*x^5 + c^7*d^15*x^2)*(d^10/c^31)^(1/6)) - 18*(c^11*d^15*x^7 - 52*c^12*d^14*x^4 - 80*c^13*d^13*x)*(d^10/c^31)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^18*x^7 - 7*c*d^17*x^4 - 8*c^2*d^16*x)) + 49008*(d^3*x^11 - 7*c*d^2*x^8 - 8*c^2*d*x^5)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) + 5*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5)*(d^10/c^31)^(1/6)*log((d^19*x^9 - 276*c*d^18*x^6 - 1608*c^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^21*d^12*x^8 + 20*c^22*d^11*x^5 - 8*c^23*d^10*x^2)*(d^10/c^31)^(2/3) + 6*sqrt(d*x^3 + c)*((c^26*d^10*x^7 - 28*c^27*d^9*x^4 - 272*c^28*d^8*x)*(d^10/c^31)^(5/6) + 4*(c^16*d^13*x^6 + 41*c^17*d^12*x^3 + 40*c^18*d^11)*

```
sqrt(d^10/c^31) - 24*(c^6*d^16*x^5 + c^7*d^15*x^2)*(d^10/c^31)^(1/6)) - 18*
(c^11*d^15*x^7 - 52*c^12*d^14*x^4 - 80*c^13*d^13*x)*(d^10/c^31)^(1/3))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(c^5*d^2*x^11 - 7*c^6*d
*x^8 - 8*c^7*x^5)*(d^10/c^31)^(1/6)*log((d^19*x^9 - 276*c*d^18*x^6 - 1608*c
^2*d^17*x^3 - 1088*c^3*d^16 + 18*(c^21*d^12*x^8 + 20*c^22*d^11*x^5 - 8*c^23
*d^10*x^2)*(d^10/c^31)^(2/3) - 6*sqrt(d*x^3 + c)*((c^26*d^10*x^7 - 28*c^27*
d^9*x^4 - 272*c^28*d^8*x)*(d^10/c^31)^(5/6) + 4*(c^16*d^13*x^6 + 41*c^17*d^
12*x^3 + 40*c^18*d^11)*sqrt(d^10/c^31) - 24*(c^6*d^16*x^5 + c^7*d^15*x^2)*(
d^10/c^31)^(1/6)) - 18*(c^11*d^15*x^7 - 52*c^12*d^14*x^4 - 80*c^13*d^13*x)*
(d^10/c^31)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 10
*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5)*(d^10/c^31)^(1/6)*log((d^11*x^9 +
 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + 18*(c^21*d^4*x^8 + 38*c^
22*d^3*x^5 + 64*c^23*d^2*x^2)*(d^10/c^31)^(2/3) + 6*sqrt(d*x^3 + c)*((c^26*
d^2*x^7 + 80*c^27*d*x^4 + 160*c^28*x)*(d^10/c^31)^(5/6) + (7*c^16*d^5*x^6 +
 152*c^17*d^4*x^3 + 64*c^18*d^3)*sqrt(d^10/c^31) + 6*(5*c^6*d^8*x^5 + 32*c^
7*d^7*x^2)*(d^10/c^31)^(1/6)) + 18*(5*c^11*d^7*x^7 + 64*c^12*d^6*x^4 + 32*c
^13*d^5*x)*(d^10/c^31)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512
*c^3)) - 10*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5)*(d^10/c^31)^(1/6)*log(
(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + 18*(c^21*d^4*
x^8 + 38*c^22*d^3*x^5 + 64*c^23*d^2*x^2)*(d^10/c^31)^(2/3) - 6*sqrt(d*x^3 +
 c)*((c^26*d^2*x^7 + 80*c^27*d*x^4 + 160*c^28*x)*(d^10/c^31)^(5/6) + (7*c^1
6*d^5*x^6 + 152*c^17*d^4*x^3 + 64*c^18*d^3)*sqrt(d^10/c^31) + 6*(5*c^6*d^8*
x^5 + 32*c^7*d^7*x^2)*(d^10/c^31)^(1/6)) + 18*(5*c^11*d^7*x^7 + 64*c^12*d^6
*x^4 + 32*c^13*d^5*x)*(d^10/c^31)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)) + 24*(2027*d^3*x^9 - 15373*c*d^2*x^6 - 7128*c^2*d*x^3 + 2
592*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \left(-8c + dx^3\right)^2 \left(c + dx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**6*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^6\,{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.460 $\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)}+\frac{2\left(c+dx^3\right)^{3/2}}{9b^2d}-\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)}+\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b^2/d-1/3*a^2*(d*x^3+c)^(3/2)/b^2/(-a*d+b*c)/(b*x^3+a)+
1/3*a*(-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7
/2)/(-a*d+b*c)^(1/2)-1/3*a*(-5*a*d+4*b*c)*(d*x^3+c)^(1/2)/b^3/(-a*d+b*c)
```

Rubi [A]
time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 91, 81, 52, 65, 214}

$$-\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2\left(a+bx^3\right)(bc-ad)}+\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}-\frac{a\sqrt{c+dx^3}(4bc-5ad)}{3b^3(bc-ad)}+\frac{2\left(c+dx^3\right)^{3/2}}{9b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(a*(4*b*c - 5*a*d)*Sqrt[c + d*x^3])/(b^3*(b*c - a*d)) + (2*(c + d*x^3)
^(3/2))/(9*b^2*d) - (a^2*(c + d*x^3)^(3/2))/(3*b^2*(b*c - a*d)*(a + b*x^3))
 + (a*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(
3*b^(7/2)*Sqrt[b*c - a*d])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{a^2(c+dx^3)^{3/2}}{3b^2(bc-ad)(a+bx^3)} + \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\,\left(-\frac{1}{2}a(2bc-3ad)+b(bc-ad)x\right)}{a+bx}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2(c+dx^3)^{3/2}}{9b^2d} - \frac{a^2(c+dx^3)^{3/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-5ad))\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right.}{6b^2(bc-ad)}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2(c+dx^3)^{3/2}}{9b^2d} - \frac{a^2(c+dx^3)^{3/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-5ad))}{}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2(c+dx^3)^{3/2}}{9b^2d} - \frac{a^2(c+dx^3)^{3/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-5ad))}{}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2(c+dx^3)^{3/2}}{9b^2d} - \frac{a^2(c+dx^3)^{3/2}}{3b^2(bc-ad)(a+bx^3)} + \frac{a(4bc-5ad)\,\text{t}}{3b}$$

Mathematica [A]
time = 0.26, size = 126, normalized size = 0.78

$$\frac{\sqrt{c+dx^3}\,(-15a^2d+2ab(c-5dx^3)+2b^2x^3(c+dx^3))}{9b^3d\,(a+bx^3)} + \frac{a(-4bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(-15*a^2*d + 2*a*b*(c - 5*d*x^3) + 2*b^2*x^3*(c + d*x^3)))
/(9*b^3*d*(a + b*x^3)) + (a*(-4*b*c + 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2)*Sqrt[-(b*c) + a*d])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 917, normalized size = 5.70

method	result

elliptic	$-\frac{a^2\sqrt{dx^3+c}}{3b^3(bx^3+a)}+\frac{2x^3\sqrt{dx^3+c}}{9b^2}+\frac{2\left(-\frac{2ad-bc}{b^3}-\frac{2c}{3b^2}\right)\sqrt{dx^3+c}}{3d}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(5ad-4bc)\left(-c\,d^2\right.}{}\right.}{}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/b^2/d-2*a/b^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2
)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_a
lpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf
(_Z^3*b+a))+a^2/b^2*(-1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)-1/6*I/b/d*2^(1/2)*su
m(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b
/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alph
```

```
a=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.32, size = 469, normalized size = 2.91

$$\left[-\frac{3\,(4\,a^2bcd-5\,a^3d^2+(4\,ab^2cd-5\,a^2bd^2)x^3)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(2\,(b^4cd-ab^3d^2)x^6+2\,ab^3c^2-17\,a^2b^2cd+15\,a^3bd^2+2\,(b^4c^2-6\,ab^3cd+5\,a^2b^2d^2)x^3)\sqrt{dx^3+c}}{18\,(ab^5cd-a^2b^4d^2+(b^6cd-ab^5d^2)x^3)},-\frac{3\,(4\,a^2bcd-5\,a^3d^2+(4\,ab^2cd-5\,a^2bd^2)x^3)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(2\,(b^4cd-ab^3d^2)x^6+2\,ab^3c^2-17\,a^2b^2cd+15\,a^3bd^2+2\,(b^4c^2-6\,ab^3cd+5\,a^2b^2d^2)x^3)\sqrt{dx^3+c}}{9\,(ab^5cd-a^2b^4d^2+(b^6cd-ab^5d^2)x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(
b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c -
a*b*d))/(b*x^3 + a)) - 2*(2*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^
2*b^2*c*d + 15*a^3*b*d^2 + 2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*s
qrt(d*x^3 + c))/(a*b^5*c*d - a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3), -1/9
*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2
*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^2*b^2*c*d + 15*a^3*b*d^2 +
2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c*d
- a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**8*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Giac [A]
time = 1.21, size = 136, normalized size = 0.84

$$-\frac{\sqrt{dx^3+c}\,a^2d}{3\left((dx^3+c)b-bc+ad\right)b^3}-\frac{(4\,abc-5\,a^2d)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^3}+\frac{2\left((dx^3+c)^{\frac{3}{2}}b^4d^2-6\,\sqrt{dx^3+c}\,ab^3d^3\right)}{9\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d*x^3 + c)*a^2*d/((((d*x^3 + c)*b - b*c + a*d)*b^3) - 1/3*(4*a*b*c
 - 5*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b^3) + 2/9*((d*x^3 + c)^(3/2)*b^4*d^2 - 6*sqrt(d*x^3 + c)*a*b^3*d^3)/
(b^6*d^3)
```

Mupad [B]
time = 6.84, size = 202, normalized size = 1.25

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2}-\frac{\sqrt{d\,x^3+c}\,\left(\frac{4\,c}{3\,b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{b^4}+\frac{2\,a\,d}{b^3}\right)}{3\,d}+\frac{a^2\,\left(\frac{2\,a\,d}{3\,(2\,b^2\,c-2\,a\,b\,d)}-\frac{2\,b\,c}{3\,(2\,b^2\,c-2\,a\,b\,d)}\right)\,\sqrt{d\,x^3+c}}{b^2\,\left(b\,x^3+a\right)}+\frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,(5\,a\,d-4\,b\,c)\,1{}\mathrm{i}}{6\,b^{7/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((4*c)/(3*b^2) - (2*
b^2*c - 2*a*b*d)/b^4 + (2*a*d)/b^3))/(3*d) + (a*log((2*b*c - a*d + b^(1/2)*
(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b
*c)*1i)/(6*b^(7/2)*(a*d - b*c)^(1/2)) + (a^2*((2*a*d)/(3*(2*b^2*c - 2*a*b*d
)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b^2*(a + b*x^3))
```

3.461 $\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=136

$$\frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)}+\frac{a(c+dx^3)^{3/2}}{3b(bc-ad)\,(a+bx^3)}-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/3*a*(d*x^3+c)^(3/2)/b/(-a*d+b*c)/(b*x^3+a)-1/3*(-3*a*d+2*b*c)*arctanh(b^(
1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)+1/3*(-3*a*d
+2*b*c)*(d*x^3+c)^(1/2)/b^2/(-a*d+b*c)
```

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 79, 52, 65, 214}

$$-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^3}\,(2bc-3ad)}{3b^2(bc-ad)}+\frac{a(c+dx^3)^{3/2}}{3b\,(a+bx^3)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(3/2
))/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{a(c+dx^3)^{3/2}}{3b(bc-ad)(a+bx^3)} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a(c+dx^3)^{3/2}}{3b(bc-ad)(a+bx^3)} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{6b^2}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a(c+dx^3)^{3/2}}{3b(bc-ad)(a+bx^3)} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x,\right.}{3b^2d}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a(c+dx^3)^{3/2}}{3b(bc-ad)(a+bx^3)} - \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

Mathematica [A]

time = 0.19, size = 98, normalized size = 0.72

$$\frac{\frac{\sqrt{b}\,(3a+2bx^3)\sqrt{c+dx^3}}{a+bx^3}+\frac{(2bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((Sqrt[b]*(3*a + 2*b*x^3)*Sqrt[c + d*x^3])/(a + b*x^3) + ((2*b*c - 3*a*d)*A
rcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d])/(3
*b^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.46, size = 897, normalized size = 6.60

method	result
elliptic	$\frac{a\sqrt{d\,x^3+c}}{3b^2(b\,x^3+a)}+\frac{2\sqrt{d\,x^3+c}}{3b^2}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(3ad-2bc)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+(-}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*
(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*
(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/
2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipti
```

```
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^
2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3
*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-a/b*(-1/3*(d*x^3+
c)^(1/2)/b/(b*x^3+a)-1/6*I/b/d*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*
I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2
)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alp
ha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.17, size = 334, normalized size = 2.46

$$\left[-\frac{\left((2\,b^2c-3\,abd)x^3+2\,abc-3\,a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(3\,ab^2c-3\,a^2bd+2\,(b^3c-ab^2d)x^3\right)\sqrt{dx^3+c}}{6\left(ab^4c-a^2b^3d+(b^5c-ab^4d)x^3\right)},\ \frac{\left((2\,b^2c-3\,abd)x^3+2\,abc-3\,a^2d\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\left(3\,ab^2c-3\,a^2bd+2\,(b^3c-ab^2d)x^3\right)\sqrt{dx^3+c}}{3\left(ab^4c-a^2b^3d+(b^5c-ab^4d)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*b^2*c - 3*a*b*d)*x^3 + 2*a*b*c - 3*a^2*d)*sqrt(b^2*c - a*b*d)*lo
g((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 +
a)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x^3)*sqrt(d*x^3 + c))/
(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3), 1/3*(((2*b^2*c - 3*a*b*d)*x^
3 + 2*a*b*c - 3*a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^
2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d
)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Giac [A]
time = 1.99, size = 102, normalized size = 0.75

$$\frac{\sqrt{dx^3+c}\,ad}{3\left((dx^3+c)b-bc+ad\right)b^2}+\frac{(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^2}+\frac{2\sqrt{dx^3+c}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x^3 + c)*a*d/(((d*x^3 + c)*b - b*c + a*d)*b^2) + 1/3*(2*b*c - 3*
a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b
^2) + 2/3*sqrt(d*x^3 + c)/b^2
```

Mupad [B]
time = 6.09, size = 152, normalized size = 1.12

$$\frac{2\sqrt{d\,x^3+c}}{3\,b^2}-\frac{a\left(\frac{2\,a\,d}{3\,(2\,b^2\,c-2\,a\,b\,d)}-\frac{2\,b\,c}{3\,(2\,b^2\,c-2\,a\,b\,d)}\right)\sqrt{d\,x^3+c}}{b\,\left(b\,x^3+a\right)}+\frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,(3\,a\,d-2\,b\,c)\,1\mathrm{i}}{6\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b^2) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/
2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*b^(5
/2)*(a*d - b*c)^(1/2)) - (a*((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(
2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

3.462 $\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

```
[Out] -1/3*d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(1/2)-1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {455, 43, 65, 214}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(b*(a + b*x^3)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx,x,x^3\right)\\ &= -\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)}+\frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6b}\\ &= -\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)}+\frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b}\\ &= -\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 80, normalized size = 1.00

$$-\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{3/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(b*(a + b*x^3)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[-(b*c) + a*d]])/(3*b^(3/2)*Sqrt[-(b*c) + a*d])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 453, normalized size = 5.66

method	result

default	$-\frac{\sqrt{d\,x^3+c}}{3b(b\,x^3+a)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c \cdots\right.}{\cdots}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\cdots}}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{\sqrt{d\,x^3+c}}{3b(b\,x^3+a)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c \cdots\right.}{\cdots}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\cdots}}\cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)-1/6*I/b/d*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2
)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2
```

```
)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**A**]
time = 2.34, size = 255, normalized size = 3.19

$$\left[\frac{(bdx^3+ad)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}\,(b^2c-abd)}{6\,(ab^3c-a^2b^2d+(b^4c-ab^3d)x^3)},\ \frac{(bdx^3+ad)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-\sqrt{dx^3+c}\,(b^2c-abd)}{3\,(ab^3c-a^2b^2d+(b^4c-ab^3d)x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sq
rt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c)*(b^2*c
- a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^3), 1/3*((b*d*x^3 + a*
d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^
3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c
- a*b^3*d)*x^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Giac [**A**]
time = 1.26, size = 79, normalized size = 0.99

$$\frac{d\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^3+c}\,d}{3\,((dx^3+c)b-bc+ad)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*b)
```

Mupad [B]
time = 5.69, size = 125, normalized size = 1.56

$$\frac{\left(\frac{2\,a\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{2\,b\,c}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{6\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d
*x^3)^(1/2))/(a + b*x^3) + (d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*
(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(6*b^(3/2)*(a*d - b*c)^(1/
2))
```

3.463 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{c+dx^3}}{3a\,(a+bx^3)}-\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\,\sqrt{bc-ad}}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^2+1/3*(-a*d+2*b*c)*arctanh(
b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(1/2)/(-a*d+b*c)^(1/2)+1/3*
(d*x^3+c)^(1/2)/a/(b*x^3+a)
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 101, 162, 65, 214}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\,\sqrt{bc-ad}}-\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}}{3a\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(3*a*(a + b*x^3)) - (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt
[c]])/(3*a^2) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
 a*d]])/(3*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} + \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,d\right.}{6a^2}$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} + \frac{(2c)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{3a^2d}$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} - \frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [A]

time = 0.46, size = 111, normalized size = 0.92

$$\frac{\frac{a\sqrt{c+dx^3}}{a+bx^3}+\frac{(-2bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^3])/(a + b*x^3) + ((-2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) - 2*Sqrt[c]*ArcT
anh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.43, size = 934, normalized size = 7.72

method	result	size
default	Expression too large to display	934
elliptic	Expression too large to display	1656

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)
*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alph
a-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-b/a*(-1/3*(d*x
^3+c)^(1/2)/b/(b*x^3+a)-1/6*I/b/d*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3
)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_
alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
```

```
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(2/3
*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(97) = 194$.
time = 1.94, size = 856, normalized size = 7.07

$$\left[-\frac{\left((2\,b^{2}c-abd)x^{3}+2\,abc-a^{2}d\right)\sqrt{b^{2}c-abd}\,\log\left(\frac{bdx^{3}+2\,bc-ad-2\,\sqrt{dx^{3}+c}\,\sqrt{b^{2}c-abd}}{bx^{3}+a}\right)-2\,\left(ab^{2}c-a^{2}bd+(b^{3}c-ab^{2}d)x^{3}\right)\sqrt{c}\,\log\left(\frac{dx^{3}-2\,\sqrt{dx^{3}+c}\,\sqrt{c}+2\,c}{x^{3}}\right)-2\,(ab^{2}c-a^{2}bd)\sqrt{dx^{3}+c}}{6\,\left(a^{3}b^{2}c-a^{4}bd+(a^{2}b^{3}c-a^{3}b^{2}d)x^{3}\right)},\ -\frac{\left((2\,b^{2}c-abd)x^{3}+2\,abc-a^{2}d\right)\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{dx^{3}+c}\,\sqrt{-b^{2}c+abd}}{bdx^{3}+bc}\right)-\left(ab^{2}c-a^{2}bd+(b^{3}c-ab^{2}d)x^{3}\right)\sqrt{c}\,\log\left(\frac{dx^{3}-2\,\sqrt{dx^{3}+c}\,\sqrt{c}+2\,c}{x^{3}}\right)-(ab^{2}c-a^{2}bd)\sqrt{dx^{3}+c}}{3\,\left(a^{3}b^{2}c-a^{4}bd+(a^{2}b^{3}c-a^{3}b^{2}d)x^{3}\right)},\ \frac{4\,\left(ab^{2}c-a^{2}bd+(b^{3}c-ab^{2}d)x^{3}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^{3}+c}\,\sqrt{-c}}{c}\right)-\left((2\,b^{2}c-abd)x^{3}+2\,abc-a^{2}d\right)\sqrt{b^{2}c-abd}\,\log\left(\frac{bdx^{3}+2\,bc-ad-2\,\sqrt{dx^{3}+c}\,\sqrt{b^{2}c-abd}}{bx^{3}+a}\right)+2\,(ab^{2}c-a^{2}bd)\sqrt{dx^{3}+c}}{6\,\left(a^{3}b^{2}c-a^{4}bd+(a^{2}b^{3}c-a^{3}b^{2}d)x^{3}\right)},\ -\frac{\left((2\,b^{2}c-abd)x^{3}+2\,abc-a^{2}d\right)\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{dx^{3}+c}\,\sqrt{-b^{2}c+abd}}{bdx^{3}+bc}\right)-2\,\left(ab^{2}c-a^{2}bd+(b^{3}c-ab^{2}d)x^{3}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^{3}+c}\,\sqrt{-c}}{c}\right)-(ab^{2}c-a^{2}bd)\sqrt{dx^{3}+c}}{3\,\left(a^{3}b^{2}c-a^{4}bd+(a^{2}b^{3}c-a^{3}b^{2}d)x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a))
- 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt
(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)
*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^3 + b*c)) - (a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3
)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - (a*b^2*c - a
^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3
), 1/6*(4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) - ((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2
*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b
*d))/(b*x^3 + a)) + 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4
*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c
 - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/
(b*d*x^3 + b*c)) - 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*a
rctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^{3}}}{x \left(a + bx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x*(a + b*x**3)**2), x)
```

Giac [A]
time = 1.32, size = 114, normalized size = 0.94

$$\frac{\sqrt{dx^3+c}\,d}{3\,((dx^3+c)b-bc+ad)a}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^2}+\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*a) - 1/3*(2*b*c - a*d)*a
rctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) +
2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c))
```

Mupad [B]
time = 8.28, size = 182, normalized size = 1.50

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2}-\frac{\left(\frac{b\,d}{3\,(b^2\,c-a\,b\,d)}-\frac{b^2\,c}{3\,a\,(b^2\,c-a\,b\,d)}\right)\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}\,2\mathrm{i}}{b\,x^3+a}\right)(a\,d-2\,b\,c)\,1\mathrm{i}}{6\,a^2\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(a + b*x^3)^2),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a^2) - (((b*d)/(3*(b^2*c - a*b*d)) - (b^2*c)/(3*a*(b^2*c - a*b*d
)))*(c + d*x^3)^(1/2))/(a + b*x^3) + (log((2*b*c - a*d + (c + d*x^3)^(1/2)*
(a*b*d - b^2*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*a^2*(
a*b*d - b^2*c)^(1/2))
```

3.464 $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)}-\frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}}-\frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}$$

```
[Out] 1/3*(-a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(1/2)-1/3*(-3*a*d+4
*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a^3/(-a*d+b
*c)^(1/2)-2/3*b*(d*x^3+c)^(1/2)/a^2/(b*x^3+a)-1/3*(d*x^3+c)^(1/2)/a/x^3/(b*
x^3+a)
```

Rubi [A]
time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 101, 156, 162, 65, 214}

$$-\frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}}-\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)}-\frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-2*b*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - Sqrt[c + d*x^3]/(3*a*x^3*(a +
b*x^3)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*Sqrt[c])
- (Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d
]])/(3*a^3*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)^2}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4bc+ad)-\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$
$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(4bc-ad)-bd(bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2(bc-ad)}$$
$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(b(4bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3}$$
$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(b(4bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3d}$$
$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}} - \frac{\sqrt{b}\,(4bc-3ad)}{}$$

Mathematica [A]
time = 0.50, size = 132, normalized size = 0.82

$$\frac{-\frac{a(a+2bx^3)\sqrt{c+dx^3}}{x^3(a+bx^3)} + \frac{\sqrt{b}\,(4bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((a*(a + 2*b*x^3)*Sqrt[c + d*x^3])/(x^3*(a + b*x^3))) + (Sqrt[b]*(4*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) +
 a*d] + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/(3*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.50, size = 978, normalized size = 6.07

method	result	size
risch	Expression too large to display	960
default	Expression too large to display	978

elliptic	Expression too large to display	1708

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/a^3*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/
2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a
lpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+b^2/a^2*(-1
/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)-1/6*I/b/d*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(
1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^
2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*
d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^
(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a
^2*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
)-2/a^3*b*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^4), x)
```

Fricas [A]
time = 1.97, size = 870, normalized size = 5.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/(
b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt
(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*
x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*a*b*
c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*
c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c - a*d))*ar
ctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + (
(4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(
d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(
a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d
)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c^2 - 3*a*b*c*
d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*
b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a))
 + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/3
*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 +
 b*c)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-c)*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c
*x^6 + a^4*c*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)**2), x)
```

Giac [A]
time = 1.54, size = 183, normalized size = 1.14

$$\frac{(4\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^3}-\frac{(4\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}}-\frac{2\,(dx^3+c)^{\frac{3}{2}}bd-2\,\sqrt{dx^3+c}\,bcd+\sqrt{dx^3+c}\,ad^2}{3\left((dx^3+c)^2b-2\,(dx^3+c)bc+bc^2+(dx^3+c)ad-acd\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqr
t(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))
```

/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*d - 2*sqrt(d*x^3 + c)*b*c*d + sqrt(d*x^3 + c)*a*d^2)/((((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)*a^2)

Mupad [B]
time = 9.69, size = 438, normalized size = 2.72

$$\frac{\left(\frac{a\left(\frac{a\left(\frac{a\left(\frac{b^2\,d^2}{2\,a^3\,c^2}-\frac{b^2\,d^2\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^2\,c^2\,\left(a^2\,d-a\,b\,c\right)}+\frac{b^2\,d\,\left(2\,a\,d-b\,c\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}-\frac{b\,d\,\left(2\,a\,d-b\,c\right)}{2\,a^3\,c^2}+\frac{b\,\left(3\,a\,d-4\,b\,c\right)\,\left(-a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2}{2\,a^3\,c^2}+\frac{b\,\left(a\,d-4\,b\,c\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^2\,c\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}-\frac{a\,d-4\,b\,c}{2\,a^2\,c}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a}-\frac{\sqrt{d\,x^3+c}}{3\,a^2\,x^3}+\frac{\ln\left(\frac{{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)\,\left(a\,d-4\,b\,c\right)}{6\,a^3\,\sqrt{c}}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,a^3\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)^2),x)

[Out] (((a*((a*((a*((b^2*d^2)/(2*a^3*c^2) - (b^2*d^2*(3*a*d - 4*b*c))/(6*a^2*c^2*(a^2*d - a*b*c)) + (b^2*d*(2*a*d - b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*d - a*b*c))))/b - (b*d*(2*a*d - b*c))/(2*a^3*c^2) + (b*(3*a*d - 4*b*c)*(2*b^2*c^2 - a^2*d^2 + 2*a*b*c*d))/(6*a^3*c^2*(a^2*d - a*b*c))))/b - (2*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(2*a^3*c^2) + (b*(a*d - 4*b*c)*(3*a*d - 4*b*c))/(6*a^2*c*(a^2*d - a*b*c))))/b - (a*d - 4*b*c)/(2*a^2*c))*(c + d*x^3)^(1/2))/(a + b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*x^3) + (log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(a*d - 4*b*c))/(6*a^3*c^(1/2)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 4*b*c)*1i)/(6*a^3*(a*d - b*c)^(1/2))

3.465 $\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,2,-1/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d
*x^3/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}} = \frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 235 vs. 2(64) = 128.
time = 10.15, size = 235, normalized size = 3.67

$$\frac{x\left(\frac{5dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(-c-dx^3+\frac{8ac^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}\right)}{a+bx^3}\right)}{24b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x*((5*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
((b*x^3)/a)])/a + (8*(-c - d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d
*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -(
(b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^
3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a
+ b*x^3)))/(24*b*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.35, size = 1468, normalized size = 22.94

method	result

elliptic	$-\frac{x\sqrt{d\,x^3+c}}{3b(b\,x^3+a)}-\frac{5i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^2)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{(-c\,d^2)^{\frac{1}{3}}}{d}}{-\frac{3(-c\,d^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}}}\,\sqrt{-}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*
_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^
(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-a/b*(1/3/a*x*(d*x^3+c)^(1/2)/(b*
x^3+a)-1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1
/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
```

```
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2))+1/18*I/a/b/d^2*2^(1/2)*sum((a*d-4*b*c)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/
3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
[Out] Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\sqrt{d\,x^3+c}}{{\left(b\,x^3+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
[Out] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3)^2, x)
```

3.466 $\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,2,-1/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d
*x^3/c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\sqrt{c+dx^3}\int \frac{x\sqrt{1+\frac{dx^3}{c}}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 153 vs. 2(64) = 128.
time = 10.08, size = 153, normalized size = 2.39

$$\frac{10ax^2(c+dx^3)+5cx^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-dx^5(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{30a^2\,(a+bx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (10*a*x^2*(c + d*x^3) + 5*c*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - d*x^5*(a + b*x^3)*Sqrt[1 + (d
*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(30*a^2*(a
 + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.38, size = 908, normalized size = 14.19

method	result	size
default	Expression too large to display	908
elliptic	Expression too large to display	908

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^2/a*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
```

```
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)))+1/18*I/a/b/d^2*2^(1/2)*sum((-a*d-2*b*c)/_alpha/(a*d-b*c)*(-c*d
^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(
1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*
d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{d\,x^3+c}}{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] int((x*(c + d*x^3)^(1/2))/(a + b*x^3)^2, x)
```

3.467 $\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] x*AppellF1(1/3,2,-1/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d*x^3/c
)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])
/(a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.
time = 10.15, size = 232, normalized size = 3.93

$$\frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a^2}+\frac{8\left(\frac{c+dx^3}{a}+\frac{16c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}\right)}{a+bx^3}\right)}{24\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)])/a^2 + (8*((c + d*x^3)/a + (16*c^2*AppellF1[1/3, 1/2, 1, 4/3, -(
(d*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
-((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(
a + b*x^3)))/(24*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.34, size = 753, normalized size = 12.76 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/b/d^2*2^(1/2)*sum((a*d-4*b*c)/_alph
```

```
a^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b
/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{(a + bx^3)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(a + b*x^3)^2,x)
```

```
[Out] int((c + d*x^3)^(1/2)/(a + b*x^3)^2, x)
```

3.468 $\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)^2}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -AppellF1(-1/3,2,-1/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x/(1+d*x^3
/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^2(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}} = -\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(62) = 124.
time = 10.10, size = 172, normalized size = 2.77

$$\frac{-20a(3a+4bx^3)\left(c+dx^3\right)+5(-8bc+9ad)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+8bdx^6(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{60a^3x\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] (-20*a*(3*a + 4*b*x^3)*(c + d*x^3) + 5*(-8*b*c + 9*a*d)*x^3*(a + b*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 8
*b*d*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^
3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.47, size = 2227, normalized size = 35.92

method	result	size
elliptic	Expression too large to display	920
risch	Expression too large to display	1819
default	Expression too large to display	2227

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1
/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
```

1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-b/a*(1/3*x^2/a*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/b/d^2*2^(1/2)*sum((-a*d-2*b*c)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1

```
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x**2*(a + b*x**3)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{x^2\,{\left(b\,x^3+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)^2),x)
[Out] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)^2), x)
```

3.469 $\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -1/2*AppellF1(-2/3,2,-1/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*(Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(64) = 128.
time = 10.19, size = 338, normalized size = 5.28

$$\frac{-5bdx^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(32ac(6ac+30bcx^3-3adx^3+10bdx^6)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3(3a+5bx^3)(c+dx^3)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}}{48a^3x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] (-5*b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)] + (a*(32*a*c*(6*a*c + 30*b*c*x^3 - 3*a*d*x^3 + 10*b*d*x^6)*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(3*a + 5*b*x^3)
*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)]
+ a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3
)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2
*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[
4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[c + d*x^
3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.46, size = 1768, normalized size = 27.62

method	result

elliptic	$-\frac{bx\sqrt{dx^3+c}}{3(bx^3+a)a^2}-\frac{\sqrt{dx^3+c}}{2a^2x^2}+\frac{5i\sqrt{3}\,(-cd^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-cd^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-cd^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-cd^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-cd^2}{d}}{-\frac{3(-cd^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}}{}}}}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1
/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2))+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-
c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b
*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(-1/2*(d*x^3+c)^(1/2)/x
^2-1/2*I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
```

```
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-b/a
*(1/3/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/b/d^2*2^(1/2)*sum((a*d-4*b*c)/_al
pha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d
^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*
d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2
)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2
*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_al
pha=RootOf(_Z^3*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^3 \left(a + bx^3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**3*(a + b*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{x^3\,{\left(b\,x^3+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)^2), x)
```

3.470 $\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4}-\frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)}+\frac{2\left(c+dx^3\right)^{5/2}}{15b^2d}-\frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)}+\frac{a(4bc-7ad}{}$$

```
[Out] -1/9*a*(-7*a*d+4*b*c)*(d*x^3+c)^(3/2)/b^3/(-a*d+b*c)+2/15*(d*x^3+c)^(5/2)/b
^2/d-1/3*a^2*(d*x^3+c)^(5/2)/b^2/(-a*d+b*c)/(b*x^3+a)+1/3*a*(-7*a*d+4*b*c)*
arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(9/2)-
1/3*a*(-7*a*d+4*b*c)*(d*x^3+c)^(1/2)/b^4
```

Rubi [A]

time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 91, 81, 52, 65, 214}

$$-\frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2\left(a+bx^3\right)(bc-ad)}+\frac{a(4bc-7ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}-\frac{a\sqrt{c+dx^3}\,(4bc-7ad)}{3b^4}-\frac{a\left(c+dx^3\right)^{3/2}(4bc-7ad)}{9b^3(bc-ad)}+\frac{2\left(c+dx^3\right)^{5/2}}{15b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(a*(4*b*c - 7*a*d)*Sqrt[c + d*x^3])/b^4 - (a*(4*b*c - 7*a*d)*(c + d*x^
3)^(3/2))/(9*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(5/2))/(15*b^2*d) - (a^2*(c
+ d*x^3)^(5/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 7*a*d)*Sqrt[b
*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{a^2(c+dx^3)^{5/2}}{3b^2(bc-ad)(a+bx^3)} + \frac{\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(-\frac{1}{2}a(2bc-5ad)+b(bc-ad)x\right)}{a+bx}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2(c+dx^3)^{5/2}}{15b^2d} - \frac{a^2(c+dx^3)^{5/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-7ad))\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{6b^2(bc-ad)}$$

$$= -\frac{a(4bc-7ad)(c+dx^3)^{3/2}}{9b^3(bc-ad)} + \frac{2(c+dx^3)^{5/2}}{15b^2d} - \frac{a^2(c+dx^3)^{5/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-7a}{}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)(c+dx^3)^{3/2}}{9b^3(bc-ad)} + \frac{2(c+dx^3)^{5/2}}{15b^2d} - \frac{a^2(c+}{3b^2(bc-a}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)(c+dx^3)^{3/2}}{9b^3(bc-ad)} + \frac{2(c+dx^3)^{5/2}}{15b^2d} - \frac{a^2(c+}{3b^2(bc-a}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)(c+dx^3)^{3/2}}{9b^3(bc-ad)} + \frac{2(c+dx^3)^{5/2}}{15b^2d} - \frac{a^2(c+}{3b^2(bc-a}$$

Mathematica [A]
time = 0.34, size = 162, normalized size = 0.86

$$\frac{\sqrt{c+dx^3}\left(105a^3d^2+6b^3x^3(c+dx^3)^2+5a^2bd(-19c+14dx^3)+2ab^2(3c^2-34cdx^3-7d^2x^6)\right)}{45b^4d(a+bx^3)} + \frac{a(4bc-7ad)\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(105*a^3*d^2 + 6*b^3*x^3*(c + d*x^3)^2 + 5*a^2*b*d*(-19*c
+ 14*d*x^3) + 2*a*b^2*(3*c^2 - 34*c*d*x^3 - 7*d^2*x^6)))/(45*b^4*d*(a + b*x
^3)) + (a*(4*b*c - 7*a*d)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[-(b*c) + a*d]])/(3*b^(9/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.41, size = 1003, normalized size = 5.31

method	result

elliptic	$\frac{a^{2}(ad-bc)\sqrt{d\,x^{3}+c}}{3b^{4}(b\,x^{3}+a)}+\frac{2d\,x^{6}\sqrt{d\,x^{3}+c}}{15b^{2}}+\frac{2\left(-\frac{2d(ad-bc)}{b^{3}}-\frac{4cd}{5b^{2}}\right)x^{3}\sqrt{d\,x^{3}+c}}{9d}+\frac{2\left(\frac{3a^{2}d^{2}-4abcd+b^{2}c^{2}}{b^{4}}-\frac{2\left(-\frac{2d(ad-b}{b^{3}}\right.}{3d}\right.}{3d}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 2/15*(d*x^3+c)^(5/2)/b^2/d-2*a/b^2*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*
d-2*b*c)/b^2-2/3*c*d/b)/d*(d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d
^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^
(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1
/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alp
ha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+a^2/b^2*(1/3*(a*d-b*c)/b^2*(d*x^3+c)
^(1/2)/(b*x^3+a)+2/3*d*(d*x^3+c)^(1/2)/b^2+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2
)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2

```
)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.22, size = 443, normalized size = 2.34

$$\left[-\frac{15\,(4\,a^2bcd-7\,a^3d^2+(4\,ab^2cd-7\,a^2bd^2)x^3)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\,(6\,b^3d^2x^9+2\,(6\,b^3cd-7\,ab^2d^2)x^6+6\,ab^2c^2-95\,a^2bcd+105\,a^3d^2+2\,(3\,b^3c^2-34\,ab^2cd+35\,a^2bd^2)x^3)\sqrt{dx^3+c}}{90\,(b^5dx^3+ab^4d)},\ \frac{15\,(4\,a^2bcd-7\,a^3d^2+(4\,ab^2cd-7\,a^2bd^2)x^3)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(6\,b^3d^2x^9+2\,(6\,b^3cd-7\,ab^2d^2)x^6+6\,ab^2c^2-95\,a^2bcd+105\,a^3d^2+2\,(3\,b^3c^2-34\,ab^2cd+35\,a^2bd^2)x^3)\sqrt{dx^3+c}}{45\,(b^5dx^3+ab^4d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/90*(15*(4*a^2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt
((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c
- a*d)/b))/(b*x^3 + a)) - 2*(6*b^3*d^2*x^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^
6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d
+ 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^5*d*x^3 + a*b^4*d), 1/45*(15*(4*a^
2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt(-(b*c - a*d)/b)
*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (6*b^3*d^2*x
^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3
*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d + 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b
^5*d*x^3 + a*b^4*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.77, size = 211, normalized size = 1.12

$$-\frac{\left(4\,ab^2c^2 - 11\,a^2bcd + 7\,a^3d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^4} - \frac{\sqrt{dx^3+c}\,a^2bcd - \sqrt{dx^3+c}\,a^3d^2}{3\,((dx^3+c)b - bc + ad)b^4} + \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}b^8d^4 - 10\,(dx^3+c)^{\frac{3}{2}}ab^7d^5 - 30\,\sqrt{dx^3+c}\,ab^7cd^5 + 45\,\sqrt{dx^3+c}\,a^2b^6d^6\right)}{45\,b^{10}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(4*a*b^2*c^2 - 11*a^2*b*c*d + 7*a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt
(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) - 1/3*(sqrt(d*x^3 + c)*a^2*b*c
*d - sqrt(d*x^3 + c)*a^3*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^4) + 2/45*(3*(
d*x^3 + c)^(5/2)*b^8*d^4 - 10*(d*x^3 + c)^(3/2)*a*b^7*d^5 - 30*sqrt(d*x^3 +
 c)*a*b^7*c*d^5 + 45*sqrt(d*x^3 + c)*a^2*b^6*d^6)/(b^10*d^5)
```

Mupad [B]
time = 7.75, size = 331, normalized size = 1.75

$$\frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,{\left(a\,d-b\,c\right)}^2}{b^4}+\frac{2\,c\,\left(\frac{2\,d\,\left(a\,d-2\,b\,c\right)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{8\,c\,d}{5\,b^2}\right)}{3\,d}+\frac{2\,a\,\left(\frac{d\,\left(a\,d-2\,b\,c\right)}{b^3}+\frac{a\,d^2}{b^3}\right)}{b}\right)}{3\,d}+\frac{2\,d\,x^6\,\sqrt{d\,x^3+c}}{15\,b^2}-\frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,d\,\left(a\,d-2\,b\,c\right)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{8\,c\,d}{5\,b^2}\right)}{9\,d}-\frac{a^2\,\left(\frac{2\,b\,c^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{a\,\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\,\sqrt{d\,x^3+c}}{b^2\,\left(b\,x^3+a\right)}+\frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,\left(7\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] ((c + d*x^3)^(1/2)*((2*(a*d - b*c)^2)/b^4 + (2*c*((2*d*(a*d - 2*b*c))/b^3 +
 (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(3*d) + (2*a*((d*(a*d - 2*b*c))/b^3 + (a
*d^2)/b^3))/b))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2))/(15*b^2) - (x^3*(c + d*
x^3)^(1/2)*((2*d*(a*d - 2*b*c))/b^3 + (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(9*
d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i -
 b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(7*a*d - 4*b*c)*1i)/(6*b^(9/2)) -
(a^2*((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b
*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/b)*(c + d*x^3)^(1/2))/(b^2*(a +
b*x^3))
```

3.471 $\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$

Optimal. Leaf size=163

$$\frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3}+\frac{(2bc-5ad)(c+dx^3)^{3/2}}{9b^2(bc-ad)}+\frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)}-\frac{(2bc-5ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\ldots\right.}{3b^{7/2}}$$

```
[Out] 1/9*(-5*a*d+2*b*c)*(d*x^3+c)^(3/2)/b^2/(-a*d+b*c)+1/3*a*(d*x^3+c)^(5/2)/b/(
-a*d+b*c)/(b*x^3+a)-1/3*(-5*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*
d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)+1/3*(-5*a*d+2*b*c)*(d*x^3+c)^(1/2)/b
^3
```

Rubi [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 79, 52, 65, 214}

$$-\frac{(2bc-5ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{\sqrt{c+dx^3}\,(2bc-5ad)}{3b^3}+\frac{(c+dx^3)^{3/2}(2bc-5ad)}{9b^2(bc-ad)}+\frac{a(c+dx^3)^{5/2}}{3b(a+bx^3)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3) + ((2*b*c - 5*a*d)*(c + d*x^3)^(3
/2))/(9*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(5/2))/(3*b*(b*c - a*d)*(a + b*x^
3)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sq
rt[b*c - a*d]])/(3*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)} + \frac{(2bc-5ad)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{(2bc-5ad)(c+dx^3)^{3/2}}{9b^2(bc-ad)} + \frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)} + \frac{(2bc-5ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\right.}{6b^2}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)(c+dx^3)^{3/2}}{9b^2(bc-ad)} + \frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)} + \frac{((2bc}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)(c+dx^3)^{3/2}}{9b^2(bc-ad)} + \frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)} + \frac{((2bc}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)(c+dx^3)^{3/2}}{9b^2(bc-ad)} + \frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)} - \frac{(2bc-}{}$$

Mathematica [A]
time = 0.25, size = 125, normalized size = 0.77

$$\frac{\sqrt{c+dx^3}\,(-15a^2d+ab(11c-10dx^3)+2b^2x^3(4c+dx^3))}{9b^3(a+bx^3)} - \frac{(2bc-5ad)\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(-15*a^2*d + a*b*(11*c - 10*d*x^3) + 2*b^2*x^3*(4*c + d*x^
3)))/(9*b^3*(a + b*x^3)) - ((2*b*c - 5*a*d)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt
[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.43, size = 983, normalized size = 6.03

method	result

elliptic	$-\frac{a(ad-bc)\sqrt{dx^3+c}}{3b^3(bx^3+a)}+\frac{2dx^3\sqrt{dx^3+c}}{9b^2}+\frac{2\left(-\frac{2d(ad-bc)}{b^3}-\frac{2cd}{3b^2}\right)\sqrt{dx^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(-5a^2 \right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*c*d/b)/d*(d*x^
3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)
*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*
d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*b+a)))-a/b*(1/3*(a*d-b*c)/b^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/3*d*(d*x^3+c)^(1
/2)/b^2+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^
(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1
/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alp
ha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*
```

```
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]

time = 2.11, size = 314, normalized size = 1.93

$$\left[-\frac{3\left((2b^2c-5abd)x^3+2abc-5a^2d\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(2b^2dx^6+2(4b^2c-5abd)x^3+11abc-15a^2d\right)\sqrt{dx^3+c}}{18(b^4x^3+ab^3)},\ -\frac{3\left((2b^2c-5abd)x^3+2abc-5a^2d\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(2b^2dx^6+2(4b^2c-5abd)x^3+11abc-15a^2d\right)\sqrt{dx^3+c}}{9(b^4x^3+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*sqrt((b*c - a*d)/b)
*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x
^3 + a)) - 2*(2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*c - 15*a^2*d
)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3), -1/9*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*
a*b*c - 5*a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c
- a*d)/b)/(b*c - a*d)) - (2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*
c - 15*a^2*d)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**5*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Giac [A]

time = 1.20, size = 173, normalized size = 1.06

$$\frac{(2b^2c^2-7abcd+5a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^3}+\frac{\sqrt{dx^3+c}\,abcd-\sqrt{dx^3+c}\,a^2d^2}{3((dx^3+c)b-bc+ad)b^3}+\frac{2\left((dx^3+c)^{\frac{3}{2}}b^4+3\sqrt{dx^3+c}\,b^4c-6\sqrt{dx^3+c}\,ab^3d\right)}{9b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/3*(sqrt(d*x^3 + c)*a*b*c*d - sqr
t(d*x^3 + c)*a^2*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^3) + 2/9*((d*x^3 + c)^
(3/2)*b^4 + 3*sqrt(d*x^3 + c)*b^4*c - 6*sqrt(d*x^3 + c)*a*b^3*d)/b^6
```

Mupad [B]
time = 7.38, size = 229, normalized size = 1.40

$$\frac{2\,d\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2}-\frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,d\,\left(a\,d-2\,b\,c\right)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{4\,c\,d}{3\,b^2}\right)}{3\,d}+\frac{a\,\left(\frac{2\,b\,c^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{a\,\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\,\sqrt{d\,x^3+c}}{b\,\left(b\,x^3+a\right)}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,\left(5\,a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*d*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((2*d*(a*d - 2*b*c
))/b^3 + (2*a*d^2)/b^3 + (4*c*d)/(3*b^2)))/(3*d) + (log((2*b*c - a*d + b^(1
/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b
*c)^(1/2)*(5*a*d - 2*b*c)*1i)/(6*b^(7/2)) + (a*((2*b*c^2)/(3*(2*b^2*c - 2*a
*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*
a*b*d))))/b)*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

3.472 $\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=94

$$\frac{d\sqrt{c+dx^3}}{b^2}-\frac{(c+dx^3)^{3/2}}{3b\,(a+bx^3)}-\frac{d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

```
[Out] -1/3*(d*x^3+c)^(3/2)/b/(b*x^3+a)-d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*
c)^(1/2))*(-a*d+b*c)^(1/2)/b^(5/2)+d*(d*x^3+c)^(1/2)/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {455, 43, 52, 65, 214}

$$-\frac{d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx^3)^{3/2}}{3b\,(a+bx^3)}+\frac{d\sqrt{c+dx^3}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (d*Sqrt[c + d*x^3])/b^2 - (c + d*x^3)^(3/2)/(3*b*(a + b*x^3)) - (d*Sqrt[b*c
 - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx,x,x^3\right)\\ &= -\frac{(c+dx^3)^{3/2}}{3b\,(a+bx^3)}+\frac{d\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^3\right)}{2b}\\ &= \frac{d\sqrt{c+dx^3}}{b^2}-\frac{(c+dx^3)^{3/2}}{3b\,(a+bx^3)}+\frac{(d(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{2b^2}\\ &= \frac{d\sqrt{c+dx^3}}{b^2}-\frac{(c+dx^3)^{3/2}}{3b\,(a+bx^3)}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{b^2}\\ &= \frac{d\sqrt{c+dx^3}}{b^2}-\frac{(c+dx^3)^{3/2}}{3b\,(a+bx^3)}-\frac{d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 94, normalized size = 1.00

$$\frac{\sqrt{c+dx^3}\,(-bc+3ad+2bdx^3)}{3b^2\,(a+bx^3)}-\frac{d\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(-(b*c) + 3*a*d + 2*b*d*x^3))/(3*b^2*(a + b*x^3)) - (d*Sqr
t[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/b^(5/
2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 466, normalized size = 4.96

method	result
default	$\frac{(ad-bc)\sqrt{d\,x^{3}+c}}{3b^{2}(b\,x^{3}+a)}+\frac{2d\sqrt{d\,x^{3}+c}}{3b^{2}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\ldots\right.}{d}\right.\ldots}{\left(-c\,d^{2}\right)^{\frac{1}{3}}\ldots}}\ldots}{\ldots}\right.\ldots}{\ldots}$
elliptic	$\frac{(ad-bc)\sqrt{d\,x^{3}+c}}{3b^{2}(b\,x^{3}+a)}+\frac{2d\sqrt{d\,x^{3}+c}}{3b^{2}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\ldots\right.}{d}\right.\ldots}{\left(-c\,d^{2}\right)^{\frac{1}{3}}\ldots}}\ldots}{\ldots}\right.\ldots}{\ldots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a*d-b*c)/b^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/3*d*(d*x^3+c)^(1/2)/b^2+1/2*I
/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(
-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*
d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),
1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_al
pha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.59, size = 234, normalized size = 2.49

$$\left[\frac{3\,(bdx^3+ad)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,(2\,bdx^3-bc+3\,ad)\sqrt{dx^3+c}}{6\,(b^3x^3+ab^2)},\ -\frac{3\,(bdx^3+ad)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(2\,bdx^3-bc+3\,ad)\sqrt{dx^3+c}}{3\,(b^3x^3+ab^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d*x^3 + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*
sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(2*b*d*x^3 - b*c +
3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2), -1/3*(3*(b*d*x^3 + a*d)*sqrt(-(b
*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) -
(2*b*d*x^3 - b*c + 3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**2*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Giac [A]
time = 1.21, size = 122, normalized size = 1.30

$$\frac{2\sqrt{dx^3+c}\,d}{3\,b^2}+\frac{(bcd-ad^2)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}-\frac{\sqrt{dx^3+c}\,bcd-\sqrt{dx^3+c}\,ad^2}{3\,((dx^3+c)b-bc+ad)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*sqrt(d*x^3 + c)*d/b^2 + (b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-
b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - 1/3*(sqrt(d*x^3 + c)*b*c*d - s
qrt(d*x^3 + c)*a*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^2)
```

Mupad [B]
time = 7.35, size = 170, normalized size = 1.81

$$\frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b^2}-\frac{\left(\frac{2\,b\,c^2}{3\,(2\,b^2\,c-2\,a\,b\,d)}+\frac{a\,\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,1{}\mathrm{i}}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*d*(c + d*x^3)^(1/2))/(3*b^2) - (((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*
((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/b)
*(c + d*x^3)^(1/2))/(a + b*x^3) + (d*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)
^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(
2*b^(5/2))
```

3.473 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=131

$$\frac{(bc-ad)\sqrt{c+dx^3}}{3ab\,(a+bx^3)}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{bc-ad}\,(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}$$

```
[Out] -2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2+1/3*(a*d+2*b*c)*arctanh(b
^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a^2/b^(3/2)+1/3*(
-a*d+b*c)*(d*x^3+c)^(1/2)/a/b/(b*x^3+a)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 100, 162, 65, 214}

$$\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}\,(bc-ad)}{3ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^3])/(3*a*b*(a + b*x^3)) - (2*c^(3/2)*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(3*a^2) + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*b^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab(a+bx^3)} + \frac{\text{Subst}\left(\int \frac{bc^2+\frac{1}{2}d(bc+ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ab}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab(a+bx^3)} + \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{((bc-ad)(2bc+ad))\text{Subs}}{}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab(a+bx^3)} + \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{((bc-ad)(2bc+ac}{}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab(a+bx^3)} - \frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{bc-ad}\,(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{3a^2b^{3/2}}$$

Mathematica [A]

time = 0.28, size = 122, normalized size = 0.93

$$\frac{\frac{a(bc-ad)\sqrt{c+dx^3}}{b(a+bx^3)}+\frac{\sqrt{-bc+ad}\,(2bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}-2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*(b*c - a*d)*Sqrt[c + d*x^3])/(b*(a + b*x^3)) + (Sqrt[-(b*c) + a*d]*(2*b
*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/b^(3/2) - 2
*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.46, size = 1036, normalized size = 7.91

method	result	size
default	Expression too large to display	1036
elliptic	Expression too large to display	1632

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*c*d/b)/d*(d
*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b
*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_
alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alp
ha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-
c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c
*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*b+a)))-b/a*(1/3*(a*d-b*c)/b^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/3*d*(d*x^3+c)
^(1/2)/b^2+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^
2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_
alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(
```

```
2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(2/9*d*x^3*(d*x^3+c)^(1/2)+
8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x), x)
```

Fricas [A]
time = 2.08, size = 686, normalized size = 5.24

$$\left[\frac{((2b^2c+abd)x^3+2abc+a^2d)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2(b^2cx^3+abc)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}(abc-a^2d)}{6(a^2b^2x^3+a^3b)},\ \frac{((2b^2c+abd)x^3+2abc+a^2d)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(b^2cx^3+abc)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+\sqrt{dx^3+c}(abc-a^2d)}{3(a^2b^2x^3+a^3b)},\ \frac{4(b^2cx^3+abc)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+((2b^2c+abd)x^3+2abc+a^2d)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\sqrt{dx^3+c}(abc-a^2d)}{6(a^2b^2x^3+a^3b)},\ \frac{((2b^2c+abd)x^3+2abc+a^2d)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+2(b^2cx^3+abc)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\sqrt{dx^3+c}(abc-a^2d)}{3(a^2b^2x^3+a^3b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt((b*c - a*d)/b)*log((b*
d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a))
 + 2*(b^2*c*x^3 + a*b*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
*c)/x^3) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/3*((
(2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(
d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b^2*c*x^3 + a*b*c)*sqrt(c
)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + sqrt(d*x^3 + c)*(a*b
*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/6*(4*(b^2*c*x^3 + a*b*c)*sqrt(-c)*arc
tan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)
*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt(
(b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*
x^3 + a^3*b), 1/3*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a
*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + 2*(b^2
*c*x^3 + a*b*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^3 +
c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{3}\right)^{\frac{3}{2}}}{x \left(a + bx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a)**2,x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x*(a + b*x**3)**2), x)
```

Giac [A]
time = 1.75, size = 155, normalized size = 1.18

$$\frac{2\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}-\frac{(2\,b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^2b}+\frac{\sqrt{dx^3+c}\,bcd-\sqrt{dx^3+c}\,ad^2}{3\,((dx^3+c)b-bc+ad)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*(2*b^2*c^2 -
a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^
2*c + a*b*d)*a^2*b) + 1/3*(sqrt(d*x^3 + c)*b*c*d - sqrt(d*x^3 + c)*a*d^2)/(
((d*x^3 + c)*b - b*c + a*d)*a*b)
```

Mupad [B]
time = 9.14, size = 214, normalized size = 1.63

$$\frac{c^{3/2}\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2}+\frac{\sqrt{d\,x^3+c}\,\left(\frac{a\left(\frac{b\,d^2}{3\,(b^2\,c-a\,b\,d)}-\frac{2\,b^2\,c\,d}{3\,a\,(b^2\,c-a\,b\,d)}\right)}{b}+\frac{b^2\,c^2}{3\,a\,(b^2\,c-a\,b\,d)}\right)}{b\,x^3+a}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\sqrt{a\,d-b\,c}\,(a\,d+2\,b\,c)\,1\mathrm{i}}{6\,a^2\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x)
```

```
[Out] (c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a^2) + ((c + d*x^3)^(1/2)*((a*((b*d^2)/(3*(b^2*c - a*b*d)) - (2*
b^2*c*d)/(3*a*(b^2*c - a*b*d))))/b + (b^2*c^2)/(3*a*(b^2*c - a*b*d))))/(a +
 b*x^3) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2
i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(a*d + 2*b*c)*1i)/(6*a^2*b^(3/2
))
```

3.474 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)}-\frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}+\frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3}-\frac{\sqrt{bc-ad}\,(4bc-ad)\tanh}{3a^3\sqrt{}}$$

```
[Out] 1/3*(-3*a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^3-1/3*(-a*d+4
*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a^
3/b^(1/2)-1/3*(-a*d+2*b*c)*(d*x^3+c)^(1/2)/a^2/(b*x^3+a)-1/3*c*(d*x^3+c)^(1
/2)/a/x^3/(b*x^3+a)
```

Rubi [A]

time = 0.17, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 100, 156, 162, 65, 214}

$$\frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3}-\frac{\sqrt{bc-ad}\,(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}}-\frac{\sqrt{c+dx^3}\,(2bc-ad)}{3a^2\left(a+bx^3\right)}-\frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/3*((2*b*c - a*d)*Sqrt[c + d*x^3])/(a^2*(a + b*x^3)) - (c*Sqrt[c + d*x^3]
)/(3*a*x^3*(a + b*x^3)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]])/(3*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)+\frac{1}{2}d(3bc-2ad)x}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)(bc-ad)+\frac{1}{2}d(bc-ad)(2bc-ad}{x(a+bx)\sqrt{c+dx}}\right.}{3a^2(bc-ad)}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{(c(4bc-3ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x,\right.}{6a^3}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{(c(4bc-3ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c}\right.}{3a^3d}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} -$$

Mathematica [A]
time = 0.57, size = 154, normalized size = 0.91

$$\frac{\frac{a\sqrt{c+dx^3}\left(-ac-2bcx^3+adx^3\right)}{x^3\left(a+bx^3\right)} + \frac{\left(4b^2c^2-5abcd+a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}} + \sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^3]*(-(a*c) - 2*b*c*x^3 + a*d*x^3))/(x^3*(a + b*x^3)) + ((4
*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c
) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) + Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[S
qrt[c + d*x^3]/Sqrt[c]])/(3*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.46, size = 1093, normalized size = 6.43

method	result	size
risch	Expression too large to display	975
default	Expression too large to display	1093

elliptic	Expression too large to display	1656

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/a^3*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*c*d/b)/d
*(d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*
d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*b+a)))+b^2/a^2*(1/3*(a*d-b*c)/b^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/3*d*(d
*x^3+c)^(1/2)/b^2+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d
*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*
d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(-1/3*c*(d*x^3+c)^(1
/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))-2
/a^3*b*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh
((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^4), x)
```

Fricas [A]
time = 2.31, size = 838, normalized size = 4.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [-1/6*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/6*(2*((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/3*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^3)^{\frac{3}{2}}}{x^4(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a)**2,x)

[Out] Integral((c + d*x**3)**(3/2)/(x**4*(a + b*x**3)**2), x)

Giac [A]
time = 1.74, size = 216, normalized size = 1.27

$$\frac{(4\,b^2c^2-5\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,a^3}-\frac{(4\,bc^2-3\,acd)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}}-\frac{2\,(dx^3+c)^{\frac{3}{2}}bcd-2\,\sqrt{dx^3+c}\,bc^2d-(dx^3+c)^{\frac{3}{2}}ad^2+2\,\sqrt{dx^3+c}\,acd^2}{3\left((dx^3+c)^2b-2\,(dx^3+c)bc+bc^2+(dx^3+c)ad-acd\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c^2 - 3*a*c*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*c*d - 2*sq

```
rt(d*x^3 + c)*b*c^2*d - (d*x^3 + c)^(3/2)*a*d^2 + 2*sqrt(d*x^3 + c)*a*c*d^2
)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)*
a^2)
```

Mupad [B]
time = 10.82, size = 531, normalized size = 3.12

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d x^3+c}+\sqrt{c}\right)}{x^6}\right)(3 a d-4 b c)}{6 a^3}-\frac{c\sqrt{d x^3+c}}{3 a^2 x^3}-\frac{\sqrt{d x^3+c}\left(\frac{3 a d-4 b c}{2 a^2}-\frac{a\left(\frac{a\left(\frac{a\left(\frac{b d^2 (a d+b c)}{a^3 c^2}-\frac{a\left(\frac{b^2 d^3}{2 a^3 c^2}-\frac{b^2 d^3 (3 a d-4 b c)}{6 a^2 c^2 (a^2 d-a b c)}\right)+\frac{b^2 d^2 (a d+b c)(3 a d-4 b c)}{3 a^3 c^2 (a^2 d-a b c)}}{b}+\frac{b (3 a d-4 b c)(a^2 d^3-b^2 c^2 d+4 a b c d^2)}{6 a^3 c^2 (a^2 d-a b c)}\right)}{b}-\frac{a^2 d^3-b^2 c^2 d+4 a b c d^2}{2 a^3 c^2}+\frac{b (3 a d-4 b c)(2 b^2 c^3-4 a^2 c d^2+2 a b c^2 d)}{6 a^3 c^2 (a^2 d-a b c)}\right)}{b}-\frac{2 b^2 c^3-4 a^2 c d^2+2 a b c^2 d}{2 a^3 c^2}+\frac{b (3 a d-4 b c)^2}{6 a^2 (a^2 d-a b c)}\right)}{b}\right)}{b x^3+a}+\frac{\ln\left(\frac{2 b c-a d+b d x^3+\sqrt{b}\sqrt{d x^3+c}\sqrt{a d-b c}\,2 i}{b x^3+a}\right)\sqrt{a d-b c}\,(a d-4 b c)\,1i}{6 a^3\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6)*(3*a*d - 4*b*c))/(6*a^3) - (c*(c + d*x^3)^(1/2))/(3*a^2*x^3) - ((c +
 d*x^3)^(1/2)*((3*a*d - 4*b*c)/(2*a^2) - (a*((a*((a*((b*d^2*(a*d + b*c))/(a
^3*c^2) - (a*((b^2*d^3)/(2*a^3*c^2) - (b^2*d^3*(3*a*d - 4*b*c))/(6*a^2*c^2*
(a^2*d - a*b*c)) + (b^2*d^2*(a*d + b*c)*(3*a*d - 4*b*c))/(3*a^3*c^2*(a^2*d
- a*b*c))))/b + (b*(3*a*d - 4*b*c)*(a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2))/(6*
a^3*c^2*(a^2*d - a*b*c))))/b - (a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2)/(2*a^3*c
^2) + (b*(3*a*d - 4*b*c)*(2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d))/(6*a^3*c^
2*(a^2*d - a*b*c))))/b - (2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d)/(2*a^3*c^2
) + (b*(3*a*d - 4*b*c)^2)/(6*a^2*(a^2*d - a*b*c))))/b))/(a + b*x^3) + (log(
(2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a
 + b*x^3))*(a*d - b*c)^(1/2)*(a*d - 4*b*c)*1i)/(6*a^3*b^(1/2))
```

3.475 $\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=65

$$\frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/4*c*x^4*AppellF1(4/3,2,-3/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x^3\left(1+\frac{dx^3}{c}\right)^{3/2}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(65) = 130.
time = 10.27, size = 338, normalized size = 5.20

$$\frac{x^4\left(\frac{d(43bc-55ad)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(-8acd(11ad+b(c+6dx^3))F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3(c+dx^3)(5bc-11ad-6bdx^3)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}\right)}{120b^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (x^4*((d*(43*b*c - 55*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-8*a*c*d*(11*a*d + b*(c + 6*d*x^3))*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*(c + d*x^3)*(5*b*c
- 11*a*d - 6*b*d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((
a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] +
 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*
AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(120*b^2*Sqrt[c
 + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.41, size = 1587, normalized size = 24.42

method	result

elliptic	$\frac{(ad-bc)x\sqrt{d\,x^3+c}}{3b^2(b\,x^3+a)}+\frac{2x\sqrt{d\,x^3+c}\,d}{5b^2}-\frac{2i\left(-\frac{11d(ad-bc)}{6b^3}-\frac{2cd}{5b^2}\right)\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2/5*d/b*x*(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-2/5*c*d/b)*3^(1/2)
/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I/b^2/d^2*2^(1
/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2
*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
)^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/
2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_al
pha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-a/b*(-1/3*(a
*d-b*c)/a/b*x*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2-1/6/b^2*d*(a*d-b*c)/
a)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d
```

```
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a
/b^2/d^2*2^(1/2)*sum((5*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d
^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(
1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*
d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**3*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,{\left(d\,x^3+c\right)}^{3/2}}{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
[Out] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2, x)
```

3.476 $\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=65

$$\frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/2*c*x^2*AppellF1(2/3,2,-3/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x\left(1+\frac{dx^3}{c}\right)^{3/2}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}} = \frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(65) = 130.
time = 10.14, size = 177, normalized size = 2.72

$$\frac{x^2\left(-10a(-bc+ad)\left(c+dx^3\right)+5c(bc+2ad)\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-d(bc-7ad)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{30a^2b\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (x^2*(-10*a*(-(b*c) + a*d)*(c + d*x^3) + 5*c*(b*c + 2*a*d)*(a + b*x^3)*Sqrt
[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - d*
(b*c - 7*a*d)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3
, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a^2*b*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.34, size = 955, normalized size = 14.69

method	result	size
default	Expression too large to display	955
elliptic	Expression too large to display	955

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a*d-b*c)/a/b*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2+1/6/b^2*d*(
a*d-b*c)/a)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
```

```
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/b^2/d^2*2^
(1/2)*sum((7*a^2*d^2-5*a*b*c*d-2*b^2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/
3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^
(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*
_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,{\left(d\,x^3+c\right)}^{3/2}}{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
[Out] int((x*(c + d*x^3)^(3/2))/(a + b*x^3)^2, x)
```

3.477 $\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$

Optimal. Leaf size=60

$$\frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] c*x*AppellF1(1/3,2,-3/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d*x^3
/c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

```
[Out] (c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 339 vs. 2(60) = 120.
time = 10.22, size = 339, normalized size = 5.65

$$\frac{x\left(d(bc+5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(-64ac(-ad^2x^3+bc(3c+dx^3))F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+24(bc-ad)x^3(c+dx^3)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}\right)}{24a^2b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

```
[Out] (x*(d*(b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d
*x^3)/c), -((b*x^3)/a)] + (a*(-64*a*c*(-(a*d^2*x^3) + b*c*(3*c + d*x^3))*Ap
pellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*(b*c - a*d)*x^3*(
c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] +
a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*
(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b
*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/
3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(24*a^2*b*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.34, size = 801, normalized size = 13.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a*d-b*c)/a/b*x*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2-1/6/b^2*d*(a*
d-b*c)/a)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1
```

```
/18*I/a/b^2/d^2*2^(1/2)*sum((5*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2/(a*d-b*c
)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha
*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d
-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^{\frac{3}{2}}}{(a + bx^3)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
[Out] Integral((c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{\left(b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(a + b*x^3)^2,x)
```

```
[Out] int((c + d*x^3)^(3/2)/(a + b*x^3)^2, x)
```

3.478 $\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -c*AppellF1(-1/3,2,-3/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x/(1+d*x
^3/c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^2(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(63) = 126.
time = 10.15, size = 190, normalized size = 3.02

$$\frac{-20a(c+dx^3)(3ac+4bcx^3-adx^3)+5c(-8bc+11ad)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2d(4bc-ad)x^6(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{60a^3x(a+bx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] (-20*a*(c + d*x^3)*(3*a*c + 4*b*c*x^3 - a*d*x^3) + 5*c*(-8*b*c + 11*a*d)*x^
3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
-((b*x^3)/a)] + 2*d*(4*b*c - a*d)*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3)*Sq
rt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.44, size = 2364, normalized size = 37.52

method	result	size
elliptic	Expression too large to display	970
risch	Expression too large to display	1854
default	Expression too large to display	2364

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(2/7*d/b*x^2*(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-4/7*c*d/b)*3^
(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
```

)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3
3^(1/2)(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d
^2+2*a*b*c*d-b^2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
3^(1/2)(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-
c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)
+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b
*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-b/a*(-1/3*(a*d-b*c)/a/b*x^2*(
d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2+1/6/b^2*d*(a*d-b*c)/a)*3^(1/2)/d*(-
c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)
(I(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/b^2/d^2*2^(1/2)*sum((7*a^2*d^2-5*a
*b*c*d-2*b^2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^
2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_
alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(-c*(d*x^3+c)^(1/2)/x+2/7*(
d*x^3+c)^(1/2)*d*x^2-9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d
*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(

```
-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d
*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{3}\right)^{\frac{3}{2}}}{x^{2} \left(a + bx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x**2*(a + b*x**3)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{x^2\,{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x)
[Out] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2), x)
```

3.479 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -1/2*c*AppellF1(-2/3,2,-3/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x^2/
(1+d*x^3/c)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -3/2, 1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^3(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 370 vs. 2(65) = 130.
time = 10.24, size = 370, normalized size = 5.69

$$\frac{-d(5bc-2ad)x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a\left(4ac(10bcx^3(3c+dx^3)+a(6c^2-15cdx^3-4d^2x^6))F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3(c+dx^3)(3ac+5bcx^3-2adx^3)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}}{48a^3x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] (-(d*(5*b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)]) + (8*a*(4*a*c*(10*b*c*x^3*(3*c + d*x^3) + a*(6*c^
2 - 15*c*d*x^3 - 4*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*
x^3)/a)] - 3*x^3*(c + d*x^3)*(3*a*c + 5*b*c*x^3 - 2*a*d*x^3)*(2*b*c*AppellF
1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1,
 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.47, size = 1902, normalized size = 29.26

method	result

elliptic	$\frac{(ad-bc)x\sqrt{d\,x^3+c}}{3a^2(b\,x^3+a)}-\frac{c\sqrt{d\,x^3+c}}{2a^2x^2}-\frac{2i\left(\frac{(ad-bc)d}{6a^2b}-\frac{dc}{4a^2}\right)\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^2)^{\frac{1}{3}}}}}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(2/5*d/b*x*(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-2/5*c*d/b)*3^(1
/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I/b^2/d^2*2
^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/
3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^
(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*
_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(-1
/2*c*(d*x^3+c)^(1/2)/x^2+2/5*d*x*(d*x^3+c)^(1/2)-9/10*I*c*3^(1/2)*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
```

```
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-b/a*(-1/3*(a*d-b*c)/a/b*x*(d*
x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2-1/6/b^2*d*(a*d-b*c)/a)*3^(1/2)/d*(-c*
d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/b^2/d^2*2^(1/2)*
sum((5*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(
1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha
-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^{\frac{3}{2}}}{x^3 (a + bx^3)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x**3*(a + b*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x^3 + c)^{3/2}}{x^3\,(b\,x^3 + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2), x)
```

3.480 $\int \frac{x^8}{(a+bx^3)^2 \sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=123

$$\frac{2\sqrt{c+dx^3}}{3b^2d}-\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}$$

```
[Out] 1/3*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+2/3*(d*x^3+c)^(1/2)/b^2/d-1/3*a^2*(d*x^3+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^3+a)
```

Rubi [A]
time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 91, 81, 65, 214}

$$-\frac{a^2\sqrt{c+dx^3}}{3b^2\left(a+bx^3\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}+\frac{2\sqrt{c+dx^3}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b^2*d) - (a^2*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)*(a
 + b*x^3)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^3\right)$$

$$= -\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}+\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2\sqrt{c+dx^3}}{3b^2d}-\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}-\frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+\ldots}}\right.}{6b^2(bc-ad)}$$

$$= \frac{2\sqrt{c+dx^3}}{3b^2d}-\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}-\frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx\ldots\right.}{3b^2d(bc-ad)}$$

$$= \frac{2\sqrt{c+dx^3}}{3b^2d}-\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\ldots\right.}{3b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A]

time = 0.37, size = 130, normalized size = 1.06

$$\frac{\frac{\sqrt{b}\,\sqrt{c+dx^3}\,(-3a^2d+2b^2cx^3+2ab(c-dx^3))}{d(bc-ad)(a+bx^3)}+\frac{a(4bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((Sqrt[b]*Sqrt[c + d*x^3]*(-3*a^2*d + 2*b^2*c*x^3 + 2*a*b*(c - d*x^3)))/(d*
(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(3*b^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 911, normalized size = 7.41

method	result
elliptic	$\frac{a^2\sqrt{d\,x^3+c}}{3b^2(ad-bc)(b\,x^3+a)}+\frac{2\sqrt{d\,x^3+c}}{3b^2d}+\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(3ad-4bc)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+}{d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b^2/d+2/3*I*a/b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^
(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d
^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)
```

```
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)
^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+a^2
/b^2*(1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*
c)^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*
_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_al
pha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*b+a)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(103) = 206$.
time = 2.31, size = 475, normalized size = 3.86

$$\left[\frac{{\left(4\,a^{2}bcd - 3\,a^{3}d^{2} + {\left(4\,ab^{2}cd - 3\,a^{2}bd^{2}\right)}x^{3}\right)}\sqrt{b^{2}c - abd}\log\left(\frac{bdx^{3} + 2\,bc - ad + 2\,\sqrt{dx^{3} + c}\sqrt{b^{2}c - abd}}{bx^{3} + a}\right) + 2\,{\left(2\,ab^{3}c^{2} - 5\,a^{2}b^{2}cd + 3\,a^{3}bd^{2} + 2\,{\left(b^{4}c^{2} - 2\,ab^{3}cd + a^{2}b^{2}d^{2}\right)}x^{3}\right)}\sqrt{dx^{3} + c}}{6\,{\left(ab^{5}c^{2}d - 2\,a^{2}b^{4}cd^{2} + a^{3}b^{3}d^{3} + {\left(b^{6}c^{2}d - 2\,ab^{5}cd^{2} + a^{2}b^{4}d^{3}\right)}x^{3}\right)}}, -\frac{{\left(4\,a^{2}bcd - 3\,a^{3}d^{2} + {\left(4\,ab^{2}cd - 3\,a^{2}bd^{2}\right)}x^{3}\right)}\sqrt{-b^{2}c + abd}\arctan\left(\frac{\sqrt{dx^{3} + c}\sqrt{-b^{2}c + abd}}{bdx^{3} + bc}\right) - {\left(2\,ab^{3}c^{2} - 5\,a^{2}b^{2}cd + 3\,a^{3}bd^{2} + 2\,{\left(b^{4}c^{2} - 2\,ab^{3}cd + a^{2}b^{2}d^{2}\right)}x^{3}\right)}\sqrt{dx^{3} + c}}{3\,{\left(ab^{5}c^{2}d - 2\,a^{2}b^{4}cd^{2} + a^{3}b^{3}d^{3} + {\left(b^{6}c^{2}d - 2\,ab^{5}cd^{2} + a^{2}b^{4}d^{3}\right)}x^{3}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(b^2*
c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*
d))/(b*x^3 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b
^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3), -1
```

```
/3*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2
*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b
^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
+ (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 0.82, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^3+c}\,a^2d}{3\,(b^3c-ab^2d)((dx^3+c)b-bc+ad)}-\frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,(b^3c-ab^2d)\sqrt{-b^2c+abd}}+\frac{2\,\sqrt{dx^3+c}}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d*x^3 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^3 + c)*b - b*c + a*d))
- 1/3*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 2/3*sqrt(d*x^3 + c)/(b^2*d)
```

Mupad [B]
time = 7.29, size = 160, normalized size = 1.30

$$\frac{2\,\sqrt{d\,x^3+c}\,\left(2\,b^2\,c-2\,a\,b\,d\right)}{3\,d\,\left(2\,b^4\,c-2\,a\,b^3\,d\right)}-\frac{2\,a^2\,\sqrt{d\,x^3+c}}{3\,b\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2)*(2*b^2*c - 2*a*b*d))/(3*d*(2*b^4*c - 2*a*b^3*d)) - (2*
a^2*(c + d*x^3)^(1/2))/(3*b*(a + b*x^3)*(2*b^2*c - 2*a*b*d)) + (a*log((a*d
- 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*
x^3))*(3*a*d - 4*b*c)*1i)/(6*b^(5/2)*(a*d - b*c)^(3/2))
```

3.481 $\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^3}}{3b(bc-ad)\,(a+bx^3)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/3*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/3*a*(d*x^3+c)^(1/2)/b/(-a*d+b*c)/(b*x^3+a)
```

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 79, 65, 214}

$$\frac{a\sqrt{c+dx^3}}{3b\,(a+bx^3)\,(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (a*Sqrt[c + d*x^3])/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd(bc-ad)}\\
&= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 100, normalized size = 1.01

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^3}}{(bc-ad)(a+bx^3)} - \frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(3
*b^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 892, normalized size = 9.01

method	result
elliptic	$-\frac{a\sqrt{d\,x^3+c}}{3(ad-bc)b(b\,x^3+a)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-ad+2bc)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2
*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-a/b*(1/3/(a*d-b*c)*(d*x^3+c)^(1/
2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^
2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.07, size = 348, normalized size = 3.52

$$\left[\frac{\left(\left(2\,b^{2}c-abd\right)x^{3}+2\,abc-a^{2}d\right)\sqrt{b^{2}c-abd}\,\log\left(\frac{bdx^{3}+2\,bc-ad-2\,\sqrt{dx^{3}+c}\,\sqrt{b^{2}c-abd}}{bx^{3}+a}\right)+2\,\left(ab^{2}c-a^{2}bd\right)\sqrt{dx^{3}+c}}{6\,\left(ab^{4}c^{2}-2\,a^{2}b^{3}cd+a^{3}b^{2}d^{2}+\left(b^{5}c^{2}-2\,ab^{4}cd+a^{2}b^{3}d^{2}\right)x^{3}\right)},\ \frac{\left(\left(2\,b^{2}c-abd\right)x^{3}+2\,abc-a^{2}d\right)\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{dx^{3}+c}\,\sqrt{-b^{2}c+abd}}{bdx^{3}+bc}\right)+\left(ab^{2}c-a^{2}bd\right)\sqrt{dx^{3}+c}}{3\,\left(ab^{4}c^{2}-2\,a^{2}b^{3}cd+a^{3}b^{2}d^{2}+\left(b^{5}c^{2}-2\,ab^{4}cd+a^{2}b^{3}d^{2}\right)x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*
d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) +
 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3), 1/3*(((2*b^2*c - a*b*d)
*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^3 + b*c)) + (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)
*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(a+bx^{3}\right)^{2}\sqrt{c+dx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.51, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^{3}+c}\,ad^{2}}{\left(b^{2}c-abd\right)\left(\left(dx^{3}+c\right)b-bc+ad\right)}+\frac{\left(2\,bcd-ad^{2}\right)\arctan\left(\frac{\sqrt{dx^{3}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\left(b^{2}c-abd\right)\sqrt{-b^{2}c+abd}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(d*x^3 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^3 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B]
time = 6.85, size = 111, normalized size = 1.12

$$\frac{2\,a\,\sqrt{d\,x^3+c}}{3\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^
3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(3/2)) + (2*a*(c +
 d*x^3)^(1/2))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

3.482 $\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}$$

```
[Out] 1/3*d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/3*(d*x^3+c)^(1/2)/(-a*d+b*c)/(b*x^3+a)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {455, 44, 65, 214}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^3}}{3\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/((b*c - a*d)*(a + b*x^3)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{3(bc-ad)(a+bx^3)} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6(bc-ad)}\\
&= -\frac{\sqrt{c+dx^3}}{3(bc-ad)(a+bx^3)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3(bc-ad)}\\
&= -\frac{\sqrt{c+dx^3}}{3(bc-ad)(a+bx^3)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 86, normalized size = 0.99

$$\frac{1}{3}\left(-\frac{\sqrt{c+dx^3}}{(bc-ad)(a+bx^3)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-(Sqrt[c + d*x^3]/((b*c - a*d)*(a + b*x^3))) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.33, size = 457, normalized size = 5.25

method	result
default	$\frac{\sqrt{d\,x^3+c}}{3(ad-bc)(b\,x^3+a)} - \frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{(-}{}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i}}}{}\right.}{}$
elliptic	$\frac{\sqrt{d\,x^3+c}}{3(ad-bc)(b\,x^3+a)} - \frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{(-}{}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(
-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha
*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2
```

```
)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*
(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b
+a))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(71) = 142$.
time = 1.90, size = 302, normalized size = 3.47

$$\left[-\frac{(bdx^3+ad)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}\,(b^2c-abd)}{6\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^3)},\,-\frac{(bdx^3+ad)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\sqrt{dx^3+c}\,(b^2c-abd)}{3\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*s
qrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c
 - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d
+ a^2*b^2*d^2)*x^3), -1/3*((b*d*x^3 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt
(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + sqrt(d*x^3 + c)*(b^2*c
- a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^3)^2\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.29, size = 93, normalized size = 1.07

$$-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx^3+c}\,d}{3\,((dx^3+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*(b*c - a
*d))
```

Mupad [B]
time = 6.37, size = 104, normalized size = 1.20

$$-\frac{2\,b\,\sqrt{d\,x^3+c}}{3\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*
x^3)/(a + b*x^3))*1i)/(6*b^(1/2)*(a*d - b*c)^(3/2)) - (2*b*(c + d*x^3)^(1/2
))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

3.483 $\int \frac{1}{x(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=132

$$\frac{b\sqrt{c+dx^3}}{3a(bc-ad)\,(a+bx^3)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

```
[Out] 1/3*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/3
*b*(d*x^3+c)^(1/2)/a/(-a*d+b*c)/(b*x^3+a)
```

Rubi [A]
time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 105, 162, 65, 214}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^3}}{3a\,(a+bx^3)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*Sqrt[c + d*x^3])/(3*a*(b*c - a*d)*(a + b*x^3)) - (2*ArcTanh[Sqrt[c + d*x
^3]/Sqrt[c]])/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)}+\frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3a(bc-ad)}\\
&= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{3a^2}-\frac{(b(2bc-3ad))\text{Su}}{}\\
&= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)}+\frac{2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3a^2d}-\frac{(b(2bc-3a}{}\\
&= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\Big(}{3a^2(bc-ad}
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 124, normalized size = 0.94

$$\frac{-\frac{ab\sqrt{c+dx^3}}{(-bc+ad)(a+bx^3)}+\frac{\sqrt{b}\,(2bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-((a*b*Sqrt[c + d*x^3])/((-(b*c) + a*d)*(a + b*x^3))) + (Sqrt[b]*(2*b*c -
3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)
^(3/2) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/(3*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.43, size = 915, normalized size = 6.93

method	result	size
default	Expression too large to display	915
elliptic	Expression too large to display	1677

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*b/a^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3
)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d
^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-
b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-b/a*(1/3/(a*d-b*c)*(d*x^3+c)^
(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-c*d^2)^(1/3)*(1/2*I*d*(
2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(
d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2
)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c
*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*
c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-2/3*arctanh((d*x^3
+c)^(1/2)/c^(1/2))/a^2/c^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x), x)
```

Fricas [A]
time = 2.94, size = 862, normalized size = 6.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(d*x^3 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b
*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*((b^2*c - a*b*d)*x^3 +
 a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))
/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c
)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3
 + b*c)) + ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqr
t(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3
*b*c*d)*x^3), 1/6*(2*sqrt(d*x^3 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^3 + a*b*c
 - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c^2 - 3*a^2*
c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c
 - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(
a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c)*
a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 +
 b*c)) + 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^3
 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 0.96, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^3+c}\,bd}{3\,(abc-a^2d)((dx^3+c)b-bc+ad)}-\frac{(2\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,(a^2bc-a^3d)\sqrt{-b^2c+abd}}+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x^3 + c)*b*d/((a*b*c - a^2*d)*((d*x^3 + c)*b - b*c + a*d)) - 1/3
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

Mupad [B]
time = 9.65, size = 162, normalized size = 1.23

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2\,\sqrt{c}}+\frac{b^2\,\sqrt{d\,x^3+c}}{3\,a\,\left(b\,x^3+a\right)\,\left(b^2\,c-a\,b\,d\right)}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,a^2\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a^2*c^(1/2)) + (b^2*(c + d*x^3)^(1/2))/(3*a*(a + b*x^3)*(b^2*c - a*b*d)) +
 (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i
 - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(3/2))
```

3.484 $\int \frac{1}{x^4(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=185

$$-\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)}-\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)}+\frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}}-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}$$

```
[Out] 1/3*(a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/3*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/3*b*(-a*d+2*b*c)*(d*x^3+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^3+a)-1/3*
(d*x^3+c)^(1/2)/a/c/x^3/(b*x^3+a)
```

Rubi [A]
time = 0.18, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 105, 156, 162, 65, 214}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}}-\frac{b\sqrt{c+dx^3}\,(2bc-ad)}{3a^2c\left(a+bx^3\right)(bc-ad)}-\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*(b*(2*b*c - a*d)*Sqrt[c + d*x^3])/(a^2*c*(b*c - a*d)*(a + b*x^3)) - Sq
rt[c + d*x^3]/(3*a*c*x^3*(a + b*x^3)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x
^3]/Sqrt[c]])/(3*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd}{x(a+bx)\sqrt{c+d}}\right.}{3a^2c(bc-a}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \frac{}{(a+}\right.}{6a^3(bc-}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \frac{}{a-}\right.}{3a^3d(bc-}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right.}{3a^3c^{3/2}}$$

Mathematica [A]
time = 0.62, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^3}\,(-a^2d+2b^2cx^3+ab(c-dx^3))}{c(-bc+ad)x^3(a+bx^3)} - \frac{b^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{3/2}}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*Sqrt[c + d*x^3]*(-(a^2*d) + 2*b^2*c*x^3 + a*b*(c - d*x^3)))/(c*(-(b*c)
+ a*d)*x^3*(a + b*x^3)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcTanh
[Sqrt[c + d*x^3]/Sqrt[c]])/c^(3/2))/(3*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.49, size = 961, normalized size = 5.19

method	result	size
risch	Expression too large to display	955
default	Expression too large to display	961

elliptic	Expression too large to display	1733

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3*I*b^2/a^3/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+b^2/a^2*(1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))+4/3/a^3*b*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^4), x)

Fricas [A]
time = 2.70, size = 1236, normalized size = 6.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)
*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*
d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*log((d*x^3 +
2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c
^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a
^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a
*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*
c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*l
og((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b*c^2 - a^3*c*d
+ (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2
*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(-c)*arcta
n(sqrt(d*x^3 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2
*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d -
 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b
*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*
c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/3*(((4*b^3*c^3 -
5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
 + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d
- a^3*d^2)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2*b*c^2 -
a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a
^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a+bx^3\right)^2 \sqrt{c+dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.71, size = 257, normalized size = 1.39

$$\frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,(a^3bc-a^4d)\sqrt{-b^2c+abd}}-\frac{2\,(dx^3+c)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^3+c}\,b^2c^2d-(dx^3+c)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^3+c}\,abcd^2-\sqrt{dx^3+c}\,a^2d^3}{3\,(a^2bc^2-a^3cd)\left((dx^3+c)^2b-2\,(dx^3+c)bc+bc^2+(dx^3+c)ad-acd\right)}-\frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^3 + c)*b^2*c^2*d - (d*x^3 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^3 + c
)*a*b*c*d^2 - sqrt(d*x^3 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^3 + c)^
2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)) - 1/3*(4*b*c +
a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B]
time = 11.55, size = 355, normalized size = 1.92

$$\frac{\sqrt{d\,x^3+c}\,\left(\frac{d\,a^2+4\,b\,c\,a}{2\,a^3\,c^2}-\frac{a\,\left(\frac{2\,c\,b^2+2\,a\,d\,b}{2\,a^3\,c^2}-\frac{a\,\left(\frac{b^2\,d}{2\,a^3\,c^2}+\frac{b\,\left(2\,c\,b^2+2\,a\,d\,b\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}-\frac{b^2\,d\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^2\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}+\frac{b\,\left(d\,a^2+4\,b\,c\,a\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}\right)}{b\,x^3+a}-\frac{\sqrt{d\,x^3+c}}{3\,a^2\,c\,x^3}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\,{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}^3}{x^6}\right)\,\left(a\,d+4\,b\,c\right)}{6\,a^3\,c^{3/2}}+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(5\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,a^3\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((c + d*x^3)^(1/2)*((a^2*d + 4*a*b*c)/(2*a^3*c^2) - (a*((2*b^2*c + 2*a*b*d)
/(2*a^3*c^2) - (a*((b^2*d)/(2*a^3*c^2) + (b*(2*b^2*c + 2*a*b*d)*(3*a*d - 4*
b*c))/(6*a^3*c^2*(a^2*d - a*b*c)) - (b^2*d*(3*a*d - 4*b*c))/(6*a^2*c^2*(a^2
*d - a*b*c))))/b + (b*(a^2*d + 4*a*b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*d
- a*b*c))))/b))/(a + b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*c*x^3) + (log((((c +
 d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(a*d + 4*b*c
))/(6*a^3*c^(3/2)) + (b^(3/2)*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*
(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b*c)*1i)/(6*a^3*(a*
d - b*c)^(3/2))
```

3.485 $\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,2,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d*
x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}} = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(64) = 128.
time = 10.14, size = 238, normalized size = 3.72

$$\frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(c+dx^3+\frac{8ac^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}\right)}{a+bx^3}\right)}{24(-bc+ad)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)])/a + (8*(c + d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^
3)/c), -((b*x^3)/a)])/(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b
*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)
/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a +
b*x^3)))/(24*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.36, size = 1207, normalized size = 18.86

method	result

	$i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\,\sqrt{-}$
elliptic	$\frac{x\sqrt{d\,x^{3}+c}}{3(ad-bc)\left(b\,x^{3}+a\right)}-$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x
-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2
)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)
/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-a/b*(-1/3*b/a/(a*d-b*c)
*x*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/a/(a*d-b*c)*3^(1/2)*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/d^2*2^(1/2)*sum((-7*a*d+4*b*c)/(
a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*
```

```
(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c
*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_a
lpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{(b\,x^3+a)^2\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.486 $\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,2,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d*
x^3+c)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}} = \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(64) = 128.
time = 10.11, size = 172, normalized size = 2.69

$$\frac{10abx^2(c+dx^3)+5(bc-3ad)x^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-bdx^5(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{30a^2(bc-ad)\,(a+bx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (10*a*b*x^2*(c + d*x^3) + 5*(b*c - 3*a*d)*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)/
c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - b*d*x^5*(a + b*
x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)
/a)])/(30*a^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.33, size = 923, normalized size = 14.42

method	result	size
default	Expression too large to display	923
elliptic	Expression too large to display	923

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/a/(a*d-b*c)*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/a/(a*d-b*c)*3^(1/2)*
(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
```

```
*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/d^2*2^(1/2)*sum((-5*a*d+2*b*c)/(
a*d-b*c)^2/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d
^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1
/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{{\left(b\,x^3+a\right)}^2\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(x/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.487 $\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,2,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d*x^3+c)
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 392 vs. 2(59) = 118.
time = 10.17, size = 392, normalized size = 6.64

$$\frac{-8acxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\left(8a(3bc-3ad+bdx^3)+bdx^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)+3bx^4\left(8a(c+dx^3)+dx^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{24a^2(bc-ad)(a+bx^3)\sqrt{c+dx^3}\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)]*(8*a*(3*b*
c - 3*a*d + b*d*x^3) + b*d*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3
, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + 3*b*x^4*(8*a*(c + d*x^3) + d*
x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c)
, -((b*x^3)/a)])*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/(24*a^2*
(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3,
-((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*
x^3)/a)])))
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.34, size = 769, normalized size = 13.03 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/a/(a*d-b*c)*x*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/a/(a*d-b*c)*3^(1/2)*(-
c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
```

```
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/d^2*2^(1/2)*su
m((-7*a*d+4*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+
2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2
)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*
c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^3+a\right)^2\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.488 $\int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,2,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/x/(d*x^3+
c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*x*Sqrt[c + d*x^3]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{1}{x^2(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\, F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(62) = 124.
time = 10.17, size = 226, normalized size = 3.65

$$\frac{20a(c+dx^3)\left(3a^2d-4b^2cx^3-3ab(c-dx^3)\right)-5(8b^2c^2-15abcd+3a^2d^2)\,x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bd(4bc-3ad)x^6(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{60a^3c(bc-ad)x\,(a+bx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (20*a*(c + d*x^3)*(3*a^2*d - 4*b^2*c*x^3 - 3*a*b*(c - d*x^3)) - 5*(8*b^2*c^
2 - 15*a*b*c*d + 3*a^2*d^2)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^6*(a
+ b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*
x^3)/a)])/(60*a^3*c*(b*c - a*d)*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.45, size = 1818, normalized size = 29.32

method	result	size
elliptic	Expression too large to display	963
default	Expression too large to display	1818
risch	Expression too large to display	1819

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*b/a^2/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^
2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(
```

```
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-b/a*(-1/3*b/a/(a*d-b*c
)*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/a/(a*d-b*c)*3^(1/2)*(-c*d^2)^(1/3)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)))+1/18*I/a/d^2*2^(1/2)*sum((-5*a*d+2*b*c)/(a*d-b*c)^2/_alpha
*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*
d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*b+a)))+1/a^2*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,{\left(b\,x^3+a\right)}^2\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.489 $\int \frac{1}{x^3(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,2,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/x^2/(
d*x^3+c)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a^2*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{1}{x^3(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\, F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 411 vs. $2(64) = 128$.

time = 10.36, size = 411, normalized size = 6.42

$$\frac{bd(5bc-3ad)x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(32ac(-10b^2cx^3(3c+dx^3)+3a^2d(2c+3dx^3)+3ab(-2c^2+7cdx^3+2d^2x^6))F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+24x^3(c+dx^3)(-3a^2d+5b^2cx^3+3ab(c-dx^3))\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}}{48a^3c(-bc+ad)x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*d*(5*b*c - 3*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)] + (a*(32*a*c*(-10*b^2*c*x^3*(3*c + d*x^3) + 3*a^2*
d*(2*c + 3*d*x^3) + 3*a*b*(-2*c^2 + 7*c*d*x^3 + 2*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*x^3*(c + d*x^3)*(-3*a^2*d + 5*
b^2*c*x^3 + 3*a*b*(c - d*x^3))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/
c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/
a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a
)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3
*c*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.

time = 0.43, size = 1512, normalized size = 23.62

method	result

elliptic	$\frac{b^2 x\sqrt{d\,x^3+c}}{3a^2(ad-bc)(b\,x^3+a)} - \frac{\sqrt{d\,x^3+c}}{2c\,a^2x^2} - \frac{2i\left(\frac{bd}{6(ad-bc)a^2}-\frac{d}{4c\,a^2}\right)\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^2)^{\frac{1}{3}}}}}{}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*b/a^2/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2
*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d
*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*
d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*
d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c
*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(-1/2/c*(d*x^3
+c)^(1/2)/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^
2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I
*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1
/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
)^(1/2)))-b/a*(-1/3*b/a/(a*d-b*c)*x*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/a/(a*d-
b*c)*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2
```

```
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a
/d^2*2^(1/2)*sum((-7*a*d+4*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(
1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha
-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + bx^3)^2 \sqrt{c + dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,{\left(b\,x^3+a\right)}^2\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.490 $\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{-2b^2c^2-a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}}-\frac{a^2}{3b^2(bc-ad)\,(a+bx^3)\,\sqrt{c+dx^3}}+\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

```
[Out] 1/3*a*(-a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2
)/(-a*d+b*c)^(5/2)+1/3*(-a^2*d^2-2*b^2*c^2)/b^2/d/(-a*d+b*c)^2/(d*x^3+c)^(1
/2)-1/3*a^2/b^2/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 91, 79, 65, 214}

$$-\frac{a^2d^2+2b^2c^2}{3b^2d\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{a^2}{3b^2\,(a+bx^3)\,\sqrt{c+dx^3}\,(bc-ad)}+\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/3*(2*b^2*c^2 + a^2*d^2)/(b^2*d*(b*c - a*d)^2*Sqrt[c + d*x^3]) - a^2/(3*b
^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + (a*(4*b*c - a*d)*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{a^2}{3b^2(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc+ad)+b(bc-ad)x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right.}{3b^2(bc-ad)}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{(a(4bc-}{}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{(a(4bc-}{}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{a(4bc-}{}$$

Mathematica [A]

time = 0.37, size = 133, normalized size = 0.89

$$\frac{-\frac{\sqrt{b}\,(2abc^2+2b^2c^2x^3+a^2d(c+dx^3))}{d(bc-ad)^2(a+bx^3)\sqrt{c+dx^3}}+\frac{a(-4bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-((Sqrt[b]*(2*a*b*c^2 + 2*b^2*c^2*x^3 + a^2*d*(c + d*x^3)))/(d*(b*c - a*d)
^2*(a + b*x^3)*Sqrt[c + d*x^3])) + (a*(-4*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2))/(3*b^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 978, normalized size = 6.52

method	result
elliptic	$-\frac{a^2\sqrt{d\,x^3+c}}{3(ad-bc)^2 b(b\,x^3+a)}-\frac{2c^2}{3d(ad-bc)^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(ad-4bc)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-\cdots}{\cdots}\right.}{\cdots}}\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b^2/d/(d*x^3+c)^(1/2)-2*a/b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*
I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/
d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2
*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
```

```
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a
*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+a^2/b^2*(-1/3*b/(a*d-b*c)
^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*d/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/2*I*b/
d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3
))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2
*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1
/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(129) = 258.
time = 2.46, size = 746, normalized size = 4.97

$$\left[-\frac{((4\,ab^2cd^2-a^2bd^3)x^6+4\,a^2bc^2d-a^3cd^2+(4\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^3)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,(2\,ab^3c^3-a^2b^2c^2d-a^3bcd^2+(2\,b^4c^3-2\,ab^3c^2d+a^2b^2cd^2-a^3bd^3)x^3)\sqrt{dx^3+c}}{6\,(ab^5c^4d-3\,a^2b^4c^3d^2+3\,a^3b^3c^2d^3-a^4b^2cd^4+(b^6c^3d^2-3\,ab^5c^2d^3+3\,a^2b^4cd^4-a^3b^3d^5)x^6+(b^6c^4d-2\,ab^5c^3d^2+2\,a^3b^3cd^4-a^4b^2d^5)x^3)},\ -\frac{((4\,ab^2cd^2-a^2bd^3)x^6+4\,a^2bc^2d-a^3cd^2+(4\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^3)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+(2\,ab^3c^3-a^2b^2c^2d-a^3bcd^2+(2\,b^4c^3-2\,ab^3c^2d+a^2b^2cd^2-a^3bd^3)x^3)\sqrt{dx^3+c}}{3\,(ab^5c^4d-3\,a^2b^4c^3d^2+3\,a^3b^3c^2d^3-a^4b^2cd^4+(b^6c^3d^2-3\,ab^5c^2d^3+3\,a^2b^4cd^4-a^3b^3d^5)x^6+(b^6c^4d-2\,ab^5c^3d^2+2\,a^3b^3cd^4-a^4b^2d^5)x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3
+ 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(2*
a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d + a^2*
b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^4*c^3*d
```

```
^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*d^3 + 3
*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2*a^3*b^
3*c*d^4 - a^4*b^2*d^5)*x^3), -1/3*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2
*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)
) + (2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d
 + a^2*b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^
4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*
d^3 + 3*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2
*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 2.27, size = 195, normalized size = 1.30

$$-\frac{\left(4\,abc-a^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^3+c\right)b^2c^2-2\,b^2c^3+2\,abc^2d+\left(dx^3+c\right)a^2d^2}{3\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}\,bc+\sqrt{dx^3+c}\,ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(4*a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)*
b^2*c^2 - 2*b^2*c^3 + 2*a*b*c^2*d + (d*x^3 + c)*a^2*d^2)/((b^3*c^2*d - 2*a*
b^2*c*d^2 + a^2*b*d^3)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*
x^3 + c)*a*d))
```

Mupad [B]
time = 7.90, size = 367, normalized size = 2.45

$$\frac{\sqrt{d\,x^3+c}\,\left(x^3\,\left(\frac{\left(\frac{3\,b\,d\,\left(a\,d+b\,c\right)-b\,d\,\left(a\,d+2\,b\,c\right)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\frac{b\,d\,\left(a\,d+b\,c\right)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\,\left(a\,d+b\,c\right)}{b\,d}+\frac{a\,b\,c\,d}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)+\frac{a\,c\,\left(\frac{3\,b\,d\,\left(a\,d+b\,c\right)-b\,d\,\left(a\,d+2\,b\,c\right)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\frac{b\,d\,\left(a\,d+b\,c\right)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)}{b\,d}\right)}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^3+a\,c}+\frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] ((c + d*x^3)^(1/2)*(x^3*((((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*
b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^
2*d - 2*a*b^2*c*d^2))*(a*d + b*c))/(b*d) + (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d
 - 2*a*b^2*c*d^2)) + (a*c*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*
b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^
2*d - 2*a*b^2*c*d^2)))/(b*d)))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (a*log((
2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a
+ b*x^3))*(a*d - 4*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(5/2))
```

3.491 $\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}}+\frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}-\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{5/2}}$$

```
[Out] -1/3*(a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*
c)^(5/2)/b^(1/2)+1/3*(a*d+2*b*c)/b/(-a*d+b*c)^2/(d*x^3+c)^(1/2)+1/3*a/b/(-a
*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 79, 53, 65, 214}

$$\frac{a}{3b\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{ad+2bc}{3b\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*b*c + a*d)/(3*b*(b*c - a*d)^2*Sqrt[c + d*x^3]) + a/(3*b*(b*c - a*d)*(a +
 b*x^3)*Sqrt[c + d*x^3]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{a}{3b(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, \right.}{6b(bc-ad)}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{(2bc+ad)\text{Sub}}{}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{(2bc+ad)\text{Sub}}{}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{(2bc+ad)\tan}{3\sqrt{b}}$$

Mathematica [A]

time = 0.28, size = 110, normalized size = 0.82

$$\frac{1}{3}\left(\frac{3ac+2bcx^3+adx^3}{(bc-ad)^2\,(a+bx^3)\,\sqrt{c+dx^3}}+\frac{(2bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,(-bc+ad)^{5/2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] ((3*a*c + 2*b*c*x^3 + a*d*x^3)/((b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3]) + ((2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(5/2)))/3

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 958, normalized size = 7.15

method	result
elliptic	$\frac{a\sqrt{d\,x^3+c}}{3(ad-bc)^2(b\,x^3+a)}+\frac{2c}{3(ad-bc)^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-ad-2bc)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}}{\cdots}\right)}{\cdots}}\cdots}{\cdots}\right)}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1

```
/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d
*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3
^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=
RootOf(_Z^3*b+a))-a/b*(-1/3*b/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*d/
(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^2
)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d
^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c
*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/
2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^
2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^
2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(114) = 228$.
time = 2.92, size = 630, normalized size = 4.70

$$\left[\frac{((2\,b^2cd + abd^2)x^6 + 2\,abc^2 + a^2cd + (2\,b^2c^2 + 3\,abcd + a^2d^2)x^3)\sqrt{b^2c - abd}\,\log\left(\frac{bdx^3 + 2\,bc - ad - 2\,\sqrt{dx^3 + c}\,\sqrt{b^2c - abd}}{bx^3 + a}\right) + 2\,(3\,ab^2c^2 - 3\,a^2bcd + (2\,b^3c^2 - ab^2cd - a^2bd^2)x^3)\sqrt{dx^3 + c}}{6\,(ab^4c^4 - 3\,a^2b^3c^3d + 3\,a^3b^2c^2d^2 - a^4bcd^3 + (b^5c^3d - 3\,ab^4c^2d^2 + 3\,a^2b^3cd^3 - a^3b^2d^4)x^6 + (b^5c^4 - 2\,ab^4c^3d + 2\,a^3b^2cd^3 - a^4bd^4)x^3)}, \frac{((2\,b^2cd + abd^2)x^6 + 2\,abc^2 + a^2cd + (2\,b^2c^2 + 3\,abcd + a^2d^2)x^3)\sqrt{-b^2c + abd}\,\arctan\left(\frac{\sqrt{dx^3 + c}\,\sqrt{-b^2c + abd}}{bdx^3 + bc}\right) + (3\,ab^2c^2 - 3\,a^2bcd + (2\,b^3c^2 - ab^2cd - a^2bd^2)x^3)\sqrt{dx^3 + c}}{3\,(ab^4c^4 - 3\,a^2b^3c^3d + 3\,a^3b^2c^2d^2 - a^4bcd^3 + (b^5c^3d - 3\,ab^4c^2d^2 + 3\,a^2b^3cd^3 - a^3b^2d^4)x^6 + (b^5c^4 - 2\,ab^4c^3d + 2\,a^3b^2cd^3 - a^4bd^4)x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b
*c*d + a^2*d^2)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqr
t(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(3*a*b^2*c^2 - 3*a^2*b*c
*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c^4 -
 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d - 3*a*b^4*c
^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4*c^3*d + 2*
```

```
a^3*b^2*c*d^3 - a^4*b*d^4)*x^3), 1/3*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^
2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^3)*sqrt(-b^2*c + a*b*d)*a
rctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c^2
- 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(
a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d
- 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4
*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 1.18, size = 181, normalized size = 1.35

$$\frac{\frac{(2\,bcd+ad^2)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{2\,(dx^3+c)bcd-2\,bc^2d+(dx^3+c)\,ad^2+2\,acd^2}{(b^2c^2-2\,abcd+a^2d^2)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}\,bc+\sqrt{dx^3+c}\,ad\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^3 + c)*b*c*d -
2*b*c^2*d + (d*x^3 + c)*a*d^2 + 2*a*c*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*x^3 + c)*a*d)))/d
```

Mupad [B]
time = 7.70, size = 247, normalized size = 1.84

$$-\frac{\sqrt{d\,x^3+c}\,\left(x^3\,\left(\frac{3\,b\,d\,(a\,d+b\,c)-b\,d\,(a\,d+2\,b\,c)}{3\,(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d)}-\frac{b\,d\,(a\,d+b\,c)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)-\frac{a\,b\,c\,d}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)}{b\,d\,x^6+(a\,d+b\,c)\,x^3+a\,c}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,(a\,d+2\,b\,c)\,1{}\mathrm{i}}{6\,\sqrt{b}\,{(a\,d-b\,c)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^
3)/(a + b*x^3))*(a*d + 2*b*c)*1i)/(6*b^(1/2)*(a*d - b*c)^(5/2)) - ((c + d*x
^3)^(1/2)*(x^3*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3
*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^2*d - 2*a*b
^2*c*d^2)) - (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)))/(a*c + x^3
*(a*d + b*c) + b*d*x^6)
```

3.492 $\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}} dx$

Optimal. Leaf size=108

$$-\frac{d}{(bc-ad)^2\sqrt{c+dx^3}}-\frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}+\frac{\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(5/2
)-d/(-a*d+b*c)^2/(d*x^3+c)^(1/2)-1/3/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {455, 44, 53, 65, 214}

$$-\frac{d}{\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{1}{3\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(d/((b*c - a*d)^2*Sqrt[c + d*x^3])) - 1/(3*(b*c - a*d)*(a + b*x^3)*Sqrt[c
+ d*x^3]) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/
(b*c - a*d)^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{3(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{2(bc-ad)}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{(bd)\text{Subst}\left(\int \frac{}{(a+}\right.}{2(}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{b\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+}\right.}{(b}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{\sqrt{b}\,d\tanh^{-1}\left(\right.}{(bc-}$$

Mathematica [A]
time = 0.26, size = 101, normalized size = 0.94

$$\frac{-2ad-b(c+3dx^3)}{3(bc-ad)^2(a+bx^3)\sqrt{c+dx^3}} - \frac{\sqrt{b}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*a*d - b*(c + 3*d*x^3))/(3*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3]) -
(Sqrt[b]*d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) +
a*d)^(5/2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.33, size = 485, normalized size = 4.49

method	result
default	$-\frac{b\sqrt{d\,x^{3}+c}}{3(ad-bc)^{2}(b\,x^{3}+a)}-\frac{2d}{3(ad-bc)^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,(-}{}\right)}{(-c}}}{}\right)}{}$
elliptic	$-\frac{b\sqrt{d\,x^{3}+c}}{3(ad-bc)^{2}(b\,x^{3}+a)}-\frac{2d}{3(ad-bc)^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,(-}{}\right)}{(-c}}}{}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*d/(a*d-b*c)^2/((x^3+c/d)*d
)^(1/2)+1/2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1
/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^
(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(
a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

<u>Maxima</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(92) = 184.
time = 3.13, size = 450, normalized size = 4.17

$$\left[\frac{3\,(bd^2x^6+(bcd+ad^2)x^3+acd)\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-2\,(3\,bdx^3+bc+2\,ad)\sqrt{dx^3+c}}{6\,((b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^6+ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^3)},\ \frac{3\,(bd^2x^6+(bcd+ad^2)x^3+acd)\sqrt{-\frac{b}{bc-ad}}\,\arctan\left(-\frac{\sqrt{dx^3+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-(3\,bdx^3+bc+2\,ad)\sqrt{dx^3+c}}{3\,((b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^6+ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d^2*x^6 + (b*c*d + a*d^2)*x^3 + a*c*d)*sqrt(b/(b*c - a*d))*log((
b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/
(b*x^3 + a)) - 2*(3*b*d*x^3 + b*c + 2*a*d)*sqrt(d*x^3 + c))/((b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^
3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3), 1/3*(3*(b*d^2*x^6 + (b*c
*d + a*d^2)*x^3 + a*c*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c
- a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (3*b*d*x^3 + b*c + 2*a*d)*sq
```

```
rt(d*x^3 + c))/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*
x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 1.44, size = 153, normalized size = 1.42

$$-\frac{bd \arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}} - \frac{3\,(dx^3+c)bd-2\,bcd+2\,ad^2}{3\,(b^2c^2-2\,abcd+a^2d^2)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}\,bc+\sqrt{dx^3+c}\,ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(3*(d*x^3 + c)*b*d - 2*b*c*d + 2*a*d^
2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*
b*c + sqrt(d*x^3 + c)*a*d))
```

Mupad [B]
time = 7.43, size = 199, normalized size = 1.84

$$-\frac{\left(\frac{3\,b\,d\,(a\,d+b\,c)-b\,d\,(a\,d+2\,b\,c)}{3\,(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d)}+\frac{b^2\,d^2\,x^3}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\sqrt{d\,x^3+c}}{b\,d\,x^6+(a\,d+b\,c)\,x^3+a\,c}+\frac{\sqrt{b}\,d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{2\,(a\,d-b\,c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (b^(1/2)*d*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2
i - b*d*x^3)/(a + b*x^3))*1i)/(2*(a*d - b*c)^(5/2)) - (((3*b*d*(a*d + b*c)
- b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (b^2*d^2
*x^3)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2))*(c + d*x^3)^(1/2))/(a*c + x^
3*(a*d + b*c) + b*d*x^6)
```

3.493 $\int \frac{1}{x(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=172

$$\frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}}+\frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}+\frac{b^{3/2}(2bc-5ad)\tanh^{-}}{3a^2(bc-}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(3/2)+1/3*b^(3/2)*(-5*a*d+2*b*c
)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(5/2)+1/
3*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/(d*x^3+c)^(1/2)+1/3*b/a/(-a*d+b*c)/(b*x^3+
a)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 105, 157, 162, 65, 214}

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}+\frac{b}{3a\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{d(2ad+bc)}{3ac\sqrt{c+dx^3}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(3*a*c*(b*c - a*d)^2*Sqrt[c + d*x^3]) + b/(3*a*(b*c - a*d
)*(a + b*x^3)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^
2*c^(3/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqr
t[b*c - a*d]])/(3*a^2*(b*c - a*d)^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$=\frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}+\frac{\text{Subst}\left(\int \frac{bc-ad+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{3a(bc-ad)}$$

$$=\frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}}+\frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}-\frac{2\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$=\frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}}+\frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}+\frac{\text{Subst}\left(\int \frac{\cdots}{x\sqrt{\cdots}}\right.}{\cdots}$$

$$=\frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}}+\frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}+\frac{2\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$=\frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}}+\frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}-\frac{2\tanh^{-1}\left(\cdots\right)}{3a^2\cdots}$$

Mathematica [A]
time = 0.76, size = 157, normalized size = 0.91

$$\frac{\frac{a\left(2a^2d^2+2abd^2x^3+b^2c\left(c+dx^3\right)\right)}{c(bc-ad)^2\left(a+bx^3\right)\sqrt{c+dx^3}}-\frac{b^{3/2}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{3/2}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((a*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3)))/(c*(b*c - a*d)^2*(a +
b*x^3)*Sqrt[c + d*x^3]) - (b^(3/2)*(2*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) - (2*ArcTanh[Sqrt[c + d*
x^3]/Sqrt[c]])/c^(3/2))/(3*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.45, size = 1002, normalized size = 5.83

method	result	size
default	Expression too large to display	1002

elliptic	Expression too large to display	1720

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+
b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^
2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d
^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)
^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*
b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*b+a)))-b/a*(-1/3*b/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3
*d/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*
d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c
*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)
))+1/a^2*(2/3/c/((x^3+c/d)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^
(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(144) = 288$.
time = 3.70, size = 1819, normalized size = 10.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 +
 (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log
((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d))
)/(b*x^3 + a)) - 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3
 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*
b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))
/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^
3*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
 a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*
b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(
-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b
*d*x^3 + b*c)) + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 -
 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3
)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + (a*b^2*
c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^
3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^
2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5
*c^2*d^3)*x^3), 1/6*(4*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a
^3*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (2*a*b^2*c^4 - 5
*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c
^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d
 - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a*b
^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/
(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3
*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b
^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^3 + b*c)) + 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^
3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a*b^2*c^3 + 2*a^3*c*
d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^3*b^2*c^5 - 2*
a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*
d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^3)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 1.28, size = 226, normalized size = 1.31

$$-\frac{\left(2 b^{3} c - 5 a b^{2} d\right) \arctan\left(\frac{\sqrt{d x^{3} + c}\, b}{\sqrt{-b^{2} c + a b d}}\right)}{3 \left(a^{2} b^{2} c^{2} - 2 a^{3} b c d + a^{4} d^{2}\right) \sqrt{-b^{2} c + a b d}} + \frac{(d x^{3} + c) b^{2} c d + 2 (d x^{3} + c) a b d^{2} - 2 a b c d^{2} + 2 a^{2} d^{3}}{3 \left(a b^{2} c^{3} - 2 a^{2} b c^{2} d + a^{3} c d^{2}\right) \left((d x^{3} + c)^{\frac{3}{2}} b - \sqrt{d x^{3} + c}\, b c + \sqrt{d x^{3} + c}\, a d\right)} + \frac{2 \arctan\left(\frac{\sqrt{d x^{3} + c}}{\sqrt{-c}}\right)}{3 a^{2} \sqrt{-c}\, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*((d*x^3 +
 c)*b^2*c*d + 2*(d*x^3 + c)*a*b*d^2 - 2*a*b*c*d^2 + 2*a^2*d^3)/((a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + s
qrt(d*x^3 + c)*a*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
)
```

Mupad [B]
time = 12.18, size = 288, normalized size = 1.67

$$\frac{\ln\left(\frac{\left(\sqrt{d x^3+c}-\sqrt{c}\right)^3 \left(\sqrt{d x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3 a^2 c^{3/2}} + \frac{\left(\frac{(2 a d+b c)^4+(2 a d+b c)^2 ((a d+2 b c) (2 a d+b c)-9 a b c d)}{9 a c (2 a d+b c)^2 (a^2 d^2-2 a b c d+b^2 c^2)} + \frac{b d x^3 (2 a d+b c)}{3 a c (a^2 d^2-2 a b c d+b^2 c^2)}\right) \sqrt{d x^3+c}}{b d x^6+(a d+b c) x^3+a c} + \frac{b^{3/2} \ln\left(\frac{2 b c-a d+b d x^3+\sqrt{b} \sqrt{d x^3+c} \sqrt{a d-b c}\, 2i}{b x^3+a}\right) (5 a d-2 b c)\, 1i}{6 a^2 (a d-b c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a^2*c^(3/2)) + ((((2*a*d + b*c)^4 + (2*a*d + b*c)^2*((a*d + 2*b*c)*(2*a*d
+ b*c) - 9*a*b*c*d))/(9*a*c*(2*a*d + b*c)^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
) + (b*d*x^3*(2*a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(c + d
*x^3)^(1/2))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (b^(3/2)*log((2*b*c - a*d
+ b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5
*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(5/2))
```

3.494 $\int \frac{1}{x^4(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{d(2b^2c^2-2abcd+3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}}-\frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}-\frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}}+\frac{(4bc+3ad)\,\text{ta}}{}$$

```
[Out] 1/3*(3*a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(5/2)-1/3*b^(5/2)*
(-7*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+
b*c)^(5/2)-1/3*d*(3*a^2*d^2-2*a*b*c*d+2*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(d*x^
3+c)^(1/2)-1/3*b*(-a*d+2*b*c)/a^2/c/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)-1/
3/a/c/x^3/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 105, 156, 157, 162, 65, 214}

$$-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}}-\frac{d(3a^2d^2-2abcd+2b^2c^2)}{3a^2c^2\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{b(2bc-ad)}{3a^2c\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}-\frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/3*(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*Sqrt[c
+ d*x^3]) - (b*(2*b*c - a*d))/(3*a^2*c*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x
^3]) - 1/(3*a*c*x^3*(a + b*x^3)*Sqrt[c + d*x^3]) + ((4*b*c + 3*a*d)*ArcTanh
[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= -\frac{1}{3acx^3(a+bx^3)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+3ad)+\frac{5bdx}{2}}{x(a+bx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)}{3ac}$$

$$= -\frac{b(2bc-ad)}{3a^2c(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{1}{3acx^3(a+bx^3)\sqrt{c+dx^3}} - \text{Subs}$$

$$= -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{}{3acx}$$

$$= -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{}{3acx}$$

$$= -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{}{3acx}$$

$$= -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{}{3acx}$$

Mathematica [A]
time = 0.96, size = 223, normalized size = 0.93

$$\frac{-\frac{a(2b^3c^2x^3(c+dx^3)+a^3d^2(c+3dx^3)+ab^2c(c^2-cdx^3-2d^2x^6)+a^2bd(-2c^2-cdx^3+3d^2x^6))}{c^2(bc-ad)^2x^3(a+bx^3)\sqrt{c+dx^3}} + \frac{b^{5/2}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}} + \frac{(4bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{5/2}}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-((a*(2*b^3*c^2*x^3*(c + d*x^3) + a^3*d^2*(c + 3*d*x^3) + a*b^2*c*(c^2 - c
*d*x^3 - 2*d^2*x^6) + a^2*b*d*(-2*c^2 - c*d*x^3 + 3*d^2*x^6)))/(c^2*(b*c -
a*d)^2*x^3*(a + b*x^3)*Sqrt[c + d*x^3])) + (b^(5/2)*(4*b*c - 7*a*d)*ArcTan[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) + ((4*b
*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/c^(5/2))/(3*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.60, size = 1067, normalized size = 4.43

method	result	size
risch	Expression too large to display	1016
default	Expression too large to display	1067
elliptic	Expression too large to display	1763

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/a^3*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a
*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d
^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1
/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*b+a))+b^2/a^2*(-1/3*b/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3
+a)-2/3*d/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)
^3*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3
^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_a
lpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alph
a*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*
d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*b+a))+1/a^2*(-2/3*d/c^2/((x^3+c/d)*d)^(1/2)-1/3*(d*x^3+c)^(1/2)/c^2/x^3
+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-2/a^3*b*(2/3/c/((x^3+c/d)*d)^(
1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^4), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(209) = 418$.
time = 4.00, size = 2384, normalized size = 9.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d -
7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(b/(b*c -
 a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b
*c - a*d)))/(b*x^3 + a)) - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^
3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b
*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2
+ 3*a^4*c*d^3)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x
^3) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2
*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^
2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*
d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a
^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3), -1/6*(2
*((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*
b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
 - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (
4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6
+ (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt
(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b^2*c^4 - 2
*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c
*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3
)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9
 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b
^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3), -1/6*(2*((4*b^4*c^3*d - 5*a*b^3
*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d -
7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2
*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c
)*sqrt(-c)/c) + ((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3
*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt
(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*
sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^
2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*
c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((
a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4
*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*
d + a^6*c^4*d^2)*x^3), -1/3*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*
```

```
c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4
*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(
b*c - a*d))/(b*d*x^3 + b*c)) + ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*
c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a
^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*
d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2*
b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2
+ 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4
*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c
^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x
^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^3)^2 (c + dx^3)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [A]
time = 1.38, size = 367, normalized size = 1.52

$$\frac{(4\,b^4c - 7\,ab^3d)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,(a^3b^2c^2 - 2\,a^4bcd + a^5d^2)\sqrt{-b^2c+abd}} - \frac{2\,(dx^3+c)^2b^3c^2d - 2\,(dx^3+c)b^3c^3d - 2\,(dx^3+c)^2ab^2cd^2 + 3\,(dx^3+c)ab^2c^2d^2 + 3\,(dx^3+c)^2a^2bd^3 - 7\,(dx^3+c)a^2bcd^3 + 2\,a^2bc^2d^3 + 3\,(dx^3+c)a^3d^4 - 2\,a^3cd^4}{3\,(a^2b^2c^4 - 2\,a^3bc^3d + a^4c^2d^2)\left((dx^3+c)^{\frac{5}{2}}b - 2\,(dx^3+c)^{\frac{3}{2}}bc + \sqrt{dx^3+c}\,bc^2 + (dx^3+c)^{\frac{3}{2}}ad - \sqrt{dx^3+c}\,acd\right)} - \frac{(4\,bc + 3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3
+ c)^2*b^3*c^2*d - 2*(d*x^3 + c)*b^3*c^3*d - 2*(d*x^3 + c)^2*a*b^2*c*d^2 +
3*(d*x^3 + c)*a*b^2*c^2*d^2 + 3*(d*x^3 + c)^2*a^2*b*d^3 - 7*(d*x^3 + c)*a^2
*b*c*d^3 + 2*a^2*b*c^2*d^3 + 3*(d*x^3 + c)*a^3*d^4 - 2*a^3*c*d^4)/((a^2*b^2
*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*((d*x^3 + c)^(5/2)*b - 2*(d*x^3 + c)^(3
/2)*b*c + sqrt(d*x^3 + c)*b*c^2 + (d*x^3 + c)^(3/2)*a*d - sqrt(d*x^3 + c)*a
*c*d)) - 1/3*(4*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)
*c^2)
```

Mupad [B]
time = 19.63, size = 2500, normalized size = 10.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)

[Out] (2*b*log(1/x^6))/(3*a^3*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a^2*c^2*x^3) + (d*l
og(1/x^6))/(2*a^2*c^(5/2)) + (2*b*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)*(
c + d*x^3)^(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/2))
)/(3*a^3*c^(3/2)) + (d*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)*(c + d*x^3)^
(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/2)))/(2*a^2*c^
(5/2)) - (b^7*c^9*x^4*(c + d*x^3)^(1/2))/(2*(2*a^9*c^6*d^5*x + 2*a^9*c^5*d^
6*x^4 + a^5*b^4*c^9*d^2*x^4 + a^6*b^3*c^8*d^3*x^4 - 3*a^7*b^2*c^7*d^4*x^4 +
a^5*b^4*c^8*d^3*x^7 - 3*a^7*b^2*c^6*d^5*x^7 - 3*a^8*b*c^7*d^4*x + a^6*b^3*
c^9*d^2*x - a^8*b*c^6*d^5*x^4 + 2*a^8*b*c^5*d^6*x^7)) - (5*a^9*d^7*x^4*(c +
d*x^3)^(1/2))/(4*(a^6*b^5*c^9*x + a^5*b^6*c^9*x^4 - 3*a^7*b^4*c^7*d^2*x^4
- a^8*b^3*c^6*d^3*x^4 + 2*a^9*b^2*c^5*d^4*x^4 - 3*a^7*b^4*c^6*d^3*x^7 + 2*a
^8*b^3*c^5*d^4*x^7 - 3*a^8*b^3*c^7*d^2*x + 2*a^9*b^2*c^6*d^3*x + a^6*b^5*c^
8*d*x^4 + a^5*b^6*c^8*d*x^7)) + (3*a^2*d^2*x*(c + d*x^3)^(1/2))/(a^2*b^2*c^
4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4
+ a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) + (2*b^2*c^2*x*(c + d*x^3)^(1/2
))/(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a
^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) - (b^(7/2)*c*log(
(a^6*b^(15/2)*c^10*36i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (
a^7*b^(13/2)*c^9*d*198i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) +
(a^12*b^(3/2)*c^4*d^6*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2))
- (a^11*b^(5/2)*c^5*d^5*126i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2
)) + (a^10*b^(7/2)*c^6*d^4*360i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(
1/2)) - (a^9*b^(9/2)*c^7*d^3*540i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)
^(1/2)) + (a^8*b^(11/2)*c^8*d^2*450i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b
*c)^(1/2)) + (a^6*b^(15/2)*c^9*d*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d
- b*c)^(1/2)) - (a^11*b^(5/2)*c^4*d^6*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^
3*(a*d - b*c)^(1/2)) + (a^10*b^(7/2)*c^5*d^5*x^3*90i)/(a*(a*d - b*c)^(1/2)
+ b*x^3*(a*d - b*c)^(1/2)) - (a^9*b^(9/2)*c^6*d^4*x^3*180i)/(a*(a*d - b*c)^
(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^8*b^(11/2)*c^7*d^3*x^3*180i)/(a*(a*d
- b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c^8*d^2*x^3*90i)/(a
*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (36*a^6*b^7*c^9*(c + d*x^3)
^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) +
(360*a^8*b^5*c^7*d^2*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(
1/2) + b*x^3*(a*d - b*c)^(1/2)) - (360*a^9*b^4*c^6*d^3*(c + d*x^3)^(1/2)*(a
*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (180*a^1
0*b^3*c^5*d^4*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b
x^3(a*d - b*c)^(1/2)) - (36*a^11*b^2*c^4*d^5*(c + d*x^3)^(1/2)*(a*d - b*c
)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (180*a^7*b^6*c^8
d(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d -
b*c)^(1/2)))*2i)/(3*a^3*(a*d - b*c)^(5/2)) + (b^(5/2)*d*log((a^6*b^(15/2)*
c^10*36i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c

```
^9*d*198i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^12*b^(3/2)*
c^4*d^6*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^11*b^(5/2
)*c^5*d^5*126i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^10*b^(
7/2)*c^6*d^4*360i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^9*b
^(9/2)*c^7*d^3*540i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^8
*b^(11/2)*c^8*d^2*450i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (
a^6*b^(15/2)*c^9*d*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2))
 - (a^11*b^(5/2)*c^4*d^6*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^
(1/2)) + (a^10*b^(7/2)*c^5*d^5*x^3*90i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d -
 b*c)^(1/2)) - (a^9*b^(9/2)*c^6*d^4*x^3*180i)/(a*(a*d - b*c)^(1/2) + b*x^3*
(a*d - b*c)^(1/2)) + (a^8*b^(11/2)*c^7*d^3*x^3*180i)/(a*(a*d - b*c)^(1/2) +
 b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c^8*d^2*x^3*90i)/(a*(a*d - b*c)^(
1/2) + b*x^3*(a*d - b*c)^(1/2)) + (36*a^6*b^7*c^9*(c + d*x^3)^(1/2)*(a*d -
b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (360*a^8*b^5*
c^7*d^2*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(
a*d - b*c)^(1/2)) - (360*a^9*b^4*c^6*d^3*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2
))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (180*a^10*b^3*c^5*d^4*
(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*
c)^(1/2)) - (36*a^11*b^2*c^4*d^5*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a
*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (180*a^7*b^6*c^8*d*(c + d*x^3)
^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)))*
7i)/(6*a^2*(a*d - b*c)^(5/2)) + (5*a^4*d^4*x^4*(c + d*x^3)^(1/2))/(2*(a^4*b
^2*c^6*x + a^3*b^3*c^6*x^4 + 2*a^5*b*c^5*d*x + 2*a^4*b^2*c^4*d^2*x^7 + 3*a^
4*b^2*c^5*d*x^4 + 2*a^5*b*c^4*d^2*x^4 + a^3*b^3*c^5*d*x^7)) - (65*a^3*d^3*x
^4*(c + d*x^3)^(1/2))/(24*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*
x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2...
```

3.495 $\int \frac{x^3}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,2,3/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/(
d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*c*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{x^3}{(a+bx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\, F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 381 vs. 2(67) = 134.
time = 10.20, size = 381, normalized size = 5.69

$$-\frac{x^4\left(-8abcdF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\left(8a+(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)+\left(8a(2ad+b(c+3dx^3))+3bdx^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{8a(bc-ad)^2(a+bx^3)\sqrt{c+dx^3}\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/8*(x^4*(-8*a*b*c*d*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
]*(8*a + (a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^
3)/c), -((b*x^3)/a)]) + (8*a*(2*a*d + b*(c + 3*d*x^3)) + 3*b*d*x^3*(a + b*x
^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/
a)])*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Ap
pellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/(a*(b*c - a*d)^2*(a
 + b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
-((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.36, size = 1593, normalized size = 23.78

method	result

elliptic	$-\frac{bx\sqrt{d\,x^3+c}}{3(ad-bc)^2(b\,x^3+a)}-\frac{2dx}{3(ad-bc)^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^2)^{\frac{1}{3}}}}\sqrt{\ldots}}{\ldots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2/3*d*x/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*
d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I*b/d^2*2^(1/2)*sum(1
/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
)),_alpha=RootOf(_Z^3*b+a)))-a/b*(1/3*b^2/a/(a*d-b*c)^2*x*(d*x^3+c)^(1/2)/(
b*x^3+a)+2/3*d^2*x/c/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-2/3*I*(1/6*b*d/a/(a*d-
b*c)^2+1/3*d^2/(a*d-b*c)^2/c)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-
```

```
1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2))+1/18*I/a/d^2*b*2^(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^3/_a
lpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*b+a)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(b\,x^3+a\right)^2\,{\left(d\,x^3+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.496 $\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=67

$$\frac{x^2 \sqrt{1+\frac{dx^3}{c}} F_1\left(\frac{2}{3};2,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2c\sqrt{c+dx^3}}$$

[Out] 1/2*x^2*AppellF1(2/3,2,3/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/(d*x^3+c)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{x^2 \sqrt{\frac{dx^3}{c}+1} F_1\left(\frac{2}{3};2,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 3/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)])/(2*a^2*c*Sqrt[c + d*x^3])

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{\left(a+bx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};2,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2c\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 216 vs. 2(67) = 134.
time = 10.18, size = 216, normalized size = 3.22

$$-\frac{x^2\left(-10a\left(2a^2d^2+2abd^2x^3+b^2c\left(c+dx^3\right)\right)+5\left(-b^2c^2+6abcd+a^2d^2\right)\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+bd(bc+2ad)x^3\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{30a^2c(bc-ad)^2\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/30*(x^2*(-10*a*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3)) + 5*(-(b^
2*c^2) + 6*a*b*c*d + a^2*d^2)*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3,
 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + b*d*(b*c + 2*a*d)*x^3*(a + b*x^
3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a
)]))/(a^2*c*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.34, size = 986, normalized size = 14.72

method	result	size
default	Expression too large to display	986
elliptic	Expression too large to display	986

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2/a/(a*d-b*c)^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/3*d^2*x^2/c/(a*d-b*c)
^2/((x^3+c/d)*d)^(1/2)-2/3*I*(-1/6*b*d/a/(a*d-b*c)^2-1/3*d^2/(a*d-b*c)^2/c)
*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
```

```
(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/d^2*b*2^(1/2)*sum((11
*a*d-2*b*c)/(a*d-b*c)^3/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(b\,x^3+a\right)^2\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(x/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.497 $\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,2,3/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/(d*x^3+
c)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*c*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}} dx = \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{1}{(a+bx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}} dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{dx^3}{c}}\, F_1\left(\frac{1}{3}; 2, \frac{3}{2}; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a^2 c\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 381 vs. 2(62) = 124.
time = 10.35, size = 381, normalized size = 6.15

$$\frac{x\left(bd(bc+2ad)x^3\sqrt{1+\frac{dx^3}{c}}\, F_1\left(\frac{4}{3}; \frac{1}{2}, 1; \frac{7}{3}; -\frac{dx^3}{c}, -\frac{bx^3}{a}\right) + \frac{a\left(64ac(3a^2d^2+2abd(-3c+dx^3)+b^2c(3c+dx^3))F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3(2a^2d^2+2abd^2x^3+b^2c(c+dx^3))\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}\right)}{24a^2c(bc-ad)^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(b*d*(b*c + 2*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)] + (a*(64*a*c*(3*a^2*d^2 + 2*a*b*d*(-3*c + d*x^3)
+ b^2*c*(3*c + d*x^3))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a
)] - 24*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3))*(2*b*c*AppellF1
[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1,
7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*a*c*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/
3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c
), -((b*x^3)/a)])))))/(24*a^2*c*(b*c - a*d)^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.34, size = 830, normalized size = 13.39 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2/a/(a*d-b*c)^2*x*(d*x^3+c)^(1/2)/(b*x^3+a)+2/3*d^2*x/c/(a*d-b*c)^2/(
(x^3+c/d)*d)^(1/2)-2/3*I*(1/6*b*d/a/(a*d-b*c)^2+1/3*d^2/(a*d-b*c)^2/c)*3^(1
/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/d^2*b*
```

```
2^(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^3/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*
x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d
^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*
d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^3+a\right)^2\,{\left(d\,x^3+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.498 $\int \frac{1}{x^2(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};2,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,2,3/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/x/(d*x^
3+c)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};2,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*c*x*Sqrt[c + d*x^3]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{1}{x^2(a+bx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};2,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 308 vs. 2(65) = 130.
time = 10.27, size = 308, normalized size = 4.74

$$\frac{-20a(4b^3c^2x^3(c+dx^3)+a^3d^2(3c+5dx^3)+3ab^2c(c^2-cdx^3-2d^2x^6)+a^2bd(-6c^2-3cdx^3+5d^2x^6))+5(-8b^3c^3+21ab^2c^2d-6a^2bcd^2+5a^3d^3)\,x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bd(4b^2c^2-6abcd+5a^2d^2)\,x^6(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{60a^3c^2(bc-ad)^2x\,(a+bx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-20*a*(4*b^3*c^2*x^3*(c + d*x^3) + a^3*d^2*(3*c + 5*d*x^3) + 3*a*b^2*c*(c^
2 - c*d*x^3 - 2*d^2*x^6) + a^2*b*d*(-6*c^2 - 3*c*d*x^3 + 5*d^2*x^6)) + 5*(-
8*b^3*c^3 + 21*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 5*a^3*d^3)*x^3*(a + b*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2
*b*d*(4*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c
]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*c^2*(b*c
- a*d)^2*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.48, size = 2383, normalized size = 36.66

method	result	size
elliptic	Expression too large to display	1019
risch	Expression too large to display	2334
default	Expression too large to display	2383

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(2/3*d*x^2/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)+2/9*I/c/(a*d-b*c)*3^(1/2)
*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
```

```
*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_a
lpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*
_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_al
pha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*b+a)))-b/a*(1/3*b^2/a/(a*d-b*c)^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/3*d^
2*x^2/c/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-2/3*I*(-1/6*b*d/a/(a*d-b*c)^2-1/3*d
^2/(a*d-b*c)^2/c)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)
^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(
x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/d^2*
b*2^(1/2)*sum((11*a*d-2*b*c)/(a*d-b*c)^3/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*
x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d
^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*
d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(-2/3*d*x^2/c^
2/((x^3+c/d)*d)^(1/2)-(d*x^3+c)^(1/2)/c^2/x-5/9*I/c^2*3^(1/2)*(-c*d^2)^(1/3
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
```

```
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + bx^3)^2 (c + dx^3)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,{\left(b\,x^3+a\right)}^2\,{\left(d\,x^3+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.499 $\int \frac{1}{x^3(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,2,3/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/x^2
/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a^2*c*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(a+bx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 515 vs. $2(67) = 134$.
time = 10.58, size = 515, normalized size = 7.69

$$\frac{-bd(5b^2c^2-6abcd+7a^2d^2)\,x^6\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(32ac\left(10b^3c^2x^3(3c+dx^3)+3a^3d^2(2c+7dx^3)+3ab^2c(2c^2-13cdx^3-4d^2x^6)+2a^2bd(-6c^2-6cdx^3+7d^2x^6)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3\left(5b^3c^2x^3(c+dx^3)+a^3d^2(3c+7dx^3)+3ab^2c(c^2-cdx^3-2d^2x^6)+a^2bd(-6c^2-3cdx^3+7d^2x^6)\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{(a+bx^3)\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}}{48a^3c^2(bc-ad)^2x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-(b*d*(5*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1
[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + (a*(32*a*c*(10*b^3*c^2*x^
3*(3*c + d*x^3) + 3*a^3*d^2*(2*c + 7*d*x^3) + 3*a*b^2*c*(2*c^2 - 13*c*d*x^3
 - 4*d^2*x^6) + 2*a^2*b*d*(-6*c^2 - 6*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(5*b^3*c^2*x^3*(c + d*x^3)
 + a^3*d^2*(3*c + 7*d*x^3) + 3*a*b^2*c*(c^2 - c*d*x^3 - 2*d^2*x^6) + a^2*b*
d*(-6*c^2 - 3*c*d*x^3 + 7*d^2*x^6))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*
x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x
^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(4
8*a^3*c^2*(b*c - a*d)^2*x^2*Sqrt[c + d*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.48, size = 1919, normalized size = 28.64

method	result	size
elliptic	Expression too large to display	863
risch	Expression too large to display	1874
default	Expression too large to display	1919

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(2/3*d*x/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(
-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I*b/d^2*2^(1/2)*su
m(1/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(-1/2/c^2*(d*x^3+c)^(1/2)/x^2-2/3*d*x
/c^2/((x^3+c/d)*d)^(1/2)+7/18*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)))-b/a*(1/3*b^2/a/(a*d-b*c)^2*x*(d*x^3+c)^(1/2)/(b*x^3
+a)+2/3*d^2*x/c/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-2/3*I*(1/6*b*d/a/(a*d-b*c)^
2+1/3*d^2/(a*d-b*c)^2/c)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(
-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/18*I/a/d^2*b*2^(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^3/_alpha^
2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha
*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d
-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
```

```
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + bx^3)^2 (c + dx^3)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**3*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(b\,x^3+a\right)}^2\,{\left(d\,x^3+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.500 $\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=134

$$\frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)}-\frac{a^2(2aB(1+m)-Ab(23+2m))(ex)^{1+m}\sqrt{a+bx^3}\ {}_2F_1\left(-\frac{5}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{be(1+m)(23+2m)\sqrt{1+\frac{bx^3}{a}}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(7/2)/b/e/(23+2*m)-a^2*(2*a*B*(1+m)-A*b*(23+2*m))
*(e*x)^(1+m)*hypergeom([-5/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1
/2)/b/e/(1+m)/(23+2*m)/(1+b*x^3/a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {470, 372, 371}

$$\frac{2B\left(a+bx^3\right)^{7/2}(ex)^{m+1}}{be(2m+23)}-\frac{a^2\sqrt{a+bx^3}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+23))\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+23)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(7/2))/(b*e*(23 + 2*m)) - (a^2*(2*a*B*(1 + m
) - A*b*(23 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-5/2, (
1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(23 + 2*m)*Sqrt[1 + (b*x^3
)/a])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{23}{2}+m\right)\right)\int (ex)^m \left(a+b\right.}{b\left(\frac{23}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{\left(a^2\left(aB(1+m)-Ab\left(\frac{23}{2}+m\right)\right)\sqrt{a+bx^3}\right.}{b\left(\frac{23}{2}+m\right)\sqrt{1+\frac{bx}{a}}}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{a^2(2aB(1+m)-Ab(23+2m))(ex)^{1+m}\sqrt{}}{be(1+m)(23+2m)}$$

Mathematica [A]
time = 0.81, size = 200, normalized size = 1.49

$$\frac{x(ex)^m\sqrt{a+bx^3}\left(\frac{a^2A\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{1+m}+\frac{a(2Ab+aB)x^3\,{}_2F_1\left(-\frac{1}{2},\frac{4+m}{3};\frac{7+m}{3};-\frac{bx^3}{a}\right)}{4+m}+bx^6\left(\frac{(Ab+2aB)\,{}_2F_1\left(-\frac{1}{2},\frac{7+m}{3};\frac{10+m}{3};-\frac{bx^3}{a}\right)}{7+m}+\frac{bBx^3\,{}_2F_1\left(-\frac{1}{2},\frac{10+m}{3};\frac{13+m}{3};-\frac{bx^3}{a}\right)}{10+m}\right)\right)}{\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^m*Sqrt[a + b*x^3]*((a^2*A*Hypergeometric2F1[-1/2, (1 + m)/3, (4 +
m)/3, -((b*x^3)/a)])/(1 + m) + (a*(2*A*b + a*B)*x^3*Hypergeometric2F1[-1/2,
 (4 + m)/3, (7 + m)/3, -((b*x^3)/a)])/(4 + m) + b*x^6*(((A*b + 2*a*B)*Hyper
geometric2F1[-1/2, (7 + m)/3, (10 + m)/3, -((b*x^3)/a)])/(7 + m) + (b*B*x^3
*Hypergeometric2F1[-1/2, (10 + m)/3, (13 + m)/3, -((b*x^3)/a)])/(10 + m)))
/Sqrt[1 + (b*x^3)/a]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (ex)^m \left(b\,x^3+a\right)^{\frac{5}{2}} \left(B\,x^3+A\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
[Out] int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(x*e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*(x*e)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 26.10, size = 388, normalized size = 2.90

$$\frac{Aa^{\frac{5}{2}}e^{m}xx^{m}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{1}{3}\\\frac{m}{3}+\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{2Aa^{\frac{3}{2}}be^{m}x^{4}x^{m}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\\frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}+\frac{A\sqrt{a}b^{2}e^{m}x^{7}x^{m}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{7}{3}\\\frac{m}{3}+\frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)}+\frac{Ba^{\frac{5}{2}}e^{m}x^{4}x^{m}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\\frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}+\frac{2Ba^{\frac{3}{2}}be^{m}x^{7}x^{m}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{7}{3}\\\frac{m}{3}+\frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)}+\frac{B\sqrt{a}b^{2}e^{m}x^{10}x^{m}\Gamma\left(\frac{m}{3}+\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{10}{3}\\\frac{m}{3}+\frac{13}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**3+a)**(5/2)*(B*x**3+A),x)
[Out] A*a**(5/2)*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 + 4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + 2*A*a**(3/2)*b*e**m*x*
*4*x**m*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + A*sqrt(a)*b**2*e**m*x**7*x**m*gamma(m
/3 + 7/3)*hyper((-1/2, m/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(m/3 + 10/3)) + B*a**(5/2)*e**m*x**4*x**m*gamma(m/3 + 4/3)*hyper((
-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 7/
3)) + 2*B*a**(3/2)*b*e**m*x**7*x**m*gamma(m/3 + 7/3)*hyper((-1/2, m/3 + 7/3
), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 10/3)) + B*sqrt(
a)*b**2*e**m*x**10*x**m*gamma(m/3 + 10/3)*hyper((-1/2, m/3 + 10/3), (m/3 +
13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 13/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(x*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^m\,{\left(b\,x^3+a\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(5/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(5/2), x)
```

3.501 $\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=132

$$\frac{2B(ex)^{1+m}\left(a+bx^3\right)^{5/2}}{be(17+2m)}-\frac{a(2aB(1+m)-Ab(17+2m))(ex)^{1+m}\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{be(1+m)(17+2m)\sqrt{1+\frac{bx^3}{a}}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(5/2)/b/e/(17+2*m)-a*(2*a*B*(1+m)-A*b*(17+2*m))*(
e*x)^(1+m)*hypergeom([-3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1/2
)/b/e/(1+m)/(17+2*m)/(1+b*x^3/a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {470, 372, 371}

$$\frac{2B\left(a+bx^3\right)^{5/2}(ex)^{m+1}}{be(2m+17)}-\frac{a\sqrt{a+bx^3}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+17))\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+17)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(5/2))/(b*e*(17 + 2*m)) - (a*(2*a*B*(1 + m)
- A*b*(17 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, (1
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(17 + 2*m)*Sqrt[1 + (b*x^3)/
a])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{2B(ex)^{1+m} \left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{17}{2}+m\right)\right) \int (ex)^m \left(a+b\right.}{b\left(\frac{17}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m} \left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{\left(a\left(aB(1+m)-Ab\left(\frac{17}{2}+m\right)\right)\sqrt{a+bx^3}\right.}{b\left(\frac{17}{2}+m\right)\sqrt{1+\frac{bx}{a}}}$$

$$= \frac{2B(ex)^{1+m} \left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{a(2aB(1+m)-Ab(17+2m))(ex)^{1+m}\sqrt{a}}{be(1+m)(17+2m)}$$

Mathematica [A]
time = 0.25, size = 149, normalized size = 1.13

$$\frac{x(ex)^m\sqrt{a+bx^3}\left(\frac{aA\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{1+m}+\frac{(Ab+aB)x^3\,{}_2F_1\left(-\frac{1}{2},\frac{4+m}{3};\frac{7+m}{3};-\frac{bx^3}{a}\right)}{4+m}+\frac{bBx^6\,{}_2F_1\left(-\frac{1}{2},\frac{7+m}{3};\frac{10+m}{3};-\frac{bx^3}{a}\right)}{7+m}\right)}{\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^m*Sqrt[a + b*x^3]*((a*A*Hypergeometric2F1[-1/2, (1 + m)/3, (4 + m)
/3, -((b*x^3)/a)])/(1 + m) + ((A*b + a*B)*x^3*Hypergeometric2F1[-1/2, (4 +
m)/3, (7 + m)/3, -((b*x^3)/a)])/(4 + m) + (b*B*x^6*Hypergeometric2F1[-1/2,
(7 + m)/3, (10 + m)/3, -((b*x^3)/a)])/(7 + m)))/Sqrt[1 + (b*x^3)/a]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(ex\right)^{m} \left(b\,x^{3}+a\right)^{\frac{3}{2}} \left(B\,x^{3}+A\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(x*e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*(x*e)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.58, size = 252, normalized size = 1.91

$$\frac{Aa^{\frac{3}{2}}e^{m}xx^{m}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{A\sqrt{a}\,be^{m}x^{4}x^{m}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}+\frac{Ba^{\frac{3}{2}}e^{m}x^{4}x^{m}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}+\frac{B\sqrt{a}\,be^{m}x^{7}x^{m}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{7}{3}\\ \frac{m}{3}+\frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 + 4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + A*sqrt(a)*b*e**m*x**4*
x**m*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + B*a**(3/2)*e**m*x**4*x**m*gamma(m/3 + 4/
3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(m/3 + 7/3)) + B*sqrt(a)*b*e**m*x**7*x**m*gamma(m/3 + 7/3)*hyper((-1/2, m/
3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(x*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^m\,{\left(b\,x^3+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(3/2),x)
[Out] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(3/2), x)
```

3.502 $\int (ex)^m \sqrt{a+bx^3} \left(A+Bx^3\right) dx$

Optimal. Leaf size=131

$$\frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)}-\frac{(2aB(1+m)-Ab(11+2m))(ex)^{1+m}\sqrt{a+bx^3}\ {}_2F_1\left(-\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{be(1+m)(11+2m)\sqrt{1+\frac{bx^3}{a}}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(3/2)/b/e/(11+2*m)-(2*a*B*(1+m)-A*b*(11+2*m))*(e*
x)^(1+m)*hypergeom([-1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1/2)/
b/e/(1+m)/(11+2*m)/(1+b*x^3/a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {470, 372, 371}

$$\frac{2B\left(a+bx^3\right)^{3/2}(ex)^{m+1}}{be(2m+11)}-\frac{\sqrt{a+bx^3}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+11))\ {}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+11)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(3/2))/(b*e*(11 + 2*m)) - ((2*a*B*(1 + m) -
A*b*(11 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, (1 +
m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(11 + 2*m)*Sqrt[1 + (b*x^3)/a]
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^m \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{11}{2}+m\right)\right)\int (ex)^m\sqrt{a+bx^3}}{b\left(\frac{11}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)} - \frac{\left(\left(aB(1+m)-Ab\left(\frac{11}{2}+m\right)\right)\sqrt{a+bx^3}\right)\int (}{b\left(\frac{11}{2}+m\right)\sqrt{1+\frac{bx^3}{a}}}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)} - \frac{(2aB(1+m)-Ab(11+2m))(ex)^{1+m}\sqrt{a+b}}{be(1+m)(11+2m)\sqrt{1}}$$

Mathematica [A]
time = 0.15, size = 110, normalized size = 0.84

$$\frac{x(ex)^m\sqrt{a+bx^3}\left(A(4+m)\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)+B(1+m)x^3\,{}_2F_1\left(-\frac{1}{2},\frac{4+m}{3};\frac{7+m}{3};-\frac{bx^3}{a}\right)\right)}{(1+m)(4+m)\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-1/2, (1 + m)/3, (4
 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-1/2, (4 + m)/3, (
7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (ex)^m \sqrt{b\,x^3+a}\,\left(B\,x^3+A\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

```
[Out] int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(x*e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(x*e)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.62, size = 122, normalized size = 0.93

$$\frac{A\sqrt{a}\,e^m x x^m \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{B\sqrt{a}\,e^m x^4 x^m \Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**3+a)**(1/2)*(B*x**3+A),x)
[Out] A*sqrt(a)*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 + 4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + B*sqrt(a)*e**m*x**4*x**
m*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(
I*pi)/a)/(3*gamma(m/3 + 7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(x*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^m\,\sqrt{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/2),x)
[Out] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/2), x)
```

3.503 $\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=131

$$\frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)}-\frac{(2aB(1+m)-Ab(5+2m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{be(1+m)(5+2m)\sqrt{a+bx^3}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(1/2)/b/e/(5+2*m)-(2*a*B*(1+m)-A*b*(5+2*m))*(e*x)
^(1+m)*hypergeom([1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/a)^(1/2)/b
/e/(1+m)/(5+2*m)/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {470, 372, 371}

$$\frac{2B\sqrt{a+bx^3}\,(ex)^{m+1}}{be(2m+5)}-\frac{\sqrt{\frac{bx^3}{a}+1}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+5))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+5)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*(e*x)^(1 + m)*Sqrt[a + b*x^3])/(b*e*(5 + 2*m)) - ((2*a*B*(1 + m) - A*b
*(5 + 2*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1 + m
)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(5 + 2*m)*Sqrt[a + b*x^3])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^m\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{5}{2}+m\right)\right)\int \frac{(ex)^m}{\sqrt{a+bx^3}}\,dx}{b\left(\frac{5}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} - \frac{\left(\left(aB(1+m)-Ab\left(\frac{5}{2}+m\right)\right)\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{bx^3}{a}}}}{b\left(\frac{5}{2}+m\right)\sqrt{a+bx^3}}$$

$$= \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} - \frac{(2aB(1+m)-Ab(5+2m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{\ }\right.}{be(1+m)(5+2m)\sqrt{a+bx^3}}$$

Mathematica [A]
time = 0.18, size = 110, normalized size = 0.84

$$\frac{x(ex)^m\sqrt{a+bx^3}\left(aB\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)+(Ab-aB)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)\right)}{ab(1+m)\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
[Out] (x*(e*x)^m*Sqrt[a + b*x^3]*(a*B*Hypergeometric2F1[-1/2, (1 + m)/3, (4 + m)/
3, -((b*x^3)/a)] + (A*b - a*B)*Hypergeometric2F1[1/2, (1 + m)/3, (4 + m)/3,
 -((b*x^3)/a)]))/(a*b*(1 + m)*Sqrt[1 + (b*x^3)/a])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m\left(B\,x^3+A\right)}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(x*e)^m/sqrt(b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*(x*e)^m/sqrt(b*x^3 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.23, size = 119, normalized size = 0.91

$$\frac{Ae^{m}xx^{m}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{3}+\frac{1}{3}\\\frac{m}{3}+\frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{Be^{m}x^{4}x^{m}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\\frac{m}{3}+\frac{7}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(1/2),x)
[Out] A*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((1/2, m/3 + 1/3), (m/3 + 4/3,), b*x**3
*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(m/3 + 4/3)) + B*e**m*x**4*x**m*gamma(m
/3 + 4/3)*hyper((1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(
3*sqrt(a)*gamma(m/3 + 7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(x*e)^m/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^3+A)\,(e\,x)^m}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/2),x)
[Out] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/2), x)
```

3.504 $\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=133

$$\frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}}+\frac{(2aB(1+m)+A(b-2bm))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{3abe(1+m)\sqrt{a+bx^3}}$$

```
[Out] 2/3*(A*b-B*a)*(e*x)^(1+m)/a/b/e/(b*x^3+a)^(1/2)+1/3*(2*a*B*(1+m)+A*(-2*b*m+
b))*(e*x)^(1+m)*hypergeom([1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/a
)^(1/2)/a/b/e/(1+m)/(b*x^3+a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {468, 372, 371}

$$\frac{\sqrt{\frac{bx^3}{a}+1}\,(ex)^{m+1}(2aB(m+1)+A(b-2bm))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{3abe(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(1 + m))/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*a*B*(1 + m) +
 A*(b - 2*b*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1
 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
```

```
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}} + \frac{(2aB(1+m)+A(b-2bm))\int \frac{(ex)^m}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}} + \frac{\left((2aB(1+m)+A(b-2bm))\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{bx^3}{a}}}\,d}{3ab\sqrt{a+bx^3}}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}} + \frac{(2aB(1+m)+A(b-2bm))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{3}\right.}{3abe(1+m)\sqrt{a+bx^3}}$$

Mathematica [A]
time = 0.31, size = 110, normalized size = 0.83

$$\frac{x(ex)^m\sqrt{1+\frac{bx^3}{a}}\left(aB\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)+(Ab-aB)\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)\right)}{ab(1+m)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(a*B*Hypergeometric2F1[1/2, (1 + m)/3, (4 +
m)/3, -((b*x^3)/a)] + (A*b - a*B)*Hypergeometric2F1[3/2, (1 + m)/3, (4 + m)
/3, -((b*x^3)/a)]))/(a*b*(1 + m)*Sqrt[a + b*x^3])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m (B\,x^3+A)}{(b\,x^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*(x*e)^m/(b*x^3 + a)^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(x*e)^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x
)

Sympy [C] Result contains complex when optimal does not.
time = 26.14, size = 119, normalized size = 0.89

$$\frac{Ae^m x x^m \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{Be^m x^4 x^m \Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(3/2),x)

[Out] A*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((3/2, m/3 + 1/3), (m/3 + 4/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(m/3 + 4/3)) + B*e**m*x**4*x**m*gamma(
m/3 + 4/3)*hyper((3/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/
(3*a**(3/2)*gamma(m/3 + 7/3))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")

```
[Out] integrate((B*x^3 + A)*(x*e)^m/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^3+A)\,(e\,x)^m}{(b\,x^3+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(3/2),x)
[Out] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(3/2), x)
```

3.505 $\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}} \, dx$

Optimal. Leaf size=133

$$\frac{2(Ab-aB)(ex)^{1+m}}{9abe\,(a+bx^3)^{3/2}}+\frac{(Ab(7-2m)+2aB(1+m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{9a^2be(1+m)\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(1+m)/a/b/e/(b*x^3+a)^(3/2)+1/9*(A*b*(7-2*m)+2*a*B*(1+m
))*(e*x)^(1+m)*hypergeom([3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/a)
^(1/2)/a^2/b/e/(1+m)/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {468, 372, 371}

$$\frac{\sqrt{\frac{bx^3}{a}+1}\,(ex)^{m+1}(2aB(m+1)+Ab(7-2m))\;{}_2F_1\left(\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{9a^2be(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(1 + m))/(9*a*b*e*(a + b*x^3)^(3/2)) + ((A*b*(7 - 2*m)
 + 2*a*B*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2,
(1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(9*a^2*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
```

```
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{1+m}}{9abe\,(a+bx^3)^{3/2}} + \frac{\left(2\left(-Ab\left(-\frac{7}{2}+m\right)+aB(1+m)\right)\right)\int \frac{(ex)^m}{(a+bx^3)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{9abe\,(a+bx^3)^{3/2}} + \frac{\left(2\left(-Ab\left(-\frac{7}{2}+m\right)+aB(1+m)\right)\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\left(1+\frac{bx^3}{a}\right)^{3/}}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{9abe\,(a+bx^3)^{3/2}} + \frac{(Ab(7-2m)+2aB(1+m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{1+}{3}\right.}{9a^2be(1+m)\sqrt{a+bx^3}}$$

Mathematica [A]
time = 0.71, size = 110, normalized size = 0.83

$$\frac{x(ex)^m\sqrt{1+\frac{bx^3}{a}}\left(aB\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)+(Ab-aB)\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)\right)}{a^2b(1+m)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(a*B*Hypergeometric2F1[3/2, (1 + m)/3, (4 +
m)/3, -((b*x^3)/a)] + (A*b - a*B)*Hypergeometric2F1[5/2, (1 + m)/3, (4 + m)
/3, -((b*x^3)/a)]))/(a^2*b*(1 + m)*Sqrt[a + b*x^3])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m (B\,x^3+A)}{(b\,x^3+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(x*e)^m/(b*x^3 + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(x*e)^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2
*b*x^3 + a^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(x*e)^m/(b*x^3 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^m}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(5/2),x)
[Out] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(5/2), x)
```

3.506 $\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

```
[Out] -1/3*(a*d+b*c)*arctanh(d^(1/2)*(b*x^3+a)^(1/2)/b^(1/2)/(d*x^3+c)^(1/2))/b^(
3/2)/d^(3/2)+1/3*(b*x^3+a)^(1/2)*(d*x^3+c)^(1/2)/b/d
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {457, 81, 65, 223, 212}

$$\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*b^(3/2)*d^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^3\right)}{6bd}\\
&= \frac{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^3}\right)}{3b^2d}\\
&= \frac{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3b^2d}\\
&= \frac{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^3}}{\sqrt{b}\,\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 1.00, size = 88, normalized size = 1.00

$$
\frac{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{d}\,\sqrt{a+bx^3}}\right)}{3b^{3/2}d^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^3])/(Sqrt[d]*Sqrt[a + b*x^3])])/(3*b^(3/2)*d^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.77, size = 256, normalized size = 2.91

$$\left[\frac{4\,\sqrt{bx^{3}+a}\,\sqrt{dx^{3}+c}\,bd+(bc+ad)\sqrt{bd}\,\log\left(8\,b^{2}d^{2}x^{6}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+8\,(b^{2}cd+abd^{2})x^{3}-4\,(2\,bdx^{3}+bc+ad)\sqrt{bx^{3}+a}\,\sqrt{dx^{3}+c}\,\sqrt{bd}\right)}{12\,b^{2}d^{2}},\ \frac{2\,\sqrt{bx^{3}+a}\,\sqrt{dx^{3}+c}\,bd+(bc+ad)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^{3}+bc+ad)\sqrt{bx^{3}+a}\,\sqrt{dx^{3}+c}\,\sqrt{-bd}}{2\,(b^{2}d^{2}x^{6}+abcd+(b^{2}cd+abd^{2})x^{3})}\right)}{6\,b^{2}d^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*
b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^3 - 4
*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(b*d)))/(b^2*d
^2), 1/6*(2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(-b*d)*ar
ctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)
/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^3)))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\sqrt{a+bx^{3}}\,\sqrt{c+dx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.69, size = 104, normalized size = 1.18

$$\frac{\frac{(bc+ad)\log\left(\left|-\sqrt{bx^3+a}\,\sqrt{bd}+\sqrt{b^2c+(bx^3+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}+\frac{\sqrt{bx^3+a}\,\sqrt{b^2c+(bx^3+a)bd-abd}}{bd}}{3\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*c + a*d)*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 +
 a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b*x^3 + a)*sqrt(b^2*c + (b*x^3 + a)
*b*d - a*b*d)/(b*d))/abs(b)
```

Mupad [B]
time = 9.25, size = 283, normalized size = 3.22

$$\frac{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{d^3\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{b\,d^2\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3}-\frac{8\,\sqrt{a}\,\sqrt{c}\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2}{3\,d^2\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^4}{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2}{d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}\right)\,\left(a\,d+b\,c\right)}{3\,b^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((((a + b*x^3)^(1/2) - a^(1/2))*((2*a*d)/3 + (2*b*c)/3))/(d^3*((c + d*x^3)^
(1/2) - c^(1/2))) + (((a + b*x^3)^(1/2) - a^(1/2))^3*((2*a*d)/3 + (2*b*c)/3
))/(b*d^2*((c + d*x^3)^(1/2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*((a + b*x^3
)^(1/2) - a^(1/2))^2)/(3*d^2*((c + d*x^3)^(1/2) - c^(1/2))^2))/(((a + b*x^3
)^(1/2) - a^(1/2))^4/((c + d*x^3)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a +
 b*x^3)^(1/2) - a^(1/2))^2)/(d*((c + d*x^3)^(1/2) - c^(1/2))^2)) - (2*atanh
((d^(1/2)*((a + b*x^3)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^3)^(1/2) - c^(1
/2))))*(a*d + b*c))/(3*b^(3/2)*d^(3/2))
```

3.507 $\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=48

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

```
[Out] 2/3*arctanh(d^(1/2)*(b*x^3+a)^(1/2)/b^(1/2)/(d*x^3+c)^(1/2))/b^(1/2)/d^(1/2
)
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {455, 65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*Sqrt[b]
*Sqrt[d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\ &= \frac{2\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3b}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^3}}{\sqrt{b}\,\sqrt{c+dx^3}}\right)}{3\sqrt{b}\,\sqrt{d}}\end{aligned}$$

Mathematica [A]
time = 0.63, size = 48, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{d}\,\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/(Sqrt[d]*Sqrt[a + b*x^3])])/(3*Sqrt[b]
*Sqrt[d])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(34) = 68.
time = 3.45, size = 194, normalized size = 4.04

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^6+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^3+4\,(2\,bdx^3+bc+ad)\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,\sqrt{bd}\right)}{6\,bd},\ -\frac{\sqrt{-bd}\arctan\left(\frac{(2\,bdx^3+bc+ad)\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,\sqrt{-bd}}{2\,(b^2d^2x^6+abcd+(b^2cd+abd^2)x^3)}\right)}{3\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(b*d)*log(8*b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c
*d + a*b*d^2)*x^3 + 4*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 +
c)*sqrt(b*d))/(b*d), -1/3*sqrt(-b*d)*arctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqr
t(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x^3))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [A]
time = 1.21, size = 54, normalized size = 1.12

$$-\frac{2\,b\log\left(\left|-\sqrt{bx^3+a}\,\sqrt{bd}+\sqrt{b^2c+(bx^3+a)bd-abd}\right|\right)}{3\,\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*b*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d)))/(sqrt(b*d)*abs(b))
```

Mupad [B]
time = 5.04, size = 49, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}\right)}{3\,\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] -(4*atan((b*((c + d*x^3)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x^3)^(1/2) - a^(1/2)))))/(3*(-b*d)^(1/2))
```

3.508 $\int \frac{1}{x\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^3}}{\sqrt{a}\,\sqrt{c+dx^3}}\right)}{3\sqrt{a}\,\sqrt{c}}$$

```
[Out] -2/3*arctanh(c^(1/2)*(b*x^3+a)^(1/2)/a^(1/2)/(d*x^3+c)^(1/2))/a^(1/2)/c^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {457, 95, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^3}}{\sqrt{a}\,\sqrt{c+dx^3}}\right)}{3\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^3\right)$$
$$= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx,x,\frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

Mathematica [A]
time = 0.58, size = 48, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^3}}{\sqrt{c}\sqrt{a+bx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
[Out] (-2*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^3])/(Sqrt[c]*Sqrt[a + b*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
[Out] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(34) = 68$.
time = 4.16, size = 204, normalized size = 4.25

$$\left[\frac{\sqrt{ac}\,\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\,(abc^2+a^2cd)x^3-4\,((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,\sqrt{ac}}{x^6}\right)}{6\,ac}, \frac{\sqrt{-ac}\,\arctan\left(\frac{((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,\sqrt{-ac}}{2\,(abcdx^6+a^2c^2+(abc^2+a^2cd)x^3)}\right)}{3\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^3 - 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x
^3 + c)*sqrt(a*c))/x^6)/(a*c), 1/3*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^3 +
 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 + a^2*c^2 +
 (a*b*c^2 + a^2*c*d)*x^3))/(a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.
time = 1.30, size = 89, normalized size = 1.85

$$-\frac{2\,\sqrt{bd}\,b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2}{2\,\sqrt{-abcd}\,b}\right)}{3\,\sqrt{-abcd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*
d)*abs(b))
```

Mupad [B]
time = 7.57, size = 136, normalized size = 2.83

$$-\frac{\ln\left(\frac{\sqrt{b\,x^3+a}-\sqrt{a}}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)-\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^3+a}-\sqrt{a}\,\sqrt{d\,x^3+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{3\,\sqrt{a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] -(log(((a + b*x^3)^(1/2) - a^(1/2))/((c + d*x^3)^(1/2) - c^(1/2))) - log(((
c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(1/2))*(b*c^(1/2) - (a^(1/2
)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^(1/2) - c^(1/2))))/((c + d*
x^3)^(1/2) - c^(1/2))))/(3*a^(1/2)*c^(1/2))
```

3.509 $\int \frac{1}{x^4\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}{3acx^3}+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^3}}{\sqrt{a}\,\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}$$

```
[Out] 1/3*(a*d+b*c)*arctanh(c^(1/2)*(b*x^3+a)^(1/2)/a^(1/2)/(d*x^3+c)^(1/2))/a^(3
/2)/c^(3/2)-1/3*(b*x^3+a)^(1/2)*(d*x^3+c)^(1/2)/a/c/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {457, 98, 95, 214}

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^3}}{\sqrt{a}\,\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sq
rt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\,\sqrt{c+dx}}\,dx,x,x^3\right)\\ &= -\frac{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}{3acx^3}-\frac{(bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx,x,x^3\right)}{6ac}\\ &= -\frac{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}{3acx^3}-\frac{(bc+ad)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx,x,\frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3ac}\\ &= -\frac{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}{3acx^3}+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^3}}{\sqrt{a}\,\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 1.00, size = 91, normalized size = 1.00

$$-\frac{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}{3acx^3}+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c+dx^3}}{\sqrt{c}\,\sqrt{a+bx^3}}\right)}{3a^{3/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sq
rt[a]*Sqrt[c + d*x^3])/(Sqrt[c]*Sqrt[a + b*x^3])])/(3*a^(3/2)*c^(3/2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**A**]
time = 5.59, size = 278, normalized size = 3.05

$$\left[\frac{\sqrt{ac}\,(bc+ad)x^3\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\,(abc^2+a^2cd)x^3+4\,((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,\sqrt{ac}}{x^6}\right)-4\,\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,ac}{12\,a^2c^2x^3},\ -\frac{\sqrt{-ac}\,(bc+ad)x^3\arctan\left(\frac{((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,\sqrt{-ac}}{2\,(abcdx^6+a^2c^2+(abc^2+a^2cd)x^3)}\right)+2\,\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,ac}{6\,a^2c^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(a*c)*(b*c + a*d)*x^3*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 +
 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^3 + 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b
*x^3 + a)*sqrt(d*x^3 + c)*sqrt(a*c))/x^6) - 4*sqrt(b*x^3 + a)*sqrt(d*x^3 +
c)*a*c)/(a^2*c^2*x^3), -1/6*(sqrt(-a*c)*(b*c + a*d)*x^3*arctan(1/2*((b*c +
a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 +
 a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^3)) + 2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*a*
c)/(a^2*c^2*x^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(71) = 142.

time = 1.74, size = 413, normalized size = 4.54

$$\frac{\sqrt{bd}\,b^4 d\left(\frac{(bc+ad)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,ab^3cd}-\frac{2\left(b^3c^2-2ab^2cd+a^2bd^2-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2bc-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2ad\right)}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2abd+\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^4\right)ab^2cd}\right)}{3\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b
))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (sqr
t(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*b*c - (sq
rt(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*d)/((b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^3 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^3 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^4)*a*b^2*c*d))/abs(b)
```

Mupad [B]

time = 10.77, size = 481, normalized size = 5.29

$$\frac{\frac{\left(\sqrt{bx^3+a}-\sqrt{a}\right)\left(\frac{cb^2}{12}+\frac{adb}{12}\right)}{a^{3/2}c^{3/2}d\left(\sqrt{dx^3+c}-\sqrt{c}\right)}-\frac{b^2}{12acd}+\frac{\left(\sqrt{bx^3+a}-\sqrt{a}\right)^2\left(\frac{a^2d^2}{12}-\frac{abcd}{4}+\frac{b^2c^2}{12}\right)}{a^2c^2d\left(\sqrt{dx^3+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{bx^3+a}-\sqrt{a}\right)^3}{\left(\sqrt{dx^3+c}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{bx^3+a}-\sqrt{a}\right)}{d\left(\sqrt{dx^3+c}-\sqrt{c}\right)}-\frac{\left(\sqrt{bx^3+a}-\sqrt{a}\right)^2(ad+bc)}{\sqrt{a}\sqrt{c}\,d\left(\sqrt{dx^3+c}-\sqrt{c}\right)^2}}+\frac{\ln\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{dx^3+c}-\sqrt{c}}\right)\left(\sqrt{a}\,bc^{3/2}+a^{3/2}\sqrt{c}\,d\right)}{6a^2c^2}-\frac{\ln\left(\frac{\left(\sqrt{c}\sqrt{bx^3+a}-\sqrt{a}\sqrt{dx^3+c}\right)\left(b\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{bx^3+a}-\sqrt{a}\right)}{\sqrt{dx^3+c}-\sqrt{c}}\right)}{\sqrt{dx^3+c}-\sqrt{c}}\right)\left(\sqrt{a}\,bc^{3/2}+a^{3/2}\sqrt{c}\,d\right)}{6a^2c^2}-\frac{d\left(\sqrt{bx^3+a}-\sqrt{a}\right)}{12ac\left(\sqrt{dx^3+c}-\sqrt{c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((((a + b*x^3)^(1/2) - a^(1/2))*((b^2*c)/12 + (a*b*d)/12))/(a^(3/2)*c^(3/2)
*d*((c + d*x^3)^(1/2) - c^(1/2))) - b^2/(12*a*c*d) + (((a + b*x^3)^(1/2) -
a^(1/2))^2*((a^2*d^2)/12 + (b^2*c^2)/12 - (a*b*c*d)/4))/(a^2*c^2*d*((c + d*
x^3)^(1/2) - c^(1/2))^2))/((((a + b*x^3)^(1/2) - a^(1/2))^3/((c + d*x^3)^(1/
2) - c^(1/2))^3 + (b*((a + b*x^3)^(1/2) - a^(1/2)))/(d*((c + d*x^3)^(1/2) -
 c^(1/2))) - (((a + b*x^3)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)
*d*((c + d*x^3)^(1/2) - c^(1/2))^2)) + (log(((a + b*x^3)^(1/2) - a^(1/2))/(
(c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d))/(6*a
^2*c^2) - (log(((c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(1/2))*(b*
c^(1/2) - (a^(1/2)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^(1/2) - c^
(1/2))))/((c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2
)*d))/(6*a^2*c^2) - (d*((a + b*x^3)^(1/2) - a^(1/2)))/(12*a*c*((c + d*x^3)^
(1/2) - c^(1/2)))
```

3.510 $\int \frac{x^4}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{x^5\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

```
[Out] 1/5*x^5*AppellF1(5/3,1/2,1/2,8/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*
x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{x^5\sqrt{\frac{bx^3}{a}+1}\,\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3, -
((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x^4}{\sqrt{1+\frac{bx^3}{a}}\,\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}$$
$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x^4}{\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$
$$= \frac{x^5\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Mathematica [A]
time = 2.30, size = 90, normalized size = 1.02

$$\frac{x^5\sqrt{\frac{a+bx^3}{a}}\,\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3, -
((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^4/(b*d*x^6 + (b*c + a*d)*x^3 + a
*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a + bx^3}\,\sqrt{c + dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
[Out] Integral(x**4/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
[Out] int(x^4/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.511 $\int \frac{x^3}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{x^4\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1/2,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*
x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{x^4\sqrt{\frac{bx^3}{a}+1}\,\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x^3}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}$$
$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x^3}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Mathematica [A]
time = 1.89, size = 90, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
[Out] (x^4*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
[Out] int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3/(b*d*x^6 + (b*c + a*d)*x^3 + a
*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.512 $\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1/2,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*
x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -
((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}$$

$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$= \frac{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Mathematica [A]
time = 1.82, size = 90, normalized size = 1.02

$$\frac{x^2\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
[Out] (x^2*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -
((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
[Out] int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x/(b*d*x^6 + (b*c + a*d)*x^3 + a*c
), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)

[Out] Integral(x/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)

[Out] int(x/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)

3.513 $\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=83

$$\frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,1/2,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)
^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1/2, 1/2, 4/3, -((
b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{1}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}dx}{\sqrt{a+bx^3}}$$
$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\operatorname{F}_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.
time = 2.08, size = 170, normalized size = 2.05

$$-\frac{8acxF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}\left(-8acF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(adF_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bcF_1\left(\frac{4}{3};\frac{3}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a
 + b*x^3]*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] + 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((
d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^6 + (b*c + a*d)*x^3 + a*c),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.514 $\int \frac{1}{x^2\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,1/2,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)
^(1/2)/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\,\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1/2, 1/2, 2/3, -(
(b*x^3)/a), -((d*x^3)/c)])/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{1}{x^2\sqrt{1+\frac{bx^3}{a}}\,\sqrt{c+dx^3}}dx}{\sqrt{a+bx^3}}$$
$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{1}{x^2\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}}dx}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 189 vs. $2(86) = 172$.
time = 2.34, size = 189, normalized size = 2.20

$$\frac{-20(a+bx^3)(c+dx^3)+5(bc+ad)x^3\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+8bdx^6\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{20acx\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-20*(a + b*x^3)*(c + d*x^3) + 5*(b*c + a*d)*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1
 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 8*
b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3
, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c*x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^8 + (b*c + a*d)*x^5 + a*c*x
^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.515 $\int \frac{1}{x^3\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$-\frac{\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,1/2,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x^
3/c)^(1/2)/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\,\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/2*(Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1/2, 1/2, 1/3,
 -((b*x^3)/a), -((d*x^3)/c)])/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{1}{x^3\sqrt{1+\frac{bx^3}{a}}\,\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}$$

$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{1}{x^3\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(88) = 176.

time = 2.49, size = 365, normalized size = 4.15

$$\frac{bdx^6\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4\left(-4ac(2ac+3bcx^3+3adx^3+2bdx^6)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3(a+bx^3)(c+dx^3)\left(adF_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bcF_1\left(\frac{4}{3};\frac{3}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{8acF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3x^3\left(adF_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bcF_1\left(\frac{4}{3};\frac{3}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}}{8acx^2\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] + (4*(-4*a*c*(2*a*c + 3*b*c*x^3 + 3*a*d*x^3
+ 2*b*d*x^6)*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*x
^3*(a + b*x^3)*(c + d*x^3)*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/(8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*x^
3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*Appel
lF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/(8*a*c*x^2*Sqrt[a +
b*x^3]*Sqrt[c + d*x^3])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^9 + (b*c + a*d)*x^6 + a*c*x
^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.516 $\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=161

$$\frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2}+\frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}-\frac{a^2(2Ab-aB)e^{7/2}}{}$$

[Out] 1/9*B*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e-1/24*a^2*(2*A*b-B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)+1/24*a*(2*A*b-B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/12*(2*A*b-B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e

Rubi [A]
time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.269$, Rules used = {470, 285, 327, 335, 281, 223, 212}

$$-\frac{a^2e^{7/2}(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}+\frac{ae^2(ex)^{3/2}\sqrt{a+bx^3}\,(2Ab-aB)}{24b^2}+\frac{(ex)^{9/2}\sqrt{a+bx^3}\,(2Ab-aB)}{12be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

Antiderivative was successfully verified.

[In] Int[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]

[Out] (a*(2*A*b - a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b^2) + ((2*A*b - a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(3/2))/(9*b*e) - (a^2*(2*A*b - a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(24*b^(5/2))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}-\frac{\left(-9Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\sqrt{a+bx^3}\,dx}{9b}$$

$$=\frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}+\frac{(a(2Ab-aB)}{}$$

$$=\frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2}+\frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be}+\frac{B(}{}$$

$$=\frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2}+\frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be}+\frac{B(}{}$$

$$=\frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2}+\frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be}+\frac{B(}{}$$

$$=\frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2}+\frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be}+\frac{B(}{}$$

$$=\frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2}+\frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be}+\frac{B(}{}$$

Mathematica [**A**]
time = 0.27, size = 122, normalized size = 0.76

$$\frac{(ex)^{7/2}\sqrt{a+bx^3}\left(6aAb-3a^2B+12Ab^2x^3+2abBx^3+8b^2Bx^6\right)}{72b^2x^2}+\frac{a^2(-2Ab+aB)(ex)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)}{24b^{5/2}x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] ((e*x)^(7/2)*Sqrt[a + b*x^3]*(6*a*A*b - 3*a^2*B + 12*A*b^2*x^3 + 2*a*b*B*x^
3 + 8*b^2*B*x^6))/(72*b^2*x^2) + (a^2*(-2*A*b + a*B)*(e*x)^(7/2)*ArcTanh[Sq
rt[a + b*x^3]/(Sqrt[b]*x^(3/2))])/(24*b^(5/2)*x^(7/2))
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.61, size = 7293, normalized size = 45.30

method	result	size
risch	Expression too large to display	1087
elliptic	Expression too large to display	1165
default	Expression too large to display	7293

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(111) = 222$.
time = 0.49, size = 292, normalized size = 1.81

$$\frac{1}{144}\left(6\left(\frac{a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{3}{2}}}+\frac{2\left(\frac{\sqrt{bx^3+a}\,a^2b}{x^{\frac{3}{2}}}+\frac{(bx^3+a)^{\frac{3}{2}}a^2}{x^{\frac{9}{2}}}\right)}{b^3-\frac{2(bx^3+a)b^2}{x^3}+\frac{(bx^3+a)^2b}{x^6}}\right)A-\left(\frac{3a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{5}{2}}}+\frac{2\left(\frac{3\sqrt{bx^3+a}\,a^3b^2}{x^{\frac{3}{2}}}+\frac{8(bx^3+a)^{\frac{3}{2}}a^3b}{x^{\frac{9}{2}}}-\frac{3(bx^3+a)^{\frac{5}{2}}a^3}{x^{\frac{15}{2}}}\right)}{b^5-\frac{3(bx^3+a)b^4}{x^3}+\frac{3(bx^3+a)^2b^3}{x^6}-\frac{(bx^3+a)^3b^2}{x^9}}\right)B\right)e^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/144*(6*(a^2*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^
3 + a)/x^(3/2)))/b^(3/2) + 2*(sqrt(b*x^3 + a)*a^2*b/x^(3/2) + (b*x^3 + a)^(
3/2)*a^2/x^(9/2))/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 + a)^2*b/x^6))*A -
(3*a^3*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 + a)/
x^(3/2)))/b^(5/2) + 2*(3*sqrt(b*x^3 + a)*a^3*b^2/x^(3/2) + 8*(b*x^3 + a)^(3
/2)*a^3*b/x^(9/2) - 3*(b*x^3 + a)^(5/2)*a^3/x^(15/2))/(b^5 - 3*(b*x^3 + a)*
b^4/x^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*B)*e^(7/2)
```

Fricas [A]
time = 2.28, size = 256, normalized size = 1.59

$$\left[-\frac{3(Ba^3-2Aa^2b)\sqrt{b}\,e^{\frac{7}{2}}\log\left(-8b^2x^6-8abx^3+4(2bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)-4(8Bb^3x^7+2(Bab^2+6Ab^3)x^4-3(Ba^2b-2Aab^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{288b^3},\ -\frac{3(Ba^3-2Aa^2b)\sqrt{-b}\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2bx^3+a}\right)e^{\frac{7}{2}}-2(8Bb^3x^7+2(Bab^2+6Ab^3)x^4-3(Ba^2b-2Aab^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{144b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(B*a^3 - 2*A*a^2*b)*sqrt(b)*e^(7/2)*log(-8*b^2*x^6 - 8*a*b*x^3 +
 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*(8*B*b^3*x^7
+ 2*(B*a*b^2 + 6*A*b^3)*x^4 - 3*(B*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^3 + a)*sq
rt(x)*e^(7/2))/b^3, -1/144*(3*(B*a^3 - 2*A*a^2*b)*sqrt(-b)*arctan(2*sqrt(b*
x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a))*e^(7/2) - 2*(8*B*b^3*x^7 + 2*(B*a*
```

```
b^2 + 6*A*b^3)*x^4 - 3*(B*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(
7/2))/b^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(141) = 282$.
time = 82.27, size = 292, normalized size = 1.81

$$\frac{Aa^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{3}{2}}}{12b\sqrt{1+\frac{bx^3}{a}}}+\frac{A\sqrt{a}\,e^{\frac{7}{2}}x^{\frac{9}{2}}}{4\sqrt{1+\frac{bx^3}{a}}}-\frac{Aa^2e^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{12b^{\frac{3}{2}}}+\frac{Abe^{\frac{7}{2}}x^{\frac{15}{2}}}{6\sqrt{a}\,\sqrt{1+\frac{bx^3}{a}}}-\frac{Ba^{\frac{5}{2}}e^{\frac{7}{2}}x^{\frac{3}{2}}}{24b^2\sqrt{1+\frac{bx^3}{a}}}-\frac{Ba^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{9}{2}}}{72b\sqrt{1+\frac{bx^3}{a}}}+\frac{5B\sqrt{a}\,e^{\frac{7}{2}}x^{\frac{15}{2}}}{36\sqrt{1+\frac{bx^3}{a}}}+\frac{Ba^3e^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{24b^{\frac{5}{2}}}+\frac{Bbe^{\frac{7}{2}}x^{\frac{21}{2}}}{9\sqrt{a}\,\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*e**(7/2)*x**(3/2)/(12*b*sqrt(1 + b*x**3/a)) + A*sqrt(a)*e**(7/2)
*x**(9/2)/(4*sqrt(1 + b*x**3/a)) - A*a**2*e**(7/2)*asinh(sqrt(b)*x**(3/2)/s
qrt(a))/(12*b**(3/2)) + A*b*e**(7/2)*x**(15/2)/(6*sqrt(a)*sqrt(1 + b*x**3/a
)) - B*a**(5/2)*e**(7/2)*x**(3/2)/(24*b**2*sqrt(1 + b*x**3/a)) - B*a**(3/2)
*e**(7/2)*x**(9/2)/(72*b*sqrt(1 + b*x**3/a)) + 5*B*sqrt(a)*e**(7/2)*x**(15/
2)/(36*sqrt(1 + b*x**3/a)) + B*a**3*e**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(a)
)/(24*b**(5/2)) + B*b*e**(7/2)*x**(21/2)/(9*sqrt(a)*sqrt(1 + b*x**3/a))
```

Giac [A]
time = 1.08, size = 201, normalized size = 1.25

$$\frac{1}{72}\left(6\sqrt{bx^3+a}\left(2x^3+\frac{a}{b}\right)Ax^{\frac{3}{2}}+\left(2\left(4x^3+\frac{a}{b}\right)x^3-\frac{3a^2}{b^2}\right)\sqrt{bx^3+a}\,Bx^{\frac{3}{2}}\right)e^{\frac{7}{2}}-\frac{(B^2a^6-4ABa^5b+4A^2a^4b^2)e^{\frac{7}{2}}\log\left(\left|-\left(Ba^3x^{\frac{3}{2}}-2Aa^2bx^{\frac{3}{2}}\right)\sqrt{b}+\sqrt{B^2a^7-4ABa^6b+4A^2a^5b^2+\left(Ba^3x^{\frac{3}{2}}-2Aa^2bx^{\frac{3}{2}}\right)^2b}\right|\right)}{24b^{\frac{5}{2}}|-Ba^3+2Aa^2b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*(6*sqrt(b*x^3 + a)*(2*x^3 + a/b)*A*x^(3/2) + (2*(4*x^3 + a/b)*x^3 - 3*
a^2/b^2)*sqrt(b*x^3 + a)*B*x^(3/2))*e^(7/2) - 1/24*(B^2*a^6 - 4*A*B*a^5*b +
 4*A^2*a^4*b^2)*e^(7/2)*log(abs(-(B*a^3*x^(3/2) - 2*A*a^2*b*x^(3/2))*sqrt(b
) + sqrt(B^2*a^7 - 4*A*B*a^6*b + 4*A^2*a^5*b^2 + (B*a^3*x^(3/2) - 2*A*a^2*b
*x^(3/2))^2*b)))/(b^(5/2)*abs(-B*a^3 + 2*A*a^2*b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}\,\sqrt{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2), x)
```

3.517 $\int (ex)^{5/2}\sqrt{a+bx^3}\,(A+Bx^3)\,dx$

Optimal. Leaf size=324

$$\frac{3a(16Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^3}}{320b^2}+\frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}-\frac{3^{3/4}a^{5/3}(16A}{}$$

```
[Out] 1/8*B*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/80*(16*A*b-7*B*a)*(e*x)^(7/2)*(b*x^
3+a)^(1/2)/b/e+3/320*a*(16*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2-1
/640*3^(3/4)*a^(5/3)*(16*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+
1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(
1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {470, 285, 327, 335, 231}

$$-\frac{3^{3/4}a^{5/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(16Ab-7aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{3ae^2\sqrt{ex}\,\sqrt{a+bx^3}\,(16Ab-7aB)}{320b^2}+\frac{(ex)^{7/2}\sqrt{a+bx^3}\,(16Ab-7aB)}{80be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (3*a*(16*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*b^2) + ((16*A*b -
 7*a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(80*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^
(3/2))/(8*b*e) - (3^(3/4)*a^(5/3)*(16*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*
x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*b^2*Sqrt[(b
^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt
[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{5/2}\sqrt{a+bx^3}\,(A+Bx^3)\,dx = \frac{B(ex)^{7/2}(a+bx^3)^{3/2}}{8be} - \frac{\left(-8Ab+\frac{7aB}{2}\right)\int (ex)^{5/2}\sqrt{a+bx^3}\,dx}{8b}$$

$$= \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be} + \frac{B(ex)^{7/2}(a+bx^3)^{3/2}}{8be} + \frac{(3a(16Ab}{}$$

$$= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2} + \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be}$$

$$= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2} + \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be}$$

$$= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2} + \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 112, normalized size = 0.35

$$\frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(-\left((a+bx^3)\sqrt{1+\frac{bx^3}{a}}(-16Ab+7aB-10bBx^3)\right)+a(-16Ab+7aB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{80b^2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(-16*A*b
+ 7*a*B - 10*b*B*x^3)) + a*(-16*A*b + 7*a*B)*Hypergeometric2F1[-1/2, 1/6, 7
/6, -((b*x^3)/a)]))/(80*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.43, size = 4175, normalized size = 12.89 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/320*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^3/(-a*b^2)^(1/3)*(96*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b^3*e*x^2+84*I*B*(-a*b^2)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b*e*x+96*I*A*(-a*b^2)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b*e-42*I*B*(-a*b^2)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*e+48*I*A*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b^2-42*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b^2*e*x^2-96*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b^3*e*x^2+40*I*B*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^3*x^6-120*B*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^3*x^6+42*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I

```
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b^2*e*x^2-192*I*A*(-a*b^2)^(1/3)*3^(1/2)*(
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*a^2*b^2*e*x+192*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*
3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a^2*b^2*e*x+64*
I*A*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^3*x^3-84*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*
3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] e^(5/2)*integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^5 + A*x^2)*sqrt(b*x^3 + a)*sqrt(x)*e^(5/2), x)
```

Sympy [C] Result contains complex when optimal does not.

time = 26.98, size = 97, normalized size = 0.30

$$\frac{A\sqrt{a}\,e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)}+\frac{B\sqrt{a}\,e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(13/6)) + B*sqrt(a)*e**(5/2)*x**(13/2)*gamma(13/6
)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,\sqrt{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(1/2), x)
```

3.518 $\int (ex)^{3/2}\sqrt{a+bx^3}\,(A+Bx^3)\,dx$

Optimal. Leaf size=581

$$\frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be}+\frac{3\left(1+\sqrt{3}\right)a(14Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^3}}{112b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}-\ 3\sqrt{\ }$$

```
[Out] 1/7*B*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/56*(14*A*b-5*B*a)*(e*x)^(5/2)*(b*x^
3+a)^(1/2)/b/e+3/112*a*(14*A*b-5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(
1/2)/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3/112*3^(1/4)*a^(4/3)*(14*A*b-
5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*
(1+3^(1/2)))^2)^(1/2)-1/224*3^(3/4)*a^(4/3)*(14*A*b-5*B*a)*e*(a^(1/3)+b^(1/
3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elli
pticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^
2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b
*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 285, 335, 314, 231, 1895}

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(14Ab-5aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{224b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\,a^{4/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(14Ab-5aB)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{112b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{3\left(1+\sqrt{3}\right)ae\sqrt{ex}\,\sqrt{a+bx^3}(14Ab-5aB)}{112b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{(ex)^{5/2}\sqrt{a+bx^3}(14Ab-5aB)}{56be}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] ((14*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(56*b*e) + (3*(1 + Sqrt[3])*
a*(14*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(112*b^(5/3)*(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)) + (B*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7*b*e) - (3*3^(
```

```
1/4)*a^(4/3)*(14*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2
]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(112*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(
3/4)*(1 - Sqrt[3])*a^(4/3)*(14*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(224*b^(5/3)*Sqrt[(b^(1/3
)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} - \frac{\left(-7Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{7b}$$

$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} + \frac{(3a(14Ab \cdots}{\cdots}$$

$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} + \frac{(3a(14Ab \cdots}{\cdots}$$

$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} - \frac{(3a(14Ab \cdots}{\cdots}$$

$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{3\left(1+\sqrt{3}\right)a(14Ab-5aB)e\sqrt{ex} \cdots}{112b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 94, normalized size = 0.16

$$\frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(5B(a+bx^3)\sqrt{1+\frac{bx^3}{a}}+(14Ab-5aB)\,{}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{35b\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (14*A
*b - 5*a*B)*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)]))/(35*b*Sqrt[1
 + (b*x^3)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.46, size = 5358, normalized size = 9.22

method	result	size
risch	Expression too large to display	1140
elliptic	Expression too large to display	1184
default	Expression too large to display	5358

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(3/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.58, size = 97, normalized size = 0.17

$$\frac{A\sqrt{a}\,e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\\frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)}+\frac{B\sqrt{a}\,e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\\frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(11/6)) + B*sqrt(a)*e**(3/2)*x**(11/2)*gamma(11/6
)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,\sqrt{b\,x^3+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(1/2), x)
```

3.519 $\int \sqrt{ex}\,\sqrt{a+bx^3}\,(A+Bx^3)\,dx$

Optimal. Leaf size=121

$$\frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{3/2}\,(a+bx^3)^{3/2}}{6be}+\frac{a(4Ab-aB)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}$$

```
[Out] 1/6*B*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/12*a*(4*A*b-B*a)*arctanh((e*x)^(3/2
)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)/b^(3/2)+1/12*(4*A*b-B*a)*(e*x)^(
3/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A]
time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {470, 285, 335, 281, 223, 212}

$$\frac{a\sqrt{e}\,(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(4Ab-aB)}{12be}+\frac{B(ex)^{3/2}\,(a+bx^3)^{3/2}}{6be}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] ((4*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(3/2)*(a +
b*x^3)^(3/2))/(6*b*e) + (a*(4*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/
2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{ex}\,\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} - \frac{\left(-6Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\,\sqrt{a+bx^3}\,dx}{6b}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))}{8}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))}{}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))}{}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))}{}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{a(4Ab-aB)\sqrt{}}{}$$

Mathematica [A]
time = 0.22, size = 95, normalized size = 0.79

$$\frac{x\sqrt{ex}\,\sqrt{a+bx^3}\,(4Ab+aB+2bBx^3)}{12b} - \frac{a(-4Ab+aB)\sqrt{ex}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)}{12b^{3/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x*Sqrt[e*x]*Sqrt[a + b*x^3]*(4*A*b + a*B + 2*b*B*x^3))/(12*b) - (a*(-4*A*b
 + a*B)*Sqrt[e*x]*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))])/(12*b^(3/2)*S
qrt[x])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 6858, normalized size = 56.68

method	result	size

risch	Expression too large to display	1055
elliptic	Expression too large to display	1096
default	Expression too large to display	6858

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(81) = 162$.
time = 0.50, size = 207, normalized size = 1.71

$$-\frac{1}{24}\left(4\left(\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{\sqrt{b}}+\frac{2\sqrt{bx^3+a}\,a}{\left(b-\frac{bx^3+a}{x^3}\right)x^{\frac{3}{2}}}\right)A-\left(\frac{a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{3}{2}}}+\frac{2\left(\frac{\sqrt{bx^3+a}\,a^2b}{x^{\frac{3}{2}}}+\frac{(bx^3+a)^{\frac{3}{2}}a^2}{x^{\frac{9}{2}}}\right)}{b^3-\frac{2(bx^3+a)b^2}{x^3}+\frac{(bx^3+a)^2b}{x^6}}\right)B\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(4*(a*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3
+ a)/x^(3/2)))/sqrt(b) + 2*sqrt(b*x^3 + a)*a/((b - (b*x^3 + a)/x^3)*x^(3/2)
))*A - (a^2*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3
+ a)/x^(3/2)))/b^(3/2) + 2*(sqrt(b*x^3 + a)*a^2*b/x^(3/2) + (b*x^3 + a)^(3/
2)*a^2/x^(9/2))/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 + a)^2*b/x^6))*B)*e^(
1/2)
```

Fricas [A]
time = 4.34, size = 206, normalized size = 1.70

$$\left[-\frac{(Ba^2-4\,Aab)\sqrt{b}\,e^{\frac{1}{2}}\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)-4\,(2\,Bb^2x^4+(Bab+4\,Ab^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{1}{2}}}{48\,b^2},\ \frac{(Ba^2-4\,Aab)\sqrt{-b}\,\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)e^{\frac{1}{2}}+2\,(2\,Bb^2x^4+(Bab+4\,Ab^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{1}{2}}}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((B*a^2 - 4*A*a*b)*sqrt(b)*e^(1/2)*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2
*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*(2*B*b^2*x^4 + (B*
a*b + 4*A*b^2)*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(1/2))/b^2, 1/24*((B*a^2 - 4*A*
a*b)*sqrt(-b)*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a))*e^(1
/2) + 2*(2*B*b^2*x^4 + (B*a*b + 4*A*b^2)*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(1/2)
)/b^2]
```

Sympy [A]
time = 4.95, size = 201, normalized size = 1.66

$$\frac{A\sqrt{a}\,(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e}+\frac{Aa\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{3\sqrt{b}}+\frac{Ba^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{12be\sqrt{1+\frac{bx^3}{a}}}+\frac{B\sqrt{a}\,(ex)^{\frac{9}{2}}}{4e^4\sqrt{1+\frac{bx^3}{a}}}-\frac{Ba^2\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{12b^{\frac{3}{2}}}+\frac{Bb(ex)^{\frac{15}{2}}}{6\sqrt{a}\,e^7\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a*sqrt(e)*asinh(sqrt(b)
*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*sqrt(b)) + B*a**(3/2)*(e*x)**(3/2)/(12
*b*e*sqrt(1 + b*x**3/a)) + B*sqrt(a)*(e*x)**(9/2)/(4*e**4*sqrt(1 + b*x**3/a
)) - B*a**2*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(12*b**(
3/2)) + B*b*(e*x)**(15/2)/(6*sqrt(a)*e**7*sqrt(1 + b*x**3/a))
```

Giac [A]
time = 2.02, size = 105, normalized size = 0.87

$$\frac{Ba^2e^{\frac{1}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{12\,b^{\frac{3}{2}}}+\frac{1}{12}\left(\sqrt{bx^3+a}\,\left(2\,x^3+\frac{a}{b}\right)Bx^{\frac{3}{2}}+4\left(\sqrt{bx^3+a}\,x^{\frac{3}{2}}-\frac{a\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{\sqrt{b}}\right)A\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*B*a^2*e^(1/2)*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3 + a)))/b^(3/2) + 1
/12*(sqrt(b*x^3 + a)*(2*x^3 + a/b)*B*x^(3/2) + 4*(sqrt(b*x^3 + a)*x^(3/2) -
 a*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3 + a)))/sqrt(b))*A)*e^(1/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,\sqrt{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2), x)
```

3.520 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=286

$$\frac{(10Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{20be}+\frac{B\sqrt{ex}\,(a+bx^3)^{3/2}}{5be}+\frac{3^{3/4}a^{2/3}(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{\cdots}}{\left(\sqrt[3]{a}+\left(\cdots\right.\right.}}\cdots}{40be\sqrt{\frac{\sqrt[3]{b}\,x\cdots}{\left(\sqrt[3]{a}+\cdots\right.}}\cdots}$$

[Out] 1/5*B*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/20*(10*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+1/40*3^(3/4)*a^(2/3)*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {470, 285, 335, 231}

$$\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(10Ab-aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40be\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\,\sqrt{a+bx^3}\,(10Ab-aB)}{20be}+\frac{B\sqrt{ex}\,(a+bx^3)^{3/2}}{5be}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]

[Out] ((10*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*b*e) + (B*Sqrt[e*x]*(a + b*x^3)^(3/2))/(5*b*e) + (3^(3/4)*a^(2/3)*(10*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{B\sqrt{ex}\,(a+bx^3)^{3/2}}{5be} - \frac{\left(-5Ab+\frac{aB}{2}\right)\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{5b}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}\,(a+bx^3)^{3/2}}{5be} + \frac{(3a(10Ab-aB))\int -}{40b}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}\,(a+bx^3)^{3/2}}{5be} + \frac{(3a(10Ab-aB))\mathrm{Subs}}{}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}\,(a+bx^3)^{3/2}}{5be} + \frac{3^{3/4}a^{2/3}(10Ab-aB)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 93, normalized size = 0.33

$$\frac{x\sqrt{a+bx^3}\left(B(a+bx^3)\sqrt{1+\frac{bx^3}{a}}+(10Ab-aB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{5b\sqrt{ex}\,\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (10*A*b - a*B)*Hype
rgeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*b*Sqrt[e*x]*Sqrt[1 + (b*x^
3)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 3721, normalized size = 13.01 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(b*x^3+a)^(1/2)*x/b^2/(-a*b^2)^(1/3)*(-120*I*A*3^(1/2)*(-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(-a*b^2)^(1/3)*a*b^2*e*x+60*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*
b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+
3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a*b*e+
60*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^3*e*x^2-6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b^
2*e*x^2-10*I*A*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*b^2-60*A*(-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^3*
e*x^2-3*I*B*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*a*b+6*B*(-(I*3^(1/2)-3)*x*b/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(
(I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b^2*e*
x^2+120*A*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2
))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x-4*I*B*3^(1/2)*((b*x^3+a)*e
```

```
*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*
b^2)^(1/3)*b^2*x^3-12*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b*e*x-60*A*(-a*b^2
)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*a*b*e+6*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*e+12*B*((b
*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(
1/2)*(-a*b^2)^(1/3)*b^2*x^3+12*I*B*3^(1/2)*(-a*...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate((B*x^3 + A)*sqrt(b*x^3 + a)/sqrt(x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*e^(-1/2)/sqrt(x), x)
```

Sympy [**C**] Result contains complex when optimal does not.

time = 2.22, size = 97, normalized size = 0.34

$$\frac{A\sqrt{a}\,\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)}+\frac{B\sqrt{a}\,x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*
pi)/a)/(3*sqrt(e)*gamma(7/6)) + B*sqrt(a)*x**(7/2)*gamma(7/6)*hyper((-1/2,
7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*e^(-1/2)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(1/2), x)
```

3.521 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=580

$$\frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4}+\frac{3\left(1+\sqrt{3}\right)(8Ab+aB)\sqrt{ex}\,\sqrt{a+bx^3}}{8b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{2A(a+bx^3)^{3/2}}{ae\sqrt{ex}}-\frac{3\sqrt[4]{3}\,\sqrt[3]{a}\,(8Ab\cdots}{\cdots}$$

```
[Out] -2*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(1/2)+1/4*(8*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^
(1/2)/a/e^4+3/8*(8*A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)
/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3/8*3^(1/4)*a^(1/3)*(8*A*b+B*a)*(a^(1/
3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1
/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b
*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2)-1/16*3^(3/4)*a^(1/3)*(8*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1
/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)
/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$,
Rules used = {464, 285, 335, 314, 231, 1895}

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}\,\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(aB+8Ab)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(aB+8Ab)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{3\left(1+\sqrt{3}\right)\sqrt{ex}\,\sqrt{a+bx^3}\,(aB+8Ab)}{8b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{(ex)^{5/2}\sqrt{a+bx^3}\,(aB+8Ab)}{4ae^4}-\frac{2A(a+bx^3)^{3/2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] ((8*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*a*e^4) + (3*(1 + Sqrt[3])*(8
*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(8*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)) - (2*A*(a + b*x^3)^(3/2))/(a*e*Sqrt[e*x]) - (3*3^(1/4)*a^(1
```

```
/3)*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(8*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*(1 - Sq
rt[3])*a^(1/3)*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} + \frac{(8Ab+aB)\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{ae^3}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} + \frac{(3(8Ab+aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}} \cdots}{8e^3}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} + \frac{(3(8Ab+aB))\mathrm{Subst}\left(\int \cdots\right)}{4e\cdots}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} - \frac{(3(8Ab+aB))\mathrm{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} + \frac{3\left(1+\sqrt{3}\right)(8Ab+aB)\sqrt{ex}\,\sqrt{a+bx^3}}{8b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

4 in optimal.
time = 10.04, size = 98, normalized size = 0.17

$$-\frac{2Ax(a+bx^3)^{3/2}}{a(ex)^{3/2}}-\frac{4\left(-4Ab-\frac{aB}{2}\right)x^4\sqrt{a+bx^3}\ {}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{5a(ex)^{3/2}\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (-2*A*x*(a + b*x^3)^(3/2))/(a*(e*x)^(3/2)) - (4*(-4*A*b - (a*B)/2)*x^4*Sqrt
[a + b*x^3]*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)])/(5*a*(e*x)^(3
/2)*Sqrt[1 + (b*x^3)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 5736, normalized size = 9.89

method	result	size
risch	Expression too large to display	1123
elliptic	Expression too large to display	1161
default	Expression too large to display	5736

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*e^(-3/2)/x^(3/2), x)

Sympy [C] Result contains complex when optimal does not.
time = 2.40, size = 100, normalized size = 0.17

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)}+\frac{B\sqrt{a}\,x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(3/2),x)

[Out] A*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a)
/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + B*sqrt(a)*x**(5/2)*gamma(5/6)*hyper((-1/
2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="giac")

[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*e^(-3/2)/x^(3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{{\left(e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(3/2),x)

[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(3/2), x)

3.522 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=118

$$\frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4}-\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}+\frac{(2Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}$$

```
[Out] -2/3*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(3/2)+1/3*(2*A*b+B*a)*arctanh((e*x)^(3/2)*
b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(1/2)+1/3*(2*A*b+B*a)*(e*x)^(3/2
)*(b*x^3+a)^(1/2)/a/e^4
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 285, 335, 281, 223, 212}

$$\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(aB+2Ab)}{3ae^4}-\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] ((2*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*a*e^4) - (2*A*(a + b*x^3)^(3
/2))/(3*a*e*(e*x)^(3/2)) + ((2*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^
(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{(ex)^{5/2}}\,dx = -\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\int \sqrt{ex}\,\sqrt{a+bx^3}\,dx}{ae^3}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,d\ldots}{2e^3}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\text{Subst}\left(\int \frac{\ldots}{\sqrt{a-\ldots}}\right.}{e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\text{Subst}\left(\int \frac{\ldots}{\sqrt{a-\ldots}}\right.}{3e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\ldots\right.}{3e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}\ldots}{e^{3/2}\sqrt{\ldots}}\right.}{3\sqrt{b}\,e^{5/2}}$$

Mathematica [A]
time = 0.22, size = 81, normalized size = 0.69

$$\frac{x\left(\sqrt{b}\,\sqrt{a+bx^3}\,(-2A+Bx^3)+(2Ab+aB)x^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)\right)}{3\sqrt{b}\,(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*(Sqrt[b]*Sqrt[a + b*x^3]*(-2*A + B*x^3) + (2*A*b + a*B)*x^(3/2)*ArcTanh[
Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))]))/(3*Sqrt[b]*(e*x)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.48, size = 6668, normalized size = 56.51

method	result	size

risch	Expression too large to display	1050
elliptic	Expression too large to display	1069
default	Expression too large to display	6668

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.50, size = 146, normalized size = 1.24

$$-\frac{1}{6}\left(2\left(\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)+\frac{2\sqrt{bx^3+a}}{x^{\frac{3}{2}}}\right)A+\left(\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{\sqrt{b}}+\frac{2\sqrt{bx^3+a}\,a}{\left(b-\frac{bx^3+a}{x^3}\right)x^{\frac{3}{2}}}\right)B\right)e^{\left(-\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sqrt(b)*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b
*x^3 + a)/x^(3/2))) + 2*sqrt(b*x^3 + a)/x^(3/2))*A + (a*log(-(sqrt(b) - sqr
t(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 + a)/x^(3/2)))/sqrt(b) + 2*sqrt
(b*x^3 + a)*a/((b - (b*x^3 + a)/x^3)*x^(3/2)))*B)*e^(-5/2)
```

Fricas [A]
time = 1.39, size = 184, normalized size = 1.56

$$\left[\frac{\left((Ba+2\,Ab)\sqrt{b}\,x^2\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)+4\,(Bbx^3-2\,Ab)\sqrt{bx^3+a}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{12\,bx^2},-\frac{\left((Ba+2\,Ab)\sqrt{-b}\,x^2\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)-2\,(Bbx^3-2\,Ab)\sqrt{bx^3+a}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{6\,bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((B*a + 2*A*b)*sqrt(b)*x^2*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 +
a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) + 4*(B*b*x^3 - 2*A*b)*sqrt(b*x^
3 + a)*sqrt(x))*e^(-5/2)/(b*x^2), -1/6*((B*a + 2*A*b)*sqrt(-b)*x^2*arctan(2
*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a)) - 2*(B*b*x^3 - 2*A*b)*sqrt
(b*x^3 + a)*sqrt(x))*e^(-5/2)/(b*x^2)]
```

Sympy [A]
time = 5.20, size = 160, normalized size = 1.36

$$-\frac{2A\sqrt{a}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2A\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3e^{\frac{5}{2}}}-\frac{2Abx^{\frac{3}{2}}}{3\sqrt{a}\,e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{B\sqrt{a}\,x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}+\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}\,e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(5/2),x)
```

```
[Out] -2*A*sqrt(a)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*sqrt(b)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*e**(5/2)) - 2*A*b*x**(3/2)/(3*sqrt(a)*e**(5/2)*
sqrt(1 + b*x**3/a)) + B*sqrt(a)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) +
B*a*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*e^(-5/2)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{(e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2), x)
```

3.523 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=283

$$\frac{(4Ab+5aB)\sqrt{ex}\,\sqrt{a+bx^3}}{10ae^4}-\frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}}+\frac{3^{3/4}(4Ab+5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right.\right.}}}{20\sqrt[3]{a}\,e^4\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}\right.}{\left(\sqrt[3]{a}+\left(1+\right.\right.}}}$$

[Out] -2/5*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(5/2)+1/10*(4*A*b+5*B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a/e^4+1/20*3^(3/4)*(4*A*b+5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(1/3)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {464, 285, 335, 231}

$$\frac{3^{3/4}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(5aB+4Ab)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{20\sqrt[3]{a}\,e^4\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\,\sqrt{a+bx^3}\,(5aB+4Ab)}{10ae^4}-\frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]

[Out] ((4*A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(10*a*e^4) - (2*A*(a + b*x^3)^(3/2))/(5*a*e*(e*x)^(5/2)) + (3^(3/4)*(4*A*b + 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(20*a^(1/3)*e^4*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{(ex)^{7/2}}\,dx = -\frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}} + \frac{(4Ab+5aB)\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{5ae^3}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\,\sqrt{a+bx^3}}{10ae^4} - \frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}} + \frac{(3(4Ab+5aB))\int \frac{}{\sqrt{ex}\,\sqrt{}}}{20e^3}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\,\sqrt{a+bx^3}}{10ae^4} - \frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}} + \frac{(3(4Ab+5aB))\text{Subst}\left(\int\right.}{10}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\,\sqrt{a+bx^3}}{10ae^4} - \frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}} + \frac{3^{3/4}(4Ab+5aB)\sqrt{ex}\left(\sqrt[3]{}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 97, normalized size = 0.34

$$\frac{2x\sqrt{a+bx^3}\left(-A(a+bx^3)\sqrt{1+\frac{bx^3}{a}}+(4Ab+5aB)x^3\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{5a(ex)^{7/2}\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]) + (4*A*b + 5*a*B
)*x^3*Hypergeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*Sq
rt[1 + (b*x^3)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.46, size = 3512, normalized size = 12.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(b*x^3+a)^(1/2)/x^2/(-a*b^2)^(1/3)/b*(-60*I*B*3^(1/2)*(-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
-a*b^2)^(1/3)*a*b*e*x^4+24*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^3*e*x^5-48*I*A*3^(1/
2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*b^2*e*x^4-24*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^3*e*x^5+48*A
*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*(-a*b^2)^(1/3)*b^2*e*x^4-24*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)
^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*b
*e*x^3+30*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
)/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/
(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x^5-12*A*(1/b^2*e*x*(-b*x+(-a*b
^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)
*b+4*I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/
2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*b-5*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-
```

```
a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1
/2)*b*x^3+24*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2
))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*b*e*x^3-30*B*(-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*a*b^2*e*x^5+60*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3
))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a*b*e*x^4-30*B*(-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
[Out] e^(-7/2)*integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*e^(-7/2)/x^(7/2), x)
```

Sympy [C] Result contains complex when optimal does not.

time = 15.70, size = 100, normalized size = 0.35

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{B\sqrt{a}\,\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(7/2),x)
```

```
[Out] A*sqrt(a)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a)
/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-1/
2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*e^(-7/2)/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{{\left(e\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(7/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(7/2), x)
```

3.524 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{9/2}}\,dx$

Optimal. Leaf size=564

$$-\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}}+\frac{3\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(2Ab+7aB)\sqrt{x}\,\sqrt{a+bx^3}}{7a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}}-\frac{3\sqrt[4]{3}\,\sqrt[3]{b}\,(2Ab}{}$$

```
[Out] -2/7*A*(b*x^3+a)^(3/2)/a/x^(7/2)-2/7*(2*A*b+7*B*a)*(b*x^3+a)^(1/2)/a/x^(1/2
)+3/7*b^(1/3)*(2*A*b+7*B*a)*(1+3^(1/2))*x^(1/2)*(b*x^3+a)^(1/2)/a/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))-3/7*3^(1/4)*b^(1/3)*(2*A*b+7*B*a)*(a^(1/3)+b^(1/3)*x
)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elliptic
E((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2),1/4*6^(1/2)+1/4*2^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(b^(1/
3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/14*3^(3
/4)*b^(1/3)*(2*A*b+7*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1
/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
1-3^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/
3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {464, 283, 335, 314, 231, 1895}

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(7aB+2Ab)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{14a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(7aB+2Ab)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}\,(7aB+2Ab)}{7a\sqrt{x}}+\frac{3\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{x}\,\sqrt{a+bx^3}\,(7aB+2Ab)}{7a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]
```

```
[Out] (-2*(2*A*b + 7*a*B)*Sqrt[a + b*x^3])/(7*a*Sqrt[x]) + (3*(1 + Sqrt[3])*b^(1/
3)*(2*A*b + 7*a*B)*Sqrt[x]*Sqrt[a + b*x^3])/(7*a*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)) - (2*A*(a + b*x^3)^(3/2))/(7*a*x^(7/2)) - (3*3^(1/4)*b^(1/3)*(2*
```

```
A*b + 7*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a
^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
+ Sqrt[3])/4])/(7*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*(1 - Sqrt[3])*b^(1
/3)*(2*A*b + 7*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(14*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{9/2}}\,dx = -\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} - \frac{\left(2\left(-Ab-\frac{7aB}{2}\right)\right)\int \frac{\sqrt{a+bx^3}}{x^{3/2}}\,dx}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} + \frac{(3b(2Ab+7aB))\int \frac{x^{3/2}}{\sqrt{a+bx}}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} + \frac{(6b(2Ab+7aB))\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} - \frac{\left(3\sqrt[3]{b}\,(2Ab+7aB)\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} + \frac{3\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(2Ab+7aB)\sqrt{x}\,\sqrt{a+bx^3}}{7a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 81, normalized size = 0.14

$$\frac{2\sqrt{a+bx^3}\left(-A(a+bx^3)-\frac{(2Ab+7aB)x^3\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((2*A*b + 7*a*B)*x^3*Hypergeometric2
F1[-1/2, -1/6, 5/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(7*a*x^(7/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.62, size = 5911, normalized size = 10.48

method	result	size
risch	Expression too large to display	1127
elliptic	Expression too large to display	1177
default	Expression too large to display	5911

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)

Sympy [C] Result contains complex when optimal does not.
time = 11.93, size = 97, normalized size = 0.17

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{7}{6},-\frac{1}{2}\\-\frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{7}{2}}\Gamma\left(-\frac{1}{6}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\\frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(9/2),x)
```

```
[Out] A*sqrt(a)*gamma(-7/6)*hyper((-7/6, -1/2), (-1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*x**(7/2)*gamma(-1/6)) + B*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6
,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(x)*gamma(5/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(9/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(9/2), x)
```

3.525 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{11/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}-\frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}}+\frac{2}{3}\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)$$

```
[Out] -2/9*A*(b*x^3+a)^(3/2)/a/x^(9/2)+2/3*B*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a)^(1
/2))*b^(1/2)-2/3*B*(b*x^3+a)^(1/2)/x^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {462, 283, 335, 281, 223, 212}

$$-\frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}}-\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}+\frac{2}{3}\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

```
[Out] (-2*B*Sqrt[a + b*x^3])/(3*x^(3/2)) - (2*A*(a + b*x^3)^(3/2))/(9*a*x^(9/2))
 + (2*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{11/2}}\,dx &= -\frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + B\int \frac{\sqrt{a+bx^3}}{x^{5/2}}\,dx \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + (bB)\int \frac{\sqrt{x}}{\sqrt{a+bx^3}}\,dx \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + (2bB)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{1}{3}(2bB)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{3/2}\right) \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{1}{3}(2bB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{a+bx^3}}\right) \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{2}{3}\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 74, normalized size = 0.94

$$-\frac{2\sqrt{a+bx^3}\,(aA+Abx^3+3aBx^3)}{9ax^{9/2}} + \frac{2}{3}\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]

[Out] (-2*Sqrt[a + b*x^3]*(a*A + A*b*x^3 + 3*a*B*x^3))/(9*a*x^(9/2)) + (2*Sqrt[b]
*B*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))])/3

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.42, size = 3759, normalized size = 47.58

method	result	size
risch	Expression too large to display	1040
elliptic	Expression too large to display	1051
default	Expression too large to display	3759

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)

[Out] -2/9*(b*x^3+a)^(1/2)/x^(9/2)/b*(18*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*x^7-3*A*
(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(x*(b*x^3+a))^(1
/2)*b^2*x^3+36*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2
)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*
b^2)^(1/3)*a*b*x^6-3*A*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))
^(1/2)*(x*(b*x^3+a))^(1/2)*a*b+18*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^
(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*
a*x^5+I*A*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2
)*(x*(b*x^3+a))^(1/2)*b^2*x^3-18*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)
^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/

```
3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*
3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/
2)-3))^(1/2))*a*b^2*x^7+3*I*B*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3)))^(1/2)*(x*(b*x^3+a))^(1/2)*a*b*x^3-18*I*B*3^(1/2)*(-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a*x^5-36*I*B*3^(1/2)*(-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*(-a*b^2)^(1/3)*a*b*x^6-18*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/
(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*
(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*x^7+18*B*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*x^7+36*B*(-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*E
llipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1
/3)*a*b*x^6-36*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3
^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/...
```

Maxima [A]

time = 0.50, size = 81, normalized size = 1.03

$$-\frac{1}{3}\left(\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)+\frac{2\sqrt{bx^3+a}}{x^{\frac{3}{2}}}\right)B-\frac{2\left(bx^3+a\right)^{\frac{3}{2}}A}{9\,ax^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(b)*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^
3 + a)/x^(3/2))) + 2*sqrt(b*x^3 + a)/x^(3/2))*B - 2/9*(b*x^3 + a)^(3/2)*A/(
a*x^(9/2))
```

Fricas [A]
time = 1.97, size = 180, normalized size = 2.28

$$\left[\frac{3\,Ba\sqrt{b}\,x^5\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)-4\,((3\,Ba+Ab)x^3+Aa)\sqrt{bx^3+a}\,\sqrt{x}}{18\,ax^5},-\frac{3\,Ba\sqrt{-b}\,x^5\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)+2\,((3\,Ba+Ab)x^3+Aa)\sqrt{bx^3+a}\,\sqrt{x}}{9\,ax^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*B*a*sqrt(b)*x^5*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqr
t(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^
3 + a)*sqrt(x))/(a*x^5), -1/9*(3*B*a*sqrt(-b)*x^5*arctan(2*sqrt(b*x^3 + a)*
sqrt(-b)*x^(3/2)/(2*b*x^3 + a)) + 2*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 +
a)*sqrt(x))/(a*x^5)]
```

Sympy [A]
time = 32.28, size = 131, normalized size = 1.66

$$-\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{9x^3}-\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{9a}-\frac{2B\sqrt{a}}{3x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3}-\frac{2Bbx^{\frac{3}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(11/2),x)
```

```
[Out] -2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(9*x**3) - 2*A*b**(3/2)*sqrt(a/(b*x**3) +
 1)/(9*a) - 2*B*sqrt(a)/(3*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*B*sqrt(b)*asinh
(sqrt(b)*x**(3/2)/sqrt(a))/3 - 2*B*b*x**(3/2)/(3*sqrt(a)*sqrt(1 + b*x**3/a)
)
```

Giac [A]
time = 0.82, size = 109, normalized size = 1.38

$$-\frac{2\,Bb\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}+\frac{2\left(3\,Bab\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,Ba\sqrt{-b}\,\sqrt{b}+A\sqrt{-b}\,b^{\frac{3}{2}}\right)}{9\,a\sqrt{-b}}-\frac{2\left(3\,Ba^3\sqrt{b+\frac{a}{x^3}}+Aa^2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}\right)}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] -2/3*B*b*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + 2/9*(3*B*a*b*arctan(sq
rt(b)/sqrt(-b)) + 3*B*a*sqrt(-b)*sqrt(b) + A*sqrt(-b)*b^(3/2))/(a*sqrt(-b))
 - 2/9*(3*B*a^3*sqrt(b + a/x^3) + A*a^2*(b + a/x^3)^(3/2))/a^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2), x)
```

3.526 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{13/2}}\,dx$

Optimal. Leaf size=269

$$\frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}}-\frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}}-\frac{3^{3/4}b(2Ab-11aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\cdots}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\cdots}$$

```
[Out] -2/11*A*(b*x^3+a)^(3/2)/a/x^(11/2)+2/55*(2*A*b-11*B*a)*(b*x^3+a)^(1/2)/a/x^
(5/2)-1/55*3^(3/4)*b*(2*A*b-11*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))^2)^(1/2)/a^(4/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)
*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {464, 283, 335, 231}

$$-\frac{3^{3/4}b\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(2Ab-11aB)F\left(\mathrm{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}\,(2Ab-11aB)}{55ax^{5/2}}-\frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
```

```
[Out] (2*(2*A*b - 11*a*B)*Sqrt[a + b*x^3])/(55*a*x^(5/2)) - (2*A*(a + b*x^3)^(3/2
))/(11*a*x^(11/2)) - (3^(3/4)*b*(2*A*b - 11*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(55*a^(4/3)*Sqrt[(b^(1/3
)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + 
b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{13/2}}\,dx = -\frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}} - \frac{\left(2\left(Ab-\frac{11aB}{2}\right)\right)\int \frac{\sqrt{a+bx^3}}{x^{7/2}}\,dx}{11a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}} - \frac{(3b(2Ab-11aB))\int \frac{1}{\sqrt{x}\,\sqrt{a+}}}{55a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}} - \frac{(6b(2Ab-11aB))\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{55a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}} - \frac{3^{3/4}b(2Ab-11aB)\sqrt{x}\left(\sqrt[3]{a}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 80, normalized size = 0.30

$$\frac{2\sqrt{a+bx^3}\left(-5A(a+bx^3)+\frac{(2Ab-11aB)x^3\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{6};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{55ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + ((2*A*b - 11*a*B)*x^3*Hypergeometric
2F1[-5/6, -1/2, 1/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(55*a*x^(11/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.44, size = 3690, normalized size = 13.72 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/55*(b*x^3+a)^(1/2)/x^(11/2)/(-a*b^2)^(1/3)*(12*I*A*3^(1/2)*(-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
```

/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
b^3*x^8+132*I*B*3^(1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*x^7-66*I*B*3^(1/2)
*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*a*b^2*x^8-5*I*A*3^(1/2)*(-a*b^2)^(1/3)*(x*(b*x^3+a))^(1/2)*(
1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a-11*I*B*3^(1/2)*
(-a*b^2)^(1/3)*(x*(b*x^3+a))^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3)))^(1/2)*a*x^3-12*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2
)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3
^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^3*x^8-24*I*A*3^(1/2)*(-a*b^2)
^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*b^2*x^7+66*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*x^8+24*A*(-a*b^2)^(1/
3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*b^2*x^7-132*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip

```
ticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^
(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*x^7-12*A*(-
a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*b*x^6+66*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
,((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*x^6+12
*I*A*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(
1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b*x^6...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 76, normalized size = 0.28

$$-\frac{2\left(3\,(11\,Bab-2\,Ab^2)\sqrt{a}\,x^6\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+\left((11\,Ba^2+3\,Aab)x^3+5\,Aa^2\right)\sqrt{bx^3+a}\,\sqrt{x}\right)}{55\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] -2/55*(3*(11*B*a*b - 2*A*b^2)*sqrt(a)*x^6*weierstrassPInverse(0, -4*b/a, 1/
x) + ((11*B*a^2 + 3*A*a*b)*x^3 + 5*A*a^2)*sqrt(b*x^3 + a)*sqrt(x))/(a^2*x^6
)
```

Sympy [C] Result contains complex when optimal does not.

time = 86.34, size = 97, normalized size = 0.36

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{11}{6},-\frac{1}{2}\\-\frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{11}{2}}\Gamma\left(-\frac{5}{6}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\\frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(13/2),x)
```

```
[Out] A*sqrt(a)*gamma(-11/6)*hyper((-11/6, -1/2), (-5/6,), b*x**3*exp_polar(I*pi)
/a)/(3*x**(11/2)*gamma(-5/6)) + B*sqrt(a)*gamma(-5/6)*hyper((-5/6, -1/2), (
1/6,), b*x**3*exp_polar(I*pi)/a)/(3*x**(5/2)*gamma(1/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^{13/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(13/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(13/2), x)
```

3.527 $\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=201

$$\frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2}+\frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be}+\frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be}+\frac{B(\ldots}{}$$

```
[Out] 1/72*(8*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e+1/12*B*(e*x)^(9/2)*(b*x^
3+a)^(5/2)/b/e-1/192*a^3*(8*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/
e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)+1/192*a^2*(8*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b
*x^3+a)^(1/2)/b^2+1/96*a*(8*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A]
time = 0.10, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {470, 285, 327, 335, 281, 223, 212}

$$-\frac{a^3e^{7/2}(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{5/2}}+\frac{a^2e^2(ex)^{3/2}\sqrt{a+bx^3}\,(8Ab-3aB)}{192b^2}+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}(8Ab-3aB)}{72be}+\frac{a(ex)^{9/2}\sqrt{a+bx^3}\,(8Ab-3aB)}{96be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a^2*(8*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b^2) + (a*(8*A*b
 - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(96*b*e) + ((8*A*b - 3*a*B)*(e*x)^(9
/2)*(a + b*x^3)^(3/2))/(72*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(5/2))/(12*b*e
) - (a^3*(8*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqr
t[a + b*x^3])])/(192*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{B(ex)^{9/2} \left(a+bx^3\right)^{5/2}}{12be} - \frac{\left(-12Ab+\frac{9aB}{2}\right) \int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \, dx}{12b}$$

$$= \frac{(8Ab-3aB)(ex)^{9/2} \left(a+bx^3\right)^{3/2}}{72be} + \frac{B(ex)^{9/2} \left(a+bx^3\right)^{5/2}}{12be} + \frac{(a(8Ab -$$

$$= \frac{a(8Ab-3aB)(ex)^{9/2} \sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)(ex)^{9/2} \left(a+bx^3\right)^{3/2}}{72be}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2} \sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2} \sqrt{a+bx^3}}{96be}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2} \sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2} \sqrt{a+bx^3}}{96be}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2} \sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2} \sqrt{a+bx^3}}{96be}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2} \sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2} \sqrt{a+bx^3}}{96be}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2} \sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2} \sqrt{a+bx^3}}{96be}$$

Mathematica [A]
time = 0.34, size = 143, normalized size = 0.71

$$\frac{e^3 \sqrt{ex} \left(\sqrt{b}\, x^{3/2} \sqrt{a+bx^3} \left(-9a^3B+6a^2b(4A+Bx^3)+16b^3x^6(4A+3Bx^3)+8ab^2x^3(14A+9Bx^3)\right)+3a^3(-8Ab+3aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\, x^{3/2}}\right)\right)}{576b^{5/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(-9*a^3*B + 6*a^2*b*(4*A +
B*x^3) + 16*b^3*x^6*(4*A + 3*B*x^3) + 8*a*b^2*x^3*(14*A + 9*B*x^3)) + 3*a^3
*(-8*A*b + 3*a*B)*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))]))/(576*b^(5/2)
*Sqrt[x])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.39, size = 7705, normalized size = 38.33

method	result	size
risch	Expression too large to display	1111
elliptic	Expression too large to display	1285
default	Expression too large to display	7705

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(145) = 290$.
time = 0.52, size = 369, normalized size = 1.84

$$\frac{1}{1152}\left(8\left(\frac{3\,a^{3}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^{3}+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^{3}+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{3}{2}}}+\frac{2\left(\frac{3\sqrt{bx^{3}+a}\,a^{3}b^{2}}{x^{\frac{3}{2}}}-\frac{8\,(bx^{3}+a)^{\frac{3}{2}}a^{3}b}{x^{\frac{9}{2}}}-\frac{3\,(bx^{3}+a)^{\frac{5}{2}}a^{3}}{x^{\frac{15}{2}}}\right)}{b^{4}-\frac{3\,(bx^{3}+a)b^{3}}{x^{3}}+\frac{3\,(bx^{3}+a)^{2}b^{2}}{x^{6}}-\frac{(bx^{3}+a)^{3}b}{x^{9}}}\right)A-3\left(\frac{3\,a^{4}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^{3}+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^{3}+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{5}{2}}}+\frac{2\left(\frac{3\sqrt{bx^{3}+a}\,a^{4}b^{3}}{x^{\frac{3}{2}}}-\frac{11\,(bx^{3}+a)^{\frac{3}{2}}a^{4}b^{2}}{x^{\frac{9}{2}}}-\frac{11\,(bx^{3}+a)^{\frac{5}{2}}a^{4}b}{x^{\frac{15}{2}}}+\frac{3\,(bx^{3}+a)^{\frac{7}{2}}a^{4}}{x^{\frac{21}{2}}}\right)}{b^{6}-\frac{4\,(bx^{3}+a)b^{5}}{x^{3}}+\frac{6\,(bx^{3}+a)^{2}b^{4}}{x^{6}}-\frac{4\,(bx^{3}+a)^{3}b^{3}}{x^{9}}+\frac{(bx^{3}+a)^{4}b^{2}}{x^{12}}}\right)B\right)e^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/1152*(8*(3*a^3*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b
*x^3 + a)/x^(3/2)))/b^(3/2) + 2*(3*sqrt(b*x^3 + a)*a^3*b^2/x^(3/2) - 8*(b*x
^3 + a)^(3/2)*a^3*b/x^(9/2) - 3*(b*x^3 + a)^(5/2)*a^3/x^(15/2))/(b^4 - 3*(b
*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3 + a)^3*b/x^9))*A - 3*(
3*a^4*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 + a)/x
^(3/2)))/b^(5/2) + 2*(3*sqrt(b*x^3 + a)*a^4*b^3/x^(3/2) - 11*(b*x^3 + a)^(3
/2)*a^4*b^2/x^(9/2) - 11*(b*x^3 + a)^(5/2)*a^4*b/x^(15/2) + 3*(b*x^3 + a)^(
7/2)*a^4/x^(21/2))/(b^6 - 4*(b*x^3 + a)*b^5/x^3 + 6*(b*x^3 + a)^2*b^4/x^6 -
 4*(b*x^3 + a)^3*b^3/x^9 + (b*x^3 + a)^4*b^2/x^12))*B)*e^(7/2)
```

Fricas [A]
time = 1.37, size = 310, normalized size = 1.54

$$\left[-\frac{3\,(3\,Ba^{4}-8\,Aa^{3}b)\sqrt{b}\,e^{\frac{7}{2}}\log\left(-8\,b^{2}x^{6}-8\,abx^{3}+4\,(2\,bx^{4}+ax)\sqrt{bx^{3}+a}\,\sqrt{b}\,\sqrt{x}-a^{2}\right)-4\,(48\,Bb^{4}x^{10}+8\,(9\,Bab^{3}+8\,Ab^{4})x^{7}+2\,(3\,Ba^{2}b^{2}+56\,Aab^{3})x^{4}-3\,(3\,Ba^{3}b-8\,Aa^{2}b^{2})x)\sqrt{bx^{3}+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{2304\,b^{3}},\ -\frac{3\,(3\,Ba^{4}-8\,Aa^{3}b)\sqrt{-b}\,\arctan\left(\frac{2\sqrt{bx^{3}+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^{3}+a}\right)e^{\frac{7}{2}}-2\,(48\,Bb^{4}x^{10}+8\,(9\,Bab^{3}+8\,Ab^{4})x^{7}+2\,(3\,Ba^{2}b^{2}+56\,Aab^{3})x^{4}-3\,(3\,Ba^{3}b-8\,Aa^{2}b^{2})x)\sqrt{bx^{3}+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{1152\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] [-1/2304*(3*(3*B*a^4 - 8*A*a^3*b)*sqrt(b)*e^(7/2)*log(-8*b^2*x^6 - 8*a*b*x^
3 + 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*(48*B*b^4*
x^10 + 8*(9*B*a*b^3 + 8*A*b^4)*x^7 + 2*(3*B*a^2*b^2 + 56*A*a*b^3)*x^4 - 3*(
```

```
3*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(7/2))/b^3, -1/1152*(
3*(3*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)/
(2*b*x^3 + a))*e^(7/2) - 2*(48*B*b^4*x^10 + 8*(9*B*a*b^3 + 8*A*b^4)*x^7 + 2
*(3*B*a^2*b^2 + 56*A*a*b^3)*x^4 - 3*(3*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^3
 + a)*sqrt(x)*e^(7/2))/b^3]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(145) = 290.
time = 0.93, size = 303, normalized size = 1.51

$$\frac{1}{576}\left(48\sqrt{bx^3+a}\left(2x^3+\frac{a}{b}\right)Aax^{\frac{3}{2}}+8\left(2\left(4x^3+\frac{a}{b}\right)x^3-\frac{3a^2}{b^2}\right)\sqrt{bx^3+a}\,Bax^{\frac{3}{2}}+8\left(2\left(4x^3+\frac{a}{b}\right)x^3-\frac{3a^2}{b^2}\right)\sqrt{bx^3+a}\,Abx^{\frac{3}{2}}+\left(2\left(4\left(6x^3+\frac{a}{b}\right)x^3-\frac{5a^2}{b^2}\right)x^3+\frac{15a^3}{b^3}\right)\sqrt{bx^3+a}\,Bbx^{\frac{3}{2}}\right)e^{\frac{7}{2}}-\frac{(9B^2a^8-48ABa^7b+64A^2a^6b^2)e^{\frac{7}{2}}\log\left(\left|-\left(3Ba^4x^{\frac{3}{2}}-8Aa^3bx^{\frac{3}{2}}\right)\sqrt{b}+\sqrt{9B^2a^9-48ABa^8b+64A^2a^7b^2+\left(3Ba^4x^{\frac{3}{2}}-8Aa^3bx^{\frac{3}{2}}\right)^2b}\right|\right)}{192b^{\frac{5}{2}}|-3Ba^4+8Aa^3b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/576*(48*sqrt(b*x^3 + a)*(2*x^3 + a/b)*A*a*x^(3/2) + 8*(2*(4*x^3 + a/b)*x^
3 - 3*a^2/b^2)*sqrt(b*x^3 + a)*B*a*x^(3/2) + 8*(2*(4*x^3 + a/b)*x^3 - 3*a^2
/b^2)*sqrt(b*x^3 + a)*A*b*x^(3/2) + (2*(4*(6*x^3 + a/b)*x^3 - 5*a^2/b^2)*x^
3 + 15*a^3/b^3)*sqrt(b*x^3 + a)*B*b*x^(3/2))*e^(7/2) - 1/192*(9*B^2*a^8 - 4
8*A*B*a^7*b + 64*A^2*a^6*b^2)*e^(7/2)*log(abs(-(3*B*a^4*x^(3/2) - 8*A*a^3*b
*x^(3/2))*sqrt(b) + sqrt(9*B^2*a^9 - 48*A*B*a^8*b + 64*A^2*a^7*b^2 + (3*B*a
^4*x^(3/2) - 8*A*a^3*b*x^(3/2))^2*b)))/(b^(5/2)*abs(-3*B*a^4 + 8*A*a^3*b))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2), x)
```

3.528 $\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=364

$$\frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^3}}{7040b^2}+\frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be}+\frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3}}{176be}$$

```
[Out] 1/176*(22*A*b-7*B*a)*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/11*B*(e*x)^(7/2)*(b*
x^3+a)^(5/2)/b/e+9/1760*a*(22*A*b-7*B*a)*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+27
/7040*a^2*(22*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2-9/14080*3^(3/4
)*a^(8/3)*(22*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(
1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(
1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2)
)*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))^2)^(1/2)/b^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {470, 285, 327, 335, 231}

$$-\frac{9\, 3^{3/4}a^{8/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(22Ab-7aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{14080b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{27a^2e^2\sqrt{ex}\,\sqrt{a+bx^3}(22Ab-7aB)}{7040b^2}+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}(22Ab-7aB)}{176be}+\frac{9a(ex)^{7/2}\sqrt{a+bx^3}(22Ab-7aB)}{1760be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (27*a^2*(22*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(7040*b^2) + (9*a*(
22*A*b - 7*a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(1760*b*e) + ((22*A*b - 7*a*B)
*(e*x)^(7/2)*(a + b*x^3)^(3/2))/(176*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^(5/2
))/(11*b*e) - (9*3^(3/4)*a^(8/3)*(22*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(14080*b^2*Sqrt[(
b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be} - \frac{\left(-11Ab+\frac{7aB}{2}\right)\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\,dx}{11b}$$

$$= \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be} + \frac{(9a(2$$

$$= \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be} + \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right.}{176be}$$

$$= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a}}{1760be}$$

$$= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a}}{1760be}$$

$$= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a}}{1760be}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 116, normalized size = 0.32

$$\frac{e^2\sqrt{ex}\,\sqrt{a+bx^3}\left(-\left(a+bx^3\right)^2\sqrt{1+\frac{bx^3}{a}}\,(-22Ab+7aB-16bBx^3)+a^2(-22Ab+7aB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{176b^2\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(-22*A*
b + 7*a*B - 16*b*B*x^3)) + a^2*(-22*A*b + 7*a*B)*Hypergeometric2F1[-3/2, 1/
6, 7/6, -((b*x^3)/a)]))/(176*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.46, size = 4619, normalized size = 12.69

method	result
risch	$\frac{\left(640B\,x^{9}b^{3}+880A\,b^{3}x^{6}+1000Ba\,b^{2}x^{6}+1672Aa\,b^{2}x^{3}+108B\,a^{2}b\,x^{3}+594A\,a^{2}b-189B\,a^{3}\right)x\sqrt{b\,x^{3}+a}\;e^{3}}{7040b^{2}\sqrt{ex}}-\frac{27a^{3}(22Ab-7Ba)\Big(\Big(}{}$
elliptic	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7040*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/(-a*b^2)^(1/3)/b^3*(-189*I*B*3^(1/2)
*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3
^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3)))^(1/2)*a^3*b+640*I*B*3^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x
)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^4*x^
9+880*I*A*3^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a
*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^4*x^6+594*I*A*3^(1/2)*(-a*b^2)^(
1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(
1/3)))^(1/2)*a^2*b^2+108*I*B*3^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(
1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a^2*b^2*x^3+167
2*I*A*3^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b^3*x^3-2640*A*(-a*b^2)^(1/3)*((b*x^
3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2
)*b^4*x^6-1782*A*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*
b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a^2*b^2+567*B*(-a*b^2)^(1/3)*((b*x^
3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2
)*a^3*b-1920*B*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^4*x^9-378*I*B*3^(1/2)*(-a*b^2)^(2/3
)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
```

```
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*a^4*e+378*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
F((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/
2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^4*e-2376*I*A*3^(
1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b^2*e*x+756*I*B*3^(1/2)*(-a*b^2)^(1/3
)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*a^4*b*e*x-3000*B*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^
2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b^3*x^6-5016*A*(-
a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3)))^(1/2)*a*b^3*x^3-324*B*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(
1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a^2*b^2*x^3+100
0*I*B*3^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b^3*x^6-1188*A*(-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x
+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b^3
*e*x^2-1188*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```
```
[Out] e^(5/2)*integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```
```
[Out] integral((B*b*x^8 + (B*a + A*b)*x^5 + A*a*x^2)*sqrt(b*x^3 + a)*sqrt(x)*e^(5
/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 74.76, size = 199, normalized size = 0.55

$$\frac{Aa^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\\frac{13}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)}+\frac{A\sqrt{a}\,be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\\frac{19}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}+\frac{Ba^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\\frac{19}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}+\frac{B\sqrt{a}\,be^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\\frac{25}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```
```
[Out] A*a**(3/2)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(13/6)) + A*sqrt(a)*b*e**(5/2)*x**(13/2)*gamma(1
3/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6))
 + B*a**(3/2)*e**(5/2)*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6)) + B*sqrt(a)*b*e**(5/2)*x**(19/2)*g
amma(19/6)*hyper((-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(
25/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```
```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(3/2),x)
```
```
[Out] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(3/2), x)
```

3.529 $\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=621

$$\frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be}+\frac{27\left(1+\sqrt{3}\right)a^2(4Ab-aB)e\sqrt{ex}\sqrt{a+bx^3}}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{(4Ab-aB)(ex)^{5/2}\left(a+bx\right.}{28be}$$

```
[Out] 1/28*(4*A*b-B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/10*B*(e*x)^(5/2)*(b*x^3+
a)^(5/2)/b/e+9/224*a*(4*A*b-B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+27/448*a^2
*(4*A*b-B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))-27/448*3^(1/4)*a^(7/3)*(4*A*b-B*a)*e*(a^(1/3)+b^(1/3)*x
)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elliptic
E((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b
^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-9/896
*3^(3/4)*a^(7/3)*(4*A*b-B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/
2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1
/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {470, 285, 335, 314, 231, 1895}

$$-\frac{9\,3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(4Ab-aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{896b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}a^{7/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(4Ab-aB)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{448b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{27\left(1+\sqrt{3}\right)a^2e\sqrt{ex}\sqrt{a+bx^3}(4Ab-aB)}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}(4Ab-aB)}{28be}+\frac{9a(ex)^{5/2}\sqrt{a+bx^3}(4Ab-aB)}{224be}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (9*a*(4*A*b - a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(224*b*e) + (27*(1 + Sqrt[3
])*a^2*(4*A*b - a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(448*b^(5/3)*(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)) + ((4*A*b - a*B)*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(2
```

```
8*b*e) + (B*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*b*e) - (27*3^(1/4)*a^(7/3)*(
4*A*b - a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCo
s[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)],
 (2 + Sqrt[3])/4])/(448*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(
1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*(1 - Sqrt[
3])*a^(7/3)*(4*A*b - a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*El
lipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)], (2 + Sqrt[3])/4])/(896*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(
1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}-\frac{\left(-10Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\,dx}{10b}$$

$$=\frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}+\frac{(9a(4Ab$$

$$=\frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be}+\frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be}$$

$$=\frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be}+\frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be}$$

$$=\frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be}+\frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be}$$

$$=\frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be}+\frac{27\left(1+\sqrt{3}\right)a^2(4Ab-aB)e\sqrt{e}}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 96, normalized size = 0.15

$$\frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(B(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}+a(4Ab-aB)\,{}_2F_1\left(-\frac{3}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{10b\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(4*
A*b - a*B)*Hypergeometric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)]))/(10*b*Sqrt[1
+ (b*x^3)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.45, size = 5790, normalized size = 9.32

method	result	size
risch	Expression too large to display	1164
elliptic	Expression too large to display	1279
default	Expression too large to display	5790

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^7 + (B*a + A*b)*x^4 + A*a*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(3/2
), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 25.09, size = 199, normalized size = 0.32

$$\frac{Aa^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)}+\frac{A\sqrt{a}\,be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)}+\frac{Ba^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)}+\frac{B\sqrt{a}\,be^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{17}{6}\\ \frac{23}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(11/6)) + A*sqrt(a)*b*e**(3/2)*x**(11/2)*gamma(1
1/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6))
 + B*a**(3/2)*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6)) + B*sqrt(a)*b*e**(3/2)*x**(17/2)*g
amma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(
23/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(3/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(3/2), x)
```

3.530 $\int \sqrt{ex} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=161

$$\frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be}+\frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}+\frac{a^2(6Ab-aB)\sqrt{e}\ \cdots}{\cdots}$$

```
[Out] 1/36*(6*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/9*B*(e*x)^(3/2)*(b*x^3+a
)^(5/2)/b/e+1/24*a^2*(6*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3
+a)^(1/2))*e^(1/2)/b^(3/2)+1/24*a*(6*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b
/e
```

Rubi [A]
time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 285, 335, 281, 223, 212}

$$\frac{a^2\sqrt{e}(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(6Ab-aB)}{36be}+\frac{a(ex)^{3/2}\sqrt{a+bx^3}(6Ab-aB)}{24be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a*(6*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b*e) + ((6*A*b - a*B)*(e*
x)^(3/2)*(a + b*x^3)^(3/2))/(36*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(9
*b*e) + (a^2*(6*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*S
qrt[a + b*x^3])])/(24*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} - \frac{\left(-9Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\,dx}{9b}$$

$$= \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} + \frac{(a(6Ab-a}{}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B}{}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B}{}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B}{}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B}{}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B}{}$$

Mathematica [A]
time = 0.29, size = 120, normalized size = 0.75

$$\frac{x\sqrt{ex}\,\sqrt{a+bx^3}\left(30aAb+3a^2B+12Ab^2x^3+14abBx^3+8b^2Bx^6\right)}{72b} - \frac{a^2(-6Ab+aB)\sqrt{ex}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)}{24b^{3/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (x*Sqrt[e*x]*Sqrt[a + b*x^3]*(30*a*A*b + 3*a^2*B + 12*A*b^2*x^3 + 14*a*b*B*
x^3 + 8*b^2*B*x^6))/(72*b) - (a^2*(-6*A*b + a*B)*Sqrt[e*x]*ArcTanh[Sqrt[a +
 b*x^3]/(Sqrt[b]*x^(3/2))])/(24*b^(3/2)*Sqrt[x])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.52, size = 7290, normalized size = 45.28

method	result	size
risch	Expression too large to display	1080
elliptic	Expression too large to display	1183
default	Expression too large to display	7290

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(111) = 222$.
time = 0.51, size = 290, normalized size = 1.80

$$-\frac{1}{144}\left(6\left(\frac{3\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{\sqrt{b}}+\frac{2\left(\frac{3\sqrt{bx^3+a}\,a^2b}{x^{\frac{3}{2}}}-\frac{5\,(bx^3+a)^{\frac{3}{2}}a^2}{x^{\frac{9}{2}}}\right)}{b^2-\frac{2\,(bx^3+a)b}{x^3}+\frac{(bx^3+a)^2}{x^6}}\right)A-\left(\frac{3\,a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{3}{2}}}+\frac{2\left(\frac{3\sqrt{bx^3+a}\,a^3b^2}{x^{\frac{3}{2}}}-\frac{8\,(bx^3+a)^{\frac{3}{2}}a^3b}{x^{\frac{9}{2}}}-\frac{3\,(bx^3+a)^{\frac{5}{2}}a^3}{x^{\frac{15}{2}}}\right)}{b^4-\frac{3\,(bx^3+a)b^3}{x^3}+\frac{3\,(bx^3+a)^2b^2}{x^6}-\frac{(bx^3+a)^3b}{x^9}}\right)B\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/144*(6*(3*a^2*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b
*x^3 + a)/x^(3/2)))/sqrt(b) + 2*(3*sqrt(b*x^3 + a)*a^2*b/x^(3/2) - 5*(b*x^3
 + a)^(3/2)*a^2/x^(9/2))/(b^2 - 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*A
 - (3*a^3*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 +
a)/x^(3/2)))/b^(3/2) + 2*(3*sqrt(b*x^3 + a)*a^3*b^2/x^(3/2) - 8*(b*x^3 + a)
^(3/2)*a^3*b/x^(9/2) - 3*(b*x^3 + a)^(5/2)*a^3/x^(15/2))/(b^4 - 3*(b*x^3 +
a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3 + a)^3*b/x^9))*B)*e^(1/2)
```

Fricas [A]
time = 2.63, size = 258, normalized size = 1.60

$$\left[-\frac{3\,(Ba^3-6\,Aa^2b)\sqrt{b}\,e^{\frac{1}{2}}\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)-4\,(8\,Bb^3x^7+2\,(7\,Bab^2+6\,Ab^3)x^4+3\,(Ba^2b+10\,Aab^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{1}{2}}}{288\,b^2},\ \frac{3\,(Ba^3-6\,Aa^2b)\sqrt{-b}\,\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2bx^3+a}\right)e^{\frac{1}{2}}+2\,(8\,Bb^3x^7+2\,(7\,Bab^2+6\,Ab^3)x^4+3\,(Ba^2b+10\,Aab^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{1}{2}}}{144\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(B*a^3 - 6*A*a^2*b)*sqrt(b)*e^(1/2)*log(-8*b^2*x^6 - 8*a*b*x^3 -
 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*(8*B*b^3*x^7
+ 2*(7*B*a*b^2 + 6*A*b^3)*x^4 + 3*(B*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^3 + a)
*sqrt(x)*e^(1/2))/b^2, 1/144*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-b)*arctan(2*sqrt(
b*x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a))*e^(1/2) + 2*(8*B*b^3*x^7 + 2*(7*
B*a*b^2 + 6*A*b^3)*x^4 + 3*(B*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^3 + a)*sqrt(x
)*e^(1/2))/b^2]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(138) = 276$.

time = 15.05, size = 335, normalized size = 2.08

$$\frac{Aa^{\frac{3}{2}}(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e}+\frac{Aa^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{12e\sqrt{1+\frac{bx^3}{a}}}+\frac{A\sqrt{a}\,b(ex)^{\frac{9}{2}}}{4e^4\sqrt{1+\frac{bx^3}{a}}}+\frac{Aa^2\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{4\sqrt{b}}+\frac{Ab^2(ex)^{\frac{15}{2}}}{6\sqrt{a}\,e^7\sqrt{1+\frac{bx^3}{a}}}+\frac{Ba^{\frac{5}{2}}(ex)^{\frac{3}{2}}}{24be\sqrt{1+\frac{bx^3}{a}}}+\frac{17Ba^{\frac{3}{2}}(ex)^{\frac{9}{2}}}{72e^4\sqrt{1+\frac{bx^3}{a}}}+\frac{11B\sqrt{a}\,b(ex)^{\frac{15}{2}}}{36e^7\sqrt{1+\frac{bx^3}{a}}}-\frac{Ba^3\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{24b^{\frac{3}{2}}}+\frac{Bb^2(ex)^{\frac{21}{2}}}{9\sqrt{a}\,e^{10}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)*(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a**(3/2)*(e*x)**(3/2)/
(12*e*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*(e*x)**(9/2)/(4*e**4*sqrt(1 + b*x**
3/a)) + A*a**2*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(4*sq
rt(b)) + A*b**2*(e*x)**(15/2)/(6*sqrt(a)*e**7*sqrt(1 + b*x**3/a)) + B*a**(5
/2)*(e*x)**(3/2)/(24*b*e*sqrt(1 + b*x**3/a)) + 17*B*a**(3/2)*(e*x)**(9/2)/(
72*e**4*sqrt(1 + b*x**3/a)) + 11*B*sqrt(a)*b*(e*x)**(15/2)/(36*e**7*sqrt(1
+ b*x**3/a)) - B*a**3*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2))
)/(24*b**(3/2)) + B*b**2*(e*x)**(21/2)/(9*sqrt(a)*e**10*sqrt(1 + b*x**3/a))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(111) = 222$.

time = 0.75, size = 272, normalized size = 1.69

$$\frac{1}{72}\left(6\sqrt{bx^3+a}\left(2x^3+\frac{a}{b}\right)Bax^{\frac{3}{2}}+6\sqrt{bx^3+a}\left(2x^3+\frac{a}{b}\right)Abx^{\frac{3}{2}}+\left(2\left(4x^3+\frac{a}{b}\right)x^3-\frac{3a^2}{b^2}\right)\sqrt{bx^3+a}\,Bbx^{\frac{3}{2}}+24\left(\sqrt{bx^3+a}\,x^{\frac{3}{2}}-\frac{a\log\left(\left|-\sqrt{b}x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{\sqrt{b}}\right)Aa\right)e^{\frac{1}{2}}-\frac{(B^2a^6+4ABa^5b+4A^2a^4b^2)e^{\frac{1}{2}}\log\left(\left|\left(Ba^3x^{\frac{3}{2}}+2Aa^2bx^{\frac{3}{2}}\right)\sqrt{b}+\sqrt{B^2a^7+4ABa^6b+4A^2a^5b^2+\left(Ba^3x^{\frac{3}{2}}+2Aa^2bx^{\frac{3}{2}}\right)^2b}\right|\right)}{24b^{\frac{3}{2}}|Ba^3+2Aa^2b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*(6*sqrt(b*x^3 + a)*(2*x^3 + a/b)*B*a*x^(3/2) + 6*sqrt(b*x^3 + a)*(2*x^
3 + a/b)*A*b*x^(3/2) + (2*(4*x^3 + a/b)*x^3 - 3*a^2/b^2)*sqrt(b*x^3 + a)*B*
b*x^(3/2) + 24*(sqrt(b*x^3 + a)*x^(3/2) - a*log(abs(-sqrt(b)*x^(3/2) + sqrt
(b*x^3 + a)))/sqrt(b))*A*a)*e^(1/2) - 1/24*(B^2*a^6 + 4*A*B*a^5*b + 4*A^2*a
^4*b^2)*e^(1/2)*log(abs((B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2))*sqrt(b) + sqrt(
B^2*a^7 + 4*A*B*a^6*b + 4*A^2*a^5*b^2 + (B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2))
^2*b)))/(b^(3/2)*abs(B*a^3 + 2*A*a^2*b))
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,\left(b\,x^3+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2), x)
```

3.531 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=324

$$\frac{9a(16Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{320be}+\frac{(16Ab-aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{80be}+\frac{B\sqrt{ex}\,(a+bx^3)^{5/2}}{8be}+\frac{9\ 3^{3/4}a^{5/3}(16Ab-a\cdots}{\cdots}$$

```
[Out] 1/80*(16*A*b-B*a)*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/8*B*(b*x^3+a)^(5/2)*(e*
x)^(1/2)/b/e+9/320*a*(16*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+9/640*3^(
3/4)*a^(5/3)*(16*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1
/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {470, 285, 335, 231}

$$\frac{9\ 3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(16Ab-aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640be\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{\sqrt{ex}\,(a+bx^3)^{3/2}(16Ab-aB)}{80be}+\frac{9a\sqrt{ex}\,\sqrt{a+bx^3}\,(16Ab-aB)}{320be}+\frac{B\sqrt{ex}\,(a+bx^3)^{5/2}}{8be}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (9*a*(16*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*b*e) + ((16*A*b - a*B)*
Sqrt[e*x]*(a + b*x^3)^(3/2))/(80*b*e) + (B*Sqrt[e*x]*(a + b*x^3)^(5/2))/(8*
b*e) + (9*3^(3/4)*a^(5/3)*(16*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*b*e*Sqrt[(b^(1/3)*x*(a^(1
/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{B\sqrt{ex}\,(a+bx^3)^{5/2}}{8be} - \frac{\left(-8Ab+\frac{aB}{2}\right)\int \frac{(a+bx^3)^{3/2}}{\sqrt{ex}}\,dx}{8b}$$

$$= \frac{(16Ab-aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{80be} + \frac{B\sqrt{ex}\,(a+bx^3)^{5/2}}{8be} + \frac{(9a(16Ab-aB))\ \ldots}{160\ldots}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{80be} + \frac{B\sqrt{ex}\ldots}{}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{80be} + \frac{B\sqrt{ex}\ldots}{}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{80be} + \frac{B\sqrt{ex}\ldots}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 96, normalized size = 0.30

$$\frac{x\sqrt{a+bx^3}\left(B(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}} + a(16Ab-aB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{8b\sqrt{ex}\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(16*A*b - a*B)*
Hypergeometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)]))/(8*b*Sqrt[e*x]*Sqrt[1 + (
b*x^3)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 4173, normalized size = 12.88 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/320*(b*x^3+a)^(1/2)*x/(-a*b^2)^(1/3)/b^2*(-54*I*B*3^(1/2)*(-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(-a*b^2)^(2/3)*a^3*e-27*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*a^2*b+108*I*B*3^
(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a^3*b*e*x-64*I*A*3^(1/2)*(1/b^2*e*x*(-b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*e*x)^(1/2)*(-a*b^2
)^(1/3)*b^3*x^3-54*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*
3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b^2*e*x^2-40*I*B*3^(1/2)*(
1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*e*x)
^(1/2)*(-a*b^2)^(1/3)*b^3*x^6-864*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(
-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b^3*e*x^2-208*I*A*3^(
1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*((b*x^3+a
)*e*x)^(1/2)*(-a*b^2)^(1/3)*a*b^2+54*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b^2*e*x^2+120*B*(-
a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3)))^(1/2)*b^3*x^6+864*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)
```

```
^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a
^2*b*e+1728*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a^2*b^2*e*x-76*I*B*3^(1/2)*(1/b^
2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*e*x)^(1/
2)*(-a*b^2)^(1/3)*a*b^2*x^3-108*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a^3*b*e*x-86
4*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^2*b*e+54*B*(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/sqrt(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*e^(-1/2)/sqrt(x)
, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.04, size = 199, normalized size = 0.61

$$\frac{Aa^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)}+\frac{A\sqrt{a}\,bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}+\frac{Ba^{\frac{3}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}+\frac{B\sqrt{a}\,bx^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(e)*gamma(7/6)) + A*sqrt(a)*b*x**(7/2)*gamma(7/6)*hyper((-1/
2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + B*a**
(3/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi
)/a)/(3*sqrt(e)*gamma(13/6)) + B*sqrt(a)*b*x**(13/2)*gamma(13/6)*hyper((-1/
2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*e^(-1/2)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(1/2), x)
```

3.532 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=614

$$\frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4}+\frac{27\left(1+\sqrt{3}\right)a(14Ab+aB)\sqrt{ex}\,\sqrt{a+bx^3}}{112b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{(14Ab+aB)(ex)^{5/2}\,(a+b\ldots}{7ae^4}$$

```
[Out] 1/7*(14*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/a/e^4-2*A*(b*x^3+a)^(5/2)/a/e/
(e*x)^(1/2)+9/56*(14*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/e^4+27/112*a*(14*
A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/e^2/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))-27/112*3^(1/4)*a^(4/3)*(14*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*(
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((
1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2
),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(
b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-9/22
4*3^(3/4)*a^(4/3)*(14*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/
2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(
a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 285, 335, 314, 231, 1895}

$$-\frac{9\,3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+14Ab)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{224b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}a^{4/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+14Ab)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{112b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{27\left(1+\sqrt{3}\right)a\sqrt{ex}\sqrt{a+bx^3}(aB+14Ab)}{112b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}(aB+14Ab)}{7ae^4}+\frac{9(ex)^{5/2}\sqrt{a+bx^3}(aB+14Ab)}{56e^4}-\frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (9*(14*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(56*e^4) + (27*(1 + Sqrt[3])
*a*(14*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(112*b^(2/3)*e^2*(a^(1/3) + (1
```

```
 + Sqrt[3])*b^(1/3)*x)) + ((14*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7
*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(a*e*Sqrt[e*x]) - (27*3^(1/4)*a^(4/3)*(14
*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(
a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
 + Sqrt[3])/4])/(112*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*(1 - Sqrt
[3])*a^(4/3)*(14*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*El
lipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)], (2 + Sqrt[3])/4])/(224*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = -\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}} + \frac{(14Ab+aB)\int (ex)^{3/2}(a+bx^3)^{3/2}\,dx}{ae^3}$$

$$= \frac{(14Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{7ae^4} - \frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}} + \frac{(9(14Ab+aB))\int (ex)}{14e^3}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{(14Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{7ae^4} - \frac{2A(a}{ae}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{(14Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{7ae^4} - \frac{2A(a}{ae}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{(14Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{7ae^4} - \frac{2A(a}{ae}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{27\left(1+\sqrt{3}\right)a(14Ab+aB)\sqrt{ex}\,\sqrt{a+b}}{112b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 84, normalized size = 0.14

$$\frac{2x\sqrt{a+bx^3}\left(-\frac{5A(a+bx^3)^2}{a}+\frac{(14Ab+aB)x^3\,{}_2F_1\left(-\frac{3}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + ((14*A*b + a*B)*x^3*Hypergeo
metric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^(3
/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 6142, normalized size = 10.00

method	result	size
risch	Expression too large to display	1140
elliptic	Expression too large to display	1233
default	Expression too large to display	6142

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*e^(-3/2)/x^(3/2)
, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.38, size = 202, normalized size = 0.33

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)}+\frac{A\sqrt{a}\,bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{Ba^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{B\sqrt{a}\,bx^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

```
[Out] A*a**(3/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + A*sqrt(a)*b*x**(5/2)*gamma(5/6)*hyper((
```

```
-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6)) + B
*a**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_polar(
I*pi)/a)/(3*e**(3/2)*gamma(11/6)) + B*sqrt(a)*b*x**(11/2)*gamma(11/6)*hyper
((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*e^(-3/2)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{{\left(e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(3/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(3/2), x)
```

3.533 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=152

$$\frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4}+\frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4}-\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}+\frac{a(4Ab+aB)\tanh^{-1}\left(\frac{\ldots}{e^{3/\ldots}}\right.}{4\sqrt{b}\,e^{5/2}}$$

```
[Out] 1/6*(4*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/a/e^4-2/3*A*(b*x^3+a)^(5/2)/a/e
/(e*x)^(3/2)+1/4*a*(4*A*b+B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a
)^(1/2))/e^(5/2)/b^(1/2)+1/4*(4*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/e^4
```

Rubi [A]
time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 285, 335, 281, 223, 212}

$$\frac{a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}\,e^{5/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+4Ab)}{6ae^4}+\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(aB+4Ab)}{4e^4}-\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] ((4*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(4*e^4) + ((4*A*b + a*B)*(e*x)^
(3/2)*(a + b*x^3)^(3/2))/(6*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(3*a*e*(e*x)^(
3/2)) + (a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*
x^3])])/(4*Sqrt[b]*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = -\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{(4Ab+aB)\int \sqrt{ex}\,(a+bx^3)^{3/2}\,dx}{ae^3}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{(3(4Ab+aB))\int \sqrt{ex}}{4e^3}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+}{3ae(e}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+}{3ae(e}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+}{3ae(e}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+}{3ae(e}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+}{3ae(e}$$

Mathematica [A]
time = 0.31, size = 100, normalized size = 0.66

$$\frac{x\left(\sqrt{b}\sqrt{a+bx^3}\,(-8aA+4Abx^3+5aBx^3+2bBx^6)+3a(4Ab+aB)x^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)\right)}{12\sqrt{b}\,(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*(Sqrt[b]*Sqrt[a + b*x^3]*(-8*a*A + 4*A*b*x^3 + 5*a*B*x^3 + 2*b*B*x^6) +
3*a*(4*A*b + a*B)*x^(3/2)*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))]))/(12*
Sqrt[b]*(e*x)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 7108, normalized size = 46.76

method	result	size
risch	Expression too large to display	1067
elliptic	Expression too large to display	1140
default	Expression too large to display	7108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(105) = 210$.
time = 0.54, size = 223, normalized size = 1.47

$$-\frac{1}{24}\left(4\left(3\,a\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)+\frac{4\sqrt{bx^3+a}\,a}{x^{\frac{3}{2}}}+\frac{2\sqrt{bx^3+a}\,ab}{\left(b-\frac{bx^3+a}{x^3}\right)x^{\frac{3}{2}}}\right)A+\left(\frac{3\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{\sqrt{b}}+\frac{2\left(\frac{3\sqrt{bx^3+a}\,a^2b}{x^{\frac{3}{2}}}-\frac{5\,(bx^3+a)^{\frac{3}{2}}a^2}{x^{\frac{9}{2}}}\right)}{b^2-\frac{2\,(bx^3+a)b}{x^3}+\frac{(bx^3+a)^2}{x^6}}\right)B\right)e^{\left(-\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(4*(3*a*sqrt(b)*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + s
qrt(b*x^3 + a)/x^(3/2))) + 4*sqrt(b*x^3 + a)*a/x^(3/2) + 2*sqrt(b*x^3 + a)*
a*b/((b - (b*x^3 + a)/x^3)*x^(3/2)))*A + (3*a^2*log(-(sqrt(b) - sqrt(b*x^3
+ a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 + a)/x^(3/2)))/sqrt(b) + 2*(3*sqrt(b*x^
3 + a)*a^2*b/x^(3/2) - 5*(b*x^3 + a)^(3/2)*a^2/x^(9/2))/(b^2 - 2*(b*x^3 + a
)*b/x^3 + (b*x^3 + a)^2/x^6))*B)*e^(-5/2)
```

Fricas [A]
time = 3.88, size = 232, normalized size = 1.53

$$\left[\frac{\left(3\,(Ba^2+4\,Aab)\sqrt{b}\,x^2\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)+4\,(2\,Bb^2x^6+(5\,Bab+4\,Ab^2)x^3-8\,Aab)\sqrt{bx^3+a}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{48\,bx^2},-\frac{\left(3\,(Ba^2+4\,Aab)\sqrt{-b}\,x^2\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)-2\,(2\,Bb^2x^6+(5\,Bab+4\,Ab^2)x^3-8\,Aab)\sqrt{bx^3+a}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{24\,bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(B*a^2 + 4*A*a*b)*sqrt(b)*x^2*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*
x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) + 4*(2*B*b^2*x^6 + (5*B*a
*b + 4*A*b^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(x))*e^(-5/2)/(b*x^2), -1/
24*(3*(B*a^2 + 4*A*a*b)*sqrt(-b)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3
/2)/(2*b*x^3 + a)) - 2*(2*B*b^2*x^6 + (5*B*a*b + 4*A*b^2)*x^3 - 8*A*a*b)*sq
rt(b*x^3 + a)*sqrt(x))*e^(-5/2)/(b*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(138) = 276$.
time = 12.90, size = 289, normalized size = 1.90

$$-\frac{2Aa^{\frac{3}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{A\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}-\frac{2A\sqrt{a}\,bx^{\frac{3}{2}}}{3e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{Aa\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{e^{\frac{5}{2}}}+\frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}+\frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{B\sqrt{a}\,bx^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4\sqrt{b}\,e^{\frac{5}{2}}}+\frac{Bb^2x^{\frac{15}{2}}}{6\sqrt{a}\,e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

```
[Out] -2*A*a**(3/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*x**(3/
2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 2*A*sqrt(a)*b*x**(3/2)/(3*e**(5/2)*sqr
t(1 + b*x**3/a)) + A*a*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/e**(5/2) + B
*a**(3/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(3/2)*x**(3/2)/(1
2*e**(5/2)*sqrt(1 + b*x**3/a)) + B*sqrt(a)*b*x**(9/2)/(4*e**(5/2)*sqrt(1 +
b*x**3/a)) + B*a**2*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*sqrt(b)*e**(5/2)) +
B*b**2*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*e^(-5/2)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{{\left(e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2), x)
```

3.534 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=314

$$\frac{9(2Ab+aB)\sqrt{ex}\,\sqrt{a+bx^3}}{20e^4}+\frac{(2Ab+aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{5ae^4}-\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}+\frac{9\,3^{3/4}a^{2/3}(2Ab+aB)\sqrt{ex}\,\Big(}{}$$

```
[Out] -2/5*A*(b*x^3+a)^(5/2)/a/e/(e*x)^(5/2)+1/5*(2*A*b+B*a)*(b*x^3+a)^(3/2)*(e*x
)^(1/2)/a/e^4+9/20*(2*A*b+B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/e^4+9/40*3^(3/4)
*a^(2/3)*(2*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2
/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))
^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^
(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))^2)^(1/2)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {464, 285, 335, 231}

$$\frac{9\,3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\,(a+bx^3)^{3/2}(aB+2Ab)}{5ae^4}+\frac{9\sqrt{ex}\,\sqrt{a+bx^3}\,(aB+2Ab)}{20e^4}-\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (9*(2*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*e^4) + ((2*A*b + a*B)*Sqrt[
e*x]*(a + b*x^3)^(3/2))/(5*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(5*a*e*(e*x)^(5
/2)) + (9*3^(3/4)*a^(2/3)*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*e^4*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = -\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}} + \frac{(2Ab+aB)\int \frac{(a+bx^3)^{3/2}}{\sqrt{ex}}\,dx}{ae^3}$$

$$= \frac{(2Ab+aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{5ae^4} - \frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}} + \frac{(9(2Ab+aB))\int \frac{\sqrt{a+b}}{\sqrt{ex}}}{10e^3}$$

$$= \frac{9(2Ab+aB)\sqrt{ex}\,\sqrt{a+bx^3}}{20e^4} + \frac{(2Ab+aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{5ae^4} - \frac{2A(a+bx^3}{5ae(ex)^{5}}$$

$$= \frac{9(2Ab+aB)\sqrt{ex}\,\sqrt{a+bx^3}}{20e^4} + \frac{(2Ab+aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{5ae^4} - \frac{2A(a+bx^3}{5ae(ex)^{5}}$$

$$= \frac{9(2Ab+aB)\sqrt{ex}\,\sqrt{a+bx^3}}{20e^4} + \frac{(2Ab+aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{5ae^4} - \frac{2A(a+bx^3}{5ae(ex)^{5}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 85, normalized size = 0.27

$$\frac{2x\sqrt{a+bx^3}\left(-\frac{A(a+bx^3)^2}{a} + \frac{5(2Ab+aB)x^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((A*(a + b*x^3)^2)/a) + (5*(2*A*b + a*B)*x^3*Hyperge
ometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^(7
/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 3966, normalized size = 12.63 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/20*(b*x^3+a)^(1/2)/x^2/(-a*b^2)^(1/3)/b*(216*A*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x^4+108*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b*e*x^4-10*I*A*3^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^2*x^3-13*I*B*3^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b*x^3+108*I*A*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*e*x^3+12*B*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^2*x^6-24*A*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b-108*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*e*x^3+108*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^3*e*x^5-4*I*B*3^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*

```
b*x-(-a*b^2)^(1/3)))^(1/2)*b^2*x^6-108*I*B*3^(1/2)*(-a*b^2)^(1/3)*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*a^2*b*e*x^4+30*A*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x
+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^2*x^3+39*B*(-a*b^2)^(1/3)*((
b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1
/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^
(1/2)*a*b*x^3-108*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^3*e*x^5-54*B*(-a*b^2)^(2/3)*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*a^2*e*x^3-216*I*A*3^(1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
[Out] e^(-7/2)*integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*e^(-7/2)/x^(7/2)
, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 24.48, size = 202, normalized size = 0.64

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{A\sqrt{a}\,b\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}+\frac{Ba^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}+\frac{B\sqrt{a}\,bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

```
[Out] A*a**(3/2)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + A*sqrt(a)*b*sqrt(x)*gamma(1/6)*hyper((
-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6)) + B*a
**(3/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi
)/a)/(3*e**(7/2)*gamma(7/6)) + B*sqrt(a)*b*x**(7/2)*gamma(7/6)*hyper((-1/2,
 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(13/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*e^(-7/2)/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{{\left(e\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(7/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(7/2), x)
```

3.535 $\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=241

$$\frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}$$

```
[Out] 1/144*a*(10*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e+1/120*(10*A*b-3*B*a)
*(e*x)^(9/2)*(b*x^3+a)^(5/2)/b/e+1/15*B*(e*x)^(9/2)*(b*x^3+a)^(7/2)/b/e-1/3
84*a^4*(10*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)
^(1/2))/b^(5/2)+1/384*a^3*(10*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^
2+1/192*a^2*(10*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A]

time = 0.12, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {470, 285, 327, 335, 281, 223, 212}

$$-\frac{a^4e^{7/2}(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}}+\frac{a^3e^2(ex)^{3/2}\sqrt{a+bx^3}\,(10Ab-3aB)}{384b^2}+\frac{a^2(ex)^{9/2}\sqrt{a+bx^3}\,(10Ab-3aB)}{192be}+\frac{(ex)^{9/2}\left(a+bx^3\right)^{5/2}(10Ab-3aB)}{120be}+\frac{a(ex)^{9/2}\left(a+bx^3\right)^{3/2}(10Ab-3aB)}{144be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (a^3*(10*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(384*b^2) + (a^2*(10
*A*b - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(192*b*e) + (a*(10*A*b - 3*a*B)*
(e*x)^(9/2)*(a + b*x^3)^(3/2))/(144*b*e) + ((10*A*b - 3*a*B)*(e*x)^(9/2)*(a
 + b*x^3)^(5/2))/(120*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(7/2))/(15*b*e) - (
a^4*(10*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
+ b*x^3])])/(384*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}-\frac{\left(-15Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\,dx}{15b}$$

$$=\frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}+\frac{(a(10A$$

$$=\frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}+\frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right.}{120be}$$

$$=\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right.}{144be}$$

$$=\frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+}}{192be}$$

$$=\frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+}}{192be}$$

$$=\frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+}}{192be}$$

$$=\frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+}}{192be}$$

$$=\frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+}}{192be}$$

Mathematica [A]
time = 0.42, size = 164, normalized size = 0.68

$$\frac{e^3\sqrt{ex}\left(\sqrt{b}\,x^{3/2}\sqrt{a+bx^3}\left(-45a^4B+30a^3b(5A+Bx^3)+96b^4x^9(5A+4Bx^3)+16ab^3x^6(85A+63Bx^3)+4a^2b^2x^3(295A+186Bx^3)\right)+15a^4(-10Ab+3aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)\right)}{5760b^{5/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(-45*a^4*B + 30*a^3*b*(5*A
+ B*x^3) + 96*b^4*x^9*(5*A + 4*B*x^3) + 16*a*b^3*x^6*(85*A + 63*B*x^3) + 4*
a^2*b^2*x^3*(295*A + 186*B*x^3)) + 15*a^4*(-10*A*b + 3*a*B)*ArcTanh[Sqrt[a
+ b*x^3]/(Sqrt[b]*x^(3/2))]))/(5760*b^(5/2)*Sqrt[x])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 8117, normalized size = 33.68

method	result	size
risch	Expression too large to display	1135
elliptic	Expression too large to display	1443
default	Expression too large to display	8117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(176) = 352.
time = 0.50, size = 443, normalized size = 1.84

$$\frac{1}{11520}\left(10\left(\frac{15\,a^{4}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^{3}+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^{3}+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{3}{2}}}+\frac{2\left(\frac{15\sqrt{bx^{3}+a}\,a^{4}b^{3}}{x^{\frac{3}{2}}}-\frac{55\,(bx^{3}+a)^{\frac{3}{2}}a^{4}b^{2}}{x^{\frac{9}{2}}}+\frac{73\,(bx^{3}+a)^{\frac{5}{2}}a^{4}b}{x^{\frac{15}{2}}}+\frac{15\,(bx^{3}+a)^{\frac{7}{2}}a^{4}}{x^{\frac{21}{2}}}\right)}{b^{5}-\frac{4\,(bx^{3}+a)b^{4}}{x^{3}}+\frac{6\,(bx^{3}+a)^{2}b^{3}}{x^{6}}-\frac{4\,(bx^{3}+a)^{3}b^{2}}{x^{9}}+\frac{(bx^{3}+a)^{4}b}{x^{12}}}\right)A-3\left(\frac{15\,a^{5}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^{3}+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^{3}+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{5}{2}}}+\frac{2\left(\frac{15\sqrt{bx^{3}+a}\,a^{5}b^{4}}{x^{\frac{3}{2}}}-\frac{70\,(bx^{3}+a)^{\frac{3}{2}}a^{5}b^{3}}{x^{\frac{9}{2}}}+\frac{128\,(bx^{3}+a)^{\frac{5}{2}}a^{5}b^{2}}{x^{\frac{15}{2}}}+\frac{70\,(bx^{3}+a)^{\frac{7}{2}}a^{5}b}{x^{\frac{21}{2}}}-\frac{15\,(bx^{3}+a)^{\frac{9}{2}}a^{5}}{x^{\frac{27}{2}}}\right)}{b^{7}-\frac{5\,(bx^{3}+a)b^{6}}{x^{3}}+\frac{10\,(bx^{3}+a)^{2}b^{5}}{x^{6}}-\frac{10\,(bx^{3}+a)^{3}b^{4}}{x^{9}}+\frac{5\,(bx^{3}+a)^{4}b^{3}}{x^{12}}-\frac{(bx^{3}+a)^{5}b^{2}}{x^{15}}}\right)B\right)e^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/11520*(10*(15*a^4*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqr
t(b*x^3 + a)/x^(3/2)))/b^(3/2) + 2*(15*sqrt(b*x^3 + a)*a^4*b^3/x^(3/2) - 55
*(b*x^3 + a)^(3/2)*a^4*b^2/x^(9/2) + 73*(b*x^3 + a)^(5/2)*a^4*b/x^(15/2) +
15*(b*x^3 + a)^(7/2)*a^4/x^(21/2))/(b^5 - 4*(b*x^3 + a)*b^4/x^3 + 6*(b*x^3
+ a)^2*b^3/x^6 - 4*(b*x^3 + a)^3*b^2/x^9 + (b*x^3 + a)^4*b/x^12))*A - 3*(15
*a^5*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 + a)/x^
(3/2)))/b^(5/2) + 2*(15*sqrt(b*x^3 + a)*a^5*b^4/x^(3/2) - 70*(b*x^3 + a)^(3
/2)*a^5*b^3/x^(9/2) + 128*(b*x^3 + a)^(5/2)*a^5*b^2/x^(15/2) + 70*(b*x^3 +
a)^(7/2)*a^5*b/x^(21/2) - 15*(b*x^3 + a)^(9/2)*a^5/x^(27/2))/(b^7 - 5*(b*x^
3 + a)*b^6/x^3 + 10*(b*x^3 + a)^2*b^5/x^6 - 10*(b*x^3 + a)^3*b^4/x^9 + 5*(b
*x^3 + a)^4*b^3/x^12 - (b*x^3 + a)^5*b^2/x^15))*B)*e^(7/2)
```

Fricas [A]
time = 2.17, size = 358, normalized size = 1.49

$$\left[-\frac{15\,(3\,Ba^{5}-10\,Aa^{4}b)\sqrt{b}\,e^{\frac{7}{2}}\log\left(-8\,b^{2}x^{6}-8\,abx^{3}+4\,(2\,bx^{4}+ax)\sqrt{bx^{3}+a}\,\sqrt{b}\,\sqrt{x}-a^{2}\right)-4\,(384\,Bb^{5}x^{13}+48\,(21\,Bab^{4}+10\,Ab^{5})x^{10}+8\,(93\,Ba^{2}b^{3}+170\,Aab^{4})x^{7}+10\,(3\,Ba^{3}b^{2}+118\,Aa^{2}b^{3})x^{4}-15\,(3\,Ba^{4}b-10\,Aa^{3}b^{2})x)\sqrt{bx^{3}+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{23040\,b^{3}},\ -\frac{15\,(3\,Ba^{5}-10\,Aa^{4}b)\sqrt{-b}\arctan\left(\frac{2\sqrt{bx^{3}+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^{3}+a}\right)e^{\frac{7}{2}}-2\,(384\,Bb^{5}x^{13}+48\,(21\,Bab^{4}+10\,Ab^{5})x^{10}+8\,(93\,Ba^{2}b^{3}+170\,Aab^{4})x^{7}+10\,(3\,Ba^{3}b^{2}+118\,Aa^{2}b^{3})x^{4}-15\,(3\,Ba^{4}b-10\,Aa^{3}b^{2})x)\sqrt{bx^{3}+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{11520\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] [-1/23040*(15*(3*B*a^5 - 10*A*a^4*b)*sqrt(b)*e^(7/2)*log(-8*b^2*x^6 - 8*a*b
*x^3 + 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*(384*B*
```

```
b^5*x^13 + 48*(21*B*a*b^4 + 10*A*b^5)*x^10 + 8*(93*B*a^2*b^3 + 170*A*a*b^4)
*x^7 + 10*(3*B*a^3*b^2 + 118*A*a^2*b^3)*x^4 - 15*(3*B*a^4*b - 10*A*a^3*b^2)
*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(7/2))/b^3, -1/11520*(15*(3*B*a^5 - 10*A*a^4*
b)*sqrt(-b)*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a))*e^(7/2
) - 2*(384*B*b^5*x^13 + 48*(21*B*a*b^4 + 10*A*b^5)*x^10 + 8*(93*B*a^2*b^3 +
 170*A*a*b^4)*x^7 + 10*(3*B*a^3*b^2 + 118*A*a^2*b^3)*x^4 - 15*(3*B*a^4*b -
10*A*a^3*b^2)*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(7/2))/b^3]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(176) = 352.
time = 1.50, size = 437, normalized size = 1.81

$$\frac{1}{5760}\left(480\sqrt{bx^3+a}\left(2x^3+\frac{a}{b}\right)Aa^2x^{\frac{3}{2}}+80\left(2\left(4x^3+\frac{a}{b}\right)x^3-\frac{3a^2}{b^2}\right)\sqrt{bx^3+a}\,Ba^2x^{\frac{3}{2}}+160\left(2\left(4x^3+\frac{a}{b}\right)x^3-\frac{3a^2}{b^2}\right)\sqrt{bx^3+a}\,Aabx^{\frac{3}{2}}+20\left(2\left(4\left(6x^3+\frac{a}{b}\right)x^3-\frac{5a^2}{b^2}\right)x^3+\frac{15a^3}{b^3}\right)\sqrt{bx^3+a}\,Babx^{\frac{3}{2}}+10\left(2\left(4\left(6x^3+\frac{a}{b}\right)x^3-\frac{5a^2}{b^2}\right)x^3+\frac{15a^3}{b^3}\right)\sqrt{bx^3+a}\,Ab^2x^{\frac{3}{2}}+\left(2\left(4\left(6\left(8x^3+\frac{a}{b}\right)x^3-\frac{7a^2}{b^2}\right)x^3+\frac{35a^3}{b^3}\right)x^3-\frac{105a^4}{b^4}\right)\sqrt{bx^3+a}\,Bb^2x^{\frac{3}{2}}\right)e^{\frac{7}{2}}-\frac{(9B^2a^{10}-60ABa^9b+100A^2a^8b^2)e^{\frac{7}{2}}\log\left(\left|-\left(3Ba^5x^{\frac{3}{2}}-10Aa^4bx^{\frac{3}{2}}\right)\sqrt{b}+\sqrt{9B^2a^{11}-60ABa^{10}b+100A^2a^9b^2+\left(3Ba^5x^{\frac{3}{2}}-10Aa^4bx^{\frac{3}{2}}\right)^2b}\right|\right)}{384b^{\frac{5}{2}}|-3Ba^5+10Aa^4b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/5760*(480*sqrt(b*x^3 + a)*(2*x^3 + a/b)*A*a^2*x^(3/2) + 80*(2*(4*x^3 + a/
b)*x^3 - 3*a^2/b^2)*sqrt(b*x^3 + a)*B*a^2*x^(3/2) + 160*(2*(4*x^3 + a/b)*x^
3 - 3*a^2/b^2)*sqrt(b*x^3 + a)*A*a*b*x^(3/2) + 20*(2*(4*(6*x^3 + a/b)*x^3 -
 5*a^2/b^2)*x^3 + 15*a^3/b^3)*sqrt(b*x^3 + a)*B*a*b*x^(3/2) + 10*(2*(4*(6*x
^3 + a/b)*x^3 - 5*a^2/b^2)*x^3 + 15*a^3/b^3)*sqrt(b*x^3 + a)*A*b^2*x^(3/2)
+ (2*(4*(6*(8*x^3 + a/b)*x^3 - 7*a^2/b^2)*x^3 + 35*a^3/b^3)*x^3 - 105*a^4/b
^4)*sqrt(b*x^3 + a)*B*b^2*x^(3/2))*e^(7/2) - 1/384*(9*B^2*a^10 - 60*A*B*a^9
*b + 100*A^2*a^8*b^2)*e^(7/2)*log(abs(-(3*B*a^5*x^(3/2) - 10*A*a^4*b*x^(3/2
))*sqrt(b) + sqrt(9*B^2*a^11 - 60*A*B*a^10*b + 100*A^2*a^9*b^2 + (3*B*a^5*x
^(3/2) - 10*A*a^4*b*x^(3/2))^2*b)))/(b^(5/2)*abs(-3*B*a^5 + 10*A*a^4*b))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2), x)
```

3.536 $\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=404

$$\frac{81a^3(4Ab-aB)e^2\sqrt{ex}\,\sqrt{a+bx^3}}{5632b^2}+\frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be}+\frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be}$$

```
[Out] 15/704*a*(4*A*b-B*a)*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/44*(4*A*b-B*a)*(e*x)
^(7/2)*(b*x^3+a)^(5/2)/b/e+1/14*B*(e*x)^(7/2)*(b*x^3+a)^(7/2)/b/e+27/1408*a
^2*(4*A*b-B*a)*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+81/5632*a^3*(4*A*b-B*a)*e^2*
(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2-27/11264*3^(3/4)*a^(11/3)*(4*A*b-B*a)*e^2*(
a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*
(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^2/(b*x^
3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {470, 285, 327, 335, 231}

$$-\frac{27\,3^{3/4}a^{11/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(4Ab-aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{11264b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{81a^3e^2\sqrt{ex}\,\sqrt{a+bx^3}\,(4Ab-aB)}{5632b^2}+\frac{27a^2(ex)^{7/2}\sqrt{a+bx^3}\,(4Ab-aB)}{1408be}+\frac{15a(ex)^{7/2}\left(a+bx^3\right)^{3/2}(4Ab-aB)}{704be}+\frac{(ex)^{7/2}\left(a+bx^3\right)^{5/2}(4Ab-aB)}{44be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (81*a^3*(4*A*b - a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(5632*b^2) + (27*a^2*(
4*A*b - a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(1408*b*e) + (15*a*(4*A*b - a*B)*
(e*x)^(7/2)*(a + b*x^3)^(3/2))/(704*b*e) + ((4*A*b - a*B)*(e*x)^(7/2)*(a +
b*x^3)^(5/2))/(44*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^(7/2))/(14*b*e) - (27*3
^(3/4)*a^(11/3)*(4*A*b - a*B)*e^2*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(11264*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{B(ex)^{7/2} \left(a+bx^3\right)^{7/2}}{14be} - \frac{\left(-14Ab+\frac{7aB}{2}\right) \int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \, dx}{14b}$$

$$= \frac{(4Ab-aB)(ex)^{7/2} \left(a+bx^3\right)^{5/2}}{44be} + \frac{B(ex)^{7/2} \left(a+bx^3\right)^{7/2}}{14be} + \frac{(15a(4A}{}$$

$$= \frac{15a(4Ab-aB)(ex)^{7/2} \left(a+bx^3\right)^{3/2}}{704be} + \frac{(4Ab-aB)(ex)^{7/2} \left(a+bx^3\right)^{5}}{44be}$$

$$= \frac{27a^2(4Ab-aB)(ex)^{7/2} \sqrt{a+bx^3}}{1408be} + \frac{15a(4Ab-aB)(ex)^{7/2} (a+bx}{704be}$$

$$= \frac{81a^3(4Ab-aB)e^2 \sqrt{ex} \sqrt{a+bx^3}}{5632b^2} + \frac{27a^2(4Ab-aB)(ex)^{7/2} \sqrt{a+}}{1408be}$$

$$= \frac{81a^3(4Ab-aB)e^2 \sqrt{ex} \sqrt{a+bx^3}}{5632b^2} + \frac{27a^2(4Ab-aB)(ex)^{7/2} \sqrt{a+}}{1408be}$$

$$= \frac{81a^3(4Ab-aB)e^2 \sqrt{ex} \sqrt{a+bx^3}}{5632b^2} + \frac{27a^2(4Ab-aB)(ex)^{7/2} \sqrt{a+}}{1408be}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.14, size = 116, normalized size = 0.29

$$\frac{e^2 \sqrt{ex} \sqrt{a+bx^3} \left(-\left(a+bx^3\right)^3 \sqrt{1+\frac{bx^3}{a}} \left(-28Ab+7aB-22bBx^3\right)+7a^3(-4Ab+aB) \, _2F_1\left(-\frac{5}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{308b^2 \sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^3*Sqrt[1 + (b*x^3)/a]*(-28*A*
b + 7*a*B - 22*b*B*x^3)) + 7*a^3*(-4*A*b + a*B)*Hypergeometric2F1[-5/2, 1/6
, 7/6, -((b*x^3)/a)]))/(308*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.33, size = 5063, normalized size = 12.53

method	result
risch	$\frac{(2816 B\,b^4 x^{12}+3584 A\,b^4 x^9+7552 B a\,b^3 x^9+10528 a\,b^3 A\,x^6+5816 B\,a^2 b^2 x^6+9968 A\,a^2 b^2 x^3+324 B\,a^3 b\,x^3+2268 A\,a^3 b-567 B\,a^4)x\sqrt{b\,x}}{39424 b^2\sqrt{ex}}$
elliptic	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*x^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^11 + (2*B*a*b + A*b^2)*x^8 + (B*a^2 + 2*A*a*b)*x^5 + A*a^
2*x^2)*sqrt(b*x^3 + a)*sqrt(x)*e^(5/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 175.61, size = 308, normalized size = 0.76

$$\frac{Aa^{\frac{5}{2}}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)}+\frac{2Aa^{\frac{3}{2}}be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}+\frac{A\sqrt{a}\,b^2e^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)}+\frac{Ba^{\frac{5}{2}}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}+\frac{2Ba^{\frac{3}{2}}be^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)}+\frac{B\sqrt{a}\,b^2e^{\frac{5}{2}}x^{\frac{25}{2}}\Gamma\left(\frac{25}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{25}{6}\\ \frac{31}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{31}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] A*a**(5/2)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(13/6)) + 2*A*a**(3/2)*b*e**(5/2)*x**(13/2)*gamm
a(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/
6)) + A*sqrt(a)*b**2*e**(5/2)*x**(19/2)*gamma(19/6)*hyper((-1/2, 19/6), (25
/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(25/6)) + B*a**(5/2)*e**(5/2)*x**(1
3/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(19/6)) + 2*B*a**(3/2)*b*e**(5/2)*x**(19/2)*gamma(19/6)*hyper((-1/2, 1
9/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(25/6)) + B*sqrt(a)*b**2*e
**(5/2)*x**(25/2)*gamma(25/6)*hyper((-1/2, 25/6), (31/6,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(31/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*x^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(5/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(5/2), x)
```

3.537 $\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=661

$$\frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be}+\frac{81\left(1+\sqrt{3}\right)a^3(26Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^3}}{11648b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{3a(26Ab-5aB)(ex)}{728be}$$

```
[Out] 3/728*a*(26*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/260*(26*A*b-5*B*a)
*(e*x)^(5/2)*(b*x^3+a)^(5/2)/b/e+1/13*B*(e*x)^(5/2)*(b*x^3+a)^(7/2)/b/e+27/
5824*a^2*(26*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+81/11648*a^3*(26*A*
b-5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))-81/11648*3^(1/4)*a^(10/3)*(26*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)
*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipt
icE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/
(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-27/
23296*3^(3/4)*a^(10/3)*(26*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+
1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3
)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 285, 335, 314, 231, 1895}

$$-\frac{27\,3^{3/4}\left(1-\sqrt{3}\right)a^{10/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(26Ab-5aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{23296b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{81\sqrt[4]{3}\,a^{10/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(26Ab-5aB)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{11648b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{81\left(1+\sqrt{3}\right)a^3e\sqrt{ex}\,\sqrt{a+bx^3}\,(26Ab-5aB)}{11648b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{27a^2(ex)^{5/2}\sqrt{a+bx^3}\,(26Ab-5aB)}{5824be}+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}(26Ab-5aB)}{260be}+\frac{3a(ex)^{5/2}\left(a+bx^3\right)^{3/2}(26Ab-5aB)}{728be}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (27*a^2*(26*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(5824*b*e) + (81*(1 +
 Sqrt[3])*a^3*(26*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(11648*b^(5/3)*
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) + (3*a*(26*A*b - 5*a*B)*(e*x)^(5/2)*(a
```

```
 + b*x^3)^(3/2))/(728*b*e) + ((26*A*b - 5*a*B)*(e*x)^(5/2)*(a + b*x^3)^(5/2
))/(260*b*e) + (B*(e*x)^(5/2)*(a + b*x^3)^(7/2))/(13*b*e) - (81*3^(1/4)*a^(
10/3)*(26*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellip
ticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^
(1/3)*x)], (2 + Sqrt[3])/4])/(11648*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (27*3^(3/
4)*(1 - Sqrt[3])*a^(10/3)*(26*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(23296*b^(5/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx = \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be} - \frac{\left(-13Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\,dx}{13b}$$

$$= \frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be} + \frac{(3a(2\ldots}{\ldots}$$

$$= \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be} + \frac{(26Ab-5aB)(ex)^{5/2}\left(a+b\ldots\right.}{260be}$$

$$= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a-\ldots\right.}{728be}$$

$$= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a-\ldots\right.}{728be}$$

$$= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a-\ldots\right.}{728be}$$

$$= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{81\left(1+\sqrt{3}\right)a^3(26Ab-5a\ldots}{11648b^{5/3}\left(\sqrt[3]{a}+\left(1-\ldots\right.\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 99, normalized size = 0.15

$$\frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(5B\left(a+bx^3\right)^3\sqrt{1+\frac{bx^3}{a}}+a^2(26Ab-5aB)\,{}_2F_1\left(-\frac{5}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{65b\sqrt{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)^3*Sqrt[1 + (b*x^3)/a] + a^2
*(26*A*b - 5*a*B)*Hypergeometric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)]))/(65*b*
Sqrt[1 + (b*x^3)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 6202, normalized size = 9.38

method	result	size
risch	Expression too large to display	1188
elliptic	Expression too large to display	1410
default	Expression too large to display	6202

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^10 + (2*B*a*b + A*b^2)*x^7 + (B*a^2 + 2*A*a*b)*x^4 + A*a^
2*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(3/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 61.29, size = 308, normalized size = 0.47

$$\frac{Aa^{\frac{5}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2}, \frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)} + \frac{2Aa^{\frac{3}{2}}be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2}, \frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{A\sqrt{a}b^{2}e^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2}, \frac{17}{6}\\ \frac{23}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)} + \frac{Ba^{\frac{5}{2}}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2}, \frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{2Ba^{\frac{3}{2}}be^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2}, \frac{17}{6}\\ \frac{23}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)} + \frac{B\sqrt{a}b^{2}e^{\frac{3}{2}}x^{\frac{23}{2}}\Gamma\left(\frac{23}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2}, \frac{23}{6}\\ \frac{29}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{29}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] A*a**(5/2)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(11/6)) + 2*A*a**(3/2)*b*e**(3/2)*x**(11/2)*gamm
a(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/
6)) + A*sqrt(a)*b**2*e**(3/2)*x**(17/2)*gamma(17/6)*hyper((-1/2, 17/6), (23
/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(23/6)) + B*a**(5/2)*e**(3/2)*x**(1
1/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(17/6)) + 2*B*a**(3/2)*b*e**(3/2)*x**(17/2)*gamma(17/6)*hyper((-1/2, 1
7/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(23/6)) + B*sqrt(a)*b**2*e
**(3/2)*x**(23/2)*gamma(23/6)*hyper((-1/2, 23/6), (29/6,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(29/6))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*x^(3/2)*e^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(5/2), x)
```

3.538 $\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=201

$$\frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be}+\frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be}+\frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be}+\frac{B(e\cdots}{}$$

```
[Out] 5/288*a*(8*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/72*(8*A*b-B*a)*(e*x)^
(3/2)*(b*x^3+a)^(5/2)/b/e+1/12*B*(e*x)^(3/2)*(b*x^3+a)^(7/2)/b/e+5/192*a^3*
(8*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)/b^
(3/2)+5/192*a^2*(8*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A]
time = 0.09, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 285, 335, 281, 223, 212}

$$\frac{5a^3\sqrt{e}\,(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}+\frac{5a^2(ex)^{3/2}\sqrt{a+bx^3}\,(8Ab-aB)}{192be}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(8Ab-aB)}{72be}+\frac{5a(ex)^{3/2}\left(a+bx^3\right)^{3/2}(8Ab-aB)}{288be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (5*a^2*(8*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b*e) + (5*a*(8*A*b -
 a*B)*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(288*b*e) + ((8*A*b - a*B)*(e*x)^(3/2)
*(a + b*x^3)^(5/2))/(72*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(7/2))/(12*b*e) +
 (5*a^3*(8*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
 + b*x^3])])/(192*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be} - \frac{\left(-12Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\,dx}{12b}$$

$$= \frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be} + \frac{(5a(8Ab-$$

$$= \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be} +$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be}$$

Mathematica [A]
time = 0.35, size = 140, normalized size = 0.70

$$\frac{\sqrt{ex}\left(\sqrt{b}\,x^{3/2}\sqrt{a+bx^3}\,(15a^3B+16b^3x^6(4A+3Bx^3)+8ab^2x^3(26A+17Bx^3)+2a^2b(132A+59Bx^3))-15a^3(-8Ab+aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)\right)}{576b^{3/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(15*a^3*B + 16*b^3*x^6*(4*A + 3
*B*x^3) + 8*a*b^2*x^3*(26*A + 17*B*x^3) + 2*a^2*b*(132*A + 59*B*x^3)) - 15*
a^3*(-8*A*b + a*B)*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))]))/(576*b^(3/2
)*Sqrt[x])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 7702, normalized size = 38.32

method	result	size
risch	Expression too large to display	1104
elliptic	Expression too large to display	1304
default	Expression too large to display	7702

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(141) = 282$.
time = 0.51, size = 364, normalized size = 1.81

$$-\frac{1}{1152}\left(8\left(\frac{15\,a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{\sqrt{b}}+\frac{2\left(\frac{15\sqrt{bx^3+a}\,a^3b^2}{x^{\frac{3}{2}}}-\frac{40\,(bx^3+a)^{\frac{3}{2}}a^3b}{x^{\frac{9}{2}}}+\frac{33\,(bx^3+a)^{\frac{5}{2}}a^3}{x^{\frac{15}{2}}}\right)}{b^3-\frac{3\,(bx^3+a)b^2}{x^3}+\frac{3\,(bx^3+a)^2b}{x^6}-\frac{(bx^3+a)^3}{x^9}}\right)A-\left(\frac{15\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{3}{2}}}+\frac{2\left(\frac{15\sqrt{bx^3+a}\,a^4b^3}{x^{\frac{3}{2}}}-\frac{55\,(bx^3+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{9}{2}}}+\frac{73\,(bx^3+a)^{\frac{5}{2}}a^4b}{x^{\frac{15}{2}}}+\frac{15\,(bx^3+a)^{\frac{7}{2}}a^4}{x^{\frac{21}{2}}}\right)}{b^5-\frac{4\,(bx^3+a)b^4}{x^3}+\frac{6\,(bx^3+a)^2b^3}{x^6}-\frac{4\,(bx^3+a)^3b^2}{x^9}+\frac{(bx^3+a)^4b}{x^{12}}}\right)B\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/1152*(8*(15*a^3*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt
(b*x^3 + a)/x^(3/2)))/sqrt(b) + 2*(15*sqrt(b*x^3 + a)*a^3*b^2/x^(3/2) - 40*
(b*x^3 + a)^(3/2)*a^3*b/x^(9/2) + 33*(b*x^3 + a)^(5/2)*a^3/x^(15/2))/(b^3 -
 3*(b*x^3 + a)*b^2/x^3 + 3*(b*x^3 + a)^2*b/x^6 - (b*x^3 + a)^3/x^9))*A - (1
5*a^4*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 + a)/x
^(3/2)))/b^(3/2) + 2*(15*sqrt(b*x^3 + a)*a^4*b^3/x^(3/2) - 55*(b*x^3 + a)^(
3/2)*a^4*b^2/x^(9/2) + 73*(b*x^3 + a)^(5/2)*a^4*b/x^(15/2) + 15*(b*x^3 + a)
^(7/2)*a^4/x^(21/2))/(b^5 - 4*(b*x^3 + a)*b^4/x^3 + 6*(b*x^3 + a)^2*b^3/x^6
 - 4*(b*x^3 + a)^3*b^2/x^9 + (b*x^3 + a)^4*b/x^12))*B)*e^(1/2)
```

Fricas [A]
time = 2.83, size = 308, normalized size = 1.53

$$\left[-\frac{15\,(Ba^4-8\,Aa^3b)\sqrt{b}\,e^{\frac{1}{2}}\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)-4\,(48\,Bb^4x^{10}+8\,(17\,Bab^3+8\,Ab^4)x^7+2\,(59\,Ba^2b^2+104\,Aab^3)x^4+3\,(5\,Ba^3b+88\,Aa^2b^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{1}{2}}}{2304\,b^2},\ \frac{15\,(Ba^4-8\,Aa^3b)\sqrt{-b}\,\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)e^{\frac{1}{2}}+2\,(48\,Bb^4x^{10}+8\,(17\,Bab^3+8\,Ab^4)x^7+2\,(59\,Ba^2b^2+104\,Aab^3)x^4+3\,(5\,Ba^3b+88\,Aa^2b^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{1}{2}}}{1152\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2304*(15*(B*a^4 - 8*A*a^3*b)*sqrt(b)*e^(1/2)*log(-8*b^2*x^6 - 8*a*b*x^3
 - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*(48*B*b^4*x
^10 + 8*(17*B*a*b^3 + 8*A*b^4)*x^7 + 2*(59*B*a^2*b^2 + 104*A*a*b^3)*x^4 + 3
*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(1/2))/b^2, 1/1152
*(15*(B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)
```

```
/(2*b*x^3 + a))*e^(1/2) + 2*(48*B*b^4*x^10 + 8*(17*B*a*b^3 + 8*A*b^4)*x^7 +
 2*(59*B*a^2*b^2 + 104*A*a*b^3)*x^4 + 3*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(
b*x^3 + a)*sqrt(x)*e^(1/2))/b^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(177) = 354$.

time = 49.81, size = 413, normalized size = 2.05

$$\frac{Aa^{\frac{5}{2}}(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e}+\frac{Aa^{\frac{5}{2}}(ex)^{\frac{3}{2}}}{8e\sqrt{1+\frac{bx^3}{a}}}+\frac{35Aa^{\frac{3}{2}}b(ex)^{\frac{9}{2}}}{72e^4\sqrt{1+\frac{bx^3}{a}}}+\frac{17A\sqrt{a}\,b^2(ex)^{\frac{15}{2}}}{36e^7\sqrt{1+\frac{bx^3}{a}}}+\frac{5Aa^3\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{24\sqrt{b}}+\frac{Ab^3(ex)^{\frac{21}{2}}}{9\sqrt{a}\,e^{10}\sqrt{1+\frac{bx^3}{a}}}+\frac{5Ba^{\frac{7}{2}}(ex)^{\frac{3}{2}}}{192be\sqrt{1+\frac{bx^3}{a}}}+\frac{133Ba^{\frac{5}{2}}(ex)^{\frac{9}{2}}}{576e^4\sqrt{1+\frac{bx^3}{a}}}+\frac{127Ba^{\frac{3}{2}}b(ex)^{\frac{15}{2}}}{288e^7\sqrt{1+\frac{bx^3}{a}}}+\frac{23B\sqrt{a}\,b^2(ex)^{\frac{21}{2}}}{72e^{10}\sqrt{1+\frac{bx^3}{a}}}-\frac{5Ba^4\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{192b^{\frac{3}{2}}}+\frac{Bb^3(ex)^{\frac{27}{2}}}{12\sqrt{a}\,e^{13}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)*(e*x)**(1/2),x)
```

```
[Out] A*a**(5/2)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a**(5/2)*(e*x)**(3/2)/
(8*e*sqrt(1 + b*x**3/a)) + 35*A*a**(3/2)*b*(e*x)**(9/2)/(72*e**4*sqrt(1 + b
*x**3/a)) + 17*A*sqrt(a)*b**2*(e*x)**(15/2)/(36*e**7*sqrt(1 + b*x**3/a)) +
5*A*a**3*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(24*sqrt(b)
) + A*b**3*(e*x)**(21/2)/(9*sqrt(a)*e**10*sqrt(1 + b*x**3/a)) + 5*B*a**(7/2
)*(e*x)**(3/2)/(192*b*e*sqrt(1 + b*x**3/a)) + 133*B*a**(5/2)*(e*x)**(9/2)/(
576*e**4*sqrt(1 + b*x**3/a)) + 127*B*a**(3/2)*b*(e*x)**(15/2)/(288*e**7*sqr
t(1 + b*x**3/a)) + 23*B*sqrt(a)*b**2*(e*x)**(21/2)/(72*e**10*sqrt(1 + b*x**
3/a)) - 5*B*a**4*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(19
2*b**(3/2)) + B*b**3*(e*x)**(27/2)/(12*sqrt(a)*e**13*sqrt(1 + b*x**3/a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(141) = 282$.

time = 2.12, size = 383, normalized size = 1.91

$$\frac{1}{576}\left(48\sqrt{bx^3+a}\left(2x^3+\frac{a}{b}\right)Ba^2x^{\frac{3}{2}}+96\sqrt{bx^3+a}\left(2x^3+\frac{a}{b}\right)Aabx^{\frac{3}{2}}+16\left(2\left(4x^3+\frac{a}{b}\right)x^3-\frac{3a^2}{b^2}\right)\sqrt{bx^3+a}\,Babx^{\frac{3}{2}}+8\left(2\left(4x^3+\frac{a}{b}\right)x^3-\frac{3a^2}{b^2}\right)\sqrt{bx^3+a}\,Ab^2x^{\frac{3}{2}}+\left(2\left(4\left(6x^3+\frac{a}{b}\right)x^3-\frac{5a^2}{b^2}\right)x^3+\frac{15a^3}{b^3}\right)\sqrt{bx^3+a}\,Bb^2x^{\frac{3}{2}}+192\left(\sqrt{bx^3+a}\,x^{\frac{3}{2}}-\frac{a\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{\sqrt{b}}\right)Aa^2\right)e^{\frac{1}{2}}-\frac{\left(25B^2a^8+240ABa^7b+576A^2a^6b^2\right)e^{\frac{1}{2}}\log\left(\left|\left(5Ba^4x^{\frac{3}{2}}+24Aa^3bx^{\frac{3}{2}}\right)\sqrt{b}+\sqrt{25B^2a^9+240ABa^8b+576A^2a^7b^2+\left(5Ba^4x^{\frac{3}{2}}+24Aa^3bx^{\frac{3}{2}}\right)^2b}\right|\right)}{192b^{\frac{3}{2}}\left|5Ba^4+24Aa^3b\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/576*(48*sqrt(b*x^3 + a)*(2*x^3 + a/b)*B*a^2*x^(3/2) + 96*sqrt(b*x^3 + a)*
(2*x^3 + a/b)*A*a*b*x^(3/2) + 16*(2*(4*x^3 + a/b)*x^3 - 3*a^2/b^2)*sqrt(b*x
^3 + a)*B*a*b*x^(3/2) + 8*(2*(4*x^3 + a/b)*x^3 - 3*a^2/b^2)*sqrt(b*x^3 + a)
*A*b^2*x^(3/2) + (2*(4*(6*x^3 + a/b)*x^3 - 5*a^2/b^2)*x^3 + 15*a^3/b^3)*sqr
t(b*x^3 + a)*B*b^2*x^(3/2) + 192*(sqrt(b*x^3 + a)*x^(3/2) - a*log(abs(-sqrt
(b)*x^(3/2) + sqrt(b*x^3 + a)))/sqrt(b))*A*a^2)*e^(1/2) - 1/192*(25*B^2*a^8
 + 240*A*B*a^7*b + 576*A^2*a^6*b^2)*e^(1/2)*log(abs((5*B*a^4*x^(3/2) + 24*A
*a^3*b*x^(3/2))*sqrt(b) + sqrt(25*B^2*a^9 + 240*A*B*a^8*b + 576*A^2*a^7*b^2
 + (5*B*a^4*x^(3/2) + 24*A*a^3*b*x^(3/2))^2*b)))/(b^(3/2)*abs(5*B*a^4 + 24*
A*a^3*b))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2), x)
```

3.539 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=364

$$\frac{27a^2(22Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{1408be}+\frac{3a(22Ab-aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{352be}+\frac{(22Ab-aB)\sqrt{ex}\,(a+bx^3)^{5/2}}{176be}+\frac{B\sqrt{e}}{}$$

```
[Out] 3/352*a*(22*A*b-B*a)*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/176*(22*A*b-B*a)*(b*
x^3+a)^(5/2)*(e*x)^(1/2)/b/e+1/11*B*(b*x^3+a)^(7/2)*(e*x)^(1/2)/b/e+27/1408
*a^2*(22*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+27/2816*3^(3/4)*a^(8/3)*(
22*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b
^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1
/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {470, 285, 335, 231}

$$\frac{27\,3^{3/4}a^{8/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(22Ab-aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2816be\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{27a^2\sqrt{ex}\,\sqrt{a+bx^3}\,(22Ab-aB)}{1408be}+\frac{\sqrt{ex}\,(a+bx^3)^{5/2}(22Ab-aB)}{176be}+\frac{3a\sqrt{ex}\,(a+bx^3)^{3/2}(22Ab-aB)}{352be}+\frac{B\sqrt{ex}\,(a+bx^3)^{7/2}}{11be}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (27*a^2*(22*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(1408*b*e) + (3*a*(22*A*b
 - a*B)*Sqrt[e*x]*(a + b*x^3)^(3/2))/(352*b*e) + ((22*A*b - a*B)*Sqrt[e*x]*
(a + b*x^3)^(5/2))/(176*b*e) + (B*Sqrt[e*x]*(a + b*x^3)^(7/2))/(11*b*e) + (
27*3^(3/4)*a^(8/3)*(22*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2816*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) +
b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{B\sqrt{ex}\,(a+bx^3)^{7/2}}{11be} - \frac{\left(-11Ab+\frac{aB}{2}\right)\int \frac{(a+bx^3)^{5/2}}{\sqrt{ex}}\,dx}{11b}$$

$$= \frac{(22Ab-aB)\sqrt{ex}\,(a+bx^3)^{5/2}}{176be} + \frac{B\sqrt{ex}\,(a+bx^3)^{7/2}}{11be} + \frac{(15a(22Ab-aB))\int}{352b}$$

$$= \frac{3a(22Ab-aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{352be} + \frac{(22Ab-aB)\sqrt{ex}\,(a+bx^3)^{5/2}}{176be} + \frac{B\sqrt{ex}}{}$$

$$= \frac{27a^2(22Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{352be} + \frac{(22}{}$$

$$= \frac{27a^2(22Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{352be} + \frac{(22}{}$$

$$= \frac{27a^2(22Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{352be} + \frac{(22}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 84, normalized size = 0.23

$$\frac{x\sqrt{a+bx^3}\left(B(a+bx^3)^3 - \frac{a^2(-22Ab+aB)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{11b\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^3 - (a^2*(-22*A*b + a*B)*Hypergeometric2F
1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(11*b*Sqrt[e*x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 4617, normalized size = 12.68

method	result
risch	$\frac{(128B\,x^9b^3+176A\,b^3x^6+376Ba\,b^2x^6+616Aa\,b^2x^3+356B\,a^2b\,x^3+1034A\,a^2b+81B\,a^3)x\sqrt{b\,x^3+a}}{1408b\sqrt{ex}}+\frac{81a^3(22Ab-Ba)\left(\frac{(-a\,b^{\cdots}}{2b^{\cdots}}\right.}{\cdots}$
elliptic	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1408*(b*x^3+a)^(1/2)*x/b^2/(-a*b^2)^(1/3)*(528*A*(-a*b^2)^(1/3)*((b*x^3+
a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*
b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*
b^4*x^6+3102*A*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a^2*b^2+243*B*(-a*b^2)^(1/3)*((b*x^3+
a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*
b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*
a^3*b+384*B*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^4*x^9-1034*I*A*3^(1/2)*(1/b^2*e*x*(-b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*e*x)^(1/2)*(-a*b^2
)^(1/3)*a^2*b^2-128*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3)))^(1/2)*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*b^4*x^9-81*I*B*3^(1/
2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*
e*x)^(1/2)*(-a*b^2)^(1/3)*a^3*b-7128*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/
3)*a^3*b^2*e*x+324*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3
```

```
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*
3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a^4*b*e*x+162*B
*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*a^4*e+1128*B*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(
1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b^3*x^6
+1848*A*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b^3*x^3+1068*B*(-a*b^2)^(1/3)*((b*x^3+a)*e
*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a^2*
b^2*x^3-176*I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3
)))^(1/2)*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*b^4*x^6-3564*A*(-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*a^3*b^3*e*x^2-3564*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/
(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*
(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b*e+162*B*(-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*a^4*b^2*e*x^2+3564*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/sqrt(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*e^(-1/2)/sqrt(x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 20.37, size = 308, normalized size = 0.85

$$\frac{Aa^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)}+\frac{2Aa^{\frac{3}{2}}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}+\frac{A\sqrt{a}\,b^{2}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{19}{6}\right)}+\frac{Ba^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}+\frac{2Ba^{\frac{3}{2}}bx^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{19}{6}\right)}+\frac{B\sqrt{a}\,b^{2}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{25}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

```
[Out] A*a**(5/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(e)*gamma(7/6)) + 2*A*a**(3/2)*b*x**(7/2)*gamma(7/6)*hyper((
-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + A*
sqrt(a)*b**2*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*a**(5/2)*x**(7/2)*gamma(7/6)*hyp
er((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6))
+ 2*B*a**(3/2)*b*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*
exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*sqrt(a)*b**2*x**(19/2)*gamma
(19/6)*hyper((-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*ga
mma(25/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*e^(-1/2)/sqrt(x), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{5/2}}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(1/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(1/2), x)
```

3.540 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=650

$$\frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4}+\frac{81\left(1+\sqrt{3}\right)a^2(20Ab+aB)\sqrt{ex}\,\sqrt{a+bx^3}}{448b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{3(20Ab+aB)(ex)^{5/2}\,(a}{28e^4}$$

```
[Out] 3/28*(20*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/e^4+1/10*(20*A*b+B*a)*(e*x)^(
5/2)*(b*x^3+a)^(5/2)/a/e^4-2*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(1/2)+27/224*a*(20
*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/e^4+81/448*a^2*(20*A*b+B*a)*(1+3^(1/2
))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-
81/448*3^(1/4)*a^(7/3)*(20*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)
+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-27/896*3^(3/4)*a^(7/3)
*(20*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(
e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x
)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 285, 335, 314, 231, 1895}

$$-\frac{27\,3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(aB+20Ab)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{896b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{81\sqrt[4]{3}\,a^{7/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(aB+20Ab)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{448b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{81\left(1+\sqrt{3}\right)a^2\sqrt{ex}\sqrt{a+bx^3}\,(aB+20Ab)}{448b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}(aB+20Ab)}{10ae^4}+\frac{3(ex)^{5/2}\left(a+bx^3\right)^{3/2}(aB+20Ab)}{28e^4}+\frac{27a(ex)^{5/2}\sqrt{a+bx^3}\,(aB+20Ab)}{224e^4}-\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (27*a*(20*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(224*e^4) + (81*(1 + Sqrt
[3])*a^2*(20*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(448*b^(2/3)*e^2*(a^(1/3
```

```
) + (1 + Sqrt[3])*b^(1/3)*x)) + (3*(20*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(
3/2))/(28*e^4) + ((20*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*a*e^4)
- (2*A*(a + b*x^3)^(7/2))/(a*e*Sqrt[e*x]) - (81*3^(1/4)*a^(7/3)*(20*A*b + a
*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3)
+ (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[
3])/4])/(448*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (27*3^(3/4)*(1 - Sqrt[3])*a^
(7/3)*(20*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF
[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3
)*x)], (2 + Sqrt[3])/4])/(896*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3
)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = -\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}} + \frac{(20Ab+aB)\int (ex)^{3/2}(a+bx^3)^{5/2}\,dx}{ae^3}$$

$$= \frac{(20Ab+aB)(ex)^{5/2}(a+bx^3)^{5/2}}{10ae^4} - \frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}} + \frac{(3(20Ab+aB))\int (ex)}{4e^3}$$

$$= \frac{3(20Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{28e^4} + \frac{(20Ab+aB)(ex)^{5/2}(a+bx^3)^{5/2}}{10ae^4} - 2A($$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{3(20Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{28e^4} + (2$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{3(20Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{28e^4} + (2$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{3(20Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{28e^4} + (2$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{81\left(1+\sqrt{3}\right)a^2(20Ab+aB)\sqrt{ex}\,\sqrt{a}}{448b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 87, normalized size = 0.13

$$\frac{2x\sqrt{a+bx^3}\left(-5A(a+bx^3)^3+\frac{a^2(20Ab+aB)x^3\,{}_2F_1\left(-\frac{5}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5a(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3)^3 + (a^2*(20*A*b + a*B)*x^3*Hypergeo
metric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*x)^
(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 6530, normalized size = 10.05

method	result	size
risch	Expression too large to display	1166
elliptic	Expression too large to display	1341
default	Expression too large to display	6530

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*e^(-3/2)/x^(3/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 20.71, size = 311, normalized size = 0.48

$$\frac{Aa^{\frac{5}{2}}\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)}+\frac{2Aa^{\frac{3}{2}}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{A\sqrt{a}\,b^{2}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}+\frac{Ba^{\frac{5}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{2Ba^{\frac{3}{2}}bx^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}+\frac{B\sqrt{a}\,b^{2}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{17}{6}\\ \frac{23}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{23}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

```
[Out] A*a**(5/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + 2*A*a**(3/2)*b*x**(5/2)*gamma(5/6)*hype
r((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))
+ A*sqrt(a)*b**2*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*
exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*a**(5/2)*x**(5/2)*gamma(5/6
)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(1
1/6)) + 2*B*a**(3/2)*b*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b
*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*sqrt(a)*b**2*x**(17/2
)*gamma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**
(3/2)*gamma(23/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*e^(-3/2)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3+A)\,{(b\,x^3+a)}^{5/2}}{{(e\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(3/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(3/2), x)
```

3.541 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=188

$$\frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}+\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9ae^4}-\frac{2A(a}{3ae}$$

```
[Out] 5/36*(6*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/e^4+1/9*(6*A*b+B*a)*(e*x)^(3/2
)*(b*x^3+a)^(5/2)/a/e^4-2/3*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(3/2)+5/24*a^2*(6*A
*b+B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(1/2
)+5/24*a*(6*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/e^4
```

Rubi [A]
time = 0.09, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 285, 335, 281, 223, 212}

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}\,e^{5/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(aB+6Ab)}{9ae^4}+\frac{5(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+6Ab)}{36e^4}+\frac{5a(ex)^{3/2}\sqrt{a+bx^3}\,(aB+6Ab)}{24e^4}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (5*a*(6*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*e^4) + (5*(6*A*b + a*B)
*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(36*e^4) + ((6*A*b + a*B)*(e*x)^(3/2)*(a +
b*x^3)^(5/2))/(9*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(3*a*e*(e*x)^(3/2)) + (5*
a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])
/(24*Sqrt[b]*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = -\frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{(6Ab+aB)\int \sqrt{ex}\,(a+bx^3)^{5/2}\,dx}{ae^3}$$

$$= \frac{(6Ab+aB)(ex)^{3/2}(a+bx^3)^{5/2}}{9ae^4} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{(5(6Ab+aB))\int \sqrt{ex}}{6e^3}$$

$$= \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}(a+bx^3)^{5/2}}{9ae^4} - \frac{2A(}{3a}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6A}{}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6A}{}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6A}{}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6A}{}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6A}{}$$

Mathematica [A]
time = 0.37, size = 126, normalized size = 0.67

$$\frac{x\left(\sqrt{b}\,\sqrt{a+bx^3}\,(4b^2x^6(3A+2Bx^3)+a^2(-48A+33Bx^3)+a(54Abx^3+26bBx^6))+15a^2(6Ab+aB)x^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)\right)}{72\sqrt{b}\,(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*(Sqrt[b]*Sqrt[a + b*x^3]*(4*b^2*x^6*(3*A + 2*B*x^3) + a^2*(-48*A + 33*B*
x^3) + a*(54*A*b*x^3 + 26*b*B*x^6)) + 15*a^2*(6*A*b + a*B)*x^(3/2)*ArcTanh[
Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))]))/(72*Sqrt[b]*(e*x)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.41, size = 7544, normalized size = 40.13

method	result	size
risch	Expression too large to display	1093
elliptic	Expression too large to display	1246
default	Expression too large to display	7544

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(132) = 264.
time = 0.49, size = 306, normalized size = 1.63

$$-\frac{1}{144}\left(6\left(15\,a^2\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)+\frac{16\sqrt{bx^3+a}\,a^2}{x^{\frac{3}{2}}}+\frac{2\left(\frac{7\sqrt{bx^3+a}\,a^2b^2}{x^{\frac{3}{2}}}-\frac{9\,(bx^3+a)^{\frac{3}{2}}a^2b}{x^{\frac{9}{2}}}\right)}{b^2-\frac{2\,(bx^3+a)b}{x^3}+\frac{(bx^3+a)^2}{x^6}}\right)A+\left(\frac{15\,a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{\sqrt{b}}+\frac{2\left(\frac{15\sqrt{bx^3+a}\,a^3b^2}{x^{\frac{3}{2}}}-\frac{40\,(bx^3+a)^{\frac{3}{2}}a^3b}{x^{\frac{9}{2}}}+\frac{33\,(bx^3+a)^{\frac{5}{2}}a^3}{x^{\frac{15}{2}}}\right)}{b^3-\frac{3\,(bx^3+a)b^2}{x^3}+\frac{3\,(bx^3+a)^2b}{x^6}-\frac{(bx^3+a)^3}{x^9}}\right)B\right)e^{\left(-\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/144*(6*(15*a^2*sqrt(b)*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b)
 + sqrt(b*x^3 + a)/x^(3/2))) + 16*sqrt(b*x^3 + a)*a^2/x^(3/2) + 2*(7*sqrt(b
*x^3 + a)*a^2*b^2/x^(3/2) - 9*(b*x^3 + a)^(3/2)*a^2*b/x^(9/2))/(b^2 - 2*(b*
x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*A + (15*a^3*log(-(sqrt(b) - sqrt(b*x^3
 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 + a)/x^(3/2)))/sqrt(b) + 2*(15*sqrt(b*
x^3 + a)*a^3*b^2/x^(3/2) - 40*(b*x^3 + a)^(3/2)*a^3*b/x^(9/2) + 33*(b*x^3 +
 a)^(5/2)*a^3/x^(15/2))/(b^3 - 3*(b*x^3 + a)*b^2/x^3 + 3*(b*x^3 + a)^2*b/x^
6 - (b*x^3 + a)^3/x^9))*B)*e^(-5/2)
```

Fricas [A]
time = 1.83, size = 286, normalized size = 1.52

$$\left[\frac{\left(15\,(Ba^3+6\,Aa^2b)\sqrt{b}\,x^2\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)+4\,(8\,Bb^3x^9+2\,(13\,Bab^2+6\,Ab^3)x^6-48\,Aa^2b+3\,(11\,Ba^2b+18\,Aab^2)x^3)\sqrt{bx^3+a}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{288\,bx^2},\ -\frac{\left(15\,(Ba^3+6\,Aa^2b)\sqrt{-b}\,x^2\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)-2\,(8\,Bb^3x^9+2\,(13\,Bab^2+6\,Ab^3)x^6-48\,Aa^2b+3\,(11\,Ba^2b+18\,Aab^2)x^3)\sqrt{bx^3+a}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{144\,bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/288*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b)*x^2*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(
2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) + 4*(8*B*b^3*x^9 + 2*
(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b + 3*(11*B*a^2*b + 18*A*a*b^2)*x^3)*
sqrt(b*x^3 + a)*sqrt(x))*e^(-5/2)/(b*x^2), -1/144*(15*(B*a^3 + 6*A*a^2*b)*s
```

```
qrt(-b)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a)) - 2*(8
*B*b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b + 3*(11*B*a^2*b + 18
*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(x))*e^(-5/2)/(b*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(180) = 360$.
time = 37.41, size = 403, normalized size = 2.14

$$-\frac{2Aa^{\frac{5}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2Aa^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}-\frac{7Aa^{\frac{3}{2}}bx^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{A\sqrt{a}\,b^2x^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{5Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4e^{\frac{5}{2}}}+\frac{Ab^3x^{\frac{15}{2}}}{6\sqrt{a}\,e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{Ba^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}+\frac{Ba^{\frac{5}{2}}x^{\frac{3}{2}}}{8e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{35Ba^{\frac{3}{2}}bx^{\frac{9}{2}}}{72e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{17B\sqrt{a}\,b^2x^{\frac{15}{2}}}{36e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{24\sqrt{b}\,e^{\frac{5}{2}}}+\frac{Bb^3x^{\frac{21}{2}}}{9\sqrt{a}\,e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

```
[Out] -2*A*a**(5/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*a**(3/2)*b*x**
(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 7*A*a**(3/2)*b*x**(3/2)/(12*e**(5/2
)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b**2*x**(9/2)/(4*e**(5/2)*sqrt(1 + b*x**3
/a)) + 5*A*a**2*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*e**(5/2)) + A*b*
*3*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a)) + B*a**(5/2)*x**(3/2)*
sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(5/2)*x**(3/2)/(8*e**(5/2)*sqrt(1 +
b*x**3/a)) + 35*B*a**(3/2)*b*x**(9/2)/(72*e**(5/2)*sqrt(1 + b*x**3/a)) + 17
*B*sqrt(a)*b**2*x**(15/2)/(36*e**(5/2)*sqrt(1 + b*x**3/a)) + 5*B*a**3*asinh
(sqrt(b)*x**(3/2)/sqrt(a))/(24*sqrt(b)*e**(5/2)) + B*b**3*x**(21/2)/(9*sqrt
(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*e^(-5/2)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{5/2}}{{\left(e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2), x)
```

3.542 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=352

$$\frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4}+\frac{3(16Ab+5aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{80e^4}+\frac{(16Ab+5aB)\sqrt{ex}\,(a+bx^3)^{5/2}}{40ae^4}-\frac{2A(}{5}$$

```
[Out] -2/5*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(5/2)+3/80*(16*A*b+5*B*a)*(b*x^3+a)^(3/2)*
(e*x)^(1/2)/e^4+1/40*(16*A*b+5*B*a)*(b*x^3+a)^(5/2)*(e*x)^(1/2)/a/e^4+27/32
0*a*(16*A*b+5*B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/e^4+27/640*3^(3/4)*a^(5/3)*(
16*A*b+5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {464, 285, 335, 231}

$$\frac{27\,3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+16Ab)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\,(a+bx^3)^{5/2}(5aB+16Ab)}{40ae^4}+\frac{3\sqrt{ex}\,(a+bx^3)^{3/2}(5aB+16Ab)}{80e^4}+\frac{27a\sqrt{ex}\sqrt{a+bx^3}(5aB+16Ab)}{320e^4}-\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (27*a*(16*A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*e^4) + (3*(16*A*b +
5*a*B)*Sqrt[e*x]*(a + b*x^3)^(3/2))/(80*e^4) + ((16*A*b + 5*a*B)*Sqrt[e*x]*
(a + b*x^3)^(5/2))/(40*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(5*a*e*(e*x)^(5/2))
 + (27*3^(3/4)*a^(5/3)*(16*A*b + 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*e^4*Sqrt[(b^(1/3)*x*(a^(1/
3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = -\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}} + \frac{(16Ab+5aB)\int \frac{(a+bx^3)^{5/2}}{\sqrt{ex}}\,dx}{5ae^3}$$

$$= \frac{(16Ab+5aB)\sqrt{ex}\,(a+bx^3)^{5/2}}{40ae^4} - \frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}} + \frac{(3(16Ab+5aB))\int \frac{(a+}{\sqrt{}}}{16e^3}$$

$$= \frac{3(16Ab+5aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{80e^4} + \frac{(16Ab+5aB)\sqrt{ex}\,(a+bx^3)^{5/2}}{40ae^4} - \frac{2A(a}{5a}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\,\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{80e^4} + \frac{(16}{}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\,\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{80e^4} + \frac{(16}{}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\,\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}\,(a+bx^3)^{3/2}}{80e^4} + \frac{(16}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 88, normalized size = 0.25

$$\frac{2x\sqrt{a+bx^3}\left(-A(a+bx^3)^3 + \frac{a^2(16Ab+5aB)x^3\,{}_2F_1\left(-\frac{5}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5a(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)^3) + (a^2*(16*A*b + 5*a*B)*x^3*Hyperg
eometric2F1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*x)
^(7/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 4422, normalized size = 12.56

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-40 b^2 B\,x^9-64 A\,b^2 x^6-140 B a b\,x^6-368 a A b\,x^3-235 a^2 B\,x^3+128 a^2 A\right)}{320 x^2 e^3 \sqrt{e x}}+\frac{81 a^2 (16 A b+5 B a)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\cdots}{\cdots}\right.}{\cdots}$
elliptic	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/320*(b*x^3+a)^(1/2)/x^2/b/(-a*b^2)^(1/3)*(810*I*B*3^(1/2)*(-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(-a*b^2)^(2/3)*a^3*e*x^3-235*I*B*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(
-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*a^2*b*x^3-4
0*I*B*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*b^3*x^9+120*B*((b*x^3+a)*e*x)^(1/2)*(1
/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*b
^3*x^9+192*A*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*b^3*x^6-384*A*((b*x^3+a)*e*x)^(1/2)*(1/
b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*a^
2*b+2592*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b^3*e*x^5+1104*A*((b*x^3+a)*e*x)^(1/
2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
```

```
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1
/3)*a*b^2*x^3+705*B*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*a^2*b*x^3+420*B*((b*x^3+a)*e*x)^
(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)
^(1/3)*a*b^2*x^6-5184*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a^2*b^2*e*x^
4-64*I*A*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*
x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*b^3*x^6+5184*A*(-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^
2)^(1/3)*a^2*b^2*e*x^4+1620*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2
)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3
^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a^3*b*e*x^4-2592
*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^2*b*e*x^3+2592*I*A*3^(1/2)*(-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(-a*b^2)^(2/3)*a^2*b*e*x^3-140*I*B*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x
*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*a*b^2*x^6
-810*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/x^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*e^(-7/2)/x^(7/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 50.41, size = 311, normalized size = 0.88

$$\frac{Aa^{\frac{5}{2}}\Gamma\left(-\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{2Aa^{\frac{3}{2}}b\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}+\frac{A\sqrt{a}\,b^{2}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)}+\frac{Ba^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}+\frac{2Ba^{\frac{3}{2}}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)}+\frac{B\sqrt{a}\,b^{2}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

```
[Out] A*a**(5/2)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + 2*A*a**(3/2)*b*sqrt(x)*gamma(1/6)*hype
r((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6)) +
A*sqrt(a)*b**2*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_p
olar(I*pi)/a)/(3*e**(7/2)*gamma(13/6)) + B*a**(5/2)*sqrt(x)*gamma(1/6)*hype
r((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6)) +
2*B*a**(3/2)*b*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_p
olar(I*pi)/a)/(3*e**(7/2)*gamma(13/6)) + B*sqrt(a)*b**2*x**(13/2)*gamma(13/
6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma
(19/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*e^(-7/2)/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{5/2}}{\left(e\,x\right)^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(7/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(7/2), x)
```

3.543 $\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=121

$$\frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}-\frac{a(4Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}$$

```
[Out] -1/12*a*(4*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)
^(1/2))/b^(5/2)+1/12*(4*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/6*
B*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A]
time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 327, 335, 281, 223, 212}

$$-\frac{ae^{7/2}(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}+\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}\,(4Ab-3aB)}{12b^2}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] ((4*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b^2) + (B*(e*x)^(9/2)
*Sqrt[a + b*x^3])/(6*b*e) - (a*(4*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*
x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(-6Ab+\frac{9aB}{2}\right)\int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}}\,dx}{6b}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{(a(4Ab-3aB)e^3)\int \cdot}{8b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{(a(4Ab-3aB)e^2)\,\mathrm{Su}}{}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{(a(4Ab-3aB)e^2)\,\mathrm{Su}}{}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{(a(4Ab-3aB)e^2)\,\mathrm{Su}}{}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{a(4Ab-3aB)e^{7/2}\,\mathrm{tan}}{12}$$

Mathematica [A]
time = 0.30, size = 99, normalized size = 0.82

$$\frac{(ex)^{7/2}\sqrt{a+bx^3}\,(4Ab-3aB+2bBx^3)}{12b^2x^2} + \frac{a(-4Ab+3aB)(ex)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)}{12b^{5/2}x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] ((e*x)^(7/2)*Sqrt[a + b*x^3]*(4*A*b - 3*a*B + 2*b*B*x^3))/(12*b^2*x^2) + (a
*(-4*A*b + 3*a*B)*(e*x)^(7/2)*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))])/(
12*b^(5/2)*x^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 6861, normalized size = 56.70

method	result	size

risch	Expression too large to display	1063
elliptic	Expression too large to display	1093
default	Expression too large to display	6861

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(83) = 166$.
time = 0.51, size = 214, normalized size = 1.77

$$-\frac{1}{24}\left(B\left(\frac{3\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{5}{2}}}-\frac{2\left(\frac{5\sqrt{bx^3+a}\,a^2b}{x^{\frac{3}{2}}}-\frac{3\,(bx^3+a)^{\frac{3}{2}}a^2}{x^{\frac{9}{2}}}\right)}{b^4-\frac{2\,(bx^3+a)b^3}{x^3}+\frac{(bx^3+a)^2b^2}{x^6}}\right)-4\,A\left(\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{bx^3+a}\,a}{\left(b^2-\frac{(bx^3+a)b}{x^3}\right)x^{\frac{3}{2}}}\right)\right)e^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(B*(3*a^2*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*
x^3 + a)/x^(3/2)))/b^(5/2) - 2*(5*sqrt(b*x^3 + a)*a^2*b/x^(3/2) - 3*(b*x^3
+ a)^(3/2)*a^2/x^(9/2))/(b^4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3 + a)^2*b^2/x^
6)) - 4*A*(a*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3
 + a)/x^(3/2)))/b^(3/2) - 2*sqrt(b*x^3 + a)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^
(3/2))))*e^(7/2)
```

Fricas [A]
time = 1.30, size = 212, normalized size = 1.75

$$\left[-\frac{(3\,Ba^2-4\,Aab)\sqrt{b}\,e^{\frac{7}{2}}\log\left(-8\,b^2x^6-8\,abx^3+4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)-4\,(2\,Bb^2x^4-(3\,Bab-4\,Ab^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{48\,b^3},-\frac{(3\,Ba^2-4\,Aab)\sqrt{-b}\,\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)e^{\frac{7}{2}}-2\,(2\,Bb^2x^4-(3\,Bab-4\,Ab^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{24\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*B*a^2 - 4*A*a*b)*sqrt(b)*e^(7/2)*log(-8*b^2*x^6 - 8*a*b*x^3 + 4*
(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*(2*B*b^2*x^4 - (
3*B*a*b - 4*A*b^2)*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(7/2))/b^3, -1/24*((3*B*a^2
 - 4*A*a*b)*sqrt(-b)*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a
))*e^(7/2) - 2*(2*B*b^2*x^4 - (3*B*a*b - 4*A*b^2)*x)*sqrt(b*x^3 + a)*sqrt(x
)*e^(7/2))/b^3]
```

Sympy [A]
time = 59.46, size = 194, normalized size = 1.60

$$\frac{A\sqrt{a}\,e^{\frac{7}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3b}-\frac{Aae^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}}-\frac{Ba^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{3}{2}}}{4b^2\sqrt{1+\frac{bx^3}{a}}}-\frac{B\sqrt{a}\,e^{\frac{7}{2}}x^{\frac{9}{2}}}{12b\sqrt{1+\frac{bx^3}{a}}}+\frac{Ba^2e^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{Be^{\frac{7}{2}}x^{\frac{15}{2}}}{6\sqrt{a}\,\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*e**(7/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*b) - A*a*e**(7/2)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - B*a**(3/2)*e**(7/2)*x**(3/2)/(4*b**
2*sqrt(1 + b*x**3/a)) - B*sqrt(a)*e**(7/2)*x**(9/2)/(12*b*sqrt(1 + b*x**3/a
)) + B*a**2*e**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*b**(5/2)) + B*e**(7
/2)*x**(15/2)/(6*sqrt(a)*sqrt(1 + b*x**3/a))
```

Giac [A]
time = 1.48, size = 90, normalized size = 0.74

$$\frac{1}{12}\sqrt{bx^3+a}\left(\frac{2\,Bx^3}{b}-\frac{3\,Bab^3-4\,Ab^4}{b^5}\right)x^{\frac{3}{2}}e^{\frac{7}{2}}-\frac{(3\,Ba^2b^3-4\,Aab^4)e^{\frac{7}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{12\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*x^3 + a)*(2*B*x^3/b - (3*B*a*b^3 - 4*A*b^4)/b^5)*x^(3/2)*e^(7/2
) - 1/12*(3*B*a^2*b^3 - 4*A*a*b^4)*e^(7/2)*log(abs(-sqrt(b)*x^(3/2) + sqrt(
b*x^3 + a)))/b^(11/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^3+A)\,(e\,x)^{7/2}}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2), x)
```

3.544 $\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=286

$$\frac{(10Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^3}}{20b^2}+\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}-\frac{a^{2/3}(10Ab-7aB)e^2\sqrt{ex}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\cdots}{\left(\sqrt[3]{a}+\cdots\right.}}}{40\sqrt[4]{3}\,b^2\sqrt{\frac{\sqrt[3]{\cdots}}{\left(\sqrt[3]{a}\cdots\right.}}}$$

```
[Out] 1/5*B*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+1/20*(10*A*b-7*B*a)*e^2*(e*x)^(1/2)*(
b*x^3+a)^(1/2)/b^2-1/120*a^(2/3)*(10*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^2/(b*x^3+a)^(1/2)/(b^
(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {470, 327, 335, 231}

$$-\frac{a^{2/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(10Ab-7aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}\,b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{e^2\sqrt{ex}\,\sqrt{a+bx^3}\,(10Ab-7aB)}{20b^2}+\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] ((10*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*b^2) + (B*(e*x)^(7/2)*
Sqrt[a + b*x^3])/(5*b*e) - (a^(2/3)*(10*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3
)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*3^(1/4)*b^
2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{\left(-5Ab+\frac{7aB}{2}\right)\int \frac{(ex)^{5/2}}{\sqrt{a+bx^3}}\,dx}{5b}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{(a(10Ab-7aB)e^3)\int -}{40b^2}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{(a(10Ab-7aB)e^2)\,\mathrm{Sul}}{}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{a^{2/3}(10Ab-7aB)e^2\sqrt{e}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 98, normalized size = 0.34

$$\frac{e^2\sqrt{ex}\left(-\left((a+bx^3)\left(-10Ab+7aB-4bBx^3\right)\right)+a(-10Ab+7aB)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{20b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (e^2*Sqrt[e*x]*(-((a + b*x^3)*(-10*A*b + 7*a*B - 4*b*B*x^3)) + a*(-10*A*b +
 7*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])
)/(20*b^2*Sqrt[a + b*x^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.33, size = 3723, normalized size = 13.02 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^3/(-a*b^2)^(1/3)*(20*I*A*(-(I*3^(1/2
)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
```

1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^3*e*x^2-14*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*(-a*b^2)^(2/3)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*e+20*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*(-a*b^2)^(2/3)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*e+4*I*B*3^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^2*x^3-40*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*(-a*b^2)^(1/3)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x-20*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^3*e*x^2+10*I*A*3^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^2+14*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b^2*e*x^2+40*A*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x-28*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+

```
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b*e*x+28*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*(-a*b^2)^(1/3
)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*a^2*b*e*x-20*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*
b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+
3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*e+14*B*(-a*b^2)^(
2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] e^(5/2)*integrate((B*x^3 + A)*x^(5/2)/sqrt(b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^5 + A*x^2)*sqrt(x)*e^(5/2)/sqrt(b*x^3 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 18.26, size = 94, normalized size = 0.33

$$\frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{6}\\\frac{13}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{13}{6}\right)}+\frac{Be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{13}{6}\\\frac{19}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (13/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(13/6)) + B*e**(5/2)*x**(13/2)*gamma(13/6)*hyper((
1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(19/6))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^(5/2)*e^(5/2)/sqrt(b*x^3 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(1/2), x)
```

3.545 $\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=543

$$\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}+\frac{\left(1+\sqrt{3}\right)(8Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^3}}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,\sqrt[3]{a}\,(8Ab-5aB)e\sqrt{ex}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{8b^{5/3}}$$

```
[Out] 1/4*B*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+1/8*(8*A*b-5*B*a)*e*(1+3^(1/2))*(e*x)
^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-1/8*3^(1/4)*
a^(1/3)*(8*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2))
)^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2))
)*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*
x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/48*a^(1/3)*(8*A*b-5*B*a)*e*(a^(1/3)+
b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/
4)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {470, 335, 314, 231, 1895}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,e\sqrt{ex}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(8Ab-5aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\,\sqrt[3]{a}\,e\sqrt{ex}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(8Ab-5aB)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{\left(1+\sqrt{3}\right)e\sqrt{ex}\,\sqrt{a+bx^3}\,(8Ab-5aB)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (B*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*b*e) + ((1 + Sqrt[3])*(8*A*b - 5*a*B)*e*
Sqrt[e*x]*Sqrt[a + b*x^3])/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x))
- (3^(1/4)*a^(1/3)*(8*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(
```

```
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (
(1 - Sqrt[3])*a^(1/3)*(8*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*3^(1/4)*b^(5/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
```

```
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}-\frac{\left(-4Ab+\frac{5aB}{2}\right)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{4b}$$

$$=\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}+\frac{(8Ab-5aB)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{4be}$$

$$=\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}-\frac{(8Ab-5aB)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{8b^{5/3}e}$$

$$=\frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}+\frac{\left(1+\sqrt{3}\right)(8Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\sqrt[3]{a}\,(8Ab}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 80, normalized size = 0.15

$$\frac{x(ex)^{3/2}\left(5B(a+bx^3)+(8Ab-5aB)\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{20b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (x*(e*x)^(3/2)*(5*B*(a + b*x^3) + (8*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(20*b*Sqrt[a + b*x^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.33, size = 4914, normalized size = 9.05

method	result	size
risch	Expression too large to display	1124
elliptic	Expression too large to display	1128
default	Expression too large to display	4914

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*e*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^3*(5*I*B*3^(1/2)*a*b^2*e*x^3-8*I*A*3^(
1/2)*(-a*b^2)^(1/3)*b^2*e*x^2-8*I*A*3^(1/2)*(-a*b^2)^(2/3)*b*e*x+5*I*B*3^(1
/2)*(-a*b^2)^(2/3)*a*e*x-24*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2
)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3
^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e-10*B*(-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*
b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*
b*e+15*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))
/(I*3^(1/2)-3))^(1/2))*a^2*b*e+5*I*B*3^(1/2)*(-a*b^2)^(1/3)*a*b*e*x^2+32*A*
(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*b*e*x-48*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b*e*x-
20*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
```

```
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*e*x+30*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a
*e*x-8*I*A*3^(1/2)*b^3*e*x^3-15*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*e*x^2+10
*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*e*x^2-8*I*A*3^(1/2)*(-a*b^2)^(1/3)*(-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*b^2*e*x^2+16*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2
))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e+5*I*B*3^(1/2)*(-a*b^2)^(1/3)
*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] e^(3/2)*integrate((B*x^3 + A)*x^(3/2)/sqrt(b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x)*sqrt(x)*e^(3/2)/sqrt(b*x^3 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.54, size = 94, normalized size = 0.17

$$\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\\frac{11}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{11}{6}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{11}{6}\\\frac{17}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{17}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6), (11/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(11/6)) + B*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((
1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(17/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^(3/2)*e^(3/2)/sqrt(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(1/2), x)
```

3.546 $\int \frac{\sqrt{ex}\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=83

$$\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}+\frac{(2Ab-aB)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

```
[Out] 1/3*(2*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2
)/b^(3/2)+1/3*B*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {470, 335, 281, 223, 212}

$$\frac{\sqrt{e}\,(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (B*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b*e) + ((2*A*b - a*B)*Sqrt[e]*ArcTanh[(S
qrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ex}\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx &= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} - \frac{\left(-3Ab+\frac{3aB}{2}\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{3b} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{be} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3be} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3be} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 78, normalized size = 0.94

$$\frac{\sqrt{ex}\left(\sqrt{b}\,Bx^{3/2}\sqrt{a+bx^3}+(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)\right)}{3b^{3/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (Sqrt[e*x]*(Sqrt[b]*B*x^(3/2)*Sqrt[a + b*x^3] + (2*A*b - a*B)*ArcTanh[Sqrt[
a + b*x^3]/(Sqrt[b]*x^(3/2))]))/(3*b^(3/2)*Sqrt[x])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.39, size = 6424, normalized size = 77.40

method	result	size
risch	Expression too large to display	1039
elliptic	Expression too large to display	1046
default	Expression too large to display	6424

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(53) = 106.
time = 0.49, size = 133, normalized size = 1.60

$$\frac{1}{6}\left(B\left(\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{bx^3+a}\,a}{\left(b^2-\frac{(bx^3+a)b}{x^3}\right)x^{\frac{3}{2}}}\right)-\frac{2\,A\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{\sqrt{b}}\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(B*(a*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 +
a)/x^(3/2)))/b^(3/2) - 2*sqrt(b*x^3 + a)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^(3/
2))) - 2*A*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 +
 a)/x^(3/2)))/sqrt(b))*e^(1/2)
```

Fricas [A]
time = 1.19, size = 159, normalized size = 1.92

$$\left[\frac{4\sqrt{bx^3+a}\,Bbx^{\frac{3}{2}}e^{\frac{1}{2}}-(Ba-2\,Ab)\sqrt{b}\,e^{\frac{1}{2}}\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)}{12\,b^2},\frac{2\sqrt{bx^3+a}\,Bbx^{\frac{3}{2}}e^{\frac{1}{2}}+(Ba-2\,Ab)\sqrt{-b}\,\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)e^{\frac{1}{2}}}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^3 + a)*B*b*x^(3/2)*e^(1/2) - (B*a - 2*A*b)*sqrt(b)*e^(1/2
)*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sq
rt(x) - a^2))/b^2, 1/6*(2*sqrt(b*x^3 + a)*B*b*x^(3/2)*e^(1/2) + (B*a - 2*A*
b)*sqrt(-b)*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a))*e^(1/2
))/b^2]
```

Sympy [A]
time = 3.15, size = 107, normalized size = 1.29

$$\frac{2A\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{3\sqrt{b}}+\frac{B\sqrt{a}\,(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3be}-\frac{Ba\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{3b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] 2*A*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*sqrt(b)) + B*
sqrt(a)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*b*e) - B*a*sqrt(e)*asinh(sqrt(b)
*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*b**(3/2))
```

Giac [A]
time = 0.80, size = 56, normalized size = 0.67

$$\frac{\sqrt{bx^3+a}\,Bx^{\frac{3}{2}}e^{\frac{1}{2}}}{3\,b}+\frac{(Ba-2\,Ab)e^{\frac{1}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^3 + a)*B*x^(3/2)*e^(1/2)/b + 1/3*(B*a - 2*A*b)*e^(1/2)*log(abs
(-sqrt(b)*x^(3/2) + sqrt(b*x^3 + a)))/b^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^3+A)\,\sqrt{e\,x}}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2), x)
```

3.547 $\int \frac{A+Bx^3}{\sqrt{ex}\,\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=249

$$\frac{B\sqrt{ex}\,\sqrt{a+bx^3}}{2be}+\frac{(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right.}{\sqrt[3]{a}+\left(1+\sqrt{3}\right.}\right.\right.}{4\sqrt[4]{3}\,\sqrt[3]{a}\,be\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] 1/2*B*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+1/12*(4*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*
((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF(
(1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b/e/(b*x^3+a
)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {470, 335, 231}

$$\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(4Ab-aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\,\sqrt[3]{a}\,be\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{B\sqrt{ex}\,\sqrt{a+bx^3}}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
```

```
[Out] (B*Sqrt[e*x]*Sqrt[a + b*x^3])/(2*b*e) + ((4*A*b - a*B)*Sqrt[e*x]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*
x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*a^(1/
3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{ex}\,\sqrt{a+bx^3}}\,dx = \frac{B\sqrt{ex}\,\sqrt{a+bx^3}}{2be} - \frac{\left(-2Ab+\frac{aB}{2}\right)\int \frac{1}{\sqrt{ex}\,\sqrt{a+bx^3}}\,dx}{2b}$$

$$= \frac{B\sqrt{ex}\,\sqrt{a+bx^3}}{2be} + \frac{(4Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{2be}$$

$$= \frac{B\sqrt{ex}\,\sqrt{a+bx^3}}{2be} + \frac{(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right.}}}{4\sqrt[4]{3}\,\sqrt[3]{a}\,be\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}-\right.}{\left(\sqrt[3]{a}+\left(1+\sqrt{\right.\right.}}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 80, normalized size = 0.32

$$\frac{Bx(a+bx^3)+(4Ab-aB)x\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)}{2b\sqrt{ex}\ \sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
```

```
[Out] (B*x*(a + b*x^3) + (4*A*b - a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/
6, 1/2, 7/6, -((b*x^3)/a)])/(2*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 3275, normalized size = 13.15 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^3+a)^(1/2)*x/b^2/(-a*b^2)^(1/3)*(8*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*
b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^3*
e*x^2-2*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x^2-16*I*A*3^(1/2)*(-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(-a*b^2)^(1/3)*b^2*e*x+4*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/
(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*
(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a*b*e*x+8
*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1
```

```
/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*b*e-8*A*(-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*E
llipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^3*e*x^2-2
*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a*e+2*B*(-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*E
llipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x^2
+16*A*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^2*e*x-4*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*a*b*e*x-8*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(
1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b*e+2*B*(-a*b^2)^(2/3)*(-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*sqrt(x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(x)*e^(-1/2)/(b*x^4 + a*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.67, size = 94, normalized size = 0.38

$$\frac{A\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)}+\frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(1/2)/(b*x**3+a)**(1/2),x)
[Out] A*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3
*sqrt(a)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (13/
6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*sqrt(e)*gamma(13/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*e^(-1/2)/(sqrt(b*x^3 + a)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{\sqrt{e\,x}\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(1/2)), x)
```

3.548 $\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=542

$$-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}+\frac{\left(1+\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\,\sqrt{a+bx^3}}{ab^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\cdots}{\left(\sqrt[3]{a}+\cdots\right.}}}{a^{2/3}b^{2/3}e^2\sqrt{\frac{\cdots}{\left(\cdots\right.}}}$$

```
[Out] -2*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(1/2)+(2*A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b
*x^3+a)^(1/2)/a/b^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3^(1/4)*(2*A*b+
B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(
2/3)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*(2*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/
3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^
(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(2/3)/e
^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {464, 335, 314, 231, 1895}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(aB+2Ab)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\,\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(aB+2Ab)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{\left(1+\sqrt{3}\right)\sqrt{ex}\,\sqrt{a+bx^3}\,(aB+2Ab)}{ab^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(a*e*Sqrt[e*x]) + ((1 + Sqrt[3])*(2*A*b + a*B)*Sqrt[
e*x]*Sqrt[a + b*x^3])/(a*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) -
 (3^(1/4)*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
```

```
1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellipt
icE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)], (2 + Sqrt[3])/4])/(a^(2/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - ((1
- Sqrt[3])*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellip
ticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^
(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*a^(2/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
```

```
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}+\frac{(2Ab+aB)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{ae^3}$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}+\frac{(2(2Ab+aB))\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{ae^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}-\frac{(2Ab+aB)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{ab^{2/3}e^4}-$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}+\frac{\left(1+\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\,\sqrt{a+bx^3}}{ab^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{\sqrt[4]{3}\,(2Ab+aB)\sqrt{e}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 83, normalized size = 0.15

$$\frac{x\left(-10A(a+bx^3)+2(2Ab+aB)x^3\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{5a(ex)^{3/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (x*(-10*A*(a + b*x^3) + 2*(2*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeomet
ric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(5*a*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 5385, normalized size = 9.94

method	result	size
risch	Expression too large to display	1119
elliptic	Expression too large to display	1132
default	Expression too large to display	5385

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(x)*e^(-3/2)/(b*x^5 + a*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.11, size = 97, normalized size = 0.18

$$\frac{A\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{6},\frac{1}{2}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)}+\frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{a}\,e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-1/6)*hyper((-1/6, 1/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*e**(3/2)*sqrt(x)*gamma(5/6)) + B*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6), (
11/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(3/2)*gamma(11/6))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*e^(-3/2)/(sqrt(b*x^3 + a)*x^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{3/2}\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(1/2)), x)
```

3.549 $\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=75

$$-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}$$

[Out] 2/3*B*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(1/2)-2/3*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(3/2)

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {462, 335, 281, 223, 212}

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]

[Out] (-2*A*Sqrt[a + b*x^3])/(3*a*e*(e*x)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx &= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{B\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{e^3}\\
&= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{(2B)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{e^4}\\
&= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{(2B)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3e^4}\\
&= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{(2B)\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3e^4}\\
&= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.23, size = 65, normalized size = 0.87

$$\frac{2x\left(-\frac{A\sqrt{a+bx^3}}{a}+\frac{Bx^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)}{\sqrt{b}}\right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*x*(-((A*Sqrt[a + b*x^3])/a) + (B*x^(3/2)*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b
]*x^(3/2))])/Sqrt[b]))/(3*(e*x)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 3397, normalized size = 45.29

method	result	size
risch	Expression too large to display	1037
elliptic	Expression too large to display	1037
default	Expression too large to display	3397

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*x^3+a)^(1/2)/x/b^2*(6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-
2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x^4-6*I*B*3
^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*
(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x^4-12*I*B*3^(1/
2)*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*e*x^3+12*I*B*3^(1/2)*(-a*b^2)^(1/3)*(-(
I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
```

2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*e*x^3+6*I*B*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*e*x^2-6*I*B*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*e*x^2-6*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x^4+6*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x^4+12*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*e*x^3-12*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*e*x^3-6*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*e*x^2+6*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2

```
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2...
```

Maxima [A]
time = 0.51, size = 68, normalized size = 0.91

$$-\frac{1}{3}\left(\frac{B\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{\sqrt{b}}+\frac{2\sqrt{bx^3+a}\,A}{ax^{\frac{3}{2}}}\right)e^{\left(-\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(B*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 + a)
/x^(3/2)))/sqrt(b) + 2*sqrt(b*x^3 + a)*A/(a*x^(3/2)))*e^(-5/2)
```

Fricas [A]
time = 0.60, size = 160, normalized size = 2.13

$$\left[\frac{\left(Ba\sqrt{b}\,x^2\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)-4\,\sqrt{bx^3+a}\,Ab\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{6\,abx^2},-\frac{\left(Ba\sqrt{-b}\,x^2\arctan\left(\frac{2\,\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)+2\,\sqrt{bx^3+a}\,Ab\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{3\,abx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(B*a*sqrt(b)*x^2*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqrt(b
*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*sqrt(b*x^3 + a)*A*b*sqrt(x))*e^(-5/2)/
(a*b*x^2), -1/3*(B*a*sqrt(-b)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)
/(2*b*x^3 + a)) + 2*sqrt(b*x^3 + a)*A*b*sqrt(x))*e^(-5/2)/(a*b*x^2)]
```

Sympy [A]
time = 5.05, size = 60, normalized size = 0.80

$$-\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ae^{\frac{5}{2}}}+\frac{2B\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}\,e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*e**(5/2)) + 2*B*asinh(sqrt(b)*x**(3/
2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

Giac [A]
time = 1.64, size = 76, normalized size = 1.01

$$-\frac{2}{3}\left(\frac{B\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\frac{A\sqrt{b+\frac{a}{x^3}}}{a}-\frac{Ba\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+A\sqrt{-b}\,\sqrt{b}}{a\sqrt{-b}}\right)e^{\left(-\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(B*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + A*sqrt(b + a/x^3)/a - (
B*a*arctan(sqrt(b)/sqrt(-b)) + A*sqrt(-b)*sqrt(b))/(a*sqrt(-b)))*e^(-5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{5/2}\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)), x)
```

3.550 $\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=246

$$-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}-\frac{(2Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right.}{\sqrt[3]{a}+\left(1+\sqrt{3}\right.}\right.\right.}{5\sqrt[4]{3}\,a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] -2/5*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(5/2)-1/15*(2*A*b-5*B*a)*(a^(1/3)+b^(1/3)*
x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1
/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipti
cF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/e^4/(b*x^
3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {464, 335, 231}

$$-\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(2Ab-5aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}\,a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(5*a*e*(e*x)^(5/2)) - ((2*A*b - 5*a*B)*Sqrt[e*x]*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b
^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4
)*a^(4/3)*e^4*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx &= -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}-\frac{(2Ab-5aB)\int\frac{1}{\sqrt{ex}\,\sqrt{a+bx^3}}\,dx}{5ae^3}\\
&= -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}-\frac{(2(2Ab-5aB))\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{5ae^4}\\
&= -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}-\frac{(2Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}}}}{5\sqrt[4]{3}\,a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\right.}{\left(\sqrt[3]{a}+\left(1+\sqrt{}\right.\right.}}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 82, normalized size = 0.33

$$-\frac{2x\left(A(a+bx^3)+(2Ab-5aB)x^3\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{5a(ex)^{7/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*x*(A*(a + b*x^3) + (2*A*b - 5*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometr
ic2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*Sqrt[a + b*x^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.50, size = 3303, normalized size = 13.43 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(b*x^3+a)^(1/2)/x^2/(-a*b^2)^(1/3)/b/a*(-8*I*A*3^(1/2)*(-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-
a*b^2)^(1/3)*b^2*e*x^4+20*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*
x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(
I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)
*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a*b*e*x^
4+4*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*b*e*x^3+3*A*(1/b^2*e*x*(-b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(
1/2)*b-I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(
1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*b+8*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*
```

```
3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3
)*b^2*e*x^4-20*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a*b*e*x^4-4*A*(-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
-a*b^2)^(2/3)*b*e*x^3+10*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1
/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e*x^5+10*B*(-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a
*b^2)^(2/3)*a*e*x^3-4*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2)
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^3*e*x^5+4*I*A*3^(1/2)*(-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*b^3*e*x^5-10*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(
1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/
2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 53, normalized size = 0.22

$$-\frac{2\left((5\,Ba-2\,Ab)\sqrt{a}\,x^3\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+\sqrt{bx^3+a}\,Aa\sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{5\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*((5*B*a - 2*A*b)*sqrt(a)*x^3*weierstrassPInverse(0, -4*b/a, 1/x) + sqr
t(b*x^3 + a)*A*a*sqrt(x))*e^(-7/2)/(a^2*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 19.38, size = 97, normalized size = 0.39

$$\frac{A\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{6},\frac{1}{2}\\\frac{1}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{B\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{6},\frac{1}{2}\\\frac{7}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-5/6)*hyper((-5/6, 1/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2), (
7/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(7/2)*gamma(7/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*e^(-7/2)/(sqrt(b*x^3 + a)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{7/2}\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(1/2)), x)
```

3.551 $\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}+\frac{(2Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

```
[Out] 1/3*(2*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/
2))/b^(5/2)-1/3*(2*A*b-3*B*a)*e^2*(e*x)^(3/2)/b^2/(b*x^3+a)^(1/2)+1/3*B*(e*
x)^(9/2)/b/e/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {470, 294, 335, 281, 223, 212}

$$\frac{e^{7/2}(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}-\frac{e^2(ex)^{3/2}(2Ab-3aB)}{3b^2\sqrt{a+bx^3}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] -1/3*((2*A*b - 3*a*B)*e^2*(e*x)^(3/2))/(b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(9/
2))/(3*b*e*Sqrt[a + b*x^3]) + ((2*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*
x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} - \frac{\left(-3Ab+\frac{9aB}{2}\right)\int \frac{(ex)^{7/2}}{(a+bx^3)^{3/2}}\,dx}{3b}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{((2Ab-3aB)e^3)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{2b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{((2Ab-3aB)e^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx}{e^3}}}\right.}{b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{((2Ab-3aB)e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx}{e^3}}}\right.}{3b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{((2Ab-3aB)e^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,\right.}{3b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{(2Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3}}{e^{3/2}\sqrt{a+b}}\right.}{3b^{5/2}}$$

Mathematica [A]
time = 0.39, size = 92, normalized size = 0.77

$$\frac{(ex)^{7/2}\left(\frac{\sqrt{b}\,x^{3/2}(-2Ab+3aB+bBx^3)}{\sqrt{a+bx^3}} + (2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)\right)}{3b^{5/2}x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] ((e*x)^(7/2)*((Sqrt[b]*x^(3/2)*(-2*A*b + 3*a*B + b*B*x^3))/Sqrt[a + b*x^3]
+ (2*A*b - 3*a*B)*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))]))/(3*b^(5/2)*x
^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.38, size = 7016, normalized size = 58.47

method	result	size
risch	Expression too large to display	1079
elliptic	Expression too large to display	1097
default	Expression too large to display	7016

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(82) = 164$.
time = 0.54, size = 175, normalized size = 1.46

$$\frac{1}{6}\left(B\left(\frac{2\left(2\,ab-\frac{3\,(bx^3+a)a}{x^3}\right)}{\frac{\sqrt{bx^3+a}\,b^3}{x^{\frac{3}{2}}}-\frac{(bx^3+a)^{\frac{3}{2}}b^2}{x^{\frac{9}{2}}}}+\frac{3\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{5}{2}}}\right)-2\,A\left(\frac{2\,x^{\frac{3}{2}}}{\sqrt{bx^3+a}\,b}+\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{3}{2}}}\right)\right)e^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(B*(2*(2*a*b - 3*(b*x^3 + a)*a/x^3)/(sqrt(b*x^3 + a)*b^3/x^(3/2) - (b*x
^3 + a)^(3/2)*b^2/x^(9/2)) + 3*a*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(
sqrt(b) + sqrt(b*x^3 + a)/x^(3/2)))/b^(5/2)) - 2*A*(2*x^(3/2)/(sqrt(b*x^3 +
 a)*b) + log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^3 + a
)/x^(3/2)))/b^(3/2)))*e^(7/2)
```

Fricas [A]
time = 2.42, size = 264, normalized size = 2.20

$$\left[-\frac{((3\,Bab-2\,Ab^2)x^3+3\,Ba^2-2\,Aab)\sqrt{b}\,e^{\frac{7}{2}}\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)-4\,(Bb^2x^4+(3\,Bab-2\,Ab^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{12\,(b^4x^3+ab^3)},\ \frac{((3\,Bab-2\,Ab^2)x^3+3\,Ba^2-2\,Aab)\sqrt{-b}\,\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)e^{\frac{7}{2}}+2\,(Bb^2x^4+(3\,Bab-2\,Ab^2)x)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{6\,(b^4x^3+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(((3*B*a*b - 2*A*b^2)*x^3 + 3*B*a^2 - 2*A*a*b)*sqrt(b)*e^(7/2)*log(-
8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) -
 a^2) - 4*(B*b^2*x^4 + (3*B*a*b - 2*A*b^2)*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(7/
2))/(b^4*x^3 + a*b^3), 1/6*(((3*B*a*b - 2*A*b^2)*x^3 + 3*B*a^2 - 2*A*a*b)*s
qrt(-b)*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a))*e^(7/2) +
2*(B*b^2*x^4 + (3*B*a*b - 2*A*b^2)*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(7/2))/(b^4
*x^3 + a*b^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.59, size = 85, normalized size = 0.71

$$\frac{\left(\frac{Bx^3}{b}+\frac{3\,Bab^3-2\,Ab^4}{b^5}\right)x^{\frac{3}{2}}e^{\frac{7}{2}}}{3\,\sqrt{bx^3+a}}+\frac{(3\,Bab^3-2\,Ab^4)e^{\frac{7}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(B*x^3/b + (3*B*a*b^3 - 2*A*b^4)/b^5)*x^(3/2)*e^(7/2)/sqrt(b*x^3 + a) +
 1/3*(3*B*a*b^3 - 2*A*b^4)*e^(7/2)*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3 +
a)))/b^(11/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2), x)
```

3.552 $\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=286

$$-\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}}+\frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}}+\frac{(4Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\;\cdots}{12\sqrt[4]{3}\,\sqrt[3]{a}\,b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\cdots\right.}}\;\cdots}$$

```
[Out] 1/2*B*(e*x)^(7/2)/b/e/(b*x^3+a)^(1/2)-1/6*(4*A*b-7*B*a)*e^2*(e*x)^(1/2)/b^2
/(b*x^3+a)^(1/2)+1/36*(4*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+
1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^2/(b*x^3+a)^(1/2)/(b^(1/3
)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {470, 294, 335, 231}

$$\frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(4Ab-7aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{12\sqrt[4]{3}\,\sqrt[3]{a}\,b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{e^2\sqrt{ex}(4Ab-7aB)}{6b^2\sqrt{a+bx^3}}+\frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] -1/6*((4*A*b - 7*a*B)*e^2*Sqrt[e*x])/(b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(7/2)
)/(2*b*e*Sqrt[a + b*x^3]) + ((4*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(12*3^(1/4)*a^(1/3)*b^
2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} - \frac{\left(-2Ab+\frac{7aB}{2}\right)\int \frac{(ex)^{5/2}}{(a+bx^3)^{3/2}}\,dx}{2b}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{((4Ab-7aB)e^3)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,d}{12b^2}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{((4Ab-7aB)e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx}{e^3}}}\right.}{6b^2}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{(4Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{12\sqrt[4]{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 87, normalized size = 0.30

$$\frac{e^2\sqrt{ex}\left(-4Ab+7aB+3bBx^3+(4Ab-7aB)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{6b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (e^2*Sqrt[e*x]*(-4*A*b + 7*a*B + 3*b*B*x^3 + (4*A*b - 7*a*B)*Sqrt[1 + (b*x^
3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(6*b^2*Sqrt[a + b*x^
3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.35, size = 3760, normalized size = 13.15

method	result

elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^3+a)\,ex}\left(-\frac{2e^3x(Ab-Ba)}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)bex}}+\frac{B\,e^2\sqrt{be\,x^4+aex}}{2b^2}+\frac{2\left(\frac{(Ab-Ba)e^3}{3b^2}-\frac{B\,e^3a}{4b^2}\right)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-\right.}{2b}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*e^2/x*(e*x)^(1/2)*(-7*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*
x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*a*b*x+4*I*A*3^(1/2)*(1/b^2*e*x*(-b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*b^2*x+8*I*A*3^
(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*b^3*x^2+8*I*A*3^(1/2)*(-(I*3^(1/2
)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*b+28*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*
b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*
x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*a*b*x-8*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
F((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/
```

```
2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1
/2)*b^3*x^2-14*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1
/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*a*b^2*x^2-14
*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*a+14*B*(-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*((b*x^3+a)*e*x)^(1/2)*a*b^2*x^2+16*A*(-a*b^2)^(1/3)*(-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*((b*x^3+a)*e*x)^(1/2)*b^2*x-28*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e
*x)^(1/2)*a*b*x-8*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-
1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*b+14
*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((B*x^3 + A)*x^(5/2)/(b*x^3 + a)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^5 + A*x^2)*sqrt(b*x^3 + a)*sqrt(x)*e^(5/2)/(b^2*x^6 + 2*a*b*x
^3 + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^(5/2)*e^(5/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(3/2), x)
```

3.553 $\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=553

$$\frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}}-\frac{\left(1+\sqrt{3}\right)(2Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^3}}{3ab^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{(2Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/}\cdots}{\left(\sqrt[3]{\cdots}\right.}}}{3^{3/4}a^{2/3}b^{5/3}\cdots}$$

```
[Out] 2/3*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^3+a)^(1/2)-1/3*(2*A*b-5*B*a)*e*(1+3^(1
/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))+
1/3*(2*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/
(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a
^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^
2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(
1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(
1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+1/18*(2*A*b-5*B*a)*e*(a^(1
/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3
^(3/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1
/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {468, 335, 314, 231, 1895}

$$\frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(2Ab-5aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[4]{3}\,a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(2Ab-5aB)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\left(1+\sqrt{3}\right)e\sqrt{ex}\,\sqrt{a+bx^3}(2Ab-5aB)}{3ab^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*Sqrt[a + b*x^3]) - ((1 + Sqrt[3])*(2*A
*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(3*a*b^(5/3)*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)) + ((2*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(
```

```
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(3/4)*a^(2/3)*b^(5/3)*Sqrt[(b^(
1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) + ((1 - Sqrt[3])*(2*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(6*3^(1/4)*a^(2/3)*b^(5/3
)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
```

```
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} + \frac{\left(2\left(-Ab+\frac{5aB}{2}\right)\right)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} - \frac{(2(2Ab-5aB))\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3abe}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} + \frac{(2Ab-5aB)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right.}{3ab^{5/3}e}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} - \frac{\left(1+\sqrt{3}\right)(2Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{3ab^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)} + \frac{(2Ab-5aB)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 77, normalized size = 0.14

$$\frac{x(ex)^{3/2}\left(5aB+(2Ab-5aB)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{5ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (x*(e*x)^(3/2)*(5*a*B + (2*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[5/6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a*b*Sqrt[a + b*x^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.32, size = 5392, normalized size = 9.75

method	result	size
elliptic	Expression too large to display	1154
default	Expression too large to display	5392

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((B*x^3 + A)*x^(3/2)/(b*x^3 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(3/2)/(b^2*x^6 + 2*a*b*x^3
 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 76.93, size = 94, normalized size = 0.17

$$\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{6},\frac{3}{2}\\\frac{11}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{11}{6}\\\frac{17}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((5/6, 3/2), (11/6,), b*x**3*exp_polar(
I*pi)/a)/(3*a**(3/2)*gamma(11/6)) + B*e**(3/2)*x**(11/2)*gamma(11/6)*hyper(
(3/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(17/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^(3/2)*e^(3/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(3/2), x)
```

3.554 $\int \frac{\sqrt{ex}\,(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

```
[Out] 2/3*B*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)/b^(3/2)+
2/3*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {463, 335, 281, 223, 212}

$$\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*Sqrt[a + b*x^3]) + (2*B*Sqrt[e]*ArcTan
h[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ex}\,(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx &= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{B\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{b}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{be}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3be}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3be}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.34, size = 80, normalized size = 0.94

$$
\frac{2\sqrt{ex}\left(\frac{\sqrt{b}\,(Ab-aB)x^{3/2}}{a\sqrt{a+bx^3}}+B\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)\right)}{3b^{3/2}\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[e*x]*((Sqrt[b]*(A*b - a*B)*x^(3/2))/(a*Sqrt[a + b*x^3]) + B*ArcTanh
[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))]))/(3*b^(3/2)*Sqrt[x])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.37, size = 3654, normalized size = 42.99

method	result	size
elliptic	Expression too large to display	1050
default	Expression too large to display	3654

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(e*x)^(1/2)/(b*x^3+a)^(1/2)/b^3*(-I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)
^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b^2*x^2-6*I*B*3^(1/2)*(-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
(b*x^3+a)*e*x)^(1/2)*a*b^2*x^2+6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)
^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*
3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/
2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*a-6*I*B*3^(1/2)*(-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*a+12*I*B*3^(1/2)*(-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
(b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*a*b*x-12*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*
```

```
b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/
2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*a*b*x+6*B*(-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*a*b^2*x^2-6*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1
+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*a*b^2*x^2-12*B*(-a*b^2)^(1/3)*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*((b*x^3+a)*e*x)^(1/2)*a*b*x+12*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2
))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*a*b*x+6*I*B*3^(1/2)*(-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/
(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*a*b^2*x^2+I*A*3^(
1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^3*x^2+6
*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2...
```

Maxima [A]

time = 0.50, size = 87, normalized size = 1.02

$$-\frac{1}{3}\left(B\left(\frac{2x^{\frac{3}{2}}}{\sqrt{bx^3+a}\,b}+\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{3}{2}}}\right)-\frac{2Ax^{\frac{3}{2}}}{\sqrt{bx^3+a}\,a}\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(B*(2*x^(3/2)/(sqrt(b*x^3 + a)*b) + log(-(sqrt(b) - sqrt(b*x^3 + a)/x^
(3/2))/(sqrt(b) + sqrt(b*x^3 + a)/x^(3/2)))/b^(3/2)) - 2*A*x^(3/2)/(sqrt(b*
x^3 + a)*a))*e^(1/2)
```

Fricas [A]
time = 6.59, size = 217, normalized size = 2.55

$$\left[-\frac{4\sqrt{bx^3+a}\,(Bab-Ab^2)x^{\frac{3}{2}}e^{\frac{1}{2}}-(Babx^3+Ba^2)\sqrt{b}\,e^{\frac{1}{2}}\log\left(-8b^2x^6-8abx^3-4(2bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)}{6(ab^3x^3+a^2b^2)},-\frac{2\sqrt{bx^3+a}\,(Bab-Ab^2)x^{\frac{3}{2}}e^{\frac{1}{2}}+(Babx^3+Ba^2)\sqrt{-b}\,\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2bx^3+a}\right)e^{\frac{1}{2}}}{3(ab^3x^3+a^2b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(b*x^3 + a)*(B*a*b - A*b^2)*x^(3/2)*e^(1/2) - (B*a*b*x^3 + B*a
^2)*sqrt(b)*e^(1/2)*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqrt(b*x
^3 + a)*sqrt(b)*sqrt(x) - a^2))/(a*b^3*x^3 + a^2*b^2), -1/3*(2*sqrt(b*x^3 +
 a)*(B*a*b - A*b^2)*x^(3/2)*e^(1/2) + (B*a*b*x^3 + B*a^2)*sqrt(-b)*arctan(2
*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a))*e^(1/2))/(a*b^3*x^3 + a^2*
b^2)]
```

Sympy [A]
time = 10.29, size = 95, normalized size = 1.12

$$\frac{2A\sqrt{e}\,x^{\frac{3}{2}}}{3a^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+B\left(\frac{2\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}}-\frac{2\sqrt{e}\,x^{\frac{3}{2}}}{3\sqrt{a}\,b\sqrt{1+\frac{bx^3}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*A*sqrt(e)*x**(3/2)/(3*a**(3/2)*sqrt(1 + b*x**3/a)) + B*(2*sqrt(e)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - 2*sqrt(e)*x**(3/2)/(3*sqrt(a)*b*sqr
t(1 + b*x**3/a)))
```

Giac [A]
time = 1.63, size = 59, normalized size = 0.69

$$-\frac{2\,Be^{\frac{1}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\,b^{\frac{3}{2}}}-\frac{2\,(Ba-Ab)x^{\frac{3}{2}}e^{\frac{1}{2}}}{3\,\sqrt{bx^3+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*B*e^(1/2)*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3 + a)))/b^(3/2) - 2/3*(
B*a - A*b)*x^(3/2)*e^(1/2)/(sqrt(b*x^3 + a)*a*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(B\,x^3+A\right)\,\sqrt{e\,x}}{{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2), x)
```

3.555 $\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=258

$$\frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}}+\frac{(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right.}{\sqrt[3]{a}+\left(1+\sqrt{3}\right.}\right.\right.}{3\sqrt[4]{3}\,a^{4/3}be\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] 2/3*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^3+a)^(1/2)+1/9*(2*A*b+B*a)*(a^(1/3)+b^
(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*E
llipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/b/e
/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {468, 335, 231}

$$\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(aB+2Ab)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}\,a^{4/3}be\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2\sqrt{ex}\,(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[e*x])/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*A*b + a*B)*Sqrt[e
*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqr
t[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3
*3^(1/4)*a^(4/3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{ex}\,\left(a+bx^3\right)^{3/2}}\,dx = \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{\left(2\left(Ab+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{ex}\,\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{(2(2Ab+aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3abe}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/\cdots}}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\cdots\right.}}\cdots}{3\sqrt[4]{3}\,a^{4/3}be\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}\cdots\right.}{\left(\sqrt[3]{a}+\left(1+\cdots\right.\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 79, normalized size = 0.31

$$\frac{2x\left(Ab-aB+(2Ab+aB)\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{3ab\sqrt{ex}\ \sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*x*(A*b - a*B + (2*A*b + a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6,
1/2, 7/6, -((b*x^3)/a)]))/(3*a*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 3565, normalized size = 13.82

method	result
elliptic	$\sqrt{(b\,x^3+a)\,ex}\left(\frac{2x(Ab-Ba)}{3ba\sqrt{\left(x^3+\frac{a}{b}\right)bex}}+\frac{2\left(\frac{B}{b}+\frac{\frac{2Ab}{3}-\frac{2Ba}{3}}{ab}\right)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{\cdots}}{\cdots}\right.}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b\cdots\right.}{2b}\right.}}}{\cdots}\right.$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(b*x^3+a)^(1/2)/b^2/(-a*b^2)^(1/3)/a*(-8*I*A*3^(1/2)*(-a*b^2)^(1/3)*(-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*b^2*x+2*I*B*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b
```

^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*((b*x^3+a)*e*x)^(1/2)*a+2*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^
(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)
^(1/2)*a*b^2*x^2+I*B*3^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b*x-4*I*B*3^(1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*((b*x^3+a)*e*x)^(1/2)*a*b*x-4*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(
-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*b^3
*x^2+4*I*A*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*b-2*B
*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*a*b^2*x^2+8*A*(-a*b^2)^(1/3)*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
3^(1/2)(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*((b*x^3+a)*e*x)^(1/2)*b^2*x+4*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a

```
)*e*x)^(1/2)*a*b*x-4*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*
x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(
I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)
*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*b
-2*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 88, normalized size = 0.34

$$-\frac{2\left(\left((Bab+2\,Ab^2)x^3+Ba^2+2\,Aab\right)\sqrt{a}\ \mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+\sqrt{bx^3+a}\,\left(Ba^2-Aab\right)\sqrt{x}\right)e^{\left(-\frac{1}{2}\right)}}{3\left(a^2b^2x^3+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*sqrt(a)*weierstrassPInverse
(0, -4*b/a, 1/x) + sqrt(b*x^3 + a)*(B*a^2 - A*a*b)*sqrt(x))*e^(-1/2)/(a^2*b
^2*x^3 + a^3*b)
```

Sympy [C] Result contains complex when optimal does not.
time = 21.68, size = 94, normalized size = 0.36

$$\frac{A\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\\frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)}+\frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{3}{2}\\\frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/6)*hyper((1/6, 3/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3
*a**(3/2)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((7/6, 3/2), (13
/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*sqrt(e)*gamma(13/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*e^(-1/2)/((b*x^3 + a)^(3/2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{\sqrt{e\,x}\,{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(3/2)), x)
```

3.556 $\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=585

$$-\frac{2A}{ae\sqrt{ex}\,\sqrt{a+bx^3}}-\frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}}+\frac{2\left(1+\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{2(4Ab-aB)\sqrt{ex}\,\left(\sqrt[3]{\right.}$$

```
[Out] -2/3*(4*A*b-B*a)*(e*x)^(5/2)/a^2/e^4/(b*x^3+a)^(1/2)-2*A/a/e/(e*x)^(1/2)/(b
*x^3+a)^(1/2)+2/3*(4*A*b-B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a^2/b
^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-2/3*(4*A*b-B*a)*(a^(1/3)+b^(1/3)
*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipt
icE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/b^(2/3)/
e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2)-1/9*(4*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*
(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(
1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b^(2/3)/e^2/(b*x^3+a)^
(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)
```

Rubi [A]
time = 0.44, antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 296, 335, 314, 231, 1895}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(4Ab-aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}\,a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(4Ab-aB)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2\left(1+\sqrt{3}\right)\sqrt{ex}\,\sqrt{a+bx^3}\,(4Ab-aB)}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{2(ex)^{5/2}(4Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{ae\sqrt{ex}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^3]) - (2*(4*A*b - a*B)*(e*x)^(5/2))/(3*a
^2*e^4*Sqrt[a + b*x^3]) + (2*(1 + Sqrt[3])*(4*A*b - a*B)*Sqrt[e*x]*Sqrt[a +
```

```
b*x^3])/(3*a^2*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) - (2*(4*A*
b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(
1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 +
Sqrt[3])/4])/(3^(3/4)*a^(5/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3
)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - ((1 - Sqrt[
3])*(4*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(3*3^(1/4)*a^(5/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/
3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{(4Ab-aB)\int \frac{(ex)^{3/2}}{(a+bx^3)^{3/2}}\,dx}{ae^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{(2(4Ab-aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{3a^2e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{(4(4Ab-aB))\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{b}{e}\cdots}}\cdots\right)}{3a^2e^4}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} - \frac{(2(4Ab-aB))\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\cdots\right.}{\sqrt{\cdots}}\cdots\right)}{3a^2b^{2/3}\cdots}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{2\left(1+\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\sqrt{a\cdots}}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 77, normalized size = 0.13

$$\frac{x\left(-10aA+2(-4Ab+aB)x^3\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{5a^2(ex)^{3/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (x*(-10*a*A + 2*(-4*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/
6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a^2*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 5563, normalized size = 9.51

method	result	size
elliptic	Expression too large to display	1177
risch	Expression too large to display	2209
default	Expression too large to display	5563

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 100, normalized size = 0.17

$$-\frac{2\left(((Bab-4\,Ab^2)x^4+(Ba^2-4\,Aab)x)\sqrt{a}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)+\sqrt{bx^3+a}\,(Ba^2-Aab)\sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{3\,(a^2b^2x^4+a^3bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*sqrt(a)*weierstrassZeta
(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x)) + sqrt(b*x^3 + a)*(B*a^2 -
 A*a*b)*sqrt(x))*e^(-3/2)/(a^2*b^2*x^4 + a^3*b*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 37.36, size = 97, normalized size = 0.17

$$\frac{A\Gamma\left(-\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{3}{2} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)} + \frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} \frac{5}{6}, \frac{3}{2} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-1/6)*hyper((-1/6, 3/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*e**(3/2)*sqrt(x)*gamma(5/6)) + B*x**(5/2)*gamma(5/6)*hyper((5/6, 3/2),
(11/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*e**(3/2)*gamma(11/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*e^(-3/2)/((b*x^3 + a)^(3/2)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)), x)
```

3.557 $\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}-\frac{2(2Ab-aB)(ex)^{3/2}}{3a^2e^4\sqrt{a+bx^3}}$$

```
[Out] -2/3*A/a/e/(e*x)^(3/2)/(b*x^3+a)^(1/2)-2/3*(2*A*b-B*a)*(e*x)^(3/2)/a^2/e^4/
(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {464, 270}

$$-\frac{2(ex)^{3/2}(2Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^3]) - (2*(2*A*b - a*B)*(e*x)^(3/2))/
(3*a^2*e^4*Sqrt[a + b*x^3])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}} - \frac{(2Ab-aB)\int \frac{\sqrt{ex}}{(a+bx^3)^{3/2}}\,dx}{ae^3}$$
$$= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}} - \frac{2(2Ab-aB)(ex)^{3/2}}{3a^2e^4\sqrt{a+bx^3}}$$

Mathematica [A]
time = 0.26, size = 44, normalized size = 0.66

$$\frac{2x(-aA-2Abx^3+aBx^3)}{3a^2(ex)^{5/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
[Out] (2*x*(-(a*A) - 2*A*b*x^3 + a*B*x^3))/(3*a^2*(e*x)^(5/2)*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.34, size = 44, normalized size = 0.66

method	result	size
gosper	$-\frac{2x(2Ab\,x^3-Ba\,x^3+Aa)}{3\sqrt{b\,x^3+a}\;a^2(ex)^{\frac{5}{2}}}$	39
default	$-\frac{2(2Ab\,x^3-Ba\,x^3+Aa)}{3x\sqrt{b\,x^3+a}\;a^2e^2\sqrt{ex}}$	44
risch	$-\frac{2A\sqrt{b\,x^3+a}}{3a^2x\,e^2\sqrt{ex}}-\frac{2(Ab-Ba)x^2}{3a^2e^2\sqrt{ex}\,\sqrt{b\,x^3+a}}$	61
elliptic	$\frac{\sqrt{(b\,x^3+a)\,ex}\left(-\frac{2x^2(Ab-Ba)}{3e^2a^2\sqrt{\left(x^3+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{be\,x^4+aex}}{3e^3a^2x^2}\right)}{\sqrt{ex}\,\sqrt{b\,x^3+a}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/x*(2*A*b*x^3-B*a*x^3+A*a)/(b*x^3+a)^(1/2)/a^2/e^2/(e*x)^(1/2)
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.88

$$-\frac{2}{3}\left(A\left(\frac{bx^{\frac{3}{2}}}{\sqrt{bx^3+a}\,a^2}+\frac{\sqrt{bx^3+a}}{a^2x^{\frac{3}{2}}}\right)-\frac{Bx^{\frac{3}{2}}}{\sqrt{bx^3+a}\,a}\right)e^{\left(-\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*(A*(b*x^(3/2)/(sqrt(b*x^3 + a)*a^2) + sqrt(b*x^3 + a)/(a^2*x^(3/2))) -
 B*x^(3/2)/(sqrt(b*x^3 + a)*a))*e^(-5/2)
```

Fricas [A]
time = 8.14, size = 51, normalized size = 0.76

$$\frac{2\left((Ba-2\,Ab)x^3-Aa\right)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\left(-\frac{5}{2}\right)}}{3\left(a^2bx^5+a^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*((B*a - 2*A*b)*x^3 - A*a)*sqrt(b*x^3 + a)*sqrt(x)*e^(-5/2)/(a^2*b*x^5 +
 a^3*x^2)
```

Sympy [A]
time = 76.34, size = 90, normalized size = 1.34

$$A\left(-\frac{2}{3a\sqrt{b}\,e^{\frac{5}{2}}x^3\sqrt{\frac{a}{bx^3}+1}}-\frac{4\sqrt{b}}{3a^2e^{\frac{5}{2}}\sqrt{\frac{a}{bx^3}+1}}\right)+\frac{2B}{3a\sqrt{b}\,e^{\frac{5}{2}}\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*(-2/(3*a*sqrt(b)*e**(5/2)*x**3*sqrt(a/(b*x**3) + 1)) - 4*sqrt(b)/(3*a**2*
e**(5/2)*sqrt(a/(b*x**3) + 1))) + 2*B/(3*a*sqrt(b)*e**(5/2)*sqrt(a/(b*x**3)
 + 1))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*e^(-5/2)/((b*x^3 + a)^(3/2)*x^(5/2)), x)
```

Mupad [B]
time = 4.75, size = 70, normalized size = 1.04

$$-\frac{\left(\frac{2\,A}{3\,a\,b\,e^2}+\frac{x^3\,\left(4\,A\,b-2\,B\,a\right)}{3\,a^2\,b\,e^2}\right)\,\sqrt{b\,x^3+a}}{x^4\,\sqrt{e\,x}+\frac{a\,x\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x)
```

```
[Out] -(((2*A)/(3*a*b*e^2) + (x^3*(4*A*b - 2*B*a))/(3*a^2*b*e^2))*(a + b*x^3)^(1/
2))/(x^4*(e*x)^(1/2) + (a*x*(e*x)^(1/2))/b)
```

3.558 $\int \frac{A+Bx^3}{(ex)^{7/2}(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}-\frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}}-\frac{2(8Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}{15\sqrt[4]{3}\,a^{7/3}e^4\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}\right.}{\left(\sqrt[3]{a}+\left(1+\right.\right.}}}$$

```
[Out] -2/5*A/a/e/(e*x)^(5/2)/(b*x^3+a)^(1/2)-2/15*(8*A*b-5*B*a)*(e*x)^(1/2)/a^2/e
^4/(b*x^3+a)^(1/2)-2/45*(8*A*b-5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(7/3)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*
x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {464, 296, 335, 231}

$$-\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(8Ab-5aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{15\sqrt[4]{3}\,a^{7/3}e^4\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{ex}\,(8Ab-5aB)}{15a^2e^4\sqrt{a+bx^3}}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^3]) - (2*(8*A*b - 5*a*B)*Sqrt[e*x])/
(15*a^2*e^4*Sqrt[a + b*x^3]) - (2*(8*A*b - 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqr
t[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(15*3^(1/4)*a^(7/3)*e
^4*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{(8Ab-5aB)\int \frac{1}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx}{5ae^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}} - \frac{(2(8Ab-5aB))\int \frac{1}{\sqrt{ex}\sqrt{a+}}}{15a^2e^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}} - \frac{(4(8Ab-5aB))\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{15a^2e^4}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 95, normalized size = 0.34

$$\frac{x\left(-2(3aA+8Abx^3-5aBx^3)+4(-8Ab+5aB)x^3\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{15a^2(ex)^{7/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (x*(-2*(3*a*A + 8*A*b*x^3 - 5*a*B*x^3) + 4*(-8*A*b + 5*a*B)*x^3*Sqrt[1 + (b
*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(15*a^2*(e*x)^(7/
2)*Sqrt[a + b*x^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 3783, normalized size = 13.37

method	result

elliptic	$\frac{\sqrt{(b\,x^3+a)\,ex}\left(-\frac{2x(Ab-Ba)}{3e^3a^2\sqrt{\left(x^3+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{be\,x^4+aex}}{5e^4a^2x^3}+\frac{2\left(-\frac{2(Ab-Ba)}{3a^2e^3}-\frac{2bA}{5a^2e^3}\right)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/15/x^2*(40*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2
))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)
*a*b*x^3+5*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
))^(1/2)*(-a*b^2)^(1/3)*a*b*x^3-20*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x
)^(1/2)*a*b^2*x^4-8*I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*b^2*x^3+32*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x
^3+a)*e*x)^(1/2)*b^3*x^4-32*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2

```
)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3
^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*b^3*x^4-3
*I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(
-a*b^2)^(1/3)*a*b+20*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3
))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*a*b^2*x^4+64*A*(
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*b^2*x^3-40*B*(-(I*3^(1/2)
-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
)*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*a*b*x^3-32*A*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a
)*e*x)^(1/2)*(-a*b^2)^(2/3)*b*x^2+20*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*
(-a*b^2)^(2/3)*a*x^2-64*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(
-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a
*b^2)^(1/3)*b^2*x^3-20*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.87, size = 109, normalized size = 0.39

$$-\frac{2\left(2\left((5\,Bab-8\,Ab^2)x^6+(5\,Ba^2-8\,Aab)x^3\right)\sqrt{a}\ \mathrm{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\left((5\,Ba^2-8\,Aab)x^3-3\,Aa^2\right)\sqrt{bx^3+a}\ \sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{15\left(a^3bx^6+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(2*((5*B*a*b - 8*A*b^2)*x^6 + (5*B*a^2 - 8*A*a*b)*x^3)*sqrt(a)*weiers
trassPInverse(0, -4*b/a, 1/x) - ((5*B*a^2 - 8*A*a*b)*x^3 - 3*A*a^2)*sqrt(b*
x^3 + a)*sqrt(x))*e^(-7/2)/(a^3*b*x^6 + a^4*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 152.28, size = 97, normalized size = 0.34

$$\frac{A\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},\frac{3}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{B\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-5/6)*hyper((-5/6, 3/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(x)*gamma(1/6)*hyper((1/6, 3/2),
(7/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*e**(7/2)*gamma(7/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*e^(-7/2)/((b*x^3 + a)^(3/2)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)), x)
```

3.559 $\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2(Ab-aB)(ex)^{9/2}}{9abe\,(a+bx^3)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(9/2)/a/b/e/(b*x^3+a)^(3/2)+2/3*B*e^(7/2)*arctanh((e*x)
^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)-2/3*B*e^2*(e*x)^(3/2)/b^2/(
b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {463, 294, 335, 281, 223, 212}

$$\frac{2(ex)^{9/2}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}+\frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(9/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*B*e^2*(e*x)^(3
/2))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(
e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{9/2}}{9abe(a+bx^3)^{3/2}} + \frac{B\int \frac{(ex)^{7/2}}{(a+bx^3)^{3/2}}\,dx}{b}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe(a+bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{(Be^3)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe(a+bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{(2Be^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe(a+bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{(2Be^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe(a+bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{(2Be^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe(a+bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

Mathematica [A]
time = 0.50, size = 99, normalized size = 0.87

$$\frac{2e^3\sqrt{ex}\left(\frac{\sqrt{b}\,x^{3/2}(-3a^2B+Ab^2x^3-4abBx^3)}{a(a+bx^3)^{3/2}} + 3B\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)\right)}{9b^{5/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*e^3*Sqrt[e*x]*((Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - 4*a*b*B*x^3))/(a
*(a + b*x^3)^(3/2)) + 3*B*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))]))/(9*b
^(5/2)*Sqrt[x])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.32, size = 7081, normalized size = 62.11

method	result	size
elliptic	Expression too large to display	1098
default	Expression too large to display	7081

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.50, size = 103, normalized size = 0.90

$$\frac{1}{9}\left(\frac{2\,Ax^{\frac{9}{2}}}{(bx^3+a)^{\frac{3}{2}}a}-\left(\frac{2\left(b+\frac{3\,(bx^3+a)}{x^3}\right)x^{\frac{9}{2}}}{(bx^3+a)^{\frac{3}{2}}b^2}+\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{b^{\frac{5}{2}}}\right)B\right)e^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(2*A*x^(9/2)/((b*x^3 + a)^(3/2)*a) - (2*(b + 3*(b*x^3 + a)/x^3)*x^(9/2)
/((b*x^3 + a)^(3/2)*b^2) + 3*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt
(b) + sqrt(b*x^3 + a)/x^(3/2)))/b^(5/2))*B)*e^(7/2)
```

Fricas [A]
time = 3.86, size = 296, normalized size = 2.60

$$\left[\frac{3\,(Bab^2x^6+2\,Ba^2bx^3+Ba^3)\sqrt{b}\,e^{\frac{7}{2}}\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)-4\,(3\,Ba^2bx+(4\,Bab^2-Ab^3)x^4)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{18\,(ab^5x^6+2\,a^2b^4x^3+a^3b^3)},\ -\frac{3\,(Bab^2x^6+2\,Ba^2bx^3+Ba^3)\sqrt{-b}\,\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)e^{\frac{7}{2}}+2\,(3\,Ba^2bx+(4\,Bab^2-Ab^3)x^4)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{9\,(ab^5x^6+2\,a^2b^4x^3+a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*(B*a*b^2*x^6 + 2*B*a^2*b*x^3 + B*a^3)*sqrt(b)*e^(7/2)*log(-8*b^2*x
^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) -
 4*(3*B*a^2*b*x + (4*B*a*b^2 - A*b^3)*x^4)*sqrt(b*x^3 + a)*sqrt(x)*e^(7/2))
/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3), -1/9*(3*(B*a*b^2*x^6 + 2*B*a^2*b*x^
3 + B*a^3)*sqrt(-b)*arctan(2*sqrt(b*x^3 + a)*sqrt(-b)*x^(3/2)/(2*b*x^3 + a)
)*e^(7/2) + 2*(3*B*a^2*b*x + (4*B*a*b^2 - A*b^3)*x^4)*sqrt(b*x^3 + a)*sqrt(
x)*e^(7/2))/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.48, size = 82, normalized size = 0.72

$$-\frac{2\,x^{\frac{3}{2}}\left(\frac{3\,Ba}{b^2}+\frac{(4\,Ba^5b^6-Aa^4b^7)x^3}{a^5b^7}\right)e^{\frac{7}{2}}}{9\,(bx^3+a)^{\frac{3}{2}}}-\frac{2\,Be^{\frac{7}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^3+a}\,\right|\right)}{3\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*x^(3/2)*(3*B*a/b^2 + (4*B*a^5*b^6 - A*a^4*b^7)*x^3/(a^5*b^7))*e^(7/2)/
(b*x^3 + a)^(3/2) - 2/3*B*e^(7/2)*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3 + a
)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^3+A)\,{(e\,x)}^{7/2}}{{(b\,x^3+a)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2), x)
```

3.560 $\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=299

$$\frac{2(Ab-aB)(ex)^{7/2}}{9abe\,(a+bx^3)^{3/2}}-\frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}}+\frac{(2Ab+7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\operatorname{ArcCos}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}\,a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(7/2)/a/b/e/(b*x^3+a)^(3/2)-2/27*(2*A*b+7*B*a)*e^2*(e*x
)^(1/2)/a/b^2/(b*x^3+a)^(1/2)+1/81*(2*A*b+7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((
a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(
a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1
-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
,1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/b^2/(b*x^3+a)^
(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)
```

Rubi [A]
time = 0.17, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {468, 294, 335, 231}

$$\frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(7aB+2Ab)F\left(\operatorname{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}\,a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2e^2\sqrt{ex}\,(7aB+2Ab)}{27ab^2\sqrt{a+bx^3}}+\frac{2(ex)^{7/2}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(7/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*(2*A*b + 7*a*B
)*e^2*Sqrt[e*x])/(27*a*b^2*Sqrt[a + b*x^3]) + ((2*A*b + 7*a*B)*e^2*Sqrt[e*x
]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[
3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*
3^(1/4)*a^(4/3)*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{7/2}}{9abe\,(a+bx^3)^{3/2}} + \frac{\left(2\left(Ab+\frac{7aB}{2}\right)\right)\int \frac{(ex)^{5/2}}{(a+bx^3)^{3/2}}dx}{9ab}$$

$$= \frac{2(Ab-aB)(ex)^{7/2}}{9abe\,(a+bx^3)^{3/2}} - \frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}} + \frac{((2Ab+7aB)e^3)\int \frac{1}{\sqrt{ex}\sqrt{a+b}}}{27ab^2}$$

$$= \frac{2(Ab-aB)(ex)^{7/2}}{9abe\,(a+bx^3)^{3/2}} - \frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}} + \frac{(2(2Ab+7aB)e^2)\,\mathrm{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{27ab^2}$$

$$= \frac{2(Ab-aB)(ex)^{7/2}}{9abe\,(a+bx^3)^{3/2}} - \frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}} + \frac{(2Ab+7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.15, size = 108, normalized size = 0.36

$$\frac{2e^2\sqrt{ex}\left(-7a^2B+Ab^2x^3-2ab(A+5Bx^3)+(2Ab+7aB)(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{27ab^2(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*e^2*Sqrt[e*x]*(-7*a^2*B + A*b^2*x^3 - 2*a*b*(A + 5*B*x^3) + (2*A*b + 7*a
*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x
^3)/a)]))/(27*a*b^2*(a + b*x^3)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 7083, normalized size = 23.69

method	result

elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^3+a)\,ex}\left(-\frac{2e^2(Ab-Ba)\sqrt{be\,x^4+aex}}{9b^4\left(x^3+\frac{a}{b}\right)^2}+\frac{2e^3x(Ab-10Ba)}{27b^2a\sqrt{\left(x^3+\frac{a}{b}\right)bex}}+\frac{2\left(\frac{B\,e^3}{b^2}+\frac{2e^3(Ab-10Ba)}{27b^2a}\right)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((B*x^3 + A)*x^(5/2)/(b*x^3 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 151, normalized size = 0.51

$$-\frac{2\left(\left((7\,Bab^2+2\,Ab^3)x^6+7\,Ba^3+2\,Aa^2b+2\,(7\,Ba^2b+2\,Aab^2)x^3\right)\sqrt{a}\,e^{\frac{5}{2}}\mathrm{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)+(7\,Ba^3+2\,Aa^2b+(10\,Ba^2b-Aab^2)x^3)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\frac{5}{2}}\right)}{27\,(a^2b^4x^6+2\,a^3b^3x^3+a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(((7*B*a*b^2 + 2*A*b^3)*x^6 + 7*B*a^3 + 2*A*a^2*b + 2*(7*B*a^2*b + 2*
A*a*b^2)*x^3)*sqrt(a)*e^(5/2)*weierstrassPInverse(0, -4*b/a, 1/x) + (7*B*a^
3 + 2*A*a^2*b + (10*B*a^2*b - A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(x)*e^(5/2)
)/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^(5/2)*e^(5/2)/(b*x^3 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3+A)\,{(e\,x)}^{5/2}}{{(b\,x^3+a)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(5/2), x)
```

3.561 $\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=596

$$\frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}}-\frac{2\left(1+\sqrt{3}\right)(4Ab+5aB)e\sqrt{ex}\,\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{2(4Ab+5aB)e\sqrt{ex}}{\cdots}$$

[Out] 2/9*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^3+a)^(3/2)+2/27*(4*A*b+5*B*a)*(e*x)^(5/2)/a^2/b/e/(b*x^3+a)^(1/2)-2/27*(4*A*b+5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a^2/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))+2/27*(4*A*b+5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+1/81*(4*A*b+5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.43, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {468, 296, 335, 314, 231, 1895}

$$\frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(5aB+4Ab)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}\,a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}(5aB+4Ab)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\,3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2\left(1+\sqrt{3}\right)e\sqrt{ex}\sqrt{a+bx^3}(5aB+4Ab)}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}+\frac{2(ex)^{5/2}(5aB+4Ab)}{27a^2be\sqrt{a+bx^3}}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]

[Out] (2*(A*b - a*B)*(e*x)^(5/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(4*A*b + 5*a*B)*(e*x)^(5/2))/(27*a^2*b*e*Sqrt[a + b*x^3]) - (2*(1 + Sqrt[3])*(4*A*b + 5*a

```
*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)) + (2*(4*A*b + 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(9*3^(3/4)*a^(5/3)*b^(5/3)*Sqrt[(b^(1
/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + ((1 - Sqrt[3])*(4*A*b + 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^(5/3)*b^(5/3
)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
```

```
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{5/2}}{9abe\,(a+bx^3)^{3/2}} + \frac{\left(2\left(2Ab+\frac{5aB}{2}\right)\right)\int \frac{(ex)^{3/2}}{(a+bx^3)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{(2(4Ab+5aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{(4(4Ab+5aB))\text{Subst}\left(\int \frac{x^4}{\sqrt{a+ \cdots}} \cdots\right)}{27a^2be}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} + \frac{(2(4Ab+5aB))\text{Subst}\left(\int \frac{\left(-1+\sqrt{\cdots}\right.}{\cdots} \cdots\right)}{27a^2b^{5 \cdots}}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{2\left(1+\sqrt{3}\right)(4Ab+5aB)e\sqrt{ex}\cdots}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\cdots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 86, normalized size = 0.14

$$\frac{x(ex)^{3/2}\left(-5a^2B+(4Ab+5aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{5}{6},\frac{5}{2};\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{10a^2b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (x*(e*x)^(3/2)*(-5*a^2*B + (4*A*b + 5*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(3/
2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 10786, normalized size = 18.10

method	result	size
elliptic	Expression too large to display	1190
default	Expression too large to display	10786

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((B*x^3 + A)*x^(3/2)/(b*x^3 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 162, normalized size = 0.27

$$-\frac{2\left(\left(\left(5\,Bab^{2}+4\,Ab^{3}\right)x^{7}+2\left(5\,Ba^{2}b+4\,Aab^{2}\right)x^{4}+\left(5\,Ba^{3}+4\,Aa^{2}b\right)x\right)\sqrt{a}\,e^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(0,-\frac{4b}{a},\mathrm{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)+\left(5\,Ba^{3}+4\,Aa^{2}b+\left(8\,Ba^{2}b+Aab^{2}\right)x^{3}\right)\sqrt{bx^{3}+a}\,\sqrt{x}\,e^{\frac{3}{2}}\right)}{27\left(a^{2}b^{4}x^{7}+2\,a^{3}b^{3}x^{4}+a^{4}b^{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(((5*B*a*b^2 + 4*A*b^3)*x^7 + 2*(5*B*a^2*b + 4*A*a*b^2)*x^4 + (5*B*a^
3 + 4*A*a^2*b)*x)*sqrt(a)*e^(3/2)*weierstrassZeta(0, -4*b/a, weierstrassPIn
verse(0, -4*b/a, 1/x)) + (5*B*a^3 + 4*A*a^2*b + (8*B*a^2*b + A*a*b^2)*x^3)*
sqrt(b*x^3 + a)*sqrt(x)*e^(3/2))/(a^2*b^4*x^7 + 2*a^3*b^3*x^4 + a^4*b^2*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^(3/2)*e^(3/2)/(b*x^3 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3+A)\,{(e\,x)}^{3/2}}{{(b\,x^3+a)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(5/2),x)
[Out] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(5/2), x)
```

3.562 $\int \frac{\sqrt{ex}\,(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2(Ab-aB)(ex)^{3/2}}{9abe\,(a+bx^3)^{3/2}}+\frac{2(2Ab+aB)(ex)^{3/2}}{9a^2be\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^3+a)^(3/2)+2/9*(2*A*b+B*a)*(e*x)^(3/2)
/a^2/b/e/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {468, 270}

$$\frac{2(ex)^{3/2}(aB+2Ab)}{9a^2be\sqrt{a+bx^3}}+\frac{2(ex)^{3/2}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(2*A*b + a*B)*
(e*x)^(3/2))/(9*a^2*b*e*Sqrt[a + b*x^3])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{\sqrt{ex}\,(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{3/2}}{9abe\,(a+bx^3)^{3/2}} + \frac{\left(2\left(3Ab+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{ex}}{(a+bx^3)^{3/2}}\,dx}{9ab}$$
$$= \frac{2(Ab-aB)(ex)^{3/2}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(2Ab+aB)(ex)^{3/2}}{9a^2be\sqrt{a+bx^3}}$$

Mathematica [A]
time = 0.35, size = 44, normalized size = 0.56

$$\frac{2x\sqrt{ex}\,(3aA+2Abx^3+aBx^3)}{9a^2\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
[Out] (2*x*Sqrt[e*x]*(3*a*A + 2*A*b*x^3 + a*B*x^3))/(9*a^2*(a + b*x^3)^(3/2))
```

Maple [A]
time = 0.30, size = 39, normalized size = 0.49

method	result	size
gosper	$\frac{2x(2Ab\,x^3+Ba\,x^3+3Aa)\sqrt{ex}}{9(b\,x^3+a)^{\frac{3}{2}}\,a^2}$	39
default	$\frac{2x(2Ab\,x^3+Ba\,x^3+3Aa)\sqrt{ex}}{9(b\,x^3+a)^{\frac{3}{2}}\,a^2}$	39
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^3+a)\,ex}\,\left(\frac{2x(Ab-Ba)\sqrt{be\,x^4+aex}}{9a\,b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{2e\,x^2(2Ab+Ba)}{9b\,a^2\sqrt{\left(x^3+\frac{a}{b}\right)bex}}\right)}{ex\sqrt{b\,x^3+a}}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/9*x*(2*A*b*x^3+B*a*x^3+3*A*a)*(e*x)^(1/2)/(b*x^3+a)^(3/2)/a^2
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.68

$$\frac{2}{9}\left(\frac{Bx^{\frac{9}{2}}}{(bx^3+a)^{\frac{3}{2}}a} - \frac{A\left(b-\frac{3\,(bx^3+a)}{x^3}\right)x^{\frac{9}{2}}}{(bx^3+a)^{\frac{3}{2}}a^2}\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*(B*x^(9/2)/((b*x^3 + a)^(3/2)*a) - A*(b - 3*(b*x^3 + a)/x^3)*x^(9/2)/((
b*x^3 + a)^(3/2)*a^2))*e^(1/2)
```

Fricas [A]
time = 4.48, size = 59, normalized size = 0.75

$$\frac{2\left(\left(Ba+2\,Ab\right)x^{4}+3\,Aax\right)\sqrt{bx^{3}+a}\,\sqrt{x}\,e^{\frac{1}{2}}}{9\left(a^{2}b^{2}x^{6}+2\,a^{3}bx^{3}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*((B*a + 2*A*b)*x^4 + 3*A*a*x)*sqrt(b*x^3 + a)*sqrt(x)*e^(1/2)/(a^2*b^2*
x^6 + 2*a^3*b*x^3 + a^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(71) = 142.
time = 71.09, size = 168, normalized size = 2.13

$$A\left(\frac{6a\sqrt{e}\,x^{\frac{3}{2}}}{9a^{\frac{7}{2}}\sqrt{1+\frac{bx^{3}}{a}}+9a^{\frac{5}{2}}bx^{3}\sqrt{1+\frac{bx^{3}}{a}}}+\frac{4b\sqrt{e}\,x^{\frac{9}{2}}}{9a^{\frac{7}{2}}\sqrt{1+\frac{bx^{3}}{a}}+9a^{\frac{5}{2}}bx^{3}\sqrt{1+\frac{bx^{3}}{a}}}\right)+\frac{2B\sqrt{e}\,x^{\frac{9}{2}}}{9a^{\frac{5}{2}}\sqrt{1+\frac{bx^{3}}{a}}+9a^{\frac{3}{2}}bx^{3}\sqrt{1+\frac{bx^{3}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*(6*a*sqrt(e)*x**(3/2)/(9*a**(7/2)*sqrt(1 + b*x**3/a) + 9*a**(5/2)*b*x**3*
sqrt(1 + b*x**3/a)) + 4*b*sqrt(e)*x**(9/2)/(9*a**(7/2)*sqrt(1 + b*x**3/a) +
 9*a**(5/2)*b*x**3*sqrt(1 + b*x**3/a))) + 2*B*sqrt(e)*x**(9/2)/(9*a**(5/2)*
sqrt(1 + b*x**3/a) + 9*a**(3/2)*b*x**3*sqrt(1 + b*x**3/a))
```

Giac [A]
time = 1.60, size = 51, normalized size = 0.65

$$\frac{2\,x^{\frac{3}{2}}\left(\frac{3\,A}{a}+\frac{\left(Ba^{5}b^{5}+2\,Aa^{4}b^{6}\right)x^{3}}{a^{6}b^{5}}\right)e^{\frac{1}{2}}}{9\left(bx^{3}+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*x^(3/2)*(3*A/a + (B*a^5*b^5 + 2*A*a^4*b^6)*x^3/(a^6*b^5))*e^(1/2)/(b*x^
3 + a)^(3/2)
```

Mupad [B]
time = 4.63, size = 73, normalized size = 0.92

$$\frac{\left(\frac{2\,A\,x\,\sqrt{e\,x}}{3\,a\,b^2}+\frac{x^4\,\sqrt{e\,x}\,\left(4\,A\,b+2\,B\,a\right)}{9\,a^2\,b^2}\right)\,\sqrt{b\,x^3+a}}{x^6+\frac{a^2}{b^2}+\frac{2\,a\,x^3}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(5/2),x)
```

```
[Out] (((2*A*x*(e*x)^(1/2))/(3*a*b^2) + (x^4*(e*x)^(1/2)*(4*A*b + 2*B*a))/(9*a^2*
b^2))*(a + b*x^3)^(1/2))/(x^6 + a^2/b^2 + (2*a*x^3)/b)
```

3.563 $\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=297

$$\frac{2(Ab-aB)\sqrt{ex}}{9abe\,(a+bx^3)^{3/2}}+\frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}}+\frac{2(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\;F}{27\sqrt[4]{3}\,a^{7/3}be\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\right.}}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^3+a)^(3/2)+2/27*(8*A*b+B*a)*(e*x)^(1/2
)/a^2/b/e/(b*x^3+a)^(1/2)+2/81*(8*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+
b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/
2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(7/3)/b/e/(b*x^3+a)^(1/2)/(b^(
1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {468, 296, 335, 231}

$$\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(aB+8Ab)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}\,a^{7/3}be\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{ex}\,(aB+8Ab)}{27a^2be\sqrt{a+bx^3}}+\frac{2\sqrt{ex}\,(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[e*x])/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(8*A*b + a*B)*Sq
rt[e*x])/(27*a^2*b*e*Sqrt[a + b*x^3]) + (2*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3
)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^
(7/3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)\sqrt{ex}}{9abe\,(a+bx^3)^{3/2}} + \frac{\left(2\left(4Ab+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} + \frac{(2(8Ab+aB))\int \frac{1}{\sqrt{ex}\,\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} + \frac{(4(8Ab+aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx\right.}{27a^2be}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} + \frac{2(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{\cdots}{(\cdots}}}{27\sqrt[4]{3}\,a^{7}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 107, normalized size = 0.36

$$\frac{2x\left(-2a^2B+8Ab^2x^3+ab(11A+Bx^3)+2(8Ab+aB)\,(a+bx^3)\,\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{27a^2b\sqrt{ex}\,(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*x*(-2*a^2*B + 8*A*b^2*x^3 + a*b*(11*A + B*x^3) + 2*(8*A*b + a*B)*(a + b*
x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(
27*a^2*b*Sqrt[e*x]*(a + b*x^3)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 7077, normalized size = 23.83

method	result

elliptic	$\frac{\sqrt{(b\,x^3+a)\,ex}\left(\frac{2(Ab-Ba)\sqrt{be\,x^4+aex}}{9ea\,b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{2x(8Ab+Ba)}{27b\,a^2\sqrt{\left(x^3+\frac{a}{b}\right)bex}}+\frac{4(8Ab+Ba)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.15, size = 145, normalized size = 0.49

$$-\frac{2\left(2\left(\left(Bab^2+8\,Ab^3\right)x^6+Ba^3+8\,Aa^2b+2\left(Ba^2b+8\,Aab^2\right)x^3\right)\sqrt{a}\ \mathrm{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)+\left(2\,Ba^3-11\,Aa^2b-\left(Ba^2b+8\,Aab^2\right)x^3\right)\sqrt{bx^3+a}\,\sqrt{x}\right)e^{\left(-\frac{1}{2}\right)}}{27\left(a^3b^3x^6+2\,a^4b^2x^3+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
[Out] -2/27*(2*((B*a*b^2 + 8*A*b^3)*x^6 + B*a^3 + 8*A*a^2*b + 2*(B*a^2*b + 8*A*a*
b^2)*x^3)*sqrt(a)*weierstrassPInverse(0, -4*b/a, 1/x) + (2*B*a^3 - 11*A*a^2
*b - (B*a^2*b + 8*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(x))*e^(-1/2)/(a^3*b^3*
x^6 + 2*a^4*b^2*x^3 + a^5*b)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(5/2)/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*e^(-1/2)/((b*x^3 + a)^(5/2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{\sqrt{e\,x}\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(5/2)), x)
```

3.564 $\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=624

$$-\frac{2A}{ae\sqrt{ex}\,\left(a+bx^3\right)^{3/2}}-\frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}}+\frac{8\left(1+\sqrt{3}\right)(10Ab-aB)\sqrt{ex}\,\sqrt{a+\cdots}}{27a^3b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\cdots\right.}$$

```
[Out] -2/9*(10*A*b-B*a)*(e*x)^(5/2)/a^2/e^4/(b*x^3+a)^(3/2)-2*A/a/e/(b*x^3+a)^(3/
2)/(e*x)^(1/2)-8/27*(10*A*b-B*a)*(e*x)^(5/2)/a^3/e^4/(b*x^3+a)^(1/2)+8/27*(
10*A*b-B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a^3/b^(2/3)/e^2/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))-8/27*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+
b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/
2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/b^(2/3)/e^2/(b*x^3+a)^(1
/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
-4/81*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2
/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/
2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)
*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 296, 335, 314, 231, 1895}

$$-\frac{4\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(10Ab-aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}\,a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{8\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(10Ab-aB)E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\ 3^{3/4}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{8\left(1+\sqrt{3}\right)\sqrt{ex}\,\sqrt{a+bx^3}\,(10Ab-aB)}{27a^3b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{8(ex)^{5/2}(10Ab-aB)}{27a^3e^4\sqrt{a+bx^3}}-\frac{2(ex)^{5/2}(10Ab-aB)}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{2A}{ae\sqrt{ex}\,\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*(a + b*x^3)^(3/2)) - (2*(10*A*b - a*B)*(e*x)^(5/2))/(
9*a^2*e^4*(a + b*x^3)^(3/2)) - (8*(10*A*b - a*B)*(e*x)^(5/2))/(27*a^3*e^4*S
```

```
qrt[a + b*x^3]) + (8*(1 + Sqrt[3])*(10*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3]
)/(27*a^3*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) - (8*(10*A*b - a
*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3)
+ (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[
3])/4])/(9*3^(3/4)*a^(8/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x
))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*(1 - Sqrt[3
])*(10*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^(8/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1
/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{(10Ab-aB)\int \frac{(ex)^{3/2}}{\left(a+bx^3\right)^{5/2}}\,dx}{ae^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{(4(10Ab-aB))\int \frac{(ex)^{3/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{9a^2e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{(8(10$$

$$(16(1$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{\quad}{\quad}$$

$$(8(10$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} - \frac{\quad}{\quad}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{8\left(1+\right.}{27a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 85, normalized size = 0.14

$$\frac{2x\left(-5a^2A+(-10Ab+aB)x^3\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{5}{6},\frac{5}{2};\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{5a^3(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*x*(-5*a^2*A + (-10*A*b + a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(5*a^3*(e*x)^(3/2)*(a + b*x^3)^
(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.43, size = 10961, normalized size = 17.57

method	result	size
elliptic	Expression too large to display	1225
risch	Expression too large to display	3336
default	Expression too large to display	10961

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 156, normalized size = 0.25

$$-\frac{2\left(4\left((Bab^2-10\,Ab^3)x^7+2\,(Ba^2b-10\,Aab^2)x^4+(Ba^3-10\,Aa^2b)x\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(0,-\frac{4b}{a},\mathrm{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)+\left(4\,Ba^3-13\,Aa^2b+(Ba^2b-10\,Aab^2)x^3\right)\sqrt{bx^3+a}\,\sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{27\,(a^3b^3x^7+2\,a^4b^2x^4+a^5bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(4*((B*a*b^2 - 10*A*b^3)*x^7 + 2*(B*a^2*b - 10*A*a*b^2)*x^4 + (B*a^3
- 10*A*a^2*b)*x)*sqrt(a)*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0,
-4*b/a, 1/x)) + (4*B*a^3 - 13*A*a^2*b + (B*a^2*b - 10*A*a*b^2)*x^3)*sqrt(b*
x^3 + a)*sqrt(x))*e^(-3/2)/(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*e^(-3/2)/((b*x^3 + a)^(5/2)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x)
[Out] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)), x)
```

3.565 $\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{2A}{3ae(ex)^{3/2}(a+bx^3)^{3/2}}-\frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4(a+bx^3)^{3/2}}-\frac{4(4Ab-aB)(ex)^{3/2}}{9a^3e^4\sqrt{a+bx^3}}$$

```
[Out] -2/3*A/a/e/(e*x)^(3/2)/(b*x^3+a)^(3/2)-2/9*(4*A*b-B*a)*(e*x)^(3/2)/a^2/e^4/
(b*x^3+a)^(3/2)-4/9*(4*A*b-B*a)*(e*x)^(3/2)/a^3/e^4/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {464, 279, 270}

$$-\frac{4(ex)^{3/2}(4Ab-aB)}{9a^3e^4\sqrt{a+bx^3}}-\frac{2(ex)^{3/2}(4Ab-aB)}{9a^2e^4(a+bx^3)^{3/2}}-\frac{2A}{3ae(ex)^{3/2}(a+bx^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^3)^(3/2)) - (2*(4*A*b - a*B)*(e*x)^(3/2)
)/(9*a^2*e^4*(a + b*x^3)^(3/2)) - (4*(4*A*b - a*B)*(e*x)^(3/2))/(9*a^3*e^4*
Sqrt[a + b*x^3])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx=-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{(4Ab-aB)\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{5/2}}\,dx}{ae^3}$$

$$=-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{(2(4Ab-aB))\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{3/2}}\,dx}{3a^2e^3}$$

$$=-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{4(4Ab-aB)(ex)^{3/2}}{9a^3e^4\sqrt{a+bx^3}}$$

Mathematica [A]
time = 0.35, size = 67, normalized size = 0.64

$$\frac{2x\left(-3a^2A-12aAbx^3+3a^2Bx^3-8Ab^2x^6+2abBx^6\right)}{9a^3(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*x*(-3*a^2*A - 12*a*A*b*x^3 + 3*a^2*B*x^3 - 8*A*b^2*x^6 + 2*a*b*B*x^6))/(
9*a^3*(e*x)^(5/2)*(a + b*x^3)^(3/2))
```

Maple [A]
time = 0.35, size = 67, normalized size = 0.64

method	result	size
gosper	$-\frac{2x\left(8A\,b^{2}x^{6}-2Bab\,x^{6}+12aAb\,x^{3}-3a^{2}B\,x^{3}+3a^{2}A\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{3}\left(ex\right)^{\frac{5}{2}}}$	62
default	$-\frac{2\left(8A\,b^{2}x^{6}-2Bab\,x^{6}+12aAb\,x^{3}-3a^{2}B\,x^{3}+3a^{2}A\right)}{9\sqrt{ex}\;e^{2}a^{3}\left(b\,x^{3}+a\right)^{\frac{3}{2}}x}$	67
risch	$-\frac{2A\sqrt{b\,x^{3}+a}}{3a^{3}x\,e^{2}\sqrt{ex}}-\frac{2\left(5A\,b^{2}x^{3}-2Bab\,x^{3}+6abA-3a^{2}B\right)x^{2}}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{3}e^{2}\sqrt{ex}}$	82
elliptic	$\frac{\sqrt{\left(b\,x^{3}+a\right)ex}\,\left(-\frac{2x(Ab-Ba)\sqrt{be\,x^{4}+aex}}{9e^{3}a^{2}b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2x^{2}(5Ab-2Ba)}{9e^{2}a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{be\,x^{4}+aex}}{3e^{3}a^{3}x^{2}}\right)}{\sqrt{ex}\;\sqrt{b\,x^{3}+a}}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(8*A*b^2*x^6-2*B*a*b*x^6+12*A*a*b*x^3-3*B*a^2*x^3+3*A*a^2)/(e*x)^(1/2)
/e^2/a^3/(b*x^3+a)^(3/2)/x
```

Maxima [A]
time = 0.29, size = 90, normalized size = 0.87

$$-\frac{2}{9}\left(\frac{B\left(b-\frac{3\,(bx^3+a)}{x^3}\right)x^{\frac{9}{2}}}{(bx^3+a)^{\frac{3}{2}}a^2}-A\left(\frac{\left(b^2-\frac{6\,(bx^3+a)b}{x^3}\right)x^{\frac{9}{2}}}{(bx^3+a)^{\frac{3}{2}}a^3}-\frac{3\,\sqrt{bx^3+a}}{a^3x^{\frac{3}{2}}}\right)\right)e^{\left(-\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/9*(B*(b - 3*(b*x^3 + a)/x^3)*x^(9/2)/((b*x^3 + a)^(3/2)*a^2) - A*((b^2 -
 6*(b*x^3 + a)*b/x^3)*x^(9/2)/((b*x^3 + a)^(3/2)*a^3) - 3*sqrt(b*x^3 + a)/(
a^3*x^(3/2))))*e^(-5/2)
```

Fricas [A]
time = 2.79, size = 84, normalized size = 0.81

$$\frac{2\,(2\,(Bab-4\,Ab^2)x^6+3\,(Ba^2-4\,Aab)x^3-3\,Aa^2)\sqrt{bx^3+a}\,\sqrt{x}\,e^{\left(-\frac{5}{2}\right)}}{9\,(a^3b^2x^8+2\,a^4bx^5+a^5x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(2*(B*a*b - 4*A*b^2)*x^6 + 3*(B*a^2 - 4*A*a*b)*x^3 - 3*A*a^2)*sqrt(b*x^
3 + a)*sqrt(x)*e^(-5/2)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*e^(-5/2)/((b*x^3 + a)^(5/2)*x^(5/2)), x)
```

Mupad [B]
time = 4.80, size = 115, normalized size = 1.11

$$-\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,A}{3\,a\,b^2\,e^2}-\frac{x^3\,\left(6\,B\,a^2-24\,A\,a\,b\right)}{9\,a^3\,b^2\,e^2}+\frac{x^6\,\left(16\,A\,b^2-4\,B\,a\,b\right)}{9\,a^3\,b^2\,e^2}\right)}{x^7\,\sqrt{e\,x}+\frac{a^2\,x\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^4\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x)
```

```
[Out] -((a + b*x^3)^(1/2)*((2*A)/(3*a*b^2*e^2) - (x^3*(6*B*a^2 - 24*A*a*b))/(9*a^
3*b^2*e^2) + (x^6*(16*A*b^2 - 4*B*a*b))/(9*a^3*b^2*e^2)))/(x^7*(e*x)^(1/2)
+ (a^2*x*(e*x)^(1/2))/b^2 + (2*a*x^4*(e*x)^(1/2))/b)
```

3.566 $\int \frac{A+Bx^3}{(ex)^{7/2}(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=320

$$-\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}-\frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}}-\frac{16(14Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{\ }\cdots\right.}{13\cdots}$$

```
[Out] -2/5*A/a/e/(e*x)^(5/2)/(b*x^3+a)^(3/2)-2/45*(14*A*b-5*B*a)*(e*x)^(1/2)/a^2/
e^4/(b*x^3+a)^(3/2)-16/135*(14*A*b-5*B*a)*(e*x)^(1/2)/a^3/e^4/(b*x^3+a)^(1/
2)-16/405*(14*A*b-5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e
*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2)*3^(3/4)/a^(10/3)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b
^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {464, 296, 335, 231}

$$-\frac{16\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}a^{10/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{16\sqrt{ex}(14Ab-5aB)}{135a^3e^4\sqrt{a+bx^3}}-\frac{2\sqrt{ex}(14Ab-5aB)}{45a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*(a + b*x^3)^(3/2)) - (2*(14*A*b - 5*a*B)*Sqrt[e*x
])/(45*a^2*e^4*(a + b*x^3)^(3/2)) - (16*(14*A*b - 5*a*B)*Sqrt[e*x])/(135*a^
3*e^4*Sqrt[a + b*x^3]) - (16*(14*A*b - 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(135*3^(1/4)*a^(10/3)*e^4
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{(14Ab-5aB)\int \frac{1}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx}{5ae^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{(8(14Ab-5aB))\int \frac{1}{\sqrt{ex}\,(\ldots}}{45a^2e^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \ldots$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \ldots$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 121, normalized size = 0.38

$$\frac{x\left(-224Ab^2x^6+a^2(-54A+110Bx^3)+a(-308Abx^3+80bBx^6)+32(-14Ab+5aB)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{135a^3(ex)^{7/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (x*(-224*A*b^2*x^6 + a^2*(-54*A + 110*B*x^3) + a*(-308*A*b*x^3 + 80*b*B*x^6
) + 32*(-14*A*b + 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(135*a^3*(e*x)^(7/2)*(a + b*x^3)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 7299, normalized size = 22.81

method	result

elliptic	$\frac{\sqrt{(b x^3+a) e x}\left(-\frac{2(Ab-Ba)\sqrt{be\, x^4+aex}}{9e^4a^2b^2\left(x^3+\frac{a}{b}\right)^2}-\frac{2x(17Ab-8Ba)}{27e^3a^3\sqrt{\left(x^3+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{be\, x^4+aex}}{5e^4a^3x^3}+\frac{2\left(-\frac{2(17Ab-8Ba)}{27a^3e^3}-\ldots\right.}{\ldots}\right.}{\ldots}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^(7/2)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 167, normalized size = 0.52

$$-\frac{2\left(16\left((5\,Bab^2-14\,Ab^3)x^9+2\,(5\,Ba^2b-14\,Aab^2)x^6+(5\,Ba^3-14\,Aa^2b)x^3\right)\sqrt{a}\;\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)-\left(8\,(5\,Ba^2b-14\,Aab^2)x^6-27\,Aa^3+11\,(5\,Ba^3-14\,Aa^2b)x^3\right)\sqrt{bx^3+a}\,\sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{135\,(a^4b^2x^9+2\,a^5bx^6+a^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/135*(16*((5*B*a*b^2 - 14*A*b^3)*x^9 + 2*(5*B*a^2*b - 14*A*a*b^2)*x^6 + (
5*B*a^3 - 14*A*a^2*b)*x^3)*sqrt(a)*weierstrassPInverse(0, -4*b/a, 1/x) - (8
*(5*B*a^2*b - 14*A*a*b^2)*x^6 - 27*A*a^3 + 11*(5*B*a^3 - 14*A*a^2*b)*x^3)*s
qrt(b*x^3 + a)*sqrt(x))*e^(-7/2)/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*e^(-7/2)/((b*x^3 + a)^(5/2)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)), x)
```

3.567 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=220

$$-\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2(a+bx^3)^{4/3}}{4b^4d}+\frac{a(a+bx^3)^{7/3}}{7b^4d}-\frac{(a+bx^3)^{10/3}}{10b^4d}+\frac{\sqrt[3]{2}\,a^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^4d}+\frac{a^{10/3}}{}$$

```
[Out] -a^3*(b*x^3+a)^(1/3)/b^4/d-1/4*a^2*(b*x^3+a)^(4/3)/b^4/d+1/7*a*(b*x^3+a)^(7
/3)/b^4/d-1/10*(b*x^3+a)^(10/3)/b^4/d+1/6*a^(10/3)*ln(-b*x^3+a)*2^(1/3)/b^4
/d-1/2*a^(10/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b^4/d+1/3*2^(1/
3)*a^(10/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/b
^4/d*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 90, 52, 59, 631, 210, 31}

$$\frac{\sqrt[3]{2}\,a^{10/3}\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^4d}+\frac{a^{10/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^4d}-\frac{a^{10/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}-\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2(a+bx^3)^{4/3}}{4b^4d}+\frac{a(a+bx^3)^{7/3}}{7b^4d}-\frac{(a+bx^3)^{10/3}}{10b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a^3*(a + b*x^3)^(1/3))/(b^4*d)) - (a^2*(a + b*x^3)^(4/3))/(4*b^4*d) + (a
*(a + b*x^3)^(7/3))/(7*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) + (2^(1/3)*a^
(10/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^4*d) + (a^(10/3)*Log[a - b*x^3])/(3*2^(2/3)*b^4*d) - (a^(10/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2\sqrt[3]{a+bx}}{b^3d} + \frac{a(a+bx)^{4/3}}{b^3d} - \frac{(a+bx)^{7/3}}{b^3d} + \frac{a^3\sqrt[3]{a+bx}}{b^3(ad-bdx)}\right)\,dx, x, x^3\right.$$

$$= -\frac{a^2(a+bx^3)^{4/3}}{4b^4d} + \frac{a(a+bx^3)^{7/3}}{7b^4d} - \frac{(a+bx^3)^{10/3}}{10b^4d} + \frac{a^3\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b^3}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2(a+bx^3)^{4/3}}{4b^4d} + \frac{a(a+bx^3)^{7/3}}{7b^4d} - \frac{(a+bx^3)^{10/3}}{10b^4d} + \frac{(2a^4)\,\text{Subst}\left(\int \frac{}{(a+}\right.}{}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2(a+bx^3)^{4/3}}{4b^4d} + \frac{a(a+bx^3)^{7/3}}{7b^4d} - \frac{(a+bx^3)^{10/3}}{10b^4d} + \frac{a^{10/3}\log\left(a-bx^3\right.}{3\ 2^{2/3}b^4d}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2(a+bx^3)^{4/3}}{4b^4d} + \frac{a(a+bx^3)^{7/3}}{7b^4d} - \frac{(a+bx^3)^{10/3}}{10b^4d} + \frac{a^{10/3}\log\left(a-bx^3\right.}{3\ 2^{2/3}b^4d}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2(a+bx^3)^{4/3}}{4b^4d} + \frac{a(a+bx^3)^{7/3}}{7b^4d} - \frac{(a+bx^3)^{10/3}}{10b^4d} + \frac{\sqrt[3]{2}\,a^{10/3}\tan^{-1}\left(\right.}{\sqrt{3}}$$

Mathematica [A]
time = 0.23, size = 202, normalized size = 0.92

$$\frac{-3\sqrt[3]{a+bx^3}\left(169a^3+37a^2bx^3+22ab^2x^6+14b^3x^9\right)+140\sqrt[3]{2}\sqrt{3}\,a^{10/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-140\sqrt[3]{2}\,a^{10/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)+70\sqrt[3]{2}\,a^{10/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{420b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-3*(a + b*x^3)^(1/3)*(169*a^3 + 37*a^2*b*x^3 + 22*a*b^2*x^6 + 14*b^3*x^9)
+ 140*2^(1/3)*Sqrt[3]*a^(10/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1
/3))/Sqrt[3]] - 140*2^(1/3)*a^(10/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(
1/3)] + 70*2^(1/3)*a^(10/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/
3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(420*b^4*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{11}\left(b\,x^3+a\right)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [A]
time = 0.51, size = 183, normalized size = 0.83

$$\frac{\frac{140\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{10}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}+\frac{70\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{140\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}-\frac{3\left(14\left(bx^3+a\right)^{\frac{10}{3}}-20\left(bx^3+a\right)^{\frac{7}{3}}a+35\left(bx^3+a\right)^{\frac{4}{3}}a^2+140\left(bx^3+a\right)^{\frac{1}{3}}a^3\right)}{d}}{420\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/420*(140*sqrt(3)*2^(1/3)*a^(10/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(
1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 70*2^(1/3)*a^(10/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 140*2^(1
/3)*a^(10/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*(14*(b*x^3 + a
)^(10/3) - 20*(b*x^3 + a)^(7/3)*a + 35*(b*x^3 + a)^(4/3)*a^2 + 140*(b*x^3 +
 a)^(1/3)*a^3)/d)/b^4
```

Fricas [A]
time = 3.16, size = 186, normalized size = 0.85

$$-\frac{140\sqrt{3}\,2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}\,a}{3a}\right)+70\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)-140\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\log\left(2^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right)+3\left(14b^3x^9+22ab^2x^6+37a^2bx^3+169a^3\right)(bx^3+a)^{\frac{1}{3}}}{420\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/420*(140*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^3*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x
^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 70*2^(1/3)*(-a)^(1/3)*a^3*log(2^
(2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)
) - 140*2^(1/3)*(-a)^(1/3)*a^3*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3))
+ 3*(14*b^3*x^9 + 22*a*b^2*x^6 + 37*a^2*b*x^3 + 169*a^3)*(b*x^3 + a)^(1/3))
/(b^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{11}\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**11*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 4.81, size = 240, normalized size = 1.09

$$\frac{a\,{\left(b\,x^3+a\right)}^{7/3}}{7\,b^4\,d}-\frac{a^3\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d}-\frac{a^2\,{\left(b\,x^3+a\right)}^{4/3}}{4\,b^4\,d}-\frac{{\left(b\,x^3+a\right)}^{10/3}}{10\,b^4\,d}-\frac{2^{1/3}\,a^{10/3}\,\ln\left({\left(b\,x^3+a\right)}^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^4\,d}-\frac{2^{1/3}\,a^{10/3}\,\ln\left(\frac{6\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d}-\frac{6\,2^{1/3}\,a^{13/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^4\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^4\,d}+\frac{2^{1/3}\,a^{10/3}\,\ln\left(\frac{6\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d}+\frac{18\,2^{1/3}\,a^{13/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^4\,d}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (a*(a + b*x^3)^(7/3))/(7*b^4*d) - (a^3*(a + b*x^3)^(1/3))/(b^4*d) - (a^2*(a
 + b*x^3)^(4/3))/(4*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) - (2^(1/3)*a^(10
/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b^4*d) - (2^(1/3)*a^(10/3)
*log((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) - (6*2^(1/3)*a^(13/3)*((3^(1/2)*1i)/
2 - 1/2))/(b^4*d))*((3^(1/2)*1i)/2 - 1/2))/(3*b^4*d) + (2^(1/3)*a^(10/3)*lo
g((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) + (18*2^(1/3)*a^(13/3)*((3^(1/2)*1i)/6
+ 1/6))/(b^4*d))*((3^(1/2)*1i)/6 + 1/6))/(b^4*d)
```

3.568 $\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=174

$$-\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{(a+bx^3)^{7/3}}{7b^3d}+\frac{\sqrt[3]{2}\,a^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}+\frac{a^{7/3}\log(a-bx^3)}{3\,2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}\right.}{2^{2/3}b}$$

```
[Out] -a^2*(b*x^3+a)^(1/3)/b^3/d-1/7*(b*x^3+a)^(7/3)/b^3/d+1/6*a^(7/3)*ln(-b*x^3+
a)*2^(1/3)/b^3/d-1/2*a^(7/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b^
3/d+1/3*2^(1/3)*a^(7/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3
)*3^(1/2))/b^3/d*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 90, 52, 59, 631, 210, 31}

$$\frac{\sqrt[3]{2}\,a^{7/3}\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}+\frac{a^{7/3}\log(a-bx^3)}{3\,2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}-\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{(a+bx^3)^{7/3}}{7b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a^2*(a + b*x^3)^(1/3))/(b^3*d)) - (a + b*x^3)^(7/3)/(7*b^3*d) + (2^(1/3)
*a^(7/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^3*d) + (a^(7/3)*Log[a - b*x^3])/(3*2^(2/3)*b^3*d) - (a^(7/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{(a+bx)^{4/3}}{b^2d}+\frac{a^2\sqrt[3]{a+bx}}{b^2(ad-bdx)}\right)\,dx, x, x^3\right)\\
&= -\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{a^2\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b^2}\\
&= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3b^2}\\
&= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{a^{7/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^3d}+\frac{a^{7/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \cdots\right.}{2^{2/3}b^3d}\\
&= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{a^{7/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right.}{2^{2/3}b^3d}\\
&= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{\sqrt[3]{2}\,a^{7/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3d}+\frac{a^{7/3}\log(a-\cdots}{3\ 2^{2/3}b^3d}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 210, normalized size = 1.21

$$
-\frac{48a^2\sqrt[3]{a+bx^3}+12abx^3\sqrt[3]{a+bx^3}+6b^2x^6\sqrt[3]{a+bx^3}-14\sqrt[3]{2}\sqrt{3}\,a^{7/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+14\sqrt[3]{2}\,a^{7/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)-7\sqrt[3]{2}\,a^{7/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{42b^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/42*(48*a^2*(a + b*x^3)^(1/3) + 12*a*b*x^3*(a + b*x^3)^(1/3) + 6*b^2*x^6*
(a + b*x^3)^(1/3) - 14*2^(1/3)*Sqrt[3]*a^(7/3)*ArcTan[(1 + (2^(2/3)*(a + b*
x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 14*2^(1/3)*a^(7/3)*Log[-2*a^(1/3) + 2^(2/3)
*(a + b*x^3)^(1/3)] - 7*2^(1/3)*a^(7/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a
+ b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^3*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{x^8\left(b\,x^3+a\right)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [A]
time = 0.51, size = 155, normalized size = 0.89

$$\frac{\frac{14\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{7}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}+\frac{7\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{14\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}-\frac{6\left(\left(bx^3+a\right)^{\frac{7}{3}}+7\left(bx^3+a\right)^{\frac{1}{3}}a^2\right)}{d}}{42\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/42*(14*sqrt(3)*2^(1/3)*a^(7/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 7*2^(1/3)*a^(7/3)*log(2^(2/3)*a^(2/3)
 + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 14*2^(1/3)*a^
(7/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*((b*x^3 + a)^(7/3) +
7*(b*x^3 + a)^(1/3)*a^2)/d)/b^3
```

Fricas [A]
time = 1.98, size = 174, normalized size = 1.00

$$-\frac{14\sqrt{3}\,2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^2\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}\,a}{3a}\right)+7\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^2\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)-14\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^2\log\left(2^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right)+6\,(b^2x^6+2\,abx^3+8\,a^2)(bx^3+a)^{\frac{1}{3}}}{42\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/42*(14*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^2*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3
 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 7*2^(1/3)*(-a)^(1/3)*a^2*log(2^(2/
3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) -
 14*2^(1/3)*(-a)^(1/3)*a^2*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*
(b^2*x^6 + 2*a*b*x^3 + 8*a^2)*(b*x^3 + a)^(1/3))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^8\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**8*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 4.65, size = 219, normalized size = 1.26

$$\frac{2^{1/3}\,{\left(-a\right)}^{7/3}\,\ln\left(6\,a^3\,{\left(b\,x^3+a\right)}^{1/3}-6\,2^{1/3}\,{\left(-a\right)}^{10/3}\right)}{3\,b^3\,d}-\frac{a^2\,{\left(b\,x^3+a\right)}^{1/3}}{b^3\,d}-\frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^3\,d}-\frac{2^{1/3}\,{\left(-a\right)}^{7/3}\,\ln\left(\frac{6\,a^3\,{\left(b\,x^3+a\right)}^{1/3}}{b^3\,d}+\frac{6\,2^{1/3}\,{\left(-a\right)}^{10/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^3\,d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^3\,d}+\frac{2^{1/3}\,{\left(-a\right)}^{7/3}\,\ln\left(\frac{6\,a^3\,{\left(b\,x^3+a\right)}^{1/3}}{b^3\,d}-\frac{18\,2^{1/3}\,{\left(-a\right)}^{10/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^3\,d}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (2^(1/3)*(-a)^(7/3)*log(6*a^3*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(10/3)))/(
3*b^3*d) - (a^2*(a + b*x^3)^(1/3))/(b^3*d) - (a + b*x^3)^(7/3)/(7*b^3*d) -
(2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) + (6*2^(1/3)*(-a)
^(10/3)*((3^(1/2)*1i)/2 + 1/2))/(b^3*d))*((3^(1/2)*1i)/2 + 1/2))/(3*b^3*d)
+ (2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) - (18*2^(1/3)*(
-a)^(10/3)*((3^(1/2)*1i)/6 - 1/6))/(b^3*d))*((3^(1/2)*1i)/6 - 1/6))/(b^3*d)
```

3.569 $\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=172

$$-\frac{a\sqrt[3]{a+bx^3}}{b^2d}-\frac{(a+bx^3)^{4/3}}{4b^2d}+\frac{\sqrt[3]{2}\,a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^2d}+\frac{a^{4/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^2d}-\frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}\right. -}{2^{2/3}b^2d}$$

```
[Out] -a*(b*x^3+a)^(1/3)/b^2/d-1/4*(b*x^3+a)^(4/3)/b^2/d+1/6*a^(4/3)*ln(-b*x^3+a)
*2^(1/3)/b^2/d-1/2*a^(4/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b^2/
d+1/3*2^(1/3)*a^(4/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*
3^(1/2))/b^2/d*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 81, 52, 59, 631, 210, 31}

$$\frac{\sqrt[3]{2}\,a^{4/3}\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^2d}+\frac{a^{4/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^2d}-\frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d}-\frac{a\sqrt[3]{a+bx^3}}{b^2d}-\frac{(a+bx^3)^{4/3}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3))/(b^2*d)) - (a + b*x^3)^(4/3)/(4*b^2*d) + (2^(1/3)*a
^(4/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^2*d) + (a^(4/3)*Log[a - b*x^3])/(3*2^(2/3)*b^2*d) - (a^(4/3)*Log[2^
(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt[3]{a+bx}}{ad-bdx}\,dx,x,x^3\right)$$
$$= -\frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{a\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx,x,x^3\right)}{3b}$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx,x,x^3\right)}{3b}$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{a^{4/3}\log(a-bx^3)}{3\,2^{2/3}b^2d} + \frac{a^{4/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a\cdots}\right.}{2^{2/3}b^2d}$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{a^{4/3}\log(a-bx^3)}{3\,2^{2/3}b^2d} - \frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d} - \cdots$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{\sqrt[3]{2}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^2d} + \frac{a^{4/3}\log(a-bx^3\cdots}{3\,2^{2/3}b^2d}$$

Mathematica [A]
time = 0.15, size = 188, normalized size = 1.09

$$-\frac{15a\sqrt[3]{a+bx^3}+3bx^3\sqrt[3]{a+bx^3}-4\sqrt[3]{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{2}\,a^{4/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)-2\sqrt[3]{2}\,a^{4/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/12*(15*a*(a + b*x^3)^(1/3) + 3*b*x^3*(a + b*x^3)^(1/3) - 4*2^(1/3)*Sqrt[
3]*a^(4/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 4*2^
(1/3)*a^(4/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 2*2^(1/3)*a^(4/
3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^
(2/3)])/(b^2*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^5(b\,x^3+a)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [A]
time = 0.52, size = 153, normalized size = 0.89

$$\frac{\frac{4\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}+\frac{2\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{4\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}-\frac{3\left(\left(bx^3+a\right)^{\frac{4}{3}}+4\left(bx^3+a\right)^{\frac{1}{3}}a\right)}{d}}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/12*(4*sqrt(3)*2^(1/3)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2*2^(1/3)*a^(4/3)*log(2^(2/3)*a^(2/3)
+ 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 4*2^(1/3)*a^(4
/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*((b*x^3 + a)^(4/3) + 4*
(b*x^3 + a)^(1/3)*a)/d)/b^2
```

Fricas [A]
time = 4.50, size = 157, normalized size = 0.91

$$-\frac{4\sqrt{3}\,2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\,a\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}\,a}{3a}\right)+2\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\,a\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)-4\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\,a\log\left(2^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right)+3\,(bx^3+5\,a)(bx^3+a)^{\frac{1}{3}}}{12\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*2^(1/3)*(-a)^(1/3)*a*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 +
a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2*2^(1/3)*(-a)^(1/3)*a*log(2^(2/3)*(-
a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 4*2^
(1/3)*(-a)^(1/3)*a*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 3*(b*x^3 +
 5*a)*(b*x^3 + a)^(1/3))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^5\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**5*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 4.66, size = 200, normalized size = 1.16

$$-\frac{\left(b\,x^3+a\right)^{4/3}}{4\,b^2\,d}-\frac{a\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d}-\frac{2^{1/3}\,a^{4/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^2\,d}-\frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{6\,a^2\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d}-\frac{6\,2^{1/3}\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^2\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^2\,d}+\frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{6\,a^2\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d}+\frac{18\,2^{1/3}\,a^{7/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^2\,d}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b^2*d) + (18*2^(1/3)*a^(7/3
)*((3^(1/2)*1i)/6 + 1/6))/(b^2*d))*((3^(1/2)*1i)/6 + 1/6))/(b^2*d) - (a*(a
+ b*x^3)^(1/3))/(b^2*d) - (2^(1/3)*a^(4/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*
a^(1/3)))/(3*b^2*d) - (2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b^2*d
) - (6*2^(1/3)*a^(7/3)*((3^(1/2)*1i)/2 - 1/2))/(b^2*d))*((3^(1/2)*1i)/2 - 1
/2))/(3*b^2*d) - (a + b*x^3)^(4/3)/(4*b^2*d)
```

3.570 $\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}+\frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\ 2^{2/3}bd}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}$$

```
[Out] -(b*x^3+a)^(1/3)/b/d+1/6*a^(1/3)*ln(-b*x^3+a)*2^(1/3)/b/d-1/2*a^(1/3)*ln(2^
(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b/d+1/3*2^(1/3)*a^(1/3)*arctan(1/3*(
a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/b/d*3^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {455, 52, 59, 631, 210, 31}

$$\frac{\sqrt[3]{2}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}-\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\ 2^{2/3}bd}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(b*d)) + (2^(1/3)*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(1/3)*Log[a - b*x^3]
)/(3*2^(2/3)*b*d) - (a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(
2/3)*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{1}{3}(2a)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{a}\,\log\left(a-bx^3\right)}{3\,2^{2/3}bd}+\frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\,\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}+\frac{a^{2/3}\cdots}{\cdots}\\
&= -\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{a}\,\log\left(a-bx^3\right)}{3\,2^{2/3}bd}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}-\frac{\left(\sqrt[3]{2}\,\sqrt[3]{a}\right)\text{Su}\cdots}{\cdots}\\
&= -\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{2}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,bd}+\frac{\sqrt[3]{a}\,\log\left(a-bx^3\right)}{3\,2^{2/3}bd}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{\cdots}\right.}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 169, normalized size = 1.13

$$\frac{-6\sqrt[3]{a+bx^3}+2\sqrt[3]{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{2}\sqrt[3]{a}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)+\sqrt[3]{2}\sqrt[3]{a}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2^(2/3)*(a +
 b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(1/3)*a^(1/3)*Log[-2*a^(1/3) + 2^(2/
3)*(a + b*x^3)^(1/3)] + 2^(1/3)*a^(1/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a
+ b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(6*b*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(b\,x^3+a\right)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [A]
time = 0.53, size = 139, normalized size = 0.93

$$\frac{\frac{2\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}+\frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{2\cdot 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}-\frac{6\left(bx^3+a\right)^{\frac{1}{3}}}{d}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
+ 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2^(1/3)*a^(1/3)*log(2^(2/3)*a^(2/3) + 2
^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 2*2^(1/3)*a^(1/3)
*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*(b*x^3 + a)^(1/3)/d)/b
```

Fricas [A]
time = 2.48, size = 144, normalized size = 0.96

$$-\frac{2\sqrt{3}\,2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-a)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)+6\left(bx^3+a\right)^{\frac{1}{3}}}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*2^(1/3)*(-a)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 + a)^
(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2^(1/3)*(-a)^(1/3)*log(2^(2/3)*(-a)^(2/3
) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 2*2^(1/3)*(
-a)^(1/3)*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*(b*x^3 + a)^(1/3)
)/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2 \sqrt[3]{a + bx^3}}{-a+bx^3}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 4.64, size = 194, normalized size = 1.29

$$\frac{2^{1/3}\,{\left(-a\right)}^{1/3}\,\ln\left(6\,a\,{\left(b\,x^3+a\right)}^{1/3}-6\,2^{1/3}\,{\left(-a\right)}^{4/3}\right)}{3\,b\,d}-\frac{{\left(b\,x^3+a\right)}^{1/3}}{b\,d}+\frac{2^{1/3}\,{\left(-a\right)}^{1/3}\,\ln\left(\frac{6\,a\,{\left(b\,x^3+a\right)}^{1/3}}{b\,d}-\frac{6\,2^{1/3}\,{\left(-a\right)}^{4/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b\,d}-\frac{2^{1/3}\,{\left(-a\right)}^{1/3}\,\ln\left(\frac{6\,a\,{\left(b\,x^3+a\right)}^{1/3}}{b\,d}+\frac{6\,2^{1/3}\,{\left(-a\right)}^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b\,d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (2^(1/3)*(-a)^(1/3)*log(6*a*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(4/3)))/(3*b
*d) - (a + b*x^3)^(1/3)/(b*d) + (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^(1
/3))/(b*d) - (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 - 1/2))/(b*d))*((3^(1/2)
*1i)/2 - 1/2))/(3*b*d) - (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^(1/3))/(b
*d) + (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/(b*d))*((3^(1/2)*1i)/2
+ 1/2))/(3*b*d)
```

3.571 $\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}\,dx$

Optimal. Leaf size=214

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}+\frac{\sqrt[3]{2}\,\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}-\frac{\log(x)}{2a^{2/3}d}+\frac{\log\left(a-bx^3\right)}{3\,2^{2/3}a^{2/3}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a-\cdots}\right.}{2a^{2/3}d}$$

```
[Out] -1/2*ln(x)/a^(2/3)/d+1/6*ln(-b*x^3+a)*2^(1/3)/a^(2/3)/d+1/2*ln(a^(1/3)-(b*x
^3+a)^(1/3))/a^(2/3)/d-1/2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/a^(2
/3)/d-1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)/d
*3^(1/2)+1/3*2^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3
^(1/2))/a^(2/3)/d*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {457, 85, 59, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}+\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}+\frac{\log\left(a-bx^3\right)}{3\,2^{2/3}a^{2/3}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}-\frac{\log(x)}{2a^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*d)) + (2^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(2/3)*d) - Log[x]/(2*a^(2/3)*d) + Log[a - b*x^3]/(3*2^(2/
3)*a^(2/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(2/3)*d) - Log[2^(1/3
)*a^(1/3) - (a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x(ad-bdx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}(2b)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx,x,x^3\right)+\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^3\right)}{3d}\\
&= -\frac{\log(x)}{2a^{2/3}d}+\frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-\cdots}\right.}{2^{2/3}\cdots}\\
&= -\frac{\log(x)}{2a^{2/3}d}+\frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}+\cdots\\
&= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}d}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}d}-\frac{\log(x)}{2a^{2/3}d}+\frac{\log\left(a-bx^3\cdots\right.}{3\ 2^{2/3}a^{2/3}\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 235, normalized size = 1.10

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)+2\sqrt[3]{2}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)+\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)-\sqrt[3]{2}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}(a+bx^3)^{2/3}\right)}{6a^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(
1/3)*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*
Log[-a^(1/3) + (a + b*x^3)^(1/3)] + 2*2^(1/3)*Log[-2*a^(1/3) + 2^(2/3)*(a +
 b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3
)] - 2^(1/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a
 + b*x^3)^(2/3)])/(a^(2/3)*d)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{x\,(-bd\,x^3+ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. 2(161) = 322.
time = 4.63, size = 1046, normalized size = 4.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/
3)*arctan(1/12*(1/2)^(2/3)*(sqrt(3)*a^3*d^5*sqrt(1/(a^4*d^6)) - 3*sqrt(3)*a
*d^2)*sqrt(-4*(1/2)^(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*(-(3*a^2*d^3*sqrt(1/(a^
4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 4*(1/2)^(2/3)*a^2*d^2*(-(
3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) + 4*(b*x^3 + a)^(2/3))*(-
(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) - 1/6*(1/2)^(2/3)*(sqrt(
3)*a^3*d^5*sqrt(1/(a^4*d^6)) - 3*sqrt(3)*a*d^2)*(b*x^3 + a)^(1/3)*(-(3*a^2*
d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) + 1/3*sqrt(3)) + 2/3*sqrt(3)*(1
/2)^(1/3)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*arctan(1/6*(1
/2)^(2/3)*(sqrt(3)*a^3*d^5*sqrt(1/(a^4*d^6)) + 3*sqrt(3)*a*d^2)*sqrt((1/2)^
(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3
))^(1/3)*sqrt(1/(a^4*d^6)) + (1/2)^(2/3)*a^2*d^2*((3*a^2*d^3*sqrt(1/(a^4*d^
6)) - 1)/(a^2*d^3))^(2/3) + (b*x^3 + a)^(2/3))*((3*a^2*d^3*sqrt(1/(a^4*d^6)
) - 1)/(a^2*d^3))^(2/3) - 1/6*(1/2)^(2/3)*(sqrt(3)*a^3*d^5*sqrt(1/(a^4*d^6)
) + 3*sqrt(3)*a*d^2)*(b*x^3 + a)^(1/3)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(
a^2*d^3))^(2/3) - 1/3*sqrt(3)) - 1/6*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d
^6)) + 1)/(a^2*d^3))^(1/3)*log(-4*(1/2)^(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*(-(
3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 4*(1/
2)^(2/3)*a^2*d^2*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) + 4*(
b*x^3 + a)^(2/3)) - 1/6*(1/2)^(1/3)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2
*d^3))^(1/3)*log(4*(1/2)^(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*((3*a^2*d^3*sqrt(1
/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 4*(1/2)^(2/3)*a^2*d^2
*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(2/3) + 4*(b*x^3 + a)^(2/3))
 + 1/3*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*log
((1/2)^(1/3)*a^3*d^4*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*s
qrt(1/(a^4*d^6)) + (b*x^3 + a)^(1/3)) + 1/3*(1/2)^(1/3)*((3*a^2*d^3*sqrt(1/
(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*log(-(1/2)^(1/3)*a^3*d^4*((3*a^2*d^3*sqrt(
1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + (b*x^3 + a)^(1/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a + bx^{3}}}{-ax + bx^{4}}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x + b*x**4), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 5.10, size = 345, normalized size = 1.61

$$\ln\left((b\,x^3+a)^{1/3}-a\,d\,\left(\frac{1}{a^2\,d^3}\right)^{1/3}\right)\,\left(\frac{1}{27\,a^2\,d^3}\right)^{1/3}+\ln\left((b\,x^3+a)^{1/3}+2^{1/3}\,a\,d\,\left(-\frac{1}{a^2\,d^3}\right)^{1/3}\right)\,\left(-\frac{2}{27\,a^2\,d^3}\right)^{1/3}-\ln\left(2^{1/3}\,a\,d\,\left(-\frac{1}{a^2\,d^3}\right)^{1/3}-2\,(b\,x^3+a)^{1/3}+2^{1/3}\,\sqrt{3}\,a\,d\,\left(-\frac{1}{a^2\,d^3}\right)^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{2}{27\,a^2\,d^3}\right)^{1/3}+\ln\left(2\,(b\,x^3+a)^{1/3}-2^{1/3}\,a\,d\,\left(-\frac{1}{a^2\,d^3}\right)^{1/3}+2^{1/3}\,\sqrt{3}\,a\,d\,\left(-\frac{1}{a^2\,d^3}\right)^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{2}{27\,a^2\,d^3}\right)^{1/3}+\ln\left(2\,(b\,x^3+a)^{1/3}+a\,d\,\left(\frac{1}{a^2\,d^3}\right)^{1/3}-\sqrt{3}\,a\,d\,\left(\frac{1}{a^2\,d^3}\right)^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{27\,a^2\,d^3}\right)^{1/3}-\ln\left(2\,(b\,x^3+a)^{1/3}+a\,d\,\left(\frac{1}{a^2\,d^3}\right)^{1/3}+\sqrt{3}\,a\,d\,\left(\frac{1}{a^2\,d^3}\right)^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{27\,a^2\,d^3}\right)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x)
```

```
[Out] log((a + b*x^3)^(1/3) - a*d*(1/(a^2*d^3))^(1/3))*(1/(27*a^2*d^3))^(1/3) + l
og((a + b*x^3)^(1/3) + 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3))*(-2/(27*a^2*d^3))^
(1/3) - log(2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3) - 2*(a + b*x^3)^(1/3) + 2^(1/3
)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-2/(27*a^2*d
^3))^(1/3) + log(2*(a + b*x^3)^(1/3) - 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3) + 2
^(1/3)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-2/(27*
a^2*d^3))^(1/3) + log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3) - 3^(1/
2)*a*d*(1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*d^3))^(1/3
) - log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3) + 3^(1/2)*a*d*(1/(a^2
*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*d^3))^(1/3)
```

3.572 $\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx$

Optimal. Leaf size=268

$$\frac{b\sqrt[3]{a+bx^3}}{3a^2d}-\frac{(a+bx^3)^{4/3}}{3a^2dx^3}-\frac{4b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}d}+\frac{\sqrt[3]{2}\,b\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}d}-\frac{2b\log(x)}{3a^{5/3}d}+b$$

```
[Out] 1/3*b*(b*x^3+a)^(1/3)/a^2/d-1/3*(b*x^3+a)^(4/3)/a^2/d/x^3-2/3*b*ln(x)/a^(5/
3)/d+1/6*b*ln(-b*x^3+a)*2^(1/3)/a^(5/3)/d+2/3*b*ln(a^(1/3)-(b*x^3+a)^(1/3))
/a^(5/3)/d-1/2*b*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/a^(5/3)/d-4/9*
b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)/d*3^(1/2)
+1/3*2^(1/3)*b*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2)
)/a^(5/3)/d*3^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {457, 105, 162, 52, 59, 631, 210, 31}

$$-\frac{4b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}d}+\frac{\sqrt[3]{2}\,b\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}d}+\frac{b\log(a-bx^3)}{3\,2^{2/3}a^{5/3}d}+\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d}-\frac{2b\log(x)}{3a^{5/3}d}-\frac{(a+bx^3)^{4/3}}{3a^2dx^3}+\frac{b\sqrt[3]{a+bx^3}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(1/3))/(3*a^2*d) - (a + b*x^3)^(4/3)/(3*a^2*d*x^3) - (4*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)
*d) + (2^(1/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(5/3)*d) - (2*b*Log[x])/(3*a^(5/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(2/3)*a^(5/3)*d) + (2*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(3*a^(5/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*a^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{4}{3}abd+\frac{1}{3}b^2dx\right)}{x(ad-bdx)}\,dx, x, x^3\right)}{3a^2d}$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} + \frac{b^2\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3a^2} + \frac{(4b)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right.}{9a^2d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3a} + \frac{(4b)\text{Subst}\left(\right.}{}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{2b\log(x)}{3a^{5/3}d} + \frac{b\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{5/3}d} - \frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,\right.}{3a^{5/3}d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{2b\log(x)}{3a^{5/3}d} + \frac{b\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{5/3}d} + \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right.}{3a^{5/3}d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{4b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}d} + \frac{\sqrt[3]{2}\,b\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a-}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{\sqrt{3}\,a^{5/3}d}$$

Mathematica [A]
time = 0.44, size = 282, normalized size = 1.05

$$-\frac{6a^{2/3}\sqrt[3]{a+bx^3}+8\sqrt{3}bx^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-6\sqrt[3]{2}\sqrt{3}bx^3\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-8bx^3\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)+6\sqrt[3]{2}bx^3\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)+4bx^3\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-3\sqrt[3]{2}bx^3\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{18a^{5/3}dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] -1/18*(6*a^(2/3)*(a + b*x^3)^(1/3) + 8*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*
x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 6*2^(1/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3
)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 8*b*x^3*Log[-a^(1/3) + (a + b*x^3)
^(1/3)] + 6*2^(1/3)*b*x^3*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] + 4*b
*x^3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 3*2^(1/
3)*b*x^3*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b
*x^3)^(2/3)])/(a^(5/3)*d*x^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{4} \left(-bd\,x^{3}+ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^4), x)
```

Fricas [A]
time = 2.44, size = 321, normalized size = 1.20

$$-\frac{6\,\sqrt{3}\,2^{\frac{1}{3}}a^{2}bx^{3}\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\,2^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^{2}}\right)^{\frac{2}{3}}+\frac{1}{3}\,\sqrt{3}\right)+3\cdot 2^{\frac{1}{3}}a^{2}bx^{3}\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{2}\left(-\frac{1}{a^{2}}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{2}{3}}\right)-6\cdot 2^{\frac{1}{3}}a^{2}bx^{3}\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}a\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\right)+8\,\sqrt{3}\left(a^{2}\right)^{\frac{1}{6}}abx^{3}\arctan\left(\frac{\left(a^{2}\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}}a+2\,\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(a^{2}\right)^{\frac{2}{3}}\right)}{3\,a^{2}}\right)+4\left(a^{2}\right)^{\frac{2}{3}}bx^{3}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}a+\left(a^{2}\right)^{\frac{1}{3}}a+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(a^{2}\right)^{\frac{2}{3}}\right)-8\left(a^{2}\right)^{\frac{2}{3}}bx^{3}\log\left(\left(bx^{3}+a\right)^{\frac{1}{3}}a-\left(a^{2}\right)^{\frac{2}{3}}\right)+6\left(bx^{3}+a\right)^{\frac{1}{3}}a^{2}}{18\,a^{3}dx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] -1/18*(6*sqrt(3)*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(2/3
)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 3*2^(1/3)*a^2*b*x^3*(
-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*a*
(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 6*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*lo
g(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 8*sqrt(3)*(a^2)^(1/6)*a*b
*x^3*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^
(1/3)*(a^2)^(2/3))/a^2) + 4*(a^2)^(2/3)*b*x^3*log((b*x^3 + a)^(2/3)*a + (a^
2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 8*(a^2)^(2/3)*b*x^3*log((b*x^
3 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2)/(a^3*d*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a + bx^{3}}}{-ax^{4}+bx^{7}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**4/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**4 + b*x**7), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 5.35, size = 455, normalized size = 1.70

$$\frac{4\ln\left(b\,{\left(b\,x^3+a\right)}^{1/3}-a^2\,d\,{\left(\frac{b^3}{a^5\,d^3}\right)}^{1/3}\right)\,{\left(\frac{b^3}{a^5\,d^3}\right)}^{1/3}}{9}+\ln\left(b\,{\left(b\,x^3+a\right)}^{1/3}+2^{1/3}\,a^2\,d\,{\left(-\frac{b^3}{a^5\,d^3}\right)}^{1/3}\right)\,{\left(-\frac{2\,b^3}{27\,a^5\,d^3}\right)}^{1/3}+\ln\left(2\,b\,{\left(b\,x^3+a\right)}^{1/3}+a^2\,d\,{\left(\frac{b^3}{a^5\,d^3}\right)}^{1/3}-\sqrt{3}\,a^2\,d\,{\left(\frac{b^3}{a^5\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{64\,b^3}{729\,a^5\,d^3}\right)}^{1/3}-\ln\left(2\,b\,{\left(b\,x^3+a\right)}^{1/3}+a^2\,d\,{\left(\frac{b^3}{a^5\,d^3}\right)}^{1/3}+\sqrt{3}\,a^2\,d\,{\left(\frac{b^3}{a^5\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{64\,b^3}{729\,a^5\,d^3}\right)}^{1/3}-\ln\left(2^{1/3}\,a^2\,d\,{\left(-\frac{b^3}{a^5\,d^3}\right)}^{1/3}-2\,b\,{\left(b\,x^3+a\right)}^{1/3}+2^{1/3}\,\sqrt{3}\,a^2\,d\,{\left(-\frac{b^3}{a^5\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{2\,b^3}{27\,a^5\,d^3}\right)}^{1/3}+\ln\left(2\,b\,{\left(b\,x^3+a\right)}^{1/3}-2^{1/3}\,a^2\,d\,{\left(-\frac{b^3}{a^5\,d^3}\right)}^{1/3}+2^{1/3}\,\sqrt{3}\,a^2\,d\,{\left(-\frac{b^3}{a^5\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{2\,b^3}{27\,a^5\,d^3}\right)}^{1/3}-\frac{b\,{\left(b\,x^3+a\right)}^{1/3}}{3\,a\,\left(d\,\left(b\,x^3+a\right)-a\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x)
```

```
[Out] (4*log(b*(a + b*x^3)^(1/3) - a^2*d*(b^3/(a^5*d^3))^(1/3))*(b^3/(a^5*d^3))^(
1/3))/9 + log(b*(a + b*x^3)^(1/3) + 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3))*(
-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(a^5*
d^3))^(1/3) - 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2
)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(
a^5*d^3))^(1/3) + 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 +
 1/2)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(
1/3) - 2*b*(a + b*x^3)^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(-b^3/(a^5*d^3))^(1/3)
*1i)*((3^(1/2)*1i)/2 + 1/2)*(-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*b*(a + b*
x^3)^(1/3) - 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(
-b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-(2*b^3)/(27*a^5*d^3))^(1
/3) - (b*(a + b*x^3)^(1/3))/(3*a*(d*(a + b*x^3) - a*d))
```

3.573 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx$

Optimal. Leaf size=283

$$-\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3}-\frac{(a+bx^3)^{4/3}}{6a^2dx^6}-\frac{11b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}d}+\frac{\sqrt[3]{2}\,b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}d}-\frac{11b^2}{18\cdots}$$

```
[Out] -2/9*b*(b*x^3+a)^(1/3)/a^2/d/x^3-1/6*(b*x^3+a)^(4/3)/a^2/d/x^6-11/18*b^2*ln
(x)/a^(8/3)/d+1/6*b^2*ln(-b*x^3+a)*2^(1/3)/a^(8/3)/d+11/18*b^2*ln(a^(1/3)-(
b*x^3+a)^(1/3))/a^(8/3)/d-1/2*b^2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/
3)/a^(8/3)/d-11/27*b^2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/
2))/a^(8/3)/d*3^(1/2)+1/3*2^(1/3)*b^2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)
^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)/d*3^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {457, 105, 154, 162, 59, 631, 210, 31}

$$-\frac{11b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}d}+\frac{\sqrt[3]{2}\,b^2\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}d}+\frac{b^2\log(a-bx^3)}{3\,2^{2/3}a^{8/3}d}+\frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d}-\frac{11b^2\log(x)}{18a^{8/3}d}-\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3}-\frac{(a+bx^3)^{4/3}}{6a^2dx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (-2*b*(a + b*x^3)^(1/3))/(9*a^2*d*x^3) - (a + b*x^3)^(4/3)/(6*a^2*d*x^6) -
(11*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[
3]*a^(8/3)*d) + (2^(1/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*d) - (11*b^2*Log[x])/(18*a^(8/3)*d) + (
b^2*Log[a - b*x^3])/(3*2^(2/3)*a^(8/3)*d) + (11*b^2*Log[a^(1/3) - (a + b*x^
3)^(1/3)])/(18*a^(8/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/
(2^(2/3)*a^(8/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{4}{3}abd-\frac{2}{3}b^2dx\right)}{x^2(ad-bdx)}\,dx, x, x^3\right)}{6a^2d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{-\frac{22}{9}a^2b^2d^2-\frac{14}{9}ab^3d^2x}{x(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{6a^3d^2}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3a^2} + \frac{\left(11b^2\right)\ldots}{\ldots}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\log(x)}{18a^{8/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{8/3}d} - \frac{\left(11b^2\right)\text{Subst}\left(\int \ldots\right)}{\ldots}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\log(x)}{18a^{8/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{8/3}d} + \frac{11b^2\log\left(\sqrt[3]{a}-\ldots\right)}{18a^{8/3}\ldots}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{8/3}d} + \frac{\sqrt[3]{2}\,b^2\tan^{-1}\left(\frac{1+\ldots}{\ldots}\right)}{\sqrt{3}\,a\ldots}$$

Mathematica [A]
time = 0.55, size = 316, normalized size = 1.12

$$-\frac{9a^{5/3}\sqrt[3]{a+bx^3}+21a^{2/3}bx^3\sqrt[3]{a+bx^3}+22\sqrt{3}\,b^2x^6\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-18\sqrt[3]{2}\sqrt{3}\,b^2x^6\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-22b^2x^6\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)+18\sqrt[3]{2}\,b^2x^6\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)+11b^2x^6\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-9\sqrt[3]{2}\,b^2x^6\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{54a^{8/3}dx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] -1/54*(9*a^(5/3)*(a + b*x^3)^(1/3) + 21*a^(2/3)*b*x^3*(a + b*x^3)^(1/3) + 2
2*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 18*
2^(1/3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sq
rt[3]] - 22*b^2*x^6*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + 18*2^(1/3)*b^2*x^6*
Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] + 11*b^2*x^6*Log[a^(2/3) + a^(1
```

```
/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(1/3)*b^2*x^6*Log[2*a^(2/3
) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^(8/3
)*d*x^6)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{x^{7}\,(-bd\,x^{3}+ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^7), x)
```

Fricas [A]
time = 3.19, size = 345, normalized size = 1.22

$$-\frac{18\,\sqrt{3}\,2^{\frac{1}{3}}a^{2}b^{2}x^{6}\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\,2^{\frac{2}{3}}(bx^{3}+a)^{\frac{1}{3}}a\left(-\frac{1}{a^{2}}\right)^{\frac{2}{3}}+\frac{1}{3}\,\sqrt{3}\right)+9\cdot 2^{\frac{1}{3}}a^{2}b^{2}x^{6}\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{2}\left(-\frac{1}{a^{2}}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^{3}+a)^{\frac{1}{3}}a\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}+(bx^{3}+a)^{\frac{2}{3}}\right)-18\cdot 2^{\frac{1}{3}}a^{2}b^{2}x^{6}\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}a\left(-\frac{1}{a^{2}}\right)^{\frac{1}{3}}+(bx^{3}+a)^{\frac{1}{3}}\right)+22\,\sqrt{3}\,(a^{2})^{\frac{1}{6}}ab^{2}x^{6}\arctan\left(\frac{(a^{2})^{\frac{1}{6}}\left(\sqrt{3}\,(a^{2})^{\frac{1}{3}}a+2\,\sqrt{3}\,(bx^{3}+a)^{\frac{1}{3}}(a^{2})^{\frac{2}{3}}\right)}{3\,a^{2}}\right)+11\,(a^{2})^{\frac{2}{3}}b^{2}x^{6}\log\left((bx^{3}+a)^{\frac{2}{3}}a+(a^{2})^{\frac{1}{3}}a+(bx^{3}+a)^{\frac{1}{3}}(a^{2})^{\frac{2}{3}}\right)-22\,(a^{2})^{\frac{2}{3}}b^{2}x^{6}\log\left((bx^{3}+a)^{\frac{1}{3}}a-(a^{2})^{\frac{2}{3}}\right)+3\,(7\,a^{2}bx^{3}+3\,a^{3})(bx^{3}+a)^{\frac{1}{3}}}{54\,a^{4}dx^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] -1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(
2/3)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 9*2^(1/3)*a^2*b^2*
x^6*(-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/
3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 18*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^
(1/3)*log(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 22*sqrt(3)*(a^2)^
(1/6)*a*b^2*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + 11*(a^2)^(2/3)*b^2*x^6*log((b*x^3 + a)
^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 22*(a^2)^(2/3)*
b^2*x^6*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x^3 + 3*a^3)*(b
*x^3 + a)^(1/3))/(a^4*d*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^{3}}}{-ax^{7}+bx^{10}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**7/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**7 + b*x**10), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 5.44, size = 490, normalized size = 1.73

$$\frac{\frac{2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}}{9\,a}-\frac{7\,b^2\,{\left(b\,x^3+a\right)}^{4/3}}{18\,a^2}}{d\,{\left(b\,x^3+a\right)}^2+a^2\,d-2\,a\,d\,\left(b\,x^3+a\right)}+\frac{11\,\ln\left(b^2\,{\left(b\,x^3+a\right)}^{1/3}-a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}\right)\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}}{27}+\ln\left(b^2\,{\left(b\,x^3+a\right)}^{1/3}+2^{1/3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}\right)\,{\left(-\frac{2\,b^6}{27\,a^8\,d^3}\right)}^{1/3}-\ln\left(2^{1/3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}-2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+2^{1/3}\,\sqrt{3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{2\,b^6}{27\,a^8\,d^3}\right)}^{1/3}+\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}-2^{1/3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}+2^{1/3}\,\sqrt{3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{2\,b^6}{27\,a^8\,d^3}\right)}^{1/3}+\frac{11\,\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}-\sqrt{3}\,a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}}{54}-\frac{11\,\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}+\sqrt{3}\,a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x)
```

```
[Out] ((2*b^2*(a + b*x^3)^(1/3))/(9*a) - (7*b^2*(a + b*x^3)^(4/3))/(18*a^2))/(d*(
a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + (11*log(b^2*(a + b*x^3)^(1/3) -
 a^3*d*(b^6/(a^8*d^3))^(1/3))*(b^6/(a^8*d^3))^(1/3))/27 + log(b^2*(a + b*x^
3)^(1/3) + 2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3))*(-(2*b^6)/(27*a^8*d^3))^(1
/3) - log(2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3) - 2*b^2*(a + b*x^3)^(1/3) +
2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-(
2*b^6)/(27*a^8*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3) - 2^(1/3)*a^3*d*(-
b^6/(a^8*d^3))^(1/3) + 2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)*1i)*((3
^(1/2)*1i)/2 - 1/2)*(-(2*b^6)/(27*a^8*d^3))^(1/3) + (11*log(2*b^2*(a + b*x^
3)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) - 3^(1/2)*a^3*d*(b^6/(a^8*d^3))^(1/3
)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^8*d^3))^(1/3))/54 - (11*log(2*b^2*(a + b*x^3
)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) + 3^(1/2)*a^3*d*(b^6/(a^8*d^3))^(1/3)
*1i)*(3^(1/2)*1i + 1)*(b^6/(a^8*d^3))^(1/3))/54
```

3.574 $\int \frac{x^7\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=268

$$-\frac{7ax^2\sqrt[3]{a+bx^3}}{18b^2d}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}+\frac{11a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{8/3}d}-\frac{\sqrt[3]{2}\,a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{8/3}d}+\frac{a^2\log\left(ad\right.}{3\ 2^{2/3}b}$$

```
[Out] -7/18*a*x^2*(b*x^3+a)^(1/3)/b^2/d-1/6*x^5*(b*x^3+a)^(1/3)/b/d+1/6*a^2*ln(-b
*d*x^3+a*d)*2^(1/3)/b^(8/3)/d+11/18*a^2*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(8/
3)/d-1/2*a^2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/b^(8/3)/d+11/27*
a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(8/3)/d*3^(1/2)-1
/3*2^(1/3)*a^2*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/
b^(8/3)/d*3^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {489, 596, 598, 337, 503}

$$\frac{11a^2\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{8/3}d}-\frac{\sqrt[3]{2}\,a^2\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{8/3}d}+\frac{a^2\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{8/3}d}+\frac{11a^2\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{18b^{8/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{8/3}d}-\frac{7ax^2\sqrt[3]{a+bx^3}}{18b^2d}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-7*a*x^2*(a + b*x^3)^(1/3))/(18*b^2*d) - (x^5*(a + b*x^3)^(1/3))/(6*b*d) +
 (11*a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*
b^(8/3)*d) - (2^(1/3)*a^2*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/
3))/Sqrt[3]])/(Sqrt[3]*b^(8/3)*d) + (a^2*Log[a*d - b*d*x^3])/(3*2^(2/3)*b^(
8/3)*d) + (11*a^2*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(18*b^(8/3)*d) - (a^2
*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(2/3)*b^(8/3)*d)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
```

```
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x^7\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^8\sqrt[3]{a+bx^3}\,F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8ad\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A]

time = 0.87, size = 327, normalized size = 1.22

$$-\frac{21ab^{2/3}x^2\sqrt[3]{a+bx^3}+9b^{5/3}x^5\sqrt[3]{a+bx^3}-22\sqrt{3}\,a^2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)+18\sqrt[3]{2}\,\sqrt{3}\,a^2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-22a^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+18\sqrt[3]{2}\,a^2\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)+11a^2\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)-9\sqrt[3]{2}\,a^2\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{54b^{8/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/54*(21*a*b^(2/3)*x^2*(a + b*x^3)^(1/3) + 9*b^(5/3)*x^5*(a + b*x^3)^(1/3)
 - 22*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/
3))] + 18*2^(1/3)*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/
3)*(a + b*x^3)^(1/3))] - 22*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 18*
2^(1/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 11*a^2*Log[b^(2
/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(1/3)*a^2*
Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^
3)^(2/3)])/(b^(8/3)*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^7(b\,x^3+a)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

Fricas [A]
time = 2.88, size = 362, normalized size = 1.35

$$-\frac{18\sqrt{3}\,2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}\,x}{3\,x}\right)-18\cdot 2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}}{x}\right)+9\cdot 2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{2}{3}}b^2x^2\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+22\sqrt{3}\,a^2(b^2)^{\frac{1}{6}}b\arctan\left(\frac{\left(\sqrt{3}\,(b^2)^{\frac{1}{3}}bx+2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{3\,b^2x}\right)-22\,a^2(b^2)^{\frac{2}{3}}\log\left(-\frac{(b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{1}{3}}b}{x}\right)+11\,a^2(b^2)^{\frac{2}{3}}\log\left(\frac{(b^2)^{\frac{1}{3}}bx^2+(bx^3+a)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}x+(bx^3+a)^{\frac{2}{3}}b}{x^2}\right)+3\,(3\,b^3x^5+7\,ab^2x^2)(bx^3+a)^{\frac{1}{3}}}{54\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3
)*(b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 18*2^(1/3)*a^2*b^2*(
-1/b^2)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 9*2
^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)
*(b*x^3 + a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 22*sqrt(3
)*a^2*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3
+ a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 22*a^2*(b^2)^(2/3)*log(-((b^
2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 11*a^2*(b^2)^(2/3)*log(((b^2)^(1/3)*
b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*
b^3*x^5 + 7*a*b^2*x^2)*(b*x^3 + a)^(1/3))/(b^4*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^7\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**7*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,{\left(b\,x^3+a\right)}^{1/3}}{a\,d-b\,d\,x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.575 $\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=233

$$-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}+\frac{4a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}d}-\frac{\sqrt[3]{2}\,a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}d}+\frac{a\log\left(ad-bdx^3\right)}{3\,2^{2/3}b^{5/3}d}+\frac{2a\log\left(\sqrt[3]{b}\right.}{3}$$

```
[Out] -1/3*x^2*(b*x^3+a)^(1/3)/b/d+1/6*a*ln(-b*d*x^3+a*d)*2^(1/3)/b^(5/3)/d+2/3*a
*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d-1/2*a*ln(2^(1/3)*b^(1/3)*x-(b*x^3+
a)^(1/3))*2^(1/3)/b^(5/3)/d+4/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3)
)*3^(1/2))/b^(5/3)/d*3^(1/2)-1/3*2^(1/3)*a*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*
x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d*3^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {489, 598, 337, 503}

$$\frac{4a\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}d}-\frac{\sqrt[3]{2}\,a\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}d}+\frac{a\log\left(ad-bdx^3\right)}{3\,2^{2/3}b^{5/3}d}+\frac{2a\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}d}-\frac{a\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{5/3}d}-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/3*(x^2*(a + b*x^3)^(1/3))/(b*d) + (4*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)*d) - (2^(1/3)*a*ArcTan[(1 + (2*2^(
1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(5/3)*d) + (a*Log[a
*d - b*d*x^3])/(3*2^(2/3)*b^(5/3)*d) + (2*a*Log[b^(1/3)*x - (a + b*x^3)^(1/
3)])/(3*b^(5/3)*d) - (a*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(2/3
)*b^(5/3)*d)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
```

```
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x^4\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^5\sqrt[3]{a+bx^3}\,F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A]

time = 0.60, size = 293, normalized size = 1.26

$$-\frac{6b^{2/3}x^2\sqrt[3]{a+bx^3}-8\sqrt{3}\,a\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)+6\sqrt[3]{2}\sqrt{3}\,a\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-8a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+6\sqrt[3]{2}\,a\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)+4a\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-3\sqrt[3]{2}\,a\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{18b^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
[Out] -1/18*(6*b^(2/3)*x^2*(a + b*x^3)^(1/3) - 8*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 6*2^(1/3)*Sqrt[3]*a*ArcTan[(Sqrt[
```

```
3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 8*a*Log[-(b^(1/3)*
x) + (a + b*x^3)^(1/3)] + 6*2^(1/3)*a*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3
)^(1/3)] + 4*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)] - 3*2^(1/3)*a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(5/3)*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4 (b\,x^3+a)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

Fricas [A]
time = 3.14, size = 338, normalized size = 1.45

$$-\frac{6\sqrt{3}\,2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}\,x}{3\,x}\right)-6\cdot 2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}}{x}\right)+3\cdot 2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{2}{3}}b^2x^2\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+6\,(bx^3+a)^{\frac{1}{3}}b^2x^2+8\sqrt{3}\,a(b^2)^{\frac{1}{6}}b\arctan\left(\frac{\left(\sqrt{3}\,(b^2)^{\frac{1}{3}}bx+2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{3\,b^2x}\right)-8\,a(b^2)^{\frac{2}{3}}\log\left(-\frac{(b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{1}{3}}b}{x}\right)+4\,a(b^2)^{\frac{2}{3}}\log\left(\frac{(b^2)^{\frac{1}{3}}bx^2+(bx^3+a)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}x+(bx^3+a)^{\frac{2}{3}}b}{x^2}\right)}{18\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*2^(1/3)*a*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(
b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 6*2^(1/3)*a*b^2*(-1/b^2
)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 3*2^(1/3)
*a*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3
+ a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 6*(b*x^3 + a)^(1/
3)*b^2*x^2 + 8*sqrt(3)*a*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x
+ 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 8*a*(b^2)
^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 4*a*(b^2)^(2/3)*log(
((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)
/x^2))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^4\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**4*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^3+a\right)}^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.576 $\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=201

$$\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}-\frac{\sqrt[3]{2}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}+\frac{\log\left(ad-bdx^3\right)}{3\,2^{2/3}b^{2/3}d}+\frac{\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\log\left(\sqrt[3]{\right.}}{}$$

```
[Out] 1/6*ln(-b*d*x^3+a*d)*2^(1/3)/b^(2/3)/d+1/2*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^
(2/3)/d-1/2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/b^(2/3)/d+1/3*arc
tan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d*3^(1/2)-1/3*2^(1
/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d*3
^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {495, 337, 503}

$$\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}-\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}+\frac{\log\left(ad-bdx^3\right)}{3\,2^{2/3}b^{2/3}d}+\frac{\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*d) -
 (2^(1/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(S
qrt[3]*b^(2/3)*d) + Log[a*d - b*d*x^3]/(3*2^(2/3)*b^(2/3)*d) + Log[b^(1/3)*
x - (a + b*x^3)^(1/3)]/(2*b^(2/3)*d) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^
(1/3)]/(2^(2/3)*b^(2/3)*d)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
```

```
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} \\ &= \frac{x^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 265, normalized size = 1.32

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)-2\sqrt[3]{2}\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}\right)-\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)+\sqrt[3]{2}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}(a+bx^3)^{2/3}\right)}{6b^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] -
2*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^
3)^(1/3))] + 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - 2*2^(1/3)*Log[-2*b^(
1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] - Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^
3)^(1/3) + (a + b*x^3)^(2/3)] + 2^(1/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)
*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(6*b^(2/3)*d)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x(b\,x^3+a)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(156) = 312$.
time = 2.21, size = 313, normalized size = 1.56

$$-\frac{2\sqrt{3}\,2^{\frac{1}{3}}b^{2}\left(-\frac{1}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^{2}}\right)^{\frac{2}{3}}+\sqrt{3}\,x}{3x}\right)-2\cdot 2^{\frac{1}{3}}b^{2}\left(-\frac{1}{b^{2}}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^{2}}\right)^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)+2^{\frac{1}{3}}b^{2}\left(-\frac{1}{b^{2}}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{2}{3}}b^{2}x^{2}\left(-\frac{1}{b^{2}}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}bx\left(-\frac{1}{b^{2}}\right)^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)+2\sqrt{3}\left(b^{2}\right)^{\frac{1}{6}}b\arctan\left(\frac{\left(\sqrt{3}\left(b^{2}\right)^{\frac{1}{3}}bx+2\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(b^{2}\right)^{\frac{2}{3}}\right)\left(b^{2}\right)^{\frac{1}{6}}}{3b^{2}x}\right)-2\left(b^{2}\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^{2}\right)^{\frac{2}{3}}x-\left(bx^{3}+a\right)^{\frac{1}{3}}b}{x}\right)+\left(b^{2}\right)^{\frac{2}{3}}\log\left(\frac{\left(b^{2}\right)^{\frac{1}{3}}bx^{2}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(b^{2}\right)^{\frac{2}{3}}x+\left(bx^{3}+a\right)^{\frac{2}{3}}b}{x^{2}}\right)}{6b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*2^(1/3)*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x
^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 2*2^(1/3)*b^2*(-1/b^2)^(1/
3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 2^(1/3)*b^2*(-
1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3
)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 2*sqrt(3)*(b^2)^(1/6)*b*ar
ctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3)
)*(b^2)^(1/6)/(b^2*x)) - 2*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1
/3)*b)/x) + (b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2
/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{x\sqrt[3]{a+bx^{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(b\,x^3+a\right)}^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.577 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx$

Optimal. Leaf size=156

$$-\frac{\sqrt[3]{a+bx^3}}{adx}-\frac{\sqrt[3]{2}\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}ad}+\frac{\sqrt[3]{b}\log(ad-bdx^3)}{3\ 2^{2/3}ad}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}ad}$$

```
[Out] -(b*x^3+a)^(1/3)/a/d/x+1/6*b^(1/3)*ln(-b*d*x^3+a*d)*2^(1/3)/a/d-1/2*b^(1/3)
*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a/d-1/3*2^(1/3)*b^(1/3)*arct
an(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a/d*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {486, 12, 503}

$$-\frac{\sqrt[3]{2}\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}ad}-\frac{\sqrt[3]{a+bx^3}}{adx}+\frac{\sqrt[3]{b}\log(ad-bdx^3)}{3\ 2^{2/3}ad}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(a*d*x)) - (2^(1/3)*b^(1/3)*ArcTan[(1 + (2*2^(1/3)*b^(1
/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a*d) + (b^(1/3)*Log[a*d - b*d*
x^3])/(3*2^(2/3)*a*d) - (b^(1/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]
)/(2^(2/3)*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
```

```
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^2(ad-bdx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{\sqrt[3]{a+bx^3}\,\sqrt[3]{1-\frac{bx^3}{a}}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{2bx^3}{a-bx^3}\right)}{adx\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [A]
time = 0.31, size = 190, normalized size = 1.22

$$-\frac{6\sqrt[3]{a+bx^3}+2\sqrt[3]{2}\,\sqrt{3}\,\sqrt[3]{b}\,x\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+2\sqrt[3]{2}\,\sqrt[3]{b}\,x\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)-\sqrt[3]{2}\,\sqrt[3]{b}\,x\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{6adx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -1/6*(6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*b^(1/3)*x*ArcTan[(Sqrt[3]*b^(
1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] + 2*2^(1/3)*b^(1/3)*x*Log[
-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] - 2^(1/3)*b^(1/3)*x*Log[2*b^(2/3)
*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a
*d*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{x^2\left(-bd\,x^3+ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(125) = 250$.
time = 186.28, size = 395, normalized size = 2.53

$$-\frac{2\,\sqrt{3}\,2^{\frac{1}{3}}(-b)^{\frac{1}{3}}\,x\arctan\left(\frac{6\,\sqrt{3}\,2^{\frac{2}{3}}\,(19\,b^2x^8+16\,abx^5+a^2x^2)\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}+6\,\sqrt{3}\,2^{\frac{1}{3}}\,(5\,b^2x^7-4\,abx^4-a^2x)\,(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{1}{3}}+\sqrt{3}\,(71\,b^3x^9+111\,ab^2x^6+33\,a^2bx^3+a^3)}{3\,(109\,b^3x^9+105\,ab^2x^6+3\,a^2bx^3-a^3)}\right)-2\cdot 2^{\frac{1}{3}}(-b)^{\frac{1}{3}}\,x\log\left(-\frac{6\cdot 2^{\frac{1}{3}}\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}bx^2+6\,(bx^3+a)^{\frac{2}{3}}bx+2^{\frac{2}{3}}\,(bx^3-a)(-b)^{\frac{2}{3}}}{bx^3-a}\right)+2^{\frac{1}{3}}(-b)^{\frac{1}{3}}\,x\log\left(\frac{3\cdot 2^{\frac{2}{3}}\,(5\,bx^4+ax)\,(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}-2^{\frac{1}{3}}\,(19\,b^2x^6+16\,abx^3+a^2)(-b)^{\frac{1}{3}}+12\,(2\,b^2x^5+abx^2)\,(bx^3+a)^{\frac{1}{3}}}{b^2x^6-2\,abx^3+a^2}\right)+18\,(bx^3+a)^{\frac{1}{3}}}{18\,adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*2^(1/3)*(-b)^(1/3)*x*arctan(1/3*(6*sqrt(3)*2^(2/3)*(19*b^2
*x^8 + 16*a*b*x^5 + a^2*x^2)*(b*x^3 + a)^(1/3)*(-b)^(2/3) + 6*sqrt(3)*2^(1/
3)*(5*b^2*x^7 - 4*a*b*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-b)^(1/3) + sqrt(3)*(
71*b^3*x^9 + 111*a*b^2*x^6 + 33*a^2*b*x^3 + a^3))/(109*b^3*x^9 + 105*a*b^2*
x^6 + 3*a^2*b*x^3 - a^3)) - 2*2^(1/3)*(-b)^(1/3)*x*log(-(6*2^(1/3)*(b*x^3 +
 a)^(1/3)*(-b)^(1/3)*b*x^2 + 6*(b*x^3 + a)^(2/3)*b*x + 2^(2/3)*(b*x^3 - a)*
(-b)^(2/3))/(b*x^3 - a)) + 2^(1/3)*(-b)^(1/3)*x*log((3*2^(2/3)*(5*b*x^4 + a
*x)*(b*x^3 + a)^(2/3)*(-b)^(2/3) - 2^(1/3)*(19*b^2*x^6 + 16*a*b*x^3 + a^2)*
(-b)^(1/3) + 12*(2*b^2*x^5 + a*b*x^2)*(b*x^3 + a)^(1/3))/(b^2*x^6 - 2*a*b*x
^3 + a^2)) + 18*(b*x^3 + a)^(1/3))/(a*d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^2+bx^5}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**2/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**2 + b*x**5), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^2\,\left(a\,d-b\,d\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)), x)
```

3.578 $\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)}\,dx$

Optimal. Leaf size=183

$$-\frac{\sqrt[3]{a+bx^3}}{4adx^4}-\frac{5b\sqrt[3]{a+bx^3}}{4a^2dx}-\frac{\sqrt[3]{2}\,b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^2d}+\frac{b^{4/3}\log\left(ad-bdx^3\right)}{3\,2^{2/3}a^2d}-\frac{b^{4/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{\right.}}{2^{2/3}a^2d}$$

```
[Out] -1/4*(b*x^3+a)^(1/3)/a/d/x^4-5/4*b*(b*x^3+a)^(1/3)/a^2/d/x+1/6*b^(4/3)*ln(-
b*d*x^3+a*d)*2^(1/3)/a^2/d-1/2*b^(4/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3)
)*2^(1/3)/a^2/d-1/3*2^(1/3)*b^(4/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))/a^2/d*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {486, 597, 12, 503}

$$-\frac{\sqrt[3]{2}\,b^{4/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^2d}+\frac{b^{4/3}\log\left(ad-bdx^3\right)}{3\,2^{2/3}a^2d}-\frac{b^{4/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^2d}-\frac{5b\sqrt[3]{a+bx^3}}{4a^2dx}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -1/4*(a + b*x^3)^(1/3)/(a*d*x^4) - (5*b*(a + b*x^3)^(1/3))/(4*a^2*d*x) - (2
^(1/3)*b^(4/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]
])/(Sqrt[3]*a^2*d) + (b^(4/3)*Log[a*d - b*d*x^3])/(3*2^(2/3)*a^2*d) - (b^(4
/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(2/3)*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
```

```
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^5(ad-bdx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{a^2+4abx^3+3b^2x^6-bx^3(a+3bx^3)\ {}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)+3bx^3(a-bx^3)\ {}_2F_1\left(\frac{2}{3},2\right.}{4a^2dx^4\left(a+bx^3\right)^{2/3}}$$

Mathematica [A]
time = 0.35, size = 214, normalized size = 1.17

$$-\frac{3a\sqrt[3]{a+bx^3}+15bx^3\sqrt[3]{a+bx^3}+4\sqrt[3]{2}\sqrt{3}\,b^{4/3}x^4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+4\sqrt[3]{2}\,b^{4/3}x^4\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)-2\sqrt[3]{2}\,b^{4/3}x^4\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{12a^2dx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -1/12*(3*a*(a + b*x^3)^(1/3) + 15*b*x^3*(a + b*x^3)^(1/3) + 4*2^(1/3)*Sqrt[
3]*b^(4/3)*x^4*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^
(1/3))] + 4*2^(1/3)*b^(4/3)*x^4*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3
```

```
)] - 2*2^(1/3)*b^(4/3)*x^4*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3
)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^2*d*x^4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{x^{5}\,(-bd\,x^{3}+ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^{3}}}{-ax^{5}+bx^{8}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**5/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**5 + b*x**8), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^5\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)), x)
```

3.579 $\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)}\,dx$

Optimal. Leaf size=210

$$-\frac{\sqrt[3]{a+bx^3}}{7adx^7}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx}-\frac{\sqrt[3]{2}\,b^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^3d}+\frac{b^{7/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^3d}-\frac{b^{7/3}\log}{}$$

```
[Out] -1/7*(b*x^3+a)^(1/3)/a/d/x^7-2/7*b*(b*x^3+a)^(1/3)/a^2/d/x^4-8/7*b^2*(b*x^3
+a)^(1/3)/a^3/d/x+1/6*b^(7/3)*ln(-b*d*x^3+a*d)*2^(1/3)/a^3/d-1/2*b^(7/3)*ln
(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a^3/d-1/3*2^(1/3)*b^(7/3)*arcta
n(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^3/d*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {486, 597, 12, 503}

$$-\frac{\sqrt[3]{2}\,b^{7/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^3d}+\frac{b^{7/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^3d}-\frac{b^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^3d}-\frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

```
[Out] -1/7*(a + b*x^3)^(1/3)/(a*d*x^7) - (2*b*(a + b*x^3)^(1/3))/(7*a^2*d*x^4) -
(8*b^2*(a + b*x^3)^(1/3))/(7*a^3*d*x) - (2^(1/3)*b^(7/3)*ArcTan[(1 + (2*2^(
1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^3*d) + (b^(7/3)*Log
[a*d - b*d*x^3])/(3*2^(2/3)*a^3*d) - (b^(7/3)*Log[2^(1/3)*b^(1/3)*x - (a +
b*x^3)^(1/3)])/(2^(2/3)*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
```

```
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^8(ad-bdx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{4a^3+10a^2bx^3+24ab^2x^6+18b^3x^9-2bx^3(2a^2+3abx^3+9b^2x^6)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right.}{}$$

Mathematica [A]
time = 0.43, size = 206, normalized size = 0.98

$$-\frac{\frac{6\sqrt[3]{a+bx^3}\,(a^2+2abx^3+8b^2x^6)}{x^7}+14\sqrt[3]{2}\,\sqrt{3}\,b^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+14\sqrt[3]{2}\,b^{7/3}\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)-7\sqrt[3]{2}\,b^{7/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{42a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

```
[Out] -1/42*((6*(a + b*x^3)^(1/3)*(a^2 + 2*a*b*x^3 + 8*b^2*x^6))/x^7 + 14*2^(1/3)
*Sqrt[3]*b^(7/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3
```

```
)^(1/3))] + 14*2^(1/3)*b^(7/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)
] - 7*2^(1/3)*b^(7/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/
3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^3*d)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{x^8\,(-bd\,x^3+ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^8+bx^{11}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**8/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**8 + b*x**11), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^8\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)), x)
```

3.580 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx$

Optimal. Leaf size=237

$$-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}-\frac{11b\sqrt[3]{a+bx^3}}{70a^2dx^7}-\frac{37b^2\sqrt[3]{a+bx^3}}{140a^3dx^4}-\frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx}-\frac{\sqrt[3]{2}\,b^{10/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^4d}+\frac{b^{10/3}\log}{3}$$

```
[Out] -1/10*(b*x^3+a)^(1/3)/a/d/x^10-11/70*b*(b*x^3+a)^(1/3)/a^2/d/x^7-37/140*b^2
*(b*x^3+a)^(1/3)/a^3/d/x^4-169/140*b^3*(b*x^3+a)^(1/3)/a^4/d/x+1/6*b^(10/3)
*ln(-b*d*x^3+a*d)*2^(1/3)/a^4/d-1/2*b^(10/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)
^(1/3))*2^(1/3)/a^4/d-1/3*2^(1/3)*b^(10/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*
x/(b*x^3+a)^(1/3))*3^(1/2))/a^4/d*3^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {486, 597, 12, 503}

$$-\frac{\sqrt[3]{2}\,b^{10/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^4d}+\frac{b^{10/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^4d}-\frac{b^{10/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^4d}-\frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx}-\frac{37b^2\sqrt[3]{a+bx^3}}{140a^3dx^4}-\frac{11b\sqrt[3]{a+bx^3}}{70a^2dx^7}-\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

```
[Out] -1/10*(a + b*x^3)^(1/3)/(a*d*x^10) - (11*b*(a + b*x^3)^(1/3))/(70*a^2*d*x^7
) - (37*b^2*(a + b*x^3)^(1/3))/(140*a^3*d*x^4) - (169*b^3*(a + b*x^3)^(1/3)
)/(140*a^4*d*x) - (2^(1/3)*b^(10/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a +
b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^4*d) + (b^(10/3)*Log[a*d - b*d*x^3])/(3*
2^(2/3)*a^4*d) - (b^(10/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(
2/3)*a^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
```

```
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^{11}(ad-bdx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{28a^4+64a^3bx^3+90a^2b^2x^6+216ab^3x^9+162b^4x^{12}-28a^3bx^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)}{}$$

Mathematica [A]
time = 0.49, size = 219, normalized size = 0.92

$$-\frac{\frac{3\sqrt[3]{a+bx^3}\left(14a^3+22a^2bx^3+37ab^2x^6+169b^3x^9\right)}{x^{10}}+140\sqrt[3]{2}\sqrt{3}\,b^{10/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+140\sqrt[3]{2}\,b^{10/3}\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)-70\sqrt[3]{2}\,b^{10/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{420a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

```
[Out] -1/420*((3*(a + b*x^3)^(1/3)*(14*a^3 + 22*a^2*b*x^3 + 37*a*b^2*x^6 + 169*b^
3*x^9))/x^10 + 140*2^(1/3)*Sqrt[3]*b^(10/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(
```

```
1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] + 140*2^(1/3)*b^(10/3)*Log[-2*b^(1/3)*
x + 2^(2/3)*(a + b*x^3)^(1/3)] - 70*2^(1/3)*b^(10/3)*Log[2*b^(2/3)*x^2 + 2^
(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^4*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{x^{11}\,(-bd\,x^3+ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^{11}+bx^{14}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**11/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**11 + b*x**14), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^{11}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)), x)
```

3.581 $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=521

$$-\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}-\frac{\sqrt[3]{2}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}d}-\frac{a^{5/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,b^{7/3}d}$$

```
[Out] -3/5*a*x*(b*x^3+a)^(1/3)/b^2/d-1/5*x^4*(b*x^3+a)^(1/3)/b/d-2/5*a^2*x*(1+b*x
^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b^2/d/(b*x^3+a)^(2/3)-1/6*
a^(5/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d+
1/6*a^(5/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d-1/3*2^(1/3)*a^(5/3)*ln(1
+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(7/3)/d+1/12*a^(5/3)*ln(2*2
^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b
*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d-1/3*2^(1/3)*a^(5/3)*arctan(1/3*(1-2*2^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d*3^(1/2)-1/6*a^(5/
3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1
/3)/b^(7/3)/d*3^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {489, 596, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt[3]{2}\,a^{5/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}d}-\frac{a^{5/3}\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,b^{7/3}d}-\frac{a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d}+\frac{a^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}b^{7/3}d}-\frac{\sqrt[3]{2}\,a^{5/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{7/3}d}+\frac{a^{5/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}b^{7/3}d}-\frac{2a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2d\,(a+bx^3)^{2/3}}-\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-3*a*x*(a + b*x^3)^(1/3))/(5*b^2*d) - (x^4*(a + b*x^3)^(1/3))/(5*b*d) - (2
^(1/3)*a^(5/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*b^(7/3)*d) - (a^(5/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3)
 + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*b^(7/3)*d) - (
2*a^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)
])/(5*b^2*d*(a + b*x^3)^(2/3)) - (a^(5/3)*Log[2^(2/3) - (a^(1/3) + b^(1/3)*
x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(7/3)*d) + (a^(5/3)*Log[1 + (2^(2/3)*(a
^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/
(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(7/3)*d) - (2^(1/3)*a^(5/3)*Log[1 + (2^(1/
3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(7/3)*d) + (a^(5/3)*Log[
```

```
2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) +
 b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*b^(7/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^6\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x^7\sqrt[3]{a+bx^3}\,F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.09, size = 234, normalized size = 0.45

$$\frac{-4(a+bx^3)(3ax+bx^4)+7abx^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{48a^4xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)\right)}}{20b^2d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-4*(a + b*x^3)*(3*a*x + b*x^4) + 7*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[
4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (48*a^4*x*AppellF1[1/3, 2/3, 1
, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/
3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)
/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))))
/(20*b^2*d*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^6(b\,x^3+a)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{6}\sqrt[3]{a+bx^{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**6*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,{\left(b\,x^3+a\right)}^{1/3}}{a\,d-b\,d\,x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.582 $\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=494

$$-\frac{x\sqrt[3]{a+bx^3}}{2bd}-\frac{\sqrt[3]{2}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d}-\frac{a^{2/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,b^{4/3}d}-\frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}}{2bd\,(a}$$

```
[Out] -1/2*x*(b*x^3+a)^(1/3)/b/d-1/2*a*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[
4/3],-b*x^3/a)/b/d/(b*x^3+a)^(2/3)-1/6*a^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)
*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(4/3)/d+1/6*a^(2/3)*ln(1+2^(2/3)*(a^(1/3)+b^
(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(
1/3)/b^(4/3)/d-1/3*2^(1/3)*a^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/3))/b^(4/3)/d+1/12*a^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+
a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(4/3)/d-1/3
*2^(1/3)*a^(2/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3
))*3^(1/2))/b^(4/3)/d*3^(1/2)-1/6*a^(2/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^
(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/b^(4/3)/d*3^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {489, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt[3]{2}\,a^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d}-\frac{a^{2/3}\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,b^{4/3}d}-\frac{a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d}+\frac{a^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}b^{4/3}d}-\frac{\sqrt[3]{2}\,a^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{4/3}d}+\frac{a^{2/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}b^{4/3}d}-\frac{ax\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2bd\,(a+bx^3)^{2/3}}-\frac{x\sqrt[3]{a+bx^3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/2*(x*(a + b*x^3)^(1/3))/(b*d) - (2^(1/3)*a^(2/3)*ArcTan[(1 - (2*2^(1/3)*
(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)*d) - (
a^(2/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt
[3]])/(2^(2/3)*Sqrt[3]*b^(4/3)*d) - (a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometr
ic2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*d*(a + b*x^3)^(2/3)) - (a^(2/3)*Lo
g[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(4/3)*d)
 + (a^(2/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (
2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(4/3)*d) -
(2^(1/3)*a^(2/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]
)/(3*b^(4/3)*d) + (a^(2/3)*Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x
^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*
b^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^3\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 5.54, size = 225, normalized size = 0.46

$$\frac{x\left(3x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{4\left(-a-bx^3+\frac{4a^3F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)\right)}\right)}{b}\right)}{8d\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x*(3*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b
*x^3)/a] + (4*(-a - b*x^3 + (4*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a),
 (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*
x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*
AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/b))/(8*d*(a + b*x^
3)^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(b\,x^3+a\right)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**3*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(b\,x^3+a\right)}^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.583 $\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=416

$$-\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\,d}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}\,d}+\frac{\log\left(1+\right.}{}$$

```
[Out] -1/6*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/
3)/d+1/6*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)/d-1/3*2^(1/3)*ln(1+2^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(1/3)/b^(1/3)/d+1/12*ln(2*2^(1
/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^
3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)/d-1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(1/3)/b^(1/3)/d*3^(1/2)-1/6*
arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)
/a^(1/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\,d}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\,d}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}\,d}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}\,d}-\frac{\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{a}\sqrt[3]{b}\,d}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

```
[Out] -((2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))
/Sqrt[3]])/(Sqrt[3]*a^(1/3)*b^(1/3)*d)) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b
^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a^(1/3)*b^(1/3)*d)
- Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)
*b^(1/3)*d) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) -
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)*b^(1
/3)*d) - (2^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)
])/(3*a^(1/3)*b^(1/3)*d) + Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x
^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^(2/3)*a
^(1/3)*b^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [A]
time = 2.55, size = 431, normalized size = 1.04

$$\frac{4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{-2\sqrt[3]{2}\sqrt[3]{a}-2\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)-4\log\left(\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)-2\log\left(-\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}\right)+\log\left(2^{2/3}a^{2/3}+2^{2/3}b^{2/3}x^2+2\sqrt[3]{2}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+4(a+bx^3)^{2/3}+2\sqrt[3]{2}\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)\right)+2\log\left(2^{2/3}a^{2/3}+2^{2/3}b^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}+\sqrt[3]{a}\left(2\,2^{2/3}\sqrt[3]{b}x-\sqrt[3]{2}\sqrt[3]{a+bx^3}\right)\right)}{6\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(-2*2^(1/3)*a^(1/3) - 2*2^(1/
3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^
(1/3))/(2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] - 4*Log[2
^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3)] - 2*Log[-(2^(1/3)*a
^(1/3)) - 2^(1/3)*b^(1/3)*x + 2*(a + b*x^3)^(1/3)] + Log[2^(2/3)*a^(2/3) +
2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 4*(a + b*x^3)
^(2/3) + 2*2^(1/3)*a^(1/3)*(2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Log
[2^(2/3)*a^(2/3) + 2^(2/3)*b^(2/3)*x^2 - 2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + (a + b*x^3)^(2/3) + a^(1/3)*(2*2^(2/3)*b^(1/3)*x - 2^(1/3)*(a + b*x^3)^
(1/3))])/(6*2^(2/3)*a^(1/3)*b^(1/3)*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{a\,d-b\,d\,x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(a*d - b*d*x^3),x)
[Out] int((a + b*x^3)^(1/3)/(a*d - b*d*x^3), x)
```

3.584 $\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

Optimal. Leaf size=496

$$-\frac{\sqrt[3]{a+bx^3}}{2adx^2}-\frac{\sqrt[3]{2}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}d}-\frac{b^{2/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a^{4/3}d}+\frac{bx\left(1+\frac{bx^3}{a}\right)^{2/3}}{2ad\,(a$$

```
[Out] -1/2*(b*x^3+a)^(1/3)/a/d/x^2+1/2*b*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3]
,[4/3],-b*x^3/a)/a/d/(b*x^3+a)^(2/3)-1/6*b^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/
3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/d+1/6*b^(2/3)*ln(1+2^(2/3)*(a^(1/3)+
b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2
^(1/3)/a^(4/3)/d-1/3*2^(1/3)*b^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^
3+a)^(1/3))/a^(4/3)/d+1/12*b^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^
3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/d-1
/3*2^(1/3)*b^(2/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/3))*3^(1/2))/a^(4/3)/d*3^(1/2)-1/6*b^(2/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(4/3)/d*3^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {486, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt[3]{2}\,b^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}d}-\frac{b^{2/3}\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a^{4/3}d}-\frac{b^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}a^{4/3}d}+\frac{b^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}a^{4/3}d}-\frac{\sqrt[3]{2}\,b^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{4/3}d}+\frac{b^{2/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\,2^{2/3}a^{4/3}d}+\frac{bx\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ad\,(a+bx^3)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(1/3)/(a*d*x^2) - (2^(1/3)*b^(2/3)*ArcTan[(1 - (2*2^(1/3)*
(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(4/3)*d) - (
b^(2/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt
[3]])/(2^(2/3)*Sqrt[3]*a^(4/3)*d) + (b*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometr
ic2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*d*(a + b*x^3)^(2/3)) - (b^(2/3)*Lo
g[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(4/3)*d)
 + (b^(2/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (
2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(4/3)*d) -
(2^(1/3)*b^(2/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]
)/(3*a^(4/3)*d) + (b^(2/3)*Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x
```

```
^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*
a^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^3(ad-bdx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2adx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 20.19, size = 231, normalized size = 0.47

$$\frac{-4a(a+bx^3)+b^2x^6\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{48a^3bx^3F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)\right)}}{8a^2dx^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] (-4*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*b*x^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*
x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)
/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)
/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))))/(8*a^2*d*x^
2*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{x^3\left(-bd\,x^3+ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^3+bx^6}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**3/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**3 + b*x**6), x)/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^3\,\left(a\,d-b\,d\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)), x)
```

3.585 $\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx$

Optimal. Leaf size=523

$$-\frac{\sqrt[3]{a+bx^3}}{5adx^5}-\frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2}-\frac{\sqrt[3]{2}\,b^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}d}-\frac{b^{5/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a^{7/3}d}+\frac{2b}{}$$

```
[Out] -1/5*(b*x^3+a)^(1/3)/a/d/x^5-3/5*b*(b*x^3+a)^(1/3)/a^2/d/x^2+2/5*b^2*x*(1+b
*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a^2/d/(b*x^3+a)^(2/3)-1/
6*b^(5/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/
d+1/6*b^(5/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/d-1/3*2^(1/3)*b^(5/3)*ln
(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(7/3)/d+1/12*b^(5/3)*ln(2
*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/d-1/3*2^(1/3)*b^(5/3)*arctan(1/3*(1-2*2^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(7/3)/d*3^(1/2)-1/6*b^(
5/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^
(1/3)/a^(7/3)/d*3^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {486, 597, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt[3]{2}\,b^{5/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}d}-\frac{b^{5/3}\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a^{7/3}d}-\frac{b^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d}+\frac{b^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{7/3}d}-\frac{\sqrt[3]{2}\,b^{5/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{7/3}d}+\frac{b^{5/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}a^{7/3}d}+\frac{2b^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2d\,(a+bx^3)^{2/3}}-\frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2}-\frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] -1/5*(a + b*x^3)^(1/3)/(a*d*x^5) - (3*b*(a + b*x^3)^(1/3))/(5*a^2*d*x^2) -
(2^(1/3)*b^(5/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*a^(7/3)*d) - (b^(5/3)*ArcTan[(1 + (2^(1/3)*(a^(1/
3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a^(7/3)*d) +
 (2*b^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/
a)])/(5*a^2*d*(a + b*x^3)^(2/3)) - (b^(5/3)*Log[2^(2/3) - (a^(1/3) + b^(1/3
)*x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(7/3)*d) + (b^(5/3)*Log[1 + (2^(2/3)*
(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x)
)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(7/3)*d) - (2^(1/3)*b^(5/3)*Log[1 + (2^(
1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*a^(7/3)*d) + (b^(5/3)*Lo
```

```
g[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3)
 + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*a^(7/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^6(ad-bdx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= -\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5adx^5\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 20.13, size = 243, normalized size = 0.46

$$\frac{-\frac{4\left(a^2+4abx^3+3b^2x^6\right)}{a^2x^5}+\frac{3b^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}+\frac{112b^2xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)\right)}}{20d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] ((-4*(a^2 + 4*a*b*x^3 + 3*b^2*x^6))/(a^2*x^5) + (3*b^3*x^4*(1 + (b*x^3)/a)^
(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])/a^3 + (112*b^2*x
*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*App
ellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2
/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^
3)/a), (b*x^3)/a]))))/(20*d*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{6}\left(-bd\,x^{3}+ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)

[Out] int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^6+bx^9}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)**(1/3)/x**6/(-b*d*x**3+a*d),x)

[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**6 + b*x**9), x)/d

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left (b\,x^3+a\right)^{1/3}}{x^6\,\left (a\,d-b\,d\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)), x)
```

3.586 $\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=223

$$-\frac{a^3(a+bx^3)^{2/3}}{2b^4d}-\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d}-\frac{(a+bx^3)^{11/3}}{11b^4d}-\frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^4d}+\frac{a^{11}}{}$$

```
[Out] -1/2*a^3*(b*x^3+a)^(2/3)/b^4/d-1/5*a^2*(b*x^3+a)^(5/3)/b^4/d+1/8*a*(b*x^3+a
)^(8/3)/b^4/d-1/11*(b*x^3+a)^(11/3)/b^4/d+1/6*a^(11/3)*ln(-b*x^3+a)*2^(2/3)
/b^4/d-1/2*a^(11/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/b^4/d-1/3*2
^(2/3)*a^(11/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2
))/b^4/d*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 90, 52, 57, 631, 210, 31}

$$-\frac{2^{2/3}a^{11/3}\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^4d}+\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^4d}-\frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^4d}-\frac{a^3(a+bx^3)^{2/3}}{2b^4d}-\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d}-\frac{(a+bx^3)^{11/3}}{11b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/2*(a^3*(a + b*x^3)^(2/3))/(b^4*d) - (a^2*(a + b*x^3)^(5/3))/(5*b^4*d) +
(a*(a + b*x^3)^(8/3))/(8*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) - (2^(2/3)*
a^(11/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^4*d) + (a^(11/3)*Log[a - b*x^3])/(3*2^(1/3)*b^4*d) - (a^(11/3)*Lo
g[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(a+bx)^{2/3}}{b^3d}+\frac{a(a+bx)^{5/3}}{b^3d}-\frac{(a+bx)^{8/3}}{b^3d}+\frac{a^3(a+bx)^{2/3}}{b^3(ad-bdx)}\right)\,dx,x,\right.\\
&= -\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d}-\frac{(a+bx^3)^{11/3}}{11b^4d}+\frac{a^3\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)}{3b^3}\\
&= -\frac{a^3(a+bx^3)^{2/3}}{2b^4d}-\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d}-\frac{(a+bx^3)^{11/3}}{11b^4d}+\frac{(2a^4)\,\text{Subst}\Big(\int}{}\\
&= -\frac{a^3(a+bx^3)^{2/3}}{2b^4d}-\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d}-\frac{(a+bx^3)^{11/3}}{11b^4d}+\frac{a^{11/3}\log(a-b}{3\sqrt[3]{2}\,b^4d}\\
&= -\frac{a^3(a+bx^3)^{2/3}}{2b^4d}-\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d}-\frac{(a+bx^3)^{11/3}}{11b^4d}+\frac{a^{11/3}\log(a-b}{3\sqrt[3]{2}\,b^4d}\\
&= -\frac{a^3(a+bx^3)^{2/3}}{2b^4d}-\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d}-\frac{(a+bx^3)^{11/3}}{11b^4d}-\frac{2^{2/3}a^{11/3}\tan^{-1}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.28, size = 202, normalized size = 0.91

$$
-\frac{3(a+bx^3)^{2/3}\left(293a^3+98a^2bx^3+65ab^2x^6+40b^3x^9\right)+440\ 2^{2/3}\sqrt{3}\,a^{11/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+440\ 2^{2/3}a^{11/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)-220\ 2^{2/3}a^{11/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{1320b^4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/1320*(3*(a + b*x^3)^(2/3)*(293*a^3 + 98*a^2*b*x^3 + 65*a*b^2*x^6 + 40*b^
3*x^9) + 440*2^(2/3)*Sqrt[3]*a^(11/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]] + 440*2^(2/3)*a^(11/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b
*x^3)^(1/3)] - 220*2^(2/3)*a^(11/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*
x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^4*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{x^{11}\left(b\,x^3+a\right)^{\frac{2}{3}}}{-bd\,x^3+ad}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [A]
time = 0.53, size = 183, normalized size = 0.82

$$-\frac{\frac{440\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{11}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}-\frac{220\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{440\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}+\frac{3\left(40\left(bx^3+a\right)^{\frac{11}{3}}-55\left(bx^3+a\right)^{\frac{8}{3}}a+88\left(bx^3+a\right)^{\frac{5}{3}}a^2+220\left(bx^3+a\right)^{\frac{2}{3}}a^3\right)}{d}}{1320\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/1320*(440*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a
^(1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 220*2^(2/3)*a^(11/3)*log(2^(2/3)
*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 440*2
^(2/3)*a^(11/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(40*(b*x^3
+ a)^(11/3) - 55*(b*x^3 + a)^(8/3)*a + 88*(b*x^3 + a)^(5/3)*a^2 + 220*(b*x^
3 + a)^(2/3)*a^3)/d)/b^4
```

Fricas [A]
time = 3.60, size = 209, normalized size = 0.94

$$-\frac{440\cdot 4^{\frac{1}{3}}\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a^3\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3a}\right)+220\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a^3\log\left(4^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{2}{3}}a-2\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a\right)-440\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a^3\log\left(-4^{\frac{2}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}a\right)+3\left(40\,b^3x^9+65\,ab^2x^6+98\,a^2bx^3+293\,a^3\right)\left(bx^3+a\right)^{\frac{2}{3}}}{1320\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^3*arctan(1/3*(4^(1/3)*sqrt(3)*(
b*x^3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 220*4^(1/3)*(-a^2)^(1/3)*a^
3*log(4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^
(1/3)*(-a^2)^(1/3)*a) - 440*4^(1/3)*(-a^2)^(1/3)*a^3*log(-4^(2/3)*(-a^2)^(2
/3) + 2*(b*x^3 + a)^(1/3)*a) + 3*(40*b^3*x^9 + 65*a*b^2*x^6 + 98*a^2*b*x^3
+ 293*a^3)*(b*x^3 + a)^(2/3))/(b^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{11}\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

[Out] -Integral(x**11*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 4.85, size = 261, normalized size = 1.17

$$\frac{a\,{\left(b\,x^3+a\right)}^{8/3}}{8\,b^4\,d}-\frac{a^3\,{\left(b\,x^3+a\right)}^{2/3}}{2\,b^4\,d}-\frac{a^2\,{\left(b\,x^3+a\right)}^{5/3}}{5\,b^4\,d}-\frac{{\left(b\,x^3+a\right)}^{11/3}}{11\,b^4\,d}+\frac{4^{1/3}\,{\left(-a\right)}^{11/3}\,\ln\left(4\,a^8\,{\left(b\,x^3+a\right)}^{1/3}+4\,2^{1/3}\,{\left(-a\right)}^{25/3}\right)}{3\,b^4\,d}-\frac{4^{1/3}\,{\left(-a\right)}^{11/3}\,\ln\left(\frac{4\,a^8\,{\left(b\,x^3+a\right)}^{1/3}}{b^8\,d^2}+\frac{2\,4^{2/3}\,{\left(-a\right)}^{25/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^8\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^4\,d}+\frac{4^{1/3}\,{\left(-a\right)}^{11/3}\,\ln\left(\frac{4\,a^8\,{\left(b\,x^3+a\right)}^{1/3}}{b^8\,d^2}+\frac{18\,4^{2/3}\,{\left(-a\right)}^{25/3}\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{b^8\,d^2}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (a*(a + b*x^3)^(8/3))/(8*b^4*d) - (a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2*
(a + b*x^3)^(5/3))/(5*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) + (4^(1/3)*(-a
)^(11/3)*log(4*a^8*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(25/3)))/(3*b^4*d) -
(4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (2*4^(2/3)*(
-a)^(25/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^8*d^2))*((3^(1/2)*1i)/2 + 1/2))/(3*
b^4*d) + (4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (18
*4^(2/3)*(-a)^(25/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^8*d^2))*((3^(1/2)*1i)/6 -
 1/6))/(b^4*d)
```

3.587 $\int \frac{x^8 (a+bx^3)^{2/3}}{ad-bdx^3} \, dx$

Optimal. Leaf size=177

$$-\frac{a^2(a+bx^3)^{2/3}}{2b^3d}-\frac{(a+bx^3)^{8/3}}{8b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{\cdots}\right.}{\sqrt[3]{2}\cdots}$$

```
[Out] -1/2*a^2*(b*x^3+a)^(2/3)/b^3/d-1/8*(b*x^3+a)^(8/3)/b^3/d+1/6*a^(8/3)*ln(-b*
x^3+a)*2^(2/3)/b^3/d-1/2*a^(8/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3
)/b^3/d-1/3*2^(2/3)*a^(8/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^
(1/3)*3^(1/2))/b^3/d*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 90, 52, 57, 631, 210, 31}

$$-\frac{2^{2/3}a^{8/3}\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^3d}-\frac{a^2(a+bx^3)^{2/3}}{2b^3d}-\frac{(a+bx^3)^{8/3}}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/2*(a^2*(a + b*x^3)^(2/3))/(b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d) - (2^(2/
3)*a^(8/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])
/(Sqrt[3]*b^3*d) + (a^(8/3)*Log[a - b*x^3])/(3*2^(1/3)*b^3*d) - (a^(8/3)*Lo
g[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{(a+bx)^{5/3}}{b^2d}+\frac{a^2(a+bx)^{2/3}}{b^2(ad-bdx)}\right)\,dx,x,x^3\right)\\
&= -\frac{(a+bx^3)^{8/3}}{8b^3d}+\frac{a^2\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)}{3b^2}\\
&= -\frac{a^2(a+bx^3)^{2/3}}{2b^3d}-\frac{(a+bx^3)^{8/3}}{8b^3d}+\frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx,x,x^3\right)}{3b^2}\\
&= -\frac{a^2(a+bx^3)^{2/3}}{2b^3d}-\frac{(a+bx^3)^{8/3}}{8b^3d}+\frac{a^{8/3}\log(a-bx^3)}{3\sqrt[3]{2}\,b^3d}+\frac{a^{8/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,\right.}{\sqrt[3]{2}\,b^3d}\\
&= -\frac{a^2(a+bx^3)^{2/3}}{2b^3d}-\frac{(a+bx^3)^{8/3}}{8b^3d}+\frac{a^{8/3}\log(a-bx^3)}{3\sqrt[3]{2}\,b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx\right.}}{\sqrt[3]{2}\,b^3d}\\
&= -\frac{a^2(a+bx^3)^{2/3}}{2b^3d}-\frac{(a+bx^3)^{8/3}}{8b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3d}+\frac{a^{8/3}\log(a}{3\sqrt[3]{2}\,b}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 210, normalized size = 1.19

$$
-\frac{15a^2(a+bx^3)^{2/3}+6abx^3(a+bx^3)^{2/3}+3b^2x^6(a+bx^3)^{2/3}+8\,2^{2/3}\sqrt{3}\,a^{8/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+8\,2^{2/3}a^{8/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)-4\,2^{2/3}a^{8/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{24b^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/24*(15*a^2*(a + b*x^3)^(2/3) + 6*a*b*x^3*(a + b*x^3)^(2/3) + 3*b^2*x^6*(
a + b*x^3)^(2/3) + 8*2^(2/3)*Sqrt[3]*a^(8/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^
3)^(1/3))/a^(1/3))/Sqrt[3]] + 8*2^(2/3)*a^(8/3)*Log[-2*a^(1/3) + 2^(2/3)*(a
 + b*x^3)^(1/3)] - 4*2^(2/3)*a^(8/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b
*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^3*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{x^8(b\,x^3+a)^{\frac{2}{3}}}{-bd\,x^3+ad}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [A]
time = 0.52, size = 155, normalized size = 0.88

$$-\frac{\frac{8\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{4\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{d}+\frac{8\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right)}{d}+\frac{3\left((bx^3+a)^{\frac{8}{3}}+4\,(bx^3+a)^{\frac{2}{3}}a^2\right)}{d}}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/24*(8*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 4*2^(2/3)*a^(8/3)*log(2^(2/3)*a^(2/3)
 + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 8*2^(2/3)*a^(
8/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*((b*x^3 + a)^(8/3) + 4
*(b*x^3 + a)^(2/3)*a^2)/d)/b^3
```

Fricas [A]
time = 5.38, size = 197, normalized size = 1.11

$$-\frac{8\cdot 4^{\frac{1}{3}}\sqrt{3}\,(-a^2)^{\frac{1}{3}}a^2\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}\,a}{3a}\right)+4\cdot 4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}a^2\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\,(bx^3+a)^{\frac{2}{3}}a-2\cdot 4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}a\right)-8\cdot 4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}a^2\log\left(-4^{\frac{2}{3}}(-a^2)^{\frac{2}{3}}+2\,(bx^3+a)^{\frac{1}{3}}a\right)+3\,(b^2x^6+2\,abx^3+5\,a^2)(bx^3+a)^{\frac{2}{3}}}{24\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/24*(8*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^2*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4*4^(1/3)*(-a^2)^(1/3)*a^2*log(
4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*
(-a^2)^(1/3)*a) - 8*4^(1/3)*(-a^2)^(1/3)*a^2*log(-4^(2/3)*(-a^2)^(2/3) + 2*
(b*x^3 + a)^(1/3)*a) + 3*(b^2*x^6 + 2*a*b*x^3 + 5*a^2)*(b*x^3 + a)^(2/3))/(
b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^8\,(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**8*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 4.91, size = 206, normalized size = 1.16

$$-\frac{\left(b\,x^3+a\right)^{8/3}}{8\,b^3\,d}-\frac{a^2\,\left(b\,x^3+a\right)^{2/3}}{2\,b^3\,d}-\frac{4^{1/3}\,a^{8/3}\,\ln\left({\left(b\,x^3+a\right)}^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^3\,d}-\frac{4^{1/3}\,a^{8/3}\,\ln\left(\frac{4\,a^6\,{\left(b\,x^3+a\right)}^{1/3}}{b^6\,d^2}-\frac{2\,4^{2/3}\,a^{19/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^6\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^3\,d}+\frac{4^{1/3}\,a^{8/3}\,\ln\left(\frac{4\,a^6\,{\left(b\,x^3+a\right)}^{1/3}}{b^6\,d^2}-\frac{18\,4^{2/3}\,a^{19/3}\,{\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{b^6\,d^2}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3)^(1/3))/(b^6*d^2) - (18*4^(2/3)*a^(1
9/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^6*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b^3*d) -
 (a^2*(a + b*x^3)^(2/3))/(2*b^3*d) - (4^(1/3)*a^(8/3)*log((a + b*x^3)^(1/3)
 - 2^(1/3)*a^(1/3)))/(3*b^3*d) - (4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3)^(1
/3))/(b^6*d^2) - (2*4^(2/3)*a^(19/3)*((3^(1/2)*1i)/2 - 1/2)^2)/(b^6*d^2))*(
(3^(1/2)*1i)/2 - 1/2))/(3*b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d)
```

3.588 $\int \frac{x^5 (a+bx^3)^{2/3}}{ad-bdx^3} \, dx$

Optimal. Leaf size=175

$$-\frac{a(a+bx^3)^{2/3}}{2b^2d}-\frac{(a+bx^3)^{5/3}}{5b^2d}-\frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^2d}+\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d}-\frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}\right.}{\sqrt[3]{2}\,b^2}$$

```
[Out] -1/2*a*(b*x^3+a)^(2/3)/b^2/d-1/5*(b*x^3+a)^(5/3)/b^2/d+1/6*a^(5/3)*ln(-b*x^
3+a)*2^(2/3)/b^2/d-1/2*a^(5/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/
b^2/d-1/3*2^(2/3)*a^(5/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1
/3)*3^(1/2))/b^2/d*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 81, 52, 57, 631, 210, 31}

$$-\frac{2^{2/3}a^{5/3}\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^2d}+\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d}-\frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^2d}-\frac{a(a+bx^3)^{2/3}}{2b^2d}-\frac{(a+bx^3)^{5/3}}{5b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/2*(a*(a + b*x^3)^(2/3))/(b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d) - (2^(2/3)
*a^(5/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^2*d) + (a^(5/3)*Log[a - b*x^3])/(3*2^(1/3)*b^2*d) - (a^(5/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)$$
$$= -\frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{a\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)}{3b}$$
$$= -\frac{a(a+bx^3)^{2/3}}{2b^2d} - \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx,x,x^3\right)}{3b}$$
$$= -\frac{a(a+bx^3)^{2/3}}{2b^2d} - \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{a^{5/3}\log(a-bx^3)}{3\sqrt[3]{2}\,b^2d} + \frac{a^{5/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,x,\right.}{\sqrt[3]{2}\,b^2d}$$
$$= -\frac{a(a+bx^3)^{2/3}}{2b^2d} - \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{a^{5/3}\log(a-bx^3)}{3\sqrt[3]{2}\,b^2d} - \frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right.}{\sqrt[3]{2}\,b^2d}$$
$$= -\frac{a(a+bx^3)^{2/3}}{2b^2d} - \frac{(a+bx^3)^{5/3}}{5b^2d} - \frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^2d} + \frac{a^{5/3}\log(a-}{3\sqrt[3]{2}\,b^2d}$$

Mathematica [A]
time = 0.18, size = 188, normalized size = 1.07

$$-\frac{21a(a+bx^3)^{2/3}+6bx^3(a+bx^3)^{2/3}+10\,2^{2/3}\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10\,2^{2/3}a^{5/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)-5\,2^{2/3}a^{5/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{30b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/30*(21*a*(a + b*x^3)^(2/3) + 6*b*x^3*(a + b*x^3)^(2/3) + 10*2^(2/3)*Sqrt
[3]*a^(5/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 10*
2^(2/3)*a^(5/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 5*2^(2/3)*a^(
5/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3
)^(2/3)])/(b^2*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^5(b\,x^3+a)^{\frac{2}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [A]
time = 0.50, size = 155, normalized size = 0.89

$$-\frac{\frac{10\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{5\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{10\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}+\frac{3\left(2\left(bx^3+a\right)^{\frac{5}{3}}+5\left(bx^3+a\right)^{\frac{2}{3}}a\right)}{d}}{30\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/30*(10*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 5*2^(2/3)*a^(5/3)*log(2^(2/3)*a^(2/3
) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 10*2^(2/3)*a
^(5/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(2*(b*x^3 + a)^(5/3)
 + 5*(b*x^3 + a)^(2/3)*a)/d)/b^2
```

Fricas [A]
time = 2.66, size = 181, normalized size = 1.03

$$-\frac{10\cdot 4^{\frac{1}{3}}\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)+5\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a\log\left(4^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{2}{3}}a-2\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a\right)-10\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a\log\left(-4^{\frac{2}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}a\right)+3\left(2\,bx^3+7\,a\right)\left(bx^3+a\right)^{\frac{2}{3}}}{30\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/30*(10*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3
 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 5*4^(1/3)*(-a^2)^(1/3)*a*log(4^(
2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a
^2)^(1/3)*a) - 10*4^(1/3)*(-a^2)^(1/3)*a*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x
^3 + a)^(1/3)*a) + 3*(2*b*x^3 + 7*a)*(b*x^3 + a)^(2/3))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^5\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**5*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 4.84, size = 221, normalized size = 1.26

$$\frac{4^{1/3}\,(-a)^{5/3}\,\ln\left(4\,a^4\,(b\,x^3+a)^{1/3}+4\,2^{1/3}\,(-a)^{13/3}\right)}{3\,b^2\,d}-\frac{a\,(b\,x^3+a)^{2/3}}{2\,b^2\,d}-\frac{(b\,x^3+a)^{5/3}}{5\,b^2\,d}-\frac{4^{1/3}\,(-a)^{5/3}\,\ln\left(\frac{4\,a^4\,(b\,x^3+a)^{1/3}}{b^4\,d^2}+\frac{2\,4^{2/3}\,(-a)^{13/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^4\,d^2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^2\,d}+\frac{4^{1/3}\,(-a)^{5/3}\,\ln\left(\frac{4\,a^4\,(b\,x^3+a)^{1/3}}{b^4\,d^2}+\frac{18\,4^{2/3}\,(-a)^{13/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^4\,d^2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*(-a)^(5/3)*log(4*a^4*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(13/3)))/(
3*b^2*d) - (a*(a + b*x^3)^(2/3))/(2*b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d) -
(4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) + (2*4^(2/3)*(-
a)^(13/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^4*d^2))*((3^(1/2)*1i)/2 + 1/2))/(3*b
^2*d) + (4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) + (18*4
^(2/3)*(-a)^(13/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^4*d^2))*((3^(1/2)*1i)/6 - 1
/6))/(b^2*d)
```

3.589 $\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=153

$$-\frac{(a+bx^3)^{2/3}}{2bd}-\frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}+\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/b/d+1/6*a^(2/3)*ln(-b*x^3+a)*2^(2/3)/b/d-1/2*a^(2/3)*l
n(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/b/d-1/3*2^(2/3)*a^(2/3)*arctan(1
/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/b/d*3^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {455, 52, 57, 631, 210, 31}

$$-\frac{2^{2/3}a^{2/3}\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}+\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}-\frac{(a+bx^3)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(b*d) - (2^(2/3)*a^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(
a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(2/3)*Log[a - b*x^
3])/(3*2^(1/3)*b*d) - (a^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2
^(1/3)*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)$$

$$=-\frac{\left(a+bx^3\right)^{2/3}}{2bd}+\frac{1}{3}(2a)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx,x,x^3\right)$$

$$=-\frac{\left(a+bx^3\right)^{2/3}}{2bd}+\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}+\frac{a^{2/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\,\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}-$$

$$=-\frac{\left(a+bx^3\right)^{2/3}}{2bd}+\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}+\frac{\left(2^{2/3}a^{2/3}\right)\text{S}}{}$$

$$=-\frac{\left(a+bx^3\right)^{2/3}}{2bd}-\frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,bd}+\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\right.}{}$$

Mathematica [A]
time = 0.16, size = 170, normalized size = 1.11

$$-\frac{3(a+bx^3)^{2/3}+2\,2^{2/3}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\,2^{2/3}a^{2/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)-2^{2/3}a^{2/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/6*(3*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2^(2/3)*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*2^(2/3)*a^(2/3)*Log[-2*a^(1/3) + 2
^(2/3)*(a + b*x^3)^(1/3)] - 2^(2/3)*a^(2/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)
*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(b\,x^3+a\right)^{\frac{2}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [A]
time = 0.51, size = 140, normalized size = 0.92

$$-\frac{\frac{2\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{2\cdot 2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}+\frac{3\left(bx^3+a\right)^{\frac{2}{3}}}{d}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*2^(2/3)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 2^(2/3)*a^(2/3)*log(2^(2/3)*a^(2/3) +
2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 2*2^(2/3)*a^(2/3
)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(b*x^3 + a)^(2/3)/d)/b
```

Fricas [A]
time = 2.91, size = 167, normalized size = 1.09

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)+4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{2}{3}}a-2\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a\right)-2\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}a\right)+3\left(bx^3+a\right)^{\frac{2}{3}}}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3 + a
)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4^(1/3)*(-a^2)^(1/3)*log(4^(2/3)*(b*
x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a^2)^(1/3
)*a) - 2*4^(1/3)*(-a^2)^(1/3)*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(1/
3)*a) + 3*(b*x^3 + a)^(2/3))/(b*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**2*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [**B**]
time = 4.83, size = 186, normalized size = 1.22

$$-\frac{\left(b\,x^3+a\right)^{2/3}}{2\,b\,d}-\frac{4^{1/3}\,a^{2/3}\,\ln\left({\left(b\,x^3+a\right)}^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b\,d}-\frac{4^{1/3}\,a^{2/3}\,\ln\left(\frac{4\,a^2\,{\left(b\,x^3+a\right)}^{1/3}}{b^2\,d^2}-\frac{2\,4^{2/3}\,a^{7/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^2\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b\,d}+\frac{4^{1/3}\,a^{2/3}\,\ln\left(\frac{4\,a^2\,{\left(b\,x^3+a\right)}^{1/3}}{b^2\,d^2}-\frac{18\,4^{2/3}\,a^{7/3}\,{\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{b^2\,d^2}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (18*4^(2/3)*a^(7
/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^2*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b*d) - (4
^(1/3)*a^(2/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b*d) - (4^(1/3)
*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (2*4^(2/3)*a^(7/3)*((3^(
1/2)*1i)/2 - 1/2)^2)/(b^2*d^2))*((3^(1/2)*1i)/2 - 1/2))/(3*b*d) - (a + b*x^
3)^(2/3)/(2*b*d)
```

3.590 $\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx$

Optimal. Leaf size=214

$$\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{\log(x)}{2\sqrt[3]{a}\,d}+\frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+\right.}}{2\sqrt[3]{a}\,d}$$

```
[Out] -1/2*ln(x)/a^(1/3)/d+1/6*ln(-b*x^3+a)*2^(2/3)/a^(1/3)/d+1/2*ln(a^(1/3)-(b*x
^3+a)^(1/3))/a^(1/3)/d-1/2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/a^(1
/3)/d+1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(1/3)/d
*3^(1/2)-1/3*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3
^(1/2))/a^(1/3)/d*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {457, 85, 57, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{2^{2/3}\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}+\frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{\log(x)}{2\sqrt[3]{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
d) - (2^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(1/3)*d) - Log[x]/(2*a^(1/3)*d) + Log[a - b*x^3]/(3*2^(1/3)*
a^(1/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(1/3)*d) - Log[2^(1/3)*a
^(1/3) - (a + b*x^3)^(1/3)]/(2^(1/3)*a^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x(ad-bdx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}(2b)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right) + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}\,d} + \frac{\log(a-bx^3)}{3\sqrt[3]{2}\,\sqrt[3]{a}\,d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{}{2^2}\right.}{}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}\,d} + \frac{\log(a-bx^3)}{3\sqrt[3]{2}\,\sqrt[3]{a}\,d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,d} - \frac{\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,\sqrt[3]{a}\,d}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,d} - \frac{2^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,d} - \frac{\log(x)}{2\sqrt[3]{a}\,d} + \frac{\log(a-bx^3}{3\sqrt[3]{2}\,\sqrt[3]{a}\,d}$$

Mathematica [**A**]
time = 0.27, size = 236, normalized size = 1.10

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right) - 2\,2^{2/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right) - \log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right) + 2^{2/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}(a+bx^3)^{2/3}\right)}{6\sqrt[3]{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(2/3)*
Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[-
a^(1/3) + (a + b*x^3)^(1/3)] - 2*2^(2/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^
3)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] +
2^(2/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*
x^3)^(2/3)])/(6*a^(1/3)*d)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x\,(-bd\,x^3+ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x), x)
```

Fricas [A]
time = 2.54, size = 530, normalized size = 2.48

$$\left[-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\,a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-3\sqrt{\frac{1}{3}}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^3+3\sqrt{\frac{1}{3}}\left(2(bx^3+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^3}\right)+4^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{2}{3}}-2\cdot 4^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}+2(bx^3+a)^{\frac{2}{3}}\right)-2\cdot 4^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}a\left(-\frac{1}{a}\right)^{\frac{2}{3}}+2(bx^3+a)^{\frac{1}{3}}\right)+a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{6ad},\ -\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\,a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+4^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{2}{3}}-2\cdot 4^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}+2(bx^3+a)^{\frac{2}{3}}\right)-2\cdot 4^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}a\left(-\frac{1}{a}\right)^{\frac{2}{3}}+2(bx^3+a)^{\frac{1}{3}}\right)-6\sqrt{\frac{1}{3}}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)+a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{6ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 +
 a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log(
(2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a -
 a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) + 4^(1
/3)*a*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)
*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2
/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a
^(1/3)))/(a*d), -1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*s
qrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 4^(1/3)*a*(-1/a)^(1/
3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3)
+ 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/
3) + 2*(b*x^3 + a)^(1/3)) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3
+ a)^(1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a
)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a
*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax+bx^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x + b*x**4), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 5.86, size = 369, normalized size = 1.72

$$\ln\left(2\,{\left(b\,x^3+a\right)}^{1/3}-2\,2^{1/3}\,a\,d^2\,{\left(-\frac{1}{a\,d^3}\right)}^{2/3}\right)\,{\left(-\frac{4}{27\,a\,d^3}\right)}^{1/3}+\ln\left({\left(b\,x^3+a\right)}^{1/3}-a\,d^2\,{\left(\frac{1}{a\,d^3}\right)}^{2/3}\right)\,{\left(\frac{1}{27\,a\,d^3}\right)}^{1/3}-\ln\left(4\,{\left(b\,x^3+a\right)}^{1/3}+2\,2^{1/3}\,a\,d^2\,{\left(-\frac{1}{a\,d^3}\right)}^{2/3}-2^{1/3}\,\sqrt{3}\,a\,d^2\,{\left(-\frac{1}{a\,d^3}\right)}^{2/3}\,2{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{4}{27\,a\,d^3}\right)}^{1/3}+\ln\left(4\,{\left(b\,x^3+a\right)}^{1/3}+2\,2^{1/3}\,a\,d^2\,{\left(-\frac{1}{a\,d^3}\right)}^{2/3}+2^{1/3}\,\sqrt{3}\,a\,d^2\,{\left(-\frac{1}{a\,d^3}\right)}^{2/3}\,2{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{4}{27\,a\,d^3}\right)}^{1/3}-\ln\left(2\,{\left(b\,x^3+a\right)}^{1/3}+a\,d^2\,{\left(\frac{1}{a\,d^3}\right)}^{2/3}-\sqrt{3}\,a\,d^2\,{\left(\frac{1}{a\,d^3}\right)}^{2/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{1}{27\,a\,d^3}\right)}^{1/3}+\ln\left(2\,{\left(b\,x^3+a\right)}^{1/3}+a\,d^2\,{\left(\frac{1}{a\,d^3}\right)}^{2/3}+\sqrt{3}\,a\,d^2\,{\left(\frac{1}{a\,d^3}\right)}^{2/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{1}{27\,a\,d^3}\right)}^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x)
```

```
[Out] log(2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3))*(-4/(27*a*d^3
))^(1/3) + log((a + b*x^3)^(1/3) - a*d^2*(1/(a*d^3))^(2/3))*(1/(27*a*d^3))^
(1/3) - log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3) - 2^(1
/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-4/(27*a*d
^3))^(1/3) + log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3) +
 2^(1/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-4/(2
7*a*d^3))^(1/3) - log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3) - 3^(1/
2)*a*d^2*(1/(a*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*d^3))^(1/3)
+ log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3) + 3^(1/2)*a*d^2*(1/(a*d
^3))^(2/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a*d^3))^(1/3)
```

3.591 $\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx$

Optimal. Leaf size=269

$$\frac{b(a+bx^3)^{2/3}}{3a^2d}-\frac{(a+bx^3)^{5/3}}{3a^2dx^3}+\frac{5b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}d}-\frac{2^{2/3}b\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}d}-\frac{5b\log(x)}{6a^{4/3}d}+$$

```
[Out] 1/3*b*(b*x^3+a)^(2/3)/a^2/d-1/3*(b*x^3+a)^(5/3)/a^2/d/x^3-5/6*b*ln(x)/a^(4/
3)/d+1/6*b*ln(-b*x^3+a)*2^(2/3)/a^(4/3)/d+5/6*b*ln(a^(1/3)-(b*x^3+a)^(1/3))
/a^(4/3)/d-1/2*b*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/a^(4/3)/d+5/9*
b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/d*3^(1/2)
-1/3*2^(2/3)*b*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2)
)/a^(4/3)/d*3^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {457, 105, 162, 52, 57, 631, 210, 31}

$$\frac{5b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}d}-\frac{2^{2/3}b\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}d}+\frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{4/3}d}+\frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^{4/3}d}-\frac{5b\log(x)}{6a^{4/3}d}-\frac{(a+bx^3)^{5/3}}{3a^2dx^3}+\frac{b(a+bx^3)^{2/3}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(2/3))/(3*a^2*d) - (a + b*x^3)^(5/3)/(3*a^2*d*x^3) + (5*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)
*d) - (2^(2/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(4/3)*d) - (5*b*Log[x])/(6*a^(4/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(1/3)*a^(4/3)*d) + (5*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(4/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*a^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2(ad-bdx)}\,dx,x,x^3\right)\\
&= -\frac{(a+bx^3)^{5/3}}{3a^2dx^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(-\frac{5}{3}abd+\frac{2}{3}b^2dx\right)}{x(ad-bdx)}\,dx,x,x^3\right)}{3a^2d}\\
&= -\frac{(a+bx^3)^{5/3}}{3a^2dx^3} + \frac{b^2\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)}{3a^2} + \frac{(5b)\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx,x,x^3\right)}{9a^2d}\\
&= \frac{b(a+bx^3)^{2/3}}{3a^2d} - \frac{(a+bx^3)^{5/3}}{3a^2dx^3} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx,x,x^3\right)}{3a} + \frac{(5b)\text{Sub}\cdots}{\cdots}\\
&= \frac{b(a+bx^3)^{2/3}}{3a^2d} - \frac{(a+bx^3)^{5/3}}{3a^2dx^3} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log(a-bx^3)}{3\sqrt[3]{2}\,a^{4/3}d} - \frac{(5b)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx\cdots\right.}{6a^{4/3}d}\\
&= \frac{b(a+bx^3)^{2/3}}{3a^2d} - \frac{(a+bx^3)^{5/3}}{3a^2dx^3} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log(a-bx^3)}{3\sqrt[3]{2}\,a^{4/3}d} + \frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx\cdots}\right.}{6a^{4/3}d}\\
&= \frac{b(a+bx^3)^{2/3}}{3a^2d} - \frac{(a+bx^3)^{5/3}}{3a^2dx^3} + \frac{5b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}d} - \frac{2^{2/3}b\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a\cdots}}{\sqrt[3]{\cdots}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}d}\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.40, size = 282, normalized size = 1.05

$$
\frac{-6\sqrt[3]{a}\,(a+bx^3)^{2/3}+10\sqrt{3}\,bx^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-6\,2^{2/3}\sqrt{3}\,bx^3\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10bx^3\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)-6\,2^{2/3}bx^3\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)-5bx^3\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)+3\,2^{2/3}bx^3\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{18a^{4/3}dx^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (-6*a^(1/3)*(a + b*x^3)^(2/3) + 10*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3)
^(1/3))/a^(1/3))/Sqrt[3]] - 6*2^(2/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3)*(a
 + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 10*b*x^3*Log[-a^(1/3) + (a + b*x^3)^(1
/3)] - 6*2^(2/3)*b*x^3*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 5*b*x^
3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + 3*2^(2/3)*
b*x^3*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^
3)^(2/3)])/(18*a^(4/3)*d*x^3)
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}}{x^{4}\left(-bd\,x^{3}+ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^4), x)
```

Fricas [A]

time = 5.85, size = 612, normalized size = 2.28

$$\left[-\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}abx^{3}\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^{3}+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-15\sqrt{\frac{1}{3}}abx^{3}\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^{3}+3\sqrt{\frac{1}{3}}\left(2(bx^{3}+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^{3}+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx^{3}+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^{3}}\right)+3\cdot 4^{\frac{1}{3}}abx^{3}\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^{3}+a)^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{2}{3}}-2\cdot 4^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}+2(bx^{3}+a)^{\frac{2}{3}}\right)-6\cdot 4^{\frac{1}{3}}abx^{3}\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}a\left(-\frac{1}{a}\right)^{\frac{2}{3}}+2(bx^{3}+a)^{\frac{1}{3}}\right)+5a^{\frac{2}{3}}bx^{3}\log\left((bx^{3}+a)^{\frac{2}{3}}+(bx^{3}+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-10a^{\frac{2}{3}}bx^{3}\log\left((bx^{3}+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+6(bx^{3}+a)^{\frac{2}{3}}a}{18a^{2}dx^{3}}, -\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}abx^{3}\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^{3}+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+3\cdot 4^{\frac{1}{3}}abx^{3}\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^{3}+a)^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{2}{3}}-2\cdot 4^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}+2(bx^{3}+a)^{\frac{2}{3}}\right)-6\cdot 4^{\frac{1}{3}}abx^{3}\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}a\left(-\frac{1}{a}\right)^{\frac{2}{3}}+2(bx^{3}+a)^{\frac{1}{3}}\right)-30\sqrt{\frac{1}{3}}a^{\frac{2}{3}}bx^{3}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx^{3}+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)+5a^{\frac{2}{3}}bx^{3}\log\left((bx^{3}+a)^{\frac{2}{3}}+(bx^{3}+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-10a^{\frac{2}{3}}bx^{3}\log\left((bx^{3}+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+6(bx^{3}+a)^{\frac{2}{3}}a}{18a^{2}dx^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(
b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 15*sqrt(1/3)*a*b*x^3*sqrt(-1
/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3
+ a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*
a)/x^3) + 3*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-
1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*
b*x^3*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + 5*a
^(2/3)*b*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) -
 10*a^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3)*a)
/(a^2*d*x^3), -1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1
/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 3*4^(1/3)*a*b*x
^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*
(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(-4
^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) - 30*sqrt(1/3)*a^(2/3)*b*x^3*a
rctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + 5*a^(2/3)*b*x^3*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 10*a^(2/3)*b
*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3)*a)/(a^2*d*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{-ax^{4}+bx^{7}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**4/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**4 + b*x**7), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 5.56, size = 490, normalized size = 1.82

$$\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}-2\,2^{1/3}\,a^3\,d^2\,{\left(-\frac{b^3}{a^4\,d^3}\right)}^{2/3}\right)\,{\left(-\frac{4\,b^3}{27\,a^4\,d^3}\right)}^{1/3}+\frac{5\,\ln\left(b^2\,{\left(b\,x^3+a\right)}^{1/3}-a^3\,d^2\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{2/3}\right)\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{1/3}}{9}-\ln\left(4\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+2\,2^{1/3}\,a^3\,d^2\,{\left(-\frac{b^3}{a^4\,d^3}\right)}^{2/3}-2^{1/3}\,\sqrt{3}\,a^3\,d^2\,{\left(-\frac{b^3}{a^4\,d^3}\right)}^{2/3}\,2{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{4\,b^3}{27\,a^4\,d^3}\right)}^{1/3}+\ln\left(4\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+2\,2^{1/3}\,a^3\,d^2\,{\left(-\frac{b^3}{a^4\,d^3}\right)}^{2/3}+2^{1/3}\,\sqrt{3}\,a^3\,d^2\,{\left(-\frac{b^3}{a^4\,d^3}\right)}^{2/3}\,2{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{4\,b^3}{27\,a^4\,d^3}\right)}^{1/3}-\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+a^3\,d^2\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{2/3}-\sqrt{3}\,a^3\,d^2\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{2/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{125\,b^3}{729\,a^4\,d^3}\right)}^{1/3}+\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+a^3\,d^2\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{2/3}+\sqrt{3}\,a^3\,d^2\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{2/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{125\,b^3}{729\,a^4\,d^3}\right)}^{1/3}-\frac{b\,{\left(b\,x^3+a\right)}^{2/3}}{3\,a\,\left(d\,\left(b\,x^3+a\right)-a\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x)
```

```
[Out] log(2*b^2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3))*(-(
4*b^3)/(27*a^4*d^3))^(1/3) + (5*log(b^2*(a + b*x^3)^(1/3) - a^3*d^2*(b^3/(a
^4*d^3))^(2/3))*(b^3/(a^4*d^3))^(1/3))/9 - log(4*b^2*(a + b*x^3)^(1/3) + 2*
2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) - 2^(1/3)*3^(1/2)*a^3*d^2*(-b^3/(a^4
*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-(4*b^3)/(27*a^4*d^3))^(1/3) + log
(4*b^2*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) + 2^(1/
3)*3^(1/2)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-(4*b
^3)/(27*a^4*d^3))^(1/3) - log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*(b^3/(a^4*d
^3))^(2/3) - 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 + 1/
2)*((125*b^3)/(729*a^4*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*
(b^3/(a^4*d^3))^(2/3) + 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)
*1i)/2 - 1/2)*((125*b^3)/(729*a^4*d^3))^(1/3) - (b*(a + b*x^3)^(2/3))/(3*a*
(d*(a + b*x^3) - a*d))
```

3.592 $\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)} dx$

Optimal. Leaf size=284

$$-\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3}-\frac{(a+bx^3)^{5/3}}{6a^2dx^6}+\frac{14b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}d}-\frac{2^{2/3}b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}d}-\frac{7b^2}{9\ldots}$$

```
[Out] -5/18*b*(b*x^3+a)^(2/3)/a^2/d/x^3-1/6*(b*x^3+a)^(5/3)/a^2/d/x^6-7/9*b^2*ln(
x)/a^(7/3)/d+1/6*b^2*ln(-b*x^3+a)*2^(2/3)/a^(7/3)/d+7/9*b^2*ln(a^(1/3)-(b*x
^3+a)^(1/3))/a^(7/3)/d-1/2*b^2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/
a^(7/3)/d+14/27*b^2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))
/a^(7/3)/d*3^(1/2)-1/3*2^(2/3)*b^2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1
/3))/a^(1/3)*3^(1/2))/a^(7/3)/d*3^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {457, 105, 154, 162, 57, 631, 210, 31}

$$\frac{14b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}d}-\frac{2^{2/3}b^2\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}d}+\frac{b^2\log(a-bx^3)}{3\sqrt[3]{2}a^{7/3}d}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{7/3}d}-\frac{7b^2\log(x)}{9a^{7/3}d}-\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3}-\frac{(a+bx^3)^{5/3}}{6a^2dx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (-5*b*(a + b*x^3)^(2/3))/(18*a^2*d*x^3) - (a + b*x^3)^(5/3)/(6*a^2*d*x^6) +
 (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt
[3]*a^(7/3)*d) - (2^(2/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/
(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*d) - (7*b^2*Log[x])/(9*a^(7/3)*d) + (b
^2*Log[a - b*x^3])/(3*2^(1/3)*a^(7/3)*d) + (7*b^2*Log[a^(1/3) - (a + b*x^3)
^(1/3)])/(9*a^(7/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^
(1/3)*a^(7/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{5/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(-\frac{5}{3}abd-\frac{1}{3}b^2dx\right)}{x^2(ad-bdx)}\,dx, x, x^3\right)}{6a^2d}$$

$$= -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{-\frac{28}{9}a^2b^2d^2-\frac{8}{9}ab^3d^2x}{x\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right)}{6a^3d^2}$$

$$= -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} + \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right)}{3a^2} + \frac{(14\ldots}{\ldots}$$

$$= -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log(a-bx^3)}{3\sqrt[3]{2}\,a^{7/3}d} - \frac{(7b^2)\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log(a-bx^3)}{3\sqrt[3]{2}\,a^{7/3}d} + \frac{7b^2\log\left(\sqrt[3]{a} - \ldots\right.}{9a^{7/3}d\ldots}$$

$$= -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} + \frac{14b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{7/3}d} - \frac{2^{2/3}b^2\tan^{-1}\left(\frac{1\ldots}{\ldots}\right.}{\sqrt{3}\ldots}$$

Mathematica [A]

time = 0.52, size = 316, normalized size = 1.11

$$\frac{-9a^{4/3}(a+bx^3)^{2/3} - 24\sqrt[3]{a}\,bx^3(a+bx^3)^{2/3} + 28\sqrt{3}\,b^2x^6\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 18\ 2^{2/3}\sqrt{3}\,b^2x^6\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 28b^2x^6\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right) - 18\ 2^{2/3}b^2x^6\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right) - 14b^2x^6\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right) + 9\ 2^{2/3}b^2x^6\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{54a^{7/3}dx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (-9*a^(4/3)*(a + b*x^3)^(2/3) - 24*a^(1/3)*b*x^3*(a + b*x^3)^(2/3) + 28*Sqr
t[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 18*2^(2/
3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]
] + 28*b^2*x^6*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 18*2^(2/3)*b^2*x^6*Log[-
2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 14*b^2*x^6*Log[a^(2/3) + a^(1/3)*(
```

```
a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + 9*2^(2/3)*b^2*x^6*Log[2*a^(2/3) + 2
^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(54*a^(7/3)*
d*x^6)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}}{x^{7}\left(-bd\,x^{3}+ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^7), x)
```

Fricas [A]
time = 3.73, size = 660, normalized size = 2.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*4^(1/3)*sqrt(3)*a*b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3
)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 42*sqrt(1/3)*a*b^2*x^6*sq
rt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b
*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3)
 + 3*a)/x^3) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/
3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 18*4^
(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1
/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3)
 + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(8*a*
b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6), -1/54*(18*4^(1/3)*sqrt(3)*a*
b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1
/3) - 1/3*sqrt(3)) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 +
a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) -
```

```
18*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 +
a)^(1/3)) - 84*sqrt(1/3)*a^(2/3)*b^2*x^6*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^
3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3)
- a^(1/3)) + 3*(8*a*b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^7+bx^{10}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**7/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**7 + b*x**10), x)/d
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a r
```

Mupad [B]
time = 5.45, size = 513, normalized size = 1.81

$$\frac{\frac{5b^2\left(bx^3+a\right)^{2/3}}{18a}-\frac{4b^2\left(bx^3+a\right)^{5/3}}{9a^2}}{d\left(bx^3+a\right)^2+a^2d-2ad\left(bx^3+a\right)}+\ln\left(2b^4\left(bx^3+a\right)^{1/3}-2\,2^{1/3}a^5d^2\left(-\frac{b^6}{a^7d^3}\right)^{2/3}\right)\left(-\frac{4b^6}{27a^7d^3}\right)^{1/3}+\frac{14\ln\left(b^4\left(bx^3+a\right)^{1/3}-a^5d^2\left(\frac{b^6}{a^7d^3}\right)^{2/3}\right)\left(\frac{b^6}{a^7d^3}\right)^{1/3}}{27}-\ln\left(4b^4\left(bx^3+a\right)^{1/3}+2\,2^{1/3}a^5d^2\left(-\frac{b^6}{a^7d^3}\right)^{2/3}-2^{1/3}\sqrt{3}\,a^5d^2\left(-\frac{b^6}{a^7d^3}\right)^{2/3}2\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{4b^6}{27a^7d^3}\right)^{1/3}+\ln\left(4b^4\left(bx^3+a\right)^{1/3}+2\,2^{1/3}a^5d^2\left(-\frac{b^6}{a^7d^3}\right)^{2/3}+2^{1/3}\sqrt{3}\,a^5d^2\left(-\frac{b^6}{a^7d^3}\right)^{2/3}2\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{4b^6}{27a^7d^3}\right)^{1/3}-\frac{7\ln\left(2b^4\left(bx^3+a\right)^{1/3}+a^5d^2\left(\frac{b^6}{a^7d^3}\right)^{2/3}-\sqrt{3}\,a^5d^2\left(\frac{b^6}{a^7d^3}\right)^{2/3}1\mathrm{i}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{b^6}{a^7d^3}\right)^{1/3}}{27}+\frac{7\ln\left(2b^4\left(bx^3+a\right)^{1/3}+a^5d^2\left(\frac{b^6}{a^7d^3}\right)^{2/3}+\sqrt{3}\,a^5d^2\left(\frac{b^6}{a^7d^3}\right)^{2/3}1\mathrm{i}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{b^6}{a^7d^3}\right)^{1/3}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x)
```

```
[Out] ((5*b^2*(a + b*x^3)^(2/3))/(18*a) - (4*b^2*(a + b*x^3)^(5/3))/(9*a^2))/(d*(
a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + log(2*b^4*(a + b*x^3)^(1/3) - 2
*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3))*(-(4*b^6)/(27*a^7*d^3))^(1/3) + (1
4*log(b^4*(a + b*x^3)^(1/3) - a^5*d^2*(b^6/(a^7*d^3))^(2/3))*(b^6/(a^7*d^3)
)^(1/3))/27 - log(4*b^4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^
3))^(2/3) - 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3)*2i)*((3^(1/2)*1i
)/2 + 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1/3) + log(4*b^4*(a + b*x^3)^(1/3) + 2*
2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3) + 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7
```

```
*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1/3) - (7*
log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) - 3^(1/2)*a^5*d
^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i + 1)*(b^6/(a^7*d^3))^(1/3))/27 + (
7*log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) + 3^(1/2)*a^5
*d^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^7*d^3))^(1/3))/27
```

3.593 $\int \frac{x^6\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=264

$$-\frac{4ax(a+bx^3)^{2/3}}{9b^2d}-\frac{x^4(a+bx^3)^{2/3}}{6bd}-\frac{14a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{7/3}d}+\frac{2^{2/3}a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}d}+\frac{a^2\log(\ldots}{3\sqrt[3]{\ldots}}$$

```
[Out] -4/9*a*x*(b*x^3+a)^(2/3)/b^2/d-1/6*x^4*(b*x^3+a)^(2/3)/b/d+1/6*a^2*ln(-b*d*
x^3+a*d)*2^(2/3)/b^(7/3)/d-1/2*a^2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^
(2/3)/b^(7/3)/d+7/9*a^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)/d-14/27*a^2*
arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d*3^(1/2)+1/3*2
^(2/3)*a^2*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7
/3)/d*3^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {489, 596, 544, 245, 384}

$$-\frac{14a^2\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{7/3}d}+\frac{2^{2/3}a^2\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}d}+\frac{a^2\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,b^{7/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^{7/3}d}+\frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{9b^{7/3}d}-\frac{4ax(a+bx^3)^{2/3}}{9b^2d}-\frac{x^4(a+bx^3)^{2/3}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-4*a*x*(a + b*x^3)^(2/3))/(9*b^2*d) - (x^4*(a + b*x^3)^(2/3))/(6*b*d) - (1
4*a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(
7/3)*d) + (2^(2/3)*a^2*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))
/Sqrt[3]])/(Sqrt[3]*b^(7/3)*d) + (a^2*Log[a*d - b*d*x^3])/(3*2^(1/3)*b^(7/3
)*d) - (a^2*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(1/3)*b^(7/3)*d)
 + (7*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(9*b^(7/3)*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
```

```
+ Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx &= \frac{(a+bx^3)^{2/3}\int \frac{x^6\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}} \\
&= \frac{x^7(a+bx^3)^{2/3}\,F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\left(1+\frac{bx^3}{a}\right)^{2/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.81, size = 325, normalized size = 1.23

$$
-\frac{24a\sqrt[3]{b}\,x(a+bx^3)^{2/3}+9b^{4/3}x^4(a+bx^3)^{2/3}+28\sqrt{3}\,a^2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-18\,2^{2/3}\sqrt{3}\,a^2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-28a^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+18\,2^{2/3}a^2\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)+14a^2\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)-9\,2^{2/3}a^2\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{54b^{7/3}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/54*(24*a*b^(1/3)*x*(a + b*x^3)^(2/3) + 9*b^(4/3)*x^4*(a + b*x^3)^(2/3) +
 28*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3)
)] - 18*2^(2/3)*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)
*(a + b*x^3)^(1/3))] - 28*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 18*2^
(2/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 14*a^2*Log[b^(2/3
)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(2/3)*a^2*Lo
g[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)
^(2/3)])/(b^(7/3)*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{6} \left(b\,x^{3}+a \right)^{\frac{2}{3}}}{-bd\,x^{3}+ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

Fricas [A]
time = 6.43, size = 701, normalized size = 2.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1
/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 42*sqrt(1/3)*a^2*b*sqrt(-1
/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4
/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/
```

```
b^(2/3)) + 2*a) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2
/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(2*4^(1/3
)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^
3 + a)^(2/3))/x^2) - 28*a^2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x)
 + 14*a^2*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d), -1/54
*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sq
rt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*lo
g(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-
1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b
*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 84*sqrt(1/3)*a^2*b^(2/3)*arct
an(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 28*a^2*b^(2/3
)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + 14*a^2*b^(2/3)*log((b^(2/3)*x^2
 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8
*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^6(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**6*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.594 $\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=229

$$-\frac{x(a+bx^3)^{2/3}}{3bd}-\frac{5a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d}+\frac{2^{2/3}a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d}+\frac{a\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,b^{4/3}d}-\frac{a\log\left(\sqrt[3]{2}\right.}{}$$

```
[Out] -1/3*x*(b*x^3+a)^(2/3)/b/d+1/6*a*ln(-b*d*x^3+a*d)*2^(2/3)/b^(4/3)/d-1/2*a*l
n(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/b^(4/3)/d+5/6*a*ln(-b^(1/3)*x+
(b*x^3+a)^(1/3))/b^(4/3)/d-5/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))
*3^(1/2))/b^(4/3)/d*3^(1/2)+1/3*2^(2/3)*a*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x
/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d*3^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {489, 544, 245, 384}

$$-\frac{5a\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d}+\frac{2^{2/3}a\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d}+\frac{a\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,b^{4/3}d}-\frac{a\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^{4/3}d}+\frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}d}-\frac{x(a+bx^3)^{2/3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/3*(x*(a + b*x^3)^(2/3))/(b*d) - (5*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^
3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)*d) + (2^(2/3)*a*ArcTan[(1 + (2*2^(1/
3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)*d) + (a*Log[a*d
 - b*d*x^3])/(3*2^(1/3)*b^(4/3)*d) - (a*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)
^(1/3)])/(2^(1/3)*b^(4/3)*d) + (5*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/
(6*b^(4/3)*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
```

```
+ Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx &= \frac{(a+bx^3)^{2/3}\int \frac{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^4(a+bx^3)^{2/3}\,F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4ad\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [A]
time = 0.59, size = 291, normalized size = 1.27

$$-\frac{6\sqrt[3]{b}\,x(a+bx^3)^{2/3}+10\sqrt{3}\,a\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-6\ 2^{2/3}\sqrt{3}\,a\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-10a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+6\ 2^{2/3}a\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)+5a\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)-3\ 2^{2/3}a\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{18b^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/18*(6*b^(1/3)*x*(a + b*x^3)^(2/3) + 10*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3)
*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 6*2^(2/3)*Sqrt[3]*a*ArcTan[(Sqrt[3
]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 10*a*Log[-(b^(1/3)*
x) + (a + b*x^3)^(1/3)] + 6*2^(2/3)*a*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3
)^(1/3)] + 5*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^
```

```
(2/3)] - 3*2^(2/3)*a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(4/3)*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{-bd\,x^{3}+ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
[Out] int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

Fricas [A]
time = 9.15, size = 653, normalized size = 2.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] [-1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)
*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 15*sqrt(1/3)*a*b*sqrt(-1/b^(2
/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x
^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/
3)) + 2*a) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*
(b*x^3 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-
1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/
3))/x^2) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x - (b*x^3
+ a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
 + (b*x^3 + a)^(2/3))/x^2))/(b^2*d), -1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1
/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3)
)/x) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3
 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(
1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^
```

```
2) - 30*sqrt(1/3)*a*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/
3))/(b^(1/3)*x)) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x -
 (b*x^3 + a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b
^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^3 (a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**3*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.595 $\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=200

$$-\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,d}+\frac{2^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,d}+\frac{\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,\sqrt[3]{b}\,d}-\frac{\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,\sqrt[3]{b}\,d}+$$

```
[Out] 1/6*ln(-b*d*x^3+a*d)*2^(2/3)/b^(1/3)/d-1/2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(
1/3))*2^(2/3)/b^(1/3)/d+1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)/d-1/3*ar
ctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)/d*3^(1/2)+1/3*2^(
2/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)/d*
3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {399, 245, 384}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,d}+\frac{2^{2/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,d}+\frac{\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,\sqrt[3]{b}\,d}-\frac{\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

```
[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)*d)
) + (2^(2/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])
/(Sqrt[3]*b^(1/3)*d) + Log[a*d - b*d*x^3]/(3*2^(1/3)*b^(1/3)*d) - Log[2^(1/
3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2^(1/3)*b^(1/3)*d) + Log[-(b^(1/3)*x) +
(a + b*x^3)^(1/3)]/(2*b^(1/3)*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx &= \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x(a+bx^3)^{2/3}\,F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [A]
time = 0.42, size = 264, normalized size = 1.32

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)+2\,2^{2/3}\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}\right)+\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)-2^{2/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}(a+bx^3)^{2/3}\right)}{6\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3)
)] - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a +
 b*x^3)^(1/3))] - 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 2*2^(2/3)*Log[-
2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + Log[b^(2/3)*x^2 + b^(1/3)*x*(a +
 b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 2^(2/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^
(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(1/3)*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

Fricas [A]
time = 6.66, size = 611, normalized size = 3.06

$$\left[-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\,b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,x-4^{\frac{1}{3}}\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3x}\right)-3\sqrt{\frac{1}{3}}\,b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2-2(bx^3+a)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2a\right)-2\cdot 4^{\frac{1}{3}}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2(bx^3+a)^{\frac{1}{3}}}{x}\right)+4^{\frac{1}{3}}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{2\cdot 4^{\frac{1}{3}}bx^2\left(-\frac{1}{b}\right)^{\frac{1}{3}}-4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2(bx^3+a)^{\frac{2}{3}}}{x^2}\right)-2b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)+b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{6bd},\ -\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\,b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,x-4^{\frac{1}{3}}\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3x}\right)-2\cdot 4^{\frac{1}{3}}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2(bx^3+a)^{\frac{1}{3}}}{x}\right)+4^{\frac{1}{3}}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{2\cdot 4^{\frac{1}{3}}bx^2\left(-\frac{1}{b}\right)^{\frac{1}{3}}-4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2(bx^3+a)^{\frac{2}{3}}}{x^2}\right)-6\sqrt{\frac{1}{3}}\,b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{1}{3}}x+2(bx^3+a)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}x}\right)-2b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)+b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{6bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] [-1/6*(2*4^(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sq
rt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 3*sqrt(1/3)*b*sqrt(-1/b^(2/3))*l
og(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (
b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) +
2*a) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 +
 a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) -
 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 2
*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2
 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(2*4^
(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-1/b)^(1/3))/x) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*
(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(
1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b
*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2
*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^
(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(a*d - b*d*x^3),x)
[Out] int((a + b*x^3)^(2/3)/(a*d - b*d*x^3), x)
```

3.596 $\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx$

Optimal. Leaf size=157

$$-\frac{(a+bx^3)^{2/3}}{2adx^2}+\frac{2^{2/3}b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,ad}+\frac{b^{2/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,ad}-\frac{b^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,ad}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/a/d/x^2+1/6*b^(2/3)*ln(-b*d*x^3+a*d)*2^(2/3)/a/d-1/2*b
^(2/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a/d+1/3*2^(2/3)*b^(2/3
)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a/d*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {486, 12, 384}

$$\frac{2^{2/3}b^{2/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,ad}+\frac{b^{2/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,ad}-\frac{b^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,ad}-\frac{(a+bx^3)^{2/3}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(a*d*x^2) + (2^(2/3)*b^(2/3)*ArcTan[(1 + (2*2^(1/3)*
b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a*d) + (b^(2/3)*Log[a*d -
b*d*x^3])/(3*2^(1/3)*a*d) - (b^(2/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1
/3)])/(2^(1/3)*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^3(ad-bdx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{(a+bx^3)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{2bx^3}{a-bx^3}\right)}{2adx^2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [A]
time = 0.32, size = 195, normalized size = 1.24

$$\frac{-3(a+bx^3)^{2/3}+2\,2^{2/3}\sqrt{3}\,b^{2/3}x^2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-2\,2^{2/3}b^{2/3}x^2\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)+2^{2/3}b^{2/3}x^2\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{6adx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] (-3*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*b^(2/3)*x^2*ArcTan[(Sqrt[3]*b^(1/
3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 2*2^(2/3)*b^(2/3)*x^2*Log[
-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 2^(2/3)*b^(2/3)*x^2*Log[2*b^(2/
3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/
(6*a*d*x^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^3\left(-bd\,x^3+ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(125) = 250.
time = 82.54, size = 434, normalized size = 2.76

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\left(-b^{2}\right)^{\frac{1}{3}}x^{2}\arctan\left(\frac{3\cdot 4^{\frac{2}{3}}\sqrt{3}\left(5b^{2}x^{7}-4abx^{4}-a^{2}x\right)\left(bx^{3}+a\right)^{\frac{2}{3}}\left(-b^{2}\right)^{\frac{2}{3}}+6\cdot 4^{\frac{1}{3}}\sqrt{3}\left(19b^{3}x^{8}+16ab^{2}x^{5}+a^{2}bx^{2}\right)\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-b^{2}\right)^{\frac{1}{3}}-\sqrt{3}\left(71b^{4}x^{9}+111ab^{3}x^{6}+33a^{2}b^{2}x^{3}+a^{3}b\right)}{3\left(109b^{4}x^{9}+105ab^{3}x^{6}+3a^{2}b^{2}x^{3}-a^{3}b\right)}\right)-2\cdot 4^{\frac{1}{3}}\left(-b^{2}\right)^{\frac{1}{3}}x^{2}\log\left(\frac{3\cdot 4^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-b^{2}\right)^{\frac{2}{3}}x^{2}-6\left(bx^{3}+a\right)^{\frac{2}{3}}bx+4^{\frac{1}{3}}\left(bx^{3}-a\right)\left(-b^{2}\right)^{\frac{1}{3}}}{bx^{3}-a}\right)+4^{\frac{1}{3}}\left(-b^{2}\right)^{\frac{1}{3}}x^{2}\log\left(-\frac{6\cdot 4^{\frac{1}{3}}\left(5b^{2}x^{4}+abx\right)\left(bx^{3}+a\right)^{\frac{2}{3}}\left(-b^{2}\right)^{\frac{1}{3}}-4^{\frac{2}{3}}\left(19b^{2}x^{6}+16abx^{3}+a^{2}\right)\left(-b^{2}\right)^{\frac{2}{3}}-24\left(2b^{3}x^{5}+ab^{2}x^{2}\right)\left(bx^{3}+a\right)^{\frac{1}{3}}}{b^{2}x^{6}-2abx^{3}+a^{2}}\right)+9\left(bx^{3}+a\right)^{\frac{2}{3}}}{18adx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] -1/18*(2*4^(1/3)*sqrt(3)*(-b^2)^(1/3)*x^2*arctan(1/3*(3*4^(2/3)*sqrt(3)*(5*
b^2*x^7 - 4*a*b*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-b^2)^(2/3) + 6*4^(1/3)*sqr
t(3)*(19*b^3*x^8 + 16*a*b^2*x^5 + a^2*b*x^2)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3)
 - sqrt(3)*(71*b^4*x^9 + 111*a*b^3*x^6 + 33*a^2*b^2*x^3 + a^3*b))/(109*b^4*
x^9 + 105*a*b^3*x^6 + 3*a^2*b^2*x^3 - a^3*b)) - 2*4^(1/3)*(-b^2)^(1/3)*x^2*
log((3*4^(2/3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x^2 - 6*(b*x^3 + a)^(2/3)*b*x
 + 4^(1/3)*(b*x^3 - a)*(-b^2)^(1/3))/(b*x^3 - a)) + 4^(1/3)*(-b^2)^(1/3)*x^
2*log(-(6*4^(1/3)*(5*b^2*x^4 + a*b*x)*(b*x^3 + a)^(2/3)*(-b^2)^(1/3) - 4^(2
/3)*(19*b^2*x^6 + 16*a*b*x^3 + a^2)*(-b^2)^(2/3) - 24*(2*b^3*x^5 + a*b^2*x^
2)*(b*x^3 + a)^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2)) + 9*(b*x^3 + a)^(2/3))/(
a*d*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{-ax^{3}+bx^{6}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**3/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**3 + b*x**6), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^3\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)), x)
```

3.597 $\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx$

Optimal. Leaf size=182

$$-\frac{(a+bx^3)^{2/3}}{5adx^5}-\frac{7b(a+bx^3)^{2/3}}{10a^2dx^2}+\frac{2^{2/3}b^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^2d}+\frac{b^{5/3}\log(ad-bdx^3)}{3\sqrt[3]{2}\,a^2d}-\frac{b^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x\right.}{\sqrt[3]{2}\,a^2}$$

```
[Out] -1/5*(b*x^3+a)^(2/3)/a/d/x^5-7/10*b*(b*x^3+a)^(2/3)/a^2/d/x^2+1/6*b^(5/3)*l
n(-b*d*x^3+a*d)*2^(2/3)/a^2/d-1/2*b^(5/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1
/3))*2^(2/3)/a^2/d+1/3*2^(2/3)*b^(5/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b
*x^3+a)^(1/3))*3^(1/2))/a^2/d*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {486, 597, 12, 384}

$$\frac{2^{2/3}b^{5/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^2d}+\frac{b^{5/3}\log(ad-bdx^3)}{3\sqrt[3]{2}\,a^2d}-\frac{b^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^2d}-\frac{7b(a+bx^3)^{2/3}}{10a^2dx^2}-\frac{(a+bx^3)^{2/3}}{5adx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] -1/5*(a + b*x^3)^(2/3)/(a*d*x^5) - (7*b*(a + b*x^3)^(2/3))/(10*a^2*d*x^2) +
 (2^(2/3)*b^(5/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]])/(Sqrt[3]*a^2*d) + (b^(5/3)*Log[a*d - b*d*x^3])/(3*2^(1/3)*a^2*d) - (b
^(5/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(1/3)*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^6(ad-bdx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{2a^2+5abx^3+3b^2x^6-4bx^3(2a+3bx^3)\ {}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)+12bx^3(a-bx^3)\ {}_2F_1\left(\frac{1}{3}\right.}{10a^2dx^5\sqrt[3]{a+bx^3}}$$

Mathematica [A]
time = 0.36, size = 216, normalized size = 1.19

$$\frac{(-2a-7bx^3)(a+bx^3)^{2/3}}{10a^2dx^5}+\frac{2^{2/3}b^{5/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}\,a^2d}-\frac{2^{2/3}b^{5/3}\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)}{3a^2d}+\frac{b^{5/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{3\sqrt[3]{2}\,a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] ((-2*a - 7*b*x^3)*(a + b*x^3)^(2/3))/(10*a^2*d*x^5) + (2^(2/3)*b^(5/3)*ArcT
an[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*a
^2*d) - (2^(2/3)*b^(5/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*
a^2*d) + (b^(5/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) +
 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*a^2*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{6} \left(-bd\,x^{3}+ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{-ax^{6}+bx^{9}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**6/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**6 + b*x**9), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^3+a)^{2/3}}{x^6\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)), x)
```

3.598 $\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx$

Optimal. Leaf size=209

$$-\frac{(a+bx^3)^{2/3}}{8adx^8}-\frac{b(a+bx^3)^{2/3}}{4a^2dx^5}-\frac{5b^2(a+bx^3)^{2/3}}{8a^3dx^2}+\frac{2^{2/3}b^{8/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^3d}+\frac{b^{8/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^3d}-\frac{b^{8/}}{}$$

```
[Out] -1/8*(b*x^3+a)^(2/3)/a/d/x^8-1/4*b*(b*x^3+a)^(2/3)/a^2/d/x^5-5/8*b^2*(b*x^3
+a)^(2/3)/a^3/d/x^2+1/6*b^(8/3)*ln(-b*d*x^3+a*d)*2^(2/3)/a^3/d-1/2*b^(8/3)*
ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a^3/d+1/3*2^(2/3)*b^(8/3)*arc
tan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^3/d*3^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {486, 597, 12, 384}

$$\frac{2^{2/3}b^{8/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^3d}+\frac{b^{8/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^3d}-\frac{b^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^3d}-\frac{5b^2(a+bx^3)^{2/3}}{8a^3dx^2}-\frac{b(a+bx^3)^{2/3}}{4a^2dx^5}-\frac{(a+bx^3)^{2/3}}{8adx^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

```
[Out] -1/8*(a + b*x^3)^(2/3)/(a*d*x^8) - (b*(a + b*x^3)^(2/3))/(4*a^2*d*x^5) - (5
*b^2*(a + b*x^3)^(2/3))/(8*a^3*d*x^2) + (2^(2/3)*b^(8/3)*ArcTan[(1 + (2*2^(
1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^3*d) + (b^(8/3)*Log
[a*d - b*d*x^3])/(3*2^(1/3)*a^3*d) - (b^(8/3)*Log[2^(1/3)*b^(1/3)*x - (a +
b*x^3)^(1/3)])/(2^(1/3)*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^9(ad-bdx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{5a^3+11a^2bx^3+15ab^2x^6+9b^3x^9-4bx^3(5a^2+6abx^3+9b^2x^6)\ {}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)}{}$$

Mathematica [A]
time = 0.43, size = 206, normalized size = 0.99

$$\frac{-\frac{3(a+bx^3)^{2/3}(a^2+2abx^3+5b^2x^6)}{x^8}+8\,2^{2/3}\sqrt{3}\,b^{8/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-8\,2^{2/3}b^{8/3}\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)+4\,2^{2/3}b^{8/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\,(a+bx^3)^{2/3}\right)}{24a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(a^2 + 2*a*b*x^3 + 5*b^2*x^6))/x^8 + 8*2^(2/3)*Sqrt[
3]*b^(8/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3
))] - 8*2^(2/3)*b^(8/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 4*2
^(2/3)*b^(8/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^
(1/3)*(a + b*x^3)^(2/3)])/(24*a^3*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{9}\left(-bd\,x^{3}+ad\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{-ax^{9}+bx^{12}}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**9/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**9 + b*x**12), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^3+a)^{2/3}}{x^9\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)), x)
```

3.599 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx$

Optimal. Leaf size=236

$$-\frac{(a+bx^3)^{2/3}}{11adx^{11}}-\frac{13b(a+bx^3)^{2/3}}{88a^2dx^8}-\frac{49b^2(a+bx^3)^{2/3}}{220a^3dx^5}-\frac{293b^3(a+bx^3)^{2/3}}{440a^4dx^2}+\frac{2^{2/3}b^{11/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^4d}+$$

```
[Out] -1/11*(b*x^3+a)^(2/3)/a/d/x^11-13/88*b*(b*x^3+a)^(2/3)/a^2/d/x^8-49/220*b^2
*(b*x^3+a)^(2/3)/a^3/d/x^5-293/440*b^3*(b*x^3+a)^(2/3)/a^4/d/x^2+1/6*b^(11/
3)*ln(-b*d*x^3+a*d)*2^(2/3)/a^4/d-1/2*b^(11/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+
a)^(1/3))*2^(2/3)/a^4/d+1/3*2^(2/3)*b^(11/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3
)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^4/d*3^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {486, 597, 12, 384}

$$\frac{2^{2/3}b^{11/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^4d}+\frac{b^{11/3}\log(ad-bdx^3)}{3\sqrt[3]{2}\,a^4d}-\frac{b^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^4d}-\frac{293b^3(a+bx^3)^{2/3}}{440a^4dx^2}-\frac{49b^2(a+bx^3)^{2/3}}{220a^3dx^5}-\frac{13b(a+bx^3)^{2/3}}{88a^2dx^8}-\frac{(a+bx^3)^{2/3}}{11adx^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

```
[Out] -1/11*(a + b*x^3)^(2/3)/(a*d*x^11) - (13*b*(a + b*x^3)^(2/3))/(88*a^2*d*x^8
) - (49*b^2*(a + b*x^3)^(2/3))/(220*a^3*d*x^5) - (293*b^3*(a + b*x^3)^(2/3)
)/(440*a^4*d*x^2) + (2^(2/3)*b^(11/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^4*d) + (b^(11/3)*Log[a*d - b*d*x^3])/(
3*2^(1/3)*a^4*d) - (b^(11/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2
^(1/3)*a^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^{12}(ad-bdx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{40a^4+85a^3bx^3+99a^2b^2x^6+135ab^3x^9+81b^4x^{12}-160a^3bx^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)-}{}$$

Mathematica [A]

time = 0.49, size = 219, normalized size = 0.93

$$\frac{-\frac{3\left(a+bx^3\right)^{2/3}\left(40a^3+65a^2bx^3+98ab^2x^6+293b^3x^9\right)}{x^{11}}+440\ 2^{2/3}\sqrt{3}\,b^{11/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-440\ 2^{2/3}b^{11/3}\log\left(-2\sqrt[3]{b}\,x+2^{2/3}\sqrt[3]{a+bx^3}\right)+220\ 2^{2/3}b^{11/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{1320a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(40*a^3 + 65*a^2*b*x^3 + 98*a*b^2*x^6 + 293*b^3*x^9)
)/x^11 + 440*2^(2/3)*Sqrt[3]*b^(11/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x
 + 2^(2/3)*(a + b*x^3)^(1/3))] - 440*2^(2/3)*b^(11/3)*Log[-2*b^(1/3)*x + 2^
```

```
(2/3)*(a + b*x^3)^(1/3)] + 220*2^(2/3)*b^(11/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)
*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(1320*a^4*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{12}\left(-bd\,x^{3}+ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{-ax^{12}+bx^{15}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**12/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**12 + b*x**15), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^{12}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)), x)
```

3.600 $\int \frac{x^7 (a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=512

$$-\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d}-\frac{x^5(a+bx^3)^{2/3}}{7bd}+\frac{2^{2/3}a^{7/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{8/3}d}+\frac{a^{7/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right.}{\sqrt[3]{2}\sqrt{3}\,b^{8/3}d}$$

```
[Out] -9/28*a*x^2*(b*x^3+a)^(2/3)/b^2/d-1/7*x^5*(b*x^3+a)^(2/3)/b/d-19/28*a^2*x^2
*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/b^2/d/(b*x^3+a)^(1/
3)+1/12*a^(7/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/b^(
8/3)/d+1/6*a^(7/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/b^(8/3)/d-1/3*2^(2/3)*a^(7/
3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(8/3)/d-1/4*a^(7/3)*
ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1
/3))*2^(2/3)/b^(8/3)/d+1/3*2^(2/3)*a^(7/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)
+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(8/3)/d*3^(1/2)+1/6*a^(7/3)*arctan(
1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/b^(8/3
)/d*3^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {489, 596, 598, 372, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{2^{2/3}a^{7/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{8/3}d}+\frac{a^{7/3}\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,b^{8/3}d}+\frac{a^{7/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}\,b^{8/3}d}-\frac{2^{2/3}a^{7/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{8/3}d}-\frac{a^{7/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\,b^{8/3}d}+\frac{a^{7/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}\,b^{8/3}d}-\frac{19a^2x^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{28b^2d\sqrt[3]{a+bx^3}}-\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d}-\frac{x^5(a+bx^3)^{2/3}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-9*a*x^2*(a + b*x^3)^(2/3))/(28*b^2*d) - (x^5*(a + b*x^3)^(2/3))/(7*b*d) +
 (2^(2/3)*a^(7/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)
^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(8/3)*d) + (a^(7/3)*ArcTan[(1 + (2^(1/3)*(a^(1
/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*b^(8/3)*d)
- (19*a^2*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x
^3)/a)])/(28*b^2*d*(a + b*x^3)^(1/3)) + (a^(7/3)*Log[((a^(1/3) - b^(1/3)*x)
^2*(a^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*b^(8/3)*d) + (a^(7/3)*Log[1 + (2^(
2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/
3)*x))/(a + b*x^3)^(1/3)]])/(3*2^(1/3)*b^(8/3)*d) - (2^(2/3)*a^(7/3)*Log[1 +
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]])/(3*b^(8/3)*d) - (a^(7/
```

```
3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^
3)^(1/3))/a^(1/3)]])/(2*2^(1/3)*b^(8/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
```

```
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x^8(a+bx^3)^{2/3}\,F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 7.18, size = 147, normalized size = 0.29

$$\frac{-5(9a^2x^2+13abx^5+4b^2x^8)+45a^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+38abx^5\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{140b^2d\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-5*(9*a^2*x^2 + 13*a*b*x^5 + 4*b^2*x^8) + 45*a^2*x^2*(1 + (b*x^3)/a)^(1/3)
*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] + 38*a*b*x^5*(1 + (b*x
^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(140*b^2*
d*(a + b*x^3)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^7(b\,x^3+a)^{\frac{2}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^7(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**7*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] int((x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.601 $\int \frac{x^4 \left(a+bx^3\right)^{2/3}}{ad-bdx^3} \, dx$

Optimal. Leaf size=485

$$-\frac{x^2(a+bx^3)^{2/3}}{4bd}+\frac{2^{2/3}a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}d}+\frac{a^{4/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,b^{5/3}d}-\frac{3ax^2\sqrt[3]{1+\frac{b}{}}}{4b}$$

[Out] -1/4*x^2*(b*x^3+a)^(2/3)/b/d-3/4*a*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/b/d/(b*x^3+a)^(1/3)+1/12*a^(4/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/b^(5/3)/d+1/6*a^(4/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/b^(5/3)/d-1/3*2^(2/3)*a^(4/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(5/3)/d-1/4*a^(4/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*2^(2/3)/b^(5/3)/d+1/3*2^(2/3)*a^(4/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d*3^(1/2)+1/6*a^(4/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/b^(5/3)/d*3^(1/2)

Rubi [A]

time = 0.29, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {489, 598, 372, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{2^{2/3}a^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}d}+\frac{a^{4/3}\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,b^{5/3}d}+\frac{a^{4/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}\,b^{5/3}d}-\frac{2^{2/3}a^{4/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{5/3}d}-\frac{a^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\,b^{5/3}d}+\frac{a^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}\,b^{5/3}d}-\frac{3ax^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4bd\sqrt[3]{a+bx^3}}-\frac{x^2(a+bx^3)^{2/3}}{4bd}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]

[Out] -1/4*(x^2*(a + b*x^3)^(2/3))/(b*d) + (2^(2/3)*a^(4/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(5/3)*d) + (a^(4/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*b^(5/3)*d) - (3*a*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(4*b*d*(a + b*x^3)^(1/3)) + (a^(4/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*b^(5/3)*d) + (a^(4/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(1/3)*b^(5/3)*d) - (2^(2/3)*a^(4/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(5/3)*d) - (a^(4/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(1/3)])/(2*2^(1/3)*b^(5/3)*d)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
```

```
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^5(a+bx^3)^{2/3}\,F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.70, size = 127, normalized size = 0.26

$$\frac{x^2\left(-5(a+bx^3)+5a\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+6bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}{20bd\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(-5*(a + b*x^3) + 5*a*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3,
 -((b*x^3)/a), (b*x^3)/a] + 6*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3
, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(20*b*d*(a + b*x^3)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4(b\,x^3+a)^{\frac{2}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^4(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**4*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] int((x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.602 $\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=457

$$\frac{2^{2/3}\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}+\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,b^{2/3}d}-\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

```
[Out] -1/2*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/d/(b*x^3+a)
^(1/3)+1/12*a^(1/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)
/b^(2/3)/d+1/6*a^(1/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/b^(2/3)/d-1/3*2^(2/3)*a
^(1/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(2/3)/d-1/4*a^(1
/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/
a^(1/3))*2^(2/3)/b^(2/3)/d+1/3*2^(2/3)*a^(1/3)*arctan(1/3*(1-2*2^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d*3^(1/2)+1/6*a^(1/3)*arc
tan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/b^
(2/3)/d*3^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {495, 372, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{2^{2/3}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}+\frac{\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,b^{2/3}d}+\frac{\sqrt[3]{a}\,\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}\,b^{2/3}d}-\frac{2^{2/3}\sqrt[3]{a}\,\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{2/3}d}-\frac{\sqrt[3]{a}\,\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\,b^{2/3}d}+\frac{\sqrt[3]{a}\,\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}\,b^{2/3}d}-\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (2^(2/3)*a^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)*d) + (a^(1/3)*ArcTan[(1 + (2^(1/3)*(a^(1/
3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*b^(2/3)*d) -
 (x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])
/(2*d*(a + b*x^3)^(1/3)) + (a^(1/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a^(1/3) +
 b^(1/3)*x))/a]])/(6*2^(1/3)*b^(2/3)*d) + (a^(1/3)*Log[1 + (2^(2/3)*(a^(1/3)
 + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b
*x^3)^(1/3)]])/(3*2^(1/3)*b^(2/3)*d) - (2^(2/3)*a^(1/3)*Log[1 + (2^(1/3)*(a^
(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]])/(3*b^(2/3)*d) - (a^(1/3)*Log[(b^(1/
3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(
1/3)]])/(2*2^(1/3)*b^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
```

```
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx &= \frac{(a+bx^3)^{2/3}\int \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^2(a+bx^3)^{2/3}\,F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 9.79, size = 63, normalized size = 0.14

$$\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), (b*x^3
)/a])/(2*d*(a + b*x^3)^(1/3))
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x(b\,x^3+a)^{\frac{2}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] int((x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.603 $\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx$

Optimal. Leaf size=483

$$-\frac{(a+bx^3)^{2/3}}{adx}+\frac{2^{2/3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}d}+\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{2/3}d}+\frac{bx^2\sqrt[3]{1+\frac{bx^3}{a}}}{2ad\sqrt[3]{}}$$

```
[Out] -(b*x^3+a)^(2/3)/a/d/x+1/2*b*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/
3],-b*x^3/a)/a/d/(b*x^3+a)^(1/3)+1/12*b^(1/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(
1/3)+b^(1/3)*x)/a)*2^(2/3)/a^(2/3)/d+1/6*b^(1/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1
/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/
3)/a^(2/3)/d-1/3*2^(2/3)*b^(1/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)
^(1/3))/a^(2/3)/d-1/4*b^(1/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3
)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*2^(2/3)/a^(2/3)/d+1/3*2^(2/3)*b^(1/3)*ar
ctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(2/3)
/d*3^(1/2)+1/6*b^(1/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^
(1/3))*3^(1/2))*2^(2/3)/a^(2/3)/d*3^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {486, 598, 372, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{2^{2/3}\sqrt[3]{b}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}d}+\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{2/3}d}+\frac{\sqrt[3]{b}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}a^{2/3}d}-\frac{2^{2/3}\sqrt[3]{b}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{2/3}d}-\frac{\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{2/3}d}+\frac{\sqrt[3]{b}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{2/3}d}+\frac{bx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2ad\sqrt[3]{a+bx^3}}-\frac{(a+bx^3)^{2/3}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)/(a*d*x)) + (2^(2/3)*b^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(
1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(2/3)*d) + (b^(1
/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]
)/(2^(1/3)*Sqrt[3]*a^(2/3)*d) + (b*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric
2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*a*d*(a + b*x^3)^(1/3)) + (b^(1/3)*Log[
((a^(1/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*a^(2/3)*d) +
 (b^(1/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^
(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(1/3)*a^(2/3)*d) - (2
^(2/3)*b^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/
(3*a^(2/3)*d) - (b^(1/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(
2/3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(1/3)])/(2*2^(1/3)*a^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
```

```
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^2(ad-bdx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{(a+bx^3)^{2/3} F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{adx\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 20.07, size = 136, normalized size = 0.28

$$\frac{15abx^3\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2\left(5a(a+bx^3)+b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}{10a^2dx\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
[Out] (15*a*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a),
(b*x^3)/a] - 2*(5*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/
3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(10*a^2*d*x*(a + b*x^3)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^2\left(-bd\,x^3+ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^2+bx^5}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**2/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**2 + b*x**5), x)/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{x^2\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)), x)
```

3.604 $\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx$

Optimal. Leaf size=512

$$-\frac{(a+bx^3)^{2/3}}{4adx^4}-\frac{3b(a+bx^3)^{2/3}}{2a^2dx}+\frac{2^{2/3}b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}d}+\frac{b^{4/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,a^{5/3}d}$$

```
[Out] -1/4*(b*x^3+a)^(2/3)/a/d/x^4-3/2*b*(b*x^3+a)^(2/3)/a^2/d/x+3/4*b^2*x^2*(1+b
*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/a^2/d/(b*x^3+a)^(1/3)+1/
12*b^(4/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/a^(5/3)/
d+1/6*b^(4/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/a^(5/3)/d-1/3*2^(2/3)*b^(4/3)*ln
(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(5/3)/d-1/4*b^(4/3)*ln(b^
(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*
2^(2/3)/a^(5/3)/d+1/3*2^(2/3)*b^(4/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(5/3)/d*3^(1/2)+1/6*b^(4/3)*arctan(1/3*(
1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/a^(5/3)/d*3
^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {486, 597, 598, 372, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{2^{2/3}b^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}d}+\frac{b^{4/3}\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,a^{5/3}d}+\frac{b^{4/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}\,a^{5/3}d}-\frac{2^{2/3}b^{4/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{5/3}d}-\frac{b^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\,a^{5/3}d}+\frac{b^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}\,a^{5/3}d}+\frac{3b^2x^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4a^2d\sqrt[3]{a+bx^3}}-\frac{3b(a+bx^3)^{2/3}}{2a^2dx}-\frac{(a+bx^3)^{2/3}}{4adx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -1/4*(a + b*x^3)^(2/3)/(a*d*x^4) - (3*b*(a + b*x^3)^(2/3))/(2*a^2*d*x) + (2
^(2/3)*b^(4/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*a^(5/3)*d) + (b^(4/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3)
 + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a^(5/3)*d) + (
3*b^2*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(4*a^2*d*(a + b*x^3)^(1/3)) + (b^(4/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a
^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*a^(5/3)*d) + (b^(4/3)*Log[1 + (2^(2/3)*
(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x)
)/(a + b*x^3)^(1/3)]])/(3*2^(1/3)*a^(5/3)*d) - (2^(2/3)*b^(4/3)*Log[1 + (2^(
1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]])/(3*a^(5/3)*d) - (b^(4/3)*Lo
```

```
g[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1
/3))/a^(1/3)])/(2*2^(1/3)*a^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
```

```
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^5(ad-bdx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{(a+bx^3)^{2/3}\,F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4adx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 20.07, size = 148, normalized size = 0.29

$$\frac{-5a(a^2+7abx^3+6b^2x^6)+35ab^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-6b^3x^9\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{20a^3dx^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] (-5*a*(a^2 + 7*a*b*x^3 + 6*b^2*x^6) + 35*a*b^2*x^6*(1 + (b*x^3)/a)^(1/3)*Ap
pellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] - 6*b^3*x^9*(1 + (b*x^3)/
a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(20*a^3*d*x^4
*(a + b*x^3)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^5\left(-bd\,x^3+ad\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^5+bx^8}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**5/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**5 + b*x**8), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^5\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)), x)
```

3.605 $\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=127

$$\frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] 2/5*(-x^3+1)^(5/3)-1/4*(-x^3+1)^(8/3)+1/11*(-x^3+1)^(11/3)-1/12*ln(x^3+1)*2
^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x
^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 90, 57, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*(1 - x^3)^(5/3))/5 - (1 - x^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + ArcTan[(1
 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^
(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-2(1-x)^{2/3}+2(1-x)^{5/3}-(1-x)^{8/3}+\frac{1}{\sqrt[3]{1-x}\,(1+x)}\right)\,dx, x\right.\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,d\right.\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{1}{2}\text{Subst}\left(\int \frac{}{2^{2/}}\right.\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1}\right.}{2\sqrt[3]{2}}\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\text{lo}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 145, normalized size = 1.14

$$\frac{1}{220}\left(1-x^3\right)^{2/3}\left(53-38x^3+5x^6-20x^9\right)+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}-\frac{\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(53 - 38*x^3 + 5*x^6 - 20*x^9))/220 + ArcTan[(1 + 2^(2/3)*
(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^(1
/3)]/(3*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3
)]/(6*2^(1/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.02, size = 509, normalized size = 4.01

method	result	size
risch	Expression too large to display	509
trager	Expression too large to display	787

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/220*(20*x^9-5*x^6+38*x^3-53)*(x^3-1)/(-x^3+1)^(1/3)+1/6*RootOf(_Z^3-4)*ln
(-(45*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)
^2*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4
)^3*x^3-15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*RootO
f(_Z^3-4)*x^3-63*(-x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)-21*(-x^3+1)^(2/3)+105*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)+14*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))+RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((72*RootOf(RootOf(_Z^3-4)^2+6*_Z*
RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+15*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+72*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+15*RootOf(_Z^3-4)*x^3+126*(-x^3+1)^(1/3)*
RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+42*(-x^
3+1)^(2/3)-168*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-35*Root
Of(_Z^3-4))/(x+1)/(x^2-x+1))
```

Maxima [A]
time = 0.66, size = 119, normalized size = 0.94

$$\frac{1}{11}\left(-x^3+1\right)^{\frac{11}{3}}-\frac{1}{4}\left(-x^3+1\right)^{\frac{8}{3}}+\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/11*(-x^3 + 1)^(11/3) - 1/4*(-x^3 + 1)^(8/3) + 1/6*sqrt(3)*2^(2/3)*arctan(
1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 2/5*(-x^3 + 1)^(5/3)
- 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) +
 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A]
time = 2.55, size = 118, normalized size = 0.93

$$-\frac{1}{220}\left(20\,x^{9}-5\,x^{6}+38\,x^{3}-53\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+\frac{1}{6}\,\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\,\sqrt{6}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/220*(20*x^9 - 5*x^6 + 38*x^3 - 53)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6
)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12
*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2
^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [A]
time = 0.88, size = 134, normalized size = 1.06

$$-\frac{1}{11}\left(x^{3}-1\right)^{3}\left(-x^{3}+1\right)^{\frac{2}{3}}-\frac{1}{4}\left(x^{3}-1\right)^{2}\left(-x^{3}+1\right)^{\frac{2}{3}}+\frac{1}{6}\,\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{2}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/11*(x^3 - 1)^3*(-x^3 + 1)^(2/3) - 1/4*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/6
*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3)))
 + 2/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/
3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

Mupad [B]
time = 5.03, size = 133, normalized size = 1.05

$$\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2\left(1-x^3\right)^{5/3}}{5}-\frac{\left(1-x^3\right)^{8/3}}{4}+\frac{\left(1-x^3\right)^{11/3}}{11}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2*(1 - x^3)^(5/3))/5 - (1 - x
^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)
*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1/3)
 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.606 $\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=128

$$-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/2*(-x^3+1)^(2/3)+1/5*(-x^3+1)^(5/3)-1/8*(-x^3+1)^(8/3)+1/12*ln(x^3+1)*2^
(2/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^
3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 90, 57, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(2/3) + (1 - x^3)^(5/3)/5 - (1 - x^3)^(8/3)/8 - ArcTan[(1 +
2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/
3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{1-x}\,(1+x)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}}-(1-x)^{2/3}+(1-x)^{5/3}-\frac{1}{\sqrt[3]{1-x}\,(1+x)}\right)dx,x,x^3\right)\\ &= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx,\right.\\ &= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{\log(1+x^3)}{6\sqrt[3]{2}}-\frac{1}{2}\text{Subst}\left(\int \frac{}{2^{2/3}}\right.\\ &= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{\log(1+x^3)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-}\right.}{2\sqrt[3]{2}}\\ &= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\log}{}\end{aligned}$$

Mathematica [A]

time = 0.12, size = 139, normalized size = 1.09

$$\frac{1}{120}\left(-3(1-x^3)^{2/3}(17-2x^3+5x^6)-20\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-20\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+10\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\,(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-3*(1 - x^3)^(2/3)*(17 - 2*x^3 + 5*x^6) - 20*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2
^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 20*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1
/3)] + 10*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)
])/120
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.07, size = 497, normalized size = 3.88

method	result	size
trager	Expression too large to display	497
risch	Expression too large to display	790

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*x^6+1/20*x^3-17/40)*(-x^3+1)^(2/3)+1/6*RootOf(_Z^3+4)*ln((-45*RootOf(
RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-6*Root
Of(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3-15*Ro
otOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-2*RootOf(_Z^3+4)*x^3
+63*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*Roo
tOf(_Z^3+4)+21*(-x^3+1)^(2/3)+105*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+
4)+36*_Z^2)+14*RootOf(_Z^3+4))/(x+1)/(x^2-x+1))+RootOf(RootOf(_Z^3+4)^2+6*_
Z*RootOf(_Z^3+4)+36*_Z^2)*ln((72*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4
)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+15*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z
^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3-72*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(
_Z^3+4)+36*_Z^2)*x^3-15*RootOf(_Z^3+4)*x^3+126*(-x^3+1)^(1/3)*RootOf(RootOf
(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)+42*(-x^3+1)^(2/3)+16
8*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+35*RootOf(_Z^3+4))/(
x+1)/(x^2-x+1))
```

Maxima [A]
time = 0.59, size = 119, normalized size = 0.93

$$-\frac{1}{8}\left(-x^3+1\right)^{\frac{8}{3}}-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/8*(-x^3 + 1)^(8/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(2/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)*log(-2^(1/3
) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

Fricas [A]
time = 2.38, size = 137, normalized size = 1.07

$$-\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\,(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-\frac{1}{40}\left(5x^{6}-2x^{3}+17\right)\left(-x^{3}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-
x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-
1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^
(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/40*(5*x^6
- 2*x^3 + 17)*(-x^3 + 1)^(2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt[3]{-(x-1)\left(x^{2}+x+1\right)}\,(x+1)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [A]
time = 0.68, size = 127, normalized size = 0.99

$$-\frac{1}{8}\left(x^{3}-1\right)^{2}\left(-x^{3}+1\right)^{\frac{2}{3}}-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)-\frac{1}{2}\left(-x^{3}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(2/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)^(2/3)
```

Mupad [B]
time = 4.73, size = 133, normalized size = 1.04

$$\frac{{\left(1-x^3\right)}^{5/3}}{5}-\frac{{\left(1-x^3\right)}^{2/3}}{2}-\frac{2^{2/3}\,\ln\left({\left(1-x^3\right)}^{1/3}-2^{1/3}\right)}{6}-\frac{{\left(1-x^3\right)}^{8/3}}{8}-\frac{2^{2/3}\,\ln\left({\left(1-x^3\right)}^{1/3}-\frac{2^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}+\frac{2^{2/3}\,\ln\left({\left(1-x^3\right)}^{1/3}-\frac{2^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (1 - x^3)^(5/3)/5 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1
/3)))/6 - (1 - x^3)^(8/3)/8 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1
/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 + (2^(2/3)*log((1 - x^3)^(1/3) - (2^
(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.607 $\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=97

$$\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] 1/5*(-x^3+1)^(5/3)-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(2/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 90, 57, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3)^(5/3)/5 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-(1-x)^{2/3}+\frac{1}{\sqrt[3]{1-x}\,(1+x)}\right)dx, x, x^3\right)\\ &= \frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\ &= \frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) -\\ &= \frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{\sqrt[3]{2}}\\ &= \frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right.}{2\sqrt[3]{2}}\end{aligned}$$

Mathematica [A]

time = 0.09, size = 127, normalized size = 1.31

$$\frac{1}{60}\left(12\left(1-x^3\right)^{5/3}+10\ 2^{2/3}\sqrt{3}\ \tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+10\ 2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-5\ 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (12*(1 - x^3)^(5/3) + 10*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] + 10*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 5*2^(2/3)*Log[
2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/60
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.78, size = 493, normalized size = 5.08

method	result
trager	$\left(-\frac{x^3}{5}+\frac{1}{5}\right)\left(-x^3+1\right)^{\frac{2}{3}}+\frac{\mathrm{RootOf}\left(\_Z^3-4\right)\ln\left(-\frac{45\,\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-4\right)^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3-4\right)+36\_Z^2\right)^2\mathrm{RootO}}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/5*x^3+1/5)*(-x^3+1)^(2/3)+1/6*RootOf(_Z^3-4)*ln(-(45*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+6*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-15*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*RootOf(_Z^3-4)*x^3-63*(-x^3+1
)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
-21*(-x^3+1)^(2/3)+105*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
+14*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))+RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z
^3-4)+36*_Z^2)*ln((72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^
2*RootOf(_Z^3-4)^2*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z
^2)*RootOf(_Z^3-4)^3*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)*x^3+15*RootOf(_Z^3-4)*x^3+126*(-x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(Ro
otOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^(2/3)-168*RootOf(Ro
otOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-35*RootOf(_Z^3-4))/(x+1)/(x^2-x
+1))
```

Maxima [A]
time = 0.61, size = 97, normalized size = 1.00

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A]
time = 2.22, size = 106, normalized size = 1.09

$$-\frac{1}{5}\left(x^{3}-1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/5*(x^3 - 1)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(s
qrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) +
2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (
-x^3 + 1)^(1/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{8}}{\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 0.93, size = 98, normalized size = 1.01

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)
))
```

Mupad [B]
time = 4.65, size = 111, normalized size = 1.14

$$\frac{2^{2/3}\ln\left(\left(1-x^{3}\right)^{1/3}-2^{1/3}\right)}{6}+\frac{\left(1-x^{3}\right)^{5/3}}{5}+\frac{2^{2/3}\ln\left(\left(1-x^{3}\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(\left(1-x^{3}\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (1 - x^3)^(5/3)/5 + (2^(2/3)*l
og((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 -
 (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i
 + 1))/12
```

3.608 $\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=98

$$-\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/2*(-x^3+1)^(2/3)+1/12*ln(x^3+1)*2^(2/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2
^(2/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 81, 57, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(2/3) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/
3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*
2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= -\frac{1}{2}(1-x^3)^{2/3} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= -\frac{1}{2}(1-x^3)^{2/3} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) -\\
&= -\frac{1}{2}(1-x^3)^{2/3} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{\sqrt[3]{2}}\\
&= -\frac{1}{2}(1-x^3)^{2/3} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right.}{2\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 126, normalized size = 1.29

$$\frac{1}{12}\left(-6(1-x^3)^{2/3} - 2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\,(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-6*(1 - x^3)^(2/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/3)*Log[2 +
2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.82, size = 671, normalized size = 6.85

method	result	size
risch	Expression too large to display	671
trager	Expression too large to display	773

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^3-1)/(-x^3+1)^(1/3)-1/6*ln((18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-12*RootOf(RootOf(_Z^3+4)^2+6*_Z*Root
Of(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*Ro
otOf(_Z^3+4)+36*_Z^2)*x^3+12*RootOf(_Z^3+4)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z
^3+4)^2+42*(-x^3+1)^(2/3)+42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36
*_Z^2)-28*RootOf(_Z^3+4))/(x+1)/(x^2-x+1))*RootOf(_Z^3+4)-ln((18*RootOf(Roo
tOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-12*RootOf
(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3-18*Root
Of(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+12*RootOf(_Z^3+4)*x^3+
21*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+42*(-x^3+1)^(2/3)+42*RootOf(RootOf(_Z^3+
4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-28*RootOf(_Z^3+4))/(x+1)/(x^2-x+1))*RootO
f(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+RootOf(RootOf(_Z^3+4)^2+6*_
Z*RootOf(_Z^3+4)+36*_Z^2)*ln((18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4
)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+15*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z
^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)*x^3+5*RootOf(_Z^3+4)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2
+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-
35*RootOf(_Z^3+4))/(x+1)/(x^2-x+1))
```

Maxima [A]
time = 0.65, size = 97, normalized size = 0.99

$$-\frac{1}{6}\,\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\,\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-\frac{1}{2}\left(-x^{3}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

Fricas [A]
time = 3.04, size = 125, normalized size = 1.28

$$-\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\,(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-
x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-
1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^
(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 +
1)^(2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\,\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 0.96, size = 98, normalized size = 1.00

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)-\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)^(2
/3)
```

Mupad [B]
time = 4.69, size = 111, normalized size = 1.13

$$-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}-\frac{\left(1-x^3\right)^{2/3}}{2}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i
+ 1))/12 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1/3) - 2^
(1/3)))/6
```

3.609 $\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1
/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {455, 57, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int \frac{}{\sqrt[3]{2}\ldots}\right.}{}$$

$$= -\frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x\ldots}\right.}{\sqrt[3]{2}}$$

$$= \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Mathematica [A]
time = 0.06, size = 104, normalized size = 1.27

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\,(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*Log[-2 + 2^(2/
3)*(1 - x^3)^(1/3)] - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(
2/3)])/(6*2^(1/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.05, size = 760, normalized size = 9.27

method	result	size
trager	Expression too large to display	760

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^3-4)*ln(-(90*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)^2*RootOf(_Z^3-4)^2*x^3+3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+3
6*_Z^2)*RootOf(_Z^3-4)^3*x^3+90*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*x^3+3*RootOf(_Z^3-4)*x^3+126*(-x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+21*(-x^3+1)^(1/3)*RootOf(_Z^
3-4)^2-42*(-x^3+1)^(2/3)-210*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)-7*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))-1/6*ln((72*RootOf(RootOf(_Z^3-4)^
2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-3*RootOf(RootOf(_Z^3-
4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-24*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+RootOf(_Z^3-4)*x^3-126*(-x^3+1)^(1/
3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-21*(
-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+42*(-x^3+1)^(2/3)+168*RootOf(RootOf(_Z^3-4)^
2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-7*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(_Z
^3-4)-ln((72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(
_Z^3-4)^2*x^3-3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf
(_Z^3-4)^3*x^3-24*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+
RootOf(_Z^3-4)*x^3-126*(-x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^
2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-21*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+42*(-x^3+
1)^(2/3)+168*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-7*RootOf(
_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z
^2)
```

Maxima [A]
time = 0.58, size = 86, normalized size = 1.05

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A]
time = 3.74, size = 90, normalized size = 1.10

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 +
 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 +
 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 0.89, size = 87, normalized size = 1.06

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

Mupad [B]
time = 4.89, size = 100, normalized size = 1.22

$$\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3) -
 (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x
^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.610 $\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=137

$$\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}\ -\right.}{2\cdot}$$

```
[Out] -1/2*ln(x)+1/12*ln(x^3+1)*2^(2/3)+1/2*ln(1-(-x^3+1)^(1/3))-1/4*ln(2^(1/3)-(
-x^3+1)^(1/3))*2^(2/3)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {457, 88, 57, 632, 210, 31, 631}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2^(2/3)*(1 -
x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(1/3)
) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
p}, x] &&  !IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{}{1}\right.\\
&= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\text{S}}{}\\
&= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} +
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 185, normalized size = 1.35

$$\frac{1}{12}\left(4\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+4\log\left(-1+\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)+2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - 2*2^(2/3)*Sqrt[3]*ArcT
an[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[-1 + (1 - x^3)^(1/3)] - 2
*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[1 + (1 - x^3)^(1/3) + (1
 - x^3)^(2/3)] + 2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3
)^(2/3)])/12
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 8.05, size = 410, normalized size = 2.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(1/8*(I*sqrt(3)*(-1)^(1
/3) - (-1)^(1/3))^3 - 3/4*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 3
*(-x^3 + 1)^(1/3) + 1) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))
```

```
- 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2))*log(3/8
*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)*(I*
sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)
^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1
)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)
)*log(-3/8*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/
3))^2)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1
/3) - (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) + 1/3*sqrt(3)*arctan(2/3*sqrt(3)*
(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/3*log(-1/24*(I*sqrt(3)*(-1)^(1/3) - (-1
)^(1/3))^3 + (-x^3 + 1)^(1/3) - 4/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1
)^(1/3) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 2.02, size = 149, normalized size = 1.09

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3
+ 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

Mupad [B]
time = 4.80, size = 256, normalized size = 1.87

$$\frac{\ln\left(6-6\,{\left(1-x^3\right)}^{1/3}\right)}{3}+\ln\left(\left({\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^3\left(1458\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2-135\,{\left(1-x^3\right)}^{1/3}\right)-{\left(1-x^3\right)}^{1/3}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)-\ln\left(-{\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^3\left(1458\,{\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2-135\,{\left(1-x^3\right)}^{1/3}\right)-{\left(1-x^3\right)}^{1/3}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)-\frac{2^{2/3}\,\ln\left(\frac{3\,{\left(1-x^3\right)}^{1/3}}{2}-\frac{3\,2^{1/3}}{2}\right)}{6}+\frac{{\left(-1\right)}^{1/3}\,2^{2/3}\,\ln\left(\frac{3\,{\left(1-x^3\right)}^{1/3}}{2}-\frac{3\,{\left(-1\right)}^{2/3}\,2^{1/3}}{2}\right)}{6}-\frac{{\left(-1\right)}^{1/3}\,2^{2/3}\,\ln\left(-\frac{{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^3\left(135\,{\left(1-x^3\right)}^{1/3}-\frac{81\,{\left(-1\right)}^{2/3}\,2^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)}{432}-{\left(1-x^3\right)}^{1/3}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] log(6 - 6*(1 - x^3)^(1/3))/3 + log(((3^(1/2)*1i)/6 - 1/6)^3*(1458*((3^(1/2)
*1i)/6 - 1/6)^2 - 135*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6 -
 1/6) - log(- ((3^(1/2)*1i)/6 + 1/6)^3*(1458*((3^(1/2)*1i)/6 + 1/6)^2 - 135
*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*log(
(3*(1 - x^3)^(1/3))/2 - (3*2^(1/3))/2))/6 + ((-1)^(1/3)*2^(2/3)*log((3*(1 -
 x^3)^(1/3))/2 - (3*(-1)^(2/3)*2^(1/3))/2))/6 - ((-1)^(1/3)*2^(2/3)*log(- (
(3^(1/2)*1i + 1)^3*(135*(1 - x^3)^(1/3) - (81*(-1)^(2/3)*2^(1/3)*(3^(1/2)*1
i + 1)^2)/4))/432 - (1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

3.611 $\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=157

$$-\frac{(1-x^3)^{2/3}}{3x^3}-\frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log(x)}{3}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}-\frac{1}{3}\log\left(1-\sqrt[3]{1-\right.$$

```
[Out] -1/3*(-x^3+1)^(2/3)/x^3+1/3*ln(x)-1/12*ln(x^3+1)*2^(2/3)-1/3*ln(1-(-x^3+1)^
(1/3))+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-2/9*arctan(1/3*(1+2*(-x^3+1)^
(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {457, 105, 162, 57, 632, 210, 31, 631}

$$-\frac{2\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{(1-x^3)^{2/3}}{3x^3}-\frac{\log(x^3+1)}{6\sqrt[3]{2}}-\frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/3*(1 - x^3)^(2/3)/x^3 - (2*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]])/(3*S
qrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 Log[x]/3 - Log[1 + x^3]/(6*2^(1/3)) - Log[1 - (1 - x^3)^(1/3)]/3 + Log[2^(
1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x^2(1+x)}\,dx,x,x^3\right) \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{2}{3}-\frac{x}{3}}{\sqrt[3]{1-x}\,x(1+x)}\,dx,x,x^3\right) \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx,x,x^3\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+
\right. \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1-x^3}\right) \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\right.}{2\sqrt[3]{}} \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} - \frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(x)}{3}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 202, normalized size = 1.29

$$\frac{1}{36}\left(-\frac{12(1-x^3)^{2/3}}{x^3} - 8\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 6\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 8\log\left(-1+\sqrt[3]{1-x^3}\right) + 6\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 4\log\left(1+\sqrt[3]{1-x^3}+(1-x^3)^{2/3}\right) - 3\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(1 - x^3)^(2/3))/x^3 - 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[
3]] + 6*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 8*L
og[-1 + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 4*
Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Log[2 + 2^(2/3)*(1 -
 x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^4), x)
```

Fricas [A]
time = 2.43, size = 187, normalized size = 1.19

$$\frac{6\sqrt{6}\,2^{\frac{1}{6}}x^3\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}x^3\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+6\cdot 2^{\frac{2}{3}}x^3\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-8\sqrt{3}\,x^3\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+4x^3\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)-8x^3\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)-12\left(-x^3+1\right)^{\frac{2}{3}}}{36\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/36*(6*sqrt(6)*2^(1/6)*x^3*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)
*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*x^3*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3)
+ (-x^3 + 1)^(2/3)) + 6*2^(2/3)*x^3*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 8*sq
rt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 4*x^3*log((-
x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*x^3*log((-x^3 + 1)^(1/3) - 1) -
12*(-x^3 + 1)^(2/3))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [A]
time = 1.72, size = 163, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{3\,x^3}+\frac{1}{9}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 2/9*sqrt(3)*arctan
(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/3*(-x^3 + 1)^(2/3)/x^3 + 1/9*log
((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 2/9*log(abs((-x^3 + 1)^(1/3) -
1))
```

Mupad [B]
time = 4.86, size = 382, normalized size = 2.43

$$\frac{2^{2/3}\ln\left(\frac{2^{1/3}\left(\frac{2^{2/3}\left(81\,2^{1/3}-75\,{\left(1-x^3\right)}^{1/3}\right)}{6}-\frac{38}{3}\right)}{18}+\frac{16\,{\left(1-x^3\right)}^{1/3}}{27}\right)}{6}-\frac{{\left(1-x^3\right)}^{2/3}}{3\,x^3}-\frac{2\,\ln\left(\frac{344\,{\left(1-x^3\right)}^{1/3}}{243}-\frac{344}{243}\right)}{9}+\ln\left({\left(\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2\,\left(\left(\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)\,\left(1458\,{\left(\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2-75\,{\left(1-x^3\right)}^{1/3}\right)-\frac{38}{3}\right)+\frac{16\,{\left(1-x^3\right)}^{1/3}}{27}\right)\,\left(\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)-\ln\left(\frac{16\,{\left(1-x^3\right)}^{1/3}}{27}-{\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2\,\left(\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)\,\left(1458\,{\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2-75\,{\left(1-x^3\right)}^{1/3}\right)+\frac{38}{3}\right)\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)+\frac{2^{2/3}\,\ln\left(\frac{16\,{\left(1-x^3\right)}^{1/3}}{27}+\frac{2^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(\frac{2^{2/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(\frac{81\,2^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}-75\,{\left(1-x^3\right)}^{1/3}\right)}{12}-\frac{38}{3}\right)}{72}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}-\frac{2^{2/3}\,\ln\left(\frac{16\,{\left(1-x^3\right)}^{1/3}}{27}-\frac{2^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(\frac{2^{2/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(\frac{81\,2^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}-75\,{\left(1-x^3\right)}^{1/3}\right)}{12}+\frac{38}{3}\right)}{72}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((2^(1/3)*((2^(2/3)*(81*2^(1/3) - 75*(1 - x^3)^(1/3)))/6 - 38/3
))/18 + (16*(1 - x^3)^(1/3))/27))/6 - (1 - x^3)^(2/3)/(3*x^3) - (2*log((344
*(1 - x^3)^(1/3))/243 - 344/243))/9 + log(((3^(1/2)*1i)/9 + 1/9)^2*(((3^(1/
2)*1i)/9 + 1/9)*(1458*((3^(1/2)*1i)/9 + 1/9)^2 - 75*(1 - x^3)^(1/3)) - 38/3
) + (16*(1 - x^3)^(1/3))/27)*((3^(1/2)*1i)/9 + 1/9) - log((16*(1 - x^3)^(1/
3))/27 - ((3^(1/2)*1i)/9 - 1/9)^2*(((3^(1/2)*1i)/9 - 1/9)*(1458*((3^(1/2)*1
i)/9 - 1/9)^2 - 75*(1 - x^3)^(1/3)) + 38/3))*((3^(1/2)*1i)/9 - 1/9) + (2^(2
/3)*log((16*(1 - x^3)^(1/3))/27 + (2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(2/3)*(3^
(1/2)*1i - 1)*((81*2^(1/3)*(3^(1/2)*1i - 1)^2)/4 - 75*(1 - x^3)^(1/3)))/12
- 38/3))/72)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((16*(1 - x^3)^(1/3))/27 -
(2^(1/3)*(3^(1/2)*1i + 1)^2*((2^(2/3)*(3^(1/2)*1i + 1)*((81*2^(1/3)*(3^(1/2
)*1i + 1)^2)/4 - 75*(1 - x^3)^(1/3)))/12 + 38/3))/72)*(3^(1/2)*1i + 1))/12
```

3.612 $\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=154

$$-\frac{1}{3}x\left(1-x^3\right)^{2/3}+\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/3*x*(-x^3+1)^(2/3)-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/
3))*2^(2/3)-1/3*ln(x+(-x^3+1)^(1/3))+2/9*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*
3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(
2/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {490, 544, 245, 384}

$$\frac{2\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{1}{3}\left(1-x^3\right)^{2/3}x-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}-\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/3*(x*(1 - x^3)^(2/3)) + (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/
(3*Sqrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(
2*2^(1/3)) - Log[x + (1 - x^3)^(1/3)]/3
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \text{Subst}\left(\int \frac{x^6}{(1+x^3)^2(1+2x^3)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{1}{3}x(1-x^3)^{2/3} + \frac{1}{3}\text{Subst}\left(\int \frac{1-x^3}{(1+x^3)(1+2x^3)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{1}{3}x(1-x^3)^{2/3} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx\right.\\
&= -\frac{1}{3}x(1-x^3)^{2/3} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{2}{9}\text{Subst}\left(\int \frac{2-x}{1-x+x}\right.\\
&= -\frac{1}{3}x(1-x^3)^{2/3} - \frac{2}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{9}\text{Subst}\left(\int\right.\\
&= -\frac{1}{3}x(1-x^3)^{2/3} + \frac{1}{9}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{2}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right.\\
&= -\frac{1}{3}x(1-x^3)^{2/3} + \frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{1}{9}\log\left(\right.
\end{aligned}$$

Mathematica [A]

time = 0.39, size = 220, normalized size = 1.43

$$\frac{1}{36}\left(-12x\left(1-x^3\right)^{2/3}+8\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{1-x^3}}\right)-6\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-8\log\left(x+\sqrt[3]{1-x^3}\right)+6\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+4\log\left(x^2-x\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)-3\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-12*x*(1 - x^3)^(2/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3
))] - 6*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] -
 8*Log[x + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]
+ 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Log[-2*x^2 +
 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [A]
time = 2.20, size = 201, normalized size = 1.31

$$-\frac{1}{3}\left(-x^3+1\right)^{\frac{2}{3}}x-\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}x-2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6\,x}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)+\frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x}\right)-\frac{2}{9}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{9}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*(-x^3 + 1)^(2/3)*x - 1/6*sqrt(6)*2^(1/6)*arctan(-1/6*2^(1/6)*(sqrt(6)*
2^(1/3)*x - 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) + 1/6*2^(2/3)*log((2^(1/3)*x + (
-x^3 + 1)^(1/3))/x) - 1/12*2^(2/3)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^(1
/3)*x + (-x^3 + 1)^(2/3))/x^2) + 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqr
```

```
t(3)*(-x^3 + 1)^(1/3))/x) - 2/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/9*log((x^
2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(1-x^3\right)}^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.613 $\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=135

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{1}{2}\log\left(x+\sqrt[3]{1\right.$$

```
[Out] 1/12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/2*ln(x+(
-x^3+1)^(1/3))-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6*a
rctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {494, 245, 384}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/
3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3)) + Log[x + (1 - x^3)^(
1/3)]/2
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \text{Subst}\left(\int \frac{x^3}{(1+x^3)(1+2x^3)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) +$$

$$= \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x,\right.$$

$$= -\frac{1}{6}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{2}{(1\cdots}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{6}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}}-\right.$$

Mathematica [A]
time = 0.27, size = 205, normalized size = 1.52

$$\frac{1}{12}\left(-4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{1-x^3}}\right)+2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+4\log\left(x+\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log\left(x^2-x\sqrt[3]{1-x^3}+(1-x^3)^{2/3}\right)+2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\,(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] + 2*2^(2/3)*Sqrt[3]
*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] + 4*Log[x + (1 - x^3)^(1
/3)] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[x^2 - x*(1 - x^
3)^(1/3) + (1 - x^3)^(2/3)] + 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3
) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(x^3/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 7.21, size = 452, normalized size = 3.35

$$\frac{1}{12}\cdot 2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)\log\left(-\frac{x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{3}-6\cdot 2^{\frac{1}{3}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}+8x-24\left(-x^{3}+1\right)^{\frac{1}{3}}}{8x}\right)-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)-2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\right)\log\left(-\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+2^{\frac{1}{3}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}-8\left(-x^{3}+1\right)^{\frac{1}{3}}\right)}{8x}\right)-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\right)\log\left(\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)-2^{\frac{1}{3}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}+8\left(-x^{3}+1\right)^{\frac{1}{3}}\right)}{8x}\right)-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{3}\log\left(\frac{x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{3}+32x+24\left(-x^{3}+1\right)^{\frac{1}{3}}}{24x}\right)-\frac{1}{6}\log\left(\frac{x^{2}-\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(-1/8*(x*(I*sqrt(3)*(-1
)^(1/3) - (-1)^(1/3))^3 - 6*2^(1/3)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
 + 8*x - 24*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-
1)^(1/3)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
))*log(-3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(
1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2^(1/3)*x*(I*sqrt(3)*(-1)^
(1/3) - (-1)^(1/3))^2 - 8*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(
-1)^(1/3) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) -
 (-1)^(1/3))^2))*log(3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(
1/3) - (-1)^(1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) - 2^(1/3)*x*(I*
sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 8*(-x^3 + 1)^(1/3))/x) - 1/3*sqrt(3)*a
rctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*log(1/24*(x*(I
*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^3 + 32*x + 24*(-x^3 + 1)^(1/3))/x) - 1/6*
log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.614 $\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=88

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {384}

$$-\frac{\mathrm{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \right. \\
&= \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \\
&= -\frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{} \\
&= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 114, normalized size = 1.30

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\,(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[2
*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^
(1/3)*(1 - x^3)^(2/3)])/2^(1/3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.60, size = 931, normalized size = 10.58

method	result	size
trager	Expression too large to display	931

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((9*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(
```

```
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(
2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*
x-5*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+9*RootOf(_Z^3-4)*x^3
+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+10*x*(-x^3+1)^
(2/3)-3*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2))/(x+1)/(x^2-x+1))-1/6*ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^
2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^
3-4)^2*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*
_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootOf(_Z^3-4)-6*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^
3-4)-ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^
3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf
(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)*RootOf(_Z^3-4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2-24*(-x^3+
1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4
)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*
_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(67) = 134$.
time = 7.58, size = 253, normalized size = 2.88

$$-\frac{1}{18}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{6}\,(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}\right)}{6\,(109x^9-105x^6+3x^3+1)}\right)+\frac{1}{18}\cdot 2^{\frac{2}{3}}\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(x^3+1)+6\,(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)-\frac{1}{36}\cdot 2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(19x^6-16x^3+1)-12\,(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 1
2*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
```

```
x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.615 $\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=105

$$-\frac{(1-x^3)^{2/3}}{2x^2}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\log(1+x^3)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/2*(-x^3+1)^(2/3)/x^2+1/12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(
1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3
)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {491, 12, 384}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\log(x^3+1)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}-\frac{(1-x^3)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(2/3)/x^2 + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[-(2^(1/3)*x) - (1 -
x^3)^(1/3)]/(2*2^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \text{Subst}\left(\int \frac{1+x^3}{x^3(1+2x^3)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{(1-x^3)^{2/3}}{2x^2} - \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{(1-x^3)^{2/3}}{2x^2} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{2}{1-\sqrt[}\right.$$

$$= -\frac{(1-x^3)^{2/3}}{2x^2} - \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x,\right.$$

$$= -\frac{(1-x^3)^{2/3}}{2x^2} + \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \text{Su}$$

$$= -\frac{(1-x^3)^{2/3}}{2x^2} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}} + \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} -$$

Mathematica [A]
time = 0.22, size = 141, normalized size = 1.34

$$\frac{1}{12}\left(-\frac{6(1-x^3)^{2/3}}{x^2}+2\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\,(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((-6*(1 - x^3)^(2/3))/x^2 + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/
3)*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/
3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 11.17, size = 931, normalized size = 8.87

method	result	size
risch	Expression too large to display	931
trager	Expression too large to display	1156

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^3-1)/x^2/(-x^3+1)^(1/3)+1/6*RootOf(_Z^3+4)*ln(-(3*RootOf(RootOf(_Z^3
+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+54*RootOf(RootOf(_Z
^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-12*(-x^3+1)^(2/
3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x-
5*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)*x^2-6*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4
)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2+RootOf(_Z^3+4)*x^3+18*R
ootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)-
RootOf(_Z^3+4)-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(x+
1)/(x^2-x+1))-1/6*ln(-(3*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^
2)*RootOf(_Z^3+4)^3*x^3-36*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_
Z^2)^2*RootOf(_Z^3+4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*
RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x-RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)*
x^2-30*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*
RootOf(_Z^3+4)*x^2-3*RootOf(_Z^3+4)*x^3+36*RootOf(RootOf(_Z^3+4)^2+6*_Z*Roo
tOf(_Z^3+4)+36*_Z^2)*x^3-10*x*(-x^3+1)^(2/3)+RootOf(_Z^3+4)-12*RootOf(RootO
f(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3+4)-l
n(-(3*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3
*x^3-36*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+
4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_
Z^2)*RootOf(_Z^3+4)^2*x-RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)*x^2-30*(-x^3+1)^(1/
3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2-
3*RootOf(_Z^3+4)*x^3+36*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2
)*x^3-10*x*(-x^3+1)^(2/3)+RootOf(_Z^3+4)-12*RootOf(RootOf(_Z^3+4)^2+6*_Z*Ro
otOf(_Z^3+4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf
(_Z^3+4)+36*_Z^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(81) = 162$.
time = 5.92, size = 307, normalized size = 2.92

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^2\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-12\sqrt{6}\,(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)\right)}{6(109x^9-105x^6+3x^3+1)}\right)-2\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^2\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}x^2-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3+1)+6(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^2\log\left(-\frac{3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(19x^6-16x^3+1)+12(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+18(-x^3+1)^{\frac{2}{3}}}{36x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^2*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/
3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3)*
(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^
6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1/3)*x^
2*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x^3
+ 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*x^2*log(-(3*2^
(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(19*x^6
- 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 18*
(-x^3 + 1)^(2/3))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(1-x^3\right)}^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.616 $\int \frac{1}{x^6 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=124

$$-\frac{(1-x^3)^{2/3}}{5x^5}+\frac{(1-x^3)^{2/3}}{5x^2}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/5*(-x^3+1)^(2/3)/x^5+1/5*(-x^3+1)^(2/3)/x^2-1/12*ln(x^3+1)*2^(2/3)+1/4*l
n(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)
^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {491, 597, 12, 384}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}-\frac{(1-x^3)^{2/3}}{5x^5}+\frac{(1-x^3)^{2/3}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/5*(1 - x^3)^(2/3)/x^5 + (1 - x^3)^(2/3)/(5*x^2) - ArcTan[(1 - (2*2^(1/3)
*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3))
+ Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \text{Subst}\left(\int \frac{(1+x^3)^2}{x^6(1+2x^3)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{x^6}+\frac{1}{1+2x^3}\right)\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{(1-x^3)^{5/3}}{5x^5} + \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{(1-x^3)^{5/3}}{5x^5} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2}{1-\sqrt[3]{}}\right.$$

$$= -\frac{(1-x^3)^{5/3}}{5x^5} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x,\right.$$

$$= -\frac{(1-x^3)^{5/3}}{5x^5} - \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Su}}{}$$

$$= -\frac{(1-x^3)^{5/3}}{5x^5} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} +$$

Mathematica [A]
time = 0.25, size = 142, normalized size = 1.15

$$\frac{1}{60}\left(-\frac{12(1-x^3)^{5/3}}{x^5}-10\ 2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+10\ 2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-5\ 2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(1 - x^3)^(5/3))/x^5 - 10*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(
2/3)*(1 - x^3)^(1/3))] + 10*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] - 5*
2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/
60
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 11.07, size = 955, normalized size = 7.70

method	result	size
risch	Expression too large to display	955
trager	Expression too large to display	1161

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(x^6-2*x^3+1)/x^5/(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)*ln((9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*
RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2
)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-5*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x
^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*R
ootOf(_Z^3-4)*x^2+9*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2)*x^3+10*x*(-x^3+1)^(2/3)-3*RootOf(_Z^3-4)-6*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/6*ln(-(6*RootO
f(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12
*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf
(_Z^3-4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2-24*(-x^3+1)^(1/3)*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z
^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-
x^3+1)^(2/3)+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-ln(-(6*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*Root
Of(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+(-x^3+1
)^(1/3)*RootOf(_Z^3-4)^2*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z
```

```
*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootO
f(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^
2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(95) = 190.
time = 8.03, size = 283, normalized size = 2.28

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}x^5\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{6}\,(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}\right)}{6(109x^9-105x^6+3x^3+1)}\right)-10\cdot 2^{\frac{2}{3}}x^5\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(x^3+1)+6(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)+5\cdot 2^{\frac{2}{3}}x^5\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(19x^6-16x^3+1)-12(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)-36(x^3-1)(-x^3+1)^{\frac{2}{3}}}{180x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] -1/180*(10*sqrt(6)*2^(1/6)*x^5*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7
 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3
 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105
*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*x^5*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2
^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*x^5*log((3*
2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(
2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 36*(x^3 - 1)*(-x^3 + 1)
^(2/3))/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(1-x^3\right)}^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.617 $\int \frac{1}{x^9\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=141

$$-\frac{(1-x^3)^{2/3}}{8x^8}+\frac{(1-x^3)^{2/3}}{20x^5}-\frac{17(1-x^3)^{2/3}}{40x^2}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\log(1+x^3)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/8*(-x^3+1)^(2/3)/x^8+1/20*(-x^3+1)^(2/3)/x^5-17/40*(-x^3+1)^(2/3)/x^2+1/
12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1
/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {491, 597, 12, 384}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\log(x^3+1)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}-\frac{(1-x^3)^{2/3}}{8x^8}+\frac{(1-x^3)^{2/3}}{20x^5}-\frac{17(1-x^3)^{2/3}}{40x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/8*(1 - x^3)^(2/3)/x^8 + (1 - x^3)^(2/3)/(20*x^5) - (17*(1 - x^3)^(2/3))/
(40*x^2) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqr
t[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2
^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \text{Subst}\left(\int \frac{(1+x^3)^3}{x^9\,(1+2x^3)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{x^9}+\frac{1}{x^6}+\frac{1}{x^3}+\frac{1}{-1-2x^3}\right)\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{(1-x^3)^{2/3}}{2x^2}-\frac{(1-x^3)^{5/3}}{5x^5}-\frac{(1-x^3)^{8/3}}{8x^8}+\text{Subst}\left(\int \frac{1}{-1-2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x}}\right.\\
&= -\frac{(1-x^3)^{2/3}}{2x^2}-\frac{(1-x^3)^{5/3}}{5x^5}-\frac{(1-x^3)^{8/3}}{8x^8}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1-\sqrt[3]{2}\,x}\,dx, x, \frac{}{\sqrt[3]{1}}\right.\\
&= -\frac{(1-x^3)^{2/3}}{2x^2}-\frac{(1-x^3)^{5/3}}{5x^5}-\frac{(1-x^3)^{8/3}}{8x^8}-\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{1}{2}\text{Subst}\left(\right.\\
&= -\frac{(1-x^3)^{2/3}}{2x^2}-\frac{(1-x^3)^{5/3}}{5x^5}-\frac{(1-x^3)^{8/3}}{8x^8}+\frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\\
&= -\frac{(1-x^3)^{2/3}}{2x^2}-\frac{(1-x^3)^{5/3}}{5x^5}-\frac{(1-x^3)^{8/3}}{8x^8}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\log\left(1+\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 154, normalized size = 1.09

$$\frac{1}{120}\left(-\frac{3(1-x^3)^{2/3}(5-2x^3+17x^6)}{x^8}+20\ 2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-20\ 2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+10\ 2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((-3*(1 - x^3)^(2/3)*(5 - 2*x^3 + 17*x^6))/x^8 + 20*2^(2/3)*Sqrt[3]*ArcTan[
(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 20*2^(2/3)*Log[2*x + 2^(2/3)*(
1 - x^3)^(1/3)] + 10*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/
3)*(1 - x^3)^(2/3)])/120
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 11.66, size = 645, normalized size = 4.57

method	result	size
risch	Expression too large to display	645
trager	Expression too large to display	783

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(17*x^9-19*x^6+7*x^3-5)/x^8/(-x^3+1)^(1/3)+1/6*RootOf(_Z^3+4)*ln(-(3*R
ootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+27
*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^
3+6*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*Roo
tOf(_Z^3+4)^2*x-2*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)*x^2+3*(-x^3+1)^(1/3)*Root
Of(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2-3*RootO
f(_Z^3+4)*x^3-27*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-5
*x*(-x^3+1)^(2/3)+RootOf(_Z^3+4)+9*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3
+4)+36*_Z^2))/(x+1)/(x^2-x+1))+RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+
36*_Z^2)*ln((9*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(
_Z^3+4)^3*x^3+36*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*Roo
tOf(_Z^3+4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^
3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x-4*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)*x^2-30*(
-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_
Z^3+4)*x^2+3*RootOf(_Z^3+4)*x^3+12*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3
+4)+36*_Z^2)*x^3+2*x*(-x^3+1)^(2/3)-3*RootOf(_Z^3+4)-12*RootOf(RootOf(_Z^3+
4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(109) = 218$.
time = 7.85, size = 320, normalized size = 2.27

$$-\frac{20\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\,x^8\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-12\sqrt{6}\,(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)\right)}{6(109x^9-105x^6+3x^3+1)}\right)-20\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x^8\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}x^2-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3+1)+6(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)+10\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x^8\log\left(-\frac{3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(19x^6-16x^3+1)+12(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+9(17x^6-2x^3+5)(-x^3+1)^{\frac{2}{3}}}{360x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(20*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^8*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(
2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3
)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*
x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 20*2^(2/3)*(-1)^(1/3)
*x^8*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x
^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 10*2^(2/3)*(-1)^(1/3)*x^8*log(
-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(1
9*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
 + 9*(17*x^6 - 2*x^3 + 5)*(-x^3 + 1)^(2/3))/x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^9 (1-x^3)^{1/3} (x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.618 $\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=271

$$-\frac{1}{4}x^2\left(1-x^3\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left((1-x)(1+\right.}{12\sqrt[3]{2}}$$

```
[Out] -1/4*x^2*(-x^3+1)^(2/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/24*ln((1-
x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/8*
ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-
x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {490, 21, 495, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{1}{4}\left(1-x^3\right)^{2/3}x^2+\frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/4*(x^2*(1 - x^3)^(2/3)) + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3
))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/
3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^
3])/4 + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(
1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (
2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)]/(4*2^(1/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
```

```
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{8}x^8F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};x^3,-x^3\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.02, size = 40, normalized size = 0.15

$$\frac{1}{4}x^2\left(-\left(1-x^3\right)^{2/3}+F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
[Out] (x^2*(-(1 - x^3)^(2/3) + AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]))/4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(-x^{3}+1\right)^{\frac{1}{3}} \left(x^{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(2/3)*x^7/(x^6 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\left(1-x^3\right)^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^7/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.619 $\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=254

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}-\frac{\log\left(1+\right.}{}$$

```
[Out] 1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/24*ln((1-x)*(1+x)^2)*2^(2/3)-1/12
*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+
1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/8*ln(-1+x+2^(2/3)*(-x^3+1)
^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2
^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {494, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3
])) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sq
rt[3]) + (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x
)^2]/(12*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*
(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)
^(1/3)]/(3*2^(1/3)) + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.02, size = 26, normalized size = 0.10

$$\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-x^3+1)^{\frac{1}{3}}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(2/3)*x^4/(x^6 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^4/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.620 $\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=233

$$\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-$$

```
[Out] 1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(
1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2
^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1
/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[
3]) + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^
(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{2}x^2 F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [A]
time = 0.75, size = 283, normalized size = 1.21

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)-2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{1-x^3}\right)+2\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2+(-1+x)\sqrt[3]{2-2x^3}+(1-x^3)^{2/3}\right)+\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2-2(-1+x)\sqrt[3]{2-2x^3}+4(1-x^3)^{2/3}\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^
3)^(1/3))] - 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(
1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)]
- 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) - 2*2^(2/
3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + Log[2^
(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1 - x
^3)^(2/3)])/(12*2^(1/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(171) = 342.
time = 7.98, size = 373, normalized size = 1.60

$$-\frac{1}{36}\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(x^{14}-2x^{11}-6x^{8}-2x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+12\sqrt{6}\,(-1)^{\frac{1}{3}}\left(x^{16}-33x^{13}+110x^{10}-110x^{7}+33x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+\sqrt{6}\,2^{\frac{1}{3}}\left(x^{18}+42x^{15}-417x^{12}+812x^{9}-417x^{6}+42x^{3}+1\right)\right)}{6\left(x^{18}-102x^{15}+447x^{12}-628x^{9}+447x^{6}-102x^{3}+1\right)}\right)-\frac{1}{72}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{8}-4x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^{12}-32x^{9}+78x^{6}-32x^{3}+1\right)-6\left(x^{10}-11x^{7}+11x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{12}+4x^{9}+6x^{6}+4x^{3}+1}\right)+\frac{1}{36}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12\left(-x^{3}+1\right)^{\frac{2}{3}}x^{2}-6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{6}+2x^{3}+1\right)}{x^{6}+2x^{3}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)
*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(
1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*
x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) -
 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) -
 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1
/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3
 + 1))/(x^6 + 2*x^3 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.621 $\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=270

$$-\frac{(1-x^3)^{2/3}}{x}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left((1-x)(1+x\right.}{12\sqrt[3]{2}}$$

```
[Out] -(-x^3+1)^(2/3)/x-1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/24*ln((1-x)*(1+
x)^2)*2^(2/3)-1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+
1)^(1/3))*2^(2/3)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/8*ln(-1+
x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {491, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{(1-x^3)^{2/3}}{x}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(2/3)/x) - ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqr
t[3]]/(2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sq
rt[3]]/(2*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2
- Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^
3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3
)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1
/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};x^3,-x^3\right)}{x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 20.04, size = 67, normalized size = 0.25

$$-\frac{(1-x^3)^{2/3}}{x}-x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)-\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(2/3)/x) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - (x^5*App
ellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(2/3)/(x^8 - x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \left(1-x^3\right)^{1/3} \left(x^3+1\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^2*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.622 $\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=289

$$-\frac{(1-x^3)^{2/3}}{4x^4}+\frac{(1-x^3)^{2/3}}{2x}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log((1$$

```
[Out] -1/4*(-x^3+1)^(2/3)/x^4+1/2*(-x^3+1)^(2/3)/x+1/4*x^2*hypergeom([1/3, 2/3],[
5/3],x^3)+1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)
^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*2^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3
*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/
3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {491, 21, 486, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-x^3)^{2/3}}{2x}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{(1-x^3)^{2/3}}{4x^4}+\frac{\log((1-x)(x+1)^2)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/4*(1 - x^3)^(2/3)/x^4 + (1 - x^3)^(2/3)/(2*x) + ArcTan[(1 - (2*2^(1/3)*(
1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*
(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + (x^2*Hypergeometri
c2F1[1/3, 2/3, 5/3, x^3])/4 + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 +
 (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(
6*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-
1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
```

```
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=-\frac{F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};x^3,-x^3\right)}{4x^4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 20.04, size = 76, normalized size = 0.26

$$\frac{5(1-x^3)^{2/3}(-1+2x^3)+15x^6F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+2x^9F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (5*(1 - x^3)^(2/3)*(-1 + 2*x^3) + 15*x^6*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x
^3] + 2*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/(20*x^4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(-x^3+1)^{\frac{1}{3}}(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(2/3)/(x^11 - x^5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,{\left(1-x^3\right)}^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^5*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.623 $\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=125

$$-\sqrt[3]{1-x^3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -(-x^3+1)^(1/3)+1/4*(-x^3+1)^(4/3)-1/7*(-x^3+1)^(7/3)+1/12*ln(x^3+1)*2^(1/3
)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)
^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 90, 59, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3) + (1 - x^3)^(4/3)/4 - (1 - x^3)^(7/3)/7 + ArcTan[(1 + 2^(2
/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(2/3))
- Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{(1-x)^{2/3}} - \sqrt[3]{1-x} + (1-x)^{4/3} - \frac{1}{(1-x)^{2/3}(1+x)}\right)dx, x, x^3\right.\\
&= -\sqrt[3]{1-x^3} + \frac{1}{4}(1-x^3)^{4/3} - \frac{1}{7}(1-x^3)^{7/3} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx,\right.\\
&= -\sqrt[3]{1-x^3} + \frac{1}{4}(1-x^3)^{4/3} - \frac{1}{7}(1-x^3)^{7/3} + \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx\right.}{2\ 2^{2/}}\\
&= -\sqrt[3]{1-x^3} + \frac{1}{4}(1-x^3)^{4/3} - \frac{1}{7}(1-x^3)^{7/3} + \frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-\right.}{2\ 2^{2/3}}\\
&= -\sqrt[3]{1-x^3} + \frac{1}{4}(1-x^3)^{4/3} - \frac{1}{7}(1-x^3)^{7/3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1}{6\ 2}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 137, normalized size = 1.10

$$\frac{1}{84}\left(3\sqrt[3]{1-x^3}\,(-25+x^3-4x^6)+14\sqrt[3]{2}\,\sqrt{3}\,\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-14\sqrt[3]{2}\,\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+7\sqrt[3]{2}\,\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\,(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (3*(1 - x^3)^(1/3)*(-25 + x^3 - 4*x^6) + 14*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(
2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 14*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3
)] + 7*2^(1/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/
84
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.86, size = 758, normalized size = 6.06

method	result	size
trager	Expression too large to display	758
risch	Expression too large to display	1074

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/7*x^6+1/28*x^3-25/28)*(-x^3+1)^(1/3)+1/6*RootOf(_Z^3+2)*ln((90*RootOf(R
ootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^3*x^3+12*Root
Of(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^4*x^3+15*Ro
otOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+2*Roo
tOf(_Z^3+2)^2*x^3-126*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*
(-x^3+1)^(1/3)-105*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*Roo
tOf(_Z^3+2)+21*(-x^3+1)^(2/3)-14*RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))-1/6*ln(
-(144*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)
^3*x^3+30*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+
2)^4*x^3-72*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^
3+2)*x^3-15*RootOf(_Z^3+2)^2*x^3+252*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z
^3+2)+36*_Z^2)*(-x^3+1)^(1/3)+168*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+
2)+36*_Z^2)*RootOf(_Z^3+2)-42*(-x^3+1)^(2/3)+35*RootOf(_Z^3+2)^2)/(x+1)/(x^
2-x+1))*RootOf(_Z^3+2)-ln(-(144*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)
+36*_Z^2)^2*RootOf(_Z^3+2)^3*x^3+30*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^
3+2)+36*_Z^2)*RootOf(_Z^3+2)^4*x^3-72*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_
Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3-15*RootOf(_Z^3+2)^2*x^3+252*RootOf(RootO
f(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*(-x^3+1)^(1/3)+168*RootOf(RootOf(_
Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-42*(-x^3+1)^(2/3)+35*R
ootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+
2)+36*_Z^2)
```

Maxima [A]
time = 0.52, size = 119, normalized size = 0.95

$$-\frac{1}{7}\left(-x^{3}+1\right)^{\frac{7}{3}}+\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^{3}+1\right)^{\frac{4}{3}}+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-\left(-x^{3}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/7*(-x^3 + 1)^(7/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(1/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)*log(-2^(1/3
) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

Fricas [A]
time = 3.57, size = 142, normalized size = 1.14

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+2\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-\frac{1}{28}\left(4\,x^{6}-x^{3}+25\right)\left(-x^{3}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 1/28*(4*x^6 - x^3 + 25)*(-x^3 + 1)^(1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [A]
time = 1.62, size = 127, normalized size = 1.02

$$-\frac{1}{7}\left(x^{3}-1\right)^{2}\left(-x^{3}+1\right)^{\frac{1}{3}}+\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^{3}+1\right)^{\frac{4}{3}}+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)-\left(-x^{3}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/7*(x^3 - 1)^2*(-x^3 + 1)^(1/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(1/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3)
```

Mupad [B]
time = 5.18, size = 135, normalized size = 1.08

$$\frac{(1-x^3)^{4/3}}{4}-(1-x^3)^{1/3}-\frac{2^{1/3}\ln\left(3\,2^{1/3}-3\,(1-x^3)^{1/3}\right)}{6}-\frac{(1-x^3)^{7/3}}{7}-\frac{2^{1/3}\ln\left(3\,(1-x^3)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}+\frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\,(1-x^3)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (1 - x^3)^(4/3)/4 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)^
(1/3)))/6 - (1 - x^3)^(7/3)/7 - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)
*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 + (2^(1/3)*log((3*2^(1/3)*(3^(1/
2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

3.624 $\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=98

$$\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] 1/4*(-x^3+1)^(4/3)-1/12*ln(x^3+1)*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(1/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 90, 59, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3)^(4/3)/4 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(1-x)^{2/3}(1+x)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\sqrt[3]{1-x}+\frac{1}{(1-x)^{2/3}(1+x)}\right)dx,x,x^3\right)\\
&= \frac{1}{4}(1-x^3)^{4/3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx,x,x^3\right)\\
&= \frac{1}{4}(1-x^3)^{4/3}-\frac{\log(1+x^3)}{6\ 2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx,x,\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\text{Subst}\left(\int\right.}{}\\
&= \frac{1}{4}(1-x^3)^{4/3}-\frac{\log(1+x^3)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,\right.}{2^{2/}}\\
&= \frac{1}{4}(1-x^3)^{4/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(1+x^3)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x\right.}{2\ 2^{2/3}}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 127, normalized size = 1.30

$$\frac{1}{12}\left(3\left(1-x^3\right)^{4/3}-2\sqrt[3]{2}\,\sqrt{3}\,\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\sqrt[3]{2}\,\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-\sqrt[3]{2}\,\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\,\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (3*(1 - x^3)^(4/3) - 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))
/Sqrt[3]] + 2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(1/3)*Log[2 + 2
^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.65, size = 546, normalized size = 5.57

method	result	size
trager	Expression too large to display	546
risch	Expression too large to display	1064

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*x^3+1/4)*(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+3
6*_Z^2)*ln((144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*Root
Of(_Z^3-2)^3*x^3-6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*Roo
tOf(_Z^3-2)^4*x^3-24*x^3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^
2)*RootOf(_Z^3-2)+x^3*RootOf(_Z^3-2)^2+168*RootOf(RootOf(_Z^3-2)^2+6*_Z*Roo
tOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)-252*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2
)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-7*RootOf(_Z^3-2)^2-42*(-x^3+1)^(1/3)*RootO
f(_Z^3-2)+42*(-x^3+1)^(2/3))/(x+1)/(x^2-x+1))+1/6*RootOf(_Z^3-2)*ln(-(180*R
ootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+
6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^4*x^3
+90*x^3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)
+3*x^3*RootOf(_Z^3-2)^2-210*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*
_Z^2)*RootOf(_Z^3-2)+252*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf
(_Z^3-2)+36*_Z^2)-7*RootOf(_Z^3-2)^2+42*(-x^3+1)^(1/3)*RootOf(_Z^3-2)-42*(-
x^3+1)^(2/3))/(x+1)/(x^2-x+1))
```

Maxima [A]
time = 0.54, size = 97, normalized size = 0.99

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)
^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A]
time = 3.14, size = 114, normalized size = 1.16

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-\frac{1}{4}\left(x^{3}-1\right)\left(-x^{3}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) +
 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 +
1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3)) - 1
/4*(x^3 - 1)*(-x^3 + 1)^(1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 1.07, size = 98, normalized size = 1.00

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^{3}+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)
^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3
)))
```

Mupad [B]
time = 4.98, size = 113, normalized size = 1.15

$$\frac{2^{1/3}\,\ln\left(\frac{{\left(1-x^3\right)}^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}+\frac{{\left(1-x^3\right)}^{4/3}}{4}+\frac{2^{1/3}\,\ln\left(3\,{\left(1-x^3\right)}^{1/3}-\frac{3\,2^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}-\frac{2^{1/3}\,\ln\left(\frac{3\,2^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+3\,{\left(1-x^3\right)}^{1/3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log((1 - x^3)^(1/3)/2 - 2^(1/3)/2))/6 + (1 - x^3)^(4/3)/4 + (2^(1/
3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1)
)/12 - (2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^
(1/2)*1i + 1))/12
```

3.625 $\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=95

$$-\sqrt[3]{1-x^3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -(-x^3+1)^(1/3)+1/12*ln(x^3+1)*2^(1/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/
3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 81, 59, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*S
qrt[3]) + Log[1 + x^3]/(6*2^(2/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= -\sqrt[3]{1-x^3}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= -\sqrt[3]{1-x^3}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\text{Subst}\left(\int \frac{}{2^{2/3}}\right.}{}\\
&= -\sqrt[3]{1-x^3}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\right.}{2^{2/3}}\\
&= -\sqrt[3]{1-x^3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 126, normalized size = 1.33

$$\frac{1}{12}\left(-12\sqrt[3]{1-x^3}+2\sqrt[3]{2}\,\sqrt{3}\,\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-2\sqrt[3]{2}\,\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+\sqrt[3]{2}\,\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-12*(1 - x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(1/3)*Log[2 +
 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.50, size = 708, normalized size = 7.45

method	result	size
trager	Expression too large to display	708
risch	Expression too large to display	1566

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*ln((-1
8*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^3*x
^3+12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^4
*x^3+9*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*
x^3-6*RootOf(_Z^3+2)^2*x^3+21*(-x^3+1)^(2/3)-21*(-x^3+1)^(1/3)*RootOf(_Z^3+
2)-21*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)+1
4*RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))-1/6*ln((-36*RootOf(RootOf(_Z^3+2)^2+6*
_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^3*x^3-30*RootOf(RootOf(_Z^3+2)^
2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^4*x^3-6*RootOf(RootOf(_Z^3+2)
^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3-5*RootOf(_Z^3+2)^2*x^3+4
2*(-x^3+1)^(2/3)-42*(-x^3+1)^(1/3)*RootOf(_Z^3+2)+42*RootOf(RootOf(_Z^3+2)^
2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)+35*RootOf(_Z^3+2)^2)/(x+1)/(x
^2-x+1))*RootOf(_Z^3+2)-ln((-36*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)
+36*_Z^2)^2*RootOf(_Z^3+2)^3*x^3-30*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^
3+2)+36*_Z^2)*RootOf(_Z^3+2)^4*x^3-6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z
^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3-5*RootOf(_Z^3+2)^2*x^3+42*(-x^3+1)^(2/3)-
42*(-x^3+1)^(1/3)*RootOf(_Z^3+2)+42*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^
3+2)+36*_Z^2)*RootOf(_Z^3+2)+35*RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))*RootOf(R
ootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)
```

Maxima [A]
time = 0.52, size = 97, normalized size = 1.02

$$\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\left(-x^3+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) - 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

Fricas [A]
time = 2.54, size = 130, normalized size = 1.37

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)-\left(-x^3+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 (-x^3 + 1)^(1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 1.54, size = 98, normalized size = 1.03

$$\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)-\left(-x^3+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) - 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3)
```

Mupad [B]
time = 4.89, size = 113, normalized size = 1.19

$$-\frac{2^{1/3}\,\ln\left(\frac{{\left(1-x^3\right)}^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}-{\left(1-x^3\right)}^{1/3}-\frac{2^{1/3}\,\ln\left(3\,{\left(1-x^3\right)}^{1/3}-\frac{3\,2^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}+\frac{2^{1/3}\,\ln\left(\frac{3\,2^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+3\,{\left(1-x^3\right)}^{1/3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1
i + 1))/12 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)*
(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log((1 - x^3)^(1/3)/2
- 2^(1/3)/2))/6
```

3.626 $\int \frac{x^2}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1
/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {455, 59, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) - Log[1
+ x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,d\right.}{2\sqrt[3]{2}}$$

$$= -\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-}\right.}{2^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [A]
time = 0.07, size = 102, normalized size = 1.23

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + \log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[-2 +
2^(2/3)*(1 - x^3)^(1/3)] + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x
^3)^(2/3)])/2^(2/3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.44, size = 531, normalized size = 6.40

method	result	size
trager	Expression too large to display	531

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*ln(-(6*RootOf(RootOf(_
Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^4*x^3-144*RootOf(RootO
f(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^3*x^3-x^3*RootOf(
_Z^3-2)^2+24*x^3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootO
f(_Z^3-2)+7*RootOf(_Z^3-2)^2-168*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2
)+36*_Z^2)*RootOf(_Z^3-2)+42*(-x^3+1)^(1/3)*RootOf(_Z^3-2)+252*(-x^3+1)^(1/
3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-42*(-x^3+1)^(2/3))/
(x+1)/(x^2-x+1))+1/6*RootOf(_Z^3-2)*ln(-(180*RootOf(RootOf(_Z^3-2)^2+6*_Z*R
ootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+6*RootOf(RootOf(_Z^3-2)^2+6*_
Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^4*x^3+90*x^3*RootOf(RootOf(_Z^3-2)
^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)+3*x^3*RootOf(_Z^3-2)^2-210*R
ootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)+252*(-x^
3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-7*RootOf(_Z
^3-2)^2+42*(-x^3+1)^(1/3)*RootOf(_Z^3-2)-42*(-x^3+1)^(2/3))/(x+1)/(x^2-x+1)
)
```

Maxima [A]
time = 0.54, size = 86, normalized size = 1.04

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A]
time = 2.51, size = 98, normalized size = 1.18

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) +
 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 +
1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 1.29, size = 87, normalized size = 1.05

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

Mupad [B]
time = 5.05, size = 102, normalized size = 1.23

$$\frac{2^{1/3}\ln\left(3\,2^{1/3}-3\,{\left(1-x^3\right)}^{1/3}\right)}{6}+\frac{2^{1/3}\ln\left(3\,{\left(1-x^3\right)}^{1/3}-\frac{3\,2^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}-\frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+3\,{\left(1-x^3\right)}^{1/3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)^(1/3)))/6 + (2^(1/3)*log(3*(1 - x^3)^(
1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log(
(3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

3.627 $\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=137

$$-\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}\right.}{}$$

```
[Out] -1/2*ln(x)+1/12*ln(x^3+1)*2^(1/3)+1/2*ln(1-(-x^3+1)^(1/3))-1/4*ln(2^(1/3)-(
-x^3+1)^(1/3))*2^(1/3)-1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {457, 88, 59, 632, 210, 31, 631}

$$-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1
 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(2
/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
p}, x] &&  !IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{2}\text{Subst}\left(\int\right.\\
&= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} -\\
&= -\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right.}{6\ 2^{2/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 185, normalized size = 1.35

$$\frac{1}{12}\left(-4\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+4\log\left(-1+\sqrt[3]{1-x^3}\right)-2\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)+\sqrt[3]{2}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[-1 + (1 - x^3)^(1/3)] -
2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[1 + (1 - x^3)^(1/3) + (
1 - x^3)^(2/3)] + 2^(1/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^
3)^(2/3)])/12
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-x^{3}+1\right)^{\frac{2}{3}}\left(x^{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x), x)
```

Fricas [A]
time = 2.83, size = 182, normalized size = 1.33

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+2\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
```

```
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((
-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 1.24, size = 149, normalized size = 1.09

$$\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)-\frac{1}{6}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/12*2^(1/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 +
 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

Mupad [B]
time = 4.92, size = 344, normalized size = 2.51

$$\frac{\ln\left(5-5\left(1-x^{3}\right)^{1/3}\right)}{3}-\frac{2^{1/3}\ln\left(6\left(1-x^{3}\right)^{1/3}-\frac{2^{1/3}\left(\frac{2^{2/3}\left(243\,2^{1/3}+243\left(1-x^{3}\right)^{1/3}\right)}{36}+9\right)}{6}\right)}{6}+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^{2}\left(243\left(1-x^{3}\right)^{1/3}+243-\sqrt{3}\,243i\right)+9\right)+6\left(1-x^{3}\right)^{1/3}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(6\left(1-x^{3}\right)^{1/3}-\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)\left(\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^{2}\left(243\left(1-x^{3}\right)^{1/3}+243+\sqrt{3}\,243i\right)+9\right)\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)+\frac{(-1)^{1/3}\,2^{1/3}\ln\left(6\left(1-x^{3}\right)^{1/3}-\frac{(-1)^{1/3}\,2^{1/3}\left(\frac{(-1)^{2/3}\,2^{2/3}\left(243\,(-1)^{1/3}\,2^{1/3}-243\left(1-x^{3}\right)^{1/3}\right)}{36}-9\right)}{6}\right)}{6}-\frac{(-1)^{1/3}\,2^{1/3}\ln\left(6\left(1-x^{3}\right)^{1/3}-\frac{(-1)^{1/3}\,2^{1/3}\left(1+\sqrt{3}\,1i\right)\left(\frac{(-1)^{2/3}\,2^{2/3}\left(1+\sqrt{3}\,1i\right)^{2}\left(243\left(1-x^{3}\right)^{1/3}+\frac{243\,(-1)^{1/3}\,2^{1/3}\left(1+\sqrt{3}\,1i\right)}{2}\right)}{144}+9\right)}{12}\right)\left(1+\sqrt{3}\,1i\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] log(5 - 5*(1 - x^3)^(1/3))/3 - (2^(1/3)*log(6*(1 - x^3)^(1/3) - (2^(1/3)*((
2^(2/3)*(243*2^(1/3) + 243*(1 - x^3)^(1/3)))/36 + 9))/6))/6 + log(((3^(1/2)
*1i)/6 - 1/6)*(((3^(1/2)*1i)/6 - 1/6)^2*(243*(1 - x^3)^(1/3) - 3^(1/2)*243i
 + 243) + 9) + 6*(1 - x^3)^(1/3))*((3^(1/2)*1i)/6 - 1/6) - log(6*(1 - x^3)^
(1/3) - ((3^(1/2)*1i)/6 + 1/6)*(((3^(1/2)*1i)/6 + 1/6)^2*(3^(1/2)*243i + 24
3*(1 - x^3)^(1/3) + 243) + 9))*((3^(1/2)*1i)/6 + 1/6) + ((-1)^(1/3)*2^(1/3)
*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(((-1)^(2/3)*2^(2/3)*(243*(-1)
^(1/3)*2^(1/3) - 243*(1 - x^3)^(1/3)))/36 - 9))/6))/6 - ((-1)^(1/3)*2^(1/3)
*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*(((-1)^(2/3)*
2^(2/3)*(3^(1/2)*1i + 1)^2*(243*(1 - x^3)^(1/3) + (243*(-1)^(1/3)*2^(1/3)*(
3^(1/2)*1i + 1))/2))/144 + 9))/12)*(3^(1/2)*1i + 1))/12
```

3.628 $\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} \, dx$

Optimal. Leaf size=158

$$-\frac{\sqrt[3]{1-x^3}}{3x^3}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(x)}{6}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)$$

[Out] -1/3*(-x^3+1)^(1/3)/x^3+1/6*ln(x)-1/12*ln(x^3+1)*2^(1/3)-1/6*ln(1-(-x^3+1)^(1/3))+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)+1/9*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {457, 105, 162, 59, 632, 210, 31, 631}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -1/3*(1 - x^3)^(1/3)/x^3 + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[3]) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[x]/6 - Log[1 + x^3]/(6*2^(2/3)) - Log[1 - (1 - x^3)^(1/3)]/6 + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4 \left(1-x^3\right)^{2/3} \left(1+x^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x^2(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt[3]{1-x^3}}{3x^3} - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{1}{3}-\frac{2x}{3}}{(1-x)^{2/3}x(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt[3]{1-x^3}}{3x^3} - \frac{1}{9}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+\dots}\right.$$
$$= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \dots$$
$$= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\dots\right.}{2\ 2^{2/\dots}}$$
$$= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(x)}{6} - \dots$$

Mathematica [A]
time = 0.19, size = 202, normalized size = 1.28

$$\frac{1}{36}\left(-\frac{12\sqrt[3]{1-x^3}}{x^3} + 4\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 6\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 4\log\left(-1+\sqrt[3]{1-x^3}\right) + 6\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 2\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right) - 3\sqrt[3]{2}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(1 - x^3)^(1/3))/x^3 + 4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[
3]] - 6*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 4*L
og[-1 + (1 - x^3)^(1/3)] + 6*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2*
Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(1/3)*Log[2 + 2^(2/3)*(1 -
 x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(-x^3+1\right)^{\frac{2}{3}} \left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^4), x)
```

Fricas [A]
time = 2.99, size = 195, normalized size = 1.23

$$-\frac{12\cdot 4^{\frac{1}{6}}\sqrt{3}\,x^{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)+3\cdot 4^{\frac{2}{3}}x^{3}\log\left(4^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)-6\cdot 4^{\frac{2}{3}}x^{3}\log\left(-4^{\frac{2}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-8\sqrt{3}\,x^{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-4\,x^{3}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)+8\,x^{3}\log\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right)+24\left(-x^{3}+1\right)^{\frac{1}{3}}}{72\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(12*4^(1/6)*sqrt(3)*x^3*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1
)^(1/3) + 4^(1/3)*sqrt(3))) + 3*4^(2/3)*x^3*log(4^(2/3)*(-x^3 + 1)^(1/3) +
2*(-x^3 + 1)^(2/3) + 2*4^(1/3)) - 6*4^(2/3)*x^3*log(-4^(2/3) + 2*(-x^3 + 1)
^(1/3)) - 8*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3))
- 4*x^3*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 8*x^3*log((-x^3 + 1)
^(1/3) - 1) + 24*(-x^3 + 1)^(1/3))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(- \left(x - 1\right) \left(x^{2} + x + 1\right)\right)^{\frac{2}{3}} \left(x + 1\right) \left(x^{2} - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [A]
time = 0.90, size = 163, normalized size = 1.03

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)-\frac{\left(-x^{3}+1\right)^{\frac{1}{3}}}{3\,x^{3}}+\frac{1}{18}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{9}\log\left(\left|\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/12*2^(1
/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3
)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/3*(-x^3 + 1)^(1/3)/x^3 + 1/18*l
og((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 1/9*log(abs((-x^3 + 1)^(1/3)
- 1))
```

Mupad [B]
time = 5.07, size = 368, normalized size = 2.33

$$\frac{2^{1/3}\ln\left(\frac{10\,{\left(1-x^3\right)}^{1/3}}{9}-\frac{2^{1/3}\,\left(\frac{2^{2/3}\,\left(243\,2^{1/3}+27\,{\left(1-x^3\right)}^{1/3}\right)}{36}-\frac{25}{3}\right)}{6}\right)}{6}-\frac{{\left(1-x^3\right)}^{1/3}}{3\,x^3}-\frac{\ln\left(\frac{31\,{\left(1-x^3\right)}^{1/3}}{243}-\frac{31}{243}\right)}{9}-\ln\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1{}\mathrm{i}}{18}\right)\,\left(\left({\left(-\frac{1}{18}+\frac{\sqrt{3}\,1{}\mathrm{i}}{18}\right)}^2\,\left(27\,{\left(1-x^3\right)}^{1/3}+81-\sqrt{3}\,81{}\mathrm{i}\right)-\frac{25}{3}\right)+\frac{10\,{\left(1-x^3\right)}^{1/3}}{9}\right)\right)\,\left(-\frac{1}{18}+\frac{\sqrt{3}\,1{}\mathrm{i}}{18}\right)+\ln\left(\frac{10\,{\left(1-x^3\right)}^{1/3}}{9}-\left(\frac{1}{18}+\frac{\sqrt{3}\,1{}\mathrm{i}}{18}\right)\,\left(\left({\left(\frac{1}{18}+\frac{\sqrt{3}\,1{}\mathrm{i}}{18}\right)}^2\,\left(27\,{\left(1-x^3\right)}^{1/3}+81+\sqrt{3}\,81{}\mathrm{i}\right)-\frac{25}{3}\right)\right)\right)\,\left(\frac{1}{18}+\frac{\sqrt{3}\,1{}\mathrm{i}}{18}\right)+\frac{2^{1/3}\,\ln\left(\frac{10\,{\left(1-x^3\right)}^{1/3}}{9}-\frac{2^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(\frac{2^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(\frac{243\,2^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+27\,{\left(1-x^3\right)}^{1/3}\right)}{144}-\frac{25}{3}\right)}{12}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}-\frac{2^{1/3}\,\ln\left(\frac{10\,{\left(1-x^3\right)}^{1/3}}{9}-\frac{2^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(\frac{2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(\frac{243\,2^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-27\,{\left(1-x^3\right)}^{1/3}\right)}{144}+\frac{25}{3}\right)}{12}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*((2^(2/3)*(243*2^(1/3) + 27*
(1 - x^3)^(1/3)))/36 - 25/3))/6))/6 - (1 - x^3)^(1/3)/(3*x^3) - log((31*(1
- x^3)^(1/3))/243 - 31/243)/9 - log((((3^(1/2)*1i)/18 - 1/18)*((((3^(1/2)*1i)
/18 - 1/18)^2*(27*(1 - x^3)^(1/3) - 3^(1/2)*81i + 81) - 25/3) + (10*(1 - x^
3)^(1/3))/9))*((3^(1/2)*1i)/18 - 1/18) + log((10*(1 - x^3)^(1/3))/9 - ((3^(1
/2)*1i)/18 + 1/18)*((((3^(1/2)*1i)/18 + 1/18)^2*(3^(1/2)*81i + 27*(1 - x^3)^
(1/3) + 81) - 25/3))*((3^(1/2)*1i)/18 + 1/18) + (2^(1/3)*log((10*(1 - x^3)^
(1/3))/9 - (2^(1/3)*(3^(1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i - 1)^2*((243*2^(
1/3)*(3^(1/2)*1i - 1))/2 + 27*(1 - x^3)^(1/3)))/144 - 25/3))/12)*(3^(1/2)*1
i - 1))/12 - (2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*(3^(1/2)*1i + 1
)*((2^(2/3)*(3^(1/2)*1i + 1)^2*((243*2^(1/3)*(3^(1/2)*1i + 1))/2 - 27*(1 -
x^3)^(1/3)))/144 + 25/3))/12)*(3^(1/2)*1i + 1))/12
```

3.629 $\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx$

Optimal. Leaf size=160

$$-\frac{1}{3}x^2\sqrt[3]{1-x^3}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{1}{6}\log\left(-x-\sqrt[3]{1-x^3}\right)-$$

```
[Out] -1/3*x^2*(-x^3+1)^(1/3)+1/12*ln(x^3+1)*2^(1/3)+1/6*ln(-x-(-x^3+1)^(1/3))-1/
4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)+1/9*arctan(1/3*(1-2*x/(-x^3+1)^(1/3
))*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*
2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {490, 598, 337, 503}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\ 2^{2/3}}-\frac{1}{3}\sqrt[3]{1-x^3}\,x^2$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/3*(x^2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3
*Sqrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sq
rt[3]) + Log[1 + x^3]/(6*2^(2/3)) + Log[-x - (1 - x^3)^(1/3)]/6 - Log[-(2^(
1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \text{Subst}\left(\int \frac{x^7}{(1+x^3)^2(1+2x^3)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{x(2+x^3)}{(1+x^3)(1+2x^3)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{3(1+x)} + \frac{-1-x}{3(1-x+x^2)} + \frac{3x}{1+2x^3}\right) dx, x, \right.$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\text{Subst}\left(\int \frac{-1-x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-}}\right.$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{}{\sqrt[3]{1}}\right.$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{18}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x}}\right.$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{18}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x}}\right.$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{1}{18}\log\left(\right.$$

Mathematica [A]
time = 0.41, size = 222, normalized size = 1.39

$$\frac{1}{36}\left(-12x^2\sqrt[3]{1-x^3} + 4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right) - 6\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) + 4\log\left(x+\sqrt[3]{1-x^3}\right) - 6\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) - 2\log\left(x^2-x\sqrt[3]{1-x^3}+(1-x^3)^{2/3}\right) + 3\sqrt[3]{2}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-12*x^2*(1 - x^3)^(1/3) + 4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1
/3))] - 6*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))]
 + 4*Log[x + (1 - x^3)^(1/3)] - 6*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)
] - 2*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 3*2^(1/3)*Log[-2*x^2
 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(-x^3+1)^{\frac{2}{3}}(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^7/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [A]
time = 3.28, size = 232, normalized size = 1.45

$$-\frac{1}{3}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}+\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\,x\right)}{6\,x}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x-2\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,x^{2}+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+2\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)+\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{9}\log\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{18}\log\left(\frac{x^{2}-\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*(-x^3 + 1)^(1/3)*x^2 + 1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/
6)*(4^(2/3)*sqrt(3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 4^(1/3)*sqrt(3)*x)/x) + 1
/12*4^(2/3)*(-1)^(1/3)*log(-(4^(2/3)*(-1)^(1/3)*x - 2*(-x^3 + 1)^(1/3))/x)
- 1/24*4^(2/3)*(-1)^(1/3)*log((2*4^(1/3)*(-1)^(2/3)*x^2 + 4^(2/3)*(-1)^(1/3
)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) + 1/9*sqrt(3)*arctan(-1/3*(
sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/9*log((x + (-x^3 + 1)^(1/3))
/x) - 1/18*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{(1-x^3)^{2/3}(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.630 $\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=139

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right)+\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{\ }\right.}{2\ 2^{2/3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(1/3)-1/2*ln(-x-(-x^3+1)^(1/3))+1/4*ln(-2^(1/3)*x-(-x^3+1
)^(1/3))*2^(1/3)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6
*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {494, 337, 503}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(2/
3)) - Log[-x - (1 - x^3)^(1/3)]/2 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*
2^(2/3))
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{}{\sqrt[3]{1-}}\right.$$

$$= -\frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx\right.$$

$$= \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(1+\frac{2}{(1-}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\right.$$

Mathematica [A]
time = 0.27, size = 206, normalized size = 1.48

$$\frac{1}{12}\left(-4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{1-x^3}}\right)+2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-4\log\left(x+\sqrt[3]{1-x^3}\right)+2\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2\log\left(x^2-x\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)-\sqrt[3]{2}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] + 2*2^(1/3)*Sqrt[3]
*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 4*Log[x + (1 - x^3)^(1
/3)] + 2*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2*Log[x^2 - x*(1 - x^
```

```
3)^(1/3) + (1 - x^3)^(2/3)] - 2^(1/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3
) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int(x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [A]
time = 2.70, size = 197, normalized size = 1.42

$$\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}\,x-4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6\,x}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2-4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+2\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x}\right)-\frac{1}{3}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{6}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] 1/6*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(4^(1/3)*sqrt(3)*x - 4^(2/3)*sqrt(3
)*(-x^3 + 1)^(1/3))/x) + 1/12*4^(2/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/
x) - 1/24*4^(2/3)*log((2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3
 + 1)^(2/3))/x^2) - 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 +
1)^(1/3))/x) - 1/3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1
)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(1-x^3\right)}^{2/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.631 $\int \frac{x}{(1-x^3)^{2/3}(1+x^3)} \, dx$

Optimal. Leaf size=88

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] 1/12*ln(x^3+1)*2^(1/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan
(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {503}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) +
Log[1 + x^3]/(6*2^(2/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x}}\right.}{3\sqrt[3]{2}}$$

$$= -\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{2^2}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}}{\sqrt[3]{1-}}\right.}{3\ 2^{2/3}}$$

Mathematica [A]
time = 0.21, size = 114, normalized size = 1.30

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\,(1-x^3)^{2/3}\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[2*x +
 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3
)*(1 - x^3)^(2/3)])/(6*2^(2/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.50, size = 954, normalized size = 10.84

method	result	size
trager	Expression too large to display	954

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*ln((6*RootOf(RootOf(_Z
^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^4*x^3-18*RootOf(RootOf(
```

```
_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^3*x^3+12*(-x^3+1)^(
2/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^2*
x+RootOf(_Z^3+2)^2*x^3-3*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^
2)*RootOf(_Z^3+2)*x^3+(-x^3+1)^(1/3)*RootOf(_Z^3+2)*x^2-24*(-x^3+1)^(1/3)*R
ootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^2+x*(-x^3+1)^(2/3)-Ro
otOf(_Z^3+2)^2+3*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootO
f(_Z^3+2))/(x+1)/(x^2-x+1))-1/6*ln(-(18*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf
(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^4*x^3+36*RootOf(RootOf(_Z^3+2)^2+6*_Z*Root
Of(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^3*x^3+24*(-x^3+1)^(2/3)*RootOf(RootOf(
_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^2*x-9*RootOf(_Z^3+2)^
2*x^3-18*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2
)*x^3-10*(-x^3+1)^(1/3)*RootOf(_Z^3+2)*x^2-48*(-x^3+1)^(1/3)*RootOf(RootOf(
_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^2-10*x*(-x^3+1)^(2/3)+3*RootOf(_Z^
3+2)^2+6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2
))/(x+1)/(x^2-x+1))*RootOf(_Z^3+2)-ln(-(18*RootOf(RootOf(_Z^3+2)^2+6*_Z*Roo
tOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^4*x^3+36*RootOf(RootOf(_Z^3+2)^2+6*_Z*R
ootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^3*x^3+24*(-x^3+1)^(2/3)*RootOf(Root
Of(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^2*x-9*RootOf(_Z^3+
2)^2*x^3-18*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^
3+2)*x^3-10*(-x^3+1)^(1/3)*RootOf(_Z^3+2)*x^2-48*(-x^3+1)^(1/3)*RootOf(Root
Of(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^2-10*x*(-x^3+1)^(2/3)+3*RootOf(
_Z^3+2)^2+6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^
3+2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(67) = 134.
time = 5.66, size = 283, normalized size = 3.22

$$-\frac{1}{18}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\,(19\,x^8-16\,x^5+x^2)(-x^3+1)^{\frac{1}{3}}-12\,\sqrt{3}\,(-1)^{\frac{1}{3}}\,(5\,x^7+4\,x^4-x)(-x^3+1)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\,(71\,x^9-111\,x^6+33\,x^3-1)\right)}{6\,(109\,x^9-105\,x^6+3\,x^3+1)}\right)+\frac{1}{36}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{3\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(-x^3+1)^{\frac{1}{3}}x^2-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(x^3+1)-6\,(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)-\frac{1}{72}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(5\,x^4-x)(-x^3+1)^{\frac{2}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(19\,x^6-16\,x^3+1)-24\,(2\,x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2\,x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(-1
)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(1/3)*(5
*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33
```

```
*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) + 1/36*4^(2/3)*(-1)^(1/3)*log(-
(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^3 + 1) -
 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/72*4^(2/3)*(-1)^(1/3)*log((6*4^(1/3)*
(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6 - 16*x
^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(x/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.632 $\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} \, dx$

Optimal. Leaf size=103

$$-\frac{\sqrt[3]{1-x^3}}{x}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -(-x^3+1)^(1/3)/x-1/12*ln(x^3+1)*2^(1/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*
2^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {491, 503}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\sqrt[3]{1-x^3}}{x}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(1/3)/x) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/
(2^(2/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[-(2^(1/3)*x) - (1 - x^3)
^(1/3)]/(2*2^(2/3))
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
```

```
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{1+x^3}{x^2\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} - \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2^{2/3}x\cdots}\right.}{3\sqrt[3]{2}\cdots}$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-\cdots}}\right.}{6\ 2^{2/3}}$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} - \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \cdots$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \cdots$$

Mathematica [A]
time = 0.29, size = 143, normalized size = 1.39

$$-\frac{\sqrt[3]{1-x^3}}{x} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{-x+2^{2/3}\sqrt[3]{1-x^3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)}{3\ 2^{2/3}} - \frac{\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(1/3)/x) - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(
2^(2/3)*Sqrt[3]) + Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[-2*
x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 14.11, size = 1162, normalized size = 11.28

method	result	size
trager	Expression too large to display	1162
risch	Expression too large to display	1386

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^3+1)^(1/3)/x-1/6*ln((62113615200*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(
_Z^3-2)+7056*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+2398446876*RootOf(RootOf(_Z^3-2)^
2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2)*RootOf(_Z^3-2)^4*x^3+43114942080*RootOf(_
Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2)*(-x^3+1)^(
2/3)*x-496908921600*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_
Z^3-2)+7056*_Z^2)^2-19187575008*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+84
*_Z*RootOf(_Z^3-2)+7056*_Z^2)+40669629000*RootOf(RootOf(_Z^3-2)^2+84*_Z*Roo
tOf(_Z^3-2)+7056*_Z^2)*RootOf(_Z^3-2)*x^3+1570411645*x^3*RootOf(_Z^3-2)^2+1
73768795928*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2)*(-x^3+1
)^(1/3)*x^2+1026546240*RootOf(_Z^3-2)*(-x^3+1)^(1/3)*x^2-1042129902*x*(-x^3
+1)^(2/3)-17007299400*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^
2)*RootOf(_Z^3-2)-656717597*RootOf(_Z^3-2)^2)/(x+1)/(x^2-x+1))*RootOf(_Z^3-
2)-14*ln((62113615200*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^
2)^2*RootOf(_Z^3-2)^3*x^3+2398446876*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_
Z^3-2)+7056*_Z^2)*RootOf(_Z^3-2)^4*x^3+43114942080*RootOf(_Z^3-2)^2*RootOf(
RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2)*(-x^3+1)^(2/3)*x-496908921
600*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2
)^2-19187575008*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-
2)+7056*_Z^2)+40669629000*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056
*_Z^2)*RootOf(_Z^3-2)*x^3+1570411645*x^3*RootOf(_Z^3-2)^2+173768795928*Root
Of(RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2)*(-x^3+1)^(1/3)*x^2+1026
546240*RootOf(_Z^3-2)*(-x^3+1)^(1/3)*x^2-1042129902*x*(-x^3+1)^(2/3)-170072
99400*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2)*RootOf(_Z^3-2
)-656717597*RootOf(_Z^3-2)^2)/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+84*_
Z*RootOf(_Z^3-2)+7056*_Z^2)+1/6*RootOf(_Z^3-2)*ln((949032*RootOf(RootOf(_Z^
3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+24612*RootOf(
RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2)*RootOf(_Z^3-2)^4*x^3-62092
8*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2)*
(-x^3+1)^(2/3)*x-7592256*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+84*_Z*Roo
tOf(_Z^3-2)+7056*_Z^2)^2-196896*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+84
*_Z*RootOf(_Z^3-2)+7056*_Z^2)-598794*RootOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_
Z^3-2)+7056*_Z^2)*RootOf(_Z^3-2)*x^3-15529*x^3*RootOf(_Z^3-2)^2-1680084*Roo
tOf(RootOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2)*(-x^3+1)^(1/3)*x^2-147
84*RootOf(_Z^3-2)*(-x^3+1)^(1/3)*x^2+5217*x*(-x^3+1)^(2/3)+79086*RootOf(Roo
tOf(_Z^3-2)^2+84*_Z*RootOf(_Z^3-2)+7056*_Z^2)*RootOf(_Z^3-2)+2051*RootOf(_Z
^3-2)^2)/(x+1)/(x^2-x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(81) = 162$.
time = 8.34, size = 272, normalized size = 2.64

$$\frac{4 \cdot 4^{\frac{1}{6}} \sqrt{3}\, x \arctan\left(\frac{4^{\frac{1}{6}}\left(6 \cdot 4^{\frac{2}{3}} \sqrt{3}\,(19\,x^8-16\,x^5+x^2)(-x^3+1)^{\frac{1}{3}}+12\,\sqrt{3}\,(5\,x^7+4\,x^4-x)(-x^3+1)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\,(71\,x^9-111\,x^6+33\,x^3-1)\right)}{6\,(109\,x^9-105\,x^6+3\,x^3+1)}\right)+2 \cdot 4^{\frac{2}{3}} x \log\left(\frac{3 \cdot 4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}} x^2+6\,(-x^3+1)^{\frac{2}{3}} x+4^{\frac{1}{3}}(x^3+1)}{x^3+1}\right)-4^{\frac{2}{3}} x \log\left(\frac{6 \cdot 4^{\frac{1}{3}}(5\,x^4-x)(-x^3+1)^{\frac{2}{3}}+4^{\frac{2}{3}}(19\,x^6-16\,x^3+1)-24\,(2\,x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2\,x^3+1}\right)-72\,(-x^3+1)^{\frac{1}{3}}}{72\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(4*4^(1/6)*sqrt(3)*x*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(19*x^8 - 1
6*x^5 + x^2)*(-x^3 + 1)^(1/3) + 12*sqrt(3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(
2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6
+ 3*x^3 + 1)) + 2*4^(2/3)*x*log((3*4^(2/3)*(-x^3 + 1)^(1/3)*x^2 + 6*(-x^3 +
 1)^(2/3)*x + 4^(1/3)*(x^3 + 1))/(x^3 + 1)) - 4^(2/3)*x*log((6*4^(1/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 72*(-x^3 + 1)^(1/3))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-\left(x - 1\right)\left(x^{2} + x + 1\right)\right)^{\frac{2}{3}} \left(x + 1\right)\left(x^{2} - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(1-x^3\right)}^{2/3}\,\left(x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.633 $\int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)} \, dx$

Optimal. Leaf size=124

$$-\frac{\sqrt[3]{1-x^3}}{4x^4}+\frac{\sqrt[3]{1-x^3}}{4x}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/4*(-x^3+1)^(1/3)/x^4+1/4*(-x^3+1)^(1/3)/x+1/12*ln(x^3+1)*2^(1/3)-1/4*ln(
-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(
1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {491, 597, 12, 503}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt[3]{1-x^3}}{4x}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\ 2^{2/3}}-\frac{\sqrt[3]{1-x^3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/4*(1 - x^3)^(1/3)/x^4 + (1 - x^3)^(1/3)/(4*x) - ArcTan[(1 - (2*2^(1/3)*x
)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(2/3)) -
Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \text{Subst}\left(\int \frac{\left(1+x^3\right)^2}{x^5\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{x^5}+\frac{x}{1+2x^3}\right)dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{4/3}}{4x^4}+\text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{4/3}}{4x^4}-\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}+\frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2^{2/3}x \cdots}\right.}{3\sqrt[3]{2} \cdots}\\
&= -\frac{\left(1-x^3\right)^{4/3}}{4x^4}-\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-\cdots}}\right.}{6\ 2^{2/3}}\\
&= -\frac{\left(1-x^3\right)^{4/3}}{4x^4}+\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}+\cdots\\
&= -\frac{\left(1-x^3\right)^{4/3}}{4x^4}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 141, normalized size = 1.14

$$\frac{1}{12}\left(-\frac{3(1-x^3)^{4/3}}{x^4}-2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+\sqrt[3]{2}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((-3*(1 - x^3)^(4/3))/x^4 - 2*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/
3)*(1 - x^3)^(1/3))] - 2*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(1/
3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 14.06, size = 1186, normalized size = 9.56

method	result	size
trager	Expression too large to display	1186
risch	Expression too large to display	1478

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(x^3-1)/x^4*(-x^3+1)^(1/3)+112*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z
^3+2)+451584*_Z^2)*ln(-(-2957791200*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_
Z^3+2)+451584*_Z^2)*RootOf(_Z^3+2)^4*x^3+4459389516288*RootOf(RootOf(_Z^3+2
)^2+672*_Z*RootOf(_Z^3+2)+451584*_Z^2)^2*RootOf(_Z^3+2)^3*x^3+172459768320*
(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+
2)+451584*_Z^2)*x+23662329600*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+2)
+451584*_Z^2)*RootOf(_Z^3+2)^4-35675116130304*RootOf(RootOf(_Z^3+2)^2+672*_
Z*RootOf(_Z^3+2)+451584*_Z^2)^2*RootOf(_Z^3+2)^3+242081125*RootOf(_Z^3+2)^2
*x^3-364979796720*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+2)+451584*_Z^2
)*RootOf(_Z^3+2)*x^3+521064951*(-x^3+1)^(1/3)*RootOf(_Z^3+2)*x^2-3449195366
40*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+2)+451584*_Z^2
)*x^2+521064951*x*(-x^3+1)^(2/3)-101233925*RootOf(_Z^3+2)^2+152627914992*Ro
otOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+2)+451584*_Z^2)*RootOf(_Z^3+2))/(x
+1)/(x^2-x+1))-1/6*ln((-19187575008*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_
Z^3+2)+451584*_Z^2)*RootOf(_Z^3+2)^4*x^3-8918779032576*RootOf(RootOf(_Z^3+2
)^2+672*_Z*RootOf(_Z^3+2)+451584*_Z^2)^2*RootOf(_Z^3+2)^3*x^3+344919536640*
(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+
2)+451584*_Z^2)*x+153500600064*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+2
)+451584*_Z^2)*RootOf(_Z^3+2)^4+71350232260608*RootOf(RootOf(_Z^3+2)^2+672*
_Z*RootOf(_Z^3+2)+451584*_Z^2)^2*RootOf(_Z^3+2)^3-1513305767*RootOf(_Z^3+2)
^2*x^3-703415608224*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+2)+451584*_Z
^2)*RootOf(_Z^3+2)*x^3-2068676142*(-x^3+1)^(1/3)*RootOf(_Z^3+2)*x^2-6898390
```

```
73280*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+2)+451584*_
Z^2)*x^2-2068676142*x*(-x^3+1)^(2/3)+199870573*RootOf(_Z^3+2)^2+92903948256
*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+2)+451584*_Z^2)*RootOf(_Z^3+2))
/(x+1)/(x^2-x+1))*RootOf(_Z^3+2)-112*ln((-19187575008*RootOf(RootOf(_Z^3+2)
^2+672*_Z*RootOf(_Z^3+2)+451584*_Z^2)*RootOf(_Z^3+2)^4*x^3-8918779032576*Ro
otOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+2)+451584*_Z^2)^2*RootOf(_Z^3+2)^3
*x^3+344919536640*(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6
72*_Z*RootOf(_Z^3+2)+451584*_Z^2)*x+153500600064*RootOf(RootOf(_Z^3+2)^2+67
2*_Z*RootOf(_Z^3+2)+451584*_Z^2)*RootOf(_Z^3+2)^4+71350232260608*RootOf(Roo
tOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+2)+451584*_Z^2)^2*RootOf(_Z^3+2)^3-1513305
767*RootOf(_Z^3+2)^2*x^3-703415608224*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf
(_Z^3+2)+451584*_Z^2)*RootOf(_Z^3+2)*x^3-2068676142*(-x^3+1)^(1/3)*RootOf(_
Z^3+2)*x^2-689839073280*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootO
f(_Z^3+2)+451584*_Z^2)*x^2-2068676142*x*(-x^3+1)^(2/3)+199870573*RootOf(_Z^
3+2)^2+92903948256*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_Z^3+2)+451584*_Z^
2)*RootOf(_Z^3+2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3+2)^2+672*_Z*RootOf(_
Z^3+2)+451584*_Z^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(95) = 190$.
time = 5.68, size = 312, normalized size = 2.52

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\,x^4\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\,(19\,x^8-16\,x^5+x^2)\,(-x^3+1)^{\frac{1}{3}}-12\sqrt{3}\,(-1)^{\frac{1}{3}}\,(5\,x^7+4\,x^4-x)\,(-x^3+1)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\,(71\,x^9-111\,x^6+33\,x^3-1)\right)}{6\,(109\,x^9-105\,x^6+3\,x^3+1)}\right)-2\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x^4\log\left(-\frac{3\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(-x^3+1)^{\frac{1}{3}}x^2-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(x^3+1)-6\,(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x^4\log\left(\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(5\,x^4-x)\,(-x^3+1)^{\frac{2}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(19\,x^6-16\,x^3+1)-24\,(2\,x^5-x^2)\,(-x^3+1)^{\frac{1}{3}}}{x^6+2\,x^3+1}\right)-18\,(x^3-1)(-x^3+1)^{\frac{1}{3}}}{72\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^4*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt
(3)*(-1)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(
1/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x
^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x
^4*log(-(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^
3 + 1) - 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 4^(2/3)*(-1)^(1/3)*x^4*log((6*4
^(1/3)*(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6
 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 18
*(x^3 - 1)*(-x^3 + 1)^(1/3))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,{\left(1-x^3\right)}^{2/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.634 $\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=291

$$-\frac{1}{2}x\sqrt[3]{1-x^3}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}}-\frac{\log\left(1+\frac{2^{2/3}(1-x}{(1-x^3)^{2/}}\right.}{6\,2}$$

```
[Out] -1/2*x*(-x^3+1)^(1/3)+1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)-1/12*l
n(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/
6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/24*ln(2*2^(1/3)+(1-x)^2/(-x^
3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3
)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/3)
*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {490, 420, 493, 298, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{1}{2}\sqrt[3]{1-x^3}\,x+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\,2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\,2^{2/3}}-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{12\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(x*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)
)/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(6*
2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/
(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(
3*2^(2/3)) - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/
(1 - x^3)^(1/3)]/(12*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx=\frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.08, size = 115, normalized size = 0.40

$$\frac{1}{2}x\sqrt[3]{1-x^3}\left(-1-\frac{4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)}{(1+x^3)\left(-4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)+x^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};x^3,-x^3\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)\right)\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x*(1 - x^3)^(1/3)*(-1 - (4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 + x
^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/3,
 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 12.19, size = 694, normalized size = 2.38

method	result	size
trager	Expression too large to display	694
risch	Expression too large to display	988

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*(-x^3+1)^(1/3)+1/4*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^
2)*ln(-(12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-
2)^4*x^3+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z
^3-2)^3*x^3+2*RootOf(_Z^3-2)^2*x^6+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z
^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^6-18*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+
3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^2*x^2+18*(-x^3+1)^(1/3)*RootOf(R
```

```
ootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^4-4*x^3*RootOf(_Z^3-2)^2-6*Ro
otOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^3-12*x^2
*(-x^3+1)^(2/3)-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-
2)+9*_Z^2)*x+2*RootOf(_Z^3-2)^2+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-
2)+9*_Z^2)*RootOf(_Z^3-2))/(x+1)^2/(x^2-x+1)^2)+1/12*RootOf(_Z^3-2)*ln(-(12
*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^3*x^3+1
8*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^2*x^
3-2*x^6*RootOf(_Z^3-2)-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2
)*x^6+18*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)
*RootOf(_Z^3-2)*x^2+6*(-x^3+1)^(1/3)*x^4+12*RootOf(_Z^3-2)*x^3+18*RootOf(Ro
otOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^3-6*x*(-x^3+1)^(1/3)-2*RootOf(
_Z^3-2)-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2))/(x+1)^2/(x^2
-x+1)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [A]
time = 5.61, size = 356, normalized size = 1.22

$$\frac{1}{36}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\,(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^{7}+33\,x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}-48\sqrt{3}\,(x^{14}-2\,x^{11}-6\,x^{8}-2\,x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\,(x^{18}+42\,x^{15}-417\,x^{12}+812\,x^{9}-417\,x^{6}+42\,x^{3}+1)\right)}{6\,(x^{18}-102\,x^{15}+447\,x^{12}-628\,x^{9}+447\,x^{6}-102\,x^{3}+1)}\right)+\frac{1}{72}\cdot 4^{\frac{2}{3}}\log\left(-\frac{12\,(-x^{3}+1)^{\frac{2}{3}}x^{2}-3\cdot 4^{\frac{2}{3}}(x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}+4^{\frac{1}{3}}(x^{6}+2\,x^{3}+1)}{x^{6}+2\,x^{3}+1}\right)-\frac{1}{144}\cdot 4^{\frac{2}{3}}\log\left(\frac{24\cdot 4^{\frac{1}{3}}(x^{8}-4\,x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}+4^{\frac{2}{3}}(x^{12}-32\,x^{9}+78\,x^{6}-32\,x^{3}+1)+12\,(x^{10}-11\,x^{7}+11\,x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}}{x^{12}+4\,x^{9}+6\,x^{6}+4\,x^{3}+1}\right)-\frac{1}{2}\,(-x^{3}+1)^{\frac{1}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] 1/36*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(x^16 - 33*x^13
 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 48*sqrt(3)*(x^14 - 2
*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*
x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x
^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/72*4^(2/3)*log(-(12*(-x^3 + 1)^
(2/3)*x^2 - 3*4^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(x^6 + 2*x^3 + 1
))/(x^6 + 2*x^3 + 1)) - 1/144*4^(2/3)*log((24*4^(1/3)*(x^8 - 4*x^5 + x^2)*(
-x^3 + 1)^(2/3) + 4^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 12*(x^10
- 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1)
) - 1/2*(-x^3 + 1)^(1/3)*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(1-x^3\right)}^{2/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.635 $\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=294

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)-\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(1\right.}{}$$

```
[Out] 1/2*x*hypergeom([1/3, 2/3],[4/3],x^3)-1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3)
)*2^(1/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(
1/3))*2^(1/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/24*ln(2*2^(1
/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/12*arcta
n(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {494, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)-\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3
])) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sq
rt[3]) + (x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/2 - Log[2^(2/3) - (1 - x
)/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3
) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*(1 -
x))/(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3
) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

& (LtQ[a, 0] || LtQ[b, 0])

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{4}x^4F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.04, size = 26, normalized size = 0.09

$$\frac{1}{4}x^4F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
[Out] (x^4*AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])/4
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(-x^3+1)^{\frac{2}{3}}(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(1/3)*x^3/(x^6 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(1-x^3\right)}^{2/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(x^3/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.636 $\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=293

$$\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\log\left(1+ \right.}{}$$

```
[Out] 1/2*x*hypergeom([1/3, 2/3],[4/3],x^3)+1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3)
)*2^(1/3)-1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(
1/3))*2^(1/3)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/24*ln(2*2^(1
/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/12*arcta
n(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {421, 251, 420, 493, 298, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[
3]) + (x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/2 + Log[2^(2/3) - (1 - x)/(
1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) -
 (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(1/3)*(1 - x))
/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) +
 (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.08, size = 111, normalized size = 0.38

$$-\frac{4xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)}{(1-x^3)^{2/3}(1+x^3)\left(-4F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)+x^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};x^3,-x^3\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};x^3,-x^3\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
[Out] (-4*x*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3])/((1 - x^3)^(2/3)*(1 + x^3)*(-4
*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, 2/3, 2, 7/3,
x^3, -x^3] - 2*AppellF1[4/3, 5/3, 1, 7/3, x^3, -x^3])))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^3+1)^{\frac{2}{3}}(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int(1/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(1/3)/(x^6 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(1-x^3\right)}^{2/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(1/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.637 $\int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=294

$$-\frac{\sqrt[3]{1-x^3}}{2x^2}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}\right.}{6\ 2^{2/3}}$$

[Out] -1/2*(-x^3+1)^(1/3)/x^2-1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/24*ln(2*2^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {491, 21, 420, 493, 298, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}-\frac{\sqrt[3]{1-x^3}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]

[Out] -1/2*(1 - x^3)^(1/3)/x^2 - ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) - Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(2/3))

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} \, dx = -\frac{F_1\left(-\frac{2}{3}; \frac{2}{3}, 1; \frac{1}{3}; x^3, -x^3\right)}{2x^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 20.09, size = 120, normalized size = 0.41

$$\frac{\sqrt[3]{1-x^3}\left(-1+\frac{4x^3 F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)}{(1+x^3)\left(-4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)+x^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};x^3,-x^3\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)\right)\right)}\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(1/3)*(-1 + (4*x^3*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 +
 x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/
3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/(2*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 35.43, size = 695, normalized size = 2.36

method	result	size
risch	Expression too large to display	695
trager	Expression too large to display	1729

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^3-1)/x^2/(-x^3+1)^(2/3)+(1/4*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^
3+2)+9*_Z^2)*ln(-(18*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*
RootOf(_Z^3+2)^2*x^3+12*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)
*RootOf(_Z^3+2)^3*x^3-3*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)
*x^6-2*RootOf(_Z^3+2)*x^6+9*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+3*_Z*R
ootOf(_Z^3+2)+9*_Z^2)*(x^6-2*x^3+1)^(2/3)*x-18*RootOf(_Z^3+2)*(x^6-2*x^3+1)
^(1/3)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^2-6*RootOf(_Z^
3+2)^2*(x^6-2*x^3+1)^(1/3)*x^2+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2
)+9*_Z^2)*x^3+4*RootOf(_Z^3+2)*x^3-3*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z
^3+2)+9*_Z^2)-2*RootOf(_Z^3+2))/(x+1)^2/(x^2-x+1)^2)+1/12*RootOf(_Z^3+2)*ln
((36*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)^2
*x^3+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3
*x^3+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^6+RootOf(_Z^3+
2)*x^6+9*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^
2)*(x^6-2*x^3+1)^(2/3)*x-6*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^2-36*Root
Of(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^3-6*RootOf(_Z^3+2)*x^3-6*
(x^6-2*x^3+1)^(2/3)*x+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)
+RootOf(_Z^3+2))/(x+1)^2/(x^2-x+1)^2))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

<u>Fricas</u> [A]
time = 5.80, size = 396, normalized size = 1.35

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}x^{2}\arctan\left(\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}(x^{16}-33x^{13}+110x^{10}-110x^{7}+33x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}+48\sqrt{3}\,(-1)^{\frac{1}{3}}(x^{14}-2x^{11}-6x^{8}-2x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\,(x^{18}+42x^{15}-417x^{12}+812x^{9}-417x^{6}+42x^{3}+1)\right)}{6\,(x^{18}-102x^{15}+447x^{12}-628x^{9}+447x^{6}-102x^{3}+1)}\right)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^{2}\log\left(\frac{24\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}(x^{8}-4x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^{12}-32x^{9}+78x^{6}-32x^{3}+1)+12\,(x^{10}-11x^{7}+11x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}}{x^{12}+4x^{9}+6x^{6}+4x^{3}+1}\right)-2\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^{2}\log\left(-\frac{12\,(-x^{3}+1)^{\frac{2}{3}}x^{2}+3\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}+4^{\frac{1}{3}}(-1)^{\frac{2}{3}}(x^{6}+2x^{3}+1)}{x^{6}+2x^{3}+1}\right)+72\,(-x^{3}+1)^{\frac{1}{3}}}{144\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/144*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^2*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt
(3)*(-1)^(2/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1
)^(1/3) + 48*sqrt(3)*(-1)^(1/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3
 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^
6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3
+ 1)) + 4^(2/3)*(-1)^(1/3)*x^2*log((24*4^(1/3)*(-1)^(2/3)*(x^8 - 4*x^5 + x^
2)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 +
 1) + 12*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x
^6 + 4*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x^2*log(-(12*(-x^3 + 1)^(2/3)*x^2 +
```

```
3*4^(2/3)*(-1)^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(-1)^(2/3)*(x^6
+ 2*x^3 + 1))/(x^6 + 2*x^3 + 1)) + 72*(-x^3 + 1)^(1/3))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(1-x^3\right)}^{2/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.638 $\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=141

$$\frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\right.}{4}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+(-x^3+1)^(2/3)-2/5*(-x^3+1)^(5/3)+1/8*(-x^3+1)^(8/3)-1/2
4*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*
(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$,
Rules used = {457, 89, 45, 641, 53, 57, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)
^(8/3)/8 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]
) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1-x}} - \frac{x^2}{\sqrt[3]{1-x}} + \frac{1}{\sqrt[3]{1-x}\,(1-x^2)}\right) dx, x, x^3\right)$$

$$= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}}\,dx, x, x^3\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1-x^2}\right.$$

$$= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}} - 2(1-x)^{2/3} + (1-x)^{5/3}\right) dx, x, x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \left(1-x^3\right)^{2/3} - \frac{2}{5}\left(1-x^3\right)^{5/3} + \frac{1}{8}\left(1-x^3\right)^{8/3} + \frac{1}{6}\text{Subst}\left(\int \frac{}{\sqrt[3]{1-}}\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \left(1-x^3\right)^{2/3} - \frac{2}{5}\left(1-x^3\right)^{5/3} + \frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{4}\text{Su}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \left(1-x^3\right)^{2/3} - \frac{2}{5}\left(1-x^3\right)^{5/3} + \frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log}{}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \left(1-x^3\right)^{2/3} - \frac{2}{5}\left(1-x^3\right)^{5/3} + \frac{1}{8}\left(1-x^3\right)^{8/3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-}}{\sqrt{3}}\right.}{2\sqrt[3]{2}\,\sqrt{3}}$$

Mathematica [A]
time = 0.21, size = 142, normalized size = 1.01

$$\frac{1}{120}\left(-\frac{3(-49+23x^3+x^6+5x^9)}{\sqrt[3]{1-x^3}} + 10\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 10\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 5\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-3*(-49 + 23*x^3 + x^6 + 5*x^9))/(1 - x^3)^(1/3) + 10*2^(2/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 10*2^(2/3)*Log[-2 + 2^(2/3)*(1
 - x^3)^(1/3)] - 5*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x
^3)^(2/3)])/120
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.59, size = 510, normalized size = 3.62

method	result	size
trager	Expression too large to display	510
risch	Expression too large to display	790

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(5*x^9+x^6+23*x^3-49)/(x^3-1)*(-x^3+1)^(2/3)+1/2*RootOf(RootOf(_Z^3-4)
^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(
_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)*x^3+15*RootOf(_Z^3-4)*x^3+126*(-x^3+1)^(1/3)*RootOf(
_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^(2
/3)-168*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-35*RootOf(_Z^3
-4))/(x+1)/(x^2-x+1))+1/12*RootOf(_Z^3-4)*ln(-(45*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+6*RootOf(RootOf(_Z^3-4)^
2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-15*RootOf(RootOf(_Z^3-4
)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*RootOf(_Z^3-4)*x^3-63*(-x^3+1)^(1/3)
*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-21*(-x
^3+1)^(2/3)+105*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+14*Roo
tOf(_Z^3-4))/(x+1)/(x^2-x+1))
```

Maxima [A]
time = 0.66, size = 128, normalized size = 0.91

$$\frac{1}{8}\left(-x^{3}+1\right)^{\frac{8}{3}}+\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{2}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)+\left(-x^{3}+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^{3}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/8*(-x^3 + 1)^(8/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(2/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3)*log(-2^(1/
3) + (-x^3 + 1)^(1/3)) + (-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Fricas [A]
time = 3.48, size = 140, normalized size = 0.99

$$\frac{10\sqrt{6}\,2^{\frac{1}{6}}\left(x^{3}-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-5\cdot 2^{\frac{2}{3}}\left(x^{3}-1\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+10\cdot 2^{\frac{2}{3}}\left(x^{3}-1\right)\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)+3\left(5x^{9}+x^{6}+23x^{3}-49\right)\left(-x^{3}+1\right)^{\frac{2}{3}}}{120\left(x^{3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/120*(10*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2
*sqrt(6)*(-x^3 + 1)^(1/3))) - 5*2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x
^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 10*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x
^3 + 1)^(1/3)) + 3*(5*x^9 + x^6 + 23*x^3 - 49)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [A]
time = 1.55, size = 136, normalized size = 0.96

$$\frac{1}{8}\left(x^{3}-1\right)^{2}\left(-x^{3}+1\right)^{\frac{2}{3}}+\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{2}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)+\left(-x^{3}+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^{3}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(2/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3
)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + (-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)
^(1/3)
```

Mupad [B]
time = 4.86, size = 148, normalized size = 1.05

$$\frac{2^{2/3}\ln\left(\frac{\left(1-x^{3}\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\left(1-x^{3}\right)^{1/3}}+\left(1-x^{3}\right)^{2/3}-\frac{2\left(1-x^{3}\right)^{5/3}}{5}+\frac{\left(1-x^{3}\right)^{8/3}}{8}+\frac{2^{2/3}\ln\left(\frac{\left(1-x^{3}\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}-\frac{2^{2/3}\ln\left(\frac{\left(1-x^{3}\right)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)^(8/3)/8 + (2^(2/3)*log((
1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24 -
(2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*
1i + 1))/24
```

3.639 $\int \frac{x^{11}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=130

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+1/2*(-x^3+1)^(2/3)-1/5*(-x^3+1)^(5/3)+1/24*ln(x^3+1)*2^(
2/3)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3*(1+2^(2/3)*(-x^
3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 89, 45, 797, 79, 57, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2
^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{x}{\sqrt[3]{1-x}} - \frac{x}{\sqrt[3]{1-x}\,(-1+x^2)}\right)dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}}\,dx, x, x^3\right)\right) - \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\,(-1+x^2)}\,dx, x, x^3\right.$$

$$= -\left(\frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}} - (1-x)^{2/3}\right)dx, x, x^3\right)\right) - \frac{1}{3}\text{Subst}\left(\int \frac{x}{(-1-x)(1\cdots}\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt[3]{1-x}}\,dx,\cdots\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{1}{4}\text{Subst}\left(\int \frac{\cdots}{2^{2/3}+\cdots}\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[3]{1-x\cdots}\right.}{4\sqrt[3]{2}}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1\cdots}{12\cdots}$$

Mathematica [A]
time = 0.17, size = 137, normalized size = 1.05

$$\frac{1}{120}\left(-\frac{12(-8+x^3+2x^6)}{\sqrt[3]{1-x^3}} - 10\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 10\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 5\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(-8 + x^3 + 2*x^6))/(1 - x^3)^(1/3) - 10*2^(2/3)*Sqrt[3]*ArcTan[(1 +
2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 10*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(
1/3)] + 5*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)
])/120
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.07, size = 677, normalized size = 5.21

method	result	size

risch	Expression too large to display	677
trager	Expression too large to display	684

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(2*x^6+x^3-8)/(-x^3+1)^(1/3)-1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(
_Z^3-4)+36*_Z^2)*ln((15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2
)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z
^2)^2*RootOf(_Z^3-4)^2*x^3-5*RootOf(_Z^3-4)*x^3-6*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+42*(-x^3
+1)^(2/3)+35*RootOf(_Z^3-4)+42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+
36*_Z^2))/(x+1)/(x^2-x+1))+1/12*ln((-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf
(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(_Z^3-4)*x^3+18*RootOf(
RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+21*(-x^3+1)^(1/3)*RootOf(
_Z^3-4)^2+42*(-x^3+1)^(2/3)+28*RootOf(_Z^3-4)-42*RootOf(RootOf(_Z^3-4)^2+6*
_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)+1/2*ln((-12*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*R
ootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-
12*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+42*(-x^3+1)^(2/3)+28*RootOf(_Z^3-
4)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1)
)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

Maxima [A]
time = 0.62, size = 119, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
 + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Fricas [A]
time = 4.26, size = 159, normalized size = 1.22

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\left(-1\right)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)+5\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-10\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-12\left(2x^6+x^3-8\right)(-x^3+1)^{\frac{2}{3}}}{120\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/120*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(
6)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 5*2^(2/3)*(-1)^(1/3)*(
x^3 - 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x
^3 + 1)^(2/3)) - 10*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) +
(-x^3 + 1)^(1/3)) - 12*(2*x^6 + x^3 - 8)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [A]
time = 1.67, size = 120, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{2}\left(-x^{3}+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^{3}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1
/3))) + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Mupad [B]
time = 5.15, size = 139, normalized size = 1.07

$$\frac{1}{2\,{\left(1-x^3\right)}^{1/3}}-\frac{2^{2/3}\,\ln\left(\frac{{\left(1-x^3\right)}^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{{\left(1-x^3\right)}^{2/3}}{2}-\frac{{\left(1-x^3\right)}^{5/3}}{5}-\frac{2^{2/3}\,\ln\left(\frac{{\left(1-x^3\right)}^{1/3}}{4}-\frac{2^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{24}+\frac{2^{2/3}\,\ln\left(\frac{{\left(1-x^3\right)}^{1/3}}{4}-\frac{2^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + (
1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^
(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24 + (2^(2/3)*log((1 - x^3)
^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

3.640 $\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=115

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+1/2*(-x^3+1)^(2/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)
-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2)
)*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {457, 89, 641, 53, 57, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(
1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3
) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1-x}} + \frac{1}{\sqrt[3]{1-x}\,(1-x^2)}\right)\,dx, x, x^3\right)$$
$$= \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1-x^2)}\,dx, x, x^3\right)$$
$$= \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx,\right.$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{\text{Subst}\left(\right.}{}$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(\right.}{}$$

Mathematica [A]
time = 0.16, size = 132, normalized size = 1.15

$$\frac{1}{24}\left(-\frac{12(-2+x^3)}{\sqrt[3]{1-x^3}} + 2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(-2 + x^3))/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(
1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2^
(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.71, size = 672, normalized size = 5.84

method	result	size
risch	Expression too large to display	672
trager	Expression too large to display	787

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x^3-2)/(-x^3+1)^(1/3)+1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*ln((15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootO
f(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*R
ootOf(_Z^3-4)^2*x^3-5*RootOf(_Z^3-4)*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+42*(-x^3+1)^(2/
3)+35*RootOf(_Z^3-4)+42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2
))/(x+1)/(x^2-x+1))-1/12*ln((-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3-4)
^2+42*(-x^3+1)^(2/3)+28*RootOf(_Z^3-4)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln((-12*RootOf(Roo
tOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*Root
Of(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+
21*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+42*(-x^3+1)^(2/3)+28*RootOf(_Z^3-4)-42*R
ootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootO
f(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

Maxima [A]
time = 0.61, size = 108, normalized size = 0.94

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)+\frac{1}{2}\left(-x^{3}+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^{3}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2*(-x^3 + 1)^(2/3)
+ 1/2/(-x^3 + 1)^(1/3)
```

Fricas [A]
time = 3.30, size = 130, normalized size = 1.13

$$\frac{2\sqrt{6}\,2^{\frac{1}{6}}\left(x^{3}-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}\left(x^{3}-1\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}\left(x^{3}-1\right)\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)+12\left(x^{3}-2\right)\left(-x^{3}+1\right)^{\frac{2}{3}}}{24\left(x^{3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*s
qrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 +
1)^(1/3)) + 12*(x^3 - 2)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 1.52, size = 109, normalized size = 0.95

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2*(-x^3 + 1)^(
2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Mupad [B]
time = 4.89, size = 128, normalized size = 1.11

$$\frac{2^{2/3}\ln\left(\frac{{\left(1-x^3\right)}^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\,{\left(1-x^3\right)}^{1/3}}+\frac{{\left(1-x^3\right)}^{2/3}}{2}+\frac{2^{2/3}\ln\left(\frac{{\left(1-x^3\right)}^{1/3}}{4}-\frac{2^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{24}-\frac{2^{2/3}\ln\left(\frac{{\left(1-x^3\right)}^{1/3}}{4}-\frac{2^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
1 - x^3)^(2/3)/2 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i -
1)^2)/16)*(3^(1/2)*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*
(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

3.641 $\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=100

$$\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(2/3)-1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 57, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(1-x)^{4/3}(1+x)}\,dx,x,x^3\right)\\ &= \frac{1}{2\sqrt[3]{1-x^3}}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx,x,x^3\right)\\ &= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx,x,\sqrt[3]{1-x^3}\right)+\\ &= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1\right.}{2\sqrt[3]{2}}\\ &= \frac{1}{2\sqrt[3]{1-x^3}}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right.}{4\sqrt[3]{2}}\end{aligned}$$

Mathematica [A]

time = 0.15, size = 126, normalized size = 1.26

$$\frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}}-2\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\,(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (12/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/3)*Log[2 +
2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.77, size = 674, normalized size = 6.74

method	result	size
trager	Expression too large to display	674
risch	Expression too large to display	775

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-x^3+1)^(2/3)/(x^3-1)+1/2*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)
+36*_Z^2)*ln((18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*Roo
tOf(_Z^3+4)^2*x^3+15*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*R
ootOf(_Z^3+4)^3*x^3+6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*
x^3+5*RootOf(_Z^3+4)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+42*(-x^3+1)^(2/
3)-42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-35*RootOf(_Z^3+4
))/(x+1)/(x^2-x+1))-1/12*ln((18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)
+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-12*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^
3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)*x^3+12*RootOf(_Z^3+4)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^
2+42*(-x^3+1)^(2/3)+42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)
-28*RootOf(_Z^3+4))/(x+1)/(x^2-x+1))*RootOf(_Z^3+4)-1/2*ln((18*RootOf(RootO
f(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-12*RootOf(R
ootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3-18*RootOf
(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+12*RootOf(_Z^3+4)*x^3+21
*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+42*(-x^3+1)^(2/3)+42*RootOf(RootOf(_Z^3+4)
^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-28*RootOf(_Z^3+4))/(x+1)/(x^2-x+1))*RootOf(
RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)
```

Maxima [A]
time = 0.71, size = 97, normalized size = 0.97

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(73) = 146$.
time = 2.67, size = 148, normalized size = 1.48

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\,(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)+12\left(-x^{3}+1\right)^{\frac{2}{3}}}{24\left(x^{3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6)
*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^3
- 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 +
 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (-x^3
 + 1)^(1/3)) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 1.73, size = 98, normalized size = 0.98

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{2\left(-x^{3}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 + 1)^
(1/3)
```

Mupad [B]
time = 4.85, size = 117, normalized size = 1.17

$$\frac{1}{2\left(1-x^3\right)^{1/3}}-\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}-\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}+\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 - (
2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1
i - 1))/24 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/
16)*(3^(1/2)*1i + 1))/24
```

3.642 $\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)} dx$

Optimal. Leaf size=100

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(2/3)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {455, 53, 57, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx,x,x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx,x,x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx,x,\sqrt[3]{1-x^3}\right)-\\
&= \frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\right.}{2\sqrt[3]{2}}\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 127, normalized size = 1.27

$$\frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}}+2\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\,(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (12/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(2/3)*Log[2 +
2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.58, size = 674, normalized size = 6.74

method	result	size
trager	Expression too large to display	674
risch	Expression too large to display	775

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-x^3+1)^(2/3)/(x^3-1)-1/12*ln((-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootO
f(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(_Z^3-4)*x^3+18*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+21*(-x^3+1)^(1/3)*RootOf
(_Z^3-4)^2+42*(-x^3+1)^(2/3)+28*RootOf(_Z^3-4)-42*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln((-12*Ro
otOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3
-12*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z
^2)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+42*(-x^3+1)^(2/3)+28*RootOf(_Z^3
-4)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1
))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+1/2*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((15*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-5*RootOf(_Z^3-4)*x^3-6*Root
Of(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+21*(-x^3+1)^(1/3)*Root
Of(_Z^3-4)^2+42*(-x^3+1)^(2/3)+35*RootOf(_Z^3-4)+42*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

Maxima [A]
time = 0.73, size = 97, normalized size = 0.97

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/3)
```

Fricas [A]
time = 2.72, size = 125, normalized size = 1.25

$$\frac{2\sqrt{6}\,2^{\frac{1}{6}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\,(-x^3+1)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}(x^3-1)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}(x^3-1)\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)-12\,(-x^3+1)^{\frac{2}{3}}}{24\,(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*s
qrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 +
1)^(1/3)) - 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 1.65, size = 98, normalized size = 0.98

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\,(-x^3+1)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right|\right)+\frac{1}{2\,(-x^3+1)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 + 1)^(
1/3)
```

Mupad [B]
time = 4.90, size = 117, normalized size = 1.17

$$\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\,(1-x^3)^{1/3}}+\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}-\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1
i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/
16)*(3^(1/2)*1i + 1))/24
```

3.643 $\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=154

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/2/(-x^3+1)^(1/3)-1/2*ln(x)+1/24*ln(x^3+1)*2^(2/3)+1/2*ln(1-(-x^3+1)^(1/3)
)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2
/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {457, 87, 162, 57, 632, 210, 31, 631}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - A
rcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[x]/2
 + Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (
1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
```

```
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}x(1+x)}\,dx,x,x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{6}\text{Subst}\left(\int \frac{2+x}{\sqrt[3]{1-x}\,x(1+x)}\,dx,x,x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx,x,x^3\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\right.\\
&= \frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx,x,\sqrt[3]{1}\right.\\
&= \frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{}\right.}{4\sqrt[3]{2}}\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}+
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 198, normalized size = 1.29

$$\frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}}+8\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+8\log\left(-1+\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-4\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)+2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (12/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - 2
*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 8*Log[-1 +
 (1 - x^3)^(1/3)] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 4*Log[1 +
 (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/
3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x), x)
```

Fricas [A]
time = 3.30, size = 226, normalized size = 1.47

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}\right)-8\sqrt{3}(x^3-1)\arctan\left(\frac{2}{3}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+4(x^3-1)\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)-8(x^3-1)\log\left((-x^3+1)^{\frac{1}{3}}-1\right)+12(-x^3+1)^{\frac{2}{3}}}{24(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6)
*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^3
- 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 +
 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (-x^3
 + 1)^(1/3)) - 8*sqrt(3)*(x^3 - 1)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/
3*sqrt(3)) + 4*(x^3 - 1)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*(
x^3 - 1)*log((-x^3 + 1)^(1/3) - 1) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 1.79, size = 160, normalized size = 1.04

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right|\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(-x^3+1)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2(-x^3+1)^{\frac{1}{3}}}-\frac{1}{6}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|(-x^3+1)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2/(-x^3 + 1)^(1/3) - 1/6*log(
(-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1
))
```

Mupad [B]
time = 5.40, size = 253, normalized size = 1.64

$$\frac{\ln\left(\frac{17}{4}-\frac{17\,(1-x^3)^{1/3}}{4}\right)}{3}+\ln\left(\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(1458\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2-\frac{459\,(1-x^3)^{1/3}}{4}\right)-\frac{63}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(1458\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2-\frac{459\,(1-x^3)^{1/3}}{4}\right)+\frac{63}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\frac{2^{2/3}\ln\left(\frac{2^{2/3}\left(\frac{81\,2^{1/3}}{4}-\frac{459\,(1-x^3)^{1/3}}{4}\right)}{12}+\frac{63}{4}\right)}{12}+\frac{1}{2\,(1-x^3)^{1/3}}+\frac{(-1)^{1/3}\,2^{2/3}\ln\left(\frac{(-1)^{1/3}\,2^{2/3}\left(\frac{81\,(-1)^{2/3}\,2^{1/3}}{4}-\frac{459\,(1-x^3)^{1/3}}{4}\right)}{12}-\frac{63}{4}\right)}{12}-\frac{(-1)^{1/3}\,2^{2/3}\ln\left(\frac{(-1)^{1/3}\,2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{459\,(1-x^3)^{1/3}}{4}-\frac{81\,(-1)^{2/3}\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)}{24}-\frac{63}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] log(17/4 - (17*(1 - x^3)^(1/3))/4)/3 + log(((3^(1/2)*1i)/6 - 1/6)*(1458*((3
^(1/2)*1i)/6 - 1/6)^2 - (459*(1 - x^3)^(1/3))/4) - 63/4)*((3^(1/2)*1i)/6 -
1/6) - log(((3^(1/2)*1i)/6 + 1/6)*(1458*((3^(1/2)*1i)/6 + 1/6)^2 - (459*(1
- x^3)^(1/3))/4) + 63/4)*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*log((2^(2/3)*((8
1*2^(1/3))/4 - (459*(1 - x^3)^(1/3))/4))/12 + 63/4))/12 + 1/(2*(1 - x^3)^(1
/3)) + ((-1)^(1/3)*2^(2/3)*log(((-1)^(1/3)*2^(2/3)*((81*(-1)^(2/3)*2^(1/3))
/4 - (459*(1 - x^3)^(1/3))/4))/12 - 63/4))/12 - ((-1)^(1/3)*2^(2/3)*log(((-
1)^(1/3)*2^(2/3)*(3^(1/2)*1i + 1)*((459*(1 - x^3)^(1/3))/4 - (81*(-1)^(2/3)
*2^(1/3)*(3^(1/2)*1i + 1)^2)/16))/24 - 63/4)*(3^(1/2)*1i + 1))/24
```

3.644 $\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=175

$$\frac{5}{6\sqrt[3]{1-x^3}}-\frac{1}{3x^3\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{6}-\frac{\log(1+x^3)}{12\sqrt[3]{2}}+\frac{1}{6}\log$$

```
[Out] 5/6/(-x^3+1)^(1/3)-1/3/x^3/(-x^3+1)^(1/3)-1/6*ln(x)-1/24*ln(x^3+1)*2^(2/3)+
1/6*ln(1-(-x^3+1)^(1/3))+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/9*arctan(
1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+
1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {457, 105, 162, 53, 57, 632, 210, 31, 631}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{1}{3x^3\sqrt[3]{1-x^3}}+\frac{5}{6\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 5/(6*(1 - x^3)^(1/3)) - 1/(3*x^3*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)
^(1/3))/Sqrt[3]]/(3*Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]
]/(2*2^(1/3)*Sqrt[3]) - Log[x]/6 - Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 -
 x^3)^(1/3)]/6 + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}x^2(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{1}{3}-\frac{4x}{3}}{(1-x)^{4/3}x(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}x}\,dx, x, x^3\right) + \frac{1}{3}\text{Subst}\left(\int \frac{}{(1-x)^{4}\cdots}\right.$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right) + \frac{1}{6}\text{Subst}\left(\cdots\right.$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x}\,dx\cdots\right.$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right.\cdots$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-\cdots}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}\cdots$$

Mathematica [A]
time = 0.33, size = 209, normalized size = 1.19

$$\frac{1}{72}\left(\frac{12\left(-2+5x^3\right)}{x^3\sqrt[3]{1-x^3}} + 8\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 6\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 8\log\left(-1+\sqrt[3]{1-x^3}\right) + 6\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 4\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right) - 3\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*(-2 + 5*x^3))/(x^3*(1 - x^3)^(1/3)) + 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3
)^(1/3))/Sqrt[3]] + 6*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/
Sqrt[3]] + 8*Log[-1 + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^
3)^(1/3)] - 4*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Log[2
+ 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/72
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^4), x)
```

Fricas [**A**]
time = 3.00, size = 238, normalized size = 1.36

$$\frac{6\sqrt{6}\,2^{\frac{1}{6}}(x^6-x^3)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\,(-x^3+1)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}(x^6-x^3)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+6\cdot 2^{\frac{2}{3}}(x^6-x^3)\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)+8\sqrt{3}\,(x^6-x^3)\arctan\left(\frac{2}{3}\sqrt{3}\,(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-4\,(x^6-x^3)\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+8\,(x^6-x^3)\log\left((-x^3+1)^{\frac{1}{3}}-1\right)-12\,(5\,x^3-2)(-x^3+1)^{\frac{2}{3}}}{72\,(x^6-x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(6)*2^(1/6)*(x^6 - x^3)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2
*sqrt(6)*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*(x^6 - x^3)*log(2^(2/3) + 2^(1/3)*(
-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 6*2^(2/3)*(x^6 - x^3)*log(-2^(1/3) +
(-x^3 + 1)^(1/3)) + 8*sqrt(3)*(x^6 - x^3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/
3) + 1/3*sqrt(3)) - 4*(x^6 - x^3)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) +
 1) + 8*(x^6 - x^3)*log((-x^3 + 1)^(1/3) - 1) - 12*(5*x^3 - 2)*(-x^3 + 1)^(
2/3))/(x^6 - x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [**A**]
time = 1.23, size = 181, normalized size = 1.03

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\,(-x^3+1)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right|\right)+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(-x^3+1)^{\frac{1}{3}}+1\right)\right)-\frac{5\,x^3-2}{6\left((-x^3+1)^{\frac{4}{3}}-(-x^3+1)^{\frac{1}{3}}\right)}-\frac{1}{18}\log\left((-x^3+1)^{\frac{2}{3}}+(-x^3+1)^{\frac{1}{3}}+1\right)+\frac{1}{9}\log\left(\left|(-x^3+1)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*(5*x^3 - 2)/((-x^3 + 1)^(4/3
) - (-x^3 + 1)^(1/3)) - 1/18*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) +
 1/9*log(abs((-x^3 + 1)^(1/3) - 1))
```

Mupad [B]
time = 5.02, size = 399, normalized size = 2.28

$$\frac{\ln\left(\frac{11(1-x^3)^{1/3}}{972}-\frac{11}{972}\right)}{9}+\frac{2^{2/3}\ln\left(\frac{2^{1/3}\left(\frac{2^{2/3}\left(\frac{81\,2^{1/3}}{4}-\frac{75(1-x^3)^{1/3}}{4}\right)}{12}-\frac{35}{12}\right)}{72}+\frac{(1-x^3)^{1/3}}{27}\right)}{12}+\ln\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1i}{18}\right)^2\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1i}{18}\right)\left(1458\left(-\frac{1}{18}+\frac{\sqrt{3}\,1i}{18}\right)^2-\frac{75(1-x^3)^{1/3}}{4}\right)-\frac{35}{12}\right)+\frac{(1-x^3)^{1/3}}{27}\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1i}{18}\right)-\ln\left(\frac{(1-x^3)^{1/3}}{27}-\left(\frac{1}{18}+\frac{\sqrt{3}\,1i}{18}\right)^2\left(\left(\frac{1}{18}+\frac{\sqrt{3}\,1i}{18}\right)\left(1458\left(\frac{1}{18}+\frac{\sqrt{3}\,1i}{18}\right)^2-\frac{75(1-x^3)^{1/3}}{4}\right)+\frac{35}{12}\right)\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1i}{18}\right)+\frac{\frac{5x^3}{6}-\frac{1}{3}}{(1-x^3)^{1/3}-(1-x^3)^{4/3}}+\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{27}+\frac{2^{1/3}(-1+\sqrt{3}\,1i)^2\left(\frac{2^{2/3}(-1+\sqrt{3}\,1i)\left(\frac{81\,2^{1/3}(-1+\sqrt{3}\,1i)^2}{16}-\frac{75(1-x^3)^{1/3}}{4}\right)}{24}-\frac{35}{12}\right)}{288}\right)(-1+\sqrt{3}\,1i)}{24}-\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{27}-\frac{2^{1/3}(1+\sqrt{3}\,1i)^2\left(\frac{2^{2/3}(1+\sqrt{3}\,1i)\left(\frac{81\,2^{1/3}(1+\sqrt{3}\,1i)^2}{16}-\frac{75(1-x^3)^{1/3}}{4}\right)}{24}+\frac{35}{12}\right)}{288}\right)(1+\sqrt{3}\,1i)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] log((11*(1 - x^3)^(1/3))/972 - 11/972)/9 + (2^(2/3)*log((2^(1/3)*((2^(2/3)*
((81*2^(1/3))/4 - (75*(1 - x^3)^(1/3))/4))/12 - 35/12))/72 + (1 - x^3)^(1/3
)/27))/12 + log(((3^(1/2)*1i)/18 - 1/18)^2*(((3^(1/2)*1i)/18 - 1/18)*(1458*
((3^(1/2)*1i)/18 - 1/18)^2 - (75*(1 - x^3)^(1/3))/4) - 35/12) + (1 - x^3)^(
1/3)/27)*((3^(1/2)*1i)/18 - 1/18) - log((1 - x^3)^(1/3)/27 - ((3^(1/2)*1i)/
18 + 1/18)^2*(((3^(1/2)*1i)/18 + 1/18)*(1458*((3^(1/2)*1i)/18 + 1/18)^2 - (
75*(1 - x^3)^(1/3))/4) + 35/12))*((3^(1/2)*1i)/18 + 1/18) + ((5*x^3)/6 - 1/
3)/((1 - x^3)^(1/3) - (1 - x^3)^(4/3)) + (2^(2/3)*log((1 - x^3)^(1/3)/27 +
(2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(2/3)*(3^(1/2)*1i - 1)*((81*2^(1/3)*(3^(1/2
)*1i - 1)^2)/16 - (75*(1 - x^3)^(1/3))/4))/24 - 35/12))/288)*(3^(1/2)*1i -
1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/27 - (2^(1/3)*(3^(1/2)*1i + 1)^2*((2^
(2/3)*(3^(1/2)*1i + 1)*((81*2^(1/3)*(3^(1/2)*1i + 1)^2)/16 - (75*(1 - x^3)^
(1/3))/4))/24 + 35/12))/288)*(3^(1/2)*1i + 1))/24
```

3.645 $\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=174

$$\frac{x^4}{2\sqrt[3]{1-x^3}}+\frac{5}{6}x\left(1-x^3\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}\,x-\cdots\right.}{4\sqrt[3]{2}}$$

```
[Out] 1/2*x^4/(-x^3+1)^(1/3)+5/6*x*(-x^3+1)^(2/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-
2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(x+(-x^3+1)^(1/3))+1/9*arctan(1/3*(
1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3
+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {481, 596, 544, 245, 384}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{5}{6}\left(1-x^3\right)^{2/3}x+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{4\sqrt[3]{2}}-\frac{1}{6}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{x^4}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^4/(2*(1 - x^3)^(1/3)) + (5*x*(1 - x^3)^(2/3))/6 + ArcTan[(1 - (2*x)/(1 -
x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[3]) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3
))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[-(2^(1/3)
*x) - (1 - x^3)^(1/3)]/(4*2^(1/3)) - Log[x + (1 - x^3)^(1/3)]/6
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{10}x^{10}F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};x^3,-x^3\right)$$

Mathematica [A]
time = 0.52, size = 227, normalized size = 1.30

$$\frac{1}{72}\left(-\frac{12x\left(-5+2x^3\right)}{\sqrt[3]{1-x^3}}+8\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{1-x^3}}\right)+6\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-8\log\left(x+\sqrt[3]{1-x^3}\right)-6\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+4\log\left(x^2-x\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)+3\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-12*x*(-5 + 2*x^3))/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2
*(1 - x^3)^(1/3))] + 6*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 -
```

```
 x^3)^(1/3))] - 8*Log[x + (1 - x^3)^(1/3)] - 6*2^(2/3)*Log[2*x + 2^(2/3)*(1
 - x^3)^(1/3)] + 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 3*2^(2/
3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/72
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(-x^3+1\right)^{\frac{4}{3}} \left(x^3+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(132) = 264.
time = 3.05, size = 271, normalized size = 1.56

$$\frac{6 \sqrt{6}\, 2^{\frac{1}{6}} (-1)^{\frac{1}{3}} (x^3-1) \arctan\left(\frac{2^{\frac{1}{6}}\left(\sqrt{6}\, 2^{\frac{1}{3}} x+2 \sqrt{6}\, (-1)^{\frac{1}{3}} (-x^3+1)^{\frac{1}{3}}\right)}{6 x}\right)+6 \cdot 2^{\frac{2}{3}} (-1)^{\frac{1}{3}} (x^3-1) \log\left(\frac{2^{\frac{1}{3}} (-1)^{\frac{2}{3}} x+(-x^3+1)^{\frac{1}{3}}}{x}\right)-3 \cdot 2^{\frac{2}{3}} (-1)^{\frac{1}{3}} (x^3-1) \log\left(-\frac{2^{\frac{2}{3}} (-1)^{\frac{1}{3}} x^2+2^{\frac{1}{3}} (-1)^{\frac{2}{3}} (-x^3+1)^{\frac{1}{3}} x-(-x^3+1)^{\frac{2}{3}}}{x^2}\right)+8 \sqrt{3}\, (x^3-1) \arctan\left(-\frac{\sqrt{3}\, x-2 \sqrt{3}\, (-x^3+1)^{\frac{1}{3}}}{3 x}\right)-8 (x^3-1) \log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right)+4 (x^3-1) \log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}} x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)+12 (2 x^4-5 x)(-x^3+1)^{\frac{2}{3}}}{72 (x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] 1/72*(6*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^
(1/3)*x + 2*sqrt(6)*(-1)^(1/3)*(-x^3 + 1)^(1/3))/x) + 6*2^(2/3)*(-1)^(1/3)*
(x^3 - 1)*log((2^(1/3)*(-1)^(2/3)*x + (-x^3 + 1)^(1/3))/x) - 3*2^(2/3)*(-1)
^(1/3)*(x^3 - 1)*log(-(2^(2/3)*(-1)^(1/3)*x^2 + 2^(1/3)*(-1)^(2/3)*(-x^3 +
1)^(1/3)*x - (-x^3 + 1)^(2/3))/x^2) + 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt
(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) - 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1
/3))/x) + 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2
) + 12*(2*x^4 - 5*x)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}} \left(x+1\right)\left(x^2-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**9/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.646 $\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=153

$$\frac{x}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{1}{2}\text{lo}$$

```
[Out] 1/2*x/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3
^(1/2))*3^(1/2)-1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(
2/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {481, 544, 245, 384}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]
 - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3])
- Log[1 + x^3]/(12*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(4*2^(1/3
)) - Log[x + (1 - x^3)^(1/3)]/2
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Mathematica [A]
time = 0.41, size = 220, normalized size = 1.44

$$\frac{1}{24}\left(\frac{12x}{\sqrt[3]{1-x^3}}+8\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{1-x^3}}\right)-2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-8\log\left(x+\sqrt[3]{1-x^3}\right)+2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+4\log\left(x^2-x\sqrt[3]{1-x^3}+(1-x^3)^{2/3}\right)-2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\,(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*x)/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/
3))] - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))]
- 8*Log[x + (1 - x^3)^(1/3)] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]
 + 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 2^(2/3)*Log[-2*x^2 +
2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(-x^3+1)^{\frac{4}{3}}(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(118) = 236$.
time = 2.96, size = 239, normalized size = 1.56

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}(x^3-1)\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}x-2\sqrt{6}\,(-x^3+1)^{\frac{1}{3}}\right)}{6x}\right)-2\cdot 2^{\frac{2}{3}}(x^3-1)\log\left(\frac{2^{\frac{1}{3}}x+(-x^3+1)^{\frac{1}{3}}}{x}\right)+2^{\frac{2}{3}}(x^3-1)\log\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)-8\sqrt{3}\,(x^3-1)\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\,(-x^3+1)^{\frac{1}{3}}}{3x}\right)+8(x^3-1)\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right)-4(x^3-1)\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)+12(-x^3+1)^{\frac{2}{3}}x}{24(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(-1/6*2^(1/6)*(sqrt(6)*2^(1/3)*x -
 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) - 2*2^(2/3)*(x^3 - 1)*log((2^(1/3)*x + (-x^
3 + 1)^(1/3))/x) + 2^(2/3)*(x^3 - 1)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^
(1/3)*x + (-x^3 + 1)^(2/3))/x^2) - 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt(3)
*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1/3)
)/x) - 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) +
 12*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.647 $\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=106

$$\frac{x}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2*x/(-x^3+1)^(1/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*
3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {482, 384}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{x}{2\sqrt[3]{1-x^3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[-(2^(1/3)*x) - (1 -
x^3)^(1/3)]/(4*2^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x^4\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{2x^3}{1+x^3}\right)}{4(1+x^3)^{4/3}}$$

Mathematica [A]
time = 0.29, size = 139, normalized size = 1.31

$$\frac{1}{24}\left(\frac{12x}{\sqrt[3]{1-x^3}}+2\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\,(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*x)/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)
*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/3)
*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.02, size = 627, normalized size = 5.92

method	result	size
risch	Expression too large to display	627
trager	Expression too large to display	634

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/(-x^3+1)^(1/3)-1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2)*ln(-(-9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3
-4)^3*x^3-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(
_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^2*x+4*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2+30*(-x^3
+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-
4)*x^2+3*RootOf(_Z^3-4)*x^3+12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+
36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)-3*RootOf(_Z^3-4)-12*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/12*RootOf(_Z^3-4)*ln((-3*
```

```
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-2
7*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x
^3+6*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*Ro
otOf(_Z^3-4)^2*x+2*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2-3*(-x^3+1)^(1/3)*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-3*Root
Of(_Z^3-4)*x^3-27*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+
5*x*(-x^3+1)^(2/3)+RootOf(_Z^3-4)+9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^
3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(79) = 158$.
time = 7.20, size = 318, normalized size = 3.00

$$-\frac{2\,\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\,\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5\,x^{7}+4\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-12\,\sqrt{6}\,(-1)^{\frac{1}{3}}\left(19\,x^{8}-16\,x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71\,x^{9}-111\,x^{6}+33\,x^{3}-1\right)\right)}{6\left(109\,x^{9}-105\,x^{6}+3\,x^{3}+1\right)}\right)-2\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{3}+1\right)+6\left(-x^{3}+1\right)^{\frac{2}{3}}x}{x^{3}+1}\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\log\left(-\frac{3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(5\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(19\,x^{6}-16\,x^{3}+1\right)+12\left(2\,x^{5}-x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{6}+2\,x^{3}+1}\right)+36\left(-x^{3}+1\right)^{\frac{2}{3}}x}{72\left(x^{3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)
*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^
(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 -
111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1
/3)*(x^3 - 1)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)
^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*(x
^3 - 1)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(
-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 +
 2*x^3 + 1)) + 36*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.648 $\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=106

$$\frac{x}{2\sqrt[3]{1-x^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2*x/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)-1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*
3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 384}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x/(2*(1 - x^3)^(1/3)) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[-(2^(1/3)*x) - (1 -
x^3)^(1/3)]/(4*2^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
```

```
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{6}\text{Subst}\left(\int \frac{2-}{1-\sqrt[3]{2}\,x}\right.\\
&= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx,x,\right.\\
&= \frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\text{Subs}}{}\\
&= \frac{x}{2\sqrt[3]{1-x^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}+\frac{\log}{}
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 140, normalized size = 1.32

$$\frac{1}{24}\left(\frac{12x}{\sqrt[3]{1-x^3}}-2\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\,(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*x)/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)
*(1 - x^3)^(1/3))] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(2/3)
*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.88, size = 944, normalized size = 8.91

method	result	size

risch	Expression too large to display	944
trager	Expression too large to display	951

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/(-x^3+1)^(1/3)-1/12*ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)
^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z
^3-4)^2*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+3
6*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootOf(_Z^3-4)-6*RootO
f(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z
^3-4)-1/2*ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootO
f(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*R
ootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2-24*(
-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_
Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)
^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)+1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*ln((9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf
(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*Ro
otOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z
^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-5*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2-24*
(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(
_Z^3-4)*x^2+9*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^
3-4)+36*_Z^2)*x^3+10*x*(-x^3+1)^(2/3)-3*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(79) = 158.

time = 6.40, size = 288, normalized size = 2.72

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}(x^3-1)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{6}\,(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}\right)}{6(109x^9-105x^6+3x^3+1)}\right)-2\cdot 2^{\frac{2}{3}}(x^3-1)\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(x^3+1)+6(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)+2^{\frac{2}{3}}(x^3-1)\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(19x^6-16x^3+1)-12(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+36(-x^3+1)^{\frac{2}{3}}x}{72(x^3-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5
*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33
*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 -
 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(x^3 - 1)*log((6*2^(1/3)*(-x^3 + 1)^(1/3
)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(x^3
 - 1)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^
3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 36*(-x^3 +
 1)^(2/3)*x)/(x^3 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.649 $\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)} dx$

Optimal. Leaf size=124

$$\frac{1}{2x^2\sqrt[3]{1-x^3}}-\frac{(1-x^3)^{2/3}}{x^2}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/x^2/(-x^3+1)^(1/3)-(-x^3+1)^(2/3)/x^2+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^
(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/
3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {483, 597, 12, 384}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{4\sqrt[3]{2}}-\frac{(1-x^3)^{2/3}}{x^2}+\frac{1}{2x^2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*x^2*(1 - x^3)^(1/3)) - (1 - x^3)^(2/3)/x^2 + ArcTan[(1 - (2*2^(1/3)*x)
/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3))
- Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = -\frac{14+56x^3-91x^6-42x^9+63x^{12}-7\left(1-x^3\right)^2\left(2+12x^3+9x^6\right)\,{}_2F_1\left(\frac{1}{3},1\right.}{}$$

Mathematica [A]
time = 0.33, size = 148, normalized size = 1.19

$$\frac{1}{24}\left(\frac{12(-1+2x^3)}{x^2\sqrt[3]{1-x^3}}+2\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*(-1 + 2*x^3))/(x^2*(1 - x^3)^(1/3)) + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3
]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)
^(1/3)] + 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3
)^(2/3)])/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 15.23, size = 777, normalized size = 6.27

method	result	size

risch	Expression too large to display	777
trager	Expression too large to display	1170

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*x^3-1)/x^2/(-x^3+1)^(1/3)-1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z
^3-4)+36*_Z^2)*ln((9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*R
ootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-5*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^
2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*Ro
otOf(_Z^3-4)*x^2+9*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootO
f(_Z^3-4)+36*_Z^2)*x^3+10*x*(-x^3+1)^(2/3)-3*RootOf(_Z^3-4)-6*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))+1/12*ln((-6*RootO
f(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-3*
(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2+2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+6*x*(-x^3+1)^(2/3)-2*RootOf(_Z^3-4
)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*
RootOf(_Z^3-4)+1/2*ln((-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z
^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)^2*RootOf(_Z^3-4)^2*x^3-3*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2+2*RootOf
(_Z^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+6*x
*(-x^3+1)^(2/3)-2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+
36*_Z^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(95) = 190.
time = 9.24, size = 340, normalized size = 2.74

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}(x^5-x^2)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)\right)}{6(109x^9-105x^6+3x^3+1)}\right)-2\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^5-x^2)\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}x^2-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3+1)+6(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^5-x^2)\log\left(-\frac{3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(19x^6-16x^3+1)+12(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+36(2x^3-1)(-x^3+1)^{\frac{2}{3}}}{72(x^5-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^5 - x^2)*arctan(1/6*2^(1/6)*(6*sqrt(
6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1
)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9
- 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^
(1/3)*(x^5 - x^2)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*
(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3
)*(x^5 - x^2)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(
1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/
(x^6 + 2*x^3 + 1)) + 36*(2*x^3 - 1)*(-x^3 + 1)^(2/3))/(x^5 - x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}} \left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.650 $\int \frac{1}{x^6(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=144

$$\frac{1}{2x^5\sqrt[3]{1-x^3}}-\frac{7(1-x^3)^{2/3}}{10x^5}-\frac{4(1-x^3)^{2/3}}{5x^2}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/x^5/(-x^3+1)^(1/3)-7/10*(-x^3+1)^(2/3)/x^5-4/5*(-x^3+1)^(2/3)/x^2-1/24*
ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3
*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {483, 597, 12, 384}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{4\sqrt[3]{2}}-\frac{7(1-x^3)^{2/3}}{10x^5}+\frac{1}{2x^5\sqrt[3]{1-x^3}}-\frac{4(1-x^3)^{2/3}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*x^5*(1 - x^3)^(1/3)) - (7*(1 - x^3)^(2/3))/(10*x^5) - (4*(1 - x^3)^(2/
3))/(5*x^2) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3
)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)
]/(4*2^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx = -\frac{28-182x^3-476x^6+819x^9+378x^{12}-567x^{15}-28\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)}{}$$

Mathematica [A]
time = 0.35, size = 152, normalized size = 1.06

$$\frac{1}{120}\left(-\frac{12\left(2+x^3-8x^6\right)}{x^5\sqrt[3]{1-x^3}}-10\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+10\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-5\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(2 + x^3 - 8*x^6))/(x^5*(1 - x^3)^(1/3)) - 10*2^(2/3)*Sqrt[3]*ArcTan[
(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] + 10*2^(2/3)*Log[2*x + 2^(2/3)*(
1 - x^3)^(1/3)] - 5*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3
)*(1 - x^3)^(2/3)])/120
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 15.59, size = 941, normalized size = 6.53

method	result	size

risch	Expression too large to display	941
trager	Expression too large to display	1160

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(8*x^6-x^3-2)/x^5/(-x^3+1)^(1/3)+1/12*RootOf(_Z^3-4)*ln(-(3*RootOf(Roo
tOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+54*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^
3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3
-4)^2*x-5*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2-6*(-x^3+1)^(1/3)*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-RootOf(_Z^3-4)*
x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1
)^(2/3)+RootOf(_Z^3-4)+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z
^2))/(x+1)/(x^2-x+1))-1/12*ln(-(3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^
3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4
)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-(-x^3+1)^(1/3)*RootOf(_
Z^3-4)^2*x^2-30*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+
36*_Z^2)*RootOf(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3-36*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-10*x*(-x^3+1)^(2/3)-RootOf(_Z^3-4)+12*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(
_Z^3-4)-1/2*ln(-(3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*Roo
tOf(_Z^3-4)^3*x^3-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2
*RootOf(_Z^3-4)^2*x^3-12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf
(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2-30
*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf
(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z
^3-4)+36*_Z^2)*x^3-10*x*(-x^3+1)^(2/3)-RootOf(_Z^3-4)+12*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^
2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(109) = 218$.

time = 7.51, size = 316, normalized size = 2.19

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}(x^8-x^5)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{6}\,(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}\right)}{6(109x^9-105x^6+3x^3+1)}\right)-10\cdot 2^{\frac{2}{3}}(x^8-x^5)\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(x^3+1)+6(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)+5\cdot 2^{\frac{2}{3}}(x^8-x^5)\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(19x^6-16x^3+1)-12(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+36(8x^6-x^3-2)(-x^3+1)^{\frac{2}{3}}}{360(x^8-x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(10*sqrt(6)*2^(1/6)*(x^8 - x^5)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3
)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6
+ 33*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x
^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(x^8 - x^5)*log((6*2^(1/3)*(-x^3 +
1)^(1/3)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(
2/3)*(x^8 - x^5)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*
x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) +
 36*(8*x^6 - x^3 - 2)*(-x^3 + 1)^(2/3))/(x^8 - x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.651 $\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} \, dx$

Optimal. Leaf size=162

$$\frac{1}{2x^8\sqrt[3]{1-x^3}}-\frac{5(1-x^3)^{2/3}}{8x^8}-\frac{13(1-x^3)^{2/3}}{20x^5}-\frac{49(1-x^3)^{2/3}}{40x^2}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{\cdots}\right.}{}$$

```
[Out] 1/2/x^8/(-x^3+1)^(1/3)-5/8*(-x^3+1)^(2/3)/x^8-13/20*(-x^3+1)^(2/3)/x^5-49/4
0*(-x^3+1)^(2/3)/x^2+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*
3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {483, 597, 12, 384}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{4\sqrt[3]{2}}-\frac{5(1-x^3)^{2/3}}{8x^8}+\frac{1}{2x^8\sqrt[3]{1-x^3}}-\frac{13(1-x^3)^{2/3}}{20x^5}-\frac{49(1-x^3)^{2/3}}{40x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*x^8*(1 - x^3)^(1/3)) - (5*(1 - x^3)^(2/3))/(8*x^8) - (13*(1 - x^3)^(2/
3))/(20*x^5) - (49*(1 - x^3)^(2/3))/(40*x^2) + ArcTan[(1 - (2*2^(1/3)*x)/(1
 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - L
og[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = -\frac{70-308x^3+1162x^6+2856x^9-4914x^{12}-2268x^{15}+3402x^{18}-70\,{}_2F_1\Big(}{}$$

Mathematica [A]
time = 0.36, size = 157, normalized size = 0.97

$$\frac{1}{120}\left(-\frac{3(5+x^3+23x^6-49x^9)}{x^8\sqrt[3]{1-x^3}}+10\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-10\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+5\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-3*(5 + x^3 + 23*x^6 - 49*x^9))/(x^8*(1 - x^3)^(1/3)) + 10*2^(2/3)*Sqrt[3
]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 10*2^(2/3)*Log[2*x +
2^(2/3)*(1 - x^3)^(1/3)] + 5*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3)
 - 2^(1/3)*(1 - x^3)^(2/3)])/120
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 11.77, size = 963, normalized size = 5.94

method	result	size

risch	Expression too large to display	963
trager	Expression too large to display	1165

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(49*x^9-23*x^6-x^3-5)/x^8/(-x^3+1)^(1/3)+1/12*ln(-(6*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2
/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x
+(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4
)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)
+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(
x+1)/(x^2-x+1))*RootOf(_Z^3-4)+1/2*ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Ro
otOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+(-x^3+1)^(1/3)*
RootOf(_Z^3-4)^2*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(
_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootOf(_Z^3-4
)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-1/2*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootO
f(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-5*(-x^3+1)^(1/3)
*RootOf(_Z^3-4)^2*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf
(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+9*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+10*x*(-x^3+1)^(2/3)-3*RootOf(_Z^
3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(123) = 246$.

time = 7.27, size = 351, normalized size = 2.17

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^{11}-x^{8}\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^{7}+4x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-12\sqrt{6}\,(-1)^{\frac{1}{3}}\left(19x^{8}-16x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^{9}-111x^{6}+33x^{3}-1\right)\right)}{6\left(109x^{9}-105x^{6}+3x^{3}+1\right)}\right)-10\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{11}-x^{8}\right)\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{3}+1\right)+6\left(-x^{3}+1\right)^{\frac{2}{3}}x}{x^{3}+1}\right)+5\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{11}-x^{8}\right)\log\left(-\frac{3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(5x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(19x^{6}-16x^{3}+1\right)+12\left(2x^{5}-x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{6}+2x^{3}+1}\right)+9\left(49x^{9}-23x^{6}-x^{3}-5\right)\left(-x^{3}+1\right)^{\frac{2}{3}}}{360\left(x^{11}-x^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^11 - x^8)*arctan(1/6*2^(1/6)*(6*sq
rt(6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*
(-1)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x
^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(
-1)^(1/3)*(x^11 - x^8)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(
2/3)*(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*(-
1)^(1/3)*(x^11 - x^8)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/
3) - 2^(1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)
^(1/3))/(x^6 + 2*x^3 + 1)) + 9*(49*x^9 - 23*x^6 - x^3 - 5)*(-x^3 + 1)^(2/3)
)/(x^11 - x^8)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9}\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^9\,{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.652 $\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=292

$$\frac{x^5}{2\sqrt[3]{1-x^3}}+\frac{3}{4}x^2\left(1-x^3\right)^{2/3}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log((}{}$$

```
[Out] 1/2*x^5/(-x^3+1)^(1/3)+3/4*x^2*(-x^3+1)^(2/3)-1/2*x^2*hypergeom([1/3, 2/3],
[5/3],x^3)-1/48*ln((1-x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1
)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3
+1)^(1/3))*2^(2/3)+1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arcta
n(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {481, 596, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}+\frac{x^5}{2\sqrt[3]{1-x^3}}+\frac{3}{4}\left(1-x^3\right)^{2/3}x^2-\frac{\log\left((1-x)(x+1)^2\right)}{24\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^5/(2*(1 - x^3)^(1/3)) + (3*x^2*(1 - x^3)^(2/3))/4 - ArcTan[(1 - (2*2^(1/3
)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(
1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (x^2*Hypergeo
metric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) - Lo
g[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/
3)]/(12*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) +
 Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{11}x^{11}F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};x^3,-x^3\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.06, size = 71, normalized size = 0.24

$$\frac{1}{20}x^2\left(-\frac{5(-3+x^3)}{\sqrt[3]{1-x^3}}-15F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)-4x^3F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^2*((-5*(-3 + x^3))/(1 - x^3)^(1/3) - 15*AppellF1[2/3, 1/3, 1, 5/3, x^3,
-x^3] - 4*x^3*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/20
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(-x^{3}+1\right)^{\frac{4}{3}} \left(x^{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)*x^10/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}} \left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**10/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{10}}{{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(x^10/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.653 $\int \frac{x^7}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=274

$$\frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}$$

```
[Out] 1/2*x^2/(-x^3+1)^(1/3)-3/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-x
)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/16
*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {481, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}+\frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\log\left((1-x)(x+1)^2\right)}{24\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^2/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3
))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (3*x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x
^3])/4 + Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/
(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) - Log[1 +
 (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 -
 x^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{8}x^8F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};x^3,-x^3\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.05, size = 66, normalized size = 0.24

$$\frac{1}{10}x^2\left(\frac{5}{\sqrt[3]{1-x^3}}-5F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)-3x^3F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - 3*x^3*A
ppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(-x^3+1)^{\frac{4}{3}}(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)*x^7/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(-(x-1)(x^2+x+1))^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\left(1-x^3\right)^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^7/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.654 $\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=274

$$\frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}-$$

```
[Out] 1/2*x^2/(-x^3+1)^(1/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/48*ln((1-x
)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/16
*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arctan(1/3*(1+2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {482, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}+\frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{\log\left((1-x)(x+1)^2\right)}{24\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^2/(2*(1 - x^3)^(1/3)) - ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3
))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3
])/4 - Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1
 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) + Log[1 + (
2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{5}x^5 F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.06, size = 66, normalized size = 0.24

$$\frac{1}{10}x^2\left(\frac{5}{\sqrt[3]{1-x^3}}-5F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)-x^3F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - x^3*App
ellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-x^3+1)^{\frac{4}{3}}(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)*x^4/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^4/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.655 $\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=274

$$\frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}$$

```
[Out] 1/2*x^2/(-x^3+1)^(1/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-x
)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/16
*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {483, 494, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}+\frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\log\left((1-x)(x+1)^2\right)}{24\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^2/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3
))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3
])/4 + Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) - Log[1 + (
2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.02, size = 45, normalized size = 0.16

$$\frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{10}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
[Out] x^2/(2*(1 - x^3)^(1/3)) - (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x}{(-x^3+1)^{\frac{4}{3}}(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)*x/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(x/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.656 $\int \frac{1}{x^2\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=292

$$\frac{1}{2x\sqrt[3]{1-x^3}}-\frac{3(1-x^3)^{2/3}}{2x}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log((1$$

```
[Out] 1/2/x/(-x^3+1)^(1/3)-3/2*(-x^3+1)^(2/3)/x-3/4*x^2*hypergeom([1/3, 2/3],[5/3
],x^3)-1/48*ln((1-x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2
/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^
(1/3))*2^(2/3)+1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3
*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arctan(1/
3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {483, 597, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{3(1-x^3)^{2/3}}{2x}+\frac{1}{2\sqrt[3]{1-x^3}\,x}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{24\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*x*(1 - x^3)^(1/3)) - (3*(1 - x^3)^(2/3))/(2*x) - ArcTan[(1 - (2*2^(1/3
)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(
1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (3*x^2*Hyperg
eometric2F1[1/3, 2/3, 5/3, x^3])/4 - Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) -
Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(
1/3)]/(12*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3))
 + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=-\frac{F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};x^3,-x^3\right)}{x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 20.06, size = 76, normalized size = 0.26

$$\frac{-2+3x^3}{2x\sqrt[3]{1-x^3}}-x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)-\frac{3}{10}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-2 + 3*x^3)/(2*x*(1 - x^3)^(1/3)) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x
^3] - (3*x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-x^{3}+1\right)^{\frac{4}{3}} \left(x^{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^11 - x^8 - x^5 + x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-\left(x - 1\right) \left(x^{2} + x + 1\right)\right)^{\frac{4}{3}} \left(x + 1\right) \left(x^{2} - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(1/(x^2*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.657 $\int \frac{1}{x^5(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=308

$$\frac{1}{2x^4\sqrt[3]{1-x^3}}-\frac{3(1-x^3)^{2/3}}{4x^4}-\frac{(1-x^3)^{2/3}}{x}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\right.$$

```
[Out] 1/2/x^4/(-x^3+1)^(1/3)-3/4*(-x^3+1)^(2/3)/x^4-(-x^3+1)^(2/3)/x-1/2*x^2*hype
rgeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-x)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3
)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^
(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^
(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3
^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^
(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {483, 597, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{(1-x^3)^{2/3}}{x}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}-\frac{3(1-x^3)^{2/3}}{4x^4}+\frac{1}{2\sqrt[3]{1-x^3}\,x^4}+\frac{\log\left((1-x)(x+1)^2\right)}{24\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*x^4*(1 - x^3)^(1/3)) - (3*(1 - x^3)^(2/3))/(4*x^4) - (1 - x^3)^(2/3)/x
 + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqr
t[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(4*2^(1/3)*
Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 + Log[(1 - x)*(1 +
 x)^2]/(24*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3
)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x
^3)^(1/3)]/(6*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^5 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\, dx = -\frac{F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};x^3,-x^3\right)}{4x^4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 20.08, size = 79, normalized size = 0.26

$$-\frac{\frac{5(1+x^3-4x^6)}{\sqrt[3]{1-x^3}}+5x^6F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+4x^9F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
[Out] -1/20*((5*(1 + x^3 - 4*x^6))/(1 - x^3)^(1/3) + 5*x^6*AppellF1[2/3, 1/3, 1, 
5/3, x^3, -x^3] + 4*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/x^4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^14 - x^11 - x^8 + x^5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^5*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.658 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=264

$$-\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{4/3}}{4b^3d^3}-\frac{(bc+2ad)\,(a+bx^3)^{7/3}}{7b^3d^2}+\frac{(a+bx^3)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{bc-ad}\ \tan^{-}}{}$$

```
[Out] -c^3*(b*x^3+a)^(1/3)/d^4+1/4*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(4/3)/b^3/
d^3-1/7*(2*a*d+b*c)*(b*x^3+a)^(7/3)/b^3/d^2+1/10*(b*x^3+a)^(10/3)/b^3/d-1/6
*c^3*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(13/3)+1/2*c^3*(-a*d+b*c)^(1/3)*ln((-a*
d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(13/3)-1/3*c^3*(-a*d+b*c)^(1/3)*arc
tan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(13/3)*3^
(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {457, 90, 52, 60, 631, 210, 31}

$$\frac{(a+bx^3)^{4/3}(a^2d^2+abcd+b^2c^2)}{4b^3d^3}-\frac{c^3\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{13/3}}-\frac{(a+bx^3)^{7/3}(2ad+bc)}{7b^3d^2}+\frac{(a+bx^3)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{bc-ad}\,\log(c+dx^3)}{6d^{13/3}}+\frac{c^3\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}-\frac{c^3\sqrt[3]{a+bx^3}}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((c^3*(a + b*x^3)^(1/3))/d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)
^(4/3))/(4*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(7/3))/(7*b^3*d^2) + (a +
b*x^3)^(10/3)/(10*b^3*d) - (c^3*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a
 + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^3*(b*
c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(13/3)) + (c^3*(b*c - a*d)^(1/3)*Log[(b
*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx}}{b^2d^3} + \frac{(-bc-2ad)(a+bx)^{4/3}}{b^2d^2} + \frac{(a+bx)^{7/3}}{b^2d}\right.\right.$$

$$= \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{7/3}}{7b^3d^2} + \frac{(a+bx^3)^{10/3}}{10b^3d} - \frac{c^3\text{Subs}}{}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{7/3}}{7b^3d^2} + \frac{(a+b}{10}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{7/3}}{7b^3d^2} + \frac{(a+b}{10}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{7/3}}{7b^3d^2} + \frac{(a+b}{10}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{7/3}}{7b^3d^2} + \frac{(a+b}{10}$$

Mathematica [A]
time = 0.57, size = 308, normalized size = 1.17

$$\frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}\left(9a^3d^3-3a^2bd^2(-5c+dx^3)+ab^2d(35c^2-5cdx^3+2d^2x^6)+b^3(-140c^3+35c^2dx^3-20cd^2x^6+14d^3x^9)\right)}{b^3} - 140\sqrt{3}\,c^3\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 140c^3\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) - 70c^3\sqrt[3]{bc-ad}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{420d^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(1/3)*(a + b*x^3)^(1/3)*(9*a^3*d^3 - 3*a^2*b*d^2*(-5*c + d*x^3) + a*b
^2*d*(35*c^2 - 5*c*d*x^3 + 2*d^2*x^6) + b^3*(-140*c^3 + 35*c^2*d*x^3 - 20*c
*d^2*x^6 + 14*d^3*x^9)))/b^3 - 140*Sqrt[3]*c^3*(b*c - a*d)^(1/3)*ArcTan[(1
- (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 140*c^3*(b*c
- a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 70*c^3*(b
*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^
3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(420*d^(13/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{11} (b\,x^{3}+a)^{\frac{1}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.45, size = 325, normalized size = 1.23

$$-\frac{140\,\sqrt{3}\,b^{3} c^{3} \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \arctan\left(-\frac{2\,\sqrt{3}\,(bx^{3}+a)^{\frac{1}{3}} d \left(\frac{bc-ad}{d}\right)^{\frac{2}{3}} - \sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right) + 70\,b^{3} c^{3} \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left((bx^{3}+a)^{\frac{2}{3}} - (bx^{3}+a)^{\frac{1}{3}} \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right) - 140\,b^{3} c^{3} \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left((bx^{3}+a)^{\frac{1}{3}} + \left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right) - 3\,(14\,b^{3} d^{3} x^{9} - 2\,(10\,b^{3} c d^{2} - a b^{2} d^{3}) x^{6} - 140\,b^{3} c^{3} + 35\,a b^{2} c^{2} d + 15\,a^{2} b c d^{2} + 9\,a^{3} d^{3} + (35\,b^{3} c^{2} d - 5\,a b^{2} c d^{2} - 3\,a^{2} b d^{3}) x^{3}) (bx^{3}+a)^{\frac{1}{3}}}{420\,b^{3} d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/420*(140*sqrt(3)*b^3*c^3*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b
*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d))
 + 70*b^3*c^3*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/
3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 140*b^3*c^3*((b*c - a*d
)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(14*b^3*d^3*x
^9 - 2*(10*b^3*c*d^2 - a*b^2*d^3)*x^6 - 140*b^3*c^3 + 35*a*b^2*c^2*d + 15*a
^2*b*c*d^2 + 9*a^3*d^3 + (35*b^3*c^2*d - 5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^3)*
(b*x^3 + a)^(1/3))/(b^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11} \sqrt[3]{a + b x^{3}}}{c + d x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

<u>Giac</u> [A]
time = 0.92, size = 379, normalized size = 1.44

$$-\frac{\left(b^{34}c^{4}d^{6}-ab^{33}c^{3}d^{7}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{34}cd^{10}-ab^{33}d^{11}\right)}+\frac{\sqrt{3}\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}c^{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^{5}}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}c^{3}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^{5}}-\frac{140\left(bx^{3}+a\right)^{\frac{1}{3}}b^{30}c^{3}d^{6}-35\left(bx^{3}+a\right)^{\frac{4}{3}}b^{29}c^{2}d^{7}+20\left(bx^{3}+a\right)^{\frac{7}{3}}b^{28}cd^{8}-35\left(bx^{3}+a\right)^{\frac{4}{3}}ab^{28}cd^{8}-14\left(bx^{3}+a\right)^{\frac{10}{3}}b^{27}d^{9}+40\left(bx^{3}+a\right)^{\frac{7}{3}}ab^{27}d^{9}-35\left(bx^{3}+a\right)^{\frac{4}{3}}a^{2}b^{27}d^{9}}{140\,b^{30}d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^34*c^4*d^6 - a*b^33*c^3*d^7)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^34*c*d^10 - a*b^33*d^11) + 1/3*sqr
t(3)*(-b*c*d^2 + a*d^3)^(1/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^5 + 1/6*(-b*c*d^2 + a*d^
3)^(1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/
3) + (-(b*c - a*d)/d)^(2/3))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^30*c^3*d^
6 - 35*(b*x^3 + a)^(4/3)*b^29*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b^28*c*d^8 - 3
5*(b*x^3 + a)^(4/3)*a*b^28*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^27*d^9 + 40*(b*x
^3 + a)^(7/3)*a*b^27*d^9 - 35*(b*x^3 + a)^(4/3)*a^2*b^27*d^9)/(b^30*d^10)
```

<u>Mupad</u> [B]
time = 4.98, size = 442, normalized size = 1.67

$$\left(\frac{3\,a^2}{4\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{4\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{4/3}-\left(\frac{3\,a}{7\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{7\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{7/3}-{\left(b\,x^3+a\right)}^{1/3}\,\left(\frac{a^3}{b^3\,d}+\frac{\left(\frac{3\,a^2}{b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)+\frac{{\left(b\,x^3+a\right)}^{10/3}}{10\,b^3\,d}-\frac{c^3\,\ln\left({\left(a\,d-b\,c\right)}^{1/3}-d^{1/3}\,{\left(b\,x^3+a\right)}^{1/3}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{13/3}}-\frac{c^3\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(b\,c^4-a\,c^3\,d\right)}{d^2}+\frac{3\,c^3\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{d^{7/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{13/3}}+\frac{c^3\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(b\,c^4-a\,c^3\,d\right)}{d^2}-\frac{9\,c^3\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{d^{7/3}}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] ((3*a^2)/(4*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(4*b^3*d))*(a + b*x^3)^(4/3) - ((3*a)/(7*b^3*d) + (b^4*c - a*b^
3*d)/(7*b^6*d^2))*(a + b*x^3)^(7/3) - (a + b*x^3)^(1/3)*(a^3/(b^3*d) + (((3
*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a*b
^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(b^3*d)) + (a + b*x^3)^(10/3)/(10*b^3*d)
 - (c^3*log((a*d - b*c)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c)^(1/3
))/(3*d^(13/3)) - (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4 - a*c^3*d))/d^2 + (3
*c^3*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(4/3))/d^(7/3))*((3^(1/2)*1i)/2 - 1
/2)*(a*d - b*c)^(1/3))/(3*d^(13/3)) + (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4
- a*c^3*d))/d^2 - (9*c^3*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(4/3))/d^(7/3))
*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(1/3))/d^(13/3)
```

3.659 $\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=220

$$\frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}}+\frac{c^2\sqrt[3]{bc-ad}}{6\ldots}$$

```
[Out] c^2*(b*x^3+a)^(1/3)/d^3-1/4*(a*d+b*c)*(b*x^3+a)^(4/3)/b^2/d^2+1/7*(b*x^3+a)
^(7/3)/b^2/d+1/6*c^2*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(10/3)-1/2*c^2*(-a*d+b*
c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)+1/3*c^2*(-a*
d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1
/2))/d^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 90, 52, 60, 631, 210, 31}

$$\frac{c^2\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}}-\frac{\left(a+bx^3\right)^{4/3}(ad+bc)}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c^2\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}+\frac{c^2\sqrt[3]{a+bx^3}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (c^2*(a + b*x^3)^(1/3))/d^3 - ((b*c + a*d)*(a + b*x^3)^(4/3))/(4*b^2*d^2) +
 (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3
)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c^2
*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c^2*(b*c - a*d)^(1/3)*Lo
g[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(-bc-ad)\sqrt[3]{a+bx}}{bd^2} + \frac{(a+bx)^{4/3}}{bd} + \frac{c^2\sqrt[3]{a+bx}}{d^2(c+dx)}\right) dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} + \frac{c^2\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} - \frac{\left(c^2(bc-ad)\right)\text{Subst}\left(\int \frac{}{(a+bx)^2}\right.}{3d^3}$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} + \frac{c^2\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{10/3}} -$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} + \frac{c^2\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{10/3}} -$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} + \frac{c^2\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}}{\sqrt[3]{b}}}{\sqrt{}}\right.}{\sqrt{3}\,d^{10/3}}$$

Mathematica [A]

time = 0.40, size = 265, normalized size = 1.20

$$\frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}\left(-3a^2d^2+abd\left(-7c+dx^3\right)+b^2\left(28c^2-7cdx^3+4d^2x^6\right)\right)}{b^2}+28\sqrt{3}\,c^2\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-28c^2\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+14c^2\sqrt[3]{bc-ad}\,\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{84d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(1/3)*(a + b*x^3)^(1/3)*(-3*a^2*d^2 + a*b*d*(-7*c + d*x^3) + b^2*(28*
c^2 - 7*c*d*x^3 + 4*d^2*x^6)))/b^2 + 28*Sqrt[3]*c^2*(b*c - a*d)^(1/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*c^2*(
b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 14*c^
2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a +
b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(84*d^(10/3))
```

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^8 (b\,x^3+a)^{\frac{1}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.61, size = 282, normalized size = 1.28

$$-\frac{28\,\sqrt{3}\,b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\,\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+14\,b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-28\,b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-3\,(4\,b^2d^2x^6+28\,b^2c^2-7\,abcd-3\,a^2d^2-(7\,b^2cd-abd^2)x^3)(bx^3+a)^{\frac{1}{3}}}{84\,b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] -1/84*(28*sqrt(3)*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*
x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d))
 + 14*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1
/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 28*b^2*c^2*(-(b*c -
a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 3*(4*b^2*d^
2*x^6 + 28*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2 - (7*b^2*c*d - a*b*d^2)*x^3)*(b*
x^3 + a)^(1/3))/(b^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [A]
time = 0.87, size = 320, normalized size = 1.45

$$\frac{\left(b^{17}c^{3}d^{4}-ab^{16}c^{2}d^{5}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^{3}+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{17}cd^{7}-ab^{16}d^{8}\right)}-\frac{\sqrt{3}\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}c^{2}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^{3}+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^{4}}-\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}c^{2}\log\left((bx^{3}+a)^{\frac{2}{3}}+(bx^{3}+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^{4}}+\frac{28\,(bx^{3}+a)^{\frac{1}{3}}b^{14}c^{2}d^{4}-7\,(bx^{3}+a)^{\frac{4}{3}}b^{13}cd^{5}+4\,(bx^{3}+a)^{\frac{7}{3}}b^{12}d^{6}-7\,(bx^{3}+a)^{\frac{4}{3}}ab^{12}d^{6}}{28\,b^{14}d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^17*c^3*d^4 - a*b^16*c^2*d^5)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 +
 a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^17*c*d^7 - a*b^16*d^8) - 1/3*sqrt(3
)*(-b*c*d^2 + a*d^3)^(1/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-
(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^4 - 1/6*(-b*c*d^2 + a*d^3)^
(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28*(28*(b*x^3 + a)^(1/3)*b^14*c^2*d^4 - 7
*(b*x^3 + a)^(4/3)*b^13*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^12*d^6 - 7*(b*x^3 + a
)^(4/3)*a*b^12*d^6)/(b^14*d^7)
```

Mupad [B]
time = 4.93, size = 336, normalized size = 1.53

$$\left(\frac{a^{2}}{b^{2}\,d}+\frac{\left(\frac{2\,a}{b^{2}\,d}+\frac{b^{3}\,c-a\,b^{2}\,d}{b^{4}\,d^{2}}\right)\,\left(b^{3}\,c-a\,b^{2}\,d\right)}{b^{2}\,d}\right)\,\left(b\,x^{3}+a\right)^{1/3}-\left(\frac{a}{2\,b^{2}\,d}+\frac{b^{3}\,c-a\,b^{2}\,d}{4\,b^{4}\,d^{2}}\right)\,\left(b\,x^{3}+a\right)^{4/3}+\frac{\left(b\,x^{3}+a\right)^{7/3}}{7\,b^{2}\,d}+\frac{c^{2}\,\ln\left(\left(a\,d-b\,c\right)^{1/3}-d^{1/3}\,\left(b\,x^{3}+a\right)^{1/3}\right)\,\left(a\,d-b\,c\right)^{1/3}}{3\,d^{10/3}}-\frac{c^{2}\,\ln\left(\frac{3\,\left(b\,x^{3}+a\right)^{1/3}\,\left(b\,c^{3}-a\,c^{2}\,d\right)}{d}-\frac{3\,c^{2}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{4/3}}{d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{1/3}}{3\,d^{10/3}}+\frac{c^{2}\,\ln\left(\frac{3\,\left(b\,x^{3}+a\right)^{1/3}\,\left(b\,c^{3}-a\,c^{2}\,d\right)}{d}+\frac{9\,c^{2}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^{4/3}}{d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^{1/3}}{d^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*b^
2*d))/(b^2*d))*(a + b*x^3)^(1/3) - (a/(2*b^2*d) + (b^3*c - a*b^2*d)/(4*b^4*
d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*log((a*d - b*c
)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c)^(1/3))/(3*d^(10/3)) - (c^2
*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d - (3*c^2*((3^(1/2)*1i)/2 + 1
/2)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/(
3*d^(10/3)) + (c^2*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d + (9*c^2*(
(3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/6 - 1/6)*(a
*d - b*c)^(1/3))/d^(10/3)
```

3.660 $\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=186

$$-\frac{c\sqrt[3]{a+bx^3}}{d^2}+\frac{(a+bx^3)^{4/3}}{4bd}-\frac{c\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}-\frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{c\sqrt[3]{bc-a}}{}$$

```
[Out] -c*(b*x^3+a)^(1/3)/d^2+1/4*(b*x^3+a)^(4/3)/b/d-1/6*c*(-a*d+b*c)^(1/3)*ln(d*
x^3+c)/d^(7/3)+1/2*c*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)
^(1/3))/d^(7/3)-1/3*c*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1
/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)*3^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 81, 52, 60, 631, 210, 31}

$$-\frac{c\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}-\frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{c\sqrt[3]{a+bx^3}}{d^2}+\frac{(a+bx^3)^{4/3}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((c*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*b*d) - (c*(b*c - a*d)^(
1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(7/3)) - (c*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(7/3)) + (c*
(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^
(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt[3]{a+bx}}{c+dx}\,dx,x,x^3\right)$$

$$= \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx,x,x^3\right)}{3d}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} + \frac{(c(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3d^2}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\,\log(c+dx^3)}{6d^{7/3}} + \frac{\left(c\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int \cdots\right.}{2\cdots}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\,\log(c+dx^3)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}\cdots\right.}{2d^{7/3}}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}} - \frac{c\sqrt[3]{bc-ad}\cdots}{6\cdots}$$

Mathematica [A]
time = 0.24, size = 226, normalized size = 1.22

$$\frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(-4bc+ad+bdx^3)}{b} - 4\sqrt{3}\,c\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 4c\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) - 2c\sqrt[3]{bc-ad}\,\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{12d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(1/3)*(a + b*x^3)^(1/3)*(-4*b*c + a*d + b*d*x^3))/b - 4*Sqrt[3]*c*(b*
c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))
/Sqrt[3]] + 4*c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^
3)^(1/3)] - 2*c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*
d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*d^(7/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^5(b\,x^3+a)^{\frac{1}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.71, size = 222, normalized size = 1.19

$$-\frac{4\sqrt{3}\,bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+2\,bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-4\,bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-3\,(bdx^3-4\,bc+ad)(bx^3+a)^{\frac{1}{3}}}{12\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*b*c*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 +
a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*b*
c*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a
*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 4*b*c*((b*c - a*d)/d)^(1/3)*log((b*
x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(b*d*x^3 - 4*b*c + a*d)*(b*x^3
+ a)^(1/3))/(b*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [A]
time = 0.85, size = 276, normalized size = 1.48

$$-\frac{\left(b^{6} c^{2} d^{2}-a b^{5} c d^{3}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{6} c d^{4}-a b^{5} d^{5}\right)}+\frac{\sqrt{3}\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}} c \arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3 d^{3}}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}} c \log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6 d^{3}}-\frac{4\left(bx^{3}+a\right)^{\frac{1}{3}} b^{4} c d^{2}-\left(bx^{3}+a\right)^{\frac{4}{3}} b^{3} d^{3}}{4 b^{4} d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^6*c^2*d^2 - a*b^5*c*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)
^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^6*c*d^4 - a*b^5*d^5) + 1/3*sqrt(3)*(-b
*c*d^2 + a*d^3)^(1/3)*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c -
a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*c
*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c
- a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b^4*c*d^2 - (b*x^3 + a)^(4/
3)*b^3*d^3)/(b^4*d^4)
```

Mupad [B]
time = 4.62, size = 298, normalized size = 1.60

$$\frac{\left(b x^{3}+a\right)^{4/3}}{4 b d}-\left(b x^{3}+a\right)^{1/3}\left(\frac{a}{b d}+\frac{b^{2} c-a b d}{b^{2} d^{2}}\right)-\frac{c \ln\left(\left(b x^{3}+a\right)^{1/3}\left(3 b c^{2}-3 a c d\right)+\frac{c\left(a d-b c\right)^{1/3}\left(9 a d^{3}-9 b c d^{2}\right)}{3 d^{7/3}}\right)\left(a d-b c\right)^{1/3}}{3 d^{7/3}}-\frac{c \ln\left(\left(b x^{3}+a\right)^{1/3}\left(3 b c^{2}-3 a c d\right)+\frac{c\left(-\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(a d-b c\right)^{1/3}\left(9 a d^{3}-9 b c d^{2}\right)}{3 d^{7/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(a d-b c\right)^{1/3}}{3 d^{7/3}}+\frac{c \ln\left(\left(b x^{3}+a\right)^{1/3}\left(3 b c^{2}-3 a c d\right)-\frac{c\left(\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(a d-b c\right)^{1/3}\left(9 a d^{3}-9 b c d^{2}\right)}{3 d^{7/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\left(a d-b c\right)^{1/3}}{3 d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*b*d) - (a + b*x^3)^(1/3)*(a/(b*d) + (b^2*c - a*b*d)/(b
^2*d^2)) - (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*(a*d - b*c)^(1
/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d - b*c)^(1/3))/(3*d^(7/3)) - (c
*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*((3^(1/2)*1i)/2 - 1/2)*(a*d
 - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 - 1/2)*(a
*d - b*c)^(1/3))/(3*d^(7/3)) + (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d)
 - (c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^
(7/3)))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/(3*d^(7/3))
```

3.661 $\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt[3]{a+bx^3}}{d}+\frac{\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}}+\frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{4/3}}-\frac{\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}\right.}{2d^{4/3}}$$

```
[Out] (b*x^3+a)^(1/3)/d+1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(4/3)-1/2*(-a*d+b*c)^(
1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(4/3)+1/3*(-a*d+b*c)^(1
/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(4
/3)*3^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {455, 52, 60, 631, 210, 31}

$$\frac{\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}}+\frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{4/3}}-\frac{\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}+\frac{\sqrt[3]{a+bx^3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(1/3)/d + ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)) + ((b*c - a*d)^(1/3)
*Log[c + d*x^3])/(6*d^(4/3)) - ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d
^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\,\log(c+dx^3)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+\right.}{2d^{4/3}}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\,\log(c+dx^3)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}} + \frac{\sqrt[3]{bc-ad}\,\log(c+dx^3)}{6d^{4/3}} - \frac{\sqrt[3]{}}{}$$

Mathematica [A]
time = 0.19, size = 205, normalized size = 1.29

$$\frac{6\sqrt[3]{d}\sqrt[3]{a+bx^3}+2\sqrt{3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+\sqrt[3]{bc-ad}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{6d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (6*d^(1/3)*(a + b*x^3)^(1/3) + 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)
*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log
[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*
(a + b*x^3)^(2/3)])/(6*d^(4/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(b\,x^3+a)^{\frac{1}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

[Out] int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.63, size = 206, normalized size = 1.30

$$-\frac{2\sqrt{3}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-6\,(bx^3+a)^{\frac{1}{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(
1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + (-(b*c
- a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^
(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^
(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 + a)^(1/3))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [A]
time = 0.73, size = 223, normalized size = 1.40

$$\frac{(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bcd-ad^2)}-\frac{\sqrt{3}\,(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^2}+\frac{(bx^3+a)^{\frac{1}{3}}}{d}-\frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c -
 a*d)/d)^(1/3)))/(b*c*d - a*d^2) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d
)/d)^(1/3))/d^2 + (b*x^3 + a)^(1/3)/d - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/
d)^(2/3))/d^2
```

Mupad [B]
time = 4.62, size = 249, normalized size = 1.57

$$\frac{(b\,x^3+a)^{1/3}}{d}+\frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a\,d^2-3\,b\,c\,d)-\frac{(a\,d-b\,c)^{1/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{4/3}}\right)\,(a\,d-b\,c)^{1/3}}{3\,d^{4/3}}-\frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a\,d^2-3\,b\,c\,d)+\frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{1/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{1/3}}{3\,d^{4/3}}+\frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a\,d^2-3\,b\,c\,d)-\frac{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^{1/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^{1/3}}{d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(1/3)/d + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) - ((a*d -
b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*(a*d - b*c)^(1/3))/(3*d^(4/3
)) - (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) + (((3^(1/2)*1i)/2 + 1/2)*(
a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*((3^(1/2)*1i)/2 + 1/2)
*(a*d - b*c)^(1/3))/(3*d^(4/3)) + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d
) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/d^(4/3
))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3))/d^(4/3)
```

3.662 $\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx$

Optimal. Leaf size=246

$$-\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}}-\frac{\sqrt[3]{a}\log(x)}{2c}-\frac{\sqrt[3]{bc-ad}\log(c+}{6c\sqrt[3]{d}}$$

```
[Out] -1/2*a^(1/3)*ln(x)/c-1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c/d^(1/3)+1/2*a^(1/3)
*ln(a^(1/3)-(b*x^3+a)^(1/3))/c+1/2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(
1/3)*(b*x^3+a)^(1/3))/c/d^(1/3)-1/3*a^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)
^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)-1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(
1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {457, 85, 59, 631, 210, 31, 60}

$$-\frac{\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}}-\frac{\sqrt[3]{a}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{\sqrt[3]{bc-ad}\log(c+dx^3)}{6c\sqrt[3]{d}}+\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}}+\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{\sqrt[3]{a}\log(x)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

```
[Out] -((a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*c)) - ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(1/3)) - (a^(1/3)*Log[x])/(2*c) - ((b
*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c*d^(1/3)) + (a^(1/3)*Log[a^(1/3) - (a +
 b*x^3)^(1/3)])/(2*c) + ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*
(a + b*x^3)^(1/3)])/(2*c*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\sqrt[3]{a}\,\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} - \frac{a^{2/}}{}$$

$$= -\frac{\sqrt[3]{a}\,\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} + \frac{\sqrt[3]{bc-ad}\,\log}{}$$

$$= -\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,c} - \frac{\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\,\log(x)}{2c}$$

Mathematica [A]
time = 0.41, size = 312, normalized size = 1.27

$$-\frac{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{d}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\sqrt[3]{d}\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+\sqrt[3]{a}\sqrt[3]{d}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)+\sqrt[3]{bc-ad}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*a^(1/3)*d^(1/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/
Sqrt[3]] + 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(
1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*a^(1/3)*d^(1/3)*Log[-a^(1/3) + (a + b
*x^3)^(1/3)] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x
^3)^(1/3)] + a^(1/3)*d^(1/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a +
 b*x^3)^(2/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a
*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(c*d^(1/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x), x)
```

Fricas [A]
time = 2.39, size = 276, normalized size = 1.12

$$-\frac{2\sqrt{3}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\left(bc-ad\right)}{3\left(bc-ad\right)}\right)+2\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-2a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-2\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1
/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*sqrt(3)
*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) + sqrt(3)*a)/a) +
a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + ((b*
c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)
^(1/3) + ((b*c - a*d)/d)^(2/3)) - 2*a^(1/3)*log((b*x^3 + a)^(1/3) - a^(1/3)
) - 2*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)))
/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x\left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x*(c + d*x**3)), x)
```

Giac [A]
time = 0.82, size = 311, normalized size = 1.26

$$-\frac{(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}-\frac{\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3c}-\frac{a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6c}+\frac{a^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3c}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3cd}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b*c^2 - a*c*d) - 1/3*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*
(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/c - 1/6*a^(1/3)*log((b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c + 1/3*a^(1/3)*log(abs((b*x^3 +
 a)^(1/3) - a^(1/3)))/c + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*s
qrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3
))/(c*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d)
```

Mupad [B]
time = 4.74, size = 1607, normalized size = 6.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x*(c + d*x^3)),x)
```

```
[Out] log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 +
 9*a^2*b^6*c^2*d^3) - (a/(27*c^3))^(1/3)*(((243*a*b^6*c^6*d^3 - 729*a^2*b^5
*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3) - (a + b*x^3)^(1/3)*(81*
a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4))*(a/(27*c^3))^(2/3) - 9*a*b^7*c^4*d^2 +
 27*a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4))*(a/(27*c^3))^(1/3) + log((a + b*
x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*
c^2*d^3) - (((243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5
)*(-(a*d - b*c)/(27*c^3*d))^(1/3) - (a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 8
1*a^2*b^5*c^4*d^4))*(-(a*d - b*c)/(27*c^3*d))^(2/3) - 9*a*b^7*c^4*d^2 + 27*
a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4)*(-(a*d - b*c)/(27*c^3*d))^(1/3))*(-(a
*d - b*c)/(27*c^3*d))^(1/3) + log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^
7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1i)/2 - 1/2)*
(-(a*d - b*c)/(27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 - 1/2)^2*((a + b*x^3)^(1/3
)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) - ((3^(1/2)*1i)/2 - 1/2)*(243*a*b
^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a*d - b*c)/(27*c
^3*d))^(1/3))*(-(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4*d^2 - 27*a^2*b^
6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(-(a*d - b*c)/(27*c
^3*d))^(1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*
a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)/(
27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 + 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5
*d^3 - 81*a^2*b^5*c^4*d^4) + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 72
9*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a*d - b*c)/(27*c^3*d))^(1/3))*(
-(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4*d^2 - 27*a^2*b^6*c^3*d^3 + 18*
a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)/(27*c^3*d))^(1/3) +
log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 +
```

```
9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1i)/2 - 1/2)*(a/(27*c^3))^(1/3)*(((3^(1/2)*
1i)/2 - 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) -
 ((3^(1/2)*1i)/2 - 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*
b^4*c^4*d^5)*(a/(27*c^3))^(1/3))*(a/(27*c^3))^(2/3) + 9*a*b^7*c^4*d^2 - 27*
a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(a/(27*c^3))^
(1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5
*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1/2)*(a/(27*c^3))^(1/3)*(((
3^(1/2)*1i)/2 + 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^
4*d^4) + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 +
486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3))*(a/(27*c^3))^(2/3) + 9*a*b^7*c^4*d
^2 - 27*a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 + 1/2)*(a/(2
7*c^3))^(1/3)
```

3.663 $\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx$

Optimal. Leaf size=340

$$\frac{d\sqrt[3]{a+bx^3}}{c^2}+\frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2}-\frac{(a+bx^3)^{4/3}}{3acx^3}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\tan$$

```
[Out] d*(b*x^3+a)^(1/3)/c^2+1/3*(-3*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2-1/3*(b*x^3+a)^
(4/3)/a/c/x^3-1/6*(-3*a*d+b*c)*ln(x)/a^(2/3)/c^2+1/6*d^(2/3)*(-a*d+b*c)^(1/
3)*ln(d*x^3+c)/c^2+1/6*(-3*a*d+b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)/c^2
-1/2*d^(2/3)*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/
c^2-1/9*(-3*a*d+b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2)
)/a^(2/3)/c^2*3^(1/2)+1/3*d^(2/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*
(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2*3^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {457, 105, 162, 52, 59, 631, 210, 31, 60}

$$-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(bc-3ad)}{3\sqrt{3}a^{2/3}c^2}+\frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2}-\frac{\log(x)(bc-3ad)}{6a^{2/3}c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^2}-\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}+\frac{d\sqrt[3]{a+bx^3}}{c^2}+\frac{\sqrt[3]{a+bx^3}(bc-3ad)}{3ac^2}-\frac{(a+bx^3)^{4/3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (d*(a + b*x^3)^(1/3))/c^2 + ((b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*a*c^2) - (
a + b*x^3)^(4/3)/(3*a*c*x^3) - ((b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*x^
3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*c^2) + (d^(2/3)*(b*c - a*d
)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3
]])/(Sqrt[3]*c^2) - ((b*c - 3*a*d)*Log[x])/(6*a^(2/3)*c^2) + (d^(2/3)*(b*c
- a*d)^(1/3)*Log[c + d*x^3])/(6*c^2) + ((b*c - 3*a*d)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(6*a^(2/3)*c^2) - (d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(\frac{1}{3}(-bc+3ad)-\frac{bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3acx^3} + \frac{d^2\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(bc-3ad)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x\right.}{9ac^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} + \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,\right.}{9c^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-a}}{}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-a}}{}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{3\sqrt{3}\,a^{2/3}c^2}$$

Mathematica [**A**]

time = 0.63, size = 351, normalized size = 1.03

$$\frac{-\frac{6c\sqrt[3]{a+bx^3}}{x^3}+\frac{2\sqrt{3}(-bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+6\sqrt{3}\,d^{2/3}\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+\frac{2(bc-3ad)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{2/3}}-6d^{2/3}\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+\frac{(-bc+3ad)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{a^{2/3}}+3d^{2/3}\sqrt[3]{bc-ad}\,\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{18c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]

[Out] ((-6*c*(a + b*x^3)^(1/3))/x^3 + (2*Sqrt[3]*(-(b*c) + 3*a*d)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + 6*Sqrt[3]*d^(2/3)*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + (2*(b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(2/3) - 6*d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + ((-(b*c) + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/a^(2/3) + 3*d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(18*c^2)

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{x^4\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x)

[Out] int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^4), x)

Fricas [A]
time = 3.70, size = 429, normalized size = 1.26

$$-\frac{6\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}a^2x^3\arctan\left(-\frac{2\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}-\sqrt{3}\left(bcd-ad^2\right)}{3\left(bcd-ad^2\right)}\right)+3\left(-bcd^2+ad^3\right)^{\frac{1}{3}}a^2x^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2+\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}d+\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\right)-6\left(-bcd^2+ad^3\right)^{\frac{1}{3}}a^2x^3\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d-\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\right)+2\sqrt{3}\left(abc-3a^2d\right)x^3\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}\left(bc-3ad\right)x^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)-2\left(-a^2\right)^{\frac{2}{3}}\left(bc-3ad\right)x^3\log\left(\left(bx^3+a\right)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)+6\left(bx^3+a\right)^{\frac{1}{3}}a^2c}{18a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="fricas")

```
[Out] -1/18*(6*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*arctan(-1/3*(2*sqrt(3)*(-
b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*d
- a*d^2)) + 3*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(2/3)*d^2 +
(-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
6*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*
d^3)^(1/3)) + 2*sqrt(3)*(a*b*c - 3*a^2*d)*x^3*sqrt(-(-a^2)^(1/3))*arctan(-1
/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt
(-(-a^2)^(1/3))/a^2) + (-a^2)^(2/3)*(b*c - 3*a*d)*x^3*log((b*x^3 + a)^(2/3)
*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*(b*c
 - 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a - (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)
*a^2*c)/(a^2*c^2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x^4 \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**4/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**4*(c + d*x**3)), x)
```

Giac [A]
time = 0.89, size = 351, normalized size = 1.03

$$\frac{(bcd - ad^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left|(bx^3 + a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3 - ac^2d)} - \frac{\sqrt{3}\,(bc - 3\,ad) \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{2}{3}}c^2} - \frac{\sqrt{3}\,(-bcd^2 + ad^3)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^2} - \frac{(bc - 3\,ad) \log\left((bx^3 + a)^{\frac{2}{3}} + (bx^3 + a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{18\,a^{\frac{2}{3}}c^2} - \frac{(-bcd^2 + ad^3)^{\frac{1}{3}} \log\left((bx^3 + a)^{\frac{2}{3}} + (bx^3 + a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2} + \frac{(bc - 3\,ad) \log\left(\left|(bx^3 + a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{2}{3}}c^2} - \frac{(bx^3 + a)^{\frac{1}{3}}}{3\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d - a*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b
*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(b*c - 3*a*d)*arctan(1
/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(2/3)*c^2) - 1/3*sqr
t(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^2 - 1/18*(b*c - 3*a*d)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^2) - 1/6
*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
- a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^2 + 1/9*(b*c - 3*a*d)*log(abs((
b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)/(c*x^3)
```

Mupad [B]
time = 9.99, size = 1917, normalized size = 5.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d
) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a
^2*c^6))^(1/3))*(-(3*a*d - b*c)^3/(a^2*c^6))^(2/3))/81 - (b^5*d^4*(27*a^3*d
^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-(3*a*d - b*c)^3/(
a^2*c^6))^(1/3))/9 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 +
 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*(-(27*a^3*
d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1/3) + log(
- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) -
81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^6)^
(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 +
17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/3 -
 (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2
- 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((a*d^3 - b*c*d^2)/(27*c^6))^(
1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(27*b^5*c^3*d^3
*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 81*a*b^4*c^4*d^3*((3
^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^
6)^(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 + (b^5*d^4*(27*a^3*d^3 + b^3*c^3
 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/
3 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d
^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^(1/2)*1i)/2 - 1/2)*((a*
d^3 - b*c*d^2)/(27*c^6))^(1/3) - log((2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d
^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*
c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(27*b^5*c^3*d^3*(a
+ b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) + 81*a*b^4*c^4*d^3*((3^(1/
2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^6)^(
1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 1
7*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/3)*(
(3^(1/2)*1i)/2 + 1/2)*((a*d^3 - b*c*d^2)/(27*c^6))^(1/3) + log((((3^(1/2)*1
i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a
^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*
a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))*(-(3*a*d
 - b*c)^3/(a^2*c^6))^(2/3))/81 + (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*
c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))/9 - (2*
b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32
*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d
^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1/3) - log((
2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 -
32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(
1/2)*1i)/2 - 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 -
5*a*b*c*d) + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 -
 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))*(-(3*a*d - b*c)^3/(a^2*c^6)
```

```
)^(2/3))/81 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*
d^2))/(3*c))*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))/9)*((3^(1/2)*1i)/2 + 1/2)*
(-(27*a^3*d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1
/3) - (a + b*x^3)^(1/3)/(3*c*x^3)
```

3.664 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx$

Optimal. Leaf size=370

$$\frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3}-\frac{(a+bx^3)^{4/3}}{6acx^6}+\frac{(b^2c^2+3abcd-9a^2d^2)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}c^3}-\frac{d^{5/3}\sqrt[3]{bc-ad}\ \tan$$

```
[Out] 1/9*(3*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^3-1/6*(b*x^3+a)^(4/3)/a/c/x^6+1/18*
(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*ln(x)/a^(5/3)/c^3-1/6*d^(5/3)*(-a*d+b*c)^(1/
3)*ln(d*x^3+c)/c^3-1/18*(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*ln(a^(1/3)-(b*x^3+a)
^(1/3))/a^(5/3)/c^3+1/2*d^(5/3)*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3
)*(b*x^3+a)^(1/3))/c^3+1/27*(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*arctan(1/3*(a^(1
/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)/c^3*3^(1/2)-1/3*d^(5/3)*(-a
*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(
1/2))/c^3*3^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {457, 105, 154, 162, 59, 631, 210, 31, 60}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(-9a^2d^2+3abcd+b^2c^2)}{9\sqrt{3}\,a^{5/3}c^3}-\frac{(-9a^2d^2+3abcd+b^2c^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3}+\frac{\log(x)(-9a^2d^2+3abcd+b^2c^2)}{18a^{5/3}c^3}-\frac{d^{5/3}\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^3}-\frac{d^{5/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^3}+\frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}+\frac{\sqrt[3]{a+bx^3}(3ad+bc)}{9ac^2x^3}-\frac{(a+bx^3)^{4/3}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((b*c + 3*a*d)*(a + b*x^3)^(1/3))/(9*a*c^2*x^3) - (a + b*x^3)^(4/3)/(6*a*c*
x^6) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(
1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*c^3) - (d^(5/3)*(b*c - a*d)^(1
/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/
(Sqrt[3]*c^3) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(5/3)*c^3)
 - (d^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 3*a*b*c
*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(5/3)*c^3) + (d^(5/
3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2
*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(\frac{2}{3}(bc+3ad)+\frac{2bdx}{3}\right)}{x^2(c+dx)}\,dx, x, x^3\right)}{6ac}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\frac{2}{9}\left(b^2c^2+3abcd-9a^2d^2\right)+\frac{2}{9}bd(bc-6ad)x}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x\right.}{6ac^2}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c^3}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}}{6}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}}{6}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{5/3}c^3}$$

Mathematica [A]
time = 0.81, size = 413, normalized size = 1.12

$$\frac{\frac{3c\sqrt[3]{a+bx^3}\left(-3ac-bcx^3+6adx^3\right)}{ax^6}+\frac{2\sqrt{3}\left(b^2c^2+3abcd-9a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}-18\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-\frac{2\left(b^2c^2+3abcd-9a^2d^2\right)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{5/3}}+18d^{5/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+\frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{a^{5/3}}-9d^{5/3}\sqrt[3]{bc-ad}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{54c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((3*c*(a + b*x^3)^(1/3)*(-3*a*c - b*c*x^3 + 6*a*d*x^3))/(a*x^6) + (2*Sqrt[3
]*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/
3))/Sqrt[3]])/a^(5/3) - 18*Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)*ArcTan[(1 - (2
*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - (2*(b^2*c^2 + 3*a
*b*c*d - 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(5/3) + 18*d^(5/3)
*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + ((b
^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (
a + b*x^3)^(2/3)])/a^(5/3) - 9*d^(5/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2
/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/
3)])/(54*c^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{7}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^7), x)
```

Fricas [A]
time = 5.03, size = 472, normalized size = 1.28

$$-\frac{18\sqrt{3}\left(bcd^2-ad^3\right)^{\frac{1}{3}}a^3dx^6\arctan\left(-\frac{2\sqrt{3}\left(bcd^2-ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}-\sqrt{3}\left(bcd-ad^2\right)}{3\left(bcd-ad^2\right)}\right)+9\left(bcd^2-ad^3\right)^{\frac{1}{3}}a^3dx^6\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2-\left(bcd^2-ad^3\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}d+\left(bcd^2-ad^3\right)^{\frac{2}{3}}\right)-18\left(bcd^2-ad^3\right)^{\frac{1}{3}}a^3dx^6\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d+\left(bcd^2-ad^3\right)^{\frac{1}{3}}\right)-2\sqrt{3}\left(ab^2c^2+3a^2bcd-9a^3d^2\right)\left(a^2\right)^{\frac{1}{6}}x^6\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)-\left(b^2c^2+3abcd-9a^2d^2\right)\left(a^2\right)^{\frac{2}{3}}x^6\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)+2\left(b^2c^2+3abcd-9a^2d^2\right)\left(a^2\right)^{\frac{2}{3}}x^6\log\left(\left(bx^3+a\right)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)+3\left(3a^3c^2+\left(a^2bc^2-6a^3cd\right)x^3\right)\left(bx^3+a\right)^{\frac{1}{3}}}{54a^3c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*arctan(-1/3*(2*sqrt(3)*
(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*d
 - a*d^2)) + 9*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(2/3)*d^2
- (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) -
18*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a
*d^3)^(1/3)) - 2*sqrt(3)*(a*b^2*c^2 + 3*a^2*b*c*d - 9*a^3*d^2)*(a^2)^(1/6)*
x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(
1/3)*(a^2)^(2/3))/a^2) - (b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*
log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) +
2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((b*x^3 + a)^(1/3)*a
 - (a^2)^(2/3)) + 3*(3*a^3*c^2 + (a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(
1/3))/(a^3*c^3*x^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**7/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**7*(c + d*x**3)), x)
```

Giac [**A**]
time = 0.87, size = 465, normalized size = 1.26

$$-\frac{\left(bcd^2-ad^3\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4-ac^3d\right)}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3c^3}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}d\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6c^3}+\frac{\sqrt{3}\left(a^{\frac{1}{3}}b^2c^2+3a^{\frac{4}{3}}bcd-9a^{\frac{7}{3}}d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{27a^2c^3}-\frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27a^{\frac{5}{3}}c^3}+\frac{\left(a^{\frac{1}{3}}b^2c^2+3a^{\frac{4}{3}}bcd-9a^{\frac{7}{3}}d^2\right)\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54a^2c^3}-\frac{\left(bx^3+a\right)^{\frac{4}{3}}b^2c+2\left(bx^3+a\right)^{\frac{1}{3}}ab^2c-6\left(bx^3+a\right)^{\frac{4}{3}}abd+6\left(bx^3+a\right)^{\frac{1}{3}}a^2bd}{18ab^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c*d^2 - a*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (
-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^
(1/3)*d*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(
-(b*c - a*d)/d)^(1/3))/c^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))
/c^3 + 1/27*sqrt(3)*(a^(1/3)*b^2*c^2 + 3*a^(4/3)*b*c*d - 9*a^(7/3)*d^2)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^2*c^3) - 1/27*(
b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(
5/3)*c^3) + 1/54*(a^(1/3)*b^2*c^2 + 3*a^(4/3)*b*c*d - 9*a^(7/3)*d^2)*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^2*c^3) - 1/18*((b
*x^3 + a)^(4/3)*b^2*c + 2*(b*x^3 + a)^(1/3)*a*b^2*c - 6*(b*x^3 + a)^(4/3)*a
*b*d + 6*(b*x^3 + a)^(1/3)*a^2*b*d)/(a*b^2*c^2*x^6)
```

Mupad [B]
time = 12.55, size = 2767, normalized size = 7.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x)
```

```
[Out] log(((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^5*(a*d - b*
c))/c^9)^(1/3) + (9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a
*b^2*c^2*d - 14*a^2*b*c*d^2))/a)*(-(d^5*(a*d - b*c))/c^9)^(2/3))/9 - (b^5*d
^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a
^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(d^5*(a
*d - b*c))/c^9)^(1/3))/3 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c
^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564
*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*(-(a*d
^6 - b*c*d^5)/(27*c^9))^(1/3) + log((((((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(1
2*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 9*a*b^4*c^4*d^3*(2
*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*
c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(2/3))/729 -
(b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 +
 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(
b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 - (b^4*d^6*(a + b*x
^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^
3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*
b*c*d^6))/(243*a^3*c^8))*(-(b^6*c^6 - 729*a^6*d^6 - 135*a^3*b^3*c^3*d^3 + 9
*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3) - (((a + b*x^3)^(1/3
)*(b^2*c + 3*a*b*d))/(9*c^2) - (b*(a + b*x^3)^(4/3)*(6*a*d - b*c))/(18*a*c^
2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) + log(- (((3^(1/2)*1i)/2 - 1/2)
*((((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d -
14*a^2*b*c*d^2))/a + 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d)*(-(d^5*(a*d - b*c))/c^9)^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(-(d^5*(a*d - b*c))/c^9)^(2/3))/9 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2
*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d -
1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(d^5*(a*d - b*c))/c^9)^(1/3))/3 - (b^4*d
^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3
*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d
- 3888*a^6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2 - 1/2)*(-(a*d^6 - b*c*d
^5)/(27*c^9))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((9*b^5*c^2*d^3*(a + b*
x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a - 81*a*
b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^5
*(a*d - b*c))/c^9)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(-(d^5*(a*d - b*c))/c^9)^(
2/3))/9 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3
*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3
*c^4))*(-(d^5*(a*d - b*c))/c^9)^(1/3))/3 + (b^4*d^6*(a + b*x^3)^(1/3)*(1458
```

*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(a*d^6 - b*c*d^5)/(27*c^9))^(1/3) + log(- (((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 9*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(2/3))/729 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2 - 1/2)*(-(b^6*c^6 - 729*a^6*d^6 - 135*a^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a - 9*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(2/3))/729 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 + (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(b^6*c^6 - 729*a^6*d^6 - 135*a^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3)

3.665 $\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=336

$$-\frac{(6bc-ad)x^2\sqrt[3]{a+bx^3}}{18bd^2}+\frac{x^5\sqrt[3]{a+bx^3}}{6d}-\frac{(9b^2c^2-3abcd-a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{5/3}d^3}+\frac{c^{5/3}\sqrt[3]{bc-ad}\,\tan}{}$$

```
[Out] -1/18*(-a*d+6*b*c)*x^2*(b*x^3+a)^(1/3)/b/d^2+1/6*x^5*(b*x^3+a)^(1/3)/d-1/6*
c^(5/3)*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^3-1/18*(-a^2*d^2-3*a*b*c*d+9*b^2*c^2
)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d^3+1/2*c^(5/3)*(-a*d+b*c)^(1/3)*ln
((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/27*(-a^2*d^2-3*a*b*c*d+9
*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d^3*3
^(1/2)+1/3*c^(5/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1
/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {489, 596, 598, 337, 503}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(-a^2d^2-3abcd+9b^2c^2)}{9\sqrt{3}\,b^{5/3}d^3}-\frac{(-a^2d^2-3abcd+9b^2c^2)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}d^3}+\frac{c^{5/3}\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^3}-\frac{c^{5/3}\sqrt[3]{bc-ad}\,\log(c+dx^3)}{6d^3}+\frac{c^{5/3}\sqrt[3]{bc-ad}\,\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3}-\frac{x^2\sqrt[3]{a+bx^3}(6bc-ad)}{18bd^2}+\frac{x^5\sqrt[3]{a+bx^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -1/18*((6*b*c - a*d)*x^2*(a + b*x^3)^(1/3))/(b*d^2) + (x^5*(a + b*x^3)^(1/3
))/(6*d) - ((9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(5/3)*d^3) + (c^(5/3)*(b*c - a*d)^(1
/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3
]])/(Sqrt[3]*d^3) - (c^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^3) - ((
9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(18*b^
(5/3)*d^3) + (c^(5/3)*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
 (a + b*x^3)^(1/3)])/(2*d^3)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x^7\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^8\sqrt[3]{a+bx^3}\,F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.11, size = 527, normalized size = 1.57

$$\frac{\frac{6dx^2\sqrt[3]{a+bx^3}(-6bc+ad+3bdx^3)}{b}-\frac{4\sqrt{3}(9b^2c^2-3abcd-a^2d^2)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{5/3}}-18\sqrt{-6-6i\sqrt{3}}c^{5/3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+\frac{4(-9b^2c^2+3abcd+a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{b^{5/3}}+18i(i+\sqrt{3})c^{5/3}\sqrt[3]{bc-ad}\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+\frac{2(9b^2c^2-3abcd-a^2d^2)\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{5/3}}+9(1-i\sqrt{3})c^{5/3}\sqrt[3]{bc-ad}\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{108d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((6*d*x^2*(a + b*x^3)^(1/3)*(-6*b*c + a*d + 3*b*d*x^3))/b - (4*Sqrt[3]*(9*b
^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a
+ b*x^3)^(1/3))])/b^(5/3) - 18*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(5/3)*(b*c - a*d)
^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I +
 Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(-9*b^2*c^2 + 3*a*b*c*d + a^2*d^
2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(5/3) + (18*I)*(I + Sqrt[3])*c^
(5/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)
*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*Log[b^(2/3)*x^2
+ b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(5/3) + 9*(1 - I*Sqrt
[3])*c^(5/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3
])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*
(a + b*x^3)^(2/3)])/(108*d^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^7 (b\,x^3+a)^{\frac{1}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

Fricas [A]
time = 4.98, size = 494, normalized size = 1.47

$$\frac{18\sqrt{3}(bc^3-ac^2d)^{\frac{1}{3}}b^3c\arctan\left(-\frac{\sqrt{3}(bc^2-acd)x+2\sqrt{3}(bc^3-ac^2d)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}}{3(bc^2-acd)x}\right)+18(bc^3-ac^2d)^{\frac{1}{3}}b^3c\log\left(\frac{(bx^3+a)^{\frac{1}{3}}c-(bc^3-ac^2d)^{\frac{1}{3}}x}{x}\right)-9(bc^3-ac^2d)^{\frac{1}{3}}b^3c\log\left(\frac{(bx^3+a)^{\frac{2}{3}}c^2+(bc^3-ac^2d)^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}cx+(bc^3-ac^2d)^{\frac{2}{3}}x^2}{x^2}\right)+2\sqrt{3}(9b^3c^2-3ab^2cd-a^2bd^2)(b^2)^{\frac{1}{6}}\arctan\left(\frac{\left(\sqrt{3}(b^2)^{\frac{1}{3}}bx+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{3b^2x}\right)-2(9b^2c^2-3abcd-a^2d^2)(b^2)^{\frac{2}{3}}\log\left(-\frac{(b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{1}{3}}b}{x}\right)+(9b^2c^2-3abcd-a^2d^2)(b^2)^{\frac{2}{3}}\log\left(\frac{(b^2)^{\frac{1}{3}}bx^2+(bx^3+a)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}x+(bx^3+a)^{\frac{2}{3}}b}{x^2}\right)+3(3b^3d^2x^5-(6b^3cd-ab^2d^2)x^2)(bx^3+a)^{\frac{1}{3}}}{54b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(18*sqrt(3)*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*arctan(-1/3*(sqrt(3)*(b*c^2
- a*c*d)*x + 2*sqrt(3)*(b*c^3 - a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 -
 a*c*d)*x)) + 18*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3 + a)^(1/3)*c - (
b*c^3 - a*c^2*d)^(1/3)*x)/x) - 9*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3
+ a)^(2/3)*c^2 + (b*c^3 - a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)*c*x + (b*c^3 - a
*c^2*d)^(2/3)*x^2)/x^2) + 2*sqrt(3)*(9*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*(
b^2)^(1/6)*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3
)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*(9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*(
b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 3*a
*b*c*d - a^2*d^2)*(b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b
^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c*d - a
*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^3*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 \sqrt[3]{a + bx^3}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.666 $\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=276

$$\frac{x^2\sqrt[3]{a+bx^3}}{3d}+\frac{(3bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}d^2}-\frac{c^{2/3}\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}+\frac{c^{2/3}\sqrt[3]{bc-ad}\cdots}{6\cdots}$$

```
[Out] 1/3*x^2*(b*x^3+a)^(1/3)/d+1/6*c^(2/3)*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^2+1/6*
(-a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)/d^2-1/2*c^(2/3)*(-a*d+b*
c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2+1/9*(-a*d+3*b*c
)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d^2*3^(1/2)-1
/3*c^(2/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x
^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {489, 598, 337, 503}

$$\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-ad)}{3\sqrt{3}\,b^{2/3}d^2}-\frac{c^{2/3}\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}+\frac{(3bc-ad)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}d^2}+\frac{c^{2/3}\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^2}-\frac{c^{2/3}\sqrt[3]{bc-ad}\,\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{x^2\sqrt[3]{a+bx^3}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3))/(3*d) + ((3*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)*d^2) - (c^(2/3)*(b*c - a*d)^(
1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[
3]])/(Sqrt[3]*d^2) + (c^(2/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^2) + (
(3*b*c - a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*b^(2/3)*d^2) - (c^(2/3
)*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*d^2)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x^4\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= \frac{x^5\sqrt[3]{a+bx^3}\,F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.76, size = 467, normalized size = 1.69

$$
\frac{12dx^2\sqrt[3]{a+bx^3}+\frac{4\sqrt{3}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{2/3}}+6\sqrt{-6-6i\sqrt{3}}\,c^{2/3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+\frac{4(3bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+6\left(1-i\sqrt{3}\right)c^{2/3}\sqrt[3]{bc-ad}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+\frac{2(-3bc+ad)\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{2/3}}+3i\left(i+\sqrt{3}\right)c^{2/3}\sqrt[3]{bc-ad}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{36d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (12*d*x^2*(a + b*x^3)^(1/3) + (4*Sqrt[3]*(3*b*c - a*d)*ArcTan[(Sqrt[3]*b^(1
/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(2/3) + 6*Sqrt[-6 - (6*I)*Sqrt
```

```
[3]]*c^(2/3)*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
 - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(3*b*c -
 a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/3) + 6*(1 - I*Sqrt[3])*c^
(2/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)
*(a + b*x^3)^(1/3)] + (2*(-3*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*
x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3) + (3*I)*(I + Sqrt[3])*c^(2/3)*(b*c
 - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c -
 a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)
])/(36*d^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(b \,x^{3}+a \right)^{\frac{1}{3}}}{d \,x^{3}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(222) = 444.
time = 4.30, size = 452, normalized size = 1.64

$$\frac{6 \left(b x^{3}+a\right)^{\frac{1}{3}} b^{2} d x^{2}+6 \sqrt{3} \left(-b c^{3}+a c^{2} d\right)^{\frac{1}{3}} b^{2} \arctan \left(-\frac{\sqrt{3} \left(b c^{2}-a c d\right) x+2 \sqrt{3} \left(-b c^{3}+a c^{2} d\right)^{\frac{2}{3}} \left(b x^{3}+a\right)^{\frac{1}{3}}}{3 \left(b c^{2}-a c d\right) x}\right)+6 \left(-b c^{3}+a c^{2} d\right)^{\frac{1}{3}} b^{2} \log \left(\frac{\left(b x^{3}+a\right)^{\frac{1}{3}} c+\left(-b c^{3}+a c^{2} d\right)^{\frac{1}{3}} x}{x}\right)-3 \left(-b c^{3}+a c^{2} d\right)^{\frac{1}{3}} b^{2} \log \left(\frac{\left(b x^{3}+a\right)^{\frac{2}{3}} c^{2}-\left(-b c^{3}+a c^{2} d\right)^{\frac{1}{3}} \left(b x^{3}+a\right)^{\frac{1}{3}} c x+\left(-b c^{3}+a c^{2} d\right)^{\frac{2}{3}} x^{2}}{x^{2}}\right)-2 \sqrt{3} \left(3 b^{2} c-a b d\right) \sqrt{-\left(-b^{2}\right)^{\frac{1}{3}}} \arctan \left(-\frac{\left(\sqrt{3} \left(-b^{2}\right)^{\frac{1}{3}} b x-2 \sqrt{3} \left(b x^{3}+a\right)^{\frac{1}{3}} \left(-b^{2}\right)^{\frac{2}{3}}\right) \sqrt{-\left(-b^{2}\right)^{\frac{1}{3}}}}{3 b^{2} x}\right)+2 \left(-b^{2}\right)^{\frac{2}{3}} \left(3 b c-a d\right) \log \left(-\frac{\left(-b^{2}\right)^{\frac{2}{3}} x-\left(b x^{3}+a\right)^{\frac{1}{3}} b}{x}\right)-\left(-b^{2}\right)^{\frac{2}{3}} \left(3 b c-a d\right) \log \left(-\frac{\left(-b^{2}\right)^{\frac{1}{3}} b x^{2}-\left(b x^{3}+a\right)^{\frac{1}{3}} \left(-b^{2}\right)^{\frac{2}{3}} x-\left(b x^{3}+a\right)^{\frac{2}{3}} b}{x^{2}}\right)}{18 b^{2} d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*sqrt(3)*(-b*c^3 + a*c^2*d)^(1/3)*b^
2*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c^3 + a*c^2*d)^(2/
3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) + 6*(-b*c^3 + a*c^2*d)^(1/3)*b^2
*log(((b*x^3 + a)^(1/3)*c + (-b*c^3 + a*c^2*d)^(1/3)*x)/x) - 3*(-b*c^3 + a*
c^2*d)^(1/3)*b^2*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x
^3 + a)^(1/3)*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) - 2*sqrt(3)*(3*b^2*c
```

```
 - a*b*d)*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqr
t(3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) + 2*(-b^2
)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) - (-b^
2)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^
(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sqrt[3]{a + bx^3}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**4*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(b\,x^3+a\right)^{1/3}}{d\,x^3+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x)
[Out] int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.667 $\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=234

$$-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d}+\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d}-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6\sqrt[3]{c}d}-\frac{\sqrt[3]{b}\log\left(\cdots\right.}{}$$

```
[Out] -1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(1/3)/d-1/2*b^(1/3)*ln(b^(1/3)*x-(b*x^3
+a)^(1/3))/d+1/2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(
1/3))/c^(1/3)/d-1/3*b^(1/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1
/2))/d*3^(1/2)+1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1
/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(1/3)/d*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {495, 337, 503}

$$\frac{\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d}-\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6\sqrt[3]{c}d}+\frac{\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d)) + ((b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(1/3)*d) - ((b*c - a*d)^(1/3)*Log[c + d
*x^3])/(6*c^(1/3)*d) - (b^(1/3)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d) +
 ((b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*c^(1/3)*d)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
```

```
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.29, size = 423, normalized size = 1.81

$$\frac{-4\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-\frac{2\sqrt{-6-6i\sqrt{3}}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{c}}-4\sqrt[3]{b}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)+\frac{2i\left(i+\sqrt{3}\right)\sqrt[3]{bc-ad}\log\left(2\sqrt[3]{bc-ad}x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{c}}+2\sqrt[3]{b}\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)+\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{bc-ad}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}\left(a+bx^3\right)^{2/3}\right)}{\sqrt[3]{c}}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (-4*Sqrt[3]*b^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(
1/3))] - (2*Sqrt[-6 - (6*I)*Sqrt[3]]*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d
)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^
3)^(1/3))])/c^(1/3) - 4*b^(1/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + ((2
*I)*(I + Sqrt[3])*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(1/3) + 2*b^(1/3)*Log[b^(2/3)*x^2 + b^(1
/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 - I*Sqrt[3])*(b*c - a*d)
^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(
1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/c^(1
/3))/(12*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x(b\,x^{3}+a)^{\frac{1}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

Fricas [A]
time = 2.94, size = 330, normalized size = 1.41

$$\frac{2\sqrt{3}\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,(bc-ad)x+2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right)-2\sqrt{3}\,(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,bx+2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}}{3\,bx}\right)+2\,(-b)^{\frac{1}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)+2\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(-\frac{x\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}-(bx^3+a)^{\frac{1}{3}}}{x}\right)-(-b)^{\frac{1}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)-\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(\frac{x^2\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}x\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*((b*c - a*d)/c)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2
*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) - 2*sq
rt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)
^(2/3))/(b*x)) + 2*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2
*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 + a)^(1/3))/x
) - (-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^
3 + a)^(2/3))/x^2) - ((b*c - a*d)/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) +
 (b*x^3 + a)^(1/3)*x*((b*c - a*d)/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{a+bx^{3}}}{c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] int((x*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.668 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx$

Optimal. Leaf size=168

$$-\frac{\sqrt[3]{a+bx^3}}{cx}-\frac{\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{4/3}}+\frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c^{4/3}}-\frac{\sqrt[3]{bc-ad}\,\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}\right.}{2c^{4/3}}$$

```
[Out] -(b*x^3+a)^(1/3)/c/x+1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(4/3)-1/2*(-a*d+b*c
)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)-1/3*(-a*d+b*
c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2
))/c^(4/3)*3^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {486, 12, 503}

$$-\frac{\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{4/3}}+\frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c^{4/3}}-\frac{\sqrt[3]{bc-ad}\,\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}}-\frac{\sqrt[3]{a+bx^3}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(c*x)) - ((b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(4/3)) + ((b*c -
 a*d)^(1/3)*Log[c + d*x^3])/(6*c^(4/3)) - ((b*c - a*d)^(1/3)*Log[((b*c - a*
d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
```

```
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^2(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{\sqrt[3]{a+bx^3}\,\sqrt[3]{1+\frac{dx^3}{c}}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{cx\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.46, size = 309, normalized size = 1.84

$$\frac{-\frac{12\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}{x}+2\sqrt{-6-6i\sqrt{3}}\,\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\,\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)+2\left(1-i\sqrt{3}\right)\sqrt[3]{bc-ad}\,\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)+i\left(i+\sqrt{3}\right)\sqrt[3]{bc-ad}\,\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{12c^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] ((-12*c^(1/3)*(a + b*x^3)^(1/3))/x + 2*Sqrt[-6 - (6*I)*Sqrt[3]]*(b*c - a*d)
^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I +
 Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 2*(1 - I*Sqrt[3])*(b*c - a*d)^(1/3)
*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + I
*(I + Sqrt[3])*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt
[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3
)*(a + b*x^3)^(2/3)])/(12*c^(4/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{2}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**2/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**2*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^3+a)^{1/3}}{x^2\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)), x)
```

3.669 $\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)} \, dx$

Optimal. Leaf size=204

$$-\frac{\sqrt[3]{a+bx^3}}{4cx^4}-\frac{(bc-4ad)\sqrt[3]{a+bx^3}}{4ac^2x}+\frac{d\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}}-\frac{d\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c^{7/3}}+\frac{d}{}$$

```
[Out] -1/4*(b*x^3+a)^(1/3)/c/x^4-1/4*(-4*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x-1/6*d*(
-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(7/3)+1/2*d*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1
/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)+1/3*d*(-a*d+b*c)^(1/3)*arctan(1/3*(1
+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {486, 597, 12, 503}

$$\frac{d\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}}-\frac{d\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c^{7/3}}+\frac{d\sqrt[3]{bc-ad}\,\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}-\frac{\sqrt[3]{a+bx^3}\,(bc-4ad)}{4ac^2x}-\frac{\sqrt[3]{a+bx^3}}{4cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -1/4*(a + b*x^3)^(1/3)/(c*x^4) - ((b*c - 4*a*d)*(a + b*x^3)^(1/3))/(4*a*c^2
*x) + (d*(b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a
+ b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(7/3)) - (d*(b*c - a*d)^(1/3)*Log[c +
 d*x^3])/(6*c^(7/3)) + (d*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/
3) - (a + b*x^3)^(1/3)])/(2*c^(7/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
```

```
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^5(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{2c(a+bx^3)\left(c-3dx^3\right)-(bc-ad)x^3(c-3dx^3)\ {}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(a+bx^3)}\right)+3(bc-ad)x}{8c^3x^4\left(a+bx^3\right)^{2/3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.58, size = 333, normalized size = 1.63

$$\frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(-ac-bcx^3+4adx^3\right)}{ax^4}-2\sqrt{-6-6i\sqrt{3}}\,d\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+2i\left(i+\sqrt{3}\right)d\sqrt[3]{bc-ad}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+\left(1-i\sqrt{3}\right)d\sqrt[3]{bc-ad}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{12c^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] ((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) - b*c*x^3 + 4*a*d*x^3))/(a*x^4) - 2*S
qrt[-6 - (6*I)*Sqrt[3]]*d*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/
```

```
(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]
+ (2*I)*(I + Sqrt[3])*d*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 +
I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + (1 - I*Sqrt[3])*d*(b*c - a*d)^(1/3)
*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x
*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(7/3
))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{5}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**5/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**5*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)), x)
```

3.670 $\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx$

Optimal. Leaf size=258

$$-\frac{\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(bc-7ad)\sqrt[3]{a+bx^3}}{28ac^2x^4}+\frac{(3b^2c^2+7abcd-28a^2d^2)\sqrt[3]{a+bx^3}}{28a^2c^3x}-\frac{d^2\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-}}{\sqrt[3]{c}\sqrt[3]{a+}}}{\sqrt{3}}\right.}{\sqrt{3}\,c^{10/3}}$$

```
[Out] -1/7*(b*x^3+a)^(1/3)/c/x^7-1/28*(-7*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^4+1/28
*(-28*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(1/3)/a^2/c^3/x+1/6*d^2*(-a*d+
b*c)^(1/3)*ln(d*x^3+c)/c^(10/3)-1/2*d^2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3
)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(10/3)-1/3*d^2*(-a*d+b*c)^(1/3)*arctan(1/3*(
1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {486, 597, 12, 503}

$$\frac{\sqrt[3]{a+bx^3}\,(-28a^2d^2+7abcd+3b^2c^2)}{28a^2c^3x}-\frac{d^2\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{10/3}}+\frac{d^2\sqrt[3]{bc-ad}\,\log(c+dx^3)}{6c^{10/3}}-\frac{d^2\sqrt[3]{bc-ad}\,\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}-\frac{\sqrt[3]{a+bx^3}\,(bc-7ad)}{28ac^2x^4}-\frac{\sqrt[3]{a+bx^3}}{7cx^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -1/7*(a + b*x^3)^(1/3)/(c*x^7) - ((b*c - 7*a*d)*(a + b*x^3)^(1/3))/(28*a*c^
2*x^4) + ((3*b^2*c^2 + 7*a*b*c*d - 28*a^2*d^2)*(a + b*x^3)^(1/3))/(28*a^2*c
^3*x) - (d^2*(b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)
*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(10/3)) + (d^2*(b*c - a*d)^(1/3)*
Log[c + d*x^3])/(6*c^(10/3)) - (d^2*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3
)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(10/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
```

```
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^8(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{8ac^3+8bc^3x^3-12ac^2dx^3-12bc^2dx^6+36acd^2x^6+36bcd^2x^9-2(bc-ad)x^3(2c^2-}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.87, size = 373, normalized size = 1.45

$$\frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(-3b^2c^2x^6+abcx^3\left(c-7dx^3\right)+a^2\left(4c^2-7cdx^3+28d^2x^6\right)\right)}{a^2x^7}+14\sqrt{-6-6i\sqrt{3}}\,d^2\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+14\left(1-i\sqrt{3}\right)d^2\sqrt[3]{bc-ad}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+7i\left(i+\sqrt{3}\right)d^2\sqrt[3]{bc-ad}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}\left(a+bx^3\right)^{2/3}\right)}{84c^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] ((-3*c^(1/3)*(a + b*x^3)^(1/3)*(-3*b^2*c^2*x^6 + a*b*c*x^3*(c - 7*d*x^3) +
a^2*(4*c^2 - 7*c*d*x^3 + 28*d^2*x^6)))/(a^2*x^7) + 14*Sqrt[-6 - (6*I)*Sqrt[
```

```
3]]*d^2*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*
d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 14*(1 - I*Sqrt[3
])*d^2*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3
)*(a + b*x^3)^(1/3)] + (7*I)*(I + Sqrt[3])*d^2*(b*c - a*d)^(1/3)*Log[2*(b*c
 - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3
)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(84*c^(10/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{8}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^{3}}}{x^{8}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**8/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**8*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^8\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)), x)
```

3.671 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx$

Optimal. Leaf size=318

$$-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(bc-10ad)\sqrt[3]{a+bx^3}}{70ac^2x^7}+\frac{(3b^2c^2+5abcd-35a^2d^2)\sqrt[3]{a+bx^3}}{140a^2c^3x^4}-\frac{(9b^3c^3+15ab^2c^2d+35a^2bcd^2-}{140a^3c^4x}$$

```
[Out] -1/10*(b*x^3+a)^(1/3)/c/x^10-1/70*(-10*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^7+1
/140*(-35*a^2*d^2+5*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(1/3)/a^2/c^3/x^4-1/140*(-
140*a^3*d^3+35*a^2*b*c*d^2+15*a*b^2*c^2*d+9*b^3*c^3)*(b*x^3+a)^(1/3)/a^3/c^
4/x-1/6*d^3*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(13/3)+1/2*d^3*(-a*d+b*c)^(1/3)*
ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(13/3)+1/3*d^3*(-a*d+b*c)^
(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/
c^(13/3)*3^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {486, 597, 12, 503}

$$\frac{\sqrt[3]{a+bx^3}(-35a^2d^2+5abcd+3b^2c^2)}{140a^2c^3x^4}-\frac{\sqrt[3]{a+bx^3}(-140a^3d^3+35a^2bcd^2+15ab^2c^2d+9b^3c^3)}{140a^3c^4x}+\frac{d^3\sqrt[3]{bc-ad}\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{13/3}}-\frac{d^3\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{13/3}}+\frac{d^3\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{13/3}}-\frac{\sqrt[3]{a+bx^3}(bc-10ad)}{70ac^2x^7}-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] -1/10*(a + b*x^3)^(1/3)/(c*x^10) - ((b*c - 10*a*d)*(a + b*x^3)^(1/3))/(70*a
*c^2*x^7) + ((3*b^2*c^2 + 5*a*b*c*d - 35*a^2*d^2)*(a + b*x^3)^(1/3))/(140*a
^2*c^3*x^4) - ((9*b^3*c^3 + 15*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 140*a^3*d^3)*
(a + b*x^3)^(1/3))/(140*a^3*c^4*x) + (d^3*(b*c - a*d)^(1/3)*ArcTan[(1 + (2*
(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(13/
3)) - (d^3*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c^(13/3)) + (d^3*(b*c - a*d
)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(13/3)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^{11}(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{56ac^4+56bc^4x^3-72ac^3dx^3-72bc^3dx^6+108ac^2d^2x^6+108bc^2d^2x^9-324acd^3x^9-}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.08, size = 419, normalized size = 1.32

$$\frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(-9b^3c^3x^9+3ab^2c^2x^6(c-5dx^3)+a^2bcx^3\left(-2c^2+5cdx^3-35d^2x^6\right)+a^3\left(-14c^3+20c^2dx^3-35cd^2x^6+140d^3x^9\right)\right)}{a^3x^{10}}-70\sqrt{-6-6i\sqrt{3}}\,d^3\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+70i\left(i+\sqrt{3}\right)d^3\sqrt[3]{bc-ad}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+35\left(1-i\sqrt{3}\right)d^3\sqrt[3]{bc-ad}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{420c^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] ((3*c^(1/3)*(a + b*x^3)^(1/3)*(-9*b^3*c^3*x^9 + 3*a*b^2*c^2*x^6*(c - 5*d*x^
3) + a^2*b*c*x^3*(-2*c^2 + 5*c*d*x^3 - 35*d^2*x^6) + a^3*(-14*c^3 + 20*c^2*
d*x^3 - 35*c*d^2*x^6 + 140*d^3*x^9)))/(a^3*x^10) - 70*Sqrt[-6 - (6*I)*Sqrt[
3]]*d^3*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*
d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (70*I)*(I + Sqrt
[3])*d^3*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3)] + 35*(1 - I*Sqrt[3])*d^3*(b*c - a*d)^(1/3)*Log[2*(b*
c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(420*c^(13/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{11}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^{3}}}{x^{11}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**11/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**11*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^{11}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)), x)
```

3.672 $\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal. Leaf size=64

$$\frac{x^7 \sqrt[3]{a+bx^3}\, F_1\left(\frac{7}{3}; -\frac{1}{3}, 1; \frac{10}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/7*x^7*(b*x^3+a)^(1/3)*AppellF1(7/3,-1/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^7 \sqrt[3]{a+bx^3}\, F_1\left(\frac{7}{3}; -\frac{1}{3}, 1; \frac{10}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -1/3, 1, 10/3, -((b*x^3)/a), -((d*x^3)
/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^6\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x^7\sqrt[3]{a+bx^3}\,F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(64) = 128.
time = 6.43, size = 281, normalized size = 4.39

$$\frac{x\left(4(a+bx^3)(-5bc+ad+2bdx^3)-\frac{(-10b^2c^2+5abcd+a^2d^2)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{16a^2c^2(-5bc+ad)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)}{40bd^2(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x*(4*(a + b*x^3)*(-5*b*c + a*d + 2*b*d*x^3) - ((-10*b^2*c^2 + 5*a*b*c*d +
a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + a*d)*AppellF1[1/3, 2/3, 1, 4/3, -
((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3,
 -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x
^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d
*x^3)/c)])))))/(40*b*d^2*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^6 (b\,x^3+a)^{\frac{1}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 \sqrt[3]{a + bx^3}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] int((x^6*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.673 $\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/4*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-1/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x
^3/a)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^3\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(64) = 128.
time = 5.54, size = 240, normalized size = 3.75

$$\frac{x\left(\frac{(-2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+4\left(a+bx^3+\frac{4a^2c^2F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)\right)}{8d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x*(((-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)])/c + 4*(a + b*x^3 + (4*a^2*c^2*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7
/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3
)/a), -((d*x^3)/c)]))))))/(8*d*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(b\,x^3+a\right)^{\frac{1}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt[3]{a + bx^3}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**3*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] int((x^3*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.674 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)^
(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(59) = 118.
time = 10.12, size = 160, normalized size = 2.71

$$\frac{4acx\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(4acF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(-3adF_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bcF_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
[Out] (4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3), x)
```

3.675 $\int \frac{\sqrt[3]{a+bx^3}}{x^3(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -1/2*(b*x^3+a)^(1/3)*AppellF1(-2/3,-1/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(1+b
*x^3/a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -1/2*((a + b*x^3)^(1/3)*AppellF1[-2/3, -1/3, 1, 1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\,(c+dx^3)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^3(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 327 vs. 2(64) = 128.
time = 10.18, size = 327, normalized size = 5.11

$$\frac{-bdx^6\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c\left(16ac(bdx^6+a(c+3dx^3))F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-4x^3(a+bx^3)(c+dx^3)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}}{8c^2x^2\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] (-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)]) + (c*(16*a*c*(b*d*x^6 + a*(c + 3*d*x^3))*AppellF1[1/3, 2/3, 1
, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(-4*a*c*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)]))))/(8*c^2*x^2*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{x^3\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**3/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**3*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^3\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^3*(c + d*x^3)), x)
```

3.676 $\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -1/5*(b*x^3+a)^(1/3)*AppellF1(-5/3,-1/3,1,-2/3,-b*x^3/a,-d*x^3/c)/c/x^5/(1+
b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$-\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -1/5*((a + b*x^3)^(1/3)*AppellF1[-5/3, -1/3, 1, -2/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c*x^5*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^6(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 289 vs. 2(64) = 128.
time = 10.24, size = 289, normalized size = 4.52

$$\frac{-\frac{4(a+bx^3)(2ac+bcx^3-5adx^3)}{ac^2x^5}+\frac{bd(-bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3}+\frac{16(b^2c^2+5abcd-10a^2d^2)xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}}{40\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] ((-4*(a + b*x^3)*(2*a*c + b*c*x^3 - 5*a*d*x^3))/(a*c^2*x^5) + (b*d*(-(b*c)
+ 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/(a*c^3) + (16*(b^2*c^2 + 5*a*b*c*d - 10*a^2*d^2)*x*AppellF1
[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/
3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7
/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{x^6\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x^6 \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**6/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**6*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^6\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^6*(c + d*x^3)), x)
```

3.677 $\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=266

$$-\frac{c^3(a+bx^3)^{2/3}}{2d^4}+\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{5/3}}{5b^3d^3}-\frac{(bc+2ad)\,(a+bx^3)^{8/3}}{8b^3d^2}+\frac{(a+bx^3)^{11/3}}{11b^3d}-\frac{c^3(bc-ad)^{2/3}}{}$$

```
[Out] -1/2*c^3*(b*x^3+a)^(2/3)/d^4+1/5*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(5/3)/
b^3/d^3-1/8*(2*a*d+b*c)*(b*x^3+a)^(8/3)/b^3/d^2+1/11*(b*x^3+a)^(11/3)/b^3/d
+1/6*c^3*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(14/3)-1/2*c^3*(-a*d+b*c)^(2/3)*ln(
(-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(14/3)-1/3*c^3*(-a*d+b*c)^(2/3)
*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(14/3
)*3^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 90, 52, 58, 631, 210, 31}

$$\frac{(a+bx^3)^{5/3}(a^2d^2+abcd+b^2c^2)}{5b^3d^3}-\frac{c^3(bc-ad)^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{14/3}}-\frac{(a+bx^3)^{8/3}(2ad+bc)}{8b^3d^2}+\frac{(a+bx^3)^{11/3}}{11b^3d}+\frac{c^3(bc-ad)^{2/3}\log(c+dx^3)}{6d^{14/3}}-\frac{c^3(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}}-\frac{c^3(a+bx^3)^{2/3}}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -1/2*(c^3*(a + b*x^3)^(2/3))/d^4 + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^
3)^(5/3))/(5*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(8/3))/(8*b^3*d^2) + (a
+ b*x^3)^(11/3)/(11*b^3*d) - (c^3*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(14/3)) + (c^3*(
b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(14/3)) - (c^3*(b*c - a*d)^(2/3)*Log[
(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(a+bx)^{2/3}}{c+dx}\,dx,x,x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(b^2c^2+abcd+a^2d^2)(a+bx)^{2/3}}{b^2d^3} + \frac{(-bc-2ad)(a+bx)^{5/3}}{b^2d^2} + \frac{(a+bx}{b^2d}\right.\right.$$

$$= \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{8/3}}{8b^3d^2} + \frac{(a+bx^3)^{11/3}}{11b^3d} - \frac{c^3\text{Su}}{}$$

$$= -\frac{c^3(a+bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{8/3}}{8b^3d^2} + \frac{(}{}$$

$$= -\frac{c^3(a+bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{8/3}}{8b^3d^2} + \frac{(}{}$$

$$= -\frac{c^3(a+bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{8/3}}{8b^3d^2} + \frac{(}{}$$

$$= -\frac{c^3(a+bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{8/3}}{8b^3d^2} + \frac{(}{}$$

Mathematica [A]
time = 0.63, size = 310, normalized size = 1.17

$$\frac{\frac{3d^{2/3}(a+bx^3)^{2/3}(18a^3d^3+3a^2bd^2(11c-4dx^3)+2ab^2d(44c^2-11cdx^3+5d^2x^6)+b^3(-220c^3+88c^2dx^3-55cd^2x^6+40d^3x^9))}{b^3} - 440\sqrt{3}c^3(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 440c^3(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) + 220c^3(bc-ad)^{2/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{1320d^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(2/3)*(a + b*x^3)^(2/3)*(18*a^3*d^3 + 3*a^2*b*d^2*(11*c - 4*d*x^3) +
2*a*b^2*d*(44*c^2 - 11*c*d*x^3 + 5*d^2*x^6) + b^3*(-220*c^3 + 88*c^2*d*x^3
- 55*c*d^2*x^6 + 40*d^3*x^9)))/b^3 - 440*Sqrt[3]*c^3*(b*c - a*d)^(2/3)*ArcT
an[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 440*c^3
*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 220
*c^3*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
 + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(1320*d^(14/3))
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{11}(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(219) = 438$.
time = 2.62, size = 455, normalized size = 1.71

$$-\frac{440\,\sqrt{3}\,b^3c^3\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\,\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+220\,b^3c^3\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{2}{3}}(bc-ad)+(bc-ad)\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\right)-440\,b^3c^3\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(-d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)\right)-3\,(40\,b^3d^3x^9-5\,(11\,b^3cd^2-2\,ab^2d^3)x^6-220\,b^3c^3+88\,ab^2c^2d+33\,a^2bcd^2+18\,a^3d^3+2\,(44\,b^3c^2d-11\,ab^2cd^2-6\,a^2bd^3)x^3)(bx^3+a)^{\frac{2}{3}}}{1320\,b^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*sqrt(3)*b^3*c^3*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*a
rctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
/d^2)^(1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 220*b^3*c^3*(-(b^2*c^2 -
2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) - 440*b^3*c^3*(-(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2
/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) - 3*(40*b^3*d^3*x^9 - 5*(11*b^3*c*d^2
- 2*a*b^2*d^3)*x^6 - 220*b^3*c^3 + 88*a*b^2*c^2*d + 33*a^2*b*c*d^2 + 18*a^3
*d^3 + 2*(44*b^3*c^2*d - 11*a*b^2*c*d^2 - 6*a^2*b*d^3)*x^3)*(b*x^3 + a)^(2/
3))/(b^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [A]
time = 1.53, size = 409, normalized size = 1.54

$$-\frac{\left(b^{37}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{36}c^3d^8\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{37}cd^{11}-ab^{36}d^{12}\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^6}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^6}-\frac{220(bx^3+a)^{\frac{2}{3}}b^{33}c^3d^7-88(bx^3+a)^{\frac{5}{3}}b^{32}c^2d^8+55(bx^3+a)^{\frac{8}{3}}b^{31}cd^9-88(bx^3+a)^{\frac{5}{3}}ab^{31}cd^9-40(bx^3+a)^{\frac{11}{3}}b^{30}d^{10}+110(bx^3+a)^{\frac{8}{3}}ab^{30}d^{10}-88(bx^3+a)^{\frac{5}{3}}a^2b^{30}d^{10}}{440b^{33}d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^37*c^4*d^7*(-(b*c - a*d)/d)^(1/3) - a*b^36*c^3*d^8*(-(b*c - a*d)/d)
^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)
^(1/3)))/(b^37*c*d^11 - a*b^36*d^12) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)
*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b
*c - a*d)/d)^(1/3))/d^6 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x^3 + a)^
(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/
d^6 - 1/440*(220*(b*x^3 + a)^(2/3)*b^33*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^32
*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^31*c*d^9 - 88*(b*x^3 + a)^(5/3)*a*b^31*c*
d^9 - 40*(b*x^3 + a)^(11/3)*b^30*d^10 + 110*(b*x^3 + a)^(8/3)*a*b^30*d^10 -
 88*(b*x^3 + a)^(5/3)*a^2*b^30*d^10)/(b^33*d^11)
```

Mupad [B]
time = 5.13, size = 490, normalized size = 1.84

$$\left(\frac{3a^2}{5b^3d}+\frac{\left(\frac{3a}{b^3d}+\frac{b^4c-ab^3d}{b^6d^2}\right)(b^4c-ab^3d)}{5b^3d}\right)(bx^3+a)^{5/3}-\left(\frac{3a}{8b^3d}+\frac{b^4c-ab^3d}{8b^6d^2}\right)(bx^3+a)^{8/3}-(bx^3+a)^{2/3}\left(\frac{a^3}{2b^3d}+\frac{\left(\frac{3a^2}{b^3d}+\frac{\left(\frac{3a}{b^3d}+\frac{b^4c-ab^3d}{b^6d^2}\right)(b^4c-ab^3d)}{b^3d}\right)(b^4c-ab^3d)}{2b^3d}\right)+\frac{(bx^3+a)^{11/3}}{11b^3d}-\frac{c^3\ln\left(\frac{(bx^3+a)^{1/3}(a^2c^6d^2-2abc^7d+b^2c^8)}{d^7}-\frac{c^6(ad-bc)^{4/3}(9ad^3-9bcd^2)}{9d^{28/3}}\right)(ad-bc)^{2/3}}{3d^{14/3}}-\frac{c^3\ln\left(\frac{c^6\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)(ad-bc)^{7/3}}{d^{22/3}}+\frac{c^6(bx^3+a)^{1/3}(ad-bc)^2}{d^7}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)(ad-bc)^{2/3}}{3d^{14/3}}+\frac{c^3\ln\left(\frac{c^6(bx^3+a)^{1/3}(ad-bc)^2}{d^7}-\frac{c^6\left(1+\sqrt{3}\,1i\right)^2(ad-bc)^{7/3}}{4d^{22/3}}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)(ad-bc)^{2/3}}{d^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] ((3*a^2)/(5*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(5*b^3*d))*(a + b*x^3)^(5/3) - ((3*a)/(8*b^3*d) + (b^4*c - a*b^
3*d)/(8*b^6*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*(a^3/(2*b^3*d) + ((
(3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a
*b^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(2*b^3*d)) + (a + b*x^3)^(11/3)/(11*b^
3*d) - (c^3*log(((a + b*x^3)^(1/3)*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))/d
^7 - (c^6*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(28/3)))*(a*d - b*c
```

```
)^(2/3))/(3*d^(14/3)) - (c^3*log((c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7
/3))/d^(22/3) + (c^6*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^7)*((3^(1/2)*1i)/2
- 1/2)*(a*d - b*c)^(2/3))/(3*d^(14/3)) + (c^3*log((c^6*(a + b*x^3)^(1/3)*(a
*d - b*c)^2)/d^7 - (c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(7/3))/(4*d^(22/3)))
*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(2/3))/d^(14/3)
```

3.678 $\int \frac{x^8 \left(a+bx^3\right)^{2/3}}{c+dx^3} \, dx$

Optimal. Leaf size=223

$$\frac{c^2(a+bx^3)^{2/3}}{2d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}+\frac{c^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{11/3}}-\frac{c^2(bc-}{}$$

[Out] 1/2*c^2*(b*x^3+a)^(2/3)/d^3-1/5*(a*d+b*c)*(b*x^3+a)^(5/3)/b^2/d^2+1/8*(b*x^3+a)^(8/3)/b^2/d-1/6*c^2*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(11/3)+1/2*c^2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/3)+1/3*c^2*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(11/3)*3^(1/2)

Rubi [A]

time = 0.17, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 90, 52, 58, 631, 210, 31}

$$\frac{c^2(bc-ad)^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{11/3}}-\frac{\left(a+bx^3\right)^{5/3}(ad+bc)}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}-\frac{c^2(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{11/3}}+\frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}}+\frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]

[Out] (c^2*(a + b*x^3)^(2/3))/(2*d^3) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^2*d^2) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(11/3)) - (c^2*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(11/3)) + (c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(11/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(-bc-ad)(a+bx)^{2/3}}{bd^2} + \frac{(a+bx)^{5/3}}{bd} + \frac{c^2(a+bx)^{2/3}}{d^2(c+dx)}\right)\,dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\,(a+bx^3)^{5/3}}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} + \frac{c^2\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c^2(a+bx^3)^{2/3}}{2d^3} - \frac{(bc+ad)\,(a+bx^3)^{5/3}}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} - \frac{(c^2(bc-ad))\,\text{Subst}\left(\int \frac{}{\sqrt[3]{c}} \right.}{3d^3}$$

$$= \frac{c^2(a+bx^3)^{2/3}}{2d^3} - \frac{(bc+ad)\,(a+bx^3)^{5/3}}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} - \frac{c^2(bc-ad)^{2/3}\log(c+dx^3}{6d^{11/3}}$$

$$= \frac{c^2(a+bx^3)^{2/3}}{2d^3} - \frac{(bc+ad)\,(a+bx^3)^{5/3}}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} - \frac{c^2(bc-ad)^{2/3}\log(c+dx^3}{6d^{11/3}}$$

$$= \frac{c^2(a+bx^3)^{2/3}}{2d^3} - \frac{(bc+ad)\,(a+bx^3)^{5/3}}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} + \frac{c^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-2}{}\right.}{\sqrt{3}\,d^{11/3}}$$

Mathematica [A]
time = 0.41, size = 266, normalized size = 1.19

$$\frac{\frac{3d^{2/3}(a+bx^3)^{2/3}(-3a^2d^2+2abd(-4c+dx^3)+b^2(20c^2-8cdx^3+5d^2x^6))}{b^2} + 40\sqrt{3}\,c^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 40c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) - 20c^2(bc-ad)^{2/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{120d^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(2/3)*(a + b*x^3)^(2/3)*(-3*a^2*d^2 + 2*a*b*d*(-4*c + d*x^3) + b^2*(2
0*c^2 - 8*c*d*x^3 + 5*d^2*x^6)))/b^2 + 40*Sqrt[3]*c^2*(b*c - a*d)^(2/3)*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 40*c^2
*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 20*
c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(120*d^(11/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^8(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(181) = 362$.
time = 3.33, size = 398, normalized size = 1.78

$$\frac{40\sqrt{3}\,b^2c^2\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)-20\,b^2c^2\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{2}{3}}(bc-ad)-(bc-ad)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\right)+40\,b^2c^2\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(-d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)\right)+3\,(5\,b^2d^2x^6+20\,b^2c^2-8\,abcd-3\,a^2d^2-2\,(4\,b^2cd-abd^2)x^3)(bx^3+a)^{\frac{2}{3}}}{120\,b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/120*(40*sqrt(3)*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arcta
n(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)
^(1/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 20*b^2*c^2*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + 40*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/d^2)^(1/3)*log(-d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3
 + a)^(1/3)*(b*c - a*d)) + 3*(5*b^2*d^2*x^6 + 20*b^2*c^2 - 8*a*b*c*d - 3*a^
2*d^2 - 2*(4*b^2*c*d - a*b*d^2)*x^3)*(b*x^3 + a)^(2/3))/(b^2*d^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [A]
time = 1.15, size = 350, normalized size = 1.57

$$\frac{\left(b^{19}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{18}c^2d^6\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{19}cd^8-ab^{18}d^9)}+\frac{\sqrt{3}\,(-bcd^2+ad^3)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^5}+\frac{20\,(bx^3+a)^{\frac{2}{3}}b^{16}c^2d^5-8\,(bx^3+a)^{\frac{5}{3}}b^{15}cd^6+5\,(bx^3+a)^{\frac{8}{3}}b^{14}d^7-8\,(bx^3+a)^{\frac{5}{3}}ab^{14}d^7}{40\,b^{16}d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^19*c^3*d^5*(-(b*c - a*d)/d)^(1/3) - a*b^18*c^2*d^6*(-(b*c - a*d)/d)^
(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b^19*c*d^8 - a*b^18*d^9) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c^
2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
- a*d)/d)^(1/3))/d^5 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^3 + a)^(2/
3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^5
 + 1/40*(20*(b*x^3 + a)^(2/3)*b^16*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^15*c*d^6
 + 5*(b*x^3 + a)^(8/3)*b^14*d^7 - 8*(b*x^3 + a)^(5/3)*a*b^14*d^7)/(b^16*d^8
)
```

Mupad [B]
time = 5.11, size = 385, normalized size = 1.73

$$\left(\frac{a^2}{2\,b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,(b^3\,c-a\,b^2\,d)}{2\,b^2\,d}\right)\,(b\,x^3+a)^{2/3}-\left(\frac{2\,a}{5\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{5\,b^4\,d^2}\right)\,(b\,x^3+a)^{5/3}+\frac{(b\,x^3+a)^{8/3}}{8\,b^2\,d}+\frac{c^2\,\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a^2\,c^4\,d^2-2\,a\,b\,c^5\,d+b^2\,c^6)}{d^5}-\frac{c^4\,(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{9\,d^{22/3}}\right)\,(a\,d-b\,c)^{2/3}}{3\,d^{11/3}}-\frac{c^2\,\ln\left(\frac{c^4\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)^2}{d^5}-\frac{c^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{7/3}}{d^{16/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{2/3}}{3\,d^{11/3}}+\frac{c^2\,\ln\left(\frac{c^4\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)^2}{d^5}-\frac{c^4\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)^2\,(a\,d-b\,c)^{7/3}}{4\,d^{16/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,(a\,d-b\,c)^{2/3}}{d^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(2*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*
b^2*d))/(2*b^2*d))*(a + b*x^3)^(2/3) - ((2*a)/(5*b^2*d) + (b^3*c - a*b^2*d)
/(5*b^4*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*log(((
a + b*x^3)^(1/3)*(b^2*c^6 + a^2*c^4*d^2 - 2*a*b*c^5*d))/d^5 - (c^4*(a*d - b
*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(22/3)))*(a*d - b*c)^(2/3))/(3*d^(11/
3)) - (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 - (c^4*((3^(1/2)*1
i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(16/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)
^(2/3))/(3*d^(11/3)) + (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 -
 (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(16/3)))*((3^(1/2)*1i)/6 -
 1/6)*(a*d - b*c)^(2/3))/d^(11/3)
```

3.679 $\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=188

$$-\frac{c(a+bx^3)^{2/3}}{2d^2}+\frac{(a+bx^3)^{5/3}}{5bd}-\frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}}+\frac{c(bc-ad)^{2/3}\log(c+dx^3)}{6d^{8/3}}-\frac{c(bc$$

```
[Out] -1/2*c*(b*x^3+a)^(2/3)/d^2+1/5*(b*x^3+a)^(5/3)/b/d+1/6*c*(-a*d+b*c)^(2/3)*l
n(d*x^3+c)/d^(8/3)-1/2*c*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^
3+a)^(1/3))/d^(8/3)-1/3*c*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a
)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 81, 52, 58, 631, 210, 31}

$$-\frac{c(bc-ad)^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}}+\frac{c(bc-ad)^{2/3}\log(c+dx^3)}{6d^{8/3}}-\frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}}-\frac{c(a+bx^3)^{2/3}}{2d^2}+\frac{(a+bx^3)^{5/3}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -1/2*(c*(a + b*x^3)^(2/3))/d^2 + (a + b*x^3)^(5/3)/(5*b*d) - (c*(b*c - a*d)
^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)) + (c*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(8/3)) - (
c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{(a+bx^3)^{5/3}}{5bd} - \frac{c\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{c(a+bx^3)^{2/3}}{2d^2} + \frac{(a+bx^3)^{5/3}}{5bd} + \frac{(c(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{c(a+bx^3)^{2/3}}{2d^2} + \frac{(a+bx^3)^{5/3}}{5bd} + \frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}} - \frac{\left(c(bc-ad)^{2/3}\right)\text{Subst}}{}$$

$$= -\frac{c(a+bx^3)^{2/3}}{2d^2} + \frac{(a+bx^3)^{5/3}}{5bd} + \frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}} - \frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc}\right.}{2}$$

$$= -\frac{c(a+bx^3)^{2/3}}{2d^2} + \frac{(a+bx^3)^{5/3}}{5bd} - \frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}} + \frac{c(bc-}{}$$

Mathematica [A]
time = 0.29, size = 228, normalized size = 1.21

$$\frac{\frac{3d^{2/3}(a+bx^3)^{2/3}(-5bc+2ad+2bdx^3)}{b} - 10\sqrt{3}\,c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 10c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) + 5c(bc-ad)^{2/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{30d^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(2/3)*(a + b*x^3)^(2/3)*(-5*b*c + 2*a*d + 2*b*d*x^3))/b - 10*Sqrt[3]*
c*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(
1/3))/Sqrt[3]] - 10*c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)] + 5*c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*
c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(30*d^(8/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(b\,x^3+a\right)^{\frac{2}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(149) = 298.
time = 3.23, size = 353, normalized size = 1.88

$$-\frac{10\sqrt{3}\,bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+5\,bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{2}{3}}(bc-ad)+(bc-ad)\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\right)-10\,bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(-d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)\right)-3\,(2\,bdx^3-5\,bc+2\,ad)(bx^3+a)^{\frac{2}{3}}}{30\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(
1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 5*b*c*(-(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2
)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)/d^2)^(1/3)) - 10*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^
2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^
(1/3)*(b*c - a*d)) - 3*(2*b*d*x^3 - 5*b*c + 2*a*d)*(b*x^3 + a)^(2/3))/(b*d^
2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(149) = 298.
time = 0.97, size = 306, normalized size = 1.63

$$-\frac{\left(b^{7}c^{2}d^{3}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{6}cd^{4}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^{3}+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{7}cd^{5}-ab^{6}d^{6}\right)}-\frac{\sqrt{3}\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}c\arctan\left(\frac{\sqrt{3}\left(2\,(bx^{3}+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^{4}}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}c\log\left((bx^{3}+a)^{\frac{2}{3}}+(bx^{3}+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^{4}}-\frac{5\,(bx^{3}+a)^{\frac{2}{3}}b^{5}cd^{3}-2\,(bx^{3}+a)^{\frac{5}{3}}b^{4}d^{4}}{10\,b^{5}d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^7*c^2*d^3*(-(b*c - a*d)/d)^(1/3) - a*b^6*c*d^4*(-(b*c - a*d)/d)^(1/
3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/
3)))/(b^7*c*d^5 - a*b^6*d^6) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c*arcta
n(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/
d)^(1/3))/d^4 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c*log((b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 - 1/10*(
5*(b*x^3 + a)^(2/3)*b^5*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^4*d^4)/(b^5*d^5)
```

Mupad [B]
time = 5.06, size = 302, normalized size = 1.61

$$\frac{\left(b\,x^3+a\right)^{5/3}}{5\,b\,d}-\left(b\,x^3+a\right)^{2/3}\left(\frac{a}{2\,b\,d}+\frac{b^2\,c-a\,b\,d}{2\,b^2\,d^2}\right)-\frac{c\,\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\left(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4\right)}{d^3}-\frac{c^2\,{\left(a\,d-b\,c\right)}^{4/3}\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{9\,d^{16/3}}\right){\left(a\,d-b\,c\right)}^{2/3}}{3\,d^{8/3}}-\frac{c\,\ln\left(\frac{c^2\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right){\left(a\,d-b\,c\right)}^{7/3}}{d^{10/3}}+\frac{c^2\left(b\,x^3+a\right)^{1/3}{\left(a\,d-b\,c\right)}^2}{d^3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right){\left(a\,d-b\,c\right)}^{2/3}}{3\,d^{8/3}}+\frac{c\,\ln\left(\frac{c^2\left(b\,x^3+a\right)^{1/3}{\left(a\,d-b\,c\right)}^2}{d^3}-\frac{c^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right){\left(a\,d-b\,c\right)}^{7/3}}{d^{10/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right){\left(a\,d-b\,c\right)}^{2/3}}{3\,d^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b*d) - (a + b*x^3)^(2/3)*(a/(2*b*d) + (b^2*c - a*b*d)/
(2*b^2*d^2)) - (c*log(((a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3
*d))/d^3 - (c^2*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(16/3)))*(a*d
 - b*c)^(2/3))/(3*d^(8/3)) - (c*log((c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)
^(7/3))/d^(10/3) + (c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^3)*((3^(1/2)*1i)
/2 - 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3)) + (c*log((c^2*(a + b*x^3)^(1/3)*(a
*d - b*c)^2)/d^3 - (c^2*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(10/3))
*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3))
```

3.680 $\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=162

$$\frac{(a+bx^3)^{2/3}}{2d}+\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}}-\frac{(bc-ad)^{2/3}\log(c+dx^3)}{6d^{5/3}}+\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-\ldots}\right.}{2d\ldots}$$

[Out] 1/2*(b*x^3+a)^(2/3)/d-1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(5/3)+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(5/3)+1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(5/3)*3^(1/2)

Rubi [A]
time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {455, 52, 58, 631, 210, 31}

$$\frac{(bc-ad)^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}}-\frac{(bc-ad)^{2/3}\log(c+dx^3)}{6d^{5/3}}+\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}+\frac{(a+bx^3)^{2/3}}{2d}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]

[Out] (a + b*x^3)^(2/3)/(2*d) + ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)) - ((b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(5/3)) + ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(5/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{(a+bx^3)^{2/3}}{2d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{(a+bx^3)^{2/3}}{2d} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6d^{5/3}} + \frac{(bc-ad)^{2/3}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, \right.}{2d^{5/3}}$$

$$= \frac{(a+bx^3)^{2/3}}{2d} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6d^{5/3}} + \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+\right.}{2d^{5/3}}$$

$$= \frac{(a+bx^3)^{2/3}}{2d} + \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}} - \frac{(bc-ad)^{2/3}\log(c+dx^3}{6d^{5/3}}$$

Mathematica [A]
time = 0.18, size = 206, normalized size = 1.27

$$\frac{3d^{2/3}(a+bx^3)^{2/3}+2\sqrt{3}(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-(bc-ad)^{2/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{6d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (3*d^(2/3)*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*(b*c - a*d)^(2/3)
*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - (b*c - a*d)^(2/3)*Log
[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*
(a + b*x^3)^(2/3)])/(6*d^(5/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(128) = 256.
time = 3.82, size = 323, normalized size = 1.99

$$\frac{2\sqrt{3}\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)-\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{2}{3}}(bc-ad)-(bc-ad)\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(-d\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)\right)+3(bx^3+a)^{\frac{2}{3}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2*s
qrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) - sq
rt(3)*(b*c - a*d))/(b*c - a*d)) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/
3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b
*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/
d^2)^(1/3)) + 2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) + 3*(b
*x^3 + a)^(2/3))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(128) = 256.
time = 1.05, size = 259, normalized size = 1.60

$$\frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bcd^2-ad^3)}+\frac{(bx^3+a)^{\frac{2}{3}}}{2d}+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^3}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^2
 - a*d^3) + 1/2*(b*x^3 + a)^(2/3)/d + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/d^3 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (
b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3
```

Mupad [B]
time = 5.05, size = 238, normalized size = 1.47

$$\frac{(b\,x^3+a)^{2/3}}{2\,d}+\frac{\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{d}-\frac{(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{9\,d^{10/3}}\right)\,(a\,d-b\,c)^{2/3}}{3\,d^{5/3}}-\frac{\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a\,d-b\,c)^2}{d}-\frac{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{7/3}}{d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{2/3}}{3\,d^{5/3}}+\frac{\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a\,d-b\,c)^2}{d}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(a\,d-b\,c)^{7/3}}{4\,d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^{2/3}}{d^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*d) + (log(((a + b*x^3)^(1/3)*(a^2*d^2 + b^2*c^2 - 2*a*
b*c*d))/d - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(10/3)))*(a*d -
b*c)^(2/3))/(3*d^(5/3)) - (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d - (((3^(
1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^(2/3))/(3*d^(5/3)) + (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d - ((3^(
1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(4/3)))*((3^(1/2)*1i)/6 - 1/6)*(a*d
- b*c)^(2/3))/d^(5/3)
```

3.681 $\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=245

$$\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,c}-\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,cd^{2/3}}-\frac{a^{2/3}\log(x)}{2c}+\frac{(bc-ad)^{2/3}\log(c+}{6cd^{2/3}}$$

```
[Out] -1/2*a^(2/3)*ln(x)/c+1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c/d^(2/3)+1/2*a^(2/3)
*ln(a^(1/3)-(b*x^3+a)^(1/3))/c-1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(
1/3)*(b*x^3+a)^(1/3))/c/d^(2/3)+1/3*a^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)
^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(
1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 85, 57, 631, 210, 31, 58}

$$\frac{a^{2/3}\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,c}+\frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{a^{2/3}\log(x)}{2c}-\frac{(bc-ad)^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,cd^{2/3}}+\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}}-\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*c) - ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(2/3)) - (a^(2/3)*Log[x])/(2*c) + ((b*c
- a*d)^(2/3)*Log[c + d*x^3])/(6*c*d^(2/3)) + (a^(2/3)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)])/(2*c*d^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3c} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6cd^{2/3}} - \frac{a^{2/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} + \cdots$$

$$= -\frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6cd^{2/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{2/3}\cdots}{\cdots}$$

$$= \frac{a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,c} - \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,cd^{2/3}} - \frac{a^{2/3}\log(x)}{2c}$$

Mathematica [A]
time = 0.41, size = 310, normalized size = 1.27

$$\frac{2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + \frac{-2\sqrt{3}\,(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 2a^{2/3}d^{2/3}\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right) - 2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) - a^{2/3}d^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right) + (bc-ad)^{2/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{d^{2/3}}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + (-
2*Sqrt[3]*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
- a*d)^(1/3))/Sqrt[3]] + 2*a^(2/3)*d^(2/3)*Log[-a^(1/3) + (a + b*x^3)^(1/3)
] - 2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]
- a^(2/3)*d^(2/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/
3)] + (b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(
a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/d^(2/3))/(6*c)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(192) = 384.
time = 3.26, size = 425, normalized size = 1.73

$$-\frac{2\sqrt{3}\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3(bc-ad)}\right)-2\sqrt{3}(a^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{1}{3}}}{3a}\right)+\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{2}{3}}(bc-ad)+(bc-ad)\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\right)+(a^2)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)-2\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(-d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)\right)-2(a^2)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2
*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) +
 sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt
(3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(1/3))/a) + (-(b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2
- 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + (a^2)^(1/3)*log((b*x^3 + a)^(2/3)*a +
(a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*(-(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) -
(b*x^3 + a)^(1/3)*(b*c - a*d)) - 2*(a^2)^(1/3)*log((b*x^3 + a)^(1/3)*a - (a
^2)^(2/3)))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{2}{3}}}{x(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x*(c + d*x**3)), x)
```

Giac [A]
time = 1.87, size = 341, normalized size = 1.39

$$-\frac{\left(bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)}+\frac{\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c}-\frac{a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,c}+\frac{a^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,c}-\frac{\sqrt{3}\,(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^2}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c*(-(b*c - a*d)/d)^(1/3) - a*d*(-(b*c - a*d)/d)^(1/3))*(-(b*c - a*d
)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^2 - a*
c*d) + 1/3*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3
))/a^(1/3))/c - 1/6*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/
3) + a^(2/3))/c + 1/3*a^(2/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c - 1/3
*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^2) + 1/6*(-b*c*d^2 +
a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/
3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

Mupad [B]
time = 4.77, size = 1963, normalized size = 8.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x*(c + d*x^3)),x)
```

```
[Out] log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*
a^3*b^7*c^2*d^2) - (a^2/(27*c^3))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4
*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 - 729
*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3)
)^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*
a*b^8*c^5*d))*(a^2/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 -
 a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - (-(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3
- 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 - 729*a^2*
b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^
3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) + 36*a
^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*
(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) - log((a + b*x^3)^(1/
3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) +
((3^(1/2)*1i)/2 + 1/2)^2*(a^2/(27*c^3))^(2/3)*(((3^(1/2)*1i)/2 + 1/2)*((a +
 b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^
5) - ((3^(1/2)*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 48
6*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(1/3) - 36*a^2*b^7*
c^4*d^2 + 54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*b^8*c^5*d))*((3^(1/
```

2)*1i)/2 + 1/2)*(a^2/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - ((3^(1/2)*1i)/2 - 1/2)^2*(a^2/(27*c^3))^(2/3)*(((3^(1/2)*1i)/2 - 1/2)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i)/2 - 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 - 1/2)*(a^2/(27*c^3))^(1/3) - log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) + ((3^(1/2)*1i)/2 + 1/2)^2*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((3^(1/2)*1i)/2 + 1/2)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) - 36*a^2*b^7*c^4*d^2 + 54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 + 1/2)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - ((3^(1/2)*1i)/2 - 1/2)^2*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((3^(1/2)*1i)/2 - 1/2)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i)/2 - 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 - 1/2)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3)

3.682 $\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx$

Optimal. Leaf size=347

$$\frac{d(a+bx^3)^{2/3}}{2c^2}+\frac{(2bc-3ad)(a+bx^3)^{2/3}}{6ac^2}-\frac{(a+bx^3)^{5/3}}{3acx^3}+\frac{(2bc-3ad)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}c^2}+\frac{\sqrt[3]{d}(bc-a\ldots}{}$$

```
[Out] 1/2*d*(b*x^3+a)^(2/3)/c^2+1/6*(-3*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2-1/3*(b*x
^3+a)^(5/3)/a/c/x^3-1/6*(-3*a*d+2*b*c)*ln(x)/a^(1/3)/c^2-1/6*d^(1/3)*(-a*d+
b*c)^(2/3)*ln(d*x^3+c)/c^2+1/6*(-3*a*d+2*b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a
^(1/3)/c^2+1/2*d^(1/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+
a)^(1/3))/c^2+1/9*(-3*a*d+2*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(
1/3)*3^(1/2))/a^(1/3)/c^2*3^(1/2)+1/3*d^(1/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(
1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2*3^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {457, 105, 162, 52, 57, 631, 210, 31, 58}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(2bc-3ad)}{3\sqrt{3}\sqrt[3]{a}c^2}+\frac{\sqrt[3]{d}(bc-ad)^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2}+\frac{d(a+bx^3)^{2/3}}{2c^2}+\frac{(a+bx^3)^{2/3}(2bc-3ad)}{6ac^2}-\frac{\sqrt[3]{d}(bc-ad)^{2/3}\log(c+dx^3)}{6c^2}+\frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2}+\frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}-\frac{\log(x)(2bc-3ad)}{6\sqrt[3]{a}c^2}-\frac{(a+bx^3)^{5/3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (d*(a + b*x^3)^(2/3))/(2*c^2) + ((2*b*c - 3*a*d)*(a + b*x^3)^(2/3))/(6*a*c^
2) - (a + b*x^3)^(5/3)/(3*a*c*x^3) + ((2*b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(
a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*c^2) + (d^(1/3)*(b
*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)
)/Sqrt[3]])/(Sqrt[3]*c^2) - ((2*b*c - 3*a*d)*Log[x])/(6*a^(1/3)*c^2) - (d^(
1/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^2) + ((2*b*c - 3*a*d)*Log[a^(1/
3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*c^2) + (d^(1/3)*(b*c - a*d)^(2/3)*Log[(
b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 58

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 105

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])

Rule 162

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(\frac{1}{3}(-2bc+3ad)-\frac{2bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{3acx^3} + \frac{d^2\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx, x, x^{\cdots}\right.}{9ac^2}$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+\cdots}}\right.}{9c^2}$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} - \frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}\,c^2} - \frac{\sqrt[3]{d}\,(b\cdots}{\cdots}$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} - \frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}\,c^2} - \frac{\sqrt[3]{d}\,(b\cdots}{\cdots}$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} + \frac{(2bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a\cdots}}{\sqrt[3]{\cdots}}}{\sqrt{3}\cdots}\right.}{3\sqrt{3}\,\sqrt[3]{a}\,c^2}$$

Mathematica [A]

time = 0.62, size = 352, normalized size = 1.01

$$\frac{-\frac{6c(a+bx^3)^{2/3}}{x^3}+\frac{2\sqrt{3}\,(2bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+6\sqrt{3}\,\sqrt[3]{d}\,(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+\frac{2(2bc-3ad)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{a}}+6\sqrt[3]{d}\,(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)+\frac{(-2bc+3ad)\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{\sqrt[3]{a}}-3\sqrt[3]{d}\,(bc-ad)^{2/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\,\sqrt[3]{bc-ad}\,\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{18c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] ((-6*c*(a + b*x^3)^(2/3))/x^3 + (2*Sqrt[3]*(2*b*c - 3*a*d)*ArcTan[(1 + (2*(
a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) + 6*Sqrt[3]*d^(1/3)*(b*c - a*d
)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3
]] + (2*(2*b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(1/3) + 6*d^(1
/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] +
((-2*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/
3)])/a^(1/3) - 3*d^(1/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*
(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(18*c^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{4}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^4), x)
```

Fricas [A]
time = 4.32, size = 1030, normalized size = 2.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/18*(3*sqrt(1/3)*(2*a*b*c - 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3
 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)
^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) -
 6*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt
(3)*(b*c - a*d) - 2*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^
3 + a)^(1/3))/(b*c - a*d)) + (2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)
^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(2*b*c - 3*a*d)*(-a
)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1
/3)*a*x^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3)^(2/3)) + 6*(b*x^3 + a)^(2/3)*a*c)/(a*c^2*x^3), 1/18*(6*sqrt(1/3
)*(2*a*b*c - 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 +
a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt(3)*(b*c - a*d) - 2*sqrt(3)*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) - (
2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3)) + 2*(2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1
/3) + (-a)^(1/3)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-
(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/
3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3
)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(1/
3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) - 6*(b*x^3
+ a)^(2/3)*a*c)/(a*c^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{2}{3}}}{x^4\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**4/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**4*(c + d*x**3)), x)
```

Giac [A]
time = 1.46, size = 400, normalized size = 1.15

$$\frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)}-\frac{(2\,bc-3\,ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{1}{3}}c^2}+\frac{\sqrt{3}\left(2\,a^{\frac{2}{3}}bc-3\,a^{\frac{5}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,ac^2}+\frac{\left(2\,a^{\frac{1}{3}}bc-3\,a^{\frac{4}{3}}d\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{2}{3}}c^2}+\frac{\sqrt{3}\,(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^2d}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2d}-\frac{(bx^3+a)^{\frac{2}{3}}}{3\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3 -
 a*c^2*d) - 1/18*(2*b*c - 3*a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*
a^(1/3) + a^(2/3))/(a^(1/3)*c^2) + 1/9*sqrt(3)*(2*a^(2/3)*b*c - 3*a^(5/3)*d
)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a*c^2) + 1/9
*(2*a^(1/3)*b*c - 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/
3)*c^2) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3
 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) - 1/6
*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
- a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c^2*d) - 1/3*(b*x^3 + a)^(2/3)/(
c*x^3)
```

Mupad [B]
time = 10.57, size = 1908, normalized size = 5.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2
- 6*a*b*c*d) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d*(a*d
- b*c)^2)/c^6)^(2/3))*((d*(a*d - b*c)^2)/c^6)^(1/3))/3 - (a*b^5*d^4*(27*a^3
*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d - b*c
)^2)/c^6)^(2/3))/9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2
+ 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/
(27*c^6))^(1/3) + log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^
2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a
*b*c*d)*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1
/3))/9 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*
d^2))/(3*c))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^
(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(6*b^4*d^3*(a + b*x
^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d
^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - 2*b*
c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9 + (a*b^5*d^4*(27
*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*(-(3*a*d -
 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(
3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 + 1/2)*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 + log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(6*b^4*d^3*(a + b*x
^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) + 3*a*b^4*c^4*d
^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - 2*b*
c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9 - (a*b^5*d^4*(27
```

```
*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*(-(3*a*d -
 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(
3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 - 1/2)*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 - (a + b*x^3)^(2/3)/(3*c*x^3) - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i
)/2 + 1/2)*(6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^
2 - 6*a*b*c*d) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)*((d*(a*d - b*c)^2)/c^6)^(2/3))*((d*(a*d - b*c)^2)/c^6)^(1/3
))/3 + (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^
2))/(3*c))*((d*(a*d - b*c)^2)/c^6)^(2/3))/9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4
*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i
)/2 + 1/2)*((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/(27*c^6))^(1/3) + log((((3^
(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(6*b^4*d^3*(a + b*x^3)^(1/3)*(a
*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) + 27*a*b^4*c^4*d^3*((3^(1/2
)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d*(a*d - b*c)^2)/c^6)^(2
/3))*((d*(a*d - b*c)^2)/c^6)^(1/3))/3 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3
 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d - b*c)^2)/c^6)^(2/3))/
9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a
*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 - 1/2)*((a^2*d^3 + b^2*c^2*d - 2*a*b*c
*d^2)/(27*c^6))^(1/3)
```

3.683 $\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)} \, dx$

Optimal. Leaf size=370

$$\frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3}-\frac{\left(a+bx^3\right)^{5/3}}{6acx^6}-\frac{\left(b^2c^2+6abcd-9a^2d^2\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}c^3}-\frac{d^{4/3}(bc-ad)^{2/3}}{}$$

```
[Out] 1/18*(6*a*d+b*c)*(b*x^3+a)^(2/3)/a/c^2/x^3-1/6*(b*x^3+a)^(5/3)/a/c/x^6+1/18
*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*ln(x)/a^(4/3)/c^3+1/6*d^(4/3)*(-a*d+b*c)^(2
/3)*ln(d*x^3+c)/c^3-1/18*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*ln(a^(1/3)-(b*x^3+a
)^(1/3))/a^(4/3)/c^3-1/2*d^(4/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/
3)*(b*x^3+a)^(1/3))/c^3-1/27*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*arctan(1/3*(a^(
1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/c^3*3^(1/2)-1/3*d^(4/3)*(-
a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^
(1/2))/c^3*3^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {457, 105, 154, 162, 57, 631, 210, 31, 58}

$$-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)\left(-9a^2d^2+6abcd+b^2c^2\right)}{9\sqrt{3}\,a^{4/3}c^3}-\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3}+\frac{\log(x)\left(-9a^2d^2+6abcd+b^2c^2\right)}{18a^{4/3}c^3}-\frac{d^{4/3}(bc-ad)^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^3}+\frac{d^{4/3}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^3}-\frac{d^{4/3}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}+\frac{\left(a+bx^3\right)^{2/3}(6ad+bc)}{18ac^2x^3}-\frac{\left(a+bx^3\right)^{5/3}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((b*c + 6*a*d)*(a + b*x^3)^(2/3))/(18*a*c^2*x^3) - (a + b*x^3)^(5/3)/(6*a*c
*x^6) - ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^
(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(
2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*c^3) + ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(4/3)*c^3
) + (d^(4/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 6*a*b*
c*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(4/3)*c^3) - (d^(4
/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3(c+dx)}\,dx,x,x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(\frac{1}{3}(bc+6ad)+\frac{bdx}{3}\right)}{x^2(c+dx)}\,dx,x,x^3\right)}{6ac}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\frac{2}{9}\left(b^2c^2+6abcd-9a^2d^2\right)+\frac{2}{9}bd(bc-3ad)x}{x\sqrt[3]{a+bx}\,(c+dx)}\,dx,x\right.}{6ac^2}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx,x,\right.}{3c^3}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc-a}{}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc-a}{}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{9\sqrt{3}\,a^{4/3}c^3}$$

Mathematica [A]
time = 0.84, size = 413, normalized size = 1.12

$$\frac{\frac{3c(a+bx^3)^{2/3}(-3ac-2bcx^3+6adx^3)}{ax^6} - \frac{2\sqrt{3}(b^2c^2+6abcd-9a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} - 18\sqrt{3}d^{4/3}(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - \frac{2(b^2c^2+6abcd-9a^2d^2)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{4/3}} - 18d^{4/3}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) + \frac{(b^2c^2+6abcd-9a^2d^2)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{a^{4/3}} + 9d^{4/3}(bc-ad)^{2/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{54c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((3*c*(a + b*x^3)^(2/3)*(-3*a*c - 2*b*c*x^3 + 6*a*d*x^3))/(a*x^6) - (2*Sqrt
[3]*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(
1/3))/Sqrt[3]])/a^(4/3) - 18*Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)*ArcTan[(1 -
(2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - (2*(b^2*c^2 + 6
*a*b*c*d - 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(4/3) - 18*d^(4/
3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (
(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) +
 (a + b*x^3)^(2/3)])/a^(4/3) + 9*d^(4/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^
(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(
2/3)])/(54*c^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^7\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^7), x)
```

Fricas [A]
time = 6.16, size = 1151, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*arc
tan(-1/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d
^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2 - a
^2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*
d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 -
a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)
^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a
*b*c*d^2 - a^2*d^3)^(2/3)) + 3*sqrt(1/3)*(a*b^2*c^2 + 6*a^2*b*c*d - 9*a^3*d
^2)*x^6*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^
(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/
3)*a^(2/3) + 3*a)/x^3) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log(
(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b^2*c^2 + 6*a
*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(3*a^2
*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)*x^3)*(b*x^3 + a)^(2/3))/(a^2*c^3*x^6), -1/54
*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*arctan(-1
/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1
/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3
)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*
a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^
3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)
*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d
^2 - a^2*d^3)^(2/3)) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b^2*c^2 + 6*a*b
*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*sqrt(1/3
)*(a*b^2*c^2 + 6*a^2*b*c*d - 9*a^3*d^2)*x^6*arctan(sqrt(1/3)*(2*(b*x^3 + a)
^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) + 3*(3*a^2*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)
*x^3)*(b*x^3 + a)^(2/3))/(a^2*c^3*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^7\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**7/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**7*(c + d*x**3)), x)
```

Giac [A]
time = 1.16, size = 493, normalized size = 1.33

$$-\frac{\left(bcd^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)}-\frac{\sqrt{3}\,(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^3}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^3}-\frac{\sqrt{3}\,(b^2c^2+6\,abcd-9\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{4}{3}}c^3}-\frac{\left(a^{\frac{1}{3}}b^2c^2+6\,a^{\frac{4}{3}}bcd-9\,a^{\frac{7}{3}}d^2\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{5}{3}}c^3}+\frac{\left(a^{\frac{2}{3}}b^2c^2+6\,a^{\frac{5}{3}}bcd-9\,a^{\frac{8}{3}}d^2\right)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^2c^3}-\frac{2\,(bx^3+a)^{\frac{5}{3}}b^2c+(bx^3+a)^{\frac{2}{3}}ab^2c-6\,(bx^3+a)^{\frac{5}{3}}abd+6\,(bx^3+a)^{\frac{2}{3}}a^2bd}{18\,ab^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c*d^2*(-(b*c - a*d)/d)^(1/3) - a*d^3*(-(b*c - a*d)/d)^(1/3))*(-(b*c
 - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^
4 - a*c^3*d) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(
b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3 + 1/
6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
 - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(b^2*c^2 + 6*
a*b*c*d - 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/(a^(4/3)*c^3) - 1/27*(a^(1/3)*b^2*c^2 + 6*a^(4/3)*b*c*d - 9*a^(7/3)*d
^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*c^3) + 1/54*(a^(2/3)*b^2
*c^2 + 6*a^(5/3)*b*c*d - 9*a^(8/3)*d^2)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*a^(1/3) + a^(2/3))/(a^2*c^3) - 1/18*(2*(b*x^3 + a)^(5/3)*b^2*c + (b*
x^3 + a)^(2/3)*a*b^2*c - 6*(b*x^3 + a)^(5/3)*a*b*d + 6*(b*x^3 + a)^(2/3)*a^
2*b*d)/(a*b^2*c^2*x^6)
```

Mupad [B]
time = 15.19, size = 2788, normalized size = 7.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x)
```

```
[Out] log((((27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b*
c)^2)/c^9)^(2/3) - (b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 +
b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^
2))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 +
 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5
*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))
/9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*
a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*(-(a^2*d^6 + b^2*c^2*d^4 -
 2*a*b*c*d^5)/(27*c^9))^(1/3) + log((((((a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2
 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/3 - (
b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2
*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2))*(-(b^2*c^2 - 9*a
^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1/3))/27 - (b^5*d^4*(729*a^6*d^6 + b^6*c^
6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*
b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b
*c*d)^3/(a^4*c^9))^(2/3))/729 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c
)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))
*(-(b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*
a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3)
 - (((a + b*x^3)^(2/3)*(b^2*c + 6*a*b*d))/(18*c^2) - (b*(a + b*x^3)^(5/3)*(
3*a*d - b*c))/(9*a*c^2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) - log((((3
^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((b^4*d^3*(a + b*x^3)^(1/3)*(a
```

```
*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d -
108*a^3*b*c*d^3))/(3*a^2*c^2) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*
a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))*(-(d^4*(a*
d - b*c)^2)/c^9)^(1/3))/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^
4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*
a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))/9 + (2*b^5*d^
7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d -
15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 + 1/2)*(-(a^2*d^6 + b^2*
c^2*d^4 - 2*a*b*c*d^5)/(27*c^9))^(1/3) + log((((3^(1/2)*1i)/2 + 1/2)*((((3^
(1/2)*1i)/2 - 1/2)*((b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 +
 b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c
^2) + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*
c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3
+ (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^
3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4)
)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))/9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d
 - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^
2*c^10))*((3^(1/2)*1i)/2 - 1/2)*(-(a^2*d^6 + b^2*c^2*d^4 - 2*a*b*c*d^5)/(27
*c^9))^(1/3) - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((b^4*d
^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*
d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) - (a*b^4*c^4*d^3*((3^(
1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2
+ 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/3)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(
a^4*c^9))^(1/3))/27 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2
- 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*
c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3
))/729 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3
+ 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 + 1/2)
*(-(b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*
a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3)
 + log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*((b^4*d^3*(a + b*x^
3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b
^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) + (a*b^4*c^4*d^3*((3^(1/2)*1i)/2 +
 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)
^3/(a^4*c^9))^(2/3))/3)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1
/3))/27 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^
3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*
a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/729 + (2*
b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^
2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 - 1/2)*(-(b^6*c^6
- 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*a^4*b^2*c^2*
d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3)
```

3.684 $\int \frac{x^6\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=334

$$-\frac{(3bc-ad)x(a+bx^3)^{2/3}}{9bd^2}+\frac{x^4(a+bx^3)^{2/3}}{6d}+\frac{(9b^2c^2-6abcd-a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}d^3}-\frac{c^{4/3}(bc-ad)^{2/3}}{}$$

```
[Out] -1/9*(-a*d+3*b*c)*x*(b*x^3+a)^(2/3)/b/d^2+1/6*x^4*(b*x^3+a)^(2/3)/d-1/6*c^(
4/3)*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^3+1/2*c^(4/3)*(-a*d+b*c)^(2/3)*ln((-a*d
+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/18*(-a^2*d^2-6*a*b*c*d+9*b^2*c
^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d^3+1/27*(-a^2*d^2-6*a*b*c*d+9*b
^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d^3*3^(
1/2)-1/3*c^(4/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3
)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {489, 596, 544, 245, 384}

$$\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(-a^2d^2-6abcd+9b^2c^2)}{9\sqrt{3}\,b^{4/3}d^3}-\frac{(-a^2d^2-6abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{4/3}d^3}-\frac{c^{4/3}(bc-ad)^{2/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^3}-\frac{c^{4/3}(bc-ad)^{2/3}\log(c+dx^3)}{6d^3}+\frac{c^{4/3}(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3}-\frac{x(a+bx^3)^{2/3}(3bc-ad)}{9bd^2}+\frac{x^4(a+bx^3)^{2/3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -1/9*((3*b*c - a*d)*x*(a + b*x^3)^(2/3))/(b*d^2) + (x^4*(a + b*x^3)^(2/3))/
(6*d) + ((9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b
*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(4/3)*d^3) - (c^(4/3)*(b*c - a*d)^(2/3)
*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])
/(Sqrt[3]*d^3) - (c^(4/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^3) + (c^(4
/3)*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)
])/(2*d^3) - ((9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x
^3)^(1/3)])/(18*b^(4/3)*d^3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx &= \frac{(a+bx^3)^{2/3}\int \frac{x^6\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^7(a+bx^3)^{2/3}\,F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.22, size = 527, normalized size = 1.58

$$\frac{\frac{6d(a+bx^3)^{2/3}(-6bcx+2adx+3bdx^4)}{b}+\frac{4\sqrt{3}\,(9b^2c^2-6abcd-a^2d^2)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)}{b^{4/3}}+18\sqrt{-6+6i\sqrt{3}}\,c^{4/3}(bc-ad)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+\frac{4(-9b^2c^2+6abcd+a^2d^2)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{b^{4/3}}-18i\left(-i+\sqrt{3}\right)c^{4/3}(bc-ad)^{2/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+\frac{2(9b^2c^2-6abcd-a^2d^2)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{4/3}}+9\left(1+i\sqrt{3}\right)c^{4/3}(bc-ad)^{2/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{108d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((6*d*(a + b*x^3)^(2/3)*(-6*b*c*x + 2*a*d*x + 3*b*d*x^4))/b + (4*Sqrt[3]*(9
*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(
a + b*x^3)^(1/3))])/b^(4/3) + 18*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(4/3)*(b*c - a*
d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I
 + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(-9*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(4/3) - (18*I)*(-I + Sqrt[3])
*c^(4/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*Log[b^(2/3)*x
^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(4/3) + 9*(1 + I*S
qrt[3])*c^(4/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqr
t[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/
3)*(a + b*x^3)^(2/3)])/(108*d^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{6} \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(276) = 552.
time = 6.84, size = 1164, normalized size = 3.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2*c*arctan(-
1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^
(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*c^2*d + a^2
*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*x - (b*x
^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(
1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2
+ (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x + (b*x^3 +
a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2
*b*d^2)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*
sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2
/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*b^(2/3
)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d - a^2*d^
2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/
3))/x^2) - 3*(3*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^(2/3))
/(b^2*d^3), -1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2
*c*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d +
a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/
3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a
^2*c*d^2)^(1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c
- a*d)*x^2 + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x
+ (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*
d^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c
*d - a^2*d^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x
^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2*b*d^2)*arc
tan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(3
*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^(2/3))/(b^2*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.685 $\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=272

$$\frac{x(a+bx^3)^{2/3}}{3d}-\frac{(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}\,d^2}+\frac{\sqrt[3]{c}\,(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}+\frac{\sqrt[3]{c}\,(bc$$

```
[Out] 1/3*x*(b*x^3+a)^(2/3)/d+1/6*c^(1/3)*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^2-1/2*c^
(1/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2+1
/6*(-2*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)/d^2-1/9*(-2*a*d+3*
b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)/d^2*3^(1/2
)+1/3*c^(1/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(
b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {489, 544, 245, 384}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-2ad)}{3\sqrt{3}\,\sqrt[3]{b}\,d^2}+\frac{\sqrt[3]{c}\,(bc-ad)^{2/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}+\frac{\sqrt[3]{c}\,(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^2}-\frac{\sqrt[3]{c}\,(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6\sqrt[3]{b}\,d^2}+\frac{x(a+bx^3)^{2/3}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*d) - ((3*b*c - 2*a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)*d^2) + (c^(1/3)*(b*c - a*d)^(
2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[
3]])/(Sqrt[3]*d^2) + (c^(1/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^2) - (
c^(1/3)*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(
1/3)])/(2*d^2) + ((3*b*c - 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6
*b^(1/3)*d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx &= \frac{(a+bx^3)^{2/3}\int \frac{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^4(a+bx^3)^{2/3}\,F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.90, size = 466, normalized size = 1.71

$$\frac{12dx(a+bx^3)^{2/3}-\frac{4\sqrt{3}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}}-6\sqrt{-6+6i\sqrt{3}}\,\sqrt[3]{c}\,(bc-ad)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\,\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)+\frac{4(3bc-2ad)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{b}}+6\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,(bc-ad)^{2/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)+\frac{2(-3bc+2ad)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{\sqrt[3]{b}}-3i\left(-i+\sqrt{3}\right)\sqrt[3]{c}\,(bc-ad)^{2/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{36d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (12*d*x*(a + b*x^3)^(2/3) - (4*Sqrt[3]*(3*b*c - 2*a*d)*ArcTan[(Sqrt[3]*b^(1
/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(1/3) - 6*Sqrt[-6 + (6*I)*Sqrt
[3]]*c^(1/3)*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
 - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(3*b*c -
```

```
 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(1/3) + 6*(1 + I*Sqrt[3])*
c^(1/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/
3)*(a + b*x^3)^(1/3)] + (2*(-3*b*c + 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a
+ b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(1/3) - (3*I)*(-I + Sqrt[3])*c^(1/3)
*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(
b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^
(2/3)])/(36*d^2)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(b \,x^{3}+a \right)^{\frac{2}{3}}}{d \,x^{3}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(219) = 438$.
time = 3.27, size = 1091, normalized size = 4.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - 2*a*b*d)*sqrt((-b
)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)
*x)*sqrt((-b)^(1/3)/b) + 2*a) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d
^2)^(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 + 2*a*b
*c^2*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*b*c -
2*a*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c - 2*a*
d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
```

```
 + a)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b
^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*
d))/x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c^3 + 2*
a*b*c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*b*c^2*d - a^
2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2
))/(b*d^2), 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(1/3)*(3*b^2*c - 2*a*b*
d)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3
))*sqrt(-(-b)^(1/3)/b)/x) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^
(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2
*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*b*c - 2*a*
d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c - 2*a*d)*(
-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a
)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c
^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/
x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c^3 + 2*a*b*
c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*b*c^2*d - a^2*c*
d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2))/(
b*d^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**3*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.686 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=233

$$\frac{b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d}-\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}d}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d}+\frac{(bc-a\ldots}{}$$

```
[Out] -1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(2/3)/d+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b
*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d-1/2*b^(2/3)*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/d+1/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/d*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(
1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d*3^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {399, 245, 384}

$$\frac{b^{2/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d}-\frac{(bc-ad)^{2/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}d}-\frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2d}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d}+\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d)
 - ((b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x
^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*d) - ((b*c - a*d)^(2/3)*Log[c + d*x^
3])/(6*c^(2/3)*d) + ((b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
(a + b*x^3)^(1/3)])/(2*c^(2/3)*d) - (b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)
^(1/3)])/(2*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
```

```
+ Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx &= \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}} \\
&= \frac{x(a+bx^3)^{2/3}\,F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.50, size = 423, normalized size = 1.82

$$
\frac{4\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)+\frac{2\sqrt{-6+6i\sqrt{3}}\,(bc-ad)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}}-4b^{2/3}\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)-\frac{2i\left(-i+\sqrt{3}\right)(bc-ad)^{2/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}}+2b^{2/3}\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)+\frac{\left(1+i\sqrt{3}\right)(bc-ad)^{2/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}}}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (4*Sqrt[3]*b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1
/3))] + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)
^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3
)^(1/3))]])/c^(2/3) - 4*b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - ((2*
I)*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/c^(2/3) + 2*b^(2/3)*Log[b^(2/3)*x^2 + b^(1
/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 + I*Sqrt[3])*(b*c - a*d)
^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(
1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/c^(2
/3))/(12*d)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(186) = 372$.
time = 5.43, size = 469, normalized size = 2.01

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3(bc-ad)x}\right)+2\sqrt{3}(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{1}{3}}}{3bx}\right)-2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(\frac{cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)}{x}\right)-2(-b^2)^{\frac{1}{3}}\log\left(-\frac{(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{1}{3}}b}{x}\right)+(-b^2)^{\frac{1}{3}}\log\left(-\frac{(-b^2)^{\frac{1}{3}}bx^2-(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{2}{3}}b}{x^2}\right)+\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)x^2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{2}{3}}(bc-ad)}{x^2}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sq
rt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3
*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*log(-(
(-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

3.687 $\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx$

Optimal. Leaf size=169

$$-\frac{(a+bx^3)^{2/3}}{2cx^2}+\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}}+\frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{5/3}}-\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc\ldots\right.}{2c\ldots}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/c/x^2+1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(5/3)-1/2*(-a
*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)+1/3*(-
a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*
3^(1/2))/c^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {486, 12, 384}

$$\frac{(bc-ad)^{2/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}}+\frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{5/3}}-\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}}-\frac{(a+bx^3)^{2/3}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(c*x^2) + ((b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(5/3)) + ((b
*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^(5/3)) - ((b*c - a*d)^(2/3)*Log[((b*c
- a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(5/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^3(c+dx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{(a+bx^3)^{2/3}\left(1+\frac{dx^3}{c}\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{2cx^2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.45, size = 309, normalized size = 1.83

$$\frac{-\frac{6c^{2/3}(a+bx^3)^{2/3}}{x^2}-2\sqrt{-6+6i\sqrt{3}}\,(bc-ad)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+2\left(1+i\sqrt{3}\right)(bc-ad)^{2/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)-i\left(-i+\sqrt{3}\right)(bc-ad)^{2/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{12c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] ((-6*c^(2/3)*(a + b*x^3)^(2/3))/x^2 - 2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*d
)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I
+ Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 2*(1 + I*Sqrt[3])*(b*c - a*d)^(2/3
)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] -
I*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sq
rt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2
/3)*(a + b*x^3)^(2/3)])/(12*c^(5/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^3\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x)
[Out] int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{2}{3}}}{x^3 (c + dx^3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**3/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**3*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{x^3\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)), x)
```

3.688 $\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx$

Optimal. Leaf size=206

$$-\frac{(a+bx^3)^{2/3}}{5cx^5}-\frac{(2bc-5ad)(a+bx^3)^{2/3}}{10ac^2x^2}-\frac{d(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{8/3}}-\frac{d(bc-ad)^{2/3}\log(c+}{6c^{8/3}}$$

```
[Out] -1/5*(b*x^3+a)^(2/3)/c/x^5-1/10*(-5*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2/x^2-1/
6*d*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(8/3)+1/2*d*(-a*d+b*c)^(2/3)*ln((-a*d+b*
c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)-1/3*d*(-a*d+b*c)^(2/3)*arctan(1
/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)*3^(1/2
)
```

Rubi [A]
time = 0.11, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {486, 597, 12, 384}

$$-\frac{d(bc-ad)^{2/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{8/3}}-\frac{d(bc-ad)^{2/3}\log(c+dx^3)}{6c^{8/3}}+\frac{d(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}}-\frac{(a+bx^3)^{2/3}(2bc-5ad)}{10ac^2x^2}-\frac{(a+bx^3)^{2/3}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -1/5*(a + b*x^3)^(2/3)/(c*x^5) - ((2*b*c - 5*a*d)*(a + b*x^3)^(2/3))/(10*a*
c^2*x^2) - (d*(b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(8/3)) - (d*(b*c - a*d)^(2/3)*Lo
g[c + d*x^3])/(6*c^(8/3)) + (d*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/
c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(8/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^6(c+dx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{c(a+bx^3)(2c-3dx^3) - 2(bc-ad)x^3(2c-3dx^3)\ {}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(a+bx^3)}\right) + 6(bc-ad)}{10c^3x^5\sqrt[3]{a+bx^3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.61, size = 334, normalized size = 1.62

$$\frac{\frac{6c^{2/3}(a+bx^3)^{2/3}(-2ac-2bcx^3+5adx^3)}{ax^5} + 10\sqrt{-6+6i\sqrt{3}}\,d(bc-ad)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right) - 10i\left(-i+\sqrt{3}\right)d(bc-ad)^{2/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right) + 5\left(1+i\sqrt{3}\right)d(bc-ad)^{2/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{60c^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] ((6*c^(2/3)*(a + b*x^3)^(2/3)*(-2*a*c - 2*b*c*x^3 + 5*a*d*x^3))/(a*x^5) + 1
0*Sqrt[-6 + (6*I)*Sqrt[3]]*d*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*
x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
```

```
)] - (10*I)*(-I + Sqrt[3])*d*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x +
(1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 5*(1 + I*Sqrt[3])*d*(b*c - a*d
)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^
(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(60
*c^(8/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{6}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x)
[Out] int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{x^{6}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**6/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**6*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{x^6\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)), x)
```

3.689 $\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx$

Optimal. Leaf size=257

$$-\frac{(a+bx^3)^{2/3}}{8cx^8}-\frac{(bc-4ad)(a+bx^3)^{2/3}}{20ac^2x^5}+\frac{(3b^2c^2+8abcd-20a^2d^2)(a+bx^3)^{2/3}}{40a^2c^3x^2}+\frac{d^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\cdots}{\cdots}\right)}{\sqrt{3}\,c^{11/3}}$$

```
[Out] -1/8*(b*x^3+a)^(2/3)/c/x^8-1/20*(-4*a*d+b*c)*(b*x^3+a)^(2/3)/a/c^2/x^5+1/40
*(-20*a^2*d^2+8*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(2/3)/a^2/c^3/x^2+1/6*d^2*(-a*
d+b*c)^(2/3)*ln(d*x^3+c)/c^(11/3)-1/2*d^2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1
/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(11/3)+1/3*d^2*(-a*d+b*c)^(2/3)*arctan(1/3
*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(11/3)*3^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {486, 597, 12, 384}

$$\frac{(a+bx^3)^{2/3}(-20a^2d^2+8abcd+3b^2c^2)}{40a^2c^3x^2}+\frac{d^2(bc-ad)^{2/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{11/3}}+\frac{d^2(bc-ad)^{2/3}\log(c+dx^3)}{6c^{11/3}}-\frac{d^2(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}}-\frac{(a+bx^3)^{2/3}(bc-4ad)}{20ac^2x^5}-\frac{(a+bx^3)^{2/3}}{8cx^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

```
[Out] -1/8*(a + b*x^3)^(2/3)/(c*x^8) - ((b*c - 4*a*d)*(a + b*x^3)^(2/3))/(20*a*c^
2*x^5) + ((3*b^2*c^2 + 8*a*b*c*d - 20*a^2*d^2)*(a + b*x^3)^(2/3))/(40*a^2*c
^3*x^2) + (d^2*(b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/
3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(11/3)) + (d^2*(b*c - a*d)^(2/3
)*Log[c + d*x^3])/(6*c^(11/3)) - (d^2*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1
/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(11/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^9(c+dx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{5ac^3+5bc^3x^3-6ac^2dx^3-6bc^2dx^6+9acd^2x^6+9bcd^2x^9-2(bc-ad)x^3(5c^2-6cdx^3}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.87, size = 374, normalized size = 1.46

$$\frac{-\frac{3c^{2/3}(a+bx^3)^{2/3}\left(-3b^2c^2x^6+2abcx^3(c-4dx^3)+a^2(5c^2-8cdx^3+20d^2x^6)\right)}{a^2x^8}-20\sqrt{-6+6i\sqrt{3}}\,d^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+20\left(1+i\sqrt{3}\right)d^2(bc-ad)^{2/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)-10i\left(-i+\sqrt{3}\right)d^2(bc-ad)^{2/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{120c^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

```
[Out] ((-3*c^(2/3)*(a + b*x^3)^(2/3)*(-3*b^2*c^2*x^6 + 2*a*b*c*x^3*(c - 4*d*x^3)
+ a^2*(5*c^2 - 8*c*d*x^3 + 20*d^2*x^6)))/(a^2*x^8) - 20*Sqrt[-6 + (6*I)*Sqr
t[3]]*d^2*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
```

```
a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 20*(1 + I*Sqrt
[3])*d^2*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3)] - (10*I)*(-I + Sqrt[3])*d^2*(b*c - a*d)^(2/3)*Log[2*
(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b
*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(120*c^(11/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{9}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x)
[Out] int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{x^{9}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**9/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**9*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{x^9\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)), x)
```

3.690 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)} \, dx$

Optimal. Leaf size=320

$$-\frac{(a+bx^3)^{2/3}}{11cx^{11}}-\frac{(2bc-11ad)(a+bx^3)^{2/3}}{88ac^2x^8}+\frac{(6b^2c^2+11abcd-44a^2d^2)(a+bx^3)^{2/3}}{220a^2c^3x^5}-\frac{(18b^3c^3+33ab^2c^2d+8\cdots}{\cdots}$$

```
[Out] -1/11*(b*x^3+a)^(2/3)/c/x^11-1/88*(-11*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2/x^8
+1/220*(-44*a^2*d^2+11*a*b*c*d+6*b^2*c^2)*(b*x^3+a)^(2/3)/a^2/c^3/x^5-1/440
*(-220*a^3*d^3+88*a^2*b*c*d^2+33*a*b^2*c^2*d+18*b^3*c^3)*(b*x^3+a)^(2/3)/a^
3/c^4/x^2-1/6*d^3*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(14/3)+1/2*d^3*(-a*d+b*c)^
(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(14/3)-1/3*d^3*(-a*d
+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(
1/2))/c^(14/3)*3^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {486, 597, 12, 384}

$$\frac{(a+bx^3)^{2/3}(-44a^2d^2+11abcd+6b^2c^2)}{220a^2c^3x^5}-\frac{(a+bx^3)^{2/3}(-220a^3d^3+88a^2bcd^2+33ab^2c^2d+18b^3c^3)}{440a^3c^4x^2}-\frac{d^3(bc-ad)^{2/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{14/3}}-\frac{d^3(bc-ad)^{2/3}\log(c+dx^3)}{6c^{14/3}}+\frac{d^3(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{14/3}}-\frac{(a+bx^3)^{2/3}(2bc-11ad)}{88ac^2x^8}-\frac{(a+bx^3)^{2/3}}{11cx^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] -1/11*(a + b*x^3)^(2/3)/(c*x^11) - ((2*b*c - 11*a*d)*(a + b*x^3)^(2/3))/(88
*a*c^2*x^8) + ((6*b^2*c^2 + 11*a*b*c*d - 44*a^2*d^2)*(a + b*x^3)^(2/3))/(22
0*a^2*c^3*x^5) - ((18*b^3*c^3 + 33*a*b^2*c^2*d + 88*a^2*b*c*d^2 - 220*a^3*d
^3)*(a + b*x^3)^(2/3))/(440*a^3*c^4*x^2) - (d^3*(b*c - a*d)^(2/3)*ArcTan[(1
 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*
c^(14/3)) - (d^3*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^(14/3)) + (d^3*(b*c
 - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^
(14/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^{12}(c+dx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{40ac^4+40bc^4x^3-45ac^3dx^3-45bc^3dx^6+54ac^2d^2x^6+54bc^2d^2x^9-81acd^3x^9-81b\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.15, size = 422, normalized size = 1.32

$$\frac{\frac{3c^{2/3}(a+bx^3)^{2/3}\left(-18b^3c^3x^9+3ab^2c^2x^6(4c-11dx^3)-2a^2bcx^3(5c^2-11cdx^3+44d^2x^6)+a^3(-40c^3+55c^2dx^3-88cd^2x^6+220d^3x^9)\right)}{a^3x^{11}}+220\sqrt{-6+6i\sqrt{3}}\,d^3(bc-ad)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)-220i\left(-i+\sqrt{3}\right)d^3(bc-ad)^{2/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+110\left(1+i\sqrt{3}\right)d^3(bc-ad)^{2/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{1320c^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] ((3*c^(2/3)*(a + b*x^3)^(2/3)*(-18*b^3*c^3*x^9 + 3*a*b^2*c^2*x^6*(4*c - 11*
d*x^3) - 2*a^2*b*c*x^3*(5*c^2 - 11*c*d*x^3 + 44*d^2*x^6) + a^3*(-40*c^3 + 5
5*c^2*d*x^3 - 88*c*d^2*x^6 + 220*d^3*x^9)))/(a^3*x^11) + 220*Sqrt[-6 + (6*I
)*Sqrt[3]]*d^3*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b
*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (220*I)*(
-I + Sqrt[3])*d^3*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 110*(1 + I*Sqrt[3])*d^3*(b*c - a*d)^(2/3)
*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x
*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(1320*c^(1
4/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{12}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3}\right)^{\frac{2}{3}}}{x^{12}\left(c + dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**12/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**12*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^{12}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)), x)
```

3.691 $\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^8(a+bx^3)^{2/3}\,F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

```
[Out] 1/8*x^8*(b*x^3+a)^(2/3)*AppellF1(8/3,-2/3,1,11/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(2/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^8(a+bx^3)^{2/3}\,F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(2/3)*AppellF1[8/3, -2/3, 1, 11/3, -((b*x^3)/a), -((d*x^3)
/c)])/(8*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^8(a+bx^3)^{2/3}\,F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 181 vs. $2(64) = 128$.
time = 6.89, size = 181, normalized size = 2.83

$$\frac{x^2\left(5c(a+bx^3)(-7bc+2ad+4bdx^3)+5ac(7bc-2ad)\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2(-14b^2c^2+7abcd+2a^2d^2)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{140bcd^2\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(5*c*(a + b*x^3)*(-7*b*c + 2*a*d + 4*b*d*x^3) + 5*a*c*(7*b*c - 2*a*d)*
(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)
] - 2*(-14*b^2*c^2 + 7*a*b*c*d + 2*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(140*b*c*d^2*(a + b*x^3
)^(1/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^7(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**7*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^7\,{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^7*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.692 $\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^5(a+bx^3)^{2/3}F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

```
[Out] 1/5*x^5*(b*x^3+a)^(2/3)*AppellF1(5/3,-2/3,1,8/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x
^3/a)^(2/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^5(a+bx^3)^{2/3}F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(2/3)*AppellF1[5/3, -2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/
c)])/(5*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^5(a+bx^3)^{2/3}\operatorname{F}_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 141 vs. $2(64) = 128$.
time = 6.65, size = 141, normalized size = 2.20

$$\frac{5cx^2(a+bx^3)-5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{F}_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2(-2bc+ad)x^5\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{F}_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{20cd\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1
, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*(-2*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(
1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*c*d*(a + b
*x^3)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(b\,x^3+a\right)^{\frac{2}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**4*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4\,{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^4*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.693 $\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^2(a+bx^3)^{2/3}\,F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

```
[Out] 1/2*x^2*(b*x^3+a)^(2/3)*AppellF1(2/3,-2/3,1,5/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x
^3/a)^(2/3)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{x^2(a+bx^3)^{2/3}\,F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^2(a+bx^3)^{2/3}\,F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [**A**]
time = 9.62, size = 65, normalized size = 1.02

$$\frac{x^2(a+bx^3)^{2/3}\,F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{a+bx^3}{a}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*((a + b*x^3)/a)^(2/3))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] int((x*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.694 $\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx$

Optimal. Leaf size=62

$$-\frac{(a+bx^3)^{2/3}\,F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

```
[Out] -(b*x^3+a)^(2/3)*AppellF1(-1/3,-2/3,1,2/3,-b*x^3/a,-d*x^3/c)/c/x/(1+b*x^3/a
)^(2/3)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{(a+bx^3)^{2/3}\,F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(2/3)*AppellF1[-1/3, -2/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c*x*(1 + (b*x^3)/a)^(2/3)))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^2(c+dx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{(a+bx^3)^{2/3}\operatorname{F}_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 138 vs. 2(62) = 124.
time = 10.06, size = 138, normalized size = 2.23

$$\frac{-10c(a+bx^3)-5(-2bc+ad)x^3\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{F}_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^6\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{F}_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{10c^2x\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] (-10*c*(a + b*x^3) - 5*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/
3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(1/
3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*c^2*x*(a + b
*x^3)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^2\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{2}{3}}}{x^2(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**2/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**2*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{x^2\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^2*(c + d*x^3)), x)
```

3.695 $\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{(a+bx^3)^{2/3} F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

```
[Out] -1/4*(b*x^3+a)^(2/3)*AppellF1(-4/3,-2/3,1,-1/3,-b*x^3/a,-d*x^3/c)/c/x^4/(1+
b*x^3/a)^(2/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{(a+bx^3)^{2/3} F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -1/4*((a + b*x^3)^(2/3)*AppellF1[-4/3, -2/3, 1, -1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c*x^4*(1 + (b*x^3)/a)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^5(c+dx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{(a+bx^3)^{2/3} F_1\left(-\frac{4}{3}; -\frac{2}{3}, 1; -\frac{1}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{4cx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 181 vs. $2(64) = 128$.
time = 10.12, size = 181, normalized size = 2.83

$$\frac{-5c(a+bx^3)\left(2bcx^3+a(c-4dx^3)\right)+5(b^2c^2-4abcd+2a^2d^2)\,x^6\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bd(bc-2ad)x^9\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{20ac^3x^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] (-5*c*(a + b*x^3)*(2*b*c*x^3 + a*(c - 4*d*x^3)) + 5*(b^2*c^2 - 4*a*b*c*d +
2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a
), -((d*x^3)/c)] + 2*b*d*(b*c - 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*AppellF1[5
/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c^3*x^4*(a + b*x^3)^(1/
3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^5\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{2}{3}}}{x^5\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**5/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**5*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^5*(c + d*x^3)), x)
```

3.696 $\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=251

$$-\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2(a+bx^3)^{4/3}}{4d^3}-\frac{(bc+ad)\,(a+bx^3)^{7/3}}{7b^2d^2}+\frac{(a+bx^3)^{10/3}}{10b^2d}-\frac{c^2(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cdots}}{\cdots}}{\cdots}\right)}{\sqrt{3}\,d^{13/3}}$$

```
[Out] -c^2*(-a*d+b*c)*(b*x^3+a)^(1/3)/d^4+1/4*c^2*(b*x^3+a)^(4/3)/d^3-1/7*(a*d+b*
c)*(b*x^3+a)^(7/3)/b^2/d^2+1/10*(b*x^3+a)^(10/3)/b^2/d-1/6*c^2*(-a*d+b*c)^(
4/3)*ln(d*x^3+c)/d^(13/3)+1/2*c^2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1
/3)*(b*x^3+a)^(1/3))/d^(13/3)-1/3*c^2*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1
/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(13/3)*3^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 90, 52, 60, 631, 210, 31}

$$-\frac{c^2(bc-ad)^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{13/3}}-\frac{(a+bx^3)^{7/3}(ad+bc)}{7b^2d^2}+\frac{(a+bx^3)^{10/3}}{10b^2d}-\frac{c^2(bc-ad)^{4/3}\log(c+dx^3)}{6d^{13/3}}+\frac{c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}-\frac{c^2\sqrt[3]{a+bx^3}(bc-ad)}{d^4}+\frac{c^2(a+bx^3)^{4/3}}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -((c^2*(b*c - a*d)*(a + b*x^3)^(1/3))/d^4) + (c^2*(a + b*x^3)^(4/3))/(4*d^3
) - ((b*c + a*d)*(a + b*x^3)^(7/3))/(7*b^2*d^2) + (a + b*x^3)^(10/3)/(10*b^
2*d) - (c^2*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*
c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^2*(b*c - a*d)^(4/3)*Log[c
 + d*x^3])/(6*d^(13/3)) + (c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^
(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(-bc-ad)(a+bx)^{4/3}}{bd^2} + \frac{(a+bx)^{7/3}}{bd} + \frac{c^2(a+bx)^{4/3}}{d^2(c+dx)}\right)\,dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^2d} + \frac{c^2\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^2d} - \frac{\left(c^2(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[3]{a}}{c}\right.}{3d^3}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4} + \frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^2d} +$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4} + \frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^2d} -$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4} + \frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^2d} -$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4} + \frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^2d} -$$

Mathematica [A]
time = 0.53, size = 308, normalized size = 1.23

$$\frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}\left(-6a^3d^3+2a^2bd^2(-10c+dx^3)+ab^2d(175c^2-40cdx^3+22d^2x^6)+b^3(-140c^3+35c^2dx^3-20cd^2x^6+14d^3x^9)\right)}{b^2} - 140\sqrt{3}\,c^2(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 140c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) - 70c^2(bc-ad)^{4/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{420d^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(1/3)*(a + b*x^3)^(1/3)*(-6*a^3*d^3 + 2*a^2*b*d^2*(-10*c + d*x^3) + a
*b^2*d*(175*c^2 - 40*c*d*x^3 + 22*d^2*x^6) + b^3*(-140*c^3 + 35*c^2*d*x^3 -
 20*c*d^2*x^6 + 14*d^3*x^9)))/b^2 - 140*Sqrt[3]*c^2*(b*c - a*d)^(4/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 140*c^2*
(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 70*c
^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a +
 b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(420*d^(13/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{8} \left(b\,x^{3}+a\right)^{\frac{4}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 4.80, size = 369, normalized size = 1.47

$$\frac{140 \, \sqrt{3} \, {\left(b^{3} c^{3} - a b^{2} c^{2} d\right)} \left(-\frac{b c - a d}{d}\right)^{\frac{1}{3}} \arctan\left(-\frac{2 \, \sqrt{3} \, {\left(b x^{3} + a\right)}^{\frac{1}{3}} d \left(-\frac{b c - a d}{d}\right)^{\frac{2}{3}} - \sqrt{3} \, {\left(b c - a d\right)}}{3 \, {\left(b c - a d\right)}}\right) + 70 \, {\left(b^{3} c^{3} - a b^{2} c^{2} d\right)} \left(-\frac{b c - a d}{d}\right)^{\frac{1}{3}} \log\left({\left(b x^{3} + a\right)}^{\frac{2}{3}} + {\left(b x^{3} + a\right)}^{\frac{1}{3}} \left(-\frac{b c - a d}{d}\right)^{\frac{1}{3}} + \left(-\frac{b c - a d}{d}\right)^{\frac{2}{3}}\right) - 140 \, {\left(b^{3} c^{3} - a b^{2} c^{2} d\right)} \left(-\frac{b c - a d}{d}\right)^{\frac{1}{3}} \log\left({\left(b x^{3} + a\right)}^{\frac{1}{3}} - \left(-\frac{b c - a d}{d}\right)^{\frac{1}{3}}\right) + 3 \, {\left(14 \, b^{3} d^{3} x^{9} - 2 \, {\left(10 \, b^{3} c d^{2} - 11 \, a b^{2} d^{3}\right)} x^{6} - 140 \, b^{3} c^{3} + 175 \, a b^{2} c^{2} d - 20 \, a^{2} b c d^{2} - 6 \, a^{3} d^{3} + {\left(35 \, b^{3} c^{2} d - 40 \, a b^{2} c d^{2} + 2 \, a^{2} b d^{3}\right)} x^{3}\right)} {\left(b x^{3} + a\right)}^{\frac{1}{3}}}{420 \, b^{2} d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/420*(140*sqrt(3)*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1
/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a
*d))/(b*c - a*d)) + 70*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3)) - 140*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(14*b^3*d^3*x^9 - 2*(10*b^3*c*d^2
- 11*a*b^2*d^3)*x^6 - 140*b^3*c^3 + 175*a*b^2*c^2*d - 20*a^2*b*c*d^2 - 6*a^
3*d^3 + (35*b^3*c^2*d - 40*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^3)*(b*x^3 + a)^(1/3
))/(b^2*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8} \left(a + b x^{3}\right)^{\frac{4}{3}}}{c + d x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [A]
time = 0.58, size = 394, normalized size = 1.57

$$-\frac{\left(b^{24}c^4d^6 - 2\,ab^{23}c^3d^7 + a^2b^{22}c^2d^8\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{23}cd^{10} - ab^{22}d^{11}\right)} + \frac{\sqrt{3}\,(bc^3 - ac^2d)(-bcd^2 + ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5} + \frac{(bc^3 - ac^2d)(-bcd^2 + ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^5} - \frac{140\,(bx^3+a)^{\frac{1}{3}}b^{21}c^3d^6 - 35\,(bx^3+a)^{\frac{4}{3}}b^{20}c^2d^7 - 140\,(bx^3+a)^{\frac{1}{3}}ab^{20}c^2d^7 + 20\,(bx^3+a)^{\frac{7}{3}}b^{19}cd^8 - 14\,(bx^3+a)^{\frac{10}{3}}b^{18}d^9 + 20\,(bx^3+a)^{\frac{7}{3}}ab^{18}d^9}{140\,b^{20}d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^24*c^4*d^6 - 2*a*b^23*c^3*d^7 + a^2*b^22*c^2*d^8)*(-(b*c - a*d)/d)^
(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^23*c*d^10 - a
*b^22*d^11) + 1/3*sqrt(3)*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*arctan
(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d
)^(1/3))/d^5 + 1/6*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3
))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^21*c^3*d^6 - 35*(b*x^3 + a)^(4/3)*b
^20*c^2*d^7 - 140*(b*x^3 + a)^(1/3)*a*b^20*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b
^19*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^18*d^9 + 20*(b*x^3 + a)^(7/3)*a*b^18*d^
9)/(b^20*d^10)
```

Mupad [B]
time = 5.12, size = 477, normalized size = 1.90

$$\left(\frac{a^2}{4\,b^2\,d} + \frac{\left(\frac{2\,a}{b^2\,d} + \frac{b^3\,c - a\,b^2\,d}{b^4\,d^2}\right)\left(b^3\,c - a\,b^2\,d\right)}{4\,b^2\,d}\right)\left(b\,x^3+a\right)^{4/3} - \left(\frac{2\,a}{7\,b^2\,d} + \frac{b^3\,c - a\,b^2\,d}{7\,b^4\,d^2}\right)\left(b\,x^3+a\right)^{7/3} + \frac{\left(b\,x^3+a\right)^{10/3}}{10\,b^2\,d} + \frac{c^2\,\ln\left(\frac{3\,\left(b\,x^3+a\right)^{1/3}\left(a^2\,c^2\,d^2 - 2\,a\,b\,c^3\,d + b^2\,c^4\right)}{d^2} - \frac{c^2\,\left(a\,d-b\,c\right)^{4/3}\left(9\,a\,d^3 - 9\,b\,c\,d^2\right)}{3\,d^{13/3}}\right)\left(a\,d-b\,c\right)^{4/3}}{3\,d^{13/3}} - \frac{\left(\frac{a^2}{b^2\,d} + \frac{\left(\frac{2\,a}{b^2\,d} + \frac{b^3\,c - a\,b^2\,d}{b^4\,d^2}\right)\left(b^3\,c - a\,b^2\,d\right)}{b^2\,d}\right)\left(b\,x^3+a\right)^{1/3}\left(b^3\,c - a\,b^2\,d\right)}{b^2\,d} - \frac{c^2\,\ln\left(\frac{3\,c^2\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^{7/3}}{d^{7/3}} + \frac{3\,c^2\left(b\,x^3+a\right)^{1/3}\left(a\,d-b\,c\right)^2}{d^2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^{4/3}}{3\,d^{13/3}} + \frac{c^2\,\ln\left(\frac{3\,c^2\left(b\,x^3+a\right)^{1/3}\left(a\,d-b\,c\right)^2}{d^2} - \frac{9\,c^2\left(-\frac{1}{6} + \frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\left(a\,d-b\,c\right)^{7/3}}{d^{7/3}}\right)\left(-\frac{1}{6} + \frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\left(a\,d-b\,c\right)^{4/3}}{d^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(4*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*
b^2*d))/(4*b^2*d))*(a + b*x^3)^(4/3) - ((2*a)/(7*b^2*d) + (b^3*c - a*b^2*d)
/(7*b^4*d^2))*(a + b*x^3)^(7/3) + (a + b*x^3)^(10/3)/(10*b^2*d) + (c^2*log(
(3*(a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))/d^2 - (c^2*(a*d
 - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(13/3)))*(a*d - b*c)^(4/3))/(3*d^
(13/3)) - ((a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b
^3*c - a*b^2*d))/(b^2*d))*(a + b*x^3)^(1/3)*(b^3*c - a*b^2*d))/(b^2*d) - (c
^2*log((3*c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(7/3) + (3*c^2*(a
 + b*x^3)^(1/3)*(a*d - b*c)^2)/d^2)*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(4/3
))/(3*d^(13/3)) + (c^2*log((3*c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^2 - (9
*c^2*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(7/3))/d^(7/3))*((3^(1/2)*1i)/6 - 1
/6)*(a*d - b*c)^(4/3))/d^(13/3)
```

3.697 $\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=211

$$\frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3}-\frac{c(a+bx^3)^{4/3}}{4d^2}+\frac{(a+bx^3)^{7/3}}{7bd}+\frac{c(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}}+\frac{c(bc-ad)^{\text{}}}{6}$$

```
[Out] c*(-a*d+b*c)*(b*x^3+a)^(1/3)/d^3-1/4*c*(b*x^3+a)^(4/3)/d^2+1/7*(b*x^3+a)^(7
/3)/b/d+1/6*c*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/d^(10/3)-1/2*c*(-a*d+b*c)^(4/3)*
ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)+1/3*c*(-a*d+b*c)^(4/3
)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(10/
3)*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 81, 52, 60, 631, 210, 31}

$$\frac{c(bc-ad)^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}}+\frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}+\frac{c\sqrt[3]{a+bx^3}(bc-ad)}{d^3}-\frac{c(a+bx^3)^{4/3}}{4d^2}+\frac{(a+bx^3)^{7/3}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (c*(b*c - a*d)*(a + b*x^3)^(1/3))/d^3 - (c*(a + b*x^3)^(4/3))/(4*d^2) + (a
+ b*x^3)^(7/3)/(7*b*d) + (c*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b
*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c*(b*c - a*
d)^(4/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c*(b*c - a*d)^(4/3)*Log[(b*c - a*d
)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{(a+bx^3)^{7/3}}{7bd} - \frac{c\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{(a+bx^3)^{7/3}}{7bd} + \frac{(c(bc-ad))\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{(a+bx^3)^{7/3}}{7bd} - \frac{(c(bc-ad)^2)\text{Subst}\left(\int \frac{}{(a+bx}\right.}{3d^3}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{(a+bx^3)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\log(c+dx^3)}{6d^{10/3}} -$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{(a+bx^3)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\log(c+dx^3)}{6d^{10/3}} -$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{(a+bx^3)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}}{\sqrt[3]{}}}{}\right.}{\sqrt{3}\,d^{10/3}}$$

Mathematica [A]
time = 0.33, size = 260, normalized size = 1.23

$$\frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}\left(4a^2d^2+abd(-35c+8dx^3)+b^2(28c^2-7cdx^3+4d^2x^6)\right)}{b} + 28\sqrt{3}c(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 28c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) + 14c(bc-ad)^{4/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{84d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
[Out] ((3*d^(1/3)*(a + b*x^3)^(1/3)*(4*a^2*d^2 + a*b*d*(-35*c + 8*d*x^3) + b^2*(2
8*c^2 - 7*c*d*x^3 + 4*d^2*x^6)))/b + 28*Sqrt[3]*c*(b*c - a*d)^(4/3)*ArcTan[
(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*c*(b*c
- a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 14*c*(b*c
 - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)
^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(84*d^(10/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(b\,x^3+a)^{\frac{4}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.96, size = 298, normalized size = 1.41

$$\frac{28\sqrt{3}\,(b^2c^2-abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+14\,(b^2c^2-abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-28\,(b^2c^2-abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)+3\,(4\,b^2d^2x^6+28\,b^2c^2-35\,abcd+4\,a^2d^2-(7\,b^2cd-8\,abd^2)x^3)(bx^3+a)^{\frac{1}{3}}}{84\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/84*(28*sqrt(3)*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*s
qrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*
c - a*d)) + 14*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2
/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 28
*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c -
a*d)/d)^(1/3)) + 3*(4*b^2*d^2*x^6 + 28*b^2*c^2 - 35*a*b*c*d + 4*a^2*d^2 - (
7*b^2*c*d - 8*a*b*d^2)*x^3)*(b*x^3 + a)^(1/3))/(b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(171) = 342.

time = 0.56, size = 348, normalized size = 1.65

$$\frac{\left(b^{10}c^3d^4-2\,ab^9c^2d^5+a^2b^8cd^6\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^9cd^7-ab^8d^8\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bc^2-acd\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bc^2-acd\right)\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^4}+\frac{28\left(bx^3+a\right)^{\frac{1}{3}}b^8c^2d^4-7\left(bx^3+a\right)^{\frac{4}{3}}b^7cd^5-28\left(bx^3+a\right)^{\frac{1}{3}}ab^7cd^5+4\left(bx^3+a\right)^{\frac{7}{3}}b^6d^6}{28\,b^7d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^10*c^3*d^4 - 2*a*b^9*c^2*d^5 + a^2*b^8*c*d^6)*(-(b*c - a*d)/d)^(1/3)
*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^9*c*d^7 - a*b^8*d^
8) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*arctan(1/3*sqrt(3
)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^
4 - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*log((b*x^3 + a)^(2/3) + (b
*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28
*(28*(b*x^3 + a)^(1/3)*b^8*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^7*c*d^5 - 28*(b*
x^3 + a)^(1/3)*a*b^7*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^6*d^6)/(b^7*d^7)
```

Mupad [B]

time = 5.06, size = 348, normalized size = 1.65

$$\frac{\left(b\,x^3+a\right)^{7/3}}{7\,b\,d}-\left(b\,x^3+a\right)^{4/3}\left(\frac{a}{4\,b\,d}+\frac{b^2\,c-a\,b\,d}{4\,b^2\,d^2}\right)-\frac{c\,\ln\left(\frac{3\,\left(b\,x^3+a\right)^{1/3}\,\left(a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3\right)}{d}-\frac{c\,\left(a\,d-b\,c\right)^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{10/3}}\right)\,\left(a\,d-b\,c\right)^{4/3}}{3\,d^{10/3}}-\frac{c\,\ln\left(\frac{3\,c\,\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)^2}{d}-\frac{3\,c\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{7/3}}{d^{4/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{4/3}}{3\,d^{10/3}}+\frac{c\,\ln\left(\frac{3\,c\,\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)^2}{d}+\frac{3\,c\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{7/3}}{d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{4/3}}{3\,d^{10/3}}+\frac{\left(b\,x^3+a\right)^{1/3}\,\left(b^2\,c-a\,b\,d\right)\,\left(\frac{a}{b\,d}+\frac{b^2\,c-a\,b\,d}{b^2\,d^2}\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(7/3)/(7*b*d) - (a + b*x^3)^(4/3)*(a/(4*b*d) + (b^2*c - a*b*d)/
(4*b^2*d^2)) - (c*log((3*(a + b*x^3)^(1/3)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2
*d))/d - (c*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(10/3)))*(a*d - b
*c)^(4/3))/(3*d^(10/3)) - (c*log((3*c*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d -
(3*c*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/2 - 1
/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + (c*log((3*c*(a + b*x^3)^(1/3)*(a*d -
b*c)^2)/d + (3*c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/
2)*1i)/2 + 1/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + ((a + b*x^3)^(1/3)*(b^2*c
 - a*b*d)*(a/(b*d) + (b^2*c - a*b*d)/(b^2*d^2)))/(b*d)
```

3.698 $\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3} dx$

Optimal. Leaf size=187

$$-\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2}+\frac{(a+bx^3)^{4/3}}{4d}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}-\frac{(bc-ad)^{4/3}\log(c+dx^3)}{6d^{7/3}}+$$

```
[Out] -(-a*d+b*c)*(b*x^3+a)^(1/3)/d^2+1/4*(b*x^3+a)^(4/3)/d-1/6*(-a*d+b*c)^(4/3)*
ln(d*x^3+c)/d^(7/3)+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3
+a)^(1/3))/d^(7/3)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(
1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)*3^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {455, 52, 60, 631, 210, 31}

$$-\frac{(bc-ad)^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}-\frac{(bc-ad)^{4/3}\log(c+dx^3)}{6d^{7/3}}+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{\sqrt[3]{a+bx^3}\,(bc-ad)}{d^2}+\frac{(a+bx^3)^{4/3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -(((b*c - a*d)*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*d) - ((b*c -
a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqr
t[3]])/(Sqrt[3]*d^(7/3)) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*d^(7/3)) +
 ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)$$
$$= \frac{(a+bx^3)^{4/3}}{4d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d}$$
$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4d} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^2}$$
$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\text{Subs}}{}$$
$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\log}{}$$
$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}} - $$

Mathematica [A]
time = 0.29, size = 221, normalized size = 1.18

$$\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(-4bc+5ad+bdx^3) - 4\sqrt{3}(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 4(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) - 2(bc-ad)^{4/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{12d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (3*d^(1/3)*(a + b*x^3)^(1/3)*(-4*b*c + 5*a*d + b*d*x^3) - 4*Sqrt[3]*(b*c -
a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqr
t[3]] + 4*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/
3)] - 2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)
*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*d^(7/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(b\,x^3+a)^{\frac{4}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.49, size = 246, normalized size = 1.32

$$\frac{4\sqrt{3}\,(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+2\,(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-4\,(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)+3\,(bdx^3-4\,bc+5\,ad)(bx^3+a)^{\frac{1}{3}}}{12\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] 1/12*(4*sqrt(3)*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(
b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d
)) + 2*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 4*(b*c - a*d)*(
-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(
b*d*x^3 - 4*b*c + 5*a*d)*(b*x^3 + a)^(1/3))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**2*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [A]
time = 0.57, size = 297, normalized size = 1.59

$$-\frac{(b^2c^2d^2-2\,abcd^3+a^2d^4)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bcd^4-ad^5)}+\frac{\sqrt{3}\,(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3}+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3}-\frac{4\,(bx^3+a)^{\frac{1}{3}}bcd^2-(bx^3+a)^{\frac{4}{3}}d^3-4\,(bx^3+a)^{\frac{1}{3}}ad^3}{4\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-(b*c - a*d)/d)^(1/3)*log(abs((
b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^4 - a*d^5) + 1/3*sqrt(3)
*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 +
a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c -
 a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b*c
*d^2 - (b*x^3 + a)^(4/3)*d^3 - 4*(b*x^3 + a)^(1/3)*a*d^3)/d^4
```

Mupad [B]
time = 4.72, size = 304, normalized size = 1.63

$$\frac{(b\,x^3+a)^{4/3}}{4\,d}+\frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2)-\frac{(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{7/3}}\right)\,(a\,d-b\,c)^{4/3}}{3\,d^{7/3}}+\frac{(b\,x^3+a)^{1/3}\,(a\,d-b\,c)}{d^2}-\frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2)+\frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{7/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{4/3}}{3\,d^{7/3}}+\frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2)-\frac{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{d^{7/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,(a\,d-b\,c)^{4/3}}{d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*d) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 - 6
*a*b*c*d) - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d - b
*c)^(4/3))/(3*d^(7/3)) + ((a + b*x^3)^(1/3)*(a*d - b*c))/d^2 - (log((a + b*
x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d) + (((3^(1/2)*1i)/2 + 1/2)*(a
*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 + 1/2)*
(a*d - b*c)^(4/3))/(3*d^(7/3)) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*
c^2 - 6*a*b*c*d) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b
*c*d^2))/d^(7/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(7/3)
```

3.699 $\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx$

Optimal. Leaf size=261

$$\frac{b\sqrt[3]{a+bx^3}}{d}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,c}+\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,cd^{4/3}}-\frac{a^{4/3}\log(x)}{2c}+\frac{(bc-}{}$$

[Out] b*(b*x^3+a)^(1/3)/d-1/2*a^(4/3)*ln(x)/c+1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c/d^(4/3)+1/2*a^(4/3)*ln(a^(1/3)-(b*x^3+a)^(1/3))/c-1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/d^(4/3)-1/3*a^(4/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)+1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d^(4/3)*3^(1/2)

Rubi [A]
time = 0.20, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {457, 86, 162, 59, 631, 210, 31, 60}

$$-\frac{a^{4/3}\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,c}+\frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{a^{4/3}\log(x)}{2c}+\frac{(bc-ad)^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,cd^{4/3}}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}}-\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{4/3}}+\frac{b\sqrt[3]{a+bx^3}}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]

[Out] (b*(a + b*x^3)^(1/3))/d - (a^(4/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*c) + ((b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(4/3)) - (a^(4/3)*Log[x])/(2*c) + ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c*d^(4/3)) + (a^(4/3)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*d^(4/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{4/3}}{x\,(c+dx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x(c+dx)}\,dx, x, x^3\right)\\
&= \frac{b\sqrt[3]{a+bx^3}}{d} + \frac{\text{Subst}\left(\int \frac{a^2d+b(-bc+2ad)x}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}\\
&= \frac{b\sqrt[3]{a+bx^3}}{d} + \frac{a^2\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,\right.}{3cd}\\
&= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6cd^{4/3}} - \frac{a^{4/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{\right.}}{2c}\\
&= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6cd^{4/3}} + \frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}\\
&= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,c} + \frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right.}{\sqrt{3}\,cd^{4/3}}
\end{aligned}$$

Mathematica [A]
time = 0.59, size = 333, normalized size = 1.28

$$\frac{6bc\sqrt[3]{d}\sqrt[3]{a+bx^3} - 2\sqrt{3}\,a^{4/3}d^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\sqrt{3}\,(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 2a^{4/3}d^{4/3}\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right) - 2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) - a^{4/3}d^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right) + (bc-ad)^{4/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{6cd^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

```
[Out] (6*b*c*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*a^(4/3)*d^(4/3)*ArcTan[(1 + (2
*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(b*c - a*d)^(4/3)*ArcTan[
(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*a^(4/3)*
d^(4/3)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 2*(b*c - a*d)^(4/3)*Log[(b*c -
a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - a^(4/3)*d^(4/3)*Log[a^(2/3) + a^(
1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + (b*c - a*d)^(4/3)*Log[(b*c -
a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x
^3)^(2/3)])/(6*c*d^(4/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{x\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x), x)
```

Fricas [A]
time = 3.81, size = 320, normalized size = 1.23

$$-\frac{2\sqrt{3}\,a^{\frac{4}{3}}d\arctan\left(\frac{2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3a}\right)+a^{\frac{4}{3}}d\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2a^{\frac{4}{3}}d\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-2\sqrt{3}\,(bc-ad)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)-6\,(bx^3+a)^{\frac{1}{3}}bc-(bc-ad)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)+2\,(bc-ad)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{6\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a^(4/3)*d*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) +
 sqrt(3)*a)/a) + a^(4/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3
) + a^(2/3)) - 2*a^(4/3)*d*log((b*x^3 + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(b*
c - a*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(
(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 6*(b*x^3 + a)^(1
/3)*b*c - (b*c - a*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3
+ a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) + 2*(b*c - a*d)*(
(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)))/(c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{4}{3}}}{x\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x*(c + d*x**3)), x)
```

Giac [A]
time = 0.79, size = 357, normalized size = 1.37

$$-\frac{\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3c}-\frac{a^{\frac{4}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6c}+\frac{a^{\frac{4}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3c}+\frac{(b^2c^2-2abcd+a^2d^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2d-acd^2)}+\frac{(bx^3+a)^{\frac{1}{3}}b}{d}-\frac{\sqrt{3}\,(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^2}-\frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/c - 1/6*a^(4/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a
^(2/3))/c + 1/3*a^(4/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c + 1/3*(b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3)
 - (-(b*c - a*d)/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (b*x^3 + a)^(1/3)*b/d - 1
/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^
3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^2) - 1/
6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^
(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

Mupad [B]
time = 6.08, size = 796, normalized size = 3.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x*(c + d*x^3)),x)
```

```
[Out] log(c*d*(-(a*d - b*c)^4/(c^3*d^4))^(1/3) + a*d*(a + b*x^3)^(1/3) - b*c*(a +
 b*x^3)^(1/3))*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) + log(c*(a^4/c^3)^(1/3) - a*(a + b*x^3)^(
1/3))*(a^4/(27*c^3))^(1/3) + (b*(a + b*x^3)^(1/3))/d - log(c*(a^4/c^3)^(1/3
) + 2*a*(a + b*x^3)^(1/3) + 3^(1/2)*c*(a^4/c^3)^(1/3)*1i)*((3^(1/2)*1i)/2 +
 1/2)*(a^4/(27*c^3))^(1/3) + log(c*(a^4/c^3)^(1/3)*1i + a*(a + b*x^3)^(1/3)
*2i + 3^(1/2)*c*(a^4/c^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a^4/(27*c^3))^(1/3
) + log((3*a^2*b^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(2*a^4*d^4 + b^4*c^4 + 6
*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + 3*a^2*b^4*c*((3^(1/2
)*1i)/2 - 1/2)*(-(a*d - b*c)^4/(c^3*d^4))^(1/3)*(2*a^5*d^5 - b^5*c^5 - 10*a
^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 6*a^4*b*c*d^4))*((3^(
1/2)*1i)/2 - 1/2)*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) - log((3*a^2*b^4*(a + b*x^3)^(1/3)*(a*
d - b*c)^2*(2*a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3
*b*c*d^3))/d - 3*a^2*b^4*c*((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)^4/(c^3*d^4)
)^(1/3)*(2*a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*
a*b^4*c^4*d - 6*a^4*b*c*d^4))*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*d^4 + b^4*c^4 +
 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3)
```

3.700 $\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx$

Optimal. Leaf size=399

$$\frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2}-\frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2}+\frac{d(a+bx^3)^{4/3}}{4c^2}+\frac{(4bc-3ad)\,(a+bx^3)^{4/3}}{12ac^2}-\frac{(a+bx^3)^{7/3}}{3acx^3}-\frac{\sqrt[3]{a}\,(4\cdots}{}$$

```
[Out] 1/3*(-3*a*d+4*b*c)*(b*x^3+a)^(1/3)/c^2-(-a*d+b*c)*(b*x^3+a)^(1/3)/c^2+1/4*d
*(b*x^3+a)^(4/3)/c^2+1/12*(-3*a*d+4*b*c)*(b*x^3+a)^(4/3)/a/c^2-1/3*(b*x^3+a
)^(7/3)/a/c/x^3-1/6*a^(1/3)*(-3*a*d+4*b*c)*ln(x)/c^2-1/6*(-a*d+b*c)^(4/3)*l
n(d*x^3+c)/c^2/d^(1/3)+1/6*a^(1/3)*(-3*a*d+4*b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3
))/c^2+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^
2/d^(1/3)-1/9*a^(1/3)*(-3*a*d+4*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))
/a^(1/3)*3^(1/2))/c^2*3^(1/2)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*
(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/d^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {457, 105, 162, 52, 59, 631, 210, 31, 60}

$$-\frac{\sqrt[3]{a}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(4bc-3ad)}{3\sqrt{3}\,c^2}-\frac{(bc-ad)^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^2\sqrt[3]{d}}+\frac{d(a+bx^3)^{4/3}}{4c^2}+\frac{(a+bx^3)^{4/3}(4bc-3ad)}{12ac^2}+\frac{\sqrt[3]{a+bx^3}(4bc-3ad)}{3c^2}-\frac{\sqrt[3]{a+bx^3}(bc-ad)}{c^2}-\frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{d}}+\frac{\sqrt[3]{a}(4bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6c^2}+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{d}}-\frac{\sqrt[3]{a}\log(x)(4bc-3ad)}{6c^2}-\frac{(a+bx^3)^{7/3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] ((4*b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*c^2) - ((b*c - a*d)*(a + b*x^3)^(1/3
))/c^2 + (d*(a + b*x^3)^(4/3))/(4*c^2) + ((4*b*c - 3*a*d)*(a + b*x^3)^(4/3)
)/(12*a*c^2) - (a + b*x^3)^(7/3)/(3*a*c*x^3) - (a^(1/3)*(4*b*c - 3*a*d)*Arc
Tan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*c^2) - (
(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/
3))/Sqrt[3]])/(Sqrt[3]*c^2*d^(1/3)) - (a^(1/3)*(4*b*c - 3*a*d)*Log[x])/(6*c
^2) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^2*d^(1/3)) + (a^(1/3)*(4*b*c
- 3*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*c^2) + ((b*c - a*d)^(4/3)*Log
[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^2(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{7/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{4/3}\left(\frac{1}{3}(-4bc+3ad)-\frac{4bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{(a+bx^3)^{7/3}}{3acx^3} + \frac{d^2\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(4bc-3ad)\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x}\,dx, x, x^3\right.}{9ac^2}$$

$$= \frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(4bc-3ad)(a+bx^3)^{4/3}}{12ac^2} - \frac{(a+bx^3)^{7/3}}{3acx^3} + \frac{(4bc-3ad)\text{Subst}\left(\int \frac{\sqrt[3]{a+}}{x}\right.}{9c^2}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(4bc-3ad)(a+bx^3)^4}{12ac^2}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(4bc-3ad)(a+bx^3)^4}{12ac^2}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(4bc-3ad)(a+bx^3)^4}{12ac^2}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(4bc-3ad)(a+bx^3)^4}{12ac^2}$$

Mathematica [A]
time = 0.77, size = 353, normalized size = 0.88

$$\frac{-\frac{6ac\sqrt[3]{a+bx^3}}{x^3}+2\sqrt{3}\sqrt[3]{a}(-4bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{6\sqrt{3}(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-2\sqrt[3]{a}(-4bc+3ad)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)+\frac{6(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{d}}+\sqrt[3]{a}(-4bc+3ad)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)-\frac{3(bc-ad)^{4/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{\sqrt[3]{d}}}{18c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] ((-6*a*c*(a + b*x^3)^(1/3))/x^3 + 2*Sqrt[3]*a^(1/3)*(-4*b*c + 3*a*d)*ArcTan
[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - (6*Sqrt[3]*(b*c - a*d)^(4/3
)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d^
(1/3) - 2*a^(1/3)*(-4*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + (6*(
b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/d^(1/3
) + a^(1/3)*(-4*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a +
 b*x^3)^(2/3)] - (3*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c
- a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/d^(1/3))/(18*c
^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{x^{4}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^4), x)
```

Fricas [A]
time = 3.97, size = 383, normalized size = 0.96

$$\frac{6\sqrt{3}(bc-ad)x^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+2\sqrt{3}(4bc-3ad)(-a)^{\frac{1}{3}}x^3\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+(4bc-3ad)(-a)^{\frac{1}{3}}x^3\log\left((bx^3+a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+3(bc-ad)x^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-2(4bc-3ad)(-a)^{\frac{1}{3}}x^3\log\left((bx^3+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-6(bc-ad)x^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-6(bx^3+a)^{\frac{1}{3}}ac}{18c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(
3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c -
 a*d)) + 2*sqrt(3)*(4*b*c - 3*a*d)*(-a)^(1/3)*x^3*arctan(1/3*(2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + (4*b*c - 3*a*d)*(-a)^(1/3)*x^3*
log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 3*(b*c
 - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(4*b*c - 3*a*d)*(-a)
^(1/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*c - a*d)*x^3*(-(b*c -
 a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 +
 a)^(1/3)*a*c)/(c^2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3}\right)^{\frac{4}{3}}}{x^{4} \left(c + dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**4/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**4*(c + d*x**3)), x)
```

Giac [A]
time = 0.77, size = 394, normalized size = 0.99

$$-\frac{\left(b^{2} c^{2}-2 \, a b c d+a^{2} d^{2}\right) \left(-\frac{b c-a d}{d}\right)^{\frac{1}{3}} \log \left(\left|\left(b x^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{b c-a d}{d}\right)^{\frac{1}{3}}\right|\right)}{3 \, \left(b c^{3}-a c^{2} d\right)}-\frac{\sqrt{3} \left(4 \, a^{\frac{1}{3}} b c-3 \, a^{\frac{4}{3}} d\right) \arctan \left(\frac{\sqrt{3} \left(2 \, \left(b x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3 \, a^{\frac{1}{3}}}\right)}{9 \, c^{2}}-\frac{\left(4 \, a^{\frac{1}{3}} b c-3 \, a^{\frac{4}{3}} d\right) \log \left(\left(b x^{3}+a\right)^{\frac{2}{3}}+\left(b x^{3}+a\right)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18 \, c^{2}}+\frac{\sqrt{3} \left(-b c d^{2}+a d^{3}\right)^{\frac{1}{3}} (b c-a d) \arctan \left(\frac{\sqrt{3} \left(2 \, \left(b x^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{b c-a d}{d}\right)^{\frac{1}{3}}\right)}{3 \, \left(-\frac{b c-a d}{d}\right)^{\frac{1}{3}}}\right)}{3 \, c^{2} d}+\frac{\left(-b c d^{2}+a d^{3}\right)^{\frac{1}{3}} (b c-a d) \log \left(\left(b x^{3}+a\right)^{\frac{2}{3}}+\left(b x^{3}+a\right)^{\frac{1}{3}} \left(-\frac{b c-a d}{d}\right)^{\frac{1}{3}}+\left(-\frac{b c-a d}{d}\right)^{\frac{2}{3}}\right)}{6 \, c^{2} d}+\frac{\left(4 \, a b c-3 \, a^{2} d\right) \log \left(\left|\left(b x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9 \, a^{\frac{2}{3}} c^{2}}-\frac{\left(b x^{3}+a\right)^{\frac{1}{3}} a}{3 \, c x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(4*a^
(1/3)*b*c - 3*a^(4/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))
/a^(1/3))/c^2 - 1/18*(4*a^(1/3)*b*c - 3*a^(4/3)*d)*log((b*x^3 + a)^(2/3) +
(b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c^2 + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(
1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)
^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c
 - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
(-(b*c - a*d)/d)^(2/3))/(c^2*d) + 1/9*(4*a*b*c - 3*a^2*d)*log(abs((b*x^3 +
a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)*a/(c*x^3)
```

Mupad [B]
time = 10.88, size = 2047, normalized size = 5.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x)

[Out] log(c^2*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3) + 3*a*d*(a + b*x^3)^(1/3) - 4*b*c*(a + b*x^3)^(1/3))*(-(27*a^4*d^3 - 64*a*b^3*c^3 + 144*a^2*b^2*c^2*d - 108*a^3*b*c*d^2)/(729*c^6))^(1/3) + log((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((a*d - b*c)^4/(c^6*d))^(1/3) - 108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2)*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4))*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^6*d))^(1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 - 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((a*d - b*c)^4/(c^6*d))^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4))*((3^(1/2)*1i)/2 - 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^6*d))^(1/3) - log((a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 + 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((a*d - b*c)^4/(c^6*d))^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 - (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3)*((3^(1/2)*1i)/2 + 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^6*d))^(1/3) + log((a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3))*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(2/3))/81 + (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3))/9)*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^4*d^3 - 64*a*b^3*c^3 + 144*a^2*b^2*c^2*d - 108*a^3*b*c*d^2)/(729*c^6))^(1/3) - log((a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*

```
(a*d - b*c)^2 + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^
2 - 3*a*b*c*d)*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3))*(-(a*(3*a*d - 4*b*c)^3)/
c^6)^(2/3))/81 - (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2
+ 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*(-(a*(3*
a*d - 4*b*c)^3)/c^6)^(1/3))/9)*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^4*d^3 - 64*a*
b^3*c^3 + 144*a^2*b^2*c^2*d - 108*a^3*b*c*d^2)/(729*c^6))^(1/3) - (a*(a + b
*x^3)^(1/3))/(3*c*x^3)
```

3.701 $\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=440

$$\frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3}+\frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3}-\frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3}-\frac{\left(a+bx^3\right)^{7/3}}{6acx^6}-\frac{(2b^2c^2}{}$$

```
[Out] d*(-a*d+b*c)*(b*x^3+a)^(1/3)/c^3+1/9*(9*a^2*d^2-12*a*b*c*d+2*b^2*c^2)*(b*x^
3+a)^(1/3)/a/c^3-1/18*(-6*a*d+b*c)*(b*x^3+a)^(4/3)/a/c^2/x^3-1/6*(b*x^3+a)^
(7/3)/a/c/x^6-1/18*(9*a^2*d^2-12*a*b*c*d+2*b^2*c^2)*ln(x)/a^(2/3)/c^3+1/6*d
^(2/3)*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^3+1/18*(9*a^2*d^2-12*a*b*c*d+2*b^2*c^
2)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)/c^3-1/2*d^(2/3)*(-a*d+b*c)^(4/3)*ln(
(-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^3-1/27*(9*a^2*d^2-12*a*b*c*d+2*
b^2*c^2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)/c^
3*3^(1/2)+1/3*d^(2/3)*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1
/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^3*3^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {457, 105, 154, 162, 52, 59, 631, 210, 31, 60}

$$\frac{\sqrt[3]{a+bx^3}\left(9a^2d^2-12abcd+2b^2c^2\right)}{9ac^3}-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)\left(9a^2d^2-12abcd+2b^2c^2\right)}{9\sqrt{3}a^{2/3}c^3}+\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3}-\frac{\log(x)\left(9a^2d^2-12abcd+2b^2c^2\right)}{18a^{2/3}c^3}+\frac{d^{2/3}(bc-ad)^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3}+\frac{d^{2/3}(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^3}-\frac{d^{2/3}(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}+\frac{d\sqrt[3]{a+bx^3}(bc-ad)}{c^3}-\frac{\left(a+bx^3\right)^{4/3}(bc-6ad)}{18ac^2x^3}-\frac{\left(a+bx^3\right)^{7/3}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] (d*(b*c - a*d)*(a + b*x^3)^(1/3))/c^3 + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^
2)*(a + b*x^3)^(1/3))/(9*a*c^3) - ((b*c - 6*a*d)*(a + b*x^3)^(4/3))/(18*a*c
^2*x^3) - (a + b*x^3)^(7/3)/(6*a*c*x^6) - ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*
d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*
a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3
)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c^3) - ((2*b^2*c^2 - 12*a*b*
c*d + 9*a^2*d^2)*Log[x])/(18*a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*Log[
c + d*x^3])/(6*c^3) + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*Log[a^(1/3) - (
a + b*x^3)^(1/3)])/(18*a^(2/3)*c^3) - (d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c -
 a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^3(c+dx)}\,dx,x,x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{4/3}\left(\frac{1}{3}(-bc+6ad)-\frac{bdx}{3}\right)}{x^2(c+dx)}\,dx,x,x^3\right)}{6ac}$$

$$= -\frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\,\left(-\frac{2}{9}\left(2b^2c^2-12abcd+9a^2d^2\right)-\frac{2}{9}bc\right.}{x(c+dx)}\right.}{6ac^2}$$

$$= -\frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/3}}{6acx^6} + \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx,x,x^3\right)}{3c^3}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3}$$

Mathematica [A]
time = 0.96, size = 413, normalized size = 0.94

$$\frac{\frac{3c\sqrt[3]{a+bx^3}\left(-3ac-7bcx^3+6adx^3\right)}{x^6} - \frac{2\sqrt{3}\left(2b^2c^2-12abcd+9a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + 18\sqrt{3}\,d^{2/3}(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + \frac{2\left(2b^2c^2-12abcd+9a^2d^2\right)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{2/3}} - 18d^{2/3}(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) + \frac{\left(-2b^2c^2+12abcd-9a^2d^2\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{a^{2/3}} + 9d^{2/3}(bc-ad)^{4/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{54c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((3*c*(a + b*x^3)^(1/3)*(-3*a*c - 7*b*c*x^3 + 6*a*d*x^3))/x^6 - (2*Sqrt[3]*
(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1
/3))/Sqrt[3]])/a^(2/3) + 18*Sqrt[3]*d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(1 - (
2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + (2*(2*b^2*c^2 -
```

```
12*a*b*c*d + 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(2/3) - 18*d^(
2/3)*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] +
 ((-2*b^2*c^2 + 12*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(
1/3) + (a + b*x^3)^(2/3)])/a^(2/3) + 9*d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c -
 a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*
x^3)^(2/3)])/(54*c^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{x^{7}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^7), x)
```

Fricas [A]
time = 8.40, size = 503, normalized size = 1.14

$$\frac{18\sqrt{3}\,(a^2bc-a^3d)(bcd^2-ad^3)^{\frac{1}{3}}x^6\arctan\left(-\frac{2\sqrt{3}\,(bcd^2-ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}-\sqrt{3}\,(bcd-ad^2)}{3\,(bcd-ad^2)}\right)-2\sqrt{3}\,(2\,ab^2c^2-12\,a^2bcd+9\,a^3d^2)(a^2)^{\frac{1}{6}}x^6\arctan\left(\frac{(a^2)^{\frac{1}{6}}\left(\sqrt{3}\,(a^2)^{\frac{1}{3}}a+2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)}{3\,a^2}\right)-(2\,b^2c^2-12\,abcd+9\,a^2d^2)(a^2)^{\frac{2}{3}}x^6\log\left((bx^3+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)+2\,(2\,b^2c^2-12\,abcd+9\,a^2d^2)(a^2)^{\frac{2}{3}}x^6\log\left((bx^3+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)+9\,(a^2bc-a^3d)(bcd^2-ad^3)^{\frac{1}{3}}x^6\log\left((bx^3+a)^{\frac{2}{3}}d^2-(bcd^2-ad^3)^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{2}{3}}\right)-18\,(a^2bc-a^3d)(bcd^2-ad^3)^{\frac{1}{3}}x^6\log\left((bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{1}{3}}\right)-3\,(3\,a^3c^2+(7\,a^2bc^2-6\,a^3cd)x^3)(bx^3+a)^{\frac{1}{3}}}{54\,a^2c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(18*sqrt(3)*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(1/3)*x^6*arctan(-1/3*
(2*sqrt(3)*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d
^2))/(b*c*d - a*d^2)) - 2*sqrt(3)*(2*a*b^2*c^2 - 12*a^2*b*c*d + 9*a^3*d^2)*
(a^2)^(1/6)*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) - (2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(
a^2)^(2/3)*x^6*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(a^2)^(2/3)) + 2*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((
b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 9*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(
1/3)*x^6*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1
/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 18*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(
```

```
1/3)*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*(3*a^3*c^2
+ (7*a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(1/3))/(a^2*c^3*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{4}{3}}}{x^7\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**7/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**7*(c + d*x**3)), x)
```

Giac [A]
time = 0.73, size = 481, normalized size = 1.09

$$\frac{(b^2c^2d-2\,abcd^2+a^2d^3)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)}-\frac{\sqrt{3}\,(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^3}-\frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^3}-\frac{\sqrt{3}\,(2\,b^2c^2-12\,abcd+9\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{2}{3}}c^3}-\frac{(2\,b^2c^2-12\,abcd+9\,a^2d^2)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{2}{3}}c^3}+\frac{(2\,b^2c^2-12\,abcd+9\,a^2d^2)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{2}{3}}c^3}-\frac{7\,(bx^3+a)^{\frac{4}{3}}b^2c-4\,(bx^3+a)^{\frac{1}{3}}ab^2c-6\,(bx^3+a)^{\frac{4}{3}}abd+6\,(bx^3+a)^{\frac{1}{3}}a^2bd}{18\,b^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x
^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) - 1/3*sqrt(3)*(-
b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3)
+ (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3 - 1/6*(-b*c*d^2 + a*d
^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*
d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(2*b^2*c^2 - 12*a*
b*c*d + 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/
3))/(a^(2/3)*c^3) - 1/54*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^3) + 1/27*(2*b^2
*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/
3)*c^3) - 1/18*(7*(b*x^3 + a)^(4/3)*b^2*c - 4*(b*x^3 + a)^(1/3)*a*b^2*c - 6
*(b*x^3 + a)^(4/3)*a*b*d + 6*(b*x^3 + a)^(1/3)*a^2*b*d)/(b^2*c^2*x^6)
```

Mupad [B]
time = 13.25, size = 2841, normalized size = 6.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x)
```

```
[Out] log(((((18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 9*a*
b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*
a*b*c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c
```

^9))^(2/3))/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))/27 - (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((729*a^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3) + log(((((18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d - b*c)^4)/c^9)^(2/3))/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 - (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3) + (((2*a*b^2*c - 3*a^2*b*d)*(a + b*x^3)^(1/3))/(9*c^2) + (b*(a + b*x^3)^(4/3)*(6*a*d - 7*b*c))/(18*c^2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d - b*c)^4)/c^9)^(2/3))/9 - (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) - 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d - b*c)^4)/c^9)^(2/3))/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 9*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(2/3))/729 - (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*

c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))
/27 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6
+ 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 17
64*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 - 1/2)*((729
*a^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^
4*b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3)
- log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(18*b^5*c^2*d^3*(a +
b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) - 9*a*b^4*c^4*d^3*((3^(1/2)*1i)/2
+ 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*
c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))
^(2/3))/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 69
39*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*
d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))/2
7 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 +
6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764
*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 + 1/2)*((729*a
^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*
b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3)

3.702 $\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=334

$$-\frac{(6bc-7ad)x^2\sqrt[3]{a+bx^3}}{18d^2}+\frac{bx^5\sqrt[3]{a+bx^3}}{6d}-\frac{(9b^2c^2-12abcd+2a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{2/3}d^3}+\frac{c^{2/3}(bc-a$$

```
[Out] -1/18*(-7*a*d+6*b*c)*x^2*(b*x^3+a)^(1/3)/d^2+1/6*b*x^5*(b*x^3+a)^(1/3)/d-1/
6*c^(2/3)*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/d^3-1/18*(2*a^2*d^2-12*a*b*c*d+9*b^2
*c^2)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)/d^3+1/2*c^(2/3)*(-a*d+b*c)^(4/3
)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/27*(2*a^2*d^2-12*a*b
*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)
/d^3*3^(1/2)+1/3*c^(2/3)*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*
x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {488, 596, 598, 337, 503}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(2a^2d^2-12abcd+9b^2c^2)}{9\sqrt{3}\,b^{2/3}d^3}-\frac{(2a^2d^2-12abcd+9b^2c^2)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{18b^{2/3}d^3}+\frac{c^{2/3}(bc-ad)^{4/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^3}-\frac{c^{2/3}(bc-ad)^{4/3}\log(c+dx^3)}{6d^3}+\frac{c^{2/3}(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3}-\frac{x^2\sqrt[3]{a+bx^3}(6bc-7ad)}{18d^2}+\frac{bx^5\sqrt[3]{a+bx^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -1/18*((6*b*c - 7*a*d)*x^2*(a + b*x^3)^(1/3))/d^2 + (b*x^5*(a + b*x^3)^(1/3
))/(6*d) - ((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(2/3)*d^3) + (c^(2/3)*(b*c - a*d)
^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqr
t[3]])/(Sqrt[3]*d^3) - (c^(2/3)*(b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*d^3) -
 ((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/
(18*b^(2/3)*d^3) + (c^(2/3)*(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(
1/3) - (a + b*x^3)^(1/3)])/(2*d^3)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x^4\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax^5\sqrt[3]{a+bx^3}\,F_1\left(\frac{5}{3};-\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 5.34, size = 526, normalized size = 1.57

$$\frac{6dx^2\sqrt[3]{a+bx^3}(-6bc+7ad+3bdx^3)-\frac{4\sqrt{3}(9b^2c^2-12abcd+2a^2d^2)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{2/3}}-18\sqrt{-6-6i\sqrt{3}}\,c^{2/3}(bc-ad)^{4/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)-\frac{4(9b^2c^2-12abcd+2a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+18i\left(i+\sqrt{3}\right)c^{2/3}(bc-ad)^{4/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+\frac{2(9b^2c^2-12abcd+2a^2d^2)\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{2/3}}+9\left(1-i\sqrt{3}\right)c^{2/3}(bc-ad)^{4/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{108d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (6*d*x^2*(a + b*x^3)^(1/3)*(-6*b*c + 7*a*d + 3*b*d*x^3) - (4*Sqrt[3]*(9*b^2
*c^2 - 12*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a
 + b*x^3)^(1/3))])/b^(2/3) - 18*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(2/3)*(b*c - a*d
)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I
+ Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (4*(9*b^2*c^2 - 12*a*b*c*d + 2*a^2
*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/3) + (18*I)*(I + Sqrt[3])
*c^(2/3)*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*Log[b^(2/3
)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3) + 9*(1 -
I*Sqrt[3])*c^(2/3)*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*
Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^
(2/3)*(a + b*x^3)^(2/3)])/(108*d^3)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4(b\,x^3+a)^{\frac{4}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

Fricas [A]
time = 6.16, size = 550, normalized size = 1.65

$$\frac{2\sqrt{3}\left(9b^3c^2-12ab^2cd+2a^2bd^2\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{3b^2x}\right)-18\sqrt{3}\left(b^3c-ab^2d\right)\left(-bc^3+ac^2d\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\left(bc^2-acd\right)x+2\sqrt{3}\left(-bc^3+ac^2d\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{3\left(bc^2-acd\right)x}\right)-2\left(9b^2c^2-12abcd+2a^2d^2\right)\left(-b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+\left(9b^2c^2-12abcd+2a^2d^2\right)\left(-b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{1}{3}}bx^2-\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)-18\left(b^3c-ab^2d\right)\left(-bc^3+ac^2d\right)^{\frac{1}{3}}\log\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}c+\left(-bc^3+ac^2d\right)^{\frac{1}{3}}x}{x}\right)+9\left(b^3c-ab^2d\right)\left(-bc^3+ac^2d\right)^{\frac{1}{3}}\log\left(\frac{\left(bx^3+a\right)^{\frac{2}{3}}c^2-\left(-bc^3+ac^2d\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}cx+\left(-bc^3+ac^2d\right)^{\frac{2}{3}}x^2}{x^2}\right)+3\left(3b^3d^2x^5-\left(6b^3cd-7ab^2d^2\right)x^2\right)\left(bx^3+a\right)^{\frac{1}{3}}}{54b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(2*sqrt(3)*(9*b^3*c^2 - 12*a*b^2*c*d + 2*a^2*b*d^2)*sqrt(-(-b^2)^(1/3)
)*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2
)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 18*sqrt(3)*(b^3*c - a*b^2*d)*(-b*c^
3 + a*c^2*d)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c
^3 + a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) - 2*(9*b^2*c^2
- 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(
1/3)*b)/x) + (9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)
^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2)
 - 18*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1/3)*log(((b*x^3 + a)^(1/3)*c +
 (-b*c^3 + a*c^2*d)^(1/3)*x)/x) + 9*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1
/3)*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)
*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c*d -
 7*a*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+bx^3\right)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(b x^3 + a\right)^{4/3}}{d x^3 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.703 $\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=277

$$\frac{bx^2\sqrt[3]{a+bx^3}}{3d}+\frac{\sqrt[3]{b}\,(3bc-4ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,d^2}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{c}\,d^2}+\frac{(bc-ad}{}$$

```
[Out] 1/3*b*x^2*(b*x^3+a)^(1/3)/d+1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(1/3)/d^2+1/
6*b^(1/3)*(-4*a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d+b*c)^(
4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(1/3)/d^2+1/9*b^(1/3)
*(-4*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(
1/2)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3
+a)^(1/3))*3^(1/2))/c^(1/3)/d^2*3^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {488, 598, 337, 503}

$$\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-4ad)}{3\sqrt{3}\,d^2}-\frac{(bc-ad)^{4/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{c}\,d^2}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6\sqrt[3]{c}\,d^2}+\frac{\sqrt[3]{b}\,(3bc-4ad)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{6d^2}-\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}\,d^2}+\frac{bx^2\sqrt[3]{a+bx^3}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (b*x^2*(a + b*x^3)^(1/3))/(3*d) + (b^(1/3)*(3*b*c - 4*a*d)*ArcTan[(1 + (2*b
^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*d^2) - ((b*c - a*d)^(4/3)
*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])
/(Sqrt[3]*c^(1/3)*d^2) + ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(1/3)*d^2)
 + (b^(1/3)*(3*b*c - 4*a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*d^2) - (
(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(
2*c^(1/3)*d^2)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
```

```
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} \\
&= \frac{ax^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.78, size = 469, normalized size = 1.69

$$\frac{12bdx^2\sqrt[3]{a+bx^3}+4\sqrt{3}\sqrt[3]{b}(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+\frac{6\sqrt{-6-6i\sqrt{3}}(bc-ad)^{4/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{c}}+4\sqrt[3]{b}(3bc-4ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)+\frac{6\left(1-i\sqrt{3}\right)(bc-ad)^{4/3}\log\left(2\sqrt[3]{bc-ad}x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{c}}-2\sqrt[3]{b}(3bc-4ad)\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)+\frac{3i\left(i+\sqrt{3}\right)(bc-ad)^{4/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{\sqrt[3]{c}}}{36d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (12*b*d*x^2*(a + b*x^3)^(1/3) + 4*Sqrt[3]*b^(1/3)*(3*b*c - 4*a*d)*ArcTan[(S
qrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + (6*Sqrt[-6 - (6*I)*S
```

```
qrt[3]]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*
d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/c^(1/3) + 4*b^(1/
3)*(3*b*c - 4*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (6*(1 - I*Sqrt[3
])*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a
 + b*x^3)^(1/3)])/c^(1/3) - 2*b^(1/3)*(3*b*c - 4*a*d)*Log[b^(2/3)*x^2 + b^(
1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((3*I)*(I + Sqrt[3])*(b*c -
 a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a
*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])
/c^(1/3))/(36*d^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

Fricas [A]
time = 3.85, size = 396, normalized size = 1.43

$$\frac{6\,(bx^3+a)^{\frac{1}{3}}bdx^2 - 6\sqrt{3}\,(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,(bc-ad)x+2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right) + 2\sqrt{3}\,(3\,bc-4\,ad)(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,bx+2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}}{3\,bx}\right) - 2\,(3\,bc-4\,ad)(-b)^{\frac{1}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right) - 6\,(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(-\frac{x\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}-(bx^3+a)^{\frac{1}{3}}}{x}\right) + (3\,bc-4\,ad)(-b)^{\frac{1}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right) + 3\,(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(\frac{x^2\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}x\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{18\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(1/3)*b*d*x^2 - 6*sqrt(3)*(b*c - a*d)*((b*c - a*d)/c)^(
1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b
*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b*c - 4*a*d)*(-b)^(1/3)
*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)^(2/3))/(b*x)) -
 2*(3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 6
*(b*c - a*d)*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 +
 a)^(1/3))/x) + (3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a
```

```
)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(b*c - a*d)*((b*c - a*d)
/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) + (b*x^3 + a)^(1/3)*x*((b*c - a*d)
/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(4/3))/(c + d*x^3),x)
[Out] int((x*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.704 $\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)} dx$

Optimal. Leaf size=254

$$-\frac{a\sqrt[3]{a+bx^3}}{cx}-\frac{b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d}+\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}d}-\frac{(bc-ad)^{4/3}\log(c+$$

```
[Out] -a*(b*x^3+a)^(1/3)/c/x-1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(4/3)/d-1/2*b^(4/
3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)
*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)/d-1/3*b^(4/3)*arctan(1/3*(1+2*b^(1/3)*x
/(b*x^3+a)^(1/3))*3^(1/2))/d*3^(1/2)+1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(
-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(4/3)/d*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {485, 598, 337, 503}

$$-\frac{b^{4/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}+\frac{(bc-ad)^{4/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}d}-\frac{b^{4/3}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{4/3}d}+\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}d}-\frac{a\sqrt[3]{a+bx^3}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3))/(c*x)) - (b^(4/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d) + ((b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c
- a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(4/3)*d)
- ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(4/3)*d) - (b^(4/3)*Log[b^(1/3)*x
 - (a + b*x^3)^(1/3)])/(2*d) + ((b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)
/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(4/3)*d)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
```

```
b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^2(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= -\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{1}{3};-\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.99, size = 457, normalized size = 1.80

$$\frac{-12a\sqrt[3]{c}\,d\sqrt[3]{a+bx^3}-4\sqrt{3}\,b^{4/3}c^{4/3}x\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)-2\sqrt{-6-6i\sqrt{3}}\,(bc-ad)^{4/3}x\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)-4b^{4/3}c^{4/3}x\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)+2i\left(i+\sqrt{3}\right)(bc-ad)^{4/3}x\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+2b^{4/3}c^{4/3}x\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)+\left(1-i\sqrt{3}\right)(bc-ad)^{4/3}x\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{12c^{4/3}dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] (-12*a*c^(1/3)*d*(a + b*x^3)^(1/3) - 4*Sqrt[3]*b^(4/3)*c^(4/3)*x*ArcTan[(Sq
rt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*Sqrt[-6 - (6*I)*Sqr
t[3]]*(b*c - a*d)^(4/3)*x*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*
```

```
d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - 4*b^(4/3)*c^(4/3
)*x*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (2*I)*(I + Sqrt[3])*(b*c - a*d)
^(4/3)*x*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3)] + 2*b^(4/3)*c^(4/3)*x*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) +
(a + b*x^3)^(2/3)] + (1 - I*Sqrt[3])*(b*c - a*d)^(4/3)*x*Log[2*(b*c - a*d)^
(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3)
+ I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(4/3)*d*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{x^2\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x)
[Out] int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{4}{3}}}{x^2\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**2/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**2*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{4/3}}{x^2\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)), x)
```

3.705 $\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx$

Optimal. Leaf size=201

$$-\frac{a\sqrt[3]{a+bx^3}}{4cx^4}-\frac{(5bc-4ad)\sqrt[3]{a+bx^3}}{4c^2x}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{7/3}}-$$

```
[Out] -1/4*a*(b*x^3+a)^(1/3)/c/x^4-1/4*(-4*a*d+5*b*c)*(b*x^3+a)^(1/3)/c^2/x+1/6*(
-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(7/3)-1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3
)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(
-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {485, 597, 12, 503}

$$-\frac{(bc-ad)^{4/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{7/3}}-\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}-\frac{\sqrt[3]{a+bx^3}\,(5bc-4ad)}{4c^2x}-\frac{a\sqrt[3]{a+bx^3}}{4cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -1/4*(a*(a + b*x^3)^(1/3))/(c*x^4) - ((5*b*c - 4*a*d)*(a + b*x^3)^(1/3))/(4
*c^2*x) - ((b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(
a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(7/3)) + ((b*c - a*d)^(4/3)*Log[c +
 d*x^3])/(6*c^(7/3)) - ((b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
 - (a + b*x^3)^(1/3)])/(2*c^(7/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^5(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{a\sqrt[3]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{4/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{4cx^4\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.69, size = 328, normalized size = 1.63

$$\frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(-ac-5bcx^3+4adx^3)}{x^4}+2\sqrt{-6-6i\sqrt{3}}(bc-ad)^{4/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+2\left(1-i\sqrt{3}\right)(bc-ad)^{4/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+i\left(i+\sqrt{3}\right)(bc-ad)^{4/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{12c^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] ((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) - 5*b*c*x^3 + 4*a*d*x^3))/x^4 + 2*Sqr
t[-6 - (6*I)*Sqrt[3]]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqr
```

```
t[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 2*
(1 - I*Sqrt[3])*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3
])*c^(1/3)*(a + b*x^3)^(1/3)] + I*(I + Sqrt[3])*(b*c - a*d)^(4/3)*Log[2*(b*
c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(12*c^(7/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{3}+a)^{\frac{4}{3}}}{x^{5}\,(d\,x^{3}+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x)
[Out] int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^{3})^{\frac{4}{3}}}{x^{5}\,(c+dx^{3})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**5/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**5*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^3+a)^{4/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)), x)
```

3.706 $\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx$

Optimal. Leaf size=250

$$-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(8bc-7ad)\sqrt[3]{a+bx^3}}{28c^2x^4}-\frac{(4b^2c^2-35abcd+28a^2d^2)\sqrt[3]{a+bx^3}}{28ac^3x}+\frac{d(bc-ad)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}}{\sqrt[3]{c}}}{}\right)}{\sqrt{3}\,c^{10/3}}$$

```
[Out] -1/7*a*(b*x^3+a)^(1/3)/c/x^7-1/28*(-7*a*d+8*b*c)*(b*x^3+a)^(1/3)/c^2/x^4-1/
28*(28*a^2*d^2-35*a*b*c*d+4*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x-1/6*d*(-a*d+b*
c)^(4/3)*ln(d*x^3+c)/c^(10/3)+1/2*d*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/
c^(1/3)-(b*x^3+a)^(1/3))/c^(10/3)+1/3*d*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-
a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$,
Rules used = {485, 597, 12, 503}

$$-\frac{\sqrt[3]{a+bx^3}\,(28a^2d^2-35abcd+4b^2c^2)}{28ac^3x}+\frac{d(bc-ad)^{4/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{10/3}}-\frac{d(bc-ad)^{4/3}\log(c+dx^3)}{6c^{10/3}}+\frac{d(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}-\frac{\sqrt[3]{a+bx^3}\,(8bc-7ad)}{28c^2x^4}-\frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -1/7*(a*(a + b*x^3)^(1/3))/(c*x^7) - ((8*b*c - 7*a*d)*(a + b*x^3)^(1/3))/(2
8*c^2*x^4) - ((4*b^2*c^2 - 35*a*b*c*d + 28*a^2*d^2)*(a + b*x^3)^(1/3))/(28*
a*c^3*x) + (d*(b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(10/3)) - (d*(b*c - a*d)^(4/3)*L
og[c + d*x^3])/(6*c^(10/3)) + (d*(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x
)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(10/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
```

```
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^8(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{12c(bc-ad)x^3(a+bx^3)(c+dx^3)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c(a+bx^3)}\right)-(4c-3dx^3)\left(c(a+bx^3)\right.\cdots}{28c^4x^7(a+bx^3)^{2/3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.87, size = 369, normalized size = 1.48

$$\frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(4b^2c^2x^6+abcx^3(8c-35dx^3)+a^2(4c^2-7cdx^3+28d^2x^6)\right)}{ax^7}-14\sqrt{-6-6i\sqrt{3}}\,d(bc-ad)^{4/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+14i\left(i+\sqrt{3}\right)d(bc-ad)^{4/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+7\left(1-i\sqrt{3}\right)d(bc-ad)^{4/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{84c^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] ((-3*c^(1/3)*(a + b*x^3)^(1/3)*(4*b^2*c^2*x^6 + a*b*c*x^3*(8*c - 35*d*x^3)
+ a^2*(4*c^2 - 7*c*d*x^3 + 28*d^2*x^6)))/(a*x^7) - 14*Sqrt[-6 - (6*I)*Sqrt[
```

```
3]]*d*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)
^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (14*I)*(I + Sqrt[3
])*d*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*
(a + b*x^3)^(1/3)] + 7*(1 - I*Sqrt[3])*d*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d
)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3
) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(84*c^(10/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{3}+a)^{\frac{4}{3}}}{x^{8}\,(d\,x^{3}+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x)
[Out] int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^{3})^{\frac{4}{3}}}{x^{8}\,(c+dx^{3})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**8/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**8*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{x^8\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)), x)
```

3.707 $\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx$

Optimal. Leaf size=318

$$-\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(11bc-10ad)\sqrt[3]{a+bx^3}}{70c^2x^7}-\frac{(2b^2c^2-40abcd+35a^2d^2)\sqrt[3]{a+bx^3}}{140ac^3x^4}+\frac{(6b^3c^3+20ab^2c^2d-175a^2\cdots}{140a\cdots}$$

```
[Out] -1/10*a*(b*x^3+a)^(1/3)/c/x^10-1/70*(-10*a*d+11*b*c)*(b*x^3+a)^(1/3)/c^2/x^
7-1/140*(35*a^2*d^2-40*a*b*c*d+2*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x^4+1/140*(
140*a^3*d^3-175*a^2*b*c*d^2+20*a*b^2*c^2*d+6*b^3*c^3)*(b*x^3+a)^(1/3)/a^2/c
^4/x+1/6*d^2*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(13/3)-1/2*d^2*(-a*d+b*c)^(4/3)
*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(13/3)-1/3*d^2*(-a*d+b*c)
^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))
/c^(13/3)*3^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {485, 597, 12, 503}

$$-\frac{\sqrt[3]{a+bx^3}\,(35a^2d^2-40abcd+2b^2c^2)}{140ac^3x^4}+\frac{\sqrt[3]{a+bx^3}\,(140a^3d^3-175a^2bcd^2+20ab^2c^2d+6b^3c^3)}{140a^2c^4x}-\frac{d^2(bc-ad)^{4/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{13/3}}+\frac{d^2(bc-ad)^{4/3}\log(c+dx^3)}{6c^{13/3}}-\frac{d^2(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{13/3}}-\frac{\sqrt[3]{a+bx^3}\,(11bc-10ad)}{70c^2x^7}-\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] -1/10*(a*(a + b*x^3)^(1/3))/(c*x^10) - ((11*b*c - 10*a*d)*(a + b*x^3)^(1/3)
)/(70*c^2*x^7) - ((2*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*(a + b*x^3)^(1/3))/
(140*a*c^3*x^4) + ((6*b^3*c^3 + 20*a*b^2*c^2*d - 175*a^2*b*c*d^2 + 140*a^3*
d^3)*(a + b*x^3)^(1/3))/(140*a^2*c^4*x) - (d^2*(b*c - a*d)^(4/3)*ArcTan[(1
+ (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c
^(13/3)) + (d^2*(b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(13/3)) - (d^2*(b*c
- a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(
13/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^{11}(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{6c(bc-ad)x^3(a+bx^3)\left(11c^2+2cdx^3-9d^2x^6\right)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c(a+bx^3)}\right)-(14c^2-12c$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.08, size = 419, normalized size = 1.32

$$\frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(6b^3c^3x^9-2ab^2c^2x^6(c-10dx^3)+a^2bcx^3\left(-22c^2+40cdx^3-175d^2x^6\right)+a^3\left(-14c^3+20c^2dx^3-35cd^2x^6+140d^3x^9\right)\right)}{a^2x^{10}}+70\sqrt{-6-6i\sqrt{3}}\,d^2(bc-ad)^{4/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+70\left(1-i\sqrt{3}\right)d^2(bc-ad)^{4/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+35i\left(i+\sqrt{3}\right)d^2(bc-ad)^{4/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{420c^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] ((3*c^(1/3)*(a + b*x^3)^(1/3)*(6*b^3*c^3*x^9 - 2*a*b^2*c^2*x^6*(c - 10*d*x^
3) + a^2*b*c*x^3*(-22*c^2 + 40*c*d*x^3 - 175*d^2*x^6) + a^3*(-14*c^3 + 20*c
^2*d*x^3 - 35*c*d^2*x^6 + 140*d^3*x^9)))/(a^2*x^10) + 70*Sqrt[-6 - (6*I)*Sq
rt[3]]*d^2*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 70*(1 - I*Sqr
t[3])*d^2*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(
1/3)*(a + b*x^3)^(1/3)] + (35*I)*(I + Sqrt[3])*d^2*(b*c - a*d)^(4/3)*Log[2*
(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b
*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(420*c^(13/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{x^{11}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}\right)^{\frac{4}{3}}}{x^{11}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**11/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**11*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{4/3}}{x^{11}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)), x)
```

3.708 $\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx$

Optimal. Leaf size=392

$$-\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}-\frac{(14bc-13ad)\sqrt[3]{a+bx^3}}{130c^2x^{10}}-\frac{(4b^2c^2-143abcd+130a^2d^2)\sqrt[3]{a+bx^3}}{910ac^3x^7}+\frac{(12b^3c^3+26ab^2c^2d-52\ldots}{18\ldots}$$

```
[Out] -1/13*a*(b*x^3+a)^(1/3)/c/x^13-1/130*(-13*a*d+14*b*c)*(b*x^3+a)^(1/3)/c^2/x
^10-1/910*(130*a^2*d^2-143*a*b*c*d+4*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x^7+1/1
820*(455*a^3*d^3-520*a^2*b*c*d^2+26*a*b^2*c^2*d+12*b^3*c^3)*(b*x^3+a)^(1/3)
/a^2/c^4/x^4-1/1820*(1820*a^4*d^4-2275*a^3*b*c*d^3+260*a^2*b^2*c^2*d^2+78*a
*b^3*c^3*d+36*b^4*c^4)*(b*x^3+a)^(1/3)/a^3/c^5/x-1/6*d^3*(-a*d+b*c)^(4/3)*l
n(d*x^3+c)/c^(16/3)+1/2*d^3*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(16/3)+1/3*d^3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*
c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(16/3)*3^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {485, 597, 12, 503}

$$-\frac{\sqrt[3]{a+bx^3}(130a^2d^2-143abcd+4b^2c^2)}{910ac^3x^7}+\frac{\sqrt[3]{a+bx^3}(455a^3d^3-520a^2bcd^2+26ab^2c^2d+12b^3c^3)}{1820a^2c^4x^4}-\frac{\sqrt[3]{a+bx^3}(1820a^4d^4-2275a^3bcd^3+260a^2b^2c^2d^2+78ab^3c^3d+36b^4c^4)}{1820a^3c^5x}+\frac{d^3(bc-ad)^{4/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{16/3}}-\frac{d^3(bc-ad)^{4/3}\log(c+dx^3)}{6c^{16/3}}+\frac{d^3(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{16/3}}-\frac{\sqrt[3]{a+bx^3}(14bc-13ad)}{130c^2x^{10}}-\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

```
[Out] -1/13*(a*(a + b*x^3)^(1/3))/(c*x^13) - ((14*b*c - 13*a*d)*(a + b*x^3)^(1/3)
)/(130*c^2*x^10) - ((4*b^2*c^2 - 143*a*b*c*d + 130*a^2*d^2)*(a + b*x^3)^(1/
3))/(910*a*c^3*x^7) + ((12*b^3*c^3 + 26*a*b^2*c^2*d - 520*a^2*b*c*d^2 + 455
*a^3*d^3)*(a + b*x^3)^(1/3))/(1820*a^2*c^4*x^4) - ((36*b^4*c^4 + 78*a*b^3*c
^3*d + 260*a^2*b^2*c^2*d^2 - 2275*a^3*b*c*d^3 + 1820*a^4*d^4)*(a + b*x^3)^(
1/3))/(1820*a^3*c^5*x) + (d^3*(b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(16/3)) - (d^3*(
b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(16/3)) + (d^3*(b*c - a*d)^(4/3)*Log[
((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(16/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^{14}(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{140a^2c^5 + 840abc^5x^3 - 686a^2c^4dx^3 + 700b^2c^5x^6 - 1316abc^4dx^6 + 612a^2c^3d^2x^6 - 63}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.64, size = 478, normalized size = 1.22

$$\frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(36b^4c^4x^{12}+6ab^3c^3x^9(-2c+13dx^3)+2a^2b^2c^2x^6(4c^2-13cdx^3+130d^2x^6)+a^3bcx^3(196c^3-286c^2dx^3+520cd^2x^6-2275d^3x^9)+a^4(140c^4-182c^3dx^3+260c^2d^2x^6-455cd^3x^9+1820d^4x^{12})\right)}{a^3x^{13}}-910\sqrt{-6-6i\sqrt{3}}\,d^3(bc-ad)^{4/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+910i\left(i+\sqrt{3}\right)d^3(bc-ad)^{4/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+455\left(1-i\sqrt{3}\right)d^3(bc-ad)^{4/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{5460c^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

```
[Out] ((-3*c^(1/3)*(a + b*x^3)^(1/3)*(36*b^4*c^4*x^12 + 6*a*b^3*c^3*x^9*(-2*c + 1
3*d*x^3) + 2*a^2*b^2*c^2*x^6*(4*c^2 - 13*c*d*x^3 + 130*d^2*x^6) + a^3*b*c*x
^3*(196*c^3 - 286*c^2*d*x^3 + 520*c*d^2*x^6 - 2275*d^3*x^9) + a^4*(140*c^4
- 182*c^3*d*x^3 + 260*c^2*d^2*x^6 - 455*c*d^3*x^9 + 1820*d^4*x^12)))/(a^3*x
^13) - 910*Sqrt[-6 - (6*I)*Sqrt[3]]*d^3*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c -
a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b
*x^3)^(1/3))] + (910*I)*(I + Sqrt[3])*d^3*(b*c - a*d)^(4/3)*Log[2*(b*c - a*
d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 455*(1 - I*Sqrt[3
])*d^3*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(
1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b
*x^3)^(2/3)])/(5460*c^(16/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{x^{14}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**14/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{x^{14}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)), x)
```

3.709 $\int \frac{x^6 \left(a+bx^3\right)^{4/3}}{c+dx^3} dx$

Optimal. Leaf size=65

$$\frac{ax^7 \sqrt[3]{a+bx^3}\, F_1\left(\frac{7}{3};-\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/7*a*x^7*(b*x^3+a)^(1/3)*AppellF1(7/3,-4/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(1+
b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{ax^7 \sqrt[3]{a+bx^3}\, F_1\left(\frac{7}{3};-\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -4/3, 1, 10/3, -((b*x^3)/a), -((d*x^
3)/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x^6\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax^7\sqrt[3]{a+bx^3}\,F_1\left(\frac{7}{3};-\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(65) = 130.
time = 8.25, size = 343, normalized size = 5.28

$$\frac{x\left(2(a+bx^3)\left(2a^2d^2+3abd(-8c+3dx^3)+b^2(20c^2-8cdx^3+5d^2x^6)\right)-\frac{(20b^3c^3-30ab^2c^2d+8a^2bcd^2+a^3d^3)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{16a^2c^2(10b^2c^2-12abcd+a^2d^2)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)}{80bd^3(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (x*(2*(a + b*x^3)*(2*a^2*d^2 + 3*a*b*d*(-8*c + 3*d*x^3) + b^2*(20*c^2 - 8*c
*d*x^3 + 5*d^2*x^6)) - ((20*b^3*c^3 - 30*a*b^2*c^2*d + 8*a^2*b*c*d^2 + a^3*
d^3)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((
d*x^3)/c)])/c + (16*a^2*c^2*(10*b^2*c^2 - 12*a*b*c*d + a^2*d^2)*AppellF1[1/
3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])))))/(80*b*d^3*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^6(b\,x^3+a)^{\frac{4}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6\,{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] int((x^6*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.710 $\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=65

$$\frac{ax^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{4}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/4*a*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-4/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(1+b
*x^3/a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{ax^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{4}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -4/3, 1, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x^3\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{4}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(65) = 130.
time = 7.70, size = 280, normalized size = 4.31

$$\frac{x\left(4(a+bx^3)(-5bc+6ad+2bdx^3)+\frac{(10b^2c^2-15abcd+4a^2d^2)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{16a^2c^2(-5bc+6ad)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)}{40d^2(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (x*(4*(a + b*x^3)*(-5*b*c + 6*a*d + 2*b*d*x^3) + ((10*b^2*c^2 - 15*a*b*c*d
+ 4*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)
/a), -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + 6*a*d)*AppellF1[1/3, 2/3, 1,
4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1
, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3,
-((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a)
, -((d*x^3)/c)])))))/(40*d^2*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^3(b\,x^3+a)^{\frac{4}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**3*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] int((x^3*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.711 $\int \frac{(a+bx^3)^{4/3}}{c+dx^3} dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}\, F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a
)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{ax\sqrt[3]{a+bx^3}\, F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = \frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 346 vs. 2(60) = 120.

time = 10.22, size = 346, normalized size = 5.77

$$\frac{x\left(\frac{b(-2bc+3ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(-4ac(2a^2d+abdx^3+b^2x^3(c+dx^3))F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bx^3(a+bx^3)(c+dx^3)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)}{8d\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*x^
3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x
^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(8*d*(a
+ b*x^3)^(2/3))
```

Maple [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

3.712 $\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx$

Optimal. Leaf size=65

$$-\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{4}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -1/2*a*(b*x^3+a)^(1/3)*AppellF1(-2/3,-4/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(1
+b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{4}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -1/2*(a*(a + b*x^3)^(1/3)*AppellF1[-2/3, -4/3, 1, 1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{x^3\,(c+dx^3)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^3(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{4}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(65) = 130.
time = 10.24, size = 341, normalized size = 5.25

$$-\frac{b(-2bc+ad)x^6\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4ac\left(-4ac(ac-2bcx^3+3adx^3+bdx^6)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3(a+bx^3)(c+dx^3)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}}{8c^2x^2\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -1/8*(b*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)] + (4*a*c*(-4*a*c*(a*c - 2*b*c*x^3 + 3*a*d*x^3
+ b*d*x^6)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(a
+ b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x
^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/(
(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]
+ x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c
*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c^2*x^2*(a + b
*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{x^3\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{4}{3}}}{x^3\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**3/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**3*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{4/3}}{x^3\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(x^3*(c + d*x^3)), x)
```

3.713 $\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx$

Optimal. Leaf size=65

$$-\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{4}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -1/5*a*(b*x^3+a)^(1/3)*AppellF1(-5/3,-4/3,1,-2/3,-b*x^3/a,-d*x^3/c)/c/x^5/(
1+b*x^3/a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{4}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -1/5*(a*(a + b*x^3)^(1/3)*AppellF1[-5/3, -4/3, 1, -2/3, -((b*x^3)/a), -((d*
x^3)/c)])/(c*x^5*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{x^6\,(c+dx^3)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^6(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{4}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 286 vs. 2(65) = 130.
time = 10.26, size = 286, normalized size = 4.40

$$\frac{-\frac{4(a+bx^3)(2ac+6bcx^3-5adx^3)}{c^2x^5}+\frac{bd(-6bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}-\frac{16a(4b^2c^2-15abcd+10a^2d^2)xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}}{40\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] ((-4*(a + b*x^3)*(2*a*c + 6*b*c*x^3 - 5*a*d*x^3))/(c^2*x^5) + (b*d*(-6*b*c
+ 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/c^3 - (16*a*(4*b^2*c^2 - 15*a*b*c*d + 10*a^2*d^2)*x*AppellF
1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c*App
ellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4
/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{x^6\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^{\frac{4}{3}}}{x^6\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**6/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**6*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{4/3}}{x^6\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(x^6*(c + d*x^3)), x)
```

3.714 $\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=290

$$-\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4}+\frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3}-\frac{(bc+3ad)\left(a+bx^3\right)^{8/3}}{8b^4d^2}+\frac{(a+
}{11}$$

```
[Out] -1/2*(a*d+b*c)*(a^2*d^2+b^2*c^2)*(b*x^3+a)^(2/3)/b^4/d^4+1/5*(3*a^2*d^2+2*a
*b*c*d+b^2*c^2)*(b*x^3+a)^(5/3)/b^4/d^3-1/8*(3*a*d+b*c)*(b*x^3+a)^(8/3)/b^4
/d^2+1/11*(b*x^3+a)^(11/3)/b^4/d+1/6*c^4*ln(d*x^3+c)/d^(14/3)/(-a*d+b*c)^(1
/3)-1/2*c^4*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(14/3)/(-a*d+b*c
)^(1/3)-1/3*c^4*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3
^(1/2))/d^(14/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 90, 58, 631, 210, 31}

$$-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)\left(a^2d^2+b^2c^2\right)}{2b^4d^4}+\frac{\left(a+bx^3\right)^{5/3}\left(3a^2d^2+2abcd+b^2c^2\right)}{5b^4d^3}-\frac{c^4\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{14/3}\sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{8/3}(3ad+bc)}{8b^4d^2}+\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{c^4\log\left(c+dx^3\right)}{6d^{14/3}\sqrt[3]{bc-ad}}-\frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*((b*c + a*d)*(b^2*c^2 + a^2*d^2)*(a + b*x^3)^(2/3))/(b^4*d^4) + ((b^2*
c^2 + 2*a*b*c*d + 3*a^2*d^2)*(a + b*x^3)^(5/3))/(5*b^4*d^3) - ((b*c + 3*a*d
)*(a + b*x^3)^(8/3))/(8*b^4*d^2) + (a + b*x^3)^(11/3)/(11*b^4*d) - (c^4*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3
]*d^(14/3)*(b*c - a*d)^(1/3)) + (c^4*Log[c + d*x^3])/(6*d^(14/3)*(b*c - a*d
)^(1/3)) - (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
4/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(bc+ad)\left(-b^2c^2-a^2d^2\right)}{b^3d^4\sqrt[3]{a+bx}} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)(a+bx)^{2/3}}{b^3d^3} -\right.\right.$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} -$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} -$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} -$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} -$$

Mathematica [A]
time = 0.65, size = 307, normalized size = 1.06

$$\frac{-3d^{2/3}\sqrt[3]{bc-ad}\left(a+bx^3\right)^{2/3}\left(81a^3d^3+9a^2bd^2\left(11c-6dx^3\right)+3ab^2d\left(44c^2-22cdx^3+15d^2x^6\right)+b^3\left(220c^3-88c^2dx^3+55cd^2x^6-40d^3x^9\right)\right)-440\sqrt{3}\,b^4c^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-440b^4c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+220b^4c^4\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{1320b^4d^{14/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(81*a^3*d^3 + 9*a^2*b*d^2*(
11*c - 6*d*x^3) + 3*a*b^2*d*(44*c^2 - 22*c*d*x^3 + 15*d^2*x^6) + b^3*(220*c
^3 - 88*c^2*d*x^3 + 55*c*d^2*x^6 - 40*d^3*x^9)) - 440*Sqrt[3]*b^4*c^4*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 440*b^4*
c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 220*b^4*c^4*Log[(b
*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
+ b*x^3)^(2/3)])/(1320*b^4*d^(14/3)*(b*c - a*d)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 4.02, size = 1004, normalized size = 3.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 + (
-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 4
40*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a
*d^3)^(1/3)) + 660*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt((-b*c*d^2 + a
*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(
2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a
*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)
/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c))
- 3*(220*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d
^5 - 81*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*
b^3*c*d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a
^2*b^2*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6 - a*b^4*d^7
), 1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 +
 (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
 440*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 +
 a*d^3)^(1/3)) + 1320*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt(-(-b*c*d^2
 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-b*
c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(2
20*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d^5 - 8
```

```
1*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*b^3*c*
d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a^2*b^2
*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6 - a*b^4*d^7)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**14/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.62, size = 454, normalized size = 1.57

$$-\frac{b^{48}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{49}cd^{11}-ab^{48}d^{12})}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^4\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^6-\sqrt{3}\,ad^7}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^4\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^6-ad^7)}-\frac{220\,(bx^3+a)^{\frac{2}{3}}b^{43}c^3d^7-88\,(bx^3+a)^{\frac{5}{3}}b^{42}c^2d^8+220\,(bx^3+a)^{\frac{2}{3}}ab^{42}c^2d^8+55\,(bx^3+a)^{\frac{8}{3}}b^{41}cd^9-176\,(bx^3+a)^{\frac{5}{3}}ab^{41}cd^9+220\,(bx^3+a)^{\frac{2}{3}}a^2b^{41}cd^9-40\,(bx^3+a)^{\frac{11}{3}}b^{40}d^{10}+165\,(bx^3+a)^{\frac{8}{3}}ab^{40}d^{10}-264\,(bx^3+a)^{\frac{5}{3}}a^2b^{40}d^{10}+220\,(bx^3+a)^{\frac{2}{3}}a^3b^{40}d^{10}}{440\,b^{44}d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^48*c^4*d^7*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^49*c*d^11 - a*b^48*d^12) - (-b*c*d^2 + a*d^3)^(2/3)*c
^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
 - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^6 - sqrt(3)*a*d^7) + 1/6*(-b*c*d^2 + a*d^3
)^(2/3)*c^4*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3
) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^6 - a*d^7) - 1/440*(220*(b*x^3 + a)^(2/3
)*b^43*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^42*c^2*d^8 + 220*(b*x^3 + a)^(2/3)*
a*b^42*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^41*c*d^9 - 176*(b*x^3 + a)^(5/3)*a*
b^41*c*d^9 + 220*(b*x^3 + a)^(2/3)*a^2*b^41*c*d^9 - 40*(b*x^3 + a)^(11/3)*b
^40*d^10 + 165*(b*x^3 + a)^(8/3)*a*b^40*d^10 - 264*(b*x^3 + a)^(5/3)*a^2*b^
40*d^10 + 220*(b*x^3 + a)^(2/3)*a^3*b^40*d^10)/(b^44*d^11)
```

Mupad [B]
time = 5.11, size = 438, normalized size = 1.51

$$\left(\frac{6\,a^2}{5\,b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{5\,b^4\,d}\right)\,{\left(b\,x^3+a\right)}^{5/3}-\left(\frac{a}{2\,b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{8\,b^8\,d^2}\right)\,{\left(b\,x^3+a\right)}^{8/3}-{\left(b\,x^3+a\right)}^{2/3}\,\left(\frac{2\,a^3}{b^4\,d}+\frac{\left(\frac{6\,a^2}{b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{b^4\,d}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{2\,b^4\,d}\right)+\frac{{\left(b\,x^3+a\right)}^{11/3}}{11\,b^4\,d}+\frac{c^4\,\ln\left(\frac{c^8\,{\left(b\,x^3+a\right)}^{1/3}}{d^7}-\frac{c^8\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{22/3}}\right)}{3\,d^{14/3}\,{\left(a\,d-b\,c\right)}^{1/3}}-\frac{\ln\left(\frac{c^8\,{\left(b\,x^3+a\right)}^{1/3}}{d^7}-\frac{c^8\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{1/3}}{4\,d^{22/3}}\right)\,\left(c^4+\sqrt{3}\,c^4\,1{}\mathrm{i}\right)}{6\,d^{14/3}\,{\left(a\,d-b\,c\right)}^{1/3}}+\frac{c^4\,\ln\left(\frac{c^8\,{\left(b\,x^3+a\right)}^{1/3}}{d^7}-\frac{c^8\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{1/3}}{4\,d^{22/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{d^{14/3}\,{\left(a\,d-b\,c\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] ((6*a^2)/(5*b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c
- a*b^4*d))/(5*b^4*d))*(a + b*x^3)^(5/3) - (a/(2*b^4*d) + (b^5*c - a*b^4*d)
/(8*b^8*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*((2*a^3)/(b^4*d) + (((6
```

```
*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c - a*b
^4*d))/(b^4*d))*(b^5*c - a*b^4*d))/(2*b^4*d)) + (a + b*x^3)^(11/3)/(11*b^4*
d) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^8*(a*d - b*c)^(1/3))/d^(22/3
)))/(3*d^(14/3)*(a*d - b*c)^(1/3)) - (log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^
8*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*(3^(1/2)*c^4*1i + c^4
))/(6*d^(14/3)*(a*d - b*c)^(1/3)) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 -
(c^8*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*((3^(1/2)*1i)/6 -
1/6))/(d^(14/3)*(a*d - b*c)^(1/3))
```

3.715 $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=244

$$\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2b^3d^3}-\frac{(bc+2ad)\,(a+bx^3)^{5/3}}{5b^3d^2}+\frac{(a+bx^3)^{8/3}}{8b^3d}+\frac{c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{11/3}\sqrt[3]{bc-ad}}-$$

```
[Out] 1/2*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(2/3)/b^3/d^3-1/5*(2*a*d+b*c)*(b*x^
3+a)^(5/3)/b^3/d^2+1/8*(b*x^3+a)^(8/3)/b^3/d-1/6*c^3*ln(d*x^3+c)/d^(11/3)/(
-a*d+b*c)^(1/3)+1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/
3)/(-a*d+b*c)^(1/3)+1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b
*c)^(1/3))*3^(1/2))/d^(11/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {457, 90, 58, 631, 210, 31}

$$\frac{(a+bx^3)^{2/3}(a^2d^2+abcd+b^2c^2)}{2b^3d^3}+\frac{c^3\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{11/3}\sqrt[3]{bc-ad}}-\frac{(a+bx^3)^{5/3}(2ad+bc)}{5b^3d^2}+\frac{(a+bx^3)^{8/3}}{8b^3d}-\frac{c^3\log(c+dx^3)}{6d^{11/3}\sqrt[3]{bc-ad}}+\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^3*d^3) - ((b*c + 2*a
*d)*(a + b*x^3)^(5/3))/(5*b^3*d^2) + (a + b*x^3)^(8/3)/(8*b^3*d) + (c^3*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3
]*d^(11/3)*(b*c - a*d)^(1/3)) - (c^3*Log[c + d*x^3])/(6*d^(11/3)*(b*c - a*d
)^(1/3)) + (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
1/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2c^2+abcd+a^2d^2}{b^2d^3\sqrt[3]{a+bx}} + \frac{(-bc-2ad)(a+bx)^{2/3}}{b^2d^2} + \frac{(a+bx)^{5/3}}{b^2d} - \right.\right.$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\,(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} -$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\,(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} -$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\,(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} -$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\,(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} +$$

Mathematica [A]
time = 0.42, size = 263, normalized size = 1.08

$$\frac{3d^{2/3}\sqrt[3]{bc-ad}\,(a+bx^3)^{2/3}\,(9a^2d^2-6abd(-2c+dx^3)+b^2(20c^2-8cdx^3+5d^2x^6))+40\sqrt{3}\,b^3c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+40b^3c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)-20b^3c^3\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\,\sqrt[3]{bc-ad}\,\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{120b^3d^{11/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(9*a^2*d^2 - 6*a*b*d*(-2*c +
 d*x^3) + b^2*(20*c^2 - 8*c*d*x^3 + 5*d^2*x^6)) + 40*Sqrt[3]*b^3*c^3*ArcTan
[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 40*b^3*c^
3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 20*b^3*c^3*Log[(b*c
- a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b
*x^3)^(2/3)])/(120*b^3*d^(11/3)*(b*c - a*d)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(b\,x^3+a)^{\frac{1}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.34, size = 873, normalized size = 3.58

$$\left[-\frac{20\,(bcd^2-ad^3)^{\frac{2}{3}}b^3c^3\log\left((bx^3+a)^{\frac{2}{3}}d^2-(bcd^2-ad^3)^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{2}{3}}\right)-40\,(bcd^2-ad^3)^{\frac{2}{3}}b^3c^3\log\left((bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{1}{3}}\right)-60\sqrt{\frac{1}{3}}\,(b^4c^4d-ab^3c^3d^2)\sqrt{-\frac{(bcd^2-ad^3)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2bd^2x^3-bcd+3ad^2-3\sqrt{\frac{1}{3}}\left(2\,(bcd^2-ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}(bcd-ad^2)-(bcd^2-ad^3)^{\frac{1}{3}}(bc-ad)\right)\sqrt{-\frac{(bcd^2-ad^3)^{\frac{1}{3}}}{bc-ad}}-3\,(bcd^2-ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\right)-3\,(20\,b^3c^3d^2-8\,ab^2c^2d^3-3\,a^2bcd^4-9\,a^3d^5+5\,(b^3cd^4-ab^2d^5)x^6-2\,(4\,b^3c^2d^3-ab^2cd^4-3\,a^2bd^5)x^3)(bx^3+a)^{\frac{2}{3}}}{120\,(b^4cd^5-ab^3d^6)},\ -\frac{20\,(bcd^2-ad^3)^{\frac{2}{3}}b^3c^3\log\left((bx^3+a)^{\frac{2}{3}}d^2-(bcd^2-ad^3)^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{2}{3}}\right)-40\,(bcd^2-ad^3)^{\frac{2}{3}}b^3c^3\log\left((bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{1}{3}}\right)+120\sqrt{\frac{1}{3}}\,(b^4c^4d-ab^3c^3d^2)\sqrt{\frac{(bcd^2-ad^3)^{\frac{1}{3}}}{bc-ad}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{1}{3}}d-(bcd^2-ad^3)^{\frac{1}{3}}\right)\sqrt{\frac{(bcd^2-ad^3)^{\frac{1}{3}}}{bc-ad}}}{d}\right)-3\,(20\,b^3c^3d^2-8\,ab^2c^2d^3-3\,a^2bcd^4-9\,a^3d^5+5\,(b^3cd^4-ab^2d^5)x^6-2\,(4\,b^3c^2d^3-ab^2cd^4-3\,a^2bd^5)x^3)(bx^3+a)^{\frac{2}{3}}}{120\,(b^4cd^5-ab^3d^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(2/3)*d^2 - (b*
c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 40*(b
*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(
1/3)) - 60*sqrt(1/3)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1
/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d
^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (
b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*
d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(20*b^3
*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d^5 + 5*(b^3*c*d^4 - a*b
^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2*b*d^5)*x^3)*(b*x^3 + a
)^(2/3))/(b^4*c*d^5 - a*b^3*d^6), -1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^
3*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d +
 (b*c*d^2 - a*d^3)^(2/3)) - 40*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 +
 a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 120*sqrt(1/3)*(b^4*c^4*d - a*b^3*c
^3*d^2)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^
3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c
 - a*d))/d) - 3*(20*b^3*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d
^5 + 5*(b^3*c*d^4 - a*b^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2
*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(b^4*c*d^5 - a*b^3*d^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.72, size = 371, normalized size = 1.52

$$\frac{b^{27}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{28}cd^8-ab^{27}d^9)}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^5-\sqrt{3}\,ad^6}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^5-ad^6)}+\frac{20\,(bx^3+a)^{\frac{2}{3}}b^{23}c^2d^5-8\,(bx^3+a)^{\frac{5}{3}}b^{22}cd^6+20\,(bx^3+a)^{\frac{2}{3}}ab^{22}cd^6+5\,(bx^3+a)^{\frac{8}{3}}b^{21}d^7-16\,(bx^3+a)^{\frac{5}{3}}ab^{21}d^7+20\,(bx^3+a)^{\frac{2}{3}}a^2b^{21}d^7}{40\,b^{24}d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^27*c^3*d^5*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b^28*c*d^8 - a*b^27*d^9) + (-b*c*d^2 + a*d^3)^(2/3)*c^3*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/(sqrt(3)*b*c*d^5 - sqrt(3)*a*d^6) - 1/6*(-b*c*d^2 + a*d^3)^(
2/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
 (-(b*c - a*d)/d)^(2/3))/(b*c*d^5 - a*d^6) + 1/40*(20*(b*x^3 + a)^(2/3)*b^2
3*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^22*c*d^6 + 20*(b*x^3 + a)^(2/3)*a*b^22*c*
d^6 + 5*(b*x^3 + a)^(8/3)*b^21*d^7 - 16*(b*x^3 + a)^(5/3)*a*b^21*d^7 + 20*(
b*x^3 + a)^(2/3)*a^2*b^21*d^7)/(b^24*d^8)
```

Mupad [B]
time = 5.09, size = 339, normalized size = 1.39

$$\left(\frac{3\,a^2}{2\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{2\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3}-\left(\frac{3\,a}{5\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{5\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^3\,d}-\frac{c^3\,\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}+\frac{b\,c^7-a\,c^6\,d}{d^{16/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}}+\frac{\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}-\frac{c^6\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{1/3}}{4\,d^{16/3}}\right)\,\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}{6\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}}-\frac{c^3\,\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}+\frac{c^6\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{16/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(2*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(2*b^3*d))*(a + b*x^3)^(2/3) - ((3*a)/(5*b^3*d) + (b^4*c - a*b^
3*d)/(5*b^6*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^3*d) - (c^3*lo
g((c^6*(a + b*x^3)^(1/3))/d^5 + (b*c^7 - a*c^6*d)/(d^(16/3)*(a*d - b*c)^(2/
3))))/(3*d^(11/3)*(a*d - b*c)^(1/3)) + (log((c^6*(a + b*x^3)^(1/3))/d^5 - (
c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(16/3)))*(3^(1/2)*c^3*1i + c
^3))/(6*d^(11/3)*(a*d - b*c)^(1/3)) - (c^3*log((c^6*(a + b*x^3)^(1/3))/d^5
+ (c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/d^(16/3))*((3^(1/2)*1i)/2
- 1/2))/(3*d^(11/3)*(a*d - b*c)^(1/3))
```

3.716 $\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=203

$$-\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2}+\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}\sqrt[3]{bc-ad}}+\frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\log\left(\sqrt[3]{bc-ad}\ldots\right)}{2d^{8/3}\sqrt[3]{\ldots}}$$

```
[Out] -1/2*(a*d+b*c)*(b*x^3+a)^(2/3)/b^2/d^2+1/5*(b*x^3+a)^(5/3)/b^2/d+1/6*c^2*ln
(d*x^3+c)/d^(8/3)/(-a*d+b*c)^(1/3)-1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x
^3+a)^(1/3))/d^(8/3)/(-a*d+b*c)^(1/3)-1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b*x^
3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 90, 58, 631, 210, 31}

$$-\frac{c^2\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}\sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^2d^2}+\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}+\frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*((b*c + a*d)*(a + b*x^3)^(2/3))/(b^2*d^2) + (a + b*x^3)^(5/3)/(5*b^2*d
) - (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(1/3)) + (c^2*Log[c + d*x^3])/(6*d^(8/3)*
(b*c - a*d)^(1/3)) - (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)
])/(2*d^(8/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{-bc-ad}{bd^2\sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{bd} + \frac{c^2}{d^2\sqrt[3]{a+bx}\,(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\,(a+bx^3)^{2/3}}{2b^2d^2} + \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{(bc+ad)\,(a+bx^3)^{2/3}}{2b^2d^2} + \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{c^2\log(c+dx^3)}{6d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2\text{Subst}\left(\int \frac{}{\frac{(bc-ad)^2}{d^{2/3}}}\right.}{}$$

$$= -\frac{(bc+ad)\,(a+bx^3)^{2/3}}{2b^2d^2} + \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{c^2\log(c+dx^3)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2\log\left(\sqrt[3]{bc-ad}\right.}{2d^{8/3}\sqrt[3]{}}$$

$$= -\frac{(bc+ad)\,(a+bx^3)^{2/3}}{2b^2d^2} + \frac{(a+bx^3)^{5/3}}{5b^2d} - \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2}{6d}$$

Mathematica [A]
time = 0.36, size = 231, normalized size = 1.14

$$\frac{-3d^{2/3}\sqrt[3]{bc-ad}\,(a+bx^3)^{2/3}(5bc+3ad-2bdx^3) - 10\sqrt{3}\,b^2c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 10b^2c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right) + 5b^2c^2\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\,\sqrt[3]{bc-ad}\,\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{30b^2d^{8/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(5*b*c + 3*a*d - 2*b*d*x^3)
 - 10*Sqrt[3]*b^2*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]] - 10*b^2*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(
1/3)] + 5*b^2*c^2*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*
x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(30*b^2*d^(8/3)*(b*c - a*d)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(b\,x^3+a)^{\frac{1}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(164) = 328$.
time = 4.39, size = 768, normalized size = 3.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c
*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-
b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)
^(1/3)) + 15*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((-b*c*d^2 + a*d^3)^
(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*
c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)
+ (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c
- a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(5
*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^2*c*d^3 - a*b*d^4)*x^3)*(b*x^
3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5), 1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2
*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)
*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b
*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) + 30*sqrt(1/3)*(b^3*c^3*d - a
*b^2*c^2*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*
(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)
^(1/3)/(b*c - a*d))/d) - 3*(5*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^
2*c*d^3 - a*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.64, size = 313, normalized size = 1.54

$$-\frac{b^{12}c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{13}cd^5-ab^{12}d^6)}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^4-\sqrt{3}\,ad^5}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^4-ad^5)}-\frac{5\,(bx^3+a)^{\frac{2}{3}}b^9cd^3-2\,(bx^3+a)^{\frac{5}{3}}b^8d^4+5\,(bx^3+a)^{\frac{2}{3}}ab^8d^4}{10\,b^{10}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^12*c^2*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^13*c*d^5 - a*b^12*d^6) - (-b*c*d^2 + a*d^3)^(2/3)*c^2
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) + 1/6*(-b*c*d^2 + a*d^3)^
(2/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) - 1/10*(5*(b*x^3 + a)^(2/3)*b^9
*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^8*d^4 + 5*(b*x^3 + a)^(2/3)*a*b^8*d^4)/(b^10
*d^5)
```

Mupad [B]
time = 5.11, size = 267, normalized size = 1.32

$$\frac{(b\,x^3+a)^{5/3}}{5\,b^2\,d}-\left(\frac{a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{2\,b^4\,d^2}\right)\,(b\,x^3+a)^{2/3}+\frac{c^2\,\ln\left(\frac{c^4\,(b\,x^3+a)^{1/3}}{d^3}+\frac{b\,c^5-a\,c^4\,d}{d^{10/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{8/3}\,(a\,d-b\,c)^{1/3}}-\frac{\ln\left(\frac{c^4\,(b\,x^3+a)^{1/3}}{d^3}-\frac{c^4\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,(a\,d-b\,c)^{1/3}}{4\,d^{10/3}}\right)\,\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)}{6\,d^{8/3}\,(a\,d-b\,c)^{1/3}}+\frac{c^2\,\ln\left(\frac{c^4\,(b\,x^3+a)^{1/3}}{d^3}-\frac{c^4\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(a\,d-b\,c)^{1/3}}{4\,d^{10/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{d^{8/3}\,(a\,d-b\,c)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b^2*d) - (a/(b^2*d) + (b^3*c - a*b^2*d)/(2*b^4*d^2))*(
a + b*x^3)^(2/3) + (c^2*log((c^4*(a + b*x^3)^(1/3))/d^3 + (b*c^5 - a*c^4*d)
/(d^(10/3)*(a*d - b*c)^(2/3))))/(3*d^(8/3)*(a*d - b*c)^(1/3)) - (log((c^4*(
a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(10
/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(8/3)*(a*d - b*c)^(1/3)) + (c^2*log((c^4
*(a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(
10/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(8/3)*(a*d - b*c)^(1/3))
```

3.717 $\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=168

$$\frac{(a+bx^3)^{2/3}}{2bd}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}\sqrt[3]{bc-ad}}-\frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/2*(b*x^3+a)^(2/3)/b/d-1/6*c*ln(d*x^3+c)/d^(5/3)/(-a*d+b*c)^(1/3)+1/2*c*ln
((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(5/3)/(-a*d+b*c)^(1/3)+1/3*c*a
rctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(5/3)/(
-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 81, 58, 631, 210, 31}

$$\frac{c\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}\sqrt[3]{bc-ad}}-\frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}+\frac{(a+bx^3)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b
*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)) - (c*Log[c +
 d*x^3])/(6*d^(5/3)*(b*c - a*d)^(1/3)) + (c*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)])/(2*d^(5/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{d}} + x^2}\,dx, x, \sqrt[3]{\cdots}\right.}{2d^2}$$

$$= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\text{Sub}\cdots}{\cdots}$$

$$= \frac{(a+bx^3)^{2/3}}{2bd} + \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc\cdots}\right.}{\cdots}$$

Mathematica [A]
time = 0.16, size = 203, normalized size = 1.21

$$\frac{3d^{2/3}\sqrt[3]{bc-ad}\,(a+bx^3)^{2/3} + 2\sqrt{3}\,bc\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 2bc\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right) - bc\log\left((bc-ad)^{2/3} - \sqrt[3]{d}\,\sqrt[3]{bc-ad}\,\sqrt[3]{a+bx^3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6bd^{5/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3) + 2*Sqrt[3]*b*c*ArcTan[(1 -
(2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*b*c*Log[(b*c
- a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - b*c*Log[(b*c - a*d)^(2/3) - d^(
1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*b
*d^(5/3)*(b*c - a*d)^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(b\,x^3+a)^{\frac{1}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(134) = 268$.
time = 5.10, size = 667, normalized size = 3.97

$$\left[-\frac{(bcd^2-ad^3)^{\frac{2}{3}}bc\log\left((bx^3+a)^{\frac{2}{3}}d^2-(bcd^2-ad^3)^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{2}{3}}\right)-2(bcd^2-ad^3)^{\frac{2}{3}}bc\log\left((bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{1}{3}}\right)-3\sqrt{\frac{1}{3}}(b^2c^2d-abcd^2)\sqrt{-\frac{(bcd^2-ad^3)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2bd^2x^3-bcd+3ad^2-3\sqrt{\frac{1}{3}}\left(2(bcd^2-ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}(bcd-ad^2)-(bcd^2-ad^3)^{\frac{1}{3}}(bc-ad)\right)\sqrt{-\frac{(bcd^2-ad^3)^{\frac{1}{3}}}{bc-ad}}-3(bcd^2-ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\right)-3(bcd^2-ad^3)(bx^3+a)^{\frac{2}{3}}}{6(b^2cd^3-abd^4)}, -\frac{(bcd^2-ad^3)^{\frac{2}{3}}bc\log\left((bx^3+a)^{\frac{2}{3}}d^2-(bcd^2-ad^3)^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{2}{3}}\right)-2(bcd^2-ad^3)^{\frac{2}{3}}bc\log\left((bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{1}{3}}\right)+6\sqrt{\frac{1}{3}}(b^2c^2d-abcd^2)\sqrt{\frac{(bcd^2-ad^3)^{\frac{1}{3}}}{bc-ad}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx^3+a)^{\frac{1}{3}}d-(bcd^2-ad^3)^{\frac{1}{3}}\right)\sqrt{\frac{(bcd^2-ad^3)^{\frac{1}{3}}}{bc-ad}}}{d}\right)-3(bcd^2-ad^3)(bx^3+a)^{\frac{2}{3}}}{6(b^2cd^3-abd^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a
*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a
*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*sqrt
(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*lo
g((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1
/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 -
a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(b*c*d^2 - a*d^3)*(b*x^3 +
 a)^(2/3))/(b^2*c*d^3 - a*b*d^4), -1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*
x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2
 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d +
(b*c*d^2 - a*d^3)^(1/3)) + 6*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt((b*c*d^
2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*
c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(b*c
*d^2 - a*d^3)*(b*x^3 + a)^(2/3))/(b^2*c*d^3 - a*b*d^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.58, size = 257, normalized size = 1.53

$$\frac{\frac{2\,bcd\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^2-ad^3}+\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^3-\sqrt{3}\,ad^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{bcd^3-ad^4}+\frac{3\,(bx^3+a)^{\frac{2}{3}}}{d}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*
d)/d)^(1/3)))/(b*c*d^2 - a*d^3) + 6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3
*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1
/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b*c*d^3 - a*d^4) + 3*(b*x^3 + a)^(2/3)/d)/b
```

Mupad [B]
time = 5.10, size = 219, normalized size = 1.30

$$\frac{(b\,x^3+a)^{2/3}}{2\,b\,d}+\frac{\ln\left(\frac{c^2\,(b\,x^3+a)^{1/3}}{d}-\frac{c^2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(a\,d-b\,c)^{1/3}}{4\,d^{4/3}}\right)\,\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{5/3}\,(a\,d-b\,c)^{1/3}}+\frac{\ln\left(\frac{c^2\,(b\,x^3+a)^{1/3}}{d}-\frac{c^2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,(a\,d-b\,c)^{1/3}}{4\,d^{4/3}}\right)\,\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{5/3}\,(a\,d-b\,c)^{1/3}}-\frac{c\,\ln\left(\frac{c^2\,(b\,x^3+a)^{1/3}}{d}+\frac{b\,c^3-a\,c^2\,d}{d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{5/3}\,(a\,d-b\,c)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)*
1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c - 3^(1/2)*c*1i))/(6*d^(5/3)*(a
*d - b*c)^(1/3)) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)*1i + 1)^2
*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c + 3^(1/2)*c*1i))/(6*d^(5/3)*(a*d - b*c)
^(1/3)) - (c*log((c^2*(a + b*x^3)^(1/3))/d + (b*c^3 - a*c^2*d)/(d^(4/3)*(a*
d - b*c)^(2/3))))/(3*d^(5/3)*(a*d - b*c)^(1/3))
```

3.718 $\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=145

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/6*ln(d*x^3+c)/d^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b
*x^3+a)^(1/3))/d^(2/3)/(-a*d+b*c)^(1/3)-1/3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+
a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {455, 58, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(6*d^(2/3)*(b*c - a*d)^(
1/3)) - Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(2/3)*(b*c
- a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\log(c+dx^3)}{6d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{d}} + x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d} - \text{Su}$$

$$= \frac{\log(c+dx^3)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,\right.}{d^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{bc-ad}} + \frac{\log(c+dx^3)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\,\sqrt[3]{a\right.}}{2d^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [A]
time = 0.12, size = 162, normalized size = 1.12

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right) + \log\left((bc-ad)^{2/3} - \sqrt[3]{d}\,\sqrt[3]{bc-ad}\,\sqrt[3]{a+bx^3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6d^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c -
a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x
^3)^(2/3)])/(6*d^(2/3)*(b*c - a*d)^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(114) = 228.
time = 4.21, size = 592, normalized size = 4.08

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(bcd-ad^2\right)\sqrt{\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2bd^2x^3-bcd+3ad^2+3\sqrt{\frac{1}{3}}\left(2\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(bcd-ad^2\right)+\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bc-ad\right)\right)\sqrt{\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}}{bc-ad}}-3\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{dx^3+c}\right)+\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2+\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}d+\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\right)-2\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d-\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\right)}{6\left(bcd^2-ad^3\right)}, \frac{6\sqrt{\frac{1}{3}}\left(bcd-ad^2\right)\sqrt{-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}}{bc-ad}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{\frac{1}{3}}d+\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}}{bc-ad}}}{d}\right)+\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2+\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}d+\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\right)-2\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d-\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\right)}{6\left(bcd^2-ad^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)
)*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2
/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d
^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c
```

```
*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + (-b*c*d^2 + a*d^3)^(2
/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*
d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^
(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)))/(b*c*d^2 - a*d^3), 1/6*(6*sqrt(1/3)*(b
*c*d - a*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*
(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)
^(1/3)/(b*c - a*d))/d) + (-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2
 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3))
 - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^
(1/3)))/(b*c*d^2 - a*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.55, size = 226, normalized size = 1.56

$$-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^2-\sqrt{3}\,ad^3}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^2-ad^3)}-\frac{\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c
- a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3)
+ 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-
(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3) - 1/3*(-(b
*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*
c - a*d)
```

Mupad [B]
time = 4.93, size = 208, normalized size = 1.43

$$\frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{9\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{2/3}\,(a\,d-b\,c)^{1/3}}+\frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)^2\,(9\,a\,d^3-9\,b\,c\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{2/3}\,(a\,d-b\,c)^{1/3}}-\frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)^2\,(9\,a\,d^3-9\,b\,c\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{2/3}\,(a\,d-b\,c)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] log(d*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(9*d^(4/3)*(a*d - b*c)^(2/3
)))/(3*d^(2/3)*(a*d - b*c)^(1/3)) + (log(d*(a + b*x^3)^(1/3) - ((3^(1/2)*1i
 - 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i
- 1))/(6*d^(2/3)*(a*d - b*c)^(1/3)) - (log(d*(a + b*x^3)^(1/3) - ((3^(1/2)*
1i + 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1
i + 1))/(6*d^(2/3)*(a*d - b*c)^(1/3))
```

3.719 $\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=244

$$\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,c}+\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{bc-ad}}-\frac{\log(x)}{2\sqrt[3]{a}\,c}-\frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}{2\sqrt[3]{a}\,\cdots}$$

```
[Out] -1/2*ln(x)/a^(1/3)/c-1/6*d^(1/3)*ln(d*x^3+c)/c/(-a*d+b*c)^(1/3)+1/2*ln(a^(1
/3)-(b*x^3+a)^(1/3))/a^(1/3)/c+1/2*d^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x
^3+a)^(1/3))/c/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/
a^(1/3)*3^(1/2))/a^(1/3)/c*3^(1/2)+1/3*d^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x
^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 88, 57, 631, 210, 31, 58}

$$\frac{\sqrt[3]{d}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{bc-ad}}+\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,c}-\frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,c}-\frac{\log(x)}{2\sqrt[3]{a}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
c) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/
Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(1/3)) - Log[x]/(2*a^(1/3)*c) - (d^(1/3)*L
og[c + d*x^3])/(6*c*(b*c - a*d)^(1/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(
2*a^(1/3)*c) + (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*c*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3c} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}\,c} - \frac{\sqrt[3]{d}\,\log(c+dx^3)}{6c\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} -$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}\,c} - \frac{\sqrt[3]{d}\,\log(c+dx^3)}{6c\sqrt[3]{bc-ad}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,c} + \frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{bc-\right.}{2c\sqrt[3]{}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,c} + \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{bc-ad}} - \frac{\log(x)}{2\sqrt[3]{a}\,c} - \frac{\sqrt[3]{}}{}$$

Mathematica [A]
time = 0.42, size = 309, normalized size = 1.27

$$\frac{\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{2\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{bc-ad}} + \frac{2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{a}} + \frac{2\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{bc-ad}} - \frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{\sqrt[3]{a}} - \frac{\sqrt[3]{d}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{\sqrt[3]{bc-ad}}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) +
(2*Sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1
/3))/Sqrt[3]])/(b*c - a*d)^(1/3) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^
(1/3) + (2*d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c
 - a*d)^(1/3) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
]/a^(1/3) - (d^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a +
 b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/(6*c)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(b\,x^3+a)^{\frac{1}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x), x)
```

Fricas [A]
time = 5.52, size = 628, normalized size = 2.57

$$\left[\frac{3\sqrt{\frac{1}{3}}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^3+3\sqrt{\frac{1}{3}}\left(2(bx^3+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^3}\right)-2\sqrt{3}\,a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(\frac{d}{bc-ad}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{2}{3}}d+(bc-ad)\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\right)+2a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left((bc-ad)\left(\frac{d}{bc-ad}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}d\right)-a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{6ac},\ -\frac{2\sqrt{3}\,a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(\frac{d}{bc-ad}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{2}{3}}d+(bc-ad)\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\right)-2a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left((bc-ad)\left(\frac{d}{bc-ad}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}d\right)-6\sqrt{\frac{1}{3}}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)+a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{6ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 +
 a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*
x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - 2*sqrt(3)*a*(d/(b*c - a*d))^(1/3)*arct
an(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3*sqrt(3)) - a*(
d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(2/
3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) + 2*a*(d/(b*c
 - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d)
 - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2
*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c), -1/6*(2*sqrt(3)*a*(d/(b*c
 - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) -
 1/3*sqrt(3)) + a*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*
(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(
1/3)) - 2*a*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (
b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A]
time = 1.29, size = 326, normalized size = 1.34

$$\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^2d-\sqrt{3}\,acd^2}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2d-acd^2)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}c}-\frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}c}+\frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b*c^2 - a*c*d) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*
x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b
*c^2*d - sqrt(3)*a*c*d^2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b
*c^2*d - a*c*d^2) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a
^(1/3))/a^(1/3))/(a^(1/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*a^(1/3) + a^(2/3))/(a^(1/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3))
)/(a^(1/3)*c)
```

Mupad [B]
time = 6.44, size = 702, normalized size = 2.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2
*d^2 + b^2*c^2 - 2*a*b*c*d) - 243*a*b^4*c^4*d^3*(d/(27*b*c^4 - 27*a*c^3*d))
^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)))/(27*b*c^4 - 27*a*c^3*d))*(d/(27*
b*c^4 - 27*a*c^3*d))^(1/3) + log((a + b*x^3)^(1/3) - a*c^2*(1/(a*c^3))^(2/3
))*(1/(27*a*c^3))^(1/3) + (log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(3^(1/2)*1i -
 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d) -
 (243*a*b^4*c^4*d^3*(3^(1/2)*1i - 1)^2*(d/(27*b*c^4 - 27*a*c^3*d))^(2/3)*(2
*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(27*b*c^4 - 27*a*c^3*d)))*(3^(1/2)*
1i - 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - (log(b^5*d^4*(a + b*x^3)^(1/
3) + (d*(3^(1/2)*1i + 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b
^2*c^2 - 2*a*b*c*d) - (243*a*b^4*c^4*d^3*(3^(1/2)*1i + 1)^2*(d/(27*b*c^4 -
27*a*c^3*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(27*b*c^4 - 27
*a*c^3*d)))*(3^(1/2)*1i + 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - log((a
+ b*x^3)^(1/3)*2i + a*c^2*(1/(a*c^3))^(2/3)*1i + 3^(1/2)*a*c^2*(1/(a*c^3))^
```

```
(2/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*c^3))^(1/3) + log((a + b*x^3)^(1/3)*
2i + a*c^2*(1/(a*c^3))^(2/3)*1i - 3^(1/2)*a*c^2*(1/(a*c^3))^(2/3))*((3^(1/2
)*1i)/2 - 1/2)*(1/(27*a*c^3))^(1/3)
```

3.720 $\int \frac{1}{x^4 \sqrt[3]{a+bx^3} (c+dx^3)} dx$

Optimal. Leaf size=296

$$-\frac{(a+bx^3)^{2/3}}{3acx^3}-\frac{(bc+3ad)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}c^2}-\frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{bc-ad}}+\frac{(bc+3ad)\log(x}{6a^{4/3}c^2}$$

```
[Out] -1/3*(b*x^3+a)^(2/3)/a/c/x^3+1/6*(3*a*d+b*c)*ln(x)/a^(4/3)/c^2+1/6*d^(4/3)*
ln(d*x^3+c)/c^2/(-a*d+b*c)^(1/3)-1/6*(3*a*d+b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3)
)/a^(4/3)/c^2-1/2*d^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^2/
(-a*d+b*c)^(1/3)-1/9*(3*a*d+b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(
1/3)*3^(1/2))/a^(4/3)/c^2*3^(1/2)-1/3*d^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^
3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {457, 105, 162, 57, 631, 210, 31, 58}

$$-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(3ad+bc)}{3\sqrt{3}a^{4/3}c^2}-\frac{(3ad+bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2}+\frac{\log(x)(3ad+bc)}{6a^{4/3}c^2}-\frac{d^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{bc-ad}}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{bc-ad}}-\frac{(a+bx^3)^{2/3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/3*(a + b*x^3)^(2/3)/(a*c*x^3) - ((b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a +
b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*c^2) - (d^(4/3)*ArcTan
[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c
^2*(b*c - a*d)^(1/3)) + ((b*c + 3*a*d)*Log[x])/(6*a^(4/3)*c^2) + (d^(4/3)*L
og[c + d*x^3])/(6*c^2*(b*c - a*d)^(1/3)) - ((b*c + 3*a*d)*Log[a^(1/3) - (a
+ b*x^3)^(1/3)])/(6*a^(4/3)*c^2) - (d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2 \sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{2/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{3}(bc+3ad)+\frac{bdx}{3}}{x\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{(a+bx^3)^{2/3}}{3acx^3} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3c^2} - \frac{(bc+3ad)\text{Subst}\left(\right.}{}$$

$$= -\frac{(a+bx^3)^{2/3}}{3acx^3} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} + \frac{d^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{bc-ad}} + \frac{d\text{Subst}\left(\int \frac{}{\frac{(bc-ad)^2}{d^{2/3}}}\right.}{}$$

$$= -\frac{(a+bx^3)^{2/3}}{3acx^3} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} + \frac{d^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{bc-ad}} - \frac{(bc+3ad)\log\left(\sqrt[3]{\right.}}{6a^{4}}$$

$$= -\frac{(a+bx^3)^{2/3}}{3acx^3} - \frac{(bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}c^2} - \frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}}{\sqrt[3]{}}}{}\right.}{\sqrt{3}\,c^2\sqrt[3]{bc-}}$$

Mathematica [A]
time = 0.82, size = 353, normalized size = 1.19

$$-\frac{\frac{6c(a+bx^3)^{2/3}}{ax^3}+\frac{2\sqrt{3}\,(bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}+\frac{6\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{bc-ad}}+\frac{2(bc+3ad)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{4/3}}+\frac{6d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{bc-ad}}-\frac{(bc+3ad)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{a^{4/3}}-\frac{3d^{4/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{\sqrt[3]{bc-ad}}}{18c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/18*((6*c*(a + b*x^3)^(2/3))/(a*x^3) + (2*Sqrt[3]*(b*c + 3*a*d)*ArcTan[(1
 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(4/3) + (6*Sqrt[3]*d^(4/3)*Ar
cTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c -
 a*d)^(1/3) + (2*(b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(4/3) +
 (6*d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)
^(1/3) - ((b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^
```

```
3)^(2/3)])/a^(4/3) - (3*d^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)
^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/c
^2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^4), x)
```

Fricas [A]
time = 3.23, size = 837, normalized size = 2.83

$$\left[\frac{6\sqrt{3} a^2 d x^3 \left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - 3a^2dx^3\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{2}{3}}d-(bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\right)+6a^2dx^3\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left((bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}d\right)+3\sqrt{\frac{1}{3}}(abc+3a^2d)x^3\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx^3-3\sqrt{\frac{1}{3}}\left(2(bx^3+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x^3}\right)+(bc+3ad)(-a)^{\frac{2}{3}}x^3\log\left((bx^3+a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(bc+3ad)(-a)^{\frac{2}{3}}x^3\log\left((bx^3+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-6(bx^3+a)^{\frac{2}{3}}ac}{18a^2c^2x^3},\ \text{[illegible]}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(3)*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3
 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c -
a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*
x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b
*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)
*d) + 3*sqrt(1/3)*(a*b*c + 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3
*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/
3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) + (b*
c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1
/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) +
(-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*a*c)/(a^2*c^2*x^3), 1/18*(6*sqrt(3)*a^2*d
*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c -
 a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*log(-(b*x^
```

```
3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*
c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*log((
b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*(a*b
*c + 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)
 - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x
^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d
)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*
a*c)/(a^2*c^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt[3]{a + bx^3}\, (c + dx^3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A]
time = 1.70, size = 383, normalized size = 1.29

$$-\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^3-\sqrt{3}\,ac^2d}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^3-ac^2d)}+\frac{(bc+3\,ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{4}{3}}c^2}-\frac{\sqrt{3}\left(a^{\frac{2}{3}}bc+3\,a^{\frac{5}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^2c^2}-\frac{\left(a^{\frac{1}{3}}bc+3\,a^{\frac{4}{3}}d\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{5}{3}}c^2}-\frac{(bx^3+a)^{\frac{2}{3}}}{3\,acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c^3 - a*c^2*d) - (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(
2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt
(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))
/(b*c^3 - a*c^2*d) + 1/18*(b*c + 3*a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c^2) - 1/9*sqrt(3)*(a^(2/3)*b*c + 3*a^(5
/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^2*c^2
) - 1/9*(a^(1/3)*b*c + 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(
a^(5/3)*c^2) - 1/3*(b*x^3 + a)^(2/3)/(a*c*x^3)
```

Mupad [B]
time = 11.36, size = 1929, normalized size = 6.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] log(- (((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^
2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*(2*a^2*d^2 +
 b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*(-(3*a*d + b*c)^3
/(a^4*c^6))^(1/3))/9 + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*
a^2*b*c*d^2))/(3*a^2*c))*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^
7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(-(27*a^3*d^3 + b^3*c^3
+ 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + log(- (-d^4/(27*b*
c^7 - 27*a*c^6*d))^(2/3)*((-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3)*((3*b^4*d^3*
(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d
 - 12*a^3*b*c*d^3))/a^2 - 243*a*b^4*c^4*d^3*(-d^4/(27*b*c^7 - 27*a*c^6*d))^
(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)) + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 +
 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c)) - (4*b^5*d^7*(a + b*x^3)^(1/3)
*(3*a*d + b*c)^2)/(27*a^2*c^5))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3) - log(
(((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2
+ 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 -
1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*
((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(1/3))/9 - (b^5*d^4*(b^
3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c))*((3^(1/2)*
1i)/2 - 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^7*(a + b*x^3
)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3
 + b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + log((((
((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4
*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 + 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2
)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*((3
^(1/2)*1i)/2 - 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(1/3))/9 + (b^5*d^4*(b^3*c
^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c))*((3^(1/2)*1i)
/2 + 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^7*(a + b*x^3)^(
1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d^3 +
b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + (log(- ((3
^(1/2)*1i - 1)^2*(((3^(1/2)*1i - 1)*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3)*((
3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a
*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - (243*a*b^4*c^4*d^3*(3^(1/2)*1i - 1)^2*(
-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/
2 + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^
2*c))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3))/4 - (4*b^5*d^7*(a + b*x^3)^(1/3
)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(3^(1/2)*1i - 1)*(-d^4/(27*b*c^7 - 27*a*c^
6*d))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(-d^4/(27*b*c^
7 - 27*a*c^6*d))^(1/3)*((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4
- 2*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - (243*a*b^4*c^4
*d^3*(3^(1/2)*1i + 1)^2*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3)*(2*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d))/4))/2 - (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d
 + 18*a^2*b*c*d^2))/(3*a^2*c))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3))/4 - (4
*b^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(3^(1/2)*1i + 1)*
(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3))/2 - (a + b*x^3)^(2/3)/(3*a*c*x^3)
```

3.721 $\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=273

$$\frac{x(a+bx^3)^{2/3}}{3bd}-\frac{(3bc+ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^2}+\frac{c^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2\sqrt[3]{bc-ad}}+\frac{c^{4/3}\log\left(c+dx^3\right)}{6d^2\sqrt[3]{bc-ad}}-$$

```
[Out] 1/3*x*(b*x^3+a)^(2/3)/b/d+1/6*c^(4/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(1/3)-1/2*
c^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(1/3)
+1/6*(a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d^2-1/9*(a*d+3*b*c)
*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d^2*3^(1/2)+1/
3*c^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/
2))/d^2/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {490, 544, 245, 384}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(ad+3bc)}{3\sqrt{3}\,b^{4/3}d^2}+\frac{c^{4/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2\sqrt[3]{bc-ad}}+\frac{(ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}d^2}+\frac{c^{4/3}\log\left(c+dx^3\right)}{6d^2\sqrt[3]{bc-ad}}-\frac{c^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2\sqrt[3]{bc-ad}}+\frac{x(a+bx^3)^{2/3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*b*d) - ((3*b*c + a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)*d^2) + (c^(4/3)*ArcTan[(1 + (
2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*d^2*
(b*c - a*d)^(1/3)) + (c^(4/3)*Log[c + d*x^3])/(6*d^2*(b*c - a*d)^(1/3)) - (
c^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d^2*(b*c
 - a*d)^(1/3)) + ((3*b*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*b
^(4/3)*d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = a^2\text{Subst}\left(\int \frac{x^6}{(1-bx^3)^2\,(c-(bc-ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} - \frac{a\text{Subst}\left(\int \frac{c+(2bc+ad)x^3}{(1-bx^3)(c+(-bc+ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{c^2\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d^2} - \frac{(3bc+ad)\text{Subs}}{}$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{c^{4/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d^2} + \frac{c^{4/3}\text{Subs}}{}$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{(3bc+ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2} - \frac{c^{4/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-a}}{\sqrt[3]{a+bx}}\right.}{3d^2\sqrt[3]{bc-ad}}$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{(3bc+ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2} - \frac{(3bc+ad)\log\left(1+\frac{b^{2/3}}{(a+bx}\right.}{18b^{4/3}d}$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} - \frac{(3bc+ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^2} + \frac{c^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-}}{\sqrt[3]{c}\sqrt[3]{a}}}{\sqrt{3}}\right.}{\sqrt{3}\,d^2\sqrt[3]{bc-ad}}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 2.77, size = 466, normalized size = 1.71

$$\frac{\frac{12dx(a+bx^3)^{2/3}}{b} - \frac{4\sqrt{3}(3bc+ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{4/3}} - \frac{6\sqrt{-6+6i\sqrt{3}}\,c^{4/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} + \frac{4(3bc+ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{b^{4/3}} + \frac{6\left(1+i\sqrt{3}\right)c^{4/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{bc-ad}} - \frac{2(3bc+ad)\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{4/3}} - \frac{3i\left(-i+\sqrt{3}\right)c^{4/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{\sqrt[3]{bc-ad}}}{36d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((12*d*x*(a + b*x^3)^(2/3))/b - (4*Sqrt[3]*(3*b*c + a*d)*ArcTan[(Sqrt[3]*b^
(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(4/3) - (6*Sqrt[-6 + (6*I)*S
qrt[3]]*c^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x
 - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(1/3) + (4*(3*b
*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(4/3) + (6*(1 + I*Sqrt[3
])*c^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^
(1/3)])/(b*c - a*d)^(1/3) - (2*(3*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a
 + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(4/3) - ((3*I)*(-I + Sqrt[3])*c^(4/
3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
```

```
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a
*d)^(1/3))/(36*d^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [A]
time = 3.22, size = 826, normalized size = 3.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*sqrt(3)*b^2*c*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*s
qrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*
d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)
/x) + 3*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))
^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 +
a)^(2/3)*c)/x^2) - 6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c + a*b*d
)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1
/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)
*sqrt(-1/b^(2/3)) + 2*a) - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3
 + a)^(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3
)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d^2), -1/18*(6*sqrt(3)*b^2*c*(-
c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(
-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d
)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + 3*b^2*c*(-c/(b*c - a
```

```
*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)
*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2) - 6*(b*x^
3 + a)^(2/3)*b*d*x - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^
(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1
/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*(3*b^2*c + a*b*d)*arctan(sqrt
(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.722 $\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=233

$$\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,d}-\frac{\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d\sqrt[3]{bc-ad}}-\frac{\sqrt[3]{c}\,\log\left(c+dx^3\right)}{6d\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{c}\,\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a-}\right.}{2d\sqrt[3]{bc-ad}}$$

```
[Out] -1/6*c^(1/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(1/3)+1/2*c^(1/3)*ln((-a*d+b*c)^(1/3)
*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(-a*d+b*c)^(1/3)-1/2*ln(-b^(1/3)*x+(b*x^3+a)^
(1/3))/b^(1/3)/d+1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^
(1/3)/d*3^(1/2)-1/3*c^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x
^3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {494, 245, 384}

$$-\frac{\sqrt[3]{c}\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d\sqrt[3]{bc-ad}}+\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,d}-\frac{\sqrt[3]{c}\,\log\left(c+dx^3\right)}{6d\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{c}\,\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)*d) -
 (c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/
Sqrt[3]])/(Sqrt[3]*d*(b*c - a*d)^(1/3)) - (c^(1/3)*Log[c + d*x^3])/(6*d*(b*
c - a*d)^(1/3)) + (c^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^
(1/3)])/(2*d*(b*c - a*d)^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*
b^(1/3)*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = a\text{Subst}\left(\int \frac{x^3}{(1-bx^3)\,(c-(bc-ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d} - \frac{c\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d} + \frac{\text{Subst}\left(\int \frac{2+\sqrt[3]{b}\,x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx \cdots}}\right)}{3d}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}\,d} + \frac{\sqrt[3]{c}\,\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{b}\,x \cdots}\right.}{\cdots}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}\,d} + \frac{\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{b}\,d} + \frac{\sqrt[3]{c}\,\log\left(\sqrt[3]{c} \cdots\right.}{3d\sqrt[3]{\cdots}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,d} - \frac{\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d\sqrt[3]{bc-ad}} - \frac{\log\left(1-\frac{\sqrt[3]{b \cdots}}{\sqrt[3]{a+\cdots}}\right.}{3\sqrt[3]{b}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.88, size = 423, normalized size = 1.82

$$\frac{\frac{4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} + \frac{2\sqrt{-6+6i\sqrt{3}}\,\sqrt[3]{c}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\,\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} - \frac{4\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{b}} - \frac{2i\left(-i+\sqrt{3}\right)\sqrt[3]{c}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{bc-ad}} + \frac{2\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{\sqrt[3]{b}} + \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{c}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{\sqrt[3]{bc-ad}}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((4*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/
b^(1/3) + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x
)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))
])/(b*c - a*d)^(1/3) - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(1/3) -
((2*I)*(-I + Sqrt[3])*c^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c
^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(1/3) + (2*Log[b^(2/3)*x^2 + b^(1/3)
*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(1/3) + ((1 + I*Sqrt[3])*c^(1/
3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a
*d)^(1/3))/(12*d)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [A]
time = 3.80, size = 761, normalized size = 3.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-
b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*
(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*b*(c/
(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(
```

```
b*c - a*d))^(1/3))/x) + 2*b*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b
*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - b*(c/(b*c - a*d))^(1/3)*log(((
b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(
b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) - 2*(-b)^(2/3)*log(((-b)^(1/3
)*x + (b*x^3 + a)^(1/3))/x) + (-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^
(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(6*sqrt(1/3)*b*sq
rt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sq
rt(-(-b)^(1/3)/b)/x) - 2*sqrt(3)*b*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3
)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) - 2*b*(c/(b*c -
 a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*
c)/x) + b*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3)
+ (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)
*c)/x^2) + 2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (-b)^(2
/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3
))/x^2))/(b*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{a + bx^3}\,(c + dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**3/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.723 $\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=148

$$\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/6*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {384}

$$\frac{\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(
Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^
(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*
(b*c - a*d)^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-}}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x}\right.}{3}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x,\right.}{2\sqrt[3]{c}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} -$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)}{(a+bx}\right.}{6c^{2/}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.07, size = 255, normalized size = 1.72

$$\frac{-2\sqrt{-6+6i\sqrt{3}}\,\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\,\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)+\left(1+i\sqrt{3}\right)\left(2\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)-\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)\right)}{12c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 + I*Sqrt[3
])*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
] - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3
)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(12*c^
(2/3)*(b*c - a*d)^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + bx^3}\,(c + dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.724 $\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=176

$$-\frac{(a+bx^3)^{2/3}}{2acx^2}-\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}}-\frac{d\log\left(c+dx^3\right)}{6c^{5/3}\sqrt[3]{bc-ad}}+\frac{d\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}\sqrt[3]{bc-ad}}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/a/c/x^2-1/6*d*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(1/3)+1/2
*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(1/3)-
1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/
c^(5/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {491, 12, 384}

$$-\frac{d\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}}-\frac{d\log\left(c+dx^3\right)}{6c^{5/3}\sqrt[3]{bc-ad}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}\sqrt[3]{bc-ad}}-\frac{(a+bx^3)^{2/3}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(a*c*x^2) - (d*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(5/3)*(b*c - a*d)^(1/3)) -
 (d*Log[c + d*x^3])/(6*c^(5/3)*(b*c - a*d)^(1/3)) + (d*Log[((b*c - a*d)^(1/
3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(5/3)*(b*c - a*d)^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-bx^3}{x^3(c-(bc-ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a} \\
&= -\frac{(a+bx^3)^{2/3}}{2acx^2} - \frac{d\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c} \\
&= -\frac{(a+bx^3)^{2/3}}{2acx^2} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}} - \frac{d\text{Subst}\left(\int \right.}{} \\
&= -\frac{(a+bx^3)^{2/3}}{2acx^2} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}}\right.}{2} \\
&= -\frac{(a+bx^3)^{2/3}}{2acx^2} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\sqrt[3]{}\right.}{6c^{5/3}\sqrt[3]{bc-ad}} \\
&= -\frac{(a+bx^3)^{2/3}}{2acx^2} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.26, size = 314, normalized size = 1.78

$$
\frac{-6c^{2/3}\sqrt[3]{bc-ad}\,(a+bx^3)^{2/3}+2\sqrt{-6+6i\sqrt{3}}\,adx^2\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\,\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)-2i\left(-i+\sqrt{3}\right)adx^2\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)+a\left(d+i\sqrt{3}\,d\right)x^2\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{12ac^{5/3}\sqrt[3]{bc-ad}\,x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-6*c^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3) + 2*Sqrt[-6 + (6*I)*Sqrt[3]
]*a*d*x^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*
I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (2*I)*(-I + Sqrt[3])*a*d*x^2*Log
[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + a*(d
+ I*Sqrt[3]*d)*x^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(
b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^
(2/3)])/(12*a*c^(5/3)*(b*c - a*d)^(1/3)*x^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \sqrt[3]{a + b x^{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.725 $\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=214

$$-\frac{(a+bx^3)^{2/3}}{5acx^5}+\frac{(3bc+5ad)\,(a+bx^3)^{2/3}}{10a^2c^2x^2}+\frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}}+\frac{d^2\log\left(c+dx^3\right)}{6c^{8/3}\sqrt[3]{bc-ad}}-\frac{d^2\log\left(\frac{\sqrt[3]{bc-\cdots}}{\sqrt[3]{c}\cdots}\right.}{2c^{8/3}\cdots}$$

```
[Out] -1/5*(b*x^3+a)^(2/3)/a/c/x^5+1/10*(5*a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c^2/x^2
+1/6*d^2*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(1/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)*x
/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(1/3)+1/3*d^2*arctan(1/3*(1+2*
(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(1/
3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {491, 597, 12, 384}

$$\frac{(a+bx^3)^{2/3}(5ad+3bc)}{10a^2c^2x^2}+\frac{d^2\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}}+\frac{d^2\log\left(c+dx^3\right)}{6c^{8/3}\sqrt[3]{bc-ad}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}\sqrt[3]{bc-ad}}-\frac{(a+bx^3)^{2/3}}{5acx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/5*(a + b*x^3)^(2/3)/(a*c*x^5) + ((3*b*c + 5*a*d)*(a + b*x^3)^(2/3))/(10*
a^2*c^2*x^2) + (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3
)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) + (d^2*Log[c + d*x^
3])/(6*c^(8/3)*(b*c - a*d)^(1/3)) - (d^2*Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
- (a + b*x^3)^(1/3)])/(2*c^(8/3)*(b*c - a*d)^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-bx^3)^2}{x^6(c-(bc-ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^6}+\frac{-bc-ad}{c^2x^3}+\frac{a^2d^2}{c^2(c-(bc-ad)x^3)}\right)dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{(bc+ad)(a+bx^3)^{2/3}}{2a^2c^2x^2} - \frac{(a+bx^3)^{5/3}}{5a^2cx^5} + \frac{d^2\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+\cdots}}\right.}{c^2}$$

$$= \frac{(bc+ad)(a+bx^3)^{2/3}}{2a^2c^2x^2} - \frac{(a+bx^3)^{5/3}}{5a^2cx^5} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \cdots\right.}{3c^{8/3}}$$

$$= \frac{(bc+ad)(a+bx^3)^{2/3}}{2a^2c^2x^2} - \frac{(a+bx^3)^{5/3}}{5a^2cx^5} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}\sqrt[3]{bc-ad}} + \frac{d^2\text{S}\cdots}{\cdots}$$

$$= \frac{(bc+ad)(a+bx^3)^{2/3}}{2a^2c^2x^2} - \frac{(a+bx^3)^{5/3}}{5a^2cx^5} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}\sqrt[3]{bc-ad}} + \frac{d^2\,\text{l}\cdots}{\cdots}$$

$$= \frac{(bc+ad)(a+bx^3)^{2/3}}{2a^2c^2x^2} - \frac{(a+bx^3)^{5/3}}{5a^2cx^5} + \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}} - \frac{d^2\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.57, size = 340, normalized size = 1.59

$$\frac{\frac{6c^{2/3}(a+bx^3)^{2/3}(-2ac+3bcx^3+5adx^3)}{a^2x^5} - \frac{10\sqrt{-6+6i\sqrt{3}}\,d^2\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} + \frac{10(1+i\sqrt{3})d^2\log\left(2\sqrt[3]{bc-ad}\,x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{bc-ad}} - \frac{5i(-i+\sqrt{3})d^2\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{\sqrt[3]{bc-ad}}}{60c^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((6*c^(2/3)*(a + b*x^3)^(2/3)*(-2*a*c + 3*b*c*x^3 + 5*a*d*x^3))/(a^2*x^5) -
 (10*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(
b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*
d)^(1/3) + (10*(1 + I*Sqrt[3])*d^2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[
3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(b*c - a*d)^(1/3) - ((5*I)*(-I + Sqrt[3])*d
^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(b*c - a
*d)^(1/3))/(60*c^(8/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6} \left(b \,x^{3}+a \right)^{\frac{1}{3}} \left(d \,x^{3}+c \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6} \sqrt[3]{a + b x^{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**6*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.726 $\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=262

$$-\frac{(a+bx^3)^{2/3}}{8acx^8}+\frac{(3bc+4ad)(a+bx^3)^{2/3}}{20a^2c^2x^5}-\frac{(9b^2c^2+12abcd+20a^2d^2)(a+bx^3)^{2/3}}{40a^3c^3x^2}-\frac{d^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx}}}{\sqrt{3}}\right.}{\sqrt{3}\,c^{11/3}\sqrt[3]{bc-ad}}$$

```
[Out] -1/8*(b*x^3+a)^(2/3)/a/c/x^8+1/20*(4*a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c^2/x^5
-1/40*(20*a^2*d^2+12*a*b*c*d+9*b^2*c^2)*(b*x^3+a)^(2/3)/a^3/c^3/x^2-1/6*d^3
*ln(d*x^3+c)/c^(11/3)/(-a*d+b*c)^(1/3)+1/2*d^3*ln((-a*d+b*c)^(1/3)*x/c^(1/3
)-(b*x^3+a)^(1/3))/c^(11/3)/(-a*d+b*c)^(1/3)-1/3*d^3*arctan(1/3*(1+2*(-a*d+
b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(11/3)/(-a*d+b*c)^(1/3)*3^
(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {491, 597, 12, 384}

$$\frac{(a+bx^3)^{2/3}(4ad+3bc)}{20a^2c^2x^5}-\frac{(a+bx^3)^{2/3}(20a^2d^2+12abcd+9b^2c^2)}{40a^3c^3x^2}-\frac{d^3\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{11/3}\sqrt[3]{bc-ad}}-\frac{d^3\log(c+dx^3)}{6c^{11/3}\sqrt[3]{bc-ad}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}\sqrt[3]{bc-ad}}-\frac{(a+bx^3)^{2/3}}{8acx^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/8*(a + b*x^3)^(2/3)/(a*c*x^8) + ((3*b*c + 4*a*d)*(a + b*x^3)^(2/3))/(20*
a^2*c^2*x^5) - ((9*b^2*c^2 + 12*a*b*c*d + 20*a^2*d^2)*(a + b*x^3)^(2/3))/(4
0*a^3*c^3*x^2) - (d^3*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x
^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(11/3)*(b*c - a*d)^(1/3)) - (d^3*Log[c + d
*x^3])/(6*c^(11/3)*(b*c - a*d)^(1/3)) + (d^3*Log[((b*c - a*d)^(1/3)*x)/c^(1
/3) - (a + b*x^3)^(1/3)])/(2*c^(11/3)*(b*c - a*d)^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
```

```
+ Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-bx^3)^3}{x^9(c-(bc-ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^9}+\frac{-2bc-ad}{c^2x^6}+\frac{b^2c^2+abcd+a^2d^2}{c^3x^3}+\frac{a^3d^3}{c^3(-c+(bc-ad)x^3)}\right)\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right.}{a^3}$$

$$= -\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\,(a+bx^3)^{5/3}}{5a^3c^2x^5}-\frac{(a+bx^3)^{8/3}}{8a^3cx^8}$$

$$= -\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\,(a+bx^3)^{5/3}}{5a^3c^2x^5}-\frac{(a+bx^3)^{8/3}}{8a^3cx^8}$$

$$= -\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\,(a+bx^3)^{5/3}}{5a^3c^2x^5}-\frac{(a+bx^3)^{8/3}}{8a^3cx^8}$$

$$= -\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\,(a+bx^3)^{5/3}}{5a^3c^2x^5}-\frac{(a+bx^3)^{8/3}}{8a^3cx^8}$$

$$= -\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\,(a+bx^3)^{5/3}}{5a^3c^2x^5}-\frac{(a+bx^3)^{8/3}}{8a^3cx^8}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.09, size = 374, normalized size = 1.43

$$\frac{-\frac{3c^{2/3}(a+bx^3)^{2/3}(9b^2c^2x^6-6abcx^3(c-2dx^3)+a^2(5c^2-8cdx^3+20d^2x^6))}{a^3x^8}+\frac{20\sqrt{-6+6i\sqrt{3}}\,d^3\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\,\sqrt[3]{bc-ad}\,x-(3i+\sqrt{3})\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}}-\frac{20i(-i+\sqrt{3})d^3\log\left(2\sqrt[3]{bc-ad}\,x+(1+i\sqrt{3})\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{bc-ad}}+\frac{10(1+i\sqrt{3})d^3\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{\sqrt[3]{bc-ad}}}{120c^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((-3*c^(2/3)*(a + b*x^3)^(2/3)*(9*b^2*c^2*x^6 - 6*a*b*c*x^3*(c - 2*d*x^3) +
 a^2*(5*c^2 - 8*c*d*x^3 + 20*d^2*x^6)))/(a^3*x^8) + (20*Sqrt[-6 + (6*I)*Sqr
t[3]]*d^3*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*
I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(1/3) - ((20*I)*(-I +
 Sqrt[3])*d^3*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^
3)^(1/3)])/(b*c - a*d)^(1/3) + (10*(1 + I*Sqrt[3])*d^3*Log[2*(b*c - a*d)^(2
/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) +
```

```
I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/(120*c^(11/3
))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9} \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9} \sqrt[3]{a + b x^{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x**9*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^9\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.727 $\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

```
[Out] 1/8*x^8*(1+b*x^3/a)^(1/3)*AppellF1(8/3,1/3,1,11/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 1/3, 1, 11/3, -((b*x^3)/a), -((d*x
^3)/c)])/(8*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^7}{\sqrt[3]{1+\frac{bx^3}{a}}(c+dx^3)}\,dx}{\sqrt[3]{a+bx^3}}$$
$$= \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(64) = 128.
time = 6.75, size = 144, normalized size = 2.25

$$\frac{5cx^2(a+bx^3)-5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2(2bc+ad)x^5\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{20bcd\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1
, 5/3, -((b*x^3)/a), -((d*x^3)/c)] - 2*(2*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(1
/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*b*c*d*(a +
b*x^3)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(b\,x^3+a)^{\frac{1}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**7/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^7}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(x^7/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.728 $\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

```
[Out] 1/5*x^5*(1+b*x^3/a)^(1/3)*AppellF1(5/3,1/3,1,8/3,-b*x^3/a,-d*x^3/c)/c/(b*x^
3+a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^4}{\sqrt[3]{1+\frac{bx^3}{a}}\,(c+dx^3)}\,dx}{\sqrt[3]{a+bx^3}}$$
$$= \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

Mathematica [A]
time = 6.46, size = 65, normalized size = 1.02

$$\frac{x^5\sqrt[3]{\frac{a+bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
[Out] (x^5*((a + b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^3+a)^{\frac{1}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**4/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(x^4/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.729 $\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

```
[Out] 1/2*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,1,5/3,-b*x^3/a,-d*x^3/c)/c/(b*x^
3+a)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}\,(c+dx^3)}\,dx}{\sqrt[3]{a+bx^3}}$$
$$= \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

Mathematica [**A**]
time = 9.21, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt[3]{\frac{a+bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*((a + b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(x/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.730 $\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

```
[Out] -(1+b*x^3/a)^(1/3)*AppellF1(-1/3,1/3,1,2/3,-b*x^3/a,-d*x^3/c)/c/x/(b*x^3+a)
^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 1/3, 1, 2/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*x*(a + b*x^3)^(1/3)))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,(c+dx^3)}\,dx}{\sqrt[3]{a+bx^3}}$$
$$= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 141 vs. $2(62) = 124$.
time = 10.07, size = 141, normalized size = 2.27

$$\frac{-10c(a+bx^3)+5(bc-ad)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^6\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{10ac^2x\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-10*c*(a + b*x^3) + 5*(b*c - a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3,
1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(1/3)*
AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a*c^2*x*(a + b*
x^3)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3} \left(c+dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x**2*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\, {\left(b\,x^3+a\right)}^{1/3}\, \left(d\,x^3+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.731 $\int \frac{1}{x^5\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

```
[Out] -1/4*(1+b*x^3/a)^(1/3)*AppellF1(-4/3,1/3,1,-1/3,-b*x^3/a,-d*x^3/c)/c/x^4/(b
*x^3+a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/4*((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 1/3, 1, -1/3, -((b*x^3)/a), -((d
*x^3)/c)])/(c*x^4*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,(c+dx^3)}\,dx}{\sqrt[3]{a+bx^3}}$$
$$= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 183 vs. 2(64) = 128.
time = 10.12, size = 183, normalized size = 2.86

$$\frac{5c(a+bx^3)\left(-ac+2bcx^3+4adx^3\right)+5\left(-b^2c^2-2abcd+2a^2d^2\right)x^6\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2bd(bc+2ad)x^9\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{20a^2c^3x^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*(a + b*x^3)*(-(a*c) + 2*b*c*x^3 + 4*a*d*x^3) + 5*(-(b^2*c^2) - 2*a*b*c
*d + 2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x
^3)/a), -((d*x^3)/c)] - 2*b*d*(b*c + 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a^2*c^3*x^4*(a + b*x
^3)^(1/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \sqrt[3]{a + bx^3} \, (c + dx^3)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^5 \, {\left(b\,x^3+a\right)}^{1/3} \, \left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.732 $\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=241

$$\frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3}-\frac{(bc+2ad)(a+bx^3)^{4/3}}{4b^3d^2}+\frac{(a+bx^3)^{7/3}}{7b^3d}+\frac{c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}(bc-ad)^{2/3}}+\frac{c}{6d}$$

```
[Out] (a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(1/3)/b^3/d^3-1/4*(2*a*d+b*c)*(b*x^3+a)
^(4/3)/b^3/d^2+1/7*(b*x^3+a)^(7/3)/b^3/d+1/6*c^3*ln(d*x^3+c)/d^(10/3)/(-a*d
+b*c)^(2/3)-1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)/(
-a*d+b*c)^(2/3)+1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^
(1/3))*3^(1/2))/d^(10/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 90, 60, 631, 210, 31}

$$\frac{\sqrt[3]{a+bx^3}(a^2d^2+abcd+b^2c^2)}{b^3d^3}+\frac{c^3\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}(bc-ad)^{2/3}}-\frac{(a+bx^3)^{4/3}(2ad+bc)}{4b^3d^2}+\frac{(a+bx^3)^{7/3}}{7b^3d}+\frac{c^3\log(c+dx^3)}{6d^{10/3}(bc-ad)^{2/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/(b^3*d^3) - ((b*c + 2*a*d
)*(a + b*x^3)^(4/3))/(4*b^3*d^2) + (a + b*x^3)^(7/3)/(7*b^3*d) + (c^3*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d^(10/3)*(b*c - a*d)^(2/3)) + (c^3*Log[c + d*x^3])/(6*d^(10/3)*(b*c - a*d)^
(2/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/
3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2c^2+abcd+a^2d^2}{b^2d^3(a+bx)^{2/3}} + \frac{(-bc-2ad)\sqrt[3]{a+bx}}{b^2d^2} + \frac{(a+bx)^{4/3}}{b^2d} - \frac{}{d^3}\right.\right.$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} - \frac{c^3}{}$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{}{6d}$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{}{6d}$$

$$c^3$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + -$$

Mathematica [A]
time = 0.48, size = 264, normalized size = 1.10

$$\frac{3\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{a+bx^3}\left(18a^2d^2+3abd(7c-2dx^3)+b^2(28c^2-7cdx^3+4d^2x^6)\right)+28\sqrt{3}\,b^3c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-28b^3c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+14b^3c^3\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{84b^3d^{10/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (3*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3)*(18*a^2*d^2 + 3*a*b*d*(7*c -
 2*d*x^3) + b^2*(28*c^2 - 7*c*d*x^3 + 4*d^2*x^6)) + 28*Sqrt[3]*b^3*c^3*ArcT
an[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*b^3*
c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 14*b^3*c^3*Log[(b*
c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a +
 b*x^3)^(2/3)])/(84*b^3*d^(10/3)*(b*c - a*d)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(b\,x^3+a)^{\frac{2}{3}}(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(201) = 402$.
time = 3.63, size = 1322, normalized size = 5.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/84*(14*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 +
a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c
- a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 28
*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1/3)*
(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 42*sqrt(1/3
)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/
3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*
sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2
- a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3))*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-
b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*
x^3 + c)) + 3*(28*b^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3
*b*c*d^4 + 18*a^4*d^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 -
 (7*b^4*c^3*d^2 - 8*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*
x^3 + a)^(1/3))/(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6), 1/84*(14*(-b^2
*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(2/3)*(b*c*d
 - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 28*(-b^2*c^2*d +
```

```
2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2)
 - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 84*sqrt(1/3)*(b^4*c^4*d -
a*b^3*c^3*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*arctan(s
qrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + 2*(-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt(-(-b^2*c^2*d +
 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 3*(28*b
^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3*b*c*d^4 + 18*a^4*d
^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 - (7*b^4*c^3*d^2 - 8
*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(1/3))/(b^
5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a + bx^{3}\right)^{\frac{2}{3}} \left(c + dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.64, size = 372, normalized size = 1.54

$$\frac{b^{24}c^{3}d^{4}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^{3}+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{25}cd^{7}-ab^{24}d^{8})}-\frac{(-bcd^{2}+ad^{3})^{\frac{1}{3}}c^{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^{3}+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^{4}-\sqrt{3}\,ad^{5}}-\frac{(-bcd^{2}+ad^{3})^{\frac{1}{3}}c^{3}\log\left((bx^{3}+a)^{\frac{2}{3}}+(bx^{3}+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^{4}-ad^{5})}+\frac{28\,(bx^{3}+a)^{\frac{1}{3}}b^{20}c^{2}d^{4}-7\,(bx^{3}+a)^{\frac{4}{3}}b^{19}cd^{5}+28\,(bx^{3}+a)^{\frac{1}{3}}ab^{19}cd^{5}+4\,(bx^{3}+a)^{\frac{7}{3}}b^{18}d^{6}-14\,(bx^{3}+a)^{\frac{4}{3}}ab^{18}d^{6}+28\,(bx^{3}+a)^{\frac{1}{3}}a^{2}b^{18}d^{6}}{28\,b^{21}d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^24*c^3*d^4*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b^25*c*d^7 - a*b^24*d^8) - (-b*c*d^2 + a*d^3)^(1/3)*c^3*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) - 1/6*(-b*c*d^2 + a*d^3)^(
1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
 (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) + 1/28*(28*(b*x^3 + a)^(1/3)*b^2
0*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^19*c*d^5 + 28*(b*x^3 + a)^(1/3)*a*b^19*c*
d^5 + 4*(b*x^3 + a)^(7/3)*b^18*d^6 - 14*(b*x^3 + a)^(4/3)*a*b^18*d^6 + 28*(
b*x^3 + a)^(1/3)*a^2*b^18*d^6)/(b^21*d^7)
```

Mupad [B]
time = 5.01, size = 331, normalized size = 1.37

$$\left(\frac{3\,a^2}{b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3}-\left(\frac{3\,a}{4\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{4\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{4/3}+\frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^3\,d}+\frac{\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}+\frac{3\,c^3\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{2\,d^{4/3}}\right)\,\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}{6\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}}-\frac{c^3\,\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}-\frac{3\,c^3\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{4/3}}\right)}{3\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}}-\frac{c^3\,\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}-\frac{3\,c^3\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{4/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x)

[Out] ((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c -
a*b^3*d))/(b^3*d))*(a + b*x^3)^(1/3) - ((3*a)/(4*b^3*d) + (b^4*c - a*b^3*d)
/(4*b^6*d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^3*d) + (log((3*c^3
*(a + b*x^3)^(1/3))/d + (3*c^3*(3^(1/2)*1i + 1)*(a*d - b*c)^(1/3))/(2*d^(4/
3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(10/3)*(a*d - b*c)^(2/3)) - (c^3*log((3*c
^3*(a + b*x^3)^(1/3))/d - (3*c^3*(a*d - b*c)^(1/3))/d^(4/3)))/(3*d^(10/3)*(
a*d - b*c)^(2/3)) - (c^3*log((3*c^3*(a + b*x^3)^(1/3))/d - (3*c^3*((3^(1/2)
*1i)/2 - 1/2)*(a*d - b*c)^(1/3))/d^(4/3))*((3^(1/2)*1i)/2 - 1/2))/(3*d^(10/
3)*(a*d - b*c)^(2/3))
```

3.733 $\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=201

$$-\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}(bc-ad)^{2/3}}-\frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}\right.}{2d^{7/3}(bc}$$

```
[Out] -(a*d+b*c)*(b*x^3+a)^(1/3)/b^2/d^2+1/4*(b*x^3+a)^(4/3)/b^2/d-1/6*c^2*ln(d*x
^3+c)/d^(7/3)/(-a*d+b*c)^(2/3)+1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a
)^(1/3))/d^(7/3)/(-a*d+b*c)^(2/3)-1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)
^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 90, 60, 631, 210, 31}

$$-\frac{c^2\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}\,(ad+bc)}{b^2d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(((b*c + a*d)*(a + b*x^3)^(1/3))/(b^2*d^2)) + (a + b*x^3)^(4/3)/(4*b^2*d)
- (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(7/3)*(b*c - a*d)^(2/3)) - (c^2*Log[c + d*x^3])/(6*d^(7/3)*(b
*c - a*d)^(2/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(7/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{-bc-ad}{bd^2(a+bx)^{2/3}} + \frac{\sqrt[3]{a+bx}}{bd} + \frac{c^2}{d^2(a+bx)^{2/3}(c+dx)}\right) dx, x, x^3 \right.$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{c^2\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\text{Subst}\left(\int \ldots\right.}{2d^7\ldots}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}\ldots\right.}{2d^{7/3}(b\ldots}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}(bc-ad)^{2/3}} - \frac{c^2}{6d^7\ldots}$$

Mathematica [A]
time = 0.37, size = 231, normalized size = 1.15

$$\frac{-3\sqrt[3]{d}\,(bc-ad)^{2/3}\sqrt[3]{a+bx^3}\,(4bc+3ad-bdx^3) - 4\sqrt{3}\,b^2c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 4b^2c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) - 2b^2c^2\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{12b^2d^{7/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-3*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3)*(4*b*c + 3*a*d - b*d*x^3) -
 4*Sqrt[3]*b^2*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1
/3))/Sqrt[3]] + 4*b^2*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)
] - 2*b^2*c^2*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)
^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*b^2*d^(7/3)*(b*c - a*d)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(b\,x^3+a)^{\frac{2}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

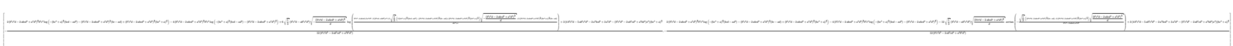
Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(164) = 328$.
time = 2.91, size = 1156, normalized size = 5.75



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a
)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c -
a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 4*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a)^(1/3)*(b*c*
d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) + 6*sqrt(1/3)*(b^3*
c^3*d - a*b^2*c^2*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)*l
og((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 + 3*sqrt(1/
3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a
)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c))
+ 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*a^3*d^4 - (b^3*c^2*d
^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))/(b^4*c^2*d^3 - 2*a*
b^3*c*d^4 + a^2*b^2*d^5), -1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3
)*b^2*c^2*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3)) - 4*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log
(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(
```

```
2/3)) - 12*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)^(1/3)*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3
+ a)^(1/3))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)) + 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*
a^3*d^4 - (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))
/(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.60, size = 312, normalized size = 1.55

$$-\frac{b^{10}c^2d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{11}cd^4-ab^{10}d^5\right)}+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^3-\sqrt{3}\,ad^4}+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^3-ad^4\right)}-\frac{4\,(bx^3+a)^{\frac{1}{3}}b^7cd^2-(bx^3+a)^{\frac{4}{3}}b^6d^3+4\,(bx^3+a)^{\frac{1}{3}}ab^6d^3}{4\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^10*c^2*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^11*c*d^4 - a*b^10*d^5) + (-b*c*d^2 + a*d^3)^(1/3)*c^2
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-b*c*d^2 + a*d^3)^
(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/(b*c*d^3 - a*d^4) - 1/4*(4*(b*x^3 + a)^(1/3)*b^7*
c*d^2 - (b*x^3 + a)^(4/3)*b^6*d^3 + 4*(b*x^3 + a)^(1/3)*a*b^6*d^3)/(b^8*d^4
)
```

Mupad [B]
time = 4.69, size = 292, normalized size = 1.45

$$\frac{{\left(b\,x^3+a\right)}^{4/3}}{4\,b^2\,d}-\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{1/3}-\frac{\ln\left(3\,c^2\,{\left(b\,x^3+a\right)}^{1/3}+\frac{\left(c^2+\sqrt{3}\,c^2\,1{}\mathrm{i}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{6\,d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(c^2+\sqrt{3}\,c^2\,1{}\mathrm{i}\right)}{6\,d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{c^2\,\ln\left(3\,c^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{c^2\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{c^2\,\ln\left(3\,c^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*b^2*d) - ((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))
*(a + b*x^3)^(1/3) - (log(3*c^2*(a + b*x^3)^(1/3) + ((3^(1/2)*c^2*1i + c^2)
*(9*a*d^3 - 9*b*c*d^2))/(6*d^(7/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*c^2*1i + c^
2))/(6*d^(7/3)*(a*d - b*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) - (c^2
*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)*(a*d - b*c)^(2/3))))/(3*d^(7/3)*(a*d - b
*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) - (c^2*((3^(1/2)*1i)/6 - 1/6)
*(9*a*d^3 - 9*b*c*d^2))/(d^(7/3)*(a*d - b*c)^(2/3)))*((3^(1/2)*1i)/6 - 1/6)
)/(d^(7/3)*(a*d - b*c)^(2/3))
```

3.734 $\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=165

$$\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)^{2/3}}+\frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}$$

```
[Out] (b*x^3+a)^(1/3)/b/d+1/6*c*ln(d*x^3+c)/d^(4/3)/(-a*d+b*c)^(2/3)-1/2*c*ln((-a
*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(4/3)/(-a*d+b*c)^(2/3)+1/3*c*arcta
n(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(4/3)/(-a*d
+b*c)^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 81, 60, 631, 210, 31}

$$\frac{c\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)^{2/3}}+\frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}+\frac{\sqrt[3]{a+bx^3}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (a + b*x^3)^(1/3)/(b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)) + (c*Log[c + d
*x^3])/(6*d^(4/3)*(b*c - a*d)^(2/3)) - (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(
a + b*x^3)^(1/3)])/(2*d^(4/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} - \frac{c\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\log(c+dx^3)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\log(c+dx^3)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}} - \frac{c\text{Sub}}{}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)^{2/3}} + \frac{c\log(c+dx^3)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc\right.}}{2}$$

Mathematica [A]
time = 0.17, size = 202, normalized size = 1.22

$$\frac{6\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{a+bx^3} + 2\sqrt{3}\,bc\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 2bc\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) + bc\log\left((bc-ad)^{2/3} - \sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6bd^{4/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (6*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) + 2*Sqrt[3]*b*c*ArcTan[(1 -
(2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*b*c*Log[(b*c
- a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + b*c*Log[(b*c - a*d)^(2/3) - d^(
1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*b
*d^(4/3)*(b*c - a*d)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(b\,x^3+a)^{\frac{2}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(133) = 266.
time = 3.01, size = 1060, normalized size = 6.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(2/3)
*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) +
 (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(-b^2*c^
2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^
2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 3*sqrt(1/3)*(b^2*c^2*d -
 a*b*c*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*log((b^2*c^2
 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*sqrt(1/3)*(2*(b*x^
3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(
b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*
sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-b^2*c^2*d + 2*a*b*
c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c)) + 6*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d^2 - 2*a*b^2*c
*d^3 + a^2*b*d^4), 1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(
-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(
1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(
1/3)) - 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(
1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 6*sqrt
(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1
/3)/d)*arctan(sqrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c -
a*d) + 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt
```

```
(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d + a^2*
d^2)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d
^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.56, size = 253, normalized size = 1.53

$$-\frac{\frac{6\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^2-\sqrt{3}\,ad^3}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{bcd^2-ad^3}-\frac{2bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd-ad^2}-\frac{6\left(bx^3+a\right)^{\frac{1}{3}}}{d}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(6*(-b*c*d^2 + a*d^3)^(1/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt
(3)*a*d^3) + (-b*c*d^2 + a*d^3)^(1/3)*b*c*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3)
 - 2*b*c*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c*d - a*d^2) - 6*(b*x^3 + a)^(1/3)/d)/b
```

Mupad [B]
time = 4.68, size = 232, normalized size = 1.41

$$\frac{(b\,x^3+a)^{1/3}}{b\,d}-\frac{c\,\ln\left(3\,c\,d\,(b\,x^3+a)^{1/3}-\frac{c\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{4/3}\,(a\,d-b\,c)^{2/3}}+\frac{\ln\left(3\,c\,d\,(b\,x^3+a)^{1/3}+\frac{(9\,a\,d^3-9\,b\,c\,d^2)\,\left(c-\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(c-\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}}+\frac{\ln\left(3\,c\,d\,(b\,x^3+a)^{1/3}+\frac{(9\,a\,d^3-9\,b\,c\,d^2)\,\left(c+\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(c+\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(1/3)/(b*d) - (c*log(3*c*d*(a + b*x^3)^(1/3) - (c*(9*a*d^3 - 9*
b*c*d^2))/(3*d^(4/3)*(a*d - b*c)^(2/3))))/(3*d^(4/3)*(a*d - b*c)^(2/3)) + (
log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c - 3^(1/2)*c*1i))/(6
*d^(4/3)*(a*d - b*c)^(2/3)))*(c - 3^(1/2)*c*1i))/(6*d^(4/3)*(a*d - b*c)^(2/
3)) + (log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c + 3^(1/2)*c*
1i))/(6*d^(4/3)*(a*d - b*c)^(2/3)))*(c + 3^(1/2)*c*1i))/(6*d^(4/3)*(a*d - b
*c)^(2/3))
```

3.735 $\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)} dx$

Optimal. Leaf size=145

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}$$

```
[Out] -1/6*ln(d*x^3+c)/d^(1/3)/(-a*d+b*c)^(2/3)+1/2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(
b*x^3+a)^(1/3))/d^(1/3)/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1-2*d^(1/3)*(b*x^3
+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(1/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {455, 60, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(6*d^(1/3)*(b*c - a*d)^(
2/3)) + Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(1/3)*(b*c
- a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\log(c+dx^3)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\log(c+dx^3)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{\sqrt[3]{d}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{d}\,(bc-ad)^{2/3}} - \frac{\log(c+dx^3)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\right.}{2\sqrt[3]{d}\,(bc-ad)}$$

Mathematica [A]
time = 0.12, size = 162, normalized size = 1.12

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right) + \log\left((bc-ad)^{2/3} - \sqrt[3]{d}\,\sqrt[3]{bc-ad}\,\sqrt[3]{a+bx^3} + d^{2/3}(a+bx^3)^{2/3}\right)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)
)/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*
c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a +
 b*x^3)^(2/3)])/(d^(1/3)*(b*c - a*d)^(2/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^3+a)^{\frac{2}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(114) = 228.
time = 2.42, size = 927, normalized size = 6.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3
)^(1/3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3
 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3
```

```
)*(b*x^3 + a)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(
d*x^3 + c)) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(2
/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d)
 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) -
 (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3), 1/6*(6*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)
*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3
))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(2/
3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d)
+ (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) + 2*(b^2*c^2
*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) -
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.58, size = 221, normalized size = 1.52

$$\frac{(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd-\sqrt{3}\,ad^2}+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6(bcd-ad^2)}-\frac{\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c -
 a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d - sqrt(3)*a*d^2) + 1
/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*
c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d - a*d^2) - 1/3*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c - a
*d)
```

Mupad [B]
time = 4.85, size = 213, normalized size = 1.47

$$\frac{\ln\left(3\,d^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{3\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{\ln\left(3\,d^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}-\frac{\ln\left(3\,d^2\,{\left(b\,x^3+a\right)}^{1/3}+\frac{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] log(3*d^2*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(3*d^(1/3)*(a*d - b*c)^
(2/3)))/(3*d^(1/3)*(a*d - b*c)^(2/3)) + (log(3*d^2*(a + b*x^3)^(1/3) - ((3^
(1/2)*1i - 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2
)*1i - 1))/(6*d^(1/3)*(a*d - b*c)^(2/3)) - (log(3*d^2*(a + b*x^3)^(1/3) + (
(3^(1/2)*1i + 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))*(3^(
1/2)*1i + 1))/(6*d^(1/3)*(a*d - b*c)^(2/3))
```

3.736 $\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=245

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}c}+\frac{d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c(bc-ad)^{2/3}}-\frac{\log(x)}{2a^{2/3}c}+\frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a\ldots}\right)}{2a^{2/3}c\ldots}$$

```
[Out] -1/2*ln(x)/a^(2/3)/c+1/6*d^(2/3)*ln(d*x^3+c)/c/(-a*d+b*c)^(2/3)+1/2*ln(a^(1
/3)-(b*x^3+a)^(1/3))/a^(2/3)/c-1/2*d^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x
^3+a)^(1/3))/c/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/
a^(1/3)*3^(1/2))/a^(2/3)/c*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x
^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 88, 59, 631, 210, 31, 60}

$$-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}c}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c}-\frac{\log(x)}{2a^{2/3}c}+\frac{d^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c(bc-ad)^{2/3}}+\frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}}-\frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*c)) + (d^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(2/3)) - Log[x]/(2*a^(2/3)*c) + (d^(2/3
)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(2/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)
]/(2*a^(2/3)*c) - (d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)])/(2*c*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{\text{Subst}}{}$$

$$= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{d^{2/3}\log\left(\sqrt[3]{bc-}\right.}{2c(b}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}c} + \frac{d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c(bc-ad)^{2/3}} - \frac{\log(x)}{2a^{2/3}c} +$$

Mathematica [A]
time = 0.51, size = 308, normalized size = 1.26

$$\frac{-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2\sqrt{3}\,d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{2/3}} + \frac{2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{2/3}} - \frac{2d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{2/3}} - \frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{a^{2/3}} + \frac{d^{2/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{2/3}}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) +
 (2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(
1/3))/Sqrt[3]])/(b*c - a*d)^(2/3) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a
^(2/3) - (2*d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*
c - a*d)^(2/3) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3
)]/a^(2/3) + (d^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(2/3))/(6*c)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(192) = 384$.
time = 2.54, size = 472, normalized size = 1.93

$$-\frac{2\sqrt{3}a^2\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)+a^2\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}d^2+(bx^3+a)^{\frac{1}{3}}(bcd-ad^2)\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}+(b^2c^2-2abcd+a^2d^2)\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{2}{3}}\right)-2a^2\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}d-(bc-ad)\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}\right)+2\sqrt{3}(a^2)^{\frac{1}{6}}a\arctan\left(\frac{(a^2)^{\frac{1}{6}}\left(\sqrt{3}(a^2)^{\frac{1}{3}}a+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)}{3a^2}\right)+(a^2)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)-2(a^2)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)}{6a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/
3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2
*d^2))^(2/3) - sqrt(3)*d)/d) + a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(
1/3)*log((b*x^3 + a)^(2/3)*d^2 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)*(-d^2/(b
^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-d^
2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 2*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*
d + a^2*d^2))^(1/3)*log((b*x^3 + a)^(1/3)*d - (b*c - a*d)*(-d^2/(b^2*c^2 -
2*a*b*c*d + a^2*d^2))^(1/3)) + 2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*(a^2)^(1/
6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) +
 (a^2)^(2/3)*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a
^2)^(2/3)) - 2*(a^2)^(2/3)*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)))/(a^2*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.83, size = 321, normalized size = 1.31

$$\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)}-\frac{(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^2-\sqrt{3}\,acd}-\frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2-acd)}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}c}-\frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}c}+\frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b*c^2 - a*c*d) - (-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*
x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b
*c^2 - sqrt(3)*a*c*d) - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3)
+ (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c^2
 - a*c*d) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/
a^(1/3))/(a^(2/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3
) + a^(2/3))/(a^(2/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/
3)*c)
```

Mupad [B]
time = 4.94, size = 1413, normalized size = 5.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] log((((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (243*a*b^6*c
^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/3))
*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*d^5
*(a + b*x^3)^(1/3))*(1/(27*a^2*c^3))^(1/3) + log(- (((81*b^6*c^5*d^3 - 162*
a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*
a*b*c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^
4*d^5))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(2/3) - 9*b^5*c
^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3) - 6*b^4*d
^5*(a + b*x^3)^(1/3))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(
1/3) + log(((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 - 1/2)^2*((81*b^6*c^5*d^
3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 - 1/2)*(243*a*b^
6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/
3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*
d^5*(a + b*x^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*c^3))^(1/3) - log(
6*b^4*d^5*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 + 1/2)*(((3^(1/2)*1i)/2 + 1/2
)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) + ((3^(1/2)*1i)
/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(
1/(27*a^2*c^3))^(1/3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c
^3))^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*c^3))^(1/3) + (log(6*b^4*d^5*
```

```
(a + b*x^3)^(1/3) + ((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((81*b^6*c^5*d^3
 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - ((3^(1/2)*1i - 1)*(-d^2/(27*b^2*c
^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5
*c^5*d^4 + 486*a^3*b^4*c^4*d^5))/2)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54
*a*b*c^4*d))^(2/3))/4 - 9*b^5*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 -
 54*a*b*c^4*d))^(1/3))/2)*(3^(1/2)*1i - 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d
^2 - 54*a*b*c^4*d))^(1/3))/2 - (log(6*b^4*d^5*(a + b*x^3)^(1/3) - ((3^(1/2)
*1i + 1)*(((3^(1/2)*1i + 1)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*
x^3)^(1/3) + ((3^(1/2)*1i + 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*
c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^
5))/2)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(2/3))/4 - 9*b^5
*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3))/2)*(3^
(1/2)*1i + 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3))/2
```

3.737 $\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)} \, dx$

Optimal. Leaf size=299

$$-\frac{\sqrt[3]{a+bx^3}}{3acx^3}+\frac{(2bc+3ad)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}c^2}-\frac{d^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{2/3}}+\frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2}$$

```
[Out] -1/3*(b*x^3+a)^(1/3)/a/c/x^3+1/6*(3*a*d+2*b*c)*ln(x)/a^(5/3)/c^2-1/6*d^(5/3
)*ln(d*x^3+c)/c^2/(-a*d+b*c)^(2/3)-1/6*(3*a*d+2*b*c)*ln(a^(1/3)-(b*x^3+a)^(
1/3))/a^(5/3)/c^2+1/2*d^(5/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/
c^2/(-a*d+b*c)^(2/3)+1/9*(3*a*d+2*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3
))/a^(1/3)*3^(1/2))/a^(5/3)/c^2*3^(1/2)-1/3*d^(5/3)*arctan(1/3*(1-2*d^(1/3)
*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {457, 105, 162, 59, 631, 210, 31, 60}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(3ad+2bc)}{3\sqrt{3}a^{5/3}c^2}-\frac{(3ad+2bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2}+\frac{\log(x)(3ad+2bc)}{6a^{5/3}c^2}-\frac{d^{5/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{2/3}}-\frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}}+\frac{d^{5/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/3*(a + b*x^3)^(1/3)/(a*c*x^3) + ((2*b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*c^2) - (d^(5/3)*ArcT
an[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]
*c^2*(b*c - a*d)^(2/3)) + ((2*b*c + 3*a*d)*Log[x])/(6*a^(5/3)*c^2) - (d^(5/
3)*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(2/3)) - ((2*b*c + 3*a*d)*Log[a^(1/3)
 - (a + b*x^3)^(1/3)])/(6*a^(5/3)*c^2) + (d^(5/3)*Log[(b*c - a*d)^(1/3) + d
^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \frac{1}{3} \text{Subst}\left(\int \frac{1}{x^2 (a+bx)^{2/3} (c+dx)} \, dx, x, x^3\right)$$
$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{3}(2bc+3ad)+\frac{2bdx}{3}}{x(a+bx)^{2/3}(c+dx)} \, dx, x, x^3\right)}{3ac}$$
$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{d^2 \text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)} \, dx, x, x^3\right)}{3c^2} - \frac{(2bc+3ad)\text{Subst}\left(\int \right.}{9ac}$$
$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2} - \frac{d^{5/3}\log(c+dx^3)}{6c^2(bc-ad)^{2/3}} + \frac{d^{5/3}\text{Subst}\left(\int \frac{}{\sqrt[3]{b}}\right.}{2c}$$
$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2} - \frac{d^{5/3}\log(c+dx^3)}{6c^2(bc-ad)^{2/3}} - \frac{(2bc+3ad)\log\left(\right.}{6a^5}$$
$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}c^2} - \frac{d^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}}{\sqrt[3]{}}}{}\right)}{\sqrt{3}\,c^2(bc-}$$

Mathematica [A]
time = 0.78, size = 355, normalized size = 1.19

$$\frac{-\frac{6c\sqrt[3]{a+bx^3}}{ax^3}+\frac{2\sqrt{3}(2bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}-\frac{6\sqrt{3}d^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{2/3}}-\frac{2(2bc+3ad)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{5/3}}+\frac{6d^{5/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{2/3}}+\frac{(2bc+3ad)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{a^{5/3}}-\frac{3d^{5/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{2/3}}}{18c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-6*c*(a + b*x^3)^(1/3))/(a*x^3) + (2*Sqrt[3]*(2*b*c + 3*a*d)*ArcTan[(1 +
(2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(5/3) - (6*Sqrt[3]*d^(5/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*
d)^(2/3) - (2*(2*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(5/3) +
(6*d^(5/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^
(2/3) + ((2*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x
^3)^(2/3)])/a^(5/3) - (3*d^(5/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d
```

```
)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(2/3))/
(18*c^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(b\,x^3+a\right)^{\frac{2}{3}} \left(d\,x^3+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 562 vs. 2(239) = 478.
time = 4.42, size = 562, normalized size = 1.88

$$-\frac{6\sqrt{3}\,a^3 d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}} x^3 \arctan\left(-\frac{2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3\,d}\right) + 3\,a^3 d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}} x^3 \log\left((bx^3+a)^{\frac{2}{3}}d^2-(bx^3+a)^{\frac{1}{3}}(bcd-ad^2)\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}+(b^2c^2-2\,abcd+a^2d^2)\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}\right) - 6\,a^3 d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}} x^3 \log\left((bx^3+a)^{\frac{1}{3}}d+(bc-ad)\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\right) - 2\sqrt{3}\,(2\,abc+3\,a^2d)x^3\sqrt{-(-a^2)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\,(-a^2)^{\frac{1}{3}}a-2\sqrt{3}\,(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}\right)\sqrt{-(-a^2)^{\frac{1}{3}}}}{3\,a^2}\right) - (-a^2)^{\frac{2}{3}}(2\,bc+3\,ad)x^3\log\left((bx^3+a)^{\frac{2}{3}}a-(-a^2)^{\frac{1}{3}}a+(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}\right) + 2\,(-a^2)^{\frac{2}{3}}(2\,bc+3\,ad)x^3\log\left((bx^3+a)^{\frac{1}{3}}a-(-a^2)^{\frac{2}{3}}\right) + 6\,(bx^3+a)^{\frac{1}{3}}a^2c}{18\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*arct
an(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(d^2/(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2))^(2/3) - sqrt(3)*d)/d) + 3*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2
*d^2))^(1/3)*x^3*log((b*x^3 + a)^(2/3)*d^2 - (b*x^3 + a)^(1/3)*(b*c*d - a*d
^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 6*a^3*d*(d^2/(b^2*c^2
 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*log((b*x^3 + a)^(1/3)*d + (b*c - a*d)*(d
^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)) - 2*sqrt(3)*(2*a*b*c + 3*a^2*d)*
x^3*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) - (-a^2)^(2/3)*(2*b*c
 + 3*a*d)*x^3*log((b*x^3 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(-a^2)^(2/3)) + 2*(-a^2)^(2/3)*(2*b*c + 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a
- (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2*c)/(a^3*c^2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a+bx^3\right)^{\frac{2}{3}} \left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.71, size = 377, normalized size = 1.26

$$-\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^3-\sqrt{3}\,ac^2d}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}d\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3-ac^2d\right)}+\frac{\sqrt{3}\left(2\,bc+3\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{5}{3}}c^2}+\frac{\left(2\,bc+3\,ad\right)\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{5}{3}}c^2}-\frac{\left(2\,bc+3\,ad\right)\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{5}{3}}c^2}-\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{3\,acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c^3 - a*c^2*d) + (-b*c*d^2 + a*d^3)^(1/3)*d*arctan(1/3*sqrt(3)
*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sq
rt(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2
/3))/(b*c^3 - a*c^2*d) + 1/9*sqrt(3)*(2*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*
(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*c^2) + 1/18*(2*b*c + 3*a*d)*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(5/3)*c^2)
- 1/9*(2*b*c + 3*a*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*c^2) -
 1/3*(b*x^3 + a)^(1/3)/(a*c*x^3)
```

Mupad [B]
time = 11.14, size = 1959, normalized size = 6.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] log(- (((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*
d))/a - 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d -
 b*c)^2))^(1/3))*(d^5/(c^6*(a*d - b*c)^2))^(2/3))/9 + (b^5*d^4*(8*b^3*c^3 -
 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5/(c^6*(a*d -
 b*c)^2))^(1/3))/3 - (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 +
 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(d^5/(27*b^2*c^8 + 27*a^2*c
^6*d^2 - 54*a*b*c^7*d))^(1/3) + log(- (((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)
```

```
*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))*(-(3*a*d + 2*b*c)^3/(
a^5*c^6))^(2/3))/81 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 1
8*a^2*b*c*d^2))/(3*a^3*c))*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))/9 - (2*b^4
*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*
c*d^2))/(9*a^3*c^4))*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*d + 54*a^2*b*
c*d^2)/(729*a^5*c^6))^(1/3) + (log(((3^(1/2)*1i - 1)*((((27*b^5*c^3*d^3*(a
+ b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - (81*a*b^4*c^4*d^3*(3^
(1/2)*1i - 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d - b*c)^2))^(
1/3))/2)*(3^(1/2)*1i - 1)^2*(d^5/(c^6*(a*d - b*c)^2))^(2/3))/36 + (b^5*d^4*
(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5
/(c^6*(a*d - b*c)^2))^(1/3))/6 + (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 +
 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(3^(1/2)*1i - 1
)*(d^5/(27*b^2*c^8 + 27*a^2*c^6*d^2 - 54*a*b*c^7*d))^(1/3))/2 - (log(((3^(1
/2)*1i + 1)*((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a
*b*c*d))/a + (81*a*b^4*c^4*d^3*(3^(1/2)*1i + 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*
b*c*d)*(d^5/(c^6*(a*d - b*c)^2))^(1/3))/2)*(3^(1/2)*1i + 1)^2*(d^5/(c^6*(a*
d - b*c)^2))^(2/3))/36 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d
+ 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5/(c^6*(a*d - b*c)^2))^(1/3))/6 - (2*b^4*d
^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*
d^2))/(9*a^3*c^4))*(3^(1/2)*1i + 1)*(d^5/(27*b^2*c^8 + 27*a^2*c^6*d^2 - 54*
a*b*c^7*d))^(1/3))/2 + log((2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3
*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4) - (((((27*b^5*c^3*d^3*
(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(
(3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3
/(a^5*c^6))^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(2
/3))/81 - (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^
2))/(3*a^3*c))*((3^(1/2)*1i)/2 - 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))
/9)*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*d + 54*
a^2*b*c*d^2)/(729*a^5*c^6))^(1/3) - log((((((27*b^5*c^3*d^3*(a + b*x^3)^(1/
3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2
+ 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/
3))*((3^(1/2)*1i)/2 - 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(2/3))/81 + (b^5*
d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*
((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))/9 - (2*b^4*d^6
*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^
2))/(9*a^3*c^4))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^
2*c^2*d + 54*a^2*b*c*d^2)/(729*a^5*c^6))^(1/3) - (a + b*x^3)^(1/3)/(3*a*c*x
^3)
```

3.738 $\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=279

$$\frac{x^2\sqrt[3]{a+bx^3}}{3bd}+\frac{(3bc+2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}d^2}-\frac{c^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2(bc-ad)^{2/3}}+\frac{c^{5/3}\log\left(c+dx^3\right)}{6d^2(bc-ad)^{2/3}}+\frac{(3b}{}$$

[Out] 1/3*x^2*(b*x^3+a)^(1/3)/b/d+1/6*c^(5/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(2/3)+1/6*(2*a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d^2-1/2*c^(5/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(2/3)+1/9*(2*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d^2*3^(1/2)-1/3*c^(5/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^2/(-a*d+b*c)^(2/3)*3^(1/2)

Rubi [A]

time = 0.15, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {490, 598, 337, 503}

$$\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(2ad+3bc)}{3\sqrt{3}\,b^{5/3}d^2}-\frac{c^{5/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2(bc-ad)^{2/3}}+\frac{(2ad+3bc)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{6b^{5/3}d^2}+\frac{c^{5/3}\log\left(c+dx^3\right)}{6d^2(bc-ad)^{2/3}}-\frac{c^{5/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{2/3}}+\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] (x^2*(a + b*x^3)^(1/3))/(3*b*d) + ((3*b*c + 2*a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)*d^2) - (c^(5/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*d^2*(b*c - a*d)^(2/3)) + (c^(5/3)*Log[c + d*x^3])/(6*d^2*(b*c - a*d)^(2/3)) + ((3*b*c + 2*a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*b^(5/3)*d^2) - (c^(5/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d^2*(b*c - a*d)^(2/3))

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Simp[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
```

```
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = a^2\text{Subst}\left(\int \frac{x^7}{(1-bx^3)^2(c-(bc-ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{a\text{Subst}\left(\int \frac{x(2c+(bc+2ad)x^3)}{(1-bx^3)(c+(-bc+ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{a\text{Subst}\left(\int \left(\frac{(3bc+2ad)x}{ad(1-bx^3)} + \frac{3bc^2x}{ad(-c+(bc-ad)x^3)}\right)\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{c^2\text{Subst}\left(\int \frac{x}{-c+(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d^2} - (3bc+2ad)\text{Subs}\cdots$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{c^{5/3}\text{Subst}\left(\int \frac{1}{-\sqrt[3]{c}+\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d^2\sqrt[3]{bc-ad}} + c^{5/3}\text{Sub}\cdots$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2} - \frac{c^{5/3}\log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-\cdots}}{\sqrt[3]{a+b\cdots}}\right)}{3d^2(bc-ad)^{2/3}}\cdots$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2} - \frac{(3bc+2ad)\log\left(1+\frac{b\cdots}{(a+\cdots}\right)}{18b^{5/\cdots}}\cdots$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}d^2} - \frac{c^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-\cdots}}{\sqrt[3]{c}\,\sqrt[3]{a\cdots}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2(bc-ad)\cdots}\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.95, size = 471, normalized size = 1.69

$$\frac{\frac{12dx^2\sqrt[3]{a+bx^3}}{b} + \frac{4\sqrt{3}(3bc+2ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)}{b^{5/3}} + \frac{6\sqrt{-6-6i\sqrt{3}}\,c^{5/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{(bc-ad)^{2/3}} + \frac{4(3bc+2ad)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{b^{5/3}} + \frac{6\left(1-i\sqrt{3}\right)c^{5/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{2/3}} - \frac{2(3bc+2ad)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{5/3}} + \frac{3i\left(i+\sqrt{3}\right)c^{5/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{2/3}}}{36d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((12*d*x^2*(a + b*x^3)^(1/3))/b + (4*Sqrt[3]*(3*b*c + 2*a*d)*ArcTan[(Sqrt[3
]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(5/3) + (6*Sqrt[-6 - (6*
I)*Sqrt[3]]*c^(5/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/
3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(2/3) + (4*
(3*b*c + 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(5/3) + (6*(1 - I*
```

```
Sqrt[3])*c^(5/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b
*x^3)^(1/3)])/(b*c - a*d)^(2/3) - (2*(3*b*c + 2*a*d)*Log[b^(2/3)*x^2 + b^(1
/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(5/3) + ((3*I)*(I + Sqrt[3]
)*c^(5/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d
)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(
b*c - a*d)^(2/3))/(36*d^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(225) = 450$.
time = 5.29, size = 558, normalized size = 2.00

$$\frac{6\sqrt{3}\,b^{3}c\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\,(bx^{3}+a)^{\frac{1}{3}}(bc-ad)\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{2}{3}}+\sqrt{3}\,cx}{3\,cx}\right)+6\,(bx^{3}+a)^{\frac{1}{3}}b^{2}dx^{2}+6\,b^{3}c\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}\log\left(\frac{(bc-ad)\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}x+(bx^{3}+a)^{\frac{1}{3}}c}{x}\right)-3\,b^{3}c\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}\log\left(\frac{(b^{2}c^{2}-2\,abcd+a^{2}d^{2})\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{2}{3}}x^{2}+(bx^{3}+a)^{\frac{2}{3}}c^{2}-(bx^{3}+a)^{\frac{1}{3}}(bc^{2}-acd)\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}x}{x^{2}}\right)-2\sqrt{3}\,(3\,b^{2}c+2\,abd)(b^{2})^{\frac{1}{6}}\arctan\left(\frac{\left(\sqrt{3}\,(b^{2})^{\frac{1}{3}}bx+2\sqrt{3}\,(bx^{3}+a)^{\frac{1}{3}}(b^{2})^{\frac{2}{3}}\right)(b^{2})^{\frac{1}{6}}}{3\,b^{2}x}\right)+2\,(b^{2})^{\frac{2}{3}}(3\,bc+2\,ad)\log\left(-\frac{(b^{2})^{\frac{2}{3}}x-(bx^{3}+a)^{\frac{1}{3}}b}{x}\right)-(b^{2})^{\frac{2}{3}}(3\,bc+2\,ad)\log\left(\frac{(b^{2})^{\frac{1}{3}}bx^{2}+(bx^{3}+a)^{\frac{1}{3}}(b^{2})^{\frac{2}{3}}x+(bx^{3}+a)^{\frac{2}{3}}b}{x^{2}}\right)}{18\,b^{3}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*b^3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a
^2*d^2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*b^
3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b*c - a*d)*(-c^2/(b^
2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x + (b*x^3 + a)^(1/3)*c)/x) - 3*b^3*c*(
-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)*x^2 + (b*x^3 + a)^(2/3)*
c^2 - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^
2))^(1/3)*x)/x^2) - 2*sqrt(3)*(3*b^2*c + 2*a*b*d)*(b^2)^(1/6)*arctan(1/3*(s
```

```
qrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/
6)/(b^2*x)) + 2*(b^2)^(2/3)*(3*b*c + 2*a*d)*log(-((b^2)^(2/3)*x - (b*x^3 +
a)^(1/3)*b)/x) - (b^2)^(2/3)*(3*b*c + 2*a*d)*log(((b^2)^(1/3)*b*x^2 + (b*x^
3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^3*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a+bx^3)^{\frac{2}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**7/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.739 $\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=234

$$-\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}+\frac{c^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d(bc-ad)^{2/3}}-\frac{c^{2/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{2/3}}-\frac{\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}+$$

```
[Out] -1/6*c^(2/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(2/3)-1/2*ln(b^(1/3)*x-(b*x^3+a)^(1/3
))/b^(2/3)/d+1/2*c^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(
-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
2/3)/d*3^(1/2)+1/3*c^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^
3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {494, 337, 503}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}+\frac{c^{2/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d(bc-ad)^{2/3}}-\frac{\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{c^{2/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{2/3}}+\frac{c^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*d)
) + (c^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)
))/Sqrt[3]])/(Sqrt[3]*d*(b*c - a*d)^(2/3)) - (c^(2/3)*Log[c + d*x^3])/(6*d*
(b*c - a*d)^(2/3)) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3)*d) + (c^
(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d*(b*c - a
*d)^(2/3))
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
```

```
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = a\text{Subst}\left(\int \frac{x^4}{(1-bx^3)(c-(bc-ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d} - \frac{c\text{Subst}\left(\int \frac{x}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}\,d} - \frac{\text{Subst}\left(\int \frac{1-\sqrt[3]{b}\,x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx}}\right.}{3\sqrt[3]{b}\,d}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{c^{2/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+2}{1+\sqrt[3]{b}\,x}\right.}{}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6b^{2/3}d} + \frac{c^{2/3}\log\left(\sqrt[3]{c}\right.}{3d(bc}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d} + \frac{c^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d(bc-ad)^{2/3}} - \frac{\log\left(1-\frac{\sqrt{}}{\sqrt[3]{a}}\right.}{3b^{2/3}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.07, size = 423, normalized size = 1.81

$$\frac{-\frac{4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{a+bx^3}}\right)}{b^{2/3}} - \frac{2\sqrt{-6-6i\sqrt{3}}\,c^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{(bc-ad)^{2/3}} - \frac{4\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{b^{2/3}} + \frac{2i\left(i+\sqrt{3}\right)c^{2/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{2/3}} + \frac{2\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{2/3}} + \frac{\left(1-i\sqrt{3}\right)c^{2/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{2/3}}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-4*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])
/b^(2/3) - (2*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*
x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
)])/(b*c - a*d)^(2/3) - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/3) +
 ((2*I)*(I + Sqrt[3])*c^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c
^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(2/3) + (2*Log[b^(2/3)*x^2 + b^(1/3)
*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3) + ((1 - I*Sqrt[3])*c^(2/
3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a
*d)^(2/3))/(12*d)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(187) = 374$.
time = 2.63, size = 530, normalized size = 2.26

$$\frac{2\sqrt{3}\,b^{2}\left(\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\,(bx^{3}+a)^{\frac{1}{3}}(bc-ad)\left(\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{2}{3}}+\sqrt{3}\,cx}{3\,cx}\right)+2\,b^{2}\left(\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)\left(\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}x-(bx^{3}+a)^{\frac{1}{3}}c}{x}\right)-b^{2}\left(\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}\log\left(\frac{(b^{2}c^{2}-2\,abcd+a^{2}d^{2})\left(\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{2}{3}}x^{2}+(bx^{3}+a)^{\frac{2}{3}}c^{2}+(bx^{3}+a)^{\frac{1}{3}}(bc^{2}-acd)\left(\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}x}{x^{2}}\right)+2\sqrt{3}\,b\sqrt{-(-b^{2})^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\,(-b^{2})^{\frac{1}{3}}bx-2\sqrt{3}\,(bx^{3}+a)^{\frac{1}{3}}(-b^{2})^{\frac{2}{3}}\right)\sqrt{-(-b^{2})^{\frac{1}{3}}}}{3\,b^{2}x}\right)-2\,(-b^{2})^{\frac{2}{3}}\log\left(-\frac{(-b^{2})^{\frac{2}{3}}x-(bx^{3}+a)^{\frac{1}{3}}b}{x}\right)+(-b^{2})^{\frac{2}{3}}\log\left(-\frac{(-b^{2})^{\frac{1}{3}}bx^{2}-(bx^{3}+a)^{\frac{1}{3}}(-b^{2})^{\frac{2}{3}}x-(bx^{3}+a)^{\frac{2}{3}}b}{x^{2}}\right)}{6\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/3*
(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^
2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 2*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2
```

```
))^(1/3)*log(-((b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x -
(b*x^3 + a)^(1/3)*c)/x) - b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*l
og(((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(
2/3)*x^2 + (b*x^3 + a)^(2/3)*c^2 + (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(c^2/(
b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x)/x^2) + 2*sqrt(3)*b*sqrt(-(-b^2)^(1
/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-
b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 2*(-b^2)^(2/3)*log(-((-b^2)^(2/3
)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(2/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*
x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**4/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.740 $\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=149

$$-\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{c}\,(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}\,(bc-ad)^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}\,(bc-ad)^{2/3}}$$

```
[Out] 1/6*ln(d*x^3+c)/c^(1/3)/(-a*d+b*c)^(2/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(1/3)/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(1/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {503}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{c}\,(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}\,(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]
/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*
d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/
3)*(b*c - a*d)^(2/3))
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Subst}\left(\int \frac{x}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}\,\sqrt[3]{bc-ad}} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{c}-\sqrt[3]{bc-}}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}}\right.}{3\sqrt[3]{c}\,\sqrt[3]{}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}\,(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x,\right.}{6\sqrt[3]{c}\,(bc-ad)^{2/3}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}\,(bc-ad)^{2/3}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{c}\,(bc-ad)^{2/3}} + $$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{c}\,(bc-ad)^{2/3}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}\,(bc-ad)^{2/3}} + \frac{\log\left(c^{2/3}+\frac{(bc-}{(a+}\right.}{6\sqrt[3]{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.14, size = 255, normalized size = 1.71

$$\frac{2\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\,\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)+\left(1-i\sqrt{3}\right)\left(2\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)-\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)\right)}{12\sqrt[3]{c}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 - I*Sqrt[3]
)*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]
 - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(12*c^(
1/3)*(b*c - a*d)^(2/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + bx^3\right)^{\frac{2}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.741 $\int \frac{1}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=173

$$-\frac{\sqrt[3]{a+bx^3}}{acx}+\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)^{2/3}}-\frac{d\log\left(c+dx^3\right)}{6c^{4/3}(bc-ad)^{2/3}}+\frac{d\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}(bc-ad)^{2/3}}$$

```
[Out] -(b*x^3+a)^(1/3)/a/c/x-1/6*d*ln(d*x^3+c)/c^(4/3)/(-a*d+b*c)^(2/3)+1/2*d*ln(
(-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)/(-a*d+b*c)^(2/3)+1/3*d*
arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(4/3
)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {491, 12, 503}

$$\frac{d\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)^{2/3}}-\frac{d\log\left(c+dx^3\right)}{6c^{4/3}(bc-ad)^{2/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(a*c*x)) + (d*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(4/3)*(b*c - a*d)^(2/3)) - (d*
Log[c + d*x^3])/(6*c^(4/3)*(b*c - a*d)^(2/3)) + (d*Log[((b*c - a*d)^(1/3)*x
)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(4/3)*(b*c - a*d)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
```

```
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)} \, dx = \frac{\text{Subst}\left(\int \frac{1-bx^3}{x^2(c-(bc-ad)x^3)} \, dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{d\text{Subst}\left(\int \frac{x}{c+(-bc+ad)x^3} \, dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x} \, dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}\sqrt[3]{bc-ad}} + \frac{d\text{Subst}(\ldots}{\ldots}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\ldots}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-\ldots}}\ldots\right)}{6c^{4/3}\ldots}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\ldots\right.}{6c^{4/3}(bc-ad\ldots}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)^{2/3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x\ldots}{\sqrt[3]{a+bx^3}}\right.}{3c^{4/3}(bc-ad)^{2/3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.33, size = 308, normalized size = 1.78

$$\frac{-12\sqrt[3]{c}\,(bc-ad)^{2/3}\sqrt[3]{a+bx^3}-2\sqrt{-6-6i\sqrt{3}}\,adx\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+2i\left(i+\sqrt{3}\right)adx\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)+a\left(d-i\sqrt{3}\,d\right)x\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{12ac^{4/3}(bc-ad)^{2/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-12*c^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) - 2*Sqrt[-6 - (6*I)*Sqrt[3
]]*a*d*x*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I
 + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (2*I)*(I + Sqrt[3])*a*d*x*Log[2*(
b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + a*(d - I*
Sqrt[3]*d)*x*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c -
a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]
)/(12*a*c^(4/3)*(b*c - a*d)^(2/3)*x)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(b\,x^{3}+a \right)^{\frac{2}{3}} \left(d\,x^{3}+c \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a + b x^{3}\right)^{\frac{2}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/(x**2*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.742 $\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} dx$

Optimal. Leaf size=215

$$-\frac{\sqrt[3]{a+bx^3}}{4acx^4}+\frac{(3bc+4ad)\sqrt[3]{a+bx^3}}{4a^2c^2x}-\frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)^{2/3}}+\frac{d^2\log\left(c+dx^3\right)}{6c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{c}}\right.}{2c^{7/3}(bc-}$$

```
[Out] -1/4*(b*x^3+a)^(1/3)/a/c/x^4+1/4*(4*a*d+3*b*c)*(b*x^3+a)^(1/3)/a^2/c^2/x+1/
6*d^2*ln(d*x^3+c)/c^(7/3)/(-a*d+b*c)^(2/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)*x/c^
(1/3)-(b*x^3+a)^(1/3))/c^(7/3)/(-a*d+b*c)^(2/3)-1/3*d^2*arctan(1/3*(1+2*(-a
*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)/(-a*d+b*c)^(2/3)*
3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {491, 597, 12, 503}

$$\frac{\sqrt[3]{a+bx^3}\,(4ad+3bc)}{4a^2c^2x}-\frac{d^2\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)^{2/3}}+\frac{d^2\log\left(c+dx^3\right)}{6c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/4*(a + b*x^3)^(1/3)/(a*c*x^4) + ((3*b*c + 4*a*d)*(a + b*x^3)^(1/3))/(4*a
^2*c^2*x) - (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(
1/3)))/Sqrt[3]])/(Sqrt[3]*c^(7/3)*(b*c - a*d)^(2/3)) + (d^2*Log[c + d*x^3])
/(6*c^(7/3)*(b*c - a*d)^(2/3)) - (d^2*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (
a + b*x^3)^(1/3)])/(2*c^(7/3)*(b*c - a*d)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
```

```
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \frac{\text{Subst}\left(\int \frac{(1-bx^3)^2}{x^5(c-(bc-ad)x^3)} \, dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^5} + \frac{-bc-ad}{c^2x^2} + \frac{a^2d^2x}{c^2(c-(bc-ad)x^3)}\right) dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{(a+bx^3)^{4/3}}{4a^2cx^4} + \frac{d^2\text{Subst}\left(\int \frac{x}{c-(bc-ad)x^3} \, dx, x, \frac{x}{\sqrt[3]{a+b}}\right.}{c^2}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{(a+bx^3)^{4/3}}{4a^2cx^4} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x} \, dx, x, \right.}{3c^{7/3}\sqrt[3]{bc-ad}}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{(a+bx^3)^{4/3}}{4a^2cx^4} - \frac{d^2 \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}} + \frac{d^2\text{Su}}{}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{(a+bx^3)^{4/3}}{4a^2cx^4} - \frac{d^2 \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}} + \frac{d^2\log}{}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{(a+bx^3)^{4/3}}{4a^2cx^4} - \frac{d^2 \tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)^{2/3}} - \frac{d^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.66, size = 340, normalized size = 1.58

$$\frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(-ac+3bcx^3+4adx^3)}{a^2x^4} + \frac{2\sqrt{-6-6i\sqrt{3}}\,d^2\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{(bc-ad)^{2/3}} + \frac{2(1-i\sqrt{3})d^2\log\left(2\sqrt[3]{bc-ad}\,x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{2/3}} + \frac{i(i+\sqrt{3})d^2\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{2/3}}}{12c^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) + 3*b*c*x^3 + 4*a*d*x^3))/(a^2*x^4) +
 (2*Sqrt[-6 - (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b
*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d
)^(2/3) + (2*(1 - I*Sqrt[3])*d^2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3]
)*c^(1/3)*(a + b*x^3)^(1/3)]])/(b*c - a*d)^(2/3) + (I*(I + Sqrt[3])*d^2*Log[
2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a +
 b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(b*c - a*d)^(2/
3))/(12*c^(7/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5} \left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5} \left(a + b x^{3}\right)^{\frac{2}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/(x**5*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.743 $\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\,(a+bx^3)^{2/3}}$$

```
[Out] 1/7*x^7*(1+b*x^3/a)^(2/3)*AppellF1(7/3,2/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(2/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^7\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\,(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^7*(1 + (b*x^3)/a)^(2/3)*AppellF1[7/3, 2/3, 1, 10/3, -((b*x^3)/a), -((d*x
^3)/c)])/(7*c*(a + b*x^3)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{x^6}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)}\,dx}{(a+bx^3)^{2/3}}$$
$$= \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\,(a+bx^3)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 249 vs. $2(64) = 128$.
time = 7.76, size = 249, normalized size = 3.89

$$\frac{x\left(-\frac{(2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{bc}+4\left(\frac{a}{b}+x^3+\frac{4a^2c^2F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{b(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right)\right)}{8d\,(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(-(((2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(b*c)) + 4*(a/b + x^3 + (4*a^2*c^2*AppellF1[1/3
, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(b*(c + d*x^3)*(-4*a*c*AppellF1
[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2
/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)]))))))/(8*d*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] int(x^6/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.744 $\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\,(a+bx^3)^{2/3}}$$

```
[Out] 1/4*x^4*(1+b*x^3/a)^(2/3)*AppellF1(4/3,2/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(b*x^
3+a)^(2/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\,(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)}\,dx}{(a+bx^3)^{2/3}}$$
$$= \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\,(a+bx^3)^{2/3}}$$

Mathematica [A]
time = 7.03, size = 65, normalized size = 1.02

$$\frac{x^4\left(\frac{a+bx^3}{a}\right)^{2/3} F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\,(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
[Out] (x^4*((a + b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^3)^{\frac{2}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**3/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x^3/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.745 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=59

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\,(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(b*x^3+a)^(
2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\,(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)}\,dx}{(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\,(a+bx^3)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.
time = 10.04, size = 161, normalized size = 2.73

$$-\frac{4acxF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(a+bx^3)^{2/3}(c+dx^3)\left(-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
[Out] (-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*x
^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^3)^{\frac{2}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.746 $\int \frac{1}{x^3 (a+bx^3)^{2/3} (c+dx^3)} \, dx$

Optimal. Leaf size=64

$$-\frac{\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2 \left(a+bx^3\right)^{2/3}}$$

```
[Out] -1/2*(1+b*x^3/a)^(2/3)*AppellF1(-2/3,2/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(b*
x^3+a)^(2/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2 \left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*((1 + (b*x^3)/a)^(2/3)*AppellF1[-2/3, 2/3, 1, 1/3, -((b*x^3)/a), -((d*
x^3)/c)])/(c*x^2*(a + b*x^3)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3} \int \frac{1}{x^3 \left(1+\frac{bx^3}{a}\right)^{2/3} (c+dx^3)} \, dx}{(a+bx^3)^{2/3}}$$
$$= -\frac{\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2 (a+bx^3)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 338 vs. $2(64) = 128$.

time = 10.17, size = 338, normalized size = 5.28

$$\frac{-bdx^6\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(-4ac(ac+2bcx^3+3adx^3+bdx^6)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3(a+bx^3)(c+dx^3)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{(c+dx^3)\left(4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}}{8ac^2x^2(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)]) + (4*c*(-4*a*c*(a*c + 2*b*c*x^3 + 3*a*d*x^3 + b*d*x^6)*Appell
F1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(a + b*x^3)*(c + d*x
^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*A
ppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*
c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*Appel
lF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3
, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(8*a*c^2*x^2*(a + b*x^3)^(2/3))
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(b\,x^3+a\right)^{\frac{2}{3}} \left(d\,x^3+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a+bx^3)^{\frac{2}{3}} (c+dx^3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.747 $\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=347

$$-\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}}+\frac{a^2(a+bx^3)^{2/3}}{2b^4d}+\frac{a(bc+ad)\,(a+bx^3)^{2/3}}{2b^4d^2}+\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2b^4d^3}-\frac{2a(a$$

```
[Out] -a^4/b^4/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/2*a^2*(b*x^3+a)^(2/3)/b^4/d+1/2*a*(a*
d+b*c)*(b*x^3+a)^(2/3)/b^4/d^2+1/2*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(2/3
)/b^4/d^3-2/5*a*(b*x^3+a)^(5/3)/b^4/d-1/5*(a*d+b*c)*(b*x^3+a)^(5/3)/b^4/d^2
+1/8*(b*x^3+a)^(8/3)/b^4/d-1/6*c^4*ln(d*x^3+c)/d^(11/3)/(-a*d+b*c)^(4/3)+1/
2*c^4*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/3)/(-a*d+b*c)^(4/3
)+1/3*c^4*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2)
)/d^(11/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 89, 45, 58, 631, 210, 31}

$$-\frac{a^4}{b^4\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{a^2(a+bx^3)^{2/3}}{2b^4d}+\frac{(a+bx^3)^{2/3}\,(a^2d^2+abcd+b^2c^2)}{2b^4d^3}+\frac{c^4\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{11/3}(bc-ad)^{4/3}}+\frac{a(a+bx^3)^{2/3}\,(ad+bc)}{2b^4d^2}-\frac{(a+bx^3)^{5/3}\,(ad+bc)}{5b^4d^2}-\frac{2a(a+bx^3)^{5/3}}{5b^4d}+\frac{(a+bx^3)^{8/3}}{8b^4d}-\frac{c^4\log(c+dx^3)}{6d^{11/3}(bc-ad)^{4/3}}+\frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{11/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(a^4/(b^4*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a^2*(a + b*x^3)^(2/3))/(2*b^4
*d) + (a*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^4*d^2) + ((b^2*c^2 + a*b*c*d +
 a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^3) - (2*a*(a + b*x^3)^(5/3))/(5*b^4*d
) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^4*d^2) + (a + b*x^3)^(8/3)/(8*b^4*
d) + (c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt
[3]])/(Sqrt[3]*d^(11/3)*(b*c - a*d)^(4/3)) - (c^4*Log[c + d*x^3])/(6*d^(11/
3)*(b*c - a*d)^(4/3)) + (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1
/3)])/(2*d^(11/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a^4}{b^3(bc-ad)(a+bx)^{4/3}} + \frac{b^2c^2+abcd+a^2d^2}{b^3d^3\sqrt[3]{a+bx}} - \frac{(bc+ad)x}{b^2d^2\sqrt[3]{a+bx}}\right.\right.$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^3} + \frac{\text{Subst}\left(\int \frac{}{\sqrt[3]{a}}\right.}{}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^3} - \frac{c^4\log(c+d}{6d^{11/3}(bc-a}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d} + \frac{a(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^4d^2} + \frac{(b^2c^2}{}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d} + \frac{a(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^4d^2} + \frac{(b^2c^2}{}$$

Mathematica [A]
time = 0.95, size = 349, normalized size = 1.01

$$\frac{\frac{3d^{2/3}\left(-81a^4d^3+9a^3bd^2\left(c-3dx^3\right)+3a^2b^2d\left(4c^2+cdx^3+3d^2x^6\right)+b^4cx^3\left(20c^2-8cdx^3+5d^2x^6\right)+ab^3\left(20c^3+4c^2dx^3-cd^2x^6-5d^3x^9\right)\right)}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{40\sqrt{3}c^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{4/3}} + \frac{40c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}} - \frac{20c^4\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{(bc-ad)^{4/3}}}{120d^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((3*d^(2/3)*(-81*a^4*d^3 + 9*a^3*b*d^2*(c - 3*d*x^3) + 3*a^2*b^2*d*(4*c^2 +
 c*d*x^3 + 3*d^2*x^6) + b^4*c*x^3*(20*c^2 - 8*c*d*x^3 + 5*d^2*x^6) + a*b^3*
(20*c^3 + 4*c^2*d*x^3 - c*d^2*x^6 - 5*d^3*x^9)))/(b^4*(b*c - a*d)*(a + b*x^
3)^(1/3)) + (40*Sqrt[3]*c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
- a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) + (40*c^4*Log[(b*c - a*d)^(1/3) +
 d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) - (20*c^4*Log[(b*c - a*d)^(2
/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/
3)])/(b*c - a*d)^(4/3))/(120*d^(11/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(291) = 582.
time = 3.01, size = 1300, normalized size = 3.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/120*(60*sqrt(1/3)*(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c
^4*d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 -
b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3)
 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)
)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(
b*x^3 + a)^(1/3))/(d*x^3 + c)) + 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a
)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*
d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*
(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5
+ 81*a^5*d^6 + 5*(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c
^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*
```

```
c^4*d^2 - 16*a*b^4*c^3*d^3 - a^2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*
d^6)*x^3)*(b*x^3 + a)^(2/3))/(a*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7
 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^3), -1/120*(120*sqrt(1/3)*
(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c^4*d^2)*x^3)*sqrt(-(-b
*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d
+ (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d)
+ 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/
3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^
(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 +
 a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c
^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5 + 81*a^5*d^6 + 5*(b^5*c^2*d^4 -
 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a
^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*c^4*d^2 - 16*a*b^4*c^3*d^3 - a^
2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(a
*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6
 + a^2*b^5*d^7)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**14/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.77, size = 431, normalized size = 1.24

$$\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d^5-2\sqrt{3}\,abcd^6+\sqrt{3}\,a^2d^7}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2d^5-2\,abcd^6+a^2d^7\right)}+\frac{c^4\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)}-\frac{a^4}{\left(b^5c-ab^4d\right)\left(bx^3+a\right)^{\frac{1}{3}}}+\frac{20\left(bx^3+a\right)^{\frac{2}{3}}b^{30}c^2d^5-8\left(bx^3+a\right)^{\frac{5}{3}}b^{29}cd^6+40\left(bx^3+a\right)^{\frac{2}{3}}ab^{29}cd^6+5\left(bx^3+a\right)^{\frac{8}{3}}b^{28}d^7-24\left(bx^3+a\right)^{\frac{5}{3}}ab^{28}d^7+60\left(bx^3+a\right)^{\frac{2}{3}}a^2b^{28}d^7}{40\,b^{32}d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(2/3)*c^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b
*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^5 - 2*sqrt(3
)*a*b*c*d^6 + sqrt(3)*a^2*d^7) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^4*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^
(2/3))/(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7) + 1/3*c^4*(-(b*c - a*d)/d)^(2/
3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^3 - 2*a*
b*c*d^4 + a^2*d^5) - a^4/((b^5*c - a*b^4*d)*(b*x^3 + a)^(1/3)) + 1/40*(20*(
b*x^3 + a)^(2/3)*b^30*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^29*c*d^6 + 40*(b*x^3
+ a)^(2/3)*a*b^29*c*d^6 + 5*(b*x^3 + a)^(8/3)*b^28*d^7 - 24*(b*x^3 + a)^(5/
3)*a*b^28*d^7 + 60*(b*x^3 + a)^(2/3)*a^2*b^28*d^7)/(b^32*d^8)
```

Mupad [B]
time = 5.19, size = 564, normalized size = 1.63

$$\left(\frac{3\,a^2}{b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{2\,b^4\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3}-\left(\frac{4\,a}{5\,b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{5\,b^8\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^4\,d}+\frac{a^4}{b^4\,{\left(b\,x^3+a\right)}^{1/3}\,\left(a\,d-b\,c\right)}+\frac{c^4\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{c^8\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{9\,d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}}-\frac{\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{{\left(c^4+\sqrt{3}\,c^4\,1{}\mathrm{i}\right)}^2\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{36\,d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(c^4+\sqrt{3}\,c^4\,1{}\mathrm{i}\right)}{6\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}}+\frac{c^4\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{c^8\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c -
a*b^4*d))/(2*b^4*d))*(a + b*x^3)^(2/3) - ((4*a)/(5*b^4*d) + (b^5*c - a*b^4*
d)/(5*b^8*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^4*d) + a^4/(b^4*
(a + b*x^3)^(1/3)*(a*d - b*c)) + (c^4*log((a + b*x^3)^(1/3)*(a*c^8*d^5 - b*
c^9*d^4) - (c^8*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*a^2*b
^2*c^2*d^13 - 36*a^3*b*c*d^14))/(9*d^(22/3)*(a*d - b*c)^(8/3))))/(3*d^(11/3
)*(a*d - b*c)^(4/3)) - (log((a + b*x^3)^(1/3)*(a*c^8*d^5 - b*c^9*d^4) - ((3
^(1/2)*c^4*1i + c^4)^2*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 5
4*a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14))/(36*d^(22/3)*(a*d - b*c)^(8/3)))*(3^
(1/2)*c^4*1i + c^4))/(6*d^(11/3)*(a*d - b*c)^(4/3)) + (c^4*log((a + b*x^3)^
(1/3)*(a*c^8*d^5 - b*c^9*d^4) - (c^8*((3^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^15 +
 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14
))/(d^(22/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(11/3)*(a*d - b
*c)^(4/3))
```

3.748 $\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=253

$$\frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}}-\frac{a(a+bx^3)^{2/3}}{2b^3d}-\frac{(bc+ad)(a+bx^3)^{2/3}}{2b^3d^2}+\frac{(a+bx^3)^{5/3}}{5b^3d}-\frac{c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}(bc-ad)^{4/3}}$$

```
[Out] a^3/b^3/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/2*a*(b*x^3+a)^(2/3)/b^3/d-1/2*(a*d+b*c
)*(b*x^3+a)^(2/3)/b^3/d^2+1/5*(b*x^3+a)^(5/3)/b^3/d+1/6*c^3*ln(d*x^3+c)/d^(
8/3)/(-a*d+b*c)^(4/3)-1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/
d^(8/3)/(-a*d+b*c)^(4/3)-1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-
a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 89, 45, 58, 631, 210, 31}

$$\frac{a^3}{b^3\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{c^3\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}(bc-ad)^{4/3}}-\frac{(a+bx^3)^{2/3}(ad+bc)}{2b^3d^2}-\frac{a(a+bx^3)^{2/3}}{2b^3d}+\frac{(a+bx^3)^{5/3}}{5b^3d}+\frac{c^3\log(c+dx^3)}{6d^{8/3}(bc-ad)^{4/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] a^3/(b^3*(b*c - a*d)*(a + b*x^3)^(1/3)) - (a*(a + b*x^3)^(2/3))/(2*b^3*d) -
 ((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^3*d^2) + (a + b*x^3)^(5/3)/(5*b^3*d)
- (c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(4/3)) + (c^3*Log[c + d*x^3])/(6*d^(8/3)*(b
*c - a*d)^(4/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(8/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^3}{b^2(bc-ad)(a+bx)^{4/3}} + \frac{-bc-ad}{b^2d^2\sqrt[3]{a+bx}} + \frac{x}{bd\sqrt[3]{a+bx}} - \frac{}{d^2(}\right.\right.$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)\,(a+bx^3)^{2/3}}{2b^3d^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}}\,dx, x,\right.}{3bd}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)\,(a+bx^3)^{2/3}}{2b^3d^2} + \frac{c^3\log(c+dx^3)}{6d^{8/3}(bc-ad)^{4/3}} + \frac{\text{Subst}}{}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a(a+bx^3)^{2/3}}{2b^3d} - \frac{(bc+ad)\,(a+bx^3)^{2/3}}{2b^3d^2} + \frac{(a+bx^3)}{5b^3d}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a(a+bx^3)^{2/3}}{2b^3d} - \frac{(bc+ad)\,(a+bx^3)^{2/3}}{2b^3d^2} + \frac{(a+bx^3)}{5b^3d}$$

Mathematica [A]
time = 0.67, size = 298, normalized size = 1.18

$$\frac{\frac{3d^{2/3}\left(18a^3d^2+b^3cx^3(-5c+2dx^3)+3a^2bd(-c+2dx^3)-ab^2(5c^2+cdx^3+2d^2x^6)\right)}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{10\sqrt{3}\,c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{4/3}} - \frac{10c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}} + \frac{5c^3\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\,\sqrt[3]{bc-ad}\,\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{4/3}}}{30d^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((3*d^(2/3)*(18*a^3*d^2 + b^3*c*x^3*(-5*c + 2*d*x^3) + 3*a^2*b*d*(-c + 2*d*
x^3) - a*b^2*(5*c^2 + c*d*x^3 + 2*d^2*x^6)))/(b^3*(b*c - a*d)*(a + b*x^3)^(
1/3)) - (10*Sqrt[3]*c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*
d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) - (10*c^3*Log[(b*c - a*d)^(1/3) + d^(
1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) + (5*c^3*Log[(b*c - a*d)^(2/3) -
 d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/
(b*c - a*d)^(4/3))/(30*d^(8/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(208) = 416.
time = 2.19, size = 1141, normalized size = 4.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^
3*d^2)*x^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b
*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*s
qrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^
3 + a)^(1/3))/(d*x^3 + c)) - 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^
(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)
*d + (b*c*d^2 - a*d^3)^(2/3)) + 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d
^3)^(2/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 3*(5*a*b^3*c
^3*d^2 - 2*a^2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 -
 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^
2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d^4 - 2*a^2*b
```

```
^4*c*d^5 + a^3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3),
1/30*(30*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^3*
d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*
x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b
*c - a*d))/d) + 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^(2/3)*log((b*
x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2
 - a*d^3)^(2/3)) - 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^(2/3)*log
((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*(5*a*b^3*c^3*d^2 - 2*a^
2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 - 2*a*b^3*c*d^
4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^2*b^2*c*d^4 +
 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d^4 - 2*a^2*b^4*c*d^5 + a^
3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [A]
time = 1.40, size = 372, normalized size = 1.47

$$-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d^4-2\sqrt{3}\,abcd^5+\sqrt{3}\,a^2d^6}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^2d^4-2\,abcd^5+a^2d^6)}-\frac{c^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)}+\frac{a^3}{(b^4c-ab^3d)(bx^3+a)^{\frac{1}{3}}}-\frac{5\,(bx^3+a)^{\frac{2}{3}}b^{13}cd^3-2\,(bx^3+a)^{\frac{5}{3}}b^{12}d^4+10\,(bx^3+a)^{\frac{2}{3}}ab^{12}d^4}{10\,b^{15}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -(-b*c*d^2 + a*d^3)^(2/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^4 - 2*sqrt(
3)*a*b*c*d^5 + sqrt(3)*a^2*d^6) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x
^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)
^(2/3))/(b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6) - 1/3*c^3*(-(b*c - a*d)/d)^(2
/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^2 - 2*a
*b*c*d^3 + a^2*d^4) + a^3/((b^4*c - a*b^3*d)*(b*x^3 + a)^(1/3)) - 1/10*(5*(
b*x^3 + a)^(2/3)*b^13*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^12*d^4 + 10*(b*x^3 + a)
^(2/3)*a*b^12*d^4)/(b^15*d^5)
```

Mupad [B]
time = 5.16, size = 493, normalized size = 1.95

$$\frac{(b\,x^3+a)^{5/3}}{5\,b^3\,d}-\left(\frac{3\,a}{2\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{2\,b^6\,d^2}\right)(b\,x^3+a)^{2/3}-\frac{a^3}{b^3\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)}-\frac{c^3\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^6\,d^4-b\,c^7\,d^3)-\frac{c^6\,(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8)}{9\,d^{16/3}\,(a\,d-b\,c)^{8/3}}\right)}{3\,d^{8/3}\,(a\,d-b\,c)^{4/3}}+\frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^6\,d^4-b\,c^7\,d^3)-\frac{\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)^2\,(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8)}{36\,d^{16/3}\,(a\,d-b\,c)^{8/3}}\right)\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}{6\,d^{8/3}\,(a\,d-b\,c)^{4/3}}-\frac{c^3\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^6\,d^4-b\,c^7\,d^3)-\frac{c^6\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^2\,(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8)}{9\,d^{16/3}\,(a\,d-b\,c)^{8/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,d^{8/3}\,(a\,d-b\,c)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x)

[Out] (a + b*x^3)^(5/3)/(5*b^3*d) - ((3*a)/(2*b^3*d) + (b^4*c - a*b^3*d)/(2*b^6*d
^2))*(a + b*x^3)^(2/3) - a^3/(b^3*(a + b*x^3)^(1/3)*(a*d - b*c)) - (c^3*log
((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*(9*a^4*d^12 + 9*b^4*c^4*d
^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(16/3)
*(a*d - b*c)^(8/3))))/(3*d^(8/3)*(a*d - b*c)^(4/3)) + (log((a + b*x^3)^(1/3
)*(a*c^6*d^4 - b*c^7*d^3) - ((3^(1/2)*c^3*1i + c^3)^2*(9*a^4*d^12 + 9*b^4*c
^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(36*d^(
16/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(8/3)*(a*d - b*c)^(4
/3)) - (c^3*log((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*((3^(1/2)*
1i)/2 - 1/2)^2*(9*a^4*d^12 + 9*b^4*c^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*
c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(16/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/
2 - 1/2))/(3*d^(8/3)*(a*d - b*c)^(4/3))

3.749 $\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=203

$$-\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}}+\frac{(a+bx^3)^{2/3}}{2b^2d}+\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)^{4/3}}-\frac{c^2\log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-\right.}{2d^{5/3}}$$

```
[Out] -a^2/b^2/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/2*(b*x^3+a)^(2/3)/b^2/d-1/6*c^2*ln(d*
x^3+c)/d^(5/3)/(-a*d+b*c)^(4/3)+1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+
a)^(1/3))/d^(5/3)/(-a*d+b*c)^(4/3)+1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a
)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(5/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 89, 58, 631, 210, 31}

$$-\frac{a^2}{b^2\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{c^2\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)^{4/3}}+\frac{(a+bx^3)^{2/3}}{2b^2d}-\frac{c^2\log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(a^2/(b^2*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a + b*x^3)^(2/3)/(2*b^2*d) +
(c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(4/3)) - (c^2*Log[c + d*x^3])/(6*d^(5/3)*(b*c
 - a*d)^(4/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*d^(5/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^{4/3}} + \frac{1}{bd\sqrt[3]{a+bx}} + \frac{c^2}{d(-bc+ad)\sqrt[3]{a+bx}\,(\ldots}\right.\right.$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} - \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x\ldots\right.}{3d(bc-ad)}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} - \frac{c^2\log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} - \frac{c^2\log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2\log\left(\sqrt[3]{bc-\ldots}\right.}{2d^{5/\ldots}}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} + \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)^{4/3}} - \ldots$$

Mathematica [A]
time = 0.53, size = 254, normalized size = 1.25

$$\frac{\frac{3d^{2/3}(-3a^2d+b^2cx^3+ab(c-dx^3))}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{3}\,c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{4/3}} + \frac{2c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}} - \frac{c^2\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\,\sqrt[3]{bc-ad}\,\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{4/3}}}{6d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((3*d^(2/3)*(-3*a^2*d + b^2*c*x^3 + a*b*(c - d*x^3)))/(b^2*(b*c - a*d)*(a +
 b*x^3)^(1/3)) + (2*Sqrt[3]*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(
b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) + (2*c^2*Log[(b*c - a*d)^(1/3
) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) - (c^2*Log[(b*c - a*d)^(2
/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/
3)])/(b*c - a*d)^(4/3))/(6*d^(5/3))
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(b\,x^3+a\right)^{\frac{4}{3}}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(167) = 334$.
time = 1.77, size = 1004, normalized size = 4.95

$$\left[-\frac{3\sqrt{\frac{1}{3}}\left(ab^3c^3d - a^2b^2c^2d^2 + (b^4c^3d - ab^3c^2d^2)x^3\right)\sqrt{\frac{(-bcd^2+ad^3)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2bd^2x^3 - bcd + 3ad^2 + 3\sqrt{\frac{1}{3}}\left(2(-bcd^2+ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}(bcd-ad^2) + (-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\right)\sqrt{\frac{(-bcd^2+ad^3)^{\frac{1}{3}}}{bc-ad}} - 3(-bcd^2+ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\right) + (b^3c^2x^3+ab^2c^2)(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}d^2 + (-bcd^2+ad^3)^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}d + (-bcd^2+ad^3)^{\frac{2}{3}}\right) - 2(b^3c^2x^3+ab^2c^2)(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{1}{3}}d - (-bcd^2+ad^3)^{\frac{1}{3}}\right) - 3\left(ab^2c^2d^2 - 4a^2bcd^3 + 3a^3d^4 + (b^3c^2d^2 - 2ab^2cd^3 + a^2bd^4)x^3\right)(bx^3+a)^{\frac{2}{3}}}{6\left(ab^4c^2d^3 - 2a^2b^3cd^4 + a^3b^2d^5 + (b^5c^2d^3 - 2ab^4cd^4 + a^2b^3d^5)x^3\right)}, \text{[illegible]}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(a*b^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*
d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c
*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) +
(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*s
qrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x
^3 + a)^(1/3))/(d*x^3 + c)) + (b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^
(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3
)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*
c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*
d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5
 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*x^3), -1/6*(6*sqrt(1/3)*(a*b
^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*d^2)*x^3)*sqrt(-(-b*c*d
```

```
^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-
b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) + (b
^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2
+ (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3))
- 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3
)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*
d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*
b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4
+ a^2*b^3*d^5)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [A]
time = 1.03, size = 325, normalized size = 1.60

$$\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d^3-2\sqrt{3}\,abcd^4+\sqrt{3}\,a^2d^5}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^2d^3-2\,abcd^4+a^2d^5)}+\frac{c^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^2d-2\,abcd^2+a^2d^3)}-\frac{a^2}{(b^3c-ab^2d)(bx^3+a)^{\frac{1}{3}}}+\frac{(bx^3+a)^{\frac{2}{3}}}{2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(2/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b
*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^3 - 2*sqrt(3
)*a*b*c*d^4 + sqrt(3)*a^2*d^5) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^
(2/3))/(b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) + 1/3*c^2*(-(b*c - a*d)/d)^(2/
3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3) - a^2/((b^3*c - a*b^2*d)*(b*x^3 + a)^(1/3)) + 1/2*(b*x^3 +
 a)^(2/3)/(b^2*d)
```

Mupad [B]
time = 4.99, size = 449, normalized size = 2.21

$$\frac{(b\,x^3+a)^{2/3}}{2\,b^2\,d}+\frac{a^2}{b^2\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)}+\frac{c^2\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{c^4\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{9\,d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)}{3\,d^{5/3}\,(a\,d-b\,c)^{4/3}}-\frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{\left(c^2+\sqrt{3}\,c^2\,1{}\mathrm{i}\right)^2\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{36\,d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)\,\left(c^2+\sqrt{3}\,c^2\,1{}\mathrm{i}\right)}{6\,d^{5/3}\,(a\,d-b\,c)^{4/3}}+\frac{c^2\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{c^4\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)^2\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{d^{5/3}\,(a\,d-b\,c)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*b^2*d) + a^2/(b^2*(a + b*x^3)^(1/3)*(a*d - b*c)) + (c^
2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*(9*a^4*d^9 + 9*b^4*c
^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(9*d^(10/
3)*(a*d - b*c)^(8/3))))/(3*d^(5/3)*(a*d - b*c)^(4/3)) - (log((a + b*x^3)^(1
/3)*(a*c^4*d^3 - b*c^5*d^2) - ((3^(1/2)*c^2*1i + c^2)^2*(9*a^4*d^9 + 9*b^4*
c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(36*d^(1
0/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(5/3)*(a*d - b*c)^(4/
3)) + (c^2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*((3^(1/2)*1
i)/6 - 1/6)^2*(9*a^4*d^9 + 9*b^4*c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^
2*d^7 - 36*a^3*b*c*d^8))/(d^(10/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/6 - 1/
6))/(d^(5/3)*(a*d - b*c)^(4/3))
```

3.750 $\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=174

$$\frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}}-\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)^{4/3}}+\frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}$$

```
[Out] a/b/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/6*c*ln(d*x^3+c)/d^(2/3)/(-a*d+b*c)^(4/3)-1
/2*c*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(2/3)/(-a*d+b*c)^(4/3)-
1/3*c*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^
(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 79, 58, 631, 210, 31}

$$-\frac{c\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)^{4/3}}+\frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}+\frac{a}{b\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] a/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(2/3)*(b*c - a*d)^(4/3)) +
(c*Log[c + d*x^3])/(6*d^(2/3)*(b*c - a*d)^(4/3)) - (c*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3(bc-ad)}$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt{}\right.}{2d^{2/3}(bc-ad)^{4/3}}$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx}\right.}{2d^{2/3}(bc-ad)^{4/3}}$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} - \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)^{4/3}} + \frac{c\log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}} -$$

Mathematica [A]
time = 0.35, size = 227, normalized size = 1.30

$$\frac{1}{6}\left(\frac{6a}{(b^2c-abd)\sqrt[3]{a+bx^3}} - \frac{2\sqrt{3}\,c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}(bc-ad)^{4/3}} - \frac{2c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{d^{2/3}(bc-ad)^{4/3}} + \frac{c\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{d^{2/3}(bc-ad)^{4/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((6*a)/((b^2*c - a*b*d)*(a + b*x^3)^(1/3)) - (2*Sqrt[3]*c*ArcTan[(1 - (2*d^
(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^
(4/3)) - (2*c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(d^(2/3)*
(b*c - a*d)^(4/3)) + (c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(
a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(d^(2/3)*(b*c - a*d)^(4/3)))
/6
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(141) = 282$.
time = 2.64, size = 872, normalized size = 5.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 + (b^3*c^2*d - a*b^2*c*d^2)*x
^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3
*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3
+ a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*
c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(
1/3))/(d*x^3 + c)) - (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3
 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 -
a*d^3)^(2/3)) + 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3 +
a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 6*(a*b*c*d^2 - a^2*d^3)*(b*x^3 + a)
^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^4*c^2*d^2 - 2*a*b
^3*c*d^3 + a^2*b^2*d^4)*x^3), 1/6*(6*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 +
 (b^3*c^2*d - a*b^2*c*d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*a
rctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c
*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) + (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)
^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3
)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2
/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 6*(a*b*c*d^2 - a^2
*d^3)*(b*x^3 + a)^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^
4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(141) = 282.
time = 0.92, size = 301, normalized size = 1.73

$$-\frac{\frac{6\,(-bcd^2+ad^3)^{\frac{2}{3}}\,bc\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d^2-2\sqrt{3}\,abcd^3+\sqrt{3}\,a^2d^4}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\,bc\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{b^2c^2d^2-2\,abcd^3+a^2d^4}+\frac{2\,bc\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^2c^2-2\,abcd+a^2d^2}-\frac{6\,a}{(bx^3+a)^{\frac{1}{3}}(bc-ad)}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^2 -
2*sqrt(3)*a*b*c*d^3 + sqrt(3)*a^2*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4) + 2*b*c*(-(b*c - a*d)/d)^(
2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2 - 2*a*b*
c*d + a^2*d^2) - 6*a/((b*x^3 + a)^(1/3)*(b*c - a*d)))/b
```

Mupad [B]
time = 4.98, size = 412, normalized size = 2.37

$$-\frac{a}{b\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)}-\frac{c\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{c^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{9\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)}{3\,d^{2/3}\,(a\,d-b\,c)^{4/3}}+\frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{(c-\sqrt{3}\,c\,1{}\mathrm{i})^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c-\sqrt{3}\,c\,1{}\mathrm{i})}{6\,d^{2/3}\,(a\,d-b\,c)^{4/3}}+\frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{(c+\sqrt{3}\,c\,1{}\mathrm{i})^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c+\sqrt{3}\,c\,1{}\mathrm{i})}{6\,d^{2/3}\,(a\,d-b\,c)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] (log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - ((c - 3^(1/2)*c*1i)^2*(9*a^4
*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d
^5))/(36*d^(4/3)*(a*d - b*c)^(8/3)))*(c - 3^(1/2)*c*1i))/(6*d^(2/3)*(a*d -
b*c)^(4/3)) - (c*log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - (c^2*(9*a^4*
d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^
5))/(9*d^(4/3)*(a*d - b*c)^(8/3))))/(3*d^(2/3)*(a*d - b*c)^(4/3)) - a/(b*(a
 + b*x^3)^(1/3)*(a*d - b*c)) + (log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d)
 - ((c + 3^(1/2)*c*1i)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54
*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(36*d^(4/3)*(a*d - b*c)^(8/3)))*(c + 3^
(1/2)*c*1i))/(6*d^(2/3)*(a*d - b*c)^(4/3))
```

3.751 $\int \frac{x^2}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=167

$$-\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}}+\frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc-ad)^{4/3}}-\frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

```
[Out] -1/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d^(1/3)*ln(d*x^3+c)/(-a*d+b*c)^(4/3)+1/2*
d^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/(-a*d+b*c)^(4/3)+1/3*d
^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/(
-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {455, 53, 58, 631, 210, 31}

$$\frac{\sqrt[3]{d}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc-ad)^{4/3}}-\frac{1}{\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a +
 b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3)) - (
d^(1/3)*Log[c + d*x^3])/(6*(b*c - a*d)^(4/3)) + (d^(1/3)*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3(bc-ad)}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d}\,\log(c+dx^3)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, \ldots\right)}{2(bc-ad)^{4/3}}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d}\,\log(c+dx^3)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+\ldots}\right)}{2(bc-ad)^{4/3}}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\,\log(c+dx^3 \ldots}{6(bc-ad)^{4/3}}$$

Mathematica [A]
time = 0.26, size = 220, normalized size = 1.32

$$-\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\,\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{6(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a +
 b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3)) + (
d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*(b*c - a*d)^
(4/3)) - (d^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*
x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*(b*c - a*d)^(4/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]

time = 1.79, size = 262, normalized size = 1.57

$$-\frac{2\sqrt{3}\left(bx^3+a\right)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\left(bx^3+a\right)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}d-(bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\right)+2\left(bx^3+a\right)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left((bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}d\right)+6\left(bx^3+a\right)^{\frac{2}{3}}}{6\left(\left(b^2c-abd\right)x^3+abc-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(b*x^3 + a)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^
3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - (b*x^3 + a)*(-d/(b*c -
 a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b
*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 2*(b*x^3 + a)*(-d
/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1
/3)*d) + 6*(b*x^3 + a)^(2/3))/((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(135) = 270.

time = 0.69, size = 285, normalized size = 1.71

$$\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d-2\sqrt{3}\,abcd^2+\sqrt{3}\,a^2d^3}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2d-2\,abcd^2+a^2d^3\right)}-\frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/
3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(
1/3))/(sqrt(3)*b^2*c^2*d - 2*sqrt(3)*a*b*c*d^2 + sqrt(3)*a^2*d^3) - 1/6*(-b
*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*
d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) -
 1/((b*x^3 + a)^(1/3)*(b*c - a*d))
```

Mupad [B]
time = 4.92, size = 389, normalized size = 2.33

$$\frac{1}{(b\,x^3+a)^{1/3}\,(a\,d-b\,c)}+\frac{d^{1/3}\,\ln\left((b\,x^3+a)^{1/3}\,(a\,d^4-b\,c\,d^3)-\frac{d^{2/3}\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{9\,(a\,d-b\,c)^{8/3}}\right)}{3\,(a\,d-b\,c)^{4/3}}-\frac{d^{1/3}\,\ln\left((b\,x^3+a)^{1/3}\,(a\,d^4-b\,c\,d^3)-\frac{d^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{9\,(a\,d-b\,c)^{8/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,(a\,d-b\,c)^{4/3}}+\frac{d^{1/3}\,\ln\left((b\,x^3+a)^{1/3}\,(a\,d^4-b\,c\,d^3)-\frac{d^{2/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{(a\,d-b\,c)^{8/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{(a\,d-b\,c)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] 1/((a + b*x^3)^(1/3)*(a*d - b*c)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 -
 b*c*d^3) - (d^(2/3)*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2
*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3))))/(3*(a*d - b*c)^(4/3
)) - (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^3) - (d^(2/3)*((3^(1/2)*
1i)/2 + 1/2)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c
^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/2 + 1/2))/(3
*(a*d - b*c)^(4/3)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^3) - (d
^(2/3)*((3^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d
^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(a*d - b*c)^(8/3))*((3^(1/2)*1i)
/6 - 1/6))/(a*d - b*c)^(4/3)
```

3.752 $\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=271

$$\frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}c}-\frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c(bc-ad)^{4/3}}-\frac{\log(x)}{2a^{4/3}c}+\frac{d^{4/3}\log(c+\cdots}{6c(bc-a\cdots}$$

```
[Out] b/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/2*ln(x)/a^(4/3)/c+1/6*d^(4/3)*ln(d*x^3+c)/
c/(-a*d+b*c)^(4/3)+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(4/3)/c-1/2*d^(4/3)*ln
((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/(-a*d+b*c)^(4/3)+1/3*arctan(1/
3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/c*3^(1/2)-1/3*d^(4/3
)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/(-a*
d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {457, 87, 162, 57, 631, 210, 31, 58}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}c}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c}-\frac{\log(x)}{2a^{4/3}c}-\frac{d^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c(bc-ad)^{4/3}}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}+\frac{b}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] b/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3)
)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(4/3)*c) - (d^(4/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(4/3
)) - Log[x]/(2*a^(4/3)*c) + (d^(4/3)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(4/3)
) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(4/3)*c) - (d^(4/3)*Log[(b*c - a*
d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\text{Subst}\left(\int \frac{-bc+ad-bdx}{x\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3ac} + \frac{d^2\text{Subst}\left(\int \frac{}{\sqrt[3]{a}}\right.}{3c(}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,\right.}{2a^{4/3}c}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right.}{2a^{4/3}c}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}c} - \frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{}}{\sqrt[3]{bc}}}{\sqrt{3}}\right.}{\sqrt{3}\,c(bc-ad)}$$

Mathematica [A]
time = 0.98, size = 350, normalized size = 1.29

$$\frac{1}{6}\left(\frac{6b}{\left(abc-a^2d\right)\sqrt[3]{a+bx^3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}c}-\frac{2\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{c(bc-ad)^{4/3}}+\frac{2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{4/3}c}-\frac{2d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{c(bc-ad)^{4/3}}-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{a^{4/3}c}+\frac{d^{4/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{c(bc-ad)^{4/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[3]*ArcTan[(1 + (2*(a +
 b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(a^(4/3)*c) - (2*Sqrt[3]*d^(4/3)*ArcTan[(
1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(c*(b*c - a*
d)^(4/3)) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/(a^(4/3)*c) - (2*d^(4/3)*
Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(c*(b*c - a*d)^(4/3)) -
 Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(a^(4/3)*c) +
 (d^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/
3) + d^(2/3)*(a + b*x^3)^(2/3)])/(c*(b*c - a*d)^(4/3)))/6
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(216) = 432.
time = 2.49, size = 975, normalized size = 3.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 3*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c -
 a^2*b*d)*x^3)*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(
2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 +
 a)^(1/3)*a^(2/3) + 3*a)/x^3) + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a
*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3
*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(
2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 + a*b*
c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a^3*d)
*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(
2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a^2*b*
d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3)
+ (b*x^3 + a)^(1/3)*d))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^
3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*
c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3)
- 1/3*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 +
```

```
 a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 +
 a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a
^3*d)*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*
d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a
^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(
2/3) + (b*x^3 + a)^(1/3)*d) + 6*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c - a^2
*b*d)*x^3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3
))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [A]
time = 0.79, size = 389, normalized size = 1.44

$$-\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^3-2\,abc^2d+a^2cd^2)}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^3-2\,\sqrt{3}\,abc^2d+\sqrt{3}\,a^2cd^2}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^3-2\,abc^2d+a^2cd^2)}+\frac{b}{(bx^3+a)^{\frac{1}{3}}(abc-a^2d)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}c}-\frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}c}+\frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{4}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - (-b*c*d^2 + a*d^3)^(2/3)*ar
ctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*
d)/d)^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2) +
1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b
*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c
*d^2) + b/((b*x^3 + a)^(1/3)*(a*b*c - a^2*d)) + 1/3*sqrt(3)*arctan(1/3*sqrt
(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(4/3)*c) - 1/6*log((b*x^3 +
 a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c) + 1/3*log(abs(
(b*x^3 + a)^(1/3) - a^(1/3)))/(a^(4/3)*c)
```

Mupad [B]
time = 5.34, size = 2500, normalized size = 9.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x)

[Out] log(9*a^7*b^14*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - (-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3) - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c^2*d^13)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3) + log(9*a^7*b^14*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - (1/(27*a^4*c^3))^(2/3)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))*(1/(27*a^4*c^3))^(1/3) - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c^2*d^13)*(1/(27*a^4*c^3))^(1/3) + (log(((3^(1/2)*1i - 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3)*((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - ((3^(1/2)*1i - 1)^2*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))/4))/2 - 9*a^7*b^14*c^11*d^4 + 90*a^8*b^13*c^10*d^5 - 405*a^9*b^12*c^9*d^6 + 1071*a^10*b^11*c^8*d^7 - 1827*a^11*b^10*c^7*d^8 + 2079*a^12*b^9*c^6*d^9 - 1575*a^13*b^8*c^5*d^10 + 765*a^14*b^7*c^4*d^11 - 216*a^15*b

^6*c^3*d^12 + 27*a^16*b^5*c^2*d^13)*(3^(1/2)*1i - 1)*(-d^4/(27*b^4*c^7 + 27
*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^
(1/3))/2 - (log(((3^(1/2)*1i + 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*
a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3)*((a + b*x^3)^
(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^
5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*
d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d
^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14
) - ((3^(1/2)*1i + 1)^2*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*
d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*(243*a^10*b^15*c^15*d^3
 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^1
2*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16
*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a
^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))/4))/2 +
 9*a^7*b^14*c^11*d^4 - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a
^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^
13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b
^5*c^2*d^13)*(3^(1/2)*1i + 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*
b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3))/2 - b/((a + b*x^
3)^(1/3)*(a^2*d - a*b*c)) + log(((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 -
297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 +
8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 -
 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 +
 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) ...

3.753 $\int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)} dx$

Optimal. Leaf size=357

$$-\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}}-\frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}}-\frac{1}{3acx^3\sqrt[3]{a+bx^3}}-\frac{(4bc+3ad)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}c^2}+\frac{d^{7/3}\,\mathrm{t}}{}$$

```
[Out] -d^2/c^2/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*(-3*a*d-4*b*c)/a^2/c^2/(b*x^3+a)^(1
/3)-1/3/a/c/x^3/(b*x^3+a)^(1/3)+1/6*(3*a*d+4*b*c)*ln(x)/a^(7/3)/c^2-1/6*d^(
7/3)*ln(d*x^3+c)/c^2/(-a*d+b*c)^(4/3)-1/6*(3*a*d+4*b*c)*ln(a^(1/3)-(b*x^3+a
)^(1/3))/a^(7/3)/c^2+1/2*d^(7/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3
))/c^2/(-a*d+b*c)^(4/3)-1/9*(3*a*d+4*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(
1/3))/a^(1/3)*3^(1/2))/a^(7/3)/c^2*3^(1/2)+1/3*d^(7/3)*arctan(1/3*(1-2*d^(1
/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {457, 105, 162, 53, 57, 631, 210, 31, 58}

$$-\frac{\mathrm{ArcTan}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(3ad+4bc)}{3\sqrt{3}\,a^{7/3}c^2}-\frac{(3ad+4bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2}+\frac{\log(x)(3ad+4bc)}{6a^{7/3}c^2}-\frac{3ad+4bc}{3a^2c^2\sqrt[3]{a+bx^3}}+\frac{d^{7/3}\mathrm{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^2(bc-ad)^{4/3}}-\frac{d^{7/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{4/3}}+\frac{d^{7/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{4/3}}-\frac{d^2}{c^2\sqrt[3]{a+bx^3}(bc-ad)}-\frac{1}{3acx^3\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(d^2/(c^2*(b*c - a*d)*(a + b*x^3)^(1/3))) - (4*b*c + 3*a*d)/(3*a^2*c^2*(a
+ b*x^3)^(1/3)) - 1/(3*a*c*x^3*(a + b*x^3)^(1/3)) - ((4*b*c + 3*a*d)*ArcTan
[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*c^2
) + (d^(7/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]])/(Sqrt[3]*c^2*(b*c - a*d)^(4/3)) + ((4*b*c + 3*a*d)*Log[x])/(6*a^(7
/3)*c^2) - (d^(7/3)*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(4/3)) - ((4*b*c + 3
*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(7/3)*c^2) + (d^(7/3)*Log[(b*c
 - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{3}(4bc+3ad)+\frac{4bdx}{3}}{x(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{d^2\text{Subst}\left(\int \frac{1}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)}{3c^2} - \frac{(4bc+3ad)\text{Sub}}{}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{d^3\text{Subst}\Big(}{}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{(4bc+3a}{6a^{7/}}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{(4bc+3a}{6a^{7/}}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{(4bc+3a}{}$$

Mathematica [A]

time = 1.12, size = 392, normalized size = 1.10

$$\frac{\frac{6c(-a^2d+4b^2cx^3+ab(c-dx^3))}{a^2(-bc+ad)x^3\sqrt[3]{a+bx^3}}-\frac{2\sqrt{3}\,(4bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}}+\frac{6\sqrt{3}\,d^{7/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{4/3}}-\frac{2(4bc+3ad)\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{7/3}}+\frac{6d^{7/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}}+\frac{(4bc+3ad)\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{a^{7/3}}-\frac{3d^{7/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\,\sqrt[3]{bc-ad}\,\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{4/3}}}{18c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((6*c*(-(a^2*d) + 4*b^2*c*x^3 + a*b*(c - d*x^3)))/(a^2*(-(b*c) + a*d)*x^3*(
a + b*x^3)^(1/3)) - (2*Sqrt[3]*(4*b*c + 3*a*d)*ArcTan[(1 + (2*(a + b*x^3)^(
1/3))/a^(1/3))/Sqrt[3]])/a^(7/3) + (6*Sqrt[3]*d^(7/3)*ArcTan[(1 - (2*d^(1/3
)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) - (2*(4
*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(7/3) + (6*d^(7/3)*Log[(
b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) + ((4*b*c
+ 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/a^(7
/3) - (3*d^(7/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x
^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(18*c^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(b\,x^3+a\right)^{\frac{4}{3}} \left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(292) = 584$.
time = 3.91, size = 1386, normalized size = 3.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*((4*a*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*
b^2*c^2 - a^3*b*c*d - 3*a^4*d^2)*x^3)*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3*s
qrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/3)
*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) - 6*sqr
t(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3
)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) + ((4*b^3*c^2 - a
*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-
a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3))
 - 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d
- 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(a^3*b
*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c
- a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c
- a*d))^(1/3)) - 6*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log
((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*(a^2*b*c^2 -
 a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)*x^3)*(b*x^3 + a)^(2/3))/((a^3*b^2*c^3
- a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/18*(6*sqrt(1/3)*((4*a
*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*b^2*c^2 - a^3*b*c*d - 3*
a^4*d^2)*x^3)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (
-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(
-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))
^(1/3) + 1/3*sqrt(3)) - ((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b
^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*
x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2
*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*
x^3 + a)^(1/3) + (-a)^(1/3)) - 3*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a
*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x
^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*(a^3*b*d^2*x^6 +
a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3)
+ (b*x^3 + a)^(1/3)*d) + 6*(a^2*b*c^2 - a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)
*x^3)*(b*x^3 + a)^(2/3))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^
5*c^2*d)*x^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^3)^{\frac{4}{3}} (c + dx^3)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [**A**]

time = 0.76, size = 486, normalized size = 1.36

$$\frac{d^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^4-2\,abc^3d+a^2c^2d^2)}+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^4-2\sqrt{3}\,abc^3d+\sqrt{3}\,a^2c^2d^2}-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}d\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^4-2\,abc^3d+a^2c^2d^2)}-\frac{4\,(bx^3+a)b^2c-3\,ab^2c-(bx^3+a)abd}{3\,(a^2bc^2-a^3cd)\left((bx^3+a)^{\frac{4}{3}}-(bx^3+a)^{\frac{1}{3}}a\right)}-\frac{\sqrt{3}\,(4\,bc+3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}c^2}-\frac{\left(4\,a^{\frac{1}{3}}bc+3\,a^{\frac{4}{3}}d\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{8}{3}}c^2}+\frac{\left(4\,a^{\frac{2}{3}}bc+3\,a^{\frac{5}{3}}d\right)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)
^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*d
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^
2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/
3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d
+ a^2*c^2*d^2) - 1/3*(4*(b*x^3 + a)*b^2*c - 3*a*b^2*c - (b*x^3 + a)*a*b*d)/
((a^2*b*c^2 - a^3*c*d)*((b*x^3 + a)^(4/3) - (b*x^3 + a)^(1/3)*a)) - 1/9*sqr
t(3)*(4*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/(a^(7/3)*c^2) - 1/9*(4*a^(1/3)*b*c + 3*a^(4/3)*d)*log(abs((b*x^3 + a)
^(1/3) - a^(1/3)))/(a^(8/3)*c^2) + 1/18*(4*a^(2/3)*b*c + 3*a^(5/3)*d)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^3*c^2)
```

Mupad [B]
time = 6.39, size = 2500, normalized size = 7.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] log((d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^
8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((a + b*x^3)^(
1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^1
6*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^
7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^
21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^
12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^
26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + (d^7/(27*b^4*c^10 + 27*a^4*
c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)
*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21
*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*
d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 258280
3260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^
6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(
d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2
 - 108*a*b^3*c^9*d))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^1
5*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 155
```

29887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3) + (b^2/(a^2*d - a*b*c) + (b*(a + b*x^3)*(a*d - 4*b*c))/(3*a^2*c*(a*d - b*c)))/(a*(a + b*x^3)^(1/3) - (a + b*x^3)^(4/3)) + log((-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + (-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) + (log(((3^(1/2)*1i - 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((3^(1/2)*1i - 1)*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3)*((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i - 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*

```
d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(4782969*a^19*b^15*c^24
*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423
490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^
10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10
+ 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^2...
```

3.754 $\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=322

$$\frac{ax^4}{b(bc-ad)\sqrt[3]{a+bx^3}}+\frac{(bc-4ad)x\left(a+bx^3\right)^{2/3}}{3b^2d(bc-ad)}-\frac{(3bc+4ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{7/3}d^2}+\frac{c^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc\ldots}}{\sqrt[3]{c}\,\sqrt[3]{a\ldots}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2(bc-ad\ldots}$$

```
[Out] a*x^4/b/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*(-4*a*d+b*c)*x*(b*x^3+a)^(2/3)/b^2/d
/(-a*d+b*c)+1/6*c^(7/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(4/3)-1/2*c^(7/3)*ln((-a
*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(4/3)+1/6*(4*a*d+3*
b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)/d^2-1/9*(4*a*d+3*b*c)*arctan(1/
3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d^2*3^(1/2)+1/3*c^(7/3)*
arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^2/(-
a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {481, 596, 544, 245, 384}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(4ad+3bc)}{3\sqrt{3}\,b^{7/3}d^2}+\frac{c^{7/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2(bc-ad)^{4/3}}+\frac{(4ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{7/3}d^2}+\frac{x\left(a+bx^3\right)^{2/3}(bc-4ad)}{3b^2d(bc-ad)}+\frac{c^{7/3}\log\left(c+dx^3\right)}{6d^2(bc-ad)^{4/3}}-\frac{c^{7/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{4/3}}+\frac{ax^4}{b\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (a*x^4)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) + ((b*c - 4*a*d)*x*(a + b*x^3)^(2
/3))/(3*b^2*d*(b*c - a*d)) - ((3*b*c + 4*a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(7/3)*d^2) + (c^(7/3)*ArcTan[(1 + (2
*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*d^2*(
b*c - a*d)^(4/3)) + (c^(7/3)*Log[c + d*x^3])/(6*d^2*(b*c - a*d)^(4/3)) - (c
^(7/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d^2*(b*c
- a*d)^(4/3)) + ((3*b*c + 4*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*
b^(7/3)*d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^9}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}\\ &= \frac{x^{10}\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{10ac\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.53, size = 506, normalized size = 1.57

$$\frac{\frac{12d(-4a^2dx+b^2cx^4+abx(c-dx^3))}{b^2(bc-ad)\sqrt[3]{a+bx^3}}-\frac{4\sqrt{3}(3bc+4ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{7/3}}-\frac{6\sqrt{-6+6i\sqrt{3}}c^{7/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{(bc-ad)^{4/3}}+\frac{4(3bc+4ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{b^{7/3}}+\frac{6(1+i\sqrt{3})c^{7/3}\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}}-\frac{2(3bc+4ad)\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{7/3}}-\frac{3i(-i+\sqrt{3})c^{7/3}\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{4/3}}}{36d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((12*d*(-4*a^2*d*x + b^2*c*x^4 + a*b*x*(c - d*x^3)))/(b^2*(b*c - a*d)*(a +
b*x^3)^(1/3)) - (4*Sqrt[3]*(3*b*c + 4*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1
/3)*x + 2*(a + b*x^3)^(1/3))])/b^(7/3) - (6*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(7/3
)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (4*(3*b*c + 4*a*d)*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(7/3) + (6*(1 + I*Sqrt[3])*c^(7/3)*L
og[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c
 - a*d)^(4/3) - (2*(3*b*c + 4*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^
(1/3) + (a + b*x^3)^(2/3)])/b^(7/3) - ((3*I)*(-I + Sqrt[3])*c^(7/3)*Log[2*(
b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*
x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))
/(36*d^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(267) = 534.
time = 3.84, size = 1329, normalized size = 4.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^2 + (3*b^4*c^2 +
a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a
)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 -
2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) - 6*sqrt(3)*(b^4*c^2
*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) + 2*(3*a*b^2*c^2 + a^2*b*c*d - 4
*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)
*x - (b*x^3 + a)^(1/3))/x) - (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*
c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^
(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/
(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^
(1/3)*c)/x) + 3*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c -
 a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c -
 a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) + 6*((b^3*c*d - a*b^2*d^2)*x^4 + (
a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*d^3 +
 (b^5*c*d^2 - a*b^4*d^3)*x^3), -1/18*(6*sqrt(3)*(b^4*c^2*x^3 + a*b^3*c^2)*(
c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c
/(b*c - a*d))^(1/3))/x) - 2*(3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c
^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1
/3))/x) + (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4
*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (
b*x^3 + a)^(2/3))/x^2) + 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*
log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - 3*(b^
4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c -
 a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x
^3 + a)^(2/3)*c)/x^2) - 6*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^
2 + (3*b^4*c^2 + a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*arctan(sqrt(1/3)*(b^(1/3)*
x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 6*((b^3*c*d - a*b^2*d^2)*x^
4 + (a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*
d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**9/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^9}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.755 $\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=260

$$\frac{ax}{b(bc-ad)\sqrt[3]{a+bx^3}}+\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d}-\frac{c^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d(bc-ad)^{4/3}}-\frac{c^{4/3}\log(c+dx^3)}{6d(bc-ad)^{4/3}}+\frac{c^{4/3}\,\mathrm{l}}{}$$

```
[Out] a*x/b/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*c^(4/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(4/3)
+1/2*c^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(-a*d+b*c)^(4
/3)-1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d+1/3*arctan(1/3*(1+2*b^(1/3
)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d*3^(1/2)-1/3*c^(4/3)*arctan(1/3*(1+2
*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(4/3)*3^
(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {481, 544, 245, 384}

$$\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d}-\frac{c^{4/3}\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d(bc-ad)^{4/3}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{4/3}d}-\frac{c^{4/3}\log(c+dx^3)}{6d(bc-ad)^{4/3}}+\frac{c^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{4/3}}+\frac{ax}{b\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (a*x)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(1 + (2*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(4/3)*d) - (c^(4/3)*ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*d*(b*c - a*d)^
(4/3)) - (c^(4/3)*Log[c + d*x^3])/(6*d*(b*c - a*d)^(4/3)) + (c^(4/3)*Log[((
b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d*(b*c - a*d)^(4/3)) -
 Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(4/3)*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^6}{\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}\\ &= \frac{x^7\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7ac\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.90, size = 466, normalized size = 1.79

$$\frac{1}{12}\left(\frac{12ax}{(b^2c-abd)\sqrt[3]{a+bx^3}}+\frac{4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{4/3}d}+\frac{2\sqrt{-6+6i\sqrt{3}}\,c^{4/3}\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{d(bc-ad)^{4/3}}-\frac{4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{b^{4/3}d}-\frac{2i\left(-i+\sqrt{3}\right)c^{4/3}\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{d(bc-ad)^{4/3}}+\frac{2\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{4/3}d}+\frac{\left(1+i\sqrt{3}\right)c^{4/3}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{d(bc-ad)^{4/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((12*a*x)/((b^2*c - a*b*d)*(a + b*x^3)^(1/3)) + (4*Sqrt[3]*ArcTan[(Sqrt[3]*
b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(b^(4/3)*d) + (2*Sqrt[-6 + (
```

```
6*I)*Sqrt[3]]*c^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(
1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(d*(b*c - a*d)^(4/3))
 - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(b^(4/3)*d) - ((2*I)*(-I + Sqr
t[3])*c^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^
3)^(1/3)])/(d*(b*c - a*d)^(4/3)) + (2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^
3)^(1/3) + (a + b*x^3)^(2/3)])/(b^(4/3)*d) + ((1 + I*Sqrt[3])*c^(4/3)*Log[2
*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a +
b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(d*(b*c - a*d)^(
4/3)))/12
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(211) = 422.
time = 2.00, size = 1127, normalized size = 4.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x + 3*sqrt(1/3)*(a*b^2*c - a^2*b*d + (b^3*c
 - a*b^2*d)*x^3)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^
(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*
x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*(b^3*c*x
^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*c
```

```
- a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c - a
*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/
3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(b*
c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1
/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*
x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d
))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d -
 a*b^3*d^2)*x^3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x - 6*sqrt(1/3)*(a*b^2*c -
 a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-
b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*sqrt(3)*(b^3*c
*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*
c - a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c -
 a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(
1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(
b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^
(1/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d
)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a
*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d
 - a*b^3*d^2)*x^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**6/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.756 $\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=172

$$-\frac{x}{(bc-ad)\sqrt[3]{a+bx^3}}+\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}}+\frac{\sqrt[3]{c}\log(c+dx^3)}{6(bc-ad)^{4/3}}-\frac{\sqrt[3]{c}\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+b\right.}{2(bc-ad)^{4/3}}$$

```
[Out] -x/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/6*c^(1/3)*ln(d*x^3+c)/(-a*d+b*c)^(4/3)-1/2*
c^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/(-a*d+b*c)^(4/3)+1/3
*c^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2
))/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {482, 12, 384}

$$\frac{\sqrt[3]{c}\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}}-\frac{x}{\sqrt[3]{a+bx^3}(bc-ad)}+\frac{\sqrt[3]{c}\log(c+dx^3)}{6(bc-ad)^{4/3}}-\frac{\sqrt[3]{c}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(x/((b*c - a*d)*(a + b*x^3)^(1/3))) + (c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3))
 + (c^(1/3)*Log[c + d*x^3])/(6*(b*c - a*d)^(4/3)) - (c^(1/3)*Log[((b*c - a*
d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*(b*c - a*d)^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{x^4\sqrt[3]{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{4ac\sqrt[3]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{4/3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.76, size = 322, normalized size = 1.87

$$\frac{1}{12}\left(-\frac{12x}{(bc-ad)\sqrt[3]{a+bx^3}}-\frac{2\sqrt{-6+6i\sqrt{3}}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\,\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{(bc-ad)^{4/3}}+\frac{2\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}}-\frac{i\left(-i+\sqrt{3}\right)\sqrt[3]{c}\,\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{4/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((-12*x)/((b*c - a*d)*(a + b*x^3)^(1/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(1
/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sq
rt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3])*
c^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/
3)])/(b*c - a*d)^(4/3) - (I*(-I + Sqrt[3])*c^(1/3)*Log[2*(b*c - a*d)^(2/3)*
x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I
 + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/12
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**3/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.757 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=179

$$\frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}-\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}$$

```
[Out] b*x/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(4/3)
+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(4
/3)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/
2))/c^(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$,
Rules used = {390, 384}

$$-\frac{d\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(1 + (2*(b*c - a*d)^(1/
3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(
4/3)) - (d*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(4/3)) + (d*Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c - a*d)^(4/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}-\frac{d\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx}{bc-ad}\\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}-\frac{d\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx,x,\frac{x}{\sqrt[3]{a+bx^3}}\right)}{bc-ad}\\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}-\frac{d\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx,x,\frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)}-\frac{dS\cdots}{\cdots}\\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}+\frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}}-\frac{d\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc\cdots}}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{\cdots}}\cdots\right)}{\cdots}\\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}+\frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/\cdots}}{(a+bx^3)^{2\cdots}}\cdots\right)}{6c^{2/3}(bc\cdots}\\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}}-\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-\cdots}}{\sqrt[3]{a+\cdots}}\cdots\right)}{3c^{2/3}(bc-ad)\cdots}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.99, size = 328, normalized size = 1.83

$$\frac{1}{12}\left(\frac{12bx}{(abc-a^2d)\sqrt[3]{a+bx^3}}+\frac{2\sqrt{-6+6i\sqrt{3}}\,d\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}}-\frac{2i\left(-i+\sqrt{3}\right)d\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{4/3}}+\frac{\left(d+i\sqrt{3}\,d\right)\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{4/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((12*b*x)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[-6 + (6*I)*Sqrt[3]]
*d*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqr
t[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(2/3)*(b*c - a*d)^(4/3)) - ((2*I)*(-I
```

```
 + Sqrt[3])*d*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^
3)^(1/3)])/(c^(2/3)*(b*c - a*d)^(4/3)) + ((d + I*Sqrt[3]*d)*Log[2*(b*c - a*
d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/
3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d)^(4/3)
))/12
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{4}{3}} \left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.758 $\int \frac{1}{x^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=229

$$\frac{b}{a(bc-ad)x^2\sqrt[3]{a+bx^3}}-\frac{(3bc-ad)\left(a+bx^3\right)^{2/3}}{2a^2c(bc-ad)x^2}+\frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}(bc-ad)^{4/3}}+\frac{d^2\log\left(c+dx^3\right)}{6c^{5/3}(bc-ad)^{4/3}}-\frac{d^2\log}{}$$

```
[Out] b/a/(-a*d+b*c)/x^2/(b*x^3+a)^(1/3)-1/2*(-a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c/(
-a*d+b*c)/x^2+1/6*d^2*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(4/3)-1/2*d^2*ln((-a*d
+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(4/3)+1/3*d^2*arc
tan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(
-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {483, 597, 12, 384}

$$-\frac{\left(a+bx^3\right)^{2/3}(3bc-ad)}{2a^2cx^2(bc-ad)}+\frac{d^2\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}(bc-ad)^{4/3}}+\frac{d^2\log\left(c+dx^3\right)}{6c^{5/3}(bc-ad)^{4/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}(bc-ad)^{4/3}}+\frac{b}{ax^2\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] b/(a*(b*c - a*d)*x^2*(a + b*x^3)^(1/3)) - ((3*b*c - a*d)*(a + b*x^3)^(2/3))
/(2*a^2*c*(b*c - a*d)*x^2) + (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(5/3)*(b*c - a*d)^(4/3)) + (d^
2*Log[c + d*x^3])/(6*c^(5/3)*(b*c - a*d)^(4/3)) - (d^2*Log[((b*c - a*d)^(1/
3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(5/3)*(b*c - a*d)^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)} \, dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}} \int \frac{1}{x^3 \left(1+\frac{bx^3}{a}\right)^{4/3} (c+dx^3)} \, dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{28c^4(a+bx^3)^2 + 168c^3dx^3(a+bx^3)^2 + 126c^2d^2x^6(a+bx^3)^2 - 28c^4(a+b\ldots}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.34, size = 358, normalized size = 1.56

$$\frac{\frac{6c^{2/3}\left(-a^2d+3b^2cx^3+ab\left(c-dx^3\right)\right)}{a^2(-bc+ad)x^2\sqrt[3]{a+bx^3}} - \frac{2\sqrt{-6+6i\sqrt{3}}\, d^2 \tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{(bc-ad)^{4/3}} + \frac{2\left(1+i\sqrt{3}\right)d^2\log\left(2\sqrt[3]{bc-ad}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}} - \frac{i\left(-i+\sqrt{3}\right)d^2\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\,x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}\left(a+bx^3\right)^{2/3}\right)}{(bc-ad)^{4/3}}}{12c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((6*c^(2/3)*(-(a^2*d) + 3*b^2*c*x^3 + a*b*(c - d*x^3)))/(a^2*(-(b*c) + a*d)
*x^2*(a + b*x^3)^(1/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c -
```

```
a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b
*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3])*d^2*Log[2*(b*c - a*d)
^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) -
(I*(-I + Sqrt[3])*d^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3
)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^
3)^(2/3)])/(b*c - a*d)^(4/3))/(12*c^(5/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a+bx^{3}\right)^{\frac{4}{3}} \left(c+dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.759 $\int \frac{1}{x^6 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)} \, dx$

Optimal. Leaf size=287

$$\frac{b}{a(bc-ad)x^5\sqrt[3]{a+bx^3}}-\frac{(6bc-ad)\left(a+bx^3\right)^{2/3}}{5a^2c(bc-ad)x^5}+\frac{\left(18b^2c^2-3abcd-5a^2d^2\right)\left(a+bx^3\right)^{2/3}}{10a^3c^2(bc-ad)x^2}-\frac{d^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\ldots}{\sqrt[3]{c}\ldots}}{\ldots}\right)}{\sqrt{3}\,c^{8/3}(bc-\ldots}$$

```
[Out] b/a/(-a*d+b*c)/x^5/(b*x^3+a)^(1/3)-1/5*(-a*d+6*b*c)*(b*x^3+a)^(2/3)/a^2/c/(
-a*d+b*c)/x^5+1/10*(-5*a^2*d^2-3*a*b*c*d+18*b^2*c^2)*(b*x^3+a)^(2/3)/a^3/c^
2/(-a*d+b*c)/x^2-1/6*d^3*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(4/3)+1/2*d^3*ln((-
a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(4/3)-1/3*d^3*
arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3
)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {483, 597, 12, 384}

$$-\frac{\left(a+bx^3\right)^{2/3}(6bc-ad)}{5a^2cx^5(bc-ad)}+\frac{\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-3abcd+18b^2c^2\right)}{10a^3c^2x^2(bc-ad)}-\frac{d^3\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{8/3}(bc-ad)^{4/3}}-\frac{d^3\log\left(c+dx^3\right)}{6c^{8/3}(bc-ad)^{4/3}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}(bc-ad)^{4/3}}+\frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] b/(a*(b*c - a*d)*x^5*(a + b*x^3)^(1/3)) - ((6*b*c - a*d)*(a + b*x^3)^(2/3))
/(5*a^2*c*(b*c - a*d)*x^5) + ((18*b^2*c^2 - 3*a*b*c*d - 5*a^2*d^2)*(a + b*x
^3)^(2/3))/(10*a^3*c^2*(b*c - a*d)*x^2) - (d^3*ArcTan[(1 + (2*(b*c - a*d)^(
1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(8/3)*(b*c - a*d)
^(4/3)) - (d^3*Log[c + d*x^3])/(6*c^(8/3)*(b*c - a*d)^(4/3)) + (d^3*Log[((b
*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(8/3)*(b*c - a*d)^(4/
3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^6 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)} \, dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}} \int \frac{1}{x^6 \left(1+\frac{bx^3}{a}\right)^{4/3} (c+dx^3)} \, dx}{a \sqrt[3]{a+bx^3}}$$

$$= \frac{56c^5(a+bx^3)^2 - 252c^4dx^3(a+bx^3)^2 - 1512c^3d^2x^6(a+bx^3)^2 - 1134c^2d^3x}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.52, size = 402, normalized size = 1.40

$$\frac{\frac{6c^{2/3}\left(-18b^3c^2x^6+3ab^2cx^3\left(-2c+dx^3\right)+a^3d\left(-2c+5dx^3\right)+a^2b\left(2c^2+cdx^3+5d^2x^6\right)\right)}{a^3(-bc+ad)x^5\sqrt[3]{a+bx^3}} + \frac{10\sqrt{-6+6i\sqrt{3}}\, d^3 \tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\, x}{\sqrt{3}\sqrt[3]{bc-ad}\, x - \left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{(bc-ad)^{4/3}} - \frac{10i\left(-i+\sqrt{3}\right) d^3 \log\left(2\sqrt[3]{bc-ad}\, x + \left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}} + \frac{5\left(1+i\sqrt{3}\right) d^3 \log\left(2(bc-ad)^{2/3}x^2 + \left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\, x\sqrt[3]{a+bx^3} + i\left(i+\sqrt{3}\right)c^{2/3}\left(a+bx^3\right)^{2/3}\right)}{(bc-ad)^{4/3}}}{60c^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((6*c^(2/3)*(-18*b^3*c^2*x^6 + 3*a*b^2*c*x^3*(-2*c + d*x^3) + a^3*d*(-2*c +
 5*d*x^3) + a^2*b*(2*c^2 + c*d*x^3 + 5*d^2*x^6)))/(a^3*(-(b*c) + a*d)*x^5*(
a + b*x^3)^(1/3)) + (10*Sqrt[-6 + (6*I)*Sqrt[3]]*d^3*ArcTan[(3*(b*c - a*d)^
(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)
^(1/3))])/(b*c - a*d)^(4/3) - ((10*I)*(-I + Sqrt[3])*d^3*Log[2*(b*c - a*d)^
(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) + (
5*(1 + I*Sqrt[3])*d^3*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3
)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^
3)^(2/3)])/(b*c - a*d)^(4/3))/(60*c^(8/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6} \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6} \left(a + b x^{3}\right)^{\frac{4}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**6*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.760 $\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)} \, dx$

Optimal. Leaf size=351

$$\frac{b}{a(bc-ad)x^8\sqrt[3]{a+bx^3}}-\frac{(9bc-ad)\left(a+bx^3\right)^{2/3}}{8a^2c(bc-ad)x^8}+\frac{(9bc-4ad)(3bc+ad)\left(a+bx^3\right)^{2/3}}{20a^3c^2(bc-ad)x^5}-\frac{(81b^3c^3-9ab^2c^2d-1}{40a^4}$$

```
[Out] b/a/(-a*d+b*c)/x^8/(b*x^3+a)^(1/3)-1/8*(-a*d+9*b*c)*(b*x^3+a)^(2/3)/a^2/c/(
-a*d+b*c)/x^8+1/20*(-4*a*d+9*b*c)*(a*d+3*b*c)*(b*x^3+a)^(2/3)/a^3/c^2/(-a*d
+b*c)/x^5-1/40*(-20*a^3*d^3-12*a^2*b*c*d^2-9*a*b^2*c^2*d+81*b^3*c^3)*(b*x^3
+a)^(2/3)/a^4/c^3/(-a*d+b*c)/x^2+1/6*d^4*ln(d*x^3+c)/c^(11/3)/(-a*d+b*c)^(4
/3)-1/2*d^4*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(11/3)/(-a*d+b
*c)^(4/3)+1/3*d^4*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3
))*3^(1/2))/c^(11/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {483, 597, 12, 384}

$$\frac{\left(a+bx^3\right)^{2/3}(9bc-4ad)(ad+3bc)}{20a^3c^2x^5(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}(9bc-ad)}{8a^2cx^8(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}\left(-20a^3d^3-12a^2bcd^2-9ab^2c^2d+81b^3c^3\right)}{40a^4c^3x^2(bc-ad)}+\frac{d^4\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{11/3}(bc-ad)^{4/3}}+\frac{d^4\log\left(c+dx^3\right)}{6c^{11/3}(bc-ad)^{4/3}}-\frac{d^4\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}(bc-ad)^{4/3}}+\frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] b/(a*(b*c - a*d)*x^8*(a + b*x^3)^(1/3)) - ((9*b*c - a*d)*(a + b*x^3)^(2/3))
/(8*a^2*c*(b*c - a*d)*x^8) + ((9*b*c - 4*a*d)*(3*b*c + a*d)*(a + b*x^3)^(2/
3))/(20*a^3*c^2*(b*c - a*d)*x^5) - ((81*b^3*c^3 - 9*a*b^2*c^2*d - 12*a^2*b*
c*d^2 - 20*a^3*d^3)*(a + b*x^3)^(2/3))/(40*a^4*c^3*(b*c - a*d)*x^2) + (d^4*
ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/
(Sqrt[3]*c^(11/3)*(b*c - a*d)^(4/3)) + (d^4*Log[c + d*x^3])/(6*c^(11/3)*(b*
c - a*d)^(4/3)) - (d^4*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3
)])/(2*c^(11/3)*(b*c - a*d)^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^9 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\, dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}} \int \frac{1}{x^9 \left(1+\frac{bx^3}{a}\right)^{4/3} (c+dx^3)}\, dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{280c^6(a+bx^3)^2 - 672c^5dx^3(a+bx^3)^2 + 3024c^4d^2x^6(a+bx^3)^2 + 18144c^3\cdots}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.62, size = 459, normalized size = 1.31

$$\frac{-\frac{3c^{2/3}\left(-81b^4c^3x^9+9ab^3c^2x^6\left(-3c+dx^3\right)+3a^2b^2cx^3\left(3c^2+cdx^3+4d^2x^6\right)+a^4d\left(5c^2-8cdx^3+20d^2x^6\right)+a^3b\left(-5c^3-c^2dx^3+4cd^2x^6+20d^3x^9\right)\right)}{a^4(-bc+ad)x^8\sqrt[3]{a+bx^3}}-\frac{20\sqrt{-6+6i\sqrt{3}}\, d^4\tan^{-1}\left(\frac{3\sqrt[3]{bc-ad}\, x}{\sqrt{3}\sqrt[3]{bc-ad}\, x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{(bc-ad)^{4/3}}+\frac{20\left(1+i\sqrt{3}\right)d^4\log\left(2\sqrt[3]{bc-ad}\, x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}}-\frac{10i\left(-i+\sqrt{3}\right)d^4\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}\, x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}\left(a+bx^3\right)^{2/3}\right)}{(bc-ad)^{4/3}}}{120c^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((-3*c^(2/3)*(-81*b^4*c^3*x^9 + 9*a*b^3*c^2*x^6*(-3*c + d*x^3) + 3*a^2*b^2*
c*x^3*(3*c^2 + c*d*x^3 + 4*d^2*x^6) + a^4*d*(5*c^2 - 8*c*d*x^3 + 20*d^2*x^6
) + a^3*b*(-5*c^3 - c^2*d*x^3 + 4*c*d^2*x^6 + 20*d^3*x^9)))/(a^4*(-(b*c) +
a*d)*x^8*(a + b*x^3)^(1/3)) - (20*Sqrt[-6 + (6*I)*Sqrt[3]]*d^4*ArcTan[(3*(b
*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(
a + b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (20*(1 + I*Sqrt[3])*d^4*Log[2*(b*c
- a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4
/3) - ((10*I)*(-I + Sqrt[3])*d^4*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt
[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3
)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(120*c^(11/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9} \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9} \left(a + b x^{3}\right)^{\frac{4}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**9*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^9\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.761 $\int \frac{x^{10}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=67

$$\frac{x^{11}\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/11*x^11*(1+b*x^3/a)^(1/3)*AppellF1(11/3,4/3,1,14/3,-b*x^3/a,-d*x^3/c)/a/c
/(b*x^3+a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^{11}\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^11*(1 + (b*x^3)/a)^(1/3)*AppellF1[11/3, 4/3, 1, 14/3, -((b*x^3)/a), -((d
*x^3)/c)])/(11*a*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^{10}}{\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^{11}\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 194 vs. 2(67) = 134.
time = 9.84, size = 194, normalized size = 2.90

$$\frac{x^2\left(5c(-5a^2d+b^2cx^3+ab(c-dx^3))+5ac(-bc+5ad)\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2(-2b^2c^2-abcd+5a^2d^2)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{20b^2cd(bc-ad)\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(5*c*(-5*a^2*d + b^2*c*x^3 + a*b*(c - d*x^3)) + 5*a*c*(-(b*c) + 5*a*d)
*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*(-2*b^2*c^2 - a*b*c*d + 5*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF
1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(20*b^2*c*d*(b*c - a*d)*(
a + b*x^3)^(1/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**10/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(x^10/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.762 $\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=67

$$\frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/8*x^8*(1+b*x^3/a)^(1/3)*AppellF1(8/3,4/3,1,11/3,-b*x^3/a,-d*x^3/c)/a/c/(b
*x^3+a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 4/3, 1, 11/3, -((b*x^3)/a), -((d*x
^3)/c)])/(8*a*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^7}{\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(67) = 134.
time = 9.61, size = 144, normalized size = 2.15

$$\frac{5acx^2-5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+(bc-2ad)x^5\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5bc(bc-ad)\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (5*a*c*x^2 - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((
b*x^3)/a), -((d*x^3)/c)] + (b*c - 2*a*d)*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1
[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(5*b*c*(b*c - a*d)*(a + b*x
^3)^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**7/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^7/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.763 $\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=67

$$\frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/5*x^5*(1+b*x^3/a)^(1/3)*AppellF1(5/3,4/3,1,8/3,-b*x^3/a,-d*x^3/c)/a/c/(b*
x^3+a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 4/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*a*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^4}{\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

Mathematica [A]
time = 9.29, size = 129, normalized size = 1.93

$$\frac{x^2\left(-5c+5c\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+dx^3\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{5c(bc-ad)\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(-5*c + 5*c*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)
/a), -((d*x^3)/c)] + d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3,
 -((b*x^3)/a), -((d*x^3)/c)]))/(5*c*(b*c - a*d)*(a + b*x^3)^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**4/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^4/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.764 $\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=67

$$\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/2*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,4/3,1,5/3,-b*x^3/a,-d*x^3/c)/a/c/(b*
x^3+a)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 4/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*a*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x}{\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.
time = 10.09, size = 141, normalized size = 2.10

$$\frac{x^2\left(-10bc+5(bc+ad)\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^3\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{10ac(-bc+ad)\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(-10*b*c + 5*(b*c + a*d)*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5
/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[
5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(10*a*c*(-(b*c) + a*d)*(a +
 b*x^3)^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.765 $\int \frac{1}{x^2(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

```
[Out] -(1+b*x^3/a)^(1/3)*AppellF1(-1/3,4/3,1,2/3,-b*x^3/a,-d*x^3/c)/a/c/x/(b*x^3+
a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 4/3, 1, 2/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*c*x*(a + b*x^3)^(1/3)))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^2\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(65) = 130.
time = 10.15, size = 193, normalized size = 2.97

$$\frac{10c(-a^2d+2b^2cx^3+ab(c-dx^3))-5(2b^2c^2-abcd+a^2d^2)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bd(-2bc+ad)x^6\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{10a^2c^2(-bc+ad)x\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (10*c*(-(a^2*d) + 2*b^2*c*x^3 + a*b*(c - d*x^3)) - 5*(2*b^2*c^2 - a*b*c*d +
 a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a)
, -((d*x^3)/c)] + 2*b*d*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[5
/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a^2*c^2*(-(b*c) + a*d)*x*
(a + b*x^3)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + bx^3)^{\frac{4}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 (b x^3 + a)^{4/3} (d x^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.766 $\int \frac{1}{x^5(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

```
[Out] -1/4*(1+b*x^3/a)^(1/3)*AppellF1(-4/3,4/3,1,-1/3,-b*x^3/a,-d*x^3/c)/a/c/x^4/
(b*x^3+a)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/4*((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 4/3, 1, -1/3, -((b*x^3)/a), -((d
*x^3)/c)])/(a*c*x^4*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^5 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}} \int \frac{1}{x^5 \left(1+\frac{bx^3}{a}\right)^{4/3} (c+dx^3)} \, dx}{a\sqrt[3]{a+bx^3}}$$
$$= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\, F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 264 vs. 2(67) = 134.
time = 10.21, size = 264, normalized size = 3.94

$$\frac{5c(-10b^3c^2x^6+ab^2cx^3(-5c+2dx^3)+a^3d(-c+4dx^3)+a^2b(c^2+cdx^3+4d^2x^6))+5(5b^3c^3-ab^2c^2d-2a^2bcd^2+2a^3d^3)\,x^6\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2bd(-5b^2c^2+abcd+2a^2d^2)\,x^9\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{20a^3c^3(-bc+ad)x^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*(-10*b^3*c^2*x^6 + a*b^2*c*x^3*(-5*c + 2*d*x^3) + a^3*d*(-c + 4*d*x^3)
 + a^2*b*(c^2 + c*d*x^3 + 4*d^2*x^6)) + 5*(5*b^3*c^3 - a*b^2*c^2*d - 2*a^2*
b*c*d^2 + 2*a^3*d^3)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -
((b*x^3)/a), -((d*x^3)/c)] - 2*b*d*(-5*b^2*c^2 + a*b*c*d + 2*a^2*d^2)*x^9*(
1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(20*a^3*c^3*(-(b*c) + a*d)*x^4*(a + b*x^3)^(1/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(b\,x^3+a\right)^{\frac{4}{3}} \left(d\,x^3+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^5), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{\frac{4}{3}} \left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(1/(x**5*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^5), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.767 $\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=90

$$-\frac{(bc+ad)x^4}{4b^2d^2}+\frac{x^8}{8bd}-\frac{a^3\log(a+bx^4)}{4b^3(bc-ad)}+\frac{c^3\log(c+dx^4)}{4d^3(bc-ad)}$$

```
[Out] -1/4*(a*d+b*c)*x^4/b^2/d^2+1/8*x^8/b/d-1/4*a^3*ln(b*x^4+a)/b^3/(-a*d+b*c)+1
/4*c^3*ln(d*x^4+c)/d^3/(-a*d+b*c)
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$-\frac{a^3\log(a+bx^4)}{4b^3(bc-ad)}-\frac{x^4(ad+bc)}{4b^2d^2}+\frac{c^3\log(c+dx^4)}{4d^3(bc-ad)}+\frac{x^8}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/4*((b*c + a*d)*x^4)/(b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^
3*(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^3}{(a+bx)(c+dx)}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d^2}+\frac{x}{bd}-\frac{a^3}{b^2(bc-ad)(a+bx)}-\frac{c^3}{d^2(-bc+ad)(c+dx)}\right) d\right.\\ &= -\frac{(bc+ad)x^4}{4b^2d^2}+\frac{x^8}{8bd}-\frac{a^3\log(a+bx^4)}{4b^3(bc-ad)}+\frac{c^3\log(c+dx^4)}{4d^3(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 92, normalized size = 1.02

$$\frac{(-bc-ad)x^4}{4b^2d^2}+\frac{x^8}{8bd}-\frac{a^3\log(a+bx^4)}{4b^3(bc-ad)}+\frac{c^3\log(c+dx^4)}{4d^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-(b*c) - a*d)*x^4)/(4*b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^
3*(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))
```

Maple [A]
time = 0.38, size = 78, normalized size = 0.87

method	result	size
default	$\frac{(-bd\,x^4+ad+bc)^2}{8b^3d^3}-\frac{c^3\ln(d\,x^4+c)}{4d^3(ad-bc)}+\frac{a^3\ln(b\,x^4+a)}{4b^3(ad-bc)}$	78
norman	$\frac{x^8}{8bd}-\frac{(ad+bc)x^4}{4b^2d^2}+\frac{a^3\ln(b\,x^4+a)}{4b^3(ad-bc)}-\frac{c^3\ln(d\,x^4+c)}{4d^3(ad-bc)}$	83
risch	$\frac{x^8}{8bd}-\frac{a\,x^4}{4b^2d}-\frac{c\,x^4}{4b\,d^2}+\frac{a^2}{8b^3d}+\frac{ac}{4b^2d^2}+\frac{c^2}{8b\,d^3}-\frac{c^3\ln(d\,x^4+c)}{4d^3(ad-bc)}+\frac{a^3\ln(-b\,x^4-a)}{4b^3(ad-bc)}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-b*d*x^4+a*d+b*c)^2/b^3/d^3-1/4*c^3/d^3/(a*d-b*c)*ln(d*x^4+c)+1/4*a^3/
b^3/(a*d-b*c)*ln(b*x^4+a)
```

Maxima [A]
time = 0.28, size = 84, normalized size = 0.93

$$-\frac{a^3\log(bx^4+a)}{4(b^4c-ab^3d)}+\frac{c^3\log(dx^4+c)}{4(bcd^3-ad^4)}+\frac{bdx^8-2(bc+ad)x^4}{8b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*a^3*log(b*x^4 + a)/(b^4*c - a*b^3*d) + 1/4*c^3*log(d*x^4 + c)/(b*c*d^3
 - a*d^4) + 1/8*(b*d*x^8 - 2*(b*c + a*d)*x^4)/(b^2*d^2)
```

Fricas [A]
time = 10.16, size = 100, normalized size = 1.11

$$\frac{(b^3cd^2-ab^2d^3)x^8-2a^3d^3\log(bx^4+a)+2b^3c^3\log(dx^4+c)-2(b^3c^2d-a^2bd^3)x^4}{8(b^4cd^3-ab^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/8*((b^3*c*d^2 - a*b^2*d^3)*x^8 - 2*a^3*d^3*log(b*x^4 + a) + 2*b^3*c^3*log
(d*x^4 + c) - 2*(b^3*c^2*d - a^2*b*d^3)*x^4)/(b^4*c*d^3 - a*b^3*d^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.82, size = 88, normalized size = 0.98

$$-\frac{a^3 \log\left(|bx^4+a|\right)}{4\left(b^4c-ab^3d\right)}+\frac{c^3 \log\left(|dx^4+c|\right)}{4\left(bcd^3-ad^4\right)}+\frac{bdx^8-2\,bcx^4-2\,adx^4}{8\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*a^3*log(abs(b*x^4 + a))/(b^4*c - a*b^3*d) + 1/4*c^3*log(abs(d*x^4 + c)
)/(b*c*d^3 - a*d^4) + 1/8*(b*d*x^8 - 2*b*c*x^4 - 2*a*d*x^4)/(b^2*d^2)
```

Mupad [**B**]
time = 5.88, size = 88, normalized size = 0.98

$$\frac{x^8}{8\,b\,d}-\frac{c^3\,\ln\left(d\,x^4+c\right)}{4\,\left(a\,d^4-b\,c\,d^3\right)}-\frac{a^3\,\ln\left(b\,x^4+a\right)}{4\,\left(b^4\,c-a\,b^3\,d\right)}-\frac{x^4\,\left(a\,d+b\,c\right)}{4\,b^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] x^8/(8*b*d) - (c^3*log(c + d*x^4))/(4*(a*d^4 - b*c*d^3)) - (a^3*log(a + b*x
^4))/(4*(b^4*c - a*b^3*d)) - (x^4*(a*d + b*c))/(4*b^2*d^2)
```

3.768 $\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=70

$$\frac{x^4}{4bd}+\frac{a^2\log(a+bx^4)}{4b^2(bc-ad)}-\frac{c^2\log(c+dx^4)}{4d^2(bc-ad)}$$

```
[Out] 1/4*x^4/b/d+1/4*a^2*ln(b*x^4+a)/b^2/(-a*d+b*c)-1/4*c^2*ln(d*x^4+c)/d^2/(-a*
d+b*c)
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\frac{a^2\log(a+bx^4)}{4b^2(bc-ad)}-\frac{c^2\log(c+dx^4)}{4d^2(bc-ad)}+\frac{x^4}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x^4/(4*b*d) + (a^2*Log[a + b*x^4])/(4*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^4
])/(4*d^2*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)\,dx,x,x^4\right)\\ &= \frac{x^4}{4bd}+\frac{a^2\log(a+bx^4)}{4b^2(bc-ad)}-\frac{c^2\log(c+dx^4)}{4d^2(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.94

$$\frac{a^2d^2\log(a+bx^4)-b(d(-bc+ad)x^4+bc^2\log(c+dx^4))}{4b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^4] - b*(d*(-(b*c) + a*d)*x^4 + b*c^2*Log[c + d*x^4]))/
(4*b^2*d^2*(b*c - a*d))
```

Maple [A]
time = 0.38, size = 65, normalized size = 0.93

method	result	size
default	$\frac{x^4}{4bd}+\frac{c^2\ln(dx^4+c)}{4d^2(ad-bc)}-\frac{a^2\ln(bx^4+a)}{4b^2(ad-bc)}$	65
norman	$\frac{x^4}{4bd}+\frac{c^2\ln(dx^4+c)}{4d^2(ad-bc)}-\frac{a^2\ln(bx^4+a)}{4b^2(ad-bc)}$	65
risch	$\frac{x^4}{4bd}-\frac{a^2\ln(-bx^4-a)}{4b^2(ad-bc)}+\frac{c^2\ln(dx^4+c)}{4d^2(ad-bc)}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4/b/d+1/4*c^2/d^2/(a*d-b*c)*ln(d*x^4+c)-1/4*a^2/b^2/(a*d-b*c)*ln(b*x^
4+a)
```

Maxima [A]
time = 0.29, size = 68, normalized size = 0.97

$$\frac{x^4}{4\,bd}+\frac{a^2\log(bx^4+a)}{4\,(b^3c-ab^2d)}-\frac{c^2\log(dx^4+c)}{4\,(bcd^2-ad^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/(b*d) + 1/4*a^2*log(b*x^4 + a)/(b^3*c - a*b^2*d) - 1/4*c^2*log(d*x^
4 + c)/(b*c*d^2 - a*d^3)
```

Fricas [A]
time = 5.17, size = 72, normalized size = 1.03

$$\frac{(b^2cd-abd^2)x^4+a^2d^2\log(bx^4+a)-b^2c^2\log(dx^4+c)}{4\,(b^3cd^2-ab^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*c*d - a*b*d^2)*x^4 + a^2*d^2*log(b*x^4 + a) - b^2*c^2*log(d*x^4 +
 c))/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.63, size = 70, normalized size = 1.00

$$\frac{x^4}{4\,bd} + \frac{a^2\log\left(|bx^4+a|\right)}{4\left(b^3c-ab^2d\right)} - \frac{c^2\log\left(|dx^4+c|\right)}{4\left(bcd^2-ad^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*x^4/(b*d) + 1/4*a^2*log(abs(b*x^4 + a))/(b^3*c - a*b^2*d) - 1/4*c^2*log
(abs(d*x^4 + c))/(b*c*d^2 - a*d^3)
```

Mupad [**B**]
time = 5.63, size = 68, normalized size = 0.97

$$\frac{a^2\,\ln\left(b\,x^4+a\right)}{4\,b^3\,c-4\,a\,b^2\,d} + \frac{c^2\,\ln\left(d\,x^4+c\right)}{4\,a\,d^3-4\,b\,c\,d^2} + \frac{x^4}{4\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (a^2*log(a + b*x^4))/(4*b^3*c - 4*a*b^2*d) + (c^2*log(c + d*x^4))/(4*a*d^3
- 4*b*c*d^2) + x^4/(4*b*d)
```

3.769 $\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=53

$$-\frac{a\log(a+bx^4)}{4b(bc-ad)}+\frac{c\log(c+dx^4)}{4d(bc-ad)}$$

```
[Out] -1/4*a*ln(b*x^4+a)/b/(-a*d+b*c)+1/4*c*ln(d*x^4+c)/d/(-a*d+b*c)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{c\log(c+dx^4)}{4d(bc-ad)}-\frac{a\log(a+bx^4)}{4b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/4*(a*Log[a + b*x^4])/(b*(b*c - a*d)) + (c*Log[c + d*x^4])/(4*d*(b*c - a*
d))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)} dx = \frac{1}{4}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)} dx, x, x^4\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)} + \frac{c}{(bc-ad)(c+dx)}\right) dx, x, x^4\right)$$
$$= -\frac{a\log(a+bx^4)}{4b(bc-ad)} + \frac{c\log(c+dx^4)}{4d(bc-ad)}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.81

$$-\frac{ad\log(a+bx^4) - bc\log(c+dx^4)}{4b^2cd - 4abd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)*(c + d*x^4)),x]
[Out] -((a*d*Log[a + b*x^4] - b*c*Log[c + d*x^4])/(4*b^2*c*d - 4*a*b*d^2))
```

Maple [A]
time = 0.38, size = 50, normalized size = 0.94

method	result	size
default	$-\frac{c\ln(d\,x^4+c)}{4(ad-bc)d}+\frac{a\ln(b\,x^4+a)}{4(ad-bc)b}$	50
norman	$-\frac{c\ln(d\,x^4+c)}{4(ad-bc)d}+\frac{a\ln(b\,x^4+a)}{4(ad-bc)b}$	50
risch	$-\frac{c\ln(-d\,x^4-c)}{4d(ad-bc)}+\frac{a\ln(b\,x^4+a)}{4(ad-bc)b}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] -1/4*c/(a*d-b*c)/d*ln(d*x^4+c)+1/4*a/(a*d-b*c)/b*ln(b*x^4+a)
```

Maxima [A]
time = 0.28, size = 49, normalized size = 0.92

$$-\frac{a\log(bx^4+a)}{4(b^2c-abd)} + \frac{c\log(dx^4+c)}{4(bcd-ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*a*log(b*x^4 + a)/(b^2*c - a*b*d) + 1/4*c*log(d*x^4 + c)/(b*c*d - a*d^2
)
```

Fricas [A]
time = 3.82, size = 42, normalized size = 0.79

$$-\frac{ad\log(bx^4+a)-bc\log(dx^4+c)}{4(b^2cd-abd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d*log(b*x^4 + a) - b*c*log(d*x^4 + c))/(b^2*c*d - a*b*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.52, size = 51, normalized size = 0.96

$$-\frac{a\log(|bx^4+a|)}{4(b^2c-abd)}+\frac{c\log(|dx^4+c|)}{4(bcd-ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*a*log(abs(b*x^4 + a))/(b^2*c - a*b*d) + 1/4*c*log(abs(d*x^4 + c))/(b*c
*d - a*d^2)
```

Mupad [B]
time = 5.10, size = 51, normalized size = 0.96

$$-\frac{a\ln(b\,x^4+a)}{4\,b^2\,c-4\,a\,b\,d}-\frac{c\ln(d\,x^4+c)}{4\,a\,d^2-4\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - (a*log(a + b*x^4))/(4*b^2*c - 4*a*b*d) - (c*log(c + d*x^4))/(4*a*d^2 - 4*
b*c*d)
```

3.770 $\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=45

$$\frac{\log(a+bx^4)}{4(bc-ad)}-\frac{\log(c+dx^4)}{4(bc-ad)}$$

```
[Out] 1/4*ln(b*x^4+a)/(-a*d+b*c)-1/4*ln(d*x^4+c)/(-a*d+b*c)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 36, 31}

$$\frac{\log(a+bx^4)}{4(bc-ad)}-\frac{\log(c+dx^4)}{4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] Log[a + b*x^4]/(4*(b*c - a*d)) - Log[c + d*x^4]/(4*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)}\,dx,x,x^4\right)$$
$$= \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^4\right)}{4(bc-ad)} - \frac{d\text{Subst}\left(\int \frac{1}{c+dx}\,dx,x,x^4\right)}{4(bc-ad)}$$
$$= \frac{\log(a+bx^4)}{4(bc-ad)} - \frac{\log(c+dx^4)}{4(bc-ad)}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.69

$$\frac{\log(a+bx^4)-\log(c+dx^4)}{4bc-4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)*(c + d*x^4)),x]
[Out] (Log[a + b*x^4] - Log[c + d*x^4])/(4*b*c - 4*a*d)
```

Maple [A]
time = 3.59, size = 42, normalized size = 0.93

method	result	size
default	$\frac{\ln(d\,x^4+c)}{4ad-4bc}-\frac{\ln(b\,x^4+a)}{4(ad-bc)}$	42
norman	$\frac{\ln(d\,x^4+c)}{4ad-4bc}-\frac{\ln(b\,x^4+a)}{4(ad-bc)}$	42
risch	$\frac{\ln(d\,x^4+c)}{4ad-4bc}-\frac{\ln(-b\,x^4-a)}{4(ad-bc)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] 1/4/(a*d-b*c)*ln(d*x^4+c)-1/4/(a*d-b*c)*ln(b*x^4+a)
```

Maxima [A]
time = 0.30, size = 41, normalized size = 0.91

$$\frac{\log(bx^4+a)}{4\,(bc-ad)} - \frac{\log(dx^4+c)}{4\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

[Out] 1/4*log(b*x^4 + a)/(b*c - a*d) - 1/4*log(d*x^4 + c)/(b*c - a*d)

Fricas [A]
time = 2.42, size = 31, normalized size = 0.69

$$\frac{\log\left(bx^4+a\right)-\log\left(dx^4+c\right)}{4\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(log(b*x^4 + a) - log(d*x^4 + c))/(b*c - a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(36) = 72.
time = 0.98, size = 138, normalized size = 3.07

$$\frac{\log\left(x^4+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4\,(ad-bc)}-\frac{\log\left(x^4+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] log(x**4 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(4*(a*d - b*c)) - log(x**4 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(4*(a*d - b*c))
```

Giac [A]
time = 1.24, size = 51, normalized size = 1.13

$$\frac{b\log\left(|bx^4+a|\right)}{4\,(b^2c-abd)}-\frac{d\log\left(|dx^4+c|\right)}{4\,(bcd-ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(b*x^4 + a))/(b^2*c - a*b*d) - 1/4*d*log(abs(d*x^4 + c))/(b*c*
d - a*d^2)
```

Mupad [B]
time = 4.99, size = 1012, normalized size = 22.49

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x^4\,\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)+\frac{\frac{x^4\,\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+x^4\,\left(384\,a^2\,b^4\,d^6+768\,a\,b^5\,c\,d^5+384\,b^6\,c^2\,d^4\right)+512\,a\,b^5\,c^2\,d^4+512\,a^2\,b^4\,c\,d^5}{4\,a\,d-4\,b\,c}+64\,a\,b^4\,c\,d^4}{4\,a\,d-4\,b\,c}+8\,b^4\,d^4\,x^4\right)\,1{}\mathrm{i}}{4\,a\,d-4\,b\,c}-\frac{\left(\frac{x^4\,\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)-\frac{x^4\,\left(384\,a^2\,b^4\,d^6+768\,a\,b^5\,c\,d^5+384\,b^6\,c^2\,d^4\right)-\frac{x^4\,\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+512\,a\,b^5\,c^2\,d^4+512\,a^2\,b^4\,c\,d^5}{4\,a\,d-4\,b\,c}+64\,a\,b^4\,c\,d^4}{4\,a\,d-4\,b\,c}-8\,b^4\,d^4\,x^4\right)\,1{}\mathrm{i}}{4\,a\,d-4\,b\,c}}{\frac{\frac{x^4\,\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)+\frac{\frac{x^4\,\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+x^4\,\left(384\,a^2\,b^4\,d^6+768\,a\,b^5\,c\,d^5+384\,b^6\,c^2\,d^4\right)+512\,a\,b^5\,c^2\,d^4+512\,a^2\,b^4\,c\,d^5}{4\,a\,d-4\,b\,c}+64\,a\,b^4\,c\,d^4}{4\,a\,d-4\,b\,c}+8\,b^4\,d^4\,x^4}{4\,a\,d-4\,b\,c}+\frac{\frac{x^4\,\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)-\frac{x^4\,\left(384\,a^2\,b^4\,d^6+768\,a\,b^5\,c\,d^5+384\,b^6\,c^2\,d^4\right)-\frac{x^4\,\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+512\,a\,b^5\,c^2\,d^4+512\,a^2\,b^4\,c\,d^5}{4\,a\,d-4\,b\,c}+64\,a\,b^4\,c\,d^4}{4\,a\,d-4\,b\,c}-8\,b^4\,d^4\,x^4}{4\,a\,d-4\,b\,c}}\right)\,2{}\mathrm{i}}{4\,a\,d-4\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + b*x^4)*(c + d*x^4)),x)

[Out] -(atan(((((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4*x^4)*1i)/(4*a*d - 4*b*c) - (((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) - (x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) - (x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) - 8*b^4*d^4*x^4)*1i)/(4*a*d - 4*b*c))/(((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4*x^4)/(4*a*d - 4*b*c) + ((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) - (x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) - (x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) - 8*b^4*d^4*x^4)/(4*a*d - 4*b*c)))*2i)/(4*a*d - 4*b*c)

3.771 $\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=62

$$\frac{\log(x)}{ac}-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}$$

```
[Out] ln(x)/a/c-1/4*b*ln(b*x^4+a)/a/(-a*d+b*c)+1/4*d*ln(d*x^4+c)/c/(-a*d+b*c)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {457, 84}

$$-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^4])/(4*a*(b*c - a*d)) + (d*Log[c + d*x^4])/(4
*c*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)\,dx,x,x^4\right)\\
&= \frac{\log(x)}{ac}-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.87

$$\frac{4bc\log(x) - 4ad\log(x) - bc\log(a + bx^4) + ad\log(c + dx^4)}{4abc^2 - 4a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (4*b*c*Log[x] - 4*a*d*Log[x] - b*c*Log[a + b*x^4] + a*d*Log[c + d*x^4])/(4*
a*b*c^2 - 4*a^2*c*d)
```

Maple [A]
time = 0.70, size = 59, normalized size = 0.95

method	result	size
default	$-\frac{d\ln(dx^4+c)}{4c(ad-bc)} + \frac{\ln(x)}{ac} + \frac{b\ln(bx^4+a)}{4a(ad-bc)}$	59
norman	$-\frac{d\ln(dx^4+c)}{4c(ad-bc)} + \frac{\ln(x)}{ac} + \frac{b\ln(bx^4+a)}{4a(ad-bc)}$	59
risch	$-\frac{d\ln(dx^4+c)}{4c(ad-bc)} + \frac{\ln(x)}{ac} + \frac{b\ln(bx^4+a)}{4a(ad-bc)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d/c/(a*d-b*c)*ln(d*x^4+c)+ln(x)/a/c+1/4*b/a/(a*d-b*c)*ln(b*x^4+a)
```

Maxima [A]
time = 0.28, size = 61, normalized size = 0.98

$$-\frac{b\log(bx^4 + a)}{4(abc - a^2d)} + \frac{d\log(dx^4 + c)}{4(bc^2 - acd)} + \frac{\log(x^4)}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*b*log(b*x^4 + a)/(a*b*c - a^2*d) + 1/4*d*log(d*x^4 + c)/(b*c^2 - a*c*d
) + 1/4*log(x^4)/(a*c)
```

Fricas [A]
time = 6.93, size = 54, normalized size = 0.87

$$-\frac{bc\log(bx^4 + a) - ad\log(dx^4 + c) - 4(bc - ad)\log(x)}{4(abc^2 - a^2cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*log(b*x^4 + a) - a*d*log(d*x^4 + c) - 4*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.08, size = 73, normalized size = 1.18

$$-\frac{b^2\log\left(|bx^4+a|\right)}{4\left(ab^2c-a^2bd\right)}+\frac{d^2\log\left(|dx^4+c|\right)}{4\left(bc^2d-acd^2\right)}+\frac{\log\left(x^4\right)}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*b^2*log(abs(b*x^4 + a))/(a*b^2*c - a^2*b*d) + 1/4*d^2*log(abs(d*x^4 +
c))/(b*c^2*d - a*c*d^2) + 1/4*log(x^4)/(a*c)
```

Mupad [**B**]
time = 5.49, size = 58, normalized size = 0.94

$$\frac{b\,\ln\left(b\,x^4+a\right)}{4\,a^2\,d-4\,a\,b\,c}+\frac{d\,\ln\left(d\,x^4+c\right)}{4\,b\,c^2-4\,a\,c\,d}+\frac{\ln\left(x\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (b*log(a + b*x^4))/(4*a^2*d - 4*a*b*c) + (d*log(c + d*x^4))/(4*b*c^2 - 4*a*
c*d) + log(x)/(a*c)
```

3.772 $\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=87

$$-\frac{1}{4acx^4}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}$$

```
[Out] -1/4/a/c/x^4-(a*d+b*c)*ln(x)/a^2/c^2+1/4*b^2*ln(b*x^4+a)/a^2/(-a*d+b*c)-1/4
*d^2*ln(d*x^4+c)/c^2/(-a*d+b*c)
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/4*1/(a*c*x^4) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^4])/(4
*a^2*(b*c - a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right.\right.\\ &= -\frac{1}{4acx^4}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 1.01

$$-\frac{1}{4acx^4}+\frac{(-bc-ad)\log(x)}{a^2c^2}-\frac{b^2\log(a+bx^4)}{4a^2(-bc+ad)}-\frac{d^2\log(c+dx^4)}{4c^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/4*1/(a*c*x^4) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^4])
/(4*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

Maple [A]
time = 0.84, size = 83, normalized size = 0.95

method	result	size
norman	$-\frac{1}{4acx^4}-\frac{b^2\ln(bx^4+a)}{4a^2(ad-bc)}+\frac{d^2\ln(dx^4+c)}{4c^2(ad-bc)}-\frac{(ad+bc)\ln(x)}{a^2c^2}$	82
default	$\frac{d^2\ln(dx^4+c)}{4c^2(ad-bc)}-\frac{1}{4acx^4}+\frac{(-ad-bc)\ln(x)}{a^2c^2}-\frac{b^2\ln(bx^4+a)}{4a^2(ad-bc)}$	83
risch	$-\frac{1}{4acx^4}-\frac{\ln(x)d}{a\,c^2}-\frac{\ln(x)b}{a^2c}-\frac{b^2\ln(bx^4+a)}{4a^2(ad-bc)}+\frac{d^2\ln(-dx^4-c)}{4(ad-bc)c^2}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d^2/c^2/(a*d-b*c)*ln(d*x^4+c)-1/4/a/c/x^4+1/a^2/c^2*(-a*d-b*c)*ln(x)-1/
4*b^2/a^2/(a*d-b*c)*ln(b*x^4+a)
```

Maxima [A]
time = 0.35, size = 87, normalized size = 1.00

$$\frac{b^2\log(bx^4+a)}{4\,(a^2bc-a^3d)}-\frac{d^2\log(dx^4+c)}{4\,(bc^3-ac^2d)}-\frac{(bc+ad)\log(x^4)}{4\,a^2c^2}-\frac{1}{4\,acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(b*x^4 + a)/(a^2*b*c - a^3*d) - 1/4*d^2*log(d*x^4 + c)/(b*c^3 -
a*c^2*d) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) - 1/4/(a*c*x^4)
```

Fricas [A]
time = 20.18, size = 99, normalized size = 1.14

$$\frac{b^2c^2x^4\log(bx^4+a)-a^2d^2x^4\log(dx^4+c)-4\,(b^2c^2-a^2d^2)x^4\log(x)-abc^2+a^2cd}{4\,(a^2bc^3-a^3c^2d)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*c^2*x^4*log(b*x^4 + a) - a^2*d^2*x^4*log(d*x^4 + c) - 4*(b^2*c^2 -
 a^2*d^2)*x^4*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.26, size = 112, normalized size = 1.29

$$\frac{b^3 \log\left(|bx^4 + a|\right)}{4\left(a^2b^2c - a^3bd\right)} - \frac{d^3 \log\left(|dx^4 + c|\right)}{4\left(bc^3d - ac^2d^2\right)} - \frac{(bc + ad)\log\left(x^4\right)}{4\,a^2c^2} + \frac{bcx^4 + adx^4 - ac}{4\,a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*b^3*log(abs(b*x^4 + a))/(a^2*b^2*c - a^3*b*d) - 1/4*d^3*log(abs(d*x^4 +
 c))/(b*c^3*d - a*c^2*d^2) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) + 1/4*(b*c*
x^4 + a*d*x^4 - a*c)/(a^2*c^2*x^4)
```

Mupad [**B**]
time = 6.21, size = 87, normalized size = 1.00

$$-\frac{b^2\,\ln\left(b\,x^4+a\right)}{4\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{d^2\,\ln\left(d\,x^4+c\right)}{4\,\left(b\,c^3-a\,c^2\,d\right)}-\frac{1}{4\,a\,c\,x^4}-\frac{\ln\left(x\right)\,\left(a\,d+b\,c\right)}{a^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - (b^2*log(a + b*x^4))/(4*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^4))/(4*(b*c
^3 - a*c^2*d)) - 1/(4*a*c*x^4) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

3.773 $\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=112

$$-\frac{(bc+ad)x^2}{2b^2d^2}+\frac{x^6}{6bd}-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}+\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}$$

```
[Out] -1/2*(a*d+b*c)*x^2/b^2/d^2+1/6*x^6/b/d-1/2*a^(5/2)*arctan(x^2*b^(1/2)/a^(1/
2))/b^(5/2)/(-a*d+b*c)+1/2*c^(5/2)*arctan(x^2*d^(1/2)/c^(1/2))/d^(5/2)/(-a*
d+b*c)
```

Rubi [A]
time = 0.18, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {476, 490, 596, 536, 211}

$$-\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}+\frac{c^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}-\frac{x^2(ad+bc)}{2b^2d^2}+\frac{x^6}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*((b*c + a*d)*x^2)/(b^2*d^2) + x^6/(6*b*d) - (a^(5/2)*ArcTan[(Sqrt[b]*x
^2)/Sqrt[a]])/(2*b^(5/2)*(b*c - a*d)) + (c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[
c]])/(2*d^(5/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
```

```
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right) \\
&= \frac{x^6}{6bd} - \frac{\text{Subst}\left(\int \frac{x^2(3ac+3(bc+ad)x^2)}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{6bd} \\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} + \frac{\text{Subst}\left(\int \frac{3ac(bc+ad)+3(b^2c^2+ad(bc+ad))x^2}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{6b^2d^2} \\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b^2(bc-ad)} + \frac{c^3\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2d^2(bc-ad)} \\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)} + \frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 104, normalized size = 0.93

$$\frac{1}{6}\left(\frac{x^2(-3bc-3ad+bdx^4)}{b^2d^2} + \frac{3a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{b^{5/2}(-bc+ad)} + \frac{3c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{d^{5/2}(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((x^2*(-3*b*c - 3*a*d + b*d*x^4))/(b^2*d^2) + (3*a^(5/2)*ArcTan[(Sqrt[b]*x^
2)/Sqrt[a]])/(b^(5/2)*(-(b*c) + a*d)) + (3*c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqr
t[c]])/(d^(5/2)*(b*c - a*d)))/6
```

Maple [A]
time = 3.42, size = 98, normalized size = 0.88

method	result
default	$-\frac{-\frac{b d \,x^{6}}{6}+\frac{(a d +b c) x^{2}}{2}}{b^{2} d^{2}}-\frac{c^{3} \arctan \left(\frac{d \,x^{2}}{\sqrt{c d}}\right)}{2 d^{2} (a d -b c) \sqrt{c d}}+\frac{a^{3} \arctan \left(\frac{b \,x^{2}}{\sqrt{a b}}\right)}{2 b^{2} (a d -b c) \sqrt{a b}}$
risch	$\frac{x^{6}}{6 b d}-\frac{a \,x^{2}}{2 b^{2} d}-\frac{c \,x^{2}}{2 b \,d^{2}}+\frac{\sqrt{-c d}\, c^{2} \ln \left(\left(-a^{6} d^{8}+a \,d^{3} c^{5} b^{5}\right) x^{2}+\left(-c d \right)^{\frac{3}{2}} a \,b^{5} c^{4} d +\left(-c d \right)^{\frac{3}{2}} b^{6} c^{5}+a^{6} d^{7} \sqrt{-c d}+b^{6} c^{6} \sqrt{-c d}\, d \right)}{4 d^{3} (a d -b c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2/d^2*(-1/6*b*d*x^6+1/2*(a*d+b*c)*x^2)-1/2*c^3/d^2/(a*d-b*c)/(c*d)^(1/
2)*arctan(d*x^2/(c*d)^(1/2))+1/2*a^3/b^2/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2
/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 100, normalized size = 0.89

$$-\frac{a^{3} \arctan \left(\frac{b x^{2}}{\sqrt{a b}}\right)}{2 \left(b^{3} c - a b^{2} d\right) \sqrt{a b}} + \frac{c^{3} \arctan \left(\frac{d x^{2}}{\sqrt{c d}}\right)}{2 \left(b c d^{2} - a d^{3}\right) \sqrt{c d}} + \frac{b d x^{6} - 3 \left(b c + a d\right) x^{2}}{6 \, b^{2} d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b*d*x^6 - 3*(b*c
 + a*d)*x^2)/(b^2*d^2)
```

Fricas [A]
time = 8.39, size = 576, normalized size = 5.14

$$\left[\frac{2 \left(b^{2} c d - a b d^{2}\right) x^{6} - 3 \, a^{2} d^{2} \sqrt{-\frac{a}{b}} \log \left(\frac{b x^{4} + 2 \, b x^{2} \sqrt{-\frac{a}{b}} - a}{b x^{4} + a}\right) - 3 \, b^{2} c^{2} \sqrt{-\frac{c}{d}} \log \left(\frac{d x^{4} - 2 \, d x^{2} \sqrt{-\frac{c}{d}} - c}{d x^{4} + c}\right) - 6 \left(b^{2} c^{2} - a^{2} d^{2}\right) x^{2}}{12 \left(b^{3} c d^{2} - a b^{2} d^{3}\right)}, \frac{2 \left(b^{2} c d - a b d^{2}\right) x^{6} - 6 \, a^{2} d^{2} \sqrt{\frac{a}{b}} \arctan \left(\frac{b x^{2} \sqrt{\frac{a}{b}}}{a}\right) - 3 \, b^{2} c^{2} \sqrt{-\frac{c}{d}} \log \left(\frac{d x^{4} - 2 \, d x^{2} \sqrt{-\frac{c}{d}} - c}{d x^{4} + c}\right) - 6 \left(b^{2} c^{2} - a^{2} d^{2}\right) x^{2}}{12 \left(b^{3} c d^{2} - a b^{2} d^{3}\right)}, \frac{2 \left(b^{2} c d - a b d^{2}\right) x^{6} + 6 \, b^{2} c^{2} \sqrt{\frac{c}{d}} \arctan \left(\frac{d x^{2} \sqrt{\frac{c}{d}}}{c}\right) - 3 \, a^{2} d^{2} \sqrt{-\frac{a}{b}} \log \left(\frac{b x^{4} + 2 \, b x^{2} \sqrt{-\frac{a}{b}} - a}{b x^{4} + a}\right) - 6 \left(b^{2} c^{2} - a^{2} d^{2}\right) x^{2}}{12 \left(b^{3} c d^{2} - a b^{2} d^{3}\right)}, \frac{\left(b^{2} c d - a b d^{2}\right) x^{6} - 3 \, a^{2} d^{2} \sqrt{\frac{a}{b}} \arctan \left(\frac{b x^{2} \sqrt{\frac{a}{b}}}{a}\right) + 3 \, b^{2} c^{2} \sqrt{\frac{c}{d}} \arctan \left(\frac{d x^{2} \sqrt{\frac{c}{d}}}{c}\right) - 3 \left(b^{2} c^{2} - a^{2} d^{2}\right) x^{2}}{6 \left(b^{3} c d^{2} - a b^{2} d^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 3*a^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^
2*sqrt(-a/b) - a)/(b*x^4 + a)) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*
sqrt(-c/d) - c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^
2*d^3), 1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 6*a^2*d^2*sqrt(a/b)*arctan(b*x^2*
sqrt(a/b)/a) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d
*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/12*(2*(b
^2*c*d - a*b*d^2)*x^6 + 6*b^2*c^2*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) - 3*a
^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) - 6*(b^
2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/6*((b^2*c*d - a*b*d^2)*x^6
 - 3*a^2*d^2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + 3*b^2*c^2*sqrt(c/d)*arct
an(d*x^2*sqrt(c/d)/c) - 3*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.10, size = 112, normalized size = 1.00

$$-\frac{a^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^3c - ab^2d)\sqrt{ab}} + \frac{c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd^2 - ad^3)\sqrt{cd}} + \frac{b^2d^2x^6 - 3\,b^2cdx^2 - 3\,abd^2x^2}{6\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b^2*d^2*x^6 - 3*
b^2*c*d*x^2 - 3*a*b*d^2*x^2)/(b^3*d^3)
```

Mupad [**B**]
time = 5.70, size = 532, normalized size = 4.75

$$\frac{\ln\left(d^{10}\,{\left(-a^5\,b^5\right)}^{5/2}+b^{20}\,c^{10}\,\sqrt{-a^5\,b^5}-a^2\,b^{23}\,c^{10}\,x^2-a^{12}\,b^{13}\,d^{10}\,x^2+2\,b^{10}\,c^5\,d^5\,{\left(-a^5\,b^5\right)}^{3/2}+2\,a^7\,b^{18}\,c^5\,d^5\,x^2\right)\,\sqrt{-a^5\,b^5}}{4\,b^6\,c-4\,a\,b^5\,d}-\frac{\ln\left(d^{10}\,{\left(-a^5\,b^5\right)}^{5/2}+b^{20}\,c^{10}\,\sqrt{-a^5\,b^5}+a^2\,b^{23}\,c^{10}\,x^2+a^{12}\,b^{13}\,d^{10}\,x^2+2\,b^{10}\,c^5\,d^5\,{\left(-a^5\,b^5\right)}^{3/2}-2\,a^7\,b^{18}\,c^5\,d^5\,x^2\right)\,\sqrt{-a^5\,b^5}}{4\,\left(b^6\,c-a\,b^5\,d\right)}-\frac{\ln\left(b^{10}\,{\left(-c^5\,d^5\right)}^{5/2}+a^{10}\,d^{20}\,\sqrt{-c^5\,d^5}+a^{10}\,c^2\,d^{23}\,x^2+b^{10}\,c^{12}\,d^{13}\,x^2+2\,a^5\,b^5\,d^{10}\,{\left(-c^5\,d^5\right)}^{3/2}-2\,a^5\,b^5\,c^7\,d^{18}\,x^2\right)\,\sqrt{-c^5\,d^5}}{4\,\left(a\,d^6-b\,c\,d^5\right)}+\frac{\ln\left(b^{10}\,{\left(-c^5\,d^5\right)}^{5/2}+a^{10}\,d^{20}\,\sqrt{-c^5\,d^5}-a^{10}\,c^2\,d^{23}\,x^2-b^{10}\,c^{12}\,d^{13}\,x^2+2\,a^5\,b^5\,d^{10}\,{\left(-c^5\,d^5\right)}^{3/2}+2\,a^5\,b^5\,c^7\,d^{18}\,x^2\right)\,\sqrt{-c^5\,d^5}}{4\,a\,d^6-4\,b\,c\,d^5}+\frac{x^6}{6\,b\,d}-\frac{x^2\,\left(a\,d+b\,c\right)}{2\,b^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(d^10*(-a^5*b^5)^(5/2) + b^20*c^10*(-a^5*b^5)^(1/2) - a^2*b^23*c^10*x^2
 - a^12*b^13*d^10*x^2 + 2*b^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^7*b^18*c^5*d^
5*x^2)*(-a^5*b^5)^(1/2))/(4*b^6*c - 4*a*b^5*d) - (log(d^10*(-a^5*b^5)^(5/2)
 + b^20*c^10*(-a^5*b^5)^(1/2) + a^2*b^23*c^10*x^2 + a^12*b^13*d^10*x^2 + 2*
b^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^7*b^18*c^5*d^5*x^2)*(-a^5*b^5)^(1/2))/(
4*(b^6*c - a*b^5*d)) - (log(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d^5)^(1
/2) + a^10*c^2*d^23*x^2 + b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*d^5)^(3
/2) - 2*a^5*b^5*c^7*d^18*x^2)*(-c^5*d^5)^(1/2))/(4*(a*d^6 - b*c*d^5)) + (lo
g(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d^5)^(1/2) - a^10*c^2*d^23*x^2 -
b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*c^7*d^18*x
^2)*(-c^5*d^5)^(1/2))/(4*a*d^6 - 4*b*c*d^5) + x^6/(6*b*d) - (x^2*(a*d + b*c
))/(2*b^2*d^2)
```

3.774 $\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=92

$$\frac{x^2}{2bd}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}$$

```
[Out] 1/2*x^2/b/d+1/2*a^(3/2)*arctan(x^2*b^(1/2)/a^(1/2))/b^(3/2)/(-a*d+b*c)-1/2*
c^(3/2)*arctan(x^2*d^(1/2)/c^(1/2))/d^(3/2)/(-a*d+b*c)
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {476, 490, 536, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)}-\frac{c^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}+\frac{x^2}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x^2/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d
)) - (c^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*d^(3/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)\\ &= \frac{x^2}{2bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+ad)x^2}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{2bd}\\ &= \frac{x^2}{2bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b(bc-ad)} - \frac{c^2\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2d(bc-ad)}\\ &= \frac{x^2}{2bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)} - \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 82, normalized size = 0.89

$$\frac{\left(-\frac{a}{b}+\frac{c}{d}\right)x^2 + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{b^{3/2}} - \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{d^{3/2}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-(a/b) + c/d)*x^2 + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/b^(3/2) - (c^
(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/d^(3/2))/(2*b*c - 2*a*d)
```

Maple [A]
time = 2.66, size = 81, normalized size = 0.88

method	result

default	$\frac{x^2}{2bd}+\frac{c^2\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2d(ad-bc)\sqrt{cd}}-\frac{a^2\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2b(ad-bc)\sqrt{ab}}$
risch	$\frac{x^2}{2bd}+\frac{\sqrt{-ab}\,a\ln\left((b^3cd^3a^3-b^6c^4)x^2+(-ab)^{\frac{3}{2}}a^3d^4+(-ab)^{\frac{3}{2}}a^2bcd^3+a^4\sqrt{-ab}\,d^4b+b^5c^4\sqrt{-ab}\right)}{4b^2(ad-bc)}-\frac{\sqrt{-ab}\,a\ln\left((b^3c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/b/d+1/2*c^2/d/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))-1/2*a
^2/b/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 80, normalized size = 0.87

$$\frac{a^2\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(b^2c-abd\right)\sqrt{ab}}-\frac{c^2\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bcd-ad^2\right)\sqrt{cd}}+\frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arcta
n(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

Fricas [A]
time = 4.47, size = 416, normalized size = 4.52

$$\left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4+2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)-2\,(bc-ad)x^2}{4\,(b^2cd-abd^2)},\ \frac{2\,ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)-bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4+2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)+2\,(bc-ad)x^2}{4\,(b^2cd-abd^2)},\ -\frac{2\,bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)+ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)-2\,(bc-ad)x^2}{4\,(b^2cd-abd^2)},\ \frac{ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)-bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)+(bc-ad)x^2}{2\,(b^2cd-abd^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + b
*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) - 2*(b*c -
a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/4*(2*a*d*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/
a) - b*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) + 2*(
b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), -1/4*(2*b*c*sqrt(c/d)*arctan(d*x^2*sqr
t(c/d)/c) + a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)
) - 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/2*(a*d*sqrt(a/b)*arctan(b*x^2
*sqrt(a/b)/a) - b*c*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) + (b*c - a*d)*x^2)/
(b^2*c*d - a*b*d^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.24, size = 80, normalized size = 0.87

$$\frac{a^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^2c - abd)\sqrt{ab}} - \frac{c^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd - ad^2)\sqrt{cd}} + \frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arcta
n(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

Mupad [**B**]
time = 5.72, size = 518, normalized size = 5.63

$$\frac{\ln\left(b^9\,c^6\,\sqrt{-a^3\,b^3}-a^3\,d^6\,{\left(-a^3\,b^3\right)}^{3/2}+a\,b^{11}\,c^6\,x^2+a^7\,b^5\,d^6\,x^2+2\,b^3\,c^3\,d^3\,{\left(-a^3\,b^3\right)}^{3/2}-2\,a^4\,b^8\,c^3\,d^3\,x^2\right)\,\sqrt{-a^3\,b^3}}{4\,b^4\,c-4\,a\,b^3\,d}-\frac{\ln\left(a^3\,d^6\,{\left(-a^3\,b^3\right)}^{3/2}-b^9\,c^6\,\sqrt{-a^3\,b^3}+a\,b^{11}\,c^6\,x^2+a^7\,b^5\,d^6\,x^2-2\,b^3\,c^3\,d^3\,{\left(-a^3\,b^3\right)}^{3/2}-2\,a^4\,b^8\,c^3\,d^3\,x^2\right)\,\sqrt{-a^3\,b^3}}{4\,\left(b^4\,c-a\,b^3\,d\right)}-\frac{\ln\left(b^6\,c^3\,{\left(-c^3\,d^3\right)}^{3/2}-a^6\,d^9\,\sqrt{-c^3\,d^3}+a^6\,c\,d^{11}\,x^2+b^6\,c^7\,d^5\,x^2-2\,a^3\,b^3\,d^3\,{\left(-c^3\,d^3\right)}^{3/2}-2\,a^3\,b^3\,c^4\,d^8\,x^2\right)\,\sqrt{-c^3\,d^3}}{4\,\left(a\,d^4-b\,c\,d^3\right)}+\frac{\ln\left(a^6\,d^9\,\sqrt{-c^3\,d^3}-b^6\,c^3\,{\left(-c^3\,d^3\right)}^{3/2}+a^6\,c\,d^{11}\,x^2+b^6\,c^7\,d^5\,x^2+2\,a^3\,b^3\,d^3\,{\left(-c^3\,d^3\right)}^{3/2}-2\,a^3\,b^3\,c^4\,d^8\,x^2\right)\,\sqrt{-c^3\,d^3}}{4\,a\,d^4-4\,b\,c\,d^3}+\frac{x^2}{2\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(b^9*c^6*(-a^3*b^3)^(1/2) - a^3*d^6*(-a^3*b^3)^(3/2) + a*b^11*c^6*x^2 +
 a^7*b^5*d^6*x^2 + 2*b^3*c^3*d^3*(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*
(-a^3*b^3)^(1/2))/(4*b^4*c - 4*a*b^3*d) - (log(a^3*d^6*(-a^3*b^3)^(3/2) - b
^9*c^6*(-a^3*b^3)^(1/2) + a*b^11*c^6*x^2 + a^7*b^5*d^6*x^2 - 2*b^3*c^3*d^3*
(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*(-a^3*b^3)^(1/2))/(4*(b^4*c - a*b
^3*d)) - (log(b^6*c^3*(-c^3*d^3)^(3/2) - a^6*d^9*(-c^3*d^3)^(1/2) + a^6*c*d
^11*x^2 + b^6*c^7*d^5*x^2 - 2*a^3*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*
d^8*x^2)*(-c^3*d^3)^(1/2))/(4*(a*d^4 - b*c*d^3)) + (log(a^6*d^9*(-c^3*d^3)^
(1/2) - b^6*c^3*(-c^3*d^3)^(3/2) + a^6*c*d^11*x^2 + b^6*c^7*d^5*x^2 + 2*a^3
*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*d^8*x^2)*(-c^3*d^3)^(1/2))/(4*a*d
^4 - 4*b*c*d^3) + x^2/(2*b*d)
```

3.775 $\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc-ad)}+\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{d}\,(bc-ad)}$$

```
[Out] -1/2*arctan(x^2*b^(1/2)/a^(1/2))*a^(1/2)/(-a*d+b*c)/b^(1/2)+1/2*arctan(x^2*
d^(1/2)/c^(1/2))*c^(1/2)/(-a*d+b*c)/d^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {476, 492, 211}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{d}\,(bc-ad)}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d)) + (Sqrt[
c]*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[d]*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)$$
$$= -\frac{a\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2(bc-ad)} + \frac{c\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2(bc-ad)}$$
$$= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc-ad)} + \frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{d}\,(bc-ad)}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.84

$$\frac{-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{\sqrt{d}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqrt
[d]*x^2)/Sqrt[c]])/Sqrt[d])/(2*b*c - 2*a*d)
```

Maple [A]
time = 0.44, size = 60, normalized size = 0.76

method	result
default	$-\frac{c\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2(ad-bc)\sqrt{cd}} + \frac{a\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2(ad-bc)\sqrt{ab}}$
risch	$\frac{\sqrt{-ab}\,\ln\left(\left(-a\,b^3cd+b^4c^2\right)x^2+(-ab)^{\frac{3}{2}}a\,d^2+(-ab)^{\frac{3}{2}}bcd+a^2\sqrt{-ab}\,d^2b+b^3c^2\sqrt{-ab}\right)}{4b(ad-bc)} - \frac{\sqrt{-ab}\,\ln\left(\left(-a\,b^3cd+b^4c^2\right)x^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))+1/2*a/(a*d-b*c)/(a*b
)^(1/2)*arctan(b*x^2/(a*b)^(1/2))
```

Maxima [A]
time = 0.54, size = 59, normalized size = 0.75

$$-\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(bc-ad)}+\frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Fricas [A]
time = 4.01, size = 325, normalized size = 4.11

$$\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)},-\frac{2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)},\frac{2\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)-\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)}{4\,(bc-ad)},-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)}{2\,(bc-ad)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + sqrt(
-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), -1/4*
(2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sq
rt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(2*sqrt(c/d)*arctan(d*x^2*sqrt
(c/d)/c) - sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)))/(b
*c - a*d), -1/2*(sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) - sqrt(c/d)*arctan(d*x
^2*sqrt(c/d)/c))/(b*c - a*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.16, size = 59, normalized size = 0.75

$$-\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(bc-ad)}+\frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Mupad [B]
time = 5.34, size = 379, normalized size = 4.80

$$\frac{\ln\left(d^2\,{\left(-a\,b\right)}^{5/2}+b^4\,c^2\,\sqrt{-a\,b}-b^5\,c^2\,x^2+2\,b^2\,c\,d\,{\left(-a\,b\right)}^{3/2}-a^2\,b^3\,d^2\,x^2+2\,a\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,b^2\,c-4\,a\,b\,d}-\frac{\ln\left(d^2\,{\left(-a\,b\right)}^{5/2}+b^4\,c^2\,\sqrt{-a\,b}+b^5\,c^2\,x^2+2\,b^2\,c\,d\,{\left(-a\,b\right)}^{3/2}+a^2\,b^3\,d^2\,x^2-2\,a\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,\left(b^2\,c-a\,b\,d\right)}-\frac{\ln\left(b^2\,{\left(-c\,d\right)}^{5/2}+a^2\,d^4\,\sqrt{-c\,d}+a^2\,d^5\,x^2+2\,a\,b\,d^2\,{\left(-c\,d\right)}^{3/2}+b^2\,c^2\,d^3\,x^2-2\,a\,b\,c\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,\left(a\,d^2-b\,c\,d\right)}+\frac{\ln\left(b^2\,{\left(-c\,d\right)}^{5/2}+a^2\,d^4\,\sqrt{-c\,d}-a^2\,d^5\,x^2+2\,a\,b\,d^2\,{\left(-c\,d\right)}^{3/2}-b^2\,c^2\,d^3\,x^2+2\,a\,b\,c\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,a\,d^2-4\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) - b^5*c^2*x^2 + 2*b^2*c*d*(-a*
b)^(3/2) - a^2*b^3*d^2*x^2 + 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*b^2*c - 4*a*
b*d) - (log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) + b^5*c^2*x^2 + 2*b^2*c
*d*(-a*b)^(3/2) + a^2*b^3*d^2*x^2 - 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*(b^2*
c - a*b*d)) - (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) + a^2*d^5*x^2 +
2*a*b*d^2*(-c*d)^(3/2) + b^2*c^2*d^3*x^2 - 2*a*b*c*d^4*x^2)*(-c*d)^(1/2))/(
4*(a*d^2 - b*c*d)) + (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) - a^2*d^5
*x^2 + 2*a*b*d^2*(-c*d)^(3/2) - b^2*c^2*d^3*x^2 + 2*a*b*c*d^4*x^2)*(-c*d)^(
1/2))/(4*a*d^2 - 4*b*c*d)
```

3.776 $\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)}$$

```
[Out] 1/2*arctan(x^2*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)/a^(1/2)-1/2*arctan(x^2*d
^(1/2)/c^(1/2))*d^(1/2)/(-a*d+b*c)/c^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {476, 400, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*
ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[c]*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{(a+bx^4)\,(c+dx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^2)\,(c+dx^2)}\,dx, x, x^2\right)$$
$$= \frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2(bc-ad)} - \frac{d\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2(bc-ad)}$$
$$= \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)} - \frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.84

$$\frac{\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{\sqrt{c}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^4)*(c + d*x^4)),x]
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]
*x^2)/Sqrt[c]])/Sqrt[c])/(2*b*c - 2*a*d)
```

Maple [A]
time = 0.44, size = 60, normalized size = 0.76

method	result
default	$\frac{d\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2(ad-bc)\sqrt{cd}} - \frac{b\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2(ad-bc)\sqrt{ab}}$
risch	$\frac{\sqrt{-cd}\,\ln\left(\left(-ac\,d^3+d^2b\,c^2\right)x^2+(-cd)^{\frac{3}{2}}ad+(-cd)^{\frac{3}{2}}bc+2\sqrt{-cd}\,b\,c^2d\right)}{4c(ad-bc)} - \frac{\sqrt{-cd}\,\ln\left(\left(-ac\,d^3+d^2b\,c^2\right)x^2-(-cd)^{\frac{3}{2}}ad-(-c\right.}{4c(ad-bc)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] 1/2*d/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))-1/2*b/(a*d-b*c)/(a*b)
^(1/2)*arctan(b*x^2/(a*b)^(1/2))
```

Maxima [A]
time = 0.55, size = 59, normalized size = 0.75

$$\frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(bc-ad)} - \frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2/
sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Fricas [A]
time = 4.28, size = 325, normalized size = 4.11

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)}{4\,(bc-ad)}, \frac{2\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)-\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)}{4\,(bc-ad)}, -\frac{2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)}{4\,(bc-ad)}, -\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)-\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)}{2\,(bc-ad)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a)) + sqrt(
-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(
2*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - sqrt(-b/a)*log((b*x^4 - 2*a*x^2*s
qrt(-b/a) - a)/(b*x^4 + a)))/(b*c - a*d), -1/4*(2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*x^2)) + sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c))
)/(b*c - a*d), -1/2*(sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - sqrt(d/c)*arct
an(c*sqrt(d/c)/(d*x^2)))/(b*c - a*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.21, size = 59, normalized size = 0.75

$$\frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2/
sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Mupad [B]
time = 5.29, size = 399, normalized size = 5.05

$$\frac{\ln \left(a^2 d^2 (-a b)^{5/2}+b^2 c^2 (-a b)^{5/2}+2 c d (-a b)^{7/2}-a^2 b^5 c^2 x^2-a^4 b^3 d^2 x^2+2 a^3 b^4 c d x^2\right) \sqrt{-a b}}{4 a^2 d-4 a b c}-\frac{\ln \left(a^2 d^2 (-a b)^{5/2}+b^2 c^2 (-a b)^{5/2}+2 c d (-a b)^{7/2}+a^2 b^5 c^2 x^2+a^4 b^3 d^2 x^2-2 a^3 b^4 c d x^2\right) \sqrt{-a b}}{4 \left(a^2 d-a b c\right)}-\frac{\ln \left(a^2 d^2 (-c d)^{5/2}+b^2 c^2 (-c d)^{5/2}+2 a b (-c d)^{7/2}+a^2 c^2 d^5 x^2+b^2 c^4 d^3 x^2-2 a b c^3 d^4 x^2\right) \sqrt{-c d}}{4 \left(b c^2-a c d\right)}+\frac{\ln \left(a^2 d^2 (-c d)^{5/2}+b^2 c^2 (-c d)^{5/2}+2 a b (-c d)^{7/2}-a^2 c^2 d^5 x^2-b^2 c^4 d^3 x^2+2 a b c^3 d^4 x^2\right) \sqrt{-c d}}{4 b c^2-4 a c d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-a*b)^(7/2) - a^2
*b^5*c^2*x^2 - a^4*b^3*d^2*x^2 + 2*a^3*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*a^2*d
- 4*a*b*c) - (log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-a*b
)^(7/2) + a^2*b^5*c^2*x^2 + a^4*b^3*d^2*x^2 - 2*a^3*b^4*c*d*x^2)*(-a*b)^(1/
2))/(4*(a^2*d - a*b*c)) - (log(a^2*d^2*(-c*d)^(5/2) + b^2*c^2*(-c*d)^(5/2)
+ 2*a*b*(-c*d)^(7/2) + a^2*c^2*d^5*x^2 + b^2*c^4*d^3*x^2 - 2*a*b*c^3*d^4*x^
2)*(-c*d)^(1/2))/(4*(b*c^2 - a*c*d)) + (log(a^2*d^2*(-c*d)^(5/2) + b^2*c^2*
(-c*d)^(5/2) + 2*a*b*(-c*d)^(7/2) - a^2*c^2*d^5*x^2 - b^2*c^4*d^3*x^2 + 2*a
*b*c^3*d^4*x^2)*(-c*d)^(1/2))/(4*b*c^2 - 4*a*c*d)
```

3.777 $\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=92

$$-\frac{1}{2acx^2}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}$$

```
[Out] -1/2/a/c/x^2-1/2*b^(3/2)*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)+1/2
*d^(3/2)*arctan(x^2*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*c)
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {476, 491, 536, 211}

$$-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*1/(a*c*x^2) - (b^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*a^(3/2)*(b*c
- a*d)) + (d^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)(c+dx^2)}\,dx,x,x^2\right)\\ &= -\frac{1}{2acx^2}+\frac{\text{Subst}\left(\int \frac{-bc-ad-bdx^2}{(a+bx^2)(c+dx^2)}\,dx,x,x^2\right)}{2ac}\\ &= -\frac{1}{2acx^2}-\frac{b^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2a(bc-ad)}+\frac{d^2\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx,x,x^2\right)}{2c(bc-ad)}\\ &= -\frac{1}{2acx^2}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 169, normalized size = 1.84

$$\frac{\frac{b}{a}-\frac{d}{c}-\frac{b^{3/2}x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{3/2}}-\frac{b^{3/2}x^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{3/2}}+\frac{d^{3/2}x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{3/2}}+\frac{d^{3/2}x^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{3/2}}}{2(-bc+ad)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (b/a - d/c - (b^(3/2)*x^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2)
- (b^(3/2)*x^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2) + (d^(3/2)*
x^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2) + (d^(3/2)*x^2*ArcTan[
1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2))/(2*(-(b*c) + a*d)*x^2)
```

Maple [A]
time = 0.46, size = 81, normalized size = 0.88

method	result

default	$-\frac{1}{2c\,x^{2}a}-\frac{d^{2}\arctan\left(\frac{d\,x^{2}}{\sqrt{cd}}\right)}{2c(ad-bc)\sqrt{cd}}+\frac{b^{2}\arctan\left(\frac{b\,x^{2}}{\sqrt{ab}}\right)}{2a(ad-bc)\sqrt{ab}}$
risch	$-\frac{1}{2c\,x^{2}a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{2}c^{3}a^{2}-2ab\,c^{4}d+b^{2}c^{5}\right)\_Z^{2}+d^{3}\right)}\_R\ln\left(\left(\left(-5c^{3}a^{7}d^{4}+18c^{4}a^{6}b\,d^{3}-26a^{5}c^{5}b^{2}d^{2}+18c^{6}a^{4}b^{3}d-5c^{7}a^{3}b^{4}\right)\_\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c/x^2/a-1/2*d^2/c/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))+1/2*
b^2/a/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))
```

Maxima [A]
time = 0.64, size = 80, normalized size = 0.87

$$-\frac{b^{2}\arctan\left(\frac{bx^{2}}{\sqrt{ab}}\right)}{2\left(abc-a^{2}d\right)\sqrt{ab}}+\frac{d^{2}\arctan\left(\frac{dx^{2}}{\sqrt{cd}}\right)}{2\left(bc^{2}-acd\right)\sqrt{cd}}-\frac{1}{2\,acx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arct
an(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

Fricas [A]
time = 7.43, size = 432, normalized size = 4.70

$$\left[-\frac{bcx^{2}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{4}+2\,ax^{2}\sqrt{-\frac{b}{a}}-a}{bx^{4}+a}\right)+adx^{2}\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{4}-2\,cx^{2}\sqrt{-\frac{d}{c}}-c}{dx^{4}+c}\right)+2\,bc-2\,ad}{4\,(abc^{2}-a^{2}cd)x^{2}},\ -\frac{2\,adx^{2}\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^{2}}\right)+bcx^{2}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{4}+2\,ax^{2}\sqrt{-\frac{b}{a}}-a}{bx^{4}+a}\right)+2\,bc-2\,ad}{4\,(abc^{2}-a^{2}cd)x^{2}},\ \frac{2\,bcx^{2}\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^{2}}\right)-adx^{2}\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{4}-2\,cx^{2}\sqrt{-\frac{d}{c}}-c}{dx^{4}+c}\right)-2\,bc+2\,ad}{4\,(abc^{2}-a^{2}cd)x^{2}},\ \frac{bcx^{2}\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^{2}}\right)-adx^{2}\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^{2}}\right)-bc+ad}{2\,(abc^{2}-a^{2}cd)x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a))
 + a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) + 2
*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), -1/4*(2*a*d*x^2*sqrt(d/c)*arctan(c
*sqrt(d/c)/(d*x^2)) + b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) -
a)/(b*x^4 + a)) + 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), 1/4*(2*b*c*x^2*
sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c
*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) - 2*b*c + 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2
), 1/2*(b*c*x^2*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqrt(d/c)*a
rctan(c*sqrt(d/c)/(d*x^2)) - b*c + a*d)/((a*b*c^2 - a^2*c*d)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.17, size = 80, normalized size = 0.87

$$-\frac{b^2\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(abc-a^2d\right)\sqrt{ab}}+\frac{d^2\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bc^2-acd\right)\sqrt{cd}}-\frac{1}{2\,acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arct
an(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

Mupad [**B**]
time = 5.35, size = 354, normalized size = 3.85

$$\frac{\ln\left(c^3\,x^2\,{\left(-a^3\,b^3\right)}^{3/2}-a^8\,b\,d^3+a^5\,b^4\,c^3+a^6\,d^3\,x^2\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{4\,a^4\,d-4\,a^3\,b\,c}-\frac{\ln\left(c^3\,x^2\,{\left(-a^3\,b^3\right)}^{3/2}+a^8\,b\,d^3-a^5\,b^4\,c^3+a^6\,d^3\,x^2\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{4\,\left(a^4\,d-a^3\,b\,c\right)}-\frac{1}{2\,a\,c\,x^2}-\frac{\ln\left(a^3\,x^2\,{\left(-c^3\,d^3\right)}^{3/2}+b^3\,c^8\,d-a^3\,c^5\,d^4+b^3\,c^6\,x^2\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{4\,\left(b\,c^4-a\,c^3\,d\right)}+\frac{\ln\left(a^3\,x^2\,{\left(-c^3\,d^3\right)}^{3/2}-b^3\,c^8\,d+a^3\,c^5\,d^4+b^3\,c^6\,x^2\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{4\,b\,c^4-4\,a\,c^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(c^3*x^2*(-a^3*b^3)^(3/2) - a^8*b*d^3 + a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3
*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(4*a^4*d - 4*a^3*b*c) - (log(c^3*x^2*(-a^3*b
^3)^(3/2) + a^8*b*d^3 - a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3*b^3)^(1/2))*(-a^3*b
^3)^(1/2))/(4*(a^4*d - a^3*b*c)) - 1/(2*a*c*x^2) - (log(a^3*x^2*(-c^3*d^3)^
(3/2) + b^3*c^8*d - a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*d^3)^
(1/2))/(4*(b*c^4 - a*c^3*d)) + (log(a^3*x^2*(-c^3*d^3)^(3/2) - b^3*c^8*d +
a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/(4*b*c^4 - 4*
a*c^3*d)
```

3.778 $\int \frac{1}{x^7(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}$$

```
[Out] -1/6/a/c/x^6+1/2*(a*d+b*c)/a^2/c^2/x^2+1/2*b^(5/2)*arctan(x^2*b^(1/2)/a^(1/
2))/a^(5/2)/(-a*d+b*c)-1/2*d^(5/2)*arctan(x^2*d^(1/2)/c^(1/2))/c^(5/2)/(-a*
d+b*c)
```

Rubi [A]
time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {476, 491, 597, 536, 211}

$$\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}-\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/6*1/(a*c*x^6) + (b*c + a*d)/(2*a^2*c^2*x^2) + (b^(5/2)*ArcTan[(Sqrt[b]*x
^2)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[
c]])/(2*c^(5/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
```

```
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6acx^6}+\frac{\text{Subst}\left(\int \frac{-3(bc+ad)-3bdx^2}{x^2(a+bx^2)(c+dx^2)}\,dx,x,x^2\right)}{6ac}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}-\frac{\text{Subst}\left(\int \frac{-3\left(b^2c^2+abcd+a^2d^2\right)-3bd(bc+ad)x^2}{(a+bx^2)(c+dx^2)}\,dx,x,x^2\right)}{6a^2c^2}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2a^2(bc-ad)}-\frac{d^3\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx,x,x^2\right.}{2c^2(bc-ad)}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}
\end{aligned}
$$

Mathematica [A]
time = 0.15, size = 193, normalized size = 1.72

$$
\frac{\frac{b}{a}-\frac{d}{c}-\frac{3b^2x^4}{a^2}+\frac{3d^2x^4}{c^2}+\frac{3b^{5/2}x^6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{5/2}}+\frac{3b^{5/2}x^6\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{5/2}}-\frac{3d^{5/2}x^6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{5/2}}-\frac{3d^{5/2}x^6\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{5/2}}}{6(-bc+ad)x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (b/a - d/c - (3*b^2*x^4)/a^2 + (3*d^2*x^4)/c^2 + (3*b^(5/2)*x^6*ArcTan[1 -
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(5/2) + (3*b^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/a^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 - (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/c^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/c^(5/2))/(6*(-(b*c) + a*d)*x^6)
```

Maple [A]
time = 0.45, size = 101, normalized size = 0.90

method	result
default	$\frac{d^{3} \arctan\left(\frac{d\,x^{2}}{\sqrt{c d}}\right)}{2 c^{2} \left(a d -b c \right) \sqrt{c d}}-\frac{1}{6 a c\,x^{6}}-\frac{-a d -b c}{2 a^{2} c^{2} x^{2}}-\frac{b^{3} \arctan\left(\frac{b\,x^{2}}{\sqrt{a b}}\right)}{2 a^{2} \left(a d -b c \right) \sqrt{a b}}$
risch	$\frac{\frac{\left(a d +b c \right) x^{4}}{2 a^{2} c^{2}}-\frac{1}{6 a c}}{x^{6}}+\frac{\left(\sum_{\_R =\mathrm{RootOf}\left(\left(d^{2} c^{5} a^{2}-2 a b\,c^{6} d +b^{2} c^{7}\right) \_Z^{2}+d^{5}\right)}\_R \ln\left(\left(\left(5 c^{5} a^{9} d^{4}-18 c^{6} a^{8} b\,d^{3}+26 c^{7} a^{7} b^{2} d^{2}-18 c^{8} a^{6} b^{3} d +5 c^{9} a^{5} b^{4}\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d^3/c^2/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))-1/6/a/c/x^6-1/2
/a^2/c^2*(-a*d-b*c)/x^2-1/2*b^3/a^2/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b
)^(1/2))
```

Maxima [A]
time = 0.52, size = 101, normalized size = 0.90

$$\frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(a^2bc - a^3d)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc^3 - ac^2d)\sqrt{cd}} + \frac{3\,(bc + ad)x^4 - ac}{6\,a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*arc
tan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*(b*c + a*d)*x^4
 - a*c)/(a^2*c^2*x^6)
```

Fricas [A]
time = 9.64, size = 592, normalized size = 5.29

$$\left[-\frac{3\,b^2c^2x^6\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+3\,a^2d^2x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)-6\,(b^2c^2-a^2d^2)x^4+2\,abc^2-2\,a^2cd}{12\,(a^2bc^3-a^3c^2d)x^6}, \frac{6\,a^2d^2x^6\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)-3\,b^2c^2x^6\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+6\,(b^2c^2-a^2d^2)x^4-2\,abc^2+2\,a^2cd}{12\,(a^2bc^3-a^3c^2d)x^6}, -\frac{6\,b^2c^2x^6\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)+3\,a^2d^2x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)-6\,(b^2c^2-a^2d^2)x^4+2\,abc^2-2\,a^2cd}{12\,(a^2bc^3-a^3c^2d)x^6}, -\frac{3\,b^2c^2x^6\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)-3\,a^2d^2x^6\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)-3\,(b^2c^2-a^2d^2)x^4+abc^2-a^2cd}{6\,(a^2bc^3-a^3c^2d)x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^
4 + a)) + 3*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*
x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3
- a^3*c^2*d)*x^6), 1/12*(6*a^2*d^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)
) - 3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 +
a)) + 6*(b^2*c^2 - a^2*d^2)*x^4 - 2*a*b*c^2 + 2*a^2*c*d)/((a^2*b*c^3 - a^3*
c^2*d)*x^6), -1/12*(6*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) + 3
*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) -
 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d
)*x^6), -1/6*(3*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - 3*a^2*d
^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - 3*(b^2*c^2 - a^2*d^2)*x^4 +
a*b*c^2 - a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^6)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.41, size = 103, normalized size = 0.92

$$\frac{b^3\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(a^2bc-a^3d)\sqrt{ab}}-\frac{d^3\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc^3-ac^2d)\sqrt{cd}}+\frac{3\,bcx^4+3\,adx^4-ac}{6\,a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*arc
tan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*b*c*x^4 + 3*a*d
*x^4 - a*c)/(a^2*c^2*x^6)
```

Mupad [B]
time = 5.53, size = 535, normalized size = 4.78

$$\frac{\ln\left(c^{10}\,{\left(-a^5\,b^5\right)}^{5/2}+a^{20}\,d^{10}\,\sqrt{-a^5\,b^5}-a^{12}\,b^{13}\,c^{10}\,x^2-a^{22}\,b^3\,d^{10}\,x^2+2\,a^{10}\,c^5\,d^5\,{\left(-a^5\,b^5\right)}^{3/2}+2\,a^{17}\,b^8\,c^5\,d^5\,x^2\right)\,\sqrt{-a^5\,b^5}}{4\,a^6\,d-4\,a^5\,b\,c}-\frac{\ln\left(c^{10}\,{\left(-a^5\,b^5\right)}^{5/2}+a^{20}\,d^{10}\,\sqrt{-a^5\,b^5}+a^{12}\,b^{13}\,c^{10}\,x^2+a^{22}\,b^3\,d^{10}\,x^2+2\,a^{10}\,c^5\,d^5\,{\left(-a^5\,b^5\right)}^{3/2}-2\,a^{17}\,b^8\,c^5\,d^5\,x^2\right)\,\sqrt{-a^5\,b^5}}{4\,\left(a^6\,d-a^5\,b\,c\right)}-\frac{\frac{1}{6\,a\,c}-\frac{x^4\,\left(a\,d+b\,c\right)}{2\,a^2\,c^2}}{x^6}-\frac{\ln\left(a^{10}\,{\left(-c^5\,d^5\right)}^{5/2}+b^{10}\,c^{20}\,\sqrt{-c^5\,d^5}+a^{10}\,c^{12}\,d^{13}\,x^2+b^{10}\,c^{22}\,d^3\,x^2+2\,a^5\,b^5\,c^{10}\,{\left(-c^5\,d^5\right)}^{3/2}-2\,a^5\,b^5\,c^{17}\,d^8\,x^2\right)\,\sqrt{-c^5\,d^5}}{4\,\left(b\,c^6-a\,c^5\,d\right)}+\frac{\ln\left(a^{10}\,{\left(-c^5\,d^5\right)}^{5/2}+b^{10}\,c^{20}\,\sqrt{-c^5\,d^5}-a^{10}\,c^{12}\,d^{13}\,x^2-b^{10}\,c^{22}\,d^3\,x^2+2\,a^5\,b^5\,c^{10}\,{\left(-c^5\,d^5\right)}^{3/2}+2\,a^5\,b^5\,c^{17}\,d^8\,x^2\right)\,\sqrt{-c^5\,d^5}}{4\,b\,c^6-4\,a\,c^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(c^10*(-a^5*b^5)^(5/2) + a^20*d^10*(-a^5*b^5)^(1/2) - a^12*b^13*c^10*x^
2 - a^22*b^3*d^10*x^2 + 2*a^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^17*b^8*c^5*d^
5*x^2)*(-a^5*b^5)^(1/2))/(4*a^6*d - 4*a^5*b*c) - (log(c^10*(-a^5*b^5)^(5/2)
 + a^20*d^10*(-a^5*b^5)^(1/2) + a^12*b^13*c^10*x^2 + a^22*b^3*d^10*x^2 + 2*
a^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^17*b^8*c^5*d^5*x^2)*(-a^5*b^5)^(1/2))/(
4*(a^6*d - a^5*b*c)) - (1/(6*a*c) - (x^4*(a*d + b*c))/(2*a^2*c^2))/x^6 - (l
og(a^10*(-c^5*d^5)^(5/2) + b^10*c^20*(-c^5*d^5)^(1/2) + a^10*c^12*d^13*x^2
+ b^10*c^22*d^3*x^2 + 2*a^5*b^5*c^10*(-c^5*d^5)^(3/2) - 2*a^5*b^5*c^17*d^8*
x^2)*(-c^5*d^5)^(1/2))/(4*(b*c^6 - a*c^5*d)) + (log(a^10*(-c^5*d^5)^(5/2) +
 b^10*c^20*(-c^5*d^5)^(1/2) - a^10*c^12*d^13*x^2 - b^10*c^22*d^3*x^2 + 2*a^
5*b^5*c^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*c^17*d^8*x^2)*(-c^5*d^5)^(1/2))/(4*
b*c^6 - 4*a*c^5*d)
```

3.779 $\int \frac{x^8}{(a+bx^4)(c+dx^4)} \, dx$

Optimal. Leaf size=457

$$\frac{x}{bd}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,d^{5/4}(bc-ad)}-\frac{c^{5/4}\tan^{-1}\left(1\right.}{2\sqrt{2}\,d^{5/4}}$$

```
[Out] x/b/d+1/4*a^(5/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)*2
^(1/2)+1/4*a^(5/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)*2
^(1/2)-1/4*c^(5/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(5/4)/(-a*d+b*c)*
2^(1/2)-1/4*c^(5/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(5/4)/(-a*d+b*c)*
2^(1/2)-1/8*a^(5/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/b^(5
/4)/(-a*d+b*c)*2^(1/2)+1/8*a^(5/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*b^(1/2))/b^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(5/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1
/2)+c^(1/2)+x^2*d^(1/2))/d^(5/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(5/4)*ln(c^(1/4)*
d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(5/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {490, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{a^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{c^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,d^{5/4}(bc-ad)}-\frac{c^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,d^{5/4}(bc-ad)}+\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,d^{5/4}(bc-ad)}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,d^{5/4}(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^4}{(a+bx^4)(c+dx^4)}\,dx}{bd}$$

$$= \frac{x}{bd} + \frac{a^2\int \frac{1}{a+bx^4}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{1}{c+dx^4}\,dx}{d(bc-ad)}$$

$$= \frac{x}{bd} + \frac{a^{3/2}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2b(bc-ad)} + \frac{a^{3/2}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2b(bc-ad)} - \frac{c^{3/2}\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2d(bc-ad)} - \ldots$$

$$= \frac{x}{bd} + \frac{a^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b^{3/2}(bc-ad)} + \frac{a^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b^{3/2}(bc-ad)} - \frac{a^{5/4}\int \ldots}{4\sqrt{2}\ldots}$$

$$= \frac{x}{bd} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\ldots\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)\ldots}$$

$$= \frac{x}{bd} - \frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{c^{5/4}\tan^{-1}\left(\ldots\right.}{2\sqrt{2}\,d^{5}\ldots}$$

Mathematica [A]
time = 0.14, size = 377, normalized size = 0.82

$$\frac{-\frac{8ax}{b}+\frac{8cx}{d}-\frac{2\sqrt{2}\,a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{5/4}}+\frac{2\sqrt{2}\,a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{b^{5/4}}+\frac{2\sqrt{2}\,c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{d^{5/4}}-\frac{2\sqrt{2}\,c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{d^{5/4}}-\frac{\sqrt{2}\,a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{b^{5/4}}+\frac{\sqrt{2}\,a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{b^{5/4}}+\frac{\sqrt{2}\,c^{5/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{d^{5/4}}-\frac{\sqrt{2}\,c^{5/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{d^{5/4}}}{8bc-8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-8*a*x)/b + (8*c*x)/d - (2*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1
/4)])/b^(5/4) + (2*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])
/d^(5/4) - (2*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/d^(5
/4) - (Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/b^(5/4) + (Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
```

```
rt[b]*x^2])/b^(5/4) + (Sqrt[2]*c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*x + Sqrt[d]*x^2])/d^(5/4) - (Sqrt[2]*c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*x + Sqrt[d]*x^2])/d^(5/4))/(8*b*c - 8*a*d)
```

Maple [A]
time = 0.44, size = 234, normalized size = 0.51

method	result
default	$\frac{x}{bd}+\frac{c\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8d(ad-bc)}-\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^{2}+(}{x^{2}-(}\right.\right.}{}$
risch	$\frac{x}{bd}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(a^{4}d^{5}-4a^{3}bc\,d^{4}+6a^{2}b^{2}c^{2}d^{3}-4a\,b^{3}c^{3}d^{2}+b^{4}c^{4}d\right)\_Z^{4}+b^{4}c^{5}\right)}\_R\ln\left(\left(\left(-a^{5}b\,d^{6}+3a^{4}b^{2}c\,d^{5}-2a^{3}b^{3}c^{2}d^{4}-2a^{2}b^{4}c^{3}d^{3}+3a\right.\right.\right.}{4bd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] x/b/d+1/8/d*c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+
(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d
)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))-1/8/b*a/(a*d-b*c)*(a/b)^(1/
4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/
b)^(1/4)*x-1))
```

Maxima [A]
time = 0.53, size = 375, normalized size = 0.82

$$\frac{\frac{2\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\,\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\,\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{8\left(b^{2}c-abd\right)}-\frac{\frac{2\sqrt{2}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\,\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\,\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,c^{\frac{5}{4}}\log\left(\sqrt{d}\,x^{2}+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}\,c^{\frac{5}{4}}\log\left(\sqrt{d}\,x^{2}-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}}{8\left(bcd-ad^{2}\right)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^
(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 - sqrt(2)
*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b^2*c - a*b*d) - 1/8*(2*sqrt(2)*c^(
3/2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c
)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2
*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*s
qrt(d)) + sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqr
t(c))/d^(1/4) - sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x
 + sqrt(c))/d^(1/4))/(b*c*d - a*d^2) + x/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1376 vs. $2(327) = 654$.
time = 5.38, size = 1376, normalized size = 3.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^
3 + a^4*b^5*d^4))^(1/4)*b*d*arctan(((a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^
5*c*d^2 - a^4*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 -
 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*x - (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2
*b^5*c*d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^
2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*sqrt(a^2*x^2 + (b^4*c^2 - 2*a*b^3
*c*d + a^2*b^2*d^2)*sqrt(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2
- 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))))/a^5) - 4*(-c^5/(b^4*c^4*d^5 - 4*a*b^3*c
^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*arctan(((b
^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*(-c^5/(b^4*c^4*
d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/4)
*x - (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c
^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3
/4)*sqrt(c^2*x^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*sqrt(-c^5/(b^4*c^4
*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))))/c^
5) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 +
 a^4*b^5*d^4))^(1/4)*b*d*log(a*x + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b
^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) + (-a^5
/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d
^4))^(1/4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2
 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) + (-c^5/(b^4*c^4*
d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)
*b*d*log(c*x + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4
*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) - (-c^5/(b^4*c^4*d^5 - 4*a*
b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*log(c
*x - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d
^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) - 4*x)/(b*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)/(d*x**4+c),x)
```

[Out] Timed out

Giac [A]
time = 1.96, size = 469, normalized size = 1.03

$$\frac{(ab^3)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\, b^3c-\sqrt{2}\, ab^2d\right)}+\frac{(ab^3)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\, b^3c-\sqrt{2}\, ab^2d\right)}-\frac{(cd^3)^{\frac{1}{4}} c \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\, bcd^2-\sqrt{2}\, ad^3\right)}-\frac{(cd^3)^{\frac{1}{4}} c \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\, bcd^2-\sqrt{2}\, ad^3\right)}+\frac{(ab^3)^{\frac{1}{4}} a \log\left(x^2+\sqrt{2}\, x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\, b^3c-\sqrt{2}\, ab^2d\right)}-\frac{(ab^3)^{\frac{1}{4}} a \log\left(x^2-\sqrt{2}\, x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\, b^3c-\sqrt{2}\, ab^2d\right)}-\frac{(cd^3)^{\frac{1}{4}} c \log\left(x^2+\sqrt{2}\, x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\, bcd^2-\sqrt{2}\, ad^3\right)}+\frac{(cd^3)^{\frac{1}{4}} c \log\left(x^2-\sqrt{2}\, x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\, bcd^2-\sqrt{2}\, ad^3\right)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) + 1/2*(a*b^3)^(1/4)*a*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^
2*d) - 1/2*(c*d^3)^(1/4)*c*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(
c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) - 1/2*(c*d^3)^(1/4)*c*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqr
t(2)*a*d^3) + 1/4*(a*b^3)^(1/4)*a*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/4*(a*b^3)^(1/4)*a*log(x^2 - sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/4*(c*d^
3)^(1/4)*c*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^2 -
sqrt(2)*a*d^3) + 1/4*(c*d^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt
(c/d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + x/(b*d)
```

Mupad [B]
time = 5.63, size = 2500, normalized size = 5.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] atan(((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*
b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a
^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^
5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4
)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^
6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(2
56*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 -
1024*a*b^8*c^3*d))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i - (-
a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*
d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c
^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 -
1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a
^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^
5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c
^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b
```

^8*c^3*d))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i)/((-a^5/(256
*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 10
24*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a
^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3
*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b^9*c
^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 -
768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c^4 + 256
*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d
))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d)) + (-a^5/(256*b^9*c^4 +
256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^
3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^
5))/(b*d) + (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3
+ 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b^9*c^8*d^4 - 76
8*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5
*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^
4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4) +
(4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))))*(-a^5/(256*b^9*c^4 + 256*a^4*b^5
*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)
*2i - 2*atan(((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1
536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3
*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-a^5/(256*b^9*c^4 + 256*
a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d)
)^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 +
512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))
*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c
^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b
*d)) - (-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2
*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 -
a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5
*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)
*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6
*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/
(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2
- 1024*a*b^8*c^3*d))^(1/4)*1i - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d)))/(
(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^
2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5
*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 -
1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a
^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^
5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/(256*b^
9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*
a*b^8*c^3*d))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i + (-a^
5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^
2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8
*d - a^8*b*c^4*d^5))/(b*d) + (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024

```
*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b
^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^
7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/(256*b^9*c^
4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^
8*c^3*d))^(1/4)*1i - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i))*(-a^5/(2
56*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 -
1024*a*b^8*c^3*d))^(1/4) + atan(((-c^5/(256*a^4...
```

3.780 $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}+\frac{c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}$$

```
[Out] -1/4*a^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)*2^(1/2
)-1/4*a^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)*2^(1/2
)+1/4*c^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(3/4)/(-a*d+b*c)*2^(1/
2)+1/4*c^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(3/4)/(-a*d+b*c)*2^(1/
2)-1/8*a^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/b^(3/4)/(
-a*d+b*c)*2^(1/2)+1/8*a^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1
/2))/b^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c
^(1/2)+x^2*d^(1/2))/d^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(3/4)*ln(c^(1/4)*d^(1/
4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(3/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {492, 303, 1176, 631, 210, 1179, 642}

$$\frac{a^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{c^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}+\frac{c^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}+\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,d^{3/4}(bc-ad)}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,d^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/
4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (a^(3/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\int \frac{x^2}{a+bx^4}\,dx}{bc-ad} + \frac{c\int \frac{x^2}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{a\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{b}\,(bc-ad)} - \frac{a\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{b}\,(bc-ad)} - \frac{c\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{d}\,(bc-ad)} + \frac{c\int \frac{\sqrt{c}+\sqrt{d}\,x}{c+dx^4}}{2\sqrt{d}\,(bc-a}$$

$$= -\frac{a\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b(bc-ad)} - \frac{a\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b(bc-ad)} - \frac{a^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}}{}}}{4\sqrt{2}\,b^{3/4}(bc-}$$

$$= -\frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{3/4}(bc-ad)} + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\right.}{4\sqrt{2}\,b^{3/4}(bc-ad)}$$

$$= \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)} - \frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)} - \frac{c^{3/4}\tan^{-1}\left(1-\right.}{2\sqrt{2}\,d^{3/4}(bc}$$

Mathematica [A]
time = 0.07, size = 340, normalized size = 0.76

$$\frac{2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-2a^{3/4}d^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-2b^{3/4}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)+2b^{3/4}c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)-a^{3/4}d^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+a^{3/4}d^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+b^{3/4}c^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)-b^{3/4}c^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,b^{3/4}d^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(3/4)*d^(3
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(3/4)*c^(3/4)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*x)/c^(1/4)] - a^(3/4)*d^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2] - b^(3/4)*c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(4*Sqrt[2]*b^(3/4)*d^(3/4)*(b*c - a*d))
```

Maple [A]
time = 0.42, size = 226, normalized size = 0.50

method	result

default	$-\frac{c\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)d\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{a\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{}}\right.\right.}{}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^4b^3d^4-4a^3b^4c\,d^3+6a^2b^5c^2d^2-4a\,b^6c^3d+b^7c^4\right)\_Z^4+a^3\right)}\_R\ln\left(\left(\left(2a^4b^3d^7-8a^3b^4c\,d^6+12a^2b^5c^2d^5-8a\,b^6c^3d^4+2b^7c^4d\right.\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*c/(a*d-b*c)/d/(c/d)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)
^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/
4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))+1/8*a/(a*d-b*c)/b/(a/b)^(1/4)*2^
(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2
)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1
/4)*x-1))
```

Maxima [A]
time = 0.51, size = 363, normalized size = 0.81

$$-\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8\,(bc-ad)}+\frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{8\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan
(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) + 1/8*c*(2*sq
rt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(
c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(
c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x
+ sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^
(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. $2(319) = 638$.
time = 3.52, size = 1374, normalized size = 3.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4
*b^3*d^4))^(1/4)*arctan(((a^2*b^2*c - a^3*b*d)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3
*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*x - sqrt(a^4
*x^2 - (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*sqrt(-a^3/(b^7*c^4 - 4*a*b
^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)))*(b^2*c - a*
b*d)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 +
 a^4*b^3*d^4))^(1/4))/a^3) + (-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b
^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*arctan(((b*c^3*d - a*c^2*d^2)*
(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 +
a^4*d^7))^(1/4)*x - sqrt(c^4*x^2 - (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3
)*sqrt(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*
d^6 + a^4*d^7)))*(b*c*d - a*d^2)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a
^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4))/c^3) - 1/4*(-a^3/(b^7*c^4
 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4
)*log(a^2*x + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a
^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3
*d^4))^(3/4)) + 1/4*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*
a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*x - (b^5*c^3 - 3*a*b^4*c^2*d +
3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c
^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/4*(-c^3/(b^4*c^4*d^3 -
4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c
^2*x + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4
*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^
(3/4)) - 1/4*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a
^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*x - (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3
*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^
2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.38, size = 453, normalized size = 1.01

$$-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,b^4c-\sqrt{2}\,ab^3d\right)}-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,b^4c-\sqrt{2}\,ab^3d\right)}+\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bcd^3-\sqrt{2}\,ad^4\right)}+\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bcd^3-\sqrt{2}\,ad^4\right)}+\frac{(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,b^4c-\sqrt{2}\,ab^3d\right)}-\frac{(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,b^4c-\sqrt{2}\,ab^3d\right)}-\frac{(cd^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bcd^3-\sqrt{2}\,ad^4\right)}+\frac{(cd^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bcd^3-\sqrt{2}\,ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2
)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d
) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^
(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*
d^4) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt
(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(c*d^3)^(3/4)*lo
g(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4
) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b*c*d^3 - sqrt(2)*a*d^4)
```

Mupad [B]
time = 5.72, size = 2553, normalized size = 5.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - 2*atan((4*b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c
*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4) + 4*a^3*b*d^3*x*(-a^
3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^
2 - 1024*a*b^6*c^3*d))^(1/4) + 2048*a^4*b^4*d^7*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4) + 2048*b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^
3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192*a*b^7*
c^3*d^4*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*
a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192*a^3*b^5*c*d^6*x*(-a^3/(25
6*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1
024*a*b^6*c^3*d))^(5/4) + 12288*a^2*b^6*c^2*d^5*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4))/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(256*b^7*c^4 + 256*a^4*b^
3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4
) - atan((b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d
^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4)*4i + a^3*b*d^3*x*(-a^3
/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2
 - 1024*a*b^6*c^3*d))^(1/4)*4i + a^4*b^4*d^7*x*(-a^3/(256*b^7*c^4 + 256*a^4
*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(
5/4)*2048i + b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*
b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4)*2048i - a*b^7*c
```

^3*d^4*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a
^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4)*8192i - a^3*b^5*c*d^6*x*(-a^3/(25
6*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1
024*a*b^6*c^3*d))^(5/4)*8192i + a^2*b^6*c^2*d^5*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4)*12288i)/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(256*b^7*c^4 + 256
*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d
))^(1/4)*2i - 2*atan((4*a^3*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 10
24*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(1/4) + 2048*b
^7*c^4*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 15
36*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) + 4*b^3*c^3*d*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(1/4) + 2048*a^4*b^3*d^8*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d
^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) -
 8192*a*b^6*c^3*d^5*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3
*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) - 8192*a^3*b^4*c*d^7
x(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2
*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) + 12288*a^2*b^5*c^2*d^6*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(5/4))/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(256*a^4*d^7
+ 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*
c*d^6))^(1/4) - atan((a^3*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024
*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(1/4)*4i + b^7*c
^4*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a
^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*2048i + b^3*c^3*d*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(1/4)*4i + a^4*b^3*d^8*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3
 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*204
8i - a*b^6*c^3*d^5*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*
d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*8192i - a^3*b^4*c*d^7
x(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2
*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*8192i + a^2*b^5*c^2*d^6*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(5/4)*12288i)/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(256*a
^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024
*a^3*b*c*d^6))^(1/4)*2i

3.781 $\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}\,(bc-ad)}+\frac{\sqrt[4]{c}\tan^{-1}\left(1+\sqrt{\ldots}\right.}{2\sqrt{2}\sqrt[4]{d}\,(bc-\ldots}$$

```
[Out] -1/4*a^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)*2^(1/2
)-1/4*a^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)*2^(1/2
)+1/4*c^(1/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(1/4)/(-a*d+b*c)*2^(1/
2)+1/4*c^(1/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(1/4)/(-a*d+b*c)*2^(1/
2)+1/8*a^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/b^(1/4)/(
-a*d+b*c)*2^(1/2)-1/8*a^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1
/2))/b^(1/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(1/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c
^(1/2)+x^2*d^(1/2))/d^(1/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(1/4)*ln(c^(1/4)*d^(1/
4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(1/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {492, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}\,(bc-ad)}+\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{d}\,(bc-ad)}+\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\sqrt[4]{d}\,(bc-ad)}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\sqrt[4]{d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/
4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (a^(1/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\int \frac{1}{a+bx^4}\,dx}{bc-ad} + \frac{c\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= -\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2(bc-ad)} - \frac{\sqrt{a}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{c}\ \cdots}{\cdots}$$

$$= -\frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{b}\,(bc-ad)} - \frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{b}\,(bc-ad)} + \frac{\sqrt[4]{a}\int \frac{\sqrt{2}\cdots}{-\frac{\sqrt{a}}{\sqrt{b}}-\cdots}}{4\sqrt{2}\sqrt[4]{b}\cdots}$$

$$= \frac{\sqrt[4]{a}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{a}\,\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{\cdots}\right)}{4\sqrt{2}\sqrt[4]{b}\,(bc-ad)}$$

$$= \frac{\sqrt[4]{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{a}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{c}\,\tan^{-1}\left(1-\cdots\right)}{2\sqrt{2}\sqrt[4]{d}\,(bc-\cdots}$$

Mathematica [A]
time = 0.06, size = 340, normalized size = 0.76

$$\frac{2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)+2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)+\sqrt[4]{a}\sqrt[4]{d}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)-\sqrt[4]{a}\sqrt[4]{d}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)-\sqrt[4]{b}\sqrt[4]{c}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)+\sqrt[4]{b}\sqrt[4]{c}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(1/4)*d^(1
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(1/4)*c^(1/4)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(1/4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*x)/c^(1/4)] + a^(1/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2] + b^(1/4)*c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(4*Sqrt[2]*b^(1/4)*d^(1/4)*(b*c - a*d))
```

Maple [A]
time = 0.42, size = 218, normalized size = 0.49

method	result

default	$-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^4d^5-4a^3bc\,d^4+6a^2b^2c^2d^3-4a\,b^3c^3d^2+b^4c^4d\right)\_Z^4+c\right)}\_R\ln\left(\left(\left(-a^5b\,d^6+3a^4b^2c\,d^5-2a^3b^3c^2d^4-2a^2b^4c^3d^3+3a\,b^5c^4\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/
2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x
+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))+1/8/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(l
n((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^
(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
)
```

Maxima [A]
time = 0.52, size = 361, normalized size = 0.80

$$-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,a^{\frac{1}{4}}\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\,a^{\frac{1}{4}}\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{8\,(bc-ad)}+\frac{\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,c^{\frac{1}{4}}\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}\,c^{\frac{1}{4}}\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}}{8\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b*c - a*d) + 1/8*(2*sqrt(2)*sqrt(c
)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*s
qrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*(2*sq
rt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt
(d)) + sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c
))/d^(1/4) - sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x +
sqrt(c))/d^(1/4))/(b*c - a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. $2(319) = 638$.
time = 4.47, size = 1238, normalized size = 2.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] (-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4))^(1/4)*arctan(((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)
*x*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4
*b*d^4))^(3/4) - (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sq
rt(x^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)))*(-a/(b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(3/4))/a) - (-c/(
b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))
^(1/4)*arctan(((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x*(-
c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^
5))^(3/4) - (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(x^
2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 +
6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)))*(-c/(b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(3/4))/c) - 1/4*(-a/(b
^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^
(1/4)*log((b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + x) + 1/4*(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(b*c - a*d)*(
-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d
^4))^(1/4) + x) + 1/4*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3
- 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log((b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*
c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x) - 1/4*(-
c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^
5))^(1/4)*log(-(b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2
*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.30, size = 437, normalized size = 0.97

$$-\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}-\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}+\frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bcd-\sqrt{2}\,ad^2\right)}+\frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bcd-\sqrt{2}\,ad^2\right)}-\frac{(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}+\frac{(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}+\frac{(cd^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bcd-\sqrt{2}\,ad^2\right)}-\frac{(cd^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bcd-\sqrt{2}\,ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) - 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) +
1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4
))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) + 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) - 1
/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^2*
c - sqrt(2)*a*b*d) + 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt
(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) - 1/4*(c*d^3
)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt(
2)*a*d^2)
```

Mupad [B]
time = 5.86, size = 2500, normalized size = 5.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - atan((a^2*d^2*x*1i + b^2*c^2*x*1i - (a^6*b*d^6*x*256i)/(256*b^5*c^4 + 256
*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d)
- (a*b^6*c^5*d*x*256i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 +
1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) + (a^5*b^2*c*d^5*x*768i)/(256*b^5*
c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^
4*c^3*d) + (a^2*b^5*c^4*d^2*x*768i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3
*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a^3*b^4*c^3*d^3*x*
512i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*
d^2 - 1024*a*b^4*c^3*d) - (a^4*b^3*c^2*d^4*x*512i)/(256*b^5*c^4 + 256*a^4*b
*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))/((-a/
(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 -
1024*a*b^4*c^3*d))^(1/4)*((a*(1024*a^6*b*d^7 + 1024*b^7*c^6*d - 6144*a*b^6*
c^5*d^2 - 6144*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a^3*b^4*c^3*d^
4 + 15360*a^4*b^3*c^2*d^5))/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d
^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - 4*b^3*c^3 - 4*a^3*d^3 + 4*a
*b^2*c^2*d + 4*a^2*b*c*d^2)))*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b
^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*2i - atan((a^2*d
^2*x*1i + b^2*c^2*x*1i - (b^6*c^6*d*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d -
1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a^5*b*c*d^
6*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*
c^2*d^3 - 1024*a^3*b*c*d^4) + (a*b^5*c^5*d^2*x*768i)/(256*a^4*d^5 + 256*b^4
*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a
^2*b^4*c^4*d^3*x*512i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 +
```

1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a^3*b^3*c^3*d^4*x*512i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) + (a^4*b^2*c^2*d^5*x*768i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))/((-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((c*(1024*a^6*b*d^7 + 1024*b^7*c^6*d - 6144*a*b^6*c^5*d^2 - 6144*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a^3*b^4*c^3*d^4 + 15360*a^4*b^3*c^2*d^5))/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - 4*b^3*c^3 - 4*a^3*d^3 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)))*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*2i - 2*atan(((x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4) + (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) + (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4))/((x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 1...

3.782 $\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\tan^{-1}\left(1+\right.}{2\sqrt{2}\sqrt[4]{c}(b}$$

```
[Out] 1/4*b^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)*2^(1/2)
+1/4*b^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)*2^(1/2)
-1/4*d^(1/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(1/4)/(-a*d+b*c)*2^(1/2
)-1/4*d^(1/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(1/4)/(-a*d+b*c)*2^(1/2
)+1/8*b^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/4)/(-
a*d+b*c)*2^(1/2)-1/8*b^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(1/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(1/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^
(1/2)+x^2*d^(1/2))/c^(1/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(1/4)*ln(c^(1/4)*d^(1/4
)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(1/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {493, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*(b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*
c - a*d)) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(1/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*S
qrt[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/
4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(1/4)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*
c - a*d)) - (d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]
)/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\int \frac{x^2}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{x^2}{c+dx^4}\,dx}{bc-ad}$$

$$= -\frac{\sqrt{b}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{b}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{d}\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2(bc-ad)} - \frac{\sqrt{d}}{}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc-ad)} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc-ad)} - \frac{\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4(bc-ad)}$$

$$= \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(bc-ad)} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{}\right.}{4\sqrt{2}\sqrt[4]{a}\,(bc-ad)}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(bc-ad)} + \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(bc-ad)} + \frac{\sqrt[4]{d}\tan^{-1}\left(1-\right.}{2\sqrt{2}\sqrt[4]{c}\,(}$$

Mathematica [A]
time = 0.08, size = 340, normalized size = 0.76

$$\frac{-2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)-2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)+\sqrt[4]{b}\sqrt[4]{c}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)-\sqrt[4]{b}\sqrt[4]{c}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)-\sqrt[4]{a}\sqrt[4]{d}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)+\sqrt[4]{a}\sqrt[4]{d}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(1/4)*c^(
1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*d^(1/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(1/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] + b^(1/4)*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] + a^(1/4)*d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(b*c - a*d))
```

Maple [A]
time = 0.43, size = 218, normalized size = 0.49

method	result

default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right.}{}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^4a^5-4c\,d^3b\,a^4+6b^2c^2d^2a^3-4b^3c^3d\,a^2+b^4c^4a\right)\_Z^4+b\right)}\_R\ln\left(\left(\left(a^6d^6-4a^5bc\,d^5+7a^4b^2c^2d^4-8a^3b^3c^3d^3+7a^2b^4c^4d^2-4\right.\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2
))/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+
1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))-1/8/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln
((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Maxima [A]
time = 0.51, size = 363, normalized size = 0.81

$$\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8\,(bc-ad)}-\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{8\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)
*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) - 1/8*d*(2*sqr
t(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c
)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c
)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x +
 sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1274 vs. $2(319) = 638$.
time = 3.85, size = 1274, normalized size = 2.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] (-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*
d^4))^(1/4)*arctan(((b^2*c - a*b*d)*x*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4) - sqrt(b^2*x^2 - (a*b^3*c
^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^
3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)))*(b*c - a*d)*(-b/(a*b^4*c^4 - 4*a
^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4))/b) - (-
d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^
4))^(1/4)*arctan(((b*c*d - a*d^2)*x*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^
2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4) - sqrt(d^2*x^2 - (b^2*c^3*d
 - 2*a*b*c^2*d^2 + a^2*c*d^3)*sqrt(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*
c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)))*(b*c - a*d)*(-d/(b^4*c^5 - 4*a*b^3
*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4))/d) + 1/4*
(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*
d^4))^(1/4)*log(b*x + (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^
3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a
^5*d^4))^(3/4)) - 1/4*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2
- 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x - (a*b^3*c^3 - 3*a^2*b^2*c^2*d +
3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d
^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) - 1/4*(-d/(b^4*c^5 - 4*a*b^3*c^4*d +
6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(d*x + (b^3*c^4
- 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d
 + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4)) + 1/4*(-d/(b^4*
c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/
4)*log(d*x - (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b
^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^
(3/4))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.48, size = 477, normalized size = 1.06

$$\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}+\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}-\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3\right)}-\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3\right)}-\frac{(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}+\frac{(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}+\frac{(cd^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3\right)}-\frac{(cd^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c - sqrt(2)*a^
2*b^2*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))
/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(c*d^3)^(3/4)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2
 - sqrt(2)*a*c*d^3) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/4*(a*b^3)^(3/4)*log(x^2
 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d)
 + 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*
b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^
(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3)
```

Mupad [B]
time = 5.50, size = 2500, normalized size = 5.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^4
*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)
*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^
2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10
 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 716
8*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^9*c^6*d^4 + 256*a^6*b^4
*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 -
768*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
 + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i + (x*(4*a*b^6*c^2*d^5
 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
 + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(
-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2
 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 40
96*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5
*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*
d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-b
/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 -
 1024*a^4*b*c*d^3))^(1/4)*1i)/((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-b
/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 -
 1024*a^4*b*c*d^3))^(3/4)*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^
3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*
d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8
192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*
```

```
b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6
 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4
*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)
 - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^
4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(2
56*a*b^9*c^6*d^4 - x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
+ 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 10
24*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*
b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*
d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768
*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d +
1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4) + 2*a*b^5*c*d^5))*(-b/(256*
a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*
a^4*b*c*d^3))^(1/4)*2i + 2*atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (
-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2
 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-b/(256*a^5*d^4 + 256*a
*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(
1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 71
68*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6
*b^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^
4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-b/(256*a^5*d^4 + 2
56*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3
))^(1/4) + (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*
a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^
(3/4)*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b
^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c
*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7
+ 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256
*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^
3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^
2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4))/((x*(4*a*b^6
*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b
^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(x*(-b/(256*a^5*
d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*
b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c
^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8
 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a
^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^
2*d^8)*1i)*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3
*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^
5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3
*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-b/(256*a^5
*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4
*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^...
```

3.783 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\tan^{-1}\left(1+\right.}{2\sqrt{2}\,c^{3/4}(bc}$$

```
[Out] 1/4*b^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
+1/4*b^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
-1/4*d^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/4*d^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/8*b^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/(-
a*d+b*c)*2^(1/2)+1/8*b^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^
(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(3/4)*ln(c^(1/4)*d^(1/4
)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {400, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{d^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}+\frac{d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*
c - a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*S
qrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/
4)])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*
c - a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]
)/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\int \frac{1}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)} + \frac{b\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)} - \frac{d\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)} - \frac{d\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}}{2\sqrt{c}\,(bc-ad}$$

$$= \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)} + \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}}-\sqrt{2}}}{4\sqrt{2}\,a^{3/4}(b}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{}\right.}{4\sqrt{2}\,a^{3/4}(bc-ad)}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{d^{3/4}\tan^{-1}\left(1-\right.}{2\sqrt{2}\,c^{3/4}(bc}$$

Mathematica [A]
time = 0.08, size = 340, normalized size = 0.76

$$\frac{-2b^{3/4}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2b^{3/4}c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)-2a^{3/4}d^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)-b^{3/4}c^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+b^{3/4}c^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)+a^{3/4}d^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)-a^{3/4}d^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

Maple [A]
time = 0.34, size = 226, normalized size = 0.50

method	result

default	$\frac{d\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)c}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^4c^3d^4-4a^3b\,c^4d^3+6a^2b^2c^5d^2-4a\,b^3c^6d+b^4c^7\right)\_Z^4+d^3\right)}\_R\ln\left(\left(\left(-a^7d^7+4c\,a^6b\,d^6-6c^2a^5b^2d^5+3a^4b^3c^3d^4+3a^3b^4c^4\right.\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^
(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4
)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))-1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(
1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/
4)*x-1))
```

Maxima [A]
time = 0.55, size = 365, normalized size = 0.81

$$\frac{\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{8\,(bc-ad)}-\frac{\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,d^{\frac{3}{4}}\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}\,d^{\frac{3}{4}}\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{8\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 -
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*sqrt(2)
*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1354 vs. $2(319) = 638$.
time = 3.90, size = 1354, normalized size = 3.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 +
 a^7*d^4))^(1/4)*arctan(((a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 -
 a^5*b*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^
6*b*c*d^3 + a^7*d^4))^(3/4)*x - (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*
d^2 - a^5*d^3)*sqrt(b^2*x^2 + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b
^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7
*d^4)))*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*
c*d^3 + a^7*d^4))^(3/4))/b^3) + (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*
c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*arctan(((b^3*c^5*d - 3*a*b^
2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*x - (b^3*c^5 - 3
*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(d^2*x^2 + (b^2*c^4 - 2*a
*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*
d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a
^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4))/d^3) + 1/4*(-b^3/(a
^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)
)^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*
a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b^3/(a^3*b^4*c^4
 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)*lo
g(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^
2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6
*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x + (b
*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*
c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2
*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (b*c^2 - a*c
*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 +
a^4*c^3*d^4))^(1/4))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.99, size = 437, normalized size = 0.97

$$\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}+\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}-\frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}-\frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}+\frac{(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}-\frac{(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}-\frac{(cd^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}+\frac{(cd^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/4*(c*d^3)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d)

Mupad [B]
time = 5.85, size = 2500, normalized size = 5.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x^4)*(c + d*x^4)),x)

[Out] - atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c

^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)
*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c
^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i)/(((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^
4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((
-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5
*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 -
4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^
3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*
d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 +
256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^
6*d))^(1/4) + ((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 +
1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a
^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))
^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^
2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^
4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c
^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*
d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 40
96*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*
b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32
*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^
3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)))*(-d^3/(256*
b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 102
4*a*b^3*c^6*d))^(1/4)*2i - atan((((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 10
24*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(
256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 -
1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*
b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^
8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^
6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 -
20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*
a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*
c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 -
16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^
3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^
(1/4)*1i - ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 153
6*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*
b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3
/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b
^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c
*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*

```
d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3...
```

3.784 $\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=460

$$-\frac{1}{acx}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{5/4}(bc-ad)}+\frac{d^{5/4}\tan^{-1}\left(\right.}{2\sqrt{2}\,c^{5}}$$

```
[Out] -1/a/c/x-1/4*b^(5/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c
)*2^(1/2)-1/4*b^(5/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c
)*2^(1/2)+1/4*d^(5/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(5/4)/(-a*d+b*
c)*2^(1/2)+1/4*d^(5/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(5/4)/(-a*d+b*
c)*2^(1/2)-1/8*b^(5/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a
^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*b^(5/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+
x^2*b^(1/2))/a^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(5/4)*ln(-c^(1/4)*d^(1/4)*x*2
^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(5/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(5/4)*ln(c^(1/
4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(5/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {491, 598, 303, 1176, 631, 210, 1179, 642}

$$\frac{b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{d^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{5/4}(bc-ad)}+\frac{d^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{5/4}(bc-ad)}+\frac{d^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{d^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{1}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(1/(a*c*x)) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(5/4)*(b*c - a*d)) - (d^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(5/4)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)) +
(b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(5/4)*(b*c - a*d)) + (d^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (d^(5/4)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= -\frac{1}{acx}+\frac{\int \frac{x^2\left(-bc-ad-bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{ac}\\
&= -\frac{1}{acx}+\frac{\int \left(-\frac{b^2cx^2}{(bc-ad)\left(a+bx^4\right)}-\frac{ad^2x^2}{(-bc+ad)\left(c+dx^4\right)}\right)dx}{ac}\\
&= -\frac{1}{acx}-\frac{b^2\int \frac{x^2}{a+bx^4}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{x^2}{c+dx^4}\,dx}{c(bc-ad)}\\
&= -\frac{1}{acx}+\frac{b^{3/2}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a(bc-ad)}-\frac{b^{3/2}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a(bc-ad)}-\frac{d^{3/2}\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2c(bc-ad)}\\
&= -\frac{1}{acx}-\frac{b\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a(bc-ad)}-\frac{b\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a(bc-ad)}-\frac{b^{5/4}\int \cdots}{4\sqrt{2}\,\cdots}\\
&= -\frac{1}{acx}-\frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\cdots\right.}{4\sqrt{2}\,a^{5/4}(bc-\cdots}\\
&= -\frac{1}{acx}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{d^{5/4}\tan\cdots}{2\sqrt{\cdots}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 385, normalized size = 0.84

$$\frac{\frac{8b}{a}-\frac{8d}{c}-\frac{2\sqrt{2}\,b^{5/4}x\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{5/4}}+\frac{2\sqrt{2}\,b^{5/4}x\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{5/4}}+\frac{2\sqrt{2}\,d^{5/4}x\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{5/4}}-\frac{2\sqrt{2}\,d^{5/4}x\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{5/4}}+\frac{\sqrt{2}\,b^{5/4}x\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{5/4}}-\frac{\sqrt{2}\,b^{5/4}x\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{5/4}}-\frac{\sqrt{2}\,d^{5/4}x\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{c^{5/4}}+\frac{\sqrt{2}\,d^{5/4}x\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{c^{5/4}}}{-8bcx+8adx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/a^(5/4) + (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(5/4) + (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
```

```
)/c^(5/4) - (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c
^(5/4) + (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/a^(5/4) - (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/a^(5/4) - (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] - Sqrt[2]*c^(1/
4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4) + (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4))/(-8*b*c*x + 8*a*d*x)
```

Maple [A]

time = 0.44, size = 237, normalized size = 0.52

method	result
default	$-\frac{d\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8c(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\cdots}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\cdots}\right)\cdots\right.}{\cdots}$
risch	$-\frac{1}{acx}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{4}a^{9}-4c\,d^{3}a^{8}b+6c^{2}d^{2}a^{7}b^{2}-4c^{3}d\,a^{6}b^{3}+a^{5}b^{4}c^{4}\right)\_Z^{4}+b^{5}\right)}\_R\ln\left(\left(\left(5a^{13}c^{5}d^{8}-38a^{12}b\,c^{6}d^{7}+128a^{11}b^{2}c^{7}d^{6}-\cdots\right.\right.\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d/c/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)
^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/
4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))+1/8*b/a/(a*d-b*c)/(a/b)^(1/4)*2^
(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2
)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1
/4)*x-1))-1/a/c/x
```

Maxima [A]

time = 0.51, size = 384, normalized size = 0.83

$$-\frac{b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8\left(abc-a^{2}d\right)}+\frac{d^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}\,x^{2}+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}\,x^{2}-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{8\left(bc^{2}-acd\right)}-\frac{1}{acx}$$

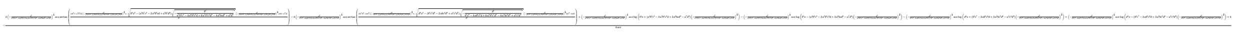
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt
(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b*c - a^2*d) + 1/8*d
^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sq
```

```
rt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^
(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^
(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^2 - a*c*d) - 1/(a*c*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1407 vs. $2(330) = 660$.
time = 4.12, size = 1407, normalized size = 3.06



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*
c*d^3 + a^9*d^4))^(1/4)*a*c*x*arctan(((a*b^5*c - a^2*b^4*d)*(-b^5/(a^5*b^4*
c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)
*x - sqrt(b^8*x^2 - (a^3*b^7*c^2 - 2*a^4*b^6*c*d + a^5*b^5*d^2)*sqrt(-b^5/(
a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4
)))*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^
3 + a^9*d^4))^(1/4)*(a*b*c - a^2*d))/b^5) - 4*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*
d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*arctan(
((b*c^2*d^4 - a*c*d^5)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 -
 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*x - sqrt(d^8*x^2 - (b^2*c^5*d^5 - 2*
a*b*c^4*d^6 + a^2*c^3*d^7)*sqrt(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c
^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4)))*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d +
6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*(b*c^2 - a*c*d))/
d^5) + (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c
*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x + (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3
*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^
2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c
^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x
- (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*
c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)
) - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*x + (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^
5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4
*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*
a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*x - (
b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4
*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) +
 4)/(a*c*x)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.16, size = 488, normalized size = 1.06

$$-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,a^2b^2c-\sqrt{2}\,a^3bd\right)}-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,a^2b^2c-\sqrt{2}\,a^3bd\right)}+\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^3d-\sqrt{2}\,ac^2d^2\right)}+\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^3d-\sqrt{2}\,ac^2d^2\right)}+\frac{(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,a^2b^2c-\sqrt{2}\,a^3bd\right)}-\frac{(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,a^2b^2c-\sqrt{2}\,a^3bd\right)}-\frac{(cd^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^3d-\sqrt{2}\,ac^2d^2\right)}+\frac{(cd^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^3d-\sqrt{2}\,ac^2d^2\right)}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c - sqrt(2)
*a^3*b*d) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4)
)/(c/d)^(1/4))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(c*d^3)^(3/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3*d
- sqrt(2)*a*c^2*d^2) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/4*(a*b^3)^(3/4)*log(x^
2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d
) - 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)
^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) - 1/(a*c*x)
```

Mupad [B]
time = 6.08, size = 2500, normalized size = 5.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] 2*atan(((-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^
2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b
^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 +
1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256
*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d
))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^1
0*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^
```

7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i - 256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a^13*b^10*c^17*d^6 + 256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^17*b^6*c^13*d^10 + 768*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i) + (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i + 256*a^11*b^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c^17*d^6 - 256*a^14*b^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13*d^10 - 768*a^18*b^5*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i))/((-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i - 256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a^13*b^10*c^17*d^6 + 256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^17*b^6*c^13*d^10 + 768*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i)*1i - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i + 256*a^11*b^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c^17*d^6 - 256*a^14*b^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13*d^10 - 768*a^18*b^5*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i)*1i))*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4) + atan((a^14*c*d^8*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*1024i + a^6*b^8*c^9*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*1024i + a^6*b^4*d^5*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*4i + a^5*b^5*c*d^4*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*

4i - a^7*b^7*c^8*d*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i - a^13*b*c^2*d^7*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i + a^8*b^6*c^7*d^2*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*6144i - a^9*b^5*c^6*d^3*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i + a^10*b^4*c^5*d^4*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*2048i - a^11*b^3*c^4*d^5*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i + a^12*b^2*c^3*d^6*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*6144i)/(b^9*c^4 + a^4*b^5*d^4 + a^3*b^6*c*d^3 + a^2*b^7*c^2*d^2 + a*b^8*c^3*d))*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 -...

3.785 $\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=462

$$-\frac{1}{3acx^3}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{7/4}(bc-ad)}+\frac{d^{7/4}\tan^{-1}}{2\sqrt{2}}$$

```
[Out] -1/3/a/c/x^3-1/4*b^(7/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d
+b*c)*2^(1/2)-1/4*b^(7/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d
+b*c)*2^(1/2)+1/4*d^(7/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*
d+b*c)*2^(1/2)+1/4*d^(7/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*
d+b*c)*2^(1/2)+1/8*b^(7/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2
))/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/8*b^(7/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(7/4)*ln(-c^(1/4)*d^(1/4)
*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(7/4)*ln(c
^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {491, 536, 217, 1179, 642, 1176, 631, 210}

$$\frac{b^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}+\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{d^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{7/4}(bc-ad)}+\frac{d^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{7/4}(bc-ad)}-\frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)}+\frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/3*1/(a*c*x^3) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*
d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b^(7/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)
) - (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqr
t[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*Log[Sqrt[c] +
Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{1}{3acx^3}+\frac{\int \frac{-3(bc+ad)-3bdx^4}{(a+bx^4)(c+dx^4)}\,dx}{3ac}$$

$$= -\frac{1}{3acx^3}-\frac{b^2\int \frac{1}{a+bx^4}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{1}{c+dx^4}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{3acx^3}-\frac{b^2\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a^{3/2}(bc-ad)}-\frac{b^2\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a^{3/2}(bc-ad)}+\frac{d^2\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2c^{3/2}(bc-ad)}+$$

$$= -\frac{1}{3acx^3}-\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a^{3/2}(bc-ad)}-\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a^{3/2}(bc-ad)}+\frac{b^{7/4}}{}$$

$$= -\frac{1}{3acx^3}+\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{}\right.}{4\sqrt{2}\,a^{7/4}(b}$$

$$= -\frac{1}{3acx^3}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{d^{7/4}\,\mathrm{t}}{2}$$

Mathematica [A]

time = 0.16, size = 406, normalized size = 0.88

$$\frac{\frac{8b}{a}-\frac{8d}{c}-\frac{6\sqrt{2}\,b^{7/4}x^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}\,b^{7/4}x^3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}\,d^{7/4}x^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{7/4}}-\frac{6\sqrt{2}\,d^{7/4}x^3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{7/4}}-\frac{3\sqrt{2}\,b^{7/4}x^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{3\sqrt{2}\,b^{7/4}x^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{3\sqrt{2}\,d^{7/4}x^3\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{c^{7/4}}-\frac{3\sqrt{2}\,d^{7/4}x^3\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{c^{7/4}}}{24(-bc+ad)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/a^(7/4) + (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/a^(7/4) + (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)])/c^(7/4) - (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/c^(7/4) - (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] + Sqrt[2]
```

*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*d^(7/4)*x^3*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (3*Sqrt[2]*d^(7/4)*x^3*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(24*(-(b*c) + a*d)*x^3)

Maple [A]
time = 0.46, size = 241, normalized size = 0.52

method	result
default	$-\frac{d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^2(ad-bc)}-\frac{1}{3ac\,x^3}+\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}}{}$
risch	$-\frac{1}{3ac\,x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(c^7d^4a^4-4a^3b\,c^8d^3+6a^2b^2c^9d^2-4a\,b^3c^{10}d+b^4c^{11}\right)\_Z^4+d^7\right)}\_R\ln\left(\left(\left(-5a^{15}c^7d^8+38a^{14}b\,c^8d^7-128a^{13}b^2\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)

[Out] -1/8/c^2*d^2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))-1/3/a/c/x^3+1/8/a^2*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))

Maxima [A]
time = 0.51, size = 390, normalized size = 0.84

$$-\frac{\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{8\,(abc-a^2d)}+\frac{\frac{2\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,d^{\frac{7}{4}}\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}\,d^{\frac{7}{4}}\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{8\,(bc^2-acd)}-\frac{1}{3acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] -1/8*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(a*b*c - a^2*d) + 1/8*(2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 + sqrt(2

```
)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 -
sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c^2 - a*c*d) - 1/3/(a*c*x^
3)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1417 vs. $2(330) = 660$.
time = 11.43, size = 1417, normalized size = 3.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/12*(12*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*
b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*arctan(((a^5*b^5*c^3 - 3*a^6*b^4*c^2*d +
 3*a^7*b^3*c*d^2 - a^8*b^2*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^
9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*x - (a^5*b^3*c^3 - 3*a^6*
b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d +
 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*sqrt(b^4*x^2 + (a^4*
b^2*c^2 - 2*a^5*b*c*d + a^6*d^2)*sqrt(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d +
 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))))/b^7) - 12*(-d^7/(b^4*c^1
1 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1
/4)*a*c*x^3*arctan(((b^3*c^8*d^2 - 3*a*b^2*c^7*d^3 + 3*a^2*b*c^6*d^4 - a^3*
c^5*d^5)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8
*d^3 + a^4*c^7*d^4))^(3/4)*x - (b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 -
 a^3*c^5*d^3)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*
b*c^8*d^3 + a^4*c^7*d^4))^(3/4)*sqrt(d^4*x^2 + (b^2*c^6 - 2*a*b*c^5*d + a^2
*c^4*d^2)*sqrt(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*
b*c^8*d^3 + a^4*c^7*d^4))))/d^7) - 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d +
 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x +
(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 +
 a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3
*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*
x - (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d
^3 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-d^7/(b^4*c^11 - 4*a*b^3*c^10
*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^3*log(
d^2*x + (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*
d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) - 3*(-d^7/(b^4*c^11 - 4*a*b^3*
c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^3*
log(d^2*x - (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*
c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) - 4)/(a*c*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out

Giac [A]
time = 1.00, size = 472, normalized size = 1.02

$$-\frac{(ab^3)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,a^2bc-\sqrt{2}\,a^3d\right)}-\frac{(ab^3)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,a^2bc-\sqrt{2}\,a^3d\right)}+\frac{(cd^3)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^3-\sqrt{2}\,ac^2d\right)}+\frac{(cd^3)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^3-\sqrt{2}\,ac^2d\right)}-\frac{(ab^3)^{\frac{1}{4}} b \log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,a^2bc-\sqrt{2}\,a^3d\right)}+\frac{(ab^3)^{\frac{1}{4}} b \log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,a^2bc-\sqrt{2}\,a^3d\right)}+\frac{(cd^3)^{\frac{1}{4}} d \log\left(x^2+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^3-\sqrt{2}\,ac^2d\right)}-\frac{(cd^3)^{\frac{1}{4}} d \log\left(x^2-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^3-\sqrt{2}\,ac^2d\right)}-\frac{1}{3acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] -1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) - 1/2*(a*b^3)^(1/4)*b*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a
^3*d) + 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/
(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) + 1/2*(c*d^3)^(1/4)*d*arctan
(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt
(2)*a*c^2*d) - 1/4*(a*b^3)^(1/4)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/4*(a*b^3)^(1/4)*b*log(x^2 - sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/4*(c*d
^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3 - s
qrt(2)*a*c^2*d) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqr
t(c/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 1/3/(a*c*x^3)

Mupad [B]
time = 6.19, size = 2500, normalized size = 5.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + b*x^4)*(c + d*x^4)),x)

[Out] - atan((a^2*b^5*d^7*x*1i + b^7*c^2*d^5*x*1i - (a^2*b^16*c^11*x*256i)/(256*a
^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 102
4*a^10*b*c*d^3) - (a^4*b^14*c^9*d^2*x*1536i)/(256*a^11*d^4 + 256*a^7*b^4*c^
4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^5*b
^13*c^8*d^3*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d +
1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^6*b^12*c^7*d^4*x*256i)/(256
*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1
024*a^10*b*c*d^3) - (a^7*b^11*c^6*d^5*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c
^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^8*
b^10*c^5*d^6*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d
+ 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^9*b^9*c^4*d^7*x*1536i)/(25

6*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^10*b^8*c^3*d^8*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^11*b^7*c^2*d^9*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^3*b^15*c^10*d*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))/((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((b^7*(1024*a^4*b^8*c^12 + 1024*a^12*c^4*d^8 - 5120*a^5*b^7*c^11*d - 5120*a^11*b*c^5*d^7 + 10240*a^6*b^6*c^10*d^2 - 11264*a^7*b^5*c^9*d^3 + 10240*a^8*b^4*c^8*d^4 - 11264*a^9*b^3*c^7*d^5 + 10240*a^10*b^2*c^6*d^6))/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + 4*a^5*b^3*d^8 + 4*b^8*c^5*d^3 - 4*a*b^7*c^4*d^4 - 4*a^4*b^4*c*d^7)))*(-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*2i - atan((a^2*b^5*d^7*x*1i + b^7*c^2*d^5*x*1i - (a^11*c^2*d^16*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^2*b^9*c^11*d^7*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^3*b^8*c^10*d^8*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^4*b^7*c^9*d^9*x*1536i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^5*b^6*c^8*d^10*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^6*b^5*c^7*d^11*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^7*b^4*c^6*d^12*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^8*b^3*c^5*d^13*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^9*b^2*c^4*d^14*x*1536i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^10*b*c^3*d^15*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))/((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*((d^7*(1024*a^4*b^8*c^12 + 1024*a^12*c^4*d^8 - 5120*a^5*b^7*c^11*d - 5120*a^11*b*c^5*d^7 + 10240*a^6*b^6*c^10*d^2 - 11264*a^7*b^5*c^9*d^3 + 10240*a^8*b^4*c^8*d^4 - 11264*a^9*b^3*c^7*d^5 + 10240*a^10*b^2*c^6*d^6))/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + 4*a^5*b^3*d^8 + 4*b^8*c^5*d^3 - 4*a*b^7*c^4*d^4 - 4*a^4*b^4*c*d^7)))*(-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*2i - 2*atan(((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024

*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4
096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 -
4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12
 + 1024*a^20*b^4*c^11*d^13) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*
a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13
*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 4505
6*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 4
0960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^1
2)*1i)*1i - 16*a^9*b^12*c^14*d^7 + 16*a^10*b^11*c^13*d^8 + 16*a^13*b^8*c^10
*d^11 - 16*a^14*b^7*c^9*d^12)*1i) + (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 -
 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 10...

3.786 $\int \frac{1}{x^6(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=479

$$-\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{d^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{9/4}(bc-ad)}-$$

```
[Out] -1/5/a/c/x^5+(a*d+b*c)/a^2/c^2/x+1/4*b^(9/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^
(1/4))/a^(9/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(9/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^
(1/4))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/4*d^(9/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c
^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)-1/4*d^(9/4)*arctan(1+d^(1/4)*x*2^(1/2)/c
^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)+1/8*b^(9/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*b^(1/2))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/8*b^(9/4)*ln(a^(1/4)*b^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(9/4)
*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(9/4)/(-a*d+b*c)*2^(1
/2)+1/8*d^(9/4)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(9/4)/(
-a*d+b*c)*2^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {491, 597, 598, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{ad+bc}{a^2c^2x}+\frac{d^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{9/4}(bc-ad)}-\frac{d^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{9/4}(bc-ad)}-\frac{d^{9/4}\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)}+\frac{d^{9/4}\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)}-\frac{1}{5acx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/5*1/(a*c*x^5) + (b*c + a*d)/(a^2*c^2*x) - (b^(9/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (b^(9/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (d^(9/4)*Arc
Tan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) - (d^
(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*
d)) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*S
qrt[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c]
- Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d))
 + (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt
[2]*c^(9/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= -\frac{1}{5acx^5}+\frac{\int \frac{-5(bc+ad)-5bdx^4}{x^2(a+bx^4)(c+dx^4)}\,dx}{5ac} \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{\int \frac{x^2\left(-5\left(b^2c^2+abcd+a^2d^2\right)-5bd(bc+ad)x^4\right)}{(a+bx^4)(c+dx^4)}\,dx}{5a^2c^2} \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{\int \left(-\frac{5b^3c^2x^2}{(bc-ad)(a+bx^4)}-\frac{5a^2d^3x^2}{(-bc+ad)(c+dx^4)}\right)dx}{5a^2c^2} \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}+\frac{b^3\int \frac{x^2}{a+bx^4}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{x^2}{c+dx^4}\,dx}{c^2(bc-ad)} \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{b^{5/2}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a^2(bc-ad)}+\frac{b^{5/2}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a^2(bc-ad)}+\frac{d^{5/2}\int}{2c} \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}+\frac{b^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a^2(bc-ad)}+\frac{b^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,d}{4a^2(bc-ad)} \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}+\frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{}\right.}{} \\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{2\sqrt{2}\,a^{9/4}(bc-ad)}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 428, normalized size = 0.89

$$
\frac{\frac{8b}{a}-\frac{8d}{c}-\frac{40b^2x^4}{a^2}+\frac{40d^2x^4}{c^2}+\frac{10\sqrt{2}\,b^{9/4}x^5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{9/4}}-\frac{10\sqrt{2}\,b^{9/4}x^5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{9/4}}-\frac{10\sqrt{2}\,d^{9/4}x^5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{9/4}}+\frac{10\sqrt{2}\,d^{9/4}x^5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{9/4}}-\frac{5\sqrt{2}\,b^{9/4}x^5\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{9/4}}+\frac{5\sqrt{2}\,b^{9/4}x^5\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{a^{9/4}}+\frac{5\sqrt{2}\,d^{9/4}x^5\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{c^{9/4}}-\frac{5\sqrt{2}\,d^{9/4}x^5\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{c^{9/4}}}{40(-bc+ad)x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (40*b^2*x^4)/a^2 + (40*d^2*x^4)/c^2 + (10*Sqrt[2]*b^(9
/4)*x^5*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*b^(9
/4)*x^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*d^(9
/4)*x^5*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) + (10*Sqrt[2]*d^(9
/4)*x^5*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) - (5*Sqrt[2]*b^(9/
4)*x^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(9/4) + (5
*Sqrt[2]*b^(9/4)*x^5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
```

```
)/a^(9/4) + (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/c^(9/4) - (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(9/4))/(40*(-(b*c) + a*d)*x^5)
```

Maple [A]

time = 0.45, size = 261, normalized size = 0.54

method	result
default	$\frac{d^2\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^2(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{1}{5ac\,x^5}-\frac{-ad-bc}{a^2c^2x}-\frac{b^2\sqrt{2}\left(\ln\right.}{}$
risch	$\frac{\frac{(ad+bc)x^4}{a^2c^2}-\frac{1}{5ac}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^4c^9a^4-4d^3c^{10}a^3b+6d^2c^{11}a^2b^2-4d\,c^{12}a\,b^3+b^4c^{13}\right)\_Z^4+d^9\right)}\_R\ln\left(\left(\left(5a^{17}c^9d^8-38a^{16}b\,c^{10}d^7+12\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d^2/c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c
/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^
(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))-1/5/a/c/x^5-1/a^2/c^2*(-a*d-b
*c)/x-1/8*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1
/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/
(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Maxima [A]

time = 0.50, size = 405, normalized size = 0.85

$$\frac{b^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8\left(a^2bc-a^3d\right)}-\frac{d^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{8\left(bc^3-ac^2d\right)}+\frac{5\,(bc+ad)x^4-ac}{5\,a^2c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b*c - a^3*d) - 1/8*
d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/s
qrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/
2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d
```

```
^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c
^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^3 - a*c^2*d) + 1/5*(5*(
b*c + a*d)*x^4 - a*c)/(a^2*c^2*x^5)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1468 vs. $2(347) = 694$.
time = 17.30, size = 1468, normalized size = 3.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/20*(20*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^1
2*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*arctan(((a^2*b^8*c - a^3*b^7*d)*(-
b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 +
 a^13*d^4))^(1/4)*x - sqrt(b^14*x^2 - (a^5*b^11*c^2 - 2*a^6*b^10*c*d + a^7*
b^9*d^2)*sqrt(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4
*a^12*b*c*d^3 + a^13*d^4)))*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*
b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(a^2*b*c - a^3*d))/b^9) - 2
0*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3
 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^5*arctan(((b*c^3*d^7 - a*c^2*d^8)*(-d^9/(b^
4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d
^4))^(1/4)*x - sqrt(d^14*x^2 - (b^2*c^7*d^9 - 2*a*b*c^6*d^10 + a^2*c^5*d^11
)*sqrt(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*
d^3 + a^4*c^9*d^4)))*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2
 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*(b*c^3 - a*c^2*d))/d^9) + 5*(-b^9/
(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^1
3*d^4))^(1/4)*a^2*c^2*x^5*log(b^7*x + (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^
9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^
2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-b^9/(a^9*b^4*c^4 - 4*a^10*
b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*
x^5*log(b^7*x - (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*
(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3
 + a^13*d^4))^(3/4)) - 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*
d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^5*log(d^7*x + (b^3*c
^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*
b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) +
 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3
 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^5*log(d^7*x - (b^3*c^10 - 3*a*b^2*c^9*d +
3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2
*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) + 20*(b*c + a*d)*x^4 -
4*a*c)/(a^2*c^2*x^5)
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.43, size = 483, normalized size = 1.01

$$\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,a^3bc-\sqrt{2}\,a^4d\right)}+\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,a^3bc-\sqrt{2}\,a^4d\right)}-\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^4-\sqrt{2}\,ac^3d\right)}-\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^4-\sqrt{2}\,ac^3d\right)}-\frac{(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,a^3bc-\sqrt{2}\,a^4d\right)}+\frac{(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,a^3bc-\sqrt{2}\,a^4d\right)}+\frac{(cd^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^4-\sqrt{2}\,ac^3d\right)}-\frac{(cd^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^4-\sqrt{2}\,ac^3d\right)}+\frac{5\,bcx^4+5\,adx^4-ac}{5\,a^2c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d)
 - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3
*d) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(
2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(c*d^3)^(3/4)*log
(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d)
 - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*
b*c^4 - sqrt(2)*a*c^3*d) + 1/5*(5*b*c*x^4 + 5*a*d*x^4 - a*c)/(a^2*c^2*x^5)
```

Mupad [B]
time = 6.01, size = 2500, normalized size = 5.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - 2*atan((1024*a^11*b^10*c^13*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 102
4*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 4*a^
11*b^6*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d +
1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) + 1024*a^21*c^3*d^10*x*(-
b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c
^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^12*b^9*c^12*d*x*(-b^9/(256*a^13
*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024
```

*a^12*b*c*d^3))^(5/4) - 4096*a^20*b*c^4*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 4*a^8*b^9*c^3*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) + 6144*a^13*b^8*c^11*d^2*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^14*b^7*c^10*d^3*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^15*b^6*c^9*d^4*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^17*b^4*c^7*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^18*b^3*c^6*d^7*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 6144*a^19*b^2*c^5*d^8*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4))/(b^16*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7 + a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3 + a^4*b^12*c^4*d^4 + a^5*b^11*c^3*d^5 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) - atan((a^11*b^10*c^13*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i + a^11*b^6*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4)*4i + a^21*c^3*d^10*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i - a^12*b^9*c^12*d*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i - a^20*b*c^4*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i + a^8*b^9*c^3*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4)*4i + a^13*b^8*c^11*d^2*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*6144i - a^14*b^7*c^10*d^3*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i + a^15*b^6*c^9*d^4*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i + a^17*b^4*c^7*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i - a^18*b^3*c^6*d^7*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i + a^19*b^2*c^5*d^8*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*6144i)/(b^16*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7 + a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3 + a^4*b^12*c^4*d^4 + a^5*b^11*c^3*d^5 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4)*2i - 2*atan((4*b^9*c^11*d^6*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^

```
4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1/4)
 + 1024*a^3*b^10*c^21*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*
c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 1024*a^13*c^
11*d^10*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 153
6*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^4*b^9*c^20*d*x*(-d^
9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11
*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^12*b*c^12*d^9*x*(-d^9/(256*b^4*c^
13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a
*b^3*c^12*d))^(5/4) + 4*a^3*b^6*c^8*d^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9
*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1
/4) + 6144*a^5*b^8*c^19*d^2*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*
a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a
^6*b^7*c^18*d^3*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d
^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 1024*a^7*b^6*c^17*
d^4*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1...
```

3.787 $\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=93

$$-\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{(c+dx^4)^{3/2}}{6bd}+\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}$$

```
[Out] 1/6*(d*x^4+c)^(3/2)/b/d+1/2*a*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1
/2))*(-a*d+b*c)^(1/2)/b^(5/2)-1/2*a*(d*x^4+c)^(1/2)/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 81, 52, 65, 214}

$$\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}-\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{(c+dx^4)^{3/2}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] -1/2*(a*Sqrt[c + d*x^4])/b^2 + (c + d*x^4)^(3/2)/(6*b*d) + (a*Sqrt[b*c - a*
d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx,x,x^4\right)\\
&= \frac{\left(c+dx^4\right)^{3/2}}{6bd}-\frac{a\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^4\right)}{4b}\\
&= -\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd}-\frac{(a(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4b^2}\\
&= -\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd}-\frac{(a(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2b^2d}\\
&= -\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd}+\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 88, normalized size = 0.95

$$\frac{\sqrt{c+dx^4}\left(-3ad+b\left(c+dx^4\right)\right)}{6b^2d}+\frac{a\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-3*a*d + b*(c + d*x^4)))/(6*b^2*d) + (a*Sqrt[-(b*c) + a*d
]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(2*b^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1011 vs. $2(73) = 146$.
time = 0.42, size = 1012, normalized size = 10.88 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(d*x^4+c)^(3/2)/b/d-a/b*(1/4/b*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^(1/2)*(-a*b)^(1/
2)*ln((d*(-a*b)^(1/2)/b+(x^2-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x^2-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/
4/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^
(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*
a*d-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2
)))*c+1/4/b*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b^2*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/
b+(x^2+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^
(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*a*d-1/4/b/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.01, size = 195, normalized size = 2.10

$$\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\,(bdx^4+bc-3\,ad)\sqrt{dx^4+c}}{12\,b^2d}, \frac{3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(bdx^4+bc-3\,ad)\sqrt{dx^4+c}}{6\,b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4
+ c)*b*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*(b*d*x^4 + b*c - 3*a*d)*sqrt(d
*x^4 + c))/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^4 + b*c - 3*a*d)*sqrt(d*x^4 +
 c))/(b^2*d)]
```

Sympy [A]
time = 7.10, size = 90, normalized size = 0.97

$$\frac{2\left(-\frac{ad^2\sqrt{c+dx^4}}{4b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d(c+dx^4)^{\frac{3}{2}}}{12b}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**4)/(4*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x
**4)/sqrt((a*d - b*c)/b))/(4*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**4)**(3
/2)/(12*b))/d**2
```

Giac [A]
time = 1.25, size = 96, normalized size = 1.03

$$-\frac{(abc-a^2d)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,b^2}+\frac{(dx^4+c)^{\frac{3}{2}}b^2d^2-3\,\sqrt{dx^4+c}\,abd^3}{6\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*b*c - a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^4 + c)*a*
b*d^3)/(b^3*d^3)
```

Mupad [B]
time = 4.69, size = 87, normalized size = 0.94

$$\frac{(d\,x^4+c)^{3/2}}{6\,b\,d}-\frac{a\,\sqrt{d\,x^4+c}}{2\,b^2}+\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\sqrt{a\,d-b\,c}}{a^2\,d-a\,b\,c}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b*d) - (a*(c + d*x^4)^(1/2))/(2*b^2) + (a*atan((a*b^(1
/2)*(c + d*x^4)^(1/2)*(a*d - b*c)^(1/2))/(a^2*d - a*b*c))*(a*d - b*c)^(1/2)
)/(2*b^(5/2))
```

3.788 $\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=120

$$\frac{x^2\sqrt{c+dx^4}}{4b}-\frac{\sqrt{a}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}$$

```
[Out] 1/4*(-2*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d^(1/2)-1/2*arcta
n(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a^(1/2)*(-a*d+b*c)^(1/2)/b^
2+1/4*x^2*(d*x^4+c)^(1/2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {476, 489, 537, 223, 212, 385, 211}

$$-\frac{\sqrt{a}\,\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}+\frac{x^2\sqrt{c+dx^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a
*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b^2) + ((b*c - 2*a*d)*ArcTanh[(Sqrt
[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx, x, x^2\right)$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\mathrm{Subst}\left(\int \frac{ac+(-bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} + \frac{(bc-2ad)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b^2} - \frac{(a(bc-ad))\mathrm{Subst}\left(\int \frac{\quad}{(a+b}\right.}{2b^2}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} + \frac{(bc-2ad)\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4b^2} - \frac{(a(bc-ad))\mathrm{Subst}\left(\int \right.}{}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+d}}\right.}{4b^2\sqrt{d}}$$

Mathematica [A]
time = 0.42, size = 138, normalized size = 1.15

$$\frac{\sqrt{d}\left(bx^2\sqrt{c+dx^4} - 2\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^4+bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}\right)\right) + (bc-2ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{d}\,x^2}\right)}{4b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[d]*(b*x^2*Sqrt[c + d*x^4] - 2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[
d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])]) + (
b*c - 2*a*d)*ArcTanh[Sqrt[c + d*x^4]/(Sqrt[d]*x^2)])/(4*b^2*Sqrt[d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1049 vs. $2(96) = 192$.
time = 0.43, size = 1050, normalized size = 8.75 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*x^2*(d*x^4+c)^(1/2)+1/4*c/d^(1/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2)))
-a/b*(1/4/(-a*b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4*d^(1/2)/b*ln((d*(-a*b)^(1/2)/b+(x^
2-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)
/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/b/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(
```

```
-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*a*d-1/4/(-a*b)^(
1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*c
-1/4/(-a*b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4*d^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+(x^2+1/
b*(-a*b)^(1/2))*d)/d^(1/2)+((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*a*d+1/4/(-a*b)^(1/2)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*c)
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^5/(b*x^4 + a), x)
```

<u>Fricas</u> [A]
time = 4.09, size = 714, normalized size = 5.95

$$\left[\frac{2\sqrt{dx^4+c}\,bdx^2-(bc-2ad)\sqrt{d}\log\left(-2dx^4+2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\sqrt{-abc+a^2d}\,d\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2-4((bc-2ad)x^6-acx^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2abx^4+a^2}\right)}{8b^2d}, \frac{2\sqrt{dx^4+c}\,bdx^2-2(bc-2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)+\sqrt{-abc+a^2d}\,d\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2-4((bc-2ad)x^6-acx^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2abx^4+a^2}\right)}{8b^2d}, \frac{2\sqrt{dx^4+c}\,bdx^2-2\sqrt{abc-a^2d}\,d\arctan\left(\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)-(bc-2ad)\sqrt{d}\log\left(-2dx^4+2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)}{8b^2d}, \frac{\sqrt{dx^4+c}\,bdx^2-(bc-2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)-\sqrt{abc-a^2d}\,d\arctan\left(\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)}{4b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sq
rt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2 - 8*a
*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c
 - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2
*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - 2*(b*c - 2*a*d)
*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + sqrt(-a*b*c + a^2*d)*d*log
(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a
^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*
d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 -
 2*sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))
 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))
```

```
/(b^2*d), 1/4*(sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(-d)*arctan(sqrt
(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)
*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 +
(a*b*c^2 - a^2*c*d)*x^2)))/(b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.789 $\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(
3/2)+1/2*(d*x^4+c)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {455, 52, 65, 214}

$$\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b) - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/
Sqrt[b*c - a*d]])/(2*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^4\right)\\ &= \frac{\sqrt{c+dx^4}}{2b}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b}\\ &= \frac{\sqrt{c+dx^4}}{2b}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2bd}\\ &= \frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 69, normalized size = 0.99

$$\frac{1}{2}\left(\frac{\sqrt{c+dx^4}}{b}-\frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[c + d*x^4]/b - (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/S
qrt[-(b*c) + a*d]])/b^(3/2))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 987 vs. 2(54) = 108.
time = 0.39, size = 988, normalized size = 14.11 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x^2-
1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*a*d-1/4/b/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*c+1/4/b*((x^2+1/b*(-a*b)^
(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4
/b^2*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x^2+1/b*(-a*b)^(1/2))*d)/d
^(1/2)+((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x^2+1/b*(-a*b)^(1/2)))*a*d-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2
*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x^2+1/b*(-a*b)^(1/2)))*c
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.30, size = 156, normalized size = 2.23

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}}{4\,b},\ -\frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^4+c}}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c))/b, -1/2*(sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqr
t(d*x^4 + c))/b]
```

Sympy [A]
time = 3.56, size = 65, normalized size = 0.93

$$\frac{2\left(\frac{d\sqrt{c+dx^4}}{4b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**4)/(4*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**4)/sqrt((a*d
 - b*c)/b))/(4*b**2*sqrt((a*d - b*c)/b)))/d
```

Giac [A]
time = 1.08, size = 66, normalized size = 0.94

$$\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,b}+\frac{\sqrt{dx^4+c}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b) + 1/2*sqrt(d*x^4 + c)/b
```

Mupad [B]
time = 4.66, size = 54, normalized size = 0.77

$$\frac{\sqrt{d\,x^4+c}}{2\,b}-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b) - (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/
2))*(a*d - b*c)^(1/2))/(2*b^(3/2))
```

3.790 $\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2\sqrt{a}\,b}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

```
[Out] 1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))*d^(1/2)/b+1/2*arctan(x^2*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2)/b/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {476, 399, 223, 212, 385, 211}

$$\frac{\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2\sqrt{a}\,b}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(
2*Sqrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(2*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx,x,x^2\right)\\
&= \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b}\\
&= \frac{d\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}\\
&= \frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2\sqrt{a}\,b}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b}
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 106, normalized size = 1.16

$$
\frac{\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}\,x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{\sqrt{a}}+\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{d}\,x^2}\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] ((Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4])
)/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[a] + Sqrt[d]*ArcTanh[Sqrt[c + d*x^4]/(Sq
rt[d]*x^2)])/(2*b)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 999 vs. $2(71) = 142$.
time = 0.35, size = 1000, normalized size = 10.99

method	result
default	$\frac{\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2} d+\frac{2d\sqrt{-ab}\,\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{4\sqrt{-ab}}+\frac{\sqrt{d}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{\sqrt{d}}+\sqrt{(\ldots}\right.}{\ldots}$
elliptic	$\frac{\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2} d+\frac{2d\sqrt{-ab}\,\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{4\sqrt{-ab}}+\frac{\sqrt{d}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{\sqrt{d}}+\sqrt{(\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(-a*b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4*d^(1/2)/b*ln((d*(-a*b)^(1/2)/b+(x^2-1/b*
(-a*b)^(1/2))*d)/d^(1/2)+((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^
2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*a*d-1/4/(-a*b)^(1/2)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b
*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*c-1/4/(
-a*b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)+1/4*d^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+(x^2+1/b*(-a*
b)^(1/2))*d)/d^(1/2)+((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1
```

```
/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*a*d+1/4/(-a*b)^(1/2)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^
2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*x/(b*x^4 + a), x)
```

Fricas [A]
time = 3.21, size = 612, normalized size = 6.73

$$\left[\frac{2\sqrt{d}\log\left(-2dx^4-2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2+4((abc-2a^2d)x^6-a^2cx^2)\sqrt{dx^4+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2abx^4+a^2}\right)}{8b}, -\frac{4\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)-\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2+4((abc-2a^2d)x^6-a^2cx^2)\sqrt{dx^4+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2abx^4+a^2}\right)}{8b}, \frac{\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}}}{2((bcd-ad^2)x^6+(bc^2-acd)x^2)}\right)+\sqrt{d}\log\left(-2dx^4-2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)}{4b}, -\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)-\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}}}{2((bcd-ad^2)x^6+(bc^2-acd)x^2)}\right)}{4b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] [1/8*(2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-(
b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4
+ c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, -1/8*(4*sqrt(-d
)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(-(b*c - a*d)/a)*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4
*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/
(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, 1/4*(sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
- 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^
6 + (b*c^2 - a*c*d)*x^2)) + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d
)*x^2 - c))/b, -1/4*(2*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt
((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((
b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] int((x*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.791 $\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx$

Optimal. Leaf size=85

$$-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}+\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}$$

```
[Out] -1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))*c^(1/2)/a+1/2*arctanh(b^(1/2)*(d*x^4+
c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 85, 65, 214}

$$\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

```
[Out] -1/2*(Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/a + (Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^4}}{x\,(a+bx^4)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx, x, x^4\right)\\ &= \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{4a} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a}\\ &= \frac{c\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2ad} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2ad}\\ &= -\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a} + \frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}} - \sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

```
[Out] ((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/
Sqrt[b] - Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(65) = 130$.
time = 0.35, size = 1039, normalized size = 12.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*(d*x^4+c)^(1/2)-1/2*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2
))-b/a*(1/4/b*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)
/b+(x^2-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)
^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d
-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*a*d-1/4/b/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*c+1/4/b*((x^2+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)-1/4/b^2*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x^2+1/b*(-a*b)^(1/
2))*d)/d^(1/2)+((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*a*d-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x), x)
```

Fricas [A]
time = 2.68, size = 383, normalized size = 4.51

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\sqrt{dx^4+c}\,\sqrt{c}+2c}{x^4}\right)}{4a}, \frac{2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\sqrt{c}\log\left(\frac{dx^4-2\sqrt{dx^4+c}\,\sqrt{c}+2c}{x^4}\right)}{4a}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-c}}{c}\right)+\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)}{4a}, \frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\sqrt{-c}\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-c}}{c}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*
sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 +
c)*sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/
```

```
c) + sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b*s
qrt((b*c - a*d)/b))/(b*x^4 + a)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt
(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqrt(d*x^
4 + c)*sqrt(-c)/c))/a]
```

Sympy [A]
time = 5.32, size = 82, normalized size = 0.96

$$\frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x/(b*x**4+a),x)
```

```
[Out] 2*(c*d*atan(sqrt(c + d*x**4)/sqrt(-c))/(4*a*sqrt(-c)) + d*(a*d - b*c)*atan(
sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*a*b*sqrt((a*d - b*c)/b)))/d
```

Giac [A]
time = 1.26, size = 79, normalized size = 0.93

$$-\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a}+\frac{c\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*a) + 1/2*c*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B]
time = 4.87, size = 199, normalized size = 2.34

$$\frac{\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{c}\left(\sqrt{d\,x^4+c}\,\left(\frac{a^2\,b\,d^4}{2}-a\,b^2\,c\,d^3+b^3\,c^2\,d^2\right)+\frac{c\,\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}}{16\,a^2}\right)}{2\,a\,\left(\frac{b^2\,c^2\,d^3}{4}-\frac{a\,b\,c\,d^4}{4}\right)}\right)}{2\,a}+\frac{\operatorname{atanh}\left(\frac{a\,b^2\,c\,d^3\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(\frac{a\,b^3\,c^2\,d^3}{4}-\frac{a^2\,b^2\,c\,d^4}{4}\right)}\right)\,\sqrt{b^2\,c-a\,b\,d}}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x*(a + b*x^4)),x)
```

```
[Out] (c^(1/2)*atanh((c^(1/2)*((c + d*x^4)^(1/2)*((a^2*b*d^4)/2 + b^3*c^2*d^2 - a
*b^2*c*d^3) + (c*(8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2))/(16*
a^2)))/(2*a*((b^2*c^2*d^3)/4 - (a*b*c*d^4)/4))))/(2*a) + (atanh((a*b^2*c*d^
3*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*((a*b^3*c^2*d^3)/4 - (a^2*b^2
*c*d^4)/4)))*(b^2*c - a*b*d)^(1/2))/(2*a*b)
```

3.792 $\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2)/
a^(3/2)-1/2*(d*x^4+c)^(1/2)/a/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {476, 486, 12, 385, 211}

$$-\frac{\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{c+dx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

```
[Out] -1/2*Sqrt[c + d*x^4]/(a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2
)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
```

```
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}+\frac{\text{Subst}\left(\int \frac{-bc+ad}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 96, normalized size = 1.26

$$
-\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}\,x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

[Out] -1/2*Sqrt[c + d*x^4]/(a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2))

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(60) = 120$.
time = 0.37, size = 1070, normalized size = 14.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/a*(-1/2/c/x^2*(d*x^4+c)^(3/2)+1/2*d/c*x^2*(d*x^4+c)^(1/2)+1/2*d^(1/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2)))-b/a*(1/4/(-a*b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4*d^(1/2)/b*ln((d*(-a*b)^(1/2)/b+(x^2-1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*a*d-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*c-1/4/(-a*b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4*d^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+(x^2+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*a*d+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*c)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^3), x)

Fricas [**A**]

time = 2.57, size = 281, normalized size = 3.70

$$\left[\frac{x^{2}\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{8}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{4}+a^{2}c^{2}-4\,((abc-2\,a^{2}d)x^{6}-a^{2}cx^{2})\sqrt{dx^{4}+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{8}+2\,abx^{4}+a^{2}}\right)-4\,\sqrt{dx^{4}+c}}{8\,ax^{2}},\ -\frac{x^{2}\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^{4}-ac)\sqrt{dx^{4}+c}\,\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^{2})x^{6}+(bc^{2}-acd)x^{2})}\right)+2\,\sqrt{dx^{4}+c}}{4\,ax^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(x^2*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c
*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)) -
4*sqrt(d*x^4 + c))/(a*x^2), -1/4*(x^2*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
- 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^
6 + (b*c^2 - a*c*d)*x^2)) + 2*sqrt(d*x^4 + c))/(a*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^{4}}}{x^{3}\,(a+bx^{4})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**3/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**3*(a + b*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(60) = 120.
time = 1.89, size = 121, normalized size = 1.59

$$\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\,\sqrt{abcd-a^{2}d^{2}}\,a}+\frac{c\sqrt{d}}{\left(\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{2}-c\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2
*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) + c*
sqrt(d)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2 - c)*a)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^4+c}}{x^3\,\left(b\,x^4+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)), x)
```

3.793 $\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{c+dx^4}}{4ax^4}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}$$

```
[Out] 1/4*(-a*d+2*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(1/2)-1/2*arctanh(b
^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)*(-a*d+b*c)^(1/2)/a^2-1/4*(
d*x^4+c)^(1/2)/a/x^4
```

Rubi [A]
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.208$, Rules used = {457, 101, 162, 65, 214}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}-\frac{\sqrt{c+dx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

```
[Out] -1/4*Sqrt[c + d*x^4]/(a*x^4) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[
c]])/(4*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^4])/Sqrt[b*c - a*d]])/(2*a^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx,x,x^4\right)$$

$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4a}$$

$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4a^2} - \frac{(2bc-ad)\text{Subst}\left(\int \right.}{8}$$

$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2a^2d} - \frac{(2bc-ad)\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} - \frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+}}{\sqrt{bc-a}}\right.}{2a^2}$$

Mathematica [A]
time = 0.28, size = 107, normalized size = 0.93

$$\frac{-\frac{a\sqrt{c+dx^4}}{x^4} - 2\sqrt{b}\,\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right) + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

```
[Out] (-((a*Sqrt[c + d*x^4])/x^4) - 2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*
Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]] + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^
4]/Sqrt[c]])/Sqrt[c])/(4*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(91) = 182$.
time = 0.42, size = 1108, normalized size = 9.63 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/4/c/x^4*(d*x^4+c)^(3/2)-1/4*d/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(
1/2))/x^2)+1/4*d/c*(d*x^4+c)^(1/2))-b/a^2*(1/2*(d*x^4+c)^(1/2)-1/2*c^(1/2)*
ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2))+b^2/a^2*(1/4/b*((x^2-1/b*(-a*b)^(1
/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b
^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x^2-1/b*(-a*b)^(1/2))*d)/d^(1
/2)+((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*
b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^
2-1/b*(-a*b)^(1/2)))*a*d-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*
b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x^2-1/b*(-a*b)^(1/2)))*c+1/4/b*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/
2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b^2*d^(1/2)*(-a*b)^(1/2)
*ln((-d*(-a*b)^(1/2)/b+(x^2+1/b*(-a*b)^(1/2))*d)/d^(1/2)+((x^2+1/b*(-a*b)^(
1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4
/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(
1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*a
*d-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)
))*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^5), x)
```

Fricas [A]
time = 3.80, size = 513, normalized size = 4.46

$$\left[\frac{2\,\sqrt{b^2c-abd}\,cx^4\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)-(2\,bc-ad)\sqrt{c}\,x^4\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\,\sqrt{c}+2\,c}{x^4}\right)-2\,\sqrt{dx^4+c}\,ac}{8\,a^2cx^4},\ \frac{4\,\sqrt{-b^2c+abd}\,cx^4\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-(2\,bc-ad)\sqrt{c}\,x^4\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\,\sqrt{c}+2\,c}{x^4}\right)-2\,\sqrt{dx^4+c}\,ac}{8\,a^2cx^4},\ -\frac{(2\,bc-ad)\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-c}}{c}\right)-\sqrt{b^2c-abd}\,cx^4\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)+\sqrt{dx^4+c}\,ac}{4\,a^2cx^4},\ \frac{2\,\sqrt{-b^2c+abd}\,cx^4\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-(2\,bc-ad)\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-c}}{c}\right)-\sqrt{dx^4+c}\,ac}{4\,a^2cx^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(b^2*c - a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4
 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*
x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c
*x^4), 1/8*(4*sqrt(-b^2*c + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^4 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*x^4 - 2*sqrt(
d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), -1/4*(
(2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - sqrt(b^2*c
- a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c -
a*b*d))/(b*x^4 + a)) + sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), 1/4*(2*sqrt(-b^2*c
 + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)
) - (2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - sqrt(d*
x^4 + c)*a*c)/(a^2*c*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^4}}{x^5\,(a+bx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**5/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**5*(a + b*x**4)), x)
```

Giac [A]
time = 1.04, size = 107, normalized size = 0.93

$$\frac{(b^2c-abd)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}}-\frac{\sqrt{dx^4+c}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c - a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a^2) - 1/4*(2*b*c - a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^
2*sqrt(-c)) - 1/4*sqrt(d*x^4 + c)/(a*x^4)
```

Mupad [B]

time = 5.39, size = 269, normalized size = 2.34

$$\frac{\operatorname{atanh}\left(\frac{b^3\,d^4\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{16\,\left(\frac{a\,b^3\,d^5}{16}-\frac{b^4\,c\,d^4}{16}\right)}\right)\,\sqrt{b^2\,c-a\,b\,d}}{2\,a^2}-\frac{\sqrt{d\,x^4+c}}{4\,a\,x^4}-\frac{\operatorname{atanh}\left(\frac{b^4\,\sqrt{c}\,d^4\,\sqrt{d\,x^4+c}}{16\,\left(\frac{b^4\,c\,d^4}{16}-\frac{3\,a\,b^3\,d^5}{32}+\frac{a^2\,b^2\,d^6}{32\,c}\right)}-\frac{3\,b^3\,d^5\,\sqrt{d\,x^4+c}}{32\,\sqrt{c}\,\left(\frac{a\,b^2\,d^6}{32\,c}-\frac{3\,b^3\,d^5}{32}+\frac{b^4\,c\,d^4}{16\,a}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^4+c}}{32\,c^{3/2}\,\left(\frac{b^2\,d^6}{32\,c}-\frac{3\,b^3\,d^5}{32\,a}+\frac{b^4\,c\,d^4}{16\,a^2}\right)}\right)\,\left(a\,d-2\,b\,c\right)}{4\,a^2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^5*(a + b*x^4)),x)
```

```
[Out] (atanh((b^3*d^4*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(16*((a*b^3*d^5)/1
6 - (b^4*c*d^4)/16)))*(b^2*c - a*b*d)^(1/2))/(2*a^2) - (c + d*x^4)^(1/2)/(4
*a*x^4) - (atanh((b^4*c^(1/2)*d^4*(c + d*x^4)^(1/2))/(16*((b^4*c*d^4)/16 -
(3*a*b^3*d^5)/32 + (a^2*b^2*d^6)/(32*c))) - (3*b^3*d^5*(c + d*x^4)^(1/2))/(
32*c^(1/2)*((a*b^2*d^6)/(32*c) - (3*b^3*d^5)/32 + (b^4*c*d^4)/(16*a))) + (b
^2*d^6*(c + d*x^4)^(1/2))/(32*c^(3/2)*((b^2*d^6)/(32*c) - (3*b^3*d^5)/(32*a
) + (b^4*c*d^4)/(16*a^2))))*(a*d - 2*b*c))/(4*a^2*c^(1/2))
```

3.794 $\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2}+\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{5/2}}$$

```
[Out] 1/2*b*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2)
/a^(5/2)-1/6*(d*x^4+c)^(1/2)/a/x^6+1/6*(-a*d+3*b*c)*(d*x^4+c)^(1/2)/a^2/c/x
^2
```

Rubi [A]
time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 486, 597, 12, 385, 211}

$$\frac{b\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{5/2}}+\frac{\sqrt{c+dx^4}\,(3bc-ad)}{6a^2cx^2}-\frac{\sqrt{c+dx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] -1/6*Sqrt[c + d*x^4]/(a*x^6) + ((3*b*c - a*d)*Sqrt[c + d*x^4])/(6*a^2*c*x^2
) + (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4
])])/(2*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{\text{Subst}\left(\int \frac{-3bc+ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} - \frac{\text{Subst}\left(\int -\frac{3bc(bc-ad)}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{6a^2c}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x\right.}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+}}\right.}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}}$$

Mathematica [**A**]
time = 0.42, size = 118, normalized size = 1.07

$$\frac{\sqrt{c+dx^4}\left(3bcx^4-a(c+dx^4)\right)}{6a^2cx^6} + \frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}\,x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] (Sqrt[c + d*x^4]*(3*b*c*x^4 - a*(c + d*x^4)))/(6*a^2*c*x^6) + (b*Sqrt[b*c -
 a*d]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*S
qrt[b*c - a*d])])/(2*a^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1092 vs. $2(90) = 180$.
time = 0.36, size = 1093, normalized size = 9.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/x^6*(d*x^4+c)^(3/2)/c-b/a^2*(-1/2/c/x^2*(d*x^4+c)^(3/2)+1/2*d/c*x^2*
(d*x^4+c)^(1/2)+1/2*d^(1/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2)))+b^2/a^2*(1/4/(
```

```
-a*b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)+1/4*d^(1/2)/b*ln((d*(-a*b)^(1/2)/b+(x^2-1/b*(-a*b
)^(1/2))*d)/d^(1/2)+((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*a*d-1/4/(-a*b)^(1/2)/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))*c-1/4/(-a*b)
^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)+1/4*d^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+(x^2+1/b*(-a*b)^(1
/2))*d)/d^(1/2)+((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*a*d+1/4/(-a*b)^(1/2)/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^7), x)
```

Fricas [A]
time = 2.69, size = 329, normalized size = 2.99

$$\left[\frac{3\,bcx^{6}\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{8}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{4}+a^{2}c^{2}+4\,((abc-2\,a^{2}d)x^{6}-a^{2}cx^{2})\sqrt{dx^{4}+c}\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{8}+2\,abx^{4}+a^{2}}\right)+4\,((3\,bc-ad)x^{4}-ac)\sqrt{dx^{4}+c}}{24\,a^{2}cx^{6}},\ \frac{3\,bcx^{6}\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^{4}-ac)\sqrt{dx^{4}+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^{2})x^{6}+(bc^{2}-acd)x^{2})}\right)+2\,((3\,bc-ad)x^{4}-ac)\sqrt{dx^{4}+c}}{12\,a^{2}cx^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/24*(3*b*c*x^6*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6
- a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a
^2)) + 4*((3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6), 1/12*(3*b*
c*x^6*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
```

```
c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)) + 2*((
3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**7/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**7*(a + b*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(90) = 180.
time = 2.67, size = 225, normalized size = 2.05

$$-\frac{\left(b^2c\sqrt{d}-abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,a^2}-\frac{3\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4bc\sqrt{d}-3\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc^2\sqrt{d}+3\,bc^3\sqrt{d}-ac^2d^{\frac{3}{2}}}{3\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2-c\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 +
c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^
2) - 1/3*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c*sqrt(d) - 3*(sqrt(d)*x^2
- sqrt(d*x^4 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^2*
sqrt(d) + 3*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2 - c)^3*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^4+c}}{x^7\,\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)),x)
```

```
[Out] int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)), x)
```

3.795 $\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=857

$$\frac{x^3\sqrt{c+dx^4}}{5b}+\frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b^2}-\frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}}{4}$$

```
[Out] 1/5*x^3*(d*x^4+c)^(1/2)/b+1/5*(-5*a*d+2*b*c)*x*(d*x^4+c)^(1/2)/b^2/d^(1/2)/
(c^(1/2)+x^2*d^(1/2))-1/4*a*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(
d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b^2-1/4*a*arctan(x*((-
a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/
b^(1/2))^(1/2)/b^2-1/5*c^(1/4)*(-5*a*d+2*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/
4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/
4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d
^(1/2))^2)^(1/2)/b^2/d^(3/4)/(d*x^4+c)^(1/2)+1/5*c^(1/4)*(-5*a^2*d^2+a*b*c*
d+b^2*c^2)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2
)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/d^(3/4)/(a*d+b
*c)/(d*x^4+c)^(1/2)+1/8*a*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(
1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(
1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^
(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(5/2)/c^(1/4)/d^(1/4)/((
-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)-1/8*a*(-a*d+b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^
(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/
2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2
)^(1/2)/b^(5/2)/c^(1/4)/d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x
^4+c)^(1/2)
```

Rubi [A]
time = 1.33, antiderivative size = 1067, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {489, 598, 311, 226, 1210, 504, 1231, 1721}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^3*Sqrt[c + d*x^4])/(5*b) + ((2*b*c - 5*a*d)*x*Sqrt[c + d*x^4])/(5*b^2*Sq
rt[d]*(Sqrt[c] + Sqrt[d]*x^2)) + ((-a)^(3/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b
*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(9/4)) + ((-a)^(3/
4)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqr
t[c + d*x^4])])/(4*b^(9/4)) - (c^(1/4)*(2*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(5*b^2*d^(3/4)*Sqrt[c + d*x^4]) + (c^(1/4)*(2*b*c - 5*a
*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(10*b^2*d^(3/4)*Sqrt[c + d*x^4]
) + (a*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
+ (a*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) +
(Sqrt[-a]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt
[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*Arc
Tan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(5/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)
*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipt
icPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqr
t[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(5/2)*c^(1/4)*d^(1/4)*(b*c
 + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
```

```
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x^3\sqrt{c+dx^4}}{5b} - \frac{\int \frac{x^2(3ac+(-2bc+5ad)x^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{5b}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} - \frac{\int \left(-\frac{(2bc-5ad)x^2}{b\sqrt{c+dx^4}} - \frac{5(-abc+a^2d)x^2}{b(a+bx^4)\sqrt{c+dx^4}}\right)dx}{5b}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{5b^2} - \frac{(a(bc-ad))\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b^2}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{\left(\sqrt{c}\,(2bc-5ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{5b^2\sqrt{d}} - \frac{\left(\sqrt{c}\,(2bc-5ad)\right)\int \frac{1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)} - \frac{\sqrt[4]{c}\,(2bc-5ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c}{\left(\sqrt{c}\right.}}}{5b^2d^{3/4}\sqrt{c}}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{(-a)^{3/4}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+}}\right.}{4b^{9/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.10, size = 141, normalized size = 0.16

$$\frac{7ax^3(c+dx^4) - 7acx^3\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right) + (2bc-5ad)x^7\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{35ab\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (7*a*x^3*(c + d*x^4) - 7*a*c*x^3*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4, 1/2, 1,
7/4, -((d*x^4)/c), -((b*x^4)/a)] + (2*b*c - 5*a*d)*x^7*Sqrt[1 + (d*x^4)/c]*
AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(35*a*b*Sqrt[c + d
*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 421, normalized size = 0.49

method	result
risch	$\frac{x^{3}\sqrt{d\,x^{4}+c}}{5b}-\frac{\frac{i(5ad-2bc)\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}\,\sqrt{d}}}{}$
elliptic	$\frac{x^{3}\sqrt{d\,x^{4}+c}}{5b}+\frac{i\left(-\frac{ad-bc}{b^{2}}-\frac{3c}{5b}\right)\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i}{}}\right.\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}\,\sqrt{d}}$

default	$\frac{\frac{x^3\sqrt{d\,x^4+c}}{5}+\frac{2ic^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{5\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,\sqrt{d}}}{b} - \frac{a\left(\frac{i\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/5*x^3*(d*x^4+c)^(1/2)+2/5*I*c^(3/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c
^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)/d
^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(
1/2))^(1/2),I)))-a/b*(I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^
(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(E
llipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/
2),I))-1/8/b^2*sum((a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2
*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(
1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/
2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1
/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/
2))),_alpha=RootOf(_Z^4*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral(x**6*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int((x^6*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.796 $\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=700

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+c^{3/4}(bc-2ad$$

```
[Out] 1/3*x*(d*x^4+c)^(1/2)/b-1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/
2))^(1/2)/(d*x^4+c)^(1/2))/b^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*(-a
*d+b*c)*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/b^2
/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/3*c^(3/4)*(-2*a*d+b*c)*(cos(2*arct
an(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(
sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4
+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*
(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2
)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^
(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)
+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1
/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1
/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^
(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(b^(1/
2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A]

time = 0.92, antiderivative size = 904, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {489, 537, 226, 418, 1231, 1721}

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x*Sqrt[c + d*x^4])/(3*b) - ((-a)^(1/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(7/4)) + ((-a)^(1/4)*Sq
```

```
rt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c +
 d*x^4])])/(4*b^(7/4)) + ((2*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(6*b^2*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4]) - (a*((Sqrt[b]*Sqrt[c])/Sqrt
[-a] + Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4
)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] -
 Sqrt[-a]*Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*
x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/
2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[
-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/
(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8
*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqr
t[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt
[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*
Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*
b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x\sqrt{c+dx^4}}{3b} - \frac{\int \frac{ac+(-2bc+3ad)x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{3b}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3b^2} - \frac{(a(bc-ad))\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b^2}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)\right.}{6b^2\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)\right.}{6b^2\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} - \frac{\sqrt[4]{-a}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{7/4}} + \frac{\sqrt[4]{-a}\,\sqrt{-bc+ad}\,\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.32, size = 241, normalized size = 0.34

$$\frac{x\left(\frac{(2bc-3ad)x^4\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a}+5\left(c+dx^4+\frac{5a^2c^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}\right)\right)}{15b\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x*(((2*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/a + 5*(c + d*x^4 + (5*a^2*c^2*AppellF1[1/4, 1/2,
1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2
, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2,
9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)
/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 368, normalized size = 0.53

method	result
risch	$\frac{x\sqrt{d\,x^4+c}}{3b}-\frac{\frac{(3ad-2bc)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-3a(ad-bc)\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\right.}{}$

elliptic	$\frac{x\sqrt{d\,x^4+c}}{3b}+\frac{\left(-\frac{ad-bc}{b^2}-\frac{c}{3b}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{a\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4b-\right.}\right.}{}$
default	$\frac{\frac{x\sqrt{d\,x^4+c}}{3}+\frac{2c\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{b}-\frac{a\left(\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{}}{}}}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3*x*(d*x^4+c)^(1/2)+2/3*c/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(
1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x
```

```
*(I/c^(1/2)*d^(1/2))^(1/2),I))-a/b*(d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1
/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Elli
pticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/((-
a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(
d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/
2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*
(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2
))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral(x**4*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int((x^4*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.797 $\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=786

$$\frac{\sqrt{d}\,x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}\,x^2\right)}+\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b}+\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b}-$$

```
[Out] x*d^(1/2)*(d*x^4+c)^(1/2)/b/(c^(1/2)+x^2*d^(1/2))+1/4*arctan(x*((a*d-b*c)/(
-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1
/2)/b+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*(
(-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b-c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(2*arc
tan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1
/2)+x^2*d^(1/2))^2)^(1/2)/b/(d*x^4+c)^(1/2)+a*c^(1/4)*d^(5/4)*(cos(2*arctan
(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(si
n(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c
)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/
2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*
d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/
2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))
/(d*x^4+c)^(1/2)+1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))
),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^
(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2
))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/((-a)^
(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A]

time = 1.02, antiderivative size = 1012, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {505, 311, 226, 1210, 504, 1231, 1721}

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^4])/(b*(Sqrt[c] + Sqrt[d]*x^2)) + (Sqrt[b*c - a*d]*
ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(
1/4)*b^(5/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(5/4)) - (c^(1/4)*d^(1/4)*(Sq
rt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*Sqrt[c + d*x^4]) + (c^(1/4)*d^(1/4)
*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipt
icF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*Sqrt[c + d*x^4]) - ((Sqrt[c]
- (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*S
qrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c
^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqr
t[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)
], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sq
rt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])
^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[
b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(
c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a
]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(
1/4)], 1/2])/(8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4
])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 505

```
Int[((x_)^2*Sqrt[(c_) + (d_.)*(x_)^4])/((a_) + (b_.)*(x_)^4), x_Symbol] :>
```

```
Dist[d/b, Int[x^2/Sqrt[c + d*x^4], x], x] + Dist[(b*c - a*d)/b, Int[x^2/((a
 + b*x^4)*Sqrt[c + d*x^4]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a
*d, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{b} + \frac{(bc-ad)\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{b} - \frac{(bc-ad)\int \frac{}{\left(\sqrt{-a}-\sqrt{b}\right.}}{2b^{3/2}}$$

$$= \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}\,x^2\right)} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{b\sqrt{c+dx^4}}$$

$$= \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\,b^{5/4}} + \frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\right.}{4\sqrt[4]{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.03, size = 65, normalized size = 0.08

$$\frac{x^3\sqrt{c+dx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^3*Sqrt[c + d*x^4]*AppellF1[3/4, -1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)
])/(3*a*Sqrt[(c + d*x^4)/c])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 299, normalized size = 0.38

method	result

default	$\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{\sum_{\_\alpha=\text{RootOf}(\_}}{}$
elliptic	$\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{\sum_{\_\alpha=\text{RootOf}(\_}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/
2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*
d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8/b^2*sum((a*
d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/(
(-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b
/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c
)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a
*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^
4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral(x**2*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int((x^2*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.798 $\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=679

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4ab\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4ab\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+\frac{c^{3/4}d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\cdots}}{(bc\cdots}$$

```
[Out] 1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2
))/a/b/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/4*(-a*d+b*c)*arctan(x*((-a*d+
b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a/b/((-a*d+b*c)/(-a)^(1/2)/
b^(1/2))^(1/2)+c^(3/4)*d^(3/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),
1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2
)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4))
)^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)
*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/
c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1
/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/d^(1/4)/
(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)+1/8*(-a*d+b*c)*(cos(2*
arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ellipt
icPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1
/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2)
)*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^
(1/2)/a/b/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1
/2)
```

Rubi [A]

time = 1.02, antiderivative size = 881, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {415, 226, 418, 1231, 1721}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

```
[Out] -1/4*(Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c
 + d*x^4])])/((-a)^(3/4)*b^(3/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c)
```

```
 + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(3/4)*b^(3/4)) +
(d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*Sqrt[c + d*x^
4]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) -
((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d
]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2])/(4*Sqrt[-a]*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) +
 ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2
)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(
1/4)*x)/c^(1/4)], 1/2])/(8*a*b*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
 + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c]
 + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2])/(8*a*b*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 415

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{(-bc+ad)\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\,\sqrt{c+dx^4}} - \frac{(-bc+ad)\int \frac{}{\left(1-\right.}}{2}$$

$$= \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\,\sqrt{c+dx^4}} + \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\right.\right.}{}$$

$$= -\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}b^{3/4}} + \frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}b^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 161, normalized size = 0.24

$$\frac{5acx\sqrt{c+dx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(5acF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(-2bcF_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

```
[Out] (5*a*c*x*Sqrt[c + d*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4
)/a)])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b*x
^4)/a)] + 2*x^4*(-2*b*c*AppellF1[5/4, -1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)
/a)] + a*d*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 273, normalized size = 0.40

method	result
default	$\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)} \frac{(ad-bc)\left(-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\cdots}{2\sqrt{\frac{-ad+bc}{b}}\cdots}\right)}{\sqrt{\frac{-ad+\cdots}{b}}}\cdots\right)}{\cdots}}{\cdots}$
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)} \frac{(ad-bc)\left(-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\cdots}{2\sqrt{\frac{-ad+bc}{b}}\cdots}\right)}{\sqrt{\frac{-ad+\cdots}{b}}}\cdots\right)}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*
d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)
-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_al
pha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2)
)^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x
^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/
d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)))
,_alpha=RootOf(_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/(b*x^4 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{a + bx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)/(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^4+c}}{b\,x^4+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(a + b*x^4),x)
```

```
[Out] int((c + d*x^4)^(1/2)/(a + b*x^4), x)
```

3.799 $\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx$

Optimal. Leaf size=809

$$-\frac{\sqrt{c+dx^4}}{ax}+\frac{\sqrt{d}\,x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a}-\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,\tan^{-1}\left(\ldots\right.}{4a}$$

```
[Out] -(d*x^4+c)^(1/2)/a/x+x*d^(1/2)*(d*x^4+c)^(1/2)/a/(c^(1/2)+x^2*d^(1/2))-1/4*
arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)/(
-a)^(1/2)/b^(1/2))^(1/2)/a-1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/
2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/a-c^(1/4)*d^(1/4)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1
/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(d*x^4+c)^(1/2)+b*c^(5/4)*
d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c
^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x
^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(a*d+b*c)/(d*x^4+c)
^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arct
an(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(
1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2
^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+
c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/b^(1/2)/((-a)^(3/2)*b^(1/
2)*c^(1/2)+a^2*d^(1/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*ar
ctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^
2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^
(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/
4)/d^(1/4)/b^(1/2)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 1.15, antiderivative size = 1031, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {486, 598, 311, 226, 1210, 504, 1231, 1721}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
```

```
[Out] -(Sqrt[c + d*x^4]/(a*x)) + (Sqrt[d]*x*Sqrt[c + d*x^4])/(a*(Sqrt[c] + Sqrt[d
]*x^2)) + (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*S
qrt[c + d*x^4])])/(4*(-a)^(5/4)*b^(1/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt
[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(5/4)*b^(1
/4)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(a*Sqrt[c +
 d*x^4]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*a*Sq
rt[c + d*x^4]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c +
 d*x^4]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4
]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*
Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[
(d^(1/4)*x)/c^(1/4)], 1/2])/(8*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c + a*
d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
Pi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[
d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx = -\frac{\sqrt{c+dx^4}}{ax} + \frac{\int \frac{x^2\left(-bc+2ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}} + \frac{(-bc+ad)x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{a} + \frac{(-bc+ad)\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\left(\sqrt{c}\,\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{a} - \frac{\left(\sqrt{c}\,\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{a} + \frac{(bc-ad)\ \ldots}{\ldots}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}\,x^2\right)} - \frac{\sqrt[4]{c}\,\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,E\Big(\ldots}{a\sqrt{c+dx^4}\ \ldots}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt[4]{b}} + \frac{\sqrt{-bc\ldots}}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.07, size = 138, normalized size = 0.17

$$\frac{-21a\left(c+dx^4\right)-7(bc-2ad)x^4\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+3bdx^8\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{21a^2x\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
```

```
[Out] (-21*a*(c + d*x^4) - 7*(b*c - 2*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4,
1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]*Ap
pellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*x*Sqrt[c + d
*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 421, normalized size = 0.52

method	result
risch	$-\frac{\sqrt{d\,x^{4}+c}}{ax}+\frac{\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}}{}$
elliptic	$-\frac{\sqrt{d\,x^{4}+c}}{ax}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\right.\right.}{a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}$

default	$-\frac{b\left(\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\text{RootOf}(\_Z^{\cdots}}}{\cdots}\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a*(I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^
2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^
(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8/b^2*s
um((a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+
2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alp
ha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d
*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alp
ha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Root
Of(_Z^4*b+a)))+1/a*(-(d*x^4+c)^(1/2)/x+2*I*d^(1/2)*c^(1/2)/(I/c^(1/2)*d^(1/
2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(
d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(
1/2)*d^(1/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{x^2 (a + bx^4)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**2/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(x**2*(a + b*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^4+c}}{x^2\,\left(b\,x^4+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^2*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/(x^2*(a + b*x^4)), x)
```

3.800 $\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx$

Optimal. Leaf size=703

$$-\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{d^{3/4}(2bc-ad)}{\cdots}$$

```
[Out] -1/3*(d*x^4+c)^(1/2)/a/x^3-1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^
(1/2))^(1/2)/(d*x^4+c)^(1/2))/a^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*
(-a*d+b*c)*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/
a^2/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/3*d^(3/4)*(-a*d+2*b*c)*(cos(2*a
rctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ellipti
cF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*
x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1
/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/
4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(
1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*
(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1
/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d
^(1/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)
^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c
^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(
1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)
*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.91, antiderivative size = 893, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {486, 537, 226, 418, 1231, 1721}

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

```
[Out] -1/3*Sqrt[c + d*x^4]/(a*x^3) - (b^(1/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(7/4)) + (b^(1/4)*Sq
```

```
rt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c +
 d*x^4])])/(4*(-a)^(7/4)) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(6*a*c^(1/4)*Sqrt[c + d*x^4]) - (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d
^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt
[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*
c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(
b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(-a)^(3/2)*c^(1/4)*(b
*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c -
 a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*E
llipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*
(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c
 - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(
b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx = -\frac{\sqrt{c+dx^4}}{3ax^3}+\frac{\int \frac{-3bc+2ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3a}-\frac{(bc-ad)\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6a\sqrt[4]{c}\,\sqrt{c+dx^4}} - \cdots (b$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6a\sqrt[4]{c}\,\sqrt{c+dx^4}} - \cdots ($$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{\sqrt[4]{b}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}}+\frac{\sqrt[4]{b}\,\sqrt{-bc+ad}\,\tan^{-1}}{4(-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.19, size = 333, normalized size = 0.47

$$\frac{-bdx^8\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\frac{a\left(25ac\left(ac+4bcx^4-adx^4+bdx^8\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-10x^4\left(a+bx^4\right)\left(c+dx^4\right)\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}{\left(a+bx^4\right)\left(-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}}{15a^2x^3\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

```
[Out] (-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]) + (a*(25*a*c*(a*c + 4*b*c*x^4 - a*d*x^4 + b*d*x^8)*AppellF1[1/4
, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 10*x^4*(a + b*x^4)*(c + d*x^4)
*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*Appell
F1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(-5*a*c*Ap
pellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF
1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1,
 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*x^3*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 370, normalized size = 0.53

method	result
risch	$-\frac{\sqrt{d\,x^4+c}}{3a\,x^3}-\frac{\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}}+(-3ad+3bc)\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\right.}{}$

elliptic	$-\frac{\sqrt{d\,x^4+c}}{3a\,x^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}(-ad+bc)\left(\ldots\right.}{\ldots}$
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{3x^3}+\frac{2d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{a}-\frac{b\left(\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}}{\sqrt{c}}\ldots}}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\ldots}}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/3*(d*x^4+c)^(1/2)/x^3+2/3*d/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*
d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Elliptic
F(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-b/a*(d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c
^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*E
llipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/
((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2
)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^
(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi
(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(
```

```
1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^4), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^4}}{x^4\,(a+bx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**4/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(x**4*(a + b*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^4+c}}{x^4\,\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^4*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/(x^4*(a + b*x^4)), x)
```

3.801 $\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=71

$$\frac{2(ex)^{5/2}\sqrt{c+dx^4}\,F_1\left(\frac{5}{8};1,-\frac{1}{2};\frac{13}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{1+\frac{dx^4}{c}}}$$

```
[Out] 2/5*(e*x)^(5/2)*AppellF1(5/8,1,-1/2,13/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2)
/a/e/(1+d*x^4/c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {477, 525, 524}

$$\frac{2(ex)^{5/2}\sqrt{c+dx^4}\,F_1\left(\frac{5}{8};1,-\frac{1}{2};\frac{13}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (2*(e*x)^(5/2)*Sqrt[c + d*x^4]*AppellF1[5/8, 1, -1/2, 13/8, -((b*x^4)/a), -
((d*x^4)/c)])/(5*a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{2\mathrm{Subst}\left(\int \frac{x^4\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\left(2\sqrt{c+dx^4}\right)\mathrm{Subst}\left(\int \frac{x^4\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}$$

$$= \frac{2(ex)^{5/2}\sqrt{c+dx^4}\,F_1\left(\frac{5}{8};1,-\frac{1}{2};\frac{13}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{1+\frac{dx^4}{c}}}$$

Mathematica [A]
time = 10.05, size = 70, normalized size = 0.99

$$\frac{2x(ex)^{3/2}\sqrt{c+dx^4}\,F_1\left(\frac{5}{8};-\frac{1}{2},1;\frac{13}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{\frac{c+dx^4}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
[Out] (2*x*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[5/8, -1/2, 1, 13/8, -((d*x^4)/c),
 -((b*x^4)/a)])/(5*a*Sqrt[(c + d*x^4)/c])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}}\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(sqrt(d*x^4 + c)*x^(3/2)/(b*x^4 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*x^(3/2)*e^(3/2)/(b*x^4 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}}\sqrt{c + dx^4}}{a + bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral((e*x)**(3/2)*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^(3/2)*e^(3/2)/(b*x^4 + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{3/2}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int(((e*x)^(3/2)*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.802 $\int \frac{\sqrt{ex}\,\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=71

$$\frac{2(ex)^{3/2}\sqrt{c+dx^4}\,F_1\left(\frac{3}{8};1,-\frac{1}{2};\frac{11}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{1+\frac{dx^4}{c}}}$$

```
[Out] 2/3*(e*x)^(3/2)*AppellF1(3/8,1,-1/2,11/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2)
/a/e/(1+d*x^4/c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {477, 525, 524}

$$\frac{2(ex)^{3/2}\sqrt{c+dx^4}\,F_1\left(\frac{3}{8};1,-\frac{1}{2};\frac{11}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (2*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[3/8, 1, -1/2, 11/8, -((b*x^4)/a), -
((d*x^4)/c)])/(3*a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ex}\,\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{2\text{Subst}\left(\int \frac{x^2\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e}\\ &= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{x^2\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}\\ &= \frac{2(ex)^{3/2}\sqrt{c+dx^4}\,F_1\left(\frac{3}{8};1,-\frac{1}{2};\frac{11}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{1+\frac{dx^4}{c}}}\end{aligned}$$

Mathematica [A]
time = 10.03, size = 70, normalized size = 0.99

$$\frac{2x\sqrt{ex}\,\sqrt{c+dx^4}\,F_1\left(\frac{3}{8};-\frac{1}{2},1;\frac{11}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (2*x*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[3/8, -1/2, 1, 11/8, -((d*x^4)/c), -
((b*x^4)/a)])/(3*a*Sqrt[(c + d*x^4)/c])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(d*x^4 + c)*sqrt(x)/(b*x^4 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(e*x)*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)*sqrt(x)*e^(1/2)/(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,x}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int(((e*x)^(1/2)*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.803 $\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx$

Optimal. Leaf size=69

$$\frac{2\sqrt{ex}\,\sqrt{c+dx^4}\,F_1\left(\frac{1}{8};1,-\frac{1}{2};\frac{9}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{1+\frac{dx^4}{c}}}$$

```
[Out] 2*AppellF1(1/8,1,-1/2,9/8,-b*x^4/a,-d*x^4/c)*(e*x)^(1/2)*(d*x^4+c)^(1/2)/a/
e/(1+d*x^4/c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {477, 441, 440}

$$\frac{2\sqrt{ex}\,\sqrt{c+dx^4}\,F_1\left(\frac{1}{8};1,-\frac{1}{2};\frac{9}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
```

```
[Out] (2*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[1/8, 1, -1/2, 9/8, -((b*x^4)/a), -((d
*x^4)/c)])/(a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
```

```
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \frac{2\text{Subst}\left(\int \frac{\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}$$

$$= \frac{2\sqrt{ex}\,\sqrt{c+dx^4}\,F_1\left(\frac{1}{8};1,-\frac{1}{2};\frac{9}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{1+\frac{dx^4}{c}}}$$

Mathematica [A]
time = 10.05, size = 68, normalized size = 0.99

$$\frac{2x\sqrt{c+dx^4}\,F_1\left(\frac{1}{8};-\frac{1}{2},1;\frac{9}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a\sqrt{ex}\,\sqrt{\frac{c+dx^4}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
[Out] (2*x*Sqrt[c + d*x^4]*AppellF1[1/8, -1/2, 1, 9/8, -((d*x^4)/c), -((b*x^4)/a)
])/(a*Sqrt[e*x]*Sqrt[(c + d*x^4)/c])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,x^4+c}}{\sqrt{ex}\,\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
[Out] int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] e^(-1/2)*integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{\sqrt{ex}\,(a + bx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/(e*x)**(1/2)/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(sqrt(e*x)*(a + b*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*e^(-1/2)/((b*x^4 + a)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^4+c}}{\sqrt{e\,x}\,\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/((e*x)^(1/2)*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/((e*x)^(1/2)*(a + b*x^4)), x)
```

3.804 $\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)}\,dx$

Optimal. Leaf size=69

$$-\frac{2\sqrt{c+dx^4}\,F_1\left(-\frac{1}{8};1,-\frac{1}{2};\frac{7}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{1+\frac{dx^4}{c}}}$$

```
[Out] -2*AppellF1(-1/8,1,-1/2,7/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2)/a/e/(e*x)^(1
/2)/(1+d*x^4/c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {477, 525, 524}

$$-\frac{2\sqrt{c+dx^4}\,F_1\left(-\frac{1}{8};1,-\frac{1}{2};\frac{7}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
```

```
[Out] (-2*Sqrt[c + d*x^4]*AppellF1[-1/8, 1, -1/2, 7/8, -((b*x^4)/a), -((d*x^4)/c)
])/(a*e*Sqrt[e*x]*Sqrt[1 + (d*x^4)/c])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)}\,dx = \frac{2\text{Subst}\left(\int \frac{\sqrt{c+\frac{dx^8}{e^4}}}{x^2\left(a+\frac{bx^8}{e^4}\right)}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{dx^8}{ce^4}}}{x^2\left(a+\frac{bx^8}{e^4}\right)}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}$$

$$= -\frac{2\sqrt{c+dx^4}\,F_1\left(-\frac{1}{8};1,-\frac{1}{2};\frac{7}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{1+\frac{dx^4}{c}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(69) = 138.

time = 10.09, size = 143, normalized size = 2.07

$$\frac{x\left(-70a(c+dx^4)-10(bc-4ad)x^4\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{7}{8};\frac{1}{2},1;\frac{15}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+14bdx^8\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{15}{8};\frac{1}{2},1;\frac{23}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}{35a^2(ex)^{3/2}\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
[Out] (x*(-70*a*(c + d*x^4) - 10*(b*c - 4*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[7
/8, 1/2, 1, 15/8, -((d*x^4)/c), -((b*x^4)/a)] + 14*b*d*x^8*Sqrt[1 + (d*x^4)
/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^4)/c), -((b*x^4)/a)]))/(35*a^2*(e*x
)^(3/2)*Sqrt[c + d*x^4])
```

Maple [F]

time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,x^4+c}}{(ex)^{\frac{3}{2}}\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

```
[Out] int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*sqrt(x)*e^(-3/2)/(b*x^6 + a*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{(ex)^{\frac{3}{2}}\,(a + bx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/(e*x)**(3/2)/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/((e*x)**(3/2)*(a + b*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)*e^(-3/2)/((b*x^4 + a)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^4+c}}{{\left(e\,x\right)}^{3/2}\,\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/((e*x)^(3/2)*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/((e*x)^(3/2)*(a + b*x^4)), x)
```

3.805 $\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=104

$$-\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{(c+dx^4)^{3/2}}{6bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/6*(d*x^4+c)^(3/2)/b/d^2-1/2*a^2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c
)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/2*(a*d+b*c)*(d*x^4+c)^(1/2)/b^2/d^2
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 90, 65, 214}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}\,(ad+bc)}{2b^2d^2}+\frac{(c+dx^4)^{3/2}}{6bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*((b*c + a*d)*Sqrt[c + d*x^4])/(b^2*d^2) + (c + d*x^4)^(3/2)/(6*b*d^2)
- (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[
b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx, x, x^4\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{(c+dx^4)^{3/2}}{6bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{(c+dx^4)^{3/2}}{6bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{(c+dx^4)^{3/2}}{6bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 91, normalized size = 0.88

$$\frac{\sqrt{c+dx^4}\,(-2bc-3ad+bdx^4)}{6b^2d^2}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2b^{5/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-2*b*c - 3*a*d + b*d*x^4))/(6*b^2*d^2) + (a^2*ArcTan[(Sqr
t[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(2*b^(5/2)*Sqrt[-(b*c) + a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(84) = 168.
time = 0.37, size = 369, normalized size = 3.55

method	result
risch	$-\frac{(-bd\,x^4+3ad+2bc)\sqrt{d\,x^4+c}}{6d^2b^2}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)\cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)\cdots}{4b^3\sqrt{-\frac{ad-bc}{b}}\cdots}$
default	$-\frac{\sqrt{d\,x^4+c}\,(-d\,x^4+2c)}{6b\,d^2}-\frac{a\sqrt{d\,x^4+c}}{2b^2d}+\frac{a^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2\cdots\right.}}{x^2-\sqrt{\cdots}}\right)}{4b\sqrt{-\cdots}}\cdots\right)}{\cdots}$
elliptic	$\frac{x^4\sqrt{d\,x^4+c}}{6bd}-\frac{c\sqrt{d\,x^4+c}}{3b\,d^2}-\frac{a\sqrt{d\,x^4+c}}{2b^2d}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\cdots}}{x^2\cdots}\right)}{4b^3\sqrt{\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b*(d*x^4+c)^(1/2)*(-d*x^4+2*c)/d^2-1/2*a/b^2/d*(d*x^4+c)^(1/2)+a^2/b^2
*(-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)
))-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)
)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: ValueError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 4.33, size = 289, normalized size = 2.78

$$\left[\frac{3\,\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)-2\,(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2d^2)x^4)\sqrt{dx^4+c}}{12\,(b^4cd^2-ab^3d^3)}, \frac{3\,\sqrt{-b^2c+abd}\,a^2d^2\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2d^2)x^4)\sqrt{dx^4+c}}{6\,(b^4cd^2-ab^3d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*
x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a
^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^4 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*
d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a+bx^4)\,\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A]
time = 1.87, size = 106, normalized size = 1.02

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,b^2}+\frac{(dx^4+c)^{\frac{3}{2}}b^2d^4-3\,\sqrt{dx^4+c}\,b^2cd^4-3\,\sqrt{dx^4+c}\,abd^5}{6\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^4 + c)*b^2*c*d^4 - 3*s
qrt(d*x^4 + c)*a*b*d^5)/(b^3*d^6)
```

Mupad [B]
time = 4.82, size = 102, normalized size = 0.98

$$\frac{{\left(d\,x^4+c\right)}^{3/2}}{6\,b\,d^2}-\left(\frac{c}{b\,d^2}+\frac{2\,a\,d^3-2\,b\,c\,d^2}{4\,b^2\,d^4}\right)\,\sqrt{d\,x^4+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b*d^2) - (c/(b*d^2) + (2*a*d^3 - 2*b*c*d^2)/(4*b^2*d^4
))*(c + d*x^4)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1
/2)))/(2*b^(5/2)*(a*d - b*c)^(1/2))
```

3.806 $\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{c+dx^4}}{2bd}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}$$

```
[Out] 1/2*a*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+1/2*(d*x^4+c)^(1/2)/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 81, 65, 214}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a
*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)\\ &= \frac{\sqrt{c+dx^4}}{2bd} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b}\\ &= \frac{\sqrt{c+dx^4}}{2bd} - \frac{a\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2bd}\\ &= \frac{\sqrt{c+dx^4}}{2bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 73, normalized size = 0.99

$$\frac{1}{2}\left(\frac{\sqrt{c+dx^4}}{bd} - \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) +
a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(58) = 116.
time = 0.36, size = 340, normalized size = 4.59

method	result

risch	$\frac{\sqrt{d\,x^4+c}}{2bd}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\,(}{}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b^2\sqrt{-\frac{ad-bc}{b}}}$
elliptic	$\frac{\sqrt{d\,x^4+c}}{2bd}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\,(}{}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b^2\sqrt{-\frac{ad-bc}{b}}}$
default	$\frac{\sqrt{d\,x^4+c}}{2bd}-\frac{a\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-a}}{}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x^4+c)^(1/2)/b/d-a/b*(-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b
*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x^2-1/b*(-a*b)^(1/2)))-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b
*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x^2+1/b*(-a*b)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.64, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}\,(b^2c-abd)}{4\,(b^3cd-ab^2d^2)},\ -\frac{\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-\sqrt{dx^4+c}\,(b^2c-abd)}{2\,(b^3cd-ab^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c
)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/2*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^4 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a+bx^4)\,\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**7/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A]
time = 1.87, size = 64, normalized size = 0.86

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^4+c}}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^4 + c)/b)/d
```

Mupad [B]
time = 4.73, size = 58, normalized size = 0.78

$$\frac{\sqrt{d\,x^4+c}}{2\,b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b*d) - (a*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)
^(1/2)))/(2*b^(3/2)*(a*d - b*c)^(1/2))
```

3.807 $\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,\sqrt{bc-ad}}$$

```
[Out] -1/2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {455, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2d}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.04, size = 51, normalized size = 1.00

$$
\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2\sqrt{b}\sqrt{-bc+ad}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]]/(2*Sqrt[b]*Sqrt[-(b*c)
 + a*d])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(39) = 78$.
time = 0.33, size = 316, normalized size = 6.20

method	result
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}$

elliptic	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{4b\sqrt{-\frac{ad-bc}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2))) -1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more detail

Fricas [A]
time = 3.26, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)}{4\,\sqrt{b^2c-abd}},\ \frac{\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)}{2\,(b^2c-abd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [1/4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a))/sqrt(b^2*c - a*b*d), 1/2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c))/(b^2*c - a*b*d)]

Sympy [A]
time = 5.49, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*b*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 1.34, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] 1/2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B]
time = 4.80, size = 40, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{2\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] atan((b*(c + d*x^4)^(1/2))/(a*b*d - b^2*c)^(1/2))/(2*(a*b*d - b^2*c)^(1/2))
```

3.808 $\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}$$

```
[Out] -1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/2*arctanh(b^(1/2)*(d*x^4+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 88, 65, 214}

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^4\right)}{4a}-\frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4a}\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2ad}-\frac{b\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2ad}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 80, normalized size = 0.94

$$-\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/Sqrt[-
(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/Sqrt[c])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(65) = 130$.
time = 0.34, size = 354, normalized size = 4.16

method	result

elliptic	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}+\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d \cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4a\sqrt{-\frac{ad-bc}{b}}} \cdots$
default	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}-\frac{b\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 \cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}} \cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)-b/a*(-1/4/b/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))-1/4/b/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x), x)
```

Fricas [A]
time = 2.80, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\sqrt{dx^4+c}\,\sqrt{c}+2c}{x^4}\right)}{4ac}, \frac{2c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)+\sqrt{c}\log\left(\frac{dx^4-2\sqrt{dx^4+c}\,\sqrt{c}+2c}{x^4}\right)}{4ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-c}}{c}\right)}{4ac}, \frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)+\sqrt{-c}\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-c}}{c}\right)}{2ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(c*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt
(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) + sqrt
(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-
c)/c))/(a*c), 1/2*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^4 + c)*
sqrt(-c)/c))/(a*c)]
```

Sympy [A]
time = 7.37, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**4)/sqrt(-c))/(2*a*sqrt(-c))
```

Giac [A]
time = 1.45, size = 71, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a}+\frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

[Out] -1/2*b*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))

Mupad [B]

time = 5.03, size = 652, normalized size = 7.67

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)}{2\,a\,\sqrt{c}}-\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{4\,\left(a^2\,d-a\,b\,c\right)}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3+\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{4\,\left(a^2\,d-a\,b\,c\right)}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3+\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}}\right)\,\sqrt{b^2\,c-a\,b\,d}\,1{}\mathrm{i}}{2\,\left(a^2\,d-a\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^4)*(c + d*x^4)^(1/2)),x)

[Out] - atanh((c + d*x^4)^(1/2)/c^(1/2))/(2*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c)))*1i)/(4*(a^2*d - a*b*c)) + ((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) + ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c)))*1i)/(4*(a^2*d - a*b*c)))/(((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) + ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i)/(2*(a^2*d - a*b*c))

3.809 $\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{c+dx^4}}{4acx^4}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}$$

```
[Out] 1/4*(a*d+2*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/2*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/4*(d
*x^4+c)^(1/2)/a/c/x^4
```

Rubi [A]
time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 105, 162, 65, 214}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{\sqrt{c+dx^4}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^4]/(a*c*x^4) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqr
t[c]])/(4*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*
c - a*d]])/(2*a^2*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5 (a+bx^4) \sqrt{c+dx^4}} dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}} dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}} dx, x, x^4\right)}{4ac}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^4\right)}{4a^2} - \frac{(2bc+ad)\text{Subst}\left(\int \right.}{8a}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}} dx, x, \sqrt{c+dx^4}\right)}{2a^2d} - \frac{(2bc+ad)\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d}}{\sqrt{bc-ad}}\right.}{2a^2\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.28, size = 109, normalized size = 0.93

$$\frac{-\frac{a\sqrt{c+dx^4}}{cx^4} + \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{c^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-((a*Sqrt[c + d*x^4])/(c*x^4)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4
])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[c
+ d*x^4]/Sqrt[c]])/c^(3/2))/(4*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(93) = 186$.
time = 0.39, size = 407, normalized size = 3.48

method	result
risch	$-\frac{\sqrt{d x^4+c}}{4ac\,x^4}+\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d x^4+c}}{x^2}\right)d}{4a\,c^{\frac{3}{2}}}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d x^4+c}}{x^2}\right)}{2a^2\sqrt{c}}-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2\right.}{b}}{}\right.}{}$
elliptic	$-\frac{\sqrt{d x^4+c}}{4ac\,x^4}+\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d x^4+c}}{x^2}\right)d}{4a\,c^{\frac{3}{2}}}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d x^4+c}}{x^2}\right)}{2a^2\sqrt{c}}-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2\right.}{b}}{}\right.}{}$
default	$\frac{-\frac{\sqrt{d x^4+c}}{4c\,x^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d x^4+c}}{x^2}\right)}{4c^{\frac{3}{2}}}}{a}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d x^4+c}}{x^2}\right)}{2a^2\sqrt{c}}+\frac{b^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x\right.}{b}}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/4/c/x^4*(d*x^4+c)^(1/2)+1/4*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(
1/2))/x^2))+1/2*b/a^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)+b^2/a
^2*(-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-
a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/
2)))-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/
2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^5), x)
```

Fricas [A]
time = 3.68, size = 565, normalized size = 4.83

$$\left[\frac{2\,bc^{2}x^{4}\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{4}+2\,bc-ad-2\,\sqrt{dx^{4}+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{4}+a}\right)+(2\,bc+ad)\sqrt{c}\,x^{4}\log\left(\frac{dx^{4}+2\,\sqrt{dx^{4}+c}\,\sqrt{c}+2\,c}{x^{4}}\right)-2\,\sqrt{dx^{4}+c}\,ac}{8\,a^{2}c^{2}x^{4}},\ -\frac{4\,bc^{2}x^{4}\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^{4}+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^{4}+bc}\right)-(2\,bc+ad)\sqrt{c}\,x^{4}\log\left(\frac{dx^{4}+2\,\sqrt{dx^{4}+c}\,\sqrt{c}+2\,c}{x^{4}}\right)+2\,\sqrt{dx^{4}+c}\,ac}{8\,a^{2}c^{2}x^{4}},\ \frac{bc^{2}x^{4}\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{4}+2\,bc-ad-2\,\sqrt{dx^{4}+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{4}+a}\right)-(2\,bc+ad)\sqrt{-c}\,x^{4}\arctan\left(\frac{\sqrt{dx^{4}+c}\,\sqrt{-c}}{c}\right)-\sqrt{dx^{4}+c}\,ac}{4\,a^{2}c^{2}x^{4}},\ -\frac{2\,bc^{2}x^{4}\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^{4}+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^{4}+bc}\right)+(2\,bc+ad)\sqrt{-c}\,x^{4}\arctan\left(\frac{\sqrt{dx^{4}+c}\,\sqrt{-c}}{c}\right)+\sqrt{dx^{4}+c}\,ac}{4\,a^{2}c^{2}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d
*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + (2*b*c + a*d)*sqr
t(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4
+ c)*a*c)/(a^2*c^2*x^4), -1/8*(4*b*c^2*x^4*sqrt(-b/(b*c - a*d))*arctan(-sqr
t(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - (2*b*c + a
*d)*sqrt(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*sqrt
(d*x^4 + c)*a*c)/(a^2*c^2*x^4), 1/4*(b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d
*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*
x^4 + a)) - (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) -
 sqrt(d*x^4 + c)*a*c)/(a^2*c^2*x^4), -1/4*(2*b*c^2*x^4*sqrt(-b/(b*c - a*d))
*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c))
+ (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + sqrt(d*x^
4 + c)*a*c)/(a^2*c^2*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5}\left(a+bx^{4}\right)\sqrt{c+dx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A]
time = 1.72, size = 104, normalized size = 0.89

$$\frac{b^2 \arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}\,c}-\frac{\sqrt{dx^4+c}}{4\,acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/4*(2*b*c + a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/4*sqrt(d*x^4 + c)/(a*c*x^4)
```

Mupad [B]
time = 5.35, size = 396, normalized size = 3.38

$$\frac{\ln\left(\sqrt{d\,x^4+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{4\,a^3\,d-4\,a^2\,b\,c}-\frac{\ln\left(\sqrt{d\,x^4+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{4\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^4+c}}{4\,a\,c\,x^4}-\frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^4+c}\,3{}\mathrm{i}}{16\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{16\,c}+\frac{5\,a\,b^3\,d^5}{32\,c^2}+\frac{a^2\,b^2\,d^6}{32\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^4+c}\,1{}\mathrm{i}}{32\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{32\,a}+\frac{b^2\,d^6}{32\,c}+\frac{3\,b^4\,c\,d^4}{16\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^4+c}\,5{}\mathrm{i}}{32\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{16\,a}+\frac{5\,b^3\,d^5}{32\,c}+\frac{a\,b^2\,d^6}{32\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)\,1{}\mathrm{i}}{4\,a^2\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (log((c + d*x^4)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(4*a^3*d - 4*a^2*b*c) - (log((c + d*x^4
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(4*(a^3*d - a^2*b*c)) - (c + d*x^4)^(1/2)/(4*a*c*x^4)
- (atan((b^4*d^4*(c + d*x^4)^(1/2)*3i)/(16*(c^3)^(1/2)*((3*b^4*d^4)/(16*c)
+ (5*a*b^3*d^5)/(32*c^2) + (a^2*b^2*d^6)/(32*c^3))) + (b^2*d^6*(c + d*x^4)^
(1/2)*1i)/(32*(c^3)^(1/2)*((5*b^3*d^5)/(32*a) + (b^2*d^6)/(32*c) + (3*b^4*c
*d^4)/(16*a^2))) + (b^3*d^5*(c + d*x^4)^(1/2)*5i)/(32*(c^3)^(1/2)*((3*b^4*d
^4)/(16*a) + (5*b^3*d^5)/(32*c) + (a*b^2*d^6)/(32*c^2))))*(a*d + 2*b*c)*1i)
/(4*a^2*(c^3)^(1/2))
```

3.810 $\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=123

$$\frac{x^2\sqrt{c+dx^4}}{4bd}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}}-\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}$$

```
[Out] -1/4*(2*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d^(3/2)+1/2*a^(3/
2)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/b^2/(-a*d+b*c)^(1/2
)+1/4*x^2*(d*x^4+c)^(1/2)/b/d
```

Rubi [A]
time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {476, 490, 537, 223, 212, 385, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}+\frac{x^2\sqrt{c+dx^4}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt
[a]*Sqrt[c + d*x^4])])/(2*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)$$
$$= \frac{x^2\sqrt{c+dx^4}}{4bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4bd}$$
$$= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b^2} - \frac{(bc+2ad)\text{Subst}\left(\int \right.}{4b}$$
$$= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2} - \frac{(bc+2ad)\text{Subst}\left(\right.}{}$$
$$= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right.}{4b^2d^{3/2}}$$

Mathematica [A]
time = 0.65, size = 140, normalized size = 1.14

$$\frac{\sqrt{d}\left(bx^2\sqrt{c+dx^4} + \frac{2a^{3/2}d\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^4+bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}\right) - (bc+2ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{d}\,x^2}\right)}{4b^2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[d]*(b*x^2*Sqrt[c + d*x^4] + (2*a^(3/2)*d*ArcTan[(a*Sqrt[d] + b*Sqrt[d
]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d])
 - (b*c + 2*a*d)*ArcTanh[Sqrt[c + d*x^4]/(Sqrt[d]*x^2)])/(4*b^2*d^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(99) = 198.
time = 0.38, size = 403, normalized size = 3.28

method	result

default	$\frac{\frac{x^2\sqrt{d\,x^4+c}}{4d}-\frac{c\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{4d^{\frac{3}{2}}}}{b}-\frac{a\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b^2\sqrt{d}}+\frac{a^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2-\cdots\right.}{b}\cdots}{\cdots}\right)}{\cdots}\cdots\right.}{\cdots}$
risch	$\frac{x^2\sqrt{d\,x^4+c}}{4bd}-\frac{a\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b^2\sqrt{d}}-\frac{\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)c}{4b\,d^{\frac{3}{2}}}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2-\cdots\right.}{b}\cdots}{\cdots}\right)}{\cdots}$
elliptic	$\frac{x^2\sqrt{d\,x^4+c}}{4bd}-\frac{a\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b^2\sqrt{d}}-\frac{\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)c}{4b\,d^{\frac{3}{2}}}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2-\cdots\right.}{b}\cdots}{\cdots}\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*x^2/d*(d*x^4+c)^(1/2)-1/4*c/d^(3/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2)
))-1/2*a/b^2*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))/d^(1/2)+a^2/b^2*(-1/4/(-a*b)^(
1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1
/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2
*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b
)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [A]
time = 3.73, size = 739, normalized size = 6.01

$$\left[\frac{2\sqrt{dx^4+c}\,bdx^2 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2+4((b^2c^2-3abcd+2a^2d^2)x^6-(abc^2-a^2cd)x^2)\sqrt{dx^4+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^8+2abx^4+a^2}\right)+(bc+2ad)\sqrt{d}\log\left(-2dx^4+2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)}{8b^2d^2}, \frac{2\sqrt{dx^4+c}\,bdx^2 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2+4((b^2c^2-3abcd+2a^2d^2)x^6-(abc^2-a^2cd)x^2)\sqrt{dx^4+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^8+2abx^4+a^2}\right)+2(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)}{8b^2d^2}, \frac{2\sqrt{dx^4+c}\,bdx^2-2ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^6+acx^2)}\right)+(bc+2ad)\sqrt{d}\log\left(-2dx^4+2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)}{8b^2d^2}, \frac{\sqrt{dx^4+c}\,bdx^2-ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^6+acx^2)}\right)+(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)}{4b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] [1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*
((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^
4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + (b*c + 2*a*d)*s
qrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1/8*(2
*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c
^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*
sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 2*(b*c + 2*a*d)*sqrt(-
d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/(b^2*d^2), 1/8*(2*sqrt(d*x^4 + c)*
b*d*x^2 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)
*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + (b*c + 2*a*d)*s
qrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1/4*(s
qrt(d*x^4 + c)*b*d*x^2 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*
d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + (b
*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x**9/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.811 $\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}$$

```
[Out] 1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b/d^(1/2)-1/2*arctan(x^2*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 494, 223, 212, 385, 211}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*S
qrt[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b}-\frac{a\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}-\frac{a\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}\\ &= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}\end{aligned}$$

Mathematica [A]
time = 0.34, size = 107, normalized size = 1.18

$$\frac{-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}\,x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{d}\,x^2}\right)}{\sqrt{d}}}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] (-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + ArcTanh[Sqrt[c + d*x^4]/(Sqrt[d]*x^2)]/Sqrt[d])/(2*b)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(71) = 142$.

time = 0.35, size = 355, normalized size = 3.90

method	result
default	$\frac{\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b\sqrt{d}} - \frac{a\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d \cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}} \cdots\right)}{\cdots}$
elliptic	$\frac{\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b\sqrt{d}} + \frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,b\sqrt{-\frac{ad-bc}{b}}} \cdots$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/b*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))/d^(1/2)-a/b*(-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [A]
time = 4.53, size = 632, normalized size = 6.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^
2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^
2*x^8 + 2*a*b*x^4 + a^2)) + 2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt
(d)*x^2 - c))/(b*d), 1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 -
3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(
-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*sqrt(-d)*arctan(sqrt(-d)*
x^2/sqrt(d*x^4 + c)))/(b*d), 1/4*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c -
 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2))
 + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b*d), 1/4*(d
*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*
sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) - 2*sqrt(-d)*arctan(sqrt(-d)*x^2/s
qrt(d*x^4 + c)))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + bx^4)\sqrt{c + dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.812 $\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2\sqrt{a}\,\sqrt{bc-ad}}$$

```
[Out] 1/2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(1/2)/(-a*d+b*c)
^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {476, 385, 211}

$$\frac{\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2\sqrt{a}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]/(2*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 74, normalized size = 1.37

$$\frac{\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}\,x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*
c - a*d])]/(2*Sqrt[a]*Sqrt[b*c - a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(42) = 84.
time = 0.33, size = 322, normalized size = 5.96

method	result
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}$

elliptic	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/
b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d
+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a
*b)^(1/2)))+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x^2+1/b*(-a*b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.
time = 4.06, size = 245, normalized size = 4.54

$$\left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2-4((bc-2ad)x^6-acx^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2abx^4+a^2}\right)}{8(abc-a^2d)}, \frac{\arctan\left(\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)}{4\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt
(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2))/(a*b*c - a^2
*d), 1/4*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c -
```

```
a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))/sqrt(a*b*c - a^
2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A]
time = 1.76, size = 72, normalized size = 1.33

$$-\frac{\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.813 $\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

```
[Out] -1/2*b*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(3/2)/(-a*d+b
*c)^(1/2)-1/2*(d*x^4+c)^(1/2)/a/c/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {476, 491, 12, 385, 211}

$$-\frac{b\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*Sqrt[c + d*x^4]/(a*c*x^2) - (b*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*S
qrt[c + d*x^4])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
```

```
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3 (a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2 (a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{\text{Subst}\left(\int \frac{bc}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2ac}\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A]
time = 0.34, size = 100, normalized size = 1.25

$$
-\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}\,x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}\sqrt{bc-ad}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*Sqrt[c + d*x^4]/(a*c*x^2) - (b*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2
+ Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(64) = 128$.

time = 0.36, size = 349, normalized size = 4.36

method	result
default	$-\frac{\sqrt{d\,x^4+c}}{2c\,x^2a}-\frac{b\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-\cdots}}{\cdots}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}\cdots\right)}{\cdots}$
risch	$-\frac{\sqrt{d\,x^4+c}}{2c\,x^2a}+\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}}{\cdots}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4a\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}\cdots$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{2c\,x^2a}+\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}}{\cdots}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4a\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x^4+c)^(1/2)/c/x^2/a-b/a*(-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/
```

```
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(64) = 128$.
time = 3.36, size = 332, normalized size = 4.15

$$\left[-\frac{\sqrt{-abc+a^2d}\,bcx^2\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((bc-2\,ad)x^6-acx^2)\sqrt{dx^4+c}\,\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)+4\,\sqrt{dx^4+c}\,(abc-a^2d)}{8\,(a^2bc^2-a^3cd)x^2},\ -\frac{\sqrt{abc-a^2d}\,bcx^2\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\,\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)+2\,\sqrt{dx^4+c}\,(abc-a^2d)}{4\,(a^2bc^2-a^3cd)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*b*c + a^2*d)*b*c*x^2*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*
x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4
*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2), -1/4*(sqrt(a
*b*c - a^2*d)*b*c*x^2*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*
sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) +
2*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A]

time = 3.03, size = 116, normalized size = 1.45

$$\frac{1}{2}\,d^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,ad}+\frac{2}{\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2-c\right)ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x^
2 - sqrt(d*x^4 + c))^2 - c)*a*d))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.814 $\int \frac{1}{x^7(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/2*b^2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(5/2)/(-a*d+
b*c)^(1/2)-1/6*(d*x^4+c)^(1/2)/a/c/x^6+1/6*(2*a*d+3*b*c)*(d*x^4+c)^(1/2)/a^
2/c^2/x^2
```

Rubi [A]
time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {476, 491, 597, 12, 385, 211}

$$\frac{b^2\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}\,(2ad+3bc)}{6a^2c^2x^2}-\frac{\sqrt{c+dx^4}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^4]/(a*c*x^6) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^4])/(6*a^2*c
^2*x^2) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*
a^(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}}\, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4 (a+bx^2) \sqrt{c+dx^2}}\, dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\, dx, x, x^2\right)}{6ac}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} - \frac{\text{Subst}\left(\int -\frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}}\, dx, x, \right.}{6a^2c^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\, dx, x, \right.}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\, dx, x, \frac{x}{\sqrt{c+}}\right.}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} + \frac{b^2 \tan^{-1}\left(\frac{\sqrt{bc-ad}\, x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.64, size = 121, normalized size = 1.05

$$\frac{\sqrt{c+dx^4}\,(-ac+3bcx^4+2adx^4)}{6a^2c^2x^6} + \frac{b^2 \tan^{-1}\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}\,x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-(a*c) + 3*b*c*x^4 + 2*a*d*x^4))/(6*a^2*c^2*x^6) + (b^2*A
rcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c
 - a*d])])/(2*a^(5/2)*Sqrt[b*c - a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(95) = 190.
time = 0.37, size = 379, normalized size = 3.30

method	result

risch	$-\frac{\sqrt{d x^{4}+c}\,(-2ad x^{4}-3bc x^{4}+ac)}{6c^{2}a^{2}x^{6}}-\frac{b^{2}\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-a}}{b}\right.}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right.}{4a^{2}\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}$
default	$-\frac{\sqrt{d x^{4}+c}\,(-2d x^{4}+c)}{6a c^{2}x^{6}}+\frac{b\sqrt{d x^{4}+c}}{2a^{2}x^{2}c}+\frac{b^{2}\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x\right.}}{x^{2}-}\right.}{4\sqrt{-ab}}\right.}{}$
elliptic	$-\frac{\sqrt{d x^{4}+c}}{6ac x^{6}}+\frac{d\sqrt{d x^{4}+c}}{3a c^{2}x^{2}}+\frac{b\sqrt{d x^{4}+c}}{2a^{2}x^{2}c}-\frac{b^{2}\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}}{}\right.}{4a^{2}\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a*(d*x^4+c)^(1/2)*(-2*d*x^4+c)/c^2/x^6+1/2*b/a^2/x^2*(d*x^4+c)^(1/2)/c
+b^2/a^2*(-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a
*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^
(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
^2-1/b*(-a*b)^(1/2)))+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+
1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^7), x)
```

Fricas [A]
time = 3.21, size = 418, normalized size = 3.63

$$\left[-\frac{3 \sqrt{-abc + a^2 d}\, b^2 c^2 x^6 \log\left(\frac{(b^2c^2 - 8\,abcd + 8\,a^2d^2)x^8 - 2\,(3\,abc^2 - 4\,a^2cd)x^4 + a^2c^2 - 4\,((bc - 2\,ad)x^6 - acx^2)\sqrt{dx^4 + c}\,\sqrt{-abc + a^2 d}}{b^2x^8 + 2\,abx^4 + a^2}\right) + 4\,(a^2bc^2 - a^3cd - (3\,ab^2c^2 - a^2bcd - 2\,a^3d^2)x^4)\sqrt{dx^4 + c}}{24\,(a^3bc^3 - a^4c^2d)x^6}, \frac{3\sqrt{abc - a^2d}\, b^2c^2x^6 \arctan\left(\frac{((bc - 2\,ad)x^4 - ac)\sqrt{dx^4 + c}\,\sqrt{abc - a^2d}}{2\,((abcd - a^2d^2)x^6 + (abc^2 - a^2cd)x^2)}\right) - 2\,(a^2bc^2 - a^3cd - (3\,ab^2c^2 - a^2bcd - 2\,a^3d^2)x^4)\sqrt{dx^4 + c}}{12\,(a^3bc^3 - a^4c^2d)x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^6*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6
 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^
2)) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*s
qrt(d*x^4 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^6), 1/12*(3*sqrt(a*b*c - a^2*d)*
b^2*c^2*x^6*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c
 - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) - 2*(a^2*b*c
^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*sqrt(d*x^4 + c))/
((a^3*b*c^3 - a^4*c^2*d)*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\,(a + bx^4)\,\sqrt{c + dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(95) = 190.
time = 3.86, size = 205, normalized size = 1.78

$$-\frac{1}{6}\,d^{\frac{5}{2}}\left(\frac{3\,b^2 \arctan\left(\frac{\left(\sqrt{d}\,x^2 - \sqrt{dx^4 + c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2d^2}}\right)}{\sqrt{abcd - a^2d^2}\,a^2d^2} + \frac{2\left(3\left(\sqrt{d}\,x^2 - \sqrt{dx^4 + c}\right)^4 b - 6\left(\sqrt{d}\,x^2 - \sqrt{dx^4 + c}\right)^2 bc - 6\left(\sqrt{d}\,x^2 - \sqrt{dx^4 + c}\right)^2 ad + 3\,bc^2 + 2\,acd\right)}{\left(\left(\sqrt{d}\,x^2 - \sqrt{dx^4 + c}\right)^2 - c\right)^3 a^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c +
 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*(
sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*
c - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d)
*x^2 - sqrt(d*x^4 + c))^2 - c)^3*a^2*d^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.815 $\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=872

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{(-a)^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{bc-ad}}-\frac{(-a)^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{-bc+ad}}+\frac{a^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}\cdots\right.}{\cdots}$$

```
[Out] -1/4*(-a)^(5/4)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2
))/b^(7/4)/(-a*d+b*c)^(1/2)-1/4*(-a)^(5/4)*arctan(x*(a*d-b*c)^(1/2)/(-a)^(1
/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(a*d-b*c)^(1/2)+1/3*x*(d*x^4+c)^(1/2)/
b/d-1/6*(3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2)
)*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/
4)/d^(5/4)/(d*x^4+c)^(1/2)+1/4*a^2*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4))
)^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*
x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^2*
d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(a*d+b*c)/(d
*x^4+c)^(1/2)+1/4*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2
*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*
x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)
+1/8*a*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^
(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-
a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2
)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x
^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*a*(c
os(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*E
llipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)
*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^
(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/
2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.80, antiderivative size = 872, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {490, 537, 226, 418, 1231, 1721}

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*Sqrt[c + d*x^4])/(3*b*d) - ((-a)^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(7/4)*Sqrt[b*c - a*d]) - ((-a)^(5/4)*
ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(
7/4)*Sqrt[-(b*c) + a*d]) + (a^2*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1
/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) + (a*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) -
 ((b*c + 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]
*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(6*b^2*c^(1/4)*d^(5
/4)*Sqrt[c + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(
Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[
c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^
(1/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
```

```
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x\sqrt{c+dx^4}}{3bd} - \frac{\int \frac{ac+(bc+3ad)x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{3bd}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} + \frac{a^2 \int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b^2} - \frac{(bc+3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3b^2d}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(bc+3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{6b^2\sqrt[4]{c}\,d^{5/4}\sqrt{c+dx^4}}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(bc+3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{6b^2\sqrt[4]{c}\,d^{5/4}\sqrt{c+dx^4}}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(-a)^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{bc-ad}} - \frac{(-a)^{5/4}\tan^{-1}\left(\frac{}{\sqrt[4]{-a}}\right.}{4b^{7/4}\sqrt{-b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.23, size = 249, normalized size = 0.29

$$\frac{x\left(-\frac{(bc+3ad)x^4\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{ad}+5\left(\frac{c}{d}+x^4+\frac{5a^2c^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{d(a+bx^4)\left(-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}\right)\right)}{15b\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(-(((b*c + 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/(a*d)) + 5*(c/d + x^4 + (5*a^2*c^2*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(d*(a + b*x^4)*(-5*a*c*AppellF1[1
/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1
/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 363, normalized size = 0.42

method	result
elliptic	$\frac{x\sqrt{d\,x^4+c}}{3bd}+\frac{\left(-\frac{a}{b^2}-\frac{c}{3db}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{a^2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\right.}{}$
risch	$\frac{x\sqrt{d\,x^4+c}}{3bd}-\frac{\frac{(3ad+bc)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{3a^2d\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}-\frac{\mathrm{arctan}}{}\right.}{}}{}$

default	$\frac{\frac{x\sqrt{d\,x^4+c}}{3d}-\frac{c\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3d\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{b}-\frac{a\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}}{b^2\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3/d*x*(d*x^4+c)^(1/2)-1/3*c/d/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)
*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Ellipti
cF(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-a/b^2/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(
1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Ell
ipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)+1/8*a^2/b^3*sum(1/_alpha^3*(-1/((-a*d
+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x
^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*
x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/
c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^
(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^8/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.816 $\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=638

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+\frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\Big(}{2\sqrt[4]{d}\,(bc+ad)\sqrt{c+dx^4}}$$

```
[Out] -1/4*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/b/((a*d
-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2)
)^(1/2)/(d*x^4+c)^(1/2))/b/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/2*c^(3/4
)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)
))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(
1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/d^(1/4)/(a*d+b*c)/(d*x^4+c)
^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2
)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^
(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)
+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2
))/(d*x^4+c)^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arc
tan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b
^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2
*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^
4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 837, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {494, 226, 418, 1231, 1721}

$$-\frac{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt[4]{-a}\,\text{ArcTan}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{-a}\,\text{ArcTan}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{3/4}\sqrt{ad-bc}}+\frac{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}-\frac{\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*((-a)^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*
x^4])])/(b^(3/4)*Sqrt[b*c - a*d]) - ((-a)^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*
x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(3/4)*Sqrt[-(b*c) + a*d]) +
```

```
((Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipt
icF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c + d*x^
4]) - (a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*
x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4
)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[-a]
*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x
^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2
])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]
*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2
)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*d^(1
/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*
(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipti
cPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt
[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{a\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}} - \frac{\int \frac{}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right.}}{}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}} - \frac{\left(\sqrt{c}\left(\sqrt{c}\right.\right.}{}$$

$$= -\frac{\sqrt[4]{-a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{-a}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{3/4}\sqrt{-bc+ad}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.06, size = 65, normalized size = 0.10

$$\frac{x^5\sqrt{\frac{c+dx^4}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)
/a)])/(5*a*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 265, normalized size = 0.42

method	result
default	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d\,x^4}}\right)}{\sqrt{\frac{-ad+bc}{b}}}}{}\right.}{}$
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d\,x^4}}\right)}{\sqrt{\frac{-ad+bc}{b}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*
d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)
-1/8*a/b^2*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*
d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2
)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1
/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2
```

```
)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x**4/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x^4/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.817 $\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=638

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-a}\,\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}}+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\Big(2}{2\sqrt[4]{c}\,(bc+ad)\sqrt{c+dx^4}}$$

```
[Out] 1/4*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a/((a*d-
b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))
^(1/2)/(d*x^4+c)^(1/2))/a/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/2*d^(3/4)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1
/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^4+c)^
(1/2)+1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x
/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)
+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(
1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+
x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2)
)/(d*x^4+c)^(1/2)+1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arct
an(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*
2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4
+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^
(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 742, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {418, 1231, 226, 1721}

$$-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{ad-bc}}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}\,\sqrt{c+dx^4}\,(ad+bc)}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a\sqrt[4]{c}\,\sqrt{c+dx^4}\,(ad+bc)}+\frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}\,(ad+bc)}+\frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}\,(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(b^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4
])])/((-a)^(3/4)*Sqrt[b*c - a*d]) - (b^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/
((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(3/4)*Sqrt[-(b*c) + a*d]) +
```

```
(((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqr
t[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(
1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[-a]*Sqrt[b]*S
qrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^
(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^
2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip
ticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]
*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)*d^(1/4)*(b*c +
 a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[
b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*Ar
cTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c +
d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a}$$

$$= \frac{\left(\sqrt{b}\,\sqrt{c}\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}\,x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a(bc+ad)} + \frac{\left(\sqrt{b}\,\sqrt{c}\left(\right.\right.}{}$$

$$= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{-bc+ad}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.04, size = 161, normalized size = 0.25

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\sqrt{c+dx^4}\left(-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*x
^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x
^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a
)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 191, normalized size = 0.30

method	result
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}(\_Z^4 b+a)} \frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^4+c}}}{\_\alpha^3}}{8b}$

elliptic	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^4+c}}}{\_\alpha^3}}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2
+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_al
pha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(
d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_al
pha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Roo
tOf(_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.818 $\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=677

$$-\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{b\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-a}\,\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}}-\frac{b\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}}-\frac{d^{3/4}(4bc+ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)}{6ac^{5/}}$$

```
[Out] -1/3*(d*x^4+c)^(1/2)/a/c/x^3-1/4*b*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^
(1/2)/(d*x^4+c)^(1/2))/a^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*b*arcta
n(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a^2/((-a*d+b*c)/
(-a)^(1/2)/b^(1/2))^(1/2)-1/6*d^(3/4)*(a*d+4*b*c)*(cos(2*arctan(d^(1/4)*x/c
^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d
^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x
^2*d^(1/2))^2)^(1/2)/a/c^(5/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*b*(cos(2*arcta
n(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(
sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2
/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(
1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/
a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)-1/
8*b*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/
4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+
x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d
^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(
d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.78, antiderivative size = 864, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {491, 537, 226, 418, 1231, 1721}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^4]/(a*c*x^3) - (b^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(7/4)*Sqrt[b*c - a*d]) - (b^(5/4)*
```

```
ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a
)^(7/4)*Sqrt[-(b*c) + a*d]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*
x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/
2])/(6*a*c^(5/4)*Sqrt[c + d*x^4]) - (b*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d
])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^4]) - (b*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt
[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*
ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^
4]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqr
t[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] -
 Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*
x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (b
*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqr
t[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)]
, 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{3acx^3} + \frac{\int \frac{-3bc-ad-bdx^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{3ac}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3} - \frac{b\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{a} - \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3ac}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right.\right.}{6ac^{5/4}\sqrt{c+dx^4}}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right.\right.}{6ac^{5/4}\sqrt{c+dx^4}}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}\sqrt{bc-ad}} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc-}}{\sqrt[4]{-a}\,\sqrt[4]{b}}\right.}{4(-a)^{7/4}\sqrt{-bc-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.21, size = 337, normalized size = 0.50

$$\frac{-bdx^8\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\frac{5a\left(-5ac(ac+4bcx^4+2adx^4+bdx^8)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4(a+bx^4)(c+dx^4)\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}{(a+bx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}}{15a^2cx^3\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]) + (5*a*(-5*a*c*(a*c + 4*b*c*x^4 + 2*a*d*x^4 + b*d*x^8)*AppellF1
[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(a + b*x^4)*(c + d*x
^4)*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*App
ellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(5*a*c*
AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 2*x^4*(2*b*c*Appel
lF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2,
1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*c*x^3*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 288, normalized size = 0.43

method	result
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{3c\,x^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3c\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{a}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}$ … arctanh(…

risch	$-\frac{\sqrt{d x^4+c}}{3ac x^3}-\frac{\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d x^4+c}}+3c\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{}{2\sqrt{}}\right)}{}}{}\right.}{}$
elliptic	$-\frac{\sqrt{d x^4+c}}{3ac x^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3ac\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d x^4+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{}{2\sqrt{}}\right)}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/3/c*(d*x^4+c)^(1/2)/x^3-1/3*d/c/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1
/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Elli
pticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8/a*sum(1/_alpha^3*(-1/((-a*d+b*c)/
b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^
(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(
1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2
)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/
(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.819 $\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=804

$$\frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b(bc-ad)}-\frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}}{\sqrt{c+dx^4}}\right.}{4b(bc-ad)}$$

```
[Out] x*(d*x^4+c)^(1/2)/b/d^(1/2)/(c^(1/2)+x^2*d^(1/2))-1/4*a*arctan(x*((a*d-b*c)
/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^
(1/2)/b/(-a*d+b*c)-1/4*a*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*
x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b/(-a*d+b*c)-c^(1/4)*(c
os(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*E
llipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2)
)*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^4+c)^(1/2)+1/2*c
^(1/4)*(2*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(
d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))
*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/d^(3/4)/
(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*a*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))
),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(
1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2)
)*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/((-a)^(
1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)-1/8*a*(cos(2*arctan(d^(1/4)
*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arc
tan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2
/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(
1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/
c^(1/4)/d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A]

time = 0.75, antiderivative size = 982, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {494, 311, 226, 1210, 504, 1231, 1721}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*Sqrt[c + d*x^4])/(b*Sqrt[d]*(Sqrt[c] + Sqrt[d]*x^2)) + ((-a)^(3/4)*ArcTa
n[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(5/4)*Sqr
t[b*c - a*d]) - ((-a)^(3/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^4])])/(4*b^(5/4)*Sqrt[-(b*c) + a*d]) - (c^(1/4)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*d^(3/4)*Sqrt[c + d*x^4]) + (c^(1/4)*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*Ar
cTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*d^(3/4)*Sqrt[c + d*x^4]) + (a*(Sqrt[c
] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (a*(Sqrt[c] + (Sqrt[-a]*Sq
rt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4
)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[-a]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt
[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sq
rt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*S
qrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^
2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sq
rt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
```

```
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{b} - \frac{a\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{a\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}} - \frac{a\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}} + \frac{\sqrt{c}\ \ldots}{}$$

$$= \frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)} - \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,E\left(2\tan^{-}\ldots\right.}{bd^{3/4}\sqrt{c+dx^4}}$$

$$= \frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{5/4}\sqrt{bc-ad}} - \frac{(-a)^{3/4}\tan\ldots}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.04, size = 65, normalized size = 0.08

$$\frac{x^7\sqrt{\frac{c+dx^4}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{7a\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^7*Sqrt[(c + d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4
)/a)])/(7*a*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 292, normalized size = 0.36

method	result

default	$\frac{i\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}\,\sqrt{d}} - \frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}\right.}\right.}{}$
elliptic	$\frac{i\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}\,\sqrt{d}} - \frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/
c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)/d^(1/2)*(EllipticF(x*(I/c^(1/2)*
d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8*a/b^2*sum(1
/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+
b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-
I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2
)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I
/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x**6/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x^6/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.820 $\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=656

$$\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}+\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}-\frac{\sqrt[4]{c}\,\sqrt[4]{d}\,\left(\sqrt{c}+\sqrt{d}\right.}{}$$

```
[Out] 1/4*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*
c)/(-a)^(1/2)/b^(1/2))^(1/2)/(-a*d+b*c)+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)
/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(-a*
d+b*c)-1/2*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2
*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*
2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(a
*d+b*c)/(d*x^4+c)^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/
4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2)
,1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((
d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/b^(1/2)/((-a)^(1/2)
*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)+1/8*(cos(2*arctan(d^(1/4)*x/c^(
1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^
(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^
(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(
-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4
)/b^(1/2)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 756, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {504, 1231, 226, 1721}

$$\frac{\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{bc-ad}}-\frac{\text{ArcTan}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{ad-bc}}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}\,\sqrt{c+dx^4}\,(ad+bc)}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\left(\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}+\sqrt{c}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}\,\sqrt{c+dx^4}\,(ad+bc)}-\frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt{-a}\,\sqrt{b}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}\,(ad+bc)}+\frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\left(\sqrt{-a}\,\sqrt{d}+\sqrt{b}\,\sqrt{c}\right)^2\Pi\left(-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt{-a}\,\sqrt{b}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}\,(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]/(4*(-a)^(1
/4)*b^(1/4)*Sqrt[b*c - a*d]) - ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^
(1/4)*Sqrt[c + d*x^4])]/(4*(-a)^(1/4)*b^(1/4)*Sqrt[-(b*c) + a*d]) - ((Sqrt[
c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
```

```
d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a]*Sqrt[
d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*
c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/
4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d])
, 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*
(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d])
, 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*
(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2\sqrt{b}} + \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2\sqrt{b}}$$

$$= -\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}\,x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2(bc+ad)} + \frac{\left(\sqrt{c}\left(\sqrt{b}\right.\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}} - \frac{\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{-bc+ad}} - \frac{\left(\sqrt{c}-\sqrt{\cdot}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.03, size = 65, normalized size = 0.10

$$\frac{x^3\sqrt{\frac{c+dx^4}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^3*Sqrt[(c + d*x^4)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)
/a)])/(3*a*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 191, normalized size = 0.29

method	result
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\,\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^4+c}}}{\_\alpha}}{8b}$

elliptic	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)} \frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}, \ldots\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^4+c}}}{\_\alpha}}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2
*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alph
a^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*
x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alph
a^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootO
f(_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x^2/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.821 $\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=833

$$-\frac{\sqrt{c+dx^4}}{acx}+\frac{\sqrt{d}\,x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{b\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)}-\frac{b\sqrt{\frac{bc-ad}{\sqrt{-a}\,\sqrt{b}}}\tan^{-1}\left(\cdots\right.}{4a(bc-a\cdots}$$

```
[Out] -(d*x^4+c)^(1/2)/a/c/x+x*d^(1/2)*(d*x^4+c)^(1/2)/a/c/(c^(1/2)+x^2*d^(1/2))-
1/4*b*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-
b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/a/(-a*d+b*c)-1/4*b*arctan(x*((-a*d+b*c)/(-a)
^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2
)/a/(-a*d+b*c)-d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arc
tan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1
/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(3
/4)/(d*x^4+c)^(1/2)+1/2*d^(1/4)*(a*d+2*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)
))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(
1/2))^2)^(1/2)/a/c^(3/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*(cos(2*arctan(d^(1/4
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*ar
ctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1
/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(-d^(1/4)*(-a)^(1/2)/c^(1/
4)+c^(1/4)*b^(1/2)/d^(1/4))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d
^(1/2))^2)^(1/2)/a/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)-1
/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4
)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^
(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(d
^(1/4)*(-a)^(1/2)/c^(1/4)+c^(1/4)*b^(1/2)/d^(1/4))*(c^(1/2)+x^2*d^(1/2))*((
d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(
1/2))/(d*x^4+c)^(1/2)
```

Rubi [A]

time = 1.05, antiderivative size = 1007, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {491, 598, 311, 226, 1210, 504, 1231, 1721}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -(Sqrt[c + d*x^4]/(a*c*x)) + (Sqrt[d]*x*Sqrt[c + d*x^4])/(a*c*(Sqrt[c] + Sq
rt[d]*x^2)) + (b^(3/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[
c + d*x^4])])/(4*(-a)^(5/4)*Sqrt[b*c - a*d]) - (b^(3/4)*ArcTan[(Sqrt[-(b*c)
 + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(5/4)*Sqrt[-(b*c)
 + a*d]) - (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqr
t[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(a*c^(3/4)*Sqrt
[c + d*x^4]) + (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*a*c^(3/4
)*Sqrt[c + d*x^4]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4
]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(S
qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*S
qrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)]
, 1/2])/(8*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[
b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*S
qrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4
)], 1/2])/(8*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{acx} + \frac{\int \frac{x^2(-bc+ad+bdx^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{ac}$$

$$= -\frac{\sqrt{c+dx^4}}{acx} + \frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}} - \frac{bcx^2}{(a+bx^4)\sqrt{c+dx^4}}\right)dx}{ac}$$

$$= -\frac{\sqrt{c+dx^4}}{acx} - \frac{b\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{a} + \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{ac}$$

$$= -\frac{\sqrt{c+dx^4}}{acx} + \frac{\sqrt{b}\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2a} - \frac{\sqrt{b}\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}\,x^2\right.}}{2a}$$

$$= -\frac{\sqrt{c+dx^4}}{acx} + \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}\,x^2\right)} - \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right.}}}{ac^{3/4}\sqrt{c+d}}$$

$$= -\frac{\sqrt{c+dx^4}}{acx} + \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt{bc-ad}} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.09, size = 141, normalized size = 0.17

$$\frac{-21a(c+dx^4)+7(-bc+ad)x^4\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+3bdx^8\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{21a^2cx\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-21*a*(c + d*x^4) + 7*(-(b*c) + a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]*A
ppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*c*x*Sqrt[c
+ d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 310, normalized size = 0.37

method	result
default	$-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^{2}\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^{3}b\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\mathrm{EllipticPi}\left(x\sqrt{\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^{4}+c}}}{\_\alpha}}{8a}$
elliptic	$-\frac{\sqrt{d\,x^{4}+c}}{acx}+\frac{i\sqrt{d}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{a\sqrt{c}\,\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}$
risch	$-\frac{\sqrt{d\,x^{4}+c}}{acx}+\frac{\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+
2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alp
ha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d
*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alp
ha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Root
```

```
Of(_Z^4*b+a))+1/a*(-1/c*(d*x^4+c)^(1/2)/x+I*d^(1/2)/c^(1/2)/(I/c^(1/2)*d^(1
/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/
(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^
(1/2)*d^(1/2))^(1/2),I)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.822 $\int \frac{x^{15}}{(a+bx^4)^2 \sqrt{c+dx^4}}\, dx$

Optimal. Leaf size=175

$$\frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}-\frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)}-\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\cdots}{\sqrt{\cdots}}\right)}{4b^{7/2}(bc-ad)^{3/2}}$$

```
[Out] -1/4*a^2*(-5*a*d+6*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b
^(7/2)/(-a*d+b*c)^(3/2)+1/4*a*x^8*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)-1/
12*(4*b^2*c^2+8*a*b*c*d-15*a^2*d^2-b*d*(-5*a*d+2*b*c)*x^4)*(d*x^4+c)^(1/2)/
b^3/d^2/(-a*d+b*c)
```

Rubi [A]
time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 100, 152, 65, 214}

$$-\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{12b^3d^2(bc-ad)}+\frac{ax^8\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x^8*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[c + d*x^4]*(4
*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2 - b*d*(2*b*c - 5*a*d)*x^4))/(12*b^3*d^2*(
b*c - a*d)) - (a^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b
*c - a*d]])/(4*b^(7/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{x^3}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\text{Subst}\left(\int \frac{x\left(2ac+\frac{1}{2}(-2bc+5ad)x\right)}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b(bc-ad)}$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4}\,(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4}{12b^3d^2(bc-ad)}$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4}\,(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4}{12b^3d^2(bc-ad)}$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4}\,(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4}{12b^3d^2(bc-ad)}$$

Mathematica [**A**]
time = 0.44, size = 175, normalized size = 1.00

$$-\frac{\sqrt{c+dx^4}\,(-15a^3d^2+2a^2bd(4c-5dx^4)+2b^3cx^4(2c-dx^4)+2ab^2(2c^2+3cdx^4+d^2x^8))}{12b^3d^2(bc-ad)(a+bx^4)} + \frac{a^2(-6bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{4b^{7/2}(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/12*(Sqrt[c + d*x^4]*(-15*a^3*d^2 + 2*a^2*b*d*(4*c - 5*d*x^4) + 2*b^3*c*x
^4*(2*c - d*x^4) + 2*a*b^2*(2*c^2 + 3*c*d*x^4 + d^2*x^8)))/(b^3*d^2*(b*c -
a*d)*(a + b*x^4)) + (a^2*(-6*b*c + 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/
Sqrt[-(b*c) + a*d]])/(4*b^(7/2)*(-(b*c) + a*d)^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 917 vs. 2(155) = 310.
time = 0.39, size = 918, normalized size = 5.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b^2*(d*x^4+c)^(1/2)*(-d*x^4+2*c)/d^2-a/b^3/d*(d*x^4+c)^(1/2)+3*a^2/b^3
*(-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*
b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)
))-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/
```

```
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)
)))-a^3/b^3*(-1/8/a*(-a*b)^(1/2)/b/(a*d-b*c)/(x^2-1/b*(-a*b)^(1/2))*((x^2-1
/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)-1/8/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)
^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-
1/b*(-a*b)^(1/2)))+1/8/a*(-a*b)^(1/2)/b/(a*d-b*c)/(x^2+1/b*(-a*b)^(1/2))*((
x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)-1/8/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x^2+1/b*(-a*b)^(1/2))))
```

<u>Maxima</u> **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

<u>Fricas</u> **[A]**
time = 3.17, size = 622, normalized size = 3.55

$$\left[\frac{3\,(6\,a^3bcd^2-5\,a^4d^3+(6\,a^2b^2cd^2-5\,a^3bd^3)x^4)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,(2\,(b^5c^2d-2\,ab^4cd^2+a^2b^3d^3)x^8-4\,ab^4c^3-4\,a^2b^3c^2d+23\,a^3b^2cd^2-15\,a^4bd^3-2\,(2\,b^5c^3+ab^4c^2d-8\,a^2b^3cd^2+5\,a^3b^2d^3)x^4)\sqrt{dx^4+c}}{24\,(ab^6c^2d^2-2\,a^2b^5cd^3+a^3b^4d^4+(b^7c^2d^2-2\,ab^6cd^3+a^2b^5d^4)x^4)},\ \frac{3\,(6\,a^3bcd^2-5\,a^4d^3+(6\,a^2b^2cd^2-5\,a^3bd^3)x^4)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)+(2\,(b^5c^2d-2\,ab^4cd^2+a^2b^3d^3)x^8-4\,ab^4c^3-4\,a^2b^3c^2d+23\,a^3b^2cd^2-15\,a^4bd^3-2\,(2\,b^5c^3+ab^4c^2d-8\,a^2b^3cd^2+5\,a^3b^2d^3)x^4)\sqrt{dx^4+c}}{12\,(ab^6c^2d^2-2\,a^2b^5cd^3+a^3b^4d^4+(b^7c^2d^2-2\,ab^6cd^3+a^2b^5d^4)x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x^4)*
sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2
*c - a*b*d))/(b*x^4 + a)) + 2*(2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*
x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d^3 - 2*(
2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqrt(d*x^4
+ c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2 - 2*a*b
^6*c*d^3 + a^2*b^5*d^4)*x^4), 1/12*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b
^2*c*d^2 - 5*a^3*b*d^3)*x^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sq
rt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + (2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b
^3*d^3)*x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d
^3 - 2*(2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqr
```

```
t(d*x^4 + c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2
 - 2*a*b^6*c*d^3 + a^2*b^5*d^4)*x^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.22, size = 180, normalized size = 1.03

$$\frac{\sqrt{dx^4+c}\,a^3d}{4\,(b^4c-ab^3d)((dx^4+c)b-bc+ad)}+\frac{(6\,a^2bc-5\,a^3d)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,(b^4c-ab^3d)\sqrt{-b^2c+abd}}+\frac{(dx^4+c)^{\frac{3}{2}}b^4d^4-3\,\sqrt{dx^4+c}\,b^4cd^4-6\,\sqrt{dx^4+c}\,ab^3d^5}{6\,b^6d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(d*x^4 + c)*a^3*d/((b^4*c - a*b^3*d)*((d*x^4 + c)*b - b*c + a*d)) +
 1/4*(6*a^2*b*c - 5*a^3*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(
(b^4*c - a*b^3*d)*sqrt(-b^2*c + a*b*d)) + 1/6*((d*x^4 + c)^(3/2)*b^4*d^4 -
3*sqrt(d*x^4 + c)*b^4*c*d^4 - 6*sqrt(d*x^4 + c)*a*b^3*d^5)/(b^6*d^6)
```

Mupad [B]
time = 5.19, size = 186, normalized size = 1.06

$$\frac{{\left(d\,x^4+c\right)}^{3/2}}{6\,b^2\,d^2}-\left(\frac{3\,c}{2\,b^2\,d^2}+\frac{a\,d-b\,c}{b^3\,d^2}\right)\,\sqrt{d\,x^4+c}+\frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\left(5\,a\,d-6\,b\,c\right)}{\sqrt{a\,d-b\,c}\,\left(5\,a^3\,d-6\,a^2\,b\,c\right)}\right)\,\left(5\,a\,d-6\,b\,c\right)}{4\,b^{7/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{a^3\,d\,\sqrt{d\,x^4+c}}{2\,\left(a\,d-b\,c\right)\,\left(2\,b^4\,\left(d\,x^4+c\right)-2\,b^4\,c+2\,a\,b^3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b^2*d^2) - ((3*c)/(2*b^2*d^2) + (a*d - b*c)/(b^3*d^2))
*(c + d*x^4)^(1/2) + (a^2*atan((a^2*b^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 6*b*
c))/((a*d - b*c)^(1/2)*(5*a^3*d - 6*a^2*b*c)))*(5*a*d - 6*b*c))/(4*b^(7/2)*
(a*d - b*c)^(3/2)) - (a^3*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^4*(c + d
*x^4) - 2*b^4*c + 2*a*b^3*d))
```

3.823 $\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{c+dx^4}}{2b^2d}-\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\,(a+bx^4)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}$$

```
[Out] 1/4*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+1/2*(d*x^4+c)^(1/2)/b^2/d-1/4*a^2*(d*x^4+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^4+a)
```

Rubi [A]

time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 91, 81, 65, 214}

$$-\frac{a^2\sqrt{c+dx^4}}{4b^2\,(a+bx^4)\,(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^4}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b^2*d) - (a^2*Sqrt[c + d*x^4])/(4*b^2*(b*c - a*d)*(a + b
*x^4)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*
d]])/(4*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= -\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)} - \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{8b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)} - \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x,\right.}{4b^2d(bc-ad)}$$

$$= \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A]

time = 0.34, size = 130, normalized size = 1.06

$$\frac{\frac{\sqrt{b}\,\sqrt{c+dx^4}\,(-3a^2d+2b^2cx^4+2ab(c-dx^4))}{d(bc-ad)(a+bx^4)}+\frac{a(4bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((Sqrt[b]*Sqrt[c + d*x^4]*(-3*a^2*d + 2*b^2*c*x^4 + 2*a*b*(c - d*x^4)))/(d*
(b*c - a*d)*(a + b*x^4)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(4*b^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(103) = 206$.
time = 0.39, size = 887, normalized size = 7.21 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x^4+c)^(1/2)/b^2/d-2*a/b^2*(-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x
^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))+a^2/b^2*(-1/8/a*(-a*b)^(1/2)/b/(a*d-b*
c)/(x^2-1/b*(-a*b)^(1/2))*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/8/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1/8/a*(-a*b)^(1/2)/b/(a
*d-b*c)/(x^2+1/b*(-a*b)^(1/2))*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)
/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/8/b*d/(a*d-b*c)/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(103) = 206.
time = 4.94, size = 475, normalized size = 3.86

$$\left[\frac{(4a^2bcd-3a^3d^2+(4ab^2cd-3a^2bd^2)x^4)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2(2ab^3c^2-5a^2b^2cd+3a^3bd^2+2(b^4c^2-2ab^3cd+a^2b^2d^2)x^4)\sqrt{dx^4+c}}{8(ab^5c^2d-2a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2ab^5cd^2+a^2b^4d^3)x^4)}, -\frac{(4a^2bcd-3a^3d^2+(4ab^2cd-3a^2bd^2)x^4)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-(2ab^3c^2-5a^2b^2cd+3a^3bd^2+2(b^4c^2-2ab^3cd+a^2b^2d^2)x^4)\sqrt{dx^4+c}}{4(ab^5c^2d-2a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2ab^5cd^2+a^2b^4d^3)x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(b^2*
c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*
d))/(b*x^4 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b
^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4), -1
/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - (2
*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b
^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
+ (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [A]
time = 1.21, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^4+c}\,a^2d}{4(b^3c-ab^2d)((dx^4+c)b-bc+ad)}-\frac{(4abc-3a^2d)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4(b^3c-ab^2d)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^4+c}}{2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(d*x^4 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^4 + c)*b - b*c + a*d))
- 1/4*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/2*sqrt(d*x^4 + c)/(b^2*d)
```

Mupad [B]
time = 5.12, size = 144, normalized size = 1.17

$$\frac{\sqrt{d\,x^4+c}}{2\,b^2\,d}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\left(3\,a\,d-4\,b\,c\right)}{\left(3\,a^2\,d-4\,a\,b\,c\right)\,\sqrt{a\,d-b\,c}}\right)\,\left(3\,a\,d-4\,b\,c\right)}{4\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}+\frac{a^2\,d\,\sqrt{d\,x^4+c}}{2\,\left(a\,d-b\,c\right)\,\left(2\,b^3\,\left(d\,x^4+c\right)-2\,b^3\,c+2\,a\,b^2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^4)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(4*b^(5/
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^3*(c
+ d*x^4) - 2*b^3*c + 2*a*b^2*d))
```

3.824 $\int \frac{x^7}{(a+bx^4)^2 \sqrt{c+dx^4}} \, dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/4*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/4*a*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A]
time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 79, 65, 214}

$$\frac{a\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{8b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4bd(bc-ad)}\\
&= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.19, size = 100, normalized size = 1.01

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)} - \frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(4
*b^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(83) = 166.
time = 0.35, size = 867, normalized size = 8.76

method	result
elliptic	$\frac{\sqrt{-ab}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8b^2(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}+\frac{ad\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-\cdots}}{}\right.}{b}\cdots}{\cdots}\right.}{\cdots}$
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2
-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1
/2)))-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2
+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1
/2))))-a/b*(-1/8/a*(-a*b)^(1/2)/b/(a*d-b*c)/(x^2-1/b*(-a*b)^(1/2))*((x^2-1/
b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)-1/8/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^
(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2
))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1
/b*(-a*b)^(1/2)))+1/8/a*(-a*b)^(1/2)/b/(a*d-b*c)/(x^2+1/b*(-a*b)^(1/2))*((x
^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)-1/8/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-
a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)
^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x^2+1/b*(-a*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.75, size = 348, normalized size = 3.52

$$\left[\frac{\left((2\,b^2c-abd)x^4+2\,abc-a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}\,(ab^2c-a^2bd)}{8\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^4\right)},\ \frac{\left((2\,b^2c-abd)x^4+2\,abc-a^2d\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)+\sqrt{dx^4+c}\,(ab^2c-a^2bd)}{4\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((2*b^2*c - a*b*d)*x^4 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*
d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) +
 2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^4), 1/4*(((2*b^2*c - a*b*d)
*x^4 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)
*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**7/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [A]
time = 1.27, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^4+c}\,ad^2}{(b^2c-abd)\left((dx^4+c)b-bc+ad\right)}+\frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(d*x^4 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^4 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B]
time = 4.93, size = 95, normalized size = 0.96

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)\,\left(a\,d-2\,b\,c\right)}{4\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{a\,d\,\sqrt{d\,x^4+c}}{2\,b\,\left(a\,d-b\,c\right)\,\left(2\,b\,\left(d\,x^4+c\right)+2\,a\,d-2\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(4*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^4)^(1/2))/(2*b*(a*d - b*c)*(2*b*(c +
 d*x^4) + 2*a*d - 2*b*c))
```

3.825 $\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{3/2}}$$

```
[Out] 1/4*d*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/4*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {455, 44, 65, 214}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}}{4\,(a+bx^4)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^4]/((b*c - a*d)*(a + b*x^4)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^4])/Sqrt[b*c - a*d]])/(4*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^4\right)\\
&= -\frac{\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{8(bc-ad)}\\
&= -\frac{\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{4(bc-ad)}\\
&= -\frac{\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 86, normalized size = 0.99

$$\frac{1}{4}\left(-\frac{\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-(Sqrt[c + d*x^4]/((b*c - a*d)*(a + b*x^4))) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^4])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(71) = 142$.
time = 0.32, size = 541, normalized size = 6.22

method	result
default	$-\frac{\sqrt{-ab}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\,\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8ab(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2-\right.}{b}\right.}{}$
elliptic	$-\frac{\sqrt{-ab}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\,\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8ab(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x^2-\right.}{b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a*(-a*b)^(1/2)/b/(a*d-b*c)/(x^2-1/b*(-a*b)^(1/2))*((x^2-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/8/b*
d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2
-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1
/2)))+1/8/a*(-a*b)^(1/2)/b/(a*d-b*c)/(x^2+1/b*(-a*b)^(1/2))*((x^2+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1
/8/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-
2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*
b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(71) = 142$.
time = 2.36, size = 302, normalized size = 3.47

$$\left[-\frac{(bdx^4+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\sqrt{dx^4+c}(b^2c-abd)}{8(ab^3c^2-2a^2b^2cd+a^3bd^2+(b^4c^2-2ab^3cd+a^2b^2d^2)x^4)}, -\frac{(bdx^4+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)+\sqrt{dx^4+c}(b^2c-abd)}{4(ab^3c^2-2a^2b^2cd+a^3bd^2+(b^4c^2-2ab^3cd+a^2b^2d^2)x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*d*x^4 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*s
qrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c
 - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d
+ a^2*b^2*d^2)*x^4), -1/4*((b*d*x^4 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt
(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(b^2*c
- a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2)*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [A]
time = 1.62, size = 93, normalized size = 1.07

$$-\frac{d\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx^4+c}\,d}{4((dx^4+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/4*sqrt(d*x^4 + c)*d/(((d*x^4 + c)*b - b*c + a*d)*(b*c - a
*d))
```

Mupad [B]
time = 4.85, size = 84, normalized size = 0.97

$$\frac{d\sqrt{d\,x^4+c}}{2\,\left(a\,d-b\,c\right)\,\left(2\,b\,\left(d\,x^4+c\right)+2\,a\,d-2\,b\,c\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b*(c + d*x^4) + 2*a*d - 2*b*c)) + (
d*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(1/2)*(a*d - b*
c)^(3/2))
```

3.826 $\int \frac{1}{x(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=132

$$\frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}+\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}$$

```
[Out] 1/4*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/4
*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A]
time = 0.20, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 105, 162, 65, 214}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}+\frac{b\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ArcTanh[Sqrt[c + d*x^4]
/Sqrt[c]]/(2*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^4\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^4\right)}{4a^2} - \frac{(b(2bc-3ad))\text{S}}{}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2a^2d} - \frac{(b(2bc-3a}{}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\right.}{4a^2(bc-ad}$$

Mathematica [A]
time = 0.36, size = 124, normalized size = 0.94

$$\frac{-\frac{ab\sqrt{c+dx^4}}{(-bc+ad)(a+bx^4)}+\frac{\sqrt{b}\,(2bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-((a*b*Sqrt[c + d*x^4])/((-(b*c) + a*d)*(a + b*x^4))) + (Sqrt[b]*(2*b*c -
3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)
^(3/2) - (2*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/Sqrt[c])/(4*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(108) = 216$.
time = 0.36, size = 900, normalized size = 6.82

method	result
elliptic	$-\frac{b\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8\sqrt{-ab}\,a(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}+\frac{d\ln\left(-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}\cdots\right)}{\cdots}$
default	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d x^4+c}}{x^2}\right)}{2a^2\sqrt{c}}-\frac{b\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2\cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)-b/a^2*(-1/4/b/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x
^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))-1/4/b/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x
^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))-b/a*(-1/8
```

```
/a*(-a*b)^(1/2)/b/(a*d-b*c)/(x^2-1/b*(-a*b)^(1/2))*((x^2-1/b*(-a*b)^(1/2))^
2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/8/b*d/(a
*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2))
)+1/8/a*(-a*b)^(1/2)/b/(a*d-b*c)/(x^2+1/b*(-a*b)^(1/2))*((x^2+1/b*(-a*b)^(1
/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/8/b
*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^
2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(
1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x), x)
```

Fricas [A]
time = 3.30, size = 862, normalized size = 6.53

$$\left[\frac{2\sqrt{dx^4+c}\,abc+\left((2b^2c^2-3abcd)x^4+2abc^2-3a^2cd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+2\left((b^2c-abd)x^4+abc-a^2d\right)\sqrt{c}\log\left(\frac{dx^4-2\sqrt{dx^4+c}\,\sqrt{c}+2c}{x^4}\right)}{8\left(a^3bc^2-a^4cd+(a^2b^2c^2-a^3bcd)x^4\right)}, \frac{\sqrt{dx^4+c}\,abc+\left((2b^2c^2-3abcd)x^4+2abc^2-3a^2cd\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)+\left((b^2c-abd)x^4+abc-a^2d\right)\sqrt{c}\log\left(\frac{dx^4-2\sqrt{dx^4+c}\,\sqrt{c}+2c}{x^4}\right)}{4\left(a^3bc^2-a^4cd+(a^2b^2c^2-a^3bcd)x^4\right)}, \frac{2\sqrt{dx^4+c}\,abc+4\left((b^2c-abd)x^4+abc-a^2d\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-c}}{c}\right)+\left((2b^2c^2-3abcd)x^4+2abc^2-3a^2cd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)}{8\left(a^3bc^2-a^4cd+(a^2b^2c^2-a^3bcd)x^4\right)}, \frac{\sqrt{dx^4+c}\,abc+\left((2b^2c^2-3abcd)x^4+2abc^2-3a^2cd\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)+2\left((b^2c-abd)x^4+abc-a^2d\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-c}}{c}\right)}{4\left(a^3bc^2-a^4cd+(a^2b^2c^2-a^3bcd)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 -
3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 +
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*((b^2*c - a*b*d)*x^4 +
 a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))
/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4
 + b*c)) + ((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqr
t(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3
*b*c*d)*x^4), 1/8*(2*sqrt(d*x^4 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^4 + a*b*c
 - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a*b
*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c
 - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)))/(
a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c)*
a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c
```

```
- a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 +
 b*c)) + 2*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^4
 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [A]
time = 0.89, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^4+c}\,bd}{4\left(abc-a^2d\right)\left(\left(dx^4+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(d*x^4 + c)*b*d/((a*b*c - a^2*d)*((d*x^4 + c)*b - b*c + a*d)) - 1/4
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

Mupad [B]
time = 5.87, size = 3017, normalized size = 22.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((2*a^6*b^2*d^5 - 3*a^5*b^
3*c*d^4 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c + d*
x^4)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a
^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a
^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*
b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*
c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)
```

^(1/2)*1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) +
((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*
(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4
 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^4)^(1
/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*
c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*
c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2
)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3
*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*
1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))/(((3*a*b
^3*d^4)/16 - (b^4*c*d^3)/8)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((((c +
 d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*(a^4*d^
2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*
b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c + d*x^4)^(1/2)*(3*
a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 -
 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*c^2 - 2
*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*
a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^
2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^
5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^4)^(1
/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*(a^4*d^2 + a^2*b^
2*c^2 - 2*a^3*b*c*d)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^
3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^4)^(1/2)*(3*a*d - 2*b*
c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b
^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d
)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*
c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d -
3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^
2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d -
b*c)^3)^(1/2)*1i)/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d
^2)) - (atan((((((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^
5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^4)^(1/2)*(64*a^7*b^2*d^5 -
256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(128*a^2*c^
(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c^(1/2)) - ((c + d*x^4
)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a
^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - ((((2*a^6*b^2*d^5 - 3*a^5*b
^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c
 + d*x^4)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 +
 320*a^5*b^4*c^2*d^3))/(128*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*
d)))/(4*a^2*c^(1/2)) + ((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 -
 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1
/2)))/(((3*a*b^3*d^4)/16 - (b^4*c*d^3)/8)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*
c*d) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^5*d^2 +
a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^4)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*
b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(128*a^2*c^(1/2)*(a

```
^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c^(1/2)) - ((c + d*x^4)^(1/2)*
(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^
4*b^4*c^2*d^3)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^4)^(1/
2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*
c^2*d^3))/(128*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c
^(1/2)) + ((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2))))*1i)/(2*a^2
*c^(1/2)) - (b*d*(c + d*x^4)^(1/2))/(2*(a^2*d - a*b*c)*(2*b*(c + d*x^4) + 2
*a*d - 2*b*c))
```

3.827 $\int \frac{1}{x^5(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=185

$$-\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)}-\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)}+\frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}}-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}$$

```
[Out] 1/4*(a*d+4*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/4*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/4*b*(-a*d+2*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^4+a)-1/4*
(d*x^4+c)^(1/2)/a/c/x^4/(b*x^4+a)
```

Rubi [A]
time = 0.17, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 105, 156, 162, 65, 214}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}}-\frac{b\sqrt{c+dx^4}\,(2bc-ad)}{4a^2c\left(a+bx^4\right)(bc-ad)}-\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(b*(2*b*c - a*d)*Sqrt[c + d*x^4])/(a^2*c*(b*c - a*d)*(a + b*x^4)) - Sq
rt[c + d*x^4]/(4*a*c*x^4*(a + b*x^4)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x
^4]/Sqrt[c]])/(4*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^3*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4(a+bx^4)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)}{4ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4}}{4acx^4(a+bx^4)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}\ldots}{x(a+bx)\sqrt{c+\ldots}}\right.}{4a^2c(bc-\ldots}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4}}{4acx^4(a+bx^4)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \ldots\right.}{8a^3(bc-\ldots}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4}}{4acx^4(a+bx^4)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \ldots\right.}{4a^3d(bc\ldots}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4}}{4acx^4(a+bx^4)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+\ldots}}{\sqrt{c}}\right.}{4a^3c^{3/2}}$$

Mathematica [A]
time = 0.65, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^4}\left(-a^2d+2b^2cx^4+ab(c-dx^4)\right)}{c(-bc+ad)x^4(a+bx^4)} - \frac{b^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{c^{3/2}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*Sqrt[c + d*x^4]*(-(a^2*d) + 2*b^2*c*x^4 + a*b*(c - d*x^4)))/(c*(-(b*c)
+ a*d)*x^4*(a + b*x^4)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcTanh
[Sqrt[c + d*x^4]/Sqrt[c]])/c^(3/2))/(4*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(157) = 314$.
time = 0.43, size = 954, normalized size = 5.16 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/4/c/x^4*(d*x^4+c)^(1/2)+1/4*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)
^(1/2))/x^2))+1/a^3*b/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)+2*b^2
/a^3*(-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^
2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*
(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(
1/2))-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^
2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*
(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(
1/2))))+b^2/a^2*(-1/8/a*(-a*b)^(1/2)/b/(a*d-b*c)/(x^2-1/b*(-a*b)^(1/2))*((x
^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)-1/8/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-
a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)
^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x^2-1/b*(-a*b)^(1/2)))+1/8/a*(-a*b)^(1/2)/b/(a*d-b*c)/(x^2+1/b*(-a*b)^(1/2)
)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)-1/8/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2
*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(
-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x^2+1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^5), x)
```

Fricas [A]
time = 4.16, size = 1236, normalized size = 6.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)
*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*
d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*log((d*x^4 +
2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c
^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^8 + (a
^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a
*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)
```

```
*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*
c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*l
og((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*(a^2*b*c^2 - a^3*c*d
+ (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2
*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(-c)*arcta
n(sqrt(d*x^4 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2
*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d -
 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*(a^2*b
*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*
c^3 - a^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/4*(((4*b^3*c^3 -
5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c))
 + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d
- a^3*d^2)*x^4)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + (a^2*b*c^2 -
a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a
^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [A]
time = 1.09, size = 257, normalized size = 1.39

$$\frac{\left(4\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(a^3bc-a^4d\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^4+c\right)^{\frac{3}{2}}b^2cd-2\sqrt{dx^4+c}\,b^2c^2d-\left(dx^4+c\right)^{\frac{3}{2}}abd^2+2\sqrt{dx^4+c}\,abcd^2-\sqrt{dx^4+c}\,a^2d^3}{4\left(a^2bc^2-a^3cd\right)\left(\left(dx^4+c\right)^2b-2\left(dx^4+c\right)bc+bc^2+\left(dx^4+c\right)ad-acd\right)}-\frac{\left(4\,bc+ad\right)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^3\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/4*(2*(d*x^4 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^4 + c)*b^2*c^2*d - (d*x^4 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^4 + c
)*a*b*c*d^2 - sqrt(d*x^4 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^4 + c)^
2*b - 2*(d*x^4 + c)*b*c + b*c^2 + (d*x^4 + c)*a*d - a*c*d)) - 1/4*(4*b*c +
a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B]
time = 7.05, size = 2500, normalized size = 13.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)

[Out] (((c + d*x^4)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 - a*c*d)) + (b*(c + d*x^4)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/((c + d*x^4)*(2*a*d - 4*b*c) + 2*b*(c + d*x^4)^2 + 2*b*c^2 - 2*a*c*d) + (atan((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^6)/32 + b^7*c^3*d^3 - (3*a*b^6*c^2*d^4)/2 + (3*a^2*b^5*c*d^5)/16)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^

```
5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(6
4*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^
4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*
c*d^2))))*(-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(4*(a^6*d^3 - a^3*
b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d*x^4)^(1/2)*(
a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*
b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((a^9*b^2*
c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c
^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^4)^(1/2)*(a*d + 4*b*c)*(128*a
^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2
*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a
*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(8*a^3*(c^3)^(1/2)) + (
(((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^
3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*
c^3*d)) - (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^
3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^4)^(1/2)
*(a*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3
*d^4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2
- 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(8
*a^3*(c^3)^(1/2)))/(((5*a^3*b^4*d^6)/32 + b^7*c^3*d^3 - (3*a*b^6*c^2*d^4)/2
 + (3*a^2*b^5*c*d^5)/16)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((((
c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b
^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3
*d)) + (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c
^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^...
```

3.828 $\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=191

$$\frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)}+\frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\,(a+bx^4)}+\frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}}-\frac{(bc+4ad)\tanh^{-}}{4b^3d}$$

```
[Out] 1/4*a^(3/2)*(-4*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1
/2))/b^3/(-a*d+b*c)^(3/2)-1/4*(4*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/
2))/b^3/d^(3/2)+1/4*(-2*a*d+b*c)*x^2*(d*x^4+c)^(1/2)/b^2/d/(-a*d+b*c)+1/4*a
*x^6*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A]

time = 0.22, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {476, 481, 596, 537, 223, 212, 385, 211}

$$\frac{a^{3/2}(5bc-4ad)\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}}-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}}+\frac{x^2\sqrt{c+dx^4}\,(bc-2ad)}{4b^2d(bc-ad)}+\frac{ax^6\sqrt{c+dx^4}}{4b\,(a+bx^4)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((b*c - 2*a*d)*x^2*Sqrt[c + d*x^4])/(4*b^2*d*(b*c - a*d)) + (a*x^6*Sqrt[c +
 d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^3*(b*c - a*d)^(3/2)) -
 ((b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^3*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\text{Subst}\left(\int \frac{x^2(3ac-2(bc-2ad)x^2)}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{-2ac(bc-2ad)-2(bc-a}{(a+bx^2)\sqrt{c+}}\right.}{8b^2d(bc-}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(a^2(5bc-4ad))\,\text{Subst}\left(\int -\right.}{4b^3(bc}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(a^2(5bc-4ad))\,\text{Subst}\left(\int \frac{}{a}\right.}{4b^3(b}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{4b^3(bc-ad)^{3/}}$$

Mathematica [A]
time = 1.66, size = 188, normalized size = 0.98

$$\frac{\frac{bx^2\sqrt{c+dx^4}\,(-2a^2d+b^2cx^4+ab(c-dx^4))}{d(bc-ad)(a+bx^4)} + \frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}\,x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{d}\,x^2}\right)}{d^{3/2}}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((b*x^2*Sqrt[c + d*x^4]*(-2*a^2*d + b^2*c*x^4 + a*b*(c - d*x^4)))/(d*(b*c -
 a*d)*(a + b*x^4)) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*x^2*(Sq
rt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(b*c - a*d)^(3/2)
 - ((b*c + 4*a*d)*ArcTanh[Sqrt[c + d*x^4]/(Sqrt[d]*x^2)])/d^(3/2))/(4*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. $2(163) = 326$.
time = 0.41, size = 1278, normalized size = 6.69 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/b^2*(1/4*x^2/d*(d*x^4+c)^(1/2)-1/4*c/d^(3/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1/
2)))-a/b^3*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))/d^(1/2)+3*a^2/b^3*(-1/4/(-a*b)^(
1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1
/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2
d(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b
)^(1/2))))-a^3/b^3*(-1/8/a/(a*d-b*c)/(x^2-1/b*(-a*b)^(1/2))*((x^2-1/b*(-a*b
)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1
/8/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d
*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a
*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x^2-1/b*(-a*b)^(1/2)))-1/8/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1/8/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b
c)/b)^(1/2)((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))-1/8/a/(a*d-b*c)/(x^2+1/
b*(-a*b)^(1/2))*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/8/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^13/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)

Fricas [A]
time = 4.15, size = 1386, normalized size = 7.26

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [1/16*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*
a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) +

```
(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b
*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2
- a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4
+ a^2)) + 4*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqr
t(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), 1/
16*(4*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2
*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + (5*a^2*b*c*d^2
 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b*c - a*d))*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^
2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)
*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*((b
^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqrt(d*x^4 + c))/(
a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), -1/8*((5*a^2*b*c*
d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*ar
ctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*
d*x^6 + a*c*x^2)) - (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b
^2*c*d - 4*a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)
*x^2 - c) - 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*s
qrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4),
1/8*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^
2*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - (5*a^2*b*c*d^
2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*arct
an(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*
x^6 + a*c*x^2)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*
x^2)*sqrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*
x^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**13/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(163) = 326.
time = 1.78, size = 337, normalized size = 1.76

$$-\frac{\left(5\,a^2bc\sqrt{d}-4\,a^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,(b^4c-ab^3d)\sqrt{abcd-a^2d^2}}+\frac{\sqrt{dx^4+c}\,x^2}{4\,b^2d}+\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2a^2bc\sqrt{d}-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2a^3d^{\frac{3}{2}}-a^2bc^2\sqrt{d}}{2\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)(b^4c-ab^3d)}+\frac{\left(bc\sqrt{d}+4\,ad^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2\right)}{8\,b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(5*a^2*b*c*sqrt(d) - 4*a^3*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*
x^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^4*c - a*b^3*d)*sq
rt(a*b*c*d - a^2*d^2)) + 1/4*sqrt(d*x^4 + c)*x^2/(b^2*d) + 1/2*((sqrt(d)*x^
2 - sqrt(d*x^4 + c))^2*a^2*b*c*sqrt(d) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^
2*a^3*d^(3/2) - a^2*b*c^2*sqrt(d))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b -
2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))
^2*a*d + b*c^2)*(b^4*c - a*b^3*d)) + 1/8*(b*c*sqrt(d) + 4*a*d^(3/2))*log((s
qrt(d)*x^2 - sqrt(d*x^4 + c))^2)/(b^3*d^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.829 $\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=141

$$\frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\,(a+bx^4)}-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

```
[Out] -1/4*(-2*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a^
(1/2)/b^2/(-a*d+b*c)^(3/2)+1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d^(
1/2)+1/4*a*x^2*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A]
time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {476, 481, 537, 223, 212, 385, 211}

$$-\frac{\sqrt{a}\,(3bc-2ad)\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}}+\frac{ax^2\sqrt{c+dx^4}}{4b\,(a+bx^4)\,(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x^2*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b^2*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)$$
$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4b(bc-ad)}$$
$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b^2} - \frac{(a(3bc-2ad))\text{Subs}}{}$$
$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2} - \frac{(a(3bc-2ad))\text{S}}{}$$
$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\Big(}{2b}$$

Mathematica [**A**]
time = 1.30, size = 152, normalized size = 1.08

$$\frac{\frac{abx^2\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)} + \frac{\sqrt{a}(-3bc+2ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^4+bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{d}\,x^2}\right)}{\sqrt{d}}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*b*x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*S
qrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*ArcTanh[Sqrt[c + d*x^4]/(Sqrt[d]*x
^2)])/Sqrt[d])/(4*b^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. 2(117) = 234.
time = 0.34, size = 1228, normalized size = 8.71 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^2*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))/d^(1/2)-2*a/b^2*(-1/4/(-a*b)^(1/2)/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/
```

```
b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1/4/(-
a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-
a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/
2))))+a^2/b^2*(-1/8/a/(a*d-b*c)/(x^2-1/b*(-a*b)^(1/2))*((x^2-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8/b/
a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*
b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(
1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^
2-1/b*(-a*b)^(1/2)))-1/8/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2
-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1/8/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))-1/8/a/(a*d-b*c)/(x^2+1/b*(-a
*b)^(1/2))*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)-1/8/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [A]
time = 4.30, size = 1077, normalized size = 7.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] [1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 + 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4
*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x
^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a*b^3*c*d - a^
```

```
2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 -
 8*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(
d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a
/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c
^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x
^4 + a^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*s
qrt(d*x^4 + c)*a*b*d*x^2 + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2
*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4
+ c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + 2*((b^2*c - a*b*d)*x^4 + a*
b*c - a^2*d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(a*
b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*
a*b*d*x^2 - 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d
)*x^2/sqrt(d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d
^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^
4*c*d - a*b^3*d^2)*x^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**9/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(117) = 234.
time = 1.27, size = 298, normalized size = 2.11

$$-\frac{\left(3\,abc\sqrt{d}-2\,a^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,(b^3c-ab^2d)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{2\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)(b^3c-ab^2d)}-\frac{\log\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2\right)}{4\,b^2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x
^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*sqrt(d
) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((s
qrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c
 + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/
4*log((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2)/(b^2*sqrt(d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.830 $\int \frac{x^5}{(a+bx^4)^2 \sqrt{c+dx^4}} dx$

Optimal. Leaf size=93

$$-\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}(bc-ad)^{3/2}}$$

```
[Out] 1/4*c*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/(-a*d+b*c)^(3/2)
/a^(1/2)-1/4*x^2*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {476, 482, 12, 385, 211}

$$\frac{c\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}(bc-ad)^{3/2}}-\frac{x^2\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (c*ArcTan[(Sqrt[b*c
- a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
```

```
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{c}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} + \frac{c\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} + \frac{c\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} + \frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}\,(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.76, size = 112, normalized size = 1.20

$$
\frac{1}{4}\left(-\frac{x^2\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)} + \frac{c\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}\,x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}\right)
$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]

[Out] (-((x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4))) + (c*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*(b*c - a*d)^(3/2)))/4

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. $2(77) = 154$.

time = 0.35, size = 1199, normalized size = 12.89

method	result
elliptic	$\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8b(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-}}{b}\right.}{b}}{}\right.}{}$ $-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}$
default	

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/b*(-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))-a/b*(-1/8/a/(a*d-b*c)/(x^2-1/b*(-a*b)^(1/2))*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(

```
a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))-1/8/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1/8/a/(-a*b)^(1/2)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(
x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))-1/8/a/(a*
d-b*c)/(x^2+1/b*(-a*b)^(1/2))*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/
b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/8/b/a*d*(-a*b)^(1/2)/(a*d-b*c
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(77) = 154$.
time = 3.86, size = 426, normalized size = 4.58

$$\left[-\frac{4\sqrt{dx^4+c}\,(abc-a^2d)x^2-(bcx^4+ac)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((bc-2\,ad)x^6-acx^2)\sqrt{dx^4+c}\,\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)}{16\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^4)}, -\frac{2\sqrt{dx^4+c}\,(abc-a^2d)x^2-(bcx^4+ac)\sqrt{abc-a^2d}\,\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\,\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)}{8\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] [-1/16*(4*sqrt(d*x^4 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2
*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(
-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d +
 a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4), -1/8*(2*sqrt(d*x^4
 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*
((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a
^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d
^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(77) = 154.
time = 3.07, size = 244, normalized size = 2.62

$$\frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{4\sqrt{abcd-a^2d^2}\,(bc-ad)}+\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{2\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)(b^2c-abd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/2*((s
qrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 +
 c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2
*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^
2*a*d + b*c^2)*(b^2*c - a*b*d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.831 $\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=104

$$\frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/4*(-2*a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(3/
2)/(-a*d+b*c)^(3/2)+1/4*b*x^2*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {476, 390, 385, 211}

$$\frac{(bc-2ad)\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}+\frac{bx^2\sqrt{c+dx^4}}{4a\,(a+bx^4)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
```

```
tQ[q, -1]) && NeQ[p, -1]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)\\ &= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)}+\frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4a(bc-ad)}\\ &= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)}+\frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)}\\ &= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.50, size = 124, normalized size = 1.19

$$-\frac{bx^2\sqrt{c+dx^4}}{4a(-bc+ad)(a+bx^4)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^4+bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(b*x^2*Sqrt[c + d*x^4])/(a*(-(b*c) + a*d)*(a + b*x^4)) + ((b*c - 2*a*d
)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[
b*c - a*d])])/(4*a^(3/2)*(b*c - a*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(88) = 176$.
time = 0.33, size = 867, normalized size = 8.34

method	result
default	$-\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8a(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\right.}{b}}{}\right.}{}$
elliptic	$-\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8a(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\right.}{b}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a/(a*d-b*c)/(x^2-1/b*(-a*b)^(1/2))*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a
*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8/b/a*d*(-a*b)^(1/2
)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2
-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(
-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1
/2)))-1/8/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)
^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/
2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-
1/b*(-a*b)^(1/2)))+1/8/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1
/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x^2+1/b*(-a*b)^(1/2)))-1/8/a/(a*d-b*c)/(x^2+1/b*(-a*b)^(1/2))*((x^
2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)-1/8/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((
x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(88) = 176$.
time = 3.48, size = 467, normalized size = 4.49

$$\left[\frac{4\,\sqrt{dx^4+c}\,(ab^2c-a^2bd)x^2-((b^2c-2\,abd)x^4+abc-2\,a^2d)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((bc-2\,ad)x^6-acx^2)\sqrt{dx^4+c}\,\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)}{16\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x^4)}, \frac{2\,\sqrt{dx^4+c}\,(ab^2c-a^2bd)x^2+((b^2c-2\,abd)x^4+abc-2\,a^2d)\sqrt{abc-a^2d}\,\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\,\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)}{8\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 - ((b^2*c - 2*a*b*d)*x^4 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a
*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))
/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*
b*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 + ((b^2*c - 2*a
*b*d)*x^4 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*
x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (
a*b*c^2 - a^2*c*d)*x^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c
^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(88) = 176$.
time = 1.61, size = 237, normalized size = 2.28

$$-\frac{1}{4}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2ad-bc^2\right)}{\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)(abcd-a^2d^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*
x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.832 $\int \frac{1}{x^3(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=149

$$-\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\,(a+bx^4)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}$$

```
[Out] -1/4*b*(-4*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/
a^(5/2)/(-a*d+b*c)^(3/2)-1/4*(-2*a*d+3*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c
)/x^2+1/4*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^2/(b*x^4+a)
```

Rubi [A]
time = 0.13, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 483, 597, 12, 385, 211}

$$-\frac{b(3bc-4ad)\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}\,(3bc-2ad)}{4a^2cx^2(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^2\,(a+bx^4)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*((3*b*c - 2*a*d)*Sqrt[c + d*x^4])/(a^2*c*(b*c - a*d)*x^2) + (b*Sqrt[c
+ d*x^4])/(4*a*(b*c - a*d)*x^2*(a + b*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sq
rt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(5/2)*(b*c - a*d)^(3/2)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2 (a+bx^2)^2 \sqrt{c+dx^2}} \, dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2 (a+bx^4)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}} \, dx, x, x^2\right)}{4a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2 (a+bx^4)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-4a}{(a+bx^2)\sqrt{c+}}\right.}{4a^2c(bc-}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2 (a+bx^4)} - \frac{(b(3bc-4ad))\text{Subst}\left(\right.}{4a}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2 (a+bx^4)} - \frac{(b(3bc-4ad))\text{Subst}\left(\right.}{}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2 (a+bx^4)} - \frac{b(3bc-4ad)\tan^{-1}\left(\right.}{4a^{5/2}(bc-}$$

Mathematica [A]
time = 0.99, size = 157, normalized size = 1.05

$$\frac{\sqrt{c+dx^4}\,(2abc-2a^2d+3b^2cx^4-2abdx^4)}{4a^2c(-bc+ad)x^2\,(a+bx^4)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^4+bx^2\sqrt{c+dx^4}}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{4a^{5/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^4 - 2*a*b*d*x^4))/(4*a^2*c*
(-(b*c) + a*d)*x^2*(a + b*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*
Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(5/2)
*(b*c - a*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1220 vs. $2(129) = 258$.
time = 0.40, size = 1221, normalized size = 8.19 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/x^2*(d*x^4+c)^(1/2)/c-b/a^2*(-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1/4/(-a*b)^(1/2)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))-b/a*(-1/8/a/(
a*d-b*c)/(x^2-1/b*(-a*b)^(1/2))*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2
)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8/b/a*d*(-a*b)^(1/2)/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1
/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))-1/
8/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b
*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+
2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*
b)^(1/2)))+1/8/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b
)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x^2+1/b*(-a*b)^(1/2)))-1/8/a/(a*d-b*c)/(x^2+1/b*(-a*b)^(1/2))*((x^2+1/b*(-
a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)-1/8/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*
(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x^2+1/b*(-a*b)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(129) = 258.
time = 2.76, size = 612, normalized size = 4.11

$$\left[-\frac{\left((3\,b^3c^2-4\,ab^2cd)x^6+(3\,ab^2c^2-4\,a^2bcd)x^2\right)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((bc-2\,ad)x^6-acx^2)\sqrt{dx^4+c}\,\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)+4\,(2\,a^2b^2c^2-4\,a^3bcd+2\,a^4d^2+(3\,ab^3c^2-5\,a^2b^2cd+2\,a^3bd^2)x^4)\sqrt{dx^4+c}}{16\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2)x^6+(a^4b^2c^3-2\,a^5bc^2d+a^6cd^2)x^2)},\ -\frac{\left((3\,b^3c^2-4\,ab^2cd)x^6+(3\,ab^2c^2-4\,a^2bcd)x^2\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\,\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)+2\,(2\,a^2b^2c^2-4\,a^3bcd+2\,a^4d^2+(3\,ab^3c^2-5\,a^2b^2cd+2\,a^3bd^2)x^4)\sqrt{dx^4+c}}{8\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2)x^6+(a^4b^2c^3-2\,a^5bc^2d+a^6cd^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*s
qrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c
```

```
^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4
+ c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*(2*a^2*b^2*c^2
- 4*a^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4
)*sqrt(d*x^4 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^
4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^2), -1/8*(((3*b^3*c^2 - 4*a*b^2*c*
d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*((
b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2
*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) + 2*(2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*
a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4)*sqrt(d*x^4 + c))
/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^4*b^2*c^3 - 2*a^5*
b*c^2*d + a^6*c*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(129) = 258.
time = 3.17, size = 418, normalized size = 2.81

$$\frac{1}{4}\, d^{\frac{5}{2}} \left(\frac{(3\,b^2c - 4\,abd) \arctan\left(\frac{\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2d^2}} \right)}{(a^2bcd^2 - a^3d^3)\sqrt{abcd - a^2d^2}} + \frac{2\left(3\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^4 b^2c - 4\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^4 abd - 6\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^2 b^2c^2 + 14\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^2 abcd - 8\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^2 a^2d^2 + 3\,b^2c^3 - 2\,abc^2d\right)}{\left(\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^6 b - 3\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^4 bc + 4\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^4 ad + 3\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^2 bc^2 - 4\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^2 acd - bc^3\right)(a^2bcd^2 - a^3d^3)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b^2*c - 4*(sq
rt(d)*x^2 - sqrt(d*x^4 + c))^4*a*b*d - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*
b^2*c^2 + 14*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*d - 8*(sqrt(d)*x^2 - s
qrt(d*x^4 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x^2 - sqrt(
d*x^4 + c))^6*b - 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c + 4*(sqrt(d)*x^2
- sqrt(d*x^4 + c))^4*a*d + 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^2 - 4*(s
qrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(b\,x^4+a\right)}^2\,\sqrt{d\,x^4+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.833 $\int \frac{1}{x^7(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=208

$$-\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6}+\frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6(a+bx^4)}+\frac{b^2(5bc-6ad)\tan^{-}}{4a^{7/2}(b}$$

```
[Out] 1/4*b^2*(-6*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))
/a^(7/2)/(-a*d+b*c)^(3/2)-1/12*(-2*a*d+5*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^6+1/12*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^4+c)^(1/2)/a^3/c^2/(-a*
d+b*c)/x^2+1/4*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^6/(b*x^4+a)
```

Rubi [A]
time = 0.22, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 483, 597, 12, 385, 211}

$$\frac{b^2(5bc-6ad)\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{12a^2cx^6(bc-ad)}+\frac{\sqrt{c+dx^4}(-4a^2d^2-8abcd+15b^2c^2)}{12a^3c^2x^2(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^6(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/12*((5*b*c - 2*a*d)*Sqrt[c + d*x^4])/(a^2*c*(b*c - a*d)*x^6) + ((15*b^2*
c^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^4])/(12*a^3*c^2*(b*c - a*d)*x^2)
+ (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x^6*(a + b*x^4)) + (b^2*(5*b*c - 6*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(7/2)*(b*
c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}} \, dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6 (a+bx^4)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4(a+bx^2)\sqrt{c+dx^2}} \, dx, x, x^2\right)}{4a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6 (a+bx^4)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8abcd}{x^2(a+b}\right.}{12a}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{}}{4a(bc-}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{}}{4a(bc-}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{}}{4a(bc-}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{}}{4a(bc-}$$

Mathematica [A]
time = 1.53, size = 201, normalized size = 0.97

$$-\frac{\sqrt{c+dx^4}\,(15b^3c^2x^8+2ab^2cx^4(5c-4dx^4)+2a^3d(c-2dx^4)-2a^2b(c^2+3cdx^4+2d^2x^8))}{12a^3c^2(-bc+ad)x^6\,(a+bx^4)} + \frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}\,x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{4a^{7/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/12*(Sqrt[c + d*x^4]*(15*b^3*c^2*x^8 + 2*a*b^2*c*x^4*(5*c - 4*d*x^4) + 2*
a^3*d*(c - 2*d*x^4) - 2*a^2*b*(c^2 + 3*c*d*x^4 + 2*d^2*x^8)))/(a^3*c^2*(-(b
*c) + a*d)*x^6*(a + b*x^4)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] + b*x^
2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(7/2)*(
b*c - a*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1251 vs. $2(184) = 368$.
time = 0.41, size = 1252, normalized size = 6.02 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/6/a^2*(d*x^4+c)^(1/2)*(-2*d*x^4+c)/c^2/x^6+1/a^3*b/x^2*(d*x^4+c)^(1/2)/c +2*b^2/a^3*(-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/ (x^2-1/b*(-a*b)^(1/2)))+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b *c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^ 2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c) /b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))+b^2/a^2*(-1/8/a/(a*d-b*c)/(x^2-1/b*(-a* b)^(1/2))*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1 /2))-(a*d-b*c)/b)^(1/2)+1/8/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/ 2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b* c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^ (1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))-1/8/a/(-a*b)^(1/2)/(-(a* d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x^ 2-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2-1/b*(-a*b)^(1/2)))+1/8/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+1/b *(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2)))-1/8/a/(a*d-b*c)/(x^2+1/b*(-a*b)^(1/2))*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d*(- a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/8/b/a*d*(-a*b)^(1/ 2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^ 2+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x^2+1/b*(-a*b)^(1/2))^2*d-2*d* (-a*b)^(1/2)/b*(x^2+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x^2+1/b*(-a*b)^(1/2))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^7), x)

Fricas [A]
time = 4.12, size = 760, normalized size = 3.65

$$\left[-\frac{3\left(\left(5\,b^{4}c^{3}-6\,ab^{3}c^{2}d\right)x^{10}+\left(5\,ab^{3}c^{3}-6\,a^{2}b^{2}c^{2}d\right)x^{6}\right)\sqrt{-abc+a^{2}d}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{8}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{4}+a^{2}c^{2}-4\left(\left(bc-2\,ad\right)x^{6}-acx^{2}\right)\sqrt{dx^{4}+c}\,\sqrt{-abc+a^{2}d}}{b^{2}x^{8}+2\,abx^{4}+a^{2}}\right)-4\left(\left(15\,ab^{4}c^{3}-23\,a^{2}b^{3}c^{2}d+4\,a^{3}b^{2}cd^{2}+4\,a^{4}bd^{3}\right)x^{8}-2\,a^{3}b^{2}c^{3}+4\,a^{4}bc^{2}d-2\,a^{5}cd^{2}+2\left(5\,a^{2}b^{3}c^{3}-8\,a^{3}b^{2}c^{2}d+a^{4}bcd^{2}+2\,a^{5}d^{3}\right)x^{4}\right)\sqrt{dx^{4}+c}}{48\left(\left(a^{4}b^{3}c^{4}-2\,a^{5}b^{2}c^{3}d+a^{6}bc^{2}d^{2}\right)x^{10}+\left(a^{5}b^{2}c^{4}-2\,a^{6}bc^{3}d+a^{7}c^{2}d^{2}\right)x^{6}\right)},\ \frac{3\left(\left(5\,b^{4}c^{3}-6\,ab^{3}c^{2}d\right)x^{10}+\left(5\,ab^{3}c^{3}-6\,a^{2}b^{2}c^{2}d\right)x^{6}\right)\sqrt{abc-a^{2}d}\,\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^{4}-ac\right)\sqrt{dx^{4}+c}\,\sqrt{abc-a^{2}d}}{2\left(\left(abcd-a^{2}d^{2}\right)x^{6}+\left(abc^{2}-a^{2}cd\right)x^{2}\right)}\right)+2\left(\left(15\,ab^{4}c^{3}-23\,a^{2}b^{3}c^{2}d+4\,a^{3}b^{2}cd^{2}+4\,a^{4}bd^{3}\right)x^{8}-2\,a^{3}b^{2}c^{3}+4\,a^{4}bc^{2}d-2\,a^{5}cd^{2}+2\left(5\,a^{2}b^{3}c^{3}-8\,a^{3}b^{2}c^{2}d+a^{4}bcd^{2}+2\,a^{5}d^{3}\right)x^{4}\right)\sqrt{dx^{4}+c}}{24\left(\left(a^{4}b^{3}c^{4}-2\,a^{5}b^{2}c^{3}d+a^{6}bc^{2}d^{2}\right)x^{10}+\left(a^{5}b^{2}c^{4}-2\,a^{6}bc^{3}d+a^{7}c^{2}d^{2}\right)x^{6}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^6)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sq
rt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*((15*a
*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^8 - 2*a^3*b^
2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d +
a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^
3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^6
), 1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2
*d)*x^6)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^
4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x
^2)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)
*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a
^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4
 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a
^7*c^2*d^2)*x^6)]
```

<u>**Sympy**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \left(a + bx^4\right)^2 \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

<u>**Giac**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(184) = 368.
time = 3.68, size = 395, normalized size = 1.90

$$\frac{1}{12}\, d^{\frac{7}{2}} \left(\frac{3 \left(5\, b^3 c - 6\, a b^2 d\right) \arctan\left(-\frac{\left(\sqrt{d}\, x^2 - \sqrt{dx^4 + c}\right)^2 b - bc + 2\, ad}{2\, \sqrt{abcd - a^2 d^2}}\right)}{\left(a^3 bcd^3 - a^4 d^4\right) \sqrt{abcd - a^2 d^2}} - \frac{6 \left(\left(\sqrt{d}\, x^2 - \sqrt{dx^4 + c}\right)^2 b^3 c - 2 \left(\sqrt{d}\, x^2 - \sqrt{dx^4 + c}\right)^2 a b^2 d - b^3 c^2\right)}{\left(a^3 bcd^3 - a^4 d^4\right) \left(\left(\sqrt{d}\, x^2 - \sqrt{dx^4 + c}\right)^4 b - 2 \left(\sqrt{d}\, x^2 - \sqrt{dx^4 + c}\right)^2 bc + 4 \left(\sqrt{d}\, x^2 - \sqrt{dx^4 + c}\right)^2 ad + bc^2\right)} - \frac{8 \left(3 \left(\sqrt{d}\, x^2 - \sqrt{dx^4 + c}\right)^4 b - 6 \left(\sqrt{d}\, x^2 - \sqrt{dx^4 + c}\right)^2 bc - 3 \left(\sqrt{d}\, x^2 - \sqrt{dx^4 + c}\right)^2 ad + 3\, bc^2 + acd\right)}{\left(\left(\sqrt{d}\, x^2 - \sqrt{dx^4 + c}\right)^2 - c\right)^3 a^3 d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x^
4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4
)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b^3*c - 2
*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d
^4)*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)
```

```
*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 3*(
sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x^2 - sq
rt(d*x^4 + c))^2 - c)^3*a^3*d^3))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^7\,{\left(b\,x^4+a\right)}^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.834 $\int \frac{x^8}{(a+bx^4)^2 \sqrt{c+dx^4}} \, dx$

Optimal. Leaf size=996

$$\frac{ax\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)}-\frac{\sqrt[4]{-a}\,(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{16b^{7/4}(bc-ad)^{3/2}}+\frac{\sqrt[4]{-a}\,(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{-b}}{\sqrt[4]{-a}\,\sqrt[4]{}}\right.}{16b^{7/4}(-bc+ad)^{3/2}}$$

```
[Out] -1/16*(-a)^(1/4)*(-3*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4
)/(d*x^4+c)^(1/2))/b^(7/4)/(-a*d+b*c)^(3/2)+1/16*(-a)^(1/4)*(-3*a*d+5*b*c)*
arctan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(a*d-b
*c)^(3/2)+1/4*a*x*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)+1/8*(-3*a*d+4*b*c)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1
/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(-a*d+b*
c)/(d*x^4+c)^(1/2)-1/16*a*d^(1/4)*(-3*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^
(1/4)*x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2
)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(-a*d+
b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-3*a*d+5*b*c)*(cos(2*arctan(d^
(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2
*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(-a)^(1/2)*(c^(1/2)+x^2*d^(1/2))*(
b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/
2)/b^2/c^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/32*(-3*a*d+5*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(
1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2
))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^
2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/32*(-3*a*
d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/
c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)
-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(
1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2
)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1
/2)
```

Rubi [A]

time = 0.92, antiderivative size = 996, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {481, 537, 226, 418, 1231, 1721}

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((-a)^(1/4)*(5*b*c -
3*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(1
6*b^(7/4)*(b*c - a*d)^(3/2)) + ((-a)^(1/4)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[-(b
*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*b^(7/4)*(-(b*c) +
a*d)^(3/2)) + ((4*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b
^2*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) - (a*((Sqrt[b]*Sqrt[c])/Sqr
t[-a] + Sqrt[d])*d^(1/4)*(5*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(16*b^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*
(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(5*b*c - 3*a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(16*b^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c +
d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(5*b*c - 3*a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*
(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]),
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*b^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b
*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c
 - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^
2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sq
rt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*b^2*c^(1/4)*d^(1/4
)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\int \frac{ac+(-4bc+3ad)x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(4bc-3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b^2(bc-ad)} - \frac{(a(5bc-3ad))\int \frac{}{(a+b}}{4b^2(bc-}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(4bc-3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F}{8b^2\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)\sqrt{c+dx^4}}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(4bc-3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F}{8b^2\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)\sqrt{c+dx^4}}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt[4]{-a}\,(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{16b^{7/4}(bc-ad)^{3/2}} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.21, size = 253, normalized size = 0.25

$$\frac{x\left(\frac{(4bc-3ad)x^4\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{ab} + \frac{5a\left(c+dx^4+\frac{5ac^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}\right)}{b(a+bx^4)}\right)}{20(bc-ad)\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(((4*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/(a*b) + (5*a*(c + d*x^4 + (5*a*c^2*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4
, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((
d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]))))/(b*(a + b*x^4))))/(20*(b*c - a*d)*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 604, normalized size = 0.61

method	result
elliptic	$-\frac{ax\sqrt{d\,x^4+c}}{4(ad-bc)b(b\,x^4+a)}+\frac{\left(\frac{1}{b^2}-\frac{ad}{4b^2(ad-bc)}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}(}\right.}{}$
default	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b^2\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d\,x^4}}\right.}{\sqrt{\frac{-ad+bc}{b}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2
)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),
I)-1/4*a/b^3*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^
2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1
/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^
(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1
/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_al
pha=RootOf(_Z^4*b+a))+a^2/b^2*(-1/4*b/a/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+
a)-1/4*d/a/(a*d-b*c)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1
/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*
d^(1/2))^(1/2),I)-1/32/b/a*sum((-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d
+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x
^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*
x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/
c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^
(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**8/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^8/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.835 $\int \frac{x^4}{(a+bx^4)^2 \sqrt{c+dx^4}} \, dx$

Optimal. Leaf size=908

$$-\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}-\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}-$$

```
[Out] -1/16*(a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2
))/(-a)^(3/4)/b^(3/4)/(-a*d+b*c)^(3/2)+1/16*(a*d+b*c)*arctan(x*(a*d-b*c)^(1
/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-b*c)^(3/2)-
1/4*x*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)-1/8*d^(3/4)*(cos(2*arctan(d^(1/4
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arc
tan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1
/2)+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/16*d^(1/4)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/
(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^
2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/16*d^(1/4)*(cos(2*arctan(d^
(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2
*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1
/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^
(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^
(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^
(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/
2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d
^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/d^(1/4)/(-a
*d+b*c)/(d*x^4+c)^(1/2)+1/32*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-
1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/
2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^
2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/d^(1/4)/(-a*d+b*c)/
(d*x^4+c)^(1/2)
```

Rubi [A]

time = 0.66, antiderivative size = 908, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {482, 537, 226, 418, 1231, 1721}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(x*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) - ((b*c + a*d)*ArcTan[(S
qrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(3/4)*b^(
3/4)*(b*c - a*d)^(3/2)) + ((b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(3/4)*b^(3/4)*(-(b*c) + a*d)^(3/2
)) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2
)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
)/c^(1/4)], 1/2])/(16*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[-a]*S
qrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4
)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(16*a*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/
4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]
*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d]
)^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(32*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c]
- Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + S
qrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a
]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a*b*c^
(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} + \frac{\int \frac{c-dx^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)} + \frac{(bc+ad)\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\right.}{8b\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^4}}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\right.}{8b\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^4}}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}} + \frac{(bc+ad)}{16(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.16, size = 238, normalized size = 0.26

$$\frac{x\left(\frac{dx^4\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a}+\frac{5\left(c+dx^4+\frac{5ac^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}\right)}{a+bx^4}\right)}{20(-bc+ad)\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*((d*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)])/a + (5*(c + d*x^4 + (5*a*c^2*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^
4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b
*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)
/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(a +
b*x^4)))/(20*(-(b*c) + a*d)*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 530, normalized size = 0.58

method	result
elliptic	$\frac{x\sqrt{d\,x^4+c}}{4(ad-bc)(b\,x^4+a)}+\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4b(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}(\_Z^4b+a)} \frac{(ad+bc)\left(\cdots\right)}{\cdots}}{\cdots}$
default	$\frac{\sum_{\_\alpha=\operatorname{RootOf}(\_Z^4b+a)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\operatorname{EllipticPi}\left(x\sqrt{\cdots}\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^4+c}}}{\_\alpha^3}}{8b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b^2*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x
^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_
alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)
/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_
alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=R
ootOf(_Z^4*b+a))-a/b*(-1/4*b/a/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*d/
a/(a*d-b*c)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/
```

```
c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^
(1/2),I)-1/32/b/a*sum((-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^
(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/
2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2
)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d
^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/
c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**4/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^4/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.836 $\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=983

$$\frac{bx\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)}+\frac{\sqrt[4]{b}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(bc-ad)^{3/2}}-\frac{\sqrt[4]{b}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{-bc+\cdots}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{\cdots}}\right.}{16(-a)^{7/4}(-bc+ad)^{3/2}}$$

```
[Out] 1/16*b^(1/4)*(-5*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d
*x^4+c)^(1/2))/(-a)^(7/4)/(-a*d+b*c)^(3/2)-1/16*b^(1/4)*(-5*a*d+3*b*c)*arct
an(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(7/4)/(a*d-b*
c)^(3/2)+1/4*b*x*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)+1/8*d^(3/4)*(cos(2*
arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ellipt
icF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d
*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)
+1/16*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/
2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x
^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+
c)^(1/2)+1/16*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(
1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1
/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2
))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/(-a*d+b*c)/
(a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/
4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1
/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/
2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(
1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)+1/32*(-5*a*d+3*b*c)*(cos(2*arctan(d
^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin
(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(
-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/
2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/
a^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)
```

Rubi [A]

time = 0.77, antiderivative size = 983, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {425, 537, 226, 418, 1231, 1721}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + (b^(1/4)*(3*b*c - 5*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(
-a)^(7/4)*(b*c - a*d)^(3/2)) - (b^(1/4)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[-(b*c)
 + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(7/4)*(-(b*c) +
a*d)^(3/2)) + (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)
*(b*c - a*d)*Sqrt[c + d*x^4]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(
1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a*c^(1/4)*
(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqr
t[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*(
-a)^(3/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt
[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[
-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(
1/4)], 1/2])/(32*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4
]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*S
qrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d
)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} - \frac{\int \frac{-3bc+4ad-bdx^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} + \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{(3bc-5ad)\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}}{4a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\Big(2\tan^-}{8a\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^4}}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\Big(2\tan^-}{8a\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^4}}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} + \frac{\sqrt[4]{b}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(bc-ad)^{3/2}} - \frac{\sqrt[4]{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.20, size = 392, normalized size = 0.40

$$\frac{-5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\left(5a(4bc-4ad+bdx^4)+bdx^4(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)+2bx^5\left(5a(c+dx^4)+dx^4(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}{20a^2(bc-ad)(a+bx^4)\sqrt{c+dx^4}\left(-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)]*(5*a*(4*b*
c - 4*a*d + b*d*x^4) + b*d*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4
, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]) + 2*b*x^5*(5*a*(c + d*x^4) + d*
x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c)
, -((b*x^4)/a)])*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/
a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))/(20*a^2*
(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4,
-((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*
x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*
x^4)/a)])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 333, normalized size = 0.34

method	result
default	$-\frac{bx\sqrt{d\,x^{4}+c}}{4a(ad-bc)(b\,x^{4}+a)}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4a(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{(-5ad+3bc}{}}{}$
elliptic	$-\frac{bx\sqrt{d\,x^{4}+c}}{4a(ad-bc)(b\,x^{4}+a)}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4a(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{(-5ad+3bc}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/a/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*d/a/(a*d-b*c)/(I/c^(1/2)
*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(
1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/b/a*sum(
(-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_
alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/
2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)
*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2
)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)
)),_alpha=RootOf(_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^4+a\right)}^2\,\sqrt{d\,x^4+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.837 $\int \frac{1}{x^4(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1046

$$-\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)}+\frac{b^{5/4}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{16(-a)^{11/4}(bc-ad)^{3/2}}-\frac{b^{5/4}(7bc}{}$$

```
[Out] 1/16*b^(5/4)*(-9*a*d+7*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d
*x^4+c)^(1/2))/(-a)^(11/4)/(-a*d+b*c)^(3/2)-1/16*b^(5/4)*(-9*a*d+7*b*c)*arc
tan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(11/4)/(a*d-
b*c)^(3/2)-1/12*(-4*a*d+7*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x^3+1/4*b*(
d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^3/(b*x^4+a)-1/24*d^(3/4)*(-4*a*d+7*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(
(d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(5/4)/(-a*d+b*c)/(d*x^4+c)^(
1/2)+1/16*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/
2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4
))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))
*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/(-a*d+b*c)/(a
*d+b*c)/(d*x^4+c)^(1/2)-1/32*b*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/
4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1
/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/
2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^3/c^(1/4)/d^(
1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/16*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(
b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/
2)/(-a)^(5/2)/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/32*b*(-9*a*d+7
*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(
1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-
a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2
)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x
^2*d^(1/2))^2)^(1/2)/a^3/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 1.39, antiderivative size = 1046, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {483, 597, 537, 226, 418, 1231, 1721}

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/12*((7*b*c - 4*a*d)*Sqrt[c + d*x^4])/(a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c
 + d*x^4])/(4*a*(b*c - a*d)*x^3*(a + b*x^4)) + (b^(5/4)*(7*b*c - 9*a*d)*Arc
Tan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(11
/4)*(b*c - a*d)^(3/2)) - (b^(5/4)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[-(b*c) + a*d
]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(11/4)*(-(b*c) + a*d)^
(3/2)) - (d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/
(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(
24*a^2*c^(5/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a
]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)
/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/
(16*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (b*(Sqrt[
b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(16*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c +
 d*x^4]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(7*b*c - 9*a*d)*(Sqrt[
c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1
/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]
), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^3*c^(1/4)*d^(1/4)*(b*c - a*d)
*(b*c + a*d)*Sqrt[c + d*x^4]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(
7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*
x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^3*c^(1/4)*d
^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3(a+bx^4)} - \frac{\int \frac{-7bc+4ad-5bdx^4}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3(a+bx^4)} + \frac{\int \frac{-21b^2c^2+20abcd+4a^2d}{(a+bx^4)\sqrt{c-}}}{12a^2c(bc}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3(a+bx^4)} - \frac{(b(7bc-9ad))\int \frac{}{(a+}}{4a^2(bc-}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3(a+bx^4)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{}\right.}{}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3(a+bx^4)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{}\right.}{}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3(a+bx^4)} + \frac{b^{5/4}(7bc-9ad)\tan^{}}{16(-a)^{1}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.37, size = 408, normalized size = 0.39

$$\frac{bd(7bc-4ad)x^8\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\frac{a\left(25ac\left(-7b^2cx^4(4c+dx^4)+4a^2d(c+2dx^4)+4ab(-c^2+5cdx^4+d^2x^8)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+10x^4(c+dx^4)(-4a^2d+7b^2cx^4+4ab(c-dx^4))\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}{(a+bx^4)\left(-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}}{60a^3c(-bc+ad)x^3\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*d*(7*b*c - 4*a*d)*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)] + (a*(25*a*c*(-7*b^2*c*x^4*(4*c + d*x^4) + 4*a^2*d
*(c + 2*d*x^4) + 4*a*b*(-c^2 + 5*c*d*x^4 + d^2*x^8))*AppellF1[1/4, 1/2, 1,
5/4, -((d*x^4)/c), -((b*x^4)/a)] + 10*x^4*(c + d*x^4)*(-4*a^2*d + 7*b^2*c*x
^4 + 4*a*b*(c - d*x^4))*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((
b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/
((a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)]
 + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*
```

```
d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(60*a^3*c*(-(b
*c) + a*d)*x^3*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 626, normalized size = 0.60 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/3/c*(d*x^4+c)^(1/2)/x^3-1/3*d/c/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^
(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*El
lipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8/a^2*sum(1/_alpha^3*(-1/((-a*d+b
*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4
+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^
2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^
(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1
/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))-b/a*(-1/4*b/a/(a*d
-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*d/a/(a*d-b*c)/(I/c^(1/2)*d^(1/2))^(1/
2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c
)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/b/a*sum((-5*a*d+3*b*c
)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2
+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_al
pha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(
d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_al
pha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Roo
tOf(_Z^4*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(b\,x^4+a\right)^2 \sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.838 $\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1146

$$\frac{\sqrt{d}\,x\sqrt{c+dx^4}}{4b(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{16\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)\,\mathrm{ta}}{16\sqrt[4]{}}$$

```
[Out] 1/16*(-a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1
/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c)^(3/2)+1/16*(-a*d+3*b*c)*arctan(x*(a*d-b*
c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(a*d-b*c)^(
3/2)-1/4*x^3*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)+1/4*x*d^(1/2)*(d*x^4+c)^(
1/2)/b/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))-1/4*c^(1/4)*d^(1/4)*(cos(2*arctan(d
^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(
2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/
(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/8*c^(1/4)*d^(
1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1
/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*
d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/(d*x^4+c)^(
1/2)-1/16*d^(1/4)*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),
1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d
*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/(-a^2*d^2+b^2*c^2)/(
d*x^4+c)^(1/2)-1/32*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)
/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4)
)),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^
(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2
))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(-a*
d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-a*d+3*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(
b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/
2)/b^(3/2)/c^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)+1/32*(-a*d+3*b*c)*(co
s(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*El
lipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*
d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(
1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2
))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^4+
c)^(1/2)
```

Rubi [A]

time = 1.13, antiderivative size = 1146, normalized size of antiderivative = 1.00, number of

steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {482, 598, 311, 226, 1210, 504, 1231, 1721}

$$-\frac{\sqrt{dx^4+c}\,x^3}{4(bc-ad)(bx^4+a)}+\frac{\sqrt{d}\sqrt{dx^4+c}\,x}{4b(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)}+\frac{(3bc-ad)\text{ArcTan}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)\text{ArcTan}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16\sqrt[4]{-a}\,b^{5/4}(ad-bc)^{3/2}}-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b(bc-ad)\sqrt{dx^4+c}}+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b(bc-ad)\sqrt{dx^4+c}}-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32\sqrt{-a}\,b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32\sqrt{-a}\,b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)) - (x^
3*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) + ((3*b*c - a*d)*ArcTan[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(1/4)*b^(5/
4)*(b*c - a*d)^(3/2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(1/4)*b^(5/4)*(-(b*c) + a*d)^(3/2
)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + S
qrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*(b*c - a
*d)*Sqrt[c + d*x^4]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
/2])/(8*b*(b*c - a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqr
t[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*b*c
^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a]*Sqr
t[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(16*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c
] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x
^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)
], 1/2])/(32*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[
c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sq
rt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b
*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
```

```
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} + \frac{\int \frac{x^2(3c+dx^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} + \frac{\int \left(\frac{dx^2}{b\sqrt{c+dx^4}} + \frac{(3bc-ad)x^2}{b(a+bx^4)\sqrt{c+dx^4}}\right)dx}{4(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} + \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)} + \frac{(3bc-ad)\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\ldots}{4b(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}\ldots}{\sqrt{\ldots}}}{\sqrt{c+\ldots}}\ldots}{4b(bc-ad)}$$

$$= \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{4b(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)} - \frac{x^3\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} - \frac{\sqrt[4]{c}\,\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x\ldots\right.}{\ldots}$$

$$= \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{4b(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)} - \frac{x^3\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} + \frac{(3bc-ad)\tan^{-1}\left(\frac{\ldots}{\sqrt[4]{\ldots}}\right.}{16\sqrt[4]{-a}\,b^{5/4}(\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.10, size = 162, normalized size = 0.14

$$\frac{-7ax^3(c+dx^4)+7cx^3(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+dx^7(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{28a(bc-ad)(a+bx^4)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-7*a*x^3*(c + d*x^4) + 7*c*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + d*x^7*(a + b*x^4)*Sqrt[1 + (d
*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(28*a*(b*
c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 556, normalized size = 0.49

method	result
elliptic	$\frac{x^3\sqrt{d x^4+c}}{4(ad-bc)(b x^4+a)}-\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right.\right.}{4b(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}$
default	$\frac{\sum_{\_\alpha=\text{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\text{arctanh}\left(\frac{2d x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\text{EllipticPi}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d x^4+c}}}{\_\alpha}}{8b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b^2*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2
+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_al
```

```
pha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(
d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_al
pha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Roo
tOf(_Z^4*b+a))-a/b*(-1/4*b/a/(a*d-b*c)*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*I*
d^(1/2)/a/(a*d-b*c)*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*
x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/
c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/32/b/
a*sum((-3*a*d+b*c)/(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2
*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(
1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/
2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1
/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/
2))),_alpha=RootOf(_Z^4*b+a)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**6/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^6/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.839 $\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1144

$$-\frac{\sqrt{d}\,x\sqrt{c+dx^4}}{4a(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)}+\frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{16(-a)^{5/4}\sqrt[4]{b}\,(bc-ad)^{3/2}}-\frac{(bc-3a\cdots}{1\cdots}$$

```
[Out] -1/16*(-3*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(
1/2))/(-a)^(5/4)/b^(1/4)/(-a*d+b*c)^(3/2)-1/16*(-3*a*d+b*c)*arctan(x*(a*d-b
*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(a*d-b*c)^
(3/2)+1/4*b*x^3*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)-1/4*x*d^(1/2)*(d*x^4
+c)^(1/2)/a/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))+1/4*c^(1/4)*d^(1/4)*(cos(2*arc
tan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE
(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^
4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(-a*d+b*c)/(d*x^4+c)^(1/2)-1/8*c^(1/4
)*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x
/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)
+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(-a*d+b*c)/(d*x^4
+c)^(1/2)+1/32*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/
4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2)
,1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*
((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/d^(1/4)/(-a*d+
b*c)/(a*d+b*c)/b^(1/2)/(d*x^4+c)^(1/2)-1/32*(-3*a*d+b*c)*(cos(2*arctan(d^(1
/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*
arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)
^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*
c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a
)^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/b^(1/2)/(d*x^4+c)^(1/2)-1/16*d
^(1/4)*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2)
)*(c^(1/2)+x^2*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+c)/(c^
(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-
1/16*d^(1/4)*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*
arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2
^(1/2))*(c^(1/2)+x^2*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+
c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^
(1/2)
```

Rubi [A]

time = 1.06, antiderivative size = 1144, normalized size of antiderivative = 1.00, number of

steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {483, 598, 311, 226, 1210, 504, 1231, 1721}

$$\frac{b\sqrt{dx^4+c}\,x^3}{4a(bc-ad)(bx^4+a)}-\frac{\sqrt{d}\sqrt{dx^4+c}\,x}{4a(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)}-\frac{(bc-3ad)\text{ArcTan}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}}-\frac{(bc-3ad)\text{ArcTan}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(ad-bc)^{3/2}}+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a(bc-ad)\sqrt{dx^4+c}}-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a(bc-ad)\sqrt{dx^4+c}}-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(Sqrt[d]*x*Sqrt[c + d*x^4])/(a*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)) +
(b*x^3*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ((b*c - 3*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(5/4
)*b^(1/4)*(b*c - a*d)^(3/2)) - ((b*c - 3*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)
/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(5/4)*b^(1/4)*(-(b*c) + a*
d)^(3/2)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt
[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*(
b*c - a*d)*Sqrt[c + d*x^4]) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt
[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1
/4)], 1/2])/(8*a*(b*c - a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[
d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)
/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/
(16*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[
-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)
], 1/2])/(16*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]
*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(
c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sq
rt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/
c^(1/4)], 1/2])/(32*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a
*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip
ticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sq
rt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*(-a)^(3/2)*Sqrt[b]*c^(1/4)
*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
```

```
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} - \frac{\int \frac{x^2(-bc+4ad+bdx^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} - \frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}} + \frac{(-bc+3ad)x^2}{(a+bx^4)\sqrt{c+dx^4}}\right)dx}{4a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} - \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{(bc-3ad)\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}}{4a(bc-ad)}$$

$$= -\frac{\sqrt{d}\,x\sqrt{c+dx^4}}{4a(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\right.}{}$$

$$= -\frac{\sqrt{d}\,x\sqrt{c+dx^4}}{4a(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} - \frac{(bc-3ad)\tan^{-1}\left(\right.}{16(-a)^{5/4}\sqrt[4]{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.13, size = 172, normalized size = 0.15

$$\frac{21abx^3(c+dx^4)+7(bc-4ad)x^3(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-3bdx^7(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{84a^2(bc-ad)(a+bx^4)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

[Out] (21*a*b*x^3*(c + d*x^4) + 7*(b*c - 4*a*d)*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] - 3*b*d*x^7*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(84*a^2*(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 359, normalized size = 0.31

method	result
default	$-\frac{b\,x^3\sqrt{d\,x^4+c}}{4a(ad-bc)(b\,x^4+a)}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\right.\right.}{4a(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}$
elliptic	$-\frac{b\,x^3\sqrt{d\,x^4+c}}{4a(ad-bc)(b\,x^4+a)}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\right.\right.}{4a(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4*b/a/(a*d-b*c)*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*I*d^(1/2)/a/(a*d-b*c)*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/32/b/a*sum((-3*a*d+b*c)/(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2

```
/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_
Z^4*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(b\,x^4+a\right)}^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^2/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.840 $\int \frac{1}{x^2(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1225

$$-\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x}+\frac{\sqrt{d}\,(5bc-4ad)x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\,(a+bx^4)}-\frac{b^{3/4}(5bc-7ad)\tan^{-1}\Big(}{16(-a)^{9/4}(bc}$$

```
[Out] -1/16*b^(3/4)*(-7*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(
d*x^4+c)^(1/2))/(-a)^(9/4)/(-a*d+b*c)^(3/2)-1/16*b^(3/4)*(-7*a*d+5*b*c)*arc
tan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(9/4)/(a*d-b
*c)^(3/2)-1/4*(-4*a*d+5*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x+1/4*b*(d*x^
4+c)^(1/2)/a/(-a*d+b*c)/x/(b*x^4+a)+1/4*(-4*a*d+5*b*c)*x*d^(1/2)*(d*x^4+c)^
(1/2)/a^2/c/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))-1/4*d^(1/4)*(-4*a*d+5*b*c)*(co
s(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*El
lipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))
*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*d+b*c)/(d*x^4+c)
^(1/2)+1/8*d^(1/4)*(-4*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2
)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4)
)),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(
1/2)/a^2/c^(3/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(-7*a*d+5*b*c)*(cos(2*arct
an(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi
(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^
2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^2*d^(1
/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2)
)^2)^(1/2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-
1/32*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1
/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^
(1/2))*b^(1/2)*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2
*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d
+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/16*b*d^(1/4)*(-7*a*d+5*b*c)*(cos(2*arctan
(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(si
n(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(c^(1/2)-
(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c
^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/16*b*d^(1/4)*(-7*a*d+5*b*c)*(
cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*
EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2
))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)
^(1/2)/a^2/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)
```

Rubi [A]

time = 1.39, antiderivative size = 1225, normalized size of antiderivative = 1.00, number of

steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {483, 597, 598, 311, 226, 1210, 504, 1231, 1721}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*((5*b*c - 4*a*d)*Sqrt[c + d*x^4])/(a^2*c*(b*c - a*d)*x) + (Sqrt[d]*(5*
b*c - 4*a*d)*x*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2
)) + (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x*(a + b*x^4)) - (b^(3/4)*(5*b*c
- 7*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/
(16*(-a)^(9/4)*(b*c - a*d)^(3/2)) - (b^(3/4)*(5*b*c - 7*a*d)*ArcTan[(Sqrt[-
(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(9/4)*(-(b*
c) + a*d)^(3/2)) - (d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)
], 1/2])/(4*a^2*c^(3/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + (d^(1/4)*(5*b*c - 4*
a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(3/4)*(b*c - a*d)*Sqr
t[c + d*x^4]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(5*b*c -
7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*(b*c - a*d)*
(b*c + a*d)*Sqrt[c + d*x^4]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^
(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + S
qrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a^2*c^(1/
4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[b]*(Sqrt[b]*Sqrt[c] + S
qrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)
/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqr
t[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(32*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4
]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[
c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(S
qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]),
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c -
a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} - \frac{\int \frac{-5bc+4ad-3bdx^4}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} + \frac{\int \frac{x^2\left(-(bc-2ad)(5bc-2ad)+\dots\right.}{(a+bx^4)\sqrt{c+\dots}}}{4a^2c(bc-\dots}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} + \frac{\int \left(\frac{d(5bc-4ad)x^2}{\sqrt{c+dx^4}} + \frac{(-5\dots}{(a+bx\dots}\right.}{4a^2c(bc-\dots}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} - \frac{(b(5bc-7ad))\int \frac{\dots}{(a+bx^{\dots}}}{4a^2(bc-\dots}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} + \frac{\left(\sqrt{b}\,(5bc-7ad)\right)\int \dots}{8a^2\dots}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{\sqrt{d}\,(5bc-4ad)x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{b\sqrt{c+d\dots}}{4a(bc-ad)x\,(\dots}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{\sqrt{d}\,(5bc-4ad)x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{b\sqrt{c+d\dots}}{4a(bc-ad)x\,(\dots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.20, size = 226, normalized size = 0.18

$$\frac{21a\left(c+dx^4\right)\left(4a^2d-5b^2cx^4-4ab\left(c-dx^4\right)\right)-7\left(5b^2c^2-12abcd+4a^2d^2\right)x^4\left(a+bx^4\right)\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+3bd(5bc-4ad)x^8\left(a+bx^4\right)\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{84a^3c(bc-ad)x\left(a+bx^4\right)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (21*a*(c + d*x^4)*(4*a^2*d - 5*b^2*c*x^4 - 4*a*b*(c - d*x^4)) - 7*(5*b^2*c^
2 - 12*a*b*c*d + 4*a^2*d^2)*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^8*(a
+ b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b
*x^4)/a)])/(84*a^3*c*(b*c - a*d)*x*(a + b*x^4)*Sqrt[c + d*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 674, normalized size = 0.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a^2*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^
2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_a
lpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/
(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_a
lpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Ro
otOf(_Z^4*b+a))-b/a*(-1/4*b/a/(a*d-b*c)*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*I
*d^(1/2)/a/(a*d-b*c)*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)
*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I
/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/32/b
/a*sum((-3*a*d+b*c)/(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(
2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^
(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1
/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(
1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1
/2))),_alpha=RootOf(_Z^4*b+a)))+1/a^2*(-1/c*(d*x^4+c)^(1/2)/x+I*d^(1/2)/c^(
1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)
*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),
I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + bx^4)^2 \sqrt{c + dx^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 (b x^4 + a)^2 \sqrt{d x^4 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.841 $\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}} \, dx$

Optimal. Leaf size=200

$$-\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)}+\frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)}+\frac{(a^2d^2(3+m)(7+m)+bc(1+m)}{}$$

```
[Out] -b*(b*c*(5+m)-2*a*d*(7+m))*(e*x)^(1+m)*(d*x^4+c)^(1/2)/d^2/e/(3+m)/(7+m)+b^
2*(e*x)^(5+m)*(d*x^4+c)^(1/2)/d/e^5/(7+m)+(a^2*d^2*(3+m)*(7+m)+b*c*(1+m)*(b
*c*(5+m)-2*a*d*(7+m)))*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-
d*x^4/c)*(1+d*x^4/c)^(1/2)/d^2/e/(1+m)/(3+m)/(7+m)/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {475, 470, 372, 371}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\left(\frac{a^2d^2(m+7)}{m+1}+\frac{bc(bc(m+5)-2ad(m+7))}{m+3}\right)\,_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{d^2e(m+7)\sqrt{c+dx^4}}-\frac{b\sqrt{c+dx^4}\,(ex)^{m+1}(bc(m+5)-2ad(m+7))}{d^2e(m+3)(m+7)}+\frac{b^2\sqrt{c+dx^4}\,(ex)^{m+5}}{de^5(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

```
[Out] -((b*(b*c*(5 + m) - 2*a*d*(7 + m))*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d^2*e*(3
 + m)*(7 + m))) + (b^2*(e*x)^(5 + m)*Sqrt[c + d*x^4])/(d*e^5*(7 + m)) + (((
a^2*d^2*(7 + m))/(1 + m) + (b*c*(b*c*(5 + m) - 2*a*d*(7 + m)))/(3 + m))*(e*
x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/4,
 -((d*x^4)/c)])/(d^2*e*(7 + m)*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} + \frac{\int \frac{(ex)^m (a^2d(7+m)-b(bc(5+m)-2ad(7+m))x^4)}{\sqrt{c+dx^4}}\,dx}{d(7+m)}$$

$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} - \Big(-a^2 -$$

$$\Big(\Big(-a^2$$

$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} -$$

$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} + \frac{\Big(a^2 + b}{}$$

Mathematica [A]
time = 7.75, size = 164, normalized size = 0.82

$$\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a^2(45+14m+m^2)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)+b(1+m)x^4\left(2a(9+m)\,{}_2F_1\left(\frac{1}{2},\frac{5+m}{4};\frac{9+m}{4};-\frac{dx^4}{c}\right)+b(5+m)x^4\,{}_2F_1\left(\frac{1}{2},\frac{9+m}{4};\frac{13+m}{4};-\frac{dx^4}{c}\right)\right)\right)}{(1+m)(5+m)(9+m)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[1/2
, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hypergeo
```

```
metric2F1[1/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hypergeo
metric2F1[1/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/((1 + m)*(5 + m)*(9
+ m)*Sqrt[c + d*x^4])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m (b\,x^4+a)^2}{\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
[Out] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^2*(x*e)^m/sqrt(d*x^4 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(x*e)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.60, size = 185, normalized size = 0.92

$$\frac{a^2e^m x x^m \Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{4}+\frac{1}{4} \\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle| \frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{abe^m x^5 x^m \Gamma\left(\frac{m}{4}+\frac{5}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{4}+\frac{5}{4} \\ \frac{m}{4}+\frac{9}{4}\end{matrix}\middle| \frac{dx^4e^{i\pi}}{c}\right)}{2\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)} + \frac{b^2e^m x^9 x^m \Gamma\left(\frac{m}{4}+\frac{9}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{4}+\frac{9}{4} \\ \frac{m}{4}+\frac{13}{4}\end{matrix}\middle| \frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] a**2*e**m*x*x**m*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x
**4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + a*b*e**m*x**5*x**m*ga
```

```
mma(m/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*pi)
/c)/(2*sqrt(c)*gamma(m/4 + 9/4)) + b**2*e**m*x**9*x**m*gamma(m/4 + 9/4)*hyp
er((1/2, m/4 + 9/4), (m/4 + 13/4,), d*x**4*exp_polar(I*pi)/c)/(4*sqrt(c)*ga
mma(m/4 + 13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^2*(x*e)^m/sqrt(d*x^4 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^m\,{\left(b\,x^4+a\right)}^2}{\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(1/2),x)
```

```
[Out] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(1/2), x)
```

3.842 $\int \frac{(ex)^m(a+bx^4)}{\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=123

$$\frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)}-\frac{(bc(1+m)-ad(3+m))(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{de(1+m)(3+m)\sqrt{c+dx^4}}$$

```
[Out] b*(e*x)^(1+m)*(d*x^4+c)^(1/2)/d/e/(3+m)-(b*c*(1+m)-a*d*(3+m))*(e*x)^(1+m)*h
ypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(1/2)/d/e/(1+m)
/(3+m)/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {470, 372, 371}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\left(\frac{a}{m+1}-\frac{bc}{d(m+3)}\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{e\sqrt{c+dx^4}}+\frac{b\sqrt{c+dx^4}\,(ex)^{m+1}}{de(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
```

```
[Out] (b*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d*e*(3 + m)) + ((a/(1 + m) - (b*c)/(d*(3
 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)])/(e*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^m\left(a+bx^4\right)}{\sqrt{c+dx^4}}\,dx = \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} - \left(-a+\frac{bc(1+m)}{d(3+m)}\right)\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx$$

$$= \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} - \frac{\left(\left(-a+\frac{bc(1+m)}{d(3+m)}\right)\sqrt{1+\frac{dx^4}{c}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$

$$= \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} + \frac{\left(a-\frac{bc(1+m)}{d(3+m)}\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}$$

Mathematica [A]
time = 1.43, size = 110, normalized size = 0.89

$$\frac{x(ex)^m\sqrt{c+dx^4}\left(bc\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)+(-bc+ad)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)\right)}{cd(1+m)\sqrt{1+\frac{dx^4}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(b*c*Hypergeometric2F1[-1/2, (1 + m)/4, (5 + m)/
4, -((d*x^4)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)
/4, -((d*x^4)/c)]))/(c*d*(1 + m)*Sqrt[1 + (d*x^4)/c])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m\left(b\,x^4+a\right)}{\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
[Out] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)*(x*e)^m/sqrt(d*x^4 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^4 + a)*(x*e)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.48, size = 119, normalized size = 0.97

$$\frac{ae^m x x^m \Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\right| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}+\frac{be^m x^5 x^m \Gamma\left(\frac{m}{4}+\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{4}+\frac{5}{4}\\ \frac{m}{4}+\frac{9}{4}\end{matrix}\right| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] a*e**m*x*x**m*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4
*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + b*e**m*x**5*x**m*gamma(m
/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*pi)/c)/(
4*sqrt(c)*gamma(m/4 + 9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^4 + a)*(x*e)^m/sqrt(d*x^4 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m\,\left(b\,x^4+a\right)}{\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(1/2),x)
[Out] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(1/2), x)
```

3.843 $\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=68

$$\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(1
/2)/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/Sqrt[c + d*x^4],x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m
)/4, -((d*x^4)/c)])/(e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}$$

Mathematica [A]
time = 0.54, size = 66, normalized size = 0.97

$$\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{4};1+\frac{1+m}{4};-\frac{dx^4}{c}\right)}{(1+m)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/Sqrt[c + d*x^4],x]
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, 1 + (1 + m
)/4, -((d*x^4)/c)])/((1 + m)*Sqrt[c + d*x^4])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(d*x^4+c)^(1/2),x)
[Out] int((e*x)^m/(d*x^4+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate((x*e)^m/sqrt(d*x^4 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x*e)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.50, size = 56, normalized size = 0.82

$$\frac{e^m x x^m \Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{4}+\frac{1}{4} \\ \frac{m}{4}+\frac{5}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(d*x**4+c)**(1/2),x)
```

```
[Out] e**m*x*x**m*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4*e
xp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e)^m/sqrt(d*x^4 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(c + d*x^4)^(1/2),x)
```

```
[Out] int((e*x)^m/(c + d*x^4)^(1/2), x)
```

3.844 $\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=81

$$\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};1,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,1,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};1,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};1,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(1+m)\sqrt{c+dx^4}}$$

Mathematica [**A**]
time = 2.88, size = 125, normalized size = 1.54

$$\frac{x(ex)^m\sqrt{c+dx^4}\left(bcF_1\left(\frac{1+m}{4};-\frac{1}{2},1;\frac{5+m}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-ad\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)\right)}{ac(bc-ad)(1+m)\sqrt{1+\frac{dx^4}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(b*c*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4, -((
d*x^4)/c), -((b*x^4)/a)] - a*d*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/4,
 -((d*x^4)/c)]))/(a*c*(b*c - a*d)*(1 + m)*Sqrt[1 + (d*x^4)/c])
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(b\,x^4+a)\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^4 + c)*(x*e)^m/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral((e*x)**m/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate((x*e)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.845 $\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=81

$$\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};2,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,2,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a^2/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};2,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^2*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};2,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(1+m)\sqrt{c+dx^4}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(81) = 162.
time = 7.91, size = 179, normalized size = 2.21

$$\frac{x(ex)^m\sqrt{c+dx^4}\left(-abcdF_1\left(\frac{1+m}{4};-\frac{1}{2},1;\frac{5+m}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+bc(bc-ad)F_1\left(\frac{1+m}{4};2,-\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+a^2d^2\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)\right)}{a^2c(bc-ad)^2(1+m)\sqrt{1+\frac{dx^4}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(-(a*b*c*d*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/
4, -((d*x^4)/c), -((b*x^4)/a)]) + b*c*(b*c - a*d)*AppellF1[(1 + m)/4, 2, -1
/2, (5 + m)/4, -((b*x^4)/a), -((d*x^4)/c)] + a^2*d^2*Hypergeometric2F1[1/2,
 (1 + m)/4, (5 + m)/4, -((d*x^4)/c)]))/(a^2*c*(b*c - a*d)^2*(1 + m)*Sqrt[1
+ (d*x^4)/c])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(b\,x^4+a)^2\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(x*e)^m/(b^2*d*x^12 + (b^2*c + 2*a*b*d)*x^8 + (2*a
*b*c + a^2*d)*x^4 + a^2*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a+bx^4)^2 \sqrt{c+dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral((e*x)**m/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.846 $\int \frac{(ex)^m}{(a+bx^4)^3 \sqrt{c+dx^4}} dx$

Optimal. Leaf size=81

$$\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};3,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(1+m)\sqrt{c+dx^4}}$$

[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,3,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/c)^(1/2)/a^3/e/(1+m)/(d*x^4+c)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};3,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

[In] Int[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]

[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4, -((b*x^4)/a), -((d*x^4)/c)])/(a^3*e*(1 + m)*Sqrt[c + d*x^4])

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};3,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(1+m)\sqrt{c+dx^4}}$$

Mathematica [A]
time = 10.08, size = 77, normalized size = 0.95

$$\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};3,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(1+m)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4, -((b*
x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*Sqrt[c + d*x^4])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(b\,x^4+a)^3\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
[Out] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate((x*e)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(x*e)^m/(b^3*d*x^16 + (b^3*c + 3*a*b^2*d)*x^12 + 3
*(a*b^2*c + a^2*b*d)*x^8 + (3*a^2*b*c + a^3*d)*x^4 + a^3*c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{{\left(b\,x^4+a\right)}^3\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(1/2)),x)
```

```
[Out] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(1/2)), x)
```

3.847 $\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}} dx$

Optimal. Leaf size=198

$$\frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}}+\frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)}-\frac{(2b^2c^2(1+m)-(3+m)\,(2a^2d^2-(bc-ad)^2(1+m)))\,(ex)^{1+m}}{2cd^2e(1+m)(3+m)\sqrt{c+dx}}$$

```
[Out] 1/2*(-a*d+b*c)^2*(e*x)^(1+m)/c/d^2/e/(d*x^4+c)^(1/2)+b^2*(e*x)^(1+m)*(d*x^4
+c)^(1/2)/d^2/e/(3+m)-1/2*(2*b^2*c^2*(1+m)-(3+m)*(2*a^2*d^2-(-a*d+b*c)^2*(1
+m)))*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4
/c)^(1/2)/c/d^2/e/(1+m)/(3+m)/(d*x^4+c)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {474, 470, 372, 371}

$$-\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\left(2b^2c^2(m+1)-(m+3)\left(2a^2d^2-(m+1)(bc-ad)^2\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{2cd^2e(m+1)(m+3)\sqrt{c+dx^4}}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}+\frac{b^2\sqrt{c+dx^4}\,(ex)^{m+1}}{d^2e(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(1 + m))/(2*c*d^2*e*Sqrt[c + d*x^4]) + (b^2*(e*x)^(1 +
 m)*Sqrt[c + d*x^4])/(d^2*e*(3 + m)) - ((2*b^2*c^2*(1 + m) - (3 + m)*(2*a^2
*d^2 - (b*c - a*d)^2*(1 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeome
tric2F1[1/2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d^2*e*(1 + m)*(3 + m
)*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(ex)^m \left(a+bx^4\right)^2}{\left(c+dx^4\right)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} - \frac{\int \frac{(ex)^m\left(-2a^2d^2+(bc-ad)^2(1+m)-2b^2cdx^4\right)}{\sqrt{c+dx^4}}\,dx}{2cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} + \frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)} - \frac{\left(-a^2d^2(1-m)-2abcd(1+m)+\frac{b^2\cdots}{}\right.}{2cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} + \frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)} - \frac{\left(\left(-a^2d^2(1-m)-2abcd(1+m)+\cdots\right.\right.}{2c\cdots}$$

$$= \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} + \frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)} + \frac{\left(a^2d^2(1-m)+2abcd(1+m)-\frac{b^2c^2(\cdots}{}\right.}{2c\cdots}$$

Mathematica [A]
time = 10.10, size = 167, normalized size = 0.84

$$\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a^2(45+14m+m^2)\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)+b(1+m)x^4\left(2a(9+m)\,{}_2F_1\left(\frac{3}{2},\frac{5+m}{4};\frac{9+m}{4};-\frac{dx^4}{c}\right)+b(5+m)x^4\,{}_2F_1\left(\frac{3}{2},\frac{9+m}{4};\frac{13+m}{4};-\frac{dx^4}{c}\right)\right)\right)}{c(1+m)(5+m)(9+m)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[3/2
, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hypergeo
```

```
metric2F1[3/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hypergeo
metric2F1[3/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/(c*(1 + m)*(5 + m)*(
9 + m)*Sqrt[c + d*x^4])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m \left(b\,x^4+a\right)^2}{\left(d\,x^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^2*(x*e)^m/(d*x^4 + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*sqrt(d*x^4 + c)*(x*e)^m/(d^2*x^8 + 2*c
*d*x^4 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m \left(a+bx^4\right)^2}{\left(c+dx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(3/2),x)
```

```
[Out] Integral((e*x)**m*(a + b*x**4)**2/(c + d*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^2*(x*e)^m/(d*x^4 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m\,{\left(b\,x^4+a\right)}^2}{{\left(d\,x^4+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x)
[Out] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2), x)
```

3.848 $\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}} \, dx$

Optimal. Leaf size=132

$$-\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}}+\frac{(ad(1-m)+bc(1+m))(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{2cde(1+m)\sqrt{c+dx^4}}$$

```
[Out] -1/2*(-a*d+b*c)*(e*x)^(1+m)/c/d/e/(d*x^4+c)^(1/2)+1/2*(a*d*(1-m)+b*c*(1+m))
*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(
1/2)/c/d/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {468, 372, 371}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\ (ex)^{m+1}(ad(1-m)+bc(m+1))\ {}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{2cde(m+1)\sqrt{c+dx^4}}-\frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```

```
[Out] -1/2*((b*c - a*d)*(e*x)^(1 + m))/(c*d*e*Sqrt[c + d*x^4]) + ((a*d*(1 - m) +
b*c*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 +
m)/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
```

```
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m\left(a+bx^4\right)}{\left(c+dx^4\right)^{3/2}}\,dx &= -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}}+\frac{(-ad(-1+m)+bc(1+m))\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx}{2cd}\\ &= -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}}+\frac{\left((-ad(-1+m)+bc(1+m))\sqrt{1+\frac{dx^4}{c}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{2cd\sqrt{c+dx^4}}\\ &= -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}}+\frac{(ad(1-m)+bc(1+m))(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};\ldots\right)}{2cde(1+m)\sqrt{c+dx^4}}\end{aligned}$$

Mathematica [A]
time = 3.51, size = 110, normalized size = 0.83

$$\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(bc\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)+(-bc+ad)\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)\right)}{cd(1+m)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(b*c*Hypergeometric2F1[1/2, (1 + m)/4, (5 +
m)/4, -((d*x^4)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[3/2, (1 + m)/4, (5 +
 m)/4, -((d*x^4)/c)]))/(c*d*(1 + m)*Sqrt[c + d*x^4])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m\left(b\,x^4+a\right)}{\left(d\,x^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

[Out] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*x^4 + a)*(x*e)^m/(d*x^4 + c)^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")

[Out] integral((b*x^4 + a)*sqrt(d*x^4 + c)*(x*e)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x
)

Sympy [C] Result contains complex when optimal does not.
time = 22.15, size = 119, normalized size = 0.90

$$\frac{ae^m xx^m \Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{4}+\frac{1}{4} \\ \frac{m}{4}+\frac{5}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{be^m x^5 x^m \Gamma\left(\frac{m}{4}+\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{4}+\frac{5}{4} \\ \frac{m}{4}+\frac{9}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(3/2),x)

[Out] a*e**m*x*x**m*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4
*exp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4)) + b*e**m*x**5*x**m*gamma(
m/4 + 5/4)*hyper((3/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*pi)/c)/
(4*c**(3/2)*gamma(m/4 + 9/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")

```
[Out] integrate((b*x^4 + a)*(x*e)^m/(d*x^4 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m\,\left(b\,x^4+a\right)}{{\left(d\,x^4+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x)
[Out] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2), x)
```

3.849 $\int \frac{(ex)^m}{\left(c+dx^4\right)^{3/2}} \, dx$

Optimal. Leaf size=71

$$\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\ {}_2F_1\left(\frac{3}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{ce(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*hypergeom([3/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(1
/2)/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {372, 371}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\ {}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{ce(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/(c + d*x^4)^(3/2),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, (5 + m
)/4, -((d*x^4)/c)])/(c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\;{}_2F_1\left(\frac{3}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{ce(1+m)\sqrt{c+dx^4}}$$

Mathematica [**A**]
time = 1.24, size = 69, normalized size = 0.97

$$\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\;{}_2F_1\left(\frac{3}{2},\frac{1+m}{4};1+\frac{1+m}{4};-\frac{dx^4}{c}\right)}{c(1+m)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/(c + d*x^4)^(3/2),x]
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, 1 + (1 + m
)/4, -((d*x^4)/c)])/(c*(1 + m)*Sqrt[c + d*x^4])
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(d\,x^4+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(d*x^4+c)^(3/2),x)
[Out] int((e*x)^m/(d*x^4+c)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="maxima")
[Out] integrate((x*e)^m/(d*x^4 + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(x*e)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.83, size = 56, normalized size = 0.79

$$\frac{e^m x x^m \Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{4}+\frac{1}{4} \\ \frac{m}{4}+\frac{5}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(d*x**4+c)**(3/2),x)
```

```
[Out] e**m*x*x**m*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4*e
xp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x*e)^m/(d*x^4 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{(d\,x^4+c)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(c + d*x^4)^(3/2),x)
```

```
[Out] int((e*x)^m/(c + d*x^4)^(3/2), x)
```

3.850 $\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};1,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,1,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};1,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^4)\,(c+dx^4)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};1,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(1+m)\sqrt{c+dx^4}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(84) = 168.
time = 10.10, size = 169, normalized size = 2.01

$$\frac{x(ex)^m\sqrt{c+dx^4}\left(b^2c^2F_1\left(\frac{1+m}{4};-\frac{1}{2},1;\frac{5+m}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+ad\left(-bc\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)+(-bc+ad)\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)\right)\right)}{ac^2(bc-ad)^2(1+m)\sqrt{1+\frac{dx^4}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(b^2*c^2*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4,
 -((d*x^4)/c), -((b*x^4)/a)] + a*d*(-(b*c*Hypergeometric2F1[1/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)]) + (-(b*c) + a*d)*Hypergeometric2F1[3/2, (1 + m)/
4, (5 + m)/4, -((d*x^4)/c)])))/(a*c^2*(b*c - a*d)^2*(1 + m)*Sqrt[1 + (d*x^4
)/c])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(b\,x^4+a)\,(d\,x^4+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")
[Out] integrate((x*e)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^4 + c)*(x*e)^m/(b*d^2*x^12 + (2*b*c*d + a*d^2)*x^8 + (b*c
^2 + 2*a*c*d)*x^4 + a*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a+bx^4)\,(c+dx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(3/2),x)
[Out] Integral((e*x)**m/((a + b*x**4)*(c + d*x**4)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")
[Out] integrate((x*e)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{(b\,x^4+a)\,(d\,x^4+c)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x)
[Out] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)), x)
```

3.851 $\int \frac{(ex)^m}{(a+bx^4)^2 (c+dx^4)^{3/2}} \, dx$

Optimal. Leaf size=84

$$\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};2,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2ce(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,2,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a^2/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};2,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2ce(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^2*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^4)^2 (c+dx^4)^{3/2}} \, dx = \frac{\sqrt{1+\frac{dx^4}{c}} \int \frac{(ex)^m}{(a+bx^4)^2 \left(1+\frac{dx^4}{c}\right)^{3/2}} \, dx}{c\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\, F_1\left(\frac{1+m}{4}; 2, \frac{3}{2}; \frac{5+m}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a^2 ce(1+m)\sqrt{c+dx^4}}$$

Mathematica [**A**]
time = 10.08, size = 77, normalized size = 0.92

$$\frac{x(ex)^m \left(1+\frac{dx^4}{c}\right)^{3/2} F_1\left(\frac{1+m}{4}; 2, \frac{3}{2}; \frac{5+m}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a^2(1+m)\left(c+dx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
```

```
[Out] (x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4, -((
b*x^4)/a), -((d*x^4)/c)])/(a^2*(1 + m)*(c + d*x^4)^(3/2))
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(b\,x^4+a\right)^2 \left(d\,x^4+c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(x*e)^m/(b^2*d^2*x^16 + 2*(b^2*c*d + a*b*d^2)*x^12
 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^8 + 2*(a*b*c^2 + a^2*c*d)*x^4 + a^2*c^
2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x*e)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\left(b\,x^4+a\right)^2\,\left(d\,x^4+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x)
```

```
[Out] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)), x)
```

3.852 $\int \frac{(ex)^m}{(a+bx^4)^3(c+dx^4)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};3,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,3,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a^3/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};3,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^3*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}} \int \frac{(ex)^m}{(a+bx^4)^3\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}$$

$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\, F_1\left(\frac{1+m}{4};3,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(1+m)\sqrt{c+dx^4}}$$

Mathematica [A]
time = 10.08, size = 77, normalized size = 0.92

$$\frac{x(ex)^m\left(1+\frac{dx^4}{c}\right)^{3/2} F_1\left(\frac{1+m}{4};3,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(1+m)\,(c+dx^4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
```

```
[Out] (x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4, -((
b*x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*(c + d*x^4)^(3/2))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(b\,x^4+a)^3\,(d\,x^4+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(x*e)^m/(b^3*d^2*x^20 + (2*b^3*c*d + 3*a*b^2*d^2)*
x^16 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^12 + (3*a*b^2*c^2 + 6*a^2*b*
c*d + a^3*d^2)*x^8 + a^3*c^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x^4), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x*e)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\left(b\,x^4+a\right)^3\,\left(d\,x^4+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x)
```

```
[Out] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)), x)
```

3.853 $\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=104

$$-\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{(c+dx^6)^{3/2}}{9bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/9*(d*x^6+c)^(3/2)/b/d^2-1/3*a^2*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c
)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/3*(a*d+b*c)*(d*x^6+c)^(1/2)/b^2/d^2
```

Rubi [A]
time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 90, 65, 214}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}\,(ad+bc)}{3b^2d^2}+\frac{(c+dx^6)^{3/2}}{9bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*((b*c + a*d)*Sqrt[c + d*x^6])/(b^2*d^2) + (c + d*x^6)^(3/2)/(9*b*d^2)
- (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[
b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx, x, x^6\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{(c+dx^6)^{3/2}}{9bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{(c+dx^6)^{3/2}}{9bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{(c+dx^6)^{3/2}}{9bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 91, normalized size = 0.88

$$\frac{\sqrt{c+dx^6}\,(-2bc-3ad+bdx^6)}{9b^2d^2}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-2*b*c - 3*a*d + b*d*x^6))/(9*b^2*d^2) + (a^2*ArcTan[(Sqr
t[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2)*Sqrt[-(b*c) + a*d])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 4.13, size = 288, normalized size = 2.77

$$\left[\frac{3\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^6+2bc-ad-2\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\left((b^3cd-ab^2d^2)x^6-2b^3c^2-ab^2cd+3a^2bd^2\right)\sqrt{dx^6+c}}{18\,(b^4cd^2-ab^3d^3)},\ \frac{3\sqrt{-b^2c+abd}\,a^2d^2\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)+\left((b^3cd-ab^2d^2)x^6-2b^3c^2-ab^2cd+3a^2bd^2\right)\sqrt{dx^6+c}}{9\,(b^4cd^2-ab^3d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*
x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 -
 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^6 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^6 - 2*b^3*c^2 - a*b^2
*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**17/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [A]
time = 1.71, size = 106, normalized size = 1.02

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^2}+\frac{(dx^6+c)^{\frac{3}{2}}b^2d^4-3\sqrt{dx^6+c}\,b^2cd^4-3\sqrt{dx^6+c}\,abd^5}{9\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*a^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/9*((d*x^6 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^6 + c)*b^2*c*d^4 - 3*s
qrt(d*x^6 + c)*a*b*d^5)/(b^3*d^6)
```

Mupad [B]
time = 4.87, size = 103, normalized size = 0.99

$$\frac{(d\,x^6+c)^{3/2}}{9\,b\,d^2}-\left(\frac{2\,c}{3\,b\,d^2}+\frac{3\,a\,d^3-3\,b\,c\,d^2}{9\,b^2\,d^4}\right)\,\sqrt{d\,x^6+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(3/2)/(9*b*d^2) - ((2*c)/(3*b*d^2) + (3*a*d^3 - 3*b*c*d^2)/(9*b
^2*d^4))*(c + d*x^6)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b
*c)^(1/2)))/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

3.854 $\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{c+dx^6}}{3bd}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

```
[Out] 1/3*a*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+1/3*(d*x^6+c)^(1/2)/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 81, 65, 214}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] Sqrt[c + d*x^6]/(3*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a
*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)\\ &= \frac{\sqrt{c+dx^6}}{3bd} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b}\\ &= \frac{\sqrt{c+dx^6}}{3bd} - \frac{a\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3bd}\\ &= \frac{\sqrt{c+dx^6}}{3bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 73, normalized size = 0.99

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^6}}{bd} - \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) +
a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 5.03, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}\,(b^2c-abd)}{6\,(b^3cd-ab^2d^2)},\ -\frac{\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)-\sqrt{dx^6+c}\,(b^2c-abd)}{3\,(b^3cd-ab^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c
)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^6 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - sqrt(d*x^6 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [A]
time = 1.42, size = 64, normalized size = 0.86

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^6+c}}{b}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^6 + c)/b)/d
```

Mupad [B]
time = 4.75, size = 58, normalized size = 0.78

$$\frac{\sqrt{d\,x^6+c}}{3\,b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(1/2)/(3*b*d) - (a*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)
^(1/2)))/(3*b^(3/2)*(a*d - b*c)^(1/2))
```

3.855 $\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -1/3*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {455, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,\sqrt{bc-ad}}$$

Mathematica [**A**]
time = 0.04, size = 51, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{3\sqrt{b}\,\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]]/(3*Sqrt[b]*Sqrt[-(b*c)
 + a*d])
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 9.26, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)}{6\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)}{3\,(b^2c-abd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b
*x^6 + a))/sqrt(b^2*c - a*b*d), 1/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A]
time = 7.94, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 1.18, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B]
time = 4.72, size = 40, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{b\sqrt{d\,x^6+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{3\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] atan((b*(c + d*x^6)^(1/2))/(a*b*d - b^2*c)^(1/2))/(3*(a*b*d - b^2*c)^(1/2))
```

3.856 $\int \frac{1}{x(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

```
[Out] -1/3*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/3*arctanh(b^(1/2)*(d*x^6+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 88, 65, 214}

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^6\right)}{6a} - \frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3ad} - \frac{b\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.13, size = 80, normalized size = 0.94

$$-\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/Sqrt[-
(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/Sqrt[c])/a
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x)

[Out] int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x), x)

Fricas [A]
time = 5.16, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2bc-ad+2\sqrt{dx^6+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+\sqrt{c}\log\left(\frac{dx^6-2\sqrt{dx^6+c}\sqrt{c}+2c}{x^6}\right)}{6ac}, \frac{2c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^6+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right)+\sqrt{c}\log\left(\frac{dx^6-2\sqrt{dx^6+c}\sqrt{c}+2c}{x^6}\right)}{6ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2bc-ad+2\sqrt{dx^6+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-c}}{c}\right)}{6ac}, \frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^6+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right)+\sqrt{-c}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-c}}{c}\right)}{3ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*(c*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(2*c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) + sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(c*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c))/(a*c), 1/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c))/(a*c)]

Sympy [A]
time = 9.45, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**6+a)/(d*x**6+c)**(1/2),x)

```
[Out] -atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**6)/sqrt(-c))/(3*a*sqrt(-c))
```

Giac [A]
time = 1.34, size = 71, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,a}+\frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B]
time = 5.19, size = 652, normalized size = 7.67

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{dx^6+c}}{\sqrt{c}}\right)}{3\,a\sqrt{c}}-\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{b^2c-abd}\left(\frac{2b^3d^2\sqrt{dx^6+c}}{27}-\frac{\sqrt{b^2c-abd}\left(\frac{2a^2b^2d^3}{9}-\frac{(8a^3b^2d^3-16a^2b^3cd^2)\sqrt{dx^6+c}\sqrt{b^2c-abd}}{36(a^2d-abc)}\right)}{6(a^2d-abc)}\right)1i}{a^2d-abc}+\frac{\sqrt{b^2c-abd}\left(\frac{2b^3d^2\sqrt{dx^6+c}}{27}+\frac{\sqrt{b^2c-abd}\left(\frac{2a^2b^2d^3}{9}+\frac{(8a^3b^2d^3-16a^2b^3cd^2)\sqrt{dx^6+c}\sqrt{b^2c-abd}}{36(a^2d-abc)}\right)}{6(a^2d-abc)}\right)1i}{a^2d-abc}}{\frac{\sqrt{b^2c-abd}\left(\frac{2b^3d^2\sqrt{dx^6+c}}{27}-\frac{\sqrt{b^2c-abd}\left(\frac{2a^2b^2d^3}{9}-\frac{(8a^3b^2d^3-16a^2b^3cd^2)\sqrt{dx^6+c}\sqrt{b^2c-abd}}{36(a^2d-abc)}\right)}{6(a^2d-abc)}\right)}{a^2d-abc}-\frac{\sqrt{b^2c-abd}\left(\frac{2b^3d^2\sqrt{dx^6+c}}{27}+\frac{\sqrt{b^2c-abd}\left(\frac{2a^2b^2d^3}{9}+\frac{(8a^3b^2d^3-16a^2b^3cd^2)\sqrt{dx^6+c}\sqrt{b^2c-abd}}{36(a^2d-abc)}\right)}{6(a^2d-abc)}\right)}{a^2d-abc}}\right)\sqrt{b^2c-abd}\,1i}{3\,(a^2d-abc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^6)^(1/2)/c^(1/2))/(3*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^
(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^
2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a
*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c)))*1i)/(a^2*d - a*b*c
) + ((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 + ((b^2*c - a*
b*d)^(1/2)*((2*a^2*b^2*d^3)/9 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*
x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c)
))*1i)/(a^2*d - a*b*c))/(((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/
2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a
^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d - a*b*c))
))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c) - ((b^2*c - a*b*d)^(1/2)*((2*b^3*d
^2*(c + d*x^6)^(1/2))/27 + ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 + ((8*
a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(3
6*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c))*(b^2*c - a*b*d
)^(1/2)*1i)/(3*(a^2*d - a*b*c))
```

3.857 $\int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{c+dx^6}}{6acx^6}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

```
[Out] 1/6*(a*d+2*b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/3*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/6*(d
*x^6+c)^(1/2)/a/c/x^6
```

Rubi [A]

time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 105, 162, 65, 214}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{\sqrt{c+dx^6}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^6]/(a*c*x^6) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqr
t[c]])/(6*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*
c - a*d]])/(3*a^2*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7 (a+bx^6) \sqrt{c+dx^6}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)\\
&= -\frac{\sqrt{c+dx^6}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6ac}\\
&= -\frac{\sqrt{c+dx^6}}{6acx^6} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a^2} - \frac{(2bc+ad)\text{Subst}\left(\int\right.}{12}\\
&= -\frac{\sqrt{c+dx^6}}{6acx^6} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3a^2d} - \frac{(2bc+ad)\text{Subst}\left(\right.}{}\\
&= -\frac{\sqrt{c+dx^6}}{6acx^6} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d}}{\sqrt{bc-ad}}\right.}{3a^2\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A]
time = 0.31, size = 109, normalized size = 0.93

$$\frac{-\frac{a\sqrt{c+dx^6}}{cx^6}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{c^{3/2}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-((a*Sqrt[c + d*x^6])/(c*x^6)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6
])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[c
+ d*x^6]/Sqrt[c]])/c^(3/2))/(6*a^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7} \left(b\,x^{6}+a \right) \sqrt{d\,x^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^7), x)
```

Fricas [A]
time = 4.08, size = 565, normalized size = 4.83

$$\left[\frac{2\,bc^{2}x^{6}\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{6}+2\,bc-ad-2\,\sqrt{dx^{6}+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{6}+a}\right)+(2\,bc+ad)\sqrt{c}\,x^{6}\log\left(\frac{dx^{6}+2\,\sqrt{dx^{6}+c}\,\sqrt{c}+2\,c}{x^{6}}\right)-2\,\sqrt{dx^{6}+c}\,ac}{12\,a^{2}c^{2}x^{6}},\ -\frac{4\,bc^{2}x^{6}\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^{6}+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^{6}+bc}\right)-(2\,bc+ad)\sqrt{c}\,x^{6}\log\left(\frac{dx^{6}+2\,\sqrt{dx^{6}+c}\,\sqrt{c}+2\,c}{x^{6}}\right)+2\,\sqrt{dx^{6}+c}\,ac}{12\,a^{2}c^{2}x^{6}},\ \frac{bc^{2}x^{6}\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{6}+2\,bc-ad-2\,\sqrt{dx^{6}+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{6}+a}\right)-(2\,bc+ad)\sqrt{-c}\,x^{6}\arctan\left(\frac{\sqrt{dx^{6}+c}\,\sqrt{-c}}{c}\right)-\sqrt{dx^{6}+c}\,ac}{6\,a^{2}c^{2}x^{6}},\ -\frac{2\,bc^{2}x^{6}\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^{6}+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^{6}+bc}\right)+(2\,bc+ad)\sqrt{-c}\,x^{6}\arctan\left(\frac{\sqrt{dx^{6}+c}\,\sqrt{-c}}{c}\right)+\sqrt{dx^{6}+c}\,ac}{6\,a^{2}c^{2}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(
d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + (2*b*c + a*d)*sq
rt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*sqrt(d*x^6
 + c)*a*c)/(a^2*c^2*x^6), -1/12*(4*b*c^2*x^6*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - (2*b*c +
 a*d)*sqrt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*sq
rt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), 1/6*(b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b
*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(
b*x^6 + a)) - (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c)
```

```
 - sqrt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), -1/6*(2*b*c^2*x^6*sqrt(-b/(b*c - a*d
))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)
) + (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + sqrt(d*
x^6 + c)*a*c)/(a^2*c^2*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \left(a+b x^6\right) \sqrt{c+d x^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [A]
time = 1.44, size = 104, normalized size = 0.89

$$\frac{b^2 \arctan\left(\frac{\sqrt{dx^6+c}\, b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\, a^2} - \frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6\,a^2\sqrt{-c}\, c} - \frac{\sqrt{dx^6+c}}{6\,acx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*b^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/6*(2*b*c + a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/6*sqrt(d*x^6 + c)/(a*c*x^6)
```

Mupad [B]
time = 5.62, size = 396, normalized size = 3.38

$$\frac{\ln\left(\sqrt{d\,x^6+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{6\,a^3\,d-6\,a^2\,b\,c}-\frac{\ln\left(\sqrt{d\,x^6+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{6\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^6+c}}{6\,a\,c\,x^6}-\frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^6+c}\,1{}\mathrm{i}}{18\,\sqrt{c^3}\,\left(\frac{b^4\,d^4}{18\,c}+\frac{5\,a\,b^3\,d^5}{108\,c^2}+\frac{a^2\,b^2\,d^6}{108\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^6+c}\,1{}\mathrm{i}}{108\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{108\,a}+\frac{b^2\,d^6}{108\,c}+\frac{b^4\,c\,d^4}{18\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^6+c}\,5{}\mathrm{i}}{108\,\sqrt{c^3}\,\left(\frac{b^4\,d^4}{18\,a}+\frac{5\,b^3\,d^5}{108\,c}+\frac{a\,b^2\,d^6}{108\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)\,1{}\mathrm{i}}{6\,a^2\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (log((c + d*x^6)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(6*a^3*d - 6*a^2*b*c) - (log((c + d*x^6
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(6*(a^3*d - a^2*b*c)) - (c + d*x^6)^(1/2)/(6*a*c*x^6)
- (atan((b^4*d^4*(c + d*x^6)^(1/2)*1i)/(18*(c^3)^(1/2)*((b^4*d^4)/(18*c) +
(5*a*b^3*d^5)/(108*c^2) + (a^2*b^2*d^6)/(108*c^3))) + (b^2*d^6*(c + d*x^6)^
(1/2)*1i)/(108*(c^3)^(1/2)*((5*b^3*d^5)/(108*a) + (b^2*d^6)/(108*c) + (b^4*
c*d^4)/(18*a^2))) + (b^3*d^5*(c + d*x^6)^(1/2)*5i)/(108*(c^3)^(1/2)*((b^4*d
^4)/(18*a) + (5*b^3*d^5)/(108*c) + (a*b^2*d^6)/(108*c^2))))*(a*d + 2*b*c)*1
i)/(6*a^2*(c^3)^(1/2))
```

3.858 $\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=123

$$\frac{x^3\sqrt{c+dx^6}}{6bd}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}}-\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}$$

```
[Out] -1/6*(2*a*d+b*c)*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b^2/d^(3/2)+1/3*a^(3/
2)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/b^2/(-a*d+b*c)^(1/2
)+1/6*x^3*(d*x^6+c)^(1/2)/b/d
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {476, 490, 537, 223, 212, 385, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}+\frac{x^3\sqrt{c+dx^6}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[c + d*x^6])/(6*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt
[a]*Sqrt[c + d*x^6])])/(3*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^3)/Sqrt[c + d*x^6]])/(6*b^2*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)$$
$$= \frac{x^3\sqrt{c+dx^6}}{6bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6bd}$$
$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b^2} - \frac{(bc+2ad)\text{Subst}\left(\int \right.}{6}$$
$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b^2} - \frac{(bc+2ad)\text{Subs}}{}$$
$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx}}\right.}{6b^2d^{3/2}}$$

Mathematica [A]
time = 0.95, size = 140, normalized size = 1.14

$$\frac{\sqrt{d}\left(bx^3\sqrt{c+dx^6}+\frac{2a^{3/2}d\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^6+bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}\right)-(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{d}\,x^3}\right)}{6b^2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] (Sqrt[d]*(b*x^3*Sqrt[c + d*x^6] + (2*a^(3/2)*d*ArcTan[(a*Sqrt[d] + b*Sqrt[d
]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d])
 - (b*c + 2*a*d)*ArcTanh[Sqrt[c + d*x^6]/(Sqrt[d]*x^3)])/(6*b^2*d^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [A]
time = 5.63, size = 739, normalized size = 6.01

$$\left[\frac{2\sqrt{dx^6+c}\,bdx^3+ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{12}-2(3abc^2-4a^2cd)x^6+a^2c^2+4((b^2c^2-3abcd+2a^2d^2)x^9-(abc^2-a^2cd)x^3)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2abx^6+a^2}\right)+(bc+2ad)\sqrt{d}\log\left(-2dx^6+2\sqrt{dx^6+c}\sqrt{d}x^3-c\right)}{12b^2d^2},\ \frac{2\sqrt{dx^6+c}\,bdx^3+ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{12}-2(3abc^2-4a^2cd)x^6+a^2c^2+4((b^2c^2-3abcd+2a^2d^2)x^9-(abc^2-a^2cd)x^3)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2abx^6+a^2}\right)+2(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^3}{\sqrt{dx^6+c}}\right)}{12b^2d^2},\ \frac{2\sqrt{dx^6+c}\,bdx^3-2ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^6-ac)\sqrt{dx^6+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^9+acx^3)}\right)+(bc+2ad)\sqrt{d}\log\left(-2dx^6+2\sqrt{dx^6+c}\sqrt{d}x^3-c\right)}{12b^2d^2},\ \frac{\sqrt{dx^6+c}\,bdx^3-ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^6-ac)\sqrt{dx^6+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^9+acx^3)}\right)+(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^3}{\sqrt{dx^6+c}}\right)}{6b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*
x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + (b*c + 2*a*d
)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2), 1/1
2*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8
*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((
b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6
+ c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*(b*c + 2*a*d)*
sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2), 1/12*(2*sqrt(d*x^
6 + c)*b*d*x^3 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6
 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + (b*c + 2
*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2),
 1/6*(sqrt(d*x^6 + c)*b*d*x^3 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c
 - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3
)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**14/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.859 $\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}$$

```
[Out] 1/3*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b/d^(1/2)-1/3*arctan(x^3*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 494, 223, 212, 385, 211}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*S
qrt[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b} - \frac{a\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b}\\
&= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}
\end{aligned}
$$

Mathematica [A]
time = 0.46, size = 107, normalized size = 1.18

$$
\frac{-\frac{\sqrt{a}\tan^{-1}\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}\,x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{d}\,x^3}\right)}{\sqrt{d}}}{3b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqr
t[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + ArcTanh[Sqrt[c + d*x^6]/(Sqrt[d]
*x^3)]/Sqrt[d])/(3*b)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(b\,x^{6}+a\right)\sqrt{d\,x^{6}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [A]
time = 2.40, size = 632, normalized size = 6.95

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{12}-2(3abc^2-4a^2cd)x^6+a^2c^2-4\left((b^2c^2-3abcd+2a^2d^2)x^9-(abc^2-a^2cd)x^3\right)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2abx^6+a^2}\right)+2\sqrt{d}\log\left(-2dx^6-2\sqrt{dx^6+c}\sqrt{d}x^3-c\right)}{12bd}, \frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{12}-2(3abc^2-4a^2cd)x^6+a^2c^2-4\left((b^2c^2-3abcd+2a^2d^2)x^9-(abc^2-a^2cd)x^3\right)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2abx^6+a^2}\right)-4\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^3}{\sqrt{dx^6+c}}\right)}{12bd}, \frac{d\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^6-ac)\sqrt{dx^6+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^9+acx^3)}\right)+\sqrt{d}\log\left(-2dx^6-2\sqrt{dx^6+c}\sqrt{d}x^3-c\right)}{6bd}, \frac{d\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^6-ac)\sqrt{dx^6+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^9+acx^3)}\right)-2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^3}{\sqrt{dx^6+c}}\right)}{6bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(
b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*s
qrt(d)*x^3 - c))/(b*d), 1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*
c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c
^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*
sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*sqrt(-d)*arctan(sqr
t(-d)*x^3/sqrt(d*x^6 + c)))/(b*d), 1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*(
(b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c
*x^3)) + sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b*d),
```

```
1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6
 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) - 2*sqrt(-d)*arctan(sqrt(-d)
*x^3/sqrt(d*x^6 + c)))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(71) = 142.
time = 2.24, size = 156, normalized size = 1.71

$$-\frac{\left(a\sqrt{-d}\,\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)\right)\mathrm{sgn}(x)}{3\sqrt{abc-a^2d}\,b\sqrt{-d}}+\frac{a\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{3\sqrt{abc-a^2d}\,b\mathrm{sgn}\left(x\right)}-\frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{3\,b\sqrt{-d}\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - sqrt(a*b*c - a^2*d
)*arctan(sqrt(d)/sqrt(-d)))*sgn(x)/(sqrt(a*b*c - a^2*d)*b*sqrt(-d)) + 1/3*a
*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*b*sgn(x
)) - 1/3*arctan(sqrt(d + c/x^6)/sqrt(-d))/(b*sqrt(-d)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.860 $\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3\sqrt{a}\,\sqrt{bc-ad}}$$

```
[Out] 1/3*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(1/2)/(-a*d+b*c)
^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {476, 385, 211}

$$\frac{\text{ArcTan}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3\sqrt{a}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])]/(3*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.28, size = 74, normalized size = 1.37

$$\frac{\tan^{-1}\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}\,x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*
c - a*d])]/(3*Sqrt[a]*Sqrt[b*c - a*d])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.
time = 2.15, size = 245, normalized size = 4.54

$$\left[-\frac{\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2-4\,((bc-2\,ad)x^9-acx^3)\sqrt{dx^6+c}\,\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)}{12\,(abc-a^2d)},\ \frac{\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\,\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right)}{6\,\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sq
rt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))/(a*b*c -
a^2*d), 1/6*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c
 - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))/sqrt(a*b*c -
 a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [A]
time = 4.52, size = 72, normalized size = 1.33

$$-\frac{\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{3\,\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.861 $\int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}$$

```
[Out] -1/3*b*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(3/2)/(-a*d+b
*c)^(1/2)-1/3*(d*x^6+c)^(1/2)/a/c/x^3
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {476, 491, 12, 385, 211}

$$-\frac{b\text{ArcTan}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^6]/(a*c*x^3) - (b*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*S
qrt[c + d*x^6])])/(3*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
```

```
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{bc}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3ac}\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3a}\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3a}\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A]
time = 0.48, size = 100, normalized size = 1.25

$$-\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\tan^{-1}\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}\,x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3a^{3/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^6]/(a*c*x^3) - (b*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3
+ Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(3*a^(3/2)*Sqrt[b*c - a*d])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(b \,x^{6}+a \right) \sqrt{d \,x^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(64) = 128.
time = 3.37, size = 332, normalized size = 4.15

$$\left[-\frac{\sqrt{-abc+a^{2}d}\, bcx^{3} \log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{12}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{6}+a^{2}c^{2}+4\,((bc-2\,ad)x^{9}-acx^{3})\sqrt{dx^{6}+c}\,\sqrt{-abc+a^{2}d}}{b^{2}x^{12}+2\,abx^{6}+a^{2}}\right)+4\,\sqrt{dx^{6}+c}\,(abc-a^{2}d)}{12\,(a^{2}bc^{2}-a^{3}cd)x^{3}}, -\frac{\sqrt{abc-a^{2}d}\, bcx^{3} \arctan\left(\frac{((bc-2\,ad)x^{6}-ac)\sqrt{dx^{6}+c}\,\sqrt{abc-a^{2}d}}{2\,((abcd-a^{2}d^{2})x^{9}+(abc^{2}-a^{2}cd)x^{3})}\right)+2\,\sqrt{dx^{6}+c}\,(abc-a^{2}d)}{6\,(a^{2}bc^{2}-a^{3}cd)x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(sqrt(-a*b*c + a^2*d)*b*c*x^3*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*
c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))
+ 4*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3), -1/6*(sqr
t(a*b*c - a^2*d)*b*c*x^3*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 +
c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))
 + 2*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(a + b x^{6}\right) \sqrt{c + d x^{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.862 $\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/3*b^2*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(5/2)/(-a*d+
b*c)^(1/2)-1/9*(d*x^6+c)^(1/2)/a/c/x^9+1/9*(2*a*d+3*b*c)*(d*x^6+c)^(1/2)/a^
2/c^2/x^3
```

Rubi [A]
time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {476, 491, 597, 12, 385, 211}

$$\frac{b^2\text{ArcTan}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}\,(2ad+3bc)}{9a^2c^2x^3}-\frac{\sqrt{c+dx^6}}{9acx^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/9*Sqrt[c + d*x^6]/(a*c*x^9) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^6])/(9*a^2*c
^2*x^3) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*
a^(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)$$

$$=-\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{9ac}$$

$$=-\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}-\frac{\text{Subst}\left(\int -\frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,\right.}{9a^2c^2}$$

$$=-\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,\right.}{3a^2}$$

$$=-\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{}{\sqrt{c-}}\right.}{3a^2}$$

$$=-\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.91, size = 121, normalized size = 1.05

$$\frac{\sqrt{c+dx^6}\left(-ac+3bcx^6+2adx^6\right)}{9a^2c^2x^9}+\frac{b^2\tan^{-1}\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}\,x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3a^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-(a*c) + 3*b*c*x^6 + 2*a*d*x^6))/(9*a^2*c^2*x^9) + (b^2*A
rcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c
 - a*d])])/(3*a^(5/2)*Sqrt[b*c - a*d])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^10), x)
```

Fricas [A]
time = 4.90, size = 416, normalized size = 3.62

$$\left[-\frac{3\sqrt{-abc+a^2d}\,b^2c^2x^9\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2-4\,((bc-2\,ad)x^9-acx^3)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)-4\,((3\,ab^2c^2-a^2bcd-2\,a^3d^2)x^6-a^2bc^2+a^3cd)\sqrt{dx^6+c}}{36\,(a^3bc^3-a^4c^2d)x^9},\ \frac{3\sqrt{abc-a^2d}\,b^2c^2x^9\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right)+2\,((3\,ab^2c^2-a^2bcd-2\,a^3d^2)x^6-a^2bc^2+a^3cd)\sqrt{dx^6+c}}{18\,(a^3bc^3-a^4c^2d)x^9}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] [-1/36*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^9*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^
9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 +
a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)
*sqrt(d*x^6 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^9), 1/18*(3*sqrt(a*b*c - a^2*d
)*b^2*c^2*x^9*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b
*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*
b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^6 + c)
)/((a^3*b*c^3 - a^4*c^2*d)*x^9)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**10*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{10}\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.863 $\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{6};1,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

```
[Out] 1/5*x^5*AppellF1(5/6,1,1/2,11/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x
^6+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{5}{6};1,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 1, 1/2, 11/6, -((b*x^6)/a), -((d*x^6
)/c)])/(5*a*Sqrt[c + d*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{x^4}{(a+bx^6)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= \frac{x^5\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{6};1,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

Mathematica [A]
time = 10.03, size = 65, normalized size = 1.02

$$\frac{x^5\sqrt{\frac{c+dx^6}{c}}\,F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{5a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] (x^5*Sqrt[(c + d*x^6)/c]*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6
)/a)])/(5*a*Sqrt[c + d*x^6])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.864 $\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

```
[Out] 1/4*x^4*AppellF1(2/3,1,1/2,5/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x^
6+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {476, 525, 524}

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^6)/a), -((d*x^6)
/c)])/(4*a*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
```

```
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{x}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= \frac{x^4\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

Mathematica [A]
time = 10.04, size = 65, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^6}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{4a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^6)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)
/a)])/(4*a*Sqrt[c + d*x^6])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(x^3/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.865 $\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

```
[Out] 1/2*x^2*AppellF1(1/3,1,1/2,4/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x^
6+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {476, 441, 440}

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^6)/a), -((d*x^6)
/c)])/(2*a*Sqrt[c + d*x^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
```

```
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}\\ &= \frac{x^2\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}\end{aligned}$$

Mathematica [A]
time = 10.03, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt{\frac{c+dx^6}{c}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{2a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] (x^2*Sqrt[(c + d*x^6)/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)
/a)])/(2*a*Sqrt[c + d*x^6])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.866 $\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{6};1,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

```
[Out] x*AppellF1(1/6,1,1/2,7/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x^6+c)^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{1}{6};1,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 1, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/c
)])/(a*Sqrt[c + d*x^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{(a+bx^6)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$

$$= \frac{x\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{6};1,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.
time = 10.14, size = 161, normalized size = 2.73

$$-\frac{7acxF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{(a+bx^6)\sqrt{c+dx^6}\left(-7acF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+3x^6\left(2bcF_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+adF_1\left(\frac{7}{6};\frac{3}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] (-7*a*c*x*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)])/((a + b*x
^6)*Sqrt[c + d*x^6]*(-7*a*c*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b*x
^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^6)/
a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -((d*x^6)/c), -((b*x^6)/a)])))
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.867 $\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{6};1,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

```
[Out] -AppellF1(-1/6,1,1/2,5/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x/(d*x^6+c)
^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{6};1,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 1, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a*x*Sqrt[c + d*x^6]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{x^2(a+bx^6)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{6};1,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.
time = 10.08, size = 141, normalized size = 2.27

$$\frac{-55a(c+dx^6)-11(bc-2ad)x^6\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+10bdx^{12}\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{55a^2cx\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-55*a*(c + d*x^6) - 11*(b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6,
 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*x^12*Sqrt[1 + (d*x^6)/c
]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -((b*x^6)/a)])/(55*a^2*c*x*Sqr
t[c + d*x^6])
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^6 + c)/(b*d*x^14 + (b*c + a*d)*x^8 + a*c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + bx^6) \sqrt{c + dx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.868 $\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

```
[Out] -1/2*AppellF1(-1/3,1,1/2,2/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x^2/(d*
x^6+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {476, 525, 524}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^6)/a), -((d*x^
6)/c)])/(a*x^2*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
```

```
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.
time = 10.08, size = 141, normalized size = 2.20

$$\frac{-20a(c+dx^6)+5(-2bc+ad)x^6\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+2bdx^{12}\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{40a^2cx^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-20*a*(c + d*x^6) + 5*(-2*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3,
 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*x^12*Sqrt[1 + (d*x^6)/c]*
AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b*x^6)/a)])/(40*a^2*c*x^2*Sqrt[
c + d*x^6])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.869 $\int \frac{1}{x^5(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

```
[Out] -1/4*AppellF1(-2/3,1,1/2,1/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x^4/(d*
x^6+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {476, 525, 524}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/4*(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^6)/a), -((d*x^
6)/c)])/(a*x^4*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
```

```
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^5 (a+bx^6) \sqrt{c+dx^6}} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3 (a+bx^3) \sqrt{c+dx^3}} \, dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}} \, dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.
time = 10.08, size = 141, normalized size = 2.20

$$\frac{-8a(c+dx^6) - 4(4bc+ad)x^6\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right) - bdx^{12}\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{32a^2cx^4\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-8*a*(c + d*x^6) - 4*(4*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] - b*d*x^12*Sqrt[1 + (d*x^6)/c]*Appe
llF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*x^6)/a)])/(32*a^2*c*x^4*Sqrt[c +
d*x^6])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(b\,x^6+a\right) \sqrt{d\,x^6+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\,(a+bx^6)\,\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**5*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^5\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^5*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.870 $\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{c+dx^6}}{3b^2d}-\frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\,(a+bx^6)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}$$

```
[Out] 1/6*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+1/3*(d*x^6+c)^(1/2)/b^2/d-1/6*a^2*(d*x^6+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^6+a)
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 91, 81, 65, 214}

$$-\frac{a^2\sqrt{c+dx^6}}{6b^2\,(a+bx^6)\,(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^6}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] Sqrt[c + d*x^6]/(3*b^2*d) - (a^2*Sqrt[c + d*x^6])/(6*b^2*(b*c - a*d)*(a + b
*x^6)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*
d]])/(6*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{17}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= -\frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} - \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+d}}\right.}{12b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} - \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,\right.}{6b^2d(bc-ad)}$$

$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right.}{6b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A]

time = 0.39, size = 130, normalized size = 1.06

$$\frac{\frac{\sqrt{b}\sqrt{c+dx^6}\,(-3a^2d+2b^2cx^6+2ab(c-dx^6))}{d(bc-ad)(a+bx^6)}+\frac{a(4bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{6b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((Sqrt[b]*Sqrt[c + d*x^6]*(-3*a^2*d + 2*b^2*c*x^6 + 2*a*b*(c - d*x^6)))/(d*
(b*c - a*d)*(a + b*x^6)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(6*b^(5/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{\left(b\,x^{6}+a\right)^{2}\sqrt{d\,x^{6}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(103) = 206.
time = 5.35, size = 475, normalized size = 3.86

$$\left[\frac{((4\,ab^2cd-3\,a^2bd^2)x^6+4\,a^2bcd-3\,a^3d^2)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,(2\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^6+2\,ab^3c^2-5\,a^2b^2cd+3\,a^3bd^2)\sqrt{dx^6+c}}{12\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3)x^6)},-\frac{((4\,ab^2cd-3\,a^2bd^2)x^6+4\,a^2bcd-3\,a^3d^2)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)-(2\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^6+2\,ab^3c^2-5\,a^2b^2cd+3\,a^3bd^2)\sqrt{dx^6+c}}{6\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3)x^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2
*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b
*d))/(b*x^6 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^
3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*
b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6), -
1/6*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*
c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - (
2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +
 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.81, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^6+c}\,a^2d}{6\,(b^3c-ab^2d)((dx^6+c)b-bc+ad)}-\frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{6\,(b^3c-ab^2d)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^6+c}}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(d*x^6 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^6 + c)*b - b*c + a*d))
- 1/6*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/3*sqrt(d*x^6 + c)/(b^2*d)
```

Mupad [**B**]
time = 5.09, size = 144, normalized size = 1.17

$$\frac{\sqrt{d\,x^6+c}}{3\,b^2\,d}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^6+c}\,\left(3\,a\,d-4\,b\,c\right)}{\left(3\,a^2\,d-4\,a\,b\,c\right)\,\sqrt{a\,d-b\,c}}\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}+\frac{a^2\,d\,\sqrt{d\,x^6+c}}{2\,\left(a\,d-b\,c\right)\,\left(3\,b^3\,\left(d\,x^6+c\right)-3\,b^3\,c+3\,a\,b^2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(1/2)/(3*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^6)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(6*b^(5/
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b^3*(c
+ d*x^6) - 3*b^3*c + 3*a*b^2*d))
```

3.871 $\int \frac{x^{11}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/6*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/6*a*(d*x^6+c)^(1/2)/b/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 79, 65, 214}

$$\frac{a\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^6\right)\\ &= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)}+\frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{12b(bc-ad)}\\ &= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)}+\frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{6bd(bc-ad)}\\ &= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 100, normalized size = 1.01

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^6}}{(bc-ad)(a+bx^6)}-\frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{6b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(6
*b^(3/2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(b\,x^6+a)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 5.08, size = 348, normalized size = 3.52

$$\left[\frac{((2\,b^2c-abd)x^6+2\,abc-a^2d)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}\,(ab^2c-a^2bd)}{12\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^6)},\ \frac{((2\,b^2c-abd)x^6+2\,abc-a^2d)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)+\sqrt{dx^6+c}\,(ab^2c-a^2bd)}{6\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((2*b^2*c - a*b*d)*x^6 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a))
+ 2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b
^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^6), 1/6*(((2*b^2*c - a*b*d
)*x^6 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-
b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b
^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2
)*x^6)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.27, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^6+c}\,ad^2}{(b^2c-abd)((dx^6+c)b-bc+ad)}+\frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(d*x^6 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^6 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B]
time = 4.99, size = 95, normalized size = 0.96

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)\,(a\,d-2\,b\,c)}{6\,b^{3/2}\,(a\,d-b\,c)^{3/2}}-\frac{a\,d\,\sqrt{d\,x^6+c}}{2\,b\,(a\,d-b\,c)\,(3\,b\,(d\,x^6+c)+3\,a\,d-3\,b\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(6*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^6)^(1/2))/(2*b*(a*d - b*c)*(3*b*(c +
 d*x^6) + 3*a*d - 3*b*c))
```

3.872 $\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{c+dx^6}}{6(bc-ad)\,(a+bx^6)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}\,(bc-ad)^{3/2}}$$

```
[Out] 1/6*d*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/6*(d*x^6+c)^(1/2)/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {455, 44, 65, 214}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}}{6\,(a+bx^6)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^6]/((b*c - a*d)*(a + b*x^6)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^6])/Sqrt[b*c - a*d]])/(6*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^6\right)\\
&= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}-\frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{12(bc-ad)}\\
&= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}-\frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{6(bc-ad)}\\
&= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}\,(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 86, normalized size = 0.99

$$
\frac{1}{6}\left(-\frac{\sqrt{c+dx^6}}{(bc-ad)\left(a+bx^6\right)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,(-bc+ad)^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (-(Sqrt[c + d*x^6]/((b*c - a*d)*(a + b*x^6))) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^6])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/6
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$
\int \frac{x^5}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.
time = 5.69, size = 302, normalized size = 3.47

$$\left[-\frac{(bdx^6+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}\,(b^2c-abd)}{12\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2)},\ -\frac{(bdx^6+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)+\sqrt{dx^6+c}\,(b^2c-abd)}{6\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*d*x^6 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d - 2*
sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*
c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2), -1/6*((b*d*x^6 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqr
t(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(b^2*c
 - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [A]
time = 1.74, size = 93, normalized size = 1.07

$$-\frac{d\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{6\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx^6+c}\,d}{6\left((dx^6+c)b-bc+ad\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/6*sqrt(d*x^6 + c)*d/(((d*x^6 + c)*b - b*c + a*d)*(b*c - a
*d))
```

Mupad [B]
time = 4.93, size = 84, normalized size = 0.97

$$\frac{d\sqrt{d\,x^6+c}}{2\,(a\,d-b\,c)\,\left(3\,b\,(d\,x^6+c)+3\,a\,d-3\,b\,c\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{6\,\sqrt{b}\,{(a\,d-b\,c)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c)) + (
d*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2)))/(6*b^(1/2)*(a*d - b*
c)^(3/2))
```

3.873 $\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=132

$$\frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}$$

```
[Out] 1/6*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-1/3*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/6
*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A]
time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 105, 162, 65, 214}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) - ArcTanh[Sqrt[c + d*x^6]
/Sqrt[c]]/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^6\right)}{6a^2} - \frac{(b(2bc-3ad))\text{S}}{}$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3a^2d} - \frac{(b(2bc-3a}{}$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\right.}{6a^2(bc-ad}$$

Mathematica [A]
time = 0.37, size = 124, normalized size = 0.94

$$\frac{-\frac{ab\sqrt{c+dx^6}}{(-bc+ad)(a+bx^6)} + \frac{\sqrt{b}\,(2bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (-((a*b*Sqrt[c + d*x^6])/((-(b*c) + a*d)*(a + b*x^6))) + (Sqrt[b]*(2*b*c -
3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)
^(3/2) - (2*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/Sqrt[c])/(6*a^2)
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(b\,x^{6}+a\right)^{2}\sqrt{d\,x^{6}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x), x)
```

Fricas [**A**]
time = 4.67, size = 862, normalized size = 6.53

$$\left[\frac{2\sqrt{dx^6+c}\,abc+\left((2b^2c^2-3abcd)x^6+2abc^2-3a^2cd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2bc-ad+2\sqrt{dx^6+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+2\left((b^2c-abd)x^6+abc-a^2d\right)\sqrt{c}\log\left(\frac{dx^6-2\sqrt{dx^6+c}\,\sqrt{c}+2c}{x^6}\right)}{12\left((a^2b^2c^2-a^3bcd)x^6+a^3bc^2-a^4cd\right)},\ \frac{\sqrt{dx^6+c}\,abc+\left((2b^2c^2-3abcd)x^6+2abc^2-3a^2cd\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^6+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right)+\left((b^2c-abd)x^6+abc-a^2d\right)\sqrt{c}\log\left(\frac{dx^6-2\sqrt{dx^6+c}\,\sqrt{c}+2c}{x^6}\right)}{6\left((a^2b^2c^2-a^3bcd)x^6+a^3bc^2-a^4cd\right)},\ \frac{2\sqrt{dx^6+c}\,abc+4\left((b^2c-abd)x^6+abc-a^2d\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-c}}{c}\right)+\left((2b^2c^2-3abcd)x^6+2abc^2-3a^2cd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2bc-ad+2\sqrt{dx^6+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)}{12\left((a^2b^2c^2-a^3bcd)x^6+a^3bc^2-a^4cd\right)},\ \frac{\sqrt{dx^6+c}\,abc+\left((2b^2c^2-3abcd)x^6+2abc^2-3a^2cd\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^6+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right)+2\left((b^2c-abd)x^6+abc-a^2d\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-c}}{c}\right)}{6\left((a^2b^2c^2-a^3bcd)x^6+a^3bc^2-a^4cd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(d*x^6 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 +
 c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*((b^2*c - a*b*d)*x^6
+ a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6)
)/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 +
c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^
6 + b*c)) + ((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sq
rt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c
^2 - a^4*c*d), 1/12*(2*sqrt(d*x^6 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^6 + a*b
```

```
*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a
*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b
*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)))
/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6
 + b*c)) + 2*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x
^6 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [A]
time = 1.45, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^6+c}\,bd}{6\,(abc-a^2d)((dx^6+c)b-bc+ad)}-\frac{(2\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{6\,(a^2bc-a^3d)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(d*x^6 + c)*b*d/((a*b*c - a^2*d)*((d*x^6 + c)*b - b*c + a*d)) - 1/6
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

Mupad [B]
time = 6.23, size = 3025, normalized size = 22.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - ((((4*a^6*b^2*d^5)/3 - 2*a
^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)
```

- ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(144*a^7*b^2
*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(216
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c
^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*
d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*
(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a
^4*b*c*d^2)) + ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*
b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + ((((4*a^6*b^2*d^5)
/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^
4*b*c*d) + ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(144
*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^
3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a
^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(
12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b
c)(-b*(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2
*d - 3*a^4*b*c*d^2)))/(((a*b^3*d^4)/18 - (b^4*c*d^3)/27)/(a^5*d^2 + a^3*b^2
*c^2 - 2*a^4*b*c*d) - ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2
- 20*a*b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - ((((4*a^6*b
^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2
- 2*a^4*b*c*d) - ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/
2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4
*c^2*d^3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^
3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(
1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*
d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2
*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2
*d^2 - 20*a*b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + ((((4*
a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^
2*c^2 - 2*a^4*b*c*d) + ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3
)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^
5*b^4*c^2*d^3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b
^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)
^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*
(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^
3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i
)/(6*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (atan((((
(((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2
+ a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a
^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) - ((c + d*x^6)^(1/
2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b
^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - (((((4*a^6*b^2*d^5)/3 - 2*a^5*b
^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))
+ ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^
3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a

^3*b*c*d)))/(6*a^2*c^(1/2)) + ((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)))/(((a*b^3*d^4)/18 - (b^4*c*d^3)/27)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) - ((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + ((((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) + ((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2))))*1i)/(3*a^2*c^(1/2)) - (b*d*(c + d*x^6)^(1/2))/(2*(a^2*d - a*b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c))

3.874 $\int \frac{1}{x^7(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=185

$$-\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)(a+bx^6)}-\frac{\sqrt{c+dx^6}}{6acx^6(a+bx^6)}+\frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}}-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}$$

```
[Out] 1/6*(a*d+4*b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/6*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/6*b*(-a*d+2*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^6+a)-1/6*
(d*x^6+c)^(1/2)/a/c/x^6/(b*x^6+a)
```

Rubi [A]
time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 105, 156, 162, 65, 214}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}}-\frac{b\sqrt{c+dx^6}(2bc-ad)}{6a^2c(a+bx^6)(bc-ad)}-\frac{\sqrt{c+dx^6}}{6acx^6(a+bx^6)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*(b*(2*b*c - a*d)*Sqrt[c + d*x^6])/(a^2*c*(b*c - a*d)*(a + b*x^6)) - Sq
rt[c + d*x^6]/(6*a*c*x^6*(a + b*x^6)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x
^6]/Sqrt[c]])/(6*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^3*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx,x,x^6\right)$$

$$= -\frac{\sqrt{c+dx^6}}{6acx^6(a+bx^6)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^6\right)}{6ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)(a+bx^6)} - \frac{\sqrt{c+dx^6}}{6acx^6(a+bx^6)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd}{x(a+bx)\sqrt{c+d}}\right.}{6a^2c(bc-a}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)(a+bx^6)} - \frac{\sqrt{c+dx^6}}{6acx^6(a+bx^6)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \frac{}{(a+}\right.}{12a^3(bc-}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)(a+bx^6)} - \frac{\sqrt{c+dx^6}}{6acx^6(a+bx^6)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \frac{}{a-}\right.}{6a^3d(bc-}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)(a+bx^6)} - \frac{\sqrt{c+dx^6}}{6acx^6(a+bx^6)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right.}{6a^3c^{3/2}}$$

Mathematica [A]
time = 0.63, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^6}\,(-a^2d+2b^2cx^6+ab(c-dx^6))}{c(-bc+ad)x^6(a+bx^6)} - \frac{b^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{c^{3/2}}}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*Sqrt[c + d*x^6]*(-(a^2*d) + 2*b^2*c*x^6 + a*b*(c - d*x^6)))/(c*(-(b*c)
+ a*d)*x^6*(a + b*x^6)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcTanh
[Sqrt[c + d*x^6]/Sqrt[c]])/c^(3/2))/(6*a^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 (b\,x^6+a)^2 \sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
[Out] int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^7), x)
```

Fricas [A]
time = 5.35, size = 1236, normalized size = 6.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] [1/12*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^
6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(c)*log((d*x^6
 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6
 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12
+ (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^
6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - ((4*b^3*c^2 - 3*
a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sq
rt(c)*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*((2*a*b^2*c^2
- a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^
4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^2 - 3*a*
b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt
(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*
b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a))
 + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/6*(((
4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^6 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2
- 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) +
 ((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a
^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 (a + bx^6)^2 \sqrt{c + dx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [A]
time = 3.26, size = 257, normalized size = 1.39

$$\frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{6\,(a^3bc-a^4d)\sqrt{-b^2c+abd}}-\frac{2\,(dx^6+c)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^6+c}\,b^2c^2d-(dx^6+c)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^6+c}\,abcd^2-\sqrt{dx^6+c}\,a^2d^3}{6\,(a^2bc^2-a^3cd)\left((dx^6+c)^2b-2\,(dx^6+c)bc+bc^2+(dx^6+c)ad-acd\right)}-\frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6\,a^3\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/6*(2*(d*x^6 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^6 + c)*b^2*c^2*d - (d*x^6 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^6 + c
)*a*b*c*d^2 - sqrt(d*x^6 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^6 + c)^
2*b - 2*(d*x^6 + c)*b*c + b*c^2 + (d*x^6 + c)*a*d - a*c*d)) - 1/6*(4*b*c +
a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B]
time = 7.29, size = 2500, normalized size = 13.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (((c + d*x^6)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^6)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
((c + d*x^6)*(3*a*d - 6*b*c) + 3*b*(c + d*x^6)^2 + 3*b*c^2 - 3*a*c*d) + (at
an((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*
c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576
*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2
*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b
*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*
a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3
```

- a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(12*(a^6*d^3 - a^3*b^3*c^3 +
3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*
b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 +
6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*
a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b^2*c
*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(21
6*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2
)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*
d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c
^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*
c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1
i)/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/((5*a^3*
b^4*d^6 + 32*b^7*c^3*d^3 - 48*a*b^6*c^2*d^4 + 6*a^2*b^5*c*d^5)/(108*(a^6*b^
2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d
- 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d
^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2
- 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b
^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)
/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^
(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*
c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a
^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^
5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
)))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3
*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32
*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(1
8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2
)*(5*a*d - 4*b*c)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c
^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3
d)) + ((-b^3(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a
^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^
2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a
^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2
*d - 3*a^5*b*c*d^2))))*(-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(6*(a
^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d
x^6)^(1/2)(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*
d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d))
+ (((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a
^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((c + d
x^6)^(1/2)(a*d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*
a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 +
a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d +
4*b*c)*1i)/(12*a^3*(c^3)^(1/2)) + ((((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b

```
^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*
(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((144*a^9*b^2*c*d^6 + 288*a
^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c
^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((c + d*x^6)^(1/2)*(a*d + 4*b*c)*(288*
a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c
^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))
*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(12*a^3*(c^3)^(1/2)
))/((5*a^3*b^4*d^6 + 32*b^7*c^3*d^3 - 48*a*b^6*c^2*d^4 + 6*a^2*b^5*c*d^5)/(
108*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((((c + d*x^6)^(1/2)*(a^
4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^
5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*...
```

3.875 $\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=141

$$\frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\,(a+bx^6)}-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

```
[Out] -1/6*(-2*a*d+3*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))*a^
(1/2)/b^2/(-a*d+b*c)^(3/2)+1/3*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b^2/d^(
1/2)+1/6*a*x^3*(d*x^6+c)^(1/2)/b/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A]
time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {476, 481, 537, 223, 212, 385, 211}

$$-\frac{\sqrt{a}\,(3bc-2ad)\text{ArcTan}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}}+\frac{ax^3\sqrt{c+dx^6}}{6b\,(a+bx^6)\,(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*x^3*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b^2*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} - \frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b^2} - \frac{(a(3bc-2ad))\text{Sub}}{}$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b^2} - \frac{(a(3bc-2ad))}{}$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} - \frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}}{3}$$

Mathematica [A]
time = 1.95, size = 152, normalized size = 1.08

$$\frac{\frac{abx^3\sqrt{c+dx^6}}{(bc-ad)(a+bx^6)} + \frac{\sqrt{a}(-3bc+2ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}x^6+bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{d}x^3}\right)}{\sqrt{d}}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*b*x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*S
qrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*ArcTanh[Sqrt[c + d*x^6]/(Sqrt[d]*x
^3)])/Sqrt[d])/(6*b^2)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(b\,x^6+a)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^14/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Fricas [A]
time = 3.16, size = 1077, normalized size = 7.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3 + 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 -
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*
x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((b^4*c*d - a
*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3
 - 8*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqr
t(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(
-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2
- 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*
b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a
*b*x^6 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12
*(2*sqrt(d*x^6 + c)*a*b*d*x^3 + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d -
2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d
*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + 2*((b^2*c - a*b*d)*x^6
 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c)
)/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12*(2*sqrt(d*x^6
 + c)*a*b*d*x^3 - 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(s
qrt(-d)*x^3/sqrt(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2
*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*
x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)))/((b^4*c*d - a*b^3*d^2)*x
^6 + a*b^3*c*d - a^2*b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**14/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(117) = 234.
time = 1.90, size = 343, normalized size = 2.43

$$-\frac{\left(3\,abc\sqrt{-d}\,\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-2\,a^2\sqrt{-d}\,d\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-2\,\sqrt{abc-a^2d}\,bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+2\,\sqrt{abc-a^2d}\,ad\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+\sqrt{abc-a^2d}\,a\sqrt{-d}\,\sqrt{d}\right)\mathrm{sgn}(x)}{6\left(\sqrt{abc-a^2d}\,b^3c\sqrt{-d}-\sqrt{abc-a^2d}\,ab^2\sqrt{-d}\,d\right)}+\frac{ac\sqrt{d+\frac{c}{x^6}}}{6\left(b^2c\mathrm{sgn}\left(x\right)-abd\mathrm{sgn}\left(x\right)\right)\left(bc+a\left(d+\frac{c}{x^6}\right)-ad\right)}+\frac{\left(3\,abc-2\,a^2d\right)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{6\left(b^3c\mathrm{sgn}\left(x\right)-ab^2d\mathrm{sgn}\left(x\right)\right)\sqrt{abc-a^2d}}-\frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{3\,b^2\sqrt{-d}\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*a*b*c*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*a^2*sqrt(-
d)*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*sqrt(a*b*c - a^2*d)*b*c*arct
an(sqrt(d)/sqrt(-d)) + 2*sqrt(a*b*c - a^2*d)*a*d*arctan(sqrt(d)/sqrt(-d)) +
 sqrt(a*b*c - a^2*d)*a*sqrt(-d)*sqrt(d))*sgn(x)/(sqrt(a*b*c - a^2*d)*b^3*c*
sqrt(-d) - sqrt(a*b*c - a^2*d)*a*b^2*sqrt(-d)*d) + 1/6*a*c*sqrt(d + c/x^6)/
((b^2*c*sgn(x) - a*b*d*sgn(x))*(b*c + a*(d + c/x^6) - a*d)) + 1/6*(3*a*b*c
- 2*a^2*d)*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/((b^3*c*sgn(x) - a
*b^2*d*sgn(x))*sqrt(a*b*c - a^2*d)) - 1/3*arctan(sqrt(d + c/x^6)/sqrt(-d))/
(b^2*sqrt(-d)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.876 $\int \frac{x^8}{(a+bx^6)^2 \sqrt{c+dx^6}} \, dx$

Optimal. Leaf size=93

$$-\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\,(a+bx^6)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6\sqrt{a}\,(bc-ad)^{3/2}}$$

```
[Out] 1/6*c*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/(-a*d+b*c)^(3/2)
/a^(1/2)-1/6*x^3*(d*x^6+c)^(1/2)/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {476, 482, 12, 385, 211}

$$\frac{c\text{ArcTan}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^3\sqrt{c+dx^6}}{6\,(a+bx^6)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*(x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (c*ArcTan[(Sqrt[b*c
- a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
```

```
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)(a+bx^6)}+\frac{\text{Subst}\left(\int \frac{c}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)(a+bx^6)}+\frac{c\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)(a+bx^6)}+\frac{c\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{6(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)(a+bx^6)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}\,(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.95, size = 112, normalized size = 1.20

$$\frac{1}{6}\left(-\frac{x^3\sqrt{c+dx^6}}{(bc-ad)(a+bx^6)}+\frac{c\tan^{-1}\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}\,x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (-((x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6))) + (c*ArcTan[(a*Sqrt[d]
+ b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[
a]*(b*c - a*d)^(3/2)))/6
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(b\,x^{6}+a\right)^{2} \sqrt{d\,x^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(77) = 154.
time = 3.83, size = 426, normalized size = 4.58

$$\left[-\frac{4\,\sqrt{dx^{6}+c}\,(abc-a^{2}d)x^{3}-(bcx^{6}+ac)\sqrt{-abc+a^{2}d}\,\log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{12}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{6}+a^{2}c^{2}+4\,((bc-2\,ad)x^{9}-acx^{3})\sqrt{dx^{6}+c}\,\sqrt{-abc+a^{2}d}}{b^{2}x^{12}+2\,abx^{6}+a^{2}}\right)}{24\,((ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2})x^{6}+a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2})}, -\frac{2\,\sqrt{dx^{6}+c}\,(abc-a^{2}d)x^{3}-(bcx^{6}+ac)\sqrt{abc-a^{2}d}\,\arctan\left(\frac{((bc-2\,ad)x^{6}-ac)\sqrt{dx^{6}+c}\,\sqrt{abc-a^{2}d}}{2\,((abcd-a^{2}d^{2})x^{9}+(abc^{2}-a^{2}cd)x^{3})}\right)}{12\,((ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2})x^{6}+a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(4*sqrt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt
(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c*
d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/12*(2*sqrt(d*
x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1
/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d
- a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/((a*b^3*c^2 - 2*a^2*b^2*c*d + a
^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(x**8/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.877 $\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=104

$$\frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\,(a+bx^6)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/6*(-2*a*d+b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(3/
2)/(-a*d+b*c)^(3/2)+1/6*b*x^3*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {476, 390, 385, 211}

$$\frac{(bc-2ad)\text{ArcTan}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}+\frac{bx^3\sqrt{c+dx^6}}{6a\,(a+bx^6)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (b*x^3*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + 2) + 1, 0] && (LtQ[p, -1] ||  !L
```

```
tQ[q, -1]) && NeQ[p, -1]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\,(a+bx^6)} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a(bc-ad)}\\
&= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\,(a+bx^6)} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{6a(bc-ad)}\\
&= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\,(a+bx^6)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.61, size = 124, normalized size = 1.19

$$-\frac{bx^3\sqrt{c+dx^6}}{6a(-bc+ad)\,(a+bx^6)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^6+bx^3\sqrt{c+dx^6}}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{6a^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*(b*x^3*Sqrt[c + d*x^6])/(a*(-(b*c) + a*d)*(a + b*x^6)) + ((b*c - 2*a*d
)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[
b*c - a*d])])/(6*a^(3/2)*(b*c - a*d)^(3/2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

[Out] int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(88) = 176.
time = 3.90, size = 467, normalized size = 4.49

$$\left[\frac{4\,\sqrt{dx^6+c}\,(ab^2c-a^2bd)x^3-((b^2c-2\,abd)x^6+abc-2\,a^2d)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2-4\,((bc-2\,ad)x^9-acx^3)\sqrt{dx^6+c}\,\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)}{24\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x^6)},\ \frac{2\,\sqrt{dx^6+c}\,(ab^2c-a^2bd)x^3+((b^2c-2\,abd)x^6+abc-2\,a^2d)\sqrt{abc-a^2d}\,\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\,\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right)}{12\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/24*(4*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 - ((b^2*c - 2*a*b*d)*x^6 + a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^6), 1/12*(2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 + ((b^2*c - 2*a*b*d)*x^6 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^6)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)

[Out] Integral(x**2/((a + b*x**6)**2*sqrt(c + d*x**6)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(88) = 176.

time = 0.89, size = 237, normalized size = 2.28

$$-\frac{1}{6}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2bc-2\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2ad-bc^2\right)}{\left(\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^4b-2\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2bc+4\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2ad+bc^2\right)(abcd-a^2d^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^3 - sqrt(d*x^6 + c))^2*b*c - 2*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^3 - sqrt(d*x^6 + c))^4*b - 2*(sqrt(d)*x^3 - sqrt(d*
x^6 + c))^2*b*c + 4*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^6+a\right)^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.878 $\int \frac{1}{x^4(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=149

$$-\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3}+\frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\,(a+bx^6)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}$$

```
[Out] -1/6*b*(-4*a*d+3*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/
a^(5/2)/(-a*d+b*c)^(3/2)-1/6*(-2*a*d+3*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b*c
)/x^3+1/6*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/x^3/(b*x^6+a)
```

Rubi [A]
time = 0.13, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 483, 597, 12, 385, 211}

$$-\frac{b(3bc-4ad)\text{ArcTan}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}\,(3bc-2ad)}{6a^2cx^3(bc-ad)}+\frac{b\sqrt{c+dx^6}}{6ax^3\,(a+bx^6)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*((3*b*c - 2*a*d)*Sqrt[c + d*x^6])/(a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c
+ d*x^6])/(6*a*(b*c - a*d)*x^3*(a + b*x^6)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sq
rt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(5/2)*(b*c - a*d)^(3/2)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2 \left(a+bx^2\right)^2 \sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-4a}{(a+bx^2)\sqrt{c+}}\right.}{6a^2c(bc-}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{(b(3bc-4ad))\text{Subst}\left(\right.}{6a}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{(b(3bc-4ad))\text{Subst}\left(\right.}{}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{b(3bc-4ad)\tan^{-1}\left(\right.}{6a^{5/2}(bc-}$$

Mathematica [A]
time = 1.18, size = 157, normalized size = 1.05

$$\frac{\sqrt{c+dx^6}\left(2abc-2a^2d+3b^2cx^6-2abdx^6\right)}{6a^2c(-bc+ad)x^3\left(a+bx^6\right)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^6+bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{5/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^6 - 2*a*b*d*x^6))/(6*a^2*c*
(-(b*c) + a*d)*x^3*(a + b*x^6)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*
Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*a^(5/2)
*(b*c - a*d)^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(129) = 258.
time = 5.93, size = 612, normalized size = 4.11

$$\left[-\frac{((3\,b^3c^2-4\,ab^2cd)x^9+(3\,ab^2c^2-4\,a^2bcd)x^3)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2+4\,((bc-2\,ad)x^9-acx^3)\sqrt{dx^6+c}\,\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)+4\,((3\,ab^3c^2-5\,a^2b^2cd+2\,a^3bd^2)x^6+2\,a^2b^2c^2-4\,a^3bcd+2\,a^4d^2)\sqrt{dx^6+c}}{24\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2)x^9+(a^4b^2c^3-2\,a^5bc^2d+a^6cd^2)x^3)},-\frac{((3\,b^3c^2-4\,ab^2cd)x^9+(3\,ab^2c^2-4\,a^2bcd)x^3)\sqrt{abc-a^2d}\,\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\,\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right)+2\,((3\,ab^3c^2-5\,a^2b^2cd+2\,a^3bd^2)x^6+2\,a^2b^2c^2-4\,a^3bcd+2\,a^4d^2)\sqrt{dx^6+c}}{12\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2)x^9+(a^4b^2c^3-2\,a^5bc^2d+a^6cd^2)x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*s
qrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*
c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6
 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 4*((3*a*b^3*c^2
 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d
^2)*sqrt(d*x^6 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (
a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^3), -1/12*(((3*b^3*c^2 - 4*a*b^2
*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2
*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d -
a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2*c*d
+ 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x^6 +
c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (a^4*b^2*c^3 - 2*a
^5*b*c^2*d + a^6*c*d^2)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.879 $\int \frac{1}{x^{10}(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=208

$$-\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9}+\frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3}+\frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\,(a+bx^6)}+\frac{b^2(5bc-6ad)\tan}{6a^{7/2}(}$$

```
[Out] 1/6*b^2*(-6*a*d+5*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))
/a^(7/2)/(-a*d+b*c)^(3/2)-1/18*(-2*a*d+5*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^9+1/18*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^6+c)^(1/2)/a^3/c^2/(-a*
d+b*c)/x^3+1/6*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/x^9/(b*x^6+a)
```

Rubi [A]
time = 0.21, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 483, 597, 12, 385, 211}

$$\frac{b^2(5bc-6ad)\text{ArcTan}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}\,(5bc-2ad)}{18a^2cx^9(bc-ad)}+\frac{\sqrt{c+dx^6}\,(-4a^2d^2-8abcd+15b^2c^2)}{18a^3c^2x^3(bc-ad)}+\frac{b\sqrt{c+dx^6}}{6ax^9\,(a+bx^6)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/18*((5*b*c - 2*a*d)*Sqrt[c + d*x^6])/(a^2*c*(b*c - a*d)*x^9) + ((15*b^2*
c^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^6])/(18*a^3*c^2*(b*c - a*d)*x^3)
+ (b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*x^9*(a + b*x^6)) + (b^2*(5*b*c - 6*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(7/2)*(b*
c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8a\cdots}{x^2(\cdots}\right.}{1\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{\cdots}{6a(bc\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{\cdots}{6a(bc\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{\cdots}{6a(bc\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{\cdots}{6a(bc\cdots}$$

Mathematica [A]
time = 2.01, size = 201, normalized size = 0.97

$$-\frac{\sqrt{c+dx^6}\left(15b^3c^2x^{12}+2ab^2cx^6(5c-4dx^6)+2a^3d(c-2dx^6)-2a^2b(c^2+3cdx^6+2d^2x^{12})\right)}{18a^3c^2(-bc+ad)x^9\left(a+bx^6\right)} + \frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}\,x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{7/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/18*(Sqrt[c + d*x^6]*(15*b^3*c^2*x^12 + 2*a*b^2*c*x^6*(5*c - 4*d*x^6) + 2
*a^3*d*(c - 2*d*x^6) - 2*a^2*b*(c^2 + 3*c*d*x^6 + 2*d^2*x^12)))/(a^3*c^2*(-
(b*c) + a*d)*x^9*(a + b*x^6)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] + b*
x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*a^(7/2)
*(b*c - a*d)^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^10), x)
```

Fricas [A]
time = 7.55, size = 760, normalized size = 3.65

$$\left[-\frac{3\left((5b^4c^3-6ab^3c^2d)x^{15}+(5ab^3c^3-6a^2b^2c^2d)x^9\right)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{12}-2(3abc^2-4a^2cd)x^6+a^2c^2-4\left((bc-2ad)x^9-acx^3\right)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2abx^6+a^2}\right)-4\left((15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3)x^{12}-2a^3b^2c^3+4a^4bc^2d-2a^5cd^2+2(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3)x^6\right)\sqrt{dx^6+c}}{72\left((a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2)x^{15}+(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2)x^9\right)}, \frac{3\left((5b^4c^3-6ab^3c^2d)x^{15}+(5ab^3c^3-6a^2b^2c^2d)x^9\right)\sqrt{abc-a^2d}\arctan\left(\frac{\left((bc-2ad)x^6-ac\right)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2\left((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3\right)}\right)+2\left((15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3)x^{12}-2a^3b^2c^3+4a^4bc^2d-2a^5cd^2+2(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3)x^6\right)\sqrt{dx^6+c}}{36\left((a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2)x^{15}+(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2)x^9\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/72*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^9)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*s
qrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*((15
*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^12 - 2*a^3
*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d
 + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3*c^4 - 2*a^5*b^2
*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*
x^9), 1/36*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*
c^2*d)*x^9)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d
*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d
)*x^3)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d
^3)*x^12 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 -
 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3
*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d
 + a^7*c^2*d^2)*x^9)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**10*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{10}\,{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.880 $\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{6};2,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

```
[Out] 1/5*x^5*AppellF1(5/6,2,1/2,11/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d
*x^6+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{5}{6};2,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 2, 1/2, 11/6, -((b*x^6)/a), -((d*x^6
)/c)])/(5*a^2*Sqrt[c + d*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{x^4}{(a+bx^6)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= \frac{x^5\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{6};2,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(64) = 128.
time = 10.13, size = 169, normalized size = 2.64

$$\frac{x^5\left(55ab(c+dx^6)+11(bc-6ad)\,(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-10bdx^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)}{330a^2(bc-ad)\,(a+bx^6)\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*(55*a*b*(c + d*x^6) + 11*(b*c - 6*a*d)*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]
*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] - 10*b*d*x^6*(a +
b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -((b*
x^6)/a)]))/(330*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^6+a)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

[Out] integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*x^6 + c)*x^4/(b^2*d*x^18 + (b^2*c + 2*a*b*d)*x^12 + (2*a*b*c + a^2*d)*x^6 + a^2*c), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)

[Out] Integral(x**4/((a + b*x**6)**2*sqrt(c + d*x**6)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\left(b\,x^6+a\right)^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)

[Out] int(x^4/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)

3.881 $\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

```
[Out] 1/4*x^4*AppellF1(2/3,2,1/2,5/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d*
x^6+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {476, 525, 524}

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^6)/a), -((d*x^6)
/c)])/(4*a^2*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
```

```
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{x}{(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= \frac{x^4\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(64) = 128.
time = 10.13, size = 168, normalized size = 2.62

$$-\frac{x^4\left(-10ab(c+dx^6)-5(bc-3ad)\,(a+bx^6)\,\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+bdx^6(a+bx^6)\,\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)}{60a^2(bc-ad)\,(a+bx^6)\,\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/60*(x^4*(-10*a*b*(c + d*x^6) - 5*(b*c - 3*a*d)*(a + b*x^6)*Sqrt[1 + (d*x
^6)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^6*(a
+ b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b*
x^6)/a)]))/(a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(x^3/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.882 $\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

```
[Out] 1/2*x^2*AppellF1(1/3,2,1/2,4/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d*
x^6+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {476, 441, 440}

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^6)/a), -((d*x^6)
/c)])/(2*a^2*Sqrt[c + d*x^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
```

```
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= \frac{x^2\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(64) = 128.
time = 10.12, size = 172, normalized size = 2.69

$$\frac{8abx^2(c+dx^6)+8(2bc-3ad)x^2(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+bdx^8(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{48a^2(bc-ad)(a+bx^6)\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
[Out] (8*a*b*x^2*(c + d*x^6) + 8*(2*b*c - 3*a*d)*x^2*(a + b*x^6)*Sqrt[1 + (d*x^6)
/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^8*(a + b
*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*x^6
)/a)])/(48*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^6+a)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
[Out] int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^6)^2 \sqrt{c+dx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(x/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(x/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.883 $\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{6};2,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

```
[Out] x*AppellF1(1/6,2,1/2,7/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d*x^6+c)
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{1}{6};2,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 2, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/c
)])/(a^2*Sqrt[c + d*x^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{(a+bx^6)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= \frac{x\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{6};2,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(59) = 118.
time = 10.20, size = 329, normalized size = 5.58

$$\frac{x\left(-2bdx^6\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-\frac{7a\left(7ac(6ad-b(6c+dx^6))F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+3bx^6(c+dx^6)\left(2bcF_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+adF_1\left(\frac{7}{6};\frac{3}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)\right)}{(a+bx^6)\left(-7acF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+3x^6\left(2bcF_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+adF_1\left(\frac{7}{6};\frac{3}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)\right)}\right)}{42a^2(-bc+ad)\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*(-2*b*d*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[7/6, 1/2, 1, 13/6, -((d*x^6)/c)
, -((b*x^6)/a)] - (7*a*(7*a*c*(6*a*d - b*(6*c + d*x^6))*AppellF1[1/6, 1/2,
1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*b*x^6*(c + d*x^6)*(2*b*c*AppellF1[7
/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 1
3/6, -((d*x^6)/c), -((b*x^6)/a)])))/((a + b*x^6)*(-7*a*c*AppellF1[1/6, 1/2,
 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2, 2, 1
3/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -((d*x^6
)/c), -((b*x^6)/a)])))))/(42*a^2*(-(b*c) + a*d)*Sqrt[c + d*x^6])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^6+a)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{6}\right)^{2} \sqrt{c + dx^{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.884 $\int \frac{1}{x^2(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{6};2,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

```
[Out] -AppellF1(-1/6,2,1/2,5/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x/(d*x^6+
c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{6};2,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 2, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a^2*x*Sqrt[c + d*x^6]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}} \int \frac{1}{x^2(a+bx^6)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\, F_1\left(-\frac{1}{6};2,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 226 vs. $2(62) = 124$.
time = 10.22, size = 226, normalized size = 3.65

$$\frac{55a(c+dx^6)\left(6a^2d-7b^2cx^6-6ab(c-dx^6)\right)-11(7b^2c^2-24abcd+12a^2d^2)x^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+10bd(7bc-6ad)x^{12}(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{330a^3c(bc-ad)x\,(a+bx^6)\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (55*a*(c + d*x^6)*(6*a^2*d - 7*b^2*c*x^6 - 6*a*b*(c - d*x^6)) - 11*(7*b^2*c
^2 - 24*a*b*c*d + 12*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[
5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*(7*b*c - 6*a*d)*x^1
2*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c)
, -((b*x^6)/a)])/(330*a^3*c*(b*c - a*d)*x*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*x^6 + c)/(b^2*d*x^20 + (b^2*c + 2*a*b*d)*x^14 + (2*a*b*c + a^2*d)*x^8 + a^2*c*x^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^6\right)^2 \sqrt{c + dx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)

[Out] Integral(1/(x**2*(a + b*x**6)**2*sqrt(c + d*x**6)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)

[Out] int(1/(x^2*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)

3.885 $\int \frac{1}{x^3(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

```
[Out] -1/2*AppellF1(-1/3,2,1/2,2/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x^2/(
d*x^6+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {476, 525, 524}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^6)/a), -((d*x^
6)/c)])/(a^2*x^2*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
```

```
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(64) = 128.
time = 10.18, size = 226, normalized size = 3.53

$$\frac{20a(c+dx^6)\left(3a^2d-4b^2cx^6-3ab(c-dx^6)\right)-5(8b^2c^2-15abcd+3a^2d^2)\,x^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+2bd(4bc-3ad)x^{12}(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{120a^3c(bc-ad)x^2\left(a+bx^6\right)\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (20*a*(c + d*x^6)*(3*a^2*d - 4*b^2*c*x^6 - 3*a*b*(c - d*x^6)) - 5*(8*b^2*c^
2 - 15*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^12*(a
 + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b
*x^6)/a)])/(120*a^3*c*(b*c - a*d)*x^2*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^6\right)^2 \sqrt{c + dx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.886 $\int \frac{1}{x^5(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

```
[Out] -1/4*AppellF1(-2/3,2,1/2,1/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x^4/(
d*x^6+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {476, 525, 524}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/4*(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^6)/a), -((d*x^
6)/c)])/(a^2*x^4*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
```

```
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}} \, dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}} \, dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\, F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 225 vs. 2(64) = 128.
time = 10.21, size = 225, normalized size = 3.52

$$\frac{8a(c+dx^6)\left(3a^2d-5b^2cx^6-3ab(c-dx^6)\right)+4(-20b^2c^2+21abcd+3a^2d^2)\,x^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+bd(-5bc+3ad)x^{12}(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{96a^3c(bc-ad)x^4\left(a+bx^6\right)\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (8*a*(c + d*x^6)*(3*a^2*d - 5*b^2*c*x^6 - 3*a*b*(c - d*x^6)) + 4*(-20*b^2*c
^2 + 21*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[1
/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*(-5*b*c + 3*a*d)*x^12*(a
 + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b
*x^6)/a)])/(96*a^3*c*(b*c - a*d)*x^4*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**5*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^5\,{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^5*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.887 $\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=104

$$-\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/12*(d*x^8+c)^(3/2)/b/d^2-1/4*a^2*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*
c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/4*(a*d+b*c)*(d*x^8+c)^(1/2)/b^2/d^2
```

Rubi [A]
time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 90, 65, 214}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}\,(ad+bc)}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*((b*c + a*d)*Sqrt[c + d*x^8])/(b^2*d^2) + (c + d*x^8)^(3/2)/(12*b*d^2)
 - (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt
[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)\\ &= \frac{1}{8}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx, x, x^8\right)\\ &= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8b^2}\\ &= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4b^2d}\\ &= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 91, normalized size = 0.88

$$\frac{\sqrt{c+dx^8}\left(-2bc-3ad+bdx^8\right)}{12b^2d^2}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{4b^{5/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-2*b*c - 3*a*d + b*d*x^8))/(12*b^2*d^2) + (a^2*ArcTan[(Sq
rt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(4*b^(5/2)*Sqrt[-(b*c) + a*d])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{23}}{(b\,x^8+a)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.41, size = 288, normalized size = 2.77

$$\left[\frac{3\,\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,((b^3cd-ab^2d^2)x^8-2\,b^3c^2-ab^2cd+3\,a^2bd^2)\sqrt{dx^8+c}}{24\,(b^4cd^2-ab^3d^3)},\ \frac{3\,\sqrt{-b^2c+abd}\,a^2d^2\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b^2c+abd}}{bdx^8+bc}\right)+((b^3cd-ab^2d^2)x^8-2\,b^3c^2-ab^2cd+3\,a^2bd^2)\sqrt{dx^8+c}}{12\,(b^4cd^2-ab^3d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*
x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^8 -
 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/12*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*
c + a*b*d)/(b*d*x^8 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^8 - 2*b^3*c^2 - a*b^
2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**23/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [A]
time = 2.76, size = 106, normalized size = 1.02

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}\,b^2}+\frac{(dx^8+c)^{\frac{3}{2}}b^2d^4-3\sqrt{dx^8+c}\,b^2cd^4-3\sqrt{dx^8+c}\,abd^5}{12\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*a^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/12*((d*x^8 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^8 + c)*b^2*c*d^4 - 3*
sqrt(d*x^8 + c)*a*b*d^5)/(b^3*d^6)
```

Mupad [B]
time = 4.68, size = 103, normalized size = 0.99

$$\frac{(d\,x^8+c)^{3/2}}{12\,b\,d^2}-\left(\frac{c}{2\,b\,d^2}+\frac{4\,a\,d^3-4\,b\,c\,d^2}{16\,b^2\,d^4}\right)\sqrt{d\,x^8+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(3/2)/(12*b*d^2) - (c/(2*b*d^2) + (4*a*d^3 - 4*b*c*d^2)/(16*b^2
*d^4))*(c + d*x^8)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c
)^(1/2)))/(4*b^(5/2)*(a*d - b*c)^(1/2))
```

3.888 $\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{c+dx^8}}{4bd}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}$$

```
[Out] 1/4*a*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+1/4*(d*x^8+c)^(1/2)/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 81, 65, 214}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] Sqrt[c + d*x^8]/(4*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a
*d]])/(4*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)\\ &= \frac{\sqrt{c+dx^8}}{4bd} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8b}\\ &= \frac{\sqrt{c+dx^8}}{4bd} - \frac{a\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4bd}\\ &= \frac{\sqrt{c+dx^8}}{4bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 73, normalized size = 0.99

$$\frac{1}{4}\left(\frac{\sqrt{c+dx^8}}{bd} - \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) +
a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.94, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}\,(b^2c-abd)}{8\,(b^3cd-ab^2d^2)},\ -\frac{\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b^2c+abd}}{bdx^8+bc}\right)-\sqrt{dx^8+c}\,(b^2c-abd)}{4\,(b^3cd-ab^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c
)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/4*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^8 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - sqrt(d*x^8 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**15/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [A]
time = 3.23, size = 64, normalized size = 0.86

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^8+c}}{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(a*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^8 + c)/b)/d
```

Mupad [B]
time = 4.73, size = 58, normalized size = 0.78

$$\frac{\sqrt{d\,x^8+c}}{4\,b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(1/2)/(4*b*d) - (a*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)
^(1/2)))/(4*b^(3/2)*(a*d - b*c)^(1/2))
```

3.889 $\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -1/4*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {455, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{8}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{4\sqrt{b}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]]/(4*Sqrt[b]*Sqrt[-(b*c)
 + a*d])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(b\,x^8+a)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.24, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)}{8\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b^2c+abd}}{bdx^8+bc}\right)}{4\,(b^2c-abd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b
*x^8 + a))/sqrt(b^2*c - a*b*d), 1/4*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A]
time = 18.50, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*b*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 2.70, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B]
time = 4.59, size = 40, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{4\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] atan((b*(c + d*x^8)^(1/2))/(a*b*d - b^2*c)^(1/2))/(4*(a*b*d - b^2*c)^(1/2))
```

3.890 $\int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}$$

```
[Out] -1/4*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/4*arctanh(b^(1/2)*(d*x^8+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 88, 65, 214}

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*a*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{8}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^8\right)}{8a} - \frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8a}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4ad} - \frac{b\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.13, size = 80, normalized size = 0.94

$$-\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/Sqrt[-
(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/Sqrt[c])/a
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x), x)
```

Fricas [A]
time = 3.14, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2bc-ad+2\sqrt{dx^8+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+\sqrt{c}\log\left(\frac{dx^8-2\sqrt{dx^8+c}\,\sqrt{c}+2c}{x^8}\right)}{8ac}, \frac{2c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)+\sqrt{c}\log\left(\frac{dx^8-2\sqrt{dx^8+c}\,\sqrt{c}+2c}{x^8}\right)}{8ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2bc-ad+2\sqrt{dx^8+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-c}}{c}\right)}{8ac}, \frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)+\sqrt{-c}\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-c}}{c}\right)}{4ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(c*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + sqrt(c)*log((d*x^8 - 2*sqrt
(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) + sqrt
(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-
c)/c))/(a*c), 1/4*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^8 + c)*
sqrt(-c)/c))/(a*c)]
```

Sympy [A]
time = 14.05, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**8)/sqrt(-c))/(4*a*sqrt(-c))
```

Giac [A]
time = 2.57, size = 71, normalized size = 0.84

$$-\frac{b \arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}\,a}+\frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{4\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B]
time = 4.81, size = 652, normalized size = 7.67

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{dx^8+c}}{\sqrt{c}}\right)}{4\,a\sqrt{c}}-\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{b^2c-a\,b\,d}\left(\frac{b^3d^2\sqrt{dx^8+c}}{4}-\frac{\sqrt{b^2c-a\,b\,d}\left(a^2b^2d^3-\frac{\left(8a^3b^2d^3-16a^2b^3c\,d^2\right)\sqrt{dx^8+c}\,\sqrt{b^2c-a\,b\,d}}{8\left(a^2d-a\,b\,c\right)}\right)}{8\left(a^2d-a\,b\,c\right)}\right)1{}\mathrm{i}}{8\left(a^2d-a\,b\,c\right)}+\frac{\sqrt{b^2c-a\,b\,d}\left(\frac{b^3d^2\sqrt{dx^8+c}}{4}+\frac{\sqrt{b^2c-a\,b\,d}\left(a^2b^2d^3+\frac{\left(8a^3b^2d^3-16a^2b^3c\,d^2\right)\sqrt{dx^8+c}\,\sqrt{b^2c-a\,b\,d}}{8\left(a^2d-a\,b\,c\right)}\right)}{8\left(a^2d-a\,b\,c\right)}\right)1{}\mathrm{i}}{8\left(a^2d-a\,b\,c\right)}}{\frac{\sqrt{b^2c-a\,b\,d}\left(\frac{b^3d^2\sqrt{dx^8+c}}{4}-\frac{\sqrt{b^2c-a\,b\,d}\left(a^2b^2d^3-\frac{\left(8a^3b^2d^3-16a^2b^3c\,d^2\right)\sqrt{dx^8+c}\,\sqrt{b^2c-a\,b\,d}}{8\left(a^2d-a\,b\,c\right)}\right)}{8\left(a^2d-a\,b\,c\right)}\right)}{8\left(a^2d-a\,b\,c\right)}-\frac{\sqrt{b^2c-a\,b\,d}\left(\frac{b^3d^2\sqrt{dx^8+c}}{4}+\frac{\sqrt{b^2c-a\,b\,d}\left(a^2b^2d^3+\frac{\left(8a^3b^2d^3-16a^2b^3c\,d^2\right)\sqrt{dx^8+c}\,\sqrt{b^2c-a\,b\,d}}{8\left(a^2d-a\,b\,c\right)}\right)}{8\left(a^2d-a\,b\,c\right)}\right)}{8\left(a^2d-a\,b\,c\right)}}\right)\sqrt{b^2c-a\,b\,d}\,1{}\mathrm{i}}{4\left(a^2d-a\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^8)^(1/2)/c^(1/2))/(4*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^
(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3
- ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/
2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8*(a^2*d - a*b*c)) + ((
b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 + ((b^2*c - a*b*d)^(1/2
)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2
*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8*(a^2*d
 - a*b*c)))/(((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*
c - a*b*d)^(1/2)*(a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*
x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c))
))/(8*(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2)
)/4 + ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*
d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2
*d - a*b*c))))/(8*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i)/(4*(a^2*d -
a*b*c))
```

3.891 $\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}$$

```
[Out] 1/8*(a*d+2*b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/4*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/8*(d
*x^8+c)^(1/2)/a/c/x^8
```

Rubi [A]

time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 105, 162, 65, 214}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}}-\frac{\sqrt{c+dx^8}}{8acx^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*Sqrt[c + d*x^8]/(a*c*x^8) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqr
t[c]])/(8*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*
c - a*d]])/(4*a^2*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)\\
&= -\frac{\sqrt{c+dx^8}}{8acx^8}-\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8ac}\\
&= -\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8a^2}-\frac{(2bc+ad)\text{Subst}\left(\int\right.}{16\ldots}\\
&= -\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{b^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{4a^2d}-\frac{(2bc+ad)\text{Subst}\left(\ldots\right.}{\ldots}\\
&= -\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\ldots}}{\sqrt{bc-a\ldots}}\right.}{4a^2\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 109, normalized size = 0.93

$$\frac{-\frac{a\sqrt{c+dx^8}}{cx^8}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{c^{3/2}}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-((a*Sqrt[c + d*x^8])/(c*x^8)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8
])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[c
+ d*x^8]/Sqrt[c]])/c^(3/2))/(8*a^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9} \left(b\,x^{8}+a \right) \sqrt{d\,x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^9), x)
```

Fricas [A]
time = 2.93, size = 565, normalized size = 4.83

$$\left[\frac{2\,bc^{2}x^{8}\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{8}+2\,bc-ad-2\,\sqrt{dx^{8}+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{8}+a}\right)+(2\,bc+ad)\sqrt{c}\,x^{8}\log\left(\frac{dx^{8}+2\,\sqrt{dx^{8}+c}\,\sqrt{c}+2\,c}{x^{8}}\right)-2\,\sqrt{dx^{8}+c}\,ac}{16\,a^{2}c^{2}x^{8}},\ -\frac{4\,bc^{2}x^{8}\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^{8}+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^{8}+bc}\right)-(2\,bc+ad)\sqrt{c}\,x^{8}\log\left(\frac{dx^{8}+2\,\sqrt{dx^{8}+c}\,\sqrt{c}+2\,c}{x^{8}}\right)+2\,\sqrt{dx^{8}+c}\,ac}{16\,a^{2}c^{2}x^{8}},\ \frac{bc^{2}x^{8}\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{8}+2\,bc-ad-2\,\sqrt{dx^{8}+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{8}+a}\right)-(2\,bc+ad)\sqrt{-c}\,x^{8}\arctan\left(\frac{\sqrt{dx^{8}+c}\,\sqrt{-c}}{c}\right)-\sqrt{dx^{8}+c}\,ac}{8\,a^{2}c^{2}x^{8}},\ -\frac{2\,bc^{2}x^{8}\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^{8}+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^{8}+bc}\right)+(2\,bc+ad)\sqrt{-c}\,x^{8}\arctan\left(\frac{\sqrt{dx^{8}+c}\,\sqrt{-c}}{c}\right)+\sqrt{dx^{8}+c}\,ac}{8\,a^{2}c^{2}x^{8}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(
d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + (2*b*c + a*d)*sq
rt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*sqrt(d*x^8
 + c)*a*c)/(a^2*c^2*x^8), -1/16*(4*b*c^2*x^8*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - (2*b*c +
 a*d)*sqrt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*sq
rt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), 1/8*(b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b
*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(
b*x^8 + a)) - (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c)
```

```
 - sqrt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), -1/8*(2*b*c^2*x^8*sqrt(-b/(b*c - a*d
))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)
) + (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + sqrt(d*
x^8 + c)*a*c)/(a^2*c^2*x^8)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**9*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [A]
time = 1.82, size = 104, normalized size = 0.89

$$\frac{b^2\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8\,a^2\sqrt{-c}\,c}-\frac{\sqrt{dx^8+c}}{8\,acx^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/8*(2*b*c + a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/8*sqrt(d*x^8 + c)/(a*c*x^8)
```

Mupad [B]
time = 5.51, size = 396, normalized size = 3.38

$$\frac{\ln\left(\sqrt{d\,x^8+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{8\,a^3\,d-8\,a^2\,b\,c}-\frac{\ln\left(\sqrt{d\,x^8+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{8\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^8+c}}{8\,a\,c\,x^8}-\frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^8+c}\,3{}\mathrm{i}}{128\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{128\,c}+\frac{5\,a\,b^3\,d^5}{256\,c^2}+\frac{a^2\,b^2\,d^6}{256\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^8+c}\,1{}\mathrm{i}}{256\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{256\,a}+\frac{b^2\,d^6}{256\,c}+\frac{3\,b^4\,c\,d^4}{128\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^8+c}\,5{}\mathrm{i}}{256\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{128\,a}+\frac{5\,b^3\,d^5}{256\,c}+\frac{a\,b^2\,d^6}{256\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)\,1{}\mathrm{i}}{8\,a^2\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] (log((c + d*x^8)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(8*a^3*d - 8*a^2*b*c) - (log((c + d*x^8
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(8*(a^3*d - a^2*b*c)) - (c + d*x^8)^(1/2)/(8*a*c*x^8)
- (atan((b^4*d^4*(c + d*x^8)^(1/2)*3i)/(128*(c^3)^(1/2)*((3*b^4*d^4)/(128*c
) + (5*a*b^3*d^5)/(256*c^2) + (a^2*b^2*d^6)/(256*c^3))) + (b^2*d^6*(c + d*x
^8)^(1/2)*1i)/(256*(c^3)^(1/2)*((5*b^3*d^5)/(256*a) + (b^2*d^6)/(256*c) + (
3*b^4*c*d^4)/(128*a^2))) + (b^3*d^5*(c + d*x^8)^(1/2)*5i)/(256*(c^3)^(1/2)*
((3*b^4*d^4)/(128*a) + (5*b^3*d^5)/(256*c) + (a*b^2*d^6)/(256*c^2))))*(a*d
+ 2*b*c)*1i)/(8*a^2*(c^3)^(1/2))
```

3.892 $\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=123

$$\frac{x^4\sqrt{c+dx^8}}{8bd}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}}-\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}$$

```
[Out] -1/8*(2*a*d+b*c)*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b^2/d^(3/2)+1/4*a^(3/
2)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/b^2/(-a*d+b*c)^(1/2
)+1/8*x^4*(d*x^8+c)^(1/2)/b/d
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {476, 490, 537, 223, 212, 385, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}+\frac{x^4\sqrt{c+dx^8}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[c + d*x^8])/(8*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt
[a]*Sqrt[c + d*x^8])])/(4*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^4)/Sqrt[c + d*x^8]])/(8*b^2*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= \frac{x^4\sqrt{c+dx^8}}{8bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8bd}$$

$$= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b^2} - \frac{(bc+2ad)\text{Subst}\left(\int\right.}{8}$$

$$= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b^2} - \frac{(bc+2ad)\text{Subs}}{}$$

$$= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right.}{8b^2d^{3/2}}$$

Mathematica [A]
time = 1.44, size = 140, normalized size = 1.14

$$\frac{\sqrt{d}\left(bx^4\sqrt{c+dx^8}+\frac{2a^{3/2}d\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^8+bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}\right)-(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{d}\,x^4}\right)}{8b^2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] (Sqrt[d]*(b*x^4*Sqrt[c + d*x^8] + (2*a^(3/2)*d*ArcTan[(a*Sqrt[d] + b*Sqrt[d
]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d])
 - (b*c + 2*a*d)*ArcTanh[Sqrt[c + d*x^8]/(Sqrt[d]*x^4)])/(8*b^2*d^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{(b\,x^8+a)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^19/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [A]
time = 3.86, size = 739, normalized size = 6.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d
*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + (b*c + 2*a*
d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^2), 1/
16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 -
8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*(
(b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^
8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*(b*c + 2*a*d
)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*d^2), 1/16*(2*sqrt(d*
x^8 + c)*b*d*x^4 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x
^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) + (b*c
+ 2*a*d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^
2), 1/8*(sqrt(d*x^8 + c)*b*d*x^4 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((
b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c
*x^4)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*
d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**19/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19}}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.893 $\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}$$

```
[Out] 1/4*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b/d^(1/2)-1/4*arctan(x^4*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 494, 223, 212, 385, 211}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*S
qrt[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\mathrm{Subst}\left(\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^4\right)\\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^4\right)}{4b} - \frac{a\mathrm{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{4b}\\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{4b} - \frac{a\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{4b}\\
&= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}
\end{aligned}
$$

Mathematica [A]
time = 0.68, size = 107, normalized size = 1.18

$$
\frac{-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}\,x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{d}\,x^4}\right)}{\sqrt{d}}}{4b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqr
t[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + ArcTanh[Sqrt[c + d*x^8]/(Sqrt[d]
*x^4)]/Sqrt[d])/(4*b)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{8}+a\right) \sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^11/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [A]
time = 3.78, size = 632, normalized size = 6.95

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^{12}-(abc^2-a^2cd)x^4)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{16}+2\,abx^8+a^2}\right)+2\sqrt{d}\log\left(-2\,dx^8-2\sqrt{dx^8+c}\sqrt{d}\,x^4-c\right)}{16\,bd}, \frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^{12}-(abc^2-a^2cd)x^4)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{16}+2\,abx^8+a^2}\right)-4\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x^4}{\sqrt{dx^8+c}}\right)}{16\,bd}, \frac{d\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\sqrt{\frac{a}{bc-ad}}}{2\,(adx^{12}+acx^4)}\right)+\sqrt{d}\log\left(-2\,dx^8-2\sqrt{dx^8+c}\sqrt{d}\,x^4-c\right)}{8\,bd}, \frac{d\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\sqrt{\frac{a}{bc-ad}}}{2\,(adx^{12}+acx^4)}\right)-2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x^4}{\sqrt{dx^8+c}}\right)}{8\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/
(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*
sqrt(d)*x^4 - c))/(b*d), 1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*
c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c
)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*sqrt(-d)*arctan(s
qrt(-d)*x^4/sqrt(d*x^8 + c)))/(b*d), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2
*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 +
a*c*x^4)) + sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b*d
```

```
), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*
x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) - 2*sqrt(-d)*arctan(sqrt
(-d)*x^4/sqrt(d*x^8 + c)))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.894 $\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4\sqrt{a}\,\sqrt{bc-ad}}$$

[Out] 1/4*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(1/2)/(-a*d+b*c)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {476, 385, 211}

$$\frac{\text{ArcTan}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4\sqrt{a}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])]/(4*Sqrt[a]*Sqrt[b*c - a*d])

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)$$
$$= \frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.35, size = 74, normalized size = 1.37

$$\frac{\tan^{-1}\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}\,x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*
c - a*d])]/(4*Sqrt[a]*Sqrt[b*c - a*d])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b\,x^8+a)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.
time = 2.75, size = 245, normalized size = 4.54

$$\left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((bc-2\,ad)x^{12}-acx^4)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)}{16\,(abc-a^2d)}, \frac{\arctan\left(\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4)}\right)}{8\,\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*s
qrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))/(a*b*c -
 a^2*d), 1/8*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*
c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4))/sqrt(a*b*c
 - a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [A]
time = 1.80, size = 72, normalized size = 1.33

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.895 $\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}$$

```
[Out] -1/4*b*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(3/2)/(-a*d+b
*c)^(1/2)-1/4*(d*x^8+c)^(1/2)/a/c/x^4
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {476, 491, 12, 385, 211}

$$-\frac{b\text{ArcTan}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^8]/(a*c*x^4) - (b*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*S
qrt[c + d*x^8])])/(4*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
```

```
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)\\ &= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{\text{Subst}\left(\int \frac{bc}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4ac}\\ &= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4a}\\ &= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4a}\\ &= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [**A**]
time = 0.57, size = 100, normalized size = 1.25

$$-\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b\tan^{-1}\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}\,x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{3/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^8]/(a*c*x^4) - (b*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4
+ Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(3/2)*Sqrt[b*c - a*d])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5} \left(b\,x^{8}+a\right) \sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(64) = 128.
time = 3.78, size = 332, normalized size = 4.15

$$\left[-\frac{\sqrt{-abc+a^{2}d}\,bcx^{4}\log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{16}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{8}+a^{2}c^{2}+4\,((bc-2\,ad)x^{12}-acx^{4})\sqrt{dx^{8}+c}\,\sqrt{-abc+a^{2}d}}{b^{2}x^{16}+2\,abx^{8}+a^{2}}\right)+4\,\sqrt{dx^{8}+c}\,(abc-a^{2}d)}{16\,(a^{2}bc^{2}-a^{3}cd)x^{4}},\ -\frac{\sqrt{abc-a^{2}d}\,bcx^{4}\arctan\left(\frac{((bc-2\,ad)x^{8}-ac)\sqrt{dx^{8}+c}\,\sqrt{abc-a^{2}d}}{2\,((abcd-a^{2}d^{2})x^{12}+(abc^{2}-a^{2}cd)x^{4})}\right)+2\,\sqrt{dx^{8}+c}\,(abc-a^{2}d)}{8\,(a^{2}bc^{2}-a^{3}cd)x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(sqrt(-a*b*c + a^2*d)*b*c*x^4*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a
*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))
 + 4*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4), -1/8*(sq
rt(a*b*c - a^2*d)*b*c*x^4*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 +
 c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4
)) + 2*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5} \left(a+bx^{8}\right) \sqrt{c+dx^{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [A]
time = 1.50, size = 116, normalized size = 1.45

$$\frac{1}{4}\,d^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,ad}+\frac{2}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2-c\right)ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x^
4 - sqrt(d*x^8 + c))^2 - c)*a*d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.896 $\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/4*b^2*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(5/2)/(-a*d+
b*c)^(1/2)-1/12*(d*x^8+c)^(1/2)/a/c/x^12+1/12*(2*a*d+3*b*c)*(d*x^8+c)^(1/2)
/a^2/c^2/x^4
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 491, 597, 12, 385, 211}

$$\frac{b^2\text{ArcTan}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}\,(2ad+3bc)}{12a^2c^2x^4}-\frac{\sqrt{c+dx^8}}{12acx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/12*Sqrt[c + d*x^8]/(a*c*x^12) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^8])/(12*a^
2*c^2*x^4) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/
(4*a^(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{12ac}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} - \frac{\text{Subst}\left(\int -\frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, \right.}{12a^2c^2}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, \right.}{4a^2}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \right.}{4a^2}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} + \frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}$$

Mathematica [A]
time = 1.50, size = 121, normalized size = 1.05

$$\frac{\sqrt{c+dx^8}\left(-ac+3bcx^8+2adx^8\right)}{12a^2c^2x^{12}} + \frac{b^2\tan^{-1}\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}\,x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-(a*c) + 3*b*c*x^8 + 2*a*d*x^8))/(12*a^2*c^2*x^12) + (b^2
*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b
*c - a*d])])/(4*a^(5/2)*Sqrt[b*c - a*d])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13}\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^13), x)
```

Fricas [A]
time = 3.01, size = 416, normalized size = 3.62

$$\left[-\frac{3\sqrt{-abc+a^2d}\,b^2c^2x^{12}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((bc-2\,ad)x^{12}-acx^4)\sqrt{dx^8+c}\,\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)-4\,((3\,ab^2c^2-a^2bcd-2\,a^3d^2)x^8-a^2bc^2+a^3cd)\sqrt{dx^8+c}}{48\,(a^3bc^3-a^4c^2d)x^{12}},\ \frac{3\sqrt{abc-a^2d}\,b^2c^2x^{12}\arctan\left(\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\,\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4)}\right)+2\,((3\,ab^2c^2-a^2bcd-2\,a^3d^2)x^8-a^2bc^2+a^3cd)\sqrt{dx^8+c}}{24\,(a^3bc^3-a^4c^2d)x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] [-1/48*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^12*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x
^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8
+ a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*
d)*sqrt(d*x^8 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12), 1/24*(3*sqrt(a*b*c - a^
2*d)*b^2*c^2*x^12*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt
(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*(
(3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^8
 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**13*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(95) = 190.
time = 4.42, size = 205, normalized size = 1.78

$$-\frac{1}{12}\,d^{\frac{5}{2}}\left(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a^2d^2}+\frac{2\left(3\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-6\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc-6\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ad+3\,bc^2+2\,acd\right)}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2-c\right)^3a^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*
(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b
*c - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d
)*x^4 - sqrt(d*x^8 + c))^2 - c)^3*a^2*d^2))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{13}\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.897 $\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=851

$$-\frac{\sqrt[4]{-a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{-a}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{-bc+ad}}+\frac{\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+\cdots}{\left(\sqrt{c}+\cdots\right.}}}{4b\sqrt[4]{c}\cdots}$$

```
[Out] -1/8*(-a)^(1/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1
/2))/b^(3/4)/(-a*d+b*c)^(1/2)-1/8*(-a)^(1/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a
)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(3/4)/(a*d-b*c)^(1/2)+1/4*(cos(2*arctan(
d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF
(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*
x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(d*x^8+c)^(1/2)-1/8
*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4
)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(
b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/
2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/8*d^(1/4)*(c
os(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)
))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^
(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1
/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/16*(cos(2*arctan(d^(1/4
)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(
2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(
-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/
2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/
b/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/16*(cos(2*arctan(d^(1/4)*x^2/
c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arct
an(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(
1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^
(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(
1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A]

time = 0.79, antiderivative size = 851, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 494, 226, 418, 1231, 1721}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*((-a)^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c +
d*x^8])])/(b^(3/4)*Sqrt[b*c - a*d]) - ((-a)^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d
]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(3/4)*Sqrt[-(b*c) + a*d]
) + ((Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*c^(1/4)*d^(1/4)*Sqrt[c
+ d*x^8]) - (a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sq
rt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(
d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((
Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[
(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(
1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c]
+ Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + S
qrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt
[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b
*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a
]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^
4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]
*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
```

```
2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^8}} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^8}} - \frac{\left(\sqrt{c}\left(\sqrt{c}\cdots\right.\right.}{\cdots}$$

$$= -\frac{\sqrt[4]{-a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{-a}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{-bc+ad}} \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.07, size = 65, normalized size = 0.08

$$\frac{x^{10}\sqrt{\frac{c+dx^8}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{10a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^10*Sqrt[(c + d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8
)/a)])/(10*a*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**9/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^9}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^9/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.898 $\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=754

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{-bc+ad}}+\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\cdots\right.}{8\cdots}$$

```
[Out] -1/8*b^(1/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2)
)/(-a)^(3/4)/(-a*d+b*c)^(1/2)-1/8*b^(1/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(
1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(3/4)/(a*d-b*c)^(1/2)+1/8*d^(1/4)*(cos(2
*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*E
llipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(
-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2
)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/8*d^(1/4)*(cos(2*arctan(d^(1/4)
*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*
arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(
1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(
1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^
(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^
2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c
^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/
2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(
a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1
/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^
(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(a*d+b
*c)/(d*x^8+c)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 754, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {476, 418, 1231, 226, 1721}

$$-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{x^2\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{ad-bc}}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}+\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}(ad+bc)}+\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(b^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x
^8])])/((-a)^(3/4)*Sqrt[b*c - a*d]) - (b^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x
^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(3/4)*Sqrt[-(b*c) + a*d]
```

```
) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)
*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^
2)/c^(1/4)], 1/2]])/(8*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[-a]*Sqr
t[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/
(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])
/(8*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*S
qrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^
2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]
*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2
*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellipt
icPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqr
t[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a*c^(1/4)*d^(1/4)*(b*c +
a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
```

```
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,\right.}{4a}$$

$$= \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}\,x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a(bc+ad)}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{-bc+ad}} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.04, size = 65, normalized size = 0.09

$$\frac{x^2\sqrt{\frac{c+dx^8}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{2a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^8)/c]*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)
/a)])/(2*a*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^8+a)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.899 $\int \frac{1}{x^7(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=878

$$-\frac{\sqrt{c+dx^8}}{6acx^6}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{bc-ad}}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{-bc+ad}}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right.}{}$$

```
[Out] -1/8*b^(5/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2)
)/(-a)^(7/4)/(-a*d+b*c)^(1/2)-1/8*b^(5/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(
1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/4)/(a*d-b*c)^(1/2)-1/6*(d*x^8+c)^(1/2
)/a/c/x^6-1/12*d^(3/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*a
rctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/
2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/
a/c^(5/4)/(d*x^8+c)^(1/2)-1/8*b*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x
^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(a*d+b*c)/(
d*x^8+c)^(1/2)-1/8*b*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4
))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2
))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/(a*d+b*c)/(d*x^8+c
)^(1/2)-1/16*b*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^
(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1
/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^
(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8
+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^
(1/2)-1/16*b*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1
/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/
2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(
1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+
c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(
1/2)
```

Rubi [A]
time = 1.15, antiderivative size = 878, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {476, 491, 537, 226, 418, 1231, 1721}

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^8]/(a*c*x^6) - (b^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a
)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(7/4)*Sqrt[b*c - a*d]) - (b^(5/4
)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8
*(-a)^(7/4)*Sqrt[-(b*c) + a*d]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
+ d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4
)], 1/2])/(12*a*c^(5/4)*Sqrt[c + d*x^8]) - (b*((Sqrt[b]*Sqrt[c])/Sqrt[-a] +
 Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[
d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b
*c + a*d)*Sqrt[c + d*x^8]) - (b*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1
/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*(b*c + a*d)*Sq
rt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[
d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b
]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTa
n[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c
+ d*x^8]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c]
+ Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/
4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
```

```
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^7 (a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} + \frac{\text{Subst}\left(\int \frac{-3bc-ad-bdx^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{6ac}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{b\text{Subst}\left(\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2a} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}}\right.}{6ac}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right.\right.}{12ac^{5/4}\sqrt{c+dx^8}}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right.\right.}{12ac^{5/4}\sqrt{c+dx^8}}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{bc-ad}} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+}}{\sqrt[4]{-a}\,\sqrt[4]{b}}\right.}{8(-a)^{7/4}\sqrt{-bc-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.12, size = 141, normalized size = 0.16

$$\frac{-5a(c+dx^8) - 5(3bc+ad)x^8\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right) - bdx^{16}\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{30a^2cx^6\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-5*a*(c + d*x^8) - 5*(3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[1/4, 1
/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] - b*d*x^16*Sqrt[1 + (d*x^8)/c]*Appe
llF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)])/(30*a^2*c*x^6*Sqrt[c +
d*x^8])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 (a + bx^8) \sqrt{c + dx^8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^7\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^7*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.900 $\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1005

$$\frac{x^2\sqrt{c+dx^8}}{2b\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^4\right)}+\frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{bc-ad}}-\frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{-bc+ad}}-\cdots$$

```
[Out] 1/8*(-a)^(3/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/
2))/b^(5/4)/(-a*d+b*c)^(1/2)-1/8*(-a)^(3/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)
^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(5/4)/(a*d-b*c)^(1/2)+1/2*x^2*(d*x^8+c)^(
1/2)/b/d^(1/2)/(c^(1/2)+x^4*d^(1/2))-1/2*c^(1/4)*(cos(2*arctan(d^(1/4)*x^2/
c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticE(sin(2*arcta
n(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1
/2)+x^4*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^8+c)^(1/2)+1/4*c^(1/4)*(cos(2*arct
an(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellipt
icF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(
(d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^8+c)^(1/2)-1/16*(co
s(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4))
)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(
1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(-a)^(1/2)*
(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/
2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x
^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^
(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))
*(-a)^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((
d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(
d*x^8+c)^(1/2)+1/8*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4
))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))
*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1
/2)+1/8*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(
1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)
/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A]

time = 0.91, antiderivative size = 1005, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {476, 494, 311, 226, 1210, 504, 1231, 1721}

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^8])/(2*b*Sqrt[d]*(Sqrt[c] + Sqrt[d]*x^4)) + ((-a)^(3/4)*A
rcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(5/
4)*Sqrt[b*c - a*d]) - ((-a)^(3/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(5/4)*Sqrt[-(b*c) + a*d]) - (c^(1/4)*(Sq
rt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[
2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*b*d^(3/4)*Sqrt[c + d*x^8]) + (c^(
1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*d^(3/4)*Sqrt[c + d*x^8]
) + (a*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*
x^2)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (a*(Sqrt[c
] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d
*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[-a]*(Sqrt[b]*Sqrt[
c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c]
+ Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(S
qrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(1
6*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[-a]*(Sqrt[b]
*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sq
rt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(
4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])
/(16*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b} - \frac{a\text{Subst}\left(\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b}$$
$$= \frac{a\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b^{3/2}} - \frac{a\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)}\right.}{4b^{3/2}}$$
$$= \frac{x^2\sqrt{c+dx^8}}{2b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^4\right)} - \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\right.}{2bd^{3/4}\sqrt{c+dx^8}}$$
$$= \frac{x^2\sqrt{c+dx^8}}{2b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^4\right)} + \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{bc-ad}} - \frac{(-a)^{3/4}\tan}{8}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.07, size = 65, normalized size = 0.06

$$\frac{x^{14}\sqrt{\frac{c+dx^8}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{14a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] (x^14*Sqrt[(c + d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^
8)/a)])/(14*a*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{(b\,x^8+a)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^8 + c)*x^13/(b*d*x^16 + (b*c + a*d)*x^8 + a*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**13/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{13}}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^13/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.901 $\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=768

$$\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{bc-ad}}-\frac{\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{-bc+ad}}-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^4\right)}{8\sqrt[4]{c}\,(bc+}$$

```
[Out] 1/8*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(1
/4)/b^(1/4)/(-a*d+b*c)^(1/2)-1/8*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1
/4)/(d*x^8+c)^(1/2))/(-a)^(1/4)/b^(1/4)/(a*d-b*c)^(1/2)-1/16*(cos(2*arctan(
d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticP
i(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2
))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*
(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^
(1/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(-a)^(1/2)/b^(1/2)/(d*x^8+c)^(1/2)+1/16*(co
s(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4))
)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^
(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x
^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*
d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(-a)^(1/2)/b^(1/2)/(d*x^8+c)^(1
/2)-1/8*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d
^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/
2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(
c^(1/2)+x^4*d^(1/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/8*d^(1/4)
*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1
/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^
4*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(
1/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 768, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {476, 504, 1231, 226, 1721}

$$\frac{\text{ArcTan}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{bc-ad}}-\frac{\text{ArcTan}\left(\frac{x^2\sqrt{ad-bc}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{ad-bc}}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}\,\sqrt{c+dx^8}\,(ad+bc)}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\left(\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}+\sqrt{c}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}\,\sqrt{c+dx^8}\,(ad+bc)}-\frac{\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16\sqrt{-a}\,\sqrt{b}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^8}\,(ad+bc)}+\frac{\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\left(\sqrt{-a}\,\sqrt{d}+\sqrt{b}\,\sqrt{c}\right)^2\Pi\left(-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16\sqrt{-a}\,\sqrt{b}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^8}\,(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])]/(8*(-a)^
(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4
```

```
)*b^(1/4)*Sqrt[c + d*x^8])]/(8*(-a)^(1/4)*b^(1/4)*Sqrt[-(b*c) + a*d]) - ((S
qrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(
c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1
/4)], 1/2])/(8*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] + (Sqrt[-a]
*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c
] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*c^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2
*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellipt
icPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*
Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*Sqrt[-a]*Sqrt[b]*c^(1/
4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt
[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*
EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[
c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*Sqrt[-a]*Sqrt[b]*c^
(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}\,x^2\right)\cdots}\right.}{4\sqrt{b}\cdots}$$

$$= -\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}\,x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx, x, x^{\cdots}\right.}{4(bc+ad)}\cdots$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}} - \frac{\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{-bc+ad}} - \frac{\left(\sqrt{c}-\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.04, size = 65, normalized size = 0.08

$$\frac{x^6\sqrt{\frac{c+dx^8}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{6a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^6*Sqrt[(c + d*x^8)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)
/a)])/(6*a*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(a+bx^{8}\right)\sqrt{c+dx^{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**5/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.902 $\int \frac{1}{x^3(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1032

$$-\frac{\sqrt{c+dx^8}}{2acx^2}+\frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{2ac\left(\sqrt{c}+\sqrt{d}\,x^4\right)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{bc-ad}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{-bc+ad}}$$

```
[Out] 1/8*b^(3/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))
/(-a)^(5/4)/(-a*d+b*c)^(1/2)-1/8*b^(3/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1
/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(5/4)/(a*d-b*c)^(1/2)-1/2*(d*x^8+c)^(1/2)
/a/c/x^2+1/2*x^2*d^(1/2)*(d*x^8+c)^(1/2)/a/c/(c^(1/2)+x^4*d^(1/2))-1/2*d^(1
/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c
^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)
+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(3/4)/(d*x^8+c)
^(1/2)+1/4*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arcta
n(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^
(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^
(3/4)/(d*x^8+c)^(1/2)-1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4)
)),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^
(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2
)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/d
^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2
)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*
x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2
)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/
2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/
2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/8*b*d^(1/4)*(cos(2*arctan(d^
(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(s
in(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/
2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/
c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/8*b*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c
^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan
(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)+(-a)^(1/
2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(a*
d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A]

time = 1.17, antiderivative size = 1032, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {476, 491, 598, 311, 226, 1210, 504, 1231, 1721}

$$-\frac{\sqrt{b}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}+\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8(-a)^{5/4}\sqrt{bc-ad}}-\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt{ad-bc}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8(-a)^{5/4}\sqrt{ad-bc}}-\frac{\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2ac^{3/4}\sqrt{dx^8+c}}+\frac{\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4ac^{3/4}\sqrt{dx^8+c}}+\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}+\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}+\frac{\sqrt{d}x^2\sqrt{dx^8+c}}{2ac\left(\sqrt{d}x^4+\sqrt{c}\right)}-\frac{\sqrt{dx^8+c}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/2*Sqrt[c + d*x^8]/(a*c*x^2) + (Sqrt[d]*x^2*Sqrt[c + d*x^8])/(2*a*c*(Sqrt
[c] + Sqrt[d]*x^4)) + (b^(3/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(
1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(5/4)*Sqrt[b*c - a*d]) - (b^(3/4)*ArcTan[(S
qrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(5/4)
*Sqrt[-(b*c) + a*d]) - (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(S
qrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(
2*a*c^(3/4)*Sqrt[c + d*x^8]) + (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d
*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
 1/2])/(4*a*c^(3/4)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqr
t[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x
^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c +
 a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*
(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellipti
cF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c + a*d)*Sqrt[c +
 d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[
d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b
]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTa
n[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*
Sqrt[c + d*x^8]) - (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c]
 + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqr
t[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*
ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c +
a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
```

```
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\text{Subst}\left(\int \frac{x^2(-bc+ad+bdx^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\text{Subst}\left(\int \left(\frac{dx^2}{\sqrt{c+dx^4}} - \frac{bcx^2}{(a+bx^4)\sqrt{c+dx^4}}\right)dx, x, x^2\right)}{2ac}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} - \frac{b\text{Subst}\left(\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2a} + \frac{d\text{Subst}\left(\int \frac{x^2}{\sqrt{c+d}}\right.}{2ac}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a} - \frac{\sqrt{b}\,\text{S}}{}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{2ac\left(\sqrt{c}+\sqrt{d}\,x^4\right)} - \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\right.}}}{2ac^{3/4}\sqrt{c}}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{2ac\left(\sqrt{c}+\sqrt{d}\,x^4\right)} + \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{bc-ad}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.10, size = 141, normalized size = 0.14

$$\frac{-21a(c+dx^8)+7(-bc+ad)x^8\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+3bdx^{16}\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{42a^2cx^2\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-21*a*(c + d*x^8) + 7*(-(b*c) + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^16*Sqrt[1 + (d*x^8)/c]*
AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)])/(42*a^2*c*x^2*Sqrt
[c + d*x^8])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.903 $\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{5}{8};1,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

```
[Out] 1/5*x^5*AppellF1(5/8,1,1/2,13/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*x
^8+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{5}{8};1,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 1, 1/2, 13/8, -((b*x^8)/a), -((d*x^8
)/c)])/(5*a*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^4}{(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= \frac{x^5\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{5}{8};1,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

Mathematica [A]
time = 10.06, size = 65, normalized size = 1.02

$$\frac{x^5\sqrt{\frac{c+dx^8}{c}}\,F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{5a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^8)/c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8
)/a)])/(5*a*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^8+a)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**4/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^4/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.904 $\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{3}{8};1,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

```
[Out] 1/3*x^3*AppellF1(3/8,1,1/2,11/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*x
^8+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{3}{8};1,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 1, 1/2, 11/8, -((b*x^8)/a), -((d*x^8
)/c)])/(3*a*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^2}{(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= \frac{x^3\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{3}{8};1,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

Mathematica [A]
time = 10.05, size = 65, normalized size = 1.02

$$\frac{x^3\sqrt{\frac{c+dx^8}{c}}\,F_1\left(\frac{3}{8};\frac{1}{2},1;\frac{11}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{3a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] (x^3*Sqrt[(c + d*x^8)/c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^8
)/a)])/(3*a*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^8+a)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^2/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.905 $\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{1}{8};1,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

```
[Out] x*AppellF1(1/8,1,1/2,9/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*x^8+c)^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{1}{8};1,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 1, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/c
)])/(a*Sqrt[c + d*x^8])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}} = \frac{x\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{1}{8};1,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.
time = 10.14, size = 161, normalized size = 2.73

$$-\frac{9acxF_1\left(\frac{1}{8};\frac{1}{2},1;\frac{9}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{(a+bx^8)\sqrt{c+dx^8}\left(-9acF_1\left(\frac{1}{8};\frac{1}{2},1;\frac{9}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+4x^8\left(2bcF_1\left(\frac{9}{8};\frac{1}{2},2;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+adF_1\left(\frac{9}{8};\frac{3}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-9*a*c*x*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)])/((a + b*x
^8)*Sqrt[c + d*x^8]*(-9*a*c*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x
^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/
a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8, -((d*x^8)/c), -((b*x^8)/a)])))
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^8+a)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.906 $\int \frac{1}{x^2(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{1+\frac{dx^8}{c}}\,F_1\left(-\frac{1}{8};1,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

```
[Out] -AppellF1(-1/8,1,1/2,7/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/x/(d*x^8+c)
^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,F_1\left(-\frac{1}{8};1,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 1, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a*x*Sqrt[c + d*x^8]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^8) \sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}} \int \frac{1}{x^2(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= -\frac{\sqrt{1+\frac{dx^8}{c}}\, F_1\left(-\frac{1}{8};1,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.
time = 10.11, size = 141, normalized size = 2.27

$$\frac{-35a(c+dx^8) - 5(bc-3ad)x^8\sqrt{1+\frac{dx^8}{c}}\, F_1\left(\frac{7}{8};\frac{1}{2},1;\frac{15}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right) + 7bdx^{16}\sqrt{1+\frac{dx^8}{c}}\, F_1\left(\frac{15}{8};\frac{1}{2},1;\frac{23}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{35a^2cx\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-35*a*(c + d*x^8) - 5*(b*c - 3*a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[7/8,
1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*x^16*Sqrt[1 + (d*x^8)/c]*
AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/c), -((b*x^8)/a)])/(35*a^2*c*x*Sqrt[
c + d*x^8])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(b\,x^8+a\right) \sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^2), x)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*x^8 + c)/(b*d*x^18 + (b*c + a*d)*x^10 + a*c*x^2), x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + bx^8) \sqrt{c + dx^8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)

[Out] Integral(1/(x**2*(a + b*x**8)*sqrt(c + d*x**8)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^2), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*x^8)*(c + d*x^8)^(1/2)),x)

[Out] int(1/(x^2*(a + b*x^8)*(c + d*x^8)^(1/2)), x)

3.907 $\int \frac{1}{x^4(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{1+\frac{dx^8}{c}}\,F_1\left(-\frac{3}{8};1,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

```
[Out] -1/3*AppellF1(-3/8,1,1/2,5/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/x^3/(d*
x^8+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,F_1\left(-\frac{3}{8};1,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/3*(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 1, 1/2, 5/8, -((b*x^8)/a), -((d*x^
8)/c)])/(a*x^3*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^4(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= -\frac{\sqrt{1+\frac{dx^8}{c}}\,F_1\left(-\frac{3}{8};1,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 141 vs. $2(64) = 128$.
time = 10.12, size = 141, normalized size = 2.20

$$\frac{-65a(c+dx^8)+13(-3bc+ad)x^8\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5bdx^{16}\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{13}{8};\frac{1}{2},1;\frac{21}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{195a^2cx^3\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-65*a*(c + d*x^8) + 13*(-3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8
, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*x^16*Sqrt[1 + (d*x^8)/c
]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -((b*x^8)/a)])/(195*a^2*c*x^3*
Sqrt[c + d*x^8])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^4), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^4\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.908 $\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{c+dx^8}}{4b^2d}-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}$$

```
[Out] 1/8*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+1/4*(d*x^8+c)^(1/2)/b^2/d-1/8*a^2*(d*x^8+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^8+a)
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 91, 81, 65, 214}

$$-\frac{a^2\sqrt{c+dx^8}}{8b^2\left(a+bx^8\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^8}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] Sqrt[c + d*x^8]/(4*b^2*d) - (a^2*Sqrt[c + d*x^8])/(8*b^2*(b*c - a*d)*(a + b
*x^8)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*
d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)$$

$$= -\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\,(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^8}}{4b^2d} - \frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\,(a+bx^8)} - \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{16b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^8}}{4b^2d} - \frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\,(a+bx^8)} - \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x,\right.}{8b^2d(bc-ad)}$$

$$= \frac{\sqrt{c+dx^8}}{4b^2d} - \frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\,(a+bx^8)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A]

time = 0.38, size = 130, normalized size = 1.06

$$\frac{\frac{\sqrt{b}\,\sqrt{c+dx^8}\,(-3a^2d+2b^2cx^8+2ab(c-dx^8))}{d(bc-ad)(a+bx^8)}+\frac{a(4bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((Sqrt[b]*Sqrt[c + d*x^8]*(-3*a^2*d + 2*b^2*c*x^8 + 2*a*b*(c - d*x^8)))/(d*
(b*c - a*d)*(a + b*x^8)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(8*b^(5/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{23}}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(103) = 206.
time = 2.14, size = 475, normalized size = 3.86

$$\left[\frac{((4\,ab^2cd-3\,a^2bd^2)x^8+4\,a^2bcd-3\,a^3d^2)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,(2\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^8+2\,ab^3c^2-5\,a^2b^2cd+3\,a^3bd^2)\sqrt{dx^8+c}}{16\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3)x^8)},-\frac{((4\,ab^2cd-3\,a^2bd^2)x^8+4\,a^2bcd-3\,a^3d^2)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b^2c+abd}}{bdx^8+bc}\right)-(2\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^8+2\,ab^3c^2-5\,a^2b^2cd+3\,a^3bd^2)\sqrt{dx^8+c}}{8\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3)x^8)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2
*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b
*d))/(b*x^8 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^
3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*
b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8), -
1/8*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*
c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - (
2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +
 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.83, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^8+c}\,a^2d}{8\,(b^3c-ab^2d)((dx^8+c)b-bc+ad)}-\frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\,(b^3c-ab^2d)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^8+c}}{4\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(d*x^8 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^8 + c)*b - b*c + a*d))
- 1/8*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/4*sqrt(d*x^8 + c)/(b^2*d)
```

Mupad [B]
time = 5.02, size = 144, normalized size = 1.17

$$\frac{\sqrt{d\,x^8+c}}{4\,b^2\,d}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^8+c}\,\left(3\,a\,d-4\,b\,c\right)}{\left(3\,a^2\,d-4\,a\,b\,c\right)\,\sqrt{a\,d-b\,c}}\right)\,\left(3\,a\,d-4\,b\,c\right)}{8\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}+\frac{a^2\,d\,\sqrt{d\,x^8+c}}{2\,\left(a\,d-b\,c\right)\,\left(4\,b^3\,\left(d\,x^8+c\right)-4\,b^3\,c+4\,a\,b^2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(1/2)/(4*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^8)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(8*b^(5/
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b^3*(c
+ d*x^8) - 4*b^3*c + 4*a*b^2*d))
```

3.909 $\int \frac{x^{15}}{(a+bx^8)^2 \sqrt{c+dx^8}} \, dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^8}}{8b(bc-ad)\,(a+bx^8)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/8*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/8*a*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 79, 65, 214}

$$\frac{a\sqrt{c+dx^8}}{8b\,(a+bx^8)\,(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)\\
&= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{16b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{8bd(bc-ad)}\\
&= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 100, normalized size = 1.01

$$
\frac{\frac{a\sqrt{b}\sqrt{c+dx^8}}{(bc-ad)(a+bx^8)} - \frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{8b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(8
*b^(3/2))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$
\int \frac{x^{15}}{(b\,x^8+a)^2\sqrt{d\,x^8+c}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.83, size = 348, normalized size = 3.52

$$\left[\frac{\left((2\,b^2c-abd)x^8+2\,abc-a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}\,(ab^2c-a^2bd)}{16\left((b^5c^2-2\,ab^4cd+a^2b^3d^2)x^8+ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)}, \frac{\left((2\,b^2c-abd)x^8+2\,abc-a^2d\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b^2c+abd}}{bdx^8+bc}\right)+\sqrt{dx^8+c}\,(ab^2c-a^2bd)}{8\left((b^5c^2-2\,ab^4cd+a^2b^3d^2)x^8+ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((2*b^2*c - a*b*d)*x^8 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a))
+ 2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*
d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2), 1/8*(((2*b^2*c - a*b*d
)*x^8 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-
b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^
5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.74, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^8+c}\,ad^2}{(b^2c-abd)((dx^8+c)b-bc+ad)}+\frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(d*x^8 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^8 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B]
time = 4.84, size = 95, normalized size = 0.96

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)\,\left(a\,d-2\,b\,c\right)}{8\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{a\,d\,\sqrt{d\,x^8+c}}{2\,b\,\left(a\,d-b\,c\right)\,\left(4\,b\,\left(d\,x^8+c\right)+4\,a\,d-4\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(8*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^8)^(1/2))/(2*b*(a*d - b*c)*(4*b*(c +
 d*x^8) + 4*a*d - 4*b*c))
```

3.910 $\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{c+dx^8}}{8(bc-ad)\,(a+bx^8)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{3/2}}$$

```
[Out] 1/8*d*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/8*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {455, 44, 65, 214}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}}{8\,(a+bx^8)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*Sqrt[c + d*x^8]/((b*c - a*d)*(a + b*x^8)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^8])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)\\
&= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{16(bc-ad)}\\
&= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{8(bc-ad)}\\
&= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 86, normalized size = 0.99

$$
\frac{1}{8}\left(-\frac{\sqrt{c+dx^8}}{(bc-ad)\left(a+bx^8\right)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,(-bc+ad)^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (-(Sqrt[c + d*x^8]/((b*c - a*d)*(a + b*x^8))) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^8])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/8
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$
\int \frac{x^7}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.
time = 2.19, size = 302, normalized size = 3.47

$$\left[-\frac{(bdx^8+ad)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}\,(b^2c-abd)}{16\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2)},-\frac{(bdx^8+ad)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b^2c+abd}}{bdx^8+bc}\right)+\sqrt{dx^8+c}\,(b^2c-abd)}{8\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*d*x^8 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d - 2*
sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*
c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2), -1/8*((b*d*x^8 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqr
t(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(b^2*c
 - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.19, size = 93, normalized size = 1.07

$$-\frac{d \arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx^8+c}\,d}{8\,((dx^8+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/8*sqrt(d*x^8 + c)*d/(((d*x^8 + c)*b - b*c + a*d)*(b*c - a
*d))
```

Mupad [B]
time = 4.80, size = 84, normalized size = 0.97

$$\frac{d\sqrt{d\,x^8+c}}{2\,(a\,d-b\,c)\,(4\,b\,(d\,x^8+c)+4\,a\,d-4\,b\,c)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{8\,\sqrt{b}\,(a\,d-b\,c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c)) + (
d*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(1/2)*(a*d - b*
c)^(3/2))
```

3.911 $\int \frac{1}{x(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=132

$$\frac{b\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}$$

```
[Out] 1/8*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-1/4*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/8
*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A]

time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 105, 162, 65, 214}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{b\sqrt{c+dx^8}}{8a\,(a+bx^8)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ArcTanh[Sqrt[c + d*x^8]
/Sqrt[c]]/(4*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^8\right)}{8a^2} - \frac{(b(2bc-3ad))\text{Su}}{}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4a^2d} - \frac{(b(2bc-3ad)}{}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\right.}{8a^2(bc-ad)^3}$$

Mathematica [A]
time = 0.38, size = 124, normalized size = 0.94

$$\frac{-\frac{ab\sqrt{c+dx^8}}{(-bc+ad)(a+bx^8)} + \frac{\sqrt{b}\,(2bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (-((a*b*Sqrt[c + d*x^8])/((-(b*c) + a*d)*(a + b*x^8))) + (Sqrt[b]*(2*b*c -
3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)
^(3/2) - (2*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/Sqrt[c])/(8*a^2)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x), x)
```

Fricas [A]
time = 2.94, size = 862, normalized size = 6.53

$$\left[\frac{2 \, \sqrt{d x^{8} + c} a b c + ((2 \, b^{2} c^{2} - 3 \, a b c d) x^{8} + 2 \, a b c^{2} - 3 \, a^{2} c d) \sqrt{\frac{b}{b c - a d}} \log\left(\frac{b d x^{8} + 2 \, b c - a d + 2 \, \sqrt{d x^{8} + c} (b c - a d) \sqrt{\frac{b}{b c - a d}}}{b x^{8} + a}\right) + 2 \, ((b^{2} c - a b d) x^{8} + a b c - a^{2} d) \sqrt{c} \log\left(\frac{d x^{8} - 2 \, \sqrt{d x^{8} + c} \sqrt{c} + 2 \, c}{x^{8}}\right)}{16 \, ((a^{2} b^{2} c^{2} - a^{3} b c d) x^{8} + a^{3} b c^{2} - a^{4} c d)}, \frac{\sqrt{d x^{8} + c} a b c + ((2 \, b^{2} c^{2} - 3 \, a b c d) x^{8} + 2 \, a b c^{2} - 3 \, a^{2} c d) \sqrt{-\frac{b}{b c - a d}} \arctan\left(-\frac{\sqrt{d x^{8} + c} (b c - a d) \sqrt{-\frac{b}{b c - a d}}}{b d x^{8} + b c}\right) + ((b^{2} c - a b d) x^{8} + a b c - a^{2} d) \sqrt{c} \log\left(\frac{d x^{8} - 2 \, \sqrt{d x^{8} + c} \sqrt{c} + 2 \, c}{x^{8}}\right)}{8 \, ((a^{2} b^{2} c^{2} - a^{3} b c d) x^{8} + a^{3} b c^{2} - a^{4} c d)}, \frac{2 \, \sqrt{d x^{8} + c} a b c + 4 \, ((b^{2} c - a b d) x^{8} + a b c - a^{2} d) \sqrt{-c} \arctan\left(\frac{\sqrt{d x^{8} + c} \sqrt{-c}}{c}\right) + ((2 \, b^{2} c^{2} - 3 \, a b c d) x^{8} + 2 \, a b c^{2} - 3 \, a^{2} c d) \sqrt{\frac{b}{b c - a d}} \log\left(\frac{b d x^{8} + 2 \, b c - a d + 2 \, \sqrt{d x^{8} + c} (b c - a d) \sqrt{\frac{b}{b c - a d}}}{b x^{8} + a}\right)}{16 \, ((a^{2} b^{2} c^{2} - a^{3} b c d) x^{8} + a^{3} b c^{2} - a^{4} c d)}, \frac{\sqrt{d x^{8} + c} a b c + ((2 \, b^{2} c^{2} - 3 \, a b c d) x^{8} + 2 \, a b c^{2} - 3 \, a^{2} c d) \sqrt{-\frac{b}{b c - a d}} \arctan\left(-\frac{\sqrt{d x^{8} + c} (b c - a d) \sqrt{-\frac{b}{b c - a d}}}{b d x^{8} + b c}\right) + 2 \, ((b^{2} c - a b d) x^{8} + a b c - a^{2} d) \sqrt{-c} \arctan\left(\frac{\sqrt{d x^{8} + c} \sqrt{-c}}{c}\right)}{8 \, ((a^{2} b^{2} c^{2} - a^{3} b c d) x^{8} + a^{3} b c^{2} - a^{4} c d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(d*x^8 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 +
 c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*((b^2*c - a*b*d)*x^8
+ a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8)
)/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 +
c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^
8 + b*c)) + ((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sq
rt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c
^2 - a^4*c*d), 1/16*(2*sqrt(d*x^8 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^8 + a*b
```

```
*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a
*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b
*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)))
/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8
 + b*c)) + 2*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x
^8 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^{8}\right)^{2}\sqrt{c+dx^{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [A]
time = 1.59, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^{8}+c}\,bd}{8\left(abc-a^{2}d\right)\left(\left(dx^{8}+c\right)b-bc+ad\right)}-\frac{\left(2\,b^{2}c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^{8}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{8\left(a^{2}bc-a^{3}d\right)\sqrt{-b^{2}c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^{8}+c}}{\sqrt{-c}}\right)}{4\,a^{2}\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^8 + c)*b*d/((a*b*c - a^2*d)*((d*x^8 + c)*b - b*c + a*d)) - 1/8
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

Mupad [B]
time = 5.82, size = 3017, normalized size = 22.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((a^6*b^2*d^5 - (3*a^5*b^
3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - (
```

(c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))/(((3*a*b^3*d^4)/128 - (b^4*c*d^3)/64)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (atan((((((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) - ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - ((((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c* d)))/(8*a^2*c^(1/2)) + ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)))/(((3*a*b^3*d^4)/128 - (b^4*c*d^3)/64)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + (((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) - ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + (((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) + ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2))))*1i)/(4*a^2*c^(1/2)) - (b*d*(c + d*x^8)^(1/2))/(2*(a^2*d - a*b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c))

3.912 $\int \frac{1}{x^9(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=185

$$-\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\,(a+bx^8)}-\frac{\sqrt{c+dx^8}}{8acx^8\,(a+bx^8)}+\frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\sqrt{\right.}{8a^3(bc-ad)^{3/}}$$

```
[Out] 1/8*(a*d+4*b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/8*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/8*b*(-a*d+2*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^8+a)-1/8*
(d*x^8+c)^(1/2)/a/c/x^8/(b*x^8+a)
```

Rubi [A]

time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 105, 156, 162, 65, 214}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}-\frac{b\sqrt{c+dx^8}\,(2bc-ad)}{8a^2c\,(a+bx^8)\,(bc-ad)}-\frac{\sqrt{c+dx^8}}{8acx^8\,(a+bx^8)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(b*(2*b*c - a*d)*Sqrt[c + d*x^8])/(a^2*c*(b*c - a*d)*(a + b*x^8)) - Sq
rt[c + d*x^8]/(8*a*c*x^8*(a + b*x^8)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x
^8]/Sqrt[c]])/(8*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^3*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \frac{1}{8}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)$$

$$= -\frac{\sqrt{c+dx^8}}{8acx^8(a+bx^8)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)}{8ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)(a+bx^8)} - \frac{\sqrt{c+dx^8}}{8acx^8(a+bx^8)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}\cdots}{x(a+bx)\sqrt{c+\cdots}}\right.}{8a^2c(bc-\cdots}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)(a+bx^8)} - \frac{\sqrt{c+dx^8}}{8acx^8(a+bx^8)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \cdots\right.}{16a^3(bc\cdots}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)(a+bx^8)} - \frac{\sqrt{c+dx^8}}{8acx^8(a+bx^8)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \cdots\right.}{8a^3d(b\cdots}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)(a+bx^8)} - \frac{\sqrt{c+dx^8}}{8acx^8(a+bx^8)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+\cdots}}{\sqrt{c}}\right.}{8a^3c^{3/2}}$$

Mathematica [A]
time = 0.64, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^8}\left(-a^2d+2b^2cx^8+ab(c-dx^8)\right)}{c(-bc+ad)x^8(a+bx^8)} - \frac{b^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{c^{3/2}}}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*Sqrt[c + d*x^8]*(-(a^2*d) + 2*b^2*c*x^8 + a*b*(c - d*x^8)))/(c*(-(b*c)
+ a*d)*x^8*(a + b*x^8)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcTanh
[Sqrt[c + d*x^8]/Sqrt[c]])/c^(3/2))/(8*a^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^9), x)
```

Fricas [A]
time = 2.52, size = 1236, normalized size = 6.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] [1/16*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^
8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(c)*log((d*x^8
 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8
 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16
+ (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^
8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - ((4*b^3*c^2 - 3*
a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sq
rt(c)*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*((2*a*b^2*c^2
- a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^
4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^2 - 3*a*
b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt
(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*
b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a))
 + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/8*(((
4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^8 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2
- 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) +
 ((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a
^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9 \left(a+b x^8\right)^2 \sqrt{c+d x^8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**9*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [A]
time = 1.31, size = 257, normalized size = 1.39

$$\frac{\left(4 \, b^{3} c - 5 \, a b^{2} d\right) \arctan\left(\frac{\sqrt{d x^{8} + c} b}{\sqrt{-b^{2} c + a b d}}\right)}{8 \, {\left(a^{3} b c - a^{4} d\right)} \sqrt{-b^{2} c + a b d}} - \frac{2 \, {\left(d x^{8} + c\right)}^{\frac{3}{2}} b^{2} c d - 2 \, \sqrt{d x^{8} + c} b^{2} c^{2} d - {\left(d x^{8} + c\right)}^{\frac{3}{2}} a b d^{2} + 2 \, \sqrt{d x^{8} + c} a b c d^{2} - \sqrt{d x^{8} + c} a^{2} d^{3}}{8 \, {\left(a^{2} b c^{2} - a^{3} c d\right)} {\left({\left(d x^{8} + c\right)}^{2} b - 2 \, {\left(d x^{8} + c\right)} b c + b c^{2} + {\left(d x^{8} + c\right)} a d - a c d\right)}} - \frac{{\left(4 \, b c + a d\right)} \arctan\left(\frac{\sqrt{d x^{8} + c}}{\sqrt{-c}}\right)}{8 \, a^{3} \sqrt{-c} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/8*(2*(d*x^8 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^8 + c)*b^2*c^2*d - (d*x^8 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^8 + c
)*a*b*c*d^2 - sqrt(d*x^8 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^8 + c)^
2*b - 2*(d*x^8 + c)*b*c + b*c^2 + (d*x^8 + c)*a*d - a*c*d)) - 1/8*(4*b*c +
a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B]
time = 7.85, size = 2500, normalized size = 13.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (((c + d*x^8)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^8)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
((c + d*x^8)*(4*a*d - 8*b*c) + 4*b*(c + d*x^8)^2 + 4*b*c^2 - 4*a*c*d) + (at
an((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*
c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b
^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*
d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6
*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^
2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3
```

*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^6)/256 + (b^7*c^3*d^3)/8 - (3*a*b^6*c^2*d^4)/16 + (3*a^2*b^5*c*d^5)/128)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^8)^(1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(16*a^3*(c^3)^(1/2)) + ((((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b

```
^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^
3*d)) - ((((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b
^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^8)^(
1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b
^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*
c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*
c)*1i)/(16*a^3*(c^3)^(1/2)))/(((5*a^3*b^4*d^6)/256 + (b^7*c^3*d^3)/8 - (3*a
*b^6*c^2*d^4)/16 + (3*a^2*b^5*c*d^5)/128)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^
7*b*c^3*d) - ((((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*
c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2
*d^2 - 2*a^5*b*c^3*d)) + ((((a^9*b^2*c*d^6)/2 +...
```

3.913 $\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=141

$$\frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)}-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

```
[Out] -1/8*(-2*a*d+3*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))*a^
(1/2)/b^2/(-a*d+b*c)^(3/2)+1/4*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b^2/d^(
1/2)+1/8*a*x^4*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A]
time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {476, 481, 537, 223, 212, 385, 211}

$$-\frac{\sqrt{a}\,(3bc-2ad)\text{ArcTan}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}}+\frac{ax^4\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a*x^4*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b^2*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} - \frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8b(bc-ad)}$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b^2} - \frac{(a(3bc-2ad))\text{Subs}}{}$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b^2} - \frac{(a(3bc-2ad))\text{S}}{}$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} - \frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\right.}{4b}$$

Mathematica [A]
time = 2.64, size = 152, normalized size = 1.08

$$\frac{\frac{abx^4\sqrt{c+dx^8}}{(bc-ad)(a+bx^8)} + \frac{\sqrt{a}(-3bc+2ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}x^8+bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{d}x^4}\right)}{\sqrt{d}}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*b*x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*S
qrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*ArcTanh[Sqrt[c + d*x^8]/(Sqrt[d]*x
^4)])/Sqrt[d])/(8*b^2)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{(b\,x^8+a)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^19/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [**A**]
time = 3.14, size = 1077, normalized size = 7.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] [1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^4 + 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 -
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d
*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((b^4*c*d -
a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^
4 - 8*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sq
rt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt
(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (
a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2
*a*b*x^8 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/
16*(2*sqrt(d*x^8 + c)*a*b*d*x^4 + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d
- 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt
(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) + 2*((b^2*c - a*b*d)*
x^8 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 -
 c))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/16*(2*sqrt(d*
x^8 + c)*a*b*d*x^4 - 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arcta
n(sqrt(-d)*x^4/sqrt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d
- 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt
(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)))/((b^4*c*d - a*b^3*d^
2)*x^8 + a*b^3*c*d - a^2*b^2*d^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(117) = 234.
time = 1.39, size = 298, normalized size = 2.11

$$-\frac{\left(3\,abc\sqrt{d}-2\,a^{2}d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{8\,(b^{3}c-ab^{2}d)\sqrt{abcd-a^{2}d^{2}}}-\frac{\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{2}abc\sqrt{d}-2\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{2}a^{2}d^{\frac{3}{2}}-abc^{2}\sqrt{d}}{4\left(\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{4}b-2\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{2}bc+4\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{2}ad+bc^{2}\right)(b^{3}c-ab^{2}d)}-\frac{\log\left(\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{2}\right)}{8\,b^{2}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x
^8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/4*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*sqrt(d
) - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((s
qrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c
 + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/
8*log((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2)/(b^2*sqrt(d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19}}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.914 $\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=93

$$-\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\,(a+bx^8)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8\sqrt{a}\,(bc-ad)^{3/2}}$$

```
[Out] 1/8*c*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/(-a*d+b*c)^(3/2)
/a^(1/2)-1/8*x^4*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A]
time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {476, 482, 12, 385, 211}

$$\frac{c\text{ArcTan}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^4\sqrt{c+dx^8}}{8\,(a+bx^8)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (c*ArcTan[(Sqrt[b*c
- a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
```

```
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx,x,x^4\right)\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)}+\frac{\text{Subst}\left(\int \frac{c}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)}+\frac{c\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)}+\frac{c\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 1.27, size = 112, normalized size = 1.20

$$\frac{1}{8}\left(-\frac{x^4\sqrt{c+dx^8}}{(bc-ad)(a+bx^8)}+\frac{c\tan^{-1}\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}\,x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (-((x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8))) + (c*ArcTan[(a*Sqrt[d]
+ b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[
a]*(b*c - a*d)^(3/2)))/8
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^11/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(77) = 154$.
time = 3.26, size = 426, normalized size = 4.58

$$\left[-\frac{4\,\sqrt{dx^{8}+c}\,(abc-a^{2}d)x^{4}-(bcx^{8}+ac)\sqrt{-abc+a^{2}d}\,\log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{16}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{8}+a^{2}c^{2}+4\,((bc-2\,ad)x^{12}-acx^{4})\sqrt{dx^{8}+c}\,\sqrt{-abc+a^{2}d}}{b^{2}x^{16}+2\,abx^{8}+a^{2}}\right)}{32\,((ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2})x^{8}+a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2})},-\frac{2\,\sqrt{dx^{8}+c}\,(abc-a^{2}d)x^{4}-(bcx^{8}+ac)\sqrt{abc-a^{2}d}\,\arctan\left(\frac{((bc-2\,ad)x^{8}-ac)\sqrt{dx^{8}+c}\,\sqrt{abc-a^{2}d}}{2\,((abcd-a^{2}d^{2})x^{12}+(abc^{2}-a^{2}cd)x^{4})}\right)}{16\,((ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2})x^{8}+a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(4*sqrt(d*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqr
t(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c
*d + a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/16*(2*sqrt(d
*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(a*b*c - a^2*d)*arctan(
1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d
 - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)))/((a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(77) = 154.
time = 3.30, size = 244, normalized size = 2.62

$$\frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{8\sqrt{abcd-a^2d^2}\,(bc-ad)}+\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{4\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ad+bc^2\right)(b^2c-abd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/4*((s
qrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^4 - sqrt(d*x^8 +
 c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2
*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^
2*a*d + b*c^2)*(b^2*c - a*b*d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.915 $\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=104

$$\frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/8*(-2*a*d+b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(3/
2)/(-a*d+b*c)^(3/2)+1/8*b*x^4*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {476, 390, 385, 211}

$$\frac{(bc-2ad)\text{ArcTan}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}+\frac{bx^4\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*x^4*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
```

```
tQ[q, -1]) && NeQ[p, -1]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx,x,x^4\right)\\
&= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)}+\frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8a(bc-ad)}\\
&= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)}+\frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{8a(bc-ad)}\\
&= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.73, size = 124, normalized size = 1.19

$$-\frac{bx^4\sqrt{c+dx^8}}{8a(-bc+ad)(a+bx^8)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^8+bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(b*x^4*Sqrt[c + d*x^8])/(a*(-(b*c) + a*d)*(a + b*x^8)) + ((b*c - 2*a*d
)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[
b*c - a*d])])/(8*a^(3/2)*(b*c - a*d)^(3/2))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b\,x^8+a)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(88) = 176$.
time = 3.20, size = 467, normalized size = 4.49

$$\left[\frac{4\sqrt{dx^8+c}\,(ab^2c-a^2bd)x^4-((b^2c-2\,abd)x^8+abc-2\,a^2d)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((bc-2\,ad)x^{12}-acx^4)\sqrt{dx^8+c}\,\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)}{32\,((a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x^8+a^3b^2c^2-2\,a^4bcd+a^5d^2)}, \frac{2\sqrt{dx^8+c}\,(ab^2c-a^2bd)x^4+((b^2c-2\,abd)x^8+abc-2\,a^2d)\sqrt{abc-a^2d}\,\arctan\left(\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\,\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4)}\right)}{16\,((a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x^8+a^3b^2c^2-2\,a^4bcd+a^5d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 - ((b^2*c - 2*a*b*d)*x^8 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 -
 a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2
)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^8 + a^3*b^2*c^2 - 2*a^4*b*
c*d + a^5*d^2), 1/16*(2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 + ((b^2*c -
 2*a*b*d)*x^8 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a
*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^1
2 + (a*b*c^2 - a^2*c*d)*x^4)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x
^8 + a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(88) = 176$.
time = 1.45, size = 237, normalized size = 2.28

$$-\frac{1}{8}d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ad-bc^2\right)}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ad+bc^2\right)(abcd-a^2d^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*
x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.916 $\int \frac{1}{x^5(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=149

$$-\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\,(a+bx^8)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

```
[Out] -1/8*b*(-4*a*d+3*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/
a^(5/2)/(-a*d+b*c)^(3/2)-1/8*(-2*a*d+3*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c
)/x^4+1/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^4/(b*x^8+a)
```

Rubi [A]
time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 483, 597, 12, 385, 211}

$$-\frac{b(3bc-4ad)\text{ArcTan}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}\,(3bc-2ad)}{8a^2cx^4(bc-ad)}+\frac{b\sqrt{c+dx^8}}{8ax^4\,(a+bx^8)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*((3*b*c - 2*a*d)*Sqrt[c + d*x^8])/(a^2*c*(b*c - a*d)*x^4) + (b*Sqrt[c
+ d*x^8])/(8*a*(b*c - a*d)*x^4*(a + b*x^8)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sq
rt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(5/2)*(b*c - a*d)^(3/2)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4 (a+bx^8)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4 (a+bx^8)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-\cdots}{(a+bx^2)\sqrt{c\cdots}}\right.}{8a^2c(bc\cdots}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4 (a+bx^8)} - \frac{(b(3bc-4ad))\text{Subs}\cdots}{\cdots}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4 (a+bx^8)} - \frac{(b(3bc-4ad))\text{Subs}\cdots}{\cdots}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4 (a+bx^8)} - \frac{b(3bc-4ad)\tan^{-1}\cdots}{8a^{5/2}(bc-\cdots}$$

Mathematica [A]
time = 1.43, size = 157, normalized size = 1.05

$$\frac{\sqrt{c+dx^8}\,(2abc-2a^2d+3b^2cx^8-2abdx^8)}{8a^2c(-bc+ad)x^4 (a+bx^8)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^8+bx^4\sqrt{c+dx^8}}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^8 - 2*a*b*d*x^8))/(8*a^2*c*
(-(b*c) + a*d)*x^4*(a + b*x^8)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*
Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(5/2)
*(b*c - a*d)^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(129) = 258$.
time = 3.01, size = 612, normalized size = 4.11

$$\left[-\frac{((3\,b^3c^2 - 4\,ab^2cd)x^{12} + (3\,ab^2c^2 - 4\,a^2bcd)x^4)\sqrt{-abc + a^2d}\,\log\left(\frac{(b^2c^2 - 8\,abcd + 8\,a^2d^2)x^{16} - 2\,(3\,abc^2 - 4\,a^2cd)x^8 + a^2c^2 + 4\,((bc - 2\,ad)x^{12} - acx^4)\sqrt{dx^8 + c}\,\sqrt{-abc + a^2d}}{b^2x^{16} + 2\,abx^8 + a^2}\right) + 4\,((3\,ab^3c^2 - 5\,a^2b^2cd + 2\,a^3bd^2)x^8 + 2\,a^2b^2c^2 - 4\,a^3bcd + 2\,a^4d^2)\sqrt{dx^8 + c}}{32\,((a^3b^3c^3 - 2\,a^4b^2c^2d + a^5bcd^2)x^{12} + (a^4b^2c^3 - 2\,a^5bc^2d + a^6cd^2)x^4)}, -\frac{((3\,b^3c^2 - 4\,ab^2cd)x^{12} + (3\,ab^2c^2 - 4\,a^2bcd)x^4)\sqrt{abc - a^2d}\,\arctan\left(\frac{((bc - 2\,ad)x^8 - ac)\sqrt{dx^8 + c}\,\sqrt{abc - a^2d}}{2\,((abcd - a^2d^2)x^{12} + (abc^2 - a^2cd)x^4)}\right) + 2\,((3\,ab^3c^2 - 5\,a^2b^2cd + 2\,a^3bd^2)x^8 + 2\,a^2b^2c^2 - 4\,a^3bcd + 2\,a^4d^2)\sqrt{dx^8 + c}}{16\,((a^3b^3c^3 - 2\,a^4b^2c^2d + a^5bcd^2)x^{12} + (a^4b^2c^3 - 2\,a^5bc^2d + a^6cd^2)x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] [-1/32*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x
^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 4*((3*a*b^3*c
^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4
*d^2)*sqrt(d*x^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12
+ (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4), -1/16*(((3*b^3*c^2 - 4*a*
b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*sqrt(a*b*c - a^2*d)*arctan
(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*
d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2
*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x
^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12 + (a^4*b^2*c^3
 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a + bx^8\right)^2\sqrt{c + dx^8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**5*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(129) = 258.

time = 2.62, size = 418, normalized size = 2.81

$$\frac{1}{8}\,d^{\frac{5}{2}}\left(\frac{\left(3\,b^2c-4\,abd\right)\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{abcd-a^2d^2}}+\frac{2\left(3\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b^2c-4\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4abd-6\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b^2c^2+14\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2abcd-8\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2a^2d^2+3\,b^2c^3-2\,abc^2d\right)}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^6b-3\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4bc+4\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4ad+3\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc^2-4\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2acd-bc^3\right)\left(a^2bcd^2-a^3d^3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b^2*c - 4*(sq
rt(d)*x^4 - sqrt(d*x^8 + c))^4*a*b*d - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*
b^2*c^2 + 14*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*d - 8*(sqrt(d)*x^4 - s
qrt(d*x^8 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/((((sqrt(d)*x^4 - sqrt(
d*x^8 + c))^6*b - 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b*c + 4*(sqrt(d)*x^4
- sqrt(d*x^8 + c))^4*a*d + 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c^2 - 4*(s
qrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.917 $\int \frac{1}{x^{13}(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=208

$$-\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}}+\frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}(a+bx^8)}+\frac{b^2(5bc-6ad)\tan}{8a^{7/2}(b}$$

```
[Out] 1/8*b^2*(-6*a*d+5*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))
/a^(7/2)/(-a*d+b*c)^(3/2)-1/24*(-2*a*d+5*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^12+1/24*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^8+c)^(1/2)/a^3/c^2/(-a
*d+b*c)/x^4+1/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^12/(b*x^8+a)
```

Rubi [A]
time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 483, 597, 12, 385, 211}

$$\frac{b^2(5bc-6ad)\text{ArcTan}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{24a^2cx^{12}(bc-ad)}+\frac{\sqrt{c+dx^8}(-4a^2d^2-8abcd+15b^2c^2)}{24a^3c^2x^4(bc-ad)}+\frac{b\sqrt{c+dx^8}}{8ax^{12}(a+bx^8)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/24*((5*b*c - 2*a*d)*Sqrt[c + d*x^8])/(a^2*c*(b*c - a*d)*x^12) + ((15*b^2
*c^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^8])/(24*a^3*c^2*(b*c - a*d)*x^4)
 + (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^12*(a + b*x^8)) + (b^2*(5*b*c - 6
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(7/2)*(
b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{13} (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}} \, dx, x, x^4\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12} (a+bx^8)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4(a+bx^2)\sqrt{c+dx^2}} \, dx, x, x^4\right)}{8a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12} (a+bx^8)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8a}{x^2(a}\right.}{2}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{(15b^2c^2-8abcd-4a^2d^2) \sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b}{8a(bc-}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{(15b^2c^2-8abcd-4a^2d^2) \sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b}{8a(bc-}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{(15b^2c^2-8abcd-4a^2d^2) \sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b}{8a(bc-}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{(15b^2c^2-8abcd-4a^2d^2) \sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b}{8a(bc-}$$

Mathematica [A]
time = 3.30, size = 201, normalized size = 0.97

$$-\frac{\sqrt{c+dx^8} \left(15b^3c^2x^{16} + 2ab^2cx^8(5c-4dx^8) + 2a^3d(c-2dx^8) - 2a^2b(c^2+3cdx^8+2d^2x^{16})\right)}{24a^3c^2(-bc+ad)x^{12} (a+bx^8)} + \frac{b^2(5bc-6ad) \tan^{-1}\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}\,x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{8a^{7/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/24*(Sqrt[c + d*x^8]*(15*b^3*c^2*x^16 + 2*a*b^2*c*x^8*(5*c - 4*d*x^8) + 2
*a^3*d*(c - 2*d*x^8) - 2*a^2*b*(c^2 + 3*c*d*x^8 + 2*d^2*x^16)))/(a^3*c^2*(-
(b*c) + a*d)*x^12*(a + b*x^8)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] + b
*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(7/2
)*(b*c - a*d)^(3/2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13} \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^13), x)
```

Fricas [A]
time = 3.99, size = 760, normalized size = 3.65

$$\left[-\frac{3\left((5b^4c^3-6ab^3c^2d)x^{20}+(5ab^3c^3-6a^2b^2c^2d)x^{12}\right)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{16}-2(3abc^2-4a^2cd)x^8+a^2c^2-4\left((bc-2ad)x^{12}-acx^4\right)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2abx^8+a^2}\right)-4\left((15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3)x^{16}+2(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3)x^8-2a^3b^2c^3+4a^4bc^2d-2a^5cd^2\right)\sqrt{dx^8+c}}{96\left((a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2)x^{20}+(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2)x^{12}\right)},\ \frac{3\left((5b^4c^3-6ab^3c^2d)x^{20}+(5ab^3c^3-6a^2b^2c^2d)x^{12}\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\left((bc-2ad)x^8-ac\right)\sqrt{dx^8+c}\sqrt{abc-a^2d}}{2\left((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4\right)}\right)+2\left((15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3)x^{16}+2(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3)x^8-2a^3b^2c^3+4a^4bc^2d-2a^5cd^2\right)\sqrt{dx^8+c}}{48\left((a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2)x^{20}+(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2)x^{12}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^12)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)
*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*((
15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^16 + 2*(
5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^8 - 2*a^3*b^2*
c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^4*b^3*c^4 - 2*a^5*b
^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2
)*x^12), 1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b
^2*c^2*d)*x^12)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sq
rt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^
2*c*d)*x^4)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^
4*b*d^3)*x^16 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^
3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^
4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*
c^3*d + a^7*c^2*d^2)*x^12)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**13*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(184) = 368.

time = 3.14, size = 395, normalized size = 1.90

$$\frac{1}{24}\,d^{\frac{7}{2}}\left(\frac{3\,(5\,b^3c-6\,ab^2d)\arctan\left(-\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(a^3bcd^3-a^4d^4)\sqrt{abcd-a^2d^2}}-\frac{6\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b^3c-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ab^2d-b^3c^2\right)}{(a^3bcd^3-a^4d^4)\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ad+bc^2\right)}-\frac{8\left(3\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-6\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc-3\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ad+3\,bc^2+acd\right)}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2-c\right)^3a^3d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x^
8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4
)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b^3*c - 2
*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d
^4)*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c)
)^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)
*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 3*(
sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x^4 - sq
rt(d*x^8 + c))^2 - c)^3*a^3*d^3))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{13}\,{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.918 $\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=924

$$-\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}-\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}+\ldots$$

```
[Out] -1/32*(a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1
/2))/(-a)^(3/4)/b^(3/4)/(-a*d+b*c)^(3/2)+1/32*(a*d+b*c)*arctan(x^2*(a*d-b*c
)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-b*c)^(3
/2)-1/8*x^2*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)-1/16*d^(3/4)*(cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Elliptic
F(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d
*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
+1/32*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(
1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2)
)*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/32*d^(1/
4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^
(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*((-a)^(1/
2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4
*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(cos(2*arcta
n(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellipti
cPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1
/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2)
)*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2
)^(1/2)/a/b/c^(1/4)/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(cos(2*arctan(d
^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi
(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2
))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*
(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^
(1/2)/a/b/c^(1/4)/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A]

time = 0.90, antiderivative size = 924, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {476, 482, 537, 226, 418, 1231, 1721}

$$\frac{\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{64ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{dx^8+c}}-\frac{(bc+ad)\text{ArcTan}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}}+\frac{(bc+ad)\text{ArcTan}\left(\frac{\sqrt{ad-bc}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{3/4}b^{3/4}(ad-bc)^{3/2}}-\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{dx^8+c}}+\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)\sqrt{dx^8+c}}+\frac{\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32ab\sqrt[4]{c}(bc-ad)\sqrt{dx^8+c}}+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{64ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{dx^8+c}}-\frac{x^2\sqrt{dx^8+c}}{8(bc-ad)\left(bx^8+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(x^2*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) - ((b*c + a*d)*ArcTan[
(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(3/4)
*b^(3/4)*(b*c - a*d)^(3/2)) + ((b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/
((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(3/4)*b^(3/4)*(-(b*c) + a*d
)^(3/2)) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[
d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(
1/4)*x^2)/c^(1/4)], 1/2])/(32*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + ((Sq
rt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/
4)], 1/2])/(32*a*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) - (d^(3/4)*(Sqrt[c]
 + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*Arc
Tan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]
) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqr
t[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)
/c^(1/4)], 1/2])/(64*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + ((S
qrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^
8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d
])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
 1/2])/(64*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{c-dx^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8(bc-ad)}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8b(bc-ad)} + \frac{(bc+ad)\text{Subst}\left(\right.}{}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\right.}{16b\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^8}}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\right.}{16b\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^8}}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} - \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}} + \frac{(bc+ad)}{32(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.12, size = 159, normalized size = 0.17

$$-\frac{x^2\left(5a(c+dx^8)-5c(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+dx^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)}{40a(bc-ad)(a+bx^8)\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/40*(x^2*(5*a*(c + d*x^8) - 5*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[
1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1 +
(d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)]))/(a*(b*c
 - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{9}}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^9}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^9/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.919 $\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=999

$$\frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)}+\frac{\sqrt[4]{b}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(bc-ad)^{3/2}}-\frac{\sqrt[4]{b}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{-bc+}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{}}\right.}{32(-a)^{7/4}(-bc+ad)^{3/2}}$$

```
[Out] 1/32*b^(1/4)*(-5*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/
(d*x^8+c)^(1/2))/(-a)^(7/4)/(-a*d+b*c)^(3/2)-1/32*b^(1/4)*(-5*a*d+3*b*c)*ar
ctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/4)/(a*
d-b*c)^(3/2)+1/8*b*x^2*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)+1/16*d^(3/4)*
(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/
4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4
*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(d
*x^8+c)^(1/2)+1/32*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)
))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/
4)*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)
+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c
)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/32*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/
4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(
2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*
c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^
(3/2)/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(-5*a*d+3*b*c)*(cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1
/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4
*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^
(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)+1/64
*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d
^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*
2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x
^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)
/(d*x^8+c)^(1/2)
```

Rubi [A]

time = 1.02, antiderivative size = 999, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = $\{476, 425, 537, 226, 418, 1231, 1721\}$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + (b^(1/4)*(3*b*c - 5
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(
32*(-a)^(7/4)*(b*c - a*d)^(3/2)) - (b^(1/4)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[-(
b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(7/4)*(-(b
*c) + a*d)^(3/2)) + (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt
[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*
a*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqr
t[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqr
t[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32
*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - S
qrt[-a]*Sqrt[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)]
, 1/2])/(32*(-a)^(3/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x
^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sq
rt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)
*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)
*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellipt
icPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqr
t[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^2*c^(1/4)*d^(1/4)*(b*c
- a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} - \frac{\text{Subst}\left(\int \frac{-3bc+4ad-bdx^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)} + \frac{(3bc-5ad)\text{Subst}}{}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\right.}{16a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\right.}{16a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} + \frac{\sqrt[4]{b}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(bc-ad)^{3/2}} - \frac{\sqrt[4]{b}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.15, size = 169, normalized size = 0.17

$$\frac{x^2\left(5ab(c+dx^8)+5(3bc-4ad)(a+bx^8)\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+bdx^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)}{40a^2(bc-ad)(a+bx^8)\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^2*(5*a*b*(c + d*x^8) + 5*(3*b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]
*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*x^8*(a + b*x^
8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a
)]))/(40*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]

time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^{8}\right)^{2} \sqrt{c+dx^{8}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.920 $\int \frac{1}{x^7 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1060

$$-\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)}+\frac{b^{5/4}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{32(-a)^{11/4}(bc-ad)^{3/2}}-\frac{b^{5/4}(7b}{}$$

```
[Out] 1/32*b^(5/4)*(-9*a*d+7*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/
(d*x^8+c)^(1/2))/(-a)^(11/4)/(-a*d+b*c)^(3/2)-1/32*b^(5/4)*(-9*a*d+7*b*c)*a
rctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(11/4)/(
a*d-b*c)^(3/2)-1/24*(-4*a*d+7*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^6+1/8
*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^6/(b*x^8+a)-1/48*d^(3/4)*(-4*a*d+7*b*c)*(
cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4
)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*
d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(5/4)/(-a*d+b*c)/(
d*x^8+c)^(1/2)+1/32*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1
/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^
(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a
)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/
4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/64*b*(-9*a*d+7*b*c)*(cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Elliptic
Pi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/
2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))
*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)
^(1/2)/a^3/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)-1/32*b*d^(1/4
)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(
d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1
/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^8+c)/
(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x
^8+c)^(1/2)-1/64*b*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/
c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^
(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2
)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^3/c^(1/4)/d^(1/4)/
(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)
```

Rubi [A]

time = 1.54, antiderivative size = 1060, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {476, 483, 597, 537, 226, 418, 1231, 1721}

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/24*((7*b*c - 4*a*d)*Sqrt[c + d*x^8])/(a^2*c*(b*c - a*d)*x^6) + (b*Sqrt[c
 + d*x^8])/(8*a*(b*c - a*d)*x^6*(a + b*x^8)) + (b^(5/4)*(7*b*c - 9*a*d)*Arc
Tan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(
11/4)*(b*c - a*d)^(3/2)) - (b^(5/4)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[-(b*c) + a
*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(11/4)*(-(b*c) + a
*d)^(3/2)) - (d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x
^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1
/2])/(48*a^2*c^(5/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[b]*Sqrt[c] - S
qrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)]
, 1/2])/(32*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (
b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + S
qrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[
(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d
)*Sqrt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(7*b*c - 9*a
*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ell
ipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[
c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^3*c^(1/4)*d^(1/4)
*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])^2*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c
] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sq
rt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64
*a^3*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^7 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6 (a+bx^8)} - \frac{\text{Subst}\left(\int \frac{-7bc+4ad-5bdx^4}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6 (a+bx^8)} + \frac{\text{Subst}\left(\int \frac{-21b^2c^2+20abc}{(a+ba}\right.}{24a}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6 (a+bx^8)} - \frac{(b(7bc-9ad))\text{Subst}\left(\right.}{8a}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6 (a+bx^8)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}\right.}{}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6 (a+bx^8)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}\right.}{}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6 (a+bx^8)} + \frac{b^{5/4}(7bc-9ad)\tan^{-1}}{32(-a)^{11/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.22, size = 225, normalized size = 0.21

$$\frac{5a(c+dx^8)(4a^2d-7b^2cx^8-4ab(c-dx^8))+5(-21b^2c^2+20abcd+4a^2d^2)x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+bd(-7bc+4ad)x^{16}(a+bx^8)\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{120a^3c(bc-ad)x^6(a+bx^8)\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (5*a*(c + d*x^8)*(4*a^2*d - 7*b^2*c*x^8 - 4*a*b*(c - d*x^8)) + 5*(-21*b^2*c
^2 + 20*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[1
/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*(-7*b*c + 4*a*d)*x^16*(a
```

```
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b
*x^8)/a)])/(120*a^3*c*(b*c - a*d)*x^6*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7} \left(b\,x^{8}+a \right)^{2} \sqrt{d\,x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7} \left(a + bx^{8}\right)^{2} \sqrt{c + dx^{8}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**7*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^7 \left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.921 $\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1164

$$\frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{8b(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)}-\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\,(a+bx^8)}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{32\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)}{32\cdots}$$

```
[Out] 1/32*(-a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^
(1/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c)^(3/2)+1/32*(-a*d+3*b*c)*arctan(x^2*(a*
d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(a*d-b*
c)^(3/2)-1/8*x^6*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)+1/8*x^2*d^(1/2)*(d*x^
8+c)^(1/2)/b/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))-1/8*c^(1/4)*d^(1/4)*(cos(2*ar
ctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Elli
pticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))
*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/1
6*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(
1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/(-a
*d+b*c)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2
/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arct
an(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2
)-(-a)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(
1/4)/(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)-1/64*(-a*d+3*b*c)*(cos(2*arctan(d^(
1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(s
in(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^
2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/
2)/b^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^8+c)^(1/2)-
1/32*d^(1/4)*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4)))
,1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((
d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/(-a^2*d^2+b^2*c^2)/
(d*x^8+c)^(1/2)+1/64*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/
c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^
(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2
)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1
/4)/(-a*d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^8+c)^(1/2)
```

Rubi [A]

time = 1.37, antiderivative size = 1164, normalized size of antiderivative = 1.00, number of

steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$,
Rules used = {476, 482, 598, 311, 226, 1210, 504, 1231, 1721}

$$-\frac{\sqrt{dx^8+c}\,x^6}{8(bc-ad)(bx^8+a)}+\frac{\sqrt{d}\sqrt{dx^8+c}\,x^2}{8b(bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)}+\frac{(3bc-ad)\text{ArcTan}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)\text{ArcTan}\left(\frac{\sqrt{ad-bc}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32\sqrt[4]{-a}\,b^{5/4}(ad-bc)^{3/2}}-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b(bc-ad)\sqrt{dx^8+c}}+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b(bc-ad)\sqrt{dx^8+c}}-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}}-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}}+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{64\sqrt{-a}\,b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{64\sqrt{-a}\,b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[d]*x^2*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)) - (
x^6*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) + ((3*b*c - a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(1/4)*b
^(5/4)*(b*c - a*d)^(3/2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/
((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(1/4)*b^(5/4)*(-(b*c) + a*d
)^(3/2)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[
c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*b*
(b*c - a*d)*Sqrt[c + d*x^8]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqr
t[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c
^(1/4)], 1/2])/(16*b*(b*c - a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] - (Sqrt[-a]*S
qrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
1/2])/(32*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] +
(Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)
/c^(1/4)], 1/2])/(32*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(
1/4)*x^2)/c^(1/4)], 1/2])/(64*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*
(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b
*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^
2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sq
rt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*Sqrt[-a]*b^(3/2)
*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
```

```
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{x^2(3c+dx^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8(bc-ad)}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{\text{Subst}\left(\int \left(\frac{dx^2}{b\sqrt{c+dx^4}} + \frac{(3bc-ad)x^2}{b(a+bx^4)\sqrt{c+dx^4}}\right) dx, x, x\right.}{8(bc-ad)}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{d\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8b(bc-ad)} + \frac{(3bc-ad)\text{Subst}\left(\right.}{}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8b(bc-ad)} - \frac{\left(\sqrt{c}\sqrt{\right.}}{}$$

$$= \frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{8b(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)} - \frac{x^6\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^4\right.}{}$$

$$= \frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{8b(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)} - \frac{x^6\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{(3bc-ad)\tan^{-1}\left(\frac{}{\sqrt[4]{-}}\right.}{32\sqrt[4]{-a}\,b^{5/4}(bc}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.12, size = 159, normalized size = 0.14

$$\frac{x^6\left(-7a(c+dx^8)+7c(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+dx^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)}{56a(bc-ad)\left(a+bx^8\right)\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^6*(-7*a*(c + d*x^8) + 7*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4,
1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1 + (d*x^
8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)]))/(56*a*(b*c
- a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(b\,x^{8}+a\right)^{2}\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{13}}{{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^13/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.922 $\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1162

$$-\frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{8a(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)}+\frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{32(-a)^{5/4}\sqrt[4]{b}\,(bc-ad)^{3/2}}-\frac{(bc-3a}{3}$$

```
[Out] -1/32*(-3*a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)
^(1/2))/(-a)^(5/4)/b^(1/4)/(-a*d+b*c)^(3/2)-1/32*(-3*a*d+b*c)*arctan(x^2*(a
*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(a*d-b
*c)^(3/2)+1/8*b*x^6*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)-1/8*x^2*d^(1/2)*
(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))+1/8*c^(1/4)*d^(1/4)*(cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(
1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/(-a*d+b*c)/(d*x^8+c)^(1/2
)-1/16*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*a
rctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/
2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/
a/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x^2/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan
(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2
)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/
2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/
2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/b^(1/2)/(d*x^8+c)^(1/2)-1/64*(-3*a*
d+b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(
1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*
(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)
/b^(1/2)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arc
tan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^
(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/
(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-3*a*d+b*c)*(cos(2*arcta
n(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellipti
cF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c
^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2
)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A]
time = 1.34, antiderivative size = 1162, normalized size of antiderivative = 1.00, number of

steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$,
Rules used = {476, 483, 598, 311, 226, 1210, 504, 1231, 1721}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(Sqrt[d]*x^2*Sqrt[c + d*x^8])/(a*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4))
+ (b*x^6*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ((b*c - 3*a*d)*Ar
cTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^
(5/4)*b^(1/4)*(b*c - a*d)^(3/2)) - ((b*c - 3*a*d)*ArcTan[(Sqrt[-(b*c) + a*d
]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(5/4)*b^(1/4)*(-(b*c
) + a*d)^(3/2)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)
/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2]
)/(8*a*(b*c - a*d)*Sqrt[c + d*x^8]) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x
^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x^2)/c^(1/4)], 1/2])/(16*a*(b*c - a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] - (Sqr
t[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[
(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(
1/4)], 1/2])/(32*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqr
t[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]
*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/
4)*x^2)/c^(1/4)], 1/2])/(32*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^
8]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt
[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[
b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcT
an[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*
c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d
])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt
[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*S
qrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(3
/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
```

```
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} - \frac{\text{Subst}\left(\int \frac{x^2(-bc+4ad+bdx^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} - \frac{\text{Subst}\left(\int \left(\frac{dx^2}{\sqrt{c+dx^4}} + \frac{(-bc+3ad)x^2}{(a+bx^4)\sqrt{c+dx^4}}\right) dx, x, x^2\right.}{8a(bc-ad)}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} - \frac{d\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)} + \frac{(bc-3ad)\text{Subst}\left(\right.}{}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)} + \frac{\left(\sqrt{c}\sqrt{\right.}}{}$$

$$= -\frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{8a(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)} + \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\right.}{}$$

$$= -\frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{8a(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)} + \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} - \frac{(bc-3ad)\tan^{-1}\left(\right.}{32(-a)^{5/4}\sqrt[4]{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.14, size = 169, normalized size = 0.15

$$\frac{x^6\left(21ab(c+dx^8)+7(bc-4ad)\,(a+bx^8)\,\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-3bdx^8(a+bx^8)\,\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)}{168a^2(bc-ad)\,(a+bx^8)\,\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^6*(21*a*b*(c + d*x^8) + 7*(b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*
AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] - 3*b*d*x^8*(a + b*x
^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)
/a)]))/(168*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(x**5/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^5/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.923 $\int \frac{1}{x^3(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1243

$$-\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2}+\frac{\sqrt{d}\,(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\,(a+bx^8)}-\frac{b^{3/4}(5bc-7ad)\tan^{-1}}{32(-a)^{9/4}}$$

```
[Out] -1/32*b^(3/4)*(-7*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)
/(d*x^8+c)^(1/2))/(-a)^(9/4)/(-a*d+b*c)^(3/2)-1/32*b^(3/4)*(-7*a*d+5*b*c)*a
rctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(9/4)/(a
*d-b*c)^(3/2)-1/8*(-4*a*d+5*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^2+1/8*b
*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^2/(b*x^8+a)+1/8*(-4*a*d+5*b*c)*x^2*d^(1/2)*
(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))-1/8*d^(1/4)*(-4*a*d+
5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1
/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*
d+b*c)/(d*x^8+c)^(1/2)+1/16*d^(1/4)*(-4*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arc
tan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(-7
*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/
4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)
*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/
2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((
d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d+b*
c)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/64*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/
c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arct
an(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(
1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^
(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/
2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/32*b*d
^(1/4)*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*ar
ctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2
*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^
8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8
+c)^(1/2)+1/32*b*d^(1/4)*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/
2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/(-a*d+b*c
)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A]

time = 1.95, antiderivative size = 1243, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {476, 483, 597, 598, 311, 226, 1210, 504, 1231, 1721}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*((5*b*c - 4*a*d)*Sqrt[c + d*x^8])/(a^2*c*(b*c - a*d)*x^2) + (Sqrt[d]*(
5*b*c - 4*a*d)*x^2*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*(Sqrt[c] + Sqrt[d]
*x^4)) + (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^2*(a + b*x^8)) - (b^(3/4)*(
5*b*c - 7*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*
x^8])])/(32*(-a)^(9/4)*(b*c - a*d)^(3/2)) - (b^(3/4)*(5*b*c - 7*a*d)*ArcTan
[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(
9/4)*(-(b*c) + a*d)^(3/2)) - (d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*
x^2)/c^(1/4)], 1/2])/(8*a^2*c^(3/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (d^(1/4)
*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d
]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(3/4)*
(b*c - a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^
(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + S
qrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32*a^2*c^(
1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqr
t[d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
1/2])/(32*a^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[b]*(
Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqr
t[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^
(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c
+ a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(
5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*
x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(5/2)*
c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+4ad-3bdx^4}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{x^2(-(bc-2ad}{(a+}\right.}{8}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \left(\frac{d(5bc-4ad}{\sqrt{c+dx}}\right.\right.}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} - \frac{(b(5bc-7ad))\text{Subs}}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} + \frac{\left(\sqrt{b}\,(5bc-7ad)\right)\text{S}}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{\sqrt{d}\,(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)} + \frac{b\sqrt{c+}}{8a(bc-ad)x}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{\sqrt{d}\,(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)} + \frac{b\sqrt{c+}}{8a(bc-ad)x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.21, size = 226, normalized size = 0.18

$$\frac{21a(c+dx^8)\left(4a^2d-5b^2cx^8-4ab(c-dx^8)\right)-7(5b^2c^2-12abcd+4a^2d^2)\,x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+3bd(5bc-4ad)x^{16}(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{168a^3c(bc-ad)x^2\left(a+bx^8\right)\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (21*a*(c + d*x^8)*(4*a^2*d - 5*b^2*c*x^8 - 4*a*b*(c - d*x^8)) - 7*(5*b^2*c^
2 - 12*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^16*(a
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((
b*x^8)/a)])/(168*a^3*c*(b*c - a*d)*x^2*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(b\,x^{8}+a \right)^{2} \sqrt{d\,x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a + bx^{8}\right)^{2} \sqrt{c + dx^{8}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.924 $\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{5}{8};2,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

```
[Out] 1/5*x^5*AppellF1(5/8,2,1/2,13/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(d
*x^8+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{5}{8};2,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 2, 1/2, 13/8, -((b*x^8)/a), -((d*x^8
)/c)])/(5*a^2*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^4}{(a+bx^8)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= \frac{x^5\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{5}{8};2,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 170 vs. $2(64) = 128$.

time = 10.14, size = 170, normalized size = 2.66

$$\frac{x^5\left(65ab(c+dx^8)+13(3bc-8ad)(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-5bdx^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{13}{8};\frac{1}{2},1;\frac{21}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)}{520a^2(bc-ad)(a+bx^8)\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*(65*a*b*(c + d*x^8) + 13*(3*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/
c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] - 5*b*d*x^8*(a +
 b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -((b
*x^8)/a)]))/(520*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]

time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^8+a)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(x**4/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^4/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.925 $\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{3}{8};2,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

```
[Out] 1/3*x^3*AppellF1(3/8,2,1/2,11/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(d
*x^8+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{3}{8};2,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 2, 1/2, 11/8, -((b*x^8)/a), -((d*x^8
)/c)])/(3*a^2*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^2}{(a+bx^8)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= \frac{x^3\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{3}{8};2,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(64) = 128.
time = 10.16, size = 170, normalized size = 2.66

$$\frac{x^3\left(33ab(c+dx^8)+11(5bc-8ad)(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{3}{8};\frac{1}{2},1;\frac{11}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+3bdx^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{11}{8};\frac{1}{2},1;\frac{19}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)}{264a^2(bc-ad)(a+bx^8)\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*(33*a*b*(c + d*x^8) + 11*(5*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/
c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^8*(a +
 b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[11/8, 1/2, 1, 19/8, -((d*x^8)/c), -((b
*x^8)/a)]))/(264*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^8+a)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^2/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.926 $\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{1}{8};2,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

```
[Out] x*AppellF1(1/8,2,1/2,9/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(d*x^8+c)
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\frac{x\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{1}{8};2,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 2, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/c
)])/(a^2*Sqrt[c + d*x^8])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{(a+bx^8)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= \frac{x\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{1}{8};2,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 328 vs. 2(59) = 118.
time = 10.23, size = 328, normalized size = 5.56

$$-\frac{x\left(bdx^8\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{9}{8};\frac{1}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+\frac{3a\left(9ac(8ad-b(8c+dx^8))F_1\left(\frac{1}{8};\frac{1}{2},1;\frac{9}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+4bx^8(c+dx^8)\left(2bcF_1\left(\frac{9}{8};\frac{1}{2},2;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+adF_1\left(\frac{9}{8};\frac{3}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)\right)}{(a+bx^8)\left(-9acF_1\left(\frac{1}{8};\frac{1}{2},1;\frac{9}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+4x^8\left(2bcF_1\left(\frac{9}{8};\frac{1}{2},2;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+adF_1\left(\frac{9}{8};\frac{3}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)\right)}\right)}{24a^2(-bc+ad)\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
[Out] -1/24*(x*(b*d*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[9/8, 1/2, 1, 17/8, -((d*x^8)
/c), -((b*x^8)/a)] + (3*a*(9*a*c*(8*a*d - b*(8*c + d*x^8))*AppellF1[1/8, 1/
2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*b*x^8*(c + d*x^8)*(2*b*c*AppellF
1[9/8, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/2, 1
, 17/8, -((d*x^8)/c), -((b*x^8)/a)])))/((a + b*x^8)*(-9*a*c*AppellF1[1/8, 1
/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8, 1/2, 2
, 17/8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8, -((d*
x^8)/c), -((b*x^8)/a)])))))/(a^2*(-(b*c) + a*d)*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^8+a)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.927 $\int \frac{1}{x^2(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{1+\frac{dx^8}{c}}\,F_1\left(-\frac{1}{8};2,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}$$

```
[Out] -AppellF1(-1/8,2,1/2,7/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/x/(d*x^8+
c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,F_1\left(-\frac{1}{8};2,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 2, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a^2*x*Sqrt[c + d*x^8]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^2(a+bx^8)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= -\frac{\sqrt{1+\frac{dx^8}{c}}\,F_1\left(-\frac{1}{8};2,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(62) = 124.
time = 10.22, size = 226, normalized size = 3.65

$$\frac{35a(c+dx^8)\left(8a^2d-9b^2cx^8-8ab(c-dx^8)\right)-5(9b^2c^2-40abcd+24a^2d^2)\,x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{7}{8};\frac{1}{2},1;\frac{15}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+7bd(9bc-8ad)x^{16}(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{15}{8};\frac{1}{2},1;\frac{23}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{280a^3c(bc-ad)x\left(a+bx^8\right)\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (35*a*(c + d*x^8)*(8*a^2*d - 9*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 5*(9*b^2*c^
2 - 40*a*b*c*d + 24*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7
/8, 1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*(9*b*c - 8*a*d)*x^16*
(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/c),
-((b*x^8)/a)])/(280*a^3*c*(b*c - a*d)*x*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^8 + c)/(b^2*d*x^26 + (b^2*c + 2*a*b*d)*x^18 + (2*a*b*c +
a^2*d)*x^10 + a^2*c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^8\right)^2 \sqrt{c + dx^8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 \left(b\,x^8 + a\right)^2 \sqrt{d\,x^8 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.928 $\int \frac{1}{x^4(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{1+\frac{dx^8}{c}}\,F_1\left(-\frac{3}{8};2,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

```
[Out] -1/3*AppellF1(-3/8,2,1/2,5/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/x^3/(
d*x^8+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,F_1\left(-\frac{3}{8};2,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/3*(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 2, 1/2, 5/8, -((b*x^8)/a), -((d*x^
8)/c)])/(a^2*x^3*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^4(a+bx^8)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$=-\frac{\sqrt{1+\frac{dx^8}{c}}\,F_1\left(-\frac{3}{8};2,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(64) = 128.
time = 10.21, size = 226, normalized size = 3.53

$$\frac{65a(c+dx^8)\left(8a^2d-11b^2cx^8-8ab(c-dx^8)\right)-13(33b^2c^2-56abcd+8a^2d^2)\,x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5bd(11bc-8ad)x^{16}(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{13}{8};\frac{1}{2},1;\frac{21}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{1560a^3c(bc-ad)x^3\left(a+bx^8\right)\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (65*a*(c + d*x^8)*(8*a^2*d - 11*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 13*(33*b^2
*c^2 - 56*a*b*c*d + 8*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1
[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*(11*b*c - 8*a*d)*x^
16*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c
), -((b*x^8)/a)])/(1560*a^3*c*(b*c - a*d)*x^3*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^8)^2 \sqrt{c + dx^8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^4 \left(b\,x^8 + a\right)^2 \sqrt{d\,x^8 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.929 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^5 \, dx$

Optimal. Leaf size=123

$$\frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}\, x^2}{16c^2}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\, x^4}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^6}{6c}-\frac{d^2(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}$$

```
[Out] 1/6*a*(c+d/x^2)^(3/2)*x^6/c-1/16*d^2*(-a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c
^(1/2))/c^(5/2)+1/16*d*(-a*d+2*b*c)*x^2*(c+d/x^2)^(1/2)/c^2+1/8*(-a*d+2*b*c
)*x^4*(c+d/x^2)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 79, 43, 44, 65, 214}

$$-\frac{d^2(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{16c^2}+\frac{x^4\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{8c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c^2) + ((2*b*c - a*d)*Sqrt[c + d/
x^2]*x^4)/(8*c) + (a*(c + d/x^2)^(3/2)*x^6)/(6*c) - (d^2*(2*b*c - a*d)*ArcT
anh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^5\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^4}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{\left(3bc-\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3}\,dx, x, \frac{1}{x^2}\right)}{6c}$$

$$= \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{(d(2bc-ad))\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+}}\right.}{16c}$$

$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} + \frac{(d^2}{}$$

$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} + \frac{(d(}{}$$

$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{d^2(}{}$$

Mathematica [A]
time = 0.14, size = 121, normalized size = 0.98

$$\frac{\sqrt{c+\frac{d}{x^2}}\,x\left(\sqrt{c}\,x\sqrt{d+cx^2}\,(6bc(d+2cx^2)+a(-3d^2+2cdx^2+8c^2x^4))-3d^2(-2bc+ad)\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)\right)}{48c^{5/2}\sqrt{d+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[d + c*x^2]*(6*b*c*(d + 2*c*x^2) + a*(-3*
d^2 + 2*c*d*x^2 + 8*c^2*x^4)) - 3*d^2*(-2*b*c + a*d)*Log[-(Sqrt[c]*x) + Sqr
t[d + c*x^2]]))/(48*c^(5/2)*Sqrt[d + c*x^2])
```

Maple [A]
time = 0.06, size = 162, normalized size = 1.32

method	result

risch	$\frac{x^{2}\left(8a\,c^{2}x^{4}+2acd\,x^{2}+12b\,c^{2}x^{2}-3a\,d^{2}+6bcd\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{48c^{2}}+\frac{\left(\frac{d^{3}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)a}{16c^{\frac{5}{2}}}-\frac{d^{2}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)b}{8c^{\frac{3}{2}}}\right)\sqrt{}}{\sqrt{c\,x^{2}+d}}$
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(8(c\,x^{2}+d)^{\frac{3}{2}}c^{\frac{3}{2}}a\,x^{3}-6(c\,x^{2}+d)^{\frac{3}{2}}\sqrt{c}\,adx+12(c\,x^{2}+d)^{\frac{3}{2}}c^{\frac{3}{2}}bx+3\sqrt{c\,x^{2}+d}\,\sqrt{c}\,a\,d^{2}x-6\sqrt{c\,x^{2}+d}\,c^{\frac{3}{2}}bdx+\right.}{48\sqrt{c\,x^{2}+d}\,c^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^5*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(1/2)*x*(8*(c*x^2+d)^(3/2)*c^(3/2)*a*x^3-6*(c*x^2+d)^(
3/2)*c^(1/2)*a*d*x+12*(c*x^2+d)^(3/2)*c^(3/2)*b*x+3*(c*x^2+d)^(1/2)*c^(1/2)
*a*d^2*x-6*(c*x^2+d)^(1/2)*c^(3/2)*b*d*x+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*
d^3-6*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d^2)/(c*x^2+d)^(1/2)/c^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(103) = 206$.
time = 0.78, size = 243, normalized size = 1.98

$$-\frac{1}{96}\left(\frac{3\,d^{3}\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}d^{3}-8\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}cd^{3}-3\sqrt{c+\frac{d}{x^{2}}}\,c^{2}d^{3}\right)}{\left(c+\frac{d}{x^{2}}\right)^{3}c^{2}-3\left(c+\frac{d}{x^{2}}\right)^{2}c^{3}+3\left(c+\frac{d}{x^{2}}\right)c^{4}-c^{5}}\right)a+\frac{1}{16}\left(\frac{d^{2}\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}d^{2}+\sqrt{c+\frac{d}{x^{2}}}\,cd^{2}\right)}{\left(c+\frac{d}{x^{2}}\right)^{2}c-2\left(c+\frac{d}{x^{2}}\right)c^{2}+c^{3}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c
^(5/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 - 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(c
+ d/x^2)*c^2*d^3)/((c + d/x^2)^3*c^2 - 3*(c + d/x^2)^2*c^3 + 3*(c + d/x^2)*
c^4 - c^5))*a + 1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2)
+ sqrt(c)))/c^(3/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2)/((c
 + d/x^2)^2*c - 2*(c + d/x^2)*c^2 + c^3))*b
```

Fricas [A]
time = 2.84, size = 242, normalized size = 1.97

$$\left[-\frac{3\left(2\,bcd^{2}-ad^{3}\right)\sqrt{c}\,\log\left(-2\,cx^{2}-2\,\sqrt{c}\,x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}-d\right)-2\left(8\,ac^{3}x^{6}+2\left(6\,bc^{3}+ac^{2}d\right)x^{4}+3\left(2\,bc^{2}d-acd^{2}\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{96\,c^{3}},\ \frac{3\left(2\,bcd^{2}-ad^{3}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cx^{2}+d}\right)+\left(8\,ac^{3}x^{6}+2\left(6\,bc^{3}+ac^{2}d\right)x^{4}+3\left(2\,bc^{2}d-acd^{2}\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{48\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(2*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*
x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^
2*d - a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3, 1/48*(3*(2*b*c*d^2 - a*d^3)
*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c^3
*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^2*d - a*c*d^2)*x^2)*sqrt((c*x^2
 + d)/x^2))/c^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(107) = 214$.
time = 69.78, size = 226, normalized size = 1.84

$$\frac{acx^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{5a\sqrt{d}\,x^5}{24\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}x^3}{48c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{5}{2}}x}{16c^2\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{16c^{\frac{5}{2}}}+\frac{bcx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3b\sqrt{d}\,x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{bd^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**5*(c+d/x**2)**(1/2),x)
```

```
[Out] a*c*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 5*a*sqrt(d)*x**5/(24*sqrt(c*x**2/
d + 1)) - a*d**(3/2)*x**3/(48*c*sqrt(c*x**2/d + 1)) - a*d**(5/2)*x/(16*c**2
*sqrt(c*x**2/d + 1)) + a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(5/2)) + b*c*
x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*sqrt(d)*x**3/(8*sqrt(c*x**2/d + 1
)) + b*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - b*d**2*asinh(sqrt(c)*x/sqrt(d)
)/(8*c**(3/2))
```

Giac [A]
time = 2.01, size = 143, normalized size = 1.16

$$\frac{1}{48}\left(2\left(4\,ax^2\operatorname{sgn}(x)+\frac{6\,bc^4\operatorname{sgn}(x)+ac^3d\operatorname{sgn}(x)}{c^4}\right)x^2+\frac{3\,(2\,bc^3d\operatorname{sgn}(x)-ac^2d^2\operatorname{sgn}(x))}{c^4}\right)\sqrt{cx^2+d}\,x+\frac{(2\,bcd^2\operatorname{sgn}(x)-ad^3\operatorname{sgn}(x))\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+d}\right|\right)}{16\,c^{\frac{5}{2}}}-\frac{(2\,bcd^2\log(|d|)-ad^3\log(|d|))\operatorname{sgn}(x)}{32\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*a*x^2*sgn(x) + (6*b*c^4*sgn(x) + a*c^3*d*sgn(x))/c^4)*x^2 + 3*(2
*b*c^3*d*sgn(x) - a*c^2*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x + 1/16*(2*b*c*d^
2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(5/2) - 1
/32*(2*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(5/2)
```

Mupad [B]
time = 5.71, size = 134, normalized size = 1.09

$$\frac{a\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16}+\frac{b\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{6\,c}-\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{5/2}}{16\,c^2}+\frac{b\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c}-\frac{b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}-\frac{a\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}}{16\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^6*(c + d/x^2)^(1/2))/16 + (b*x^4*(c + d/x^2)^(1/2))/8 + (a*x^6*(c + d/
x^2)^(3/2))/(6*c) - (a*x^6*(c + d/x^2)^(5/2))/(16*c^2) + (b*x^4*(c + d/x^2)
^(3/2))/(8*c) - (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16*c^(5/2)
) - (b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

3.930 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^3 \, dx$

Optimal. Leaf size=90

$$\frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}+\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}$$

```
[Out] 1/4*a*(c+d/x^2)^(3/2)*x^4/c+1/8*d*(-a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1
/2))/c^(3/2)+1/8*(-a*d+4*b*c)*x^2*(c+d/x^2)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 43, 65, 214}

$$\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}+\frac{x^2\sqrt{c+\frac{d}{x^2}}\,(4bc-ad)}{8c}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

```
[Out] ((4*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(8*c) + (a*(c + d/x^2)^(3/2)*x^4)/(4*c)
 + (d*(4*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^3\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c} - \frac{\left(2bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2}\,dx, x, \frac{1}{x^2}\right)}{4c}$$

$$= \frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c} - \frac{(d(4bc-ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+d\ldots}}\right.}{16c}$$

$$= \frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c} - \frac{(4bc-ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x\ldots\right.}{8c}$$

$$= \frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c} + \frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right.}{8c^{3/2}}$$

Mathematica [A]

time = 0.10, size = 97, normalized size = 1.08

$$\frac{\sqrt{c+\frac{d}{x^2}}\, x\left(\sqrt{c}\, x\sqrt{d+cx^2}\,(4bc+a(d+2cx^2))+d(-4bc+ad)\log\left(-\sqrt{c}\, x+\sqrt{d+cx^2}\right)\right)}{8c^{3/2}\sqrt{d+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[d + c*x^2]*(4*b*c + a*(d + 2*c*x^2)) + d
*(-4*b*c + a*d)*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]]))/(8*c^(3/2)*Sqrt[d + c
*x^2])
```

Maple [A]
time = 0.05, size = 122, normalized size = 1.36

method	result
risch	$\frac{x^2(2cx^2a+ad+4bc)\sqrt{\frac{cx^2+d}{x^2}}}{8c}+\frac{\left(-\frac{d^2\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)a}{8c^{\frac{3}{2}}}+\frac{d\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)b}{2\sqrt{c}}\right)\sqrt{\frac{cx^2+d}{x^2}}\,x}{\sqrt{cx^2+d}}$
default	$\frac{\sqrt{\frac{cx^2+d}{x^2}}\,x\left(2\sqrt{c}\,(cx^2+d)^{\frac{3}{2}}ax-\sqrt{c}\,\sqrt{cx^2+d}\,adx+4c^{\frac{3}{2}}\sqrt{cx^2+d}\,bx-\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)ad^2+4\ln\left(\sqrt{c}\right.\right.}{8\sqrt{cx^2+d}\,c^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^3*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((c*x^2+d)/x^2)^(1/2)*x*(2*c^(1/2)*(c*x^2+d)^(3/2)*a*x-c^(1/2)*(c*x^2+d
)^(1/2)*a*d*x+4*c^(3/2)*(c*x^2+d)^(1/2)*b*x-ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a
*d^2+4*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d)/(c*x^2+d)^(1/2)/c^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(74) = 148$.
time = 0.52, size = 159, normalized size = 1.77

$$\frac{1}{16}\left(\frac{d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2+\sqrt{c+\frac{d}{x^2}}\,cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c-2\left(c+\frac{d}{x^2}\right)c^2+c^3}\right)a+\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}\,x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3
/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c -
2*(c + d/x^2)*c^2 + c^3))*a + 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c +
d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c))*b
```

Fricas [A]
time = 3.40, size = 191, normalized size = 2.12

$$\left[-\frac{\left(4\,bcd-ad^{2}\right)\sqrt{c}\,\log\left(-2\,cx^{2}+2\,\sqrt{c}\,x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}-d\right)-2\,\left(2\,ac^{2}x^{4}+\left(4\,bc^{2}+acd\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{16\,c^{2}},-\frac{\left(4\,bcd-ad^{2}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cx^{2}+d}\right)-\left(2\,ac^{2}x^{4}+\left(4\,bc^{2}+acd\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{8\,c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*b*c*d - a*d^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*x^2
+ d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2 + d)/x
^2))/c^2, -1/8*((4*b*c*d - a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2
+ d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2
+ d)/x^2))/c^2]
```

Sympy [A]
time = 42.19, size = 144, normalized size = 1.60

$$\frac{acx^{5}}{4\sqrt{d}\,\sqrt{\frac{cx^{2}}{d}+1}}+\frac{3a\sqrt{d}\,x^{3}}{8\sqrt{\frac{cx^{2}}{d}+1}}+\frac{ad^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^{2}}{d}+1}}-\frac{ad^{2}\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}+\frac{b\sqrt{d}\,x\sqrt{\frac{cx^{2}}{d}+1}}{2}+\frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3*(c+d/x**2)**(1/2),x)
```

```
[Out] a*c*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*sqrt(d)*x**3/(8*sqrt(c*x**2/d
 + 1)) + a*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - a*d**2*asinh(sqrt(c)*x/sqr
t(d))/(8*c**(3/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + b*d*asinh(sqrt(c)*x
/sqrt(d))/(2*sqrt(c))
```

Giac [A]
time = 1.37, size = 105, normalized size = 1.17

$$\frac{1}{8}\left(2\,ax^{2}\mathrm{sgn}\left(x\right)+\frac{4\,bc^{2}\mathrm{sgn}\left(x\right)+acd\mathrm{sgn}\left(x\right)}{c^{2}}\right)\sqrt{cx^{2}+d}\,x-\frac{\left(4\,bcd\mathrm{sgn}\left(x\right)-ad^{2}\mathrm{sgn}\left(x\right)\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^{2}+d}\right|\right)}{8\,c^{\frac{3}{2}}}+\frac{\left(4\,bcd\log\left(\left|d\right|\right)-ad^{2}\log\left(\left|d\right|\right)\right)\mathrm{sgn}\left(x\right)}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*a*x^2*sgn(x) + (4*b*c^2*sgn(x) + a*c*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*
x - 1/8*(4*b*c*d*sgn(x) - a*d^2*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d
)))/c^(3/2) + 1/16*(4*b*c*d*log(abs(d)) - a*d^2*log(abs(d)))*sgn(x)/c^(3/2)
```

Mupad [B]
time = 5.31, size = 93, normalized size = 1.03

$$\frac{a\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{b\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}+\frac{a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c}+\frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}}-\frac{a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^4*(c + d/x^2)^(1/2))/8 + (b*x^2*(c + d/x^2)^(1/2))/2 + (a*x^4*(c + d/x
^2)^(3/2))/(8*c) + (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2)) - (a*
d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

3.931 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x \, dx$

Optimal. Leaf size=84

$$-\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

```
[Out] 1/2*a*(c+d/x^2)^(3/2)*x^2/c+1/2*(a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2)
)/c^(1/2)-1/2*(a*d+2*b*c)*(c+d/x^2)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 79, 52, 65, 214}

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(ad+2bc)}{2c}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

```
[Out] -1/2*((2*b*c + a*d)*Sqrt[c + d/x^2])/c + (a*(c + d/x^2)^(3/2)*x^2)/(2*c) +
((2*b*c + a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*Sqrt[c])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} - \frac{(2bc+ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)}{4c}$$

$$= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} - \frac{1}{4}(2bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\right.$$

$$= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} - \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x\right.}{2d}$$

$$= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

Mathematica [A]
time = 0.13, size = 72, normalized size = 0.86

$$\frac{1}{2}\sqrt{c+\frac{d}{x^2}}\left(-2b+ax^2-\frac{(2bc+ad)x\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)}{\sqrt{c}\,\sqrt{d+cx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-2*b + a*x^2 - ((2*b*c + a*d)*x*Log[-(Sqrt[c]*x) + Sqrt[d
 + c*x^2]])/(Sqrt[c]*Sqrt[d + c*x^2])))/2
```

Maple [A]
time = 0.06, size = 127, normalized size = 1.51

method	result
risch	$\frac{(a\,x^2-2b)\sqrt{\frac{c\,x^2+d}{x^2}}}{2}+\frac{\left(\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)a d}{2\sqrt{c}}+\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{c}\,b\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{\sqrt{c\,x^2+d}}$
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(2c^{\frac{3}{2}}\sqrt{c\,x^2+d}\,b\,x^2+\sqrt{c}\,\sqrt{c\,x^2+d}\,a d\,x^2-2\sqrt{c}\,(c\,x^2+d)^{\frac{3}{2}}b+\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)a\,d^2x+2\ln\left(\sqrt{c}\right.\right.}{2\sqrt{c\,x^2+d}\,d\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((c*x^2+d)/x^2)^(1/2)*(2*c^(3/2)*(c*x^2+d)^(1/2)*b*x^2+c^(1/2)*(c*x^2+d
)^(1/2)*a*d*x^2-2*c^(1/2)*(c*x^2+d)^(3/2)*b+ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a
*d^2*x+2*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d*x)/(c*x^2+d)^(1/2)/d/c^(1/2)
```

Maxima [A]
time = 0.56, size = 108, normalized size = 1.29

$$\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}\,x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)a-\frac{1}{2}\left(\sqrt{c}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^2}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/
x^2) + sqrt(c)))/sqrt(c))*a - 1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c))/
(sqrt(c + d/x^2) + sqrt(c))) + 2*sqrt(c + d/x^2))*b
```

Fricas [A]
time = 2.30, size = 155, normalized size = 1.85

$$\left[\frac{(2\,bc+ad)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\,(acx^2-2\,bc)\sqrt{\frac{cx^2+d}{x^2}}}{4\,c},-\frac{(2\,bc+ad)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(acx^2-2\,bc)\sqrt{\frac{cx^2+d}{x^2}}}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*b*c + a*d)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x
^2) - d) + 2*(a*c*x^2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c, -1/2*((2*b*c + a*d
)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (a*c*x^
2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [A]
time = 54.64, size = 107, normalized size = 1.27

$$\frac{a\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{ad\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2\sqrt{c}}+b\sqrt{c}\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)-\frac{bcx}{\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}-\frac{b\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + a*d*asinh(sqrt(c)*x/sqrt(d))/(2*sqrt(c))
 + b*sqrt(c)*asinh(sqrt(c)*x/sqrt(d)) - b*c*x/(sqrt(d)*sqrt(c*x**2/d + 1))
- b*sqrt(d)/(x*sqrt(c*x**2/d + 1))
```

Giac [A]
time = 1.79, size = 92, normalized size = 1.10

$$\frac{1}{2}\sqrt{cx^2+d}\,ax\mathrm{sgn}(x)+\frac{2\,b\sqrt{c}\,d\mathrm{sgn}(x)}{\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d}-\frac{\left(2\,bc^{\frac{3}{2}}\mathrm{sgn}(x)+a\sqrt{c}\,d\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*x*sgn(x) + 2*b*sqrt(c)*d*sgn(x)/((sqrt(c)*x - sqrt(c*
x^2 + d))^2 - d) - 1/4*(2*b*c^(3/2)*sgn(x) + a*sqrt(c)*d*sgn(x))*log((sqrt(
c)*x - sqrt(c*x^2 + d))^2)/c
```

Mupad [B]
time = 5.10, size = 68, normalized size = 0.81

$$\frac{a\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}-b\,\sqrt{c+\frac{d}{x^2}}+b\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^2*(c + d/x^2)^(1/2))/2 - b*(c + d/x^2)^(1/2) + b*c^(1/2)*atanh((c + d/
x^2)^(1/2)/c^(1/2)) + (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2))
```

3.932 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx$

Optimal. Leaf size=59

$$-a\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

```
[Out] -1/3*b*(c+d/x^2)^(3/2)/d+a*arctanh((c+d/x^2)^(1/2)/c^(1/2))*c^(1/2)-a*(c+d/
x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 81, 52, 65, 214}

$$-a\sqrt{c+\frac{d}{x^2}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

```
[Out] -(a*Sqrt[c + d/x^2]) - (b*(c + d/x^2)^(3/2))/(3*d) + a*Sqrt[c]*ArcTanh[Sqrt
[c + d/x^2]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}-\frac{1}{2}a\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx,x,\frac{1}{x^2}\right)\\
&= -a\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}-\frac{1}{2}(ac)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)\\
&= -a\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}-\frac{(ac)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+\frac{d}{x^2}}\right)}{d}\\
&= -a\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 82, normalized size = 1.39

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(3adx^2+b(d+cx^2)+\frac{3a\sqrt{c}\,dx^3\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)}{\sqrt{d+cx^2}}\right)}{3dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

```
[Out] -1/3*(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d + c*x^2) + (3*a*Sqrt[c]*d*x^3*Log[-
(Sqrt[c]*x) + Sqrt[d + c*x^2]])/Sqrt[d + c*x^2]))/(d*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(47) = 94$.
time = 0.06, size = 110, normalized size = 1.86

method	result	size
risch	$-\frac{(3ad\,x^2+c\,x^2b+bd)\sqrt{\frac{c\,x^2+d}{x^2}}}{3x^2d}+\frac{a\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{\sqrt{c\,x^2+d}}$	84
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(3c^{\frac{3}{2}}\sqrt{c\,x^2+d}\,a\,x^4-3\sqrt{c}\,(c\,x^2+d)^{\frac{3}{2}}a\,x^2+3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)acd\,x^3-\sqrt{c}\,(c\,x^2+d)^{\frac{3}{2}}b\right)}{3x^2\sqrt{c\,x^2+d}\,d\sqrt{c}}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((c*x^2+d)/x^2)^(1/2)*(3*c^(3/2)*(c*x^2+d)^(1/2)*a*x^4-3*c^(1/2)*(c*x^2
+d)^(3/2)*a*x^2+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c*d*x^3-c^(1/2)*(c*x^2+d)
^(3/2)*b)/x^2/(c*x^2+d)^(1/2)/d/c^(1/2)
```

Maxima [A]
time = 0.49, size = 67, normalized size = 1.14

$$-\frac{1}{2}\left(\sqrt{c}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^2}}\right)a-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))
+ 2*sqrt(c + d/x^2))*a - 1/3*b*(c + d/x^2)^(3/2)/d
```

Fricas [A]
time = 2.50, size = 166, normalized size = 2.81

$$\left[\frac{3\,a\sqrt{c}\,dx^2\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,\left((bc+3\,ad)x^2+bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{6\,dx^2},-\frac{3\,a\sqrt{-c}\,dx^2\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left((bc+3\,ad)x^2+bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,dx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(c)*d*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2)
- d) - 2*((b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2), -1/3*(3*
a*sqrt(-c)*d*x^2*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (
(b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2)]
```

Sympy [A]
time = 25.26, size = 75, normalized size = 1.27

$$\frac{a\left(-\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{c+\frac{d}{x^2}}\right)}{2}+\frac{b\left(\begin{cases}-\frac{\sqrt{c}}{x^2} & \text{for}\ d=0\\ -\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x,x)
```

```
[Out] a*(-2*c*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - 2*sqrt(c + d/x**2))/2 +
b*Piecewise((-sqrt(c)/x**2, Eq(d, 0)), (-2*(c + d/x**2)**(3/2)/(3*d), True)
)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(47) = 94.
time = 2.05, size = 163, normalized size = 2.76

$$-\frac{1}{2}\,a\sqrt{c}\,\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2\right)\mathrm{sgn}(x)+\frac{2\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}\mathrm{sgn}(x)+3\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4a\sqrt{c}\,d\mathrm{sgn}(x)-6\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2a\sqrt{c}\,d^2\mathrm{sgn}(x)+bc^{\frac{3}{2}}d^2\mathrm{sgn}(x)+3\,a\sqrt{c}\,d^3\mathrm{sgn}(x)\right)}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/3*(3*(sqrt(c
)*x - sqrt(c*x^2 + d))^4*b*c^(3/2)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))
^4*a*sqrt(c)*d*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^2*sgn
(x) + b*c^(3/2)*d^2*sgn(x) + 3*a*sqrt(c)*d^3*sgn(x))/((sqrt(c)*x - sqrt(c*x
^2 + d))^2 - d)^3
```

Mupad [B]
time = 5.21, size = 57, normalized size = 0.97

$$a\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-a\sqrt{c+\frac{d}{x^2}}-\frac{b\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)}{3\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x,x)
```

```
[Out] a*c^(1/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - a*(c + d/x^2)^(1/2) - (b*(c +
d/x^2)^(1/2)*(d + c*x^2))/(3*d*x^2)
```

3.933 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

```
[Out] 1/3*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^2-1/5*b*(c+d/x^2)^(5/2)/d^2
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

```
[Out] ((b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^2) - (b*(c + d/x^2)^(5/2))/(5*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int (a+bx)\sqrt{c+dx}\,dx,x,\frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)\sqrt{c+dx}}{d}+\frac{b(c+dx)^{3/2}}{d}\right)dx,x,\frac{1}{x^2}\right)\right)$$
$$= \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

Mathematica [A]
time = 0.09, size = 47, normalized size = 1.02

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)\,(-3bd+2bcx^2-5adx^2)}{15d^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(-3*b*d + 2*b*c*x^2 - 5*a*d*x^2))/(15*d^2*x^4)
```

Maple [A]
time = 0.06, size = 48, normalized size = 1.04

method	result	size
gosper	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(5ad\,x^{2}-2c\,x^{2}b+3bd\right)\left(c\,x^{2}+d\right)}{15d^{2}x^{4}}$	48
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(5ad\,x^{2}-2c\,x^{2}b+3bd\right)\left(c\,x^{2}+d\right)}{15d^{2}x^{4}}$	48
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(5x^{4}acd-2x^{4}b\,c^{2}+5a\,d^{2}x^{2}+bcd\,x^{2}+3b\,d^{2}\right)}{15x^{4}d^{2}}$	62
trager	$-\frac{\left(5x^{4}acd-2x^{4}b\,c^{2}+5a\,d^{2}x^{2}+bcd\,x^{2}+3b\,d^{2}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{15x^{4}d^{2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/15*((c*x^2+d)/x^2)^(1/2)*(5*a*d*x^2-2*b*c*x^2+3*b*d)*(c*x^2+d)/d^2/x^4
```

Maxima [A]
time = 0.39, size = 49, normalized size = 1.07

$$-\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/15*b*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2)*c/d^2) - 1/3*a*(c +
d/x^2)^(3/2)/d
```

Fricas [A]
time = 2.69, size = 60, normalized size = 1.30

$$\frac{\left(\left(2\,bc^2-5\,acd\right)x^4-3\,bd^2-\left(bcd+5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/15*((2*b*c^2 - 5*a*c*d)*x^4 - 3*b*d^2 - (b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^
2 + d)/x^2)/(d^2*x^4)
```

Sympy [A]
time = 4.81, size = 58, normalized size = 1.26

$$-\frac{a\left(\begin{cases}\frac{\sqrt{c}}{x^2} & \text{for}\ d=0\\ \frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}-\frac{b\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**3,x)
```

```
[Out] -a*Piecewise((sqrt(c)/x**2, Eq(d, 0)), (2*(c + d/x**2)**(3/2)/(3*d), True))
/2 - b*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(38) = 76.
time = 2.35, size = 250, normalized size = 5.43

$$\frac{2\left(15\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}\mathrm{sgn}(x)+30\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^6bc^{\frac{5}{2}}\mathrm{sgn}(x)-30\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}d\mathrm{sgn}(x)+10\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4bc^{\frac{5}{2}}d\mathrm{sgn}(x)+20\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4ac^{\frac{3}{2}}d^2\mathrm{sgn}(x)+10\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2bc^{\frac{5}{2}}d^2\mathrm{sgn}(x)-10\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2ac^{\frac{3}{2}}d^3\mathrm{sgn}(x)-2\,bc^{\frac{5}{2}}d^3\mathrm{sgn}(x)+5\,ac^{\frac{3}{2}}d^4\mathrm{sgn}(x)\right)}{15\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(3/2)*sgn(x) + 30*(sqrt(c)*x -
 sqrt(c*x^2 + d))^6*b*c^(5/2)*sgn(x) - 30*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a
*c^(3/2)*d*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d*sgn(x) +
 20*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^2*sgn(x) + 10*(sqrt(c)*x -
sqrt(c*x^2 + d))^2*b*c^(5/2)*d^2*sgn(x) - 10*(sqrt(c)*x - sqrt(c*x^2 + d))^
2*a*c^(3/2)*d^3*sgn(x) - 2*b*c^(5/2)*d^3*sgn(x) + 5*a*c^(3/2)*d^4*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

Mupad [B]
time = 4.82, size = 91, normalized size = 1.98

$$\frac{\sqrt{c+\frac{d}{x^2}}\,\left(b\,c^2+a\,d\,c\right)}{5\,d^2}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(5\,a\,d^2+b\,c\,d\right)}{15\,d^2\,x^2}-\frac{c\,\sqrt{c+\frac{d}{x^2}}\,\left(8\,a\,d+b\,c\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^3,x)
```

```
[Out] ((c + d/x^2)^(1/2)*(b*c^2 + a*c*d))/(5*d^2) - (b*(c + d/x^2)^(1/2))/(5*x^4)
 - ((c + d/x^2)^(1/2)*(5*a*d^2 + b*c*d))/(15*d^2*x^2) - (c*(c + d/x^2)^(1/2
)*(8*a*d + b*c))/(15*d^2)
```

3.934 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$

Optimal. Leaf size=74

$$-\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

```
[Out] -1/3*c*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^3+1/5*(-a*d+2*b*c)*(c+d/x^2)^(5/2)/d^3-
1/7*b*(c+d/x^2)^(7/2)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(2bc-ad)}{5d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

```
[Out] -1/3*(c*(b*c - a*d)*(c + d/x^2)^(3/2))/d^3 + ((2*b*c - a*d)*(c + d/x^2)^(5/
2))/(5*d^3) - (b*(c + d/x^2)^(7/2))/(7*d^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int x(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c(bc-ad)\sqrt{c+dx}}{d^2} + \frac{(-2bc+ad)(c+dx)^{3/2}}{d^2} + \frac{b(c+dx)^{5/}}{d^2}\right.\right.\right.$$
$$= -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3} + \frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

Mathematica [A]
time = 0.12, size = 69, normalized size = 0.93

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)\,(-15bd^2+12bcdx^2-21ad^2x^2-8bc^2x^4+14acdx^4)}{105d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(-15*b*d^2 + 12*b*c*d*x^2 - 21*a*d^2*x^2 - 8*b
*c^2*x^4 + 14*a*c*d*x^4))/(105*d^3*x^6)
```

Maple [A]
time = 0.07, size = 70, normalized size = 0.95

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(14x^4acd-8x^4b\,c^2-21a\,d^2x^2+12bcd\,x^2-15b\,d^2\right)\left(c\,x^2+d\right)}{105d^3x^6}$	70
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(14x^4acd-8x^4b\,c^2-21a\,d^2x^2+12bcd\,x^2-15b\,d^2\right)\left(c\,x^2+d\right)}{105d^3x^6}$	70
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(14a\,c^2d\,x^6-8b\,c^3x^6-7ac\,d^2x^4+4b\,c^2d\,x^4-21a\,d^3x^2-3bc\,d^2x^2-15b\,d^3\right)}{105x^6d^3}$	87
trager	$\frac{\left(14a\,c^2d\,x^6-8b\,c^3x^6-7ac\,d^2x^4+4b\,c^2d\,x^4-21a\,d^3x^2-3bc\,d^2x^2-15b\,d^3\right)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{105x^6d^3}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*((c*x^2+d)/x^2)^(1/2)*(14*a*c*d*x^4-8*b*c^2*x^4-21*a*d^2*x^2+12*b*c*d
*x^2-15*b*d^2)*(c*x^2+d)/d^3/x^6
```

Maxima [A]
time = 0.38, size = 84, normalized size = 1.14

$$-\frac{1}{105}\,b\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right)-\frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/105*b*(15*(c + d/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d
/x^2)^(3/2)*c^2/d^3) - 1/15*a*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2
)*c/d^2)
```

Fricas [A]
time = 2.66, size = 85, normalized size = 1.15

$$-\frac{\left(2\left(4\,bc^3-7\,ac^2d\right)x^6-\left(4\,bc^2d-7\,acd^2\right)x^4+15\,bd^3+3\left(bcd^2+7\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/105*(2*(4*b*c^3 - 7*a*c^2*d)*x^6 - (4*b*c^2*d - 7*a*c*d^2)*x^4 + 15*b*d^
3 + 3*(b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^3*x^6)
```

Sympy [A]
time = 3.97, size = 78, normalized size = 1.05

$$-\frac{a\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{b\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**5,x)
```

```
[Out] -a*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - b*(c**2*(c + d
/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(62) = 124.
time = 3.69, size = 310, normalized size = 4.19

$$\frac{4\left(105\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{5}{2}}\mathrm{sgn}(x)+280\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{7}{2}}\mathrm{sgn}(x)-175\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{5}{2}}d\,\mathrm{sgn}(x)+140\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{7}{2}}d\,\mathrm{sgn}(x)+70\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{5}{2}}d^2\mathrm{sgn}(x)+84\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{7}{2}}d^2\mathrm{sgn}(x)-42\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{5}{2}}d^3\mathrm{sgn}(x)-28\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{7}{2}}d^3\mathrm{sgn}(x)+49\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{5}{2}}d^4\mathrm{sgn}(x)+4\,bc^{\frac{7}{2}}d^4\mathrm{sgn}(x)-7\,ac^{\frac{5}{2}}d^5\mathrm{sgn}(x)\right)}{105\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(5/2)*sgn(x) + 280*(sqrt(c)
*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*sgn(x) - 175*(sqrt(c)*x - sqrt(c*x^2 + d)
)^8*a*c^(5/2)*d*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d*sg
n(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(5/2)*d^2*sgn(x) + 84*(sqrt(c
)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c*x^2
+ d))^4*a*c^(5/2)*d^3*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)
*d^3*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^4*sgn(x) + 4*b
*c^(7/2)*d^4*sgn(x) - 7*a*c^(5/2)*d^5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d)
)^2 - d)^7
```

Mupad [B]
time = 4.92, size = 126, normalized size = 1.70

$$\frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{15\,d^2}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{15\,d\,x^2}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4}+\frac{4\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^5,x)
```

```
[Out] (2*a*c^2*(c + d/x^2)^(1/2))/(15*d^2) - (b*(c + d/x^2)^(1/2))/(7*x^6) - (a*(
c + d/x^2)^(1/2))/(5*x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*c*(c
 + d/x^2)^(1/2))/(15*d*x^2) - (b*c*(c + d/x^2)^(1/2))/(35*d*x^4) + (4*b*c^2
*(c + d/x^2)^(1/2))/(105*d^2*x^2)
```

3.935 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$

Optimal. Leaf size=104

$$\frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

```
[Out] 1/3*c^2*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^4-1/5*c*(-2*a*d+3*b*c)*(c+d/x^2)^(5/2)
/d^4+1/7*(-a*d+3*b*c)*(c+d/x^2)^(7/2)/d^4-1/9*b*(c+d/x^2)^(9/2)/d^4
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-ad)}{7d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-2ad)}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(5/2))/(5*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^4) - (b*(c + d/x^
2)^(9/2))/(9*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{c^2(bc-ad)\sqrt{c+dx}}{d^3} + \frac{c(3bc-2ad)(c+dx)^{3/2}}{d^3} + \frac{(-3bc+$$
$$= \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4} - \frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4} -$$

Mathematica [A]
time = 0.14, size = 93, normalized size = 0.89

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)\,(-35bd^3+30bcd^2x^2-45ad^3x^2-24bc^2dx^4+36acd^2x^4+16bc^3x^6-24ac^2dx^6)}{315d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(-35*b*d^3 + 30*b*c*d^2*x^2 - 45*a*d^3*x^2 - 2
4*b*c^2*d*x^4 + 36*a*c*d^2*x^4 + 16*b*c^3*x^6 - 24*a*c^2*d*x^6))/(315*d^4*x
^8)
```

Maple [A]
time = 0.09, size = 94, normalized size = 0.90

method	result	size
gosper	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,(24a\,c^2d\,x^6-16b\,c^3x^6-36ac\,d^2x^4+24b\,c^2d\,x^4+45a\,d^3x^2-30bc\,d^2x^2+35b\,d^3)(c\,x^2+d)}{315d^4x^8}$	94
default	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,(24a\,c^2d\,x^6-16b\,c^3x^6-36ac\,d^2x^4+24b\,c^2d\,x^4+45a\,d^3x^2-30bc\,d^2x^2+35b\,d^3)(c\,x^2+d)}{315d^4x^8}$	94
risch	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,(24a\,c^3d\,x^8-16b\,c^4x^8-12a\,c^2d^2x^6+8b\,c^3d\,x^6+9ac\,d^3x^4-6b\,c^2d^2x^4+45a\,d^4x^2+5bc\,d^3x^2+35b\,d^4)}{315x^8d^4}$	111
trager	$-\frac{(24a\,c^3d\,x^8-16b\,c^4x^8-12a\,c^2d^2x^6+8b\,c^3d\,x^6+9ac\,d^3x^4-6b\,c^2d^2x^4+45a\,d^4x^2+5bc\,d^3x^2+35b\,d^4)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{315x^8d^4}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*((c*x^2+d)/x^2)^(1/2)*(24*a*c^2*d*x^6-16*b*c^3*x^6-36*a*c*d^2*x^4+24
*b*c^2*d*x^4+45*a*d^3*x^2-30*b*c*d^2*x^2+35*b*d^3)*(c*x^2+d)/d^4/x^8
```

Maxima [A]
time = 0.37, size = 118, normalized size = 1.13

$$-\frac{1}{315}\,b\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right)-\frac{1}{105}\,a\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/315*b*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7/2)*c/d^4 + 189*(c +
 d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4) - 1/105*a*(15*(c + d
/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d/x^2)^(3/2)*c^2/d^3
)
```

Fricas [A]
time = 2.47, size = 109, normalized size = 1.05

$$\frac{\left(8\left(2\,bc^4-3\,ac^3d\right)x^8-4\left(2\,bc^3d-3\,ac^2d^2\right)x^6-35\,bd^4+3\left(2\,bc^2d^2-3\,acd^3\right)x^4-5\left(bcd^3+9\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(2*b*c^4 - 3*a*c^3*d)*x^8 - 4*(2*b*c^3*d - 3*a*c^2*d^2)*x^6 - 35*b
*d^4 + 3*(2*b*c^2*d^2 - 3*a*c*d^3)*x^4 - 5*(b*c*d^3 + 9*a*d^4)*x^2)*sqrt((c
*x^2 + d)/x^2)/(d^4*x^8)
```

Sympy [A]
time = 5.70, size = 112, normalized size = 1.08

$$-\frac{a\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{b\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**7,x)
```

```
[Out] -a*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**
(7/2)/7)/d**3 - b*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)
/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(88) = 176.
time = 4.38, size = 370, normalized size = 3.56

$$\frac{16\left(210\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{7}{2}}\mathrm{sgn}(x)+630\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{9}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{7}{2}}d\,\mathrm{sgn}(x)+378\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{9}{2}}d\,\mathrm{sgn}(x)+63\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{7}{2}}d^2\mathrm{sgn}(x)+168\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{9}{2}}d^2\mathrm{sgn}(x)-42\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{7}{2}}d^3\mathrm{sgn}(x)-72\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{9}{2}}d^3\mathrm{sgn}(x)+108\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{7}{2}}d^4\mathrm{sgn}(x)+18\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{9}{2}}d^4\mathrm{sgn}(x)-27\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{7}{2}}d^5\mathrm{sgn}(x)-2\,bc^{\frac{9}{2}}d^5\mathrm{sgn}(x)+3\,ac^{\frac{7}{2}}d^6\mathrm{sgn}(x)\right)}{315\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(7/2)*sgn(x) + 630*(sqrt(c
)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 +
d))^10*a*c^(7/2)*d*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d
*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(7/2)*d^2*sgn(x) + 168*(sq
rt(c)*x - sqrt(c*x^2 + d))^6*b*c^(9/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c*
x^2 + d))^6*a*c^(7/2)*d^3*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(
9/2)*d^3*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^4*sgn(x)
+ 18*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(9/2)*d^4*sgn(x) - 27*(sqrt(c)*x -
 sqrt(c*x^2 + d))^2*a*c^(7/2)*d^5*sgn(x) - 2*b*c^(9/2)*d^5*sgn(x) + 3*a*c^(
7/2)*d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^9
```

Mupad [B]
time = 5.22, size = 168, normalized size = 1.62

$$\frac{16\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{8\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6}+\frac{4\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2}+\frac{2\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^4}-\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^7,x)
```

```
[Out] (16*b*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(9*x^8) - (8
*a*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*(c + d/x^2)^(1/2))/(7*x^6) - (a*c*
(c + d/x^2)^(1/2))/(35*d*x^4) - (b*c*(c + d/x^2)^(1/2))/(63*d*x^6) + (4*a*c
^2*(c + d/x^2)^(1/2))/(105*d^2*x^2) + (2*b*c^2*(c + d/x^2)^(1/2))/(105*d^2*
x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(315*d^3*x^2)
```

3.936 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$

Optimal. Leaf size=134

$$-\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}-\frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}+\frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^5}-\frac{b(c$$

```
[Out] -1/3*c^3*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^5+1/5*c^2*(-3*a*d+4*b*c)*(c+d/x^2)^(5
/2)/d^5-3/7*c*(-a*d+2*b*c)*(c+d/x^2)^(7/2)/d^5+1/9*(-a*d+4*b*c)*(c+d/x^2)^(
9/2)/d^5-1/11*b*(c+d/x^2)^(11/2)/d^5
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$-\frac{c^3\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^5}+\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-3ad)}{5d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(4bc-ad)}{9d^5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

```
[Out] -1/3*(c^3*(b*c - a*d)*(c + d/x^2)^(3/2))/d^5 + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (3*c*(2*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^5) + ((4*b
*c - a*d)*(c + d/x^2)^(9/2))/(9*d^5) - (b*(c + d/x^2)^(11/2))/(11*d^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int x^3(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c^3(bc-ad)\sqrt{c+dx}}{d^4} - \frac{c^2(4bc-3ad)(c+dx)^{3/2}}{d^4} + \frac{3c(2bc-\right.\right.\right.$$

$$= -\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^5} + \frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5} - \frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7}}{7d^5}$$

Mathematica [A]
time = 0.16, size = 113, normalized size = 0.84

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)\,(11adx^2(-35d^3+30cd^2x^2-24c^2dx^4+16c^3x^6)+b(-315d^4+280cd^3x^2-240c^2d^2x^4+192c^3dx^6-128c^4x^8))}{3465d^5x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(11*a*d*x^2*(-35*d^3 + 30*c*d^2*x^2 - 24*c^2*d
*x^4 + 16*c^3*x^6) + b*(-315*d^4 + 280*c*d^3*x^2 - 240*c^2*d^2*x^4 + 192*c^
3*d*x^6 - 128*c^4*x^8)))/(3465*d^5*x^10)
```

Maple [A]
time = 0.12, size = 118, normalized size = 0.88

method	result
gosper	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,(176a\,c^3d\,x^8-128b\,c^4x^8-264a\,c^2d^2x^6+192b\,c^3d\,x^6+330ac\,d^3x^4-240b\,c^2d^2x^4-385a\,d^4x^2+280bc\,d^3x^2-315b\,d^4)\,(c\,x^2+}{3465d^5x^{10}}$
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,(176a\,c^3d\,x^8-128b\,c^4x^8-264a\,c^2d^2x^6+192b\,c^3d\,x^6+330ac\,d^3x^4-240b\,c^2d^2x^4-385a\,d^4x^2+280bc\,d^3x^2-315b\,d^4)\,(c\,x^2+}{3465d^5x^{10}}$
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,(176a\,c^4d\,x^{10}-128b\,c^5x^{10}-88a\,c^3d^2x^8+64b\,c^4d\,x^8+66a\,c^2d^3x^6-48b\,c^3d^2x^6-55ac\,d^4x^4+40b\,c^2d^3x^4-385a\,d^5x^2-35bc}{3465x^{10}d^5}$
trager	$\frac{(176a\,c^4d\,x^{10}-128b\,c^5x^{10}-88a\,c^3d^2x^8+64b\,c^4d\,x^8+66a\,c^2d^3x^6-48b\,c^3d^2x^6-55ac\,d^4x^4+40b\,c^2d^3x^4-385a\,d^5x^2-35bc\,d^4x^2-315b\,d}{3465x^{10}d^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(1/2)*(176*a*c^3*d*x^8-128*b*c^4*x^8-264*a*c^2*d^2*x
^6+192*b*c^3*d*x^6+330*a*c*d^3*x^4-240*b*c^2*d^2*x^4-385*a*d^4*x^2+280*b*c*
d^3*x^2-315*b*d^4)*(c*x^2+d)/d^5/x^10
```

Maxima [A]
time = 0.37, size = 152, normalized size = 1.13

$$-\frac{1}{3465}\,b\left(\frac{315\,\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^5}-\frac{1540\,\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^5}+\frac{2970\,\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^5}-\frac{2772\,\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^5}+\frac{1155\,\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4}{d^5}\right)-\frac{1}{315}\,a\left(\frac{35\,\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\,\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\,\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\,\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/3465*b*(315*(c + d/x^2)^(11/2)/d^5 - 1540*(c + d/x^2)^(9/2)*c/d^5 + 2970
*(c + d/x^2)^(7/2)*c^2/d^5 - 2772*(c + d/x^2)^(5/2)*c^3/d^5 + 1155*(c + d/x
^2)^(3/2)*c^4/d^5) - 1/315*a*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7
/2)*c/d^4 + 189*(c + d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4)
```

Fricas [A]
time = 2.96, size = 133, normalized size = 0.99

$$-\frac{\left(16\,\left(8\,bc^5-11\,ac^4d\right)x^{10}-8\,\left(8\,bc^4d-11\,ac^3d^2\right)x^8+6\,\left(8\,bc^3d^2-11\,ac^2d^3\right)x^6+315\,bd^5-5\,\left(8\,bc^2d^3-11\,acd^4\right)x^4+35\,\left(bcd^4+11\,ad^5\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{3465\,d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/3465*(16*(8*b*c^5 - 11*a*c^4*d)*x^10 - 8*(8*b*c^4*d - 11*a*c^3*d^2)*x^8
+ 6*(8*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 + 315*b*d^5 - 5*(8*b*c^2*d^3 - 11*a*c*
d^4)*x^4 + 35*(b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^10)
```

Sympy [A]
time = 5.77, size = 146, normalized size = 1.09

$$-\frac{a\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}-\frac{b\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**9,x)
```

```
[Out] -a*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d
/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - b*(c**4*(c + d/x**2)**(3/2)
/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c +
 d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(114) = 228.
time = 5.04, size = 430, normalized size = 3.21

$$\frac{32\left(3465\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{9}{2}}\mathrm{sgn}(x)+11088\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{11}{2}}\mathrm{sgn}(x)-4851\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{9}{2}}d\,\mathrm{sgn}(x)+7392\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{11}{2}}d\,\mathrm{sgn}(x)+231\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{9}{2}}d^2\mathrm{sgn}(x)+2640\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{11}{2}}d^2\mathrm{sgn}(x)-165\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{9}{2}}d^3\mathrm{sgn}(x)-1320\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{11}{2}}d^3\mathrm{sgn}(x)+1815\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{9}{2}}d^4\mathrm{sgn}(x)+440\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{11}{2}}d^4\mathrm{sgn}(x)-605\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{9}{2}}d^5\mathrm{sgn}(x)-88\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{11}{2}}d^5\mathrm{sgn}(x)+121\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{9}{2}}d^6\mathrm{sgn}(x)+8\,bc^{\frac{11}{2}}d^6\mathrm{sgn}(x)-11\,ac^{\frac{9}{2}}d^7\mathrm{sgn}(x)\right)}{3465\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(9/2)*sgn(x) + 11088*(sq
rt(c)*x - sqrt(c*x^2 + d))^12*b*c^(11/2)*sgn(x) - 4851*(sqrt(c)*x - sqrt(c*
x^2 + d))^12*a*c^(9/2)*d*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c
^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^2*sgn(x
) + 2640*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(11/2)*d^2*sgn(x) - 165*(sqrt(
c)*x - sqrt(c*x^2 + d))^8*a*c^(9/2)*d^3*sgn(x) - 1320*(sqrt(c)*x - sqrt(c*x
^2 + d))^6*b*c^(11/2)*d^3*sgn(x) + 1815*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c
^(9/2)*d^4*sgn(x) + 440*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^4*sgn(
x) - 605*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(9/2)*d^5*sgn(x) - 88*(sqrt(c)
*x - sqrt(c*x^2 + d))^2*b*c^(11/2)*d^5*sgn(x) + 121*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*a*c^(9/2)*d^6*sgn(x) + 8*b*c^(11/2)*d^6*sgn(x) - 11*a*c^(9/2)*d^7*
sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^11
```

Mupad [B]
time = 5.61, size = 210, normalized size = 1.57

$$\frac{16\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{128\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{3465\,d^5}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{99\,d\,x^8}+\frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^4}-\frac{8\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3\,x^2}+\frac{8\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{693\,d^2\,x^6}-\frac{16\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^4}+\frac{64\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{3465\,d^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^9,x)
```

```
[Out] (16*a*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(11*x^10) -
(a*(c + d/x^2)^(1/2))/(9*x^8) - (128*b*c^5*(c + d/x^2)^(1/2))/(3465*d^5) -
(a*c*(c + d/x^2)^(1/2))/(63*d*x^6) - (b*c*(c + d/x^2)^(1/2))/(99*d*x^8) + (
2*a*c^2*(c + d/x^2)^(1/2))/(105*d^2*x^4) - (8*a*c^3*(c + d/x^2)^(1/2))/(315
*d^3*x^2) + (8*b*c^2*(c + d/x^2)^(1/2))/(693*d^2*x^6) - (16*b*c^3*(c + d/x^
2)^(1/2))/(1155*d^3*x^4) + (64*b*c^4*(c + d/x^2)^(1/2))/(3465*d^4*x^2)
```

3.937 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^{10}\, dx$

Optimal. Leaf size=150

$$-\frac{16d^3(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3465c^5}+\frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4}-\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3}+\frac{(11bc-8}{}$$

```
[Out] -16/3465*d^3*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^3/c^5+8/1155*d^2*(-8*a*d+11*
b*c)*(c+d/x^2)^(3/2)*x^5/c^4-2/231*d*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^7/c^
3+1/99*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^9/c^2+1/11*a*(c+d/x^2)^(3/2)*x^11/
c
```

Rubi [A]
time = 0.05, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 270}

$$-\frac{16d^3x^3\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{3465c^5}+\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{1155c^4}-\frac{2dx^7\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{231c^3}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{99c^2}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

```
[Out] (-16*d^3*(11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^3)/(3465*c^5) + (8*d^2*(11*b*
c - 8*a*d)*(c + d/x^2)^(3/2)*x^5)/(1155*c^4) - (2*d*(11*b*c - 8*a*d)*(c + d
/x^2)^(3/2)*x^7)/(231*c^3) + ((11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^9)/(99*c
^2) + (a*(c + d/x^2)^(3/2)*x^11)/(11*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^{10}\,dx=\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c}+\frac{(11bc-8ad)\int\sqrt{c+\frac{d}{x^2}}\,x^8\,dx}{11c}$$

$$=\frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c}-\frac{(2d(11bc-8ad))\int\sqrt{c+}}{33c^2}$$

$$=-\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3}+\frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right.}{11c}$$

$$=\frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4}-\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3}+\frac{(11bc-}{}$$

$$=-\frac{16d^3(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3465c^5}+\frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4}-\frac{2d(}{}$$

Mathematica [A]
time = 0.06, size = 108, normalized size = 0.72

$$\frac{\sqrt{c+\frac{d}{x^2}}\,x(d+cx^2)\left(11bc(-16d^3+24cd^2x^2-30c^2dx^4+35c^3x^6)+a(128d^4-192cd^3x^2+240c^2d^2x^4-280c^3dx^6+315c^4x^8)\right)}{3465c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(11*b*c*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*x
^4 + 35*c^3*x^6) + a*(128*d^4 - 192*c*d^3*x^2 + 240*c^2*d^2*x^4 - 280*c^3*d
*x^6 + 315*c^4*x^8)))/(3465*c^5)
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.75

method	result
gosper	$\frac{\sqrt{\frac{cx^2+d}{x^2}}\,x(315ax^8c^4-280ac^3dx^6+385bc^4x^6+240ac^2d^2x^4-330bc^3dx^4-192acd^3x^2+264bc^2d^2x^2+128ad^4-176bcd^3)(cx^2+d}{3465c^5}$

default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x(315a\,x^8c^4-280a\,c^3d\,x^6+385b\,c^4x^6+240a\,c^2d^2x^4-330b\,c^3d\,x^4-192ac\,d^3x^2+264b\,c^2d^2x^2+128a\,d^4-176bc\,d^3)\,(c\,x^2+d)}{3465c^5}$
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x(315a\,c^5x^{10}+35a\,c^4d\,x^8+385b\,c^5x^8-40a\,c^3d^2x^6+55b\,c^4d\,x^6+48a\,c^2d^3x^4-66b\,c^3d^2x^4-64ac\,d^4x^2+88b\,c^2d^3x^2+128a\,d^5-}{3465c^5}$
trager	$\frac{(315a\,c^5x^{10}+35a\,c^4d\,x^8+385b\,c^5x^8-40a\,c^3d^2x^6+55b\,c^4d\,x^6+48a\,c^2d^3x^4-66b\,c^3d^2x^4-64ac\,d^4x^2+88b\,c^2d^3x^2+128a\,d^5-176bc\,d^4)\,x\,\sqrt{}}{3465c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^10*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(1/2)*x*(315*a*c^4*x^8-280*a*c^3*d*x^6+385*b*c^4*x^6
+240*a*c^2*d^2*x^4-330*b*c^3*d*x^4-192*a*c*d^3*x^2+264*b*c^2*d^2*x^2+128*a*
d^4-176*b*c*d^3)*(c*x^2+d)/c^5
```

Maxima [A]
time = 0.40, size = 158, normalized size = 1.05

$$\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5-105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)b}{315\,c^4}+\frac{\left(315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-1540\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+2970\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-2772\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5+1155\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^4x^3\right)a}{3465\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)^(7/2)*d*x^7 + 189*(c + d/
x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^3)*b/c^4 + 1/3465*(315*(c
+ d/x^2)^(11/2)*x^11 - 1540*(c + d/x^2)^(9/2)*d*x^9 + 2970*(c + d/x^2)^(7/2
)*d^2*x^7 - 2772*(c + d/x^2)^(5/2)*d^3*x^5 + 1155*(c + d/x^2)^(3/2)*d^4*x^3
)*a/c^5
```

Fricas [A]
time = 3.40, size = 131, normalized size = 0.87

$$\frac{(315\,ac^5x^{11}+35\,(11\,bc^5+ac^4d)x^9+5\,(11\,bc^4d-8\,ac^3d^2)x^7-6\,(11\,bc^3d^2-8\,ac^2d^3)x^5+8\,(11\,bc^2d^3-8\,acd^4)x^3-16\,(11\,bcd^4-8\,ad^5)x)\sqrt{\frac{cx^2+d}{x^2}}}{3465\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + a*c^4*d)*x^9 + 5*(11*b*c^4*d - 8*a*
c^3*d^2)*x^7 - 6*(11*b*c^3*d^2 - 8*a*c^2*d^3)*x^5 + 8*(11*b*c^2*d^3 - 8*a*c
*d^4)*x^3 - 16*(11*b*c*d^4 - 8*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(146) = 292$.
time = 7.30, size = 1386, normalized size = 9.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**10*(c+d/x**2)**(1/2),x)
```

```
[Out] 315*a*c**9*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860
*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**
5*d**20) + 1295*a*c**8*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*
x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**
2 + 3465*c**5*d**20) + 1990*a*c**7*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/(3465
*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c*
*6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**6*d**(39/2)*x**12*sqrt(c*x**2/
d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**
4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 343*a*c**5*d**(41/2)*x**10*s
qrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**
7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**4*d**(43/
2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 280*a*c*
*3*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**
17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20)
+ 560*a*c**2*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 448*a*c*d**(49/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 128*a*d**(51/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 35*b*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d
**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 11
0*b*c**6*d**(21/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*
d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 114*b*c**5*d**(23/2)*x
**10*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**
5*d**11*x**2 + 315*c**4*d**12) + 40*b*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12) - 5*b*c**3*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 +
 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*b*c**2*d*
*(29/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 +
 945*c**5*d**11*x**2 + 315*c**4*d**12) - 40*b*c*d**(31/2)*x**2*sqrt(c*x**2/
d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 31
5*c**4*d**12) - 16*b*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945
*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
```

Giac [A]

time = 1.09, size = 175, normalized size = 1.17

$$\frac{16\left(11\,bcd^{\frac{9}{2}}-8\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^5}+\frac{315\,(cx^2+d)^{\frac{11}{2}}a\mathrm{sgn}(x)+385\,(cx^2+d)^{\frac{9}{2}}bc\mathrm{sgn}(x)-1540\,(cx^2+d)^{\frac{9}{2}}ad\mathrm{sgn}(x)-1485\,(cx^2+d)^{\frac{7}{2}}bcd\mathrm{sgn}(x)+2970\,(cx^2+d)^{\frac{7}{2}}ad^2\mathrm{sgn}(x)+2079\,(cx^2+d)^{\frac{5}{2}}bcd^2\mathrm{sgn}(x)-2772\,(cx^2+d)^{\frac{5}{2}}ad^3\mathrm{sgn}(x)-1155\,(cx^2+d)^{\frac{3}{2}}bcd^3\mathrm{sgn}(x)+1155\,(cx^2+d)^{\frac{3}{2}}ad^4\mathrm{sgn}(x)}{3465\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 16/3465*(11*b*c*d^(9/2) - 8*a*d^(11/2))*sgn(x)/c^5 + 1/3465*(315*(c*x^2 + d
)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1540*(c*x^2 + d)^(9/
2)*a*d*sgn(x) - 1485*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 2970*(c*x^2 + d)^(7/2
)*a*d^2*sgn(x) + 2079*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 2772*(c*x^2 + d)^(
5/2)*a*d^3*sgn(x) - 1155*(c*x^2 + d)^(3/2)*b*c*d^3*sgn(x) + 1155*(c*x^2 + d
)^(3/2)*a*d^4*sgn(x))/c^5
```

Mupad [B]
time = 4.59, size = 117, normalized size = 0.78

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{a\,x^{11}}{11}+\frac{x\,\left(128\,a\,d^5-176\,b\,c\,d^4\right)}{3465\,c^5}+\frac{x^9\,\left(385\,b\,c^5+35\,a\,d\,c^4\right)}{3465\,c^5}-\frac{d\,x^7\,\left(8\,a\,d-11\,b\,c\right)}{693\,c^2}+\frac{2\,d^2\,x^5\,\left(8\,a\,d-11\,b\,c\right)}{1155\,c^3}-\frac{8\,d^3\,x^3\,\left(8\,a\,d-11\,b\,c\right)}{3465\,c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^11)/11 + (x*(128*a*d^5 - 176*b*c*d^4))/(3465*c^5) +
 (x^9*(385*b*c^5 + 35*a*c^4*d))/(3465*c^5) - (d*x^7*(8*a*d - 11*b*c))/(693*
c^2) + (2*d^2*x^5*(8*a*d - 11*b*c))/(1155*c^3) - (8*d^3*x^3*(8*a*d - 11*b*c
))/(3465*c^4))
```

3.938 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^8 \, dx$

Optimal. Leaf size=117

$$\frac{8d^2(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{315c^4}-\frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{105c^3}+\frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{21c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}$$

```
[Out] 8/315*d^2*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x^3/c^4-4/105*d*(-2*a*d+3*b*c)*(c+
d/x^2)^(3/2)*x^5/c^3+1/21*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x^7/c^2+1/9*a*(c+d
/x^2)^(3/2)*x^9/c
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 270}

$$\frac{8d^2x^3\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{315c^4}-\frac{4dx^5\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{105c^3}+\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{21c^2}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

```
[Out] (8*d^2*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(315*c^4) - (4*d*(3*b*c - 2*a
*d)*(c + d/x^2)^(3/2)*x^5)/(105*c^3) + ((3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x
^7)/(21*c^2) + (a*(c + d/x^2)^(3/2)*x^9)/(9*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^8\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}+\frac{(9bc-6ad)\int \sqrt{c+\frac{d}{x^2}}\,x^6\,dx}{9c}$$

$$= \frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{21c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}-\frac{(4d(3bc-2ad))\int \sqrt{c+\frac{d}{x^2}}\,x$$

$$= -\frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{105c^3}+\frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{21c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x$$

$$= \frac{8d^2(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{315c^4}-\frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{105c^3}+\frac{(3bc-2ad)}{2$$

Mathematica [A]
time = 0.05, size = 88, normalized size = 0.75

$$\frac{\sqrt{c+\frac{d}{x^2}}\,x(d+cx^2)\left(24bcd^2-16ad^3-36bc^2dx^2+24acd^2x^2+45bc^3x^4-30ac^2dx^4+35ac^3x^6\right)}{315c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(24*b*c*d^2 - 16*a*d^3 - 36*b*c^2*d*x^2 + 24
*a*c*d^2*x^2 + 45*b*c^3*x^4 - 30*a*c^2*d*x^4 + 35*a*c^3*x^6))/(315*c^4)
```

Maple [A]
time = 0.06, size = 89, normalized size = 0.76

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(35x^6a\,c^3-30x^4a\,c^2d+45x^4b\,c^3+24ac\,d^2x^2-36b\,c^2d\,x^2-16a\,d^3+24bc\,d^2\right)\left(c\,x^2+d\right)}{315c^4}$	89
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(35x^6a\,c^3-30x^4a\,c^2d+45x^4b\,c^3+24ac\,d^2x^2-36b\,c^2d\,x^2-16a\,d^3+24bc\,d^2\right)\left(c\,x^2+d\right)}{315c^4}$	89
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(35a\,x^8c^4+5a\,c^3d\,x^6+45b\,c^4x^6-6a\,c^2d^2x^4+9b\,c^3d\,x^4+8ac\,d^3x^2-12b\,c^2d^2x^2-16a\,d^4+24bc\,d^3\right)}{315c^4}$	106

trager	$\frac{(35a\,x^8c^4+5a\,c^3d\,x^6+45b\,c^4x^6-6a\,c^2d^2x^4+9b\,c^3d\,x^4+8ac\,d^3x^2-12b\,c^2d^2x^2-16a\,d^4+24bc\,d^3)\,x\,\sqrt{-\frac{-c\,x^2-d}{x^2}}}{315c^4}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^8*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(1/2)*x*(35*a*c^3*x^6-30*a*c^2*d*x^4+45*b*c^3*x^4+24*
a*c*d^2*x^2-36*b*c^2*d*x^2-16*a*d^3+24*b*c*d^2)*(c*x^2+d)/c^4
```

Maxima [A]
time = 0.36, size = 124, normalized size = 1.06

$$\frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)b}{105\,c^3}+\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5-105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)a}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^
2)^(3/2)*d^2*x^3)*b/c^3 + 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)
^(7/2)*d*x^7 + 189*(c + d/x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^
3)*a/c^4
```

Fricas [A]
time = 3.84, size = 107, normalized size = 0.91

$$\frac{(35\,ac^4x^9+5\,(9\,bc^4+ac^3d)x^7+3\,(3\,bc^3d-2\,ac^2d^2)x^5-4\,(3\,bc^2d^2-2\,acd^3)x^3+8\,(3\,bcd^3-2\,ad^4)x)\sqrt{\frac{cx^2+d}{x^2}}}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + a*c^3*d)*x^7 + 3*(3*b*c^3*d - 2*a*c^2*d^
2)*x^5 - 4*(3*b*c^2*d^2 - 2*a*c*d^3)*x^3 + 8*(3*b*c*d^3 - 2*a*d^4)*x)*sqrt(
(c*x^2 + d)/x^2)/c^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(112) = 224$.
time = 4.38, size = 910, normalized size = 7.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**8*(c+d/x**2)**(1/2),x)
```

```
[Out] 35*a*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6
*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**6*d**(21/2)*
x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c*
*5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**5*d**(23/2)*x**10*sqrt(c*x**2/d
+ 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*
c**4*d**12) + 40*a*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x*
*6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 5*a*c**3
*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**
4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*a*c**2*d**(29/2)*x**4*sqrt(c
*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**
2 + 315*c**4*d**12) - 40*a*c*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(315*c**7*d*
*9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 16*
a*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 +
945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(9/2)*x**10*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 33*b*c
**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x
**2 + 105*c**3*d**6) + 17*b*c**3*d**(13/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**
5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**2*d**(15/2)*x**4
*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**
6) + 12*b*c*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**
4*d**5*x**2 + 105*c**3*d**6) + 8*b*d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)
```

Giac [A]
time = 1.45, size = 140, normalized size = 1.20

$$-\frac{8\left(3\,bcd^{\frac{7}{2}}-2\,ad^{\frac{9}{2}}\right)\mathrm{sgn}(x)}{315\,c^4}+\frac{35\,(cx^2+d)^{\frac{9}{2}}a\mathrm{sgn}(x)+45\,(cx^2+d)^{\frac{7}{2}}bc\mathrm{sgn}(x)-135\,(cx^2+d)^{\frac{7}{2}}ad\mathrm{sgn}(x)-126\,(cx^2+d)^{\frac{5}{2}}bcd\mathrm{sgn}(x)+189\,(cx^2+d)^{\frac{5}{2}}ad^2\mathrm{sgn}(x)+105\,(cx^2+d)^{\frac{3}{2}}bcd^2\mathrm{sgn}(x)-105\,(cx^2+d)^{\frac{3}{2}}ad^3\mathrm{sgn}(x)}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -8/315*(3*b*c*d^(7/2) - 2*a*d^(9/2))*sgn(x)/c^4 + 1/315*(35*(c*x^2 + d)^(9/
2)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 135*(c*x^2 + d)^(7/2)*a*d*s
gn(x) - 126*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 189*(c*x^2 + d)^(5/2)*a*d^2*sg
n(x) + 105*(c*x^2 + d)^(3/2)*b*c*d^2*sgn(x) - 105*(c*x^2 + d)^(3/2)*a*d^3*s
gn(x))/c^4
```

Mupad [B]
time = 4.52, size = 97, normalized size = 0.83

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{a\,x^9}{9}-\frac{x\,\left(16\,a\,d^4-24\,b\,c\,d^3\right)}{315\,c^4}+\frac{x^7\,\left(45\,b\,c^4+5\,a\,d\,c^3\right)}{315\,c^4}-\frac{d\,x^5\,\left(2\,a\,d-3\,b\,c\right)}{105\,c^2}+\frac{4\,d^2\,x^3\,\left(2\,a\,d-3\,b\,c\right)}{315\,c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^9)/9 - (x*(16*a*d^4 - 24*b*c*d^3))/(315*c^4) + (x^7
*(45*b*c^4 + 5*a*c^3*d))/(315*c^4) - (d*x^5*(2*a*d - 3*b*c))/(105*c^2) + (4
*d^2*x^3*(2*a*d - 3*b*c))/(315*c^3))
```

3.939 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^6 \, dx$

Optimal. Leaf size=84

$$-\frac{2d(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{105c^3}+\frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}$$

```
[Out] -2/105*d*(-4*a*d+7*b*c)*(c+d/x^2)^(3/2)*x^3/c^3+1/35*(-4*a*d+7*b*c)*(c+d/x^
2)^(3/2)*x^5/c^2+1/7*a*(c+d/x^2)^(3/2)*x^7/c
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 270}

$$-\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{105c^3}+\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{35c^2}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

```
[Out] (-2*d*(7*b*c - 4*a*d)*(c + d/x^2)^(3/2)*x^3)/(105*c^3) + ((7*b*c - 4*a*d)*(
c + d/x^2)^(3/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(3/2)*x^7)/(7*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^6\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}+\frac{(7bc-4ad)\int\sqrt{c+\frac{d}{x^2}}\,x^4\,dx}{7c}$$

$$= \frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}-\frac{(2d(7bc-4ad))\int\sqrt{c+\frac{d}{x^2}}}{35c^2}$$

$$= -\frac{2d(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{105c^3}+\frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.76

$$\frac{\sqrt{c+\frac{d}{x^2}}\,x(d+cx^2)\left(-14bcd+8ad^2+21bc^2x^2-12acdx^2+15ac^2x^4\right)}{105c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(-14*b*c*d + 8*a*d^2 + 21*b*c^2*x^2 - 12*a*c
*d*x^2 + 15*a*c^2*x^4))/(105*c^3)
```

Maple [A]
time = 0.05, size = 65, normalized size = 0.77

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(15a\,c^2x^4-12acd\,x^2+21b\,c^2x^2+8a\,d^2-14bcd\right)\left(c\,x^2+d\right)}{105c^3}$	65
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(15a\,c^2x^4-12acd\,x^2+21b\,c^2x^2+8a\,d^2-14bcd\right)\left(c\,x^2+d\right)}{105c^3}$	65
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(15x^6a\,c^3+3x^4a\,c^2d+21x^4b\,c^3-4ac\,d^2x^2+7b\,c^2d\,x^2+8a\,d^3-14bc\,d^2\right)}{105c^3}$	82
trager	$\frac{\left(15x^6a\,c^3+3x^4a\,c^2d+21x^4b\,c^3-4ac\,d^2x^2+7b\,c^2d\,x^2+8a\,d^3-14bc\,d^2\right)x\sqrt{-\frac{-c\,x^2-d}{x^2}}}{105c^3}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^6*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*((c*x^2+d)/x^2)^(1/2)*x*(15*a*c^2*x^4-12*a*c*d*x^2+21*b*c^2*x^2+8*a*d
^2-14*b*c*d)*(c*x^2+d)/c^3
```

Maxima [A]
time = 0.37, size = 90, normalized size = 1.07

$$\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)b}{15\,c^2}+\frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)a}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3)*b/c^2 + 1/105*(1
5*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^2)^(3/2)
*d^2*x^3)*a/c^3
```

Fricas [A]
time = 2.93, size = 82, normalized size = 0.98

$$\frac{\left(15\,ac^3x^7+3\,(7\,bc^3+ac^2d)x^5+(7\,bc^2d-4\,acd^2)x^3-2\,(7\,bcd^2-4\,ad^3)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*a*c^3*x^7 + 3*(7*b*c^3 + a*c^2*d)*x^5 + (7*b*c^2*d - 4*a*c*d^2)*x
^3 - 2*(7*b*c*d^2 - 4*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(78) = 156.
time = 4.05, size = 422, normalized size = 5.02

$$\frac{15ac^5d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{33ac^4d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{17ac^3d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{3ac^2d^{\frac{15}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{12acd^{\frac{17}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{8ad^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{b\sqrt{d}\,x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{bd^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c}-\frac{2bd^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**6*(c+d/x**2)**(1/2),x)
```

```
[Out] 15*a*c**5*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*
d**5*x**2 + 105*c**3*d**6) + 33*a*c**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**3*d**(13/
2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) + 3*a*c**2*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
+ 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c*d**(17/2)*x**2*sqrt(c*x**2/d
 + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(1
9/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
```

```
*d**6) + b*sqrt(d)*x***4*sqrt(c*x**2/d + 1)/5 + b*d**(3/2)*x***2*sqrt(c*x**2/
d + 1)/(15*c) - 2*b*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

Giac [A]
time = 1.29, size = 105, normalized size = 1.25

$$\frac{2\left(7\,bcd^{\frac{5}{2}}-4\,ad^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{105\,c^3}+\frac{15\,(cx^2+d)^{\frac{7}{2}}a\mathrm{sgn}(x)+21\,(cx^2+d)^{\frac{5}{2}}bc\mathrm{sgn}(x)-42\,(cx^2+d)^{\frac{5}{2}}ad\mathrm{sgn}(x)-35\,(cx^2+d)^{\frac{3}{2}}bcd\mathrm{sgn}(x)+35\,(cx^2+d)^{\frac{3}{2}}ad^2\mathrm{sgn}(x)}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*b*c*d^(5/2) - 4*a*d^(7/2))*sgn(x)/c^3 + 1/105*(15*(c*x^2 + d)^(7/2
)*a*sgn(x) + 21*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 42*(c*x^2 + d)^(5/2)*a*d*sgn
(x) - 35*(c*x^2 + d)^(3/2)*b*c*d*sgn(x) + 35*(c*x^2 + d)^(3/2)*a*d^2*sgn(x)
)/c^3
```

Mupad [B]
time = 4.49, size = 77, normalized size = 0.92

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{a\,x^7}{7}+\frac{x\,\left(8\,a\,d^3-14\,b\,c\,d^2\right)}{105\,c^3}+\frac{x^5\,\left(21\,b\,c^3+3\,a\,d\,c^2\right)}{105\,c^3}-\frac{d\,x^3\,\left(4\,a\,d-7\,b\,c\right)}{105\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^7)/7 + (x*(8*a*d^3 - 14*b*c*d^2))/(105*c^3) + (x^5*
(21*b*c^3 + 3*a*c^2*d))/(105*c^3) - (d*x^3*(4*a*d - 7*b*c))/(105*c^2))
```

3.940 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^4 \, dx$

Optimal. Leaf size=53

$$\frac{(5bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{15c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{5c}$$

```
[Out] 1/15*(-2*a*d+5*b*c)*(c+d/x^2)^(3/2)*x^3/c^2+1/5*a*(c+d/x^2)^(3/2)*x^5/c
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {464, 270}

$$\frac{x^3\left(c+\frac{d}{x^2}\right)^{3/2}(5bc-2ad)}{15c^2}+\frac{ax^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

```
[Out] ((5*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(15*c^2) + (a*(c + d/x^2)^(3/2)*x^5
)/(5*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\, x^4\, dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{5c}+\frac{(5bc-2ad)\int \sqrt{c+\frac{d}{x^2}}\, x^2\, dx}{5c}\\ &= \frac{(5bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{15c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{5c}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.79

$$\frac{\sqrt{c+\frac{d}{x^2}}\,x(d+cx^2)\left(5bc-2ad+3acx^2\right)}{15c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(5*b*c - 2*a*d + 3*a*c*x^2))/(15*c^2)
```

Maple [A]
time = 0.05, size = 43, normalized size = 0.81

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(3c\,x^2a-2ad+5bc\right)\left(c\,x^2+d\right)}{15c^2}$	43
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(3c\,x^2a-2ad+5bc\right)\left(c\,x^2+d\right)}{15c^2}$	43
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(3a\,c^2x^4+acd\,x^2+5b\,c^2x^2-2a\,d^2+5bcd\right)}{15c^2}$	57
trager	$\frac{\left(3a\,c^2x^4+acd\,x^2+5b\,c^2x^2-2a\,d^2+5bcd\right)x\sqrt{-\frac{-c\,x^2-d}{x^2}}}{15c^2}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^4*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/15*((c*x^2+d)/x^2)^(1/2)*x*(3*a*c*x^2-2*a*d+5*b*c)*(c*x^2+d)/c^2
```

Maxima [A]
time = 0.47, size = 55, normalized size = 1.04

$$\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3\,c}+\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)a}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*b*(c + d/x^2)^(3/2)*x^3/c + 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^
2)^(3/2)*d*x^3)*a/c^2
```

Fricas [A]
time = 2.08, size = 57, normalized size = 1.08

$$\frac{\left(3\,ac^{2}x^{5}+\left(5\,bc^{2}+acd\right)x^{3}+\left(5\,bcd-2\,ad^{2}\right)x\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{15\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 + a*c*d)*x^3 + (5*b*c*d - 2*a*d^2)*x)*sqrt((c*
x^2 + d)/x^2)/c^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(46) = 92.
time = 3.80, size = 119, normalized size = 2.25

$$\frac{a\sqrt{d}\,x^{4}\sqrt{\frac{cx^{2}}{d}+1}}{5}+\frac{ad^{\frac{3}{2}}x^{2}\sqrt{\frac{cx^{2}}{d}+1}}{15c}-\frac{2ad^{\frac{5}{2}}\sqrt{\frac{cx^{2}}{d}+1}}{15c^{2}}+\frac{b\sqrt{d}\,x^{2}\sqrt{\frac{cx^{2}}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^{2}}{d}+1}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/(1
5*c) - 2*a*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*sqrt(d)*x**2*sqrt(c*x*
*2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c)
```

Giac [A]
time = 1.64, size = 72, normalized size = 1.36

$$-\frac{\left(5\,bcd^{\frac{3}{2}}-2\,ad^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c^{2}}+\frac{3\,(cx^{2}+d)^{\frac{5}{2}}a\mathrm{sgn}(x)+5\,(cx^{2}+d)^{\frac{3}{2}}bc\mathrm{sgn}(x)-5\,(cx^{2}+d)^{\frac{3}{2}}ad\mathrm{sgn}(x)}{15\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(5*b*c*d^(3/2) - 2*a*d^(5/2))*sgn(x)/c^2 + 1/15*(3*(c*x^2 + d)^(5/2)*
a*sgn(x) + 5*(c*x^2 + d)^(3/2)*b*c*sgn(x) - 5*(c*x^2 + d)^(3/2)*a*d*sgn(x))
/c^2
```

Mupad [B]
time = 4.44, size = 54, normalized size = 1.02

$$\sqrt{c+\frac{d}{x^{2}}}\,\left(\frac{a\,x^{5}}{5}-\frac{x\,\left(2\,a\,d^{2}-5\,b\,c\,d\right)}{15\,c^{2}}+\frac{x^{3}\,\left(5\,b\,c^{2}+a\,d\,c\right)}{15\,c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^5)/5 - (x*(2*a*d^2 - 5*b*c*d))/(15*c^2) + (x^3*(5*b
*c^2 + a*c*d))/(15*c^2))
```

3.941 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^2 \, dx$

Optimal. Leaf size=66

$$b\sqrt{c+\frac{d}{x^2}}\, x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{3c}-b\sqrt{d}\, \tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\, x}\right)$$

```
[Out] 1/3*a*(c+d/x^2)^(3/2)*x^3/c-b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))*d^(1/2)+b*
x*(c+d/x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {462, 248, 283, 223, 212}

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}+bx\sqrt{c+\frac{d}{x^2}}-b\sqrt{d}\, \tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

```
[Out] b*Sqrt[c + d/x^2]*x + (a*(c + d/x^2)^(3/2)*x^3)/(3*c) - b*Sqrt[d]*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^2\,dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}+b\int \sqrt{c+\frac{d}{x^2}}\,dx \\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2}\,dx,x,\frac{1}{x}\right) \\
&= b\sqrt{c+\frac{d}{x^2}}\,x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-(bd)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right) \\
&= b\sqrt{c+\frac{d}{x^2}}\,x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-(bd)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right) \\
&= b\sqrt{c+\frac{d}{x^2}}\,x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 1.03

$$
\frac{1}{3}\sqrt{c+\frac{d}{x^2}}\,x\left(3b+a\left(\frac{d}{c}+x^2\right)-\frac{3b\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(3*b + a*(d/c + x^2) - (3*b*Sqrt[d]*ArcTanh[Sqrt[d + c*x
^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/3
```

Maple [A]
time = 0.05, size = 83, normalized size = 1.26

method	result	size
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(3 \sqrt{d}\, \ln\left(\frac{2 d+2 \sqrt{d}\, \sqrt{c\,x^{2}+d}}{x}\right) b c-a \left(c\,x^{2}+d\right)^{\frac{3}{2}}-3 \sqrt{c\,x^{2}+d}\, b c\right)}{3 \sqrt{c\,x^{2}+d}\, c}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^2*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((c*x^2+d)/x^2)^(1/2)*x*(3*d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)
*b*c-a*(c*x^2+d)^(3/2)-3*(c*x^2+d)^(1/2)*b*c)/(c*x^2+d)^(1/2)/c
```

Maxima [A]
time = 0.65, size = 75, normalized size = 1.14

$$\frac{a\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}} x^{3}}{3\,c}+\frac{1}{2}\left(2\,\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}\,\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(c + d/x^2)^(3/2)*x^3/c + 1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sqr
t(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

Fricas [A]
time = 3.18, size = 156, normalized size = 2.36

$$\left[\frac{3\,bc\sqrt{d}\,\log\left(-\frac{cx^{2}-2\,\sqrt{d}\,x\sqrt{\frac{cx^{2}+d}{x^{2}}}+2\,d}{x^{2}}\right)+2\,\left(acx^{3}+\left(3\,bc+ad\right)x\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{6\,c}, \frac{3\,bc\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cx^{2}+d}\right)+\left(acx^{3}+\left(3\,bc+ad\right)x\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{3\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/
x^2) + 2*(a*c*x^3 + (3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c, 1/3*(3*b*c*s
```

```
qrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (a*c*x^3 + (
3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [A]
time = 3.77, size = 107, normalized size = 1.62

$$\frac{a\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}+\frac{b\sqrt{c}\,x}{\sqrt{1+\frac{d}{cx^2}}}-b\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)+\frac{bd}{\sqrt{c}\,x\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + a*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c) +
 b*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - b*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) +
b*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(54) = 108.
time = 2.55, size = 116, normalized size = 1.76

$$\frac{bd\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\operatorname{sgn}(x)}{\sqrt{-d}}-\frac{\left(3\,bcd\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+3\,bc\sqrt{-d}\,\sqrt{d}\,+a\sqrt{-d}\,d^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3\,c\sqrt{-d}}+\frac{(cx^2+d)^{\frac{3}{2}}ac^2\operatorname{sgn}(x)+3\,\sqrt{cx^2+d}\,bc^3\operatorname{sgn}(x)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b*d*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/3*(3*b*c*d*arctan(
sqrt(d)/sqrt(-d)) + 3*b*c*sqrt(-d)*sqrt(d) + a*sqrt(-d)*d^(3/2))*sgn(x)/(c*
sqrt(-d)) + 1/3*((c*x^2 + d)^(3/2)*a*c^2*sgn(x) + 3*sqrt(c*x^2 + d)*b*c^3*s
gn(x))/c^3
```

Mupad [B]
time = 4.72, size = 80, normalized size = 1.21

$$b\,x\,\sqrt{c+\frac{d}{x^2}}+\frac{a\,x\,\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)}{3\,c}+\frac{b\,\sqrt{d}\,\mathrm{asin}\left(\frac{\sqrt{d}\,1{}\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c+\frac{d}{x^2}}\,1{}\mathrm{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] b*x*(c + d/x^2)^(1/2) + (a*x*(c + d/x^2)^(1/2)*(d + c*x^2))/(3*c) + (b*d^(1
/2)*asin((d^(1/2)*1i)/(c^(1/2)*x))*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x^2
) + 1)^(1/2))
```

3.942 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, dx$

Optimal. Leaf size=85

$$-\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}-\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2\sqrt{d}}$$

```
[Out] a*(c+d/x^2)^(3/2)*x/c-1/2*(2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^
(1/2)-1/2*(2*a*d+b*c)*(c+d/x^2)^(1/2)/c/x
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {382, 464, 201, 223, 212}

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(2ad+bc)}{2cx}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

```
[Out] -1/2*((b*c + 2*a*d)*Sqrt[c + d/x^2])/(c*x) + (a*(c + d/x^2)^(3/2)*x)/c - ((
b*c + 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt[d])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx &= -\text{Subst}\left(\int \frac{(a+bx^2)\sqrt{c+dx^2}}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}+\frac{(-bc-2ad)\text{Subst}\left(\int \sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}+\frac{1}{2}(-bc-2ad)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\right.\\
&= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}+\frac{1}{2}(-bc-2ad)\text{Subst}\left(\int \frac{1}{1-dx^2}\,d\right.\\
&= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}-\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2\sqrt{d}}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 75, normalized size = 0.88

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(-b+2ax^2-\frac{(bc+2ad)x^2\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{d+cx^2}}\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*(-b + 2*a*x^2 - ((b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]
/Sqrt[d]])/(Sqrt[d]*Sqrt[d + c*x^2])))/(2*x)
```

Maple [A]
time = 0.07, size = 135, normalized size = 1.59

method	result
risch	$-\frac{b\sqrt{\frac{c\,x^2+d}{x^2}}}{2x}+\frac{\left(a\sqrt{c\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)bc}{2\sqrt{d}}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{\sqrt{c\,x^2+d}}$
default	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(2d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a\,x^2+\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)bc\,x^2-2\sqrt{c\,x^2+d}\,ad\,x^2-\sqrt{c\right.}{2x\sqrt{c\,x^2+d}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((c*x^2+d)/x^2)^(1/2)/x*(2*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)
*a*x^2+d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c*x^2-2*(c*x^2+d)^(1/2
)*a*d*x^2-(c*x^2+d)^(1/2)*b*c*x^2+(c*x^2+d)^(3/2)*b)/(c*x^2+d)^(1/2)/d
```

Maxima [A]
time = 0.54, size = 133, normalized size = 1.56

$$\frac{1}{2}\left(2\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)a-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,cx}{\left(c+\frac{d}{x^2}\right)x^2-d}-\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(
c + d/x^2)*x + sqrt(d))))*a - 1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2 -
 d) - c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sq
rt(d))*b
```

Fricas [A]
time = 3.05, size = 164, normalized size = 1.93

$$\left[\frac{(bc+2\,ad)\sqrt{d}\,x\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,adx^2-bd)\sqrt{\frac{cx^2+d}{x^2}}}{4\,dx},\ \frac{(bc+2\,ad)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(2\,adx^2-bd)\sqrt{\frac{cx^2+d}{x^2}}}{2\,dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^
2) + 2*d)/x^2) + 2*(2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x), 1/2*((b*
c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d))
+ (2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x)]
```

Sympy [A]
time = 5.71, size = 107, normalized size = 1.26

$$\frac{a\sqrt{c}\,x}{\sqrt{1+\frac{d}{cx^2}}}-a\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)+\frac{ad}{\sqrt{c}\,x\sqrt{1+\frac{d}{cx^2}}}-\frac{b\sqrt{c}\,\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) + a
*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x)
- b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*sqrt(d))
```

Giac [A]
time = 1.49, size = 76, normalized size = 0.89

$$\frac{2\,\sqrt{cx^2+d}\,ac\mathrm{sgn}(x)+\frac{(bc^2\mathrm{sgn}(x)+2\,acd\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{\sqrt{cx^2+d}\,bc\mathrm{sgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(c*x^2 + d)*a*c*sgn(x) + (b*c^2*sgn(x) + 2*a*c*d*sgn(x))*arctan(
sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - sqrt(c*x^2 + d)*b*c*sgn(x)/x^2)/c
```

Mupad [B]
time = 5.11, size = 97, normalized size = 1.14

$$a\,x\,\sqrt{c+\frac{d}{x^2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{2\,x}-\frac{b\,c\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{2\,\sqrt{d}}+\frac{a\,\sqrt{d}\,\mathrm{asin}\left(\frac{\sqrt{d}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] a*x*(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2))/(2*x) - (b*c*log((c + d/x^2)^
(1/2) + d^(1/2)/x))/(2*d^(1/2)) + (a*d^(1/2)*asin((d^(1/2)*1i)/(c^(1/2)*x))
*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x^2) + 1)^(1/2))
```

3.943 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$

Optimal. Leaf size=91

$$\frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{3/2}}$$

```
[Out] -1/4*b*(c+d/x^2)^(3/2)/d/x+1/8*c*(-4*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(
1/2))/d^(3/2)+1/8*(-4*a*d+b*c)*(c+d/x^2)^(1/2)/d/x
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 342, 201, 223, 212}

$$\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{3/2}}+\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-4ad)}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

```
[Out] ((b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d*x) - (b*(c + d/x^2)^(3/2))/(4*d*x) + (
c*(b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(-bc+4ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx}{4d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}-\frac{(-bc+4ad)\text{Subst}\left(\int \sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{4d}$$

$$= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(c(bc-4ad))\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\right.}{8d}$$

$$= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(c(bc-4ad))\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\right.}{8d}$$

$$= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{3/2}}$$

Mathematica [A]

time = 0.16, size = 91, normalized size = 1.00

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(-\sqrt{d}\,(2bd+bcx^2+4adx^2)+\frac{c(bc-4ad)x^4\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)}{8d^{3/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-(Sqrt[d]*(2*b*d + b*c*x^2 + 4*a*d*x^2)) + (c*(b*c - 4*a*
d)*x^4*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/(8*d^(3/2)*x^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(75) = 150.
time = 0.07, size = 175, normalized size = 1.92

method	result
risch	$-\frac{(4adx^2+cx^2b+2bd)\sqrt{\frac{cx^2+d}{x^2}}}{8x^3d}+\frac{\left(-\frac{c\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{cx^2+d}}{x}\right)a}{2\sqrt{d}}+\frac{c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{cx^2+d}}{x}\right)b}{8d^{\frac{3}{2}}}\right)\sqrt{\frac{cx^2+d}{x^2}}\,x}{\sqrt{cx^2+d}}$
default	$-\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(4d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{cx^2+d}}{x}\right)acx^4-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{cx^2+d}}{x}\right)bc^2x^4-4\sqrt{cx^2+d}\,acdx^4+\right.}{8x^3\sqrt{cx^2+d}\,d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*((c*x^2+d)/x^2)^(1/2)/x^3*(4*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/
x)*a*c*x^4-d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c^2*x^4-4*(c*x^2+d
)^(1/2)*a*c*d*x^4+(c*x^2+d)^(1/2)*b*c^2*x^4+4*(c*x^2+d)^(3/2)*a*d*x^2-(c*x^
2+d)^(3/2)*b*c*x^2+2*(c*x^2+d)^(3/2)*b*d)/(c*x^2+d)^(1/2)/d^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(75) = 150.
time = 0.54, size = 193, normalized size = 2.12

$$-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,cx}{\left(c+\frac{d}{x^2}\right)x^2-d}-\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}}\right)a-\frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2 - d) - c*log((sqrt(c + d/x^2)*
x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d))*a - 1/16*(c^2*log((sqr
t(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*((c +
d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2)^2*d*x^4 - 2*(c
 + d/x^2)*d^2*x^2 + d^3))*b
```

Fricas [A]
time = 2.24, size = 194, normalized size = 2.13

$$\left[-\frac{(bc^2-4\,acd)\sqrt{d}\,x^3\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,bd^2+(bcd+4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^2x^3}, -\frac{(bc^2-4\,acd)\sqrt{-d}\,x^3\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(2\,bd^2+(bcd+4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,d^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2
 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 +
d)/x^2))/(d^2*x^3), -1/8*((b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-d)*x*
sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2))/(d^2*x^3)]
```

Sympy [A]
time = 8.38, size = 144, normalized size = 1.58

$$-\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2\sqrt{d}}-\frac{bc^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}}-\frac{3b\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8d^{\frac{3}{2}}}-\frac{bd}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**2,x)
```

```
[Out] -a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x) - a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*s
qrt(d)) - b*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*b*sqrt(c)/(8*x**3*sqr
t(1 + d/(c*x**2))) + b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - b*d/(
4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

Giac [A]
time = 3.06, size = 130, normalized size = 1.43

$$-\frac{\frac{(bc^3\operatorname{sgn}(x)-4\,ac^2d\operatorname{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d}+\frac{(cx^2+d)^{\frac{3}{2}}bc^3\operatorname{sgn}(x)+4\,(cx^2+d)^{\frac{3}{2}}ac^2d\operatorname{sgn}(x)+\sqrt{cx^2+d}\,bc^3d\operatorname{sgn}(x)-4\,\sqrt{cx^2+d}\,ac^2d^2\operatorname{sgn}(x)}{c^2dx^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/8*((b*c^3*sgn(x) - 4*a*c^2*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(s
qrt(-d)*d) + ((c*x^2 + d)^(3/2)*b*c^3*sgn(x) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*
sgn(x) + sqrt(c*x^2 + d)*b*c^3*d*sgn(x) - 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x
))/(c^2*d*x^4))/c
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2, x)
```

3.944 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$

Optimal. Leaf size=123

$$\frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x}-\frac{c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{16d^{5/2}}$$

```
[Out] -1/6*b*(c+d/x^2)^(3/2)/d/x^3-1/16*c^2*(-2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x
^2)^(1/2))/d^(5/2)+1/8*(-2*a*d+b*c)*(c+d/x^2)^(1/2)/d/x^3+1/16*c*(-2*a*d+b*
c)*(c+d/x^2)^(1/2)/d^2/x
```

Rubi [A]
time = 0.05, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {470, 342, 285, 327, 223, 212}

$$-\frac{c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-2ad)}{16d^2x}+\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-2ad)}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

```
[Out] ((b*c - 2*a*d)*Sqrt[c + d/x^2])/(8*d*x^3) - (b*(c + d/x^2)^(3/2))/(6*d*x^3)
 + (c*(b*c - 2*a*d)*Sqrt[c + d/x^2])/(16*d^2*x) - (c^2*(b*c - 2*a*d)*ArcTan
h[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{(-3bc+6ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx}{6d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} - \frac{(-3bc+6ad)\mathrm{Subst}\left(\int x^2\sqrt{c+dx^2}\,dx, x, \frac{1}{x}\right)}{6d}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{(c(bc-2ad))\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx, x, \right.}{8d}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x} - \frac{(c^2(bc-2ad))\,\mathrm{S}}{}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x} - \frac{(c^2(bc-2ad))\,\mathrm{S}}{}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x} - \frac{c^2(bc-2ad)\tan}{1}$$

Mathematica [A]
time = 0.19, size = 124, normalized size = 1.01

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(-8bd^2-2bcdx^2-12ad^2x^2+3bc^2x^4-6acdx^4\right)}{48d^2x^5} - \frac{c^2(bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{16d^{5/2}\sqrt{d+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-8*b*d^2 - 2*b*c*d*x^2 - 12*a*d^2*x^2 + 3*b*c^2*x^4 - 6*a
*c*d*x^4))/(48*d^2*x^5) - (c^2*(b*c - 2*a*d)*Sqrt[c + d/x^2]*x*ArcTanh[Sqrt
[d + c*x^2]/Sqrt[d]])/(16*d^(5/2)*Sqrt[d + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(103) = 206.
time = 0.08, size = 220, normalized size = 1.79

method	result
risch	$-\frac{(6x^4acd-3x^4bc^2+12ad^2x^2+2bcdx^2+8bd^2)\sqrt{\frac{cx^2+d}{x^2}}}{48x^5d^2}+\frac{\left(\frac{c^2\ln\left(\frac{2d+2\sqrt{d}\sqrt{cx^2+d}}{x}\right)a}{8d^{\frac{3}{2}}}-\frac{c^3\ln\left(\frac{2d+2\sqrt{d}\sqrt{cx^2+\ldots}}{x}\right.}{16d^{\frac{5}{2}}}\right.}{\sqrt{cx^2+d}}$
default	$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(6d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\sqrt{cx^2+d}}{x}\right)ac^2x^6-3\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{cx^2+d}}{x}\right)bc^3x^6-6\sqrt{cx^2+d}\,ac^2dx^6\ldots\right.}{48x^5\sqrt{cx^2\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(1/2)/x^5*(6*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/
x)*a*c^2*x^6-3*d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c^3*x^6-6*(c*x
^2+d)^(1/2)*a*c^2*d*x^6+3*(c*x^2+d)^(1/2)*b*c^3*x^6+6*(c*x^2+d)^(3/2)*a*c*d
*x^4-3*(c*x^2+d)^(3/2)*b*c^2*x^4-12*(c*x^2+d)^(3/2)*a*d^2*x^2+6*(c*x^2+d)^(
3/2)*b*c*d*x^2-8*(c*x^2+d)^(3/2)*b*d^2)/(c*x^2+d)^(1/2)/d^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(103) = 206$.

time = 0.51, size = 277, normalized size = 2.25

$$-\frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)a+\frac{1}{96}\left(\frac{3c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5-8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}\,c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3d^2x^6-3\left(c+\frac{d}{x^2}\right)^2d^3x^4+3\left(c+\frac{d}{x^2}\right)d^4x^2-d^5}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/16*(c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))
/d^(3/2) + 2*((c + d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c + d/
x^2)^2*d*x^4 - 2*(c + d/x^2)*d^2*x^2 + d^3))*a + 1/96*(3*c^3*log((sqrt(c +
d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c + d/x^
2)^(5/2)*c^3*x^5 - 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*c^3*d^
2*x)/((c + d/x^2)^3*d^2*x^6 - 3*(c + d/x^2)^2*d^3*x^4 + 3*(c + d/x^2)*d^4*x
^2 - d^5))*b
```

Fricas [A]

time = 2.45, size = 244, normalized size = 1.98

$$\left[-\frac{3\,(bc^3-2\,ac^2d)\sqrt{d}\,x^5\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\,(3\,(bc^2d-2\,acd^2)x^4-8\,bd^3-2\,(bcd^2+6\,ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{96\,d^3x^5},\ \frac{3\,(bc^3-2\,ac^2d)\sqrt{-d}\,x^5\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(3\,(bc^2d-2\,acd^2)x^4-8\,bd^3-2\,(bcd^2+6\,ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{48\,d^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c
*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(
b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^5), 1/48*(3*(b*c^3 -
2*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)
) + (3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(b*c*d^2 + 6*a*d^3)*x^2)*sqr
t((c*x^2 + d)/x^2))/(d^3*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(107) = 214$.
time = 19.95, size = 226, normalized size = 1.84

$$- \frac{a c^{\frac{3}{2}}}{8 d x \sqrt{1 + \frac{d}{c x^{2}}}} - \frac{3 a \sqrt{c}}{8 x^{3} \sqrt{1 + \frac{d}{c x^{2}}}} + \frac{a c^{2} \operatorname{asinh}{\left(\frac{\sqrt{d}}{\sqrt{c} x} \right)}}{8 d^{\frac{3}{2}}} - \frac{a d}{4 \sqrt{c} x^{5} \sqrt{1 + \frac{d}{c x^{2}}}} + \frac{b c^{\frac{5}{2}}}{16 d^{2} x \sqrt{1 + \frac{d}{c x^{2}}}} + \frac{b c^{\frac{3}{2}}}{48 d x^{3} \sqrt{1 + \frac{d}{c x^{2}}}} - \frac{5 b \sqrt{c}}{24 x^{5} \sqrt{1 + \frac{d}{c x^{2}}}} - \frac{b c^{3} \operatorname{asinh}{\left(\frac{\sqrt{d}}{\sqrt{c} x} \right)}}{16 d^{\frac{5}{2}}} - \frac{b d}{6 \sqrt{c} x^{7} \sqrt{1 + \frac{d}{c x^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**4,x)
```

```
[Out] -a*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*a*sqrt(c)/(8*x**3*sqrt(1 + d/(
c*x**2))) + a*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - a*d/(4*sqrt(c)
*x**5*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(16*d**2*x*sqrt(1 + d/(c*x**2))) +
 b*c**(3/2)/(48*d*x**3*sqrt(1 + d/(c*x**2))) - 5*b*sqrt(c)/(24*x**5*sqrt(1
+ d/(c*x**2))) - b*c**3*asinh(sqrt(d)/(sqrt(c)*x))/(16*d**(5/2)) - b*d/(6*s
qrt(c)*x**7*sqrt(1 + d/(c*x**2)))
```

Giac [A]
time = 1.84, size = 153, normalized size = 1.24

$$\frac{\frac{3 \, {\left(b c^{4} \mathrm{sgn}\left(x\right) - 2 \, a c^{3} d \mathrm{sgn}\left(x\right)\right)} \arctan\left(\frac{\sqrt{c x^{2} + d}}{\sqrt{-d}}\right)}{\sqrt{-d} d^{2}} + \frac{3 \, {\left(c x^{2} + d\right)}^{\frac{5}{2}} b c^{4} \mathrm{sgn}\left(x\right) - 6 \, {\left(c x^{2} + d\right)}^{\frac{5}{2}} a c^{3} d \mathrm{sgn}\left(x\right) - 8 \, {\left(c x^{2} + d\right)}^{\frac{3}{2}} b c^{4} d \mathrm{sgn}\left(x\right) - 3 \, \sqrt{c x^{2} + d} b c^{4} d^{2} \mathrm{sgn}\left(x\right) + 6 \, \sqrt{c x^{2} + d} a c^{3} d^{3} \mathrm{sgn}\left(x\right)}{c^{3} d^{2} x^{6}}}{48 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(3*(b*c^4*sgn(x) - 2*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/
(sqrt(-d)*d^2) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) - 6*(c*x^2 + d)^(5/2)*a*
c^3*d*sgn(x) - 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4
*d^2*sgn(x) + 6*sqrt(c*x^2 + d)*a*c^3*d^3*sgn(x))/(c^3*d^2*x^6))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\,\sqrt{c+\frac{d}{x^2}}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4,x)
[Out] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4, x)
```

3.945 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^5 \, dx$

Optimal. Leaf size=123

$$\frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c}+\frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{24c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c}+\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}$$

```
[Out] 1/24*(-a*d+6*b*c)*(c+d/x^2)^(3/2)*x^4/c+1/6*a*(c+d/x^2)^(5/2)*x^6/c+1/16*d^
2*(-a*d+6*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+1/16*d*(-a*d+6*b*c)
*x^2*(c+d/x^2)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 43, 65, 214}

$$\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}\,(6bc-ad)}{16c}+\frac{x^4\left(c+\frac{d}{x^2}\right)^{3/2}(6bc-ad)}{24c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

```
[Out] (d*(6*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c) + ((6*b*c - a*d)*(c + d/x^2)^(
3/2)*x^4)/(24*c) + (a*(c + d/x^2)^(5/2)*x^6)/(6*c) + (d^2*(6*b*c - a*d)*Arc
Tanh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^4}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{\left(3bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)}{6c}$$

$$= \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{(d(6bc-ad))\text{Subst}\left(\int \sqrt{}\right.}{16c}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c}$$

Mathematica [A]
time = 0.15, size = 123, normalized size = 1.00

$$\frac{\sqrt{c+\frac{d}{x^2}}\, x\left(\sqrt{c}\, x\sqrt{d+cx^2}\,(6bc(5d+2cx^2)+a(3d^2+14cdx^2+8c^2x^4))+3d^2(-6bc+ad)\log\left(-\sqrt{c}\, x+\sqrt{d+cx^2}\right)\right)}{48c^{3/2}\sqrt{d+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[d + c*x^2]*(6*b*c*(5*d + 2*c*x^2) + a*(3
*d^2 + 14*c*d*x^2 + 8*c^2*x^4)) + 3*d^2*(-6*b*c + a*d)*Log[-(Sqrt[c]*x) + S
qrt[d + c*x^2]]))/(48*c^(3/2)*Sqrt[d + c*x^2])
```

Maple [A]
time = 0.06, size = 162, normalized size = 1.32

method	result
risch	$\frac{x^2(8ac^2x^4+14acdx^2+12bc^2x^2+3ad^2+30bcd)\sqrt{\frac{cx^2+d}{x^2}}}{48c}+\frac{\left(-\frac{d^3\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)a}{16c^{\frac{3}{2}}}+\frac{3d^2\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)}{8\sqrt{c}}\right.}{\sqrt{cx^2+d}}$
default	$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}x^3\left(8(cx^2+d)^{\frac{5}{2}}\sqrt{c}\,ax-2(cx^2+d)^{\frac{3}{2}}\sqrt{c}\,adx+12(cx^2+d)^{\frac{3}{2}}c^{\frac{3}{2}}bx-3\sqrt{cx^2+d}\,\sqrt{c}\,ad^2x+18\sqrt{cx^2+d}\,c^{\frac{3}{2}}bd\right.}{48(cx^2+d)^{\frac{3}{2}}c^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)*x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(3/2)*x^3*(8*(c*x^2+d)^(5/2)*c^(1/2)*a*x-2*(c*x^2+d)^(
3/2)*c^(1/2)*a*d*x+12*(c*x^2+d)^(3/2)*c^(3/2)*b*x-3*(c*x^2+d)^(1/2)*c^(1/2)
*a*d^2*x+18*(c*x^2+d)^(1/2)*c^(3/2)*b*d*x-3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a
*d^3+18*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d^2)/(c*x^2+d)^(3/2)/c^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(103) = 206.
time = 0.53, size = 240, normalized size = 1.95

$$\frac{1}{96}\left(\frac{3\,d^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}cd^3-3\sqrt{c+\frac{d}{x^2}}\,c^2d^3\right)}{\left(c+\frac{d}{x^2}\right)^3c-3\left(c+\frac{d}{x^2}\right)^2c^2+3\left(c+\frac{d}{x^2}\right)c^3-c^4}\right)a-\frac{1}{16}\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-3\sqrt{c+\frac{d}{x^2}}\,cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2-2\left(c+\frac{d}{x^2}\right)c+c^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="maxima")
```

```
[Out] 1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^
(3/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 + 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(c +
 d/x^2)*c^2*d^3)/((c + d/x^2)^3*c - 3*(c + d/x^2)^2*c^2 + 3*(c + d/x^2)*c^3
 - c^4))*a - 1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) +
 sqrt(c)))/sqrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*d^2)/
((c + d/x^2)^2 - 2*(c + d/x^2)*c + c^2))*b
```

Fricas [A]
time = 3.04, size = 243, normalized size = 1.98

$$\left[-\frac{3\,(6\,bcd^2-ad^3)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(8\,ac^3x^6+2\,(6\,bc^3+7\,ac^2d)x^4+3\,(10\,bc^2d+acd^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{96\,c^2},-\frac{3\,(6\,bcd^2-ad^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(8\,ac^3x^6+2\,(6\,bc^3+7\,ac^2d)x^4+3\,(10\,bc^2d+acd^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{48\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*
x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b
*c^2*d + a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^2, -1/48*(3*(6*b*c*d^2 - a*
d^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (8*a
*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b*c^2*d + a*c*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2))/c^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(105) = 210$.
time = 76.37, size = 253, normalized size = 2.06

$$\frac{ac^2x^7}{6\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}+\frac{11ac\sqrt{d}\,x^5}{24\sqrt{\frac{cx^2}{d}+1}}+\frac{17ad^{\frac{3}{2}}x^3}{48\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{5}{2}}x}{16c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{16c^{\frac{3}{2}}}+\frac{bc^2x^5}{4\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}+\frac{3bc\sqrt{d}\,x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd^{\frac{3}{2}}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{bd^{\frac{3}{2}}x}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{3bd^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**5,x)
```

```
[Out] a*c**2*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 11*a*c*sqrt(d)*x**5/(24*sqrt(c
*x**2/d + 1)) + 17*a*d**(3/2)*x**3/(48*sqrt(c*x**2/d + 1)) + a*d**(5/2)*x/(
16*c*sqrt(c*x**2/d + 1)) - a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(3/2)) +
b*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*c*sqrt(d)*x**3/(8*sqrt(c*x
**2/d + 1)) + b*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + b*d**(3/2)*x/(8*sqrt(c*x*
*2/d + 1)) + 3*b*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c))
```

Giac [A]
time = 1.90, size = 144, normalized size = 1.17

$$\frac{1}{48}\left(2\left(4\,acx^2\mathrm{sgn}(x)+\frac{6\,bc^5\mathrm{sgn}(x)+7\,ac^4d\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\,(10\,bc^4d\mathrm{sgn}(x)+ac^3d^2\mathrm{sgn}(x))}{c^4}\right)\sqrt{cx^2+d}\,x-\frac{(6\,bcd^2\mathrm{sgn}(x)-ad^3\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+d}\right|\right)}{16\,c^{\frac{3}{2}}}+\frac{(6\,bcd^2\log\left(|d|\right)-ad^3\log\left(|d|\right))\mathrm{sgn}(x)}{32\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*a*c*x^2*sgn(x) + (6*b*c^5*sgn(x) + 7*a*c^4*d*sgn(x))/c^4)*x^2 +
3*(10*b*c^4*d*sgn(x) + a*c^3*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x - 1/16*(6*b
*c*d^2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(3/2
) + 1/32*(6*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(3/2)
```

Mupad [B]
time = 5.78, size = 130, normalized size = 1.06

$$\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{6}+\frac{5\,b\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8}+\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{5/2}}{16\,c}+\frac{3\,b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}}-\frac{a\,c\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16}-\frac{3\,b\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{a\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}}{16\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x^6*(c + d/x^2)^(3/2))/6 + (5*b*x^4*(c + d/x^2)^(3/2))/8 + (a*x^6*(c + d
/x^2)^(5/2))/(16*c) + (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16*c
^(3/2)) + (3*b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(1/2)) - (a*c*x^6
*(c + d/x^2)^(1/2))/16 - (3*b*c*x^4*(c + d/x^2)^(1/2))/8
```

3.946 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^3 \, dx$

Optimal. Leaf size=115

$$-\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c}+\frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c}+\frac{3d(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}$$

```
[Out] 1/8*(a*d+4*b*c)*(c+d/x^2)^(3/2)*x^2/c+1/4*a*(c+d/x^2)^(5/2)*x^4/c+3/8*d*(a*
d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(1/2)-3/8*d*(a*d+4*b*c)*(c+d/x^
2)^(1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 43, 52, 65, 214}

$$\frac{x^2\left(c+\frac{d}{x^2}\right)^{3/2}(ad+4bc)}{8c}-\frac{3d\sqrt{c+\frac{d}{x^2}}\,(ad+4bc)}{8c}+\frac{3d(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

```
[Out] (-3*d*(4*b*c + a*d)*Sqrt[c + d/x^2])/(8*c) + ((4*b*c + a*d)*(c + d/x^2)^(3/
2)*x^2)/(8*c) + (a*(c + d/x^2)^(5/2)*x^4)/(4*c) + (3*d*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d/x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} - \frac{(4bc+ad)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)}{8c}$$

$$= \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} - \frac{(3d(4bc+ad))\text{Subst}\left(\int \right.}{16c}$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c}$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c}$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c}$$

Mathematica [A]
time = 0.16, size = 90, normalized size = 0.78

$$\frac{1}{8}\sqrt{c+\frac{d}{x^2}}\left(-8bd+4bcx^2+5adx^2+2acx^4-\frac{3d(4bc+ad)x\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)}{\sqrt{c}\,\sqrt{d+cx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-8*b*d + 4*b*c*x^2 + 5*a*d*x^2 + 2*a*c*x^4 - (3*d*(4*b*c
+ a*d)*x*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]])/(Sqrt[c]*Sqrt[d + c*x^2])))/8
```

Maple [A]
time = 0.07, size = 174, normalized size = 1.51

method	result

risch	$\frac{(2acx^4+5adx^2+4cx^2b-8bd)\sqrt{\frac{cx^2+d}{x^2}}}{8}+\frac{\left(\frac{3d^2\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)a}{8\sqrt{c}}+\frac{3d\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)\sqrt{c}\,b}{2}\right)\sqrt{\frac{cx^2+c}{x^2}}}{\sqrt{cx^2+d}}$
default	$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}x^2\left(8c^{\frac{3}{2}}(cx^2+d)^{\frac{3}{2}}bx^2+12c^{\frac{3}{2}}\sqrt{cx^2+d}\,bdx^2+2\sqrt{c}\,(cx^2+d)^{\frac{3}{2}}adx^2-8\sqrt{c}\,(cx^2+d)^{\frac{5}{2}}b+3\sqrt{c}\,\sqrt{cx^2+d}\,a\,c\right.}{8(cx^2+d)^{\frac{3}{2}}d\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((c*x^2+d)/x^2)^(3/2)*x^2*(8*c^(3/2)*(c*x^2+d)^(3/2)*b*x^2+12*c^(3/2)*(
c*x^2+d)^(1/2)*b*d*x^2+2*c^(1/2)*(c*x^2+d)^(3/2)*a*d*x^2-8*c^(1/2)*(c*x^2+d
)^(5/2)*b+3*c^(1/2)*(c*x^2+d)^(1/2)*a*d^2*x^2+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2
))*a*d^3*x+12*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d^2*x)/(c*x^2+d)^(3/2)/d/c^
(1/2)
```

Maxima [A]
time = 0.54, size = 171, normalized size = 1.49

$$-\frac{1}{16}\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-3\sqrt{c+\frac{d}{x^2}}\,cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2-2\left(c+\frac{d}{x^2}\right)c+c^2}\right)a+\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}\,cx^2-3\sqrt{c}\,d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)-4\sqrt{c+\frac{d}{x^2}}\,d\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/s
qrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)
^2 - 2*(c + d/x^2)*c + c^2))*a + 1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c)*d
*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(c +
d/x^2)*d)*b
```

Fricas [A]
time = 2.58, size = 203, normalized size = 1.77

$$\left[\frac{3\,(4\,bcd+ad^2)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\,(2\,ac^2x^4-8\,bcd+(4\,bc^2+5\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c},\,-\frac{3\,(4\,bcd+ad^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(2\,ac^2x^4-8\,bcd+(4\,bc^2+5\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*b*c*d + a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2
 + d)/x^2) - d) + 2*(2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5*a*c*d)*x^2)*sqrt(
(c*x^2 + d)/x^2))/c, -1/8*(3*(4*b*c*d + a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2
*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5
*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(104) = 208$.
time = 70.57, size = 216, normalized size = 1.88

$$\frac{ac^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3ac\sqrt{d}\,x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{3}{2}}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{ad^{\frac{3}{2}}x}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8\sqrt{c}}+\frac{3b\sqrt{c}\,d\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2}+\frac{bc\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2}-\frac{bc\sqrt{d}\,x}{\sqrt{\frac{cx^2}{d}+1}}-\frac{bd^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**3,x)
```

```
[Out] a*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*c*sqrt(d)*x**3/(8*sqrt(c*x
**2/d + 1)) + a*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + a*d**(3/2)*x/(8*sqrt(c*x*
*2/d + 1)) + 3*a*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c)) + 3*b*sqrt(c)*d*
asinh(sqrt(c)*x/sqrt(d))/2 + b*c*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 - b*c*sqrt(
d)*x/sqrt(c*x**2/d + 1) - b*d**(3/2)/(x*sqrt(c*x**2/d + 1))
```

Giac [A]
time = 1.56, size = 126, normalized size = 1.10

$$\frac{2\,b\sqrt{c}\,d^2\operatorname{sgn}(x)}{\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d}+\frac{1}{8}\left(2\,acx^2\operatorname{sgn}(x)+\frac{4\,bc^3\operatorname{sgn}(x)+5\,ac^2d\operatorname{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}\,x-\frac{3\left(4\,bc^{\frac{3}{2}}d\operatorname{sgn}(x)+a\sqrt{c}\,d^2\operatorname{sgn}(x)\right)\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2\right)}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 2*b*sqrt(c)*d^2*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d) + 1/8*(2*a*c*x
^2*sgn(x) + (4*b*c^3*sgn(x) + 5*a*c^2*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*x - 3/
16*(4*b*c^(3/2)*d*sgn(x) + a*sqrt(c)*d^2*sgn(x))*log((sqrt(c)*x - sqrt(c*x^
2 + d))^2)/c
```

Mupad [B]
time = 5.70, size = 105, normalized size = 0.91

$$\frac{5\,a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8}-b\,d\,\sqrt{c+\frac{d}{x^2}}+\frac{3\,b\,\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2}+\frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}}-\frac{3\,a\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{b\,c\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (5*a*x^4*(c + d/x^2)^(3/2))/8 - b*d*(c + d/x^2)^(1/2) + (3*b*c^(1/2)*d*atan
h((c + d/x^2)^(1/2)/c^(1/2)))/2 + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2))
)/(8*c^(1/2)) - (3*a*c*x^4*(c + d/x^2)^(1/2))/8 + (b*c*x^2*(c + d/x^2)^(1/2
))/2
```

3.947 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx$

Optimal. Leaf size=110

$$-\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}}-\frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c}+\frac{1}{2}\sqrt{c}\,(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

```
[Out] -1/6*(3*a*d+2*b*c)*(c+d/x^2)^(3/2)/c+1/2*a*(c+d/x^2)^(5/2)*x^2/c+1/2*(3*a*d
+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))*c^(1/2)-1/2*(3*a*d+2*b*c)*(c+d/x^2
)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 79, 52, 65, 214}

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3ad+2bc)}{6c}-\frac{1}{2}\sqrt{c+\frac{d}{x^2}}\,(3ad+2bc)+\frac{1}{2}\sqrt{c}\,(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

```
[Out] -1/2*((2*b*c + 3*a*d)*Sqrt[c + d/x^2]) - ((2*b*c + 3*a*d)*(c + d/x^2)^(3/2)
)/(6*c) + (a*(c + d/x^2)^(5/2)*x^2)/(2*c) + (Sqrt[c]*(2*b*c + 3*a*d)*ArcTan
h[Sqrt[c + d/x^2]/Sqrt[c]])/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c} - \frac{\left(bc+\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)}{2c}\\
&= -\frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c} - \frac{1}{4}(2bc+3ad)\text{Subst}\left(\int \frac{\sqrt{c}\cdots}{\cdots}\right.\\
&= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c} - \frac{1}{4}\cdots\\
&= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c} - \cdots\\
&= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c} + \frac{1}{2}\cdots
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 108, normalized size = 0.98

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(\sqrt{d+cx^2}\,(-6adx^2+3acx^4-2b(d+4cx^2))-3\sqrt{c}\,(2bc+3ad)x^3\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)\right)}{6x^2\sqrt{d+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(Sqrt[d + c*x^2]*(-6*a*d*x^2 + 3*a*c*x^4 - 2*b*(d + 4*c*x^
2)) - 3*Sqrt[c]*(2*b*c + 3*a*d)*x^3*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]]))/(
6*x^2*Sqrt[d + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(90) = 180$.
time = 0.07, size = 216, normalized size = 1.96

method	result
risch	$\frac{(3ac\,x^4-6ad\,x^2-8c\,x^2b-2bd)\sqrt{\frac{c\,x^2+d}{x^2}}}{6x^2}+\frac{\left(\frac{3\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)ad}{2}+c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)b\right)\sqrt{\frac{c\,x^2}{x^2}}}{\sqrt{c\,x^2+d}}$
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(4c^{\frac{5}{2}}(c\,x^2+d)^{\frac{3}{2}}b\,x^4+6c^{\frac{5}{2}}\sqrt{c\,x^2+d}\,bd\,x^4+6c^{\frac{3}{2}}(c\,x^2+d)^{\frac{3}{2}}ad\,x^4-4c^{\frac{3}{2}}(c\,x^2+d)^{\frac{5}{2}}b\,x^2+9c^{\frac{3}{2}}\sqrt{c\,x^2+d}\,a\,d^2x^4-6\right.}{6(c\,x^2+d)^{\frac{3}{2}}c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((c*x^2+d)/x^2)^(3/2)*(4*c^(5/2)*(c*x^2+d)^(3/2)*b*x^4+6*c^(5/2)*(c*x^2
+d)^(1/2)*b*d*x^4+6*c^(3/2)*(c*x^2+d)^(3/2)*a*d*x^4-4*c^(3/2)*(c*x^2+d)^(5/
2)*b*x^2+9*c^(3/2)*(c*x^2+d)^(1/2)*a*d^2*x^4-6*c^(1/2)*(c*x^2+d)^(5/2)*a*d*
x^2+9*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c*d^3*x^3+6*ln(c^(1/2)*x+(c*x^2+d)^(1
/2))*b*c^2*d^2*x^3-2*c^(1/2)*(c*x^2+d)^(5/2)*b*d)/(c*x^2+d)^(3/2)/d^2/c^(1/
2)
```

Maxima [A]
time = 0.53, size = 134, normalized size = 1.22

$$\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}\,cx^2-3\sqrt{c}\,d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)-4\sqrt{c+\frac{d}{x^2}}\,d\right)a-\frac{1}{6}\left(3c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+6\sqrt{c+\frac{d}{x^2}}\,c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c)*d*log((sqrt(c + d/x^2) - sqrt(c))/
(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(c + d/x^2)*d)*a - 1/6*(3*c^(3/2)*log(
(sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)^(3
/2) + 6*sqrt(c + d/x^2)*c)*b
```

Fricas [A]
time = 3.26, size = 195, normalized size = 1.77

$$\left[\frac{3\,(2\,bc+3\,ad)\sqrt{c}\,x^2\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\,(3\,acx^4-2\,(4\,bc+3\,ad)x^2-2\,bd)\sqrt{\frac{cx^2+d}{x^2}}}{12\,x^2},\ -\frac{3\,(2\,bc+3\,ad)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(3\,acx^4-2\,(4\,bc+3\,ad)x^2-2\,bd)\sqrt{\frac{cx^2+d}{x^2}}}{6\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*b*c + 3*a*d)*sqrt(c)*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x
^2 + d)/x^2) - d) + 2*(3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 - 2*b*d)*sqrt((c*x
^2 + d)/x^2))/x^2, -1/6*(3*(2*b*c + 3*a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)*x^2
*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 -
2*b*d)*sqrt((c*x^2 + d)/x^2))/x^2]
```

Sympy [A]
time = 22.32, size = 187, normalized size = 1.70

$$\frac{3a\sqrt{c}\,d\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2}+\frac{ac\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2}-\frac{ac\sqrt{d}\,x}{\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}}+bc^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)-\frac{bc^2x}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{bc\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}+bd\left(\begin{cases}-\frac{\sqrt{c}}{2x^2} & \text{for}\ d=0\\ -\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x,x)
```

```
[Out] 3*a*sqrt(c)*d*asinh(sqrt(c)*x/sqrt(d))/2 + a*c*sqrt(d)*x*sqrt(c*x**2/d + 1)
/2 - a*c*sqrt(d)*x/sqrt(c*x**2/d + 1) - a*d**(3/2)/(x*sqrt(c*x**2/d + 1)) +
 b*c**(3/2)*asinh(sqrt(c)*x/sqrt(d)) - b*c**2*x/(sqrt(d)*sqrt(c*x**2/d + 1)
) - b*c*sqrt(d)/(x*sqrt(c*x**2/d + 1)) + b*d*Piecewise((-sqrt(c)/(2*x**2),
Eq(d, 0)), (-(c + d/x**2)**(3/2)/(3*d), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(90) = 180.
time = 1.79, size = 225, normalized size = 2.05

$$\frac{1}{2}\sqrt{cx^2+d}\,acx\operatorname{sgn}(x)-\frac{1}{4}\left(2\,bc^{\frac{3}{2}}\operatorname{sgn}(x)+3\,a\sqrt{c}\,d\operatorname{sgn}(x)\right)\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2\right)+\frac{2\left(6\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}d\operatorname{sgn}(x)+3\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4a\sqrt{c}\,d^2\operatorname{sgn}(x)-6\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2bc^{\frac{3}{2}}d^2\operatorname{sgn}(x)-6\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2a\sqrt{c}\,d^3\operatorname{sgn}(x)+4\,bc^{\frac{3}{2}}d^3\operatorname{sgn}(x)+3\,a\sqrt{c}\,d^4\operatorname{sgn}(x)\right)}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*c*x*sgn(x) - 1/4*(2*b*c^(3/2)*sgn(x) + 3*a*sqrt(c)*d*
sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2) + 2/3*(6*(sqrt(c)*x - sqrt(c*x
^2 + d))^4*b*c^(3/2)*d*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c)
*d^2*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(3/2)*d^2*sgn(x) - 6*(s
qrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^3*sgn(x) + 4*b*c^(3/2)*d^3*sgn(x)
 + 3*a*sqrt(c)*d^4*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^3
```

Mupad [B]
time = 5.65, size = 95, normalized size = 0.86

$$b\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{b\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{3}-a\,d\,\sqrt{c+\frac{d}{x^2}}-b\,c\,\sqrt{c+\frac{d}{x^2}}+\frac{3\,a\,\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2}+\frac{a\,c\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] b*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (b*(c + d/x^2)^(3/2))/3 - a*d*
(c + d/x^2)^(1/2) - b*c*(c + d/x^2)^(1/2) + (3*a*c^(1/2)*d*atanh((c + d/x^2
)^(1/2)/c^(1/2)))/2 + (a*c*x^2*(c + d/x^2)^(1/2))/2
```

3.948 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=76

$$-ac\sqrt{c+\frac{d}{x^2}}-\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}+ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

```
[Out] -1/3*a*(c+d/x^2)^(3/2)-1/5*b*(c+d/x^2)^(5/2)/d+a*c^(3/2)*arctanh((c+d/x^2)^
(1/2)/c^(1/2))-a*c*(c+d/x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 81, 52, 65, 214}

$$ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-ac\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

```
[Out] -(a*c*Sqrt[c + d/x^2]) - (a*(c + d/x^2)^(3/2))/3 - (b*(c + d/x^2)^(5/2))/(5
*d) + a*c^(3/2)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}-\frac{1}{2}a\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx,x,\frac{1}{x^2}\right)\\
&= -\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}-\frac{1}{2}(ac)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx,x,\frac{1}{x^2}\right)\\
&= -ac\sqrt{c+\frac{d}{x^2}}-\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}-\frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+d}}\right.\\
&= -ac\sqrt{c+\frac{d}{x^2}}-\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}-\frac{\left(ac^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,\right.}{d}\\
&= -ac\sqrt{c+\frac{d}{x^2}}-\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}+ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right.
\end{aligned}
$$

Mathematica [A]

time = 0.21, size = 91, normalized size = 1.20

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(-\frac{3b(d+cx^2)^2}{d}-5ax^2(d+4cx^2)-\frac{15ac^{3/2}x^5\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)}{\sqrt{d+cx^2}}\right)}{15x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[c + d/x^2]*((-3*b*(d + c*x^2)^2)/d - 5*a*x^2*(d + 4*c*x^2) - (15*a*c^
(3/2)*x^5*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]])/Sqrt[d + c*x^2]))/(15*x^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(60) = 120$.
time = 0.08, size = 153, normalized size = 2.01

method	result
risch	$-\frac{\left(20x^4acd+3x^4b\,c^2+5a\,d^2x^2+6bcd\,x^2+3b\,d^2\right)\sqrt{\frac{c\,x^2+d}{x^2}}}{15x^4d}+\frac{c^{\frac{3}{2}}a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{\sqrt{c\,x^2+d}}$
default	$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(-10\left(c\,x^2+d\right)^{\frac{3}{2}}c^{\frac{5}{2}}a\,x^6+10\left(c\,x^2+d\right)^{\frac{5}{2}}c^{\frac{3}{2}}a\,x^4-15\sqrt{c\,x^2+d}\,c^{\frac{5}{2}}ad\,x^6-15\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)a\,c^2d^2x^5+5\right.}{15x^2\left(c\,x^2+d\right)^{\frac{3}{2}}d^2\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*((c*x^2+d)/x^2)^(3/2)/x^2*(-10*(c*x^2+d)^(3/2)*c^(5/2)*a*x^6+10*(c*x^
2+d)^(5/2)*c^(3/2)*a*x^4-15*(c*x^2+d)^(1/2)*c^(5/2)*a*d*x^6-15*ln(c^(1/2)*x
+(c*x^2+d)^(1/2))*a*c^2*d^2*x^5+5*c^(1/2)*(c*x^2+d)^(5/2)*a*d*x^2+3*c^(1/2)
*(c*x^2+d)^(5/2)*b*d)/(c*x^2+d)^(3/2)/d^2/c^(1/2)
```

Maxima [A]
time = 0.49, size = 80, normalized size = 1.05

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d}-\frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+6\sqrt{c+\frac{d}{x^2}}\,c\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/5*b*(c + d/x^2)^(5/2)/d - 1/6*(3*c^(3/2)*log((sqrt(c + d/x^2) - sqrt(c))
/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)^(3/2) + 6*sqrt(c + d/x^2)*c)*
a
```

Fricas [A]
time = 3.93, size = 213, normalized size = 2.80

$$\left[\frac{15\,ac^{\frac{3}{2}}dx^4\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,\left((3\,bc^2+20\,acd)x^4+3\,bd^2+(6\,bcd+5\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{30\,dx^4}, -\frac{15\,a\sqrt{-c}\,cdx^4\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left((3\,bc^2+20\,acd)x^4+3\,bd^2+(6\,bcd+5\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,dx^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*a*c^(3/2)*d*x^4*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2
) - d) - 2*((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d^2 + (6*b*c*d + 5*a*d^2)*x^2)*s
qrt((c*x^2 + d)/x^2))/(d*x^4), -1/15*(15*a*sqrt(-c)*c*d*x^4*arctan(sqrt(-c)
*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + ((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d
^2 + (6*b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^4)]
```

Sympy [A]
time = 23.39, size = 73, normalized size = 0.96

$$-\frac{ac^2\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-ac\sqrt{c+\frac{d}{x^2}}-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x,x)
```

```
[Out] -a*c**2*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - a*c*sqrt(c + d/x**2) - a
*(c + d/x**2)**(3/2)/3 - b*(c + d/x**2)**(5/2)/(5*d)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(60) = 120.
time = 4.86, size = 254, normalized size = 3.34

$$-\frac{1}{2}\,ac^{\frac{3}{2}}\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2\right)\operatorname{sgn}(x)+\frac{2\left(15\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^8bc^{\frac{5}{2}}\operatorname{sgn}(x)+30\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}d\operatorname{sgn}(x)-90\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}d^2\operatorname{sgn}(x)+30\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4bc^{\frac{5}{2}}d^2\operatorname{sgn}(x)+110\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4ac^{\frac{3}{2}}d^3\operatorname{sgn}(x)-70\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2ac^{\frac{3}{2}}d^4\operatorname{sgn}(x)+3\,bc^{\frac{5}{2}}d^4\operatorname{sgn}(x)+20\,ac^{\frac{3}{2}}d^5\operatorname{sgn}(x)\right)}{15\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/15*(15*(sqrt
(c)*x - sqrt(c*x^2 + d))^8*b*c^(5/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 +
d))^8*a*c^(3/2)*d*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(3/2)*d^2
*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d^2*sgn(x) + 110*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^3*sgn(x) - 70*(sqrt(c)*x - sqrt(c*
```

```
x^2 + d))^2*a*c^(3/2)*d^4*sgn(x) + 3*b*c^(5/2)*d^4*sgn(x) + 20*a*c^(3/2)*d^
5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

Mupad [B]
time = 5.83, size = 72, normalized size = 0.95

$$a\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{a\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{3}-a\,c\,\sqrt{c+\frac{d}{x^2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}\,{\left(c\,x^2+d\right)}^2}{5\,d\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x,x)
```

```
[Out] a*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (a*(c + d/x^2)^(3/2))/3 - a*c*
(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2)*(d + c*x^2)^2)/(5*d*x^4)
```

3.949 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

```
[Out] 1/5*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^2-1/7*b*(c+d/x^2)^(7/2)/d^2
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {455, 45}

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

```
[Out] ((b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^2) - (b*(c + d/x^2)^(7/2))/(7*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int (a+bx)(c+dx)^{3/2}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)(c+dx)^{3/2}}{d}+\frac{b(c+dx)^{5/2}}{d}\right)dx,x,\frac{1}{x^2}\right)\right)\\ &= \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 49, normalized size = 1.07

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)^2\,(5bd-2bcx^2+7adx^2)}{35d^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

```
[Out] -1/35*(Sqrt[c + d/x^2]*(d + c*x^2)^2*(5*b*d - 2*b*c*x^2 + 7*a*d*x^2))/(d^2*
x^6)
```

Maple [A]
time = 0.08, size = 48, normalized size = 1.04

method	result	size
gosper	$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}(7ad\,x^2-2c\,x^2b+5bd)\,(c\,x^2+d)}{35d^2x^4}$	48
default	$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}(7ad\,x^2-2c\,x^2b+5bd)\,(c\,x^2+d)}{35d^2x^4}$	48
risch	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,(7a\,c^2d\,x^6-2b\,c^3x^6+14ac\,d^2x^4+b\,c^2d\,x^4+7a\,d^3x^2+8bc\,d^2x^2+5b\,d^3)}{35x^6d^2}$	86
trager	$-\frac{(7a\,c^2d\,x^6-2b\,c^3x^6+14ac\,d^2x^4+b\,c^2d\,x^4+7a\,d^3x^2+8bc\,d^2x^2+5b\,d^3)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{35x^6d^2}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*((c*x^2+d)/x^2)^(3/2)*(7*a*d*x^2-2*b*c*x^2+5*b*d)*(c*x^2+d)/d^2/x^4
```

Maxima [A]
time = 0.27, size = 49, normalized size = 1.07

$$-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d}-\frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2}-\frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/5*a*(c + d/x^2)^(5/2)/d - 1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^
(5/2)*c/d^2)*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(38) = 76$.
time = 2.70, size = 84, normalized size = 1.83

$$\frac{\left((2\,bc^3 - 7\,ac^2d)x^6 - (bc^2d + 14\,acd^2)x^4 - 5\,bd^3 - (8\,bcd^2 + 7\,ad^3)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/35*((2*b*c^3 - 7*a*c^2*d)*x^6 - (b*c^2*d + 14*a*c*d^2)*x^4 - 5*b*d^3 - (8
*b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^2*x^6)
```

Sympy [A]
time = 6.20, size = 138, normalized size = 3.00

$$-\frac{ac\left(\begin{cases}\frac{\sqrt{c}}{x^2} & \text{for } d=0\\ \frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2} - \frac{a\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} + \frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d} - \frac{bc\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} + \frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} - \frac{b\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3} - \frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5} + \frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**3,x)
```

```
[Out] -a*c*Piecewise((sqrt(c)/x**2, Eq(d, 0)), (2*(c + d/x**2)**(3/2)/(3*d), True
))/2 - a*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d - b*c*(-c*(c
+ d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - b*(c**2*(c + d/x**2)**(3
/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(38) = 76$.
time = 5.19, size = 370, normalized size = 8.04

$$\frac{2\left(35\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{5}{2}}\mathrm{sgn}(x)+70\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{7}{2}}\mathrm{sgn}(x)-70\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{5}{2}}d\,\mathrm{sgn}(x)+70\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{7}{2}}d\,\mathrm{sgn}(x)+105\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{5}{2}}d^2\mathrm{sgn}(x)+140\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{7}{2}}d^2\mathrm{sgn}(x)-140\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{5}{2}}d^3\mathrm{sgn}(x)+28\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{7}{2}}d^3\mathrm{sgn}(x)+77\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{5}{2}}d^4\mathrm{sgn}(x)+14\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{7}{2}}d^4\mathrm{sgn}(x)-14\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{5}{2}}d^5\mathrm{sgn}(x)-2\,bc^{\frac{7}{2}}d^5\mathrm{sgn}(x)+7\,ac^{\frac{5}{2}}d^6\mathrm{sgn}(x)\right)}{35\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(5/2)*sgn(x) + 70*(sqrt(c)*x
- sqrt(c*x^2 + d))^10*b*c^(7/2)*sgn(x) - 70*(sqrt(c)*x - sqrt(c*x^2 + d))^1
0*a*c^(5/2)*d*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*d*sgn(x
) + 105*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2)*d^2*sgn(x) + 140*(sqrt(c)
*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d^2*sgn(x) - 140*(sqrt(c)*x - sqrt(c*x^2
+ d))^6*a*c^(5/2)*d^3*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)
*d^3*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(5/2)*d^4*sgn(x) + 14*
```

```
(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)*d^4*sgn(x) - 14*(sqrt(c)*x - sqrt
(c*x^2 + d))^2*a*c^(5/2)*d^5*sgn(x) - 2*b*c^(7/2)*d^5*sgn(x) + 7*a*c^(5/2)*
d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^7
```

Mupad [B]
time = 5.33, size = 122, normalized size = 2.65

$$\frac{2\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{5\,d}-\frac{2\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{5\,x^2}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{8\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x)
```

```
[Out] (2*b*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (a*c^2*(c + d/x^2)^(1/2))/(5*d) - (2
*a*c*(c + d/x^2)^(1/2))/(5*x^2) - (a*d*(c + d/x^2)^(1/2))/(5*x^4) - (8*b*c*
(c + d/x^2)^(1/2))/(35*x^4) - (b*d*(c + d/x^2)^(1/2))/(7*x^6) - (b*c^2*(c +
 d/x^2)^(1/2))/(35*d*x^2)
```

3.950 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5} dx$

Optimal. Leaf size=74

$$-\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

```
[Out] -1/5*c*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^3+1/7*(-a*d+2*b*c)*(c+d/x^2)^(7/2)/d^3-
1/9*b*(c+d/x^2)^(9/2)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

```
[Out] -1/5*(c*(b*c - a*d)*(c + d/x^2)^(5/2))/d^3 + ((2*b*c - a*d)*(c + d/x^2)^(7/
2))/(7*d^3) - (b*(c + d/x^2)^(9/2))/(9*d^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int x(a+bx)(c+dx)^{3/2}\,dx,x,\frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int\left(\frac{c(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{(-2bc+ad)(c+dx)^{5/2}}{d^2}+\frac{b(c+dx)^{7}}{d^2}\right.\right.\right.$$
$$= -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

Mathematica [A]
time = 0.18, size = 71, normalized size = 0.96

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)^2\,(9adx^2(-5d+2cx^2)+b(-35d^2+20cdx^2-8c^2x^4))}{315d^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(9*a*d*x^2*(-5*d + 2*c*x^2) + b*(-35*d^2 + 2
0*c*d*x^2 - 8*c^2*x^4)))/(315*d^3*x^8)
```

Maple [A]
time = 0.09, size = 70, normalized size = 0.95

method	result	size
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(18x^4acd-8x^4b\,c^2-45a\,d^2x^2+20bcd\,x^2-35b\,d^2\right)\left(c\,x^2+d\right)}{315d^3x^6}$	70
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(18x^4acd-8x^4b\,c^2-45a\,d^2x^2+20bcd\,x^2-35b\,d^2\right)\left(c\,x^2+d\right)}{315d^3x^6}$	70
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(18a\,c^3d\,x^8-8b\,c^4x^8-9a\,c^2d^2x^6+4b\,c^3d\,x^6-72ac\,d^3x^4-3b\,c^2d^2x^4-45a\,d^4x^2-50bc\,d^3x^2-35b\,d^4\right)}{315x^8d^3}$	111
trager	$\frac{\left(18a\,c^3d\,x^8-8b\,c^4x^8-9a\,c^2d^2x^6+4b\,c^3d\,x^6-72ac\,d^3x^4-3b\,c^2d^2x^4-45a\,d^4x^2-50bc\,d^3x^2-35b\,d^4\right)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{315x^8d^3}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(3/2)*(18*a*c*d*x^4-8*b*c^2*x^4-45*a*d^2*x^2+20*b*c*d
*x^2-35*b*d^2)*(c*x^2+d)/d^3/x^6
```

Maxima [A]
time = 0.29, size = 84, normalized size = 1.14

$$-\frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2}-\frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)a-\frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3}-\frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3}+\frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^(5/2)*c/d^2)*a - 1/315*(35*(
c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x^2)^(5/2)*c^
2/d^3)*b
```

Fricas [A]
time = 3.12, size = 109, normalized size = 1.47

$$-\frac{\left(2\left(4\,bc^4-9\,ac^3d\right)x^8-\left(4\,bc^3d-9\,ac^2d^2\right)x^6+35\,bd^4+3\left(bc^2d^2+24\,acd^3\right)x^4+5\left(10\,bcd^3+9\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,d^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*(4*b*c^4 - 9*a*c^3*d)*x^8 - (4*b*c^3*d - 9*a*c^2*d^2)*x^6 + 35*b*
d^4 + 3*(b*c^2*d^2 + 24*a*c*d^3)*x^4 + 5*(10*b*c*d^3 + 9*a*d^4)*x^2)*sqrt((
c*x^2 + d)/x^2)/(d^3*x^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(65) = 130.
time = 7.23, size = 194, normalized size = 2.62

$$-\frac{ac\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{a\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^2}-\frac{bc\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{b\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**5,x)
```

```
[Out] -a*c*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - a*(c**2*(c +
 d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**2
 - b*c*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**
2)**(7/2)/7)/d**3 - b*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(
5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(62) = 124.
time = 6.39, size = 430, normalized size = 5.81

$$\frac{4\left(315\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{7}{2}}\mathrm{sgn}(x)+840\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{9}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{7}{2}}d\,\mathrm{sgn}(x)+1260\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{9}{2}}d\,\mathrm{sgn}(x)+315\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{7}{2}}d^2\mathrm{sgn}(x)+1764\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{9}{2}}d^2\mathrm{sgn}(x)-819\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{7}{2}}d^3\mathrm{sgn}(x)+504\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{9}{2}}d^3\mathrm{sgn}(x)+441\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{7}{2}}d^4\mathrm{sgn}(x)+144\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{9}{2}}d^4\mathrm{sgn}(x)-9\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{7}{2}}d^5\mathrm{sgn}(x)-36\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{9}{2}}d^5\mathrm{sgn}(x)+81\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{7}{2}}d^6\mathrm{sgn}(x)+4\,bc^{\frac{9}{2}}d^6\mathrm{sgn}(x)-9\,ac^{\frac{7}{2}}d^7\mathrm{sgn}(x)\right)}{315\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(7/2)*sgn(x) + 840*(sqrt(c)
*x - sqrt(c*x^2 + d))^12*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 + d
))^12*a*c^(7/2)*d*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*
d*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(7/2)*d^2*sgn(x) + 1764
*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d^2*sgn(x) - 819*(sqrt(c)*x - sq
rt(c*x^2 + d))^8*a*c^(7/2)*d^3*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 + d))^6
*b*c^(9/2)*d^3*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(7/2)*d^4*s
gn(x) + 144*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(9/2)*d^4*sgn(x) - 9*(sqrt(
c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^5*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*b*c^(9/2)*d^5*sgn(x) + 81*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(7/2
)*d^6*sgn(x) + 4*b*c^(9/2)*d^6*sgn(x) - 9*a*c^(7/2)*d^7*sgn(x))/((sqrt(c)*x
 - sqrt(c*x^2 + d))^2 - d)^9
```

Mupad [B]
time = 5.76, size = 164, normalized size = 2.22

$$\frac{2\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2}-\frac{8\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3}-\frac{8\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{10\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d\,x^4}+\frac{4\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x)
```

```
[Out] (2*a*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (8*b*c^4*(c + d/x^2)^(1/2))/(315*d^3
) - (8*a*c*(c + d/x^2)^(1/2))/(35*x^4) - (a*d*(c + d/x^2)^(1/2))/(7*x^6) -
(10*b*c*(c + d/x^2)^(1/2))/(63*x^6) - (b*d*(c + d/x^2)^(1/2))/(9*x^8) - (a*
c^2*(c + d/x^2)^(1/2))/(35*d*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(105*d*x^4) +
 (4*b*c^3*(c + d/x^2)^(1/2))/(315*d^2*x^2)
```

3.951 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7} dx$

Optimal. Leaf size=104

$$\frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

```
[Out] 1/5*c^2*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^4-1/7*c*(-2*a*d+3*b*c)*(c+d/x^2)^(7/2)
/d^4+1/9*(-a*d+3*b*c)*(c+d/x^2)^(9/2)/d^4-1/11*b*(c+d/x^2)^(11/2)/d^4
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(3bc-ad)}{9d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-2ad)}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(7/2))/(7*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(9/2))/(9*d^4) - (b*(c + d/x^
2)^(11/2))/(11*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)(c+dx)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{c^2(bc-ad)(c+dx)^{3/2}}{d^3} + \frac{c(3bc-2ad)(c+dx)^{5/2}}{d^3} + \frac{(-3bc-$$

$$= \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4} - \frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4} -$$

Mathematica [A]
time = 0.21, size = 94, normalized size = 0.90

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)^2\left(-11adx^2(35d^2-20cdx^2+8c^2x^4)-3b(105d^3-70cd^2x^2+40c^2dx^4-16c^3x^6)\right)}{3465d^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-11*a*d*x^2*(35*d^2 - 20*c*d*x^2 + 8*c^2*x^
4) - 3*b*(105*d^3 - 70*c*d^2*x^2 + 40*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4*x
^10)
```

Maple [A]
time = 0.14, size = 94, normalized size = 0.90

method	result
gosper	$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(88a\,c^2d\,x^6-48b\,c^3x^6-220ac\,d^2x^4+120b\,c^2d\,x^4+385a\,d^3x^2-210bc\,d^2x^2+315b\,d^3\right)\left(c\,x^2+d\right)}{3465d^4x^8}$
default	$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(88a\,c^2d\,x^6-48b\,c^3x^6-220ac\,d^2x^4+120b\,c^2d\,x^4+385a\,d^3x^2-210bc\,d^2x^2+315b\,d^3\right)\left(c\,x^2+d\right)}{3465d^4x^8}$
risch	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(88a\,c^4d\,x^{10}-48b\,c^5x^{10}-44a\,c^3d^2x^8+24b\,c^4d\,x^8+33a\,c^2d^3x^6-18b\,c^3d^2x^6+550ac\,d^4x^4+15b\,c^2d^3x^4+385a\,d^5x^2+420bc\right.}{3465x^{10}d^4}$
trager	$-\frac{\left(88a\,c^4d\,x^{10}-48b\,c^5x^{10}-44a\,c^3d^2x^8+24b\,c^4d\,x^8+33a\,c^2d^3x^6-18b\,c^3d^2x^6+550ac\,d^4x^4+15b\,c^2d^3x^4+385a\,d^5x^2+420bc\,d^4x^2+315b\,d\right.}{3465x^{10}d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3465*((c*x^2+d)/x^2)^(3/2)*(88*a*c^2*d*x^6-48*b*c^3*x^6-220*a*c*d^2*x^4+
120*b*c^2*d*x^4+385*a*d^3*x^2-210*b*c*d^2*x^2+315*b*d^3)*(c*x^2+d)/d^4/x^8
```

Maxima [A]
time = 0.31, size = 118, normalized size = 1.13

$$-\frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3}-\frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3}+\frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)a-\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4}-\frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/315*(35*(c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x
^2)^(5/2)*c^2/d^3)*a - 1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)
^(9/2)*c/d^4 + 495*(c + d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^
4)*b
```

Fricas [A]
time = 2.42, size = 134, normalized size = 1.29

$$\frac{\left(8\left(6\,bc^5-11\,ac^4d\right)x^{10}-4\left(6\,bc^4d-11\,ac^3d^2\right)x^8+3\left(6\,bc^3d^2-11\,ac^2d^3\right)x^6-315\,bd^5-5\left(3\,bc^2d^3+110\,acd^4\right)x^4-35\left(12\,bcd^4+11\,ad^5\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{3465\,d^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/3465*(8*(6*b*c^5 - 11*a*c^4*d)*x^10 - 4*(6*b*c^4*d - 11*a*c^3*d^2)*x^8 +
3*(6*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 - 315*b*d^5 - 5*(3*b*c^2*d^3 + 110*a*c*d
^4)*x^4 - 35*(12*b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^10)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(92) = 184.
time = 8.13, size = 262, normalized size = 2.52

$$-\frac{ac\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{a\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^3}-\frac{bc\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}-\frac{b\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**7,x)
```

```
[Out] -a*c*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)
**(7/2)/7)/d**3 - a*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/
2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**3 - b*c*(-c**3
*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(
7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - b*(c**4*(c + d/x**2)**(3/2)/3 - 4*c*
*3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)*
*(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(88) = 176.
time = 6.64, size = 490, normalized size = 4.71

$$\frac{16\left(2310\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}ac^{\frac{9}{2}}\mathrm{sgn}(x)+6930\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}bc^{\frac{11}{2}}\mathrm{sgn}(x)-1155\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{9}{2}}d\,\mathrm{sgn}(x)+12474\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{11}{2}}d\,\mathrm{sgn}(x)+231\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{9}{2}}d^2\mathrm{sgn}(x)+15246\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{11}{2}}d^2\mathrm{sgn}(x)-4851\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{9}{2}}d^3\mathrm{sgn}(x)+4950\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{11}{2}}d^3\mathrm{sgn}(x)+2475\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{9}{2}}d^4\mathrm{sgn}(x)+990\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{11}{2}}d^4\mathrm{sgn}(x)+495\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{9}{2}}d^5\mathrm{sgn}(x)-330\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{11}{2}}d^5\mathrm{sgn}(x)+605\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{9}{2}}d^6\mathrm{sgn}(x)+66\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{11}{2}}d^6\mathrm{sgn}(x)-121\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{9}{2}}d^7\mathrm{sgn}(x)-6\,bc^{\frac{11}{2}}d^7\mathrm{sgn}(x)+11\,ac^{\frac{9}{2}}d^8\mathrm{sgn}(x)\right)}{3465\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + d))^16*a*c^(9/2)*sgn(x) + 6930*(sqr
t(c)*x - sqrt(c*x^2 + d))^14*b*c^(11/2)*sgn(x) - 1155*(sqrt(c)*x - sqrt(c*x
^2 + d))^14*a*c^(9/2)*d*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c
^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(9/2)*d^2*sgn(x
) + 15246*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(11/2)*d^2*sgn(x) - 4851*(sq
rt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^3*sgn(x) + 4950*(sqrt(c)*x - sqrt
(c*x^2 + d))^8*b*c^(11/2)*d^3*sgn(x) + 2475*(sqrt(c)*x - sqrt(c*x^2 + d))^8
*a*c^(9/2)*d^4*sgn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(11/2)*d^4*
sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(9/2)*d^5*sgn(x) - 330*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^5*sgn(x) + 605*(sqrt(c)*x - sqrt(
c*x^2 + d))^4*a*c^(9/2)*d^6*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c
^(11/2)*d^6*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(9/2)*d^7*sgn(
x) - 6*b*c^(11/2)*d^7*sgn(x) + 11*a*c^(9/2)*d^8*sgn(x))/((sqrt(c)*x - sqrt(
c*x^2 + d))^2 - d)^11
```

Mupad [B]

time = 6.31, size = 206, normalized size = 1.98

$$\frac{16\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4}-\frac{8\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3}-\frac{10\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{4\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d\,x^4}+\frac{4\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^2\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{231\,d\,x^6}+\frac{2\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{385\,d^2\,x^4}-\frac{8\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x)
```

```
[Out] (16*b*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(315*
d^3) - (10*a*c*(c + d/x^2)^(1/2))/(63*x^6) - (a*d*(c + d/x^2)^(1/2))/(9*x^8
) - (4*b*c*(c + d/x^2)^(1/2))/(33*x^8) - (b*d*(c + d/x^2)^(1/2))/(11*x^10)
- (a*c^2*(c + d/x^2)^(1/2))/(105*d*x^4) + (4*a*c^3*(c + d/x^2)^(1/2))/(315*
d^2*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*b*c^3*(c + d/x^2)^(1/
2))/(385*d^2*x^4) - (8*b*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2)
```

3.952 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=134

$$-\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}-\frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{3d^5}+\frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}-\frac{b(c$$

```
[Out] -1/5*c^3*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^5+1/7*c^2*(-3*a*d+4*b*c)*(c+d/x^2)^(7
/2)/d^5-1/3*c*(-a*d+2*b*c)*(c+d/x^2)^(9/2)/d^5+1/11*(-a*d+4*b*c)*(c+d/x^2)^
(11/2)/d^5-1/13*b*(c+d/x^2)^(13/2)/d^5
```

Rubi [A]
time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$-\frac{c^3\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^5}+\frac{c^2\left(c+\frac{d}{x^2}\right)^{7/2}(4bc-3ad)}{7d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{11/2}(4bc-ad)}{11d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{9/2}(2bc-ad)}{3d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

```
[Out] -1/5*(c^3*(b*c - a*d)*(c + d/x^2)^(5/2))/d^5 + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(7/2))/(7*d^5) - (c*(2*b*c - a*d)*(c + d/x^2)^(9/2))/(3*d^5) + ((4*b*c
 - a*d)*(c + d/x^2)^(11/2))/(11*d^5) - (b*(c + d/x^2)^(13/2))/(13*d^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int x^3(a+bx)(c+dx)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c^3(bc-ad)(c+dx)^{3/2}}{d^4} - \frac{c^2(4bc-3ad)(c+dx)^{5/2}}{d^4} + \frac{3c(2bc-\right.\right.\right.$$
$$= -\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5} + \frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5} - \frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{3d^5}$$

Mathematica [A]
time = 0.26, size = 115, normalized size = 0.86

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)^2\,(13adx^2(-105d^3+70cd^2x^2-40c^2dx^4+16c^3x^6)+b(-1155d^4+840cd^3x^2-560c^2d^2x^4+320c^3dx^6-128c^4x^8))}{15015d^5x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(13*a*d*x^2*(-105*d^3 + 70*c*d^2*x^2 - 40*c^
2*d*x^4 + 16*c^3*x^6) + b*(-1155*d^4 + 840*c*d^3*x^2 - 560*c^2*d^2*x^4 + 32
0*c^3*d*x^6 - 128*c^4*x^8)))/(15015*d^5*x^12)
```

Maple [A]
time = 0.26, size = 118, normalized size = 0.88

method	result
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(208a\,c^3d\,x^8-128b\,c^4x^8-520a\,c^2d^2x^6+320b\,c^3d\,x^6+910ac\,d^3x^4-560b\,c^2d^2x^4-1365a\,d^4x^2+840bc\,d^3x^2-1155b\,d^4\right)\left(c\,x^2+a\right.}{15015d^5x^{10}}$
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(208a\,c^3d\,x^8-128b\,c^4x^8-520a\,c^2d^2x^6+320b\,c^3d\,x^6+910ac\,d^3x^4-560b\,c^2d^2x^4-1365a\,d^4x^2+840bc\,d^3x^2-1155b\,d^4\right)\left(c\,x^2+a\right.}{15015d^5x^{10}}$
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(208a\,c^5d\,x^{12}-128b\,c^6x^{12}-104a\,c^4d^2x^{10}+64b\,c^5d\,x^{10}+78a\,c^3d^3x^8-48b\,c^4d^2x^8-65a\,c^2d^4x^6+40b\,c^3d^3x^6-1820ac\,d^5x^4-3\right.}{15015x^{12}d^5}$
trager	$\frac{\left(208a\,c^5d\,x^{12}-128b\,c^6x^{12}-104a\,c^4d^2x^{10}+64b\,c^5d\,x^{10}+78a\,c^3d^3x^8-48b\,c^4d^2x^8-65a\,c^2d^4x^6+40b\,c^3d^3x^6-1820ac\,d^5x^4-35b\,c^2d^4x^4-1\right.}{15015x^{12}d^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15015*((c*x^2+d)/x^2)^(3/2)*(208*a*c^3*d*x^8-128*b*c^4*x^8-520*a*c^2*d^2*
x^6+320*b*c^3*d*x^6+910*a*c*d^3*x^4-560*b*c^2*d^2*x^4-1365*a*d^4*x^2+840*b*
c*d^3*x^2-1155*b*d^4)*(c*x^2+d)/d^5/x^10
```

Maxima [A]
time = 0.29, size = 152, normalized size = 1.13

$$-\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4}-\frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)a-\frac{1}{15015}\left(\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{d^5}-\frac{5460\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}c}{d^5}+\frac{10010\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c^2}{d^5}-\frac{8580\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^3}{d^5}+\frac{3003\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^4}{d^5}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)^(9/2)*c/d^4 + 495*(c
+ d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^4)*a - 1/15015*(1155*(
c + d/x^2)^(13/2)/d^5 - 5460*(c + d/x^2)^(11/2)*c/d^5 + 10010*(c + d/x^2)^(
9/2)*c^2/d^5 - 8580*(c + d/x^2)^(7/2)*c^3/d^5 + 3003*(c + d/x^2)^(5/2)*c^4/
d^5)*b
```

Fricas [A]
time = 2.79, size = 157, normalized size = 1.17

$$-\frac{\left(16\left(8\,bc^6-13\,ac^5d\right)x^{12}-8\left(8\,bc^5d-13\,ac^4d^2\right)x^{10}+6\left(8\,bc^4d^2-13\,ac^3d^3\right)x^8+1155\,bd^6-5\left(8\,bc^3d^3-13\,ac^2d^4\right)x^6+35\left(bc^2d^4+52\,acd^5\right)x^4+105\left(14\,bcd^5+13\,ad^6\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15015\,d^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/15015*(16*(8*b*c^6 - 13*a*c^5*d)*x^12 - 8*(8*b*c^5*d - 13*a*c^4*d^2)*x^1
0 + 6*(8*b*c^4*d^2 - 13*a*c^3*d^3)*x^8 + 1155*b*d^6 - 5*(8*b*c^3*d^3 - 13*a
*c^2*d^4)*x^6 + 35*(b*c^2*d^4 + 52*a*c*d^5)*x^4 + 105*(14*b*c*d^5 + 13*a*d^
6)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^12)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(121) = 242$.
time = 8.99, size = 326, normalized size = 2.43

$$-\frac{ac\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}-\frac{a\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^4}-\frac{bc\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5}-\frac{b\left(-\frac{c^5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+c^4\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}-\frac{10c^3\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{10c^2\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}-\frac{5c\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{13}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**9,x)
```

```
[Out] -a*c*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c +
 d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - a*(c**4*(c + d/x**2)**(3/
2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c
 + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**4 - b*c*(c**4*(c + d/x**2
)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 -
4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5 - b*(-c**5*(c + d
/x**2)**(3/2)/3 + c**4*(c + d/x**2)**(5/2) - 10*c**3*(c + d/x**2)**(7/2)/7
+ 10*c**2*(c + d/x**2)**(9/2)/9 - 5*c*(c + d/x**2)**(11/2)/11 + (c + d/x**2
)**(13/2)/13)/d**5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(114) = 228$.
time = 9.16, size = 550, normalized size = 4.10

$$\frac{32\left(15015\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{18}ac^{\frac{11}{2}}\mathrm{sgn}(x)+48048\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}bc^{\frac{13}{2}}\mathrm{sgn}(x)-3003\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}ac^{\frac{11}{2}}d\,\mathrm{sgn}(x)+96096\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}bc^{\frac{13}{2}}d\,\mathrm{sgn}(x)-6006\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{11}{2}}d^2\mathrm{sgn}(x)+109824\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{13}{2}}d^2\mathrm{sgn}(x)-28314\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{11}{2}}d^3\mathrm{sgn}(x)+37752\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{13}{2}}d^3\mathrm{sgn}(x)+13728\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{11}{2}}d^4\mathrm{sgn}(x)+5720\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{13}{2}}d^4\mathrm{sgn}(x)+5720\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{11}{2}}d^5\mathrm{sgn}(x)-2288\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{13}{2}}d^5\mathrm{sgn}(x)+3718\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{11}{2}}d^6\mathrm{sgn}(x)+624\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{13}{2}}d^6\mathrm{sgn}(x)-1014\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{11}{2}}d^7\mathrm{sgn}(x)-104\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{13}{2}}d^7\mathrm{sgn}(x)+169\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{11}{2}}d^8\mathrm{sgn}(x)+8bc^{\frac{13}{2}}d^8\mathrm{sgn}(x)-13ac^{\frac{11}{2}}d^9\mathrm{sgn}(x)\right)}{15015\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + d))^18*a*c^(11/2)*sgn(x) + 48048*
(sqrt(c)*x - sqrt(c*x^2 + d))^16*b*c^(13/2)*sgn(x) - 3003*(sqrt(c)*x - sqrt
(c*x^2 + d))^16*a*c^(11/2)*d*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + d))^1
4*b*c^(13/2)*d*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(11/2)*d^
2*sgn(x) + 109824*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c^(13/2)*d^2*sgn(x) -
28314*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(11/2)*d^3*sgn(x) + 37752*(sqrt(
c)*x - sqrt(c*x^2 + d))^10*b*c^(13/2)*d^3*sgn(x) + 13728*(sqrt(c)*x - sqrt(
c*x^2 + d))^10*a*c^(11/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8
*b*c^(13/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(11/2)*d^
5*sgn(x) - 2288*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(13/2)*d^5*sgn(x) + 371
8*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(11/2)*d^6*sgn(x) + 624*(sqrt(c)*x -
sqrt(c*x^2 + d))^4*b*c^(13/2)*d^6*sgn(x) - 1014*(sqrt(c)*x - sqrt(c*x^2 + d
))^4*a*c^(11/2)*d^7*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(13/2)
*d^7*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(11/2)*d^8*sgn(x) + 8
*b*c^(13/2)*d^8*sgn(x) - 13*a*c^(11/2)*d^9*sgn(x))/((sqrt(c)*x - sqrt(c*x^2
 + d))^2 - d)^13
```

Mupad [B]
time = 6.81, size = 248, normalized size = 1.85

$$\frac{16\,a\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4}-\frac{128\,b\,c^6\,\sqrt{c+\frac{d}{x^2}}}{15015\,d^5}-\frac{4\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{14\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{143\,x^{10}}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{13\,x^{12}}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{231\,d\,x^6}+\frac{2\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{385\,d^2\,x^4}-\frac{8\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{429\,d\,x^8}+\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{3003\,d^2\,x^6}-\frac{16\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{5005\,d^3\,x^4}+\frac{64\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{15015\,d^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x)
```

```
[Out] (16*a*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (128*b*c^6*(c + d/x^2)^(1/2))/(15
015*d^5) - (4*a*c*(c + d/x^2)^(1/2))/(33*x^8) - (a*d*(c + d/x^2)^(1/2))/(11
*x^10) - (14*b*c*(c + d/x^2)^(1/2))/(143*x^10) - (b*d*(c + d/x^2)^(1/2))/(1
3*x^12) - (a*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*a*c^3*(c + d/x^2)^(1/2
))/(385*d^2*x^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2) - (b*c^2*(c +
 d/x^2)^(1/2))/(429*d*x^8) + (8*b*c^3*(c + d/x^2)^(1/2))/(3003*d^2*x^6) - (
16*b*c^4*(c + d/x^2)^(1/2))/(5005*d^3*x^4) + (64*b*c^5*(c + d/x^2)^(1/2))/(
15015*d^4*x^2)
```

3.953 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{12} \, dx$

Optimal. Leaf size=150

$$-\frac{16d^3(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{15015c^5}+\frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{3003c^4}-\frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{429c^3}+\frac{(13bc-$$

```
[Out] -16/15015*d^3*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^5/c^5+8/3003*d^2*(-8*a*d+13
*b*c)*(c+d/x^2)^(5/2)*x^7/c^4-2/429*d*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^9/c
^3+1/143*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^11/c^2+1/13*a*(c+d/x^2)^(5/2)*x^
13/c
```

Rubi [A]
time = 0.05, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 270}

$$-\frac{16d^3x^5\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{15015c^5}+\frac{8d^2x^7\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{3003c^4}-\frac{2dx^9\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{429c^3}+\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{143c^2}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

```
[Out] (-16*d^3*(13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^5)/(15015*c^5) + (8*d^2*(13*b
*c - 8*a*d)*(c + d/x^2)^(5/2)*x^7)/(3003*c^4) - (2*d*(13*b*c - 8*a*d)*(c +
d/x^2)^(5/2)*x^9)/(429*c^3) + ((13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^11)/(14
3*c^2) + (a*(c + d/x^2)^(5/2)*x^13)/(13*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{13}}{13c} + \frac{(13bc-8ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx}{13c}$$
$$= \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{13}}{13c} - \frac{(6d(13bc-8ad))\int (c}{143c^2}$$
$$= -\frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{429c^3} + \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a(c+}{}$$
$$= \frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{3003c^4} - \frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{429c^3} + \frac{(13bc}{}$$
$$= -\frac{16d^3(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{15015c^5} + \frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{3003c^4} - \frac{2c}{}$$

Mathematica [A]
time = 0.07, size = 110, normalized size = 0.73

$$\frac{\sqrt{c+\frac{d}{x^2}}\,x(d+cx^2)^2\left(13bc(-16d^3+40cd^2x^2-70c^2dx^4+105c^3x^6)+a(128d^4-320cd^3x^2+560c^2d^2x^4-840c^3dx^6+1155c^4x^8)\right)}{15015c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(13*b*c*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d
*x^4 + 105*c^3*x^6) + a*(128*d^4 - 320*c*d^3*x^2 + 560*c^2*d^2*x^4 - 840*c^
3*d*x^6 + 1155*c^4*x^8)))/(15015*c^5)
```

Maple [A]
time = 0.08, size = 115, normalized size = 0.77

method	result
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3\left(1155a\,x^8c^4-840a\,c^3d\,x^6+1365b\,c^4x^6+560a\,c^2d^2x^4-910b\,c^3d\,x^4-320ac\,d^3x^2+520b\,c^2d^2x^2+128a\,d^4-208bc\,d^3\right)\left(c\,x^2+c\right.}{15015c^5}$
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3\left(1155a\,x^8c^4-840a\,c^3d\,x^6+1365b\,c^4x^6+560a\,c^2d^2x^4-910b\,c^3d\,x^4-320ac\,d^3x^2+520b\,c^2d^2x^2+128a\,d^4-208bc\,d^3\right)\left(c\,x^2+c\right.}{15015c^5}$
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(1155a\,c^6x^{12}+1470a\,c^5d\,x^{10}+1365b\,c^6x^{10}+35a\,c^4d^2x^8+1820b\,c^5d\,x^8-40a\,c^3d^3x^6+65b\,c^4d^2x^6+48a\,c^2d^4x^4-78x^4c^3b\,d^3\right.}{15015c^5}$

trager	$\frac{(1155a\,c^6x^{12}+1470a\,c^5d\,x^{10}+1365b\,c^6x^{10}+35a\,c^4d^2x^8+1820b\,c^5d\,x^8-40a\,c^3d^3x^6+65b\,c^4d^2x^6+48a\,c^2d^4x^4-78x^4c^3b\,d^3-64ac\,d^5x^2}{15015c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)*x^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15015*((c*x^2+d)/x^2)^(3/2)*x^3*(1155*a*c^4*x^8-840*a*c^3*d*x^6+1365*b*c^
4*x^6+560*a*c^2*d^2*x^4-910*b*c^3*d*x^4-320*a*c*d^3*x^2+520*b*c^2*d^2*x^2+1
28*a*d^4-208*b*c*d^3)*(c*x^2+d)/c^5
```

Maxima [A]
time = 0.31, size = 158, normalized size = 1.05

$$\frac{\left(105\,\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\,\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\,\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-231\,\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)b}{1155\,c^4}+\frac{\left(1155\,\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}x^{13}-5460\,\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}dx^{11}+10010\,\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}d^2x^9-8580\,\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^3x^7+3003\,\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^4x^5\right)a}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="maxima")
```

```
[Out] 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/x^2)^(9/2)*d*x^9 + 495*(c
+ d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^3*x^5)*b/c^4 + 1/15015*(11
55*(c + d/x^2)^(13/2)*x^13 - 5460*(c + d/x^2)^(11/2)*d*x^11 + 10010*(c + d/
x^2)^(9/2)*d^2*x^9 - 8580*(c + d/x^2)^(7/2)*d^3*x^7 + 3003*(c + d/x^2)^(5/2
)*d^4*x^5)*a/c^5
```

Fricas [A]
time = 2.67, size = 155, normalized size = 1.03

$$\frac{(1155\,ac^6x^{13}+105\,(13\,bc^6+14\,ac^5d)x^{11}+35\,(52\,bc^5d+ac^4d^2)x^9+5\,(13\,bc^4d^2-8\,ac^3d^3)x^7-6\,(13\,bc^3d^3-8\,ac^2d^4)x^5+8\,(13\,bc^2d^4-8\,acd^5)x^3-16\,(13\,bcd^5-8\,ad^6)x)\sqrt{\frac{cx^2+d}{x^2}}}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*a*c^6*x^13 + 105*(13*b*c^6 + 14*a*c^5*d)*x^11 + 35*(52*b*c^5*
d + a*c^4*d^2)*x^9 + 5*(13*b*c^4*d^2 - 8*a*c^3*d^3)*x^7 - 6*(13*b*c^3*d^3 -
 8*a*c^2*d^4)*x^5 + 8*(13*b*c^2*d^4 - 8*a*c*d^5)*x^3 - 16*(13*b*c*d^5 - 8*a
*d^6)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3351 vs. $2(146) = 292$.
time = 5.94, size = 3351, normalized size = 22.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**12,x)

[Out] 693*a*c**12*d**(51/2)*x**22*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 3528*a*c**11*d**(53/2)*x**20*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 7175*a*c**10*d**(55/2)*x**18*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 7290*a*c**9*d**(57/2)*x**16*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 315*a*c**9*d**(35/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 3699*a*c**8*d**(59/2)*x**14*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 1295*a*c**8*d**(37/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 756*a*c**7*d**(61/2)*x**12*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 1990*a*c**7*d**(39/2)*x**14*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 63*a*c**6*d**(63/2)*x**10*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 1358*a*c**6*d**(41/2)*x**12*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 630*a*c**5*d**(65/2)*x**8*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 343*a*c**5*d**(43/2)*x**10*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 1680*a*c**4*d**(67/2)*x**6*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 35*a*c**4*d**(45/2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 2016*a*c**3*d**(69/2)*x**4*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 280*a*c**3*d**(47/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 1152*a*c**2*d**(71/2)*x**2*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 560*a*c**2*d**(49/2)*x**4

```
*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c
**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 256*a*c*d**(73/
2)*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90
090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*
c**6*d**30) + 448*a*c*d**(51/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x*
*8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2
+ 3465*c**5*d**20) + 128*a*d**(53/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x*
*8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2
+ 3465*c**5*d**20) + 315*b*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c
**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6
*d**19*x**2 + 3465*c**5*d**20) + 1295*b*c**9*d**(35/2)*x**16*sqrt(c*x**2/d
+ 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4
+ 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1990*b*c**8*d**(37/2)*x**14*sq
rt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7
*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1358*b*c**7*d**(39
/2)*x**12*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6
+ 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*b*c
**7*d**(21/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10
*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 343*b*c**6*d**(41/2)*x**10*
sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c*
*7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*b*c**6*d**(2
3/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 9
45*c**5*d**11*x**2 + 315*c**4*d**12) + 35*b*c**5*d**(43/2)*x**8*sqrt(c*x**2
/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**1...
```

Giac [A]
time = 1.11, size = 175, normalized size = 1.17

$$\frac{16\left(13\,bcd^{\frac{11}{2}}-8\,ad^{\frac{13}{2}}\right)\mathrm{sgn}(x)}{15015\,c^5}+\frac{1155\,(cx^2+d)^{\frac{13}{2}}a\mathrm{sgn}(x)+1365\,(cx^2+d)^{\frac{11}{2}}bc\mathrm{sgn}(x)-5460\,(cx^2+d)^{\frac{11}{2}}ad\mathrm{sgn}(x)-5005\,(cx^2+d)^{\frac{9}{2}}bcd\mathrm{sgn}(x)+10010\,(cx^2+d)^{\frac{9}{2}}ad^2\mathrm{sgn}(x)+6435\,(cx^2+d)^{\frac{7}{2}}bcd^2\mathrm{sgn}(x)-8580\,(cx^2+d)^{\frac{7}{2}}ad^3\mathrm{sgn}(x)-3003\,(cx^2+d)^{\frac{5}{2}}bcd^3\mathrm{sgn}(x)+3003\,(cx^2+d)^{\frac{5}{2}}ad^4\mathrm{sgn}(x)}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="giac")
```

```
[Out] 16/15015*(13*b*c*d^(11/2) - 8*a*d^(13/2))*sgn(x)/c^5 + 1/15015*(1155*(c*x^2
 + d)^(13/2)*a*sgn(x) + 1365*(c*x^2 + d)^(11/2)*b*c*sgn(x) - 5460*(c*x^2 +
d)^(11/2)*a*d*sgn(x) - 5005*(c*x^2 + d)^(9/2)*b*c*d*sgn(x) + 10010*(c*x^2 +
 d)^(9/2)*a*d^2*sgn(x) + 6435*(c*x^2 + d)^(7/2)*b*c*d^2*sgn(x) - 8580*(c*x^
2 + d)^(7/2)*a*d^3*sgn(x) - 3003*(c*x^2 + d)^(5/2)*b*c*d^3*sgn(x) + 3003*(c
*x^2 + d)^(5/2)*a*d^4*sgn(x))/c^5
```

Mupad [B]
time = 4.66, size = 137, normalized size = 0.91

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{x\,\left(128\,a\,d^6-208\,b\,c\,d^5\right)}{15015\,c^5}+\frac{x^{11}\,\left(1365\,b\,c^6+1470\,a\,d\,c^5\right)}{15015\,c^5}+\frac{a\,c\,x^{13}}{13}+\frac{d\,x^9\,\left(a\,d+52\,b\,c\right)}{429\,c}-\frac{d^2\,x^7\,\left(8\,a\,d-13\,b\,c\right)}{3003\,c^2}+\frac{2\,d^3\,x^5\,\left(8\,a\,d-13\,b\,c\right)}{5005\,c^3}-\frac{8\,d^4\,x^3\,\left(8\,a\,d-13\,b\,c\right)}{15015\,c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x*(128*a*d^6 - 208*b*c*d^5))/(15015*c^5) + (x^11*(1365*
b*c^6 + 1470*a*c^5*d))/(15015*c^5) + (a*c*x^13)/13 + (d*x^9*(a*d + 52*b*c))
/(429*c) - (d^2*x^7*(8*a*d - 13*b*c))/(3003*c^2) + (2*d^3*x^5*(8*a*d - 13*b
*c))/(5005*c^3) - (8*d^4*x^3*(8*a*d - 13*b*c))/(15015*c^4))
```

3.954 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10} \, dx$

Optimal. Leaf size=117

$$\frac{8d^2(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{3465c^4}-\frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{693c^3}+\frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{11c}$$

```
[Out] 8/3465*d^2*(-6*a*d+11*b*c)*(c+d/x^2)^(5/2)*x^5/c^4-4/693*d*(-6*a*d+11*b*c)*
(c+d/x^2)^(5/2)*x^7/c^3+1/99*(-6*a*d+11*b*c)*(c+d/x^2)^(5/2)*x^9/c^2+1/11*a
*(c+d/x^2)^(5/2)*x^11/c
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 270}

$$\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{3465c^4}-\frac{4dx^7\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{693c^3}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{99c^2}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

```
[Out] (8*d^2*(11*b*c - 6*a*d)*(c + d/x^2)^(5/2)*x^5)/(3465*c^4) - (4*d*(11*b*c -
6*a*d)*(c + d/x^2)^(5/2)*x^7)/(693*c^3) + ((11*b*c - 6*a*d)*(c + d/x^2)^(5/
2)*x^9)/(99*c^2) + (a*(c + d/x^2)^(5/2)*x^11)/(11*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{10}\,dx=\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{11c}+\frac{(11bc-6ad)\int\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx}{11c}$$
$$=\frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{11c}-\frac{(4d(11bc-6ad))\int\left(c\right.}{99c^2}$$
$$=-\frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{693c^3}+\frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{99c^2}+\frac{a\left(c+\right.}{1}$$
$$=\frac{8d^2(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{3465c^4}-\frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{693c^3}+\frac{(11bc}{}$$

Mathematica [A]
time = 0.06, size = 89, normalized size = 0.76

$$\frac{\sqrt{c+\frac{d}{x^2}}\,x(d+cx^2)^2\left(11bc(8d^2-20cdx^2+35c^2x^4)+3a(-16d^3+40cd^2x^2-70c^2dx^4+105c^3x^6)\right)}{3465c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(11*b*c*(8*d^2 - 20*c*d*x^2 + 35*c^2*x^4)
+ 3*a*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d*x^4 + 105*c^3*x^6)))/(3465*c^4)
```

Maple [A]
time = 0.08, size = 91, normalized size = 0.78

method	result
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^3\left(315x^6a\,c^3-210x^4a\,c^2d+385x^4b\,c^3+120ac\,d^2x^2-220b\,c^2d\,x^2-48a\,d^3+88bc\,d^2\right)\left(c\,x^2+d\right)}{3465c^4}$
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^3\left(315x^6a\,c^3-210x^4a\,c^2d+385x^4b\,c^3+120ac\,d^2x^2-220b\,c^2d\,x^2-48a\,d^3+88bc\,d^2\right)\left(c\,x^2+d\right)}{3465c^4}$
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(315a\,c^5x^{10}+420a\,c^4d\,x^8+385b\,c^5x^8+15a\,c^3d^2x^6+550b\,c^4d\,x^6-18a\,c^2d^3x^4+33b\,c^3d^2x^4+24ac\,d^4x^2-44b\,c^2d^3x^2-48a\,d^5\right.}{3465c^4}$
trager	$\frac{\left(315a\,c^5x^{10}+420a\,c^4d\,x^8+385b\,c^5x^8+15a\,c^3d^2x^6+550b\,c^4d\,x^6-18a\,c^2d^3x^4+33b\,c^3d^2x^4+24ac\,d^4x^2-44b\,c^2d^3x^2-48a\,d^5+88bc\,d^4\right)x\,\sqrt{}}{3465c^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)*x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(3/2)*x^3*(315*a*c^3*x^6-210*a*c^2*d*x^4+385*b*c^3*x
^4+120*a*c*d^2*x^2-220*b*c^2*d*x^2-48*a*d^3+88*b*c*d^2)*(c*x^2+d)/c^4
```

Maxima [A]
time = 0.28, size = 124, normalized size = 1.06

$$\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)b}{315\,c^3}+\frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)a}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="maxima")
```

```
[Out] 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^
2)^(5/2)*d^2*x^5)*b/c^3 + 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/
x^2)^(9/2)*d*x^9 + 495*(c + d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^
3*x^5)*a/c^4
```

Fricas [A]
time = 1.72, size = 132, normalized size = 1.13

$$\frac{\left(315\,ac^5x^{11}+35\,(11\,bc^5+12\,ac^4d)x^9+5\,(110\,bc^4d+3\,ac^3d^2)x^7+3\,(11\,bc^3d^2-6\,ac^2d^3)x^5-4\,(11\,bc^2d^3-6\,acd^4)x^3+8\,(11\,bcd^4-6\,ad^5)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3465\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="fricas")
```

```
[Out] 1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + 12*a*c^4*d)*x^9 + 5*(110*b*c^4*d +
3*a*c^3*d^2)*x^7 + 3*(11*b*c^3*d^2 - 6*a*c^2*d^3)*x^5 - 4*(11*b*c^2*d^3 - 6
*a*c*d^4)*x^3 + 8*(11*b*c*d^4 - 6*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2304 vs. $2(112) = 224$.
time = 4.65, size = 2304, normalized size = 19.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**10,x)
```

```
[Out] 315*a*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 1295*a*c**9*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16
*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x*
*2 + 3465*c**5*d**20) + 1990*a*c**8*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/(346
5*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c
```

```
**6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**7*d**(39/2)*x**12*sqrt(c*x**2
/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x*
*4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**7*d**(21/2)*x**14*s
qrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**1
1*x**2 + 315*c**4*d**12) + 343*a*c**6*d**(41/2)*x**10*sqrt(c*x**2/d + 1)/(3
465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860
*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*a*c**6*d**(23/2)*x**12*sqrt(c*x**
2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 +
315*c**4*d**12) + 35*a*c**5*d**(43/2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d*
*16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19
*x**2 + 3465*c**5*d**20) + 114*a*c**5*d**(25/2)*x**10*sqrt(c*x**2/d + 1)/(3
15*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d*
*12) + 280*a*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 +
 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 34
65*c**5*d**20) + 40*a*c**4*d**(27/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9
*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 560*a
*c**3*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*
d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**2
0) - 5*a*c**3*d**(29/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c
**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 448*a*c**2*d**(49/
2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 30*a*c**
2*d**(31/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x*
*4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 128*a*c*d**(51/2)*sqrt(c*x**2/
d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**
4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 40*a*c*d**(33/2)*x**2*sqrt(c
*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**
2 + 315*c**4*d**12) - 16*a*d**(35/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6
 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 35*b*c**8*
d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**
4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*b*c**7*d**(21/2)*x**12*sqrt
(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x
**2 + 315*c**4*d**12) + 114*b*c**6*d**(23/2)*x**10*sqrt(c*x**2/d + 1)/(315*
c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12
) + 40*b*c**5*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c
**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(11/2
)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) - 5*b*c**4*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6
+ 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 33*b*c**4*d
**(13/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 +
 105*c**3*d**6) - 30*b*c**3*d**(29/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**
9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 17*b
*c**3*d**(15/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5
*x**2 + 105*c**3*d**6) - 40*b*c**2*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(315*c
**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
```

```
 + 3*b*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**
4*d**5*x**2 + 105*c**3*d**6) - 16*b*c*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**
7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) +
 12*b*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d*
*5*x**2 + 105*c**3*d**6) + 8*b*d**(21/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*
x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)
```

Giac [A]
time = 1.46, size = 140, normalized size = 1.20

$$-\frac{8\left(11\,bcd^{\frac{9}{2}}-6\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^4}+\frac{315\,(cx^2+d)^{\frac{11}{2}}a\mathrm{sgn}(x)+385\,(cx^2+d)^{\frac{9}{2}}bc\mathrm{sgn}(x)-1155\,(cx^2+d)^{\frac{9}{2}}ad\mathrm{sgn}(x)-990\,(cx^2+d)^{\frac{7}{2}}bcd\mathrm{sgn}(x)+1485\,(cx^2+d)^{\frac{7}{2}}ad^2\mathrm{sgn}(x)+693\,(cx^2+d)^{\frac{5}{2}}bcd^2\mathrm{sgn}(x)-693\,(cx^2+d)^{\frac{5}{2}}ad^3\mathrm{sgn}(x)}{3465\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="giac")
```

```
[Out] -8/3465*(11*b*c*d^(9/2) - 6*a*d^(11/2))*sgn(x)/c^4 + 1/3465*(315*(c*x^2 + d
)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1155*(c*x^2 + d)^(9/
2)*a*d*sgn(x) - 990*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 1485*(c*x^2 + d)^(7/2)
*a*d^2*sgn(x) + 693*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 693*(c*x^2 + d)^(5/2
)*a*d^3*sgn(x))/c^4
```

Mupad [B]
time = 4.57, size = 118, normalized size = 1.01

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{x^9\,\left(385\,b\,c^5+420\,a\,d\,c^4\right)}{3465\,c^4}-\frac{x\,\left(48\,a\,d^5-88\,b\,c\,d^4\right)}{3465\,c^4}+\frac{a\,c\,x^{11}}{11}+\frac{d\,x^7\,\left(3\,a\,d+110\,b\,c\right)}{693\,c}-\frac{d^2\,x^5\,\left(6\,a\,d-11\,b\,c\right)}{1155\,c^2}+\frac{4\,d^3\,x^3\,\left(6\,a\,d-11\,b\,c\right)}{3465\,c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x^9*(385*b*c^5 + 420*a*c^4*d))/(3465*c^4) - (x*(48*a*d^
5 - 88*b*c*d^4))/(3465*c^4) + (a*c*x^11)/11 + (d*x^7*(3*a*d + 110*b*c))/(69
3*c) - (d^2*x^5*(6*a*d - 11*b*c))/(1155*c^2) + (4*d^3*x^3*(6*a*d - 11*b*c))
/(3465*c^3))
```

3.955 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx$

Optimal. Leaf size=84

$$-\frac{2d(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{315c^3}+\frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{63c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{9c}$$

```
[Out] -2/315*d*(-4*a*d+9*b*c)*(c+d/x^2)^(5/2)*x^5/c^3+1/63*(-4*a*d+9*b*c)*(c+d/x^
2)^(5/2)*x^7/c^2+1/9*a*(c+d/x^2)^(5/2)*x^9/c
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 270}

$$-\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2}(9bc-4ad)}{315c^3}+\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}(9bc-4ad)}{63c^2}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

```
[Out] (-2*d*(9*b*c - 4*a*d)*(c + d/x^2)^(5/2)*x^5)/(315*c^3) + ((9*b*c - 4*a*d)*(
c + d/x^2)^(5/2)*x^7)/(63*c^2) + (a*(c + d/x^2)^(5/2)*x^9)/(9*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{9c} + \frac{(9bc-4ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} x^6\,dx}{9c}$$
$$= \frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{63c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{9c} - \frac{(2d(9bc-4ad))\int \left(c+\right.}{63c^2}$$
$$= -\frac{2d(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{315c^3} + \frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{63c^2} + \frac{a\left(c+\frac{d}{x^2}\right.}{9c}$$

Mathematica [A]
time = 0.05, size = 66, normalized size = 0.79

$$\frac{\sqrt{c+\frac{d}{x^2}}\,x(d+cx^2)^2\left(9bc(-2d+5cx^2)+a(8d^2-20cdx^2+35c^2x^4)\right)}{315c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(9*b*c*(-2*d + 5*c*x^2) + a*(8*d^2 - 20*c*
d*x^2 + 35*c^2*x^4)))/(315*c^3)
```

Maple [A]
time = 0.07, size = 67, normalized size = 0.80

method	result	size
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3\left(35a\,c^2x^4-20acd\,x^2+45b\,c^2x^2+8a\,d^2-18bcd\right)\left(c\,x^2+d\right)}{315c^3}$	67
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3\left(35a\,c^2x^4-20acd\,x^2+45b\,c^2x^2+8a\,d^2-18bcd\right)\left(c\,x^2+d\right)}{315c^3}$	67
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(35a\,x^8c^4+50a\,c^3d\,x^6+45b\,c^4x^6+3a\,c^2d^2x^4+72b\,c^3d\,x^4-4ac\,d^3x^2+9b\,c^2d^2x^2+8a\,d^4-18bc\,d^3\right)}{315c^3}$	106
trager	$\frac{\left(35a\,x^8c^4+50a\,c^3d\,x^6+45b\,c^4x^6+3a\,c^2d^2x^4+72b\,c^3d\,x^4-4ac\,d^3x^2+9b\,c^2d^2x^2+8a\,d^4-18bc\,d^3\right)x\sqrt{-\frac{-c\,x^2-d}{x^2}}}{315c^3}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)*x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(3/2)*x^3*(35*a*c^2*x^4-20*a*c*d*x^2+45*b*c^2*x^2+8*a
*d^2-18*b*c*d)*(c*x^2+d)/c^3
```

Maxima [A]
time = 0.29, size = 90, normalized size = 1.07

$$\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)b}{35\,c^2}+\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)a}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="maxima")
```

```
[Out] 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^2)^(5/2)*d*x^5)*b/c^2 + 1/315*(3
5*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^2)^(5/2)
*d^2*x^5)*a/c^3
```

Fricas [A]
time = 2.64, size = 106, normalized size = 1.26

$$\frac{\left(35\,ac^4x^9+5\,(9\,bc^4+10\,ac^3d)x^7+3\,(24\,bc^3d+ac^2d^2)x^5+(9\,bc^2d^2-4\,acd^3)x^3-2\,(9\,bcd^3-4\,ad^4)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="fricas")
```

```
[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + 10*a*c^3*d)*x^7 + 3*(24*b*c^3*d + a*c^2*
d^2)*x^5 + (9*b*c^2*d^2 - 4*a*c*d^3)*x^3 - 2*(9*b*c*d^3 - 4*a*d^4)*x)*sqrt(
(c*x^2 + d)/x^2)/c^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1340 vs. 2(78) = 156.
time = 3.58, size = 1340, normalized size = 15.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**8,x)
```

```
[Out] 35*a*c**8*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6
*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**7*d**(21/2)*
x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c*
*5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**6*d**(23/2)*x**10*sqrt(c*x**2/d
+ 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*
c**4*d**12) + 40*a*c**5*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x*
*6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*a*c**
5*d**(11/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x*
*2 + 105*c**3*d**6) - 5*a*c**4*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*
d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 3
```

```
3*a*c**4*d**(13/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d
**5*x**2 + 105*c**3*d**6) - 30*a*c**3*d**(29/2)*x**4*sqrt(c*x**2/d + 1)/(31
5*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**
12) + 17*a*c**3*d**(15/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210
*c**4*d**5*x**2 + 105*c**3*d**6) - 40*a*c**2*d**(31/2)*x**2*sqrt(c*x**2/d +
 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c
**4*d**12) + 3*a*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 16*a*c*d**(33/2)*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12) + 12*a*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 2
10*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(21/2)*sqrt(c*x**2/d + 1)/(105*
c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 15*b*c**6*d**(9/2)*x
**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
*d**6) + 33*b*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 +
210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*b*c**4*d**(13/2)*x**6*sqrt(c*x**2/
d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**3
*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2
 + 105*c**3*d**6) + 12*b*c**2*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*b*c*d**(19/2)*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + b*d**(
3/2)*x**4*sqrt(c*x**2/d + 1)/5 + b*d**(5/2)*x**2*sqrt(c*x**2/d + 1)/(15*c)
- 2*b*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

Giac [A]
time = 0.85, size = 105, normalized size = 1.25

$$\frac{2\left(9\,bcd^{\frac{7}{2}}-4\,ad^{\frac{9}{2}}\right)\mathrm{sgn}(x)}{315\,c^3}+\frac{35\,(cx^2+d)^{\frac{9}{2}}a\mathrm{sgn}(x)+45\,(cx^2+d)^{\frac{7}{2}}bc\mathrm{sgn}(x)-90\,(cx^2+d)^{\frac{7}{2}}ad\mathrm{sgn}(x)-63\,(cx^2+d)^{\frac{5}{2}}bcd\mathrm{sgn}(x)+63\,(cx^2+d)^{\frac{5}{2}}ad^2\mathrm{sgn}(x)}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="giac")
```

```
[Out] 2/315*(9*b*c*d^(7/2) - 4*a*d^(9/2))*sgn(x)/c^3 + 1/315*(35*(c*x^2 + d)^(9/2
)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 90*(c*x^2 + d)^(7/2)*a*d*sgn
(x) - 63*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 63*(c*x^2 + d)^(5/2)*a*d^2*sgn(x)
)/c^3
```

Mupad [B]
time = 4.55, size = 97, normalized size = 1.15

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{x\,\left(8\,a\,d^4-18\,b\,c\,d^3\right)}{315\,c^3}+\frac{x^7\,\left(45\,b\,c^4+50\,a\,d\,c^3\right)}{315\,c^3}+\frac{a\,c\,x^9}{9}+\frac{d\,x^5\,\left(a\,d+24\,b\,c\right)}{105\,c}-\frac{d^2\,x^3\,\left(4\,a\,d-9\,b\,c\right)}{315\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x*(8*a*d^4 - 18*b*c*d^3))/(315*c^3) + (x^7*(45*b*c^4 +
50*a*c^3*d))/(315*c^3) + (a*c*x^9)/9 + (d*x^5*(a*d + 24*b*c))/(105*c) - (d^
2*x^3*(4*a*d - 9*b*c))/(315*c^2))
```

3.956 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx$

Optimal. Leaf size=53

$$\frac{(7bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{7c}$$

```
[Out] 1/35*(-2*a*d+7*b*c)*(c+d/x^2)^(5/2)*x^5/c^2+1/7*a*(c+d/x^2)^(5/2)*x^7/c
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {464, 270}

$$\frac{x^5\left(c+\frac{d}{x^2}\right)^{5/2}(7bc-2ad)}{35c^2}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

```
[Out] ((7*b*c - 2*a*d)*(c + d/x^2)^(5/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(5/2)*x^7
)/(7*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{7c}+\frac{(7bc-2ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx}{7c}\\ &= \frac{(7bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{7c}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.83

$$\frac{\sqrt{c+\frac{d}{x^2}}\,x(d+cx^2)^2\,(7bc-2ad+5acx^2)}{35c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(7*b*c - 2*a*d + 5*a*c*x^2))/(35*c^2)
```

Maple [A]
time = 0.06, size = 45, normalized size = 0.85

method	result	size
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3\,(5c\,x^2 a-2ad+7bc)\,(c\,x^2+d)}{35c^2}$	45
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3\,(5c\,x^2 a-2ad+7bc)\,(c\,x^2+d)}{35c^2}$	45
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\;x\,(5x^6 a\,c^3+8x^4 a\,c^2 d+7x^4 b\,c^3+ac\,d^2 x^2+14b\,c^2 d\,x^2-2a\,d^3+7bc\,d^2)}{35c^2}$	81
trager	$\frac{(5x^6 a\,c^3+8x^4 a\,c^2 d+7x^4 b\,c^3+ac\,d^2 x^2+14b\,c^2 d\,x^2-2a\,d^3+7bc\,d^2)\,x\sqrt{-\frac{-c\,x^2-d}{x^2}}}{35c^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)*x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*((c*x^2+d)/x^2)^(3/2)*x^3*(5*a*c*x^2-2*a*d+7*b*c)*(c*x^2+d)/c^2
```

Maxima [A]
time = 0.29, size = 55, normalized size = 1.04

$$\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5\,c}+\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)a}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b*(c + d/x^2)^(5/2)*x^5/c + 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^
2)^(5/2)*d*x^5)*a/c^2
```

Fricas [A]
time = 3.18, size = 80, normalized size = 1.51

$$\frac{\left(5\,ac^3x^7+\left(7\,bc^3+8\,ac^2d\right)x^5+\left(14\,bc^2d+acd^2\right)x^3+\left(7\,bcd^2-2\,ad^3\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a*c^3*x^7 + (7*b*c^3 + 8*a*c^2*d)*x^5 + (14*b*c^2*d + a*c*d^2)*x^3
+ (7*b*c*d^2 - 2*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(46) = 92$.
time = 2.73, size = 498, normalized size = 9.40

$$\frac{15ac^6d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{33ac^5d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{17ac^4d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{3ac^3d^{\frac{15}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{12ac^2d^{\frac{17}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{8acd^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{ad^{\frac{3}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{5}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c}-\frac{2ad^{\frac{7}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}+\frac{bc\sqrt{d}\,x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{2bd^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{bd^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**6,x)
```

```
[Out] 15*a*c**6*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*
d**5*x**2 + 105*c**3*d**6) + 33*a*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**4*d**(13/
2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) + 3*a*c**3*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
+ 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c**2*d**(17/2)*x**2*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*c*
d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105
*c**3*d**6) + a*d**(3/2)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(5/2)*x**2*sqrt(c
*x**2/d + 1)/(15*c) - 2*a*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*c*sqrt(
d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*b*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/5 + b*d
**(5/2)*sqrt(c*x**2/d + 1)/(5*c)
```

Giac [A]
time = 1.41, size = 72, normalized size = 1.36

$$-\frac{\left(7\,bcd^{\frac{5}{2}}-2\,ad^{\frac{7}{2}}\right)\mathrm{sgn}\left(x\right)}{35\,c^2}+\frac{5\,\left(cx^2+d\right)^{\frac{7}{2}}a\mathrm{sgn}\left(x\right)+7\,\left(cx^2+d\right)^{\frac{5}{2}}bc\mathrm{sgn}\left(x\right)-7\,\left(cx^2+d\right)^{\frac{5}{2}}ad\mathrm{sgn}\left(x\right)}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="giac")
```

```
[Out] -1/35*(7*b*c*d^(5/2) - 2*a*d^(7/2))*sgn(x)/c^2 + 1/35*(5*(c*x^2 + d)^(7/2)*
a*sgn(x) + 7*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 7*(c*x^2 + d)^(5/2)*a*d*sgn(x))
/c^2
```

Mupad [B]
time = 4.63, size = 77, normalized size = 1.45

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{x^5\,\left(7\,b\,c^3+8\,a\,d\,c^2\right)}{35\,c^2}-\frac{x\,\left(2\,a\,d^3-7\,b\,c\,d^2\right)}{35\,c^2}+\frac{a\,c\,x^7}{7}+\frac{d\,x^3\,\left(a\,d+14\,b\,c\right)}{35\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x^2)*(c + d/x^2)^(3/2),x)
[Out] (c + d/x^2)^(1/2)*((x^5*(7*b*c^3 + 8*a*c^2*d))/(35*c^2) - (x*(2*a*d^3 - 7*b
*c*d^2))/(35*c^2) + (a*c*x^7)/7 + (d*x^3*(a*d + 14*b*c))/(35*c))
```

3.957 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx$

Optimal. Leaf size=86

$$bd\sqrt{c+\frac{d}{x^2}}\,x+\frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2}x^3+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{5c}-bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)$$

```
[Out] 1/3*b*(c+d/x^2)^(3/2)*x^3+1/5*a*(c+d/x^2)^(5/2)*x^5/c-b*d^(3/2)*arctanh(d^(
1/2)/x/(c+d/x^2)^(1/2))+b*d*x*(c+d/x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {462, 342, 283, 223, 212}

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+bdx\sqrt{c+\frac{d}{x^2}}+\frac{1}{3}bx^3\left(c+\frac{d}{x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

```
[Out] b*d*Sqrt[c + d/x^2]*x + (b*(c + d/x^2)^(3/2)*x^3)/3 + (a*(c + d/x^2)^(5/2)*
x^5)/(5*c) - b*d^(3/2)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} + b\int \left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - b\text{Subst}\left(\int \frac{(c+dx^2)^{3/2}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - (bd)\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2}\,dx, x, \right.$$

$$= bd\sqrt{c+\frac{d}{x^2}}\,x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - \left(bd^2\right)\text{Subst}\left(\int \right.$$

$$= bd\sqrt{c+\frac{d}{x^2}}\,x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - \left(bd^2\right)\text{Subst}\left(\int \right.$$

$$= bd\sqrt{c+\frac{d}{x^2}}\,x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - bd^{3/2}\tanh^{-1}\left(\right.$$

Mathematica [A]
time = 0.12, size = 81, normalized size = 0.94

$$\frac{1}{15}\sqrt{c+\frac{d}{x^2}}\,x\left(\frac{3a(d+cx^2)^2}{c} + 5b(4d+cx^2) - \frac{15bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((3*a*(d + c*x^2)^2)/c + 5*b*(4*d + c*x^2) - (15*b*d^(3/
2)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/15
```

Maple [A]
time = 0.05, size = 99, normalized size = 1.15

method	result	size
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(-3 a \left(c\,x^{2}+d\right)^{\frac{5}{2}}+15 d^{\frac{3}{2}} \ln \left(\frac{2 d+2 \sqrt{d}\, \sqrt{c\,x^{2}+d}}{x}\right) b c-5 \left(c\,x^{2}+d\right)^{\frac{3}{2}} b c-15 \sqrt{c\,x^{2}+d}\, b c d\right)}{15 \left(c\,x^{2}+d\right)^{\frac{3}{2}} c}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)*x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*((c*x^2+d)/x^2)^(3/2)*x^3*(-3*a*(c*x^2+d)^(5/2)+15*d^(3/2)*ln(2*(d^(1
/2)*(c*x^2+d)^(1/2)+d)/x)*b*c-5*(c*x^2+d)^(3/2)*b*c-15*(c*x^2+d)^(1/2)*b*c*
d)/(c*x^2+d)^(3/2)/c
```

Maxima [A]
time = 0.51, size = 91, normalized size = 1.06

$$\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}} x^5}{5\,c}+\frac{1}{6}\left(2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}} x^3+6\sqrt{c+\frac{d}{x^2}}\,dx+3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/5*a*(c + d/x^2)^(5/2)*x^5/c + 1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c + d
/x^2)*d*x + 3*d^(3/2)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x
+ sqrt(d))))*b
```

Fricas [A]
time = 3.33, size = 203, normalized size = 2.36

$$\left[\frac{15\,bcd^{\frac{3}{2}}\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(3\,ac^2x^5+(5\,bc^2+6\,acd)x^3+(20\,bcd+3\,ad^2)x)\sqrt{\frac{cx^2+d}{x^2}}}{30\,c},\ \frac{15\,bc\sqrt{-d}\,d\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(3\,ac^2x^5+(5\,bc^2+6\,acd)x^3+(20\,bcd+3\,ad^2)x)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b*c*d^(3/2)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d
)/x^2) + 2*(3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20*b*c*d + 3*a*d^2)*x)
*sqrt((c*x^2 + d)/x^2))/c, 1/15*(15*b*c*sqrt(-d)*d*arctan(sqrt(-d)*x*sqrt((
c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20
*b*c*d + 3*a*d^2)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(75) = 150$.
time = 2.50, size = 184, normalized size = 2.14

$$\frac{ac\sqrt{d}\,x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{2ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c}+\frac{b\sqrt{c}\,dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{bc\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3}-bd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)+\frac{bd^2}{\sqrt{c}\,x\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**4,x)
```

```
[Out] a*c*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*a*d**(3/2)*x**2*sqrt(c*x**2/d + 1
)/5 + a*d**(5/2)*sqrt(c*x**2/d + 1)/(5*c) + b*sqrt(c)*d*x/sqrt(1 + d/(c*x**
2)) + b*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)
/3 - b*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) + b*d**2/(sqrt(c)*x*sqrt(1 + d/(
c*x**2)))
```

Giac [A]
time = 2.03, size = 140, normalized size = 1.63

$$\frac{bd^2\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\operatorname{sgn}(x)}{\sqrt{-d}}-\frac{\left(15\,bcd^2\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+20\,bc\sqrt{-d}\,d^{\frac{3}{2}}+3\,a\sqrt{-d}\,d^{\frac{5}{2}}\right)\operatorname{sgn}(x)}{15\,c\sqrt{-d}}+\frac{3\,(cx^2+d)^{\frac{5}{2}}ac^4\operatorname{sgn}(x)+5\,(cx^2+d)^{\frac{3}{2}}bc^5\operatorname{sgn}(x)+15\,\sqrt{cx^2+d}\,bc^5d\operatorname{sgn}(x)}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] b*d^2*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/15*(15*b*c*d^2*a
rctan(sqrt(d)/sqrt(-d)) + 20*b*c*sqrt(-d)*d^(3/2) + 3*a*sqrt(-d)*d^(5/2))*s
gn(x)/(c*sqrt(-d)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^4*sgn(x) + 5*(c*x^2 + d)
^(3/2)*b*c^5*sgn(x) + 15*sqrt(c*x^2 + d)*b*c^5*d*sgn(x))/c^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

3.958 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^2 \, dx$

Optimal. Leaf size=121

$$-\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{1}{2}\sqrt{d}\,(3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)$$

```
[Out] 1/3*(2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x/c+1/3*a*(c+d/x^2)^(5/2)*x^3/c-1/2*(2*a*
d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))*d^(1/2)-1/2*d*(2*a*d+3*b*c)*(c+
d/x^2)^(1/2)/c/x
```

Rubi [A]
time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {464, 248, 283, 201, 223, 212}

$$\frac{x\left(c+\frac{d}{x^2}\right)^{3/2}(2ad+3bc)}{3c}-\frac{d\sqrt{c+\frac{d}{x^2}}\,(2ad+3bc)}{2cx}-\frac{1}{2}\sqrt{d}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

```
[Out] -1/2*(d*(3*b*c + 2*a*d)*Sqrt[c + d/x^2])/(c*x) + ((3*b*c + 2*a*d)*(c + d/x^
2)^(3/2)*x)/(3*c) + (a*(c + d/x^2)^(5/2)*x^3)/(3*c) - (Sqrt[d]*(3*b*c + 2*a
*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}+\frac{(3bc+2ad)\int\left(c+\frac{d}{x^2}\right)^{3/2}dx}{3c}$$

$$=\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{(3bc+2ad)\mathrm{Subst}\left(\int\frac{(c+dx^2)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)}{3c}$$

$$=\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{(d(3bc+2ad))\mathrm{Subst}\left(\int\sqrt{c\ldots\right.}{c}$$

$$=-\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\ldots$$

$$=-\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\ldots$$

$$=-\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\ldots$$

Mathematica [A]
time = 0.13, size = 105, normalized size = 0.87

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(\sqrt{d+cx^2}\left(-3bd+6bcx^2+8adx^2+2acx^4\right)-3\sqrt{d}\,(3bc+2ad)x^2\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)\right)}{6x\sqrt{d+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*(Sqrt[d + c*x^2]*(-3*b*d + 6*b*c*x^2 + 8*a*d*x^2 + 2*a*c*x
^4) - 3*Sqrt[d]*(3*b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(6*x
*Sqrt[d + c*x^2])
```

Maple [A]
time = 0.08, size = 170, normalized size = 1.40

method	result

risch	$-\frac{bd\sqrt{\frac{cx^2+d}{x^2}}}{2x}+\frac{\left(\frac{cax^2\sqrt{cx^2+d}}{3}+\frac{4ad\sqrt{cx^2+d}}{3}+\sqrt{cx^2+d}\,bc-d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{cx^2+d}}{x}\right)a-\frac{3\sqrt{d}}{}\right.}{\sqrt{cx^2+d}}$
default	$-\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}x\left(6d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{cx^2+d}}{x}\right)ax^2+9d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{cx^2+d}}{x}\right)bcx^2-2(cx^2+d)^{\frac{3}{2}}adx^2-3(cx^2+d\right.}{6(cx^2+d)^{\frac{3}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((c*x^2+d)/x^2)^(3/2)*x*(6*d^(5/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)
*a*x^2+9*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c*x^2-2*(c*x^2+d)^(3
/2)*a*d*x^2-3*(c*x^2+d)^(3/2)*b*c*x^2+3*(c*x^2+d)^(5/2)*b-6*(c*x^2+d)^(1/2)
*a*d^2*x^2-9*(c*x^2+d)^(1/2)*b*c*d*x^2)/(c*x^2+d)^(3/2)/d
```

Maxima [A]
time = 0.49, size = 163, normalized size = 1.35

$$\frac{1}{6}\left(2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3+6\sqrt{c+\frac{d}{x^2}}\,dx+3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)a+\frac{1}{4}\left(4\sqrt{c+\frac{d}{x^2}}\,cx-\frac{2\sqrt{c+\frac{d}{x^2}}\,cdx}{(c+\frac{d}{x^2})x^2-d}+3\,c\sqrt{d}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c + d/x^2)*d*x + 3*d^(3/2)*log((sqrt(
c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*a + 1/4*(4*sqrt(c +
 d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d) + 3*c*sqrt(d)*l
og((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

Fricas [A]
time = 3.41, size = 190, normalized size = 1.57

$$\left[\frac{3\,(3\,bc+2\,ad)\sqrt{d}\,x\log\left(-\frac{cx^2-2\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,acx^4+2\,(3\,bc+4\,ad)x^2-3\,bd)\sqrt{\frac{cx^2+d}{x^2}}}{12\,x},\ \frac{3\,(3\,bc+2\,ad)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(2\,acx^4+2\,(3\,bc+4\,ad)x^2-3\,bd)\sqrt{\frac{cx^2+d}{x^2}}}{6\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 +
d)/x^2) + 2*d)/x^2) + 2*(2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d)*sqrt((c
```

```
*x^2 + d)/x^2))/x, 1/6*(3*(3*b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt
((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d
)*sqrt((c*x^2 + d)/x^2))/x]
```

Sympy [A]
time = 3.58, size = 202, normalized size = 1.67

$$\frac{a\sqrt{c}\,dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{ac\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3}-ad^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)+\frac{ad^2}{\sqrt{c}\,x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}}-\frac{b\sqrt{c}\,d\sqrt{1+\frac{d}{cx^2}}}{2x}+\frac{b\sqrt{c}\,d}{x\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**2,x)
```

```
[Out] a*sqrt(c)*d*x/sqrt(1 + d/(c*x**2)) + a*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3
+ a*d**(3/2)*sqrt(c*x**2/d + 1)/3 - a*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) +
 a*d**2/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) + b*c**(3/2)*x/sqrt(1 + d/(c*x**2)
) - b*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x) + b*sqrt(c)*d/(x*sqrt(1 + d/(c*x
**2))) - 3*b*c*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x))/2
```

Giac [A]
time = 1.32, size = 115, normalized size = 0.95

$$\frac{2\,(cx^2+d)^{\frac{3}{2}}ac\operatorname{sgn}(x)+6\,\sqrt{cx^2+d}\,bc^2\operatorname{sgn}(x)+6\,\sqrt{cx^2+d}\,acd\operatorname{sgn}(x)-\frac{3\,\sqrt{cx^2+d}\,bcd\operatorname{sgn}(x)}{x^2}+\frac{3\,(3\,bc^2d\operatorname{sgn}(x)+2\,acd^2\operatorname{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*(c*x^2 + d)^(3/2)*a*c*sgn(x) + 6*sqrt(c*x^2 + d)*b*c^2*sgn(x) + 6*sq
rt(c*x^2 + d)*a*c*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c*d*sgn(x)/x^2 + 3*(3*b*c^
2*d*sgn(x) + 2*a*c*d^2*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

3.959 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} dx$

Optimal. Leaf size=112

$$-\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x}-\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}-\frac{3c(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8\sqrt{d}}$$

```
[Out] -1/4*(4*a*d+b*c)*(c+d/x^2)^(3/2)/c/x+a*(c+d/x^2)^(5/2)*x/c-3/8*c*(4*a*d+b*c
)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(1/2)-3/8*(4*a*d+b*c)*(c+d/x^2)^(1/2
)/x
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {382, 464, 201, 223, 212}

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(4ad+bc)}{4cx}-\frac{3\sqrt{c+\frac{d}{x^2}}\,(4ad+bc)}{8x}-\frac{3c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

```
[Out] (-3*(b*c + 4*a*d)*Sqrt[c + d/x^2])/(8*x) - ((b*c + 4*a*d)*(c + d/x^2)^(3/2)
)/(4*c*x) + (a*(c + d/x^2)^(5/2)*x)/c - (3*c*(b*c + 4*a*d)*ArcTanh[Sqrt[d]/
(Sqrt[c + d/x^2]*x)])/(8*Sqrt[d])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx = -\text{Subst}\left(\int \frac{(a+bx^2)\,(c+dx^2)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} + \frac{(-bc-4ad)\text{Subst}\left(\int (c+dx^2)^{3/2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} - \frac{1}{4}(3(bc+4ad))\text{Subst}\left(\int \sqrt{c+\right.$$

$$= -\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x} - \frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} - \frac{1}{8}(3c($$

$$= -\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x} - \frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} - \frac{1}{8}(3c($$

$$= -\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x} - \frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} - \frac{3c(bc}{}$$

Mathematica [A]
time = 0.19, size = 92, normalized size = 0.82

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(-2bd-5bcx^2-4adx^2+8acx^4-\frac{3c(bc+4ad)x^4\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{d+cx^2}}\right)}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2),x]
[Out] (Sqrt[c + d/x^2]*(-2*b*d - 5*b*c*x^2 - 4*a*d*x^2 + 8*a*c*x^4 - (3*c*(b*c + 4*a*d)*x^4*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(Sqrt[d]*Sqrt[d + c*x^2])))/(8*x^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(94) = 188.
time = 0.08, size = 213, normalized size = 1.90

method	result
risch	$-\frac{(4ad\,x^2+5c\,x^2b+2bd)\sqrt{\frac{c\,x^2+d}{x^2}}}{8x^3}+\frac{\left(ca\sqrt{c\,x^2+d}-\frac{3c\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a}{2}-\frac{3\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+\cdots}}{x}\right.}{8\sqrt{d}}\right.}{\sqrt{c\,x^2+d}}$
default	$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(12d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)ac\,x^4+3d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)b\,c^2x^4-4(c\,x^2+d)^{\frac{3}{2}}acd\,x^4-(c\,x^2+\cdots\right.}{8x(c\,x^2+d)^{\frac{3}{2}}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/8*((c*x^2+d)/x^2)^(3/2)/x*(12*d^(5/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*a*c*x^4+3*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c^2*x^4-4*(c*x^2+d)^(3/2)*a*c*d*x^4-(c*x^2+d)^(3/2)*b*c^2*x^4+4*(c*x^2+d)^(5/2)*a*d*x^2+(c*x^2+d)^(5/2)*b*c*x^2-12*(c*x^2+d)^(1/2)*a*c*d^2*x^4-3*(c*x^2+d)^(1/2)*b*c^2*d*x^4+2*(c*x^2+d)^(5/2)*b*d)/(c*x^2+d)^(3/2)/d^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(94) = 188.
time = 0.50, size = 207, normalized size = 1.85

$$\frac{1}{4}\left(4\sqrt{c+\frac{d}{x^2}}\,cx-\frac{2\sqrt{c+\frac{d}{x^2}}\,cdx}{\left(c+\frac{d}{x^2}\right)x^2-d}+3\,c\sqrt{d}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)a+\frac{1}{16}\left(\frac{3\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3-3\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4-2\left(c+\frac{d}{x^2}\right)dx^2+d^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*sqrt(c + d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d)
+ 3*c*sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d
))))*a + 1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x +
 sqrt(d)))/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)*c^2
*d*x)/((c + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*b
```

Fricas [A]
time = 4.02, size = 216, normalized size = 1.93

$$\left[\frac{3\,(bc^2 + 4\,acd)\sqrt{d}\,x^3 \log\left(-\frac{cx^2 - 2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}} + 2\,d}{x^2}\right) + 2\,(8\,acdx^4 - 2\,bd^2 - (5\,bcd + 4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,dx^3}, \frac{3\,(bc^2 + 4\,acd)\sqrt{-d}\,x^3 \arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + (8\,acdx^4 - 2\,bd^2 - (5\,bcd + 4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,dx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b*c^2 + 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^
2 + d)/x^2) + 2*d)/x^2) + 2*(8*a*c*d*x^4 - 2*b*d^2 - (5*b*c*d + 4*a*d^2)*x^
2)*sqrt((c*x^2 + d)/x^2))/(d*x^3), 1/8*(3*(b*c^2 + 4*a*c*d)*sqrt(-d)*x^3*ar
ctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c*d*x^4 - 2*b*d^2
 - (5*b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(100) = 200$.
time = 5.92, size = 216, normalized size = 1.93

$$\frac{ac^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}} - \frac{a\sqrt{c}\,d\sqrt{1+\frac{d}{cx^2}}}{2x} + \frac{a\sqrt{c}\,d}{x\sqrt{1+\frac{d}{cx^2}}} - \frac{3ac\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2} - \frac{bc^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{bc^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}} - \frac{3b\sqrt{c}\,d}{8x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8\sqrt{d}} - \frac{bd^2}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2),x)
```

```
[Out] a*c**(3/2)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x)
+ a*sqrt(c)*d/(x*sqrt(1 + d/(c*x**2))) - 3*a*c*sqrt(d)*asinh(sqrt(d)/(sqrt(
c)*x))/2 - b*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - b*c**(3/2)/(8*x*sqrt(1 +
 d/(c*x**2))) - 3*b*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asin
h(sqrt(d)/(sqrt(c)*x))/(8*sqrt(d)) - b*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x
**2)))
```

Giac [A]
time = 2.28, size = 145, normalized size = 1.29

$$\frac{8\,\sqrt{cx^2+d}\,ac^2\operatorname{sgn}(x) + \frac{3\,(bc^3\operatorname{sgn}(x) + 4\,ac^2d\operatorname{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}} - \frac{5\,(cx^2+d)^{\frac{3}{2}}bc^3\operatorname{sgn}(x) + 4\,(cx^2+d)^{\frac{3}{2}}ac^2d\operatorname{sgn}(x) - 3\,\sqrt{cx^2+d}\,bc^3d\operatorname{sgn}(x) - 4\,\sqrt{cx^2+d}\,ac^2d^2\operatorname{sgn}(x)}{c^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(c*x^2 + d)*a*c^2*sgn(x) + 3*(b*c^3*sgn(x) + 4*a*c^2*d*sgn(x))*a
rctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - (5*(c*x^2 + d)^(3/2)*b*c^3*sgn(x
) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^3*d*sgn(x) -
 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x))/(c^2*x^4))/c
```

Mupad [B]
time = 5.86, size = 78, normalized size = 0.70

$$\frac{a\,x\,{\left(c\,x^2+d\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{d}{c\,x^2}\right)}{{\left(\frac{d}{c}+x^2\right)}^{3/2}}-\frac{b\,{\left(c\,x^2+d\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{d}{c\,x^2}\right)}{x\,{\left(\frac{d}{c}+x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x*(d + c*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -d/(c*x^2)))/(d/c + x^2
)^(3/2) - (b*(d + c*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -d/(c*x^2)))/(x*
(d/c + x^2)^(3/2))
```

3.960 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=123

$$\frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx}+\frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{16d^{3/2}}$$

```
[Out] 1/24*(-6*a*d+b*c)*(c+d/x^2)^(3/2)/d/x-1/6*b*(c+d/x^2)^(5/2)/d/x+1/16*c^2*(-
6*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)+1/16*c*(-6*a*d+b*c)*(
c+d/x^2)^(1/2)/d/x
```

Rubi [A]
time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 342, 201, 223, 212}

$$\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{3/2}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-6ad)}{24dx}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-6ad)}{16dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

```
[Out] (c*(b*c - 6*a*d)*Sqrt[c + d/x^2])/(16*d*x) + ((b*c - 6*a*d)*(c + d/x^2)^(3/
2))/(24*d*x) - (b*(c + d/x^2)^(5/2))/(6*d*x) + (c^2*(b*c - 6*a*d)*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{(-bc+6ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx}{6d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}-\frac{(-bc+6ad)\text{Subst}\left(\int \left(c+dx^2\right)^{3/2}\,dx,x,\frac{1}{x}\right)}{6d}$$

$$= \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{(c(bc-6ad))\text{Subst}\left(\int \sqrt{c+dx^2}\,dx,\right.}{8d}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx}+\frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{(c^2(bc-6ad}{}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx}+\frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{(c^2(bc-6ad}{}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx}+\frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{c^2(bc-6ad)}{}$$

Mathematica [A]
time = 0.23, size = 117, normalized size = 0.95

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(-\sqrt{d}\,(6adx^2(2d+5cx^2)+b(8d^2+14cdx^2+3c^2x^4))+\frac{3c^2(bc-6ad)x^6\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)}{48d^{3/2}x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-(Sqrt[d]*(6*a*d*x^2*(2*d + 5*c*x^2) + b*(8*d^2 + 14*c*d*
x^2 + 3*c^2*x^4))) + (3*c^2*(b*c - 6*a*d)*x^6*ArcTanh[Sqrt[d + c*x^2]/Sqrt[
d]])/Sqrt[d + c*x^2]))/(48*d^(3/2)*x^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(103) = 206.
time = 0.08, size = 259, normalized size = 2.11

method	result
risch	$-\frac{(30x^4acd+3x^4bc^2+12ad^2x^2+14bcdx^2+8bd^2)\sqrt{\frac{cx^2+d}{x^2}}}{48x^5d}+\frac{\left(-\frac{3c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{cx^2+d}}{x}\right)a}{8\sqrt{d}}+\frac{c^3\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{cx^2}}{x}\right.}{16d^{\frac{3}{2}}}\right.}{\sqrt{cx^2+d}}$
default	$-\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(-6(cx^2+d)^{\frac{3}{2}}ac^2dx^6+(cx^2+d)^{\frac{3}{2}}bc^3x^6+18d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{cx^2+d}}{x}\right)ac^2x^6-3d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{cx^2}}{x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*((c*x^2+d)/x^2)^(3/2)/x^3*(-6*(c*x^2+d)^(3/2)*a*c^2*d*x^6+(c*x^2+d)^(
3/2)*b*c^3*x^6+18*d^(5/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*a*c^2*x^6-3*d
^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c^3*x^6+6*(c*x^2+d)^(5/2)*a*c*
d*x^4-(c*x^2+d)^(5/2)*b*c^2*x^4-18*(c*x^2+d)^(1/2)*a*c^2*d^2*x^6+3*(c*x^2+d
)^(1/2)*b*c^3*d*x^6+12*(c*x^2+d)^(5/2)*a*d^2*x^2-2*(c*x^2+d)^(5/2)*b*c*d*x^
2+8*(c*x^2+d)^(5/2)*b*d^2)/(c*x^2+d)^(3/2)/d^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(103) = 206.
time = 0.52, size = 275, normalized size = 2.24

$$\frac{1}{16}\left(\frac{3c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3-3\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4-2\left(c+\frac{d}{x^2}\right)dx^2+d^2}\right)a-\frac{1}{96}\left(\frac{3c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}\,c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6-3\left(c+\frac{d}{x^2}\right)^2d^2x^4+3\left(c+\frac{d}{x^2}\right)d^3x^2-d^4}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))
)/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)*c^2*d*x)/((c
 + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*a - 1/96*(3*c^3*log((sqrt(c +
 d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*(3*(c + d/x
^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*c^3*d
^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d^2*x^4 + 3*(c + d/x^2)*d^3*x^
2 - d^4))*b
```

Fricas [A]
time = 3.40, size = 246, normalized size = 2.00

$$\left[-\frac{3\,(bc^3-6\,ac^2d)\sqrt{d}\,x^5\log\left(-\frac{cx^2-2\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(3\,(bc^2d+10\,acd^2)x^4+8\,bd^3+2\,(7\,bcd^2+6\,ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{96\,d^2x^5},-\frac{3\,(bc^3-6\,ac^2d)\sqrt{-d}\,x^5\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(3\,(bc^2d+10\,acd^2)x^4+8\,bd^3+2\,(7\,bcd^2+6\,ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{48\,d^2x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 6*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c
*x^2 + d)/x^2) + 2*d)/x^2) + 2*(3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*
(7*b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5), -1/48*(3*(b*c^
3 - 6*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2
+ d)) + (3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*(7*b*c*d^2 + 6*a*d^3)*x
^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(102) = 204$.
time = 11.38, size = 253, normalized size = 2.06

$$-\frac{ac^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{ac^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}}-\frac{3a\sqrt{c}\,d}{8x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8\sqrt{d}}-\frac{ad^2}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}-\frac{bc^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}}-\frac{17bc^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{11b\sqrt{c}\,d}{24x^5\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{16d^{\frac{3}{2}}}-\frac{bd^2}{6\sqrt{c}\,x^7\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**2,x)
```

```
[Out] -a*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - a*c**(3/2)/(8*x*sqrt(1 + d/(c*x**2
))) - 3*a*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*a*c**2*asinh(sqrt(d)/
(sqrt(c)*x))/(8*sqrt(d)) - a*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2))) - b
*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*b*c**(3/2)/(48*x**3*sqrt(1 + d
/(c*x**2))) - 11*b*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + b*c**3*asinh(
sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - b*d**2/(6*sqrt(c)*x**7*sqrt(1 + d/(c*x
**2)))
```

Giac [A]
time = 1.08, size = 173, normalized size = 1.41

$$-\frac{\frac{3\,\left(bc^{4}\mathrm{sgn}\left(x\right)-6\,ac^{3}d\mathrm{sgn}\left(x\right)\right)\arctan\left(\frac{\sqrt{cx^{2}+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d}+\frac{3\,\left(cx^{2}+d\right)^{\frac{5}{2}}bc^{4}\mathrm{sgn}\left(x\right)+30\,\left(cx^{2}+d\right)^{\frac{5}{2}}ac^{3}d\mathrm{sgn}\left(x\right)+8\,\left(cx^{2}+d\right)^{\frac{3}{2}}bc^{4}d\mathrm{sgn}\left(x\right)-48\,\left(cx^{2}+d\right)^{\frac{3}{2}}ac^{3}d^{2}\mathrm{sgn}\left(x\right)-3\,\sqrt{cx^{2}+d}\,bc^{4}d^{2}\mathrm{sgn}\left(x\right)+18\,\sqrt{cx^{2}+d}\,ac^{3}d^{3}\mathrm{sgn}\left(x\right)}{c^{3}dx^{6}}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/48*(3*(b*c^4*sgn(x) - 6*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))
/(sqrt(-d)*d) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) + 30*(c*x^2 + d)^(5/2)*a*
c^3*d*sgn(x) + 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 48*(c*x^2 + d)^(3/2)*a*
c^3*d^2*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4*d^2*sgn(x) + 18*sqrt(c*x^2 + d)*a*
c^3*d^3*sgn(x))/(c^3*d*x^6))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2, x)
```

3.961 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=159

$$\frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{c^2(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{128d^2x}-\frac{c^3(3bc-8ad)\,\mathrm{ta}}{12}$$

```
[Out] 1/48*(-8*a*d+3*b*c)*(c+d/x^2)^(3/2)/d/x^3-1/8*b*(c+d/x^2)^(5/2)/d/x^3-1/128
*c^3*(-8*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)+1/64*c*(-8*a
*d+3*b*c)*(c+d/x^2)^(1/2)/d/x^3+1/128*c^2*(-8*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^
2/x
```

Rubi [A]

time = 0.06, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {470, 342, 285, 327, 223, 212}

$$-\frac{c^3(3bc-8ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{128d^{5/2}}+\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(3bc-8ad)}{128d^2x}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(3bc-8ad)}{64dx^3}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-8ad)}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

```
[Out] (c*(3*b*c - 8*a*d)*Sqrt[c + d/x^2])/(64*d*x^3) + ((3*b*c - 8*a*d)*(c + d/x^
2)^(3/2))/(48*d*x^3) - (b*(c + d/x^2)^(5/2))/(8*d*x^3) + (c^2*(3*b*c - 8*a*
d)*Sqrt[c + d/x^2])/(128*d^2*x) - (c^3*(3*b*c - 8*a*d)*ArcTanh[Sqrt[d]/(Sqr
t[c + d/x^2]*x)])/(128*d^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{(-3bc+8ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx}{8d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} - \frac{(-3bc+8ad)\text{Subst}\left(\int x^2(c+dx^2)^{3/2}\,dx,x,\frac{1}{x}\right)}{8d}$$

$$= \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{(c(3bc-8ad))\text{Subst}\left(\int x^2\sqrt{c+dx^2}\right.}{16d}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3} + \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{(c^2(3bc-}{}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3} + \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{c^2(3bc-8}{1}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3} + \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{c^2(3bc-8}{1}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3} + \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{c^2(3bc-8}{1}$$

Mathematica [A]
time = 0.23, size = 150, normalized size = 0.94

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(\sqrt{d}\sqrt{d+cx^2}\left(8adx^2(8d^2+14cdx^2+3c^2x^4)+b(48d^3+72cd^2x^2+6c^2dx^4-9c^3x^6)\right)+3c^3(3bc-8ad)x^8\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)\right)}{384d^{5/2}x^7\sqrt{d+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

```
[Out] -1/384*(Sqrt[c + d/x^2]*(Sqrt[d]*Sqrt[d + c*x^2]*(8*a*d*x^2*(8*d^2 + 14*c*d
*x^2 + 3*c^2*x^4) + b*(48*d^3 + 72*c*d^2*x^2 + 6*c^2*d*x^4 - 9*c^3*x^6)) +
3*c^3*(3*b*c - 8*a*d)*x^8*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(d^(5/2)*x^7*S
qrt[d + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(135) = 270$.
time = 0.10, size = 302, normalized size = 1.90

method	result
risch	$-\frac{(24a\,c^2d\,x^6-9b\,c^3x^6+112ac\,d^2x^4+6b\,c^2d\,x^4+64a\,d^3x^2+72bc\,d^2x^2+48b\,d^3)\sqrt{\frac{c\,x^2+d}{x^2}}}{384x^7d^2}+\frac{\left(\frac{c^3\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a}{16d^{\frac{3}{2}}}-\right.}{}$
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(-8(c\,x^2+d)^{\frac{3}{2}}a\,c^3d\,x^8+3(c\,x^2+d)^{\frac{3}{2}}b\,c^4x^8+24d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a\,c^3x^8-9d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+}}{x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*(c+d/x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*((c*x^2+d)/x^2)^(3/2)/x^5*(-8*(c*x^2+d)^(3/2)*a*c^3*d*x^8+3*(c*x^2+d)
^(3/2)*b*c^4*x^8+24*d^(5/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*a*c^3*x^8-9
*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c^4*x^8+8*(c*x^2+d)^(5/2)*a*
c^2*d*x^6-3*(c*x^2+d)^(5/2)*b*c^3*x^6-24*(c*x^2+d)^(1/2)*a*c^3*d^2*x^8+9*(c
*x^2+d)^(1/2)*b*c^4*d*x^8+16*(c*x^2+d)^(5/2)*a*c*d^2*x^4-6*(c*x^2+d)^(5/2)*
b*c^2*d*x^4-64*(c*x^2+d)^(5/2)*a*d^3*x^2+24*(c*x^2+d)^(5/2)*b*c*d^2*x^2-48*
(c*x^2+d)^(5/2)*b*d^3)/(c*x^2+d)^(3/2)/d^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(135) = 270$.

time = 0.52, size = 354, normalized size = 2.23

$$-\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}\,c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6-3\left(c+\frac{d}{x^2}\right)^2d^2x^4+3\left(c+\frac{d}{x^2}\right)d^3x^2-d^4}\right)a+\frac{1}{256}\left(\frac{3\,c^4\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^4x^7-11\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^4dx^5-11\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4d^2x^3+3\sqrt{c+\frac{d}{x^2}}\,c^4d^3x\right)}{\left(c+\frac{d}{x^2}\right)^4d^2x^8-4\left(c+\frac{d}{x^2}\right)^3d^3x^6+6\left(c+\frac{d}{x^2}\right)^2d^4x^4-4\left(c+\frac{d}{x^2}\right)d^5x^2+d^6}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(3*c^3*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)
))/d^(3/2) + 2*(3*(c + d/x^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x^3
 - 3*sqrt(c + d/x^2)*c^3*d^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d^2*
x^4 + 3*(c + d/x^2)*d^3*x^2 - d^4))*a + 1/256*(3*c^4*log((sqrt(c + d/x^2)*x
 - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c + d/x^2)^(7/2)
*c^4*x^7 - 11*(c + d/x^2)^(5/2)*c^4*d*x^5 - 11*(c + d/x^2)^(3/2)*c^4*d^2*x^
3 + 3*sqrt(c + d/x^2)*c^4*d^3*x)/((c + d/x^2)^4*d^2*x^8 - 4*(c + d/x^2)^3*d
^3*x^6 + 6*(c + d/x^2)^2*d^4*x^4 - 4*(c + d/x^2)*d^5*x^2 + d^6))*b
```

Fricas [A]

time = 3.04, size = 298, normalized size = 1.87

$$\left[-\frac{3\,(3\,bc^4-8\,ac^3d)\sqrt{d}\,x^7\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\,(3\,(3\,bc^3d-8\,ac^2d^2)x^6-48\,bd^4-2\,(3\,bc^2d^2+56\,acd^3)x^4-8\,(9\,bcd^3+8\,ad^4)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{768\,d^3x^7},\ \frac{3\,(3\,bc^4-8\,ac^3d)\sqrt{-d}\,x^7\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(3\,(3\,bc^3d-8\,ac^2d^2)x^6-48\,bd^4-2\,(3\,bc^2d^2+56\,acd^3)x^4-8\,(9\,bcd^3+8\,ad^4)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{384\,d^3x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(d)*x^7*log(-(c*x^2 + 2*sqrt(d)*x*sqrt
((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(3*b*c^3*d - 8*a*c^2*d^2)*x^6 - 48*b*d
^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*a*d^4)*x^2)*sqrt((
c*x^2 + d)/x^2))/(d^3*x^7), 1/384*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(-d)*x^7*arc
tan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(3*b*c^3*d - 8*a*c^2
*d^2)*x^6 - 48*b*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*
a*d^4)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^7)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(141) = 282$.
time = 33.35, size = 287, normalized size = 1.81

$$-\frac{ac^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}} - \frac{17ac^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{11a\sqrt{c}\,d}{24x^5\sqrt{1+\frac{d}{cx^2}}} + \frac{ac^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{16d^{\frac{3}{2}}} - \frac{ad^2}{6\sqrt{c}\,x^7\sqrt{1+\frac{d}{cx^2}}} + \frac{3bc^{\frac{7}{2}}}{128d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{5}{2}}}{128dx^3\sqrt{1+\frac{d}{cx^2}}} - \frac{13bc^{\frac{3}{2}}}{64x^5\sqrt{1+\frac{d}{cx^2}}} - \frac{5b\sqrt{c}\,d}{16x^7\sqrt{1+\frac{d}{cx^2}}} - \frac{3bc^4\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{128d^{\frac{5}{2}}} - \frac{bd^2}{8\sqrt{c}\,x^9\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**4,x)
```

```
[Out] -a*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*a*c**(3/2)/(48*x**3*sqrt(1 +
 d/(c*x**2))) - 11*a*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + a*c**3*asin
h(sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - a*d**2/(6*sqrt(c)*x**7*sqrt(1 + d/(c
*x**2))) + 3*b*c**(7/2)/(128*d**2*x*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(128
*d*x**3*sqrt(1 + d/(c*x**2))) - 13*b*c**(3/2)/(64*x**5*sqrt(1 + d/(c*x**2))
) - 5*b*sqrt(c)*d/(16*x**7*sqrt(1 + d/(c*x**2))) - 3*b*c**4*asinh(sqrt(d)/(
sqrt(c)*x))/(128*d**(5/2)) - b*d**2/(8*sqrt(c)*x**9*sqrt(1 + d/(c*x**2)))
```

Giac [**A**]
time = 1.30, size = 214, normalized size = 1.35

$$\frac{\frac{3\left(3bc^5\operatorname{sgn}(x)-8ac^4d\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d^2} + \frac{9(cx^2+d)^{\frac{7}{2}}bc^5\operatorname{sgn}(x)-24(cx^2+d)^{\frac{7}{2}}ac^4d\operatorname{sgn}(x)-33(cx^2+d)^{\frac{5}{2}}bc^5d\operatorname{sgn}(x)-40(cx^2+d)^{\frac{5}{2}}ac^4d^2\operatorname{sgn}(x)-33(cx^2+d)^{\frac{3}{2}}bc^5d^2\operatorname{sgn}(x)+88(cx^2+d)^{\frac{3}{2}}ac^4d^3\operatorname{sgn}(x)+9\sqrt{cx^2+d}\,bc^5d^3\operatorname{sgn}(x)-24\sqrt{cx^2+d}\,ac^4d^4\operatorname{sgn}(x)}{c^4d^2x^8}}{384c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/384*(3*(3*b*c^5*sgn(x) - 8*a*c^4*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d
))/(sqrt(-d)*d^2) + (9*(c*x^2 + d)^(7/2)*b*c^5*sgn(x) - 24*(c*x^2 + d)^(7/2
)*a*c^4*d*sgn(x) - 33*(c*x^2 + d)^(5/2)*b*c^5*d*sgn(x) - 40*(c*x^2 + d)^(5/
2)*a*c^4*d^2*sgn(x) - 33*(c*x^2 + d)^(3/2)*b*c^5*d^2*sgn(x) + 88*(c*x^2 + d
)^(3/2)*a*c^4*d^3*sgn(x) + 9*sqrt(c*x^2 + d)*b*c^5*d^3*sgn(x) - 24*sqrt(c*x
^2 + d)*a*c^4*d^4*sgn(x))/(c^4*d^2*x^8))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x)
[Out] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4, x)
```

3.962 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=90

$$\frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c}-\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}$$

```
[Out] -1/8*d*(-3*a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(5/2)+1/8*(-3*a*d+
4*b*c)*x^2*(c+d/x^2)^(1/2)/c^2+1/4*a*x^4*(c+d/x^2)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 44, 65, 214}

$$-\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}+\frac{x^2\sqrt{c+\frac{d}{x^2}}\,(4bc-3ad)}{8c^2}+\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

```
[Out] ((4*b*c - 3*a*d)*Sqrt[c + d/x^2]*x^2)/(8*c^2) + (a*Sqrt[c + d/x^2]*x^4)/(4*
c) - (d*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x^3\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} - \frac{\left(2bc-\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{4c}$$

$$= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} + \frac{(d(4bc-3ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \right.}{16c^2}$$

$$= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} + \frac{(4bc-3ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\right.}{8c^2}$$

$$= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} - \frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}$$

Mathematica [A]
time = 0.11, size = 97, normalized size = 1.08

$$\frac{\sqrt{c}\,x(d+cx^2)\left(4bc-3ad+2acx^2\right)+d(4bc-3ad)\sqrt{d+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)}{8c^{5/2}\sqrt{c+\frac{d}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c]*x*(d + c*x^2)*(4*b*c - 3*a*d + 2*a*c*x^2) + d*(4*b*c - 3*a*d)*Sqrt
[d + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]])/(8*c^(5/2)*Sqrt[c + d/x^2]
*x)
```

Maple [A]
time = 0.06, size = 129, normalized size = 1.43

method	result

risch	$\frac{(2cx^2a-3ad+4bc)(cx^2+d)}{8c^2\sqrt{\frac{cx^2+d}{x^2}}}+\frac{\left(\frac{3d^2\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)a}{8c^{\frac{5}{2}}}-\frac{d\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)b}{2c^{\frac{3}{2}}}\right)\sqrt{cx^2+d}}{\sqrt{\frac{cx^2+d}{x^2}}\,x}$
default	$\frac{\sqrt{cx^2+d}\left(2c^{\frac{5}{2}}\sqrt{cx^2+d}\,ax^3-3c^{\frac{3}{2}}\sqrt{cx^2+d}\,adx+4c^{\frac{5}{2}}\sqrt{cx^2+d}\,bx+3\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)acd^2-4\ln\left(\right.\right.}{8\sqrt{\frac{cx^2+d}{x^2}}\,xc^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^3/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*x^2+d)^(1/2)*(2*c^(5/2)*(c*x^2+d)^(1/2)*a*x^3-3*c^(3/2)*(c*x^2+d)^(1
/2)*a*d*x+4*c^(5/2)*(c*x^2+d)^(1/2)*b*x+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c
*d^2-4*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c^2*d)/((c*x^2+d)/x^2)^(1/2)/x/c^(7/
2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(74) = 148$.
time = 0.49, size = 178, normalized size = 1.98

$$\frac{1}{4}\,b\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,d}{\left(c+\frac{d}{x^2}\right)c-c^2}+\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{1}{16}\,a\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-5\sqrt{c+\frac{d}{x^2}}\,cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c^2-2\left(c+\frac{d}{x^2}\right)c^3+c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2) -
 sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/16*a*(3*d^2*log((sqrt(c
 + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(5/2) + 2*(3*(c + d/x^2
)^(3/2)*d^2 - 5*sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c^2 - 2*(c + d/x^2)*c
^3 + c^4))
```

Fricas [A]
time = 3.25, size = 192, normalized size = 2.13

$$\left[-\frac{(4\,bcd-3\,ad^2)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(2\,ac^2x^4+(4\,bc^2-3\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^3},\ \frac{(4\,bcd-3\,ad^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(2\,ac^2x^4+(4\,bc^2-3\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*b*c*d - 3*a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^
2 + d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 +
d)/x^2))/c^3, 1/8*((4*b*c*d - 3*a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c
*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt(
(c*x^2 + d)/x^2))/c^3]
```

Sympy [A]
time = 24.85, size = 150, normalized size = 1.67

$$\frac{ax^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{a\sqrt{d}\,x^3}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{3ad^{\frac{3}{2}}x}{8c^2\sqrt{\frac{cx^2}{d}+1}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{\frac{5}{2}}}+\frac{b\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2c}-\frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3/(c+d/x**2)**(1/2),x)
```

```
[Out] a*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) - a*sqrt(d)*x**3/(8*c*sqrt(c*x**2/d +
 1)) - 3*a*d**(3/2)*x/(8*c**2*sqrt(c*x**2/d + 1)) + 3*a*d**2*asinh(sqrt(c)*
x/sqrt(d))/(8*c**(5/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - b*d*asinh(
sqrt(c)*x/sqrt(d))/(2*c**(3/2))
```

Giac [A]
time = 1.43, size = 111, normalized size = 1.23

$$\frac{1}{8}\sqrt{cx^2+d}\,x\left(\frac{2\,ax^2}{c\mathrm{sgn}\,(x)}+\frac{4\,bc^2\mathrm{sgn}(x)-3\,acd\mathrm{sgn}(x)}{c^3}\right)-\frac{(4\,bcd\log\left(|d|\right)-3\,ad^2\log\left(|d|\right))\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}}+\frac{(4\,bcd-3\,ad^2)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+d}\,\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^2 + d)*x*(2*a*x^2/(c*sgn(x)) + (4*b*c^2*sgn(x) - 3*a*c*d*sgn(x
))/c^3) - 1/16*(4*b*c*d*log(abs(d)) - 3*a*d^2*log(abs(d)))*sgn(x)/c^(5/2) +
 1/8*(4*b*c*d - 3*a*d^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(5/2)*sg
n(x))
```

Mupad [B]
time = 5.35, size = 99, normalized size = 1.10

$$\frac{5\,a\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8\,c}-\frac{3\,a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c^2}+\frac{b\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2\,c}-\frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{3/2}}+\frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (5*a*x^4*(c + d/x^2)^(1/2))/(8*c) - (3*a*x^4*(c + d/x^2)^(3/2))/(8*c^2) + (
b*x^2*(c + d/x^2)^(1/2))/(2*c) - (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*
c^(3/2)) + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(5/2))
```

3.963 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=59

$$\frac{a\sqrt{c+\frac{d}{x^2}}\,x^2}{2c}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}$$

```
[Out] 1/2*(-a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+1/2*a*x^2*(c+d/x^
2)^(1/2)/c
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {457, 79, 65, 214}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c + d/x^2]*x^2)/(2*c) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt
[c]])/(2*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x^2\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^2}{2c}-\frac{\left(bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)}{2c}\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^2}{2c}-\frac{\left(bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+\frac{d}{x^2}}\right)}{cd}\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^2}{2c}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 1.36

$$
\frac{a\sqrt{c}\,x(d+cx^2)+(-2bc+ad)\sqrt{d+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)}{2c^{3/2}\sqrt{c+\frac{d}{x^2}}\,x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c]*x*(d + c*x^2) + (-2*b*c + a*d)*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x)
+ Sqrt[d + c*x^2]])/(2*c^(3/2)*Sqrt[c + d/x^2]*x)
```

Maple [A]
time = 0.06, size = 90, normalized size = 1.53

method	result	size
default	$\frac{\sqrt{c\,x^{2}+d}\,\left(c^{\frac{3}{2}}\sqrt{c\,x^{2}+d}\,a x+2 b \ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right) c^{2}-\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right) a c d\right)}{2 \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\,c^{\frac{5}{2}}}$	90
risch	$\frac{a\left(c\,x^{2}+d\right)}{2 c \sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{\left(-\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right) a d}{2 c^{\frac{3}{2}}}+\frac{b \ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)}{\sqrt{c}}\right)\sqrt{c\,x^{2}+d}}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2+d)^(1/2)*(c^(3/2)*(c*x^2+d)^(1/2)*a*x+2*b*ln(c^(1/2)*x+(c*x^2+d)
^(1/2))*c^2-ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c*d)/((c*x^2+d)/x^2)^(1/2)/x/c^
(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(47) = 94.
time = 0.51, size = 109, normalized size = 1.85

$$\frac{1}{4}\,a\left(\frac{2\,\sqrt{c+\frac{d}{x^{2}}}\,d}{\left(c+\frac{d}{x^{2}}\right)c-c^{2}}+\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{b\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2) -
 sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/2*b*log((sqrt(c + d/x^2
) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c)
```

Fricas [A]
time = 3.74, size = 146, normalized size = 2.47

$$\left[\frac{2\,acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{4\,c^2},\ \frac{acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*c*x^2*sqrt((c*x^2 + d)/x^2) - (2*b*c - a*d)*sqrt(c)*log(-2*c*x^2
+ 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/c^2, 1/2*(a*c*x^2*sqrt((c*x^2 +
 d)/x^2) - (2*b*c - a*d)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)
/(c*x^2 + d)))/c^2]
```

Sympy [A]
time = 29.77, size = 66, normalized size = 1.12

$$\frac{a\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2c}-\frac{ad\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - a*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(
3/2)) + b*asinh(sqrt(c)*x/sqrt(d))/sqrt(c)
```

Giac [A]
time = 0.86, size = 79, normalized size = 1.34

$$\frac{\sqrt{cx^2+d}\,ax}{2\,c\mathrm{sgn}\,(x)}+\frac{(2\,bc\log\,(|d|)-ad\log\,(|d|))\mathrm{sgn}(x)}{4\,c^{\frac{3}{2}}}-\frac{(2\,bc-ad)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+d}\right|\right)}{2\,c^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*x/(c*sgn(x)) + 1/4*(2*b*c*log(abs(d)) - a*d*log(abs(d
)))*sgn(x)/c^(3/2) - 1/2*(2*b*c - a*d)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)
))/(c^(3/2)*sgn(x))
```

Mupad [B]
time = 5.08, size = 59, normalized size = 1.00

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{a\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2\,c}-\frac{a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) + (a*x^2*(c + d/x^2)^(1/2))/(2
*c) - (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(3/2))
```

3.964 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\, x} \, dx$

Optimal. Leaf size=43

$$-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] a*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(1/2)-b*(c+d/x^2)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {457, 81, 65, 214}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

```
[Out] -((b*Sqrt[c + d/x^2])/d) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/Sqrt[c]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d} \\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 76, normalized size = 1.77

$$\frac{-b\sqrt{c}\,(d+cx^2) - adx\sqrt{d+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)}{\sqrt{c}\,d\sqrt{c+\frac{d}{x^2}}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
[Out] (-(b*Sqrt[c]*(d + c*x^2)) - a*d*x*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[d
 + c*x^2]])/(Sqrt[c]*d*Sqrt[c + d/x^2]*x^2)
```

Maple [A]
time = 0.05, size = 69, normalized size = 1.60

method	result	size
default	$\frac{\sqrt{c\,x^{2}+d}\,\left(a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)dx-b\sqrt{c\,x^{2}+d}\,\sqrt{c}\right)}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x^{2}d\sqrt{c}}$	69
risch	$-\frac{b\left(c\,x^{2}+d\right)}{d\,x^{2}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{c\,x^{2}+d}}{\sqrt{c}\,\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/x/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+d)^(1/2)*(a*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*d*x-b*(c*x^2+d)^(1/2)*c^(1
/2))/((c*x^2+d)/x^2)^(1/2)/x^2/d/c^(1/2)
```

Maxima [A]
time = 0.52, size = 54, normalized size = 1.26

$$-\frac{a\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{2\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c)
 - b*sqrt(c + d/x^2)/d
```

Fricas [A]
time = 2.88, size = 130, normalized size = 3.02

$$\left[\frac{a\sqrt{c}\,d\log\left(-2\,cx^{2}-2\,\sqrt{c}\,x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}-d\right)-2\,bc\sqrt{\frac{cx^{2}+d}{x^{2}}}}{2\,cd},-\frac{a\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}\,x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cx^{2}+d}\right)+bc\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) -
 2*b*c*sqrt((c*x^2 + d)/x^2))/(c*d), -(a*sqrt(-c)*d*arctan(sqrt(-c)*x^2*sqr
t((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*c*sqrt((c*x^2 + d)/x^2))/(c*d)]
```

Sympy [A]
time = 8.98, size = 63, normalized size = 1.47

$$-\frac{a \operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{c}}\sqrt{c+\frac{d}{x^2}}}\right)}{c\sqrt{-\frac{1}{c}}}+\frac{b\left(\begin{cases}-\frac{1}{\sqrt{c}\,x^2} & \text{for } d=0\\ -\frac{2\sqrt{c+\frac{d}{x^2}}}{d} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*atan(1/(sqrt(-1/c)*sqrt(c + d/x**2)))/(c*sqrt(-1/c)) + b*Piecewise((-1/(
sqrt(c)*x**2), Eq(d, 0)), (-2*sqrt(c + d/x**2)/d, True))/2
```

Giac [A]
time = 0.78, size = 66, normalized size = 1.53

$$-\frac{a\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2\right)}{2\sqrt{c}\operatorname{sgn}(x)}+\frac{2b\sqrt{c}}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)/(sqrt(c)*sgn(x)) + 2*b*sqrt(c)/
(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)*sgn(x))
```

Mupad [B]
time = 4.85, size = 35, normalized size = 0.81

$$\frac{a \operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x*(c + d/x^2)^(1/2)),x)
```

```
[Out] (a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) - (b*(c + d/x^2)^(1/2))/d
```

3.965 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\, x^3} \, dx$

Optimal. Leaf size=43

$$\frac{(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

```
[Out] -1/3*b*(c+d/x^2)^(3/2)/d^2+(-a*d+b*c)*(c+d/x^2)^(1/2)/d^2
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d/x^2])/d^2 - (b*(c + d/x^2)^(3/2))/(3*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^3}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{-bc+ad}{d\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d}\right)dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

Mathematica [A]
time = 0.07, size = 39, normalized size = 0.91

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(3adx^2+b(d-2cx^2))}{3d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
```

```
[Out] -1/3*(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d - 2*c*x^2)))/(d^2*x^2)
```

Maple [A]
time = 0.05, size = 47, normalized size = 1.09

method	result	size
trager	$-\frac{(3ad\,x^2-2c\,x^2b+bd)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{3x^2d^2}$	44
gosper	$-\frac{(3ad\,x^2-2c\,x^2b+bd)(c\,x^2+d)}{3\sqrt{\frac{c\,x^2+d}{x^2}}\,d^2x^4}$	47
default	$-\frac{(3ad\,x^2-2c\,x^2b+bd)(c\,x^2+d)}{3\sqrt{\frac{c\,x^2+d}{x^2}}\,d^2x^4}$	47
risch	$-\frac{(3ad\,x^2-2c\,x^2b+bd)(c\,x^2+d)}{3\sqrt{\frac{c\,x^2+d}{x^2}}\,d^2x^4}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/x^3/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(3*a*d*x^2-2*b*c*x^2+b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/d^2/x^4
```

Maxima [A]
time = 0.27, size = 48, normalized size = 1.12

$$-\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\sqrt{c+\frac{d}{x^2}}\,c}{d^2}\right)-\frac{a\sqrt{c+\frac{d}{x^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*b*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2) - a*sqrt(c + d/x^2
)/d
```

Fricas [A]
time = 3.25, size = 39, normalized size = 0.91

$$\frac{\left((2\,bc-3\,ad)x^2-bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b*c - 3*a*d)*x^2 - b*d)*sqrt((c*x^2 + d)/x^2)/(d^2*x^2)
```

Sympy [A]
time = 3.07, size = 138, normalized size = 3.21

$$\frac{\begin{cases} \frac{-\frac{a}{x^2}-\frac{b}{2x^4}}{\sqrt{c}} & \text{for}\ d=0 \\ \frac{\frac{2ac}{\sqrt{c+\frac{d}{x^2}}}+2a\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)+\frac{2bc\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)}{d}+\frac{2b\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{otherwise} \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**3/(c+d/x**2)**(1/2),x)
```

```
[Out] Piecewise(((-a/x**2 - b/(2*x**4))/sqrt(c), Eq(d, 0)), ((2*a*c/sqrt(c + d/x*
*2) + 2*a*(-c/sqrt(c + d/x**2) - sqrt(c + d/x**2)) + 2*b*c*(-c/sqrt(c + d/x
**2) - sqrt(c + d/x**2))/d + 2*b*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x*
*2) - (c + d/x**2)**(3/2)/3)/d)/d, True))/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(37) = 74.
time = 0.83, size = 124, normalized size = 2.88

$$\frac{2\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4 a\sqrt{c}+6\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2 bc^{\frac{3}{2}}-6\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2 a\sqrt{c}\,d-2\,bc^{\frac{3}{2}}d+3\,a\sqrt{c}\,d^2\right)}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^3\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c) + 6*(sqrt(c)*x - sqrt(c*x^
2 + d))^2*b*c^(3/2) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d - 2*b*c
^(3/2)*d + 3*a*sqrt(c)*d^2)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^3*sgn(x)
)
```

Mupad [B]
time = 4.56, size = 35, normalized size = 0.81

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(b\,d+3\,a\,d\,x^2-2\,b\,c\,x^2\right)}{3\,d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^3*(c + d/x^2)^(1/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(b*d + 3*a*d*x^2 - 2*b*c*x^2))/(3*d^2*x^2)
```

3.966 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\, x^5} \, dx$

Optimal. Leaf size=72

$$-\frac{c(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

```
[Out] 1/3*(-a*d+2*b*c)*(c+d/x^2)^(3/2)/d^3-1/5*b*(c+d/x^2)^(5/2)/d^3-c*(-a*d+b*c)
*(c+d/x^2)^(1/2)/d^3
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{3d^3}-\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

```
[Out] -((c*(b*c - a*d)*Sqrt[c + d/x^2])/d^3) + ((2*b*c - a*d)*(c + d/x^2)^(3/2))/
(3*d^3) - (b*(c + d/x^2)^(5/2))/(5*d^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^5}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)}{\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c(bc-ad)}{d^2\sqrt{c+dx}} + \frac{(-2bc+ad)\sqrt{c+dx}}{d^2} + \frac{b(c+dx)^{3/2}}{d^2}\right)dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{c(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3} + \frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 0.83

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(-5adx^2(d-2cx^2)+b(-3d^2+4cdx^2-8c^2x^4))}{15d^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

```
[Out] (Sqrt[c + d/x^2]*(-5*a*d*x^2*(d - 2*c*x^2) + b*(-3*d^2 + 4*c*d*x^2 - 8*c^2*x^4)))/(15*d^3*x^4)
```

Maple [A]
time = 0.06, size = 70, normalized size = 0.97

method	result	size
trager	$\frac{\left(10x^4acd-8x^4b\,c^2-5a\,d^2x^2+4bcd\,x^2-3b\,d^2\right)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{15x^4d^3}$	67
gosper	$\frac{\left(10x^4acd-8x^4b\,c^2-5a\,d^2x^2+4bcd\,x^2-3b\,d^2\right)\left(c\,x^2+d\right)}{15\sqrt{\frac{c\,x^2+d}{x^2}}\,d^3x^6}$	70
default	$\frac{\left(10x^4acd-8x^4b\,c^2-5a\,d^2x^2+4bcd\,x^2-3b\,d^2\right)\left(c\,x^2+d\right)}{15\sqrt{\frac{c\,x^2+d}{x^2}}\,d^3x^6}$	70
risch	$\frac{\left(10x^4acd-8x^4b\,c^2-5a\,d^2x^2+4bcd\,x^2-3b\,d^2\right)\left(c\,x^2+d\right)}{15\sqrt{\frac{c\,x^2+d}{x^2}}\,d^3x^6}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/x^5/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/15*(10*a*c*d*x^4-8*b*c^2*x^4-5*a*d^2*x^2+4*b*c*d*x^2-3*b*d^2)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/d^3/x^6

Maxima [A]
time = 0.29, size = 83, normalized size = 1.15

$$-\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}\,c^2}{d^3}\right)-\frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\sqrt{c+\frac{d}{x^2}}\,c}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] -1/15*b*(3*(c + d/x^2)^(5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c + d/x^2)*c^2/d^3) - 1/3*a*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2)

Fricas [A]
time = 3.91, size = 62, normalized size = 0.86

$$-\frac{\left(2\left(4\,bc^2-5\,acd\right)x^4+3\,bd^2-\left(4\,bcd-5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] -1/15*(2*(4*b*c^2 - 5*a*c*d)*x^4 + 3*b*d^2 - (4*b*c*d - 5*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^3*x^4)

Sympy [A]
time = 5.47, size = 204, normalized size = 2.83

$$\frac{\begin{cases}\frac{-\frac{a}{2x^4}-\frac{b}{3x^6}}{\sqrt{c}} & \text{for}\ d=0\\ \frac{\frac{2ac\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)}{d}+\frac{2a\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}+\frac{2bc\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{2b\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}}{d} & \text{otherwise}\end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/x**5/(c+d/x**2)**(1/2),x)

[Out] Piecewise(((-a/(2*x**4) - b/(3*x**6))/sqrt(c), Eq(d, 0)), ((2*a*c*(-c/sqrt(c + d/x**2) - sqrt(c + d/x**2))/d + 2*a*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d + 2*b*c*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d**2 + 2*b*(-c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5)/d**2)/d, True))/2

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(62) = 124.
time = 1.32, size = 180, normalized size = 2.50

$$\frac{4\left(15\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}+40\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4bc^{\frac{5}{2}}-35\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4ac^{\frac{3}{2}}d-20\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2bc^{\frac{5}{2}}d+25\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2ac^{\frac{3}{2}}d^2+4\,bc^{\frac{5}{2}}d^2-5\,ac^{\frac{3}{2}}d^3\right)}{15\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^5\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(3/2) + 40*(sqrt(c)*x - sqrt(c
*x^2 + d))^4*b*c^(5/2) - 35*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d - 2
0*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(5/2)*d + 25*(sqrt(c)*x - sqrt(c*x^2
+ d))^2*a*c^(3/2)*d^2 + 4*b*c^(5/2)*d^2 - 5*a*c^(3/2)*d^3)/(((sqrt(c)*x - s
qrt(c*x^2 + d))^2 - d)^5*sgn(x))
```

Mupad [B]
time = 4.68, size = 58, normalized size = 0.81

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(8\,b\,c^2\,x^4-10\,a\,c\,d\,x^4-4\,b\,c\,d\,x^2+5\,a\,d^2\,x^2+3\,b\,d^2\right)}{15\,d^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^5*(c + d/x^2)^(1/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(3*b*d^2 + 5*a*d^2*x^2 + 8*b*c^2*x^4 - 10*a*c*d*x^4 - 4
*b*c*d*x^2))/(15*d^3*x^4)
```

3.967 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^7}\,dx$

Optimal. Leaf size=101

$$\frac{c^2(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

```
[Out] -1/3*c*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)/d^4+1/5*(-a*d+3*b*c)*(c+d/x^2)^(5/2)/
d^4-1/7*b*(c+d/x^2)^(7/2)/d^4+c^2*(-a*d+b*c)*(c+d/x^2)^(1/2)/d^4
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-ad)}{5d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

```
[Out] (c^2*(b*c - a*d)*Sqrt[c + d/x^2])/d^4 - (c*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2
))/(3*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (b*(c + d/x^2)^(7/
2))/(7*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^7}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^2(a+bx)}{\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{c^2(bc-ad)}{d^3\sqrt{c+dx}} + \frac{c(3bc-2ad)\sqrt{c+dx}}{d^3} + \frac{(-3bc+ad)(c+dx)^{3/2}}{d^3} +\right.\right.\right.$$

$$= \frac{c^2(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^4} - \frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4} - \frac{b\left(c+\frac{d}{x^2}\right.}{7d^4}$$

Mathematica [A]
time = 0.12, size = 93, normalized size = 0.92

$$\frac{(d+cx^2)\left(-15bd^3+18bcd^2x^2-21ad^3x^2-24bc^2dx^4+28acd^2x^4+48bc^3x^6-56ac^2dx^6\right)}{105d^4\sqrt{c+\frac{d}{x^2}}\,x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

```
[Out] ((d + c*x^2)*(-15*b*d^3 + 18*b*c*d^2*x^2 - 21*a*d^3*x^2 - 24*b*c^2*d*x^4 +
28*a*c*d^2*x^4 + 48*b*c^3*x^6 - 56*a*c^2*d*x^6))/(105*d^4*Sqrt[c + d/x^2]*x
^8)
```

Maple [A]
time = 0.06, size = 94, normalized size = 0.93

method	result	size
trager	$-\frac{\left(56a\,c^2d\,x^6-48b\,c^3x^6-28ac\,d^2x^4+24b\,c^2d\,x^4+21a\,d^3x^2-18bc\,d^2x^2+15b\,d^3\right)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{105x^6d^4}$	91
gosper	$-\frac{\left(56a\,c^2d\,x^6-48b\,c^3x^6-28ac\,d^2x^4+24b\,c^2d\,x^4+21a\,d^3x^2-18bc\,d^2x^2+15b\,d^3\right)\left(c\,x^2+d\right)}{105\sqrt{\frac{c\,x^2+d}{x^2}}\,d^4x^8}$	94
default	$-\frac{\left(56a\,c^2d\,x^6-48b\,c^3x^6-28ac\,d^2x^4+24b\,c^2d\,x^4+21a\,d^3x^2-18bc\,d^2x^2+15b\,d^3\right)\left(c\,x^2+d\right)}{105\sqrt{\frac{c\,x^2+d}{x^2}}\,d^4x^8}$	94
risch	$-\frac{\left(56a\,c^2d\,x^6-48b\,c^3x^6-28ac\,d^2x^4+24b\,c^2d\,x^4+21a\,d^3x^2-18bc\,d^2x^2+15b\,d^3\right)\left(c\,x^2+d\right)}{105\sqrt{\frac{c\,x^2+d}{x^2}}\,d^4x^8}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/x^7/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -1/105*(56*a*c^2*d*x^6-48*b*c^3*x^6-28*a*c*d^2*x^4+24*b*c^2*d*x^4+21*a*d^3*x^2-18*b*c*d^2*x^2+15*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/d^4/x^8

Maxima [A]
time = 0.29, size = 118, normalized size = 1.17

$$-\frac{1}{35}\,b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^4}-\frac{21\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^4}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^4}-\frac{35\sqrt{c+\frac{d}{x^2}}\,c^3}{d^4}\right)-\frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}\,c^2}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] -1/35*b*(5*(c + d/x^2)^(7/2)/d^4 - 21*(c + d/x^2)^(5/2)*c/d^4 + 35*(c + d/x^2)^(3/2)*c^2/d^4 - 35*sqrt(c + d/x^2)*c^3/d^4) - 1/15*a*(3*(c + d/x^2)^(5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c + d/x^2)*c^2/d^3)

Fricas [A]
time = 3.01, size = 86, normalized size = 0.85

$$\frac{\left(8\left(6\,bc^3-7\,ac^2d\right)x^6-4\left(6\,bc^2d-7\,acd^2\right)x^4-15\,bd^3+3\left(6\,bcd^2-7\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] 1/105*(8*(6*b*c^3 - 7*a*c^2*d)*x^6 - 4*(6*b*c^2*d - 7*a*c*d^2)*x^4 - 15*b*d^3 + 3*(6*b*c*d^2 - 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^6)

Sympy [A]
time = 7.80, size = 269, normalized size = 2.66

$$\frac{\begin{cases}\frac{-\frac{a}{3x^6}-\frac{b}{4x^8}}{\sqrt{c}} & \text{for } d=0\\ \frac{\frac{2ac\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{2a\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{2bc\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^3}+\frac{2b\left(\frac{c^4}{\sqrt{c+\frac{d}{x^2}}}+4c^3\sqrt{c+\frac{d}{x^2}}-2c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}}{d} & \text{otherwise}\end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/x**7/(c+d/x**2)**(1/2),x)

[Out] Piecewise(((-a/(3*x**6) - b/(4*x**8))/sqrt(c), Eq(d, 0)), ((2*a*c*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d**2 + 2*a*(-c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5)/d**2 + 2*b*c*(-c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5)/d**3 + 2*b*(c**4/sqrt(c + d/x**2) + 4*c**3*sqrt(c + d/x**2) - 2*c**2*(c + d/x**2)**(3/2) + 4*c*(c + d/x**2)**(5/2)/5 - (c + d/x**2)**(7/2)/7)/d**3)/d, True))/2

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(87) = 174$.
time = 2.08, size = 236, normalized size = 2.34

$$\frac{16\left(70\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^8 ac^{\frac{5}{2}}+210\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^6 bc^{\frac{7}{2}}-175\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^6 ac^{\frac{5}{2}}d-126\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4 bc^{\frac{7}{2}}d+147\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4 ac^{\frac{5}{2}}d^2+42\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2 bc^{\frac{7}{2}}d^2-49\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2 ac^{\frac{5}{2}}d^3-6\,bc^{\frac{7}{2}}d^3+7\,ac^{\frac{5}{2}}d^4\right)}{105\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^7\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2) + 210*(sqrt(c)*x - sqr
t(c*x^2 + d))^6*b*c^(7/2) - 175*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(5/2)*d
 - 126*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d + 147*(sqrt(c)*x - sqrt(
c*x^2 + d))^4*a*c^(5/2)*d^2 + 42*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)*
d^2 - 49*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^3 - 6*b*c^(7/2)*d^3 +
7*a*c^(5/2)*d^4)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^7*sgn(x))
```

Mupad [B]
time = 4.72, size = 102, normalized size = 1.01

$$\frac{\sqrt{c+\frac{d}{x^2}}\,\left(48\,b\,c^3-56\,a\,c^2\,d\right)}{105\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{7\,d\,x^6}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(24\,b\,c^2-28\,a\,c\,d\right)}{105\,d^3\,x^2}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(7\,a\,d-6\,b\,c\right)}{35\,d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^7*(c + d/x^2)^(1/2)),x)
```

```
[Out] ((c + d/x^2)^(1/2)*(48*b*c^3 - 56*a*c^2*d))/(105*d^4) - (b*(c + d/x^2)^(1/2
))/(7*d*x^6) - ((c + d/x^2)^(1/2)*(24*b*c^2 - 28*a*c*d))/(105*d^3*x^2) - ((
c + d/x^2)^(1/2)*(7*a*d - 6*b*c))/(35*d^2*x^4)
```

3.968 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=82

$$-\frac{2d(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^3}+\frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x^3}{15c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c}$$

```
[Out] -2/15*d*(-4*a*d+5*b*c)*x*(c+d/x^2)^(1/2)/c^3+1/15*(-4*a*d+5*b*c)*x^3*(c+d/x
^2)^(1/2)/c^2+1/5*a*x^5*(c+d/x^2)^(1/2)/c
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 277, 197}

$$-\frac{2dx\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{15c^3}+\frac{x^3\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{15c^2}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

```
[Out] (-2*d*(5*b*c - 4*a*d)*Sqrt[c + d/x^2]*x)/(15*c^3) + ((5*b*c - 4*a*d)*Sqrt[c
 + d/x^2]*x^3)/(15*c^2) + (a*Sqrt[c + d/x^2]*x^5)/(5*c)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c} + \frac{(5bc-4ad)\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx}{5c}$$

$$= \frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x^3}{15c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c} - \frac{(2d(5bc-4ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{15c^2}$$

$$= -\frac{2d(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^3} + \frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x^3}{15c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.68

$$\frac{\sqrt{c+\frac{d}{x^2}}\,x(5bc(-2d+cx^2)+a(8d^2-4cdx^2+3c^2x^4))}{15c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
[Out] (Sqrt[c + d/x^2]*x*(5*b*c*(-2*d + c*x^2) + a*(8*d^2 - 4*c*d*x^2 + 3*c^2*x^4
)))/(15*c^3)
```

Maple [A]
time = 0.05, size = 67, normalized size = 0.82

method	result	size
trager	$\frac{\left(3a\,c^2x^4-4acd\,x^2+5b\,c^2x^2+8a\,d^2-10bcd\right)x\sqrt{-\frac{-c\,x^2-d}{x^2}}}{15c^3}$	62
gosper	$\frac{\left(3a\,c^2x^4-4acd\,x^2+5b\,c^2x^2+8a\,d^2-10bcd\right)\left(c\,x^2+d\right)}{15x\sqrt{\frac{c\,x^2+d}{x^2}}\;c^3}$	67
default	$\frac{\left(3a\,c^2x^4-4acd\,x^2+5b\,c^2x^2+8a\,d^2-10bcd\right)\left(c\,x^2+d\right)}{15x\sqrt{\frac{c\,x^2+d}{x^2}}\;c^3}$	67

risch	$\frac{(3a\,c^{2}x^{4}-4acd\,x^{2}+5b\,c^{2}x^{2}+8a\,d^{2}-10bcd)\,(c\,x^{2}+d)}{15x\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\;c^{3}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^4/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/x*(3*a*c^2*x^4-4*a*c*d*x^2+5*b*c^2*x^2+8*a*d^2-10*b*c*d)*(c*x^2+d)/((c
*x^2+d)/x^2)^(1/2)/c^3
```

Maxima [A]
time = 0.28, size = 85, normalized size = 1.04

$$\frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-3\sqrt{c+\frac{d}{x^2}}\,dx\right)b}{3\,c^2}+\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3+15\sqrt{c+\frac{d}{x^2}}\,d^2x\right)a}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)*b/c^2 + 1/15*(3*(c + d/
x^2)^(5/2)*x^5 - 10*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)*a/c
^3
```

Fricas [A]
time = 2.22, size = 59, normalized size = 0.72

$$\frac{\left(3\,ac^2x^5+(5\,bc^2-4\,acd)x^3-2\,(5\,bcd-4\,ad^2)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 - 4*a*c*d)*x^3 - 2*(5*b*c*d - 4*a*d^2)*x)*sqrt
((c*x^2 + d)/x^2)/c^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(76) = 152.
time = 1.59, size = 338, normalized size = 4.12

$$\frac{3ac^4d^{\frac{9}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{2ac^3d^{\frac{11}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{3ac^2d^{\frac{13}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{12acd^{\frac{15}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{8ad^{\frac{17}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{b\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3c}-\frac{2bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4/(c+d/x**2)**(1/2),x)
```

```
[Out] 3*a*c**4*d**(9/2)*x**8*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5
*x**2 + 15*c**3*d**6) + 2*a*c**3*d**(11/2)*x**6*sqrt(c*x**2/d + 1)/(15*c**5
*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 3*a*c**2*d**(13/2)*x**4*sq
rt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 1
2*a*c*d**(15/2)*x**2*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x
**2 + 15*c**3*d**6) + 8*a*d**(17/2)*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 +
 30*c**4*d**5*x**2 + 15*c**3*d**6) + b*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c
) - 2*b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c**2)
```

Giac [A]
time = 0.96, size = 99, normalized size = 1.21

$$\frac{2\left(5\,bcd^{\frac{3}{2}}-4\,ad^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c^3}-\frac{(bcd-ad^2)\sqrt{cx^2+d}}{c^3\mathrm{sgn}\,(x)}+\frac{3\,(cx^2+d)^{\frac{5}{2}}a+5\,(cx^2+d)^{\frac{3}{2}}bc-10\,(cx^2+d)^{\frac{3}{2}}ad}{15\,c^3\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*b*c*d^(3/2) - 4*a*d^(5/2))*sgn(x)/c^3 - (b*c*d - a*d^2)*sqrt(c*x^2
+ d)/(c^3*sgn(x)) + 1/15*(3*(c*x^2 + d)^(5/2)*a + 5*(c*x^2 + d)^(3/2)*b*c -
 10*(c*x^2 + d)^(3/2)*a*d)/(c^3*sgn(x))
```

Mupad [B]
time = 5.31, size = 53, normalized size = 0.65

$$\frac{x\,\sqrt{c+\frac{d}{x^2}}\,\left(3\,a\,c^2\,x^4+5\,b\,c^2\,x^2-4\,a\,c\,d\,x^2-10\,b\,c\,d+8\,a\,d^2\right)}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (x*(c + d/x^2)^(1/2)*(8*a*d^2 + 3*a*c^2*x^4 + 5*b*c^2*x^2 - 10*b*c*d - 4*a*
c*d*x^2))/(15*c^3)
```

3.969 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=51

$$\frac{(3bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c}$$

```
[Out] 1/3*(-2*a*d+3*b*c)*x*(c+d/x^2)^(1/2)/c^2+1/3*a*x^3*(c+d/x^2)^(1/2)/c
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {464, 197}

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,(3bc-2ad)}{3c^2}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] ((3*b*c - 2*a*d)*Sqrt[c + d/x^2]*x)/(3*c^2) + (a*Sqrt[c + d/x^2]*x^3)/(3*c)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c} + \frac{(3bc-2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{3c}$$
$$= \frac{(3bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.67

$$\frac{\sqrt{c+\frac{d}{x^2}}\,x(3bc-2ad+acx^2)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(3*b*c - 2*a*d + a*c*x^2))/(3*c^2)
```

Maple [A]
time = 0.04, size = 44, normalized size = 0.86

method	result	size
trager	$\frac{(c\,x^2a-2ad+3bc)x\sqrt{-\frac{-c\,x^2-d}{x^2}}}{3c^2}$	39
gosper	$\frac{(c\,x^2a-2ad+3bc)(c\,x^2+d)}{3x\sqrt{\frac{c\,x^2+d}{x^2}}\,c^2}$	44
default	$\frac{(c\,x^2a-2ad+3bc)(c\,x^2+d)}{3x\sqrt{\frac{c\,x^2+d}{x^2}}\,c^2}$	44
risch	$\frac{(c\,x^2a-2ad+3bc)(c\,x^2+d)}{3x\sqrt{\frac{c\,x^2+d}{x^2}}\,c^2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^2/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/x*(a*c*x^2-2*a*d+3*b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/c^2
```

Maxima [A]
time = 0.28, size = 49, normalized size = 0.96

$$\frac{b\sqrt{c+\frac{d}{x^2}}\,x}{c}+\frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-3\,\sqrt{c+\frac{d}{x^2}}\,dx\right)a}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*sqrt(c + d/x^2)*x/c + 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)
*a/c^2
```

Fricas [A]
time = 3.53, size = 36, normalized size = 0.71

$$\frac{(acx^3+(3\,bc-2\,ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*c*x^3 + (3*b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

Sympy [A]
time = 1.25, size = 70, normalized size = 1.37

$$\frac{a\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3c}-\frac{2ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2}+\frac{b\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c) - 2*a*d**(3/2)*sqrt(c*x**2/d + 1)/(
3*c**2) + b*sqrt(d)*sqrt(c*x**2/d + 1)/c
```

Giac [A]
time = 1.02, size = 66, normalized size = 1.29

$$-\frac{\left(3\,bc\sqrt{d}\;-2\,ad^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{3\,c^2}+\frac{(cx^2+d)^{\frac{3}{2}}a}{3\,c^2\mathrm{sgn}\,(x)}+\frac{\sqrt{cx^2+d}\,(bc-ad)}{c^2\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b*c*sqrt(d) - 2*a*d^(3/2))*sgn(x)/c^2 + 1/3*(c*x^2 + d)^(3/2)*a/(c^
2*sgn(x)) + sqrt(c*x^2 + d)*(b*c - a*d)/(c^2*sgn(x))
```

Mupad [B]
time = 4.92, size = 67, normalized size = 1.31

$$\frac{a\,x^3\,\sqrt{c+\frac{d}{x^2}}\,\left(c-\frac{2\,d}{x^2}\right)}{3\,c^2}+\frac{b\,x\,\sqrt{\frac{c\,x^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d}+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^3*(c + d/x^2)^(1/2)*(c - (2*d)/x^2))/(3*c^2) + (b*x*((c*x^2)/d + 1)^(1
/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1))
```

3.970 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=47

$$\frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{\sqrt{d}}$$

```
[Out] -b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(1/2)+a*x*(c+d/x^2)^(1/2)/c
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {382, 462, 223, 212}

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c + d/x^2]*x)/c - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/Sqrt[d]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
```

```
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx &= -\text{Subst}\left(\int \frac{a+bx^2}{x^2\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{\sqrt{d}}
\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 71, normalized size = 1.51

$$\frac{a\sqrt{d}\,(d+cx^2)-bc\sqrt{d+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{c\sqrt{d}\,\sqrt{c+\frac{d}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[d]*(d + c*x^2) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d
]])/(c*Sqrt[d]*Sqrt[c + d/x^2]*x)
```

Maple [A]
time = 0.04, size = 73, normalized size = 1.55

method	result	size
default	$\frac{\sqrt{c\,x^{2}+d}\,\left(a\sqrt{c\,x^{2}+d}\,\sqrt{d}-b\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)c\right)}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,xc\sqrt{d}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+d)^(1/2)*(a*(c*x^2+d)^(1/2)*d^(1/2)-b*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+
d)/x)*c)/((c*x^2+d)/x^2)^(1/2)/x/c/d^(1/2)
```

Maxima [A]
time = 0.49, size = 58, normalized size = 1.23

$$\frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c}+\frac{b\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] a*sqrt(c + d/x^2)*x/c + 1/2*b*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d
/x^2)*x + sqrt(d)))/sqrt(d)
```

Fricas [A]
time = 4.28, size = 131, normalized size = 2.79

$$\left[\frac{2\,adx\sqrt{\frac{cx^2+d}{x^2}}+bc\sqrt{d}\,\log\left(-\frac{cx^2-2\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)}{2\,cd},\ \frac{adx\sqrt{\frac{cx^2+d}{x^2}}+bc\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*d*x*sqrt((c*x^2 + d)/x^2) + b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x
*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c*d), (a*d*x*sqrt((c*x^2 + d)/x^2) + b
*c*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c*d)]
```

Sympy [A]
time = 1.31, size = 39, normalized size = 0.83

$$\frac{a\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c}-\frac{b\,\text{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*sqrt(c*x**2/d + 1)/c - b*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(39) = 78.
time = 0.97, size = 80, normalized size = 1.70

$$-\frac{\left(bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+a\sqrt{-d}\,\sqrt{d}\right)\text{sgn}(x)}{c\sqrt{-d}}+\frac{\frac{b\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}+\frac{\sqrt{cx^2+d}\,a}{c}}{\text{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*c*arctan(sqrt(d)/sqrt(-d)) + a*sqrt(-d)*sqrt(d))*sgn(x)/(c*sqrt(-d)) +
(b*arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) + sqrt(c*x^2 + d)*a/c)/sgn(x)
```

Mupad [B]
time = 5.00, size = 65, normalized size = 1.38

$$\frac{a\,x\,\sqrt{\frac{c\,x^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d}+1}\,+1\right)}-\frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x*((c*x^2)/d + 1)^(1/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1))
 - (b*log((c + d/x^2)^(1/2) + d^(1/2)/x))/d^(1/2)
```

3.971 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\, x^2}\, dx$

Optimal. Leaf size=61

$$-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\, x}\right)}{2d^{3/2}}$$

```
[Out] 1/2*(-2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)-1/2*b*(c+d/x^2)
^(1/2)/d/x
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {470, 342, 223, 212}

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

```
[Out] -1/2*(b*Sqrt[c + d/x^2])/(d*x) + ((b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d
/x^2]*x)])/(2*d^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx &= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}+\frac{(-bc+2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx}{2d}\\ &= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{(-bc+2ad)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{2d}\\ &= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{(-bc+2ad)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d}\\ &= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 80, normalized size = 1.31

$$\frac{-b\sqrt{d}\,(d+cx^2)+(bc-2ad)x^2\sqrt{d+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{c+\frac{d}{x^2}}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

[Out] (-(b*Sqrt[d]*(d + c*x^2)) + (b*c - 2*a*d)*x^2*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(2*d^(3/2)*Sqrt[c + d/x^2]*x^3)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(49) = 98$.
time = 0.05, size = 105, normalized size = 1.72

method	result	size
default	$-\frac{\sqrt{c\,x^{2}+d}\,\left(2a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)d^{2}x^{2}-\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)bcd\,x^{2}+\sqrt{c\,x^{2}+d}\,d^{\frac{3}{2}}b\right)}{2\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x^{3}d^{\frac{5}{2}}}$	105
risch	$-\frac{b(c\,x^{2}+d)}{2d\,x^{3}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{\left(-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)}{\sqrt{d}}+\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)bc}{2d^{\frac{3}{2}}}\right)\sqrt{c\,x^{2}+d}}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$	117

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b/x^2+a)/x^2/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(c*x^2+d)^(1/2)*(2*a*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*d^2*x^2-ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c*d*x^2+(c*x^2+d)^(1/2)*d^(3/2)*b)/((c*x^2+d)/x^2)^(1/2)/x^3/d^(5/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(49) = 98$.
time = 0.49, size = 121, normalized size = 1.98

$$-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^{2}}}\,cx}{\left(c+\frac{d}{x^{2}}\right)dx^{2}-d^{2}}+\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)b+\frac{a\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*b + 1/2*a*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d)

Fricas [A]
time = 3.26, size = 144, normalized size = 2.36

$$\left[-\frac{(bc-2ad)\sqrt{d}\,x\log\left(-\frac{cx^2-2\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2d}{x^2}\right)+2bd\sqrt{\frac{cx^2+d}{x^2}}}{4d^2x}, -\frac{(bc-2ad)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+bd\sqrt{\frac{cx^2+d}{x^2}}}{2d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c - 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x
^2) + 2*d)/x^2) + 2*b*d*sqrt((c*x^2 + d)/x^2))/(d^2*x), -1/2*((b*c - 2*a*d)
*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*d*sqrt
((c*x^2 + d)/x^2))/(d^2*x)]
```

Sympy [A]
time = 2.25, size = 66, normalized size = 1.08

$$-\frac{a\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{\sqrt{d}}-\frac{b\sqrt{c}\,\sqrt{1+\frac{d}{cx^2}}}{2dx}+\frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**2/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d
*x) + b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(3/2))
```

Giac [A]
time = 0.92, size = 64, normalized size = 1.05

$$-\frac{\frac{(bc^2-2acd)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d}+\frac{\sqrt{cx^2+d}\,bc}{dx^2}}{2\,c\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((b*c^2 - 2*a*c*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d) + sqr
t(c*x^2 + d)*b*c/(d*x^2))/(c*sgn(x))
```

Mupad [B]
time = 5.53, size = 94, normalized size = 1.54

$$\begin{cases} -\frac{3\,a\,x^2+b}{3\,\sqrt{c}\,x^3} & \text{if}\ \ d=0\\ \frac{b\,c\,\ln\left(2\,\sqrt{c+\frac{d}{x^2}}+\frac{2\,\sqrt{d}}{x}\right)}{2\,d^{3/2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{2\,d\,x}-\frac{a\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}} & \text{if}\ \ d\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^2*(c + d/x^2)^(1/2)),x)
```

```
[Out] piecewise(d == 0, -(b + 3*a*x^2)/(3*c^(1/2)*x^3), d ~= 0, - (a*log((c + d/x
^2)^(1/2) + d^(1/2)/x))/d^(1/2) - (b*(c + d/x^2)^(1/2))/(2*d*x) + (b*c*log(
2*(c + d/x^2)^(1/2) + (2*d^(1/2))/x))/(2*d^(3/2)))
```

3.972 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\, x^4}\, dx$

Optimal. Leaf size=93

$$-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{5/2}}$$

```
[Out] -1/8*c*(-4*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)-1/4*b*(c+d
/x^2)^(1/2)/d/x^3+1/8*(-4*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^2/x
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 342, 327, 223, 212}

$$-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{5/2}}+\frac{\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{8d^2x}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

```
[Out] -1/4*(b*Sqrt[c + d/x^2])/(d*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^2
*x) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx = -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(-3bc+4ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx}{4d}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}-\frac{(-3bc+4ad)\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{4d}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{(c(3bc-4ad))\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{8d^2}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{(c(3bc-4ad))\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}}\right.}{8d^2}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{5/2}}$$

Mathematica [A]
time = 0.15, size = 101, normalized size = 1.09

$$\frac{-\sqrt{d}\,(d+cx^2)\,(2bd-3bcx^2+4adx^2)-c(3bc-4ad)x^4\sqrt{d+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{c+\frac{d}{x^2}}\,x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

```
[Out] (-(Sqrt[d]*(d + c*x^2)*(2*b*d - 3*b*c*x^2 + 4*a*d*x^2)) - c*(3*b*c - 4*a*d)
*x^4*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(8*d^(5/2)*Sqrt[c +
d/x^2]*x^5)
```

Maple [A]
time = 0.06, size = 146, normalized size = 1.57

method	result
risch	$-\frac{(c\,x^2+d)\,(4ad\,x^2-3c\,x^2b+2bd)}{8d^2x^5\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{\left(\frac{c\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a}{2d^{\frac{3}{2}}}-\frac{3c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)b}{8d^{\frac{5}{2}}}\right)\sqrt{c\,x^2+d}}{\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$
default	$-\frac{\sqrt{c\,x^2+d}\,\left(-4\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)ac\,d^2x^4+3\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)b\,c^2d\,x^4+4\sqrt{c\,x^2+d}\,d^{\frac{5}{2}}a\,x^2 \ldots\right.}{8\sqrt{\frac{c\,x^2+d}{x^2}}\,x^5d^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/x^4/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^2+d)^(1/2)*(-4*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*a*c*d^2*x^4+3*
ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c^2*d*x^4+4*(c*x^2+d)^(1/2)*d^(5/2)*a
*x^2-3*(c*x^2+d)^(1/2)*d^(3/2)*b*c*x^2+2*(c*x^2+d)^(1/2)*d^(5/2)*b)/((c*x^2
+d)/x^2)^(1/2)/x^5/d^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(77) = 154.
time = 0.51, size = 200, normalized size = 2.15

$$-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,cx}{\left(c+\frac{d}{x^2}\right)dx^2-d^2}+\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)a+\frac{1}{16}b\left(\frac{3c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3-5\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2d^2x^4-2\left(c+\frac{d}{x^2}\right)d^3x^2+d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/x
^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*a + 1/16*b*(3*c^2*
log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) +
2*(3*(c + d/x^2)^(3/2)*c^2*x^3 - 5*sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2)^2*
d^2*x^4 - 2*(c + d/x^2)*d^3*x^2 + d^4))
```

Fricas [A]
time = 3.22, size = 201, normalized size = 2.16

$$\left[-\frac{(3\,bc^2-4\,acd)\sqrt{d}\,x^3\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,bd^2-(3\,bcd-4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^3x^3},\ \frac{(3\,bc^2-4\,acd)\sqrt{-d}\,x^3\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(2\,bd^2-(3\,bcd-4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,d^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x
^2 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^2)*sqrt((c*x^
2 + d)/x^2))/(d^3*x^3), 1/8*((3*b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-
d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^
2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^3)]
```

Sympy [A]
time = 4.48, size = 150, normalized size = 1.61

$$-\frac{a\sqrt{c}\,\sqrt{1+\frac{d}{cx^2}}}{2dx}+\frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2d^{\frac{3}{2}}}+\frac{3bc^{\frac{3}{2}}}{8d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{b\sqrt{c}}{8dx^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8d^{\frac{5}{2}}}-\frac{b}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**4/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d*x) + a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2
*d**(3/2)) + 3*b*c**(3/2)/(8*d**2*x*sqrt(1 + d/(c*x**2))) + b*sqrt(c)/(8*d*
x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(5/2
)) - b/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

Giac [A]
time = 0.82, size = 125, normalized size = 1.34

$$\frac{\frac{(3\,bc^3-4\,ac^2d)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d^2}+\frac{3\,(cx^2+d)^{\frac{3}{2}}bc^3-4\,(cx^2+d)^{\frac{3}{2}}ac^2d-5\,\sqrt{cx^2+d}\,bc^3d+4\,\sqrt{cx^2+d}\,ac^2d^2}{c^2d^2x^4}}{8\,\mathrm{csgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((3*b*c^3 - 4*a*c^2*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^2)
+ (3*(c*x^2 + d)^(3/2)*b*c^3 - 4*(c*x^2 + d)^(3/2)*a*c^2*d - 5*sqrt(c*x^2 +
 d)*b*c^3*d + 4*sqrt(c*x^2 + d)*a*c^2*d^2)/(c^2*d^2*x^4))/(c*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{x^2}}{x^4\sqrt{c+\frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)),x)
```

```
[Out] int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)), x)
```

3.973 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{3d(4bc-5ad)}{8c^3\sqrt{c+\frac{d}{x^2}}}+\frac{(4bc-5ad)x^2}{8c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}-\frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}$$

```
[Out] -3/8*d*(-5*a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(7/2)+3/8*d*(-5*a*
d+4*b*c)/c^3/(c+d/x^2)^(1/2)+1/8*(-5*a*d+4*b*c)*x^2/c^2/(c+d/x^2)^(1/2)+1/4
*a*x^4/c/(c+d/x^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 44, 53, 65, 214}

$$-\frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}+\frac{3d(4bc-5ad)}{8c^3\sqrt{c+\frac{d}{x^2}}}+\frac{x^2(4bc-5ad)}{8c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

```
[Out] (3*d*(4*b*c - 5*a*d))/(8*c^3*Sqrt[c + d/x^2]) + ((4*b*c - 5*a*d)*x^2)/(8*c^
2*Sqrt[c + d/x^2]) + (a*x^4)/(4*c*Sqrt[c + d/x^2]) - (3*d*(4*b*c - 5*a*d)*A
rcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x^3(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{\left(2bc-\frac{5ad}{2}\right)\text{Subst}\left(\int \frac{1}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{4c}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{(3(4bc-5ad))\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{8c^2}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} + \frac{(3d(4bc-5ad))\text{Subst}\left(\int\right.}{16c}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} + \frac{(3(4bc-5ad))\text{Subst}\left(\int\right.}{8}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{3d(4bc-5ad)\tanh^{-1}\left(\right.}{8c^{7/2}}$$

Mathematica [A]
time = 0.16, size = 112, normalized size = 0.95

$$\frac{\sqrt{c}\,x(4bc(3d+cx^2)+a(-15d^2-5cdx^2+2c^2x^4))+3d(4bc-5ad)\sqrt{d+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)}{8c^{7/2}\sqrt{c+\frac{d}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*x*(4*b*c*(3*d + c*x^2) + a*(-15*d^2 - 5*c*d*x^2 + 2*c^2*x^4)) + 3*
d*(4*b*c - 5*a*d)*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]])/(8*c
^(7/2)*Sqrt[c + d/x^2]*x)
```

Maple [A]
time = 0.09, size = 140, normalized size = 1.19

method	result
default	$\frac{(c x^2+d)\left(2c^{\frac{7}{2}} a x^5-5c^{\frac{5}{2}} ad x^3+4c^{\frac{7}{2}} b x^3-15c^{\frac{3}{2}} a d^2x+12c^{\frac{5}{2}} bdx+15\sqrt{c x^2+d}\ \ln\left(\sqrt{c}\ x+\sqrt{c x^2+d}\right)ac d^2-12\sqrt{c x^2+}\right.}{8\left(\frac{c x^2+d}{x^2}\right)^{\frac{3}{2}} x^3 c^{\frac{9}{2}}}$
risch	$\frac{(2c x^2a-7ad+4bc)(c x^2+d)}{8c^3\sqrt{\frac{c x^2+d}{x^2}}}+\frac{\left(-\frac{d^2xa}{c^3\sqrt{c x^2+d}}+\frac{dxb}{c^2\sqrt{c x^2+d}}+\frac{15d^2\ln\left(\sqrt{c}\ x+\sqrt{c x^2+d}\right)a}{8c^{\frac{7}{2}}}-\frac{3d\ln\left(\sqrt{c}\ x+\sqrt{c x\right.}}{2c^{\frac{5}{2}}}\right.}{\sqrt{\frac{c x^2+d}{x^2}}\ x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^3/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*x^2+d)*(2*c^(7/2)*a*x^5-5*c^(5/2)*a*d*x^3+4*c^(7/2)*b*x^3-15*c^(3/2)
*a*d^2*x+12*c^(5/2)*b*d*x+15*(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*
a*c*d^2-12*(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c^2*d)/((c*x^2+d
)/x^2)^(3/2)/x^3/c^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(98) = 196.
time = 0.50, size = 215, normalized size = 1.82

$$-\frac{1}{16}a\left(\frac{2\left(15\left(c+\frac{d}{x^2}\right)^2d^2-25\left(c+\frac{d}{x^2}\right)cd^2+8c^2d^2\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3-2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4+\sqrt{c+\frac{d}{x^2}}\,c^5}+\frac{15\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right)+\frac{1}{4}b\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)d-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2-\sqrt{c+\frac{d}{x^2}}\,c^3}+\frac{3\,d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*a*(2*(15*(c + d/x^2)^2*d^2 - 25*(c + d/x^2)*c*d^2 + 8*c^2*d^2)/((c +
d/x^2)^(5/2)*c^3 - 2*(c + d/x^2)^(3/2)*c^4 + sqrt(c + d/x^2)*c^5) + 15*d^2*
log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(7/2)) + 1/4
*b*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2)*c^
3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(5/
2))
```

Fricas [A]
time = 4.58, size = 304, normalized size = 2.58

$$\left[-\frac{3\left(4\,bcd^2-5\,ad^3+\left(4\,bc^2d-5\,acd^2\right)x^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(2\,ac^3x^6+\left(4\,bc^3-5\,ac^2d\right)x^4+3\left(4\,bc^2d-5\,acd^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\left(c^5x^2+c^4d\right)},\ \frac{3\left(4\,bcd^2-5\,ad^3+\left(4\,bc^2d-5\,acd^2\right)x^2\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(2\,ac^3x^6+\left(4\,bc^3-5\,ac^2d\right)x^4+3\left(4\,bc^2d-5\,acd^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{8\left(c^5x^2+c^4d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(c)*log(-
2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(2*a*c^3*x^6 + (4*b*
c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2)
)/(c^5*x^2 + c^4*d), 1/8*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*
x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a
*c^3*x^6 + (4*b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(
(c*x^2 + d)/x^2))/(c^5*x^2 + c^4*d)]
```

Sympy [A]
time = 38.19, size = 177, normalized size = 1.50

$$a\left(\frac{x^5}{4c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{5\sqrt{d}\,x^3}{8c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{15d^{\frac{3}{2}}x}{8c^3\sqrt{\frac{cx^2}{d}+1}}+\frac{15d^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{\frac{7}{2}}}\right)+b\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}\,x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{3d\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**5/(4*c*sqrt(d)*sqrt(c*x**2/d + 1)) - 5*sqrt(d)*x**3/(8*c**2*sqrt(c*x*
*2/d + 1)) - 15*d**(3/2)*x/(8*c**3*sqrt(c*x**2/d + 1)) + 15*d**2*asinh(sqrt
(c)*x/sqrt(d))/(8*c**(7/2))) + b*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3
*sqrt(d)*x/(2*c**2*sqrt(c*x**2/d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c*
*(5/2)))
```

Giac [A]
time = 0.94, size = 144, normalized size = 1.22

$$\frac{\left(x^2\left(\frac{2\,ax^2}{c\operatorname{sgn}(x)}+\frac{4\,bc^4\operatorname{sgn}(x)-5\,ac^3d\operatorname{sgn}(x)}{c^5}\right)+\frac{3\,(4\,bc^3d\operatorname{sgn}(x)-5\,ac^2d^2\operatorname{sgn}(x))}{c^5}\right)x}{8\,\sqrt{cx^2+d}}-\frac{3\,(4\,bcd\log{(|d|)}-5\,ad^2\log{(|d|)})\operatorname{sgn}(x)}{16\,c^{\frac{7}{2}}}+\frac{3\,(4\,bcd-5\,ad^2)\log{\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+d}\right|\right)}}{8\,c^{\frac{7}{2}}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(x^2*(2*a*x^2/(c*sgn(x)) + (4*b*c^4*sgn(x) - 5*a*c^3*d*sgn(x))/c^5) + 3
*(4*b*c^3*d*sgn(x) - 5*a*c^2*d^2*sgn(x))/c^5)*x/sqrt(c*x^2 + d) - 3/16*(4*b
*c*d*log(abs(d)) - 5*a*d^2*log(abs(d)))*sgn(x)/c^(7/2) + 3/8*(4*b*c*d - 5*a
*d^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(7/2)*sgn(x))
```

Mupad [B]
time = 6.34, size = 134, normalized size = 1.14

$$\frac{a\,x^4}{4\,c\,\sqrt{c+\frac{d}{x^2}}}-\frac{15\,a\,d^2}{8\,c^3\,\sqrt{c+\frac{d}{x^2}}}+\frac{b\,x^2}{2\,c\,\sqrt{c+\frac{d}{x^2}}}-\frac{3\,b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}}+\frac{15\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{7/2}}+\frac{3\,b\,d}{2\,c^2\,\sqrt{c+\frac{d}{x^2}}}-\frac{5\,a\,d\,x^2}{8\,c^2\,\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x^4)/(4*c*(c + d/x^2)^(1/2)) - (15*a*d^2)/(8*c^3*(c + d/x^2)^(1/2)) + (b
*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2
*c^(5/2)) + (15*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(7/2)) + (3*b*
d)/(2*c^2*(c + d/x^2)^(1/2)) - (5*a*d*x^2)/(8*c^2*(c + d/x^2)^(1/2))
```

3.974 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}$$

```
[Out] 1/2*(-3*a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(5/2)+1/2*(3*a*d-2*b*
c)/c^2/(c+d/x^2)^(1/2)+1/2*a*x^2/c/(c+d/x^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 79, 53, 65, 214}

$$\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

```
[Out] -1/2*(2*b*c - 3*a*d)/(c^2*Sqrt[c + d/x^2]) + (a*x^2)/(2*c*Sqrt[c + d/x^2])
+ ((2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*c^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{\left(bc-\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{1}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{2c}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{4c^2}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{2c^2d}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} + \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Mathematica [**A**]
time = 0.13, size = 88, normalized size = 1.02

$$\frac{\sqrt{c}\,x(-2bc+3ad+acx^2)+(-2bc+3ad)\sqrt{d+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)}{2c^{5/2}\sqrt{c+\frac{d}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*x*(-2*b*c + 3*a*d + a*c*x^2) + (-2*b*c + 3*a*d)*Sqrt[d + c*x^2]*Lo
g[-(Sqrt[c]*x) + Sqrt[d + c*x^2]])/(2*c^(5/2)*Sqrt[c + d/x^2]*x)
```

Maple [**A**]
time = 0.08, size = 114, normalized size = 1.33

method	result

default	$\frac{(c\,x^2+d)\left(c^{\frac{5}{2}}a\,x^3+3c^{\frac{3}{2}}adx-2c^{\frac{5}{2}}bx-3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{c\,x^2+d}\,acd+2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{c\,x^2+ }\right.}{2\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^3c^{\frac{7}{2}}}$
risch	$\frac{a(c\,x^2+d)}{2c^2\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{\left(\frac{xad}{c^2\sqrt{c\,x^2+d}}-\frac{xb}{c\sqrt{c\,x^2+d}}-\frac{3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)ad}{2c^{\frac{5}{2}}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)b}{c^{\frac{3}{2}}}\right)\sqrt{c\,x}}{\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2+d)*(c^(5/2)*a*x^3+3*c^(3/2)*a*d*x-2*c^(5/2)*b*x-3*ln(c^(1/2)*x+(
c*x^2+d)^(1/2))*(c*x^2+d)^(1/2)*a*c*d+2*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^
2+d)^(1/2)*b*c^2)/((c*x^2+d)/x^2)^(3/2)/x^3/c^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(70) = 140$.
time = 0.52, size = 144, normalized size = 1.67

$$\frac{1}{4}\,a\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)d-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2-\sqrt{c+\frac{d}{x^2}}\,c^3}+\frac{3\,d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)-\frac{1}{2}\,b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2)
*c^3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^
(5/2)) - 1/2*b*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))
)/c^(3/2) + 2/(sqrt(c + d/x^2)*c))
```

Fricas [A]
time = 2.98, size = 249, normalized size = 2.90

$$\left[-\frac{(2\,bcd-3\,ad^2+(2\,bc^2-3\,acd)x^2)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(ac^2x^4-(2\,bc^2-3\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{4\,(c^4x^2+c^3d)},\ -\frac{(2\,bcd-3\,ad^2+(2\,bc^2-3\,acd)x^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(ac^2x^4-(2\,bc^2-3\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{2\,(c^4x^2+c^3d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*b*c*d - 3*a*d^2 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(c)*log(-2*c*x^2 +
 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(a*c^2*x^4 - (2*b*c^2 - 3*a*c
*d)*x^2)*sqrt((c*x^2 + d)/x^2))/(c^4*x^2 + c^3*d), -1/2*((2*b*c*d - 3*a*d^2
 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x
^2)/(c*x^2 + d)) - (a*c^2*x^4 - (2*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 + d)/x
^2))/(c^4*x^2 + c^3*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(73) = 146$.
time = 18.64, size = 264, normalized size = 3.07

$$a\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}\,x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{3d\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right)+b\left(-\frac{2c^3x^2\sqrt{1+\frac{d}{cx^2}}}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}-\frac{c^3x^2\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^3x^2\log\left(\sqrt{1+\frac{d}{cx^2}}+1\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}-\frac{c^2d\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^2d\log\left(\sqrt{1+\frac{d}{cx^2}}+1\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*sqrt(d)*x/(2*c**2*sqrt(c*x**2/
d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(5/2))) + b*(-2*c**3*x**2*sqrt
(1 + d/(c*x**2))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**3*x**2*log(d/(c*x**2
))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) + 2*c**3*x**2*log(sqrt(1 + d/(c*x**2))
+ 1)/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**2*d*log(d/(c*x**2))/(2*c**(9/2)*
x**2 + 2*c**(7/2)*d) + 2*c**2*d*log(sqrt(1 + d/(c*x**2)) + 1)/(2*c**(9/2)*x
**2 + 2*c**(7/2)*d))
```

Giac [A]
time = 1.06, size = 105, normalized size = 1.22

$$\frac{x\left(\frac{ax^2}{c\operatorname{sgn}(x)}-\frac{2\,bc^2\operatorname{sgn}(x)-3\,acd\operatorname{sgn}(x)}{c^3}\right)}{2\sqrt{cx^2+d}}+\frac{(2\,bc\log(|d|)-3\,ad\log(|d|))\operatorname{sgn}(x)}{4\,c^{\frac{5}{2}}}-\frac{(2\,bc-3\,ad)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+d}\right|\right)}{2\,c^{\frac{5}{2}}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*(a*x^2/(c*sgn(x)) - (2*b*c^2*sgn(x) - 3*a*c*d*sgn(x))/c^3)/sqrt(c*x^2
 + d) + 1/4*(2*b*c*log(abs(d)) - 3*a*d*log(abs(d)))*sgn(x)/c^(5/2) - 1/2*(2
*b*c - 3*a*d)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(5/2)*sgn(x))
```

Mupad [B]
time = 5.61, size = 90, normalized size = 1.05

$$\frac{b\operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{b}{c\sqrt{c+\frac{d}{x^2}}}+\frac{a\,x^2}{2\,c\sqrt{c+\frac{d}{x^2}}}-\frac{3\,a\,d\operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}}+\frac{3\,a\,d}{2\,c^2\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - b/(c*(c + d/x^2)^(1/2)) + (a
*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2
*c^(5/2)) + (3*a*d)/(2*c^2*(c + d/x^2)^(1/2))
```

3.975 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} dx$

Optimal. Leaf size=52

$$\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

```
[Out] a*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+(-a*d+b*c)/c/d/(c+d/x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {457, 79, 65, 214}

$$\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

```
[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c^
(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{2c}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{cd}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 75, normalized size = 1.44

$$\frac{\sqrt{c}\,(bc-ad)x-ad\sqrt{d+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{d+cx^2}\right)}{c^{3/2}d\sqrt{c+\frac{d}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

```
[Out] (Sqrt[c]*(b*c - a*d)*x - a*d*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[d + c*
x^2]])/(c^(3/2)*d*Sqrt[c + d/x^2]*x)
```

Maple [A]
time = 0.05, size = 75, normalized size = 1.44

method	result	size
default	$\frac{(c\,x^2+d)\left(c^{\frac{5}{2}}bx-c^{\frac{3}{2}}adx+\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{c\,x^2+d}\,acd\right)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^3c^{\frac{5}{2}}d}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/(c+d/x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+d)*(c^(5/2)*b*x-c^(3/2)*a*d*x+ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^2+d
)^(1/2)*a*c*d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^(5/2)/d
```

Maxima [A]
time = 0.52, size = 69, normalized size = 1.33

$$-\frac{1}{2}\,a\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}\,c}\right)+\frac{b}{\sqrt{c+\frac{d}{x^2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2
) + 2/(sqrt(c + d/x^2)*c)) + b/(sqrt(c + d/x^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.
time = 3.50, size = 200, normalized size = 3.85

$$\left[\frac{2\,(bc^2-acd)x^2\sqrt{\frac{cx^2+d}{x^2}}+(acdx^2+ad^2)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{2\,(c^3dx^2+c^2d^2)},\ \frac{(bc^2-acd)x^2\sqrt{\frac{cx^2+d}{x^2}}-(acdx^2+ad^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{c^3dx^2+c^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) + (a*c*d*x^2 + a*d^2)*sqr
t(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/(c^3*d*x^2 +
c^2*d^2), ((b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) - (a*c*d*x^2 + a*d^2)*
sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^3*d*x^2
 + c^2*d^2)]
```

Sympy [A]
time = 7.57, size = 49, normalized size = 0.94

$$-\frac{a\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{c\sqrt{-c}}-\frac{ad-bc}{cd\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x,x)
```

```
[Out] -a*atan(sqrt(c + d/x**2)/sqrt(-c))/(c*sqrt(-c)) - (a*d - b*c)/(c*d*sqrt(c +
 d/x**2))
```

Giac [A]
time = 1.48, size = 69, normalized size = 1.33

$$\frac{a\log\left(|d|\right)\operatorname{sgn}(x)}{2\,c^{\frac{3}{2}}}+\frac{(bc\operatorname{sgn}(x)-ad\operatorname{sgn}(x))x}{\sqrt{cx^2+d}\,cd}-\frac{a\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+d}\right|\right)}{c^{\frac{3}{2}}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*log(abs(d))*sgn(x)/c^(3/2) + (b*c*sgn(x) - a*d*sgn(x))*x/(sqrt(c*x^2
+ d)*c*d) - a*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(3/2)*sgn(x))
```

Mupad [B]
time = 5.06, size = 54, normalized size = 1.04

$$\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{a}{c\,\sqrt{c+\frac{d}{x^2}}}+\frac{b\,\sqrt{x^2}}{d\,\sqrt{c\,x^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x*(c + d/x^2)^(3/2)),x)
```

```
[Out] (a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - a/(c*(c + d/x^2)^(1/2)) + (b
*(x^2)^(1/2))/(d*(d + c*x^2)^(1/2))
```

3.976 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3} \, dx$

Optimal. Leaf size=42

$$-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

```
[Out] (a*d-b*c)/d^2/(c+d/x^2)^(1/2)-b*(c+d/x^2)^(1/2)/d^2
```

Rubi [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
```

```
[Out] -((b*c - a*d)/(d^2*Sqrt[c + d/x^2])) - (b*Sqrt[c + d/x^2])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{-bc+ad}{d(c+dx)^{3/2}} + \frac{b}{d\sqrt{c+dx}}\right)\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}} - \frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

Mathematica [A]
time = 0.07, size = 36, normalized size = 0.86

$$\frac{adx^2 - b(d+2cx^2)}{d^2\sqrt{c+\frac{d}{x^2}}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
[Out] (a*d*x^2 - b*(d + 2*c*x^2))/(d^2*Sqrt[c + d/x^2]*x^2)
```

Maple [A]
time = 0.06, size = 46, normalized size = 1.10

method	result	size
gosper	$\frac{(ad\,x^2-2c\,x^2b-bd)\,(c\,x^2+d)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} d^2 x^4}$	46
default	$\frac{(ad\,x^2-2c\,x^2b-bd)\,(c\,x^2+d)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} d^2 x^4}$	46
trager	$\frac{(ad\,x^2-2c\,x^2b-bd)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{d^2(c\,x^2+d)}$	49
risch	$-\frac{b(c\,x^2+d)}{d^2x^2\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{ad-bc}{d^2\sqrt{\frac{c\,x^2+d}{x^2}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/(c+d/x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
[Out] (a*d*x^2-2*b*c*x^2-b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^2/x^4
```

Maxima [A]
time = 0.28, size = 46, normalized size = 1.10

$$-b\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2}+\frac{c}{\sqrt{c+\frac{d}{x^2}}\,d^2}\right)+\frac{a}{\sqrt{c+\frac{d}{x^2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] -b*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2)) + a/(sqrt(c + d/x^2)*d)
```

Fricas [A]
time = 2.10, size = 46, normalized size = 1.10

$$-\frac{((2\,bc-ad)x^2+bd)\sqrt{\frac{cx^2+d}{x^2}}}{cd^2x^2+d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
[Out] -((2*b*c - a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2)/(c*d^2*x^2 + d^3)
```

Sympy [A]
time = 0.63, size = 68, normalized size = 1.62

$$\begin{cases}\frac{a}{d\sqrt{c+\frac{d}{x^2}}}-\frac{2bc}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b}{dx^2\sqrt{c+\frac{d}{x^2}}} & \text{for}\ d\neq 0\\ \frac{-\frac{a}{2x^2}-\frac{b}{4x^4}}{c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**3,x)
[Out] Piecewise((a/(d*sqrt(c + d/x**2)) - 2*b*c/(d**2*sqrt(c + d/x**2)) - b/(d*x*
*2*sqrt(c + d/x**2)), Ne(d, 0)), ((-a/(2*x**2) - b/(4*x**4))/c**(3/2), True
))
```

Giac [A]
time = 1.78, size = 66, normalized size = 1.57

$$\frac{2\,b\sqrt{c}}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)d\mathrm{sgn}\left(x\right)}-\frac{(bc-ad)x}{\sqrt{cx^2+d}\,d^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 2*b*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)*d*sgn(x)) - (b*c - a*d)*
x/(sqrt(c*x^2 + d)*d^2*sgn(x))
```

Mupad [B]
time = 4.52, size = 46, normalized size = 1.10

$$\frac{x\,\sqrt{c+\frac{d}{x^2}}\,\left(x^2\,\left(\frac{a}{d}-\frac{2\,b\,c}{d^2}\right)-\frac{b}{d}\right)}{c\,x^3+d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^3*(c + d/x^2)^(3/2)),x)
```

```
[Out] (x*(c + d/x^2)^(1/2)*(x^2*(a/d - (2*b*c)/d^2) - b/d))/(d*x + c*x^3)
```

3.977 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5}\,dx$

Optimal. Leaf size=68

$$\frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

```
[Out] -1/3*b*(c+d/x^2)^(3/2)/d^3+c*(-a*d+b*c)/d^3/(c+d/x^2)^(1/2)+(-a*d+2*b*c)*(c
+d/x^2)^(1/2)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{d^3}+\frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

```
[Out] (c*(b*c - a*d))/(d^3*Sqrt[c + d/x^2]) + ((2*b*c - a*d)*Sqrt[c + d/x^2])/d^3
 - (b*(c + d/x^2)^(3/2))/(3*d^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)}{(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c(bc-ad)}{d^2(c+dx)^{3/2}} + \frac{-2bc+ad}{d^2\sqrt{c+dx}} + \frac{b\sqrt{c+dx}}{d^2}\right)dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

Mathematica [A]
time = 0.10, size = 60, normalized size = 0.88

$$\frac{-3adx^2(d+2cx^2)+b(-d^2+4cdx^2+8c^2x^4)}{3d^3\sqrt{c+\frac{d}{x^2}}\,x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
[Out] (-3*a*d*x^2*(d + 2*c*x^2) + b*(-d^2 + 4*c*d*x^2 + 8*c^2*x^4))/(3*d^3*Sqrt[c
 + d/x^2]*x^4)
```

Maple [A]
time = 0.07, size = 69, normalized size = 1.01

method	result	size
gosper	$-\frac{\left(6x^4acd-8x^4b\,c^2+3a\,d^2x^2-4bcd\,x^2+b\,d^2\right)\left(c\,x^2+d\right)}{3\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^3x^6}$	69
default	$-\frac{\left(6x^4acd-8x^4b\,c^2+3a\,d^2x^2-4bcd\,x^2+b\,d^2\right)\left(c\,x^2+d\right)}{3\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^3x^6}$	69
trager	$-\frac{\left(6x^4acd-8x^4b\,c^2+3a\,d^2x^2-4bcd\,x^2+b\,d^2\right)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{3x^2d^3\left(c\,x^2+d\right)}$	75
risch	$-\frac{\left(c\,x^2+d\right)\left(3ad\,x^2-5c\,x^2b+bd\right)}{3d^3x^4\sqrt{\frac{c\,x^2+d}{x^2}}}-\frac{(ad-bc)c}{d^3\sqrt{\frac{c\,x^2+d}{x^2}}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/(c+d/x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(6*a*c*d*x^4-8*b*c^2*x^4+3*a*d^2*x^2-4*b*c*d*x^2+b*d^2)*(c*x^2+d)/((c*
x^2+d)/x^2)^(3/2)/d^3/x^6
```

Maxima [A]
time = 0.29, size = 81, normalized size = 1.19

$$-\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3}-\frac{6\,\sqrt{c+\frac{d}{x^2}}\,c}{d^3}-\frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}\,d^3}\right)-a\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2}+\frac{c}{\sqrt{c+\frac{d}{x^2}}\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/3*b*((c + d/x^2)^(3/2)/d^3 - 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d
/x^2)*d^3)) - a*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2))
```

Fricas [A]
time = 4.17, size = 73, normalized size = 1.07

$$\frac{\left(2\,(4\,bc^2-3\,acd)x^4-bd^2+(4\,bcd-3\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,(cd^3x^4+d^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(4*b*c^2 - 3*a*c*d)*x^4 - b*d^2 + (4*b*c*d - 3*a*d^2)*x^2)*sqrt((c*x
^2 + d)/x^2)/(c*d^3*x^4 + d^4*x^2)
```

Sympy [A]
time = 4.43, size = 61, normalized size = 0.90

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d^3}-\frac{c(ad-bc)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c+\frac{d}{x^2}}\,(ad-2bc)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**5,x)
```

```
[Out] -b*(c + d/x**2)**(3/2)/(3*d**3) - c*(a*d - b*c)/(d**3*sqrt(c + d/x**2)) - s
qrt(c + d/x**2)*(a*d - 2*b*c)/d**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(60) = 120$.
time = 2.08, size = 188, normalized size = 2.76

$$\frac{(bc^2 - acd)x}{\sqrt{cx^2 + d}\, d^3 \mathrm{sgn}(x)} - \frac{2\left(3\left(\sqrt{c}\,x - \sqrt{cx^2 + d}\right)^4 bc^{\frac{3}{2}} - 3\left(\sqrt{c}\,x - \sqrt{cx^2 + d}\right)^4 a\sqrt{c}\,d - 12\left(\sqrt{c}\,x - \sqrt{cx^2 + d}\right)^2 bc^{\frac{3}{2}} d + 6\left(\sqrt{c}\,x - \sqrt{cx^2 + d}\right)^2 a\sqrt{c}\,d^2 + 5\,bc^{\frac{3}{2}} d^2 - 3\,a\sqrt{c}\,d^3\right)}{3\left(\left(\sqrt{c}\,x - \sqrt{cx^2 + d}\right)^2 - d\right)^3 d^2 \mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] (b*c^2 - a*c*d)*x/(sqrt(c*x^2 + d)*d^3*sgn(x)) - 2/3*(3*(sqrt(c)*x - sqrt(c
*x^2 + d))^4*b*c^(3/2) - 3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c)*d - 12
*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(3/2)*d + 6*(sqrt(c)*x - sqrt(c*x^2 +
d))^2*a*sqrt(c)*d^2 + 5*b*c^(3/2)*d^2 - 3*a*sqrt(c)*d^3)/(((sqrt(c)*x - sqr
t(c*x^2 + d))^2 - d)^3*d^2*sgn(x))
```

Mupad [B]
time = 4.64, size = 66, normalized size = 0.97

$$-\frac{\sqrt{c + \frac{d}{x^2}}\,\left(-8\,b\,c^2\,x^4 + 6\,a\,c\,d\,x^4 - 4\,b\,c\,d\,x^2 + 3\,a\,d^2\,x^2 + b\,d^2\right)}{3\,d^3\,x^2\,\left(c\,x^2 + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^5*(c + d/x^2)^(3/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(b*d^2 + 3*a*d^2*x^2 - 8*b*c^2*x^4 + 6*a*c*d*x^4 - 4*b*
c*d*x^2))/(3*d^3*x^2*(d + c*x^2))
```

3.978 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx$

Optimal. Leaf size=100

$$-\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}-\frac{c(3bc-2ad)\sqrt{c+\frac{d}{x^2}}}{d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

```
[Out] 1/3*(-a*d+3*b*c)*(c+d/x^2)^(3/2)/d^4-1/5*b*(c+d/x^2)^(5/2)/d^4-c^2*(-a*d+b*
c)/d^4/(c+d/x^2)^(1/2)-c*(-2*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^4
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$-\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-ad)}{3d^4}-\frac{c\sqrt{c+\frac{d}{x^2}}\,(3bc-2ad)}{d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

```
[Out] -((c^2*(b*c - a*d))/(d^4*Sqrt[c + d/x^2])) - (c*(3*b*c - 2*a*d)*Sqrt[c + d/
x^2])/d^4 + ((3*b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (b*(c + d/x^2)^(5/2
))/(5*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^2(a+bx)}{(c+dx)^{3/2}}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{c^2(bc-ad)}{d^3(c+dx)^{3/2}}+\frac{c(3bc-2ad)}{d^3\sqrt{c+dx}}+\frac{(-3bc+ad)\sqrt{c+dx}}{d^3}+\frac{b(c+dx}{d^3}\right.\right.\right.\\ &= -\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}-\frac{c(3bc-2ad)\sqrt{c+\frac{d}{x^2}}}{d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 81, normalized size = 0.81

$$\frac{-5adx^2(d^2-4cdx^2-8c^2x^4)-3b(d^3-2cd^2x^2+8c^2dx^4+16c^3x^6)}{15d^4\sqrt{c+\frac{d}{x^2}}\,x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
[Out] (-5*a*d*x^2*(d^2 - 4*c*d*x^2 - 8*c^2*x^4) - 3*b*(d^3 - 2*c*d^2*x^2 + 8*c^2*
d*x^4 + 16*c^3*x^6))/(15*d^4*Sqrt[c + d/x^2]*x^6)
```

Maple [A]
time = 0.08, size = 94, normalized size = 0.94

method	result	size
gosper	$\frac{\left(40a\,c^2d\,x^6-48b\,c^3x^6+20ac\,d^2x^4-24b\,c^2d\,x^4-5a\,d^3x^2+6bc\,d^2x^2-3b\,d^3\right)\left(c\,x^2+d\right)}{15\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^4x^8}$	94
default	$\frac{\left(40a\,c^2d\,x^6-48b\,c^3x^6+20ac\,d^2x^4-24b\,c^2d\,x^4-5a\,d^3x^2+6bc\,d^2x^2-3b\,d^3\right)\left(c\,x^2+d\right)}{15\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^4x^8}$	94
risch	$\frac{\left(c\,x^2+d\right)\left(25x^4acd-33x^4b\,c^2-5a\,d^2x^2+9bcd\,x^2-3b\,d^2\right)}{15d^4x^6\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{c^2\left(ad-bc\right)}{d^4\sqrt{\frac{c\,x^2+d}{x^2}}}$	99
trager	$\frac{\left(40a\,c^2d\,x^6-48b\,c^3x^6+20ac\,d^2x^4-24b\,c^2d\,x^4-5a\,d^3x^2+6bc\,d^2x^2-3b\,d^3\right)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{15x^4d^4\left(c\,x^2+d\right)}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/(c+d/x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(40*a*c^2*d*x^6-48*b*c^3*x^6+20*a*c*d^2*x^4-24*b*c^2*d*x^4-5*a*d^3*x^2
+6*b*c*d^2*x^2-3*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^4/x^8
```

Maxima [A]
time = 0.28, size = 116, normalized size = 1.16

$$-\frac{1}{5}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\sqrt{c+\frac{d}{x^2}}\,c^2}{d^4}+\frac{5\,c^3}{\sqrt{c+\frac{d}{x^2}}\,d^4}\right)-\frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3}-\frac{6\sqrt{c+\frac{d}{x^2}}\,c}{d^3}-\frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}\,d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/5*b*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c + d/x
^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4)) - 1/3*a*((c + d/x^2)^(3/2)/d^3 -
 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d/x^2)*d^3))
```

Fricas [A]
time = 3.00, size = 98, normalized size = 0.98

$$-\frac{\left(8\left(6\,bc^3-5\,ac^2d\right)x^6+4\left(6\,bc^2d-5\,acd^2\right)x^4+3\,bd^3-\left(6\,bcd^2-5\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\left(cd^4x^6+d^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(6*b*c^3 - 5*a*c^2*d)*x^6 + 4*(6*b*c^2*d - 5*a*c*d^2)*x^4 + 3*b*d^
3 - (6*b*c*d^2 - 5*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(c*d^4*x^6 + d^5*x^4)
```

Sympy [A]
time = 5.29, size = 90, normalized size = 0.90

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d^4}+\frac{c^2(ad-bc)}{d^4\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}(ad-3bc)}{3d^4}-\frac{\sqrt{c+\frac{d}{x^2}}\,(-2acd+3bc^2)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**7,x)
```

```
[Out] -b*(c + d/x**2)**(5/2)/(5*d**4) + c**2*(a*d - b*c)/(d**4*sqrt(c + d/x**2))
- (c + d/x**2)**(3/2)*(a*d - 3*b*c)/(3*d**4) - sqrt(c + d/x**2)*(-2*a*c*d +
 3*b*c**2)/d**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(88) = 176.
time = 2.98, size = 303, normalized size = 3.03

$$-\frac{(bc^3-ac^2d)x}{\sqrt{cx^2+d}\,d^4\mathrm{sgn}\,(x)}+\frac{2\left(15\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^8bc^{\frac{5}{2}}-15\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}d-90\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^6bc^{\frac{5}{2}}d+90\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}d^2+240\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4bc^{\frac{5}{2}}d^2-160\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4ac^{\frac{3}{2}}d^3-150\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2bc^{\frac{5}{2}}d^3+110\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2ac^{\frac{3}{2}}d^4+33\,bc^{\frac{5}{2}}d^4-25\,ac^{\frac{3}{2}}d^5\right)}{15\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d\right)^5d^3\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -(b*c^3 - a*c^2*d)*x/(sqrt(c*x^2 + d)*d^4*sgn(x)) + 2/15*(15*(sqrt(c)*x - s
qrt(c*x^2 + d))^8*b*c^(5/2) - 15*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(3/2)*
d - 90*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(5/2)*d + 90*(sqrt(c)*x - sqrt(c
*x^2 + d))^6*a*c^(3/2)*d^2 + 240*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*
d^2 - 160*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^3 - 150*(sqrt(c)*x -
sqrt(c*x^2 + d))^2*b*c^(5/2)*d^3 + 110*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^
(3/2)*d^4 + 33*b*c^(5/2)*d^4 - 25*a*c^(3/2)*d^5)/(((sqrt(c)*x - sqrt(c*x^2
+ d))^2 - d)^5*d^3*sgn(x))
```

Mupad [B]
time = 4.84, size = 91, normalized size = 0.91

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(48\,b\,c^3\,x^6-40\,a\,c^2\,d\,x^6+24\,b\,c^2\,d\,x^4-20\,a\,c\,d^2\,x^4-6\,b\,c\,d^2\,x^2+5\,a\,d^3\,x^2+3\,b\,d^3\right)}{15\,d^4\,x^4\,\left(c\,x^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^7*(c + d/x^2)^(3/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(3*b*d^3 + 5*a*d^3*x^2 + 48*b*c^3*x^6 - 20*a*c*d^2*x^4
- 40*a*c^2*d*x^6 - 6*b*c*d^2*x^2 + 24*b*c^2*d*x^4))/(15*d^4*x^4*(d + c*x^2)
)
```

3.979 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} dx$

Optimal. Leaf size=126

$$\frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{c^2(4bc-3ad)\sqrt{c+\frac{d}{x^2}}}{d^5}-\frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^5}+\frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

```
[Out] -c*(-a*d+2*b*c)*(c+d/x^2)^(3/2)/d^5+1/5*(-a*d+4*b*c)*(c+d/x^2)^(5/2)/d^5-1/
7*b*(c+d/x^2)^(7/2)/d^5+c^3*(-a*d+b*c)/d^5/(c+d/x^2)^(1/2)+c^2*(-3*a*d+4*b*
c)*(c+d/x^2)^(1/2)/d^5
```

Rubi [A]
time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(4bc-3ad)}{d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-ad)}{5d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

```
[Out] (c^3*(b*c - a*d))/(d^5*Sqrt[c + d/x^2]) + (c^2*(4*b*c - 3*a*d)*Sqrt[c + d/x
^2])/d^5 - (c*(2*b*c - a*d)*(c + d/x^2)^(3/2))/d^5 + ((4*b*c - a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (b*(c + d/x^2)^(7/2))/(7*d^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9}\,dx = -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{x^3(a+bx)}{(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{c^3(bc-ad)}{d^4(c+dx)^{3/2}} - \frac{c^2(4bc-3ad)}{d^4\sqrt{c+dx}} + \frac{3c(2bc-ad)\sqrt{c+dx}}{d^4} + \frac{(-4bc+\right.\right.\right.$$

$$= \frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}} + \frac{c^2(4bc-3ad)\sqrt{c+\frac{d}{x^2}}}{d^5} - \frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^5} + \frac{(4bc-ad)\left(c+\right.}{5d^5}$$

Mathematica [A]
time = 0.15, size = 104, normalized size = 0.83

$$\frac{-7adx^2(d^3-2cd^2x^2+8c^2dx^4+16c^3x^6)+b(-5d^4+8cd^3x^2-16c^2d^2x^4+64c^3dx^6+128c^4x^8)}{35d^5\sqrt{c+\frac{d}{x^2}}\,x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

```
[Out] (-7*a*d*x^2*(d^3 - 2*c*d^2*x^2 + 8*c^2*d*x^4 + 16*c^3*x^6) + b*(-5*d^4 + 8*
c*d^3*x^2 - 16*c^2*d^2*x^4 + 64*c^3*d*x^6 + 128*c^4*x^8))/(35*d^5*Sqrt[c +
d/x^2]*x^8)
```

Maple [A]
time = 0.09, size = 118, normalized size = 0.94

method	result	size
gosper	$-\frac{\left(112a\,c^3d\,x^8-128b\,c^4x^8+56a\,c^2d^2x^6-64b\,c^3d\,x^6-14ac\,d^3x^4+16b\,c^2d^2x^4+7a\,d^4x^2-8bc\,d^3x^2+5b\,d^4\right)\left(c\,x^2+d\right)}{35\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^5x^{10}}$	118
default	$-\frac{\left(112a\,c^3d\,x^8-128b\,c^4x^8+56a\,c^2d^2x^6-64b\,c^3d\,x^6-14ac\,d^3x^4+16b\,c^2d^2x^4+7a\,d^4x^2-8bc\,d^3x^2+5b\,d^4\right)\left(c\,x^2+d\right)}{35\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^5x^{10}}$	118
trager	$-\frac{\left(112a\,c^3d\,x^8-128b\,c^4x^8+56a\,c^2d^2x^6-64b\,c^3d\,x^6-14ac\,d^3x^4+16b\,c^2d^2x^4+7a\,d^4x^2-8bc\,d^3x^2+5b\,d^4\right)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{35x^6d^5\left(c\,x^2+d\right)}$	124

risch	$-\frac{(c\,x^2+d)\,(77a\,c^2d\,x^6-93b\,c^3x^6-21ac\,d^2x^4+29b\,c^2d\,x^4+7a\,d^3x^2-13bc\,d^2x^2+5b\,d^3)}{35d^5x^8\sqrt{\frac{c\,x^2+d}{x^2}}}-\frac{c^3(ad-bc)}{d^5\sqrt{\frac{c\,x^2+d}{x^2}}}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/(c+d/x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(112*a*c^3*d*x^8-128*b*c^4*x^8+56*a*c^2*d^2*x^6-64*b*c^3*d*x^6-14*a*c
*d^3*x^4+16*b*c^2*d^2*x^4+7*a*d^4*x^2-8*b*c*d^3*x^2+5*b*d^4)*(c*x^2+d)/((c*
x^2+d)/x^2)^(3/2)/d^5/x^10
```

Maxima [A]
time = 0.28, size = 151, normalized size = 1.20

$$-\frac{1}{35}\,b\left(\frac{5\,\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^5}-\frac{28\,\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^5}+\frac{70\,\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^5}-\frac{140\,\sqrt{c+\frac{d}{x^2}}\,c^3}{d^5}-\frac{35\,c^4}{\sqrt{c+\frac{d}{x^2}}\,d^5}\right)-\frac{1}{5}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\,\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\,\sqrt{c+\frac{d}{x^2}}\,c^2}{d^4}+\frac{5\,c^3}{\sqrt{c+\frac{d}{x^2}}\,d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/35*b*(5*(c + d/x^2)^(7/2)/d^5 - 28*(c + d/x^2)^(5/2)*c/d^5 + 70*(c + d/x
^2)^(3/2)*c^2/d^5 - 140*sqrt(c + d/x^2)*c^3/d^5 - 35*c^4/(sqrt(c + d/x^2)*d
^5)) - 1/5*a*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c
 + d/x^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4))
```

Fricas [A]
time = 3.07, size = 121, normalized size = 0.96

$$\frac{\left(16\,(8\,bc^4-7\,ac^3d)x^8+8\,(8\,bc^3d-7\,ac^2d^2)x^6-5\,bd^4-2\,(8\,bc^2d^2-7\,acd^3)x^4+(8\,bcd^3-7\,ad^4)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,(cd^5x^8+d^6x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/35*(16*(8*b*c^4 - 7*a*c^3*d)*x^8 + 8*(8*b*c^3*d - 7*a*c^2*d^2)*x^6 - 5*b*
d^4 - 2*(8*b*c^2*d^2 - 7*a*c*d^3)*x^4 + (8*b*c*d^3 - 7*a*d^4)*x^2)*sqrt((c*
x^2 + d)/x^2)/(c*d^5*x^8 + d^6*x^6)
```

Sympy [A]
time = 6.45, size = 122, normalized size = 0.97

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7d^5}-\frac{c^3(ad-bc)}{d^5\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}(ad-4bc)}{5d^5}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}(-3acd+6bc^2)}{3d^5}-\frac{\sqrt{c+\frac{d}{x^2}}\cdot(3ac^2d-4bc^3)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**9,x)
```

```
[Out] -b*(c + d/x**2)**(7/2)/(7*d**5) - c**3*(a*d - b*c)/(d**5*sqrt(c + d/x**2))
- (c + d/x**2)**(5/2)*(a*d - 4*b*c)/(5*d**5) - (c + d/x**2)**(3/2)*(-3*a*c*
d + 6*b*c**2)/(3*d**5) - sqrt(c + d/x**2)*(3*a*c**2*d - 4*b*c**3)/d**5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(112) =$ 224.
time = 4.85, size = 414, normalized size = 3.29

$$\frac{(bc^4 - ac^3d)x}{\sqrt{cx^2+d}\,d^5\,\mathrm{sgn}(x)} - \frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{7}{2}} - 35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{5}{2}}d - 280\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{7}{2}}d + 280\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{5}{2}}d^2 + 1015\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{7}{2}}d^2 - 1015\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{5}{2}}d^3 - 2240\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{7}{2}}d^3 + 1680\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{5}{2}}d^4 + 1673\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{7}{2}}d^4 - 1337\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{5}{2}}d^5 - 616\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{7}{2}}d^5 + 504\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{5}{2}}d^6 + 93bc^{\frac{7}{2}}d^6 - 77ac^{\frac{5}{2}}d^7\right)}{35\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^7 d^4\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] (b*c^4 - a*c^3*d)*x/(sqrt(c*x^2 + d)*d^5*sgn(x)) - 2/35*(35*(sqrt(c)*x - sq
rt(c*x^2 + d))^12*b*c^(7/2) - 35*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(5/2)
*d - 280*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(7/2)*d + 280*(sqrt(c)*x - sq
rt(c*x^2 + d))^10*a*c^(5/2)*d^2 + 1015*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^
(7/2)*d^2 - 1015*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2)*d^3 - 2240*(sqrt
(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d^3 + 1680*(sqrt(c)*x - sqrt(c*x^2 + d
))^6*a*c^(5/2)*d^4 + 1673*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d^4 - 1
337*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(5/2)*d^5 - 616*(sqrt(c)*x - sqrt(c
*x^2 + d))^2*b*c^(7/2)*d^5 + 504*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*
d^6 + 93*b*c^(7/2)*d^6 - 77*a*c^(5/2)*d^7)/(((sqrt(c)*x - sqrt(c*x^2 + d))^
2 - d)^7*d^4*sgn(x))
```

Mupad [B]
time = 4.92, size = 154, normalized size = 1.22

$$\frac{c\sqrt{c+\frac{d}{x^2}}\,(21\,a\,d-29\,b\,c)}{35\,d^4\,x^2}-\frac{b\sqrt{c+\frac{d}{x^2}}}{7\,d^2\,x^6}-\frac{\sqrt{c+\frac{d}{x^2}}\,(7\,a\,d^2-13\,b\,c\,d)}{35\,d^4\,x^4}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(x^2\left(\frac{58\,b\,c^4-42\,a\,c^3\,d}{35\,d^5}+\frac{2\,c^3\,(77\,a\,d-93\,b\,c)}{35\,d^5}\right)+\frac{c^2\,(77\,a\,d-93\,b\,c)}{35\,d^4}\right)}{c\,x^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^9*(c + d/x^2)^(3/2)),x)
```

```
[Out] (c*(c + d/x^2)^(1/2)*(21*a*d - 29*b*c))/(35*d^4*x^2) - (b*(c + d/x^2)^(1/2)
)/(7*d^2*x^6) - ((c + d/x^2)^(1/2)*(7*a*d^2 - 13*b*c*d))/(35*d^4*x^4) - ((c
 + d/x^2)^(1/2)*(x^2*((58*b*c^4 - 42*a*c^3*d)/(35*d^5) + (2*c^3*(77*a*d - 9
3*b*c))/(35*d^5)) + (c^2*(77*a*d - 93*b*c))/(35*d^4)))/(d + c*x^2)
```

3.980 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}}-\frac{8d(5bc-6ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^4}+\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] 4/15*d*(-6*a*d+5*b*c)*x/c^3/(c+d/x^2)^(1/2)+1/15*(-6*a*d+5*b*c)*x^3/c^2/(c+
d/x^2)^(1/2)+1/5*a*x^5/c/(c+d/x^2)^(1/2)-8/15*d*(-6*a*d+5*b*c)*x*(c+d/x^2)^
(1/2)/c^4
```

Rubi [A]
time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 277, 198, 197}

$$-\frac{8dx\sqrt{c+\frac{d}{x^2}}\,(5bc-6ad)}{15c^4}+\frac{4dx(5bc-6ad)}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{x^3(5bc-6ad)}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

```
[Out] (4*d*(5*b*c - 6*a*d)*x)/(15*c^3*Sqrt[c + d/x^2]) - (8*d*(5*b*c - 6*a*d)*Sqr
t[c + d/x^2]*x)/(15*c^4) + ((5*b*c - 6*a*d)*x^3)/(15*c^2*Sqrt[c + d/x^2]) +
 (a*x^5)/(5*c*Sqrt[c + d/x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx &= \frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}+\frac{(5bc-6ad)\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{5c}\\
&= \frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}-\frac{(4d(5bc-6ad))\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{15c^2}\\
&= \frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}-\frac{(8d(5bc-6ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{15c^3}\\
&= \frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}}-\frac{8d(5bc-6ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^4}+\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 0.72

$$\frac{5bc(-8d^2-4cdx^2+c^2x^4)+3a(16d^3+8cd^2x^2-2c^2dx^4+c^3x^6)}{15c^4\sqrt{c+\frac{d}{x^2}}\,x}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]

[Out] (5*b*c*(-8*d^2 - 4*c*d*x^2 + c^2*x^4) + 3*a*(16*d^3 + 8*c*d^2*x^2 - 2*c^2*d*x^4 + c^3*x^6))/(15*c^4*Sqrt[c + d/x^2]*x)

Maple [A]
time = 0.08, size = 91, normalized size = 0.82

method	result	size
gosper	$\frac{\left(3 x^{6} a \,c^{3}-6 x^{4} a \,c^{2} d +5 x^{4} b \,c^{3}+24 a c \,d^{2} x^{2}-20 b \,c^{2} d \,x^{2}+48 a \,d^{3}-40 b c \,d^{2}\right) \left(c \,x^{2}+d \right)}{15 \left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} c^{4}}$	91
default	$\frac{\left(3 x^{6} a \,c^{3}-6 x^{4} a \,c^{2} d +5 x^{4} b \,c^{3}+24 a c \,d^{2} x^{2}-20 b \,c^{2} d \,x^{2}+48 a \,d^{3}-40 b c \,d^{2}\right) \left(c \,x^{2}+d \right)}{15 \left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} c^{4}}$	91
trager	$\frac{\left(3 x^{6} a \,c^{3}-6 x^{4} a \,c^{2} d +5 x^{4} b \,c^{3}+24 a c \,d^{2} x^{2}-20 b \,c^{2} d \,x^{2}+48 a \,d^{3}-40 b c \,d^{2}\right) x \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{15 \left(c \,x^{2}+d \right) c^{4}}$	95
risch	$\frac{\left(3 a \,c^{2} x^{4}-9 a c d \,x^{2}+5 b \,c^{2} x^{2}+33 a \,d^{2}-25 b c d \right) \left(c \,x^{2}+d \right)}{15 c^{4} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}+\frac{\left(a d -b c \right) d^{2}}{c^{4} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}$	99

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b/x^2+a)*x^4/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/15*(3*a*c^3*x^6-6*a*c^2*d*x^4+5*b*c^3*x^4+24*a*c*d^2*x^2-20*b*c^2*d*x^2+48*a*d^3-40*b*c*d^2)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^4

Maxima [A]
time = 0.29, size = 128, normalized size = 1.15

$$\frac{1}{3} \, b {\left(\frac{{\left(c + \frac{d}{x^{2}}\right)}^{\frac{3}{2}} x^{3} - 6 \, \sqrt{c + \frac{d}{x^{2}}} d x}{c^{3}} - \frac{3 \, d^{2}}{\sqrt{c + \frac{d}{x^{2}}} c^{3} x}\right)} + \frac{1}{5} \, a {\left(\frac{5 \, d^{3}}{\sqrt{c + \frac{d}{x^{2}}} c^{4} x} + \frac{{\left(c + \frac{d}{x^{2}}\right)}^{\frac{5}{2}} x^{5} - 5 \, {\left(c + \frac{d}{x^{2}}\right)}^{\frac{3}{2}} d x^{3} + 15 \, \sqrt{c + \frac{d}{x^{2}}} d^{2} x}{c^{4}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="maxima")

[Out] 1/3*b*(((c + d/x^2)^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c + d/x^2)*c^3*x)) + 1/5*a*(5*d^3/(sqrt(c + d/x^2)*c^4*x) + ((c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)/c^4)

Fricas [A]
time = 2.99, size = 95, normalized size = 0.86

$$\frac{{\left(3 \, a c^{3} x^{7} + {\left(5 \, b c^{3} - 6 \, a c^{2} d\right)} x^{5} - 4 \, {\left(5 \, b c^{2} d - 6 \, a c d^{2}\right)} x^{3} - 8 \, {\left(5 \, b c d^{2} - 6 \, a d^{3}\right)} x\right)} \sqrt{\frac{c x^{2} + d}{x^{2}}}}{15 \, {\left(c^{5} x^{2} + c^{4} d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^3*x^7 + (5*b*c^3 - 6*a*c^2*d)*x^5 - 4*(5*b*c^2*d - 6*a*c*d^2)*x
^3 - 8*(5*b*c*d^2 - 6*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/(c^5*x^2 + c^4*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(107) = 214$.
time = 3.03, size = 561, normalized size = 5.05

$$a\left(\frac{c^5d^{\frac{19}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}+\frac{5c^3d^{\frac{23}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}+\frac{30c^2d^{\frac{25}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}+\frac{40cd^{\frac{27}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}+\frac{16d^{\frac{29}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}\right)+b\left(\frac{c^3d^{\frac{9}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{3c^2d^{\frac{11}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{12cd^{\frac{13}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{8d^{\frac{15}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(c**5*d**(19/2)*x**10*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**1
0*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 5*c**3*d**(23/2)*x**6*sqrt(c*
x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5
*c**4*d**12) + 30*c**2*d**(25/2)*x**4*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6
+ 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 40*c*d**(27/2)*
x**2*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d*
*11*x**2 + 5*c**4*d**12) + 16*d**(29/2)*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**
6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12)) + b*(c**3*d**(
9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*
d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4
*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d + 1)/(3*c**5*
d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(c*x**2/d + 1
)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6))
```

Giac [A]
time = 1.68, size = 146, normalized size = 1.32

$$\frac{8\left(5\,bcd^2-6\,ad^3\right)\mathrm{sgn}(x)}{15\,c^4\sqrt{d}}-\frac{bcd^2-ad^3}{\sqrt{cx^2+d}\;c^4\mathrm{sgn}\left(x\right)}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}ac^{16}+5\left(cx^2+d\right)^{\frac{3}{2}}bc^{17}-15\left(cx^2+d\right)^{\frac{3}{2}}ac^{16}d-30\sqrt{cx^2+d}\;bc^{17}d+45\sqrt{cx^2+d}\;ac^{16}d^2}{15\,c^{20}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 8/15*(5*b*c*d^2 - 6*a*d^3)*sgn(x)/(c^4*sqrt(d)) - (b*c*d^2 - a*d^3)/(sqrt(c
*x^2 + d)*c^4*sgn(x)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^16 + 5*(c*x^2 + d)^(3
/2)*b*c^17 - 15*(c*x^2 + d)^(3/2)*a*c^16*d - 30*sqrt(c*x^2 + d)*b*c^17*d +
45*sqrt(c*x^2 + d)*a*c^16*d^2)/(c^20*sgn(x))
```

Mupad [B]
time = 5.77, size = 79, normalized size = 0.71

$$\frac{3\,a\,c^3\,x^6+5\,b\,c^3\,x^4-6\,a\,c^2\,d\,x^4-20\,b\,c^2\,d\,x^2+24\,a\,c\,d^2\,x^2-40\,b\,c\,d^2+48\,a\,d^3}{15\,c^4\,x\,\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (48*a*d^3 + 3*a*c^3*x^6 + 5*b*c^3*x^4 - 40*b*c*d^2 + 24*a*c*d^2*x^2 - 6*a*c
^2*d*x^4 - 20*b*c^2*d*x^2)/(15*c^4*x*(c + d/x^2)^(1/2))
```

3.981 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{2(3bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^3}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -1/3*(-4*a*d+3*b*c)*x/c^2/(c+d/x^2)^(1/2)+1/3*a*x^3/c/(c+d/x^2)^(1/2)+2/3*(
-4*a*d+3*b*c)*x*(c+d/x^2)^(1/2)/c^3
```

Rubi [A]

time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 198, 197}

$$\frac{2x\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{3c^3}-\frac{x(3bc-4ad)}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] -1/3*((3*b*c - 4*a*d)*x)/(c^2*Sqrt[c + d/x^2]) + (2*(3*b*c - 4*a*d)*Sqrt[c
+ d/x^2]*x)/(3*c^3) + (a*x^3)/(3*c*Sqrt[c + d/x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}} + \frac{(3bc-4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{3c}$$

$$= -\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}} + \frac{(2(3bc-4ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{3c^2}$$

$$= -\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}} + \frac{2(3bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^3} + \frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.72

$$\frac{3bc(2d+cx^2)+a(-8d^2-4cdx^2+c^2x^4)}{3c^3\sqrt{c+\frac{d}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] (3*b*c*(2*d + c*x^2) + a*(-8*d^2 - 4*c*d*x^2 + c^2*x^4))/(3*c^3*Sqrt[c + d/
x^2]*x)
```

Maple [A]
time = 0.07, size = 66, normalized size = 0.84

method	result	size
gosper	$\frac{\left(a\,c^2x^4-4acd\,x^2+3b\,c^2x^2-8a\,d^2+6bcd\right)\left(c\,x^2+d\right)}{3\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}c^3x^3}$	66

default	$\frac{(a\,c^{2}x^{4}-4acd\,x^{2}+3b\,c^{2}x^{2}-8a\,d^{2}+6bcd)\,(c\,x^{2}+d)}{3\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}c^{3}x^{3}}$	66
trager	$\frac{(a\,c^{2}x^{4}-4acd\,x^{2}+3b\,c^{2}x^{2}-8a\,d^{2}+6bcd)x\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{3(c\,x^{2}+d)c^{3}}$	70
risch	$\frac{(c\,x^{2}a-5ad+3bc)\,(c\,x^{2}+d)}{3c^{3}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}-\frac{(ad-bc)d}{c^{3}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)*x^2/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a*c^2*x^4-4*a*c*d*x^2+3*b*c^2*x^2-8*a*d^2+6*b*c*d)*(c*x^2+d)/((c*x^2+d
)/x^2)^(3/2)/c^3/x^3
```

Maxima [A]
time = 0.29, size = 90, normalized size = 1.14

$$b\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x}{c^{2}}+\frac{d}{\sqrt{c+\frac{d}{x^{2}}}\,c^{2}x}\right)+\frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}x^{3}-6\,\sqrt{c+\frac{d}{x^{2}}}\,dx}{c^{3}}-\frac{3\,d^{2}}{\sqrt{c+\frac{d}{x^{2}}}\,c^{3}x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) + 1/3*a*(((c + d/x^2)
^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c + d/x^2)*c^3*x))
```

Fricas [A]
time = 4.06, size = 70, normalized size = 0.89

$$\frac{(ac^{2}x^{5}+(3\,bc^{2}-4\,acd)x^{3}+2\,(3\,bcd-4\,ad^{2})x)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{3\,(c^{4}x^{2}+c^{3}d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*c^2*x^5 + (3*b*c^2 - 4*a*c*d)*x^3 + 2*(3*b*c*d - 4*a*d^2)*x)*sqrt((c
*x^2 + d)/x^2)/(c^4*x^2 + c^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(70) = 140$.
time = 2.95, size = 267, normalized size = 3.38

$$a\left(\frac{c^{3}d^{\frac{9}{2}}x^{6}\sqrt{\frac{cx^{2}}{d}+1}}{3c^{5}d^{4}x^{4}+6c^{4}d^{5}x^{2}+3c^{3}d^{6}}-\frac{3c^{2}d^{\frac{11}{2}}x^{4}\sqrt{\frac{cx^{2}}{d}+1}}{3c^{5}d^{4}x^{4}+6c^{4}d^{5}x^{2}+3c^{3}d^{6}}-\frac{12cd^{\frac{13}{2}}x^{2}\sqrt{\frac{cx^{2}}{d}+1}}{3c^{5}d^{4}x^{4}+6c^{4}d^{5}x^{2}+3c^{3}d^{6}}-\frac{8d^{\frac{15}{2}}\sqrt{\frac{cx^{2}}{d}+1}}{3c^{5}d^{4}x^{4}+6c^{4}d^{5}x^{2}+3c^{3}d^{6}}\right)+b\left(\frac{x^{2}}{c\sqrt{d}\sqrt{\frac{cx^{2}}{d}+1}}+\frac{2\sqrt{d}}{c^{2}\sqrt{\frac{cx^{2}}{d}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(c**3*d**(9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6)) + b*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)))
```

Giac [A]
time = 2.27, size = 106, normalized size = 1.34

$$-\frac{2\,(3\,bcd-4\,ad^2)\mathrm{sgn}(x)}{3\,c^3\sqrt{d}}+\frac{bcd-ad^2}{\sqrt{cx^2+d}\,c^3\mathrm{sgn}\,(x)}+\frac{(cx^2+d)^{\frac{3}{2}}ac^6+3\,\sqrt{cx^2+d}\,bc^7-6\,\sqrt{cx^2+d}\,ac^6d}{3\,c^9\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*c*d - 4*a*d^2)*sgn(x)/(c^3*sqrt(d)) + (b*c*d - a*d^2)/(sqrt(c*x^2 + d)*c^3*sgn(x)) + 1/3*((c*x^2 + d)^(3/2)*a*c^6 + 3*sqrt(c*x^2 + d)*b*c^7 - 6*sqrt(c*x^2 + d)*a*c^6*d)/(c^9*sgn(x))
```

Mupad [B]
time = 5.20, size = 81, normalized size = 1.03

$$\frac{b\,c^2\,x^4+3\,b\,c\,d\,x^2+2\,b\,d^2}{c^2\,x^3\,\left(c+\frac{d}{x^2}\right)^{3/2}}-\frac{-a\,c^2\,x^4+4\,a\,c\,d\,x^2+8\,a\,d^2}{3\,c^3\,x\,\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (2*b*d^2 + b*c^2*x^4 + 3*b*c*d*x^2)/(c^2*x^3*(c + d/x^2)^(3/2)) - (8*a*d^2 - a*c^2*x^4 + 4*a*c*d*x^2)/(3*c^3*x*(c + d/x^2)^(1/2))
```

3.982 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}} \, dx$

Optimal. Leaf size=45

$$-\frac{bc-2ad}{c^2\sqrt{c+\frac{d}{x^2}}\,x}+\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] (2*a*d-b*c)/c^2/x/(c+d/x^2)^(1/2)+a*x/c/(c+d/x^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {382, 464, 197}

$$\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{bc-2ad}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] -((b*c - 2*a*d)/(c^2*Sqrt[c + d/x^2]*x)) + (a*x)/(c*Sqrt[c + d/x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{a+bx^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{ax}{c\sqrt{c+\frac{d}{x^2}}} + \frac{(-bc+2ad)\text{Subst}\left(\int \frac{1}{(c+dx^2)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{bc-2ad}{c^2\sqrt{c+\frac{d}{x^2}}\,x} + \frac{ax}{c\sqrt{c+\frac{d}{x^2}}}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.73

$$\frac{-bc+2ad+acx^2}{c^2\sqrt{c+\frac{d}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(c + d/x^2)^(3/2),x]
[Out] (-(b*c) + 2*a*d + a*c*x^2)/(c^2*Sqrt[c + d/x^2]*x)
```

Maple [A]
time = 0.06, size = 43, normalized size = 0.96

method	result	size
gosper	$\frac{(c\,x^2a+2ad-bc)(c\,x^2+d)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}c^2x^3}$	43
default	$\frac{(c\,x^2a+2ad-bc)(c\,x^2+d)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}c^2x^3}$	43
trager	$\frac{x(c\,x^2a+2ad-bc)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{(c\,x^2+d)c^2}$	47
risch	$\frac{a(c\,x^2+d)}{c^2\sqrt{\frac{c\,x^2+d}{x^2}}\,x}+\frac{ad-bc}{c^2\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] (a*c*x^2+2*a*d-b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/c^2/x^3
```

Maxima [A]
time = 0.29, size = 53, normalized size = 1.18

$$a\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x}{c^2}+\frac{d}{\sqrt{c+\frac{d}{x^2}}\,c^2x}\right)-\frac{b}{\sqrt{c+\frac{d}{x^2}}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] a*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) - b/(sqrt(c + d/x^2)*
c*x)
```

Fricas [A]
time = 4.10, size = 47, normalized size = 1.04

$$\frac{(acx^3-(bc-2\,ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{c^3x^2+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a*c*x^3 - (b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/(c^3*x^2 + c^2*d)
```

Sympy [A]
time = 2.84, size = 65, normalized size = 1.44

$$a\left(\frac{x^2}{c\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}+\frac{2\sqrt{d}}{c^2\sqrt{\frac{cx^2}{d}+1}}\right)-\frac{b}{c\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)
)) - b/(c*sqrt(d)*sqrt(c*x**2/d + 1))
```

Giac [A]
time = 1.83, size = 62, normalized size = 1.38

$$\frac{(bc-2\,ad)\mathrm{sgn}(x)}{c^2\sqrt{d}}+\frac{\sqrt{cx^2+d}\,a}{c^2\mathrm{sgn}\,(x)}-\frac{bc-ad}{\sqrt{cx^2+d}\,c^2\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (b*c - 2*a*d)*sgn(x)/(c^2*sqrt(d)) + sqrt(c*x^2 + d)*a/(c^2*sgn(x)) - (b*c
- a*d)/(sqrt(c*x^2 + d)*c^2*sgn(x))
```

Mupad [B]
time = 4.90, size = 38, normalized size = 0.84

$$\frac{(c\,x^2+d)\,\left(a\,c\,x^2+2\,a\,d-b\,c\right)}{c^2\,x^3\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(c + d/x^2)^(3/2),x)
```

```
[Out] ((d + c*x^2)*(2*a*d - b*c + a*c*x^2))/(c^2*x^3*(c + d/x^2)^(3/2))
```

3.983 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} dx$

Optimal. Leaf size=59

$$\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\, x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\, x}\right)}{d^{3/2}}$$

```
[Out] -b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)+(-a*d+b*c)/c/d/x/(c+d/x^2)^(1
/2)
```

Rubi [A]

time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {463, 342, 223, 212}

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

```
[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]*x) - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x
)])/d^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx &= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}+\frac{b\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx}{d}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}-\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{d}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}-\frac{b\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{d}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{d^{3/2}}\end{aligned}$$

Mathematica [A]

time = 0.07, size = 71, normalized size = 1.20

$$\frac{\sqrt{d}\,(bc-ad)-bc\sqrt{d+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{cd^{3/2}\sqrt{c+\frac{d}{x^2}}\,x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]

[Out] (Sqrt[d]*(b*c - a*d) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(c*d^(3/2)*Sqrt[c + d/x^2]*x)

Maple [A]
time = 0.05, size = 79, normalized size = 1.34

method	result	size
default	$-\frac{(c\,x^2+d)\left(a\,d^{\frac{5}{2}}-d^{\frac{3}{2}}bc+\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)\sqrt{c\,x^2+d}\,bcd\right)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^3c\,d^{\frac{5}{2}}}$	79

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b/x^2+a)/(c+d/x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -(c*x^2+d)*(a*d^(5/2)-d^(3/2)*b*c+ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*(c*x^2+d)^(1/2)*b*c*d)/((c*x^2+d)/x^2)^(3/2)/x^3/c/d^(5/2)

Maxima [A]
time = 0.50, size = 80, normalized size = 1.36

$$\frac{1}{2}\,b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}\,dx}\right)-\frac{a}{\sqrt{c+\frac{d}{x^2}}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")

[Out] 1/2*b*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2/(sqrt(c + d/x^2)*d*x)) - a/(sqrt(c + d/x^2)*c*x)

Fricas [A]
time = 3.48, size = 195, normalized size = 3.31

$$\left[\frac{2\,(bcd-ad^2)x\sqrt{\frac{cx^2+d}{x^2}}+(bc^2x^2+bcd)\sqrt{d}\,\log\left(-\frac{cx^2-2\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2d}{x^2}\right)}{2\,(c^2d^2x^2+cd^3)},\frac{(bcd-ad^2)x\sqrt{\frac{cx^2+d}{x^2}}+(bc^2x^2+bcd)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{c^2d^2x^2+cd^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*sqrt(
d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c^2*d^2*x^
2 + c*d^3), ((b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*
sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^2*d^2*x^2
 + c*d^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(46) = 92.
time = 4.41, size = 206, normalized size = 3.49

$$-\frac{a}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+b\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**2,x)
```

```
[Out] -a/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + b*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7
/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**
(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2
*d**(9/2)) + d**3*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*l
og(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(51) = 102.
time = 1.73, size = 108, normalized size = 1.83

$$\frac{b\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d\mathrm{sgn}\,(x)}-\frac{\left(bc\sqrt{d}\,\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+bc\sqrt{-d}-a\sqrt{-d}\,d\right)\mathrm{sgn}(x)}{c\sqrt{-d}\,d^{\frac{3}{2}}}+\frac{bc-ad}{\sqrt{cx^2+d}\,cd\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d*sgn(x)) - (b*c*sqrt(d)*arcta
n(sqrt(d)/sqrt(-d)) + b*c*sqrt(-d) - a*sqrt(-d)*d)*sgn(x)/(c*sqrt(-d)*d^(3/
2)) + (b*c - a*d)/(sqrt(c*x^2 + d)*c*d*sgn(x))
```

Mupad [B]
time = 5.11, size = 60, normalized size = 1.02

$$\frac{b}{d\,x\,\sqrt{c+\frac{d}{x^2}}}-\frac{a}{c\,x\,\sqrt{c+\frac{d}{x^2}}}-\frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^2*(c + d/x^2)^(3/2)),x)
```

```
[Out] b/(d*x*(c + d/x^2)^(1/2)) - a/(c*x*(c + d/x^2)^(1/2)) - (b*log((c + d/x^2)^
(1/2) + d^(1/2)/x))/d^(3/2)
```

3.984 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4} dx$

Optimal. Leaf size=92

$$-\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3}-\frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}\,x}+\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^{5/2}}$$

```
[Out] 1/2*(-2*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)-1/2*b/d/x^3/(
c+d/x^2)^(1/2)+1/2*(2*a*d-3*b*c)/d^2/x/(c+d/x^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 342, 294, 223, 212}

$$\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{5/2}}-\frac{3bc-2ad}{2d^2x\sqrt{c+\frac{d}{x^2}}}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

```
[Out] -1/2*b/(d*Sqrt[c + d/x^2]*x^3) - (3*b*c - 2*a*d)/(2*d^2*Sqrt[c + d/x^2]*x)
+ ((3*b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*d^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{(-3bc+2ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx}{2d}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3} - \frac{(-3bc+2ad)\text{Subst}\left(\int \frac{x^2}{(c+dx^2)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2d}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3} - \frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}\,x} + \frac{(3bc-2ad)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{2d^2}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3} - \frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}\,x} + \frac{(3bc-2ad)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^2}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3} - \frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}\,x} + \frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^{5/2}}$$

Mathematica [A]
time = 0.16, size = 91, normalized size = 0.99

$$\frac{\sqrt{d}\,(2adx^2-b(d+3cx^2))+(3bc-2ad)x^2\sqrt{d+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{c+\frac{d}{x^2}}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

```
[Out] (Sqrt[d]*(2*a*d*x^2 - b*(d + 3*c*x^2)) + (3*b*c - 2*a*d)*x^2*Sqrt[d + c*x^2
]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(2*d^(5/2)*Sqrt[c + d/x^2]*x^3)
```

Maple [A]
time = 0.08, size = 131, normalized size = 1.42

method	result
default	$-\frac{(c\,x^2+d)\left(2\sqrt{c\,x^2+d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a\,d^2x^2-3\sqrt{c\,x^2+d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)bcd\,x^2-2d^{\frac{5}{2}}a}{2\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^5d^{\frac{7}{2}}}$
risch	$-\frac{b(c\,x^2+d)}{2d^2x^3\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{\left(-\frac{bc}{d^2\sqrt{c\,x^2+d}}+\frac{a}{d\sqrt{c\,x^2+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a}{d^{\frac{3}{2}}}+\frac{3\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)}{2d^{\frac{5}{2}}}\right.}{\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/(c+d/x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^2+d)*(2*(c*x^2+d)^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*a*d^2
*x^2-3*(c*x^2+d)^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c*d*x^2-2*d^(5
/2)*a*x^2+3*d^(3/2)*b*c*x^2+d^(5/2)*b)/((c*x^2+d)/x^2)^(3/2)/x^5/d^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(76) = 152$.
time = 0.50, size = 162, normalized size = 1.76

$$-\frac{1}{4}\,b\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)cx^2-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3-\sqrt{c+\frac{d}{x^2}}\,d^3x}+\frac{3\,c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right)+\frac{1}{2}\,a\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}\,dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(3/2)*d^2*x^3 - sqrt(c
 + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x
 + sqrt(d)))/d^(5/2)) + 1/2*a*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c +
d/x^2)*x + sqrt(d)))/d^(3/2) + 2/(sqrt(c + d/x^2)*d*x))
```

Fricas [A]
time = 3.71, size = 248, normalized size = 2.70

$$\left[-\frac{((3\,bc^2-2\,acd)x^3+(3\,bcd-2\,ad^2)x)\sqrt{d}\,\log\left(-\frac{cx^2-2\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(bd^2+(3\,bcd-2\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{4\,(cd^3x^3+d^4x)},-\frac{((3\,bc^2-2\,acd)x^3+(3\,bcd-2\,ad^2)x)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(bd^2+(3\,bcd-2\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{2\,(cd^3x^3+d^4x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/4*(((3*b*c^2 - 2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(d)*log(-(c*x^
2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(b*d^2 + (3*b*c*d - 2
*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^3*x^3 + d^4*x), -1/2*(((3*b*c^2 -
2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^
2 + d)/x^2)/(c*x^2 + d)) + (b*d^2 + (3*b*c*d - 2*a*d^2)*x^2)*sqrt((c*x^2 +
d)/x^2))/(c*d^3*x^3 + d^4*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(76) = 152$.
time = 7.16, size = 262, normalized size = 2.85

$$a\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right)+b\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2d^{\frac{5}{2}}}-\frac{1}{2\sqrt{c}\,dx^3\sqrt{1+\frac{d}{cx^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**4,x)
```

```
[Out] a*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x*
*2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sq
rt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + d**3*log(c*x**2/d)/(2*c
*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(
7/2)*x**2 + 2*d**(9/2))) + b*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) +
3*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 +
d/(c*x**2))))
```

Giac [A]
time = 1.00, size = 107, normalized size = 1.16

$$-\frac{(3\,bc-2\,ad)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{2\,\sqrt{-d}\,d^2\operatorname{sgn}(x)}-\frac{3\,(cx^2+d)bc-2\,(cx^2+d)ad-2\,bcd+2\,ad^2}{2\left((cx^2+d)^{\frac{3}{2}}-\sqrt{cx^2+d}\,d\right)d^2\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*c - 2*a*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^2*sgn(x))
 - 1/2*(3*(c*x^2 + d)*b*c - 2*(c*x^2 + d)*a*d - 2*b*c*d + 2*a*d^2)/(((c*x^2
 + d)^(3/2) - sqrt(c*x^2 + d)*d)*d^2*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{x^2}}{x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)),x)
[Out] int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)), x)
```

3.985 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6} dx$

Optimal. Leaf size=123

$$-\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5}-\frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3}+\frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x}-\frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{7/2}}$$

```
[Out] -3/8*c*(-4*a*d+5*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(7/2)-1/4*b/d/x^
5/(c+d/x^2)^(1/2)+1/4*(4*a*d-5*b*c)/d^2/x^3/(c+d/x^2)^(1/2)+3/8*(-4*a*d+5*b
*c)*(c+d/x^2)^(1/2)/d^3/x
```

Rubi [A]
time = 0.05, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {470, 342, 294, 327, 223, 212}

$$-\frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{7/2}}+\frac{3\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{8d^3x}-\frac{5bc-4ad}{4d^2x^3\sqrt{c+\frac{d}{x^2}}}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

```
[Out] -1/4*b/(d*Sqrt[c + d/x^2]*x^5) - (5*b*c - 4*a*d)/(4*d^2*Sqrt[c + d/x^2]*x^3
) + (3*(5*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^3*x) - (3*c*(5*b*c - 4*a*d)*Ar
cTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5}+\frac{(-5bc+4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx}{4d}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5}-\frac{(-5bc+4ad)\text{Subst}\left(\int \frac{x^4}{(c+dx^2)^{3/2}}\,dx,x,\frac{1}{x}\right)}{4d}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5}-\frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3}+\frac{(3(5bc-4ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{4d^2}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5}-\frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3}+\frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x}-\frac{(3c(5bc-4ad))\text{Su}}{}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5}-\frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3}+\frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x}-\frac{(3c(5bc-4ad))\text{Su}}{}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5}-\frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3}+\frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x}-\frac{3c(5bc-4ad)\tanh}{8d}$$

Mathematica [A]
time = 0.17, size = 113, normalized size = 0.92

$$\frac{\sqrt{d}\left(-4adx^2(d+3cx^2)+b(-2d^2+5cdx^2+15c^2x^4)\right)-3c(5bc-4ad)x^4\sqrt{d+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{c+\frac{d}{x^2}}\,x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

```
[Out] (Sqrt[d]*(-4*a*d*x^2*(d + 3*c*x^2) + b*(-2*d^2 + 5*c*d*x^2 + 15*c^2*x^4)) -
 3*c*(5*b*c - 4*a*d)*x^4*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/
(8*d^(7/2)*Sqrt[c + d/x^2]*x^5)
```

Maple [A]
time = 0.09, size = 157, normalized size = 1.28

method	result
default	$\frac{(c\,x^2+d)\left(12\sqrt{c\,x^2+d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a c\,d^2x^4-15\sqrt{c\,x^2+d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)b\,c^2d\,x^4-12d}{8\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^7d^{\frac{9}{2}}}$
risch	$-\frac{(c\,x^2+d)\,(4ad\,x^2-7c\,x^2b+2bd)}{8d^3x^5\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{\left(-\frac{ca}{d^2\sqrt{c\,x^2+d}}+\frac{c^2b}{d^3\sqrt{c\,x^2+d}}+\frac{3c\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a}{2d^{\frac{5}{2}}}-\frac{15c^2\ln\left(2d+2\sqrt{\right.}\right.}{}\right.}{\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)/(c+d/x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*x^2+d)*(12*(c*x^2+d)^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*a*c*d
^2*x^4-15*(c*x^2+d)^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c^2*d*x^4-1
2*d^(5/2)*a*c*x^4+15*d^(3/2)*b*c^2*x^4-4*d^(7/2)*a*x^2+5*d^(5/2)*b*c*x^2-2*
d^(7/2)*b)/((c*x^2+d)/x^2)^(3/2)/x^7/d^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(103) = 206$.
time = 0.50, size = 243, normalized size = 1.98

$$\frac{1}{16}\,b\left(\frac{2\left(15\left(c+\frac{d}{x^2}\right)^2c^2x^4-25\left(c+\frac{d}{x^2}\right)c^2dx^2+8\,c^2d^2\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5-2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^4x^3+\sqrt{c+\frac{d}{x^2}}\,d^5x}+\frac{15\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{7}{2}}}\right)-\frac{1}{4}\,a\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)cx^2-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3-\sqrt{c+\frac{d}{x^2}}\,d^3x}+\frac{3\,c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/16*b*(2*(15*(c + d/x^2)^2*c^2*x^4 - 25*(c + d/x^2)*c^2*d*x^2 + 8*c^2*d^2)
/((c + d/x^2)^(5/2)*d^3*x^5 - 2*(c + d/x^2)^(3/2)*d^4*x^3 + sqrt(c + d/x^2)
*d^5*x) + 15*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqr
t(d)))/d^(7/2)) - 1/4*a*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(3/2)
*d^2*x^3 - sqrt(c + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(d))/(
sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2))
```

Fricas [A]
time = 2.98, size = 314, normalized size = 2.55

$$\left[-\frac{3\left((5\,bc^3-4\,ac^2d)x^5+(5\,bc^2d-4\,acd^2)x^3\right)\sqrt{d}\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\,(5\,bc^2d-4\,acd^2)x^4-2\,bd^3+(5\,bcd^2-4\,ad^3)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,(cd^4x^5+d^5x^3)},\ \frac{3\left((5\,bc^3-4\,ac^2d)x^5+(5\,bc^2d-4\,acd^2)x^3\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(3\,(5\,bc^2d-4\,acd^2)x^4-2\,bd^3+(5\,bcd^2-4\,ad^3)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{8\,(cd^4x^5+d^5x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2*d - 4*a*c*d^2)*x^3)*sqrt(d)
*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(5*b*c^
2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*x^2)*sqrt((c*x^2 + d
)/x^2))/(c*d^4*x^5 + d^5*x^3), 1/8*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2
*d - 4*a*c*d^2)*x^3)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^
2 + d)) + (3*(5*b*c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*
x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^4*x^5 + d^5*x^3)]
```

Sympy [A]
time = 12.49, size = 180, normalized size = 1.46

$$a\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2d^{\frac{5}{2}}}-\frac{1}{2\sqrt{c}\,dx^3\sqrt{1+\frac{d}{cx^2}}}\right)+b\left(\frac{15c^{\frac{3}{2}}}{8d^3x\sqrt{1+\frac{d}{cx^2}}}+\frac{5\sqrt{c}}{8d^2x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{15c^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8d^{\frac{7}{2}}}-\frac{1}{4\sqrt{c}\,dx^5\sqrt{1+\frac{d}{cx^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**6,x)
```

```
[Out] a*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) + 3*c*asinh(sqrt(d)/(sqrt(c)*
x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 + d/(c*x**2)))) + b*(15*c**(3
/2)/(8*d**3*x*sqrt(1 + d/(c*x**2))) + 5*sqrt(c)/(8*d**2*x**3*sqrt(1 + d/(c*
x**2))) - 15*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(7/2)) - 1/(4*sqrt(c)*d*
x**5*sqrt(1 + d/(c*x**2))))
```

Giac [A]
time = 0.75, size = 148, normalized size = 1.20

$$\frac{3\,(5\,bc^2-4\,acd)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{8\,\sqrt{-d}\,d^3\mathrm{sgn}\left(x\right)}+\frac{bc^2-acd}{\sqrt{cx^2+d}\,d^3\mathrm{sgn}\left(x\right)}+\frac{7\,(cx^2+d)^{\frac{3}{2}}bc^2-4\,(cx^2+d)^{\frac{3}{2}}acd-9\,\sqrt{cx^2+d}\,bc^2d+4\,\sqrt{cx^2+d}\,acd^2}{8\,c^2d^3x^4\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 3/8*(5*b*c^2 - 4*a*c*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^3*sgn(
x)) + (b*c^2 - a*c*d)/(sqrt(c*x^2 + d)*d^3*sgn(x)) + 1/8*(7*(c*x^2 + d)^(3/
2)*b*c^2 - 4*(c*x^2 + d)^(3/2)*a*c*d - 9*sqrt(c*x^2 + d)*b*c^2*d + 4*sqrt(c
*x^2 + d)*a*c*d^2)/(c^2*d^3*x^4*sgn(x))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{x^2}}{x^6\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)),x)
[Out] int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)), x)
```

3.986 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx$

Optimal. Leaf size=105

$$\frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} (ex)^{1+m} F_1\left(\frac{1}{2}(-1-m); -p, -q; \frac{1-m}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)}{e(1+m)}$$

```
[Out] (a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1+m)*AppellF1(-1/2-1/2*m,-p,-q,1/2-1/2*m,-b/
a/x^2,-d/c/x^2)/e/(1+m)/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {511, 525, 524}

$$\frac{x(ex)^m \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} F_1\left(\frac{1}{2}(-m-1); -p, -q; \frac{1-m}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(-1 - m)/2, -p, -q, (1 - m)
/2, -(b/(a*x^2)), -(d/(c*x^2))])/((1 + m)*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))
^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b/x^n)^p*((
c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q},
 x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\left(a+bx^2\right)^p \left(c+dx^2\right)^q\,dx, x, \frac{1}{x}\right)\right.$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m}\left(1+\right.\right.\right.$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} \left(\frac{1}{x}\right)^m (ex)^m\right.\right.$$

$$= \frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x(ex)^m F_1\left(\frac{1}{2}(-1-m); -p,\right.}{1+m}$$

Mathematica [A]
time = 0.12, size = 115, normalized size = 1.10

$$\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x(ex)^m \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} F_1\left(\frac{1}{2}(1+m-2p-2q); -p, -q; \frac{1}{2}(3+m-2p-2q); -\frac{ax^2}{b}, -\frac{cx^2}{d}\right)}{1+m-2p-2q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(1 + m - 2*p - 2*q)/2, -p,
-q, (3 + m - 2*p - 2*q)/2, -((a*x^2)/b), -((c*x^2)/d)])/((1 + m - 2*p - 2*q
)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q*(e*x)^m,x)
```

```
[Out] int((b/x^2+a)^p*(c+d/x^2)^q*(e*x)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**m,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="giac")
```

```
[Out] integrate((x*e)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^m\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.987 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 \, dx$

Optimal. Leaf size=84

$$\frac{1}{5}\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}x^5F_1\left(-\frac{5}{2};-p,-q;-\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] 1/5*(a+b/x^2)^p*(c+d/x^2)^q*x^5*AppellF1(-5/2,-p,-q,-3/2,-b/a/x^2,-d/c/x^2)
/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {509, 525, 524}

$$\frac{1}{5}x^5\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{5}{2};-p,-q;-\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[-5/2, -p, -q, -3/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/(5*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 509

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = -\text{Subst}\left(\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^6}\,dx, x, \frac{1}{x}\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right) \text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^6}\,dx, x,\right.\right.$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right) \text{Subst}\left(\int \frac{(1\right.\right.$$
$$= \frac{1}{5}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^5 F_1\left(-\frac{5}{2}; -p, -q;\right.$$

Mathematica [A]
time = 0.10, size = 106, normalized size = 1.26

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^5 \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} F_1\left(\frac{5}{2}-p-q; -p, -q; \frac{7}{2}-p-q; -\frac{ax^2}{b}, -\frac{cx^2}{d}\right)}{-5+2p+2q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[5/2 - p - q, -p, -q, 7/2 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q*x^4,x)
```

```
[Out] int((b/x^2+a)^p*(c+d/x^2)^q*x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="fricas")
```

```
[Out] integral(x^4*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int(x^4*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.988 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$

Optimal. Leaf size=100

$$\frac{b^2\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,3;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(1+p)}$$

```
[Out] 1/2*b^2*(a+b/x^2)^(1+p)*(c+d/x^2)^q*AppellF1(1+p,3,-q,2+p,(a+b/x^2)/a,-d*(a
+b/x^2)/(-a*d+b*c))/a^3/(1+p)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A]

time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {457, 142, 141}

$$\frac{b^2\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,3;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]
```

```
[Out] (b^2*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 3, 2 + p, -((d*(
a + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a^3*(1 + p)*((b*(c + d/x^2))/(
b*c - a*d))^q)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^3}\right.\right.$$
$$= \frac{b^2\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,3;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+}{c}\right.}{2a^3(1+p)}$$

Mathematica [A]
time = 0.09, size = 100, normalized size = 1.00

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^4\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q} F_1\left(2-p-q;-p,-q;3-p-q;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(-2+p+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*x^4*AppellF1[2 - p - q, -p, -q, 3 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-2 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)
/d)^q)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q*x^3,x)
[Out] int((b/x^2+a)^p*(c+d/x^2)^q*x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(a+\frac{b}{x^2}\right)}^p\,{\left(c+\frac{d}{x^2}\right)}^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int(x^3*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.989 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}x^3F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] 1/3*(a+b/x^2)^p*(c+d/x^2)^q*x^3*AppellF1(-3/2,-p,-q,-1/2,-b/a/x^2,-d/c/x^2)
/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {509, 525, 524}

$$\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[-3/2, -p, -q, -1/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/(3*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 509

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = -\text{Subst}\left(\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^4}\,dx, x, \frac{1}{x}\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right) \text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^4}\,dx, x,\right.\right.$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right) \text{Subst}\left(\int \frac{(1}{}\right.\right.$$
$$= \frac{1}{3}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^3 F_1\left(-\frac{3}{2}; -p, -q;\right.$$

Mathematica [A]
time = 0.09, size = 106, normalized size = 1.26

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} F_1\left(\frac{3}{2}-p-q; -p, -q; \frac{5}{2}-p-q; -\frac{ax^2}{b}, -\frac{cx^2}{d}\right)}{-3+2p+2q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[3/2 - p - q, -p, -q, 5/2 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q*x^2,x)
```

```
[Out] int((b/x^2+a)^p*(c+d/x^2)^q*x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int(x^2*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.990 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$

Optimal. Leaf size=98

$$-\frac{b\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,2;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(1+p)}$$

```
[Out] -1/2*b*(a+b/x^2)^(1+p)*(c+d/x^2)^q*AppellF1(1+p,2,-q,2+p,(a+b/x^2)/a,-d*(a+
b/x^2)/(-a*d+b*c))/a^2/(1+p)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A]

time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {457, 142, 141}

$$-\frac{b\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
```

```
[Out] -1/2*(b*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 2, 2 + p, -((
d*(a + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(a^2*(1 + p)*((b*(c + d/x^2))/
(b*c - a*d))^q)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^2}\,d\right.\right.$$

$$= -\frac{b\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,2;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+}{a}\right.}{2a^2(1+p)}$$

Mathematica [A]
time = 0.09, size = 100, normalized size = 1.02

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^2\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q} F_1\left(1-p-q;-p,-q;2-p-q;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(-1+p+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
```

```
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*x^2*AppellF1[1 - p - q, -p, -q, 2 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)
/d)^q)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q*x,x)
```

```
[Out] int((b/x^2+a)^p*(c+d/x^2)^q*x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="fricas")
```

```
[Out] integral(x*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int(x*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.991 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$

Optimal. Leaf size=79

$$\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} xF_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] (a+b/x^2)^p*(c+d/x^2)^q*x*AppellF1(-1/2,-p,-q,1/2,-b/a/x^2,-d/c/x^2)/((1+b/
a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {382, 525, 524}

$$x\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = -\text{Subst}\left(\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^2} dx, x, \frac{1}{x}\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right) \text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^2} dx, x, \frac{1}{x}\right.\right.$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right) \text{Subst}\left(\int \frac{(1+\right.\right.$$
$$= \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} xF_1\left(-\frac{1}{2}; -p, -q; \frac{1}{2}; -\right.$$

Mathematica [A]
time = 0.09, size = 104, normalized size = 1.32

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} F_1\left(\frac{1}{2}-p-q; -p, -q; \frac{3}{2}-p-q; -\frac{ax^2}{b}, -\frac{cx^2}{d}\right)}{-1+2p+2q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q,x)
```

```
[Out] int((b/x^2+a)^p*(c+d/x^2)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.992 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx$

Optimal. Leaf size=97

$$\frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,1;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(1+p)}$$

```
[Out] 1/2*(a+b/x^2)^(1+p)*(c+d/x^2)^q*AppellF1(1+p,1,-q,2+p,(a+b/x^2)/a,-d*(a+b/x
^2)/(-a*d+b*c))/a/(1+p)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {457, 142, 141}

$$\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,1;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
```

```
[Out] ((a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d*(a +
b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a*(1 + p)*((b*(c + d/x^2))/(b*c -
a*d))^q)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x}\,dx, x, \right.\right.\\ &= \frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,1;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 95, normalized size = 0.98

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}F_1\left(-p-q;-p,-q;1-p-q;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(p+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
```

```
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-p - q, -p, -q, 1 - p - q, -((a*
x^2)/b), -((c*x^2)/d)])/((p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{x^2}+a\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q/x,x)
```

```
[Out] int((b/x^2+a)^p*(c+d/x^2)^q/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x,x)
[Out] Integral((a + b/x**2)**p*(c + d/x**2)**q/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x,x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x, x)
```

3.993 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

```
[Out] -(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(1/2,-p,-q,3/2,-b/a/x^2,-d/c/x^2)/((1+b/a/
x^2)^p)/((1+d/c/x^2)^q)/x
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {509, 441, 440}

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/2, -p, -q, 3/2, -(b/(a*x^2)), -(d
/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 509

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = -\text{Subst}\left(\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q\,dx, x, \frac{1}{x}\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q\,dx, x, \frac{1}{x}\right)\right.$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \left(1+\frac{bx^2}{a}\right.\right.\right.$$
$$= -\frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

Mathematica [A]
time = 0.11, size = 106, normalized size = 1.29

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} F_1\left(-\frac{1}{2}-p-q;-p,-q;\frac{1}{2}-p-q;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(1+2p+2q)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-1/2 - p - q, -p, -q, 1/2 - p - q,
-((a*x^2)/b), -((c*x^2)/d)])/((1 + 2*p + 2*q)*x*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q/x^2,x)
```

```
[Out] int((b/x^2+a)^p*(c+d/x^2)^q/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^2, x)
```

3.994 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$

Optimal. Leaf size=85

$$-\frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}{}_2F_1\left(1+p,-q;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(1+p)}$$

```
[Out] -1/2*(a+b/x^2)^(1+p)*(c+d/x^2)^q*hypergeom([-q, 1+p],[2+p],-d*(a+b/x^2)/(-a
*d+b*c))/b/(1+p)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 72, 71}

$$-\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
```

```
[Out] -1/2*((a + b/x^2)^(1 + p)*(c + d/x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p,
 -((d*(a + b/x^2))/(b*c - a*d))])/(b*(1 + p)*((b*(c + d/x^2))/(b*c - a*d))^
q)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx&=-\left(\frac{1}{2}\text{Subst}\left(\int(a+bx)^p(c+dx)^q\,dx,x,\frac{1}{x^2}\right)\right)\\&=-\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q\right.\right.\\&=-\frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}{}_2F_1\left(1+p,-q;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 110, normalized size = 1.29

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{ax^2}{b}\right)^{-p}(d+cx^2)\left(1+\frac{cx^2}{d}\right)^p{}_2F_1\left(-p,-1-p-q;-p-q;\frac{(bc-ad)x^2}{b(d+cx^2)}\right)}{2d(1+p+q)x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*(d + c*x^2)*(1 + (c*x^2)/d)^p*Hypergeomet
ric2F1[-p, -1 - p - q, -p - q, ((b*c - a*d)*x^2)/(b*(d + c*x^2))])/(d*(1 +
p + q)*x^2*(1 + (a*x^2)/b)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{x^2}+a\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q/x^3,x)
[Out] int((b/x^2+a)^p*(c+d/x^2)^q/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x)
```

```
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^3, x)
```

3.995 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$

Optimal. Leaf size=84

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

```
[Out] -1/3*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(3/2,-p,-q,5/2,-b/a/x^2,-d/c/x^2)/((1+
b/a/x^2)^p)/((1+d/c/x^2)^q)/x^3
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {509, 525, 524}

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
```

```
[Out] -1/3*((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/2, -p, -q, 5/2, -(b/(a*x^2)),
-(d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x^3)
```

Rule 509

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx = -\text{Subst}\left(\int x^2\left(a+bx^2\right)^p \left(c+dx^2\right)^q\,dx,x,\frac{1}{x}\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q\,dx,x,\frac{1}{x}\right.\right.$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int x^2\left(1+\right.\right.\right.$$

$$= -\frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

Mathematica [A]
time = 0.15, size = 106, normalized size = 1.26

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} F_1\left(-\frac{3}{2}-p-q;-p,-q;-\frac{1}{2}-p-q;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(3+2p+2q)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-3/2 - p - q, -p, -q, -1/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((3 + 2*p + 2*q)*x^3*(1 + (a*x^2)/b)^p*(1 + (
c*x^2)/d)^q))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q/x^4,x)
```

```
[Out] int((b/x^2+a)^p*(c+d/x^2)^q/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x)
```

```
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^4, x)
```

3.996 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx$

Optimal. Leaf size=91

$$\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{7/2}F_1\left(-\frac{7}{4};-p,-q;-\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{7e}$$

```
[Out] 2/7*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(7/2)*AppellF1(-7/4,-p,-q,-3/4,-b/a/x^2,-
d/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {510, 525, 524}

$$\frac{2(ex)^{7/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{7}{4};-p,-q;-\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2),x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(7/2)*AppellF1[-7/4, -p, -q, -3/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(7*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx = -\frac{2\text{Subst}\left(\int \frac{(a+be^2x^4)^p(c+de^2x^4)^q}{x^8}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q}{x^8}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p}{x^8}\right.}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{7/2}F_1\left(-\frac{7}{4}; -p, -q; -\right.}{7e}$$

Mathematica [A]
time = 0.12, size = 111, normalized size = 1.22

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x(ex)^{5/2}\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}F_1\left(\frac{7}{4}-p-q; -p, -q; \frac{11}{4}-p-q; -\frac{ax^2}{b}, -\frac{cx^2}{d}\right)}{-7+4p+4q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2),x]
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^(5/2)*AppellF1[7/4 - p - q, -p, -q,
 11/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-7 + 4*p + 4*q)*(1 + (a*x^2)/
b)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q*(e*x)^(5/2),x)
[Out] int((b/x^2+a)^p*(c+d/x^2)^q*(e*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((a + b/x^2)^p*(c + d/x^2)^q*x^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^(5/2)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q*e^(5/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^{5/2}\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.997 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} (ex)^{5/2} F_1\left(-\frac{5}{4};-p,-q;-\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

```
[Out] 2/5*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2)*AppellF1(-5/4,-p,-q,-1/4,-b/a/x^2,-
d/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {510, 525, 524}

$$\frac{2(ex)^{5/2} \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} F_1\left(-\frac{5}{4};-p,-q;-\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2)*AppellF1[-5/4, -p, -q, -1/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(5*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q(ex)^{3/2}\,dx=-\frac{2\text{Subst}\left(\int\frac{(a+be^2x^4)^p(c+de^2x^4)^q}{x^6}\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}$$

$$=-\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int\frac{\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q}{x^6}\,dx,x,\frac{1}{\sqrt{ex}}\right.}{e}$$

$$=-\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int\frac{\left(1+\frac{be^2x^4}{a}\right)}{}\right.}{e}$$

$$=\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{5/2}F_1\left(-\frac{5}{4};-p,-q;\right.}{5e}$$

Mathematica [A]
time = 0.11, size = 111, normalized size = 1.22

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x(ex)^{3/2}\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}F_1\left(\frac{5}{4}-p-q;-p,-q;\frac{9}{4}-p-q;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-5+4p+4q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^(3/2)*AppellF1[5/4 - p - q, -p, -q,
 9/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 4*p + 4*q)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int\left(\frac{b}{x^2}+a\right)^p\left(c+\frac{d}{x^2}\right)^q(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
[Out] int((b/x^2+a)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="maxima")
[Out] e^(3/2)*integrate((a + b/x^2)^p*(c + d/x^2)^q*x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="fricas")
[Out] integral(x^(3/2)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q*e^(3/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^{3/2}\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.998 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx$

Optimal. Leaf size=91

$$\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{3/2}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

```
[Out] 2/3*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2)*AppellF1(-3/4,-p,-q,1/4,-b/a/x^2,-d
/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {510, 525, 524}

$$\frac{2(ex)^{3/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2)*AppellF1[-3/4, -p, -q, 1/4, -(b/
(a*x^2)), -(d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\, dx = -\frac{2\text{Subst}\left(\int \frac{(a+be^2x^4)^p(c+de^2x^4)^q}{x^4}\, dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q}{x^4}\, dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p(1}{x^4}\right.}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{3/2}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};\right.}{3e}$$

Mathematica [A]
time = 0.11, size = 111, normalized size = 1.22

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\sqrt{ex}\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}F_1\left(\frac{3}{4}-p-q;-p,-q;\frac{7}{4}-p-q;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-3+4p+4q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*Sqrt[e*x]*AppellF1[3/4 - p - q, -p, -q, 7
/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 4*p + 4*q)*(1 + (a*x^2)/b)^
p*(1 + (c*x^2)/d)^q)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
[Out] int((b/x^2+a)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((a + b/x^2)^p*(c + d/x^2)^q*sqrt(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q*e^(1/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*sqrt(x)*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,x}\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int((e*x)^(1/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.999 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}} \, dx$

Optimal. Leaf size=89

$$\frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} \sqrt{ex}\, F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

```
[Out] 2*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(-1/4,-p,-q,3/4,-b/a/x^2,-d/c/x^2)*(e*x)^
(1/2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {510, 525, 524}

$$\frac{2\sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x]*AppellF1[-1/4, -p, -q, 3/4, -(b/(a
*x^2)), -(d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = -\frac{2\text{Subst}\left(\int \frac{(a+be^2x^4)^p(c+de^2x^4)^q}{x^2}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q}{x^2}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^2}\right.}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\sqrt{ex}\,F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{c}\right.}{e}$$

Mathematica [A]
time = 0.11, size = 111, normalized size = 1.25

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}F_1\left(\frac{1}{4}-p-q;-p,-q;\frac{5}{4}-p-q;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(-1+4p+4q)\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/4 - p - q, -p, -q, 5/4 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 4*p + 4*q)*Sqrt[e*x]*(1 + (a*x^2)/b)^
p*(1 + (c*x^2)/d)^q)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{x^2}+a\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

```
[Out] int((b/x^2+a)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate((a + b/x^2)^p*(c + d/x^2)^q/sqrt(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q*e^(-1/2)/sqrt(x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*e^(-1/2)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(1/2),x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(1/2), x)
```

3.1000 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}} \, dx$

Optimal. Leaf size=89

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

```
[Out] -2*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(1/4,-p,-q,5/4,-b/a/x^2,-d/c/x^2)/e/((1+
b/a/x^2)^p)/((1+d/c/x^2)^q)/(e*x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {510, 441, 440}

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/4, -p, -q, 5/4, -(b/(a*x^2)), -(
d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*Sqrt[e*x])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
```

`Q[m]`

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = -\frac{2\mathrm{Subst}\left(\int \left(a+be^2x^4\right)^p \left(c+de^2x^4\right)^q\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right)\mathrm{Subst}\left(\int \left(1+\frac{be^2x^4}{a}\right)^p \left(c+de^2x^4\right)^q\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right)\mathrm{Subst}\left(\int \left(1+\frac{be^2x^4}{a}\right)^p \left(1+\frac{de}{}\right.\right.}{e}$$

$$= -\frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

Mathematica [A]
time = 0.12, size = 111, normalized size = 1.25

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q} F_1\left(-\frac{1}{4}-p-q;-p,-q;\frac{3}{4}-p-q;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(1+4p+4q)(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/4 - p - q, -p, -q, 3/4 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((1 + 4*p + 4*q)*(e*x)^(3/2)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{x^2}+a\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

```
[Out] int((b/x^2+a)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate((a + b/x^2)^p*(c + d/x^2)^q/x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q*e^(-3/2)/x^(3/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*e^(-3/2)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2), x)
```

3.1001 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

```
[Out] -2/3*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(3/4,-p,-q,7/4,-b/a/x^2,-d/c/x^2)/e/((
1+b/a/x^2)^p)/((1+d/c/x^2)^q)/(e*x)^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {510, 525, 524}

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/4, -p, -q, 7/4, -(b/(a*x^2)), -(
d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*(e*x)^(3/2))
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx=-\frac{2\text{Subst}\left(\int x^2\left(a+be^2x^4\right)^p\left(c+de^2x^4\right)^q\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}$$

$$=-\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\frac{be^2x^4}{a}\right)^p\left(c+de^2x^4\right)^q\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}$$

$$=-\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int x^2\left(1+\frac{be^2x^4}{a}\right)^p\left(1-\right.\right.}{e}$$

$$=-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

Mathematica [A]
time = 0.13, size = 111, normalized size = 1.22

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}F_1\left(-\frac{3}{4}-p-q;-p,-q;\frac{1}{4}-p-q;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(3+4p+4q)(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-3/4 - p - q, -p, -q, 1/4 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((3 + 4*p + 4*q)*(e*x)^(5/2)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{x^2}+a\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+a)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

```
[Out] int((b/x^2+a)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="maxima")
[Out] e^(-5/2)*integrate((a + b/x^2)^p*(c + d/x^2)^q/x^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q*e^(-5/2)/x^(5/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*e^(-5/2)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q}{{\left(e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2), x)
```

3.1002 $\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}\,dx$

Optimal. Leaf size=135

$$-\frac{5}{64}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{1}{24}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}+\frac{1}{4}$$

```
[Out] -5/64*arccosh(x^(1/2))-5/96*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1/
24*x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/4*x^(7/2)*(-1+x^(1/2))^(1
/2)*(1+x^(1/2))^(1/2)-5/64*x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {286, 329, 336, 54}

$$\frac{1}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{7/2}-\frac{1}{24}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{5}{96}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{5}{64}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{5}{64}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

```
[Out] (-5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/64 - (5*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]]*x^(3/2))/96 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
5/2))/24 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2))/4 - (5*ArcCosh[Sq
rt[x]])/64
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 286

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Dist[2*a1*a2*n*(p/(m + 2*n*p + 1)), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
```

```
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}\,dx = \frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2} - \frac{1}{8}\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{1}{24}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2} + \frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2} -$$

$$= -\frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{24}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}$$

$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}$$

$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}$$

$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}$$

Mathematica [A]
time = 1.55, size = 99, normalized size = 0.73

$$\frac{1}{192}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\,\sqrt{x}\left(-15-15\sqrt{x}-10x-10x^{3/2}-8x^2-8x^{5/2}+48x^3+48x^{7/2}\right)-30\tanh^{-1}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(-15 - 15*Sqrt[x] - 10*x - 10*x
^(3/2) - 8*x^2 - 8*x^(5/2) + 48*x^3 + 48*x^(7/2)) - 30*ArcTanh[Sqrt[(-1 + S
qrt[x])/(1 + Sqrt[x])]])/192
```

Maple [A]
time = 0.35, size = 75, normalized size = 0.56

method	result
derivativedivides	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(-48\sqrt{x-1}\,x^{\frac{7}{2}}+8x^{\frac{5}{2}}\sqrt{x-1}+10x^{\frac{3}{2}}\sqrt{x-1}+15\sqrt{x}\,\sqrt{x-1}+\right.}{192\sqrt{x-1}}$
default	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(-48\sqrt{x-1}\,x^{\frac{7}{2}}+8x^{\frac{5}{2}}\sqrt{x-1}+10x^{\frac{3}{2}}\sqrt{x-1}+15\sqrt{x}\,\sqrt{x-1}+\right.}{192\sqrt{x-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(-48*(x-1)^(1/2)*x^(7/2)+8*x^(5
/2)*(x-1)^(1/2)+10*x^(3/2)*(x-1)^(1/2)+15*x^(1/2)*(x-1)^(1/2)+15*ln(x^(1/2)
+(x-1)^(1/2)))/(x-1)^(1/2)
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.42

$$\frac{1}{4}(x-1)^{\frac{3}{2}}x^{\frac{5}{2}}+\frac{5}{24}(x-1)^{\frac{3}{2}}x^{\frac{3}{2}}+\frac{5}{32}(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{5}{64}\sqrt{x-1}\,\sqrt{x}-\frac{5}{64}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/4*(x - 1)^(3/2)*x^(5/2) + 5/24*(x - 1)^(3/2)*x^(3/2) + 5/32*(x - 1)^(3/2)
*sqrt(x) + 5/64*sqrt(x - 1)*sqrt(x) - 5/64*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A]
time = 1.30, size = 62, normalized size = 0.46

$$\frac{1}{192}\left(48x^{3}-8x^{2}-10x-15\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\frac{5}{128}\log\left(2\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/192*(48*x^3 - 8*x^2 - 10*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) -
 1) + 5/128*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

Giac [A]
time = 1.38, size = 162, normalized size = 1.20

$$\frac{1}{6720}\left(\left(2\left(\left(4\left(5\left(6\left(7\sqrt{x}-50\right)\left(\sqrt{x}+1\right)+1219\right)\left(\sqrt{x}+1\right)-12463\right)\left(\sqrt{x}+1\right)+64233\right)\left(\sqrt{x}+1\right)-53963\right)\left(\sqrt{x}+1\right)+59465\right)\left(\sqrt{x}+1\right)-23205\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{840}\left(\left(2\left(\left(4\left(5\left(6\sqrt{x}-37\right)\left(\sqrt{x}+1\right)+661\right)\left(\sqrt{x}+1\right)-4551\right)\left(\sqrt{x}+1\right)+4781\right)\left(\sqrt{x}+1\right)-6335\right)\left(\sqrt{x}+1\right)+2835\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{5}{32}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6720*((2*((4*(5*(6*(7*sqrt(x) - 50)*(sqrt(x) + 1) + 1219)*(sqrt(x) + 1) -
 12463)*(sqrt(x) + 1) + 64233)*(sqrt(x) + 1) - 53963)*(sqrt(x) + 1) + 59465
)*(sqrt(x) + 1) - 23205)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/840*((2*((
4*(5*(6*sqrt(x) - 37)*(sqrt(x) + 1) + 661)*(sqrt(x) + 1) - 4551)*(sqrt(x) +
 1) + 4781)*(sqrt(x) + 1) - 6335)*(sqrt(x) + 1) + 2835)*sqrt(sqrt(x) + 1)*s
qrt(sqrt(x) - 1) + 5/32*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B]
time = 52.03, size = 831, normalized size = 6.16

$$-\frac{5\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)}{16}+\frac{-\frac{235\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{48\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{1723\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{48\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{72283\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{16\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{848801\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{16\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{4181067\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{16\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{10994181\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{13}}{16\left(\sqrt{\sqrt{x}+1}-1\right)^{13}}+\frac{17457599\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{15}}{16\left(\sqrt{\sqrt{x}+1}-1\right)^{15}}+\frac{17457599\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{17}}{16\left(\sqrt{\sqrt{x}+1}-1\right)^{17}}+\frac{10994181\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{19}}{16\left(\sqrt{\sqrt{x}+1}-1\right)^{19}}+\frac{4181067\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{21}}{16\left(\sqrt{\sqrt{x}+1}-1\right)^{21}}+\frac{848801\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{23}}{16\left(\sqrt{\sqrt{x}+1}-1\right)^{23}}+\frac{72283\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{25}}{16\left(\sqrt{\sqrt{x}+1}-1\right)^{25}}+\frac{1723\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{27}}{48\left(\sqrt{\sqrt{x}+1}-1\right)^{27}}-\frac{235\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{29}}{48\left(\sqrt{\sqrt{x}+1}-1\right)^{29}}+\frac{5\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{31}}{16\left(\sqrt{\sqrt{x}+1}-1\right)^{31}}+\frac{5\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)}{16\left(\sqrt{\sqrt{x}+1}-1\right)}}{1+\frac{120\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{560\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{1820\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{4368\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{8008\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{11440\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}}+\frac{12870\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{16}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{16}}-\frac{11440\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{18}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{18}}+\frac{8008\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{20}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{20}}-\frac{4368\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{22}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{22}}+\frac{1820\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{24}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{24}}-\frac{560\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{26}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{26}}+\frac{120\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{28}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{28}}-\frac{16\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{30}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{30}}+\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{32}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{32}}-\frac{16\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\sqrt{x}+1}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

```
[Out] ((1723*((x^(1/2) - 1)^(1/2) - 1i)^5)/(48*((x^(1/2) + 1)^(1/2) - 1)^5) - (23
5*((x^(1/2) - 1)^(1/2) - 1i)^3)/(48*((x^(1/2) + 1)^(1/2) - 1)^3) + (72283*(
(x^(1/2) - 1)^(1/2) - 1i)^7)/(16*((x^(1/2) + 1)^(1/2) - 1)^7) + (848801*((x
^(1/2) - 1)^(1/2) - 1i)^9)/(16*((x^(1/2) + 1)^(1/2) - 1)^9) + (4181067*((x^
(1/2) - 1)^(1/2) - 1i)^11)/(16*((x^(1/2) + 1)^(1/2) - 1)^11) + (10994181*((
x^(1/2) - 1)^(1/2) - 1i)^13)/(16*((x^(1/2) + 1)^(1/2) - 1)^13) + (17457599*
((x^(1/2) - 1)^(1/2) - 1i)^15)/(16*((x^(1/2) + 1)^(1/2) - 1)^15) + (1745759
9*((x^(1/2) - 1)^(1/2) - 1i)^17)/(16*((x^(1/2) + 1)^(1/2) - 1)^17) + (10994
181*((x^(1/2) - 1)^(1/2) - 1i)^19)/(16*((x^(1/2) + 1)^(1/2) - 1)^19) + (418
1067*((x^(1/2) - 1)^(1/2) - 1i)^21)/(16*((x^(1/2) + 1)^(1/2) - 1)^21) + (84
8801*((x^(1/2) - 1)^(1/2) - 1i)^23)/(16*((x^(1/2) + 1)^(1/2) - 1)^23) + (72
283*((x^(1/2) - 1)^(1/2) - 1i)^25)/(16*((x^(1/2) + 1)^(1/2) - 1)^25) + (172
3*((x^(1/2) - 1)^(1/2) - 1i)^27)/(48*((x^(1/2) + 1)^(1/2) - 1)^27) - (235*(
(x^(1/2) - 1)^(1/2) - 1i)^29)/(48*((x^(1/2) + 1)^(1/2) - 1)^29) + (5*((x^(1
/2) - 1)^(1/2) - 1i)^31)/(16*((x^(1/2) + 1)^(1/2) - 1)^31) + (5*((x^(1/2) -
 1)^(1/2) - 1i))/(16*((x^(1/2) + 1)^(1/2) - 1)))/((120*((x^(1/2) - 1)^(1/2)
 - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (16*((x^(1/2) - 1)^(1/2) - 1i)^2)/(
```

```
(x^(1/2) + 1)^(1/2) - 1)^2 - (560*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2) +
 1)^(1/2) - 1)^6 + (1820*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2)
 - 1)^8 - (4368*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10
 + (8008*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (114
40*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + (12870*((x
^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)^16 - (11440*((x^(1/2)
 - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (8008*((x^(1/2) - 1)^(
1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (4368*((x^(1/2) - 1)^(1/2) -
1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + (1820*((x^(1/2) - 1)^(1/2) - 1i)^24)
/((x^(1/2) + 1)^(1/2) - 1)^24 - (560*((x^(1/2) - 1)^(1/2) - 1i)^26)/((x^(1/
2) + 1)^(1/2) - 1)^26 + (120*((x^(1/2) - 1)^(1/2) - 1i)^28)/((x^(1/2) + 1)^
(1/2) - 1)^28 - (16*((x^(1/2) - 1)^(1/2) - 1i)^30)/((x^(1/2) + 1)^(1/2) - 1
)^30 + ((x^(1/2) - 1)^(1/2) - 1i)^32/((x^(1/2) + 1)^(1/2) - 1)^32 + 1) - (5
*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)))/16
```

3.1003 $\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{1}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}-\frac{1}{8}\cosh$$

```
[Out] -1/8*arccosh(x^(1/2))-1/12*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/3
*x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1/8*x^(1/2)*(-1+x^(1/2))^(1/2
)*(1+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {286, 329, 336, 54}

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{1}{12}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{1}{8}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{1}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

```
[Out] -1/8*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]) - (Sqrt[-1 + Sqrt[x]]*S
qrt[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2
))/3 - ArcCosh[Sqrt[x]]/8
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 286

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Dist[2*a1*a2*n*(p/(m + 2*n*p + 1)), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
```

```
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}\,dx &= \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}-\frac{1}{6}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx\\
&= -\frac{1}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}\\
&= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{1}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}\\
&= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{1}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}\\
&= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{1}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}
\end{aligned}$$

Mathematica [A]
time = 1.15, size = 87, normalized size = 0.84

$$\frac{1}{24}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\,\sqrt{x}\left(-3-3\sqrt{x}-2x-2x^{3/2}+8x^2+8x^{5/2}\right)-6\tanh^{-1}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(-3 - 3*Sqrt[x] - 2*x - 2*x^(3/
2) + 8*x^2 + 8*x^(5/2)) - 6*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/24
```

Maple [A]
time = 0.33, size = 65, normalized size = 0.62

method	result
derivativedivides	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(-8x^{\frac{5}{2}}\sqrt{x-1}+2x^{\frac{3}{2}}\sqrt{x-1}+3\sqrt{x}\,\sqrt{x-1}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right.\right.}{24\sqrt{x-1}}$
default	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(-8x^{\frac{5}{2}}\sqrt{x-1}+2x^{\frac{3}{2}}\sqrt{x-1}+3\sqrt{x}\,\sqrt{x-1}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right.\right.}{24\sqrt{x-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(-8*x^(5/2)*(x-1)^(1/2)+2*x^(3/2
)*(x-1)^(1/2)+3*x^(1/2)*(x-1)^(1/2)+3*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

Maxima [A]
time = 0.30, size = 47, normalized size = 0.45

$$\frac{1}{3}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}}+\frac{1}{4}\,(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{1}{8}\,\sqrt{x-1}\,\sqrt{x}-\frac{1}{8}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(x - 1)^(3/2)*x^(3/2) + 1/4*(x - 1)^(3/2)*sqrt(x) + 1/8*sqrt(x - 1)*sqr
t(x) - 1/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A]
time = 1.27, size = 57, normalized size = 0.55

$$\frac{1}{24}\,(8\,x^2-2\,x-3)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\frac{1}{16}\,\log\left(2\,\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/24*(8*x^2 - 2*x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/16*l
og(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

Giac [A]
time = 2.84, size = 127, normalized size = 1.22

$$\frac{1}{120}\left(\left(2\left(\left(4\left(5\sqrt{x}-26\right)\left(\sqrt{x}+1\right)+321\right)\left(\sqrt{x}+1\right)-451\right)\left(\sqrt{x}+1\right)+745\right)\left(\sqrt{x}+1\right)-405\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{60}\left(\left(2\left(3\left(4\sqrt{x}-17\right)\left(\sqrt{x}+1\right)+133\right)\left(\sqrt{x}+1\right)-295\right)\left(\sqrt{x}+1\right)+195\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{4}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*((2*((4*(5*sqrt(x) - 26)*(sqrt(x) + 1) + 321)*(sqrt(x) + 1) - 451)*(s
qrt(x) + 1) + 745)*(sqrt(x) + 1) - 405)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)
 + 1/60*((2*(3*(4*sqrt(x) - 17)*(sqrt(x) + 1) + 133)*(sqrt(x) + 1) - 295)*(
sqrt(x) + 1) + 195)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/4*log(sqrt(sqrt
(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B]
time = 31.39, size = 632, normalized size = 6.08

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)}{2}-\frac{\frac{35\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{6\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{757\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{2\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{7339\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{2\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{41929\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{3\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{25661\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{25661\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{13}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}}+\frac{41929\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{15}}{3\left(\sqrt{\sqrt{x}+1}-1\right)^{15}}+\frac{7339\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{17}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{17}}+\frac{757\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{19}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{19}}+\frac{35\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{21}}{6\left(\sqrt{\sqrt{x}+1}-1\right)^{21}}-\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{23}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{23}}-\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{2\left(\sqrt{\sqrt{x}+1}-1\right)}}{1+\frac{66\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{220\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{495\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}}+\frac{495\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{16}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{16}}-\frac{220\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{18}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{18}}+\frac{66\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{20}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{20}}-\frac{12\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{22}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{22}}+\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{24}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{24}}-\frac{12\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\sqrt{x}+1}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

```
[Out] - atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1))/2 - ((35*((x^
(1/2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (757*((x^(1/2)
- 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) + (7339*((x^(1/2) - 1)^
(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (41929*((x^(1/2) - 1)^(1/2
) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25661*((x^(1/2) - 1)^(1/2) -
1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25661*((x^(1/2) - 1)^(1/2) - 1i)^13
)/((x^(1/2) + 1)^(1/2) - 1)^13 + (41929*((x^(1/2) - 1)^(1/2) - 1i)^15)/(3*(
(x^(1/2) + 1)^(1/2) - 1)^15) + (7339*((x^(1/2) - 1)^(1/2) - 1i)^17)/(2*((x^
(1/2) + 1)^(1/2) - 1)^17) + (757*((x^(1/2) - 1)^(1/2) - 1i)^19)/(2*((x^(1/2
) + 1)^(1/2) - 1)^19) + (35*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6*((x^(1/2) + 1
)^(1/2) - 1)^21) - ((x^(1/2) - 1)^(1/2) - 1i)^23/(2*((x^(1/2) + 1)^(1/2) -
1)^23) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1)^(1/2) - 1)))/((66*((x
^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (12*((x^(1/2) - 1)
^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (220*((x^(1/2) - 1)^(1/2) - 1
i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^
(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1
)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2)
- 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14
+ (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)^16 - (220*(
```

```
(x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (66*((x^(1/2)
- 1)^(1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (12*((x^(1/2) - 1)^(1/2
) - 1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + ((x^(1/2) - 1)^(1/2) - 1i)^24/((
x^(1/2) + 1)^(1/2) - 1)^24 + 1)
```

3.1004 $\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/4*arccosh(x^(1/2))+1/2*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1/4*
x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {286, 329, 336, 54}

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{1}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] -1/4*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]) + (Sqrt[-1 + Sqrt[x]]*S
qrt[1 + Sqrt[x]]*x^(3/2))/2 - ArcCosh[Sqrt[x]]/4
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 286

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Dist[2*a1*a2*n*(p/(m + 2*n*p + 1)), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
```

```
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}\,dx = \frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$
$$= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} -$$
$$= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} -$$
$$= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} -$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 404 vs. 2(73) = 146.
time = 1.39, size = 404, normalized size = 5.53

$$\frac{-4\sqrt{1+\sqrt{x}}\left(-18816+28224\sqrt{x}+55360x+17296x^{3/2}+7240x^2-1096x^{5/2}-4752x^3-1136x^{7/2}\right)-4\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\left(32592+74488\sqrt{x}+38632x+6992x^{3/2}-104x^2-6079x^{5/2}-3120x^3-194x^{7/2}\right)+\sqrt{3}\left(-4\sqrt{-1+\sqrt{x}}\left(-18816-52416\sqrt{x}-41472x-10928x^{3/2}-1192x^2+3832x^{5/2}+3408x^3+656x^{7/2}\right)-4\left(10864-10872\sqrt{x}-41440x-23268x^{3/2}-6678x^2-1148x^{5/2}+3416x^3+1800x^{7/2}+112x^4\right)\right)}{-12416+13312\sqrt{x}+49408x+24960x^{3/2}+1552x^2+\sqrt{3}\,\sqrt{1+\sqrt{x}}\left(7168-11264\sqrt{x}-22016x-5248x^{3/2}\right)+\sqrt{-1+\sqrt{x}}\left(21504+60416\sqrt{x}+47104x+9088x^{3/2}+\sqrt{3}\,\sqrt{1+\sqrt{x}}\left(-12416-28672\sqrt{x}-14400x-896x^{3/2}\right)\right)}+\tanh^{-1}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] (-4*Sqrt[1 + Sqrt[x]]*(-18816 + 28224*Sqrt[x] + 55360*x + 17296*x^(3/2) + 7
240*x^2 - 1096*x^(5/2) - 4752*x^3 - 1136*x^(7/2)) - 4*Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]]*(32592 + 74488*Sqrt[x] + 38632*x + 6992*x^(3/2) - 104*x^2 -
 6079*x^(5/2) - 3120*x^3 - 194*x^(7/2)) + Sqrt[3]*(-4*Sqrt[-1 + Sqrt[x]]*(-
18816 - 52416*Sqrt[x] - 41472*x - 10928*x^(3/2) - 1192*x^2 + 3832*x^(5/2) +
 3408*x^3 + 656*x^(7/2)) - 4*(10864 - 10872*Sqrt[x] - 41440*x - 23268*x^(3/
2) - 6678*x^2 - 1148*x^(5/2) + 3416*x^3 + 1800*x^(7/2) + 112*x^4)))/(-12416
 + 13312*Sqrt[x] + 49408*x + 24960*x^(3/2) + 1552*x^2 + Sqrt[3]*Sqrt[1 + Sq
rt[x]]*(7168 - 11264*Sqrt[x] - 22016*x - 5248*x^(3/2)) + Sqrt[-1 + Sqrt[x]]
```

```
*(21504 + 60416*Sqrt[x] + 47104*x + 9088*x^(3/2) + Sqrt[3]*Sqrt[1 + Sqrt[x]
]*(-12416 - 28672*Sqrt[x] - 14400*x - 896*x^(3/2)))) + ArcTanh[(-1 + Sqrt[-
1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Maple [A]
time = 0.40, size = 52, normalized size = 0.71

method	result	size
derivativedivides	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(-2x^{\frac{3}{2}}\sqrt{x-1}+\sqrt{x}\,\sqrt{x-1}+\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{4\sqrt{x-1}}$	52
default	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(-2x^{\frac{3}{2}}\sqrt{x-1}+\sqrt{x}\,\sqrt{x-1}+\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{4\sqrt{x-1}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(-2*x^(3/2)*(x-1)^(1/2)+x^(1/2)*(
x-1)^(1/2)+ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.51

$$\frac{1}{2}\,(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{1}{4}\,\sqrt{x-1}\,\sqrt{x}-\frac{1}{4}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(x - 1)^(3/2)*sqrt(x) + 1/4*sqrt(x - 1)*sqrt(x) - 1/4*log(2*sqrt(x - 1)
 + 2*sqrt(x))
```

Fricas [A]
time = 0.98, size = 52, normalized size = 0.71

$$\frac{1}{4}\,(2\,x-1)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\frac{1}{8}\,\log\left(2\,\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/4*(2*x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/8*log(2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
[Out] Integral(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.
time = 1.36, size = 92, normalized size = 1.26

$$\frac{1}{12}\left(\left(2\left(3\sqrt{x}-10\right)\left(\sqrt{x}+1\right)+43\right)\left(\sqrt{x}+1\right)-39\right)\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\frac{1}{3}\left(\left(2\sqrt{x}-5\right)\left(\sqrt{x}+1\right)+9\right)\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\frac{1}{2}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/12*((2*(3*sqrt(x) - 10)*(sqrt(x) + 1) + 43)*(sqrt(x) + 1) - 39)*sqrt(sqrt
(x) + 1)*sqrt(sqrt(x) - 1) + 1/3*((2*sqrt(x) - 5)*(sqrt(x) + 1) + 9)*sqrt(s
qrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
[Out] int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2), x)
```

3.1005 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$

Optimal. Leaf size=37

$$\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arccosh(x^(1/2))+x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.107$, Rules used = {286, 336, 54}

$$\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - ArcCosh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 286

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Dist[2*a1*a2*n*(p/(m + 2*n*p + 1)), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx$$
$$= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \cosh^{-1}\left(\sqrt{x}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 264 vs. 2(37) = 74.
time = 1.13, size = 264, normalized size = 7.14

$$4\left(\frac{4\sqrt{1+\sqrt{x}}\,(-12-24\sqrt{x}+x+5x^{3/2})+\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(-84-10\sqrt{x}+28x+7x^{3/2})+\sqrt{3}\left(28+70\sqrt{x}+18x-14x^{3/2}-4x^2-4\sqrt{-1+\sqrt{x}}\,(-12-8\sqrt{x}+5x+3x^{3/2})\right)}{56-16\sqrt{3}\,\sqrt{1+\sqrt{x}}\,(2+3\sqrt{x})+\sqrt{-1+\sqrt{x}}\left(96-8\sqrt{3}\,\sqrt{1+\sqrt{x}}\,(7+2\sqrt{x})+80\sqrt{x}\right)+112\sqrt{x}+28x}+\tanh^{-1}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

```
[Out] 4*((4*Sqrt[1 + Sqrt[x]]*(-12 - 24*Sqrt[x] + x + 5*x^(3/2)) + Sqrt[-1 + Sqrt
[x]]*Sqrt[1 + Sqrt[x]]*(-84 - 10*Sqrt[x] + 28*x + 7*x^(3/2)) + Sqrt[3]*(28
+ 70*Sqrt[x] + 18*x - 14*x^(3/2) - 4*x^2 - 4*Sqrt[-1 + Sqrt[x]]*(-12 - 8*Sq
rt[x] + 5*x + 3*x^(3/2))))/(56 - 16*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(2 + 3*Sqrt[x
]) + Sqrt[-1 + Sqrt[x]]*(96 - 8*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(7 + 2*Sqrt[x]) +
 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) + ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqr
t[3] - Sqrt[1 + Sqrt[x]])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.
time = 0.33, size = 72, normalized size = 1.95

method	result
derivativedivides	$\sqrt{-1+\sqrt{x}}\,\left(\sqrt{x}+1\right)^{\frac{3}{2}} - \sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1} - \frac{\sqrt{\left(\sqrt{x}+1\right)\left(-1+\sqrt{x}\right)}\,\ln(}{\sqrt{\sqrt{x}+1}\,\sqrt{-1+}}$
default	$\sqrt{-1+\sqrt{x}}\,\left(\sqrt{x}+1\right)^{\frac{3}{2}} - \sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1} - \frac{\sqrt{\left(\sqrt{x}+1\right)\left(-1+\sqrt{x}\right)}\,\ln(}{\sqrt{\sqrt{x}+1}\,\sqrt{-1+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(3/2)-(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)-(
(x^(1/2)+1)*(-1+x^(1/2)))^(1/2)/(x^(1/2)+1)^(1/2)/(-1+x^(1/2))^(1/2)*ln(x^(
1/2)+(x-1)^(1/2))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.70

$$\sqrt{x-1}\sqrt{x}-\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="maxima
")
```

```
[Out] sqrt(x - 1)*sqrt(x) - log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A]
time = 2.63, size = 46, normalized size = 1.24

$$\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="fricas
")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(2*sqrt(x)*sqrt(sqrt(x
) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/sqrt(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.
time = 2.74, size = 57, normalized size = 1.54

$$\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\left(\sqrt{x}-2\right)+2\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*(sqrt(x) - 2) + 2*sqrt(sqrt(x) + 1)*sqr
t(sqrt(x) - 1) + 2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [**B**]
time = 5.07, size = 41, normalized size = 1.11

$$\sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}-\ln\left(\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2),x)
```

```
[Out] x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2) - log((x^(1/2) - 1)^(1/2)*(
x^(1/2) + 1)^(1/2) + x^(1/2))
```

3.1006 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}}-2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+2\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*arccosh(x^(1/2))+2*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(1/2)-2*x^(1/2)
*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {333, 286, 336, 54}

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\sqrt{\sqrt{x}-1}\,\sqrt{x}\,\sqrt{\sqrt{x}+1}+2\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]
*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*ArcCosh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 286

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Dist[2*a1*a2*n*(p/(m + 2*n*p + 1)), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 333

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2
*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a
1*a2*c^(2*n)*(m + 1))), Int[(c*x)^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p
```

```
, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IG
tQ[2*n, 0] && LtQ[m, -1] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx \\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + \int \frac{}{\sqrt{-}} \\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + 2\text{Subst} \\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + 2\cosh\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 184 vs. 2(67) = 134.
time = 0.95, size = 184, normalized size = 2.75

$$\frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\,\sqrt{1+\sqrt{x}}-\sqrt{x}\right)}{\left(-3-2\sqrt{-1+\sqrt{x}}+2\sqrt{3}\,\sqrt{1+\sqrt{x}}+\sqrt{3}\,\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}-2\sqrt{x}\right)\sqrt{x}} - 8\tanh^{-1}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

```
[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sq
rt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x]))/((-3 - 2*Sqrt[-1 + Sqrt[x]]
+ 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]
] - 2*Sqrt[x])*Sqrt[x]) - 8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sq
rt[1 + Sqrt[x]])]
```

Maple [A]
time = 0.32, size = 47, normalized size = 0.70

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(\ln\left(\sqrt{x}+\sqrt{x-1}\right)\sqrt{x}-\sqrt{x-1}\right)}{\sqrt{x}\,\sqrt{x-1}}$	47
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(\ln\left(\sqrt{x}+\sqrt{x-1}\right)\sqrt{x}-\sqrt{x-1}\right)}{\sqrt{x}\,\sqrt{x-1}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(ln(x^(1/2)+(x-1)^(1/2))*x^(1/2)-(x-
1)^(1/2))/x^(1/2)/(x-1)^(1/2)
```

Maxima [A]
time = 0.50, size = 27, normalized size = 0.40

$$-\frac{2\sqrt{x-1}}{\sqrt{x}}+2\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="maxima
")
```

```
[Out] -2*sqrt(x - 1)/sqrt(x) + 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A]
time = 2.94, size = 55, normalized size = 0.82

$$-\frac{x\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)+2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="fricas
")
```

```
[Out] -(x*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1) + 2*sqrt(x
)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 6.25, size = 129, normalized size = 1.93

$$8\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)-\frac{\frac{5\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^2}{2\left(\sqrt{\sqrt{x}+1}-1\right)^2}+\frac{1}{2}}{\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}}-\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{2\left(\sqrt{\sqrt{x}+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(3/2),x)
```

```
[Out] 8*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) - ((5*((x^(1/
2) - 1)^(1/2) - 1i)^2)/(2*((x^(1/2) + 1)^(1/2) - 1)^2) + 1/2)/((((x^(1/2) -
1)^(1/2) - 1i)^3/((x^(1/2) + 1)^(1/2) - 1)^3 + ((x^(1/2) - 1)^(1/2) - 1i)/(
(x^(1/2) + 1)^(1/2) - 1)) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1)^(1
/2) - 1))
```

3.1007 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx$

Optimal. Leaf size=31

$$\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{3x^{3/2}}$$

```
[Out] 2/3*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{271\}$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{3x^{3/2}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 421 vs. $2(31) = 62$.
time = 1.77, size = 421, normalized size = 13.58

$$\frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{x}}-\sqrt{x}\right)\left(8\left(7+12\sqrt{-1+\sqrt{x}}-4\sqrt{3}\sqrt{1+\sqrt{x}}-7\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)+4\left(49+8\sqrt{-1+\sqrt{x}}-24\sqrt{3}\sqrt{1+\sqrt{x}}+3\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)\sqrt{x}+2\left(-23-144\sqrt{-1+\sqrt{x}}+32\sqrt{3}\sqrt{1+\sqrt{x}}+77\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)x+2\left(-140-106\sqrt{-1+\sqrt{x}}+62\sqrt{3}\sqrt{1+\sqrt{x}}+21\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)x^{3/2}-73x^2\right)}{12\left(-3-2\sqrt{-1+\sqrt{x}}+2\sqrt{3}\sqrt{1+\sqrt{x}}+\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}-2\sqrt{x}\right)^3 x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

```
[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sq
rt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x])*(8*(7 + 12*Sqrt[-1 + Sqrt[x]]
 - 4*Sqrt[3]*Sqrt[1 + Sqrt[x]] - 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt
[x]]) + 4*(49 + 8*Sqrt[-1 + Sqrt[x]] - 24*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 3*Sqr
t[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 2*(-23 - 144*Sqrt[-1 +
 Sqrt[x]] + 32*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 77*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]])*x + 2*(-140 - 106*Sqrt[-1 + Sqrt[x]] + 62*Sqrt[3]*Sqrt[1 +
 Sqrt[x]] + 21*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(3/2) - 73*x
^2))/(12*(-3 - 2*Sqrt[-1 + Sqrt[x]] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]
*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]] - 2*Sqrt[x])^3*x^(3/2))
```

Maple [A]
time = 0.35, size = 23, normalized size = 0.74

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(x-1)}{3x^{\frac{3}{2}}}$	23
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(x-1)}{3x^{\frac{3}{2}}}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)/x^(3/2)
```

Maxima [A]
time = 0.50, size = 10, normalized size = 0.32

$$\frac{2\,(x-1)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="maxima
")
```

```
[Out] 2/3*(x - 1)^(3/2)/x^(3/2)
```

Fricas [A]
time = 1.97, size = 30, normalized size = 0.97

$$\frac{2\left((x-1)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+x^2\right)}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="fricas
")
```

```
[Out] 2/3*((x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x^2)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(19) = 38.
time = 2.05, size = 48, normalized size = 1.55

$$\frac{16\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+16\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 16/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 16)/((sqrt(sqrt(x) + 1)
 - sqrt(sqrt(x) - 1))^4 + 4)^3
```

Mupad [B]
time = 5.26, size = 31, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{3}-\frac{2\,\sqrt{\sqrt{x}+1}}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(5/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/3 - (2*(x^(1/2) + 1)^(1/2))
/3))/x^(3/2)
```

3.1008 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{15x^{3/2}}$$

```
[Out] 2/5*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(5/2)+4/15*(-1+x^(1/2))^(3/2)*(1
+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {278, 271}

$$\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 278

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}} + \frac{2}{5}\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx$$
$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}} + \frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{15x^{3/2}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 693 vs. 2(63) = 126.
time = 6.72, size = 693, normalized size = 11.00

$$\frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{x}}-\sqrt{x}\right)\left(-384\left(97-168\sqrt{-1+\sqrt{x}}-56\sqrt{3}\sqrt{1+\sqrt{x}}+97\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)-192\left(-499-1112\sqrt{-1+\sqrt{x}}+344\sqrt{3}\sqrt{1+\sqrt{x}}+545\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)\sqrt{x}+32\left(9925+1656\sqrt{-1+\sqrt{x}}-4616\sqrt{3}\sqrt{1+\sqrt{x}}+535\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)x+16\left(2243-20096\sqrt{-1+\sqrt{x}}+2720\sqrt{3}\sqrt{1+\sqrt{x}}+10385\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)x^{3/2}+8\left(-33645-39152\sqrt{-1+\sqrt{x}}+15056\sqrt{3}\sqrt{1+\sqrt{x}}+13135\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)x^2+8\left(-21349-18772\sqrt{-1+\sqrt{x}}+6180\sqrt{3}\sqrt{1+\sqrt{x}}+6379\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)x^{5/2}+4\left(-22053-18788\sqrt{-1+\sqrt{x}}+8788\sqrt{3}\sqrt{1+\sqrt{x}}+5745\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)x^3+2\left(-19920-7252\sqrt{-1+\sqrt{x}}+4188\sqrt{3}\sqrt{1+\sqrt{x}}+715\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)x^{7/2}-2477x^4\right)}{240\left(-3-2\sqrt{-1+\sqrt{x}}+2\sqrt{3}\sqrt{1+\sqrt{x}}+\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}-2\sqrt{x}\right)^5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

```
[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sq
rt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x])*(-384*(97 - 168*Sqrt[-1 + Sqr
t[x]] - 56*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 97*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1
 + Sqrt[x]]) - 192*(-499 - 1112*Sqrt[-1 + Sqrt[x]] + 344*Sqrt[3]*Sqrt[1 + S
qrt[x]] + 545*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 32*(9
925 + 1656*Sqrt[-1 + Sqrt[x]] - 4616*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 535*Sqrt[3
]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x + 16*(2243 - 20096*Sqrt[-1 + Sqrt
[x]] + 2720*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 10385*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]])*x^(3/2) + 8*(-33645 - 39152*Sqrt[-1 + Sqrt[x]] + 15056*Sqr
t[3]*Sqrt[1 + Sqrt[x]] + 13135*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]
)*x^2 + 8*(-21349 - 18772*Sqrt[-1 + Sqrt[x]] + 6180*Sqrt[3]*Sqrt[1 + Sqrt[x
]] + 6379*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(5/2) + 4*(-22053
 - 18788*Sqrt[-1 + Sqrt[x]] + 8788*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 5745*Sqrt[3]
*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^3 + 2*(-19920 - 7252*Sqrt[-1 + Sqr
t[x]] + 4188*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 715*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqr
t[1 + Sqrt[x]])*x^(7/2) - 2477*x^4))/(240*(-3 - 2*Sqrt[-1 + Sqrt[x]] + 2*Sq
rt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]] - 2*
Sqrt[x])^5*x^(5/2))
```

Maple [A]
time = 0.32, size = 28, normalized size = 0.44

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(x-1)(2x+3)}{15x^{\frac{5}{2}}}$	28

default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(x-1)(2x+3)}{15x^{\frac{5}{2}}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(2*x+3)/x^(5/2)
```

Maxima [A]
time = 0.52, size = 21, normalized size = 0.33

$$\frac{4\,(x-1)^{\frac{3}{2}}}{15\,x^{\frac{3}{2}}}+\frac{2\,(x-1)^{\frac{3}{2}}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="maxima
")
```

```
[Out] 4/15*(x - 1)^(3/2)/x^(3/2) + 2/5*(x - 1)^(3/2)/x^(5/2)
```

Fricas [A]
time = 1.35, size = 37, normalized size = 0.59

$$\frac{2\left(2\,x^3+(2\,x^2+x-3)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\right)}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="fricas
")
```

```
[Out] 2/15*(2*x^3 + (2*x^2 + x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1))/
x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(7/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(7/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(39) = 78$.
time = 1.46, size = 90, normalized size = 1.43

$$\frac{128\left(15\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}-20\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+80\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+64\right)}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 128/15*(15*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^12 - 20*(sqrt(sqrt(x) +
1) - sqrt(sqrt(x) - 1))^8 + 80*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 +
64)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^5
```

Mupad [B]
time = 5.07, size = 43, normalized size = 0.68

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{2x\sqrt{\sqrt{x}+1}}{15}-\frac{2\sqrt{\sqrt{x}+1}}{5}+\frac{4x^2\sqrt{\sqrt{x}+1}}{15}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(7/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/15 - (2*(x^(1/2) + 1)^(1/2)
)/5 + (4*x^2*(x^(1/2) + 1)^(1/2))/15))/x^(5/2)
```

3.1009 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{3/2}}$$

```
[Out] 2/7*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(7/2)+8/35*(-1+x^(1/2))^(3/2)*(1
+x^(1/2))^(3/2)/x^(5/2)+16/105*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {278, 271}

$$\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{3/2}}+\frac{8\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{35x^{5/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(7*x^(7/2)) + (8*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(35*x^(5/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
qrt[x])^(3/2))/(105*x^(3/2))
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 278

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}} + \frac{4}{7}\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx$$

$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}} + \frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}} + \frac{8}{35}\int$$

$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}} + \frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}} + \frac{16(-}{}$$

Mathematica [A]
time = 10.04, size = 41, normalized size = 0.44

$$\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}\left(15+12x+8x^2\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(15 + 12*x + 8*x^2))/(105*x^(7/
2))
```

Maple [A]
time = 0.33, size = 33, normalized size = 0.35

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(x-1)\left(8x^{2}+12x+15\right)}{105x^{\frac{7}{2}}}$	33
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(x-1)\left(8x^{2}+12x+15\right)}{105x^{\frac{7}{2}}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(8*x^2+12*x+15)/x^(7/2)
```

Maxima [A]
time = 0.54, size = 31, normalized size = 0.33

$$\frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{3}{2}}} + \frac{8\,(x-1)^{\frac{3}{2}}}{35\,x^{\frac{5}{2}}} + \frac{2\,(x-1)^{\frac{3}{2}}}{7\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="maxima
")
```

```
[Out] 16/105*(x - 1)^(3/2)/x^(3/2) + 8/35*(x - 1)^(3/2)/x^(5/2) + 2/7*(x - 1)^(3/
2)/x^(7/2)
```

Fricas [**A**]
time = 2.15, size = 44, normalized size = 0.47

$$\frac{2\left(8\,x^4+\left(8\,x^3+4\,x^2+3\,x-15\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\right)}{105\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(8*x^4 + (8*x^3 + 4*x^2 + 3*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sq
rt(x) - 1))/x^4
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**A**]
time = 2.25, size = 111, normalized size = 1.18

$$\frac{4096\left(35\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16}-70\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}+168\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+224\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+128\right)}{105\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 4096/105*(35*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^16 - 70*(sqrt(sqrt(x)
+ 1) - sqrt(sqrt(x) - 1))^12 + 168*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^
8 + 224*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 128)/((sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 4)^7
```

Mupad [B]
time = 5.04, size = 55, normalized size = 0.59

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{35}-\frac{2\,\sqrt{\sqrt{x}+1}}{7}+\frac{8\,x^2\,\sqrt{\sqrt{x}+1}}{105}+\frac{16\,x^3\,\sqrt{\sqrt{x}+1}}{105}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(9/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/35 - (2*(x^(1/2) + 1)^(1/2)
)/7 + (8*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/105))/
x^(7/2)
```

3.1010 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx$

Optimal. Leaf size=125

$$\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{5/2}}+\frac{32\left(-1+\sqrt{x}\right.}{}$$

```
[Out] 2/9*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(9/2)+4/21*(-1+x^(1/2))^(3/2)*(1
+x^(1/2))^(3/2)/x^(7/2)+16/105*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(5/2)
+32/315*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {278, 271}

$$\frac{32\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{315x^{3/2}}+\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{5/2}}+\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{21x^{7/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(9*x^(9/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(21*x^(7/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
qrt[x])^(3/2))/(105*x^(5/2)) + (32*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)
)/(315*x^(3/2))
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 278

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}} + \frac{2}{3}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx$$

$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}} + \frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}} + \frac{8}{21}\int$$

$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}} + \frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}} + \frac{16(-}{}$$

$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}} + \frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}} + \frac{16(-}{}$$

Mathematica [A]
time = 10.06, size = 46, normalized size = 0.37

$$\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}(35+30x+24x^2+16x^3)}{315x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(35 + 30*x + 24*x^2 + 16*x^3))/
(315*x^(9/2))
```

Maple [A]
time = 0.32, size = 38, normalized size = 0.30

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(x-1)(16x^3+24x^2+30x+35)}{315x^{\frac{9}{2}}}$	38
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(x-1)(16x^3+24x^2+30x+35)}{315x^{\frac{9}{2}}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
[Out] 2/315*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(16*x^3+24*x^2+30*x+35)/x^
(9/2)
```

Maxima [A]
time = 0.50, size = 41, normalized size = 0.33

$$\frac{32\,(x-1)^{\frac{3}{2}}}{315\,x^{\frac{3}{2}}} + \frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{5}{2}}} + \frac{4\,(x-1)^{\frac{3}{2}}}{21\,x^{\frac{7}{2}}} + \frac{2\,(x-1)^{\frac{3}{2}}}{9\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="maxim
a")
```

```
[Out] 32/315*(x - 1)^(3/2)/x^(3/2) + 16/105*(x - 1)^(3/2)/x^(5/2) + 4/21*(x - 1)^
(3/2)/x^(7/2) + 2/9*(x - 1)^(3/2)/x^(9/2)
```

Fricas [A]
time = 1.70, size = 49, normalized size = 0.39

$$\frac{2\left(16\,x^5+\left(16\,x^4+8\,x^3+6\,x^2+5\,x-35\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\right)}{315\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="frica
s")
```

```
[Out] 2/315*(16*x^5 + (16*x^4 + 8*x^3 + 6*x^2 + 5*x - 35)*sqrt(x)*sqrt(sqrt(x) +
1)*sqrt(sqrt(x) - 1))/x^5
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(11/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8856 deep
```

Giac [A]
time = 1.41, size = 132, normalized size = 1.06

$$\frac{16384\left(315\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{20}-756\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16}+1344\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}+2304\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+2304\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+1024\right)}{315\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="giac"
)
```

```
[Out] 16384/315*(315*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^20 - 756*(sqrt(sqrt(
x) + 1) - sqrt(sqrt(x) - 1))^16 + 1344*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))^12 + 2304*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 2304*(sqrt(sqrt(x
) + 1) - sqrt(sqrt(x) - 1))^4 + 1024)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))^4 + 4)^9
```

Mupad [B]
time = 5.02, size = 67, normalized size = 0.54

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{2x\sqrt{\sqrt{x}+1}}{63}-\frac{2\sqrt{\sqrt{x}+1}}{9}+\frac{4x^2\sqrt{\sqrt{x}+1}}{105}+\frac{16x^3\sqrt{\sqrt{x}+1}}{315}+\frac{32x^4\sqrt{\sqrt{x}+1}}{315}\right)}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(11/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/63 - (2*(x^(1/2) + 1)^(1/2)
)/9 + (4*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/315 +
(32*x^4*(x^(1/2) + 1)^(1/2))/315))/x^(9/2)
```

3.1011 $\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$

Optimal. Leaf size=104

$$\frac{5}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{5}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}+\frac{5}{8}\cosh^{-1}$$

```
[Out] 5/8*arccosh(x^(1/2))+5/12*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/3*
x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+5/8*x^(1/2)*(-1+x^(1/2))^(1/2)
*(1+x^(1/2))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {329, 336, 54}

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}+\frac{5}{12}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{5}{8}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+\frac{5}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 + (5*Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/
2))/3 + (5*ArcCosh[Sqrt[x]])/8
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
```

```
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}+\frac{5}{6}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= \frac{5}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}+\frac{5}{8}$$

$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{5}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{3}$$

$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{5}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{3}$$

$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{5}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{3}$$

Mathematica [A]
time = 1.17, size = 88, normalized size = 0.85

$$\frac{1}{24}\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\,\sqrt{x}\left(15+15\sqrt{x}+10x+10x^{3/2}+8x^2+8x^{5/2}\right)+\frac{5}{4}\tanh^{-1}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(15 + 15*Sqrt[x] + 10*x + 10*x^
(3/2) + 8*x^2 + 8*x^(5/2)))/24 + (5*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x
])]])/4
```

Maple [A]
time = 0.33, size = 65, normalized size = 0.62

method	result
derivativedivides	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(8x^{\frac{5}{2}}\sqrt{x-1}+10x^{\frac{3}{2}}\sqrt{x-1}+15\sqrt{x}\,\sqrt{x-1}+15\ln\left(\sqrt{x}+\sqrt{x-\right.\right.}{24\sqrt{x-1}}$

default	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(8x^{\frac{5}{2}}\sqrt{x-1}+10x^{\frac{3}{2}}\sqrt{x-1}+15\sqrt{x}\,\sqrt{x-1}+15\ln\left(\sqrt{x}+\sqrt{x-1}\right.\right.}{24\sqrt{x-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-1+x^(1/2))^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(8*x^(5/2)*(x-1)^(1/2)+10*x^(3/2)
*(x-1)^(1/2)+15*x^(1/2)*(x-1)^(1/2)+15*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

Maxima [A]
time = 0.29, size = 47, normalized size = 0.45

$$\frac{1}{3}\sqrt{x-1}\,x^{\frac{5}{2}}+\frac{5}{12}\sqrt{x-1}\,x^{\frac{3}{2}}+\frac{5}{8}\sqrt{x-1}\,\sqrt{x}+\frac{5}{8}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*sqrt(x - 1)*x^(5/2) + 5/12*sqrt(x - 1)*x^(3/2) + 5/8*sqrt(x - 1)*sqrt(x
) + 5/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A]
time = 1.47, size = 57, normalized size = 0.55

$$\frac{1}{24}\left(8x^{2}+10x+15\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-\frac{5}{16}\log\left(2\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/24*(8*x^2 + 10*x + 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/16
*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{5}{2}}}{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(5/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Giac [A]
time = 2.05, size = 76, normalized size = 0.73

$$\frac{1}{24}\left(\left(2\left(\left(4\left(\sqrt{x}+1\right)\left(\sqrt{x}-4\right)+45\right)\left(\sqrt{x}+1\right)-55\right)\left(\sqrt{x}+1\right)+85\right)\left(\sqrt{x}+1\right)-33\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{5}{4}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*((4*(sqrt(x) + 1)*(sqrt(x) - 4) + 45)*(sqrt(x) + 1) - 55)*(sqrt(x)
 + 1) + 85)*(sqrt(x) + 1) - 33)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/4*l
og(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B]
time = 27.09, size = 632, normalized size = 6.08

$$\frac{5\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-i}{\sqrt{\sqrt{x}+1}-1}\right)}{2}-\frac{-\frac{175\left(\sqrt{\sqrt{x}-1}-i\right)^3}{6\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{311\left(\sqrt{\sqrt{x}-1}-i\right)^5}{2\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{8361\left(\sqrt{\sqrt{x}-1}-i\right)^7}{2\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{42259\left(\sqrt{\sqrt{x}-1}-i\right)^9}{3\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{25295\left(\sqrt{\sqrt{x}-1}-i\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{25295\left(\sqrt{\sqrt{x}-1}-i\right)^{13}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}}+\frac{42259\left(\sqrt{\sqrt{x}-1}-i\right)^{15}}{3\left(\sqrt{\sqrt{x}+1}-1\right)^{15}}+\frac{8361\left(\sqrt{\sqrt{x}-1}-i\right)^{17}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{17}}+\frac{311\left(\sqrt{\sqrt{x}-1}-i\right)^{19}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{19}}-\frac{175\left(\sqrt{\sqrt{x}-1}-i\right)^{21}}{6\left(\sqrt{\sqrt{x}+1}-1\right)^{21}}+\frac{5\left(\sqrt{\sqrt{x}-1}-i\right)^{23}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{23}}+\frac{5\left(\sqrt{\sqrt{x}-1}-i\right)}{2\left(\sqrt{\sqrt{x}+1}-1\right)}}{1+\frac{66\left(\sqrt{\sqrt{x}-1}-i\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{220\left(\sqrt{\sqrt{x}-1}-i\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{495\left(\sqrt{\sqrt{x}-1}-i\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{792\left(\sqrt{\sqrt{x}-1}-i\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\sqrt{x}-1}-i\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{792\left(\sqrt{\sqrt{x}-1}-i\right)^{14}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}}+\frac{495\left(\sqrt{\sqrt{x}-1}-i\right)^{16}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{16}}-\frac{220\left(\sqrt{\sqrt{x}-1}-i\right)^{18}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{18}}+\frac{66\left(\sqrt{\sqrt{x}-1}-i\right)^{20}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{20}}-\frac{12\left(\sqrt{\sqrt{x}-1}-i\right)^{22}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{22}}+\frac{\left(\sqrt{\sqrt{x}-1}-i\right)^{24}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{24}}-\frac{12\left(\sqrt{\sqrt{x}-1}-i\right)^2}{\left(\sqrt{\sqrt{x}+1}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] (5*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)))/2 - ((311*(
(x^(1/2) - 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) - (175*((x^(1/
2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (8361*((x^(1/2) -
1)^(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (42259*((x^(1/2) - 1)^(
1/2) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25295*((x^(1/2) - 1)^(1/2)
 - 1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25295*((x^(1/2) - 1)^(1/2) - 1i)
^13)/((x^(1/2) + 1)^(1/2) - 1)^13 + (42259*((x^(1/2) - 1)^(1/2) - 1i)^15)/(
3*((x^(1/2) + 1)^(1/2) - 1)^15) + (8361*((x^(1/2) - 1)^(1/2) - 1i)^17)/(2*(
(x^(1/2) + 1)^(1/2) - 1)^17) + (311*((x^(1/2) - 1)^(1/2) - 1i)^19)/(2*((x^(
1/2) + 1)^(1/2) - 1)^19) - (175*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6*((x^(1/2)
 + 1)^(1/2) - 1)^21) + (5*((x^(1/2) - 1)^(1/2) - 1i)^23)/(2*((x^(1/2) + 1)^
(1/2) - 1)^23) + (5*((x^(1/2) - 1)^(1/2) - 1i))/(2*((x^(1/2) + 1)^(1/2) - 1
)))/((66*((x^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (12*((
x^(1/2) - 1)^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (220*((x^(1/2) -
1)^(1/2) - 1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/2) - 1)^(1/2) -
 1i)^8)/((x^(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(1/2) - 1i)^10)/(
(x^(1/2) + 1)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2)
 + 1)^(1/2) - 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1
/2) - 1)^14 + (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)
^16 - (220*((x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (6
6*((x^(1/2) - 1)^(1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (12*((x^(1/
2) - 1)^(1/2) - 1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + ((x^(1/2) - 1)^(1/2)
 - 1i)^24/((x^(1/2) + 1)^(1/2) - 1)^24 + 1)
```

3.1012 $\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

Optimal. Leaf size=73

$$\frac{3}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 3/4*arccosh(x^(1/2))+1/2*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+3/4*x
^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {329, 336, 54}

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{3}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+\frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (3*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]]*x^(3/2))/2 + (3*ArcCosh[Sqrt[x]])/4
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
```

```
0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= \frac{3}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{3}{8}\int$$

$$= \frac{3}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{3}{4}\mathrm{Su}$$

$$= \frac{3}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{3}{4}\mathrm{co}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 406 vs. 2(73) = 146.
time = 1.46, size = 406, normalized size = 5.56

$$\frac{-4\sqrt{1+\sqrt{x}}\left(-29568+50496\sqrt{x}+98112x+21840x^{3/2}-2264x^2-3368x^{5/2}-4752x^3-1136x^{7/2}\right)-4\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(51216+120600\sqrt{x}+56904x-4016x^{3/2}-6344x^2-6467x^{5/2}-3120x^3-194x^{7/2}\right)+\sqrt{3}\left(-4\sqrt{-1+\sqrt{x}}\left(-29568-84416\sqrt{x}-64000x-7152x^{3/2}+5624x^2+5144x^{5/2}+3408x^3+656x^{7/2}\right)-4\left(17072-20632\sqrt{x}-73312x-36244x^{3/2}-510x^2+2452x^{5/2}+3640x^3+1800x^{7/2}+112x^4\right)\right)}{-12416+13312\sqrt{x}+49408x+24960x^{3/2}+1552x^2+\sqrt{3}\sqrt{1+\sqrt{x}}\left(7168-11264\sqrt{x}-22016x-5248x^{3/2}\right)+\sqrt{-1+\sqrt{x}}\left(21504+60416\sqrt{x}+47104x+9088x^{3/2}+\sqrt{3}\sqrt{1+\sqrt{x}}\left(-12416-28672\sqrt{x}-14400x-896x^{3/2}\right)\right)}-3\tanh^{-1}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (-4*Sqrt[1 + Sqrt[x]]*(-29568 + 50496*Sqrt[x] + 98112*x + 21840*x^(3/2) - 2
264*x^2 - 3368*x^(5/2) - 4752*x^3 - 1136*x^(7/2)) - 4*Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]]*(51216 + 120600*Sqrt[x] + 56904*x - 4016*x^(3/2) - 6344*x^2
 - 6467*x^(5/2) - 3120*x^3 - 194*x^(7/2)) + Sqrt[3]*(-4*Sqrt[-1 + Sqrt[x]]*
(-29568 - 84416*Sqrt[x] - 64000*x - 7152*x^(3/2) + 5624*x^2 + 5144*x^(5/2)
+ 3408*x^3 + 656*x^(7/2)) - 4*(17072 - 20632*Sqrt[x] - 73312*x - 36244*x^(3
/2) - 510*x^2 + 2452*x^(5/2) + 3640*x^3 + 1800*x^(7/2) + 112*x^4)))/(-12416
 + 13312*Sqrt[x] + 49408*x + 24960*x^(3/2) + 1552*x^2 + Sqrt[3]*Sqrt[1 + Sq
rt[x]]*(7168 - 11264*Sqrt[x] - 22016*x - 5248*x^(3/2)) + Sqrt[-1 + Sqrt[x]]
*(21504 + 60416*Sqrt[x] + 47104*x + 9088*x^(3/2) + Sqrt[3]*Sqrt[1 + Sqrt[x]
]*(-12416 - 28672*Sqrt[x] - 14400*x - 896*x^(3/2)))) - 3*ArcTanh[(-1 + Sqrt
[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Maple [A]
time = 0.32, size = 55, normalized size = 0.75

method	result	size

derivativedivides	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(2x^{\frac{3}{2}}\sqrt{x-1}+3\sqrt{x}\,\sqrt{x-1}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{4\sqrt{x-1}}$	55
default	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(2x^{\frac{3}{2}}\sqrt{x-1}+3\sqrt{x}\,\sqrt{x-1}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{4\sqrt{x-1}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-1+x^(1/2))^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(2*x^(3/2)*(x-1)^(1/2)+3*x^(1/2)*(
x-1)^(1/2)+3*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.51

$$\frac{1}{2}\sqrt{x-1}\,x^{\frac{3}{2}}+\frac{3}{4}\sqrt{x-1}\,\sqrt{x}+\frac{3}{4}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*sqrt(x - 1)*x^(3/2) + 3/4*sqrt(x - 1)*sqrt(x) + 3/4*log(2*sqrt(x - 1) +
 2*sqrt(x))
```

Fricas [A]
time = 1.10, size = 52, normalized size = 0.71

$$\frac{1}{4}(2x+3)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-\frac{3}{8}\log\left(2\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/4*(2*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 3/8*log(2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(3/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Giac [A]
time = 1.77, size = 59, normalized size = 0.81

$$\frac{1}{4}\left(\left(2\left(\sqrt{x}+1\right)\left(\sqrt{x}-2\right)+9\right)\left(\sqrt{x}+1\right)-5\right)\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-\frac{3}{2}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*((2*(sqrt(x) + 1)*(sqrt(x) - 2) + 9)*(sqrt(x) + 1) - 5)*sqrt(sqrt(x) +
1)*sqrt(sqrt(x) - 1) - 3/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B]
time = 18.76, size = 429, normalized size = 5.88

$$3\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)+\frac{\frac{23\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{333\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{671\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{671\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{333\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{23\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{13}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}}-\frac{3\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{15}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{15}}-\frac{3\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)}{\sqrt{\sqrt{x}+1}-1}}{1+\frac{28\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{56\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{70\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{56\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{28\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{8\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}}+\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{16}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{16}}-\frac{8\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\sqrt{x}+1}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] 3*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) + ((23*((x^(1
/2) - 1)^(1/2) - 1i)^3)/((x^(1/2) + 1)^(1/2) - 1)^3 + (333*((x^(1/2) - 1)^(
1/2) - 1i)^5)/((x^(1/2) + 1)^(1/2) - 1)^5 + (671*((x^(1/2) - 1)^(1/2) - 1i)
^7)/((x^(1/2) + 1)^(1/2) - 1)^7 + (671*((x^(1/2) - 1)^(1/2) - 1i)^9)/((x^(1
/2) + 1)^(1/2) - 1)^9 + (333*((x^(1/2) - 1)^(1/2) - 1i)^11)/((x^(1/2) + 1)^
(1/2) - 1)^11 + (23*((x^(1/2) - 1)^(1/2) - 1i)^13)/((x^(1/2) + 1)^(1/2) - 1
)^13 - (3*((x^(1/2) - 1)^(1/2) - 1i)^15)/((x^(1/2) + 1)^(1/2) - 1)^15 - (3*
((x^(1/2) - 1)^(1/2) - 1i))/((x^(1/2) + 1)^(1/2) - 1))/((28*((x^(1/2) - 1)^
(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (8*((x^(1/2) - 1)^(1/2) - 1i)^
2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (56*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2
) + 1)^(1/2) - 1)^6 + (70*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2
) - 1)^8 - (56*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10
+ (28*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (8*((x^
(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + ((x^(1/2) - 1)^(1
/2) - 1i)^16/((x^(1/2) + 1)^(1/2) - 1)^16 + 1)
```

3.1013 $\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$

Optimal. Leaf size=35

$$\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] arccosh(x^(1/2))+x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {329, 336, 54}

$$\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
```

```
, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx &= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx \\
&= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx, x, \sqrt{x}\right) \\
&= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 265 vs. 2(35) = 70.
time = 1.16, size = 265, normalized size = 7.57

$$
\frac{4\left(4\sqrt{1+\sqrt{x}}\left(-12-24\sqrt{x}+x+5x^{3/2}\right)+\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\left(-84-10\sqrt{x}+28x+7x^{3/2}\right)+\sqrt{3}\left(28+70\sqrt{x}+18x-14x^{3/2}-4x^2-4\sqrt{-1+\sqrt{x}}\left(-12-8\sqrt{x}+5x+3x^{3/2}\right)\right)\right)}{56-16\sqrt{3}\,\sqrt{1+\sqrt{x}}\left(2+3\sqrt{x}\right)+\sqrt{-1+\sqrt{x}}\left(96-8\sqrt{3}\,\sqrt{1+\sqrt{x}}\left(7+2\sqrt{x}\right)+80\sqrt{x}\right)+112\sqrt{x}+28x}-4\tanh^{-1}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (4*(4*Sqrt[1 + Sqrt[x]]*(-12 - 24*Sqrt[x] + x + 5*x^(3/2)) + Sqrt[-1 + Sqrt
[x]]*Sqrt[1 + Sqrt[x]]*(-84 - 10*Sqrt[x] + 28*x + 7*x^(3/2)) + Sqrt[3]*(28
+ 70*Sqrt[x] + 18*x - 14*x^(3/2) - 4*x^2 - 4*Sqrt[-1 + Sqrt[x]]*(-12 - 8*Sq
rt[x] + 5*x + 3*x^(3/2)))))/(56 - 16*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(2 + 3*Sqrt[
x]) + Sqrt[-1 + Sqrt[x]]*(96 - 8*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(7 + 2*Sqrt[x])
+ 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) - 4*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(
Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Maple [**A**]
time = 0.41, size = 41, normalized size = 1.17

method	result	size
derivativedivides	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(\sqrt{x}\,\sqrt{x-1}+\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{\sqrt{x-1}}$	41
default	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,\left(\sqrt{x}\,\sqrt{x-1}+\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{\sqrt{x-1}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-1+x^(1/2))^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] (-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(x^(1/2)*(x-1)^(1/2)+ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)

Maxima [A]
time = 0.29, size = 24, normalized size = 0.69

$$\sqrt{x-1}\,\sqrt{x}+\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] sqrt(x - 1)*sqrt(x) + log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A]
time = 0.59, size = 46, normalized size = 1.31

$$\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-\frac{1}{2}\,\log\left(2\,\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 1/2*log(2*sqrt(x)*sqrt(sqrt(x
) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.91, size = 39, normalized size = 1.11

$$\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2\,\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*log(sqrt(sqrt(x) + 1) - sqr
t(sqrt(x) - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
[Out] int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)), x)
```

3.1014 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$2\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*arccosh(x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {336, 54}

$$2\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

```
[Out] 2*ArcCosh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,x,\sqrt{x}\right)\\ &= 2\cosh^{-1}\left(\sqrt{x}\right) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. $2(8) = 16$.
time = 0.82, size = 38, normalized size = 4.75

$$-8\tanh^{-1}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

```
[Out] -8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(6) = 12$.
time = 0.39, size = 40, normalized size = 5.00

method	result	size
derivativedivides	$\frac{2\sqrt{\left(\sqrt{x}+1\right)\left(-1+\sqrt{x}\right)}\,\ln\left(\sqrt{x}+\sqrt{x-1}\right)}{\sqrt{\sqrt{x}+1}\,\sqrt{-1+\sqrt{x}}}$	40
default	$\frac{2\sqrt{\left(\sqrt{x}+1\right)\left(-1+\sqrt{x}\right)}\,\ln\left(\sqrt{x}+\sqrt{x-1}\right)}{\sqrt{\sqrt{x}+1}\,\sqrt{-1+\sqrt{x}}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((x^(1/2)+1)*(-1+x^(1/2)))^(1/2)/(x^(1/2)+1)^(1/2)/(-1+x^(1/2))^(1/2)*ln(
x^(1/2)+(x-1)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 0.29, size = 16, normalized size = 2.00

$$2\,\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.
time = 1.99, size = 27, normalized size = 3.38

$$-\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] -log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(6) = 12.
time = 1.61, size = 20, normalized size = 2.50

$$-4\,\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] -4*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B]
time = 5.29, size = 6, normalized size = 0.75

$$2\,\mathrm{acosh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] 2*acosh(x^(1/2))
```

3.1015 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

```
[Out] 2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {271}

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(29) = 58.
time = 0.88, size = 146, normalized size = 5.03

$$\frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\,\sqrt{1+\sqrt{x}}-\sqrt{x}\right)}{\left(3+2\sqrt{-1+\sqrt{x}}-2\sqrt{3}\,\sqrt{1+\sqrt{x}}-\sqrt{3}\,\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}+2\sqrt{x}\right)\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

```
[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sq
rt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x]))/((3 + 2*Sqrt[-1 + Sqrt[x]] -
 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]
 + 2*Sqrt[x])*Sqrt[x])
```

Maple [A]
time = 0.33, size = 20, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$	20
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.53, size = 10, normalized size = 0.34

$$\frac{2\sqrt{x-1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2*sqrt(x - 1)/sqrt(x)
```

Fricas [A]
time = 2.10, size = 25, normalized size = 0.86

$$\frac{2\left(\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2*(sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
[Out] Integral(1/(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Giac [A]
time = 2.74, size = 25, normalized size = 0.86

$$\frac{16}{\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
[Out] 16/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)
```

Mupad [B]
time = 5.56, size = 19, normalized size = 0.66

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
[Out] (2*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2)
```

3.1016 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\, x^{5/2}}\, dx$

Optimal. Leaf size=63

$$\frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3x^{3/2}}+\frac{4\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3\sqrt{x}}$$

```
[Out] 2/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2)+4/3*(-1+x^(1/2))^(1/2)*(1+
x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {278, 271}

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x])
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 278

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3x^{3/2}} + \frac{2}{3}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx$$

$$= \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3x^{3/2}} + \frac{4\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3\sqrt{x}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 407 vs. 2(63) = 126.

time = 1.82, size = 407, normalized size = 6.46

$$\frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{x}}-\sqrt{x}\right)\left(8\left(-7-12\sqrt{-1+\sqrt{x}}+4\sqrt{3}\sqrt{1+\sqrt{x}}+7\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)-4\left(49+8\sqrt{-1+\sqrt{x}}-24\sqrt{3}\sqrt{1+\sqrt{x}}+3\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)\sqrt{x}+2\left(-61+16\sqrt{3}\sqrt{1+\sqrt{x}}+7\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)x+\left(-56-28\sqrt{-1+\sqrt{x}}+20\sqrt{3}\sqrt{1+\sqrt{x}}+6\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\right)x^{3/2}-11x^2\right)}{12\left(-3-2\sqrt{-1+\sqrt{x}}+2\sqrt{3}\sqrt{1+\sqrt{x}}+\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}-2\sqrt{x}\right)^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

```
[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sq
rt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x])*(8*(-7 - 12*Sqrt[-1 + Sqrt[x]
] + 4*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqr
t[x]]) - 4*(49 + 8*Sqrt[-1 + Sqrt[x]] - 24*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 3*Sq
rt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 2*(-61 + 16*Sqrt[3]*S
qrt[1 + Sqrt[x]] + 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x + (-56
 - 28*Sqrt[-1 + Sqrt[x]] + 20*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 6*Sqrt[3]*Sqrt[-1
 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(3/2) - 11*x^2))/(12*(-3 - 2*Sqrt[-1 + Sqr
t[x]] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + S
qrt[x]] - 2*Sqrt[x])^3*x^(3/2))
```

Maple [**A**]

time = 0.33, size = 25, normalized size = 0.40

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(2x+1)}{3x^{\frac{3}{2}}}$	25
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(2x+1)}{3x^{\frac{3}{2}}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(2*x+1)/x^(3/2)
```

Maxima [A]
time = 0.51, size = 21, normalized size = 0.33

$$\frac{4\sqrt{x-1}}{3\sqrt{x}}+\frac{2\sqrt{x-1}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 4/3*sqrt(x - 1)/sqrt(x) + 2/3*sqrt(x - 1)/x^(3/2)
```

Fricas [A]
time = 1.81, size = 34, normalized size = 0.54

$$\frac{2\left((2x+1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2x^{2}\right)}{3x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/3*((2*x + 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x^2)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Giac [A]
time = 1.67, size = 48, normalized size = 0.76

$$\frac{128\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 128/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)/((sqrt(sqrt(x) + 1)
 - sqrt(sqrt(x) - 1))^4 + 4)^3
```

Mupad [B]
time = 5.50, size = 33, normalized size = 0.52

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{4x}{3}+\frac{2\sqrt{x}}{3}+\frac{4x^{3/2}}{3}+\frac{2}{3}\right)}{x^{3/2}\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((4*x)/3 + (2*x^(1/2))/3 + (4*x^(3/2))/3 + 2/3))/(x^(3
/2)*(x^(1/2) + 1)^(1/2))
```

3.1017 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{7/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{8\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15x^{3/2}}+\frac{16\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15\sqrt{x}}$$

```
[Out] 2/5*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2)+8/15*(-1+x^(1/2))^(1/2)*(1
+x^(1/2))^(1/2)/x^(3/2)+16/15*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {278, 271}

$$\frac{8\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15x^{3/2}}+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}}+\frac{16\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(5*x^(5/2)) + (8*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(15*x^(3/2)) + (16*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]
])/(15*Sqrt[x])
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 278

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{5x^{5/2}} + \frac{4}{5}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx$$

$$= \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{5x^{5/2}} + \frac{8\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{15x^{3/2}} + \frac{8}{15}\int -$$

$$= \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{5x^{5/2}} + \frac{8\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{15x^{3/2}} + \frac{16\sqrt{-1}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 693 vs. 2(94) = 188.
time = 6.93, size = 693, normalized size = 7.37

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

```
[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sq
rt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x])*(384*(97 - 168*Sqrt[-1 + Sqrt
[x]] - 56*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 97*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1
+ Sqrt[x]]) + 192*(-499 - 1112*Sqrt[-1 + Sqrt[x]] + 344*Sqrt[3]*Sqrt[1 + Sq
rt[x]] + 545*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 32*(-7
985 - 5016*Sqrt[-1 + Sqrt[x]] + 3496*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 1405*Sqrt[
3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x + 16*(-12223 - 2144*Sqrt[-1 + Sq
rt[x]] + 4160*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 515*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]])*x^(3/2) + 8*(-14415 - 3248*Sqrt[-1 + Sqrt[x]] + 5264*Sqrt[
3]*Sqrt[1 + Sqrt[x]] + 1405*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x
^2 + 8*(-6511 - 2908*Sqrt[-1 + Sqrt[x]] + 1740*Sqrt[3]*Sqrt[1 + Sqrt[x]] +
1141*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(5/2) + 4*(-5007 - 369
2*Sqrt[-1 + Sqrt[x]] + 1852*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 1155*Sqrt[3]*Sqrt[-
1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^3 + 2*(-4080 - 1468*Sqrt[-1 + Sqrt[x]] +
852*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 145*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqr
t[x]])*x^(7/2) - 503*x^4))/(240*(-3 - 2*Sqrt[-1 + Sqrt[x]] + 2*Sqrt[3]*Sqrt
[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]] - 2*Sqrt[x])^5
*x^(5/2))
```

Maple [A]
time = 0.40, size = 30, normalized size = 0.32

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(8x^2+4x+3)}{15x^{\frac{5}{2}}}$	30
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{\sqrt{x}+1}\,(8x^2+4x+3)}{15x^{\frac{5}{2}}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(-1+x^(1/2))^(1/2)*(x^(1/2)+1)^(1/2)*(8*x^2+4*x+3)/x^(5/2)
```

Maxima [A]
time = 0.54, size = 31, normalized size = 0.33

$$\frac{16\sqrt{x-1}}{15\sqrt{x}}+\frac{8\sqrt{x-1}}{15\,x^{\frac{3}{2}}}+\frac{2\sqrt{x-1}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 16/15*sqrt(x - 1)/sqrt(x) + 8/15*sqrt(x - 1)/x^(3/2) + 2/5*sqrt(x - 1)/x^(5
/2)
```

Fricas [A]
time = 2.47, size = 39, normalized size = 0.41

$$\frac{2\left(8\,x^3+(8\,x^2+4\,x+3)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\right)}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/15*(8*x^3 + (8*x^2 + 4*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)
)/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{7}{2}}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Giac [A]
time = 1.06, size = 69, normalized size = 0.73

$$\frac{4096\left(5\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+10\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+8\right)}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 4096/15*(5*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 10*(sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 8)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 +
4)^5
```

Mupad [B]
time = 5.66, size = 43, normalized size = 0.46

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{8\,x}{15}+\frac{16\,x^2}{15}+\frac{2\,\sqrt{x}}{5}+\frac{8\,x^{3/2}}{15}+\frac{16\,x^{5/2}}{15}+\frac{2}{5}\right)}{x^{5/2}\,\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((8*x)/15 + (16*x^2)/15 + (2*x^(1/2))/5 + (8*x^(3/2))/
15 + (16*x^(5/2))/15 + 2/5))/(x^(5/2)*(x^(1/2) + 1)^(1/2))
```

3.1018 $\int x^2(-a+bx^n)^p (a+bx^n)^p \, dx$

Optimal. Leaf size=78

$$\frac{1}{3}x^3(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] 1/3*x^3*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 3/2/n],[1+3/2/n],b^2*x^(2*n
)/a^2)/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {373, 372, 371}

$$\frac{1}{3}x^3(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^3*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[3/(2*n), -p, 1 + 3/(2*n
), (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 373

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p]), Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\,dx = \left((-a+bx^n)^p\,(a+bx^n)^p\,\left(-a^2+b^2x^{2n}\right)^{-p}\right)\int x^2\left(-a^2+b^2x^{2n}\right)^p\,dx$$
$$= \left((-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int x^2\left(1-\frac{b^2x^{2n}}{a^2}\right)^p\,dx$$
$$= \frac{1}{3}x^3(-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Mathematica [A]
time = 0.05, size = 80, normalized size = 1.03

$$\frac{1}{3}x^3(-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^3*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{3/(2*n), -p}, {1 + 3/(
2*n)}, (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int x^2(b\,x^n-a)^p\,(a+b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^n-a)^p*(a+b*x^n)^p,x)
```

```
[Out] int(x^2*(b*x^n-a)^p*(a+b*x^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a+b*x**n)**p*(a+b*x**n)**p,x)
[Out] Integral(x**2*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(a+b\,x^n\right)}^p\,{\left(b\,x^n-a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^n)^p*(b*x^n - a)^p,x)
[Out] int(x^2*(a + b*x^n)^p*(b*x^n - a)^p, x)
```

3.1019 $\int x(-a+bx^n)^p (a+bx^n)^p \, dx$

Optimal. Leaf size=70

$$\frac{1}{2}x^2(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] 1/2*x^2*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],b^2*x^(2*n)/a^
2)/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {373, 372, 371}

$$\frac{1}{2}x^2(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^2*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1),
 (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 373

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p]), Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int x(-a+bx^n)^p (a+bx^n)^p \, dx &= \left((-a+bx^n)^p (a+bx^n)^p \left(-a^2+b^2x^{2n}\right)^{-p}\right) \int x\left(-a^2+b^2x^{2n}\right)^p \, dx \\ &= \left((-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int x\left(1-\frac{b^2x^{2n}}{a^2}\right)^p \, dx \\ &= \frac{1}{2}x^2(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 1.03

$$\frac{1}{2}x^2(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
[Out] (x^2*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{n^(-1), -p}, {1 + n^(-
1)}, (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int x(b\,x^n-a)^p\,(a+b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n-a)^p*(a+b*x^n)^p,x)
[Out] int(x*(b*x^n-a)^p*(a+b*x^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(-a+bx^n)^p (a+bx^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x**n)**p*(a+b*x**n)**p,x)
[Out] Integral(x*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+b\,x^n\right)}^p\,{\left(b\,x^n-a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n)^p*(b*x^n - a)^p,x)
[Out] int(x*(a + b*x^n)^p*(b*x^n - a)^p, x)
```

3.1020 $\int(-a+bx^n)^p(a+bx^n)^p\,dx$

Optimal. Leaf size=73

$$x(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] x*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)
/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {259, 252, 251}

$$x(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))
/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \left((-a+bx^n)^p (a+bx^n)^p \left(-a^2+b^2x^{2n}\right)^{-p}\right) \int \left(-a^2+b^2x^{2n}\right)^p \, dx$$
$$= \left((-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \, dx$$
$$= x(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 1.00

$$x(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x^n)^p*(a + b*x^n)^p,x]
[Out] (x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n),
 (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (b\,x^n-a)^p \left(a+b\,x^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n-a)^p*(a+b*x^n)^p,x)
[Out] int((b*x^n-a)^p*(a+b*x^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-a + bx^n)^p (a + bx^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p,x)
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,x^n)^p\,(b\,x^n - a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(b*x^n - a)^p,x)
[Out] int((a + b*x^n)^p*(b*x^n - a)^p, x)
```

3.1021 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x}\,dx$

Optimal. Leaf size=72

$$-\frac{(-a+bx^n)^p\,(a+bx^n)^p\,(a^2-b^2x^{2n})\;{}_2F_1\left(1,1+p;2+p;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}$$

[Out] -1/2*(-a+b*x^n)^p*(a+b*x^n)^p*(a^2-b^2*x^(2*n))*hypergeom([1, 1+p],[2+p],1-b^2*x^(2*n)/a^2)/a^2/n/(1+p)

Rubi [A]
time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {273, 127, 272, 67}

$$-\frac{(a^2-b^2x^{2n})\,(bx^n-a)^p\,(a+bx^n)^p\;{}_2F_1\left(1,p+1;p+2;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]

[Out] -1/2*((-a + b*x^n)^p*(a + b*x^n)^p*(a^2 - b^2*x^(2*n))*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(a^2*n*(1 + p))

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*
d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 273

```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(
p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a1 + b1*
x)^p*(a2 + b2*x)^p, x], x, x^n], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x]
&& EqQ[a2*b1 + a1*b2, 0] && IntegerQ[Simplify[(m + 1)/(2*n)]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+bx)^p(a+bx)^p}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{\left((-a+bx^n)^p\,(a+bx^n)^p\,(-a^2+b^2x^{2n})^{-p}\right)\text{Subst}\left(\int \frac{(-a^2+b^2x^2)^p}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{\left((-a+bx^n)^p\,(a+bx^n)^p\,(-a^2+b^2x^{2n})^{-p}\right)\text{Subst}\left(\int \frac{(-a^2+b^2x)^p}{x}\,dx,x,x^{2n}\right)}{2n}\\
&= -\frac{(-a+bx^n)^p\,(a+bx^n)^p\,(a^2-b^2x^{2n})\;{}_2F_1\left(1,1+p;2+p;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 73, normalized size = 1.01

$$\frac{(-a+bx^n)^p\,(a+bx^n)^p\,(-a^2+b^2x^{2n})\;{}_2F_1\left(1,1+p;2+p;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]
[Out] ((-a + b*x^n)^p*(a + b*x^n)^p*(-a^2 + b^2*x^(2*n))*Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(2*a^2*n*(1 + p))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n-a)^p\,(a+b\,x^n)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n-a)^p*(a+b*x^n)^p/x,x)
[Out] int((b*x^n-a)^p*(a+b*x^n)^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p/x,x)
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^p\,(b\,x^n-a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^n)^p*(b*x^n - a)^p)/x,x)
[Out] int(((a + b*x^n)^p*(b*x^n - a)^p)/x, x)
```

3.1022 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x^2}\,dx$

Optimal. Leaf size=76

$$-\frac{(-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

```
[Out] -(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, -1/2/n],[1-1/2/n],b^2*x^(2*n)/a^2)
/x/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {373, 372, 371}

$$-\frac{(bx^n-a)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```

```
[Out] -(((-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[-1/2*1/n, -p, 1 - 1/(2*n)
, (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 373

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p]), Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} dx = \left((-a+bx^n)^p (a+bx^n)^p \left(-a^2+b^2x^{2n}\right)^{-p}\right) \int \frac{\left(-a^2+b^2x^{2n}\right)^p}{x^2} dx$$
$$= \left((-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int \frac{\left(1-\frac{b^2x^{2n}}{a^2}\right)^p}{x^2} dx$$
$$= -\frac{(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 1.03

$$-\frac{(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```

```
[Out] -(((-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{-1/2*1/n, -p}, {1 - 1/(2
*n)}, (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n-a)^p\,(a+b\,x^n)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n-a)^p*(a+b*x^n)^p/x^2,x)
```

```
[Out] int((b*x^n-a)^p*(a+b*x^n)^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a + bx^n)^p (a + bx^n)^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p/x**2,x)
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,x^n)^p\,(b\,x^n - a)^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^n)^p*(b*x^n - a)^p)/x^2,x)
[Out] int(((a + b*x^n)^p*(b*x^n - a)^p)/x^2, x)
```

3.1023 $\int \frac{1+x^6}{x(1-x^6)}\,dx$

Optimal. Leaf size=15

$$\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

```
[Out] ln(x)-1/3*ln(-x^6+1)
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {457, 78}

$$\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(x*(1 - x^6)),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{1+x}{(1-x)x}\,dx,x,x^6\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)\,dx,x,x^6\right)\\ &= \log(x)-\frac{1}{3}\log\left(1-x^6\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\log(x) - \frac{1}{3}\log\left(1 - x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(x*(1 - x^6)),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(13) = 26$.
time = 0.32, size = 36, normalized size = 2.40

method	result	size
risch	$\ln(x) - \frac{\ln(x^6-1)}{3}$	12
meijerg	$-\frac{\ln(-x^6+1)}{3} + \ln(x) + \frac{i\pi}{6}$	18
default	$-\frac{\ln(x+1)}{3} - \frac{\ln(x^2+x+1)}{3} - \frac{\ln(x-1)}{3} + \ln(x) - \frac{\ln(x^2-x+1)}{3}$	36
norman	$-\frac{\ln(x+1)}{3} - \frac{\ln(x^2+x+1)}{3} - \frac{\ln(x-1)}{3} + \ln(x) - \frac{\ln(x^2-x+1)}{3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/x/(-x^6+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(x+1)-1/3*ln(x^2+x+1)-1/3*ln(x-1)+ln(x)-1/3*ln(x^2-x+1)
```

Maxima [A]
time = 0.30, size = 15, normalized size = 1.00

$$-\frac{1}{3}\log\left(x^6 - 1\right) + \frac{1}{6}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -1/3*log(x^6 - 1) + 1/6*log(x^6)
```

Fricas [A]
time = 2.63, size = 11, normalized size = 0.73

$$-\frac{1}{3}\log\left(x^6 - 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="fricas")
[Out] -1/3*log(x^6 - 1) + log(x)
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.67

$$\log(x) - \frac{\log(x^6 - 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/x/(-x**6+1),x)
[Out] log(x) - log(x**6 - 1)/3
```

Giac [A]
time = 0.76, size = 16, normalized size = 1.07

$$\frac{1}{6}\log(x^6) - \frac{1}{3}\log(|x^6 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="giac")
[Out] 1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

Mupad [B]
time = 0.09, size = 11, normalized size = 0.73

$$\ln(x) - \frac{\ln(x^6 - 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6 + 1)/(x*(x^6 - 1)),x)
[Out] log(x) - log(x^6 - 1)/3
```

3.1024 $\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x$

Optimal. Leaf size=22

$$\frac{(ex)^{1+m}(a+bx^n)^{1+p}}{e}$$

```
[Out] (e*x)^(1+m)*(a+b*x^n)^(1+p)/e
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {460}

$$\frac{(ex)^{m+1}(a+bx^n)^{p+1}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

```
[Out] ((e*x)^(1 + m)*(a + b*x^n)^(1 + p))/e
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n)\ dx = \frac{(ex)^{1+m}(a+bx^n)^{1+p}}{e}$$

Mathematica [A]
time = 0.16, size = 18, normalized size = 0.82

$$x(ex)^m(a+bx^n)^{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

```
[Out] x*(e*x)^m*(a + b*x^n)^(1 + p)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (ex)^m (a+b\,x^n)^p (a(1+m)+b(np+m+n+1)\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x)
```

```
[Out] int((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x)
```

Maxima [A]
time = 0.35, size = 35, normalized size = 1.59

$$\left(bxe^{(m\log(x)+n\log(x)+m)}+axe^{(m\log(x)+m)}\right)(bx^n+a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="max
ima")
```

```
[Out] (b*x*e^(m*log(x) + n*log(x) + m) + a*x*e^(m*log(x) + m))*(b*x^n + a)^p
```

Fricas [A]
time = 1.69, size = 34, normalized size = 1.55

$$\left(bxx^ne^{(m\log(x)+m)}+axe^{(m\log(x)+m)}\right)(bx^n+a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="fri
cas")
```

```
[Out] (b*x*x^n*e^(m*log(x) + m) + a*x*e^(m*log(x) + m))*(b*x^n + a)^p
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.
time = 2.26, size = 36, normalized size = 1.64

$$ax(ex)^m\,(a+bx^n)^p+bxx^n(ex)^m\,(a+bx^n)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*x**n)**p*(a*(1+m)+b*(n*p+m+n+1)*x**n),x)
```

```
[Out] a*x*(e*x)**m*(a + b*x**n)**p + b*x*x**n*(e*x)**m*(a + b*x**n)**p
```

Giac [A]
time = 0.94, size = 38, normalized size = 1.73

$$(bx^n+a)^pbxx^mx^ne^m+(bx^n+a)^paxx^me^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="gia
c")
```

```
[Out] (b*x^n + a)^p*b*x*x^m*x^n*e^m + (b*x^n + a)^p*a*x*x^m*e^m
```

Mupad [B]
time = 4.94, size = 31, normalized size = 1.41

$$\left(a\,x\,{\left(e\,x\right)}^m+b\,x^{n+1}\,{\left(e\,x\right)}^m\right)\,{\left(a+b\,x^n\right)}^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a*(m + 1) + b*x^n*(m + n + n*p + 1))*(a + b*x^n)^p,x)
```

```
[Out] (a*x*(e*x)^m + b*x^(n + 1)*(e*x)^m)*(a + b*x^n)^p
```

3.1025 $\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=114

$$\frac{b(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)e(1+m)}-\frac{d(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)e(1+m)}$$

```
[Out] b*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(-a*d+b*c)/e/(
1+m)-d*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/(-a*d+b*c
)/e/(1+m)
```

Rubi [A]

time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {522, 371}

$$\frac{b(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a
)])/(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 +
m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 522

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx &= \frac{b\int \frac{(ex)^m}{a+bx^n}\,dx}{bc-ad}-\frac{d\int \frac{(ex)^m}{c+dx^n}\,dx}{bc-ad}\\ &= \frac{b(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)e(1+m)}-\frac{d(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)e(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 88, normalized size = 0.77

$$\frac{x(ex)^m \left(-bc\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)+ad\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{dx^n}{c}\right)\right)}{ac(-bc+ad)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
[Out] (x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/
a)]) + a*d*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)]))/(
a*c*(-(b*c) + a*d)*(1 + m))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a+b*x^n)/(c+d*x^n),x)
[Out] int((e*x)^m/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] integrate((x*e)^m/((b*x^n + a)*(d*x^n + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral((x*e)^m/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a+bx^n)\,(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral((e*x)**m/((a + b*x**n)*(c + d*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((x*e)^m/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int((e*x)^m/((a + b*x^n)*(c + d*x^n)), x)
```

3.1026 $\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=89

$$\frac{bx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a(bc-ad)}-\frac{dx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

```
[Out] 1/3*b*x^3*hypergeom([1, 3/n],[(3+n)/n],-b*x^n/a)/a/(-a*d+b*c)-1/3*d*x^3*hyp
ergeom([1, 3/n],[(3+n)/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {522, 371}

$$\frac{bx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a(bc-ad)}-\frac{dx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*(b*c - a*d)
) - (d*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c -
a*d))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 522

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx &= \frac{b\int \frac{x^2}{a+bx^n}\,dx}{bc-ad}-\frac{d\int \frac{x^2}{c+dx^n}\,dx}{bc-ad}\\ &= \frac{bx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a(bc-ad)}-\frac{dx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 78, normalized size = 0.88

$$\frac{bcx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)-adx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{dx^n}{c}\right)}{3abc^2-3a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)] - a*d*x^3*Hyper
geometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*a*b*c^2 - 3*a^2*c*d)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(x^2/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(x**2/((a + b*x**n)*(c + d*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a+b\,x^n\right)\,\left(c+d\,x^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(x^2/((a + b*x^n)*(c + d*x^n)), x)
```

3.1027 $\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=89

$$\frac{bx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)}-\frac{dx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

```
[Out] 1/2*b*x^2*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a/(-a*d+b*c)-1/2*d*x^2*hyp
ergeom([1, 2/n],[(2+n)/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A]

time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {522, 371}

$$\frac{bx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)}-\frac{dx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*(b*c - a*d)
) - (d*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c -
a*d))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 522

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx&=\frac{b\int \frac{x}{a+bx^n}\,dx}{bc-ad}-\frac{d\int \frac{x}{c+dx^n}\,dx}{bc-ad}\\&=\frac{bx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)}-\frac{dx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 78, normalized size = 0.88

$$\frac{bcx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)-adx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{dx^n}{c}\right)}{2abc^2-2a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^n)*(c + d*x^n)),x]
[Out] (b*c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)] - a*d*x^2*Hyper
geometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*a*b*c^2 - 2*a^2*c*d)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,x^{n}\right)\left(c+d\,x^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^n)/(c+d*x^n),x)
[Out] int(x/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral(x/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^{n}\right)\left(c+dx^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(x/((a + b*x**n)*(c + d*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(x/((a + b*x^n)*(c + d*x^n)), x)
```

3.1028 $\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=72

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

```
[Out] b*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a*d+b*c)-d*x*hypergeom([1, 1/n
],[1+1/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {400, 251}

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)
) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
a*d))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx &= \frac{b\int \frac{1}{a+bx^n}\,dx}{bc-ad}-\frac{d\int \frac{1}{c+dx^n}\,dx}{bc-ad}\\ &= \frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 64, normalized size = 0.89

$$\frac{x\left(-bc\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+ad\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{ac(-bc+ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(1/((a + b*x^n)*(c + d*x^n)), x)
```

3.1029 $\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=63

$$\frac{\log(x)}{ac}-\frac{b\log(a+bx^n)}{a(bc-ad)n}+\frac{d\log(c+dx^n)}{c(bc-ad)n}$$

```
[Out] ln(x)/a/c-b*ln(a+b*x^n)/a/(-a*d+b*c)/n+d*ln(c+d*x^n)/c/(-a*d+b*c)/n
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {457, 84}

$$-\frac{b\log(a+bx^n)}{an(bc-ad)}+\frac{d\log(c+dx^n)}{cn(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^n])/(a*(b*c - a*d)*n) + (d*Log[c + d*x^n])/(c
*(b*c - a*d)*n)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx&=\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx,x,x^n\right)}{n}\\&=\frac{\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx,x,x^n\right)}{n}\\&=\frac{\log(x)}{ac}-\frac{b\log(a+bx^n)}{a(bc-ad)n}+\frac{d\log(c+dx^n)}{c(bc-ad)n}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 0.92

$$\frac{bc\log(x^n) - ad\log(x^n) - bc\log(a+bx^n) + ad\log(c+dx^n)}{abc^2n - a^2cdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*Log[x^n] - a*d*Log[x^n] - b*c*Log[a + b*x^n] + a*d*Log[c + d*x^n])/(a*
b*c^2*n - a^2*c*d*n)
```

Maple [A]
time = 0.39, size = 64, normalized size = 1.02

method	result	size
derivativedivides	$\frac{-\frac{d\ln(c+d\,x^n)}{c(ad-bc)}+\frac{b\ln(a+b\,x^n)}{a(ad-bc)}+\frac{\ln(x^n)}{ac}}{n}$	64
default	$\frac{-\frac{d\ln(c+d\,x^n)}{c(ad-bc)}+\frac{b\ln(a+b\,x^n)}{a(ad-bc)}+\frac{\ln(x^n)}{ac}}{n}$	64
norman	$\frac{\ln(x)}{ac}+\frac{b\ln(a+b\,\mathrm{e}^{n\ln(x)})}{(ad-bc)an}-\frac{d\ln(c+d\,\mathrm{e}^{n\ln(x)})}{cn(ad-bc)}$	68
risch	$-\frac{\ln(x)b}{(ad-bc)a}+\frac{\ln(x)d}{c(ad-bc)}+\frac{b\ln(x^n+\frac{a}{b})}{(ad-bc)an}-\frac{d\ln(x^n+\frac{c}{d})}{cn(ad-bc)}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-d/c/(a*d-b*c)*ln(c+d*x^n)+b/a/(a*d-b*c)*ln(a+b*x^n)+1/a/c*ln(x^n))
```

Maxima [A]
time = 0.31, size = 69, normalized size = 1.10

$$-\frac{b\log\left(\frac{bx^n+a}{b}\right)}{abcn - a^2dn} + \frac{d\log\left(\frac{dx^n+c}{d}\right)}{bc^2n - acdn} + \frac{\log(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -b*log((b*x^n + a)/b)/(a*b*c*n - a^2*d*n) + d*log((d*x^n + c)/d)/(b*c^2*n -
 a*c*d*n) + log(x)/(a*c)
```

Fricas [A]
time = 1.03, size = 58, normalized size = 0.92

$$-\frac{bc\log(bx^n+a) - ad\log(dx^n+c) - (bc-ad)n\log(x)}{(abc^2 - a^2cd)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(b*c*log(b*x^n + a) - a*d*log(d*x^n + c) - (b*c - a*d)*n*log(x))/((a*b*c^2
 - a^2*c*d)*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(46) = 92$.

time = 1.38, size = 332, normalized size = 5.27

$$\begin{cases} \frac{\frac{\log(x)}{c} - \frac{\log\left(\frac{c}{d}+x^n\right)}{cn}}{a} & \text{for } b = 0 \\ \frac{\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^n\right)}{an}}{c} & \text{for } d = 0 \\ \frac{-\frac{x^{-n}}{cn} + \frac{d\log\left(x^{-n}+\frac{d}{c}\right)}{c^2n}}{b} & \text{for } a = 0 \\ \frac{cn\log(x)}{ac^2n+acdnx^n} - \frac{c\log\left(\frac{c}{d}+x^n\right)}{ac^2n+acdnx^n} + \frac{c}{ac^2n+acdnx^n} + \frac{dnx^n\log(x)}{ac^2n+acdnx^n} - \frac{dx^n\log\left(\frac{c}{d}+x^n\right)}{ac^2n+acdnx^n} & \text{for } b = \frac{ad}{c} \\ \frac{-\frac{x^{-n}}{an} + \frac{b\log\left(x^{-n}+\frac{b}{a}\right)}{a^2n}}{d} & \text{for } c = 0 \\ \frac{\log(x)}{(a+b)(c+d)} & \text{for } n = 0 \\ \frac{adn\log(x)}{a^2cdn-abc^2n} - \frac{ad\log\left(\frac{c}{d}+x^n\right)}{a^2cdn-abc^2n} - \frac{bcn\log(x)}{a^2cdn-abc^2n} + \frac{bc\log\left(\frac{a}{b}+x^n\right)}{a^2cdn-abc^2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((log(x)/c - log(c/d + x**n)/(c*n))/a, Eq(b, 0)), ((log(x)/a - lo
g(a/b + x**n)/(a*n))/c, Eq(d, 0)), ((-1/(c*n*x**n) + d*log(x**(-n) + d/c)/(
c**2*n))/b, Eq(a, 0)), (c*n*log(x)/(a*c**2*n + a*c*d*n*x**n) - c*log(c/d +
x**n)/(a*c**2*n + a*c*d*n*x**n) + c/(a*c**2*n + a*c*d*n*x**n) + d*n*x**n*lo
g(x)/(a*c**2*n + a*c*d*n*x**n) - d*x**n*log(c/d + x**n)/(a*c**2*n + a*c*d*n
*x**n), Eq(b, a*d/c)), ((-1/(a*n*x**n) + b*log(x**(-n) + b/a)/(a**2*n))/d,
Eq(c, 0)), (log(x)/((a + b)*(c + d)), Eq(n, 0)), (a*d*n*log(x)/(a**2*c*d*n
- a*b*c**2*n) - a*d*log(c/d + x**n)/(a**2*c*d*n - a*b*c**2*n) - b*c*n*log(x
)/(a**2*c*d*n - a*b*c**2*n) + b*c*log(a/b + x**n)/(a**2*c*d*n - a*b*c**2*n)
, True))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x), x)
```

Mupad [B]
time = 5.72, size = 162, normalized size = 2.57

$$\frac{b\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^n+b\,c\,n\,x^n}{d\,x\,\left(a^2\,d\,n-a\,b\,c\,n\right)}\right)}{a^2\,d\,n-a\,b\,c\,n}+\frac{d\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^n+b\,c\,n\,x^n}{b\,x\,\left(b\,c^2\,n-a\,c\,d\,n\right)}\right)}{b\,c^2\,n-a\,c\,d\,n}+\frac{\ln\left(x\right)\,\left(n-1\right)}{a\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] (b*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b*c*n*x^n)/(d*x*(a^2*d*n - a*b*
c*n))))/(a^2*d*n - a*b*c*n) + (d*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b
*c*n*x^n)/(b*x*(b*c^2*n - a*c*d*n))))/(b*c^2*n - a*c*d*n) + (log(x)*(n - 1)
)/(a*c*n)
```

3.1030 $\int \frac{1}{x^2(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=90

$$-\frac{b\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)x}+\frac{d\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)x}$$

```
[Out] -b*hypergeom([1, -1/n],[(-1+n)/n],-b*x^n/a)/a/(-a*d+b*c)/x+d*hypergeom([1,
-1/n],[(-1+n)/n],-d*x^n/c)/c/(-a*d+b*c)/x
```

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {522, 371}

$$\frac{d\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{cx(bc-ad)}-\frac{b\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((b*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a*(b*c - a
*d)*x)) + (d*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*
(b*c - a*d)*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 522

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^n\right)\left(c+dx^n\right)}\,dx &= \frac{b\int \frac{1}{x^2(a+bx^n)}\,dx}{bc-ad}-\frac{d\int \frac{1}{x^2(c+dx^n)}\,dx}{bc-ad}\\ &= -\frac{b\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)x}+\frac{d\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)x}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 74, normalized size = 0.82

$$\frac{bc\,{}_2F_1\left(1,-\frac{1}{n};\frac{-1+n}{n};-\frac{bx^n}{a}\right)-ad\,{}_2F_1\left(1,-\frac{1}{n};\frac{-1+n}{n};-\frac{dx^n}{c}\right)}{ac(-bc+ad)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((b*x^n)/a)] - a*d*Hypergeo
metric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)])/(a*c*(-(b*c) + a*d)*x)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\,x^n\right)\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(1/x^2/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^2*x^(2*n) + (b*c + a*d)*x^2*x^n + a*c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^n\right)\left(c+dx^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**n)*(c + d*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(a+b\,x^n\right)\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(1/(x^2*(a + b*x^n)*(c + d*x^n)), x)
```

3.1031 $\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=95

$$-\frac{b\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)x^2}+\frac{d\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)x^2}$$

```
[Out] -1/2*b*hypergeom([1, -2/n],[(-2+n)/n],-b*x^n/a)/a/(-a*d+b*c)/x^2+1/2*d*hype
rgeom([1, -2/n],[(-2+n)/n],-d*x^n/c)/c/(-a*d+b*c)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {522, 371}

$$\frac{d\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)}-\frac{b\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -1/2*(b*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(a*(b*c - a
*d)*x^2) + (d*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*c*
(b*c - a*d)*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 522

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^n\right)\left(c+dx^n\right)}\,dx &= \frac{b\int \frac{1}{x^3(a+bx^n)}\,dx}{bc-ad}-\frac{d\int \frac{1}{x^3(c+dx^n)}\,dx}{bc-ad}\\ &= -\frac{b\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)x^2}+\frac{d\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)x^2}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 77, normalized size = 0.81

$$\frac{bc\,{}_2F_1\left(1,-\frac{2}{n};\frac{-2+n}{n};-\frac{bx^n}{a}\right)-ad\,{}_2F_1\left(1,-\frac{2}{n};\frac{-2+n}{n};-\frac{dx^n}{c}\right)}{2ac(-bc+ad)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*Hypergeometric2F1[1, -2/n, (-2 + n)/n, -((b*x^n)/a)] - a*d*Hypergeomet
ric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)])/(2*a*c*(-(b*c) + a*d)*x^2)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\,(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(1/x^3/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^3*x^(2*n) + (b*c + a*d)*x^3*x^n + a*c*x^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\left(a+b\,x^n\right)\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(1/(x^3*(a + b*x^n)*(c + d*x^n)), x)
```

3.1032 $\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=175

$$\frac{b(ex)^{1+m}}{a(bc-ad)en\,(a+bx^n)}+\frac{b(ad(1+m-2n)-bc(1+m-n))(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2e(1+m)n}+\frac{d^2(ex)^{1+\cdots}}{c\cdots}$$

```
[Out] b*(e*x)^(1+m)/a/(-a*d+b*c)/e/n/(a+b*x^n)+b*(a*d*(1+m-2*n)-b*c*(1+m-n))*(e*x
)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/e/(1+
m)/n+d^2*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/(-a*d+b
*c)^2/e/(1+m)
```

Rubi [A]
time = 0.21, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {518, 611, 371}

$$\frac{b(ex)^{m+1}(ad(m-2n+1)-bc(m-n+1))\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^2e(m+1)n(bc-ad)^2}+\frac{d^2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)^2}+\frac{b(ex)^{m+1}}{aen(bc-ad)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)) + (b*(a*d*(1 + m - 2*n) -
 b*c*(1 + m - n))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*e*(1 + m)*n) + (d^2*(e*x)^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)^2
*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 518

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 611

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} - \frac{\int \frac{(ex)^m(bc(1+m-n)+adn+bd(1+m-n)x^n)}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}$$

$$= \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} - \frac{\int \left(\frac{b(-ad(1+m-2n)+bc(1+m-n))(ex)^m}{(bc-ad)(a+bx^n)} + \frac{ad^2n(ex)^m}{(-bc+ad)(c+dx^n)}\right) dx}{a(bc-ad)n}$$

$$= \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} + \frac{d^2\int \frac{(ex)^m}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1+m-2n)-bc(1+m-n)) \ldots}{a(bc-ad)^2n}$$

$$= \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} + \frac{b(ad(1+m-2n)-bc(1+m-n))(ex)^{1+m}\,{}_2F_1(1, \ldots}{a^2(bc-ad)^2e(1+m)n}$$

Mathematica [A]
time = 0.18, size = 141, normalized size = 0.81

$$\frac{x(ex)^m\left(\frac{b^2c-abd}{a^2n+abnx^n} + \frac{b(ad(1+m-2n)-bc(1+m-n))\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^2(1+m)n} + \frac{d^2\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{dx^n}{c}\right)}{c+cm}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x*(e*x)^m*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 + m - 2*n) - b
*c*(1 + m - n))*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)
])/(a^2*(1 + m)*n) + (d^2*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -(
(d*x^n)/c)])/(c + c*m)))/(b*c - a*d)^2
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(e^(m*log(x) + m)/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*x^n), x) + b*x*e^(m*log(x) + m)/(a^2*b*c*n - a
^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) - ((m*e^m - (n - 1)*e^m)*b^2*c - (m*e
^m - (2*n - 1)*e^m)*a*b*d)*integrate(x^m/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a
^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((x*e)^m/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*
b*c + a^2*d)*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((x*e)^m/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x)
[Out] int((e*x)^m/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1033 $\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=142

$$\frac{bx^3}{a(bc-ad)n\,(a+bx^n)}+\frac{b(ad(3-2n)-bc(3-n))x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a^2(bc-ad)^2n}+\frac{d^2x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)^2}$$

```
[Out] b*x^3/a/(-a*d+b*c)/n/(a+b*x^n)+1/3*b*(a*d*(3-2*n)-b*c*(3-n))*x^3*hypergeom(
[1, 3/n],[(3+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+1/3*d^2*x^3*hypergeom([1, 3
/n],[(3+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2
```

Rubi [A]
time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {518, 611, 371}

$$\frac{bx^3(ad(3-2n)-bc(3-n))\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^2n(bc-ad)^2}+\frac{d^2x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)^2}+\frac{bx^3}{an(bc-ad)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x^3)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(3 - 2*n) - b*c*(3 - n))*x^
3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^2*(b*c - a*d)^2*
n) + (d^2*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c
 - a*d)^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 518

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 611

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{bx^3}{a(bc-ad)n(a+bx^n)}-\frac{\int \frac{x^2(bc(3-n)+adn+bd(3-n)x^n)}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}\\ &= \frac{bx^3}{a(bc-ad)n(a+bx^n)}-\frac{\int \left(\frac{b(-ad(3-2n)+bc(3-n))x^2}{(bc-ad)(a+bx^n)}+\frac{ad^2nx^2}{(-bc+ad)(c+dx^n)}\right)dx}{a(bc-ad)n}\\ &= \frac{bx^3}{a(bc-ad)n(a+bx^n)}+\frac{d^2\int \frac{x^2}{c+dx^n}\,dx}{(bc-ad)^2}+\frac{(b(ad(3-2n)-bc(3-n)))\int \frac{x^2}{a+bx^n}\,d}{a(bc-ad)^2n}\\ &= \frac{bx^3}{a(bc-ad)n(a+bx^n)}+\frac{b(ad(3-2n)-bc(3-n))x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a^2(bc-ad)^2n}+\frac{d}{}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 135, normalized size = 0.95

$$\frac{x^3\left(bc(ad(3-2n)+bc(-3+n))\,(a+bx^n)\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)+a\left(3bc(bc-ad)+ad^2n(a+bx^n)\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{dx^n}{c}\right)\right)\right)}{3a^2c(bc-ad)^2n\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x^3*(b*c*(a*d*(3 - 2*n) + b*c*(-3 + n))*(a + b*x^n)*Hypergeometric2F1[1, 3
/n, (3 + n)/n, -((b*x^n)/a)] + a*(3*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n)*H
ypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])))/(3*a^2*c*(b*c - a*d)^2
*n*(a + b*x^n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] b*x^3/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + d^2*integrate(x
^2/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*x^n), x) - (a*b*d*(2*n - 3) - b^2*c*(n - 3))*integrate(x^2/(a^2*b^2*c^2*n
- 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)
*x^n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c
+ a^2*d)*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a+b\,x^n)^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^n)^2*(c + d*x^n)),x)
[Out] int(x^2/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1034 $\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=143

$$\frac{bx^2}{a(bc-ad)n\,(a+bx^n)}+\frac{b(2ad(1-n)-bc(2-n))x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2n}+\frac{d^2x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)^2}$$

```
[Out] b*x^2/a/(-a*d+b*c)/n/(a+b*x^n)+1/2*b*(2*a*d*(1-n)-b*c*(2-n))*x^2*hypergeom(
[1, 2/n],[(2+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+1/2*d^2*x^2*hypergeom([1, 2
/n],[(2+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2
```

Rubi [A]
time = 0.13, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {518, 611, 371}

$$\frac{bx^2(2ad(1-n)-bc(2-n))\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^2n(bc-ad)^2}+\frac{d^2x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)^2}+\frac{bx^2}{an(bc-ad)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x^2)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(2*a*d*(1 - n) - b*c*(2 - n))*x^
2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^2*(b*c - a*d)^2*
n) + (d^2*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c
 - a*d)^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 518

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 611

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{bx^2}{a(bc-ad)n(a+bx^n)} - \frac{\int \frac{x(bc(2-n)+adn+bd(2-n)x^n)}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}\\ &= \frac{bx^2}{a(bc-ad)n(a+bx^n)} - \frac{\int \left(\frac{b(-2ad(1-n)+bc(2-n))x}{(bc-ad)(a+bx^n)} + \frac{ad^2nx}{(-bc+ad)(c+dx^n)}\right)dx}{a(bc-ad)n}\\ &= \frac{bx^2}{a(bc-ad)n(a+bx^n)} + \frac{d^2\int \frac{x}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(2ad(1-n)-bc(2-n)))\int \frac{x}{a+bx^n}\,d}{a(bc-ad)^2n}\\ &= \frac{bx^2}{a(bc-ad)n(a+bx^n)} + \frac{b(2ad(1-n)-bc(2-n))x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2n} + \frac{d}{\ }\end{aligned}$$

Mathematica [A]
time = 0.14, size = 134, normalized size = 0.94

$$\frac{x^2\left(bc(bc(-2+n)-2ad(-1+n))(a+bx^n)\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)+a\left(2bc(bc-ad)+ad^2n(a+bx^n)\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{dx^n}{c}\right)\right)\right)}{2a^2c(bc-ad)^2n(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x^2*(b*c*(b*c*(-2 + n) - 2*a*d*(-1 + n))*(a + b*x^n)*Hypergeometric2F1[1,
2/n, (2 + n)/n, -((b*x^n)/a)] + a*(2*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n)*
Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])))/(2*a^2*c*(b*c - a*d)^
2*n*(a + b*x^n))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+b\,x^n)^2(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(x/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(x/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3)*x^n), x) + b*x^2/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*
n)*x^n) - (2*a*b*d*(n - 1) - b^2*c*(n - 2))*integrate(x/(a^2*b^2*c^2*n - 2*
a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^n
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(x/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^n)^2*(c + d*x^n)),x)
[Out] int(x/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1035 $\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=122

$$\frac{bx}{a(bc-ad)n\left(a+bx^n\right)}+\frac{b(ad(1-2n)-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}$$

```
[Out] b*x/a/(-a*d+b*c)/n/(a+b*x^n)+b*(a*d*(1-2*n)-b*c*(1-n))*x*hypergeom([1, 1/n]
,[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+d^2*x*hypergeom([1, 1/n],[1+1/n],-d*x
^n/c)/c/(-a*d+b*c)^2
```

Rubi [A]
time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {425, 536, 251}

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1 - n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n)
 + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
 a*d)^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{bx}{a(bc-ad)n(a+bx^n)} - \frac{\int \frac{adn+b(c-cn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}\\ &= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{d^2\int \frac{1}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1-2n)-bc(1-n)))\int \frac{1}{a+bx^n}\,d}{a(bc-ad)^2n}\\ &= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(1-2n)-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} + \end{aligned}$$

Mathematica [A]
time = 0.14, size = 108, normalized size = 0.89

$$\frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1-2n)+bc(-1+n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n}+\frac{d^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
[Out] (x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n))
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)^2/(c+d*x^n),x)
[Out] int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a^2
*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a
^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x
^n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x^n\right)}^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1036 $\int \frac{1}{x(a+bx^n)^2(c+dx^n)} dx$

Optimal. Leaf size=101

$$\frac{b}{a(bc-ad)n\,(a+bx^n)}+\frac{\log(x)}{a^2c}-\frac{b(bc-2ad)\log\left(a+bx^n\right)}{a^2(bc-ad)^2n}-\frac{d^2\log\left(c+dx^n\right)}{c(bc-ad)^2n}$$

[Out] b/a/(-a*d+b*c)/n/(a+b*x^n)+ln(x)/a^2/c-b*(-2*a*d+b*c)*ln(a+b*x^n)/a^2/(-a*d+b*c)^2/n-d^2*ln(c+d*x^n)/c/(-a*d+b*c)^2/n

Rubi [A]

time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$-\frac{b(bc-2ad)\log\left(a+bx^n\right)}{a^2n(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log\left(c+dx^n\right)}{cn(bc-ad)^2}+\frac{b}{an(bc-ad)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*(a + b*x^n)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a + b*x^n])/(a^2*(b*c - a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2*n)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2cx}+\frac{b^2}{a(-bc+ad)(a+bx)^2}+\frac{b^2(-bc+2ad)}{a^2(-bc+ad)^2(a+bx)}-\frac{d^3}{c(bc-ad)^2(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= \frac{b}{a(bc-ad)n\left(a+bx^n\right)}+\frac{\log(x)}{a^2c}-\frac{b(bc-2ad)\log\left(a+bx^n\right)}{a^2(bc-ad)^2n}-\frac{d^2\log\left(c+dx^n\right)}{c(bc-ad)^2n}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 107, normalized size = 1.06

$$-\frac{b}{a(-bc+ad)n\left(a+bx^n\right)}+\frac{\log\left(x^n\right)}{a^2cn}+\frac{b(-bc+2ad)\log\left(a+bx^n\right)}{a^2(-bc+ad)^2n}-\frac{d^2\log\left(c+dx^n\right)}{c(bc-ad)^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] -(b/(a*(-(b*c) + a*d)*n*(a + b*x^n))) + Log[x^n]/(a^2*c*n) + (b*(-(b*c) + 2
*a*d)*Log[a + b*x^n])/(a^2*(-(b*c) + a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*(b
*c - a*d)^2*n)
```

Maple [A]
time = 0.40, size = 100, normalized size = 0.99

method	result
derivativedivides	$\frac{\frac{\ln\left(x^{n}\right)}{a^{2}c}-\frac{b}{a\left(ad-bc\right)\left(a+b\,x^{n}\right)}+\frac{b\left(2ad-bc\right)\ln\left(a+b\,x^{n}\right)}{\left(ad-bc\right)^{2}a^{2}}-\frac{d^{2}\ln\left(c+d\,x^{n}\right)}{\left(ad-bc\right)^{2}c}}{n}$
default	$\frac{\frac{\ln\left(x^{n}\right)}{a^{2}c}-\frac{b}{a\left(ad-bc\right)\left(a+b\,x^{n}\right)}+\frac{b\left(2ad-bc\right)\ln\left(a+b\,x^{n}\right)}{\left(ad-bc\right)^{2}a^{2}}-\frac{d^{2}\ln\left(c+d\,x^{n}\right)}{\left(ad-bc\right)^{2}c}}{n}$
norman	$\frac{\frac{b^{2}e^{n\ln(x)}}{n\,a^{2}\left(ad-bc\right)}+\frac{\ln(x)}{ac}+\frac{b\ln(x)e^{n\ln(x)}}{a^{2}c}}{a+b\,e^{n\ln(x)}}+\frac{b(2ad-bc)\ln\left(a+b\,e^{n\ln(x)}\right)}{(a^{2}d^{2}-2abcd+b^{2}c^{2})a^{2}n}-\frac{d^{2}\ln\left(c+d\,e^{n\ln(x)}\right)}{cn(a^{2}d^{2}-2abcd+b^{2}c^{2})}$
risch	$\frac{\ln(x)d^{2}}{c(a^{2}d^{2}-2abcd+b^{2}c^{2})}-\frac{2\ln(x)bd}{(a^{2}d^{2}-2abcd+b^{2}c^{2})a}+\frac{\ln(x)b^{2}c}{(a^{2}d^{2}-2abcd+b^{2}c^{2})a^{2}}-\frac{b}{(ad-bc)an(a+b\,x^{n})}-\frac{d^{2}\ln\left(x^{n}+\right.}{cn(a^{2}d^{2}-2abc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/a^2/c*ln(x^n)-b/a/(a*d-b*c)/(a+b*x^n)+b*(2*a*d-b*c)/(a*d-b*c)^2/a^2*
ln(a+b*x^n)-d^2/(a*d-b*c)^2/c*ln(c+d*x^n))
```

Maxima [A]
time = 0.31, size = 151, normalized size = 1.50

$$-\frac{d^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^3n-2\,abc^2dn+a^2cd^2n}-\frac{\left(b^2c-2\,abd\right)\log\left(\frac{bx^n+a}{b}\right)}{a^2b^2c^2n-2\,a^3bcdn+a^4d^2n}+\frac{b}{a^2bcn-a^3dn+\left(ab^2cn-a^2bdn\right)x^n}+\frac{\log\left(x\right)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -d^2*log((d*x^n + c)/d)/(b^2*c^3*n - 2*a*b*c^2*d*n + a^2*c*d^2*n) - (b^2*c
- 2*a*b*d)*log((b*x^n + a)/b)/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n) +
 b/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + log(x)/(a^2*c)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(101) = 202$.
time = 0.50, size = 224, normalized size = 2.22

$$\frac{ab^2c^2 - a^2bcd + (b^3c^2 - 2\,ab^2cd + a^2bd^2)nx^n \log\left(x\right) + (ab^2c^2 - 2\,a^2bcd + a^3d^2)n \log\left(x\right) - (ab^2c^2 - 2\,a^2bcd + (b^3c^2 - 2\,ab^2cd)x^n) \log\left(bx^n + a\right) - (a^2bd^2x^n + a^3d^2) \log\left(dx^n + c\right)}{(a^2b^3c^3 - 2\,a^3b^2c^2d + a^4bcd^2)nx^n + (a^3b^2c^3 - 2\,a^4bc^2d + a^5cd^2)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a*b^2*c^2 - a^2*b*c*d + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*n*x^n*log(x) +
 (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*n*log(x) - (a*b^2*c^2 - 2*a^2*b*c*d +
(b^3*c^2 - 2*a*b^2*c*d)*x^n)*log(b*x^n + a) - (a^2*b*d^2*x^n + a^3*d^2)*log
(d*x^n + c))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*n*x^n + (a^3*b^
2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*n)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(a+b\,x^n\right)}^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(1/(x*(a + b*x^n)^2*(c + d*x^n)), x)
```

3.1037 $\int \frac{1}{x^2(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=142

$$\frac{b}{a(bc-ad)nx\,(a+bx^n)}-\frac{b(bc(1+n)-ad(1+2n))\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2nx}-\frac{d^2\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2x}$$

```
[Out] b/a/(-a*d+b*c)/n/x/(a+b*x^n)-b*(b*c*(1+n)-a*d*(1+2*n))*hypergeom([1, -1/n],
[(-1+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n/x-d^2*hypergeom([1, -1/n],[(-1+n)/n
],-d*x^n/c)/c/(-a*d+b*c)^2/x
```

Rubi [A]
time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {518, 611, 371}

$$-\frac{b(bc(n+1)-ad(2n+1))\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2nx(bc-ad)^2}-\frac{d^2\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{cx(bc-ad)^2}+\frac{b}{anx(bc-ad)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*x*(a + b*x^n)) - (b*(b*c*(1 + n) - a*d*(1 + 2*n))*Hyperg
eometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n*x
) - (d^2*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*(b*c
 - a*d)^2*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 518

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 611

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}\,dx = \frac{b}{a(bc-ad)nx\,(a+bx^n)} - \frac{\int \frac{adn-bc(1+n)-bd(1+n)x^n}{x^2(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}$$

$$= \frac{b}{a(bc-ad)nx\,(a+bx^n)} - \frac{\int \left(\frac{b(-bc(1+n)+ad(1+2n))}{(bc-ad)x^2(a+bx^n)} + \frac{ad^2n}{(-bc+ad)x^2(c+dx^n)}\right)dx}{a(bc-ad)n}$$

$$= \frac{b}{a(bc-ad)nx\,(a+bx^n)} + \frac{d^2\int \frac{1}{x^2(c+dx^n)}\,dx}{(bc-ad)^2} + \frac{(b(bc(1+n)-ad(1+2n)))\int}{a(bc-ad)^2n}$$

$$= \frac{b}{a(bc-ad)nx\,(a+bx^n)} - \frac{b(bc(1+n)-ad(1+2n))\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2nx}$$

Mathematica [A]
time = 0.16, size = 133, normalized size = 0.94

$$\frac{bc(-bc(1+n)+ad(1+2n))\,(a+bx^n)\,{}_2F_1\left(1,-\frac{1}{n};\frac{-1+n}{n};-\frac{bx^n}{a}\right) - a\left(bc(-bc+ad)+ad^2n(a+bx^n)\,{}_2F_1\left(1,-\frac{1}{n};\frac{-1+n}{n};-\frac{dx^n}{c}\right)\right)}{a^2c(bc-ad)^2nx\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*c*(-(b*c*(1 + n)) + a*d*(1 + 2*n))*(a + b*x^n)*Hypergeometric2F1[1, -n^(
-1), (-1 + n)/n, -((b*x^n)/a)] - a*(b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x^n
)*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)]))/(a^2*c*(b*c - a
*d)^2*n*x*(a + b*x^n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a+b\,x^n)^2 (c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(1/x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2*x^n + (b^2*c^3 - 2
*a*b*c^2*d + a^2*c*d^2)*x^2), x) - (a*b*d*(2*n + 1) - b^2*c*(n + 1))*integr
ate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^2*x^n + (a^2*b^2*c^2
*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^2), x) + b/((a*b^2*c*n - a^2*b*d*n)*x*x^n
 + (a^2*b*c*n - a^3*d*n)*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^2*x^(3*n) + a^2*c*x^2 + (b^2*c + 2*a*b*d)*x^2*x^(2*n) +
 (2*a*b*c + a^2*d)*x^2*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x)
[Out] int(1/(x^2*(a + b*x^n)^2*(c + d*x^n)), x)
```

3.1038 $\int \frac{1}{x^3(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=145

$$\frac{b}{a(bc-ad)nx^2\left(a+bx^n\right)}+\frac{b(2ad(1+n)-bc(2+n))\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2nx^2}-\frac{d^2\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)^2x^2}$$

```
[Out] b/a/(-a*d+b*c)/n/x^2/(a+b*x^n)+1/2*b*(2*a*d*(1+n)-b*c*(2+n))*hypergeom([1,
-2/n],[(-2+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n/x^2-1/2*d^2*hypergeom([1, -2/
n],[(-2+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2/x^2
```

Rubi [A]
time = 0.15, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {518, 611, 371}

$$\frac{b(2ad(n+1)-bc(n+2))\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2nx^2(bc-ad)^2}-\frac{d^2\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)^2}+\frac{b}{anx^2(bc-ad)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*x^2*(a + b*x^n)) + (b*(2*a*d*(1 + n) - b*c*(2 + n))*Hype
rgeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a^2*(b*c - a*d)^2*n*
x^2) - (d^2*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*c*(b
*c - a*d)^2*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 518

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 611

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)} dx = \frac{b}{a(bc-ad)nx^2 (a+bx^n)} - \frac{\int \frac{adn-bc(2+n)-bd(2+n)x^n}{x^3(a+bx^n)(c+dx^n)} dx}{a(bc-ad)n}$$
$$= \frac{b}{a(bc-ad)nx^2 (a+bx^n)} - \frac{\int \left(\frac{b(2ad(1+n)-bc(2+n))}{(bc-ad)x^3(a+bx^n)} + \frac{ad^2n}{(-bc+ad)x^3(c+dx^n)}\right) dx}{a(bc-ad)n}$$
$$= \frac{b}{a(bc-ad)nx^2 (a+bx^n)} + \frac{d^2 \int \frac{1}{x^3(c+dx^n)} dx}{(bc-ad)^2} - \frac{(b(2ad(1+n)-bc(2+n))) \int}{a(bc-ad)^2 n}$$
$$= \frac{b}{a(bc-ad)nx^2 (a+bx^n)} + \frac{b(2ad(1+n)-bc(2+n))\, {}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right.}{2a^2(bc-ad)^2nx^2}$$

Mathematica [A]
time = 0.15, size = 136, normalized size = 0.94

$$\frac{bc(2ad(1+n)-bc(2+n))(a+bx^n)\, {}_2F_1\left(1,-\frac{2}{n};\frac{-2+n}{n};-\frac{bx^n}{a}\right) - a\left(2bc(-bc+ad)+ad^2n(a+bx^n)\, {}_2F_1\left(1,-\frac{2}{n};\frac{-2+n}{n};-\frac{dx^n}{c}\right)\right)}{2a^2c(bc-ad)^2nx^2(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*c*(2*a*d*(1 + n) - b*c*(2 + n))*(a + b*x^n)*Hypergeometric2F1[1, -2/n, (
-2 + n)/n, -((b*x^n)/a)] - a*(2*b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x^n)*Hy
pergeometric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)]))/(2*a^2*c*(b*c - a*d)^2
*n*x^2*(a + b*x^n))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a+b x^n)^2 (c+d x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(1/x^3/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^3*x^n + (b^2*c^3 - 2
*a*b*c^2*d + a^2*c*d^2)*x^3), x) + (b^2*c*(n + 2) - 2*a*b*d*(n + 1))*integr
ate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^3*x^n + (a^2*b^2*c^2
*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^3), x) + b/((a*b^2*c*n - a^2*b*d*n)*x^2*x
^n + (a^2*b*c*n - a^3*d*n)*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^3*x^(3*n) + a^2*c*x^3 + (b^2*c + 2*a*b*d)*x^3*x^(2*n) +
 (2*a*b*c + a^2*d)*x^3*x^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,(a+b\,x^n)^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x)
[Out] int(1/(x^3*(a + b*x^n)^2*(c + d*x^n)), x)
```

3.1039 $\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$

Optimal. Leaf size=130

$$-\frac{(bc-ad)^3x^n}{d^4n}+\frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{2n}}{2d^3n}-\frac{b^2(bc-3ad)x^{3n}}{3d^2n}+\frac{b^3x^{4n}}{4dn}+\frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}$$

```
[Out] -(-a*d+b*c)^3*x^n/d^4/n+1/2*b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)*x^(2*n)/d^3/n-1
/3*b^2*(-3*a*d+b*c)*x^(3*n)/d^2/n+1/4*b^3*x^(4*n)/d/n+c*(-a*d+b*c)^3*ln(c+d
*x^n)/d^5/n
```

Rubi [A]

time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {457, 78}

$$\frac{bx^{2n}(3a^2d^2-3abcd+b^2c^2)}{2d^3n}-\frac{b^2x^{3n}(bc-3ad)}{3d^2n}+\frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}-\frac{x^n(bc-ad)^3}{d^4n}+\frac{b^3x^{4n}}{4dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] -(((b*c - a*d)^3*x^n)/(d^4*n)) + (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2*
n))/(2*d^3*n) - (b^2*(b*c - 3*a*d)*x^(3*n))/(3*d^2*n) + (b^3*x^(4*n))/(4*d*
n) + (c*(b*c - a*d)^3*Log[c + d*x^n])/(d^5*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^3}{c+dx}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(-bc+ad)^3}{d^4} + \frac{b(b^2c^2-3abcd+3a^2d^2)x}{d^3} - \frac{b^2(bc-3ad)x^2}{d^2} + \frac{b^3x^3}{d} + \frac{c(bc-ad)^3}{d^4(c+dx)}\right) dx, x, x^n\right.}{n}$$
$$= -\frac{(bc-ad)^3x^n}{d^4n} + \frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{2n}}{2d^3n} - \frac{b^2(bc-3ad)x^{3n}}{3d^2n} + \frac{b^3x^{4n}}{4dn} + \frac{c(bc-}{}$$

Mathematica [A]
time = 0.13, size = 134, normalized size = 1.03

$$\frac{dx^n(12a^3d^3 + 18a^2bd^2(-2c+dx^n) + 6ab^2d(6c^2-3cdx^n+2d^2x^{2n}) + b^3(-12c^3+6c^2dx^n-4cd^2x^{2n}+3d^3x^{3n})) + 12c(bc-ad)^3\log(c+dx^n)}{12d^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (d*x^n*(12*a^3*d^3 + 18*a^2*b*d^2*(-2*c + d*x^n) + 6*a*b^2*d*(6*c^2 - 3*c*d
*x^n + 2*d^2*x^(2*n)) + b^3*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*d^
3*x^(3*n))) + 12*c*(b*c - a*d)^3*Log[c + d*x^n])/(12*d^5*n)
```

Maple [A]
time = 0.38, size = 188, normalized size = 1.45

method	result
norman	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)e^{n\ln(x)}}{d^4n} + \frac{b^3e^{4n\ln(x)}}{4dn} + \frac{b(3a^2d^2-3abcd+b^2c^2)e^{2n\ln(x)}}{2d^3n} + \frac{b^2(3ad-bc)e^{3n\ln(x)}}{3d^2n} - \frac{c(a^3d^3-3a^2b}{}$
risch	$\frac{b^3x^{4n}}{4dn} + \frac{b^2x^{3n}a}{dn} - \frac{b^3x^{3n}c}{3d^2n} + \frac{3b\,x^{2n}a^2}{2dn} - \frac{3b^2x^{2n}ac}{2d^2n} + \frac{b^3x^{2n}c^2}{2d^3n} + \frac{x^na^3}{dn} - \frac{3x^na^2bc}{d^2n} + \frac{3x^na\,b^2c^2}{d^3n} - \frac{x^nb^3c^3}{d^4n} - \frac{c\ln(x^n+}{d^2n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/n*exp(n*ln(x))+1/4*b^3/
d/n*exp(n*ln(x))^4+1/2*b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3/n*exp(n*ln(x))^2
+1/3*b^2*(3*a*d-b*c)/d^2/n*exp(n*ln(x))^3-c*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)/d^5/n*ln(c+d*exp(n*ln(x)))
```

Maxima [A]
time = 0.31, size = 231, normalized size = 1.78

$$a^3\left(\frac{x^n}{dn} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right) + \frac{1}{12}b^3\left(\frac{12c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n} + \frac{3d^3x^{4n} - 4cd^2x^{3n} + 6c^2dx^{2n} - 12c^3x^n}{d^4n}\right) - \frac{1}{2}ab^2\left(\frac{6c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2d^2x^{3n} - 3cdx^{2n} + 6c^2x^n}{d^3n}\right) + \frac{3}{2}a^2b\left(\frac{2c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2n} - 2cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^3*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/12*b^3*(12*c^4*log((d*x^
n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n) + 6*c^2*d*x^(2*n) - 12
*c^3*x^n)/(d^4*n)) - 1/2*a*b^2*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x
^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 3/2*a^2*b*(2*c^2*log((d*x^n
+ c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Fricas [A]
time = 3.41, size = 177, normalized size = 1.36

$$\frac{3\,b^{3} d^{4} x^{4\,n} - 4\,{\left(b^{3} c d^{3} - 3\,a b^{2} d^{4}\right)} x^{3\,n} + 6\,{\left(b^{3} c^{2} d^{2} - 3\,a b^{2} c d^{3} + 3\,a^{2} b d^{4}\right)} x^{2\,n} - 12\,{\left(b^{3} c^{3} d - 3\,a b^{2} c^{2} d^{2} + 3\,a^{2} b c d^{3} - a^{3} d^{4}\right)} x^{n} + 12\,{\left(b^{3} c^{4} - 3\,a b^{2} c^{3} d + 3\,a^{2} b c^{2} d^{2} - a^{3} c d^{3}\right)} \log\left(d x^{n} + c\right)}{12\,d^{5} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^3*d^4*x^(4*n) - 4*(b^3*c*d^3 - 3*a*b^2*d^4)*x^(3*n) + 6*(b^3*c^2*
d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*x^(2*n) - 12*(b^3*c^3*d - 3*a*b^2*c^2*d^
2 + 3*a^2*b*c*d^3 - a^3*d^4)*x^n + 12*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^
2*d^2 - a^3*c*d^3)*log(d*x^n + c))/(d^5*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(114) = 228$.
time = 70.04, size = 320, normalized size = 2.46

$$\begin{cases} \frac{\left(a + b\right)^{3} \log{\left(x \right)}}{c} & \text{for}\: d = 0 \wedge n = 0 \\ \frac{\left(a + b\right)^{3} \log{\left(x \right)}}{c + d} & \text{for}\: n = 0 \\ \frac{\frac{a^{3} x^{2 n}}{2 n} + \frac{a^{2} b x^{3 n}}{n} + \frac{3 a b^{2} x^{4 n}}{4 n} + \frac{b^{3} x^{5 n}}{5 n}}{c} & \text{for}\: d = 0 \\ - \frac{a^{3} c \log{\left(\frac{c}{d} + x^{n} \right)}}{d^{2} n} + \frac{a^{3} x^{n}}{d n} + \frac{3 a^{2} b c^{2} \log{\left(\frac{c}{d} + x^{n} \right)}}{d^{3} n} - \frac{3 a^{2} b c x^{n}}{d^{2} n} + \frac{3 a^{2} b x^{2 n}}{2 d n} - \frac{3 a b^{2} c^{3} \log{\left(\frac{c}{d} + x^{n} \right)}}{d^{4} n} + \frac{3 a b^{2} c^{2} x^{n}}{d^{3} n} - \frac{3 a b^{2} c x^{2 n}}{2 d^{2} n} + \frac{a b^{2} x^{3 n}}{d n} + \frac{b^{3} c^{4} \log{\left(\frac{c}{d} + x^{n} \right)}}{d^{5} n} - \frac{b^{3} c^{3} x^{n}}{d^{4} n} + \frac{b^{3} c^{2} x^{2 n}}{2 d^{3} n} - \frac{b^{3} c x^{3 n}}{3 d^{2} n} + \frac{b^{3} x^{4 n}}{4 d n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**3*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a + b)**3*log(x)/(c
 + d), Eq(n, 0)), ((a**3*x**(2*n)/(2*n) + a**2*b*x**(3*n)/n + 3*a*b**2*x**(
4*n)/(4*n) + b**3*x**(5*n)/(5*n))/c, Eq(d, 0)), (-a**3*c*log(c/d + x**n)/(d
**2*n) + a**3*x**n/(d*n) + 3*a**2*b*c**2*log(c/d + x**n)/(d**3*n) - 3*a**2*
b*c*x**n/(d**2*n) + 3*a**2*b*x**(2*n)/(2*d*n) - 3*a*b**2*c**3*log(c/d + x**
n)/(d**4*n) + 3*a*b**2*c**2*x**n/(d**3*n) - 3*a*b**2*c*x**(2*n)/(2*d**2*n)
+ a*b**2*x**(3*n)/(d*n) + b**3*c**4*log(c/d + x**n)/(d**5*n) - b**3*c**3*x*
*n/(d**4*n) + b**3*c**2*x**(2*n)/(2*d**3*n) - b**3*c*x**(3*n)/(3*d**2*n) +
b**3*x**(4*n)/(4*d*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3*x^(2*n - 1)/(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}\left(a+b\,x^n\right)^3}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

3.1040 $\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n} \, dx$

Optimal. Leaf size=90

$$\frac{(bc-ad)^2 x^n}{d^3 n} - \frac{b(bc-2ad)x^{2n}}{2d^2 n} + \frac{b^2 x^{3n}}{3dn} - \frac{c(bc-ad)^2 \log(c+dx^n)}{d^4 n}$$

```
[Out] (-a*d+b*c)^2*x^n/d^3/n-1/2*b*(-2*a*d+b*c)*x^(2*n)/d^2/n+1/3*b^2*x^(3*n)/d/n
-c*(-a*d+b*c)^2*ln(c+d*x^n)/d^4/n
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {457, 78}

$$-\frac{c(bc-ad)^2 \log(c+dx^n)}{d^4 n} + \frac{x^n(bc-ad)^2}{d^3 n} - \frac{bx^{2n}(bc-2ad)}{2d^2 n} + \frac{b^2 x^{3n}}{3dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] ((b*c - a*d)^2*x^n)/(d^3*n) - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2*n) + (b^2*x^
(3*n))/(3*d*n) - (c*(b*c - a*d)^2*Log[c + d*x^n])/(d^4*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^2}{c+dx}\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^3} - \frac{b(bc-2ad)x}{d^2} + \frac{b^2x^2}{d} - \frac{c(bc-ad)^2}{d^3(c+dx)}\right)\,dx,x,x^n\right)}{n}$$
$$= \frac{(bc-ad)^2x^n}{d^3n} - \frac{b(bc-2ad)x^{2n}}{2d^2n} + \frac{b^2x^{3n}}{3dn} - \frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}$$

Mathematica [A]
time = 0.08, size = 87, normalized size = 0.97

$$\frac{dx^n(6a^2d^2+6abd(-2c+dx^n)+b^2(6c^2-3cdx^n+2d^2x^{2n}))-6c(bc-ad)^2\log(c+dx^n)}{6d^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] (d*x^n*(6*a^2*d^2 + 6*a*b*d*(-2*c + d*x^n) + b^2*(6*c^2 - 3*c*d*x^n + 2*d^2
*x^(2*n))) - 6*c*(b*c - a*d)^2*Log[c + d*x^n])/(6*d^4*n)
```

Maple [A]
time = 0.37, size = 118, normalized size = 1.31

method	result	size
norman	$\frac{(a^2d^2-2abcd+b^2c^2)\,\mathrm{e}^{n\ln(x)}}{d^3n}+\frac{b^2\mathrm{e}^{3n\ln(x)}}{3dn}+\frac{b(2ad-bc)\,\mathrm{e}^{2n\ln(x)}}{2d^2n}-\frac{c(a^2d^2-2abcd+b^2c^2)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{d^4n}$	118
risch	$\frac{b^2x^{3n}}{3dn}+\frac{b\,x^{2n}a}{dn}-\frac{b^2x^{2n}c}{2d^2n}+\frac{x^na^2}{dn}-\frac{2x^nabc}{d^2n}+\frac{x^nb^2c^2}{d^3n}-\frac{c\ln(x^n+\frac{c}{d})a^2}{d^2n}+\frac{2c^2\ln(x^n+\frac{c}{d})ab}{d^3n}-\frac{c^3\ln(x^n+\frac{c}{d})b^2}{d^4n}$	161

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(x))+1/3*b^2/d/n*exp(n*ln(x))^3
+1/2*b*(2*a*d-b*c)/d^2/n*exp(n*ln(x))^2-c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^4/n
*ln(c+d*exp(n*ln(x)))
```

Maxima [A]
time = 0.31, size = 150, normalized size = 1.67

$$a^2\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right)-\frac{1}{6}b^2\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right)+ab\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) - 1/6*b^2*(6*c^3*log((d*x^n
+ c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + a*
b*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Fricas [A]
time = 6.77, size = 108, normalized size = 1.20

$$\frac{2\,b^2d^3x^{3\,n}-3\,(b^2cd^2-2\,abd^3)x^{2\,n}+6\,(b^2c^2d-2\,abcd^2+a^2d^3)x^n-6\,(b^2c^3-2\,abc^2d+a^2cd^2)\log\left(dx^n+c\right)}{6\,d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^2*d^3*x^(3*n) - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^(2*n) + 6*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*x^n - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x
^n + c))/(d^4*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(75) = 150.
time = 29.97, size = 202, normalized size = 2.24

$$\begin{cases}\frac{(a+b)^2\log(x)}{c} & \text{for}\ d=0\wedge n=0\\ \frac{(a+b)^2\log(x)}{c+d} & \text{for}\ n=0\\ \frac{\frac{a^2x^{2n}}{2n}+\frac{2abx^{3n}}{3n}+\frac{b^2x^{4n}}{4n}}{c} & \text{for}\ d=0\\ -\frac{a^2c\log\left(\frac{c}{d}+x^n\right)}{d^2n}+\frac{a^2x^n}{dn}+\frac{2abc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{2abcx^n}{d^2n}+\frac{abx^{2n}}{dn}-\frac{b^2c^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{b^2c^2x^n}{d^3n}-\frac{b^2cx^{2n}}{2d^2n}+\frac{b^2x^{3n}}{3dn} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**2*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a + b)**2*log(x)/(c
 + d), Eq(n, 0)), ((a**2*x**(2*n)/(2*n) + 2*a*b*x**(3*n)/(3*n) + b**2*x**(4
*n)/(4*n))/c, Eq(d, 0)), (-a**2*c*log(c/d + x**n)/(d**2*n) + a**2*x**n/(d*n
) + 2*a*b*c**2*log(c/d + x**n)/(d**3*n) - 2*a*b*c*x**n/(d**2*n) + a*b*x**(2
*n)/(d*n) - b**2*c**3*log(c/d + x**n)/(d**4*n) + b**2*c**2*x**n/(d**3*n) -
b**2*c*x**(2*n)/(2*d**2*n) + b**2*x**(3*n)/(3*d*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2*x^(2*n - 1)/(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}\,{\left(a+b\,x^n\right)}^2}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
[Out] int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

3.1041 $\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx$

Optimal. Leaf size=60

$$-\frac{(bc-ad)x^n}{d^2n}+\frac{bx^{2n}}{2dn}+\frac{c(bc-ad)\log(c+dx^n)}{d^3n}$$

```
[Out] -(-a*d+b*c)*x^n/d^2/n+1/2*b*x^(2*n)/d/n+c*(-a*d+b*c)*ln(c+d*x^n)/d^3/n
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {457, 78}

$$\frac{c(bc-ad)\log(c+dx^n)}{d^3n}-\frac{x^n(bc-ad)}{d^2n}+\frac{bx^{2n}}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] -(((b*c - a*d)*x^n)/(d^2*n)) + (b*x^(2*n))/(2*d*n) + (c*(b*c - a*d)*Log[c + 
 d*x^n])/(d^3*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)}{c+dx}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-bc+ad}{d^2}+\frac{bx}{d}+\frac{c(bc-ad)}{d^2(c+dx)}\right)\,dx, x, x^n\right)}{n}$$
$$= -\frac{(bc-ad)x^n}{d^2n}+\frac{bx^{2n}}{2dn}+\frac{c(bc-ad)\log\left(c+dx^n\right)}{d^3n}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.83

$$\frac{dx^n(-2bc+2ad+bdx^n)+2c(bc-ad)\log\left(c+dx^n\right)}{2d^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] (d*x^n*(-2*b*c + 2*a*d + b*d*x^n) + 2*c*(b*c - a*d)*Log[c + d*x^n])/(2*d^3*
n)
```

Maple [A]
time = 0.36, size = 65, normalized size = 1.08

method	result	size
norman	$\frac{(ad-bc)\mathrm{e}^{n\ln(x)}}{d^2n}+\frac{b\,\mathrm{e}^{2n\ln(x)}}{2dn}-\frac{c(ad-bc)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{d^3n}$	65
risch	$\frac{b\,x^{2n}}{2dn}+\frac{x^n a}{dn}-\frac{x^n bc}{d^2n}-\frac{c\ln\left(x^n+\frac{c}{d}\right)a}{d^2n}+\frac{c^2\ln\left(x^n+\frac{c}{d}\right)b}{d^3n}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(a*d-b*c)/n*exp(n*ln(x))+1/2*b/d/n*exp(n*ln(x))^2-c*(a*d-b*c)/d^3/n*l
n(c+d*exp(n*ln(x)))
```

Maxima [A]
time = 0.30, size = 83, normalized size = 1.38

$$a\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right)+\frac{1}{2}\,b\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/2*b*(2*c^2*log((d*x^n + c)
/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Fricas [A]
time = 5.15, size = 56, normalized size = 0.93

$$\frac{bd^2x^{2n} - 2(bcd - ad^2)x^n + 2(bc^2 - acd)\log(dx^n + c)}{2d^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^2*x^(2*n) - 2*(b*c*d - a*d^2)*x^n + 2*(b*c^2 - a*c*d)*log(d*x^n +
c))/(d^3*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(48) = 96$.
time = 12.47, size = 105, normalized size = 1.75

$$\begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d = 0 \wedge n = 0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n = 0 \\ \frac{\frac{ax^{2n}}{2n} + \frac{bx^{3n}}{3n}}{c} & \text{for } d = 0 \\ -\frac{ac\log\left(\frac{c}{d} + x^n\right)}{d^2n} + \frac{ax^n}{dn} + \frac{bc^2\log\left(\frac{c}{d} + x^n\right)}{d^3n} - \frac{bcx^n}{d^2n} + \frac{bx^{2n}}{2dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a + b)*log(x)/(c + d),
 Eq(n, 0)), ((a*x**(2*n)/(2*n) + b*x**(3*n)/(3*n))/c, Eq(d, 0)), (-a*c*log(
c/d + x**n)/(d**2*n) + a*x**n/(d*n) + b*c**2*log(c/d + x**n)/(d**3*n) - b*c
*x**n/(d**2*n) + b*x**(2*n)/(2*d*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)*x^(2*n - 1)/(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2n-1}\,(a+b\,x^n)}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n), x)
```

3.1042 $\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=54

$$-\frac{a\log(a+bx^n)}{b(bc-ad)n}+\frac{c\log(c+dx^n)}{d(bc-ad)n}$$

```
[Out] -a*ln(a+b*x^n)/b/(-a*d+b*c)/n+c*ln(c+d*x^n)/d/(-a*d+b*c)/n
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {457, 78}

$$\frac{c\log(c+dx^n)}{dn(bc-ad)}-\frac{a\log(a+bx^n)}{bn(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((a*Log[a + b*x^n])/(b*(b*c - a*d)*n)) + (c*Log[c + d*x^n])/(d*(b*c - a*d)
*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)} + \frac{c}{(bc-ad)(c+dx)}\right)\,dx, x, x^n\right)}{n}$$
$$= -\frac{a\log(a+bx^n)}{b(bc-ad)n} + \frac{c\log(c+dx^n)}{d(bc-ad)n}$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 0.81

$$-\frac{ad\log(a+bx^n) - bc\log(c+dx^n)}{b^2cdn - abd^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
[Out] -((a*d*Log[a + b*x^n] - b*c*Log[c + d*x^n])/(b^2*c*d*n - a*b*d^2*n))
```

Maple [A]
time = 0.40, size = 59, normalized size = 1.09

method	result	size
norman	$\frac{a\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{(ad-bc)bn} - \frac{c\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{dn(ad-bc)}$	59
risch	$\frac{\ln(x)}{bd} + \frac{\ln(x)c}{d(ad-bc)} - \frac{\ln(x)a}{(ad-bc)b} - \frac{c\ln\left(x^n+\frac{c}{d}\right)}{dn(ad-bc)} + \frac{a\ln\left(x^n+\frac{a}{b}\right)}{(ad-bc)bn}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
[Out] a/(a*d-b*c)/b/n*ln(a+b*exp(n*ln(x)))-c/d/n/(a*d-b*c)*ln(c+d*exp(n*ln(x)))
```

Maxima [A]
time = 0.32, size = 60, normalized size = 1.11

$$-\frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2cn - abdn} + \frac{c\log\left(\frac{dx^n+c}{d}\right)}{bcdn - ad^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -a*log((b*x^n + a)/b)/(b^2*c*n - a*b*d*n) + c*log((d*x^n + c)/d)/(b*c*d*n -
 a*d^2*n)
```

Fricas [A]
time = 2.75, size = 45, normalized size = 0.83

$$-\frac{ad\log(bx^n+a)-bc\log(dx^n+c)}{(b^2cd-abd^2)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(a*d*log(b*x^n + a) - b*c*log(d*x^n + c))/((b^2*c*d - a*b*d^2)*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2\,n-1}}{\left(a+b\,x^n\right)\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

3.1043 $\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=75

$$\frac{a}{b(bc-ad)n\,(a+bx^n)}+\frac{c\log\left(a+bx^n\right)}{(bc-ad)^2n}-\frac{c\log\left(c+dx^n\right)}{(bc-ad)^2n}$$

```
[Out] a/b/(-a*d+b*c)/n/(a+b*x^n)+c*ln(a+b*x^n)/(-a*d+b*c)^2/n-c*ln(c+d*x^n)/(-a*d
+b*c)^2/n
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 78}

$$\frac{a}{bn(bc-ad)\,(a+bx^n)}+\frac{c\log\left(a+bx^n\right)}{n(bc-ad)^2}-\frac{c\log\left(c+dx^n\right)}{n(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (c
*Log[c + d*x^n])/((b*c - a*d)^2*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^2}+\frac{bc}{(bc-ad)^2(a+bx)}-\frac{cd}{(bc-ad)^2(c+dx)}\right)dx, x, x^n\right)}{n}$$
$$= \frac{a}{b(bc-ad)n(a+bx^n)}+\frac{c\log(a+bx^n)}{(bc-ad)^2n}-\frac{c\log(c+dx^n)}{(bc-ad)^2n}$$

Mathematica [A]
time = 0.09, size = 75, normalized size = 1.00

$$\frac{a}{b(bc-ad)n(a+bx^n)}+\frac{c\log(a+bx^n)}{(bc-ad)^2n}-\frac{c\log(c+dx^n)}{(bc-ad)^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (c
*Log[c + d*x^n])/((b*c - a*d)^2*n)
```

Maple [A]
time = 0.40, size = 107, normalized size = 1.43

method	result	size
risch	$-\frac{a}{(ad-bc)bn(a+b\,x^n)}-\frac{c\ln\left(x^n+\frac{c}{d}\right)}{n(a^2d^2-2abcd+b^2c^2)}+\frac{c\ln\left(x^n+\frac{a}{b}\right)}{n(a^2d^2-2abcd+b^2c^2)}$	107
norman	$\frac{e^{n\ln(x)}}{(ad-bc)n\left(a+b\,e^{n\ln(x)}\right)}+\frac{c\ln\left(a+b\,e^{n\ln(x)}\right)}{n(a^2d^2-2abcd+b^2c^2)}-\frac{c\ln\left(c+d\,e^{n\ln(x)}\right)}{n(a^2d^2-2abcd+b^2c^2)}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -a/(a*d-b*c)/b/n/(a+b*x^n)-c/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(x^n+c/d)+c/n/
(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(x^n+a/b)
```

Maxima [A]
time = 0.31, size = 121, normalized size = 1.61

$$\frac{c\log\left(\frac{bx^n+a}{b}\right)}{b^2c^2n-2\,abcdn+a^2d^2n}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{b^2c^2n-2\,abcdn+a^2d^2n}+\frac{a}{ab^2cn-a^2bdn+(b^3cn-ab^2dn)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c*log((b*x^n + a)/b)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) - c*log((d*x^n +
 c)/d)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) + a/(a*b^2*c*n - a^2*b*d*n + (
b^3*c*n - a*b^2*d*n)*x^n)
```

Fricas [A]
time = 3.53, size = 120, normalized size = 1.60

$$\frac{abc - a^2d + (b^2cx^n + abc)\log(bx^n + a) - (b^2cx^n + abc)\log(dx^n + c)}{(b^4c^2 - 2\,ab^3cd + a^2b^2d^2)nx^n + (ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a*b*c - a^2*d + (b^2*c*x^n + a*b*c)*log(b*x^n + a) - (b^2*c*x^n + a*b*c)*l
og(d*x^n + c))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^n + (a*b^3*c^2 -
2*a^2*b^2*c*d + a^3*b*d^2)*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a+b\,x^n\right)}^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1044 $\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx$

Optimal. Leaf size=105

$$\frac{a}{2b(bc-ad)n\,(a+bx^n)^2}-\frac{c}{(bc-ad)^2n\,(a+bx^n)}-\frac{cd\log\left(a+bx^n\right)}{(bc-ad)^3n}+\frac{cd\log\left(c+dx^n\right)}{(bc-ad)^3n}$$

```
[Out] 1/2*a/b/(-a*d+b*c)/n/(a+b*x^n)^2-c/(-a*d+b*c)^2/n/(a+b*x^n)-c*d*ln(a+b*x^n)
/(-a*d+b*c)^3/n+c*d*ln(c+d*x^n)/(-a*d+b*c)^3/n
```

Rubi [A]
time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {457, 78}

$$\frac{a}{2bn(bc-ad)\,(a+bx^n)^2}-\frac{c}{n(bc-ad)^2\,(a+bx^n)}-\frac{cd\log\left(a+bx^n\right)}{n(bc-ad)^3}+\frac{cd\log\left(c+dx^n\right)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*n*(a + b*x^n)^2) - c/((b*c - a*d)^2*n*(a + b*x^n)) - (c*
d*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*n
)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^3(c+dx)}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^3} + \frac{bc}{(bc-ad)^2(a+bx)^2} - \frac{bcd}{(bc-ad)^3(a+bx)} + \frac{cd^2}{(bc-ad)^3(c+dx)}\right) dx, x\right.}{n}$$

$$= \frac{a}{2b(bc-ad)n\,(a+bx^n)^2} - \frac{c}{(bc-ad)^2n\,(a+bx^n)} - \frac{cd\log(a+bx^n)}{(bc-ad)^3n} + \frac{cd\log(c}{(bc-}$$

Mathematica [A]
time = 0.12, size = 100, normalized size = 0.95

$$\frac{-abc-a^2d-2b^2cx^n}{2b(bc-ad)^2n\,(a+bx^n)^2} - \frac{cd\log(a+bx^n)}{(bc-ad)^3n} + \frac{cd\log(c+dx^n)}{(bc-ad)^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] (-(a*b*c) - a^2*d - 2*b^2*c*x^n)/(2*b*(b*c - a*d)^2*n*(a + b*x^n)^2) - (c*d
*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*n)
```

Maple [A]
time = 0.41, size = 157, normalized size = 1.50

method	result	si
risch	$-\frac{2b^2c\,x^n+a^2d+abc}{2(ad-bc)^2bn(a+b\,x^n)^2} - \frac{cd\ln\left(x^n+\frac{c}{d}\right)}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)} + \frac{cd\ln\left(x^n+\frac{a}{b}\right)}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$	15
norman	$\frac{-\frac{bc\,e^{n\ln(x)}}{\left(a^2d^2-2abcd+b^2c^2\right)n}+\frac{a\left(-abd-b^2c\right)}{2\left(a^2d^2-2abcd+b^2c^2\right)b^2n}}{\left(a+b\,e^{n\ln(x)}\right)^2} + \frac{cd\ln\left(a+b\,e^{n\ln(x)}\right)}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)} - \frac{cd\ln\left(c+d\,e^{n\ln(x)}\right)}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*b^2*c*x^n+a^2*d+a*b*c)/(a*d-b*c)^2/b/n/(a+b*x^n)^2-c*d/n/(a^3*d^3-3
*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+c/d)+c*d/n/(a^3*d^3-3*a^2*b*c*d^
2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+a/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(103) = 206.
time = 0.31, size = 243, normalized size = 2.31

$$-\frac{cd\log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n} + \frac{cd\log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n} - \frac{2\,b^2cx^n+abc+a^2d}{2\left(a^2b^3c^2n-2\,a^3b^2cdn+a^4bd^2n+(b^5c^2n-2\,ab^4cdn+a^2b^3d^2n)x^{2n}+2\,(ab^4c^2n-2\,a^2b^3cdn+a^3b^2d^2n)x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -c*d*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^
3*d^3*n) + c*d*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*
d^2*n - a^3*d^3*n) - 1/2*(2*b^2*c*x^n + a*b*c + a^2*d)/(a^2*b^3*c^2*n - 2*a
^3*b^2*c*d*n + a^4*b*d^2*n + (b^5*c^2*n - 2*a*b^4*c*d*n + a^2*b^3*d^2*n)*x^
(2*n) + 2*(a*b^4*c^2*n - 2*a^2*b^3*c*d*n + a^3*b^2*d^2*n)*x^n)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(103) = 206$.
time = 2.23, size = 267, normalized size = 2.54

$$-\frac{ab^2c^2 - a^3d^2 + 2\,(b^3c^2 - ab^2cd)x^n + 2\,(b^3cdx^{2\,n} + 2\,ab^2cdx^n + a^2bcd)\log\left(bx^n + a\right) - 2\,(b^3cdx^{2\,n} + 2\,ab^2cdx^n + a^2bcd)\log\left(dx^n + c\right)}{2\,((b^6c^3 - 3\,ab^5c^2d + 3\,a^2b^4cd^2 - a^3b^3d^3)nx^{2\,n} + 2\,(ab^5c^3 - 3\,a^2b^4c^2d + 3\,a^3b^3cd^2 - a^4b^2d^3)nx^n + (a^2b^4c^3 - 3\,a^3b^3c^2d + 3\,a^4b^2cd^2 - a^5bd^3)n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*c^2 - a^3*d^2 + 2*(b^3*c^2 - a*b^2*c*d)*x^n + 2*(b^3*c*d*x^(2*n
) + 2*a*b^2*c*d*x^n + a^2*b*c*d)*log(b*x^n + a) - 2*(b^3*c*d*x^(2*n) + 2*a*
b^2*c*d*x^n + a^2*b*c*d)*log(d*x^n + c))/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*
b^4*c*d^2 - a^3*b^3*d^3)*n*x^(2*n) + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3
*b^3*c*d^2 - a^4*b^2*d^3)*n*x^n + (a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^
2*c*d^2 - a^5*b*d^3)*n)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a+b\,x^n\right)}^3\,\left(c+d\,x^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

3.1045 $\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$

Optimal. Leaf size=158

$$\frac{c(bc-ad)^3x^n}{d^5n}-\frac{(bc-ad)^3x^{2n}}{2d^4n}+\frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{3n}}{3d^3n}-\frac{b^2(bc-3ad)x^{4n}}{4d^2n}+\frac{b^3x^{5n}}{5dn}-\frac{c^2(bc-ad)^3\log(c+$$

```
[Out] c*(-a*d+b*c)^3*x^n/d^5/n-1/2*(-a*d+b*c)^3*x^(2*n)/d^4/n+1/3*b*(3*a^2*d^2-3*
a*b*c*d+b^2*c^2)*x^(3*n)/d^3/n-1/4*b^2*(-3*a*d+b*c)*x^(4*n)/d^2/n+1/5*b^3*x
^(5*n)/d/n-c^2*(-a*d+b*c)^3*ln(c+d*x^n)/d^6/n
```

Rubi [A]
time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {457, 90}

$$\frac{bx^{3n}(3a^2d^2-3abcd+b^2c^2)}{3d^3n}-\frac{b^2x^{4n}(bc-3ad)}{4d^2n}-\frac{c^2(bc-ad)^3\log(c+dx^n)}{d^6n}+\frac{cx^n(bc-ad)^3}{d^5n}-\frac{x^{2n}(bc-ad)^3}{2d^4n}+\frac{b^3x^{5n}}{5dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (c*(b*c - a*d)^3*x^n)/(d^5*n) - ((b*c - a*d)^3*x^(2*n))/(2*d^4*n) + (b*(b^2
*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n))/(3*d^3*n) - (b^2*(b*c - 3*a*d)*x^(4*
n))/(4*d^2*n) + (b^3*x^(5*n))/(5*d*n) - (c^2*(b*c - a*d)^3*Log[c + d*x^n])/
(d^6*n)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^3}{c+dx}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{c(bc-ad)^3}{d^5} + \frac{(-bc+ad)^3x}{d^4} + \frac{b(b^2c^2-3abcd+3a^2d^2)x^2}{d^3} - \frac{b^2(bc-3ad)x^3}{d^2} + \frac{b^3x^4}{d} - \frac{c^2(bc-a}{d^5(c+c}\right.\right.}{n}$$
$$= \frac{c(bc-ad)^3x^n}{d^5n} - \frac{(bc-ad)^3x^{2n}}{2d^4n} + \frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{3n}}{3d^3n} - \frac{b^2(bc-3ad)x^{4n}}{4d^2n}$$

Mathematica [A]
time = 0.15, size = 185, normalized size = 1.17

$$\frac{dx^n(30a^3d^3(-2c+dx^n)+30a^2bd^2(6c^2-3cdx^n+2d^2x^{2n})+15ab^2d(-12c^3+6c^2dx^n-4cd^2x^{2n}+3d^3x^{3n})+b^3(60c^4-30c^3dx^n+20c^2d^2x^{2n}-15cd^3x^{3n}+12d^4x^{4n}))-60c^2(bc-ad)^3\log(c+dx^n)}{60d^6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (d*x^n*(30*a^3*d^3*(-2*c + d*x^n) + 30*a^2*b*d^2*(6*c^2 - 3*c*d*x^n + 2*d^2
*x^(2*n)) + 15*a*b^2*d*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*d^3*x^(
3*n)) + b^3*(60*c^4 - 30*c^3*d*x^n + 20*c^2*d^2*x^(2*n) - 15*c*d^3*x^(3*n)
+ 12*d^4*x^(4*n))) - 60*c^2*(b*c - a*d)^3*Log[c + d*x^n])/(60*d^6*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(150) = 300$.
time = 0.37, size = 342, normalized size = 2.16

method	result
risch	$\frac{b^3x^{5n}}{5dn} + \frac{3b^2x^{4n}a}{4dn} - \frac{b^3x^{4n}c}{4d^2n} + \frac{b\,x^{3n}a^2}{dn} - \frac{b^2x^{3n}ac}{d^2n} + \frac{b^3x^{3n}c^2}{3d^3n} + \frac{x^{2n}a^3}{2dn} - \frac{3x^{2n}a^2bc}{2d^2n} + \frac{3x^{2n}a\,b^2c^2}{2d^3n} - \frac{x^{2n}b^3c^3}{2d^4n} - \frac{c\,x^na^3}{d^2n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^3/d/n*(x^n)^5+3/4*b^2/d/n*(x^n)^4*a-1/4*b^3/d^2/n*(x^n)^4*c+b/d/n*(x^
n)^3*a^2-b^2/d^2/n*(x^n)^3*a*c+1/3*b^3/d^3/n*(x^n)^3*c^2+1/2/d/n*(x^n)^2*a^
3-3/2/d^2/n*(x^n)^2*a^2*b*c+3/2/d^3/n*(x^n)^2*a*b^2*c^2-1/2/d^4/n*(x^n)^2*b
^3*c^3-c/d^2/n*x^n*a^3+3*c^2/d^3/n*x^n*a^2*b-3*c^3/d^4/n*x^n*a*b^2+c^4/d^5/
n*x^n*b^3+c^2/d^3/n*ln(x^n+c/d)*a^3-3*c^3/d^4/n*ln(x^n+c/d)*a^2*b+3*c^4/d^5
/n*ln(x^n+c/d)*a*b^2-c^5/d^6/n*ln(x^n+c/d)*b^3
```

Maxima [A]
time = 0.31, size = 286, normalized size = 1.81

$$-\frac{1}{60}\,b^3\left(\frac{60\,c^5\log\left(\frac{dx^n+c}{d}\right)}{d^6n} - \frac{12\,d^4x^{5\,n} - 15\,cd^3x^{4\,n} + 20\,c^2d^2x^{3\,n} - 30\,c^3dx^{2\,n} + 60\,c^4x^n}{d^5n}\right) + \frac{1}{4}\,ab^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n} + \frac{3\,d^3x^{4\,n} - 4\,cd^2x^{3\,n} + 6\,c^2dx^{2\,n} - 12\,c^3x^n}{d^4n}\right) - \frac{1}{2}\,a^2b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2\,d^2x^{3\,n} - 3\,cdx^{2\,n} + 6\,c^2x^n}{d^3n}\right) + \frac{1}{2}\,a^3\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2\,n} - 2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -1/60*b^3*(60*c^5*log((d*x^n + c)/d)/(d^6*n) - (12*d^4*x^(5*n) - 15*c*d^3*x
^(4*n) + 20*c^2*d^2*x^(3*n) - 30*c^3*d*x^(2*n) + 60*c^4*x^n)/(d^5*n)) + 1/4
*a*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n
) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/2*a^2*b*(6*c^3*log((d*x^n +
c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1/2*
a^3*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Fricas [A]
time = 2.66, size = 230, normalized size = 1.46

$$\frac{12\,b^3d^5x^{5\,n}-15\,(b^3cd^4-3\,ab^2d^5)x^{4\,n}+20\,(b^3c^2d^3-3\,ab^2cd^4+3\,a^2bd^5)x^{3\,n}-30\,(b^3c^3d^2-3\,ab^2c^2d^3+3\,a^2bcd^4-a^3d^5)x^{2\,n}+60\,(b^3c^4d-3\,ab^2c^3d^2+3\,a^2bc^2d^3-a^3cd^4)x^n-60\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\log\left(dx^n+c\right)}{60\,d^6n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^3*d^5*x^(5*n) - 15*(b^3*c*d^4 - 3*a*b^2*d^5)*x^(4*n) + 20*(b^3*c
^2*d^3 - 3*a*b^2*c*d^4 + 3*a^2*b*d^5)*x^(3*n) - 30*(b^3*c^3*d^2 - 3*a*b^2*c
^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^(2*n) + 60*(b^3*c^4*d - 3*a*b^2*c^3*d^2
 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^n - 60*(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b
*c^3*d^2 - a^3*c^2*d^3)*log(d*x^n + c))/(d^6*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(138) = 276$.
time = 55.06, size = 401, normalized size = 2.54

$$\begin{cases} \frac{(a+b)^3\log(x)}{c} & \text{for } d=0\wedge n=0 \\ \frac{(a+b)^3\log(x)}{c+d} & \text{for } n=0 \\ \frac{\frac{a^3x^{3n}}{3n}+\frac{3a^2bx^{4n}}{4n}+\frac{3ab^2x^{5n}}{5n}+\frac{b^3x^{6n}}{6n}}{c} & \text{for } d=0 \\ \frac{a^3c^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{a^3cx^n}{d^2n}+\frac{a^3x^{2n}}{2dn}-\frac{3a^2bc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{3a^2bc^2x^n}{d^3n}-\frac{3a^2bcx^{2n}}{2d^2n}+\frac{a^2bx^{3n}}{dn}+\frac{3ab^2c^4\log\left(\frac{c}{d}+x^n\right)}{d^5n}-\frac{3ab^2c^3x^n}{d^4n}+\frac{3ab^2c^2x^{2n}}{2d^3n}-\frac{ab^2cx^{3n}}{d^2n}+\frac{3ab^2x^{4n}}{4dn}-\frac{b^3c^5\log\left(\frac{c}{d}+x^n\right)}{d^6n}+\frac{b^3c^4x^n}{d^5n}-\frac{b^3c^3x^{2n}}{2d^4n}+\frac{b^3c^2x^{3n}}{3d^3n}-\frac{b^3cx^{4n}}{4d^2n}+\frac{b^3x^{5n}}{5dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**3*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a + b)**3*log(x)/(c
 + d), Eq(n, 0)), ((a**3*x**(3*n)/(3*n) + 3*a**2*b*x**(4*n)/(4*n) + 3*a*b**
2*x**(5*n)/(5*n) + b**3*x**(6*n)/(6*n))/c, Eq(d, 0)), (a**3*c**2*log(c/d +
x**n)/(d**3*n) - a**3*c*x**n/(d**2*n) + a**3*x**(2*n)/(2*d*n) - 3*a**2*b*c*
*3*log(c/d + x**n)/(d**4*n) + 3*a**2*b*c**2*x**n/(d**3*n) - 3*a**2*b*c*x**(
2*n)/(2*d**2*n) + a**2*b*x**(3*n)/(d*n) + 3*a*b**2*c**4*log(c/d + x**n)/(d*
*5*n) - 3*a*b**2*c**3*x**n/(d**4*n) + 3*a*b**2*c**2*x**(2*n)/(2*d**3*n) - a
*b**2*c*x**(3*n)/(d**2*n) + 3*a*b**2*x**(4*n)/(4*d*n) - b**3*c**5*log(c/d +
 x**n)/(d**6*n) + b**3*c**4*x**n/(d**5*n) - b**3*c**3*x**(2*n)/(2*d**4*n) +
 b**3*c**2*x**(3*n)/(3*d**3*n) - b**3*c*x**(4*n)/(4*d**2*n) + b**3*x**(5*n)
/(5*d*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
[Out] integrate((b*x^n + a)^3*x^(3*n - 1)/(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}\,{\left(a+b\,x^n\right)}^3}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

3.1046 $\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx$

Optimal. Leaf size=118

$$-\frac{c(bc-ad)^2x^n}{d^4n}+\frac{(bc-ad)^2x^{2n}}{2d^3n}-\frac{b(bc-2ad)x^{3n}}{3d^2n}+\frac{b^2x^{4n}}{4dn}+\frac{c^2(bc-ad)^2\log\left(c+dx^n\right)}{d^5n}$$

```
[Out] -c*(-a*d+b*c)^2*x^n/d^4/n+1/2*(-a*d+b*c)^2*x^(2*n)/d^3/n-1/3*b*(-2*a*d+b*c)
*x^(3*n)/d^2/n+1/4*b^2*x^(4*n)/d/n+c^2*(-a*d+b*c)^2*ln(c+d*x^n)/d^5/n
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 90}

$$\frac{c^2(bc-ad)^2\log\left(c+dx^n\right)}{d^5n}-\frac{cx^n(bc-ad)^2}{d^4n}+\frac{x^{2n}(bc-ad)^2}{2d^3n}-\frac{bx^{3n}(bc-2ad)}{3d^2n}+\frac{b^2x^{4n}}{4dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] -((c*(b*c - a*d)^2*x^n)/(d^4*n)) + ((b*c - a*d)^2*x^(2*n))/(2*d^3*n) - (b*(
b*c - 2*a*d)*x^(3*n))/(3*d^2*n) + (b^2*x^(4*n))/(4*d*n) + (c^2*(b*c - a*d)^
2*Log[c + d*x^n])/(d^5*n)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^2}{c+dx}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2}{d^4} + \frac{(-bc+ad)^2x}{d^3} - \frac{b(bc-2ad)x^2}{d^2} + \frac{b^2x^3}{d} + \frac{c^2(bc-ad)^2}{d^4(c+dx)}\right)dx, x, x^n\right)}{n}$$
$$= -\frac{c(bc-ad)^2x^n}{d^4n} + \frac{(bc-ad)^2x^{2n}}{2d^3n} - \frac{b(bc-2ad)x^{3n}}{3d^2n} + \frac{b^2x^{4n}}{4dn} + \frac{c^2(bc-ad)^2\log(c-}{d^5n}$$

Mathematica [A]
time = 0.11, size = 125, normalized size = 1.06

$$\frac{dx^n(6a^2d^2(-2c+dx^n) + 4abd(6c^2-3cdx^n+2d^2x^{2n}) + b^2(-12c^3+6c^2dx^n-4cd^2x^{2n}+3d^3x^{3n})) + 12c^2(bc-ad)^2\log(c+dx^n)}{12d^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] (d*x^n*(6*a^2*d^2*(-2*c + d*x^n) + 4*a*b*d*(6*c^2 - 3*c*d*x^n + 2*d^2*x^(2*
n)) + b^2*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*d^3*x^(3*n))) + 12*c
^2*(b*c - a*d)^2*Log[c + d*x^n])/(12*d^5*n)
```

Maple [A]
time = 0.37, size = 157, normalized size = 1.33

method	result
norman	$\frac{b^2 e^{4n\ln(x)}}{4dn} + \frac{(a^2d^2-2abcd+b^2c^2)e^{2n\ln(x)}}{2d^3n} + \frac{b(2ad-bc)e^{3n\ln(x)}}{3d^2n} - \frac{c(a^2d^2-2abcd+b^2c^2)e^{n\ln(x)}}{d^4n} + \frac{c^2(a^2d^2-2abcd+b^2c^2)\ln(c+}{d^5n}$
risch	$\frac{b^2x^{4n}}{4dn} + \frac{2b\,x^{3n}a}{3dn} - \frac{b^2x^{3n}c}{3d^2n} + \frac{x^{2n}a^2}{2dn} - \frac{x^{2n}abc}{d^2n} + \frac{x^{2n}b^2c^2}{2d^3n} - \frac{c\,x^na^2}{d^2n} + \frac{2c^2x^nab}{d^3n} - \frac{c^3x^nb^2}{d^4n} + \frac{c^2\ln(x^n+\frac{c}{d})a^2}{d^3n} - \frac{2c^3\ln(x}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^2/d/n*exp(n*ln(x))^4+1/2/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(x
))^2+1/3*b*(2*a*d-b*c)/d^2/n*exp(n*ln(x))^3-c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d
^4/n*exp(n*ln(x))+c^2/d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*ln(c+d*exp(n*ln(x))
)
```

Maxima [A]
time = 0.33, size = 192, normalized size = 1.63

$$\frac{1}{12}b^2\left(\frac{12c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n} + \frac{3d^3x^{4n}-4cd^2x^{3n}+6c^2dx^{2n}-12c^3x^n}{d^4n}\right) - \frac{1}{3}ab\left(\frac{6c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2d^2x^{3n}-3cdx^{2n}+6c^2x^n}{d^3n}\right) + \frac{1}{2}a^2\left(\frac{2c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2n}-2cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] 1/12*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3
*n) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/3*a*b*(6*c^3*log((d*x^n +
c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1/2*
a^2*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Fricas [A]
time = 2.61, size = 146, normalized size = 1.24

$$\frac{3\,b^2d^4x^{4\,n}-4\,(b^2cd^3-2\,abd^4)x^{3\,n}+6\,(b^2c^2d^2-2\,abcd^3+a^2d^4)x^{2\,n}-12\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)x^n+12\,(b^2c^4-2\,abc^3d+a^2c^2d^2)\log\left(dx^n+c\right)}{12\,d^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*d^4*x^(4*n) - 4*(b^2*c*d^3 - 2*a*b*d^4)*x^(3*n) + 6*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*x^(2*n) - 12*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*
d^3)*x^n + 12*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*log(d*x^n + c))/(d^5*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(99) = 198$.
time = 27.03, size = 258, normalized size = 2.19

$$\begin{cases} \frac{(a+b)^2\log(x)}{c} & \text{for}\ d=0\wedge n=0 \\ \frac{(a+b)^2\log(x)}{c+d} & \text{for}\ n=0 \\ \frac{\frac{a^2x^{3n}}{3n}+\frac{abx^{4n}}{2n}+\frac{b^2x^{5n}}{5n}}{c} & \text{for}\ d=0 \\ \frac{a^2c^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{a^2cx^n}{d^2n}+\frac{a^2x^{2n}}{2dn}-\frac{2abc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{2abc^2x^n}{d^3n}-\frac{abcx^{2n}}{d^2n}+\frac{2abx^{3n}}{3dn}+\frac{b^2c^4\log\left(\frac{c}{d}+x^n\right)}{d^5n}-\frac{b^2c^3x^n}{d^4n}+\frac{b^2c^2x^{2n}}{2d^3n}-\frac{b^2cx^{3n}}{3d^2n}+\frac{b^2x^{4n}}{4dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**2*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a + b)**2*log(x)/(c
 + d), Eq(n, 0)), ((a**2*x**(3*n)/(3*n) + a*b*x**(4*n)/(2*n) + b**2*x**(5*n
)/(5*n))/c, Eq(d, 0)), (a**2*c**2*log(c/d + x**n)/(d**3*n) - a**2*c*x**n/(d
**2*n) + a**2*x**(2*n)/(2*d*n) - 2*a*b*c**3*log(c/d + x**n)/(d**4*n) + 2*a*
b*c**2*x**n/(d**3*n) - a*b*c*x**(2*n)/(d**2*n) + 2*a*b*x**(3*n)/(3*d*n) + b
**2*c**4*log(c/d + x**n)/(d**5*n) - b**2*c**3*x**n/(d**4*n) + b**2*c**2*x**
(2*n)/(2*d**3*n) - b**2*c*x**(3*n)/(3*d**2*n) + b**2*x**(4*n)/(4*d*n), True
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
[Out] integrate((b*x^n + a)^2*x^(3*n - 1)/(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}\,{\left(a+b\,x^n\right)}^2}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

3.1047 $\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx$

Optimal. Leaf size=86

$$\frac{c(bc-ad)x^n}{d^3n}-\frac{(bc-ad)x^{2n}}{2d^2n}+\frac{bx^{3n}}{3dn}-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}$$

```
[Out] c*(-a*d+b*c)*x^n/d^3/n-1/2*(-a*d+b*c)*x^(2*n)/d^2/n+1/3*b*x^(3*n)/d/n-c^2*(
-a*d+b*c)*ln(c+d*x^n)/d^4/n
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {457, 78}

$$-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}+\frac{cx^n(bc-ad)}{d^3n}-\frac{x^{2n}(bc-ad)}{2d^2n}+\frac{bx^{3n}}{3dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] (c*(b*c - a*d)*x^n)/(d^3*n) - ((b*c - a*d)*x^(2*n))/(2*d^2*n) + (b*x^(3*n))
/(3*d*n) - (c^2*(b*c - a*d)*Log[c + d*x^n])/(d^4*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)}{c+dx}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{c(bc-ad)}{d^3} + \frac{(-bc+ad)x}{d^2} + \frac{bx^2}{d} - \frac{c^2(bc-ad)}{d^3(c+dx)}\right) dx, x, x^n\right)}{n}$$
$$= \frac{c(bc-ad)x^n}{d^3n} - \frac{(bc-ad)x^{2n}}{2d^2n} + \frac{bx^{3n}}{3dn} - \frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}$$

Mathematica [A]
time = 0.07, size = 76, normalized size = 0.88

$$\frac{dx^n(3ad(-2c+dx^n)+b(6c^2-3cdx^n+2d^2x^{2n}))+6c^2(-bc+ad)\log(c+dx^n)}{6d^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
[Out] (d*x^n*(3*a*d*(-2*c + d*x^n) + b*(6*c^2 - 3*c*d*x^n + 2*d^2*x^(2*n))) + 6*c
^2*(-(b*c) + a*d)*Log[c + d*x^n])/(6*d^4*n)
```

Maple [A]
time = 0.34, size = 91, normalized size = 1.06

method	result	size
norman	$\frac{b\,\mathrm{e}^{3n\ln(x)}}{3dn} + \frac{(ad-bc)\mathrm{e}^{2n\ln(x)}}{2d^2n} - \frac{c(ad-bc)\mathrm{e}^{n\ln(x)}}{d^3n} + \frac{c^2(ad-bc)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{d^4n}$	91
risch	$\frac{b\,x^{3n}}{3dn} + \frac{x^{2n}a}{2dn} - \frac{x^{2n}bc}{2d^2n} - \frac{c\,x^n a}{d^2n} + \frac{c^2x^n b}{d^3n} + \frac{c^2\ln\left(x^n+\frac{c}{d}\right)a}{d^3n} - \frac{c^3\ln\left(x^n+\frac{c}{d}\right)b}{d^4n}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
[Out] 1/3*b/d/n*exp(n*ln(x))^3+1/2/d^2*(a*d-b*c)/n*exp(n*ln(x))^2-c*(a*d-b*c)/d^3
/n*exp(n*ln(x))+c^2/d^4*(a*d-b*c)/n*ln(c+d*exp(n*ln(x)))
```

Maxima [A]
time = 0.37, size = 112, normalized size = 1.30

$$-\frac{1}{6}\,b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2\,d^2x^{3\,n} - 3\,cdx^{2\,n} + 6\,c^2x^n}{d^3n}\right) + \frac{1}{2}\,a\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2\,n} - 2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -1/6*b*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) +
 6*c^2*x^n)/(d^3*n)) + 1/2*a*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n)
 - 2*c*x^n)/(d^2*n))
```

Fricas [A]
time = 2.79, size = 82, normalized size = 0.95

$$\frac{2\,bd^3x^{3\,n}-3\,(bcd^2-ad^3)x^{2\,n}+6\,(bc^2d-acd^2)x^n-6\,(bc^3-ac^2d)\log\left(dx^n+c\right)}{6\,d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d^3*x^(3*n) - 3*(b*c*d^2 - a*d^3)*x^(2*n) + 6*(b*c^2*d - a*c*d^2)*
x^n - 6*(b*c^3 - a*c^2*d)*log(d*x^n + c))/(d^4*n)
```

Sympy [A]
time = 12.60, size = 139, normalized size = 1.62

$$\begin{cases} \frac{(a+b)\log\left(x\right)}{c} & \text{for}\: d=0\wedge n=0 \\ \frac{(a+b)\log\left(x\right)}{c+d} & \text{for}\: n=0 \\ \frac{\frac{ax^{3n}}{3n}+\frac{bx^{4n}}{4n}}{c} & \text{for}\: d=0 \\ \frac{ac^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{acx^n}{d^2n}+\frac{ax^{2n}}{2dn}-\frac{bc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{bc^2x^n}{d^3n}-\frac{bcx^{2n}}{2d^2n}+\frac{bx^{3n}}{3dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a + b)*log(x)/(c + d),
 Eq(n, 0)), ((a*x**(3*n)/(3*n) + b*x**(4*n)/(4*n))/c, Eq(d, 0)), (a*c**2*lo
g(c/d + x**n)/(d**3*n) - a*c*x**n/(d**2*n) + a*x**(2*n)/(2*d*n) - b*c**3*lo
g(c/d + x**n)/(d**4*n) + b*c**2*x**n/(d**3*n) - b*c*x**(2*n)/(2*d**2*n) + b
*x**(3*n)/(3*d*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)*x^(3*n - 1)/(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}\,\left(a+b\,x^n\right)}{c+d\,x^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n),x)
[Out] int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n), x)
```

3.1048 $\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=71

$$\frac{x^n}{bdn}+\frac{a^2\log\left(a+bx^n\right)}{b^2(bc-ad)n}-\frac{c^2\log\left(c+dx^n\right)}{d^2(bc-ad)n}$$

```
[Out] x^n/b/d/n+a^2*ln(a+b*x^n)/b^2/(-a*d+b*c)/n-c^2*ln(c+d*x^n)/d^2/(-a*d+b*c)/n
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 84}

$$\frac{a^2\log\left(a+bx^n\right)}{b^2n(bc-ad)}-\frac{c^2\log\left(c+dx^n\right)}{d^2n(bc-ad)}+\frac{x^n}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] x^n/(b*d*n) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)*n) - (c^2*Log[c + d*x^n
])/(d^2*(b*c - a*d)*n)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{\left(a+bx^n\right)\left(c+dx^n\right)}\,dx&=\frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx,x,x^n\right)}{n}\\&=\frac{\text{Subst}\left(\int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)\,dx,x,x^n\right)}{n}\\&=\frac{x^n}{bdn}+\frac{a^2\log\left(a+bx^n\right)}{b^2(bc-ad)n}-\frac{c^2\log\left(c+dx^n\right)}{d^2(bc-ad)n}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.93

$$\frac{a^2d^2\log(a+bx^n)+b(d(bc-ad)x^n-bc^2\log(c+dx^n))}{b^2d^2(bc-ad)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^n] + b*(d*(b*c - a*d)*x^n - b*c^2*Log[c + d*x^n]))/(b^
2*d^2*(b*c - a*d)*n)
```

Maple [A]
time = 0.40, size = 78, normalized size = 1.10

method	result	size
norman	$\frac{e^{n\ln(x)}}{bdn}+\frac{c^2\ln\left(c+d\,e^{n\ln(x)}\right)}{d^2n(ad-bc)}-\frac{a^2\ln\left(a+b\,e^{n\ln(x)}\right)}{(ad-bc)b^2n}$	78
risch	$-\frac{\ln(x)a}{b^2d}-\frac{\ln(x)c}{b\,d^2}+\frac{x^n}{bdn}-\frac{\ln(x)c^2}{d^2(ad-bc)}+\frac{\ln(x)a^2}{(ad-bc)b^2}+\frac{c^2\ln\left(x^n+\frac{c}{d}\right)}{d^2n(ad-bc)}-\frac{a^2\ln\left(x^n+\frac{a}{b}\right)}{(ad-bc)b^2n}$	137

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d/n*exp(n*ln(x))+c^2/d^2/n/(a*d-b*c)*ln(c+d*exp(n*ln(x)))-a^2/(a*d-b*c)
/b^2/n*ln(a+b*exp(n*ln(x)))
```

Maxima [A]
time = 0.34, size = 81, normalized size = 1.14

$$\frac{a^2\log\left(\frac{bx^n+a}{b}\right)}{b^3cn-ab^2dn}-\frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{bcd^2n-ad^3n}+\frac{x^n}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*log((b*x^n + a)/b)/(b^3*c*n - a*b^2*d*n) - c^2*log((d*x^n + c)/d)/(b*c*
d^2*n - a*d^3*n) + x^n/(b*d*n)
```

Fricas [A]
time = 3.29, size = 74, normalized size = 1.04

$$\frac{a^2d^2\log(bx^n+a)-b^2c^2\log(dx^n+c)+(b^2cd-abd^2)x^n}{(b^3cd^2-ab^2d^3)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a^2*d^2*log(b*x^n + a) - b^2*c^2*log(d*x^n + c) + (b^2*c*d - a*b*d^2)*x^n)
/((b^3*c*d^2 - a*b^2*d^3)*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{\left(a+b\,x^n\right)\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

3.1049 $\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=95

$$-\frac{a^2}{b^2(bc-ad)n\,(a+bx^n)}-\frac{a(2bc-ad)\log\left(a+bx^n\right)}{b^2(bc-ad)^2n}+\frac{c^2\log\left(c+dx^n\right)}{d(bc-ad)^2n}$$

```
[Out] -a^2/b^2/(-a*d+b*c)/n/(a+b*x^n)-a*(-a*d+2*b*c)*ln(a+b*x^n)/b^2/(-a*d+b*c)^2
/n+c^2*ln(c+d*x^n)/d/(-a*d+b*c)^2/n
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 90}

$$-\frac{a^2}{b^2n(bc-ad)\left(a+bx^n\right)}-\frac{a(2bc-ad)\log\left(a+bx^n\right)}{b^2n(bc-ad)^2}+\frac{c^2\log\left(c+dx^n\right)}{dn(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] -(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/(
b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2*n)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^2}+\frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)}+\frac{c^2}{(bc-ad)^2(c+dx)}\right)dx, x, x^n\right)}{n}$$
$$= -\frac{a^2}{b^2(bc-ad)n(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2n} + \frac{c^2\log(c+dx^n)}{d(bc-ad)^2n}$$

Mathematica [A]
time = 0.10, size = 93, normalized size = 0.98

$$-\frac{a^2}{b^2(bc-ad)n(a+bx^n)} + \frac{a(-2bc+ad)\log(a+bx^n)}{b^2(bc-ad)^2n} + \frac{c^2\log(c+dx^n)}{d(-bc+ad)^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] -(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) + (a*(-2*b*c + a*d)*Log[a + b*x^n])/
(b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(-(b*c) + a*d)^2*n)
```

Maple [A]
time = 0.40, size = 125, normalized size = 1.32

method	result
norman	$\frac{a^2}{(ad-bc)b^2n(a+b\,e^{n\ln(x)})}+\frac{c^2\ln(c+d\,e^{n\ln(x)})}{dn(a^2d^2-2abcd+b^2c^2)}+\frac{a(ad-2bc)\ln(a+b\,e^{n\ln(x)})}{(a^2d^2-2abcd+b^2c^2)b^2n}$
risch	$\frac{\ln(x)}{b^2d}-\frac{\ln(x)c^2}{d(a^2d^2-2abcd+b^2c^2)}-\frac{\ln(x)a^2d}{(a^2d^2-2abcd+b^2c^2)b^2}+\frac{2\ln(x)ac}{(a^2d^2-2abcd+b^2c^2)b}+\frac{a^2}{(ad-bc)b^2n(a+b\,x^n)}+\frac{c^2\ln(x^n+\frac{c}{d}}{dn(a^2d^2-2abcd+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/(a*d-b*c)/b^2/n/(a+b*exp(n*ln(x)))+c^2/d/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*
ln(c+d*exp(n*ln(x)))+a*(a*d-2*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n*ln(a+b
*exp(n*ln(x)))
```

Maxima [A]
time = 0.29, size = 147, normalized size = 1.55

$$\frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^2dn-2\,abcd^2n+a^2d^3n} - \frac{a^2}{ab^3cn-a^2b^2dn+(b^4cn-ab^3dn)x^n} - \frac{(2\,abc-a^2d)\log\left(\frac{bx^n+a}{b}\right)}{b^4c^2n-2\,ab^3cdn+a^2b^2d^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c^2*log((d*x^n + c)/d)/(b^2*c^2*d*n - 2*a*b*c*d^2*n + a^2*d^3*n) - a^2/(a*b
^3*c*n - a^2*b^2*d*n + (b^4*c*n - a*b^3*d*n)*x^n) - (2*a*b*c - a^2*d)*log((
b*x^n + a)/b)/(b^4*c^2*n - 2*a*b^3*c*d*n + a^2*b^2*d^2*n)
```

Fricas [A]
time = 2.75, size = 166, normalized size = 1.75

$$-\frac{a^2bcd - a^3d^2 + (2\,a^2bcd - a^3d^2 + (2\,ab^2cd - a^2bd^2)x^n)\log\left(bx^n + a\right) - (b^3c^2x^n + ab^2c^2)\log\left(dx^n + c\right)}{(b^5c^2d - 2\,ab^4cd^2 + a^2b^3d^3)nx^n + (ab^4c^2d - 2\,a^2b^3cd^2 + a^3b^2d^3)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*d^2)*
x^n)*log(b*x^n + a) - (b^3*c^2*x^n + a*b^2*c^2)*log(d*x^n + c))/((b^5*c^2*d
 - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*n*x^n + (a*b^4*c^2*d - 2*a^2*b^3*c*d^2 + a^
3*b^2*d^3)*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{{\left(a+b\,x^n\right)}^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1050 $\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx$

Optimal. Leaf size=120

$$-\frac{a^2}{2b^2(bc-ad)n\left(a+bx^n\right)^2}+\frac{a(2bc-ad)}{b^2(bc-ad)^2n\left(a+bx^n\right)}+\frac{c^2\log\left(a+bx^n\right)}{(bc-ad)^3n}-\frac{c^2\log\left(c+dx^n\right)}{(bc-ad)^3n}$$

```
[Out] -1/2*a^2/b^2/(-a*d+b*c)/n/(a+b*x^n)^2+a*(-a*d+2*b*c)/b^2/(-a*d+b*c)^2/n/(a+
b*x^n)+c^2*ln(a+b*x^n)/(-a*d+b*c)^3/n-c^2*ln(c+d*x^n)/(-a*d+b*c)^3/n
```

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 90}

$$-\frac{a^2}{2b^2n(bc-ad)\left(a+bx^n\right)^2}+\frac{a(2bc-ad)}{b^2n(bc-ad)^2\left(a+bx^n\right)}+\frac{c^2\log\left(a+bx^n\right)}{n(bc-ad)^3}-\frac{c^2\log\left(c+dx^n\right)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] -1/2*a^2/(b^2*(b*c - a*d)*n*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c -
a*d)^2*n*(a + b*x^n)) + (c^2*Log[a + b*x^n])/((b*c - a*d)^3*n) - (c^2*Log[c
 + d*x^n])/((b*c - a*d)^3*n)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3\,(c+dx^n)}\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^2}{(a+bx)^3(c+dx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\mathrm{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^3} + \frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)^2} + \frac{bc^2}{(bc-ad)^3(a+bx)} - \frac{c^2 d}{(bc-ad)^3(c+dx)}\right) dx, x$$
$$= -\frac{a^2}{2b^2(bc-ad)n\,(a+bx^n)^2} + \frac{a(2bc-ad)}{b^2(bc-ad)^2n\,(a+bx^n)} + \frac{c^2\log(a+bx^n)}{(bc-ad)^3n} - \frac{c^2\,\mathrm{lo}}{(b}$$

Mathematica [A]
time = 0.19, size = 94, normalized size = 0.78

$$\frac{\frac{a(-bc+ad)(-3abc+a^2d-4b^2cx^n+2abdx^n)}{b^2(a+bx^n)^2} + 2c^2\log(a+bx^n) - 2c^2\log(c+dx^n)}{2(bc-ad)^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] ((a*(-(b*c) + a*d)*(-3*a*b*c + a^2*d - 4*b^2*c*x^n + 2*a*b*d*x^n))/(b^2*(a
+ b*x^n)^2) + 2*c^2*Log[a + b*x^n] - 2*c^2*Log[c + d*x^n])/(2*(b*c - a*d)^3
*n)
```

Maple [A]
time = 0.41, size = 169, normalized size = 1.41

method	result	siz
risch	$-\frac{a(2abd\,x^n-4b^2c\,x^n+a^2d-3abc)}{2(ad-bc)^2b^2n(a+b\,x^n)^2} + \frac{c^2\ln(x^n+\frac{c}{d})}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)} - \frac{c^2\ln(x^n+\frac{a}{b})}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$	16
norman	$\frac{\frac{(-ad+2bc)a\,e^{n\ln(x)}}{nb(a^2d^2-2abcd+b^2c^2)} + \frac{a^2(-ad+3bc)}{2(a^2d^2-2abcd+b^2c^2)b^2n}}{(a+b\,e^{n\ln(x)})^2} + \frac{c^2\ln(c+d\,e^{n\ln(x)})}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)} - \frac{c^2\ln(a+b\,e^{n\ln(x)})}{n(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*(2*a*b*d*x^n-4*b^2*c*x^n+a^2*d-3*a*b*c)/(a*d-b*c)^2/b^2/n/(a+b*x^n)^
2+c^2/n/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+c/d)-c^2/n/(a^
3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+a/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(118) = 236.
time = 0.30, size = 262, normalized size = 2.18

$$\frac{c^2\log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n} - \frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n} + \frac{3\,a^2bc - a^3d + 2\,(2\,ab^2c - a^2bd)x^n}{2\,(a^2b^4c^2n - 2\,a^3b^3cdn + a^4b^2d^2n + (b^6c^2n - 2\,ab^5cdn + a^2b^4d^2n)x^{2\,n} + 2\,(ab^5c^2n - 2\,a^2b^4cdn + a^3b^3d^2n)x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c^2*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^3
*d^3*n) - c^2*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d
^2*n - a^3*d^3*n) + 1/2*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^n)/(
a^2*b^4*c^2*n - 2*a^3*b^3*c*d*n + a^4*b^2*d^2*n + (b^6*c^2*n - 2*a*b^5*c*d*
n + a^2*b^4*d^2*n)*x^(2*n) + 2*(a*b^5*c^2*n - 2*a^2*b^4*c*d*n + a^3*b^3*d^2
*n)*x^n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(118) = 236$.
time = 2.41, size = 301, normalized size = 2.51

$$\frac{3\,a^2b^2c^2-4\,a^3bcd+a^4d^2+2\,(2\,ab^3c^2-3\,a^2b^2cd+a^3bd^2)x^n+2\,(b^4c^2x^{2\,n}+2\,ab^3c^2x^n+a^2b^2c^2)\log\left(bx^n+a\right)-2\,(b^4c^2x^{2\,n}+2\,ab^3c^2x^n+a^2b^2c^2)\log\left(dx^n+c\right)}{2\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)nx^{2\,n}+2\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3)nx^n+(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3)n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*d
 + a^3*b*d^2)*x^n + 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log
(b*x^n + a) - 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log(d*x^n
 + c))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*n*x^(2*n)
 + 2*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*n*x^n +
(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{{\left(a+b\,x^n\right)}^3\,\left(c+d\,x^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
[Out] int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

3.1051 $\int x^{13}(b+cx)^{13}(b+2cx)\,dx$

Optimal. Leaf size=14

$$\frac{1}{14}x^{14}(b+cx)^{14}$$

```
[Out] 1/14*x^14*(c*x+b)^14
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {75}

$$\frac{1}{14}x^{14}(b+cx)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

```
[Out] (x^14*(b + c*x)^14)/14
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx = \frac{1}{14}x^{14}(b+cx)^{14}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(14) = 28.
time = 0.00, size = 172, normalized size = 12.29

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
```

```
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(12) = 24$.
time = 0.28, size = 155, normalized size = 11.07

method	result
gosper	$\frac{1}{14}b^{14}x^{14} + b^{13}c\,x^{15} + \frac{13}{2}b^{12}c^2x^{16} + 26b^{11}c^3x^{17} + \frac{143}{2}b^{10}c^4x^{18} + 143b^9c^5x^{19} + \frac{429}{2}b^8c^6x^{20} + \frac{1716}{7}b^7c^7x$
default	$\frac{1}{14}b^{14}x^{14} + b^{13}c\,x^{15} + \frac{13}{2}b^{12}c^2x^{16} + 26b^{11}c^3x^{17} + \frac{143}{2}b^{10}c^4x^{18} + 143b^9c^5x^{19} + \frac{429}{2}b^8c^6x^{20} + \frac{1716}{7}b^7c^7x$
norman	$\frac{1}{14}b^{14}x^{14} + b^{13}c\,x^{15} + \frac{13}{2}b^{12}c^2x^{16} + 26b^{11}c^3x^{17} + \frac{143}{2}b^{10}c^4x^{18} + 143b^9c^5x^{19} + \frac{429}{2}b^8c^6x^{20} + \frac{1716}{7}b^7c^7x$
risch	$\frac{1}{14}b^{14}x^{14} + b^{13}c\,x^{15} + \frac{13}{2}b^{12}c^2x^{16} + 26b^{11}c^3x^{17} + \frac{143}{2}b^{10}c^4x^{18} + 143b^9c^5x^{19} + \frac{429}{2}b^8c^6x^{20} + \frac{1716}{7}b^7c^7x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(c*x+b)^13*(2*c*x+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*b^14*x^14+b^13*c*x^15+13/2*b^12*c^2*x^16+26*b^11*c^3*x^17+143/2*b^10*c
^4*x^18+143*b^9*c^5*x^19+429/2*b^8*c^6*x^20+1716/7*b^7*c^7*x^21+429/2*b^6*c
^8*x^22+143*b^5*c^9*x^23+143/2*b^4*c^10*x^24+26*b^3*c^11*x^25+13/2*b^2*c^12
*x^26+b*c^13*x^27+1/14*c^14*x^28
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(12) = 24$.
time = 0.33, size = 154, normalized size = 11.00

$$\frac{1}{14}c^{14}x^{28} + bc^{13}x^{27} + \frac{13}{2}b^2c^{12}x^{26} + 26\,b^3c^{11}x^{25} + \frac{143}{2}b^4c^{10}x^{24} + 143\,b^5c^9x^{23} + \frac{429}{2}b^6c^8x^{22} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^8c^6x^{20} + 143\,b^9c^5x^{19} + \frac{143}{2}b^{10}c^4x^{18} + 26\,b^{11}c^3x^{17} + \frac{13}{2}b^{12}c^2x^{16} + b^{13}cx^{15} + \frac{1}{14}b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(12) = 24$.
time = 1.40, size = 154, normalized size = 11.00

$$\frac{1}{14}c^{14}x^{28} + bc^{13}x^{27} + \frac{13}{2}b^2c^{12}x^{26} + 26\,b^3c^{11}x^{25} + \frac{143}{2}b^4c^{10}x^{24} + 143\,b^5c^9x^{23} + \frac{429}{2}b^6c^8x^{22} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^8c^6x^{20} + 143\,b^9c^5x^{19} + \frac{143}{2}b^{10}c^4x^{18} + 26\,b^{11}c^3x^{17} + \frac{13}{2}b^{12}c^2x^{16} + b^{13}cx^{15} + \frac{1}{14}b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="fricas")
```

[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(10) = 20$.
time = 0.03, size = 175, normalized size = 12.50

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**13*(c*x+b)**13*(2*c*x+b),x)

[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**17 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 + 1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*x**27 + c**14*x**28/14

Giac [A]
time = 0.75, size = 13, normalized size = 0.93

$$\frac{1}{14}\left(cx^2 + bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="giac")

[Out] 1/14*(c*x^2 + b*x)^14

Mupad [B]
time = 0.16, size = 154, normalized size = 11.00

$$\frac{b^{14}\,x^{14}}{14} + b^{13}\,c\,x^{15} + \frac{13\,b^{12}\,c^2\,x^{16}}{2} + 26\,b^{11}\,c^3\,x^{17} + \frac{143\,b^{10}\,c^4\,x^{18}}{2} + 143\,b^9\,c^5\,x^{19} + \frac{429\,b^8\,c^6\,x^{20}}{2} + \frac{1716\,b^7\,c^7\,x^{21}}{7} + \frac{429\,b^6\,c^8\,x^{22}}{2} + 143\,b^5\,c^9\,x^{23} + \frac{143\,b^4\,c^{10}\,x^{24}}{2} + 26\,b^3\,c^{11}\,x^{25} + \frac{13\,b^2\,c^{12}\,x^{26}}{2} + b\,c^{13}\,x^{27} + \frac{c^{14}\,x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^13*(b + c*x)^13*(b + 2*c*x),x)

[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2

3.1052 $\int x^{27}(b+cx^2)^{13}\left(b+2cx^2\right)\,dx$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[Out] 1/28*x^28*(c*x^2+b)^14
```

Rubi [A]
time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {457, 75}

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx, x, x^2\right)\\ &= \frac{1}{28}x^{28}\left(b+cx^2\right)^{14}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(16) = 32.
time = 0.00, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 0.34, size = 157, normalized size = 9.81

method	result
gosper	$\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}b\,c$
default	$\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}b\,c$
risch	$\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}b\,c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^27*(c*x^2+b)^13*(2*c*x^2+b),x,method=_RETURNVERBOSE)
```

```
[Out] 429/4*b^8*c^6*x^40+858/7*b^7*c^7*x^42+429/4*b^6*c^8*x^44+143/2*b^5*c^9*x^46
+143/4*b^4*c^10*x^48+13*b^3*c^11*x^50+13/4*b^2*c^12*x^52+1/2*b*c^13*x^54+1/
28*c^14*x^56+1/28*b^14*x^28+1/2*b^13*c*x^30+13/4*b^12*c^2*x^32+13*b^11*c^3*
x^34+143/4*b^10*c^4*x^36+143/2*b^9*c^5*x^38
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 0.34, size = 156, normalized size = 9.75

$$\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8c^6x^{40}+\frac{143}{2}\,b^9c^5x^{38}+\frac{143}{4}\,b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}\,b^{12}c^2x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 1.12, size = 156, normalized size = 9.75

$$\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8c^6x^{40}+\frac{143}{2}\,b^9c^5x^{38}+\frac{143}{4}\,b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}\,b^{12}c^2x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="fricas")

[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 + 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(12) = 24$.
time = 0.03, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4} + \frac{bc^{13}x^{54}}{2} + \frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**27*(c*x**2+b)**13*(2*c*x**2+b),x)

[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x**34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 + 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c**13*x**54/2 + c**14*x**56/28

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 0.59, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="giac")

[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 + 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28

Mupad [B]
time = 4.93, size = 156, normalized size = 9.75

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}c\,x^{30}}{2} + \frac{13\,b^{12}c^2x^{32}}{4} + 13\,b^{11}c^3x^{34} + \frac{143\,b^{10}c^4x^{36}}{4} + \frac{143\,b^9c^5x^{38}}{2} + \frac{429\,b^8c^6x^{40}}{4} + \frac{858\,b^7c^7x^{42}}{7} + \frac{429\,b^6c^8x^{44}}{4} + \frac{143\,b^5c^9x^{46}}{2} + \frac{143\,b^4c^{10}x^{48}}{4} + 13\,b^3c^{11}x^{50} + \frac{13\,b^2c^{12}x^{52}}{4} + \frac{b\,c^{13}x^{54}}{2} + \frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^27*(b + c*x^2)^13*(b + 2*c*x^2),x)

[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4

3.1053 $\int x^{41}(b+cx^3)^{13}\left(b+2cx^3\right)\,dx$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

[Out] 1/42*x^42*(c*x^3+b)^14

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {457, 75}

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx &= \frac{1}{3}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^3\right)\\ &= \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(16) = 32.
time = 0.00, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 0.33, size = 157, normalized size = 9.81

method	result
gosper	$\frac{1}{42}b^{14}x^{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{572}{7}b^{7}c^{7}x$
default	$\frac{1}{42}b^{14}x^{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{572}{7}b^{7}c^{7}x$
risch	$\frac{1}{42}b^{14}x^{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{572}{7}b^{7}c^{7}x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^41*(c*x^3+b)^13*(2*c*x^3+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*b^14*x^42+1/3*b^13*c*x^45+13/6*b^12*c^2*x^48+26/3*b^11*c^3*x^51+143/6*
b^10*c^4*x^54+143/3*b^9*c^5*x^57+143/2*b^8*c^6*x^60+572/7*b^7*c^7*x^63+143/
2*b^6*c^8*x^66+143/3*b^5*c^9*x^69+143/6*b^4*c^10*x^72+26/3*b^3*c^11*x^75+13
/6*b^2*c^12*x^78+1/3*b*c^13*x^81+1/42*c^14*x^84
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 0.31, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 0.94, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="fricas")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(12) = 24$.

time = 0.03, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^{2}x^{48}}{6} + \frac{26b^{11}c^{3}x^{51}}{3} + \frac{143b^{10}c^{4}x^{54}}{6} + \frac{143b^{9}c^{5}x^{57}}{3} + \frac{143b^{8}c^{6}x^{60}}{2} + \frac{572b^{7}c^{7}x^{63}}{7} + \frac{143b^{6}c^{8}x^{66}}{2} + \frac{143b^{5}c^{9}x^{69}}{3} + \frac{143b^{4}c^{10}x^{72}}{6} + \frac{26b^{3}c^{11}x^{75}}{3} + \frac{13b^{2}c^{12}x^{78}}{6} + \frac{bc^{13}x^{81}}{3} + \frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**41*(c*x**3+b)**13*(2*c*x**3+b),x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

time = 0.60, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^{2}c^{12}x^{78} + \frac{26}{3}b^{3}c^{11}x^{75} + \frac{143}{6}b^{4}c^{10}x^{72} + \frac{143}{3}b^{5}c^{9}x^{69} + \frac{143}{2}b^{6}c^{8}x^{66} + \frac{572}{7}b^{7}c^{7}x^{63} + \frac{143}{2}b^{8}c^{6}x^{60} + \frac{143}{3}b^{9}c^{5}x^{57} + \frac{143}{6}b^{10}c^{4}x^{54} + \frac{26}{3}b^{11}c^{3}x^{51} + \frac{13}{6}b^{12}c^{2}x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Mupad [B]

time = 4.77, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{42}}{42} + \frac{b^{13}\,c\,x^{45}}{3} + \frac{13\,b^{12}\,c^{2}\,x^{48}}{6} + \frac{26\,b^{11}\,c^{3}\,x^{51}}{3} + \frac{143\,b^{10}\,c^{4}\,x^{54}}{6} + \frac{143\,b^{9}\,c^{5}\,x^{57}}{3} + \frac{143\,b^{8}\,c^{6}\,x^{60}}{2} + \frac{572\,b^{7}\,c^{7}\,x^{63}}{7} + \frac{143\,b^{6}\,c^{8}\,x^{66}}{2} + \frac{143\,b^{5}\,c^{9}\,x^{69}}{3} + \frac{143\,b^{4}\,c^{10}\,x^{72}}{6} + \frac{26\,b^{3}\,c^{11}\,x^{75}}{3} + \frac{13\,b^{2}\,c^{12}\,x^{78}}{6} + \frac{b\,c^{13}\,x^{81}}{3} + \frac{c^{14}\,x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^41*(b + c*x^3)^13*(b + 2*c*x^3),x)
```

```
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

3.1054 $\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$

Optimal. Leaf size=21

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[Out] 1/14*x^(14*n)*(b+c*x^n)^14/n
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {457, 75}

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx &= \frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^n\right)}{n}\\ &= \frac{x^{14n}(b+cx^n)^{14}}{14n}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 21, normalized size = 1.00

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(19) = 38$.
time = 0.33, size = 230, normalized size = 10.95

method	result
risch	$\frac{c^{14} x^{28 n}}{14 n}+\frac{b\,c^{13} x^{27 n}}{n}+\frac{13 b^{2} c^{12} x^{26 n}}{2 n}+\frac{26 b^{3} c^{11} x^{25 n}}{n}+\frac{143 b^{4} c^{10} x^{24 n}}{2 n}+\frac{143 b^{5} c^{9} x^{23 n}}{n}+\frac{429 b^{6} c^{8} x^{22 n}}{2 n}+\frac{1716 b^{7} c^{7} x^{21 n}}{7 n}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^11
/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^8/
n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^5/n
*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^2/n*
(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(19) = 38$.
time = 0.29, size = 229, normalized size = 10.90

$$\frac{c^{14} x^{28 n}}{14 n}+\frac{b c^{13} x^{27 n}}{n}+\frac{13 b^{2} c^{12} x^{26 n}}{2 n}+\frac{26 b^{3} c^{11} x^{25 n}}{n}+\frac{143 b^{4} c^{10} x^{24 n}}{2 n}+\frac{143 b^{5} c^{9} x^{23 n}}{n}+\frac{429 b^{6} c^{8} x^{22 n}}{2 n}+\frac{1716 b^{7} c^{7} x^{21 n}}{7 n}+\frac{429 b^{8} c^{6} x^{20 n}}{2 n}+\frac{143 b^{9} c^{5} x^{19 n}}{n}+\frac{143 b^{10} c^{4} x^{18 n}}{2 n}+\frac{26 b^{11} c^{3} x^{17 n}}{n}+\frac{13 b^{12} c^{2} x^{16 n}}{2 n}+\frac{b^{13} c x^{15 n}}{n}+\frac{b^{14} x^{14 n}}{14 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(19) = 38$.
time = 1.25, size = 189, normalized size = 9.00

$$\frac{c^{14} x^{28 n}+14 b c^{13} x^{27 n}+91 b^{2} c^{12} x^{26 n}+364 b^{3} c^{11} x^{25 n}+1001 b^{4} c^{10} x^{24 n}+2002 b^{5} c^{9} x^{23 n}+3003 b^{6} c^{8} x^{22 n}+3432 b^{7} c^{7} x^{21 n}+3003 b^{8} c^{6} x^{20 n}+2002 b^{9} c^{5} x^{19 n}+1001 b^{10} c^{4} x^{18 n}+364 b^{11} c^{3} x^{17 n}+91 b^{12} c^{2} x^{16 n}+14 b^{13} c x^{15 n}+b^{14} x^{14 n}}{14 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="fricas")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+14*n)*(b+c*x**n)**13*(b+2*c*x**n),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)*(c*x^n + b)^13*x^(14*n - 1), x)
```

Mupad [B]
time = 5.21, size = 229, normalized size = 10.90

$$\frac{b^{14}\,x^{14\,n}}{14\,n}+\frac{c^{14}\,x^{28\,n}}{14\,n}+\frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n}+\frac{26\,b^{11}\,c^3\,x^{17\,n}}{n}+\frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n}+\frac{143\,b^9\,c^5\,x^{19\,n}}{n}+\frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n}+\frac{1716\,b^7\,c^7\,x^{21\,n}}{7\,n}+\frac{429\,b^6\,c^8\,x^{22\,n}}{2\,n}+\frac{143\,b^5\,c^9\,x^{23\,n}}{n}+\frac{143\,b^4\,c^{10}\,x^{24\,n}}{2\,n}+\frac{26\,b^3\,c^{11}\,x^{25\,n}}{n}+\frac{13\,b^2\,c^{12}\,x^{26\,n}}{2\,n}+\frac{b^{13}\,c\,x^{15\,n}}{n}+\frac{b\,c^{13}\,x^{27\,n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(14*n - 1)*(b + c*x^n)^13*(b + 2*c*x^n),x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

3.1055 $\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx$

Optimal. Leaf size=13

$$x^m(a+bx^n)^p$$

```
[Out] x^m*(a+b*x^n)^p
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = $\{460\}$

$$x^m(a+bx^n)^p$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

```
[Out] x^m*(a + b*x^n)^p
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx = x^m(a+bx^n)^p$$

Mathematica [A]
time = 0.14, size = 13, normalized size = 1.00

$$x^m(a+bx^n)^p$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

```
[Out] x^m*(a + b*x^n)^p
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^{-1+m}(a+b\,x^n)^{-1+p}\left(am+b(np+m)\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x)
```

```
[Out] int(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x)
```

Maxima [A]
time = 0.42, size = 16, normalized size = 1.23

$$e^{(p \log(bx^n+a)+m \log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="maxim
a")
```

```
[Out] e^(p*log(b*x^n + a) + m*log(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.
time = 1.38, size = 32, normalized size = 2.46

$$\left(bxx^{m-1}x^n + axx^{m-1}\right)(bx^n + a)^{p-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="frica
s")
```

```
[Out] (b*x*x^(m - 1)*x^n + a*x*x^(m - 1))*(b*x^n + a)^(p - 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*(a+b*x**n)**(-1+p)*(a*m+b*(n*p+m)*x**n),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(13) = 26.
time = 0.57, size = 70, normalized size = 5.38

$$bxx^n e^{(p \log(bx^n+a)+m \log(x)-\log(bx^n+a)-\log(x))} + axe^{(p \log(bx^n+a)+m \log(x)-\log(bx^n+a)-\log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="giac"
)
```

```
[Out] b*x*x^n*e^(p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x)) + a*x*e^(
p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x))
```

Mupad [B]
time = 4.81, size = 25, normalized size = 1.92

$$\left(a\,x^m+b\,x^{m+n}\right)\,{\left(a+b\,x^n\right)}^{p-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)*(a*m + b*x^n*(m + n*p))*(a + b*x^n)^(p - 1),x)
[Out] (a*x^m + b*x^(m + n))*(a + b*x^n)^(p - 1)
```

3.1056 $\int \frac{b+2cx}{x(b+cx)}\,dx$

Optimal. Leaf size=8

$$\log(x(b+cx))$$

```
[Out] ln(x*(c*x+b))
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {78}

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(x*(b + c*x)),x]
[Out] Log[x] + Log[b + c*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx}{x(b+cx)}\,dx &= \int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx\\ &= \log(x)+\log(b+cx)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.12

$$\log(x)+\log(b+cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(x*(b + c*x)),x]
[Out] Log[x] + Log[b + c*x]
```

Maple [A]
time = 0.31, size = 9, normalized size = 1.12

method	result	size
default	$\ln\left(x(cx+b)\right)$	9
norman	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
risch	$\ln\left(c\,x^{2}+bx\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/x/(c*x+b),x,method=_RETURNVERBOSE)
[Out] ln(x*(c*x+b))
```

Maxima [A]
time = 0.31, size = 9, normalized size = 1.12

$$\log\left(cx+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="maxima")
[Out] log(c*x + b) + log(x)
```

Fricas [A]
time = 2.98, size = 10, normalized size = 1.25

$$\log\left(cx^{2}+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="fricas")
[Out] log(c*x^2 + b*x)
```

Sympy [A]
time = 0.04, size = 8, normalized size = 1.00

$$\log\left(bx+cx^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x)
[Out] log(b*x + c*x**2)
```

Giac [A]
time = 0.58, size = 11, normalized size = 1.38

$$\log\left(|cx+b|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="giac")
```

```
[Out] log(abs(c*x + b)) + log(abs(x))
```

Mupad [B]
time = 4.66, size = 8, normalized size = 1.00

$$\ln\left(x\left(b+c\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(x*(b + c*x)),x)
```

```
[Out] log(x*(b + c*x))
```

3.1057 $\int \frac{b+2cx^2}{x(b+cx^2)}\,dx$

Optimal. Leaf size=15

$$\log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

```
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {457, 78}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx,x,x^2\right)\\ &= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\log(x) + \frac{1}{2}\log\left(b + cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Maple [A]
time = 0.45, size = 14, normalized size = 0.93

method	result	size
default	$\ln(x)+\frac{\ln(c\,x^2+b)}{2}$	14
norman	$\ln(x)+\frac{\ln(c\,x^2+b)}{2}$	14
risch	$\ln(x)+\frac{\ln(c\,x^2+b)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x/(c*x^2+b),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Maxima [A]
time = 0.31, size = 17, normalized size = 1.13

$$\frac{1}{2}\log\left(cx^2 + b\right) + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

Fricas [A]
time = 2.26, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(cx^2 + b\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Sympy [**A**]
time = 0.07, size = 12, normalized size = 0.80

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x/(c*x**2+b),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

Giac [**A**]
time = 0.56, size = 18, normalized size = 1.20

$$\frac{1}{2}\log\left(x^2\right) + \frac{1}{2}\log\left(\left|cx^2 + b\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

Mupad [**B**]
time = 0.06, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^2 + b\right)}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(x*(b + c*x^2)),x)
```

```
[Out] log(b + c*x^2)/2 + log(x)
```

3.1058 $\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx$

Optimal. Leaf size=15

$$\log(x) + \frac{1}{3}\log\left(b + cx^3\right)$$

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {457, 78}

$$\frac{1}{3}\log\left(b + cx^3\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^3\right)\\ &= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\log(x) + \frac{1}{3}\log\left(b + cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Maple [A]
time = 0.32, size = 14, normalized size = 0.93

method	result	size
default	$\ln(x) + \frac{\ln(c x^3+b)}{3}$	14
norman	$\ln(x) + \frac{\ln(c x^3+b)}{3}$	14
risch	$\ln(x) + \frac{\ln(c x^3+b)}{3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x/(c*x^3+b),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Maxima [A]
time = 0.31, size = 17, normalized size = 1.13

$$\frac{1}{3}\log\left(cx^3 + b\right) + \frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

Fricas [A]
time = 3.14, size = 13, normalized size = 0.87

$$\frac{1}{3}\log\left(cx^3 + b\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Sympy [A]
time = 0.08, size = 12, normalized size = 0.80

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x/(c*x**3+b),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

Giac [A]
time = 0.57, size = 15, normalized size = 1.00

$$\frac{1}{3}\log\left(\left|cx^3 + b\right|\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

Mupad [B]
time = 4.63, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^3 + b\right)}{3} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(x*(b + c*x^3)),x)
```

```
[Out] log(b + c*x^3)/3 + log(x)
```

3.1059 $\int \frac{b+2cx^n}{x(b+cx^n)}\,dx$

Optimal. Leaf size=15

$$\log(x)+\frac{\log\left(b+cx^n\right)}{n}$$

```
[Out] ln(x)+ln(b+c*x^n)/n
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {457, 78}

$$\frac{\log\left(b+cx^n\right)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+2cx^n}{x\left(b+cx^n\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx,x,x^n\right)}{n}\\
&= \log(x)+\frac{\log\left(b+cx^n\right)}{n}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.27

$$\frac{\log(x^n) + \log(n(b + cx^n))}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

```
[Out] (Log[x^n] + Log[n*(b + c*x^n)])/n
```

Maple [A]
time = 0.33, size = 17, normalized size = 1.13

method	result	size
derivativedivides	$\frac{\ln(x^n(b+c x^n))}{n}$	17
default	$\frac{\ln(x^n(b+c x^n))}{n}$	17
norman	$\ln(x) + \frac{\ln(b+c\,e^{n\ln(x)})}{n}$	18
risch	$\ln(x) + \frac{\ln\left(x^n+\frac{b}{c}\right)}{n}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/x/(b+c*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*ln(x^n*(b+c*x^n))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.
time = 0.43, size = 47, normalized size = 3.13

$$b\left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right) + \frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="maxima")
```

```
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Fricas [A]
time = 2.57, size = 17, normalized size = 1.13

$$\frac{n\log(x) + \log(cx^n + b)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(c*x^n + b))/n
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 0.28, size = 29, normalized size = 1.93

$$\begin{cases} \log(x) & \text{for } c = 0 \wedge n = 0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n = 0 \\ \log(x) & \text{for } c = 0 \\ \log(x) + \frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/x/(b+c*x**n),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0
)), (log(x), Eq(c, 0)), (log(x) + log(b/c + x**n)/n, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)/((c*x^n + b)*x), x)
```

Mupad [B]
time = 4.72, size = 15, normalized size = 1.00

$$\ln(x) + \frac{\ln(b + c\,x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^n)/(x*(b + c*x^n)),x)
```

```
[Out] log(x) + log(b + c*x^n)/n
```

3.1060 $\int \frac{b+2cx}{x^8(b+cx)^8} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{7x^7(b+cx)^7}$$

[Out] -1/7/x^7/(c*x+b)^7

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {75}

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(x^8*(b + c*x)^8),x]

[Out] -1/7*1/(x^7*(b + c*x)^7)

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{b+2cx}{x^8(b+cx)^8} \, dx = -\frac{1}{7x^7(b+cx)^7}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(x^8*(b + c*x)^8),x]

[Out] -1/7*1/(x^7*(b + c*x)^7)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(12) = 24$.
time = 0.32, size = 177, normalized size = 12.64

method	result
gosper	$-\frac{1}{7x^{7}(cx+b)^{7}}$
norman	$-\frac{1}{7x^{7}(cx+b)^{7}}$
risch	$-\frac{1}{7x^{7}(cx+b)^{7}}$
default	$\frac{132c^{7}}{b^{13}(cx+b)}+\frac{66c^{7}}{b^{12}(cx+b)^{2}}+\frac{30c^{7}}{b^{11}(cx+b)^{3}}+\frac{12c^{7}}{b^{10}(cx+b)^{4}}+\frac{4c^{7}}{b^{9}(cx+b)^{5}}+\frac{c^{7}}{b^{8}(cx+b)^{6}}+\frac{c^{7}}{7b^{7}(cx+b)^{7}}-\frac{1}{7b^{7}x^{7}}-\frac{132c^{6}}{b^{13}x}+\frac{6}{b^{1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/x^8/(c*x+b)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^
7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^6+1/7*c^7/b^7/(c*x+b)^7-1/7
/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4/b
^9*c^2/x^5+1/b^8*c/x^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(12) = 24$.
time = 0.32, size = 81, normalized size = 5.79

$$-\frac{1}{7\left(c^{7}x^{14}+7\,bc^{6}x^{13}+21\,b^{2}c^{5}x^{12}+35\,b^{3}c^{4}x^{11}+35\,b^{4}c^{3}x^{10}+21\,b^{5}c^{2}x^{9}+7\,b^{6}cx^{8}+b^{7}x^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(12) = 24$.
time = 1.75, size = 81, normalized size = 5.79

$$-\frac{1}{7\left(c^{7}x^{14}+7\,bc^{6}x^{13}+21\,b^{2}c^{5}x^{12}+35\,b^{3}c^{4}x^{11}+35\,b^{4}c^{3}x^{10}+21\,b^{5}c^{2}x^{9}+7\,b^{6}cx^{8}+b^{7}x^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(14) = 28$.
time = 0.43, size = 87, normalized size = 6.21

$$-\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x**8/(c*x+b)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

Giac [**A**]
time = 0.62, size = 13, normalized size = 0.93

$$-\frac{1}{7\left(cx^2 + bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

Mupad [**B**]
time = 7.09, size = 12, normalized size = 0.86

$$-\frac{1}{7\,x^7\,\left(b+c\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(x^8*(b + c*x)^8),x)
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

3.1061 $\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[Out] -1/14/x^14/(c*x^2+b)^7
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {457, 75}

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx,x,x^2\right)\\&=-\frac{1}{14x^{14}\left(b+cx^2\right)^7}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(14) = 28$.
time = 0.31, size = 197, normalized size = 12.31

method	result
gosper	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
norman	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
risch	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
default	$-\frac{c^8\left(-\frac{66b}{c\left(c\,x^2+b\right)^2}-\frac{4b^4}{c\left(c\,x^2+b\right)^5}-\frac{12b^3}{c\left(c\,x^2+b\right)^4}-\frac{132}{c\left(c\,x^2+b\right)}-\frac{b^6}{7c\left(c\,x^2+b\right)^7}-\frac{30b^2}{c\left(c\,x^2+b\right)^3}-\frac{b^5}{c\left(c\,x^2+b\right)^6}\right)}{2b^{13}}-\frac{1}{14b^7x^{14}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x^15/(c*x^2+b)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c^8/b^13*(-66*b/c/(c*x^2+b)^2-4*b^4/c/(c*x^2+b)^5-12*b^3/c/(c*x^2+b)^4
-132/c/(c*x^2+b)-1/7*b^6/c/(c*x^2+b)^7-30*b^2/c/(c*x^2+b)^3-b^5/c/(c*x^2+b)
^6)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^
3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.
time = 0.31, size = 81, normalized size = 5.06

$$-\frac{1}{14\,\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.
time = 0.96, size = 81, normalized size = 5.06

$$-\frac{1}{14\,(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.
time = 0.65, size = 87, normalized size = 5.44

$$-\frac{1}{14b^7x^{14}+98b^6cx^{16}+294b^5c^2x^{18}+490b^4c^3x^{20}+490b^3c^4x^{22}+294b^2c^5x^{24}+98bc^6x^{26}+14c^7x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x**15/(c*x**2+b)**8,x)
```

```
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

Giac [A]
time = 0.69, size = 15, normalized size = 0.94

$$-\frac{1}{14\,(cx^4+bx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

Mupad [B]
time = 2.32, size = 14, normalized size = 0.88

$$-\frac{1}{14\,x^{14}\,(c\,x^2+b)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x)
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

3.1062 $\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[Out] -1/21/x^21/(c*x^3+b)^7
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {457, 75}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx,x,x^3\right)\\ &= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(14) = 28$.
time = 0.32, size = 197, normalized size = 12.31

method	result
gosper	$-\frac{1}{21x^{21}\left(c\,x^3+b\right)^7}$
risch	$-\frac{1}{21x^{21}\left(c\,x^3+b\right)^7}$
default	$-\frac{c^8\left(-\frac{66b}{c\left(c\,x^3+b\right)^2}-\frac{4b^4}{c\left(c\,x^3+b\right)^5}-\frac{12b^3}{c\left(c\,x^3+b\right)^4}-\frac{132}{c\left(c\,x^3+b\right)}-\frac{b^6}{7c\left(c\,x^3+b\right)^7}-\frac{30b^2}{c\left(c\,x^3+b\right)^3}-\frac{b^5}{c\left(c\,x^3+b\right)^6}\right)}{3b^{13}}-\frac{1}{21b^7x^{21}}-\frac{44c^6}{b^{13}x^3}+\frac{22c^5}{b^{12}x^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x^22/(c*x^3+b)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c^8/b^13*(-66*b/c/(c*x^3+b)^2-4*b^4/c/(c*x^3+b)^5-12*b^3/c/(c*x^3+b)^4
-132/c/(c*x^3+b)-1/7*b^6/c/(c*x^3+b)^7-30*b^2/c/(c*x^3+b)^3-b^5/c/(c*x^3+b)
^6)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^
3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.
time = 0.33, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

time = 0.89, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.
time = 0.90, size = 87, normalized size = 5.44

$$-\frac{1}{21b^{7}x^{21}+147b^{6}cx^{24}+441b^{5}c^{2}x^{27}+735b^{4}c^{3}x^{30}+735b^{3}c^{4}x^{33}+441b^{2}c^{5}x^{36}+147bc^{6}x^{39}+21c^{7}x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x**22/(c*x**3+b)**8,x)
```

```
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

Giac [A]
time = 0.62, size = 15, normalized size = 0.94

$$-\frac{1}{21\left(cx^{6}+bx^{3}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

Mupad [B]
time = 9.98, size = 14, normalized size = 0.88

$$-\frac{1}{21\,x^{21}\,\left(c\,x^{3}+b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x)
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

3.1063 $\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} \, dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

```
[Out] -1/7/n/(x^(7*n))/(b+c*x^n)^7
```

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {457, 75}

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-7n}(b+2cx^n)}{\left(b+cx^n\right)^8} \, dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} \, dx, x, x^n\right)}{n} \\
&= -\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(21) = 42.
time = 0.38, size = 203, normalized size = 9.67

method	result
risch	$-\frac{132c^6x^{-n}}{b^{13}n}+\frac{66c^5x^{-2n}}{b^{12}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{c\,x^{-6n}}{b^8n}-\frac{x^{-7n}}{7b^7n}+\frac{c^7\left(924x^{6n}c^6+6006b\,c^5x^{5n}+16380b^2c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(21) = 42.
time = 0.38, size = 612, normalized size = 29.14

$$-\frac{1}{105}b\left(\frac{360360\,c^{13}x^{13n}+2342340\,bc^{12}x^{12n}+6426420\,b^2c^{11}x^{11n}+9579570\,b^3c^{10}x^{10n}+8270262\,b^4c^9x^{9n}+4018014\,b^5c^8x^{8n}+934362\,b^6c^7x^{7n}+45045\,b^7c^6x^{6n}-5005\,b^8c^5x^{5n}+1001\,b^9c^4x^{4n}-273\,b^{10}c^3x^{3n}+91\,b^{11}c^2x^{2n}-35\,b^{12}cx^n+15\,b^{13}}{b^{14}c^7nx^{14n}+7\,b^{15}c^6nx^{13n}+21\,b^{16}c^5nx^{12n}+35\,b^{17}c^4nx^{11n}+35\,b^{18}c^3nx^{10n}+21\,b^{19}c^2nx^{9n}+7\,b^{20}cnx^{8n}+b^{21}nx^{7n}}+\frac{360360\,c^7\log\left(x\right)}{b^{15}}-\frac{360360\,c^7\log\left(\frac{cx^n+b}{c}\right)}{b^{15}n}\right)+\frac{1}{105}c\left(\frac{360360\,c^{12}x^{12n}+2342340\,bc^{11}x^{11n}+6426420\,b^2c^{10}x^{10n}+9579570\,b^3c^9x^{9n}+8270262\,b^4c^8x^{8n}+4018014\,b^5c^7x^{7n}+934362\,b^6c^6x^{6n}+45045\,b^7c^5x^{5n}-5005\,b^8c^4x^{4n}+1001\,b^9c^3x^{3n}-273\,b^{10}c^2x^{2n}+91\,b^{11}cx^n-35\,b^{12}}{b^{13}c^7nx^{13n}+7\,b^{14}c^6nx^{12n}+21\,b^{15}c^5nx^{11n}+35\,b^{16}c^4nx^{10n}+35\,b^{17}c^3nx^{9n}+21\,b^{18}c^2nx^{8n}+7\,b^{19}cnx^{7n}+b^{20}nx^{6n}}+\frac{360360\,c^6\log\left(x\right)}{b^{14}}-\frac{360360\,c^6\log\left(\frac{cx^n+b}{c}\right)}{b^{14}n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
```

```
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(21) = 42$.
time = 1.22, size = 105, normalized size = 5.00

$$-\frac{1}{7\left(c^{7} n x^{14 n}+7 b c^{6} n x^{13 n}+21 b^{2} c^{5} n x^{12 n}+35 b^{3} c^{4} n x^{11 n}+35 b^{4} c^{3} n x^{10 n}+21 b^{5} c^{2} n x^{9 n}+7 b^{6} c n x^{8 n}+b^{7} n x^{7 n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*
c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x
^(8*n) + b^7*n*x^(7*n))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-7*n)*(b+2*c*x**n)/(b+c*x**n)**8,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)*x^(-7*n - 1)/(c*x^n + b)^8, x)
```

Mupad [**B**]
time = 4.99, size = 105, normalized size = 5.00

$$-\frac{1}{7\, x^{7 n}\, \left(b^{7}\, n+c^{7}\, n\, x^{7 n}+7\, b^{6}\, c\, n\, x^{n}+7\, b\, c^{6}\, n\, x^{6 n}+21\, b^{5}\, c^{2}\, n\, x^{2 n}+35\, b^{4}\, c^{3}\, n\, x^{3 n}+35\, b^{3}\, c^{4}\, n\, x^{4 n}+21\, b^{2}\, c^{5}\, n\, x^{5 n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^n)/(x^(7*n + 1)*(b + c*x^n)^8),x)
```

```
[Out] -1/(7*x^(7*n)*(b^7*n + c^7*n*x^(7*n) + 7*b^6*c*n*x^n + 7*b*c^6*n*x^(6*n) +
21*b^5*c^2*n*x^(2*n) + 35*b^4*c^3*n*x^(3*n) + 35*b^3*c^4*n*x^(4*n) + 21*b^2
*c^5*n*x^(5*n)))
```

3.1064 $\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{\tanh^{-1}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/24*(x^16+1)^(3/2)+1/8*arctanh(1/2*(x^16+1)^(1/2)*2^(1/2))*2^(1/2)-1/8*(x
^16+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 81, 52, 65, 212}

$$-\frac{1}{24}\left(x^{16}+1\right)^{3/2}-\frac{\sqrt{x^{16}+1}}{8}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

```
[Out] -1/8*Sqrt[1 + x^16] - (1 + x^16)^(3/2)/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]
/(4*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx &= \frac{1}{16}\text{Subst}\left(\int \frac{x\sqrt{1+x}}{1-x}\,dx,x,x^{16}\right)\\
&= -\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{16}\text{Subst}\left(\int \frac{\sqrt{1+x}}{1-x}\,dx,x,x^{16}\right)\\
&= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{1+x}}\,dx,x,x^{16}\right)\\
&= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+x^{16}}\right)\\
&= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{\tanh^{-1}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 0.88

$$\frac{1}{24}\left(-4-x^{16}\right)\sqrt{1+x^{16}}+\frac{\tanh^{-1}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

```
[Out] ((-4 - x^16)*Sqrt[1 + x^16])/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]/(4*Sqrt[2
])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.69, size = 85, normalized size = 1.63

method	result	size
risch	$-\frac{(x^{16}+4)\sqrt{x^{16}+1}}{24}-\frac{\mathrm{RootOf}(\_Z^2-2)\ln\left(\frac{\mathrm{RootOf}(\_Z^2-2)x^{16}+3\,\mathrm{RootOf}(\_Z^2-2)-4\sqrt{x^{16}+1}}{(x-1)(x+1)(x^2+1)(x^4+1)(x^8+1)}\right)}{16}$	85
trager	$\left(-\frac{x^{16}}{24}-\frac{1}{6}\right)\sqrt{x^{16}+1}+\frac{\mathrm{RootOf}(\_Z^2-2)\ln\left(-\frac{\mathrm{RootOf}(\_Z^2-2)x^{16}+3\,\mathrm{RootOf}(\_Z^2-2)+4\sqrt{x^{16}+1}}{(x-1)(x+1)(x^2+1)(x^4+1)(x^8+1)}\right)}{16}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^31*(x^16+1)^(1/2)/(-x^16+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(x^16+4)*(x^16+1)^(1/2)-1/16*RootOf(_Z^2-2)*ln((RootOf(_Z^2-2)*x^16+3
*RootOf(_Z^2-2)-4*(x^16+1)^(1/2))/(x-1)/(x+1)/(x^2+1)/(x^4+1)/(x^8+1))
```

Maxima [A]
time = 0.56, size = 53, normalized size = 1.02

$$-\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}}-\frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^{16}+1}}{\sqrt{2}+\sqrt{x^{16}+1}}\right)-\frac{1}{8}\sqrt{x^{16}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="maxima")
```

```
[Out] -1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(-(sqrt(2) - sqrt(x^16 + 1))/(sqrt
(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

Fricas [A]
time = 2.46, size = 46, normalized size = 0.88

$$-\frac{1}{24}\left(x^{16}+4\right)\sqrt{x^{16}+1}+\frac{1}{16}\sqrt{2}\log\left(\frac{x^{16}+2\sqrt{2}\sqrt{x^{16}+1}+3}{x^{16}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="fricas")
```

```
[Out] -1/24*(x^16 + 4)*sqrt(x^16 + 1) + 1/16*sqrt(2)*log((x^16 + 2*sqrt(2)*sqrt(x
^16 + 1) + 3)/(x^16 - 1))
```

Sympy [A]
time = 112.71, size = 76, normalized size = 1.46

$$-\frac{(x^{16}+1)^{\frac{3}{2}}}{24}-\frac{\sqrt{x^{16}+1}}{8}-\frac{\begin{cases}-\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}\sqrt{x^{16}+1}}{2}\right)}{2} & \text{for } x^{16}>1\\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{x^{16}+1}}{2}\right)}{2} & \text{for } x^{16}<1\end{cases}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**31*(x**16+1)**(1/2)/(-x**16+1),x)
```

```
[Out] -(x**16 + 1)**(3/2)/24 - sqrt(x**16 + 1)/8 - Piecewise((-sqrt(2)*acoth(sqrt
(2)*sqrt(x**16 + 1)/2)/2, x**16 > 1), (-sqrt(2)*atanh(sqrt(2)*sqrt(x**16 +
1)/2)/2, x**16 < 1))/4
```

Giac [A]
time = 1.15, size = 56, normalized size = 1.08

$$-\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}}-\frac{1}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{x^{16}+1}\right|}{2\left(\sqrt{2}+\sqrt{x^{16}+1}\right)}\right)-\frac{1}{8}\sqrt{x^{16}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="giac")
```

```
[Out] -1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(x^16
+ 1))/(sqrt(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

Mupad [B]
time = 4.82, size = 37, normalized size = 0.71

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{x^{16}+1}}{2}\right)}{8}-\frac{\sqrt{x^{16}+1}}{8}-\frac{(x^{16}+1)^{3/2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^31*(x^16 + 1)^(1/2))/(x^16 - 1),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(x^16 + 1)^(1/2))/2))/8 - (x^16 + 1)^(1/2)/8 - (x^1
6 + 1)^(3/2)/24
```

3.1065 $\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{b}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))*c^(1/2)/a^(1/2)-2*ar
ctanh(d^(1/2)*(a+b/x)^(1/2)/b^(1/2)/(c+d/x)^(1/2))*d^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {457, 132, 65, 223, 212, 12, 95, 214}

$$\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{b}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

```
[Out] (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a
] - (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])])/Sq
rt[b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx = -\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(c\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)\right) - d\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x\right.$$

$$= -\left((2c)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)\right) - \frac{(2d)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,\right.}{b}$$

$$= \frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{(2d)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{b}$$

$$= \frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{b}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 348 vs. $2(93) = 186$.
time = 1.54, size = 348, normalized size = 3.74

$$\frac{2\sqrt{c}\,\sqrt{c+\frac{d}{x}}\,\sqrt{b+ax}\left(\sqrt{b+ax}-\sqrt{\frac{a}{c}}\,\sqrt{d+cx}\right)\left(bc-2\sqrt{\frac{a}{c}}\,c\sqrt{b+ax}\,\sqrt{d+cx}+a(d+2cx)\right)\left(\sqrt{a}\,\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{c}\left(-ax+\sqrt{\frac{a}{c}}\,\sqrt{b+ax}\,\sqrt{d+cx}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{d}}\right)+\sqrt{b}\,\sqrt{c}\,\log\left(\sqrt{b+ax}-\sqrt{\frac{a}{c}}\,\sqrt{d+cx}\right)\right)}{\sqrt{b}\,\sqrt{a+\frac{b}{x}}\,\sqrt{d+cx}\left(bc\left(-\sqrt{\frac{a}{c}}\,c\sqrt{b+ax}+3a\sqrt{d+cx}\right)+a\left(-3\sqrt{\frac{a}{c}}\,cd\sqrt{b+ax}-4\sqrt{\frac{a}{c}}\,c^2x\sqrt{b+ax}+ad\sqrt{d+cx}+4acx\sqrt{d+cx}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[c + d/x]*Sqrt[b + a*x]*(Sqrt[b + a*x] - Sqrt[a/c]*Sqrt[d +
c*x])*(b*c - 2*Sqrt[a/c]*c*Sqrt[b + a*x]*Sqrt[d + c*x] + a*(d + 2*c*x))*(Sq
rt[a]*Sqrt[d]*ArcTanh[(Sqrt[c]*(-(a*x) + Sqrt[a/c]*Sqrt[b + a*x]*Sqrt[d + c
*x]))/(Sqrt[a]*Sqrt[b]*Sqrt[d])] + Sqrt[b]*Sqrt[c]*Log[Sqrt[b + a*x] - Sqrt
[a/c]*Sqrt[d + c*x]]))/(Sqrt[b]*Sqrt[a + b/x]*Sqrt[d + c*x]*(b*c*(-(Sqrt[a/
c]*c*Sqrt[b + a*x]) + 3*a*Sqrt[d + c*x]) + a*(-3*Sqrt[a/c]*c*d*Sqrt[b + a*x
```

] - 4*Sqrt[a/c]*c^2*x*Sqrt[b + a*x] + a*d*Sqrt[d + c*x] + 4*a*c*x*Sqrt[d + c*x])))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(69) = 138$.
time = 0.08, size = 143, normalized size = 1.54

method	result
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\sqrt{\frac{cx+d}{x}}\left(\ln\left(\frac{adx+bcx+2\sqrt{bd}\,\sqrt{(cx+d)(ax+b)}+2bd}{x}\right)\sqrt{ac}\,d-\ln\left(\frac{2acx+2\sqrt{(cx+d)(ax+b)}}{2\sqrt{ac}}\right.\right.}{\sqrt{bd}\,\sqrt{ac}\,\sqrt{(cx+d)(ax+b)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^(1/2)/x/(a+1/x*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] -((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(ln((a*d*x+b*c*x+2*(b*d)^(1/2)*((c*x+d)*(a*x+b))^(1/2)+2*b*d)/x)*(a*c)^(1/2)*d-ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*(b*d)^(1/2)*c)/(b*d)^(1/2)/(a*c)^(1/2)/((c*x+d)*(a*x+b))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(69) = 138$.
time = 1.32, size = 757, normalized size = 8.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), -sqrt(-c/a)*arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c

```
 + a*d)) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt((a*x + b)/x)*sqrt(
(c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), sqrt(-d/b)*arctan(1/2*(2*b*d*
x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*
x^2 + b*d^2 + (b*c*d + a*d^2)*x)) + 1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*
c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x), -sqrt(-c/a)*
arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c
+ a*d)) + sqrt(-d/b)*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt
((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*x^2 + b*d^2 + (b*c*d + a*d^2)*x))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + \frac{d}{x}}}{x\sqrt{a + \frac{b}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**(1/2)/x/(a+b/x)**(1/2),x)
[Out] Integral(sqrt(c + d/x)/(x*sqrt(a + b/x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+\frac{d}{x}}}{x\,\sqrt{a+\frac{b}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)),x)
[Out] int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)), x)
```

3.1066 $\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=252

$$-\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{64bd^4n}+\frac{5(bc-ad)(7bc+ad)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{96bd^3n}-\frac{(7bc+ad)\,(a+}{2}$$

```
[Out] 5/64*(-a*d+b*c)^3*(a*d+7*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*
x^n)^(1/2))/b^(3/2)/d^(9/2)/n+5/96*(-a*d+b*c)*(a*d+7*b*c)*(a+b*x^n)^(3/2)*(
c+d*x^n)^(1/2)/b/d^3/n-1/24*(a*d+7*b*c)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b/d
^2/n+1/4*(a+b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b/d/n-5/64*(-a*d+b*c)^2*(a*d+7*b*c
)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^4/n
```

Rubi [A]
time = 0.16, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {457, 81, 52, 65, 223, 212}

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n}-\frac{5(bc-ad)^2(ad+7bc)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{64bd^4n}+\frac{5(bc-ad)(ad+7bc)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{96bd^3n}-\frac{(ad+7bc)\,(a+bx^n)^{5/2}\,\sqrt{c+dx^n}}{24bd^2n}+\frac{(a+bx^n)^{7/2}\,\sqrt{c+dx^n}}{4bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(64*b*d^4*
n) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(96*b*
d^3*n) - ((7*b*c + a*d)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(24*b*d^2*n) + (
(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(4*b*d*n) + (5*(b*c - a*d)^3*(7*b*c + a*
d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(3/2
)*d^(9/2)*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx,x,x^n\right)}{n}$$

$$= \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn} - \frac{(7bc+ad)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx,x,x^n\right)}{8bdn}$$

$$= -\frac{(7bc+ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn} + \frac{(5(bc-ad)(7b}{}$$

$$= \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(7bc+ad)(a+bx^n)^{5/2}\sqrt{c+dx^{}}}{24bd^2n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3}}{96bd^3n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3}}{96bd^3n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3}}{96bd^3n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3}}{96bd^3n}$$

<u>Mathematica</u> [A]
time = 1.46, size = 223, normalized size = 0.88

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)(15a^3d^3+a^2bd^2(-191c+118dx^n)+ab^2d(265c^2-172cdx^n+136d^2x^{2n})+b^3(-105c^3+70c^2dx^n-56cd^2x^{2n}+48d^3x^{3n}))+15(bc-ad)^{7/2}(7bc+ad)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{192b^2d^{9/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 11
8*d*x^n) + a*b^2*d*(265*c^2 - 172*c*d*x^n + 136*d^2*x^(2*n)) + b^3*(-105*c^
3 + 70*c^2*d*x^n - 56*c*d^2*x^(2*n) + 48*d^3*x^(3*n))) + 15*(b*c - a*d)^(7/
2)*(7*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a
+ b*x^n])/Sqrt[b*c - a*d]])/(192*b^2*d^(9/2)*n*Sqrt[c + d*x^n])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}(a+b\,x^n)^{\frac{5}{2}}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
[Out] int(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A]
time = 1.46, size = 607, normalized size = 2.41

$$\left[-\frac{15\left(7b^4c^4-20ab^3c^3d+18a^2b^2c^2d^2-4a^3bcd^3-a^4d^4\right)\sqrt{bd}\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2-4\left(2\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}+8\left(b^2cd+abd^2\right)x^n\right)-4\left(48b^4d^4x^{3n}-105b^4c^3d+265ab^3c^2d^2-191a^2b^2cd^3+15a^3bd^4-8\left(7b^4cd^3-17ab^3d^4\right)x^{2n}+2\left(35b^4c^2d^2-86ab^3cd^3+59a^2b^2d^4\right)x^n\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{768\,b^2d^5n},\ -\frac{15\left(7b^4c^4-20ab^3c^3d+18a^2b^2c^2d^2-4a^3bcd^3-a^4d^4\right)\sqrt{-bd}\arctan\left(\frac{\left(2\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{2\left(b^2d^2x^{2n}+abcd+\left(b^2cd+abd^2\right)x^n\right)}\right)-2\left(48b^4d^4x^{3n}-105b^4c^3d+265ab^3c^2d^2-191a^2b^2cd^3+15a^3bd^4-8\left(7b^4cd^3-17ab^3d^4\right)x^{2n}+2\left(35b^4c^2d^2-86ab^3cd^3+59a^2b^2d^4\right)x^n\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{384\,b^2d^5n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
[Out] [-1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^
2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*
x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3
*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3
- 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d
^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^5*n), -1/384*(15*(7*b^4*c^
4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(-b*
d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n +
a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) -
 2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*
d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2
*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c
))/(b^2*d^5*n)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{2\,n-1}\,{\left(a+b\,x^n\right)}^{5/2}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

3.1067 $\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=199

$$\frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8bd^3n}-\frac{(5bc+ad)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{12bd^2n}+\frac{(a+bx^n)^{5/2}\,\sqrt{c+dx^n}}{3bdn}-\frac{(b}{}$$

[Out] -1/8*(-a*d+b*c)^2*(a*d+5*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)/d^(7/2)/n-1/12*(a*d+5*b*c)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b/d^2/n+1/3*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b/d/n+1/8*(-a*d+b*c)*(a*d+5*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^3/n

Rubi [A]
time = 0.11, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {457, 81, 52, 65, 223, 212}

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n}+\frac{(bc-ad)(ad+5bc)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8bd^3n}-\frac{(ad+5bc)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{12bd^2n}+\frac{(a+bx^n)^{5/2}\,\sqrt{c+dx^n}}{3bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b*d^3*n) - ((5*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b*d^2*n) + ((a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)^2*(5*b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(8*b^(3/2)*d^(7/2)*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} - \frac{(5bc+ad)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{6bdn}$$

$$= -\frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} + \frac{((bc-ad)(5b}{}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n}$$

Mathematica [A]
time = 0.63, size = 178, normalized size = 0.89

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(3a^2d^2+2abd(-11c+7dx^n)+b^2(15c^2-10cdx^n+8d^2x^{2n})\right)-3(bc-ad)^{5/2}(5bc+ad)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{24b^2d^{7/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x^
n) + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(5/2)*(5*b*
c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n]
)/Sqrt[b*c - a*d]])/(24*b^2*d^(7/2)*n*Sqrt[c + d*x^n])
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}(a+b\,x^n)^{\frac{3}{2}}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A]
time = 0.97, size = 469, normalized size = 2.36

$$\left[\frac{3\,(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2-4\left(2\,\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}+8\,(b^2cd+abd^2)x^n\right)+4\,(8\,b^3d^3x^{2\,n}+15\,b^3c^2d-22\,ab^2cd^2+3\,a^2bd^3-2\,(5\,b^3cd^2-7\,ab^2d^3)x^n)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{96\,b^2d^4n},\ \frac{3\,(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right)+2\,(8\,b^3d^3x^{2\,n}+15\,b^3c^2d-22\,ab^2cd^2+3\,a^2bd^3-2\,(5\,b^3cd^2-7\,ab^2d^3)x^n)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{48\,b^2d^4n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^
n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a
*b*d^2)*x^n) + 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2
*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)
)/(b^2*d^4*n), 1/48*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3
)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqr
t(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^
2)*x^n)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d
^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^2*d^4*n)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}\,{\left(a+b\,x^n\right)}^{3/2}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

3.1068 $\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=146

$$-\frac{(3bc+ad)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}+\frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}$$

```
[Out] 1/4*(-a*d+b*c)*(a*d+3*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n
)^(1/2))/b^(3/2)/d^(5/2)/n+1/2*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b/d/n-1/4*(a
*d+3*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^2/n
```

Rubi [A]

time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {457, 81, 52, 65, 223, 212}

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}-\frac{(ad+3bc)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] -1/4*((3*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b*d^2*n) + ((a + b*x^
n)^(3/2)*Sqrt[c + d*x^n])/(2*b*d*n) + ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(S
qrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(3/2)*d^(5/2)*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} - \frac{(3bc+ad)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{4bdn}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+a}{}$$

$$((bc-ad)(3bc+a$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{}{}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+a}{}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{(bc-ad)(3bc+ac}{}$$

Mathematica [A]
time = 0.38, size = 141, normalized size = 0.97

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)(-3bc+ad+2bdx^n)+(bc-ad)^{3/2}(3bc+ad)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{4b^2d^{5/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c + a*d + 2*b*d*x^n) + (b*c -
a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^2*d^(5/2)*n*Sqrt[c + d*x^n])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
[Out] int(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A]
time = 1.25, size = 359, normalized size = 2.46

$$\left[-\frac{(3\,b^2c^2 - 2\,abcd - a^2d^2)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2\,n} + b^2c^2 + 6\,abcd + a^2d^2 - 4\left(2\,\sqrt{bd}\,bdx^n + (bc + ad)\sqrt{bd}\right)\sqrt{bx^n + a}\,\sqrt{dx^n + c} + 8\,(b^2cd + abd^2)x^n\right) - 4\,(2\,b^2d^2x^n - 3\,b^2cd + abd^2)\sqrt{bx^n + a}\,\sqrt{dx^n + c}}{16\,b^2d^3n}, -\frac{(3\,b^2c^2 - 2\,abcd - a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,\sqrt{-bd}\,bdx^n + (bc+ad)\sqrt{-bd}\right)\sqrt{bx^n + a}\,\sqrt{dx^n + c}}{2\,(b^2d^2x^{2\,n} + abcd + (b^2cd + abd^2)x^n)}\right) - 2\,(2\,b^2d^2x^n - 3\,b^2cd + abd^2)\sqrt{bx^n + a}\,\sqrt{dx^n + c}}{8\,b^2d^3n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
[Out] [-1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) +
 b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(
b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(2*b
^2*d^2*x^n - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3
*n), -1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*sqrt(
-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^
2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x^n - 3*
b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}\sqrt{a + bx^n}}{\sqrt{c + dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
[Out] Integral(x**(2*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}\,\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

3.1069 $\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn}-\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

```
[Out] -(a*d+b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)
/d^(3/2)/n+(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d/n
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 81, 65, 223, 212}

$$\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b*d*n) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*d^(3/2)*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{2bdn}$$

$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{b^2dn}$$

$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2dn}$$

$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

Mathematica [A]
time = 0.23, size = 123, normalized size = 1.38

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n) - \sqrt{bc-ad}\,(bc+ad)\sqrt{\frac{b\,(c+dx^n)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{b^2d^{3/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[(
b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*
d]])/(b^2*d^(3/2)*n*Sqrt[c + d*x^n])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}}{\sqrt{a+b\,x^{n}}\,\sqrt{c+d\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Fricas [A]
time = 1.03, size = 281, normalized size = 3.16

$$\left[\frac{4\,\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}\,bd+(bc+ad)\sqrt{bd}\,\log\left(8\,b^{2}d^{2}x^{2\,n}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}-4\left(2\,\sqrt{bd}\,bdx^{n}+(bc+ad)\sqrt{bd}\right)\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}+8\,(b^{2}cd+abd^{2})x^{n}\right)}{4\,b^{2}d^{2}n},\ \frac{2\,\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}\,bd+(bc+ad)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,\sqrt{-bd}\,bdx^{n}+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}}{2\,(b^{2}d^{2}x^{2\,n}+abcd+(b^{2}cd+abd^{2})x^{n})}\right)}{2\,b^{2}d^{2}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*b
^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (
b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^
2)*x^n))/(b^2*d^2*n), 1/2*(2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a
*d)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*s
qrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*
d^2)*x^n)))/(b^2*d^2*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{a+bx^{n}}\,\sqrt{c+dx^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{\sqrt{a+b\,x^n}\,\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

3.1070 $\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=91

$$\frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}\,n}$$

```
[Out] 2*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)/n/d^(1/2
)+2*a*(c+d*x^n)^(1/2)/b/(-a*d+b*c)/n/(a+b*x^n)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 79, 65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}\,n}+\frac{2a\sqrt{c+dx^n}}{bn(bc-ad)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*a*Sqrt[c + d*x^n])/(b*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (2*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*Sqrt[d]*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}\\ &= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{bn}\\ &= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{b^2n}\\ &= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2n}\\ &= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}\,n}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 105, normalized size = 1.15

$$\frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{\log\left(bc+ad+2bdx^n+2\sqrt{b}\sqrt{d}\sqrt{a+bx^n}\sqrt{c+dx^n}\right)}{b^{3/2}\sqrt{d}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*a*Sqrt[c + d*x^n])/(b*(b*c - a*d)*n*Sqrt[a + b*x^n]) + Log[b*c + a*d + 2
*b*d*x^n + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^n]*Sqrt[c + d*x^n]]/(b^(3/2)*Sqrt
[d]*n)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}}{\left(a+b\,x^{n}\right)^{\frac{3}{2}} \sqrt{c+d\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(75) = 150.
time = 1.09, size = 408, normalized size = 4.48

$$\left[\frac{4\,\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}\,abd+\left((b^{2}c-abd)\sqrt{bd}\,x^{n}+(abc-a^{2}d)\sqrt{bd}\right)\log\left(8\,b^{2}d^{2}x^{2\,n}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+4\left(2\,\sqrt{bd}\,bdx^{n}+(bc+ad)\sqrt{bd}\right)\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}+8\,(b^{2}cd+abd^{2})x^{n}\right)}{2\left((b^{4}cd-ab^{3}d^{2})nx^{n}+(ab^{3}cd-a^{2}b^{2}d^{2})n\right)}, \frac{2\,\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}\,abd-\left((b^{2}c-abd)\sqrt{-bd}\,x^{n}+(abc-a^{2}d)\sqrt{-bd}\right)\arctan\left(\frac{\left(2\,\sqrt{-bd}\,bdx^{n}+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}}{2\,(b^{2}d^{2}x^{2\,n}+abcd+(b^{2}cd+abd^{2})x^{n})}\right)}{(b^{4}cd-ab^{3}d^{2})nx^{n}+(ab^{3}cd-a^{2}b^{2}d^{2})n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d + ((b^2*c - a*b*d)*sqrt(b*d)*
x^n + (a*b*c - a^2*d)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*
d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n +
a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^4*c*d - a*b^3*d^2)*n*x
^n + (a*b^3*c*d - a^2*b^2*d^2)*n), (2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d
 - ((b^2*c - a*b*d)*sqrt(-b*d)*x^n + (a*b*c - a^2*d)*sqrt(-b*d))*arctan(1/2
*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n
 + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^4*c*d - a
*b^3*d^2)*n*x^n + (a*b^3*c*d - a^2*b^2*d^2)*n)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a+b\,x^n\right)}^{3/2}\,\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

3.1071 $\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=95

$$\frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2n\sqrt{a+bx^n}}$$

```
[Out] 2/3*a*(c+d*x^n)^(1/2)/b/(-a*d+b*c)/n/(a+b*x^n)^(3/2)-2/3*(-a*d+3*b*c)*(c+d*
x^n)^(1/2)/b/(-a*d+b*c)^2/n/(a+b*x^n)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 79, 37}

$$\frac{2a\sqrt{c+dx^n}}{3bn(bc-ad)\,(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3bn(bc-ad)^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*a*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)*n*(a + b*x^n)^(3/2)) - (2*(3*b*c - a
*d)*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)^2*n*Sqrt[a + b*x^n])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{(3bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{3b(bc-ad)n}$$

$$= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}} - \frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2n\sqrt{a+bx^n}}$$

Mathematica [A]
time = 0.65, size = 57, normalized size = 0.60

$$\frac{2\sqrt{c+dx^n}\,(-2ac-3bcx^n+adx^n)}{3(bc-ad)^2n\,(a+bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*Sqrt[c + d*x^n]*(-2*a*c - 3*b*c*x^n + a*d*x^n))/(3*(b*c - a*d)^2*n*(a +
b*x^n)^(3/2))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}}{(a+b\,x^n)^{\frac{5}{2}}\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Fricas [A]
time = 0.88, size = 135, normalized size = 1.42

$$-\frac{2\left(2\,ac+\left(3\,bc-ad\right)x^{n}\right)\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}}{3\left(\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)nx^{2\,n}+2\left(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2}\right)nx^{n}+\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*a*c + (3*b*c - a*d)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)/((b^4*c^2
- 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3
*b*d^2)*n*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a+b\,x^n\right)}^{5/2}\,\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

3.1072 $\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=358

$$\frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{128b^2d^5n}-\frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)(a+bx^n)^{3/}}{192b^2d^4n}$$

```
[Out] -1/128*(-a*d+b*c)^3*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*arctanh(d^(1/2)*(a+b*
x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(11/2)/n-1/192*(-a*d+b*c)*(3*
a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b^2/d^4/n+1/
240*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b^2/d
^3/n-3/40*(a*d+3*b*c)*(a+b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/5*x^n*(a+
b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b/d/n+1/128*(-a*d+b*c)^2*(3*a^2*d^2+14*a*b*c*d
+63*b^2*c^2)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d^5/n
```

Rubi [A]
time = 0.25, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {457, 92, 81, 52, 65, 223, 212}

$$\frac{(bc-ad)^2\left(3a^2d^2+14abcd+63b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{128b^2d^5n}-\frac{(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n}+\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n}-\frac{(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{128b^{5/2}d^{11/2}n}-\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n}+\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c
 + d*x^n])/(128*b^2*d^5*n) - ((b*c - a*d)*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*
d^2)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(192*b^2*d^4*n) + ((63*b^2*c^2 + 14
*a*b*c*d + 3*a^2*d^2)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(240*b^2*d^3*n) -
(3*(3*b*c + a*d)*(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(40*b^2*d^2*n) + (x^n*(
a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(5*b*d*n) - ((b*c - a*d)^3*(63*b^2*c^2 +
14*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c +
 d*x^n])])/(128*b^(5/2)*d^(11/2)*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(-ac-\frac{3}{2}(3bc+ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{5bdn}$$

$$= -\frac{3(3bc+ad)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn} + \frac{(63b^2c^2+1\cdots}{\cdots}$$

$$= \frac{(63b^2c^2+14abcd+3a^2d^2)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{3(3bc+ad)(a+bx^n)^{7/2}\sqrt{\cdots}}{40b^2d^2n}$$

$$= -\frac{(bc-ad)(63b^2c^2+14abcd+3a^2d^2)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} + \frac{(63b^2c^2+14abc\cdots}{\cdots}$$

$$= \frac{(bc-ad)^2(63b^2c^2+14abcd+3a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)(63b^2c^2\cdots}{\cdots}$$

$$= \frac{(bc-ad)^2(63b^2c^2+14abcd+3a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)(63b^2c^2\cdots}{\cdots}$$

$$= \frac{(bc-ad)^2(63b^2c^2+14abcd+3a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)(63b^2c^2\cdots}{\cdots}$$

$$= \frac{(bc-ad)^2(63b^2c^2+14abcd+3a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)(63b^2c^2\cdots}{\cdots}$$

Mathematica [A]

time = 1.95, size = 274, normalized size = 0.77

$$\frac{\sqrt{c+dx^n}\left(-\frac{24(3bc+ad)(a+bx^n)^4}{bd}+64x^n(a+bx^n)^4+\frac{5(bc-ad)^3(63b^2c^2+14abcd+3a^2d^2)\left(-\frac{2d(a+bx^n)}{-bc+ad}-\frac{4d^2(a+bx^n)^2}{3(bc-ad)^2}-\frac{16d^3(a+bx^n)^3}{15(-bc+ad)^3}-\frac{2\sqrt{d}\sqrt{a+bx^n}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}}\right)}{4bd^5}\right)}{320bdn\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (Sqrt[c + d*x^n]*((-24*(3*b*c + a*d)*(a + b*x^n)^4)/(b*d) + 64*x^n*(a + b*x
^n)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((-2*d*(a +
b*x^n))/(-(b*c) + a*d) - (4*d^2*(a + b*x^n)^2)/(3*(b*c - a*d)^2) - (16*d^3*
(a + b*x^n)^3)/(15*(-(b*c) + a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(
Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*
x^n))/(b*c - a*d)]))))/(4*b*d^5)))/(320*b*d*n*Sqrt[a + b*x^n])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}(a+b\,x^{n})^{\frac{5}{2}}}{\sqrt{c+d\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A]
time = 1.16, size = 771, normalized size = 2.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b
^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b
^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(384*b
^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3
- 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^(3*n)
 + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^(2*n) - 2*(315*b
^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^n)*s
```

```
qrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n), 1/3840*(15*(63*b^5*c^5 - 175*a
*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a
^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d
))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d +
a*b*d^2)*x^n)) + 2*(384*b^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^
2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d
^4 - 7*a*b^4*d^5)*x^(3*n) + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^
3*d^5)*x^(2*n) - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4
 - 15*a^3*b^2*d^5)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3\,n-1}\,{\left(a+b\,x^n\right)}^{5/2}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

3.1073 $\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=291

$$-\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{64b^2d^4n}+\frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+}}{96b^2d^3n}$$

```
[Out] 1/64*(-a*d+b*c)^2*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*arctanh(d^(1/2)*(a+b*x^
n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(9/2)/n+1/96*(3*a^2*d^2+10*a*b*
c*d+35*b^2*c^2)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b^2/d^3/n-1/24*(3*a*d+7*b*c
)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/4*x^n*(a+b*x^n)^(5/2)*(c+d*x^
n)^(1/2)/b/d/n-1/64*(-a*d+b*c)*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*(a+b*x^n)^
(1/2)*(c+d*x^n)^(1/2)/b^2/d^4/n
```

Rubi [A]
time = 0.20, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {457, 92, 81, 52, 65, 223, 212}

$$-\frac{(bc-ad)\left(3a^2d^2+10abcd+35b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{64b^2d^4n}+\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n}+\frac{(bc-ad)^2\left(3a^2d^2+10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{64b^{5/2}d^{9/2}n}-\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n}+\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] -1/64*((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sq
rt[c + d*x^n])/(b^2*d^4*n) + ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*(a + b*
x^n)^(3/2)*Sqrt[c + d*x^n])/(96*b^2*d^3*n) - ((7*b*c + 3*a*d)*(a + b*x^n)^(
5/2)*Sqrt[c + d*x^n])/(24*b^2*d^2*n) + (x^n*(a + b*x^n)^(5/2)*Sqrt[c + d*x^
n])/(4*b*d*n) + ((b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTan
h[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(5/2)*d^(9/2)
*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(-ac-\frac{1}{2}(7bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{4bdn}$$

$$= -\frac{(7bc+3ad)\,(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{(35b^2c^2 +}{}$$

$$= \frac{(35b^2c^2+10abcd+3a^2d^2)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(7bc+3ad)\,(a+bx^n)^{5/2}}{24b^2d^2n}$$

$$= -\frac{(bc-ad)\,(35b^2c^2+10abcd+3a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{(35b^2c^2+10abc}{}$$

$$= -\frac{(bc-ad)\,(35b^2c^2+10abcd+3a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{(35b^2c^2+10abc}{}$$

$$= -\frac{(bc-ad)\,(35b^2c^2+10abcd+3a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{(35b^2c^2+10abc}{}$$

$$= -\frac{(bc-ad)\,(35b^2c^2+10abcd+3a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{(35b^2c^2+10abc}{}$$

Mathematica [A]
time = 0.76, size = 241, normalized size = 0.83

$$\frac{-b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\,(9a^3d^3+3a^2bd^2(5c-2dx^n)-ab^2d(145c^2-92cdx^n+72d^2x^{2n})+b^3(105c^3-70c^2dx^n+56cd^2x^{2n}-48d^3x^{3n}))+3(bc-ad)^{5/2}\,(35b^2c^2+10abcd+3a^2d^2)\sqrt{\frac{b\,(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{192b^3d^{9/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2*
d*x^n) - a*b^2*d*(145*c^2 - 92*c*d*x^n + 72*d^2*x^(2*n)) + b^3*(105*c^3 - 7
0*c^2*d*x^n + 56*c*d^2*x^(2*n) - 48*d^3*x^(3*n)))) + 3*(b*c - a*d)^(5/2)*(3
5*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSi
nh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(192*b^3*d^(9/2)*n*Sqrt[c +
d*x^n])
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}(a+b\,x^{n})^{\frac{3}{2}}}{\sqrt{c+d\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A]
time = 3.02, size = 607, normalized size = 2.09

$$\left[\frac{3\left(35 b^{4} c^{4}-60 a b^{3} c^{3} d+18 a^{2} b^{2} c^{2} d^{2}+4 a^{3} b c d^{3}+3 a^{4} d^{4}\right) \sqrt{b d} \log\left(8 b^{2} d^{2} x^{2 n}+b^{2} c^{2}+6 a b c d+a^{2} d^{2}+4\left(2 \sqrt{b d}\, b d x^{n}+(b c+a d) \sqrt{b d}\right) \sqrt{b x^{n}+a} \sqrt{d x^{n}+c}+8\left(b^{2} c d+a b d^{2}\right) x^{n}\right)+4\left(48 b^{4} d^{4} x^{3 n}-105 b^{4} c^{3} d+145 a b^{3} c^{2} d^{2}-15 a^{2} b^{2} c d^{3}-9 a^{3} b d^{4}-8\left(7 b^{4} c d^{3}-9 a b^{3} d^{4}\right) x^{2 n}+2\left(35 b^{4} c^{2} d^{2}-46 a b^{3} c d^{3}+3 a^{2} b^{2} d^{4}\right) x^{n}\right) \sqrt{b x^{n}+a} \sqrt{d x^{n}+c}}{768 b^{3} d^{5} n}, -\frac{3\left(35 b^{4} c^{4}-60 a b^{3} c^{3} d+18 a^{2} b^{2} c^{2} d^{2}+4 a^{3} b c d^{3}+3 a^{4} d^{4}\right) \sqrt{-b d} \arctan\left(\frac{\left(2 \sqrt{-b d}\, b d x^{n}+(b c+a d) \sqrt{-b d}\right) \sqrt{b x^{n}+a} \sqrt{d x^{n}+c}}{2\left(b^{2} d^{2} x^{2 n}+a b c d+\left(b^{2} c d+a b d^{2}\right) x^{n}\right)}\right)-2\left(48 b^{4} d^{4} x^{3 n}-105 b^{4} c^{3} d+145 a b^{3} c^{2} d^{2}-15 a^{2} b^{2} c d^{3}-9 a^{3} b d^{4}-8\left(7 b^{4} c d^{3}-9 a b^{3} d^{4}\right) x^{2 n}+2\left(35 b^{4} c^{2} d^{2}-46 a b^{3} c d^{3}+3 a^{2} b^{2} d^{4}\right) x^{n}\right) \sqrt{b x^{n}+a} \sqrt{d x^{n}+c}}{384 b^{3} d^{5} n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3
 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d
^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d
*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^
3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 -
 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)
*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^5*n), -1/384*(3*(35*b^4*c^4 -
 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(-b*d
)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a
)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) -
2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^
3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2
 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^3*d^5*n)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3\,n-1}\,{\left(a+b\,x^n\right)}^{3/2}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
```

```
[Out] int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

3.1074 $\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=221

$$\frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8b^2d^3n}-\frac{(5bc+3ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n}+\frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn}$$

```
[Out] -1/8*(-a*d+b*c)*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*arctanh(d^(1/2)*(a+b*x^n)^(1/
2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(7/2)/n-1/12*(3*a*d+5*b*c)*(a+b*x^n)^
(3/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/3*x^n*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b/d
/n+1/8*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d^
3/n
```

Rubi [A]
time = 0.15, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {457, 92, 81, 52, 65, 223, 212}

$$\frac{(a^2d^2+2abcd+5b^2c^2)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8b^2d^3n}-\frac{(bc-ad)\,(a^2d^2+2abcd+5b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n}-\frac{(3ad+5bc)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n}+\frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b^2*
d^3*n) - ((5*b*c + 3*a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b^2*d^2*n)
 + (x^n*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)*(5*b^2*
c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[
c + d*x^n])])/(8*b^(5/2)*d^(7/2)*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,x,x^n\right)}{n}$$

$$= \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-ac-\frac{1}{2}(5bc+3ad)x\right)}{\sqrt{c+dx}}\,dx,x,x^n\right)}{3bdn}$$

$$= -\frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn} + \frac{(5b^2c^2+2ab}{}$$

$$= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n}$$

$$= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n}$$

$$= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n}$$

$$= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n}$$

Mathematica [A]
time = 0.42, size = 191, normalized size = 0.86

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(-3a^2d^2+2abd(-2c+dx^n)+b^2(15c^2-10cdx^n+8d^2x^{2n})\right)-3(bc-ad)^{3/2}(5b^2c^2+2abcd+a^2d^2)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{24b^3d^{7/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*a^2*d^2 + 2*a*b*d*(-2*c + d*x^n)
 + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(3/2)*(5*b^2*
c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[
d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(24*b^3*d^(7/2)*n*Sqrt[c + d*x^n])
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A]
time = 5.30, size = 471, normalized size = 2.13

$$\left[-\frac{3\,(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}+8\,(b^2cd+abd^2)x^n\right)-4\,(8\,b^3d^3x^{2\,n}+15\,b^3c^2d-4\,ab^2cd^2-3\,a^2bd^3-2\,(5\,b^3cd^2-ab^2d^3)x^n)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{96\,b^3d^4n}, \frac{3\,(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right)+2\,(8\,b^3d^3x^{2\,n}+15\,b^3c^2d-4\,ab^2cd^2-3\,a^2bd^3-2\,(5\,b^3cd^2-ab^2d^3)x^n)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{48\,b^3d^4n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n
 + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*
b*d^2)*x^n) - 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b
*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^3*d^4*n), 1/48*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt
(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^
n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n
)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*
(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^4*n)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Integral(x**(3*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3\,n-1}\,\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

3.1075 $\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=150

$$-\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n}+\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn}-\frac{(4abcd-3(bc+ad)^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}$$

```
[Out] -1/4*(4*a*b*c*d-3*(a*d+b*c)^2)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d
*x^n)^(1/2))/b^(5/2)/d^(5/2)/n-3/4*(a*d+b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2
)/b^2/d^2/n+1/2*x^n*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d/n
```

Rubi [A]
time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {457, 92, 81, 65, 223, 212}

$$-\frac{(4abcd-3(ad+bc)^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}-\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n}+\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(4*b^2*d^2*n) + (x^n*Sqrt[
a + b*x^n]*Sqrt[c + d*x^n])/(2*b*d*n) - ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcT
anh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(5/2)*d^(5/2
)*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} + \frac{\text{Subst}\left(\int \frac{-ac-\frac{3}{2}(bc+ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{2bdn}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(b}{}$$

$$(4abcd-3(b$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - $$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(b}{}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(b}{}$$

Mathematica [A]
time = 0.30, size = 157, normalized size = 1.05

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\,(-3bc-3ad+2bdx^n)+\sqrt{bc-ad}\,(3b^2c^2+2abcd+3a^2d^2)\sqrt{\frac{b\,(c+dx^n)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{4b^3d^{5/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c - 3*a*d + 2*b*d*x^n) + Sqrt[
b*c - a*d]*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c -
a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^3*d^(5/2)*n*
Sqrt[c + d*x^n])
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}}{\sqrt{a+b\,x^n}\,\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Fricas [A]
time = 7.32, size = 361, normalized size = 2.41

$$\left[\frac{(3b^2c^2+2abcd+3a^2d^2)\sqrt{bd}\,\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2+4\left(2\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}+8(b^2cd+abd^2)x^n\right)+4(2b^2d^2x^n-3b^2cd-3abd^2)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{16b^3d^3n}, -\frac{(3b^2c^2+2abcd+3a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{\left(2\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{2(b^2d^2x^{2n}+abcd+(b^2cd+abd^2)x^n)}\right)-2(2b^2d^2x^n-3b^2cd-3abd^2)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{8b^3d^3n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(2*
b^2*d^2*x^n - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*
d^3*n), -1/8*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*
sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c
)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x^n
 - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^3*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{\sqrt{a+bx^n}\,\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Integral(x**(3*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{\sqrt{a+b\,x^n}\,\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

3.1076 $\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=133

$$-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}}+\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn}-\frac{(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}$$

```
[Out] -(3*a*d+b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/
2)/d^(3/2)/n-2*a^2*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)/n/(a+b*x^n)^(1/2)+(a+b*x^
n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d/n
```

Rubi [A]

time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {457, 91, 81, 65, 223, 212}

$$-\frac{2a^2\sqrt{c+dx^n}}{b^2n(bc-ad)\sqrt{a+bx^n}}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}+\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(b^2*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (Sqrt[a + b*
x^n]*Sqrt[c + d*x^n])/(b^2*d*n) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a +
b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*d^(3/2)*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{2}a(bc-ad)+\frac{1}{2}b(bc-ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{b^2(bc-ad)n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\text{Subst}\left(\int \ldots\right.}{2\ldots}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\text{Subst}\left(\int \ldots\right.}{b^3\ldots}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\tanh^{-1}\left(\ldots\right.}{b^{5/2}d^{3/2}n\ldots}$$

Mathematica [A]
time = 0.50, size = 185, normalized size = 1.39

$$\frac{-b\sqrt{d}\,(c+dx^n)\left(-3a^2d+b^2cx^n+ab(c-dx^n)\right)+\sqrt{bc-ad}\,\left(b^2c^2+2abcd-3a^2d^2\right)\sqrt{a+bx^n}\,\sqrt{\frac{b\,(c+dx^n)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{b^3d^{3/2}(-bc+ad)n\sqrt{a+bx^n}\,\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-(b*Sqrt[d]*(c + d*x^n)*(-3*a^2*d + b^2*c*x^n + a*b*(c - d*x^n))) + Sqrt[b
*c - a*d]*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[(b*(c + d*
x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(b^3
*d^(3/2)*(-(b*c) + a*d)*n*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}}{\left(a+b\,x^{n}\right)^{\frac{3}{2}}\sqrt{c+d\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
[Out] int(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(113) = 226.
time = 2.98, size = 540, normalized size = 4.06

$$\left[\frac{4\,(ab^2cd-3\,a^2bd^2+(b^3cd-ab^2d^2)x^n)\sqrt{bx^n+a}\,\sqrt{dx^n+c}+\left((b^3c^2+2\,ab^2cd-3\,a^2bd^2)\sqrt{bd}\,x^n+(ab^2c^2+2\,a^2bcd-3\,a^3d^2)\sqrt{bd}\right)\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2-4\left(2\,\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}+8\,(b^2cd+abd^2)x^n\right)}{4\,((b^5cd^2-ab^4d^3)nx^n+(ab^4cd^2-a^2b^3d^3)n)}, \frac{2\,(ab^2cd-3\,a^2bd^2+(b^3cd-ab^2d^2)x^n)\sqrt{bx^n+a}\,\sqrt{dx^n+c}+\left((b^3c^2+2\,ab^2cd-3\,a^2bd^2)\sqrt{-bd}\,x^n+(ab^2c^2+2\,a^2bcd-3\,a^3d^2)\sqrt{-bd}\right)\arctan\left(\frac{\left(2\,\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right)}{2\,((b^5cd^2-ab^4d^3)nx^n+(ab^4cd^2-a^2b^3d^3)n)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
[Out] [1/4*(4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n +
a)*sqrt(d*x^n + c) + ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(b*d)*x^n +
 (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b
^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^5*c*d
^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2 - a^2*b^3*d^3)*n), 1/2*(2*(a*b^2*c*d -
 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) +
 ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(-b*d)*x^n + (a*b^2*c^2 + 2*a^2
*b*c*d - 3*a^3*d^2)*sqrt(-b*d))*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a
*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d
+ (b^2*c*d + a*b*d^2)*x^n)))/((b^5*c*d^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2
- a^2*b^3*d^3)*n)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{{\left(a+b\,x^n\right)}^{3/2}\,\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

3.1077 $\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=147

$$-\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}\,n}$$

```
[Out] 2*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/n/d^(1/2
)-2/3*a^2*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)/n/(a+b*x^n)^(3/2)+4/3*a*(-2*a*d+3*
b*c)*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)^2/n/(a+b*x^n)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {457, 91, 79, 65, 223, 212}

$$-\frac{2a^2\sqrt{c+dx^n}}{3b^2n(bc-ad)\,(a+bx^n)^{3/2}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}\,n}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2n(bc-ad)^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)*n*(a + b*x^n)^(3/2)) + (4*a*(3*
b*c - 2*a*d)*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)^2*n*Sqrt[a + b*x^n]) + (2*
ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*Sqrt
[d]*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-ad)+\frac{3}{2}b(bc-ad)x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{3b^2(bc-ad)n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{}{\sqrt{a-}}\right.}{}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{}{\sqrt{c}}\right.}{}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{1}{1-\frac{d}{}}\right.}{}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{b}}\right.}{b^{5/2}\sqrt{}}$$

Mathematica [A]
time = 0.82, size = 136, normalized size = 0.93

$$\frac{2a\sqrt{c+dx^n}\,(-3a^2d+6b^2cx^n+ab(5c-4dx^n))}{3b^2(bc-ad)^2n\,(a+bx^n)^{3/2}} + \frac{\log\left(bc+ad+2bdx^n+2\sqrt{b}\,\sqrt{d}\,\sqrt{a+bx^n}\,\sqrt{c+dx^n}\right)}{b^{5/2}\sqrt{d}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*a*Sqrt[c + d*x^n]*(-3*a^2*d + 6*b^2*c*x^n + a*b*(5*c - 4*d*x^n)))/(3*b^2
*(b*c - a*d)^2*n*(a + b*x^n)^(3/2)) + Log[b*c + a*d + 2*b*d*x^n + 2*Sqrt[b]
*Sqrt[d]*Sqrt[a + b*x^n]*Sqrt[c + d*x^n]]/(b^(5/2)*Sqrt[d]*n)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}}{(a+b\,x^n)^{\frac{5}{2}}\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(123) = 246$.
time = 4.44, size = 769, normalized size = 5.23

$$\left[\frac{4\,(5\,a^2b^2cd-3\,a^3bd^2+2\,(3\,ab^3cd-2\,a^2b^2d^2)x^n)\sqrt{bx^n+a}\,\sqrt{dx^n+c}+3\left((b^4c^2-2\,ab^3cd+a^2b^2d^2)\sqrt{bd}\,x^{2n}+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)\sqrt{bd}\,x^n+(a^2b^2c^2-2\,a^3bcd+a^4d^2)\sqrt{bd}\right)\log\left(8\,b^2d^2x^{2n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}+8\,(b^2cd+abd^2)x^n\right)}{6\,((b^7c^2d-2\,ab^6cd^2+a^2b^5d^3)nx^{2n}+2\,(ab^6c^2d-2\,a^2b^5cd^2+a^3b^4d^3)nx^n+(a^2b^5c^2d-2\,a^3b^4cd^2+a^4b^3d^3)n)},\ \frac{2\,(5\,a^2b^2cd-3\,a^3bd^2+2\,(3\,ab^3cd-2\,a^2b^2d^2)x^n)\sqrt{bx^n+a}\,\sqrt{dx^n+c}-3\left((b^4c^2-2\,ab^3cd+a^2b^2d^2)\sqrt{-bd}\,x^{2n}+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)\sqrt{-bd}\,x^n+(a^2b^2c^2-2\,a^3bcd+a^4d^2)\sqrt{-bd}\right)\arctan\left(\frac{\left(2\,\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{2\,(b^2d^2x^{2n}+abcd+(b^2cd+abd^2)x^n)}\right)}{3\,((b^7c^2d-2\,ab^6cd^2+a^2b^5d^3)nx^{2n}+2\,(ab^6c^2d-2\,a^2b^5cd^2+a^3b^4d^3)nx^n+(a^2b^5c^2d-2\,a^3b^4cd^2+a^4b^3d^3)n)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^n)
*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
*sqrt(b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(b*d)*x^
n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^7*
c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5
*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^
3)*n), 1/3*(2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2
)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) - 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^
2*d^2)*sqrt(-b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(
-b*d)*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b*d))*arctan(1/2*(2
*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n +
c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^7*c^2*d - 2*
a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a
^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3)*n)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{{\left(a+b\,x^n\right)}^{5/2}\,\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

3.1078 $\int x^p(b+cx)^p(b+2cx)\,dx$

Optimal. Leaf size=20

$$\frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

[Out] x^(1+p)*(c*x+b)^(1+p)/(1+p)

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {75}

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

[In] Int[x^p*(b + c*x)^p*(b + 2*c*x),x]

[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^p(b+cx)^p(b+2cx)\,dx = \frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.00

$$\frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

Antiderivative was successfully verified.

[In] Integrate[x^p*(b + c*x)^p*(b + 2*c*x),x]

[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)

Maple [A]
time = 0.32, size = 21, normalized size = 1.05

method	result	size
gosper	$\frac{x^{1+p}(cx+b)^{1+p}}{1+p}$	21
risch	$\frac{x(cx+b)x^p(cx+b)^p}{1+p}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(c*x+b)^p*(2*c*x+b),x,method=_RETURNVERBOSE)
[Out] x^(1+p)*(c*x+b)^(1+p)/(1+p)
```

Maxima [A]
time = 0.33, size = 29, normalized size = 1.45

$$\frac{(cx^2+bx)e^{(p\log(cx+b)+p\log(x))}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="maxima")
[Out] (c*x^2 + b*x)*e^(p*log(c*x + b) + p*log(x))/(p + 1)
```

Fricas [A]
time = 2.10, size = 25, normalized size = 1.25

$$\frac{(cx^2+bx)(cx+b)^p x^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="fricas")
[Out] (c*x^2 + b*x)*(c*x + b)^p*x^p/(p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.
time = 1.01, size = 46, normalized size = 2.30

$$\begin{cases} \frac{bx x^p(b+cx)^p}{p+1} + \frac{cx^2x^p(b+cx)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**p*(c*x+b)**p*(2*c*x+b),x)
```

```
[Out] Piecewise((b*x*x**p*(b + c*x)**p/(p + 1) + c*x**2*x**p*(b + c*x)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

Giac [A]
time = 1.12, size = 35, normalized size = 1.75

$$\frac{(cx+b)^p cx^2 x^p + (cx+b)^p bx x^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="giac")
```

```
[Out] ((c*x + b)^p*c*x^2*x^p + (c*x + b)^p*b*x*x^p)/(p + 1)
```

Mupad [B]
time = 4.80, size = 22, normalized size = 1.10

$$\frac{x\,x^p\,(b+c\,x)^p\,(b+c\,x)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(b + c*x)^p*(b + 2*c*x),x)
```

```
[Out] (x*x^p*(b + c*x)^p*(b + c*x))/(p + 1)
```

3.1079 $\int x^{-1+2(1+p)}(b+cx^2)^p\,(b+2cx^2)\;dx$

Optimal. Leaf size=27

$$\frac{x^{2(1+p)}(b+cx^2)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*x^(2+2*p)*(c*x^2+b)^(1+p)/(1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {460}

$$\frac{x^{2(p+1)}(b+cx^2)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

```
[Out] (x^(2*(1 + p))*(b + c*x^2)^(1 + p))/(2*(1 + p))
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\frac{x^{2(1+p)}\left(b+cx^2\right)^{1+p}}{2(1+p)}$$

Mathematica [A]
time = 0.06, size = 26, normalized size = 0.96

$$\frac{x^{2+2p}(b+cx^2)^{1+p}}{2+2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

```
[Out] (x^(2 + 2*p)*(b + c*x^2)^(1 + p))/(2 + 2*p)
```

Maple [A]
time = 0.34, size = 26, normalized size = 0.96

method	result	size
gosper	$\frac{x^{2p+2}(c\,x^{2}+b)^{1+p}}{2p+2}$	26
risch	$\frac{x(c\,x^{2}+b)x^{1+2p}(c\,x^{2}+b)^{p}}{2p+2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x,method=_RETURNVERBOSE)
[Out] 1/2*x^(2*p+2)*(c*x^2+b)^(1+p)/(1+p)
```

Maxima [A]
time = 0.35, size = 35, normalized size = 1.30

$$\frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2\,p\log(x))}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Fricas [A]
time = 1.65, size = 32, normalized size = 1.19

$$\frac{(cx^3+bx)(cx^2+b)^p x^{2\,p+1}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="fricas")
[Out] 1/2*(c*x^3 + b*x)*(c*x^2 + b)^p*x^(2*p + 1)/(p + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+2*p)*(c*x**2+b)**p*(2*c*x**2+b),x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.
time = 0.68, size = 52, normalized size = 1.93

$$\frac{(cx^2+b)^p cx^3 e^{(2\,p\log(x)+\log(x))} + (cx^2+b)^p bxe^{(2\,p\log(x)+\log(x))}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="giac")
```

```
[Out] 1/2*((c*x^2 + b)^p*c*x^3*e^(2*p*log(x) + log(x)) + (c*x^2 + b)^p*b*x*e^(2*p
*log(x) + log(x)))/(p + 1)
```

Mupad [B]
time = 4.88, size = 47, normalized size = 1.74

$$\left(c\,x^2+b\right)^p\,\left(\frac{c\,x^{2\,p+1}\,x^3}{2\,p+2}+\frac{b\,x\,x^{2\,p+1}}{2\,p+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*p + 1)*(b + c*x^2)^p*(b + 2*c*x^2),x)
```

```
[Out] (b + c*x^2)^p*((c*x^(2*p + 1)*x^3)/(2*p + 2) + (b*x*x^(2*p + 1))/(2*p + 2))
```

3.1080 $\int x^{-1+3(1+p)}(b+cx^3)^p\,(b+2cx^3)\;dx$

Optimal. Leaf size=27

$$\frac{x^{3(1+p)}(b+cx^3)^{1+p}}{3(1+p)}$$

```
[Out] 1/3*x^(3+3*p)*(c*x^3+b)^(1+p)/(1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {460}

$$\frac{x^{3(p+1)}(b+cx^3)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
[Out] (x^(3*(1 + p))*(b + c*x^3)^(1 + p))/(3*(1 + p))
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{x^{3(1+p)}\left(b+cx^3\right)^{1+p}}{3(1+p)}$$

Mathematica [A]
time = 0.07, size = 26, normalized size = 0.96

$$\frac{x^{3+3p}(b+cx^3)^{1+p}}{3+3p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
[Out] (x^(3 + 3*p)*(b + c*x^3)^(1 + p))/(3 + 3*p)
```

Maple [A]
time = 0.32, size = 26, normalized size = 0.96

method	result	size
gosper	$\frac{x^{3+3p}\left(c\,x^{3}+b\right)^{1+p}}{3+3p}$	26
risch	$\frac{x\left(c\,x^{3}+b\right)x^{2+3p}\left(c\,x^{3}+b\right)^{p}}{3+3p}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x,method=_RETURNVERBOSE)
[Out] 1/3*x^(3+3*p)*(c*x^3+b)^(1+p)/(1+p)
```

Maxima [A]
time = 0.34, size = 35, normalized size = 1.30

$$\frac{(cx^6 + bx^3)e^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)\right)}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Fricas [A]
time = 1.31, size = 32, normalized size = 1.19

$$\frac{(cx^4 + bx)(cx^3 + b)^p x^{3\,p+2}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="fricas")
[Out] 1/3*(c*x^4 + b*x)*(c*x^3 + b)^p*x^(3*p + 2)/(p + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+3*p)*(c*x**3+b)**p*(2*c*x**3+b),x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(25) = 50$.
time = 0.68, size = 56, normalized size = 2.07

$$\frac{(cx^3+b)^p cx^4 e^{(3\,p\log(x)+2\,\log(x))} + (cx^3+b)^p bx e^{(3\,p\log(x)+2\,\log(x))}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="giac")
```

```
[Out] 1/3*((c*x^3 + b)^p*c*x^4*e^(3*p*log(x) + 2*log(x)) + (c*x^3 + b)^p*b*x*e^(3
*p*log(x) + 2*log(x)))/(p + 1)
```

Mupad [**B**]
time = 4.90, size = 47, normalized size = 1.74

$$(c\,x^3+b)^p\,\left(\frac{c\,x^{3\,p+2}\,x^4}{3\,p+3}+\frac{b\,x\,x^{3\,p+2}}{3\,p+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*p + 2)*(b + c*x^3)^p*(b + 2*c*x^3),x)
```

```
[Out] (b + c*x^3)^p*((c*x^(3*p + 2)*x^4)/(3*p + 3) + (b*x*x^(3*p + 2))/(3*p + 3))
```

3.1081 $\int x^{-1+n(1+p)}(b+cx^n)^p(b+2cx^n)\,dx$

Optimal. Leaf size=27

$$\frac{x^{n(1+p)}(b+cx^n)^{1+p}}{n(1+p)}$$

```
[Out] x^(n*(1+p))*(b+c*x^n)^(1+p)/n/(1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{460\}$

$$\frac{x^{n(p+1)}(b+cx^n)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

```
[Out] (x^(n*(1 + p))*(b + c*x^n)^(1 + p))/(n*(1 + p))
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+n(1+p)}(b+cx^n)^p(b+2cx^n)\,dx = \frac{x^{n(1+p)}(b+cx^n)^{1+p}}{n(1+p)}$$

Mathematica [A]
time = 0.11, size = 26, normalized size = 0.96

$$\frac{x^{n+np}(b+cx^n)^{1+p}}{n+np}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

```
[Out] (x^(n + n*p)*(b + c*x^n)^(1 + p))/(n + n*p)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^{-1+n(1+p)}(b+c\,x^{n})^{p}\,(b+2c\,x^{n})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x)
[Out] int(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x)
```

Maxima [A]
time = 0.36, size = 39, normalized size = 1.44

$$\frac{(cx^{2\,n}+bx^{n})e^{(np\log(x)+p\log(cx^{n}+b))}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="maxima")
[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

Fricas [A]
time = 2.86, size = 35, normalized size = 1.30

$$\frac{(cxx^{n}+bx)(cx^{n}+b)^{p}x^{np+n-1}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="fricas")
[Out] (c*x*x^n + b*x)*(c*x^n + b)^p*x^(n*p + n - 1)/(n*p + n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n*(1+p))*(b+c*x**n)**p*(b+2*c*x**n),x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.
time = 0.65, size = 66, normalized size = 2.44

$$\frac{(cx^{n}+b)^{p}cxx^{n}e^{(np\log(x)+n\log(x)-\log(x))}+(cx^{n}+b)^{p}bxe^{(np\log(x)+n\log(x)-\log(x))}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="giac")
```

```
[Out] ((c*x^n + b)^p*c*x*x^n*e^(n*p*log(x) + n*log(x) - log(x)) + (c*x^n + b)^p*b
*x*e^(n*p*log(x) + n*log(x) - log(x)))/(n*p + n)
```

Mupad [B]
time = 4.86, size = 54, normalized size = 2.00

$$\left(\frac{b\,x\,x^{n\,(p+1)-1}}{n\,\left(p+1\right)}+\frac{c\,x\,x^n\,x^{n\,(p+1)-1}}{n\,\left(p+1\right)}\right)\,{\left(b+c\,x^n\right)}^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n*(p + 1) - 1)*(b + c*x^n)^p*(b + 2*c*x^n),x)
```

```
[Out] ((b*x*x^(n*(p + 1) - 1))/(n*(p + 1)) + (c*x*x^n*x^(n*(p + 1) - 1))/(n*(p +
1)))*(b + c*x^n)^p
```

Chapter 4

Appendix

Local contents

4.1 Download section 5516
4.2 Listing of Grading functions 5516

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```